LUISS Guido Carli
LIBERA UNIVERSITA' INTERNAZIONALE DEGLI STUDI SOCIALI

Department of Economics and Finance, Chair of International Finance

JOBLESS RECOVERY: A GENERAL EQUILIBRIUM MODEL WITH A COLLATERAL CONSTRAINT

Supervisor

Prof. Pierpaolo Benigno

Candidate

Martina Fazio

ID 640881

Co-supervisor

Prof. Pietro Reichlin

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To those I shared this journey with,

making it interesting and worthwhile.

Abstract

This paper develops a general equilibrium model with the aim of analyzing the jobless recovery phenomenon, i.e. output growth occurring without improved employment conditions. A real business cycle model is taken into consideration, whereby a number of modifications are rendered necessary so as to allow for the model to reproduce this fact. In particular, the introduction of wage rigidities and a credit constraint considerably enhance the ability of the model to illustrate this phenomenon. The collateral quality of physical capital induces a less labor-intensive output production. Credit conditions in the economy turn out as an important factor for macroeconomic fluctuations.

Contents

1 Introduction

A recurring phenomenon has been observed in the recovery from a few crisis episodes from the past decade and from the 2008 financial crisis in particular. At a time when output has already recovered its pre-crisis level, employment tends to lag behind, so that the recovery takes place without a reduction in unemployment, i.e. a jobless recovery occurs. Many different explanations have been provided for this fact, most of which rely on wage rigidities assumptions and other labor market imperfections. An alternative theory primarily focuses on credit constraints rather than on labor market issues. According to this theory, firms can borrow against their physical capital (their tangible assets), while they are not able to obtain a loan on the basis of their human capital (their employees). After a crisis, firms need liquidity in order to start reinvesting and restructuring. At the same time, banks are less prone to lend and raise their requirements for granting loans. Indeed, they are aware of the higher probability of dealing with a distressed firm, that might not be able to repay its debt. As a consequence, all firms, whether distressed or not, are required to post a higher amount of collateral, so that the creditor is protected against the possibility of default on the part of the debtor. Capital intensive projects are hence favored over labor intensive ones, because of the characteristic of inalienability of human capital. Production is adjusted in such a way that a lower level of employment is needed to produce a unit of output. This is the mechanism that causes output to recover, and unemployment to remain high.

This paper develops a general equilibrium model to analyze this phenomenon. In the first part, a frictionless neoclassical growth model for a decentralized equilibrium is presented, following Shimer (2012). Successively, a credit constraint on the resources of the firm is introduced and finally, rigid wages are assumed. In the end a number of conclusions will be drawn from the results obtained.

2 Literature Review

Throughout time, the jobless recovery phenomenon has been analyzed from a wide range of points of view, looking for its determinants and considering its consequences and possible remedies. The main cause for high unemployment levels - after a recovery from a crisis and in general - was typically considered to be the structure and rigidities of labor markets[1]. This was also the most common explanation for the difference observed in the rates of unemployment in Europe and the United States. Most economists believed that it was the lack of rigidities in the US labor market to ensure a low rate of unemployment, while the higher European rate was the consequence of its job market's frictions. This kind of reasoning has some serious drawbacks: first of all, a European labor market as a whole does not even exist, so that a comparison with the unified American market for jobs makes little sense. In Europe, there remain important linguistic and cultural barriers to a unified labor market, and each country still has its own market with its own structure and characteristics. As a consequence, the great geographical mobility of the American labor market is clearly not present in the Old Continent.

Moreover, looking at the average unemployment levels alone may not provide an accurate picture: some European countries do exhibit higher unemployment rates than the US, but some others do not, and not all of the countries with more flexible labor markets also exhibit lower unemployment levels (see e.g. the United Kingdom). This hints at the possibility that labor market rigidities may not be a sufficient explanation[2].

When high unemployment levels became a feature of the flexible American labor market during the recovery from the 2008 financial crisis, it was clear that a different approach to explain this fact was needed.

Acemoglu (2001) notices how neither institutional changes nor macroeconomic factors are thoroughly convincing explanations for the European high and

[1] See for example Layard, Nickell, and Jackman (2005); Nickell (1997)

[2] The relevance of labor market rigidities for actual unemployment levels is not put into question, though.

persistent unemployment level. In particular, there were not many major institutional reforms in the 1970's that could motivate the surge in unemployment occurring after the 1980's. At the same time, macroeconomic shocks of different types were never large enough to explain such high and persistent levels of unemployment. Therefore, he proposes a credit market explanation for this fact, showing that European firms belonging to credit-dependent sectors have a lower share of employment with respect to comparable American firms, although growth in the two cases is approximately the same. This is so, due to access to credit market being more facilitated in the U.S. with respect to Europe.

Using the same idea that unemployment may be worsened by frictions that lay outside the labor market, Dromel, Kolakez, and Lehmann (2010) develop a model with a matching equilibrium à la Mortensen and Pissarides (1999) where they include a number of credit constraints. These not only have a significant effect on the level of steady state unemployment but also on its persistence, and on the time that it takes for the economy to converge to its steady state after a shock.

Calvo, Coricelli, and Ottonello (2012) also provide a financial market explanation for jobless recoveries. Their paper adopts a cohesive approach to analyze both the phenomena of jobless and wageless recoveries, two possible outcomes of a financial crisis. When the value of a firm's collateral experiences a sudden drop, countries with low inflation and rigid wages face a jobless recovery, while countries where inflation levels are high, and real wages can shift to absorb the consequences of a shock, are more likely to suffer a wageless recovery. Evidence of these occurrences is provided in their paper through an extensive empirical analysis of financial crises over the past thirty years. They perform a cross country regression on a split sample of high and low inflation countries, considering both developed and emerging economies. Expectations of jobless recovery in low inflation environments and wageless recovery in those showing high inflation are confirmed by data, although the reliability of some of these results can be questioned due to the small size of the sample. At the same time, data show that in the case of non-financial recessions, neither of the two phenomena affect the recovery process. A simple partial equilibrium model also supports the empirical analysis, adding an exogenous collateral constraint to the standard maximization problem of the firm[3]. During a financial crisis, the constraint becomes binding at optimum; however, while it has a full impact on the amount of labor costs the firm can bear, it only partially influences the amount of capital used for production. This is so because, up to a certain extent, capital can be considered as its own collateral. In fact, investors find capital intensive projects considerably less risky than those being labor intensive. The reason for this is that in case of default of the borrower, creditors will have more chances to recover their funds if these are invested in capital, than if they are invested in labor. The principle of inalienability of human capital puts capital and labor costs on two different levels, so that labor costs result in being more restricted than capital by the constraint on collateral.

The concept of human capital inalienability is inherited from a long-standing strand of literature. Hart and Moore (1994), for example, propose a model for debt financed projects, showing that in a number of cases, a profitable investment opportunity will be missed because firms are subject to credit constraints. In fact, an entrepreneur always has the option to repudiate a debt contract by withdrawing human capital from the project. In that case, the investor will only be able to retaliate by liquidating the physical assets of the project, and this gives rise to an upper bound for the total amount of indebtedness of the entrepreneur.

Almeida and Campello (2007) provide further insight into the influence of financial frictions on real investment decisions. They define a "credit multiplier" for which pledgeable assets support borrowing for further investment in pledgeable assets. The relationship between sensitivity to cash flows of the project and tangibility of the assets of the firm is non-linear: at a low level of tangibility, the firm is most likely to be credit constrained, so sensitivity to investment-cash flows is extremely high; on the contrary, at a high level of tangibility, the firm is not subject to any credit constraint, and tangibility has no impact on the sensitivity to project cash flows.

[3]This is the kind of constraint that will be used in the analysis that follows.

A seminal paper by Kiyotaki and Moore (1995) shows how credit market frictions give rise to a credit multiplier that amplifies business cycle fluctuations. These "credit cycles" are characterized by major spillover effects, higher persistence and amplification of shock. A key assumption is that a creditor cannot force the borrower to repay his debt except for the secured part of the loan: this naturally gives rise to credit constraints. By assuming a dynamic economy with just one production factor, where some firms are credit constrained and some are not, they show how a one period drop in productivity brings about a negative spiral that causes the shock to propagate for many subsequent periods. Constrained firms will be the ones to suffer the most and, because of the shock, they will miss profitable investment opportunities, earning therefore a lower revenue. As a consequence, they will invest less in the next period and this same effect will be repeated over and again. Price effects contribute to worsening this situation: expectations of a fall in the price of the production factor will cause capital losses for constrained firms and this will further reduce their investment possibility. Amplification and persistence are in place through both a static and an inter-temporal multiplier.

Along the same line, Bernanke, Gertler, and Gilchrist (1999) propose a principal-agent model with macroeconomic relevance. They identify the solvency of the borrower as one of the sources for macroeconomic fluctuations, and modify a classic real business cycle model to account for an informational asymmetry between entrepreneurs and investors. This inefficiency produces agency costs, due to which internal financing is less expensive than external financing. The net worth of the borrowers is negatively related to agency costs, so that whenever the first rises, the latter shrinks. Moreover, there is a link between aggregate economic conditions and the balance sheet of the borrowers, so that when there is a recession, the net worth of the debtor is reduced and agency costs rise, causing investment fluctuations and shock persistence. At the same time, shocks that affect the balance sheet of the borrower have also a macroeconomic propagation through a change in agency costs.

3 Model Setup

The proposed model is a classic real business cycle model, where two sources of frictions are introduced to reproduce the jobless recovery phenomenon: wage rigidity and a collateral constraint on the borrowing of the firm. In fact, given a credit constraint, if wages were fully flexible, they would adjust at a lower level so as to avoid a jobless recovery, causing a "wageless recovery" instead. Since these two frictions need to be incorporated into the model, it can be shown that there is no equivalence between the "planner's problem" and the competitive equilibrium[4]. For this reason, a decentralized equilibrium needs to be analyzed[5], where households maximize their utility function by choosing the optimal path for consumption and leisure subject to an inter-temporal budget constraint, while firms maximize their expected profits, subject to a collateral constraint. The problem will be analyzed in three steps: the standard model without constraints and with fully flexible wages is first considered, then a credit constraint will be included, and only at the end wage rigidities will also be assumed.

3.1 Baseline model

A representative household is considered. The number of components of the household is normalized to 1 and they live infinitely. They discount utility from future consumption at the rate of β. Labor is indivisible. A fraction n_t of the household members is employed and consumes $C_{e,t}$, while a fraction $1-n_t$ is unemployed and consumes $C_{u,t}$. Total consumption for the household is thus

$$C_t = n_t C_{e,t} + (1-n_t) C_{u,t} \tag{1}$$

The utility function for employed and unemployed individuals differ, as the latter do not have to bear any disutility from working.

[4] I.e. the welfare theorem does not apply here.

[5] The planner's problem solution can be found in the Appendix.

In particular,

$$U(C_{u,t}) = \begin{cases} \frac{C_{u,t}^{1-\sigma}}{1-\sigma} & if\ \sigma \neq 1 \\ \log C_{u,t} & if\ \sigma = 1 \end{cases}$$

$$U(C_{e,t}) = \begin{cases} \frac{C_{e,t}^{1-\sigma}(1+(\sigma-1)\gamma)^{\sigma}}{1-\sigma} & if\ \sigma \neq 1 \\ \log C_{e,t} - \gamma & if\ \sigma = 1 \end{cases}$$

The parameter σ measures risk aversion and is the inverse of the inter-temporal elasticity of substitution. It also determines the complementarity between labor and consumption, hence defining whether the employed will consume more than unemployed individuals. $\gamma > 0$ is the disutility that the employed must bear from working.

The problem for the representative family can be stated as:

$$\max \quad \sum_{t=0}^{\infty} \beta^t \left(n_t \frac{C_{e,t}^{1-\sigma}(1+(\sigma-1)\gamma)^{\sigma}}{1-\sigma} + (1-n_t)\frac{C_{u,t}^{1-\sigma}}{1-\sigma} \right)$$

$$s.to \qquad a_t + w_t n_t = \frac{q_0^{t+1}}{q_0^t} a_{t+1} + (n_t C_{e,t} + (1-n_t) C_{u,t})$$

The budget constraint simply states that resources at time t (on the left-hand side of the equation) can either be consumed, or saved and invested for the next period. In particular, q_0^t is the time 0 price of a unit of consumption at time t, a_t is the amount of assets of the household at time t, set at time t-1, given an initial endowment at time 0 of a_0, and w_t is the wage.

Form the Lagrangian as

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left\{ \left(n_t \frac{C_{e,t}^{1-\sigma}\left(1+(\sigma-1)\gamma\right)^{\sigma}}{1-\sigma} + (1-n_t)\frac{C_{u,t}^{1-\sigma}}{1-\sigma} \right) + \lambda_t \left[a_t - (n_t C_{e,t} + (1-n_t) C_{u,t}) + w_t n_t - \frac{q_0^{t+1}}{q_0^t} a_{t+1} \right] \right\}$$

Derive first order conditions by computing derivatives of the Lagrangian with respect to $C_{e,t}$, $C_{u,t}$, n_t and a_{t+1}.

$$\frac{\partial \mathcal{L}}{\partial C_{e,t}} = \beta^t \left[n_t C_{e,t}^{-\sigma} \left(1+(\sigma-1)\gamma\right)^{\sigma} - \lambda_t n_t \right] = 0$$

$$\Rightarrow \quad \left(\frac{C_{e,t}}{1+(\sigma-1)\gamma} \right)^{-\sigma} = \lambda_t \tag{2}$$

$$\frac{\partial \mathcal{L}}{\partial C_{u,t}} = \beta^t \left[(1-n_t) C_{u,t}^{-\sigma} - \lambda_t (1-n_t) \right] = 0$$

$$\Rightarrow \quad C_{u,t}^{-\sigma} = \lambda_t \tag{3}$$

$$\frac{\partial \mathcal{L}}{\partial a_{t+1}} = \beta^{t+1} \lambda_{t+1} - \beta^t \lambda_t \frac{q_0^{t+1}}{q_0^t} = 0$$

$$\Rightarrow \quad \beta \frac{\lambda_{t+1}}{\lambda_t} = \frac{q_0^{t+1}}{q_0^t} \tag{4}$$

$$\frac{\partial \mathcal{L}}{\partial n_t} = \frac{C_{e,t}^{1-\sigma}\left(1+(\sigma-1)\gamma\right)^{\sigma}}{1-\sigma} - \frac{C_{u,t}^{1-\sigma}}{1-\sigma} - \lambda_t C_{e,t} + \lambda_t C_{u,t} + \lambda_t w_t = 0$$

$$\Rightarrow \quad w_t = \frac{\sigma}{\sigma-1} \left(C_{e,t} - C_{u,t} \right) \tag{5}$$

By exploiting expression (1), we can express consumption for both the two categories of individuals in terms of aggregate consumption:

$$C_{e,t} = \frac{1+(\sigma-1)\gamma}{1+(\sigma-1)\gamma n_t} Ct; \quad C_{u,t} = \frac{C_t}{1+(\sigma-1)\gamma n_t}$$

This establishes λ_t, the Lagrangian multiplier, as the marginal utility of aggregate consumption, i.e.

$$\lambda_t = \left(\frac{C_t}{1 + (\sigma - 1)\gamma n_t} \right)^{-\sigma} \tag{6}$$

If equations (2) and (3) are plugged into the first order condition for employment (5), what emerges is that the wage is equal to the marginal rate of substitution between consumption and leisure:

$$w_t = \sigma\gamma\lambda_t^{-\frac{1}{\sigma}} \tag{7}$$

Let us now analyze the maximization problem for a representative firm, whose aim is to maximize the present value of its profits by deciding a time path for capital and employment. The production function is a Cobb-Douglas, with labor augmenting technology evolving overtime according to $A_{t+1} = (1+g)A_t$

$$\max_{K_{t+1}, n_t} \quad \sum_{t=0}^{\infty} q_0^t \left[K_t^{\alpha} (A_t n_t)^{1-\alpha} + (1-\delta) K_t - K_{t+1} - w_t n_t \right]$$

$$\frac{\partial \Pi}{\partial K_{t+1}} = q_0^{t+1} \left[\alpha K_{t+1}^{\alpha-1} (A_{t+1} n_{t+1})^{1-\alpha} + (1-\delta) \right] - q_0^t = 0$$

$$\Rightarrow \quad q_0^{t+1} \left[\alpha \kappa_{t+1}^{\alpha-1} + (1-\delta) \right] = q_0^t \tag{8}$$

$$\frac{\partial \Pi}{\partial n_t} = q_0^t \left[(1-\alpha) K_t^{\alpha} A_t^{1-\alpha} n_t^{-\alpha} - w_t \right] = 0$$

$$\Rightarrow \quad (1-\alpha)\kappa_t^{\alpha} A_t = w_t \tag{9}$$

where $\kappa_t = \frac{K_t}{A_t N_t}$ is capital per efficiency unit of labor.

It is now possible to combine the result for the household and the firm to obtain a Euler equation and an employment equilibrium condition for the whole economy. In particular, plug equation (4) into (8) and combine (7) with (9) to obtain:

$$\lambda_t = \beta\lambda_{t+1}\left(\alpha\kappa_{t+1}^{\alpha-1} + 1 - \delta\right) \tag{10}$$

$$(1-\alpha)A_t\kappa_t^{\alpha} = \sigma\gamma\lambda_t^{-\frac{1}{\sigma}} \tag{11}$$

The resource constraint for the entire economy is

$$K_{t+1} = K_t^{\alpha}(A_t n_t)^{1-\alpha} + (1-\delta)K_t - C_t \tag{12}$$

Now eliminate κ_t from (10) using (11) and n_t and C_t from (12) exploiting (6) and (11) to obtain two relationships for the marginal utility of consumption and capital:

$$\lambda_t = \beta\lambda_{t+1}\left[\alpha\left(\frac{(1-\alpha)A_{t+1}\lambda_{t+1}^{\frac{1}{\sigma}}}{\sigma\gamma}\right)^{\frac{1-\alpha}{\alpha}} + 1 - \delta\right]$$

$$K_{t+1} = \left[\left(\frac{(1-\alpha)A_t\lambda_t^{\frac{1}{\sigma}}}{\sigma\gamma}\right)^{\frac{1-\alpha}{\alpha}}\left(\frac{1-\alpha+\alpha\sigma}{\sigma}\right) + 1 - \delta\right]K_t - \lambda_t^{-\frac{1}{\sigma}}$$

These two equations completely describe the dynamics of the economy. From here, an equilibrium for the model can be found, by solving a linear system of two first order difference equations in two unknowns $(\lambda\, and\, K)$. This is possible after log-linearizing the two equations around a balanced growth path where labor augmenting technology, capital, output and consumption (of both employed and unemployed individuals) all grow at rate g, employment is constant, and the marginal utility of consumption λ_t grows at $(1+g)^{-\sigma} - 1$. The log-linearized expressions result in:

$$\hat{\Lambda}_{t+1} = \frac{\alpha\sigma}{1-\alpha+\alpha\sigma-\beta(1-\alpha)(1-\delta)(1+g)^{-\sigma}}\hat{\Lambda}_t$$

$$\hat{k}_{t+1} = \frac{(1-\alpha+\alpha\sigma)(1+g)^{\sigma} - \beta(1-\delta)(1-\alpha)}{\beta\alpha\sigma(1+g)}\hat{k}_t + \frac{(1-\alpha+\alpha\sigma)(1+g)^{\sigma} - \beta(1-\delta)(1-\alpha) - \beta\sigma\alpha^2(1+g)}{\beta\alpha^2\sigma^2(1+g)}\hat{\Lambda}_t$$

The model is then calibrated to match the US economy figures. The recent financial crisis is reproduced in this setup as a shock that destroys collateral value, i.e. a shock to the stock of capital. But for every reasonable calibration, the dynamics of unemployment and output is not able to replicate the facts observed in reality, nor the jobless recovery phenomenon.

If one assumes a shock for which capital stock starts 10% below the steady state trend, this causes employment to increase on impact, which is the opposite of what happened in reaction to the crisis. Although to a different extent, this happens for any possible value of the complementarity between consumption and labor parameter, σ. For this reason, some modification needs to be implemented into the model, in order to make it more accurate. In particular, it is crucial that wages respond less to shocks, so that they do not absorb them entirely. Rigid wages are going to be the key to reverse the reaction of employment to the shock.

3.2 Credit constraint

Let's now analyze the case in which the problem for the household is left unchanged, while firms face a constraint on the amount of borrowing they can undertake, and because of this, they will only be allowed to borrow up to the point where they can post collateral. In particular, the resources needed for production in period t are $K_{t+1} + w_t n_t$, but they can be no higher than the total amount of collateral that the firm owns. This is equal to the exogenous credit constraint $Z_t > 0$ (the extrinsic collateral), plus the proportion of total capital that serves as intrinsic collateral, $(1-\theta)K_{t+1}$, $(0 \leq \theta \leq 1)$. The parameter θ measures the extent to which capital enters in the credit constraint, i.e. if $\theta = 0$ then capital is its own collateral and is not subject to the credit constraint at all (only human capital will be restricted), while if $\theta = 1$ then capital doesn't have the quality of collateral, and is entirely subject to the constraint, to the same extent that human capital is. The problem of the firm becomes:

$$Max \quad \sum_{t=0}^{\infty} q_0^t \left[K_t^{\alpha} (A_t n_t)^{1-\alpha} + (1-\delta) K_t - K_{t+1} - w_t n_t\right]$$

$$s.to \qquad \theta K_{t+1} + w_t n_t \leq Z_t$$

$$\mathcal{L} = \sum_{t=0}^{\infty} q_0^t \left\{K_t^{\alpha} (A_t n_t)^{1-\alpha} + (1-\delta) K_t - K_{t+1} - w_t n_t + \mu_t \left[Z_t - \theta K_{t+1} - w_t n_t\right]\right\}$$

The interesting case is when the constraint is binding at optimum, i.e. it holds with equality (otherwise we are back to the previous circumstance). In this case, first order conditions are

$$\frac{\partial \mathcal{L}}{\partial K_{t+1}} = q_0^{t+1} \left[\alpha K_{t+1}^{\alpha-1} (A_{t+1} n_{t+1})^{1-\alpha} + (1-\delta)\right] - q_0^t \left[1 + \mu_t \theta\right] = 0$$

$$\Rightarrow \quad q_0^{t+1} \left(\alpha \kappa_{t+1}^{\alpha-1} + 1 - \delta\right) = q_0^t (1 + \mu_t \theta) \tag{13}$$

$$\frac{\partial \mathcal{L}}{\partial n_t} = q_0^t \left[(1-\alpha) K_t^{\alpha} A_t^{1-\alpha} n_t^{-\alpha} - w_t (1 + \mu_t)\right] = 0$$

$$\Rightarrow \quad (1-\alpha) \kappa_t^{\alpha} A_t = w_t (1 + \mu_t) \tag{14}$$

$$\frac{\partial \mathcal{L}}{\partial \mu_t} : \quad \theta K_{t+1} + w_t n_t = Z_t \tag{15}$$

Put the two constrained f.o.c.s together so as to eliminate μ_t.

$$q_0^{t+1} \left(\alpha \kappa_{t+1}^{\alpha-1} + 1 - \delta\right) = q_0^t \left[(1-\theta) + \theta (1-\alpha) \kappa_t^{\alpha} \left(\frac{A_t}{w_t}\right)\right]$$

Use household Euler equation (4) to eliminate prices

$$\beta\lambda_{t+1}\left(\alpha\kappa_{t+1}^{\alpha-1}+1-\delta\right)=\lambda_t\left[(1-\theta)+\theta(1-\alpha)\kappa_t^{\alpha}\left(\frac{A_t}{w_t}\right)\right] \tag{16}$$

Now use credit constraint (15) to eliminate employment, and equation (7) to eliminate the wage from equation (16)

$$\beta\lambda_{t+1}\left(\alpha\left(\frac{A_{t+1}\lambda_{t+1}^{\frac{1}{\sigma}}}{\sigma\gamma}\frac{(Z_{t+1}-\theta K_{t+2})}{K_{t+1}}\right)^{1-\alpha}+1-\delta\right)=$$
$$\lambda_t\left[(1-\theta)+\theta(1-\alpha)\left(\frac{A_t\lambda_t^{\frac{1}{\sigma}}}{\sigma\gamma}\frac{(Z_t-\theta K_{t+1})}{K_t}\right)^{-\alpha}\left(\frac{A_t\lambda_t^{\frac{1}{\sigma}}}{\sigma\gamma}\right)\right] \tag{17}$$

Eliminate wage, employment and consumption from the resource feasibility of the economy (12) which is left unchanged from the baseline model

$$K_{t+1}=\left[\left(\frac{A_t\lambda_t^{\frac{1}{\sigma}}}{\sigma\gamma}\frac{(Z_t-\theta K_{t+1})}{K_t}\right)^{1-\alpha}+(1-\delta)\right]K_t-\lambda_t^{-\frac{1}{\sigma}}-\frac{(\sigma-1)}{\sigma}(Z_t-\theta K_{t+1}) \tag{18}$$

These are the two fundamental relationships of the model: after their log-linearization, it is possible to solve for an equilibrium. In particular, when the model is calibrated and solved by means of a statistical software, it is possible to see what the reaction of the variables to different kinds of shocks is.

3.3 Rigid wages

A key feature of the neoclassical growth model is that the reaction of employment following a negative shock on the stock of capital is an increase on impact from its steady state value. This occurs for any reasonable parametrization of the model, because the destruction of capital causes households to feel poorer, so they start working more to recover their previous levels of wealth. Labor productivity falls and employment increases.

In order to have this tendency reversed, the introduction of some form of rigidities for wages is needed. In particular, it is assumed that households have no bargaining power as far as wages are concerned. They need to take whatever wage level is proposed to them, and supply labor at that price. Thus, wages thus grow according to an exogenous path:

$$w_{t+1} = (1+g)\,w_t$$

This equation will substitute the wage equation (7) that would result from families maximizing their employment level. The maximization problem for firms, on the contrary, is left unchanged.

As t goes to infinity, the wage equation has no solution and tends to explode. However, it can be made steady-state compatible by dividing it for the law of growth of labor-augmenting technology. This results in:

$$\frac{w_t}{A_t} = \frac{w_0}{A_0}$$

If this result is plugged into equation (9), then the expressions for the unconstrained rigid wage model can be easily derived.

$$K_{t+1} = \left[\left(\frac{(1-\alpha)A_0}{w_0}\right)^{\frac{1-\alpha}{\alpha}} + 1 - \delta\right] K_t - \left[1 + (\sigma-1)\gamma\left(\frac{(1-\alpha)A_0}{w_0}\right)^{\frac{1}{\sigma}} \frac{K_t}{A_t}\right] \lambda_t^{-\frac{1}{\sigma}}$$

$$\lambda_t = \beta\lambda_{t+1}\left[\alpha\left(\frac{(1-\alpha)A_0}{w_0}\right)^{\frac{1-\alpha}{\alpha}} + 1 - \delta\right]$$

It can be proven that a shock on capital has now a different impact on the considered variables. In particular, in the previous circumstance, wages were falling on impact of the shock, while now they are forced to keep growing at the rate of g. The reduction in capital causes an equal reduction in consumption, output, investment and employment, after which all the variables except for the latter start growing at the rate of g again, while employment remains at a permanently depressed level, so that a jobless recovery occurs.

Let us now introduce rigid wages in the constrained model. The two main equations become:

$$\beta\lambda_{t+1}\left(\alpha\left(\frac{A_0}{w_0}\frac{(Z_{t+1}-\theta K_{t+2})}{K_{t+1}}\right)^{1-\alpha}+1-\delta\right)=$$
$$\lambda_t\left[(1-\theta)+\theta(1-\alpha)\left(\frac{(Z_t-\theta K_{t+1})}{K_t}\right)^{-\alpha}\left(\frac{A_0}{w_0}\right)^{1-\alpha}\right]$$

$$K_{t+1}=\left[\left(\frac{A_0}{w_0}\frac{(Z_t-\theta K_{t+1})}{K_t}\right)^{1-\alpha}+(1-\delta)\right]K_t-$$
$$\lambda_t^{-\frac{1}{\sigma}}\left[1+(\sigma-1)\gamma\left(\frac{A_0}{w_0}\right)\left(\frac{Z_t-\theta K_{t+1}}{A_t}\right)\right]$$

The introduction of wage rigidities in the constrained model produces various consequences. Whenever $\theta<1$, it can be proven that a neutral shock (i.e. a shock that does not favor either labor or capital) such as a shock on technology, will support a more capital-intensive production. Output and capital grow more than employment, which tends to lag behind. On the contrary, when wages are flexible, they fall in consequence of a shock, thus bearing all the adjustment.

4 Main results and graphic output

Flexible wages

This section shows the outcome after all different specifications of the model analyzed so far are calibrated and solved using a statistical software. In particular, two different types of shocks are considered. On the one hand, a negative shock on capital is taken into account, as in Shimer (2012); on the other hand, for the constrained model, a shock on the collateral constraint is also specified, as in Calvo et al. (2012). In both cases, the two shocks considered involve the shocked variable to start 10% below its steady state trend. Furthermore, in all figures shown below, σ is set equal to one and one period corresponds to one month.

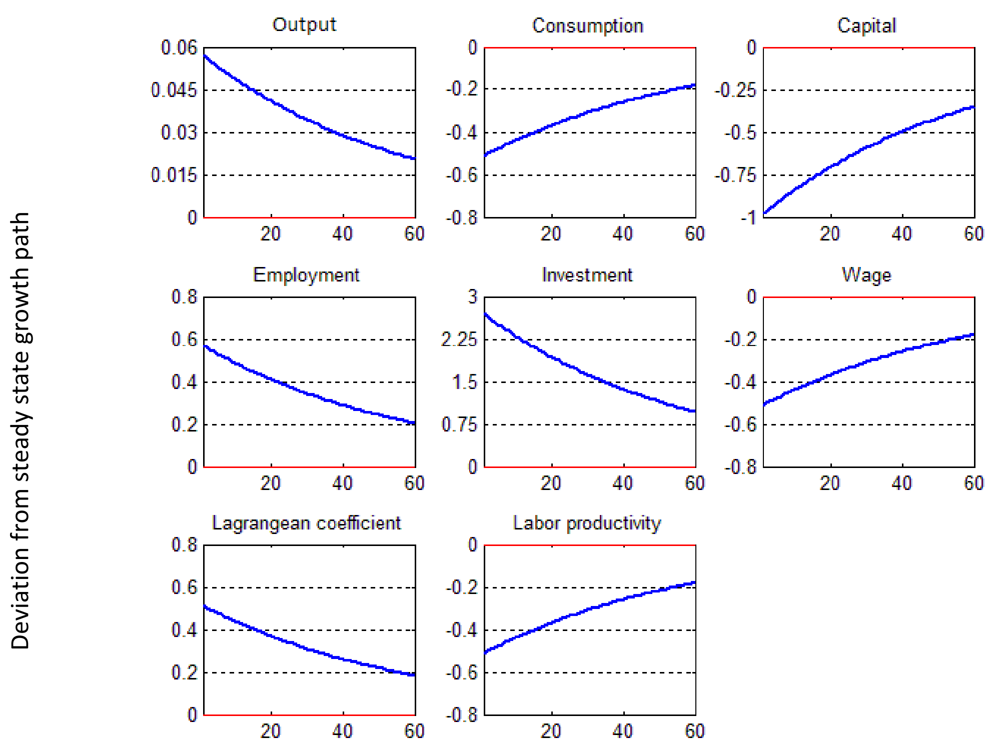

Figure 1: Shock on capital, decentralized model

Figure 1 shows the baseline model: it displays the reaction of considered variables when capital starts below its steady state path. Investment increases, so as to make up for the lower capital stock, which then starts accumulating at a faster pace. Hence, savings take over consumption, which decreases quite substantially. As already mentioned, the reaction of employment is one of increase, so that output also grows slightly and capital can be expected to recover in a reasonable amount of time.

The constrained model includes two different specifications: one in which the collateral constraint is only binding for human capital (i.e. $\theta = 0$) and capital serves as its own collateral, and another in which capital is subject to the constraint as well, even though only in minimum part ($\theta = 0.1$). As figures 2 to 5 show, there is quite a substantial difference between the two cases.

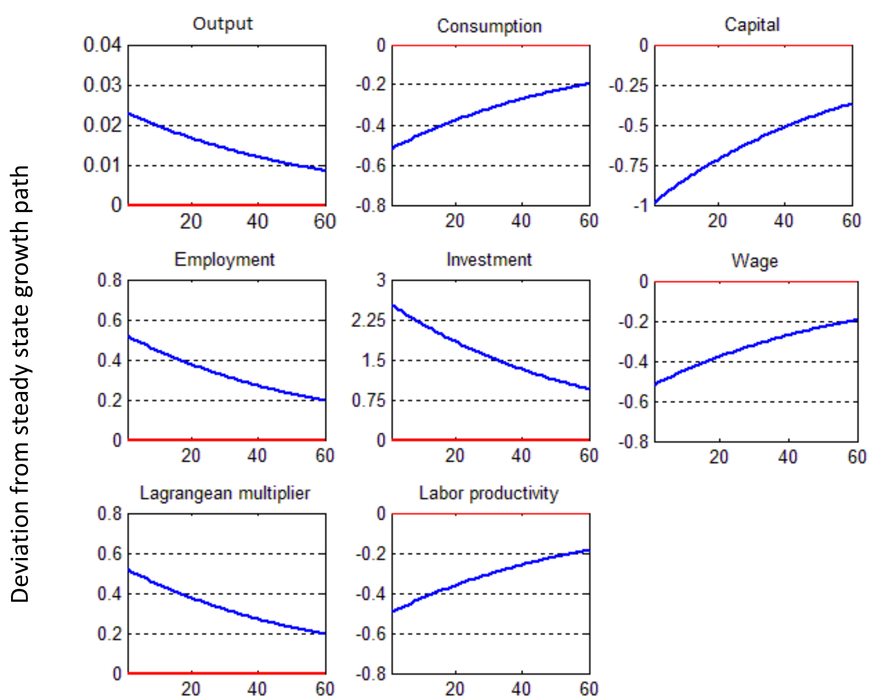

Figure 2: Shock on capital, model with binding credit constraint on employment

Let us consider the shock on capital first. When capital is not subject to the constraint, the variables of the model react only slightly differently with respect to the unconstrained case. In particular, wages still decline in consequence to the shock, and employment still increases, but to a lesser extent. Also, income barely increases in this case, but the main tendency of all the variables is left unchanged.

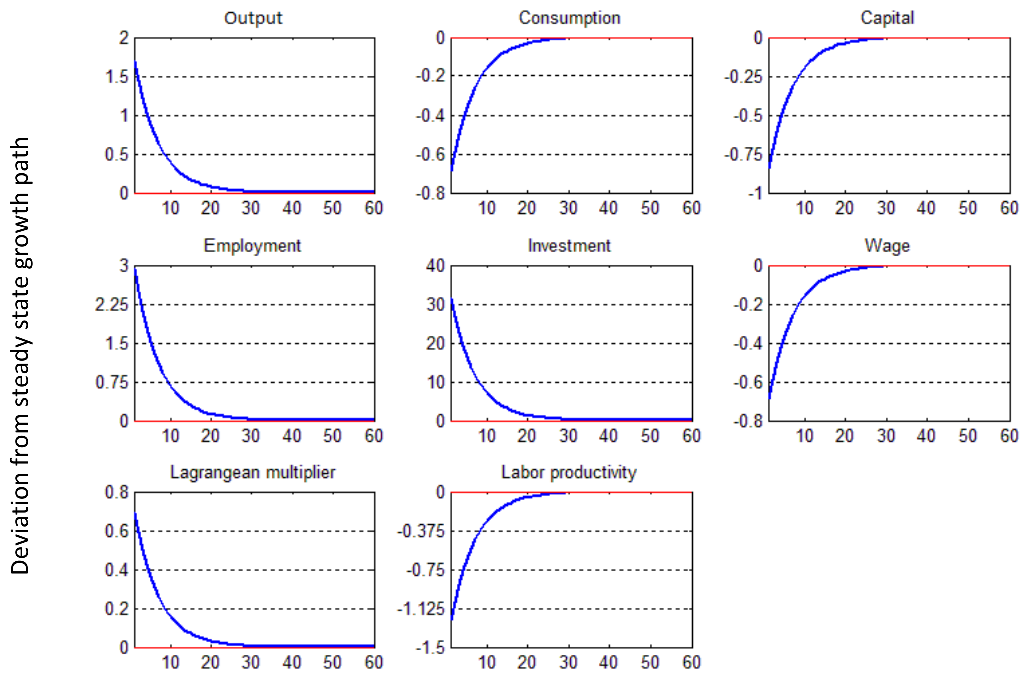

Figure 3: Shock on capital, model with binding credit constraint on employment and capital

Figure 3 shows that in case of constrained capital, the reaction of all variables to the same shock is much stronger and vanishes more quickly. Even though the direction of the response to the shock remains the same, variables now move a lot more than they did in the previous case. Employment, for example, moves six times as much as it did in the case where capital was not subject to the constraint, while investment increases over ten times as much as it did. Moreover, over a period of approximately three years, the shock is completely absorbed and the variables start growing at their steady-state pace again. As a matter of fact, the higher the value of θ, the faster the rate of convergence after a shock on capital. Apparently, the collateral constraint Z has a strong disciplining effect over capital, which is forced to rapidly recover so that it is possible for the constraint not to be violated and to remain in place.

When the shock hits the collateral constraint, the reaction of the variables is considerably different to previous cases (figures 4 and 5). Whether capital is subject to the constraint or not, both income and employment shrink quite substantially.

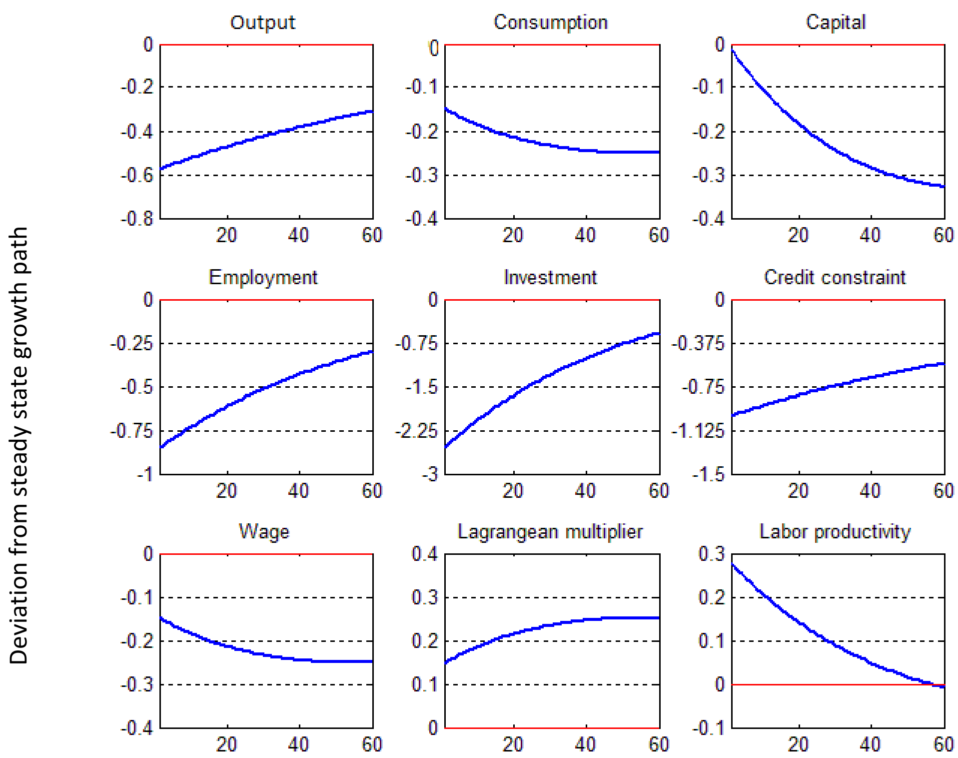

Figure 4: Shock on credit constraint, model with binding credit constraint on employment

Investment and consumption are also reduced, and the consumption trend is downward sloping: this means that the economy is moving further away from the previous steady state, before recovering its normal growth pace. Similarly, capital does not jump immediately on impact of the shock; rather, it tends to move away from its steady state for many periods after. This kind of shock will thus be much more persistent than a simple transitory shock on capital. Wages also tend to shift down on impact of the shock and then continue to further decline, while employment starts to recover.

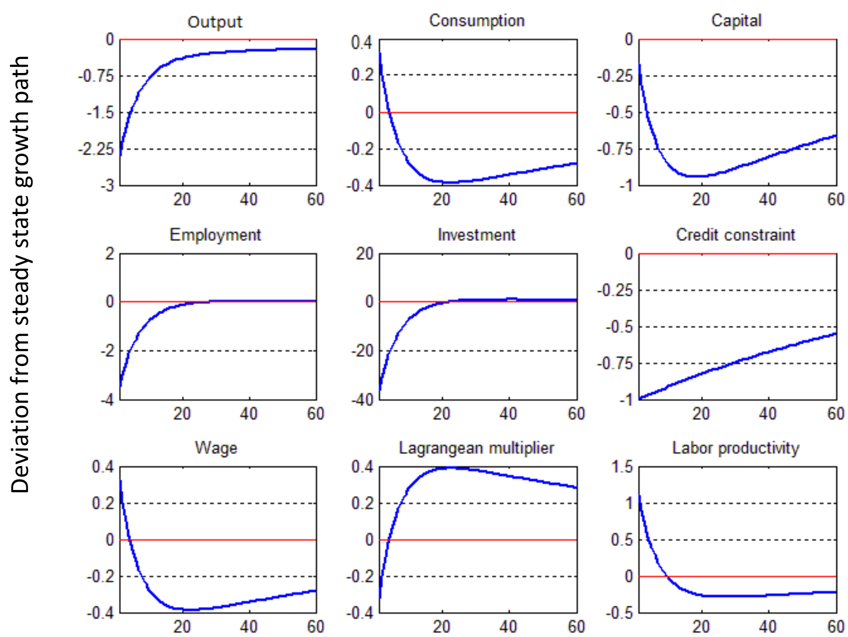

Figure 5: Shock on credit constraint, model with binding credit constraint on employment and capital

When the credit constraint is imposed on capital as well, the reaction of the variables tends to be stronger and more non-linear. Employment, output and investment all slump when the shock hits the economy; successively, they start to recover but instead of returning to the steady-state growth path immediately, they exceed it before finally recovering their normal pace. This creates a sort of "overshooting" in the variables previously mentioned. Wage increases on impact, but then rapidly shrinks below the steady-state growth path, before starting to recover.

Rigid wages

The introduction of rigid wages in the otherwise frictionless model gives rise to the jobless recovery phenomenon. In particular, figure 6 below shows how after the shock on capital, all variables fall by exactly the same amount as the initial shock. After this, they continue growing at their usual level (there is no recovery back towards previous steady-state, the shock has now permanent effects). This means that output starts growing again at the rate of g, while employment remains constant, and will never catch up with output.[6]

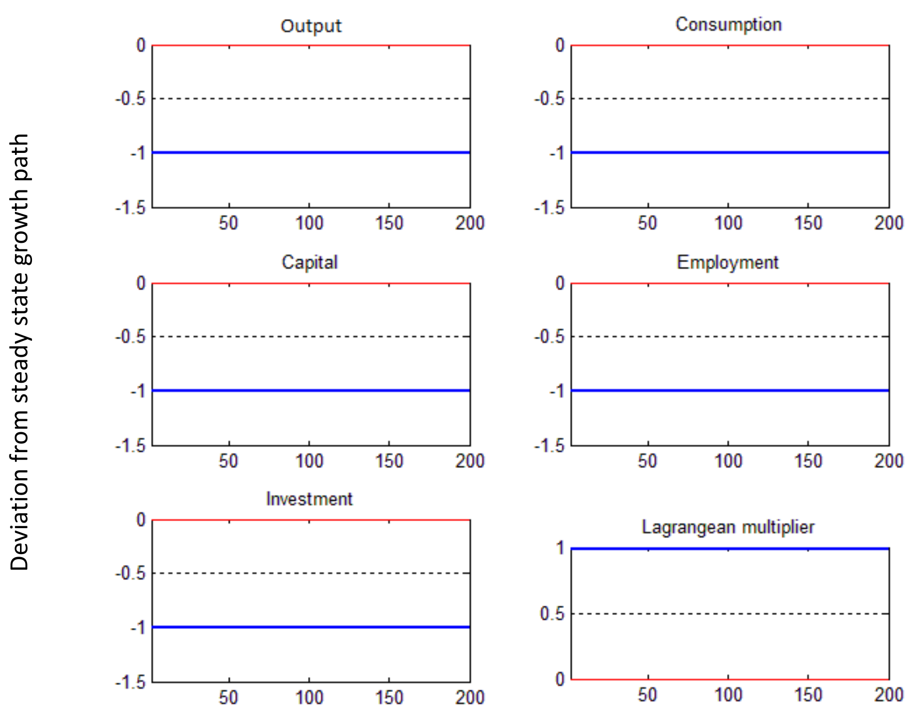

Figure 6: Shock on capital, rigid wage model

[6]The graph shows steady-state compatible variables, this is why they all look flat

On the contrary, the introduction of rigid wages in the constrained model does not create a jobless recovery situation: employment and income still increase on impact of the negative shock on capital, even though wage remains constant. This can be due to the specific calibration of the model, and although not shown here, it might be the case that different assumptions on the preference parameter σ would lead to a different outcome.

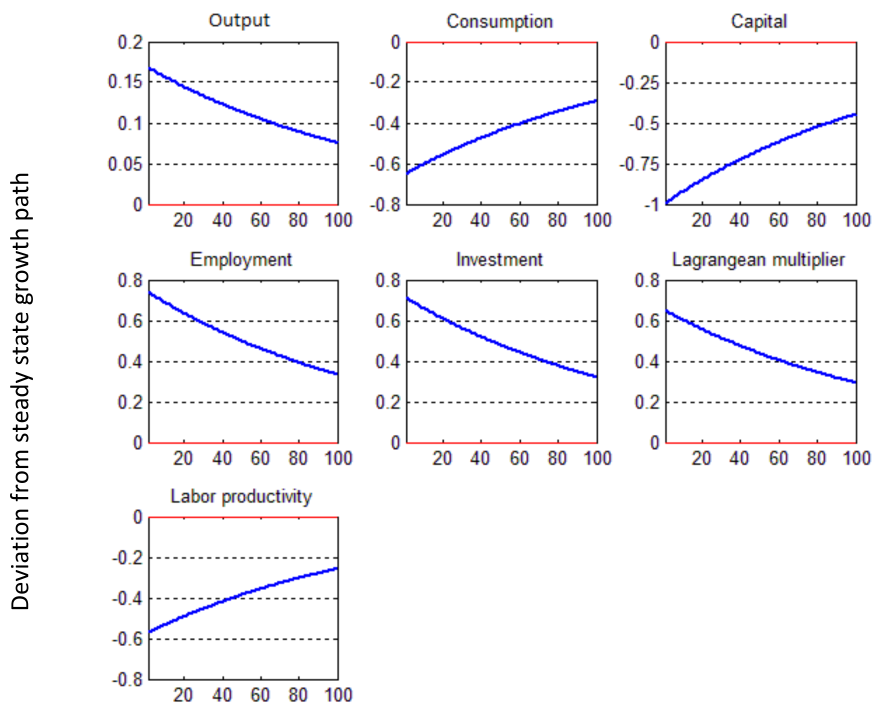

Figure 7: Shock on capital, model with wage rigidities and constraint on employment

Finally, when the shock on credit constraint is considered together with rigid wages, a situation of jobless recovery takes place. In fact, both employment and output fall on impact of the shock on collateral value, but employment falls more. Although employment starts growing faster than output, it does not catch up throughout five years. Moreover, at the new equilibrium, employment will remain constant whereas output will start to grow at the rate of g again. Capital starts deteriorating after the impact of the shock, while investment slowly begins to recover. Therefore, when the shock hits the economy, a unit of output contains more capital than employment. Only very slowly will this configuration shift towards a less capital and more labor intensive production. Consumption falls and continues falling for many periods after the shock: a perfect storm takes place.

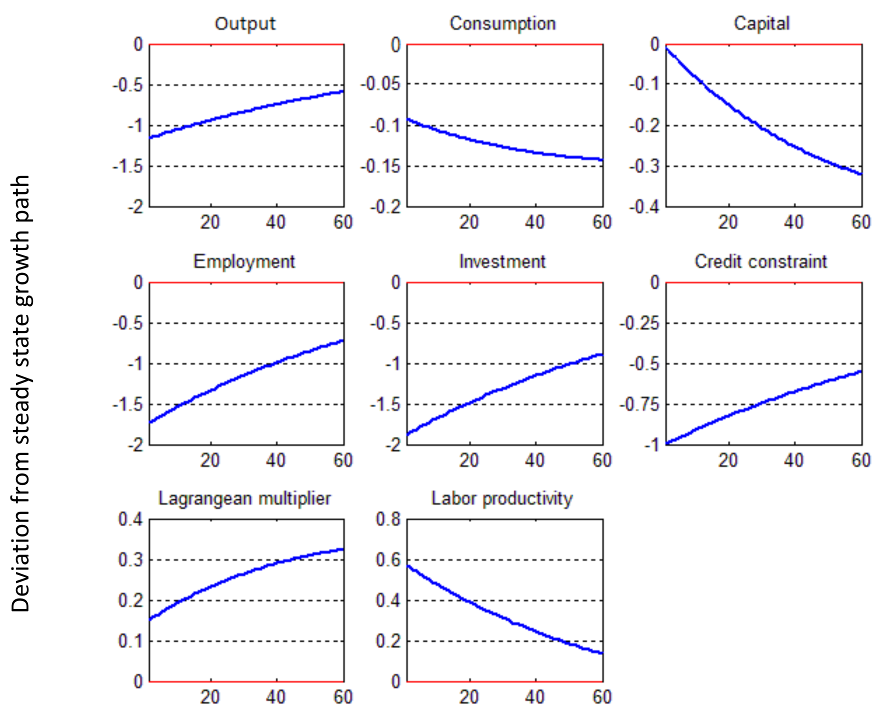

Figure 8: Shock on credit constraint, model with wage rigidities and constraint on employment

5 Conclusions

A simple frictionless real business cycle model is not able to reproduce the major facts observed in the recessions of the past decade. However, introducing some forms of constraints and rigidities substantially improves the explanatory ability of such a model. Rigid wages alone are enough to reproduce a situation of jobless recovery, although on a much smaller scale than that which has been observed during the past crisis. A negative shock on a collateral constraint, combined with inflexible wages, performs this task better. Indeed, the root of the 2008 financial crisis can be traced back to the bursting of the subprime mortgage bubble. After that, the value of collateral owned by firms and banks plunged to record low levels. Due to this, a shock that hits the collateral value first, and then propagates, eroding the stock of capital, is more accurate and truthful to reality.

This shows how the current labor market condition is not exclusively nor primarily the consequence of labor market rigidities and frictions. Rather, the credit market situation has a strong impact on the performance of other markets, and major spillover effects exist from one sector to the other. Moreover, credit conditions give rise to fluctuations in the business cycles and constitute a major source of shock amplification and propagation.

The main conclusion that can be drawn from this is that the old adage that finance and real economy are separate and distinct from one another is no longer valid or meaningful; a more in-depth scrutinization and supervision of the former is needed if one desires that business runs smoothly in the latter.

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A Appendices

A.1 The Planner's Problem

Households

A non-stochastic real business cycle model is considered. Let us analyze the case where a planner wants to maximize the utility of a representative family by finding the optimal allocation of consumption, employment and capital over time, taking into account a resource feasibility constraint (i.e. households can consume no more than what is produced inside the economy.)

The number of components of the household is normalized to 1 and they live infinitely. They discount utility from future consumption at the rate of β. Labor is indivisible. A fraction N_t of the household members is employed and consumes $C_{e,t}$, while a fraction $1-N_t$ is unemployed and consume $C_{u,t}$. Total consumption for the household is thus

$$C_t = N_t C_{e,t} + (1-N_t) C_{u,t} \tag{19}$$

The two groups have different utility functions, since employed people have to suffer a disutility from working (represented by the parameter $\gamma > 0$), while unemployed individuals have not. Therefore, at equilibrium, the amount that is consumed by each household member will depend on his/her employment status. The utility function for the whole household, and the social planner's objective function, is the equally weighted sum of the utility of its components:

$$\sum_{t=0}^{\infty} \beta^t \left(N_t \frac{C_{e,t}^{1-\sigma}(1+(\sigma-1)\gamma)^{\sigma}}{1-\sigma} + (1-N_t)\frac{C_{u,t}^{1-\sigma}}{1-\sigma} \right)$$

Firms

Firms use capital and labor to produce output. In each period, output together with undepreciated capital will be used for both consumption and investment.

The production function is a Cobb-Douglas, with labor augmenting technology evolving overtime according to:

$$A_{t+1} = (1+g)A_t \tag{20}$$

The law of motion for the capital stock, which constraints the maximization problem of the planner is:

$$K_{t+1} = K_t^{\alpha}(A_t N_t)^{1-\alpha} + (1-\delta)K_t - N_t C_{e,t} - (1-N_t)C_{u,t}$$

The maximization problem

Construct the Lagrangian for the problem

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left\{ \left(N_t \frac{C_{e,t}^{1-\sigma}(1+(\sigma-1)\gamma)^{\sigma}}{1-\sigma} + (1-N_t)\frac{C_{u,t}^{1-\sigma}}{1-\sigma} \right) \right.$$
$$\left. +\lambda_t \left[K_t^{\alpha}(A_t N_t)^{1-\alpha} + (1-\delta)K_t - N_t C_{e,t} - (1-N_t)C_{u,t} - K_{t+1} \right] \right\}$$

Derive first order conditions for $C_{e,t}$, $C_{u,t}$, N_t, K_{t+1} and λ_t

$$\frac{\partial \mathcal{L}}{\partial C_{e,t}}: \quad \beta^t \left[N_t C_{e,t}^{-\sigma}(1+(\sigma-1)\gamma)^{\sigma} - \lambda_t N_t \right] = 0 \tag{21}$$

$$\frac{\partial \mathcal{L}}{\partial C_{u,t}}: \quad \beta^t \left[(1-N_t)C_{u,t}^{-\sigma} - \lambda_t (1-N_t) \right] = 0 \tag{22}$$

$$\frac{\partial \mathcal{L}}{\partial N_t}: \ \frac{C_{e,t}^{1-\sigma}(1+(\sigma-1)\gamma)^{\sigma}}{1-\sigma} - \frac{C_{u,t}^{1-\sigma}}{1-\sigma} + \lambda_t \left[(1-\alpha)K_t^{\alpha}A_t^{1-\alpha}N_t^{-\alpha} - C_{e,t} + C_{u,t} \right] = 0 \tag{23}$$

$$\frac{\partial \mathcal{L}}{\partial K_{t+1}}: \quad \beta^{t+1}\lambda_{t+1}\left[\alpha K_{t+1}^{\alpha-1}\left(A_{t+1}N_{t+1}\right)^{1-\alpha}+(1-\delta)\right]-\beta^t\lambda_t=0 \tag{24}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_t}: \quad K_t^{\alpha}\left(A_tN_t\right)^{1-\alpha}+(1-\delta)K_t-N_tC_{e,t}-(1-N_t)C_{u,t}-K_{t+1}=0 \tag{25}$$

Consumption: From the first two equations, derive an expression which equates the Lagrangian multiplier to the marginal utility of consumption for the two groups.

$\Rightarrow \quad \left(\frac{C_{e,t}}{1+(\sigma-1)\gamma}\right)^{-\sigma}=\lambda_t$

$\Rightarrow \quad C_{u,t}^{-\sigma}=\lambda_t$

Putting these two results together, a relationship between consumption for employed and unemployed individuals is obtained.

$C_{e,t}=[1+(\sigma-1)\gamma]C_{u,t}$

From this, it emerges that when $\sigma>1$, employed individuals consume more than unemployed ones. Indeed, this parameter determines the complementarity between consumption and work (other than risk-aversion and the inter-temporal elasticity of substitution).

Recalling relation (19), one can easily express consumption for employed and unemployed in terms of total consumption. In particular, $C_{u,t}=\dfrac{C_t-N_tC_{e,t}}{1-N_t}$.

By plugging this into the relationship between consumption of employed and unemployed, it can be concluded that:

$$C_{e,t}=\frac{1+(\sigma-1)\gamma}{1+(\sigma-1)\gamma N_t}Ct; \quad \textit{and} \quad C_{u,t}=\frac{C_t}{1+(\sigma-1)\gamma N_t} \tag{26}$$

Then, combining this last expression with equation (22), it is easy to see that

$$\lambda_t=\left(\frac{C_t}{1+(-1)\gamma}\right)^{-\sigma} \tag{27}$$

Employment: Manipulate equation (23) so as to eliminate consumption and substitute it with its marginal utility λ_t by exploiting the first order conditions for consumption.

$$\Rightarrow \quad \frac{C_{e,t}}{1-\sigma}\left(\frac{C_{e,t}}{1+(\sigma-1)\gamma}\right)^{-\sigma} - \frac{C_{u,t}}{1-\sigma}(C_{u,t})^{-\sigma} + \lambda_t\left[(1-\alpha)\left(\frac{K_t}{A_t N_t}\right)^{\alpha} A_t + (C_{u,t}-C_{e,t})\right] = 0$$

$\frac{\sigma}{1-\sigma}(C_{e,t}-C_{u,t}) + (1-\alpha)A_t\kappa_t^{\alpha} = 0$

where $\kappa_t = \frac{K_t}{A_t N_t}$ is capital per efficiency unit of labor. From relationships (21) and (22), observe that $C_{e,t}-C_{u,t} = \lambda_t^{-\frac{1}{\sigma}}(\sigma-1)\gamma$. Plug this into the previous equation to get the optimal condition for employment, which equates the marginal product of labor (on the left-hand side of the equal sign) to the marginal rate of substitution between consumption and leisure (on the right-hand side)

$$(1-\alpha)A_t\kappa_t^{\alpha} = \sigma\gamma\lambda_t^{-\frac{1}{\sigma}} \tag{28}$$

Capital: From the first derivative of the Lagrangian with respect to capital, the Euler equation is obtained:

$$\lambda_t = \beta\lambda_{t+1}\left(\alpha\kappa_{t+1}^{\alpha-1} + 1 - \delta\right) \tag{29}$$

Where $\left(\alpha\kappa_{t+1}^{\alpha-1} + 1 - \delta\right)$ is the marginal utility of capital.

Lagrangian multiplier: Equation (25) gives back the resource constraint of the economy. By plugging in definition (19), this results in:

$$K_{t+1} = K_t^{\alpha}(A_t N_t)^{1-\alpha} + (1-\delta)K_t - C_t \tag{30}$$

Putting it all together: Now use equation (28) into (29) to get rid of κ_{t+1}.

$$\kappa_{t+1} = \left(\frac{\sigma\gamma}{(1-\alpha)A_{t+1}\lambda_{t+1}^{\frac{1}{\sigma}}} \right)^{\frac{1}{\alpha}}$$

$$\lambda_t = \beta\lambda_{t+1} \left[\alpha \left(\frac{(1-\alpha)A_{t+1}\lambda_{t+1}^{\frac{1}{\sigma}}}{\sigma\gamma} \right)^{\frac{1-\alpha}{\alpha}} + 1 - \delta \right] \tag{31}$$

Next, eliminate C_t and N_t from the resource feasibility constraint (30) so as to obtain a relationship between capital and marginal utility of consumption exclusively. To do this, use equation (28) to obtain a formula for employment, then use this one into expression (27) and rewrite the latter so as to express consumption in terms of the Lagrangian multiplier only.

$$\Rightarrow \quad N_t = \left(\frac{(1-\alpha)A_t\lambda_t^{\frac{1}{\sigma}}}{\sigma\gamma} \right)^{\frac{1}{\alpha}} \frac{K_t}{A_t}$$

$$C_t = \lambda_t^{-\frac{1}{\sigma}} [1 + (\sigma - 1)\gamma N_t]$$

$$\Rightarrow \quad C_t = \lambda_t^{-\frac{1}{\sigma}} \left[1 + (\sigma-1)\gamma \left(\frac{(1-\alpha)A_t\lambda_t^{\frac{1}{\sigma}}}{\sigma\gamma} \right)^{\frac{1}{\alpha}} \frac{K_t}{A_t} \right]$$

Rewrite (30) using these two to obtain:

$$K_{t+1} = K_t^{\alpha} A_t^{1-\alpha} \left(\frac{(1-\alpha)A_t\lambda_t^{\frac{1}{\sigma}}}{\sigma\gamma} \right)^{\frac{1-\alpha}{\alpha}} \left(\frac{K_t}{A_t} \right)^{1-\alpha} + (1-\delta) K_t -$$
$$\lambda_t^{-\frac{1}{\sigma}} \left[1 + (\sigma-1)\gamma \left(\frac{(1-\alpha)A_t\lambda_t^{\frac{1}{\sigma}}}{\sigma\gamma} \right)^{\frac{1}{\alpha}} \frac{K_t}{A_t} \right]$$

$$K_{t+1}=K_t\left(\frac{(1-\alpha)A_t\lambda_t^{\frac{1}{\sigma}}}{\sigma\gamma}\right)^{\frac{1-\alpha}{\alpha}}+(1-\delta)K_t-\lambda_t^{-\frac{1}{\sigma}}-$$

$$(\sigma-1)\gamma\left(\frac{(1-\alpha)A_t\lambda_t^{\frac{1}{\sigma}}}{\sigma\gamma}\right)^{\frac{1}{\alpha}}\frac{K_t}{A_t}\lambda_t^{-\frac{1}{\sigma}}$$

$$K_{t+1}=K_t\left(\frac{(1-\alpha)A_t\lambda_t^{\frac{1}{\sigma}}}{\sigma\gamma}\right)^{\frac{1}{\alpha}}\left[\frac{\sigma\gamma}{(1-\alpha)A_t\lambda_t^{\frac{1}{\sigma}}}-\frac{(\sigma-1)\gamma}{A_t\lambda_t^{\frac{1}{\sigma}}}\right]+(1-\delta)K_t-\lambda_t^{-\frac{1}{\sigma}}$$

$$K_{t+1}=K_t\left(\frac{(1-\alpha)A_t\lambda_t^{\frac{1}{\sigma}}}{\sigma\gamma}\right)^{\frac{1}{\alpha}}\left[\frac{\sigma\gamma-(\sigma-1)\gamma(1-\alpha)}{(1-\alpha)A_t\lambda_t^{\frac{1}{\sigma}}}\right]+(1-\delta)K_t-\lambda_t^{-\frac{1}{\sigma}}$$

$$K_{t+1}=K_t\left(\frac{(1-\alpha)A_t\lambda_t^{\frac{1}{\sigma}}}{\sigma\gamma}\right)^{\frac{1}{\alpha}}\left[\frac{\gamma\sigma}{(1-\alpha)A_t\lambda_t^{\frac{1}{\sigma}}}\cdot\frac{1-\alpha+\alpha\sigma}{\sigma}\right]+(1-\delta)K_t-\lambda_t^{-\frac{1}{\sigma}}$$

$$K_{t+1}=\left[\left(\frac{(1-\alpha)A_t\lambda_t^{\frac{1}{\sigma}}}{\sigma\gamma}\right)^{\frac{1-\alpha}{\alpha}}\left(\frac{1-\alpha+\alpha\sigma}{\sigma}\right)+1-\delta\right]K_t-\lambda_t^{-\frac{1}{\sigma}} \qquad (32)$$

Log-linearization around steady state

In order to solve the model, it is sufficient to log-linearize equations (31) and (32) around their balanced growth path. However, given the law of motion for the technology shock (20), it is clear that such a variable does not exist at steady state, and the same happens for consumption, capital and the marginal utility of consumption. Therefore, the two main relationships of the model need to be modified before log-linearization, so that they will only contain stable variables, that can be defined at steady state. In particular, while consumption and capital grow at the same rate as the technology shock, g, the marginal utility of consumption λ_t grows at $(1+g)^{-\sigma}-1$. Therefore, we will define:

$\frac{K_t}{A_t} = k_t; \qquad \frac{\lambda_t^{-\frac{1}{\sigma}}}{A_t} = \Lambda_t^{-\frac{1}{\sigma}}; \qquad \frac{\lambda_t}{A_t^{-\sigma}} = \Lambda_t$

Let us rewrite equations (31) and (32) in terms of these new variables:

$$\frac{\lambda_t}{A_t^{-\sigma}} \cdot A_t^{-\sigma} = \beta \frac{\lambda_{t+1}}{A_{t+1}^{-\sigma}} \cdot A_{t+1}^{-\sigma} \left[\alpha \left(\frac{1-\alpha}{\sigma\gamma} \right)^{\frac{1-\alpha}{\alpha}} \left(\frac{\lambda_{t+1}^{-\frac{1}{\sigma}}}{A_{t+1}} \right)^{\frac{\alpha-1}{\alpha}} + 1 - \delta \right]$$

$$\Lambda_t = \beta \Lambda_{t+1} \left(\frac{A_{t+1}}{A_t} \right)^{-\sigma} \left[\alpha \left(\frac{(1-\alpha)\Lambda_{t+1}^{\frac{1}{\sigma}}}{\sigma\gamma} \right)^{\frac{1-\alpha}{\alpha}} + 1 - \delta \right]$$

Exploiting the law of movement for the technology shock 20, one obtains

$$\Lambda_t = \frac{\beta}{(1+g)^{\sigma}} \Lambda_{t+1} \left[\alpha \left(\frac{(1-\alpha)\Lambda_{t+1}^{\frac{1}{\sigma}}}{\sigma\gamma} \right)^{\frac{1-\alpha}{\alpha}} + 1 - \delta \right] \tag{33}$$

$$\frac{K_{t+1}}{A_{t+1}} \cdot A_{t+1} = \left[\left(\frac{(1-\alpha)A_t \lambda_t^{\frac{1}{\sigma}}}{\sigma\gamma} \right)^{\frac{1-\alpha}{\alpha}} \left(\frac{1-\alpha+\alpha\sigma}{\sigma} \right) + 1 - \delta \right] \frac{K_t}{A_t} \cdot A_t - \frac{\lambda_t^{-\frac{1}{\sigma}}}{A_t} \cdot A_t$$

$$k_{t+1}(1+g) = \left[\left(\frac{(1-\alpha)\Lambda_t^{\frac{1}{\sigma}}}{\sigma\gamma} \right)^{\frac{1-\alpha}{\alpha}} \left(\frac{1-\alpha+\alpha\sigma}{\sigma} \right) + 1 - \delta \right] k_t - \Lambda_t^{-\frac{1}{\sigma}} \tag{34}$$

These two equations, with the newly defined variables Λ_t and k_t can be log linearized; but let's first, analyze their behavior at steady state.

$$\Lambda = \frac{\beta}{(1+g)^{\sigma}} \Lambda \left[\alpha \left(\frac{(1-\alpha)\Lambda^{\frac{1}{\sigma}}}{\sigma\gamma} \right)^{\frac{1-\alpha}{\alpha}} + 1 - \delta \right]$$

$$\frac{(1+g)^{\sigma}}{\beta} = \alpha \left(\frac{(1-\alpha)\Lambda^{\frac{1}{\sigma}}}{\sigma\gamma} \right)^{\frac{1-\alpha}{\alpha}} + (1-\delta)$$

$$\left(\frac{(1-\alpha)\Lambda^{\frac{1}{\sigma}}}{\sigma\gamma} \right)^{\frac{1-\alpha}{\alpha}} = \frac{(1+g)^{\sigma}}{\beta\alpha} - \frac{(1-\delta)}{\alpha} \tag{35}$$

$$\frac{k(1+g)}{k} = \left[\left(\frac{(1-\alpha)\Lambda^{\frac{1}{\sigma}}}{\sigma\gamma}\right)^{\frac{1-\alpha}{\alpha}} \left(\frac{1-\alpha+\alpha\sigma}{\sigma}\right) + 1 - \delta\right] - \frac{\Lambda^{-\frac{1}{\sigma}}}{k}$$

$$\frac{\Lambda^{-\frac{1}{\sigma}}}{k} = \left[\left(\frac{(1-\alpha)\Lambda^{\frac{1}{\sigma}}}{\sigma\gamma}\right)^{\frac{1-\alpha}{\alpha}} \left(\frac{1-\alpha+\alpha\sigma}{\sigma}\right) + 1 - \delta\right] - (1+g)$$

Substitute $\left(\frac{(1-\alpha)\Lambda^{\frac{1}{\sigma}}}{\sigma\gamma}\right)^{\frac{1-\alpha}{\alpha}}$ using the previous relationship

$$\frac{\Lambda^{-\frac{1}{\sigma}}}{k} = \left[\left(\frac{(1+g)^{\sigma} - \beta(1-\delta)}{\beta\alpha}\right) \left(\frac{1-\alpha+\alpha\sigma}{\sigma}\right) + 1 - \delta\right] - (1+g)$$

$$\frac{\Lambda^{-\frac{1}{\sigma}}}{k} = \frac{(1-\alpha+\alpha\sigma)(1+g)^{\sigma} - \beta(1-\delta)(1-\alpha)}{\beta\alpha\sigma} - (1+g) \tag{36}$$

Now rewrite equation (33) and (34) in terms of log deviation of the variables from their steady state. In particular, set $\tilde{x}_t = \ln x_t$, (where x stands for the variable in consideration), $\tilde{x} = \ln x$ and $\hat{x}_t = \tilde{x}_t - \tilde{x}$.

$$\frac{\Lambda_{t+1}}{\Lambda_t} = \frac{(1+g)^{\sigma}}{\beta} \left[\alpha \left(\frac{(1-\alpha)\Lambda_{t+1}^{\frac{1}{\sigma}}}{\sigma\gamma}\right)^{\frac{1-\alpha}{\alpha}} + 1 - \delta\right]^{-1}$$

Substitute each variable with the exponential of its logarithm (which will leave the relation unchanged)

$$e^{\tilde{\Lambda}_{t+1} - \tilde{\Lambda}_t} = \frac{(1+g)^{\sigma}}{\beta} \left[\alpha \left(\frac{1-\alpha}{\sigma\gamma}\right)^{\frac{1-\alpha}{\alpha}} e^{\frac{1-\alpha}{\alpha\sigma}\tilde{\Lambda}_{t+1}} + 1 - \delta\right]^{-1}$$

Now apply Taylor expansion $[f(x) \simeq f(x_0) + f'(x_0)(x - x_0)]$ around steady state values of the variables.

$$1+\left[\left(\tilde{\Lambda}_{t+1}-\tilde{\Lambda}\right)-\left(\tilde{\Lambda}_t-\tilde{\Lambda}\right)\right]=\tfrac{(1+g)^{\sigma}}{\beta}\left[\alpha\left(\tfrac{(1-\alpha)\Lambda^{\frac{1}{\sigma}}}{\sigma\gamma}\right)^{\frac{1-\alpha}{\alpha}}+1-\delta\right]^{-1}-$$

$$\tfrac{(1+g)^{\sigma}}{\beta}\left[\alpha\left(\tfrac{(1-\alpha)\Lambda^{\frac{1}{\sigma}}}{\sigma\gamma}\right)^{\frac{1-\alpha}{\alpha}}+1-\delta\right]^{-2}\left(\tfrac{1-\alpha}{\alpha\sigma}\right)\cdot\alpha\left(\tfrac{(1-\alpha)\Lambda^{\frac{1}{\sigma}}}{\sigma\gamma}\right)^{\frac{1-\alpha}{\alpha}}\left[\tilde{\Lambda}_{t+1}-\tilde{\Lambda}\right]$$

Plug in steady state relation (35)

$$\tfrac{\beta}{(1+g)^{\sigma}}\left(1+\hat{\Lambda}_{t+1}-\hat{\Lambda}_t\right)=\left[\alpha\left(\tfrac{(1+g)^{\sigma}-\beta(1-\delta)}{\beta\alpha}\right)+1-\delta\right]^{-1}-$$
$$\tfrac{(1-\alpha)}{\sigma}\left[\tfrac{(1+g)^{\sigma}-\beta(1-\delta)}{\beta}+1-\delta\right]^{-2}\left(\tfrac{(1+g)^{\sigma}-\beta(1-\delta)}{\beta\alpha}\right)\hat{\Lambda}_{t+1}$$

$$\tfrac{\beta}{(1+g)^{\sigma}}\left(1+\hat{\Lambda}_{t+1}-\hat{\Lambda}_t\right)=\tfrac{\beta}{(1+g)^{\sigma}}-\tfrac{(1-\alpha)}{\sigma}\left[\tfrac{\beta}{(1+g)^{\sigma}}\right]^{2}\left(\tfrac{(1+g)^{\sigma}-\beta(1-\delta)}{\beta\alpha}\right)\hat{\Lambda}_{t+1}$$

$$1+\hat{\Lambda}_{t+1}-\hat{\Lambda}_t=1-\tfrac{1-\alpha}{\alpha\sigma}\cdot\tfrac{(1+g)^{\sigma}-\beta(1-\delta)}{(1+g)^{\sigma}}\hat{\Lambda}_{t+1}$$

$$\hat{\Lambda}_{t+1}\left[1+\tfrac{1-\alpha}{\alpha\sigma}\left(1-\beta(1-\delta)(1+g)^{-\sigma}\right)\right]=\hat{\Lambda}_t$$

$$\hat{\Lambda}_{t+1}\left[\tfrac{\alpha\sigma+(1-\alpha)\left[1-\beta(1-\delta)(1+g)^{-\sigma}\right]}{\alpha\sigma}\right]=\hat{\Lambda}_t$$

$$\hat{\Lambda}_{t+1}=\frac{\alpha\sigma}{1-\alpha+\alpha\sigma-\beta(1-\alpha)(1-\delta)(1+g)^{-\sigma}}\hat{\Lambda}_t$$

$$\hat{\Lambda}_{t+1}=b_{\Lambda\Lambda}\hat{\Lambda}_t$$

Now repeat the same steps for equation (34)

$$\tfrac{k_{t+1}}{k_t}(1+g)=\left(\tfrac{(1-\alpha)\Lambda_t^{\frac{1}{\sigma}}}{\sigma\gamma}\right)^{\frac{1-\alpha}{\alpha}}\left(\tfrac{1-\alpha+\alpha\sigma}{\sigma}\right)+1-\delta-\tfrac{\Lambda_t^{-\frac{1}{\sigma}}}{k_t}$$

$$(1+g)\,e^{\tilde{k}_{t+1}-\tilde{k}_t}=\left(\tfrac{1-\alpha+\alpha\sigma}{\sigma}\right)\left(\frac{1-\alpha}{\sigma\gamma}\right)^{\frac{1-\alpha}{\alpha}}e^{\frac{1-\alpha}{\alpha\sigma}\tilde{\Lambda}_t}+1-\delta-e^{-\left(\frac{1}{\sigma}\tilde{\Lambda}_t+\tilde{k}_t\right)}$$

$$(1+g)+(1+g)\left[\left(\tilde{k}_{t+1}-\tilde{k}\right)-\left(\tilde{k}_t-\tilde{k}\right)\right]=\left(\tfrac{1-\alpha+\alpha\sigma}{\sigma}\right)\left(\frac{(1-\alpha)\Lambda^{\frac{1}{\sigma}}}{\sigma\gamma}\right)^{\frac{1-\alpha}{\alpha}}+$$
$$\tfrac{1-\alpha}{\alpha\sigma}\left(\tfrac{1-\alpha+\alpha\sigma}{\sigma}\right)\left(\frac{(1-\alpha)\Lambda^{\frac{1}{\sigma}}}{\sigma\gamma}\right)^{\frac{1-\alpha}{\alpha}}\left[\tilde{\Lambda}_t-\tilde{\Lambda}\right]+(1-\delta)-\tfrac{\Lambda^{-\frac{1}{\sigma}}}{K}-\tfrac{\Lambda^{-\frac{1}{\sigma}}}{K}\left[-(\tilde{k}_t-\tilde{k})-\tfrac{1}{\sigma}(\tilde{\Lambda}_t-\tilde{\Lambda})\right]$$

$$(1+g)\left(1+\hat{k}_{t+1}-\hat{k}_t\right)=\left(\tfrac{1-\alpha+\alpha\sigma}{\sigma}\right)\left(\tfrac{(1+g)^{\sigma}-\beta(1-\delta)}{\beta\alpha}\right)\left[1+\tfrac{1-\alpha}{\alpha\sigma}\hat{\Lambda}_t\right]+(1-\delta)-$$
$$\tfrac{(1-\alpha+\alpha\sigma)(1+g)^{\sigma}-\beta(1-\delta)(1-\alpha)-\beta\alpha\sigma(1+g)}{\beta\alpha\sigma}\left[1-\tfrac{1}{\sigma}\hat{\Lambda}_t-\hat{k}_t\right]$$

$$(1+g)+(1+g)\,\hat{k}_{t+1}=\left[(1+g)+\tfrac{(1-\alpha+\alpha\sigma)(1+g)^{\sigma}-\beta(1-\delta)(1-\alpha)}{\beta\alpha\sigma}-(1+g)\right]\hat{k}_t+$$
$$\left[\tfrac{(1-\alpha)(1-\alpha+\alpha\sigma)}{\sigma^2}\left(\tfrac{(1+g)^{\sigma}-\beta(1-\delta)}{\beta\alpha^2}\right)+\tfrac{(1-\alpha+\alpha\sigma)(1+g)^{\sigma}-\beta(1-\delta)(1-\alpha)-\beta\alpha\sigma(1+g)}{\beta\alpha\sigma^2}\right]\hat{\Lambda}_t+$$
$$(1-\delta)+(1+g)-(1-\delta)$$

$$\hat{k}_{t+1}=\frac{(1-\alpha+\alpha\sigma)\,(1+g)^{\sigma}-\beta(1-\delta)(1-\alpha)}{\beta\alpha\sigma(1+g)}\hat{k}_t+$$
$$\frac{(1-\alpha+\alpha\sigma)\,(1+g)^{\sigma}-\beta(1-\delta)(1-\alpha)-\beta\sigma\alpha^2(1+g)}{\beta\alpha^2\sigma^2(1+g)}\hat{\Lambda}_t$$

$$\hat{k}_{t+1}=b_{kk}\hat{k}_t+b_{k\lambda}\hat{\Lambda}_t$$

A.2 Dynare code

Baseline model

```
var y c k n a i w lambda y_n;
varexo eps_a eps_k;
parameters beta gamma sigma delta alpha rho g n_ss k_ss i_ss y_ss c_ss w_ss
lambda_ss y_n_ss;
    alpha = 0.33;
    i_y = 0.015;
    k_y = 3.2;
    delta = i_y/k_y;
    beta = 0.996;
    rho = 0.99;
    sigma = 1;
    g=0.0018;
    n_ss=0.95;
    k_ss = ((1/beta-(1-delta))/alpha)^(1/(alpha-1))*n_ss;
    i_ss = delta*k_ss;
    y_ss=k_ss^alpha*n_ss^(1-alpha);
    c_ss = k_ss^(alpha)*n_ss^(1-alpha)-i_ss;
    w_ss = (1-alpha)*(k_ss/n_ss)^alpha;
    gamma = w_ss/(sigma*c_ss-(sigma-1)*w_ss*n_ss);
    lambda_ss = (w_ss/(sigma*gamma))^(-sigma);
    y_n_ss = y_ss/n_ss;
model;
    exp(lambda) = (exp(c)/(1+(sigma-1)*gamma*exp(n)))^(-sigma);
    exp(w) = sigma*gamma*exp(lambda)^(-(1/sigma));
    exp(w) = (1-alpha)*exp(a)*(exp(k)/(exp(n)*exp(a)))^alpha;
    exp(lambda)= beta*exp(lambda(+1))*(alpha*(exp(k(+1))/(exp(n(+1))*exp(a(+1))))^(alpha-
1)+1-delta);
```

```
    exp(k) = exp(i)+(1-delta)*exp(-eps_k)*exp(k(-1));
    exp(y) = exp(k)^alpha*(exp(n)*exp(a))^(1-alpha);
    exp(i) = exp(y)-exp(c);
    a = g + rho*a(-1)+eps_a;
    exp(y_n) = exp(y)/exp(n);
end;
initval;
    i = log(i_ss);
    y = log(y_ss);
    k = log(k_ss);
    c = log(c_ss);
    n = log(n_ss);
    a = 0;
    w = log(w_ss);
    lambda = log(lambda_ss);
    y_n = log(y_n_ss);
end;
shocks;
    var eps_a = 1;
    var eps_k = 1;
end;
resid(1);
steady;
check;
stoch_simul(order = 1,irf=60);
```

Rigid wages[7]

```
model;
    exp(lambda) = (exp(c)/(1+(sigma-1)*gamma*exp(n)))^(-sigma);
    exp(w)/exp(a) = (1-alpha)*(exp(k)/(exp(n)*exp(a)))^alpha;
    exp(w)/exp(a) = w_ss;
    exp(lambda)= beta*exp(lambda(+1))*(alpha*(exp(k(+1))/(exp(a(+1))*exp(n(+1))))^(alpha-
1)+1-delta);
    exp(k) = exp(i)+(1-delta)*exp(-eps_k)*exp(k(-1));
    exp(y) = exp(k)^alpha*(exp(n)*exp(a))^(1-alpha);
    exp(i) = exp(y)-exp(c);
    a = g + rho*a(-1)+eps_a;
    exp(y_n) = exp(y)/exp(n);
end;
```

Credit constraint

```
model;
    sigma*gamma*exp(lambda)^(-1/sigma) = exp(w);
    beta*exp(lambda(+1))*(alpha*(exp(k(+1))/(exp(a(+1))*exp(n(+1))))^(alpha-1)+1-
delta) = exp(lambda)*(1-theta+theta*(1-alpha)*(exp(k)/(exp(a)*exp(n)))^alpha*(exp(a)/exp(w)));
    theta*exp(k)+exp(w)*exp(n)=exp(z);
    exp(lambda) = (exp(c)/(1+(sigma-1)*gamma*exp(n)))^(-sigma);
    exp(k) = exp(i)+(1-delta)*exp(-eps_k)*exp(k(-1));
    exp(y) = exp(k)^alpha*(exp(n)*exp(a))^(1-alpha);
    exp(i)=exp(y)-exp(c);
    a = g + rho_a*a(-1)+eps_a;
    exp(z) = exp(-eps_z)*exp(rho_z*z(-1));
    exp(y_n)=exp(y)/exp(n);
end;
```

[7] The rest of the code remains unchanged with respect to previous case.

Credit constraint and rigid wages

```
model;
    exp(w)/exp(a) = w_ss;
    beta*exp(lambda(+1))*(alpha*(exp(k(+1))/(exp(a(+1))*exp(n(+1))))^(alpha-1)+1-
delta) = exp(lambda)*(1-theta+theta*(1-alpha)*(exp(k)/(exp(a)*exp(n)))^alpha*(exp(a)/exp(w)));
    exp(n)=(exp(z)-theta*exp(k))/exp(w);
    exp(lambda) = (exp(c)/(1+(sigma-1)*gamma*exp(n)))^(-sigma);
    exp(k) = exp(i)+(1-delta)*exp(-eps_k)*exp(k(-1));
    exp(y) = exp(k)^alpha*(exp(n)*exp(a))^(1-alpha);
    exp(i)=exp(y)-exp(c);
    a = g + rho_a*a(-1)+eps_a;
    exp(z) = exp(-eps_z)*exp(rho_z*z(-1));
    exp(y_n)=exp(y)/exp(n);
end;
```