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COURSE L-33 “ECONOMICS AND BUSINESS”

An Analysis of Solvency II Standard Formula for Calculation of SCR, possible corrections and a comparison with an internal model.

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EXECUTIVE SUMMARY

In this work I will investigate and analyze Solvency II directives, with a particular focus on the calculation of Solvency Capital Requirements (SCR). This is a really interesting field of study, given its relative newness and the important implication it has in the financial sector. This article highlights strengths and weaknesses of the standard model for the evaluation of SCR, showing possible corrections for mattering much problems entailed. Lastly, in the last section I will examine internal models; in particular a partial internal model, developed by Gatzer and Martin (2012), will be deeply analyze and compared to the standard model, showing differences between them in order to finally draw some conclusions.

The one advantage of playing with fire is that one never gets even singed.
It is the people who don't know how to play with it who get burned up.

Oscar Wilde

Summary:

Introduction.

Oscar Wilde said that playing with fire involve no risks, on the condition that you know how to play with. Insurance companies know for sure how to cope with risk.

Insurance companies, by their nature, bear risks. Indeed the main activity of insurance companies is to spread risk among individuals facing similar exposures. Insurance companies have done this for centuries, since the Hammurabi Code, that ruled a first primitive form of insurance for insolvent debtors, passing through the insurance contract for shipping that was underwritten in London during XVII° century. And nowadays insurance companies do the same activities,with different methods and acting in an environment completely changed from the past.

In the present day insurance companies can cope with the risks they bear, partly thanks to their ability to forecast frequency and magnitude of losses they promise to cover, and partly by composing their portfolio of assets, thanks to the proceeds of their activities.

Today more than ever, insurance companies plays a key role in financial markets and in the overall economy. Skipper (1997) described main features of insurance companies and how they support economic growth, indeed they reduce angst of consumers and stimulate financial stability, promote a more efficient management of risks and capital allocation. Insurance services also facilitate trade and commerce, thanks to their role as financial intermediaries they made many economic activities possible also to small savers. Insurance companies are indeed classified, along with pension funds, as Contractual Savings Institutions. Insurers does not worry much of losing funds quickly, since they can predict with reasonable accuracy how much cash outflows they will face in coming years. This fact has important implications on the kind of assets hold by insurers in their portfolio: insurance companies, especially life insurance companies, tend to invest in long-term securities such as corporate and government bonds, stock, and mortgages. Insurance, therefore, expose also themselves to risks similar to those faced by other investors, such as changes in value of their assets with respect to the value to their contractual liabilities[1].

This means that insurance companies' absorbing capacity of losses is limited, also their customers bear some risks associated with the investment choices of insurers.

Insolvency of insurance companies can potentially have devastating effects on consumers and on the society as a whole; this is due to exceptionally high costs associated with the "default" of an insurance company, especially in comparison to insolvency in other industry.

[1] Mishkin, F. (2013), *The economics of Money, Banking, and Financial Markets*, tenth edition, Pearson Education, Edinburgh.

Indeed. whenever an insurance company default and became insolvent, its inability to cover losses suffered by policyholders, can undermine the very economic existence of policyholders[2].

This extent of the effects caused by a default of insurance companies can be mitigate in several ways by insurance companies, such as reinsurance and guaranty funds, which enable customers of failed insurers to transfer part of their claims to other participating insurers. Unfortunately, also these countermeasure can fail whether too many insurance companies have subscribed insurance contracts for the same risk (Kopcke and Randall 1991).

Thus we see that insurance contracts' quality is strictly related to the soundness of the financial position of the offering insurance company, and in particular to the solvency level of the insurer.

However, due to one of the major problems characterizing financial markets, asymmetric information, policyholders are not always able to know the effective solvency status of their insurer. This problem of adverse selection along with the possible "catastrophic" risks associated with the default of an insurance company call for a strict regulation of this sector, primary to ensure protection to consumers and economic stability.

European Union, after years of transition, have finally enacted Solvency II directive. The way that brought EU to the development of Solvency II framework has been very long. It started in 1998 in response to a financial market that was becoming always more complex. Objectives of Solvency II, as well as of the the previous supervision framework developed, is to harmonize regulation frameworks across European Union, enhancing consumers protection and improving the stability of the entire financial system.

The analysis of this paper proceeds from general to specific. In the first chapter we will introduce insurance companies, how they work and the most important characteristics of this industry.

In the second chapter we will spell out what Solvency II directive prescribes, here we will see this supervision framework in details, focusing mainly on the calculation with the standard model of Standard Capital Requirements (SCR). We will analyze its strengths and, in the third chapter its weaknesses. In the third chapter we will also try to comprehend how these problems highlighted by many economists can be solved. Finally we will observe how internal models are developed and in the last section there is a comparison between the standard model and a partial internal model.

[2]Holzmüller, I.(2009) The United States RBC Standards, Solvency II and the Swiss Solvency Test: A Comparative Assessment. The Geneva Papers on Risk and Insurance.

1. *Insurance Companies and the use of "surplus"*

1.1 What is an insurance company

Insurance is legally defined as: *"A contract whereby, for specified consideration, one party undertakes to compensate the other for a loss relating to a particular subject as a result of the occurrence of designated hazards"*.

Numerous people agree to pay a specified amount of money to protect from some kind of risks.

The term insurance gives us a too broad definition, since we are interested only in those insurance which takes the form of a contract and are consequently subject to several regulations by either statutes or by court decisions. Indeed not all risks can be covered by an insurance contract, as well as all contracts which violates a statue or are contrary to public policy will be unenforceable.

Generally speaking,insurances work in a simple way: when a person enters into a contract of insurance, it pays a premium, a specified amount of money, to the insurer in turn, promises to cover specific future losses of the insured.

A key point from a legal point of view is that a contract can be considered as a proper insurance if and only if it distributes risk among a large number of persons. This is done primary by *insurance companies*, one of the most important player in todays economic world.

Insurance companies help consumers to manage the risks in which they may occur in everyday life. These companies collect premiums, which are technically liabilities, from all their customers and then will pay out only qualified clients. In this way insurance companies pool different types of risk and redistribute them, and create value. Insurances gather risks according to their nature, in this way they uniform the risks covered by a specific kind of policy. Through this extremely useful process, insurers are able to forecast their potential losses and set premiums accordingly. This is a crucial task for insurers, which needs to make sure they are able to satisfy the responsibilities assumed towards their clients, and thus survive in their industry. In order to achieve this objective, insures not only has to evaluate correctly risk and set premiums accordingly, as we have already said, but they also have to calculate the amount of capital that should be kept in order to face possible losses during the year.

The main forms of insurance policies includes:

I. *Property and Causality (PC) insurance:* contracts providing protection against damage to or loss of property caused by various perils; legal liability resulting from injuries to other persons or damage to their property; losses resulting from various sources of business interruption; losses due to accident or illness.

II. *Life and Health (LH) insurance:* contracts that pay off in lump sums or annuities upon the insured's death, disability or retirement.[3]

Obviously, not all insurance companies offer the same contracts, and they developed different lines of insurance. This is a difference of great importance in this industry, also in the calculation of capital requirements as we will see later.

But how does insurance companies actually works in practice?

1.2 Capital cycle in insurance companies

Insurers by selling insurance contracts, are risky taker, therefore the main goal of insurance companies is to diversify risks, the reason is straightforward. The different polices undertaken by the company entails different degrees of risk, this risks are obviously not perfectly correlated, therefore this implies that the total risk of the portfolio is smaller than the the risk of single polices.

At a first glance this may resemble as a quite simple task, instead insurance companies must evaluate properly each risk they will take on.

Indeed managing risk is a crucial activity for an insurance company. An efficient risk management can reduce costs of financial distress and those related to income taxes, furthermore it prevents from the necessity of costly regulatory interventions. Insurers has to deal with three main categories of risks:

- *Underwriting Risk:* It represents the risk that the premiums settled are too low and therefore will not be sufficient to cover the cost of coverage.
- *Market Risk:* it is the potential losses arising from changes in the value of financial instruments or other assets hold by the company, due to changes in financial variables like interest rates or stock prices.
- *Regulation Risk:* it entails all risks related to regulation, including rate intervention, assessment risk, reinsurance requirements and so on.[4]

Obviously the activities of an insurance company are not restricted to the mere valuation of risk, even though it is one of the most important task; insurers have to underwriting insurance contracts, collecting polices, billing them and obviously invest.

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[3] Analysis and Valuation of Insurance Companies; D. Nissim, 2010.

[4] Analysis and Valuation of Insurance Companies, D.Nissim; 2010

But before going on an analyze deeply how capital flows inside insurance companies, it is necessary to briefly describe the production cycle of this companies, because it influences greatly the flow of capital. Indeed the production cycle of insurance companies is a really singular one and represents a main characteristic of this industry: insurance companies are characterized by the *reverse production cycle.* As someone says, this means the “insurer sells promises”. Indeed insurers collect premiums, and therefore money, well before setting claims or other commitments. This is in contrast to what happens in the usual production cycle.

This peculiarity has three important consequences on insurance companies.

The first one is that their particular production cycle allows to this companies to carry extraordinary and constant amount of *technical reserves*.[5] Technical reserves are defined as the amounts set aside from profits in order to cover the possible future claims, and they usually include unearned premium reserves and outstanding claim reserves.

This huge liquidity implies that asset management in insurance companies must be an activity of primary importance.

The second consequence is the fact that insurance company are able to increase their productivity without the injection of new capital. This is particularly important in such a special industry: indeed since insurances do not produce tangible goods, but provide only safeguard against risk, they do not have productivity constraints. These two facts combined cause that an insurance company, at least in theory, is always profitable.

Finally, the reverse production cycle implies a completely different use of capital with respect to common companies: while in common companies equity has a crucial role in production, in insurance companies, the production is financed through the profits flowed from the sale of insurance contracts. Therefore in this industry equity is perceived by customers as an index of solidity and security.

As we previously said, insurance company has to deal with huge and constant amounts of capital, this implies that investing activity and asset management assume an essential role in the activity of the company. Usually, in this sector capital is referred to as “surplus”. Obviously having such a great amount of surplus has some benefits, like the ones previously stated, but it implies also some costs. Specifically, costs associated with surplus are at least two: firstly costs described by Merton and Perold (1993) related to information and agency costs; secondly those with comes as a consequence of double taxation, in US.

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[5] United Nation Conference on Trade and Development; 1971

Then, investing activities assume a key role in insurances' asset management, indeed investments in this sector are regulated by several laws, since they act directly on technical reserves and could therefore mine the overall stability of the company. But there is another important point, which is frequently undervalued because insurance companies face such strict regulation, it is the great importance they carry in Europe. In fact collecting long-term savings of millions of Europeans, it is the largest institutional investor on EU stock exchanges. An appropriate prudential framework for insurance is therefore of extreme importance.

We can consider any insurer as an investor, and *a*s any investor, an insurer's portfolio typically consists of various assets of different types with different risk profiles. The assets portfolio of an insurer include all types of financial products (e.g. simple deposits, stocks, bonds, investment funds units, real estate investments and various types of derivative products and so forth).

When a company generates profits, it invests the return in order to collateralize policies, this means that "the shareholders' investment is subject to a layer of taxes not encountered in direct investment in securities or in ordinary mutual funds. To survive, the insurance company has to recover these tax costs". [6]

So, investments are done in particular to achieve some objective, as strength the capital structure or improve the solvency of the company.

We can therefore distinguish two different types of investment:

- Investments made to accomplish some law requirements with respect to insureds needs;
- Investments made by the company which stems from internal decisions.

The first type of investment are the so called "prudence investment"[7], made exclusively to give more credibility to the company in the eyes of consumers and to protect the company from insolvency.

Investment strategy results as a crucial point for insurance company, indeed with no surprise there is a flourish literature on this topic. Many influential authors developed some models which enable them to construct a proper efficient frontier for property-liability companies. Among these authors, Cummins and Nye (1981) not only developed developed an efficient frontier for insurance companies, but, calculating risk aversion coefficient and ruin probabilities, they also offered a decision-making model to invent in the more appropriate portfolio.

Cummins and Nye started from the "*Portfolio Diversification Model*", this is a very simple model

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[6]Myers and Read (2001) "Capital Allocation for Insurance Companies".

[7] Caldei (2010), "Sistemi Organizzativi delle imprese di assicurazione: il caso Gruppo Generali"

which minimizes the variance of return on equity for a particular expected return value on equity, creating by plotting the various point founded, the efficient frontier[8]. For simplicity, this model assumes that the company has only two lines of insurance and it can invests only in two different types of assets.

This implies that the profits made by companies are the sum of the premiums received by the two lines of insurance plus the revenues from investments in both risky and risk-free asset.

Accordingly we can write:

$$\tilde{\pi} = P_1\tilde{r}_1 + P_2\tilde{r}_2 + A_m\tilde{r}_m + A_f r_f$$

where:

P *= premiums;*

r *= rate of underwriting returns;*

A *= dollar amount invested in risky/risk-free asset.*

The rate of return on equity is obtained simply by dividing both sides of the equation by the equity at beginning of the period, so we get:

$$\tilde{r}_p = \frac{P_1}{K_0}\tilde{r}_1 + \frac{P_2}{K_0}\tilde{r}_2 + \frac{A_m}{K_0}\tilde{r}_m + \frac{A_f}{K_0}r_f$$

or equally:

$$\tilde{r}_p = w_1\tilde{r}_1 + w_2\tilde{r}_2 + w_3\tilde{r}_m + w_4 r_f$$

From here the model minuses the variance of expected return subject to

$$E(\tilde{r}_p) = E(\tilde{r}_p)^*,$$

and

$$w_3 + w_4 = w_1 + w_2 + 1.$$

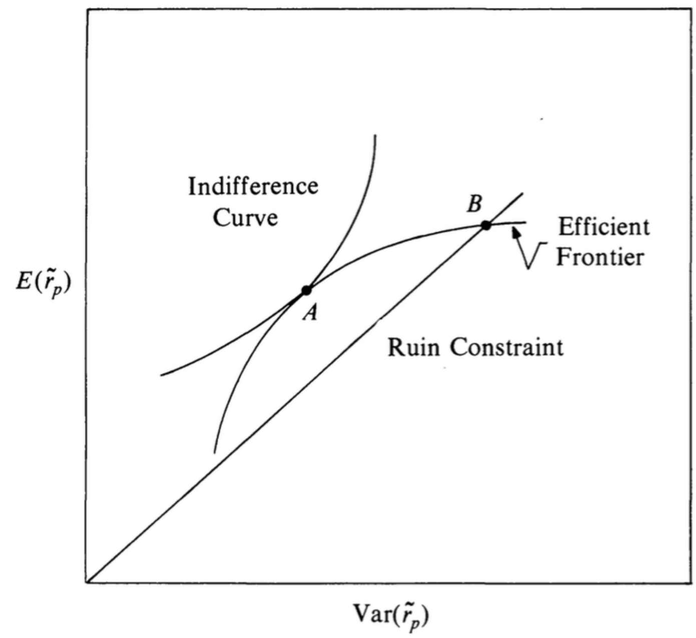
This is the so called *balance sheet constraint*, rearranged. In this way, we obtain an efficient frontier, but this model has an important limitation: indeed it doesn’t give us a guidance on selecting the efficient point on this frontier. To do so, we implement the previous model by relating it to the *utility theory* and *collective risk theory*, as done by Cummins and Nye. The utility theory aims at

[8] Investment Science, Luenberger D. Oxford University Press.

maximize the expect utility of net worth for a firm[9]. According to the previous model, if we plot the efficient front described by this model, we find the optimal point where this frontier is tangent to the firm's indifference curve, this result stems from the fact that "the appropriate objective for a firm in a utility maximization context is the maximization of the expected utility of net worth".[10]

In *Figure 1.1* is represented an ideal efficient frontier; we see from the graph that the point A, the tangency point with indifference curve, is where utility is maximized. Therefore an hypothetic insurance company with the efficient frontier described by the graph, should invest returns from premiums in order to achieve point A and maximizes in this way its utility.

Figure 1.1 *Hypothetical Efficient Frontier and the utility maximization point.*



However this point of efficiency is not always attainable, especially because of the laws regulating the investments of insurance companies.

Indeed investments in this sector are strictly regulated as a result of their so close relation to reserves, and therefore to the security of insureds.

Indeed the proceeds from investments and the returns deriving from premiums constitutes, as we have already said, the technical reserves of an insurance company.

Reserves are the most important indicator of the solvency of a company. Solvency is defined as the ability of an enterprise to meet butte liabilities and obligations.Nowadays insurance companies are regulated for what concerns reserves by the precepts of Solvency II.

We will talk about the regulation in this sector, and in particular of Solvency II in the next chapter.

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[9]Fishburn, Nonlinear preference and utility theory, 1988.

[10] Cummins (1981)Portfolio Optimization Model for Property-Liability Insurance Companies: an analysis and some extensions.

2. *Solvency II and Standard Capital Requirements*

2.1 An introduction to Solvency II

Regulation in the insurance industry became more and more necessary, especially in recent years, since with globalization, and consequently with the increasing importance of international trading, also the amount of relevant risks grew enormously. All these factors raised the urgency for regulations aimed at improving the risk management of companies, to meliorate the solvability of companies and thus preserve the stability of the whole economy, always more interconnected.
In the previous chapter we described insurance companies and how they work and we outlined the main features that characterize this industry, among them we recognized:

- providing cover from many risks by pooling them and in this way lower significantly the cost of claims for the insured;
- receiving payment for their service well before having to provide it (reverse production cycle);
- possessing, like most financial institutions, a total capital which is very small in relation to its total liabilities. [11]

We have talked extensively about the reverse production cycle which characterize insurance companies, this aspect is of particular importance also because historically is the aspect of insurance industry which made supervision necessary in this sector. Indeed as we have already said, basically the insurer sells promises, and therefore is essential for public authorities regulate and supervise insurers. The European Commission sets the supervisory regime for solvency of insurance companies grounding it on the main features of the industry.
Indeed it states that:
"*Authorities should supervise insurance undertakings and ensure that they valued their liabilities correctly and kept secure and sufficiently liquid assets to match them*".
The European commission groups the objectives of its supervision on insurance in three part:

- ensure that an undertaking assesses its liabilities correctly *(adequate provisions)*;
- lay down rules regulating investment policy *(secure and liquid assets)*;
- require a minimum level of capital *(solvency margin)*.

[11] European Commission, 2003, "The foundations of solvency supervision in the insurance industry: the European approach and its alternatives".

This is the basis of the European solvency regime, which focuses firstly on establishment of prudent provisions and secondly on fairly strict regulation concerning the assets that may be held to match liabilities.

The Solvency II project was started in 1998 by the European Commission with the aim of renew the insurance supervision regime and harmonize it with the banking system within the member states of the European Union.

Before the current insurance supervision regime, the main elements were settled by the Directives 73/239/CEE and 88/357/CEE, which regulates the Non-Life insurance lines, and the Directives 79/267/CEE and 90/619/CEE, for Life insurance lines. Even though new directives were issued subsequently, in 1991 and 1992, to renew the supervision regime, the variations in European economic environment, in particular the implementation of the European Single Market, made necessary a rearrangement of the supervisory regulations.

The first step was made in 1994 with the foundation of the Müller-Commission, composed of several European supervisory authorities, with the task of set guidelines to enhance the European solvency supervision. From these guidelines was traced the Solvency I, which accomplished the recommendation of the Müller Commission, but has some critical shortfall.

Solvency I involved stricter requirements to assure solvency not only at the end of the fiscal year, as prescribed by previous regulations, but at every single point in time.

Moreover from now on, was introduced a solvency margin with the objective of cover unexpected losses and future risks. The minimum amount for the guarantee fund was set to three millions Euro both for Non-Life-Insurance companies, instead for Life-Insurance the guaranty fund has to be one third of the solvency margin and minimum three million Euro.

The main shortfall of Solvency I is that exactly as the Directives issued previously, continue to target only the liability-side of the balance sheet, this causes an underestimation of the risks associated with investment activities. Therefore there is no interconnection between assets and liabilities, from this fact stems often problems related to guaranteed returns as Kidwell et al. said[12].

This problem is strictly related with another weakness of this provision: the not-close-to-market-valuation of assets and liabilities and the resulting insufficient provision for interconnected risks.

The European Union has now to face another challenge: create a new solvency regime which entails also investment directives concerning technical reserves, and which should be able to provide

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[12] Kidwell, Blackwell, Whidbee, Peterson; "Financial Institutions, Markets and Money".

supervision authorities with effective qualitative and quantitative tools to assess and regulate the solvency of an insurance company.

Therefore in 1998 the European Commission started to work on a new and more effective "regulation framework" with the aim of *"enhancing consumer confidence by promoting full financial market integration while ensuring high levels of consumer protection"* (European Commission, 1998, p.1)[13]. This new framework, known as Solvency II, was developed in two stages: a first phase in which were conducted several studies and was sketched a general framework, and the final phase in which the general guidelines were implemented into specific rules. During the first stage, the KPMG study recommended a three-pillar structure of insurance supervision. A structure identical to that adopted by the Basel II, the supervision model for banks.

This structure so similar to that of Basel II, should provide an understanding of risk, based on the overall solvability. It involves quantitative requirements, supervisory activities, supervisory reporting and public disclosure. Each of these activities constitute one of the three pillar that we will see in detail later in this chapter.

Besides the study conducted by the KPMG group, a more intense and structured study was carried on, the Sharma report. The Sharma Report highlighted the central aspects to be implemented in the second stage, they are summed up in Key Insight 1. .

After knowing how the fundamental characteristics of the model shall look like, a specification of the drafted general conditions was performed within Phase II. In this stage the

KEY INSIGHT 1

- **I. Evaluation of the overall solvability of the insurance company.**
- **II. A three-pillar structure, similar to that of the Basel II**
- **III. A two-step differentiation of equity requirements distinguishes between the economic capital, also called Solvency Capital Requirements (SCR), and the Minimum Capital Requirements (MCR).**
- **IV. A risk-based standard approach is given. Additionally, internal models to detect the equity requirements are admitted.**

[13] European Commission, 1998, Financial Services: Building a Framework for Action, Communication of the Commission, Working Paper, 1998-10-28, Brussels.

European Commission used four workgroups to develop and implement the rules. These groups were composed by members either of the Committee of European Insurance and of the CEIOPS (Occupational Pensions Supervisors).

Each groups focused on a specific topic and consequently developed specific proposal which were submitted to the European Parliament for decision making.

The four topics on which the four groups worked on are:

- Life/Non-Life Insurance → *Pillar I*;

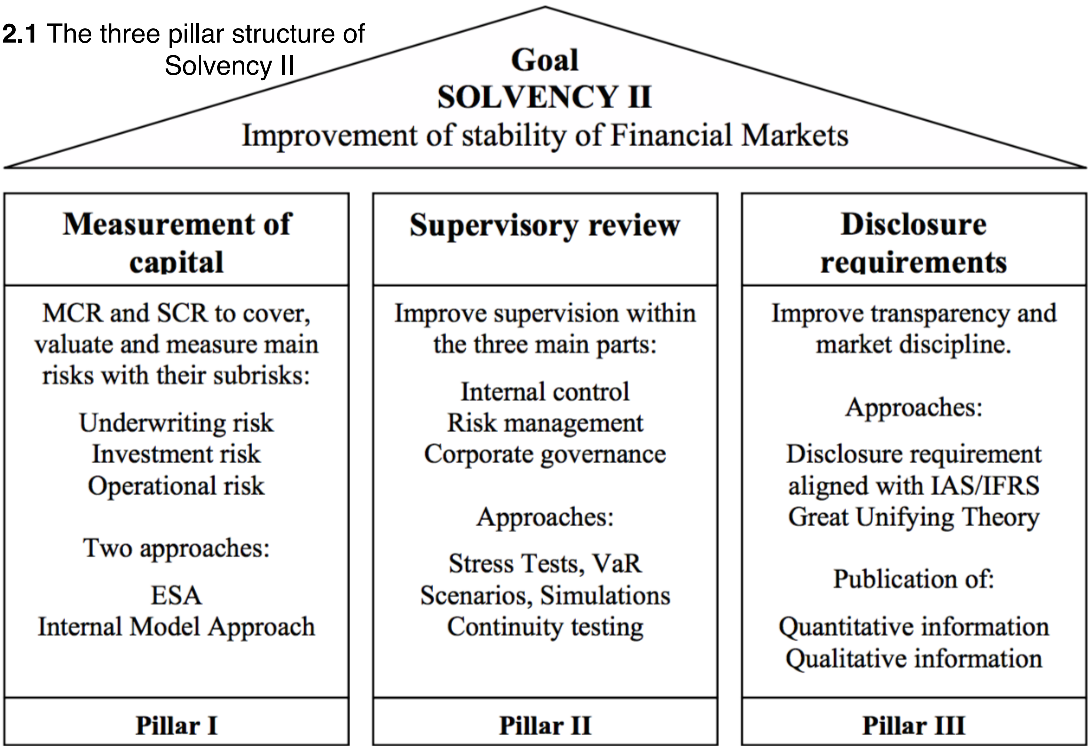
- Qualitative Financial Supervision → *Pillar II*;

- Market transparency → *Pillar III*;

- Sector overlapping questions.

Now we will present Solvency II in detail analyzing each pillar; Figure 2.1 presented below is a useful summary for what we are going to discuss[14]:

Figure 2.1 The three pillar structure of Solvency II



Solvency II aims at keep the probability that the insurer defaults below a certain threshold, 0,5% within one year; therefore capital requirements are necessary. Pillar I deals exactly with this

[14] Leurent, E. and Voigt, T. “Basel II and Solvency II–Impact Analysis of two supervision models on Financial Institutions”

problem. It contains the basic calculation principles of Solvency II and the quantitative regulations for insurance companies capital requirements. Along with technical provisions, it also provides rules for determining the minimum capital required and the target capital, which is usually higher than the former. Pillar one is the most important for the scope of this work, therefore we will discuss it more deeply than the other two. However, for the sake of the work itself, the first pillar will be presented after the other two pillars.

Pillars two focuses on the qualitative elements of supervision; it is based mainly on the Sharma report and provides qualitative elements of supervision. Among this belong the principles of internal control, proper risk management, and the formulation of combined principles and instruments of supervisory control[15]. The second pillar adopts the principles outlined in the Basel II and adjusts them to the insurance industry. Indeed to following the principles expressed in Basel II insurance companies must have a qualified measure to define a proper amount of equity with respect to the risk profile evaluated through the methods provided by Pillar I. The supervisory authorities play a key role in the implementation of the rules imposed by Solvency II: whenever a company opt for internal assessment tools they have to judge whether these are efficient or not. Moreover authorities have to evaluate the quality of strategies adopted by companies to maintain equity. Solvency II includes some regulation more than Basel II, specific for assure solvability of insurance companies, like stricter definitions of investment management and intervention rights with regard to target capital configuration.[16]

Pillar three concerns market transparency and disclosure requirements. Its aim is to promote market discipline in the insurance sector and encourage a corporate management more aware of risks and thus more effective. Also in this case the work of the European Commission is strongly influenced by the Basel II, but keeping in mind that unlike the banking sector, in the insurance sector, companies are much less interdependent.[17] An aspect that European Commission takes into consideration, is the impact that the new regulation has on the existing reporting standards. Indeed the disclosure requirements of the third pillar should be adjusted to the developments of IAS/IFRS to avoid a duplication of work within the reporting. This is the reason why there is a strong

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[15]Linder, U., and V. Ronkainen, 2004, Solvency II—Towards a New Insurance Supervisory System in the EU, *Scandinavian Actuarial Journal*, 104(6): 462– 474.

[16]European Commission, "Considerations on the design of a future prudential supervisory system", MARKT 2535/02, 2002;

[17]Leurent E. and Voigt T., 2007, Basel II and Solvency II–Impact Analysis of two supervision models on Financial Institutions

relationship between Solvency II and the IASB (International Accounting Standard Boards). IASB was indeed heard as consultant, with the purpose to develop new standards for the reporting procedures of technical provisions, on the basis of one of the main change introduced by Solvency II, a market orientated consideration of assets and liabilities. Even if this harmonization implies an increase in expenses for companies, IAS/IFRS developed in this way undoubtedly fortify Solvency II's objectives[18].

The first pillar is probably the one which introduce the main innovation respect to the previous solvency supervision regime: the market oriented evaluation of assets. This has great implications in the calculation of equity requirements for insurance companies. Pillar I introduces two different requirements, the Minimum Capital Requirement, or *MCR,* and the Solvency Capital Requirement, or *SCR.* The calculation of the capital requirements could be performed either on basis of a standard model or through an internal approach, developed by the company itself and subsequently investigated and validated by the supervision authority. This is to ensure that also small firms can implement a type of risk management adequate to their business and within their possibilities. About the standard approach, CEIOPS states that "*it should be a holistic method that contains all basic risk for both assets and liabilities"*. Now, as we already said in the first chapter, the relevant risks that should be taken into specific consideration can be grouped in four categories as the European Commission suggests[19] , and they are listed below

Underwriting Risk	it stems especially from premium calculations and claims reserves;
Credit Risk	arising from debtors default;
Market Risk	resulting from the fluctuation of all relevant market prices, including stocks, bonds, and exchange rates;
Operational Risk	it results from inadequate or failed internal processes, people and systems, or from external events.

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Now we will present briefly the two requirements introduced by Solvency II, then we will focus on how to calculate the Solvency Capital Requirement and the issues related to it.

[18] Rob Jones, Wolfgang Rief, Yann Le Pallec, "The Impact of SOLVENCY II on the European Insurance Market", in: "Reactions Supplement" (Standard & Poors).

[19]European Commission, 2004b, p. 22; International Actuarial Association, 2004, pp. 29–34

[20]Basel Committee on Banking Supervision, 2001,

- *Minimum Capital Requirement (MCR):* It defines the amount of capital on their shortfall the activities of an insurance company would have an unacceptably high risk for the insuree. If the capital of the company falls below the MCR ultimate supervisory actions shall be initiated. *"The MCR should be considered as a simple, robust and objective indicator"*[21]. Simply the MCR is the absolute minimum amount of capital the company is allowed to hold in comparison to its level of risk: whenever an insurance company acquires a new client, it has to adjust its MCR by taking into account the risks associated with the new client, which entered in company's portfolio.

- *Solvency Capital Requirement (SCR):* SCR should warrant a capitalization that gives an insurance company the opportunity to absorb huge unexpected losses to provide the insuree an appropriate security that payments will be made if necessary. The SCR should represent that amount of capital, which is needed to fulfill all obligations within a certain time horizon to a particular confidence level. Usually it is much higher than the MCR, and the gap between the two provides space for intervention by the supervision authority to restructure or sale a part of the insurance in order to recover or simply to protect the insurees.

In the next part we will analyze the calculus of SCR as recommended by the principles outlined in Solvency II. Thus what follows is an analysis of the standard model that all companies have to apply, unless they have a valid internal model available.

2.2 Analysis of the European Standard Model for the calculation of SCR

The European standard model takes the name of *European Standard Approach* (ESA), and was provided by the *Committee of insurance and Occupational Pension Supervision* (CEIOPS). In general insurance companies are encouraged to implement internal models to assess their risks as accurately as possible, but, since the implementation of such internal models is quite costly and complex, the European Commission supported by CEIOPS has established a scenario based standard model which all insurance companies will be allowed to use in order to approximate their capital requirements, that is the ESA. Its current state is given by the Technical Specifications of the 5th *Quantitative Impact Study (QIS)*.

[21]Committee of European Insurance and Occupational Pensions Supervisors (CEIOPS): "Answers to the European Commission on the second wave of Calls for Advice in the framework of the SOLVENCY II project", CEIOPS-DOC-07/05, 2005

CEIOPS want to realize a framework which takes into consideration all main risk types, and can be potentially applied by all insurance companies no matters whether they cover only Life or Non-Life line or both. Moreover, its aim is to provide a simple and correct way to calculate SCR.

ESA is constructed as a "*k-factor*" (or multi-factor) model, in this way companies are able to estimate each risk component separately. The standard model assume a lognormal distribution with a mean of 1 and a standard deviation parameter which is set by the company itself. he model pays a particular attention to market and underwriting risk, because these can impact seriously on the calculation of SCR and hide many insidious in they calculation. Indeed for these two kinds of risk, ESA provides companies more than one possible scenario to allow them to find the one who fits better their individual incidents and thus correctly calculate SCR. Moreover, ESA tries to cover not only activities already undertaken by the company, but also a forecast of the new activities that the company will do during the economic year.

The Solvency Capital Requirement is evaluated at the 99,5% *Value at Risk (Var)* of the *Available Capital*. Var is a statistical measure which quantify the financial risk over a whole firm or an investment portfolio in a specified time period. In particular the time period specified by ESA is one year, and it sets a confidence interval of 99,5%. Roughly speaking, the Value at Risk is the amount of capital required at time $t=0$ to ensure, with a certain degree of certainty, that the company will cover all losses which may occur during the next year until $t=1$. The degree of certainty is arbitrary, however usually is set at 99,5%, even though sometimes it could be much lower, for example 95%, i.e. to fit better the characteristic of a single line of business. The *Available Capital (AC)* is defined as the difference between the market value of assets and liabilities (therefore it roughly measures the capital the company owns to cover future losses). The market value of assets can be determined easily, but for liabilities the issue is more complex. Indeed, since there no liquid markets for liabilities, and the standard models can not be applied directly, CEIOPS was almost forced to introduce a new way to derive the market value of liabilities. The so called Technical Provisions give an approximate value of the liabilities in the Balance Sheet of insurance Companies. Technical Provision consist of two elements:

- *Best Estimate Liabilities (BEL)*: the article 77 of Solvency II defines it as: "the probability-weighted average of future cash-flows, taking account of the time value of money (expected present value of future cash-flows), using the relevant risk-free interest rate term structure"; and
- *Risk Margin (RM):* also RM is defined in art. 77 of Solvency II directive, however, a

striking definition is given directly by CEIOPS as: “loading for non-hedgeable risk and has to ensure that the value of technical provisions is equivalent to the amount that (re)insurance undertakings would be expected to require to take over and meet the (re)insurance obligations” [22]. In QIS the RM is defined as:

$$RM := \sum_{t\geq 0} \frac{CoC \cdot SCR_t}{(1+i_{t+1})^{t+1}} \qquad \textbf{Eq. 2.2.1}$$

where *“i”* is the risk-free interest rate at a specified point in time and *“CoC”* is the cost of capital rate.

Mathematically SCR is defined as:

$$P(AC_1 > 0 | AC_0 = x) \geq 99.5\%$$

The same relation can be expressed with an equivalent formula provided by Bauer et al. which solve a serious problem of the previous formula: it is markedly difficult to apply in practical computation. Therefore the following formula helps the calculation of SCR[23]:

$$SCR^{VaR} := argmin_x \left\{ P\left(AC_0 - \frac{AC_1}{1+i_1} > x \right) \leq 0.005 \right\} \qquad \textbf{Eq. 2.2.2}$$

It appears clear from this equations, a mutual dependence between Available Capital and SCR. Indeed SCR is the Var of Available Capital, but in Risk Margin, Available Capital depends on SCR. To solve this problem CEIOPS suggests to get rid off Risk Margin in calculating capital requirements. This implies that Risk Margin is quite constant, and consequently the Net Asset Value can work as an approximation of Available Capital[24]:

$$NAV_t := A_t - BEL_t \qquad \textbf{Eq. 2.2.3}$$

where A_t denotes the market value of assets and BEL_t the Best Estimate Liabilities at time t. For simplicity, we will refer to the latter as the liabilities only in what follows.

[22] CEIOPS, 2008a. QIS4 Technical Specifications. Available at: http://www.ceiops.eu/media/docman/ Technical%20Specifications %20QIS4.doc.

[23] Bauer, D., Bergmann, D., Reuß, A., 2009b. Solvency II and Nested Simulations – a Least-Squares Monte Carlo Approach. Working Paper, Georgia State University and Ulm University.

[24] This is true especially in stressed scenarios.

The ESA model is based on some basic economic principles, which should be briefly analyzed before starting the proper calculation. Firstly is the *going concern assumption*, one of the many connection point of this model with IAS/IFRS regulation. Secondly, since Capital Requirements are based on the Balance Sheet, both asset and liabilities, as we have already stated before, should be measured at their *market value* (economic value). For what concern liabilities, the *prudence principle* must not be forgotten.

The basic formula for SCR is:

SCR = BSCR - RPS- NL_PS.

In which: ***BSCR*** = stands for *"Basic Solvency Capital Requirement"* .

RPS and NL_PS = *are the absorbing capacity respectively for Life and Non-Life insurance companies.*

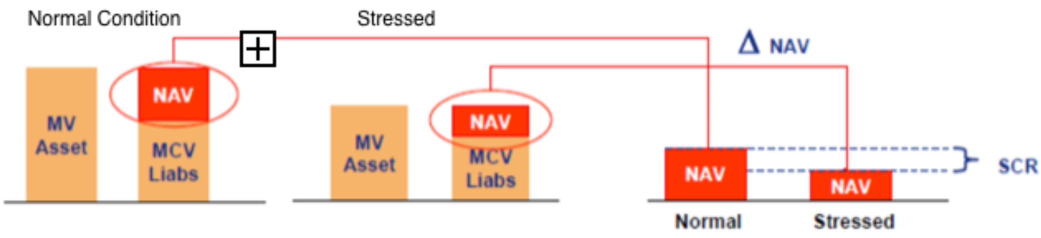
The most important element in this formula is represented by the Basic Solvency Capital Requirement (BSCR), that is the aggregated amount of the sub-SCR for each class of risk and their respective sub-categories; risk category taken into account by the standard model are six and are listed in later in this chapter in the Key Insight box. In particular there is a specific procedure to be followed, that can be divided in three steps:

1. Determination of capital requirements for each risk category, which can be calculated using different approaches; among others it is important to highlight followings:

Scenario Testing Approach: in which SCR is calculated as the difference between the best estimation of NAV (Net Asset Value) and the stressed value of NAV (the value of the 99,5 percentile).CEIOPS conduced several analysis and at a first moment it concludes that "a stress of 45%" was reasonable "for global equities". However recently, for purposes of QIS5, the European Commission sets at 35%. We will see in Chapter 4 how this approach actually works and we will present stress factors for each security category. This approach is described in Figure 2.2;[25]

[25] Un modello stocastico per il calcolo del Fair Value della Riserva Sinistri R.C.Auto in presenza dell'Indennizzo Diretto e valutazione del requisito patrimoniale in ottica Solvency II, Paola Fersini, 2014.

Figure 2.2 Scenario Testing Approach.



Factor Based Formula: using this approach, capital requirements are calculated using variables representing the exposure of the company to risks:

$$\mathbf{SCR = f\,(G_1, G_2, G_3, \ldots\, G_n)}$$

2. Summation, through correlation coefficients, for each risk category, of the capital requirements of their sub-categories.

3. Summation, through correlation coefficients, of the capital requirements of the main categories of risk.[26]

Summarizing, once every sub-SCR is derived, the total SCR is obtained simply by adding up them all; in the following graph, we have a useful representation of the composition of the SCR.

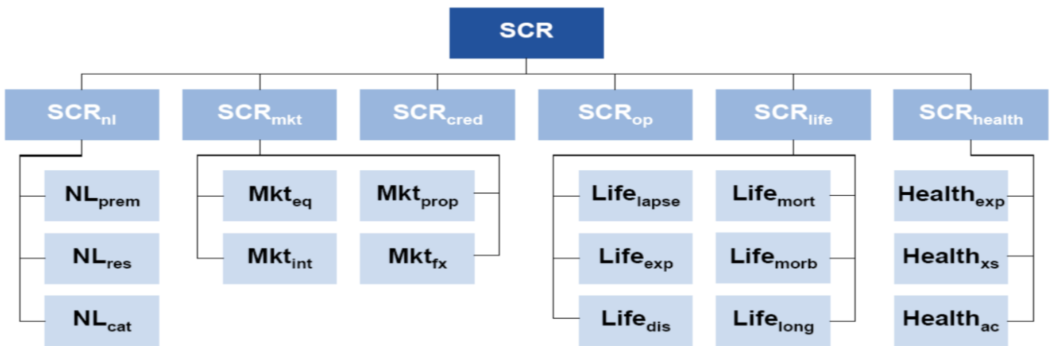


Table 2.1: This *summary of the risk modules taken into consideration by ESA is provided by CEIOPS.*

The BSCR could be seen as a sort of gross SCR obtained without the adjustments for the absorbing capacities.

Technically the BSCR is calculated using the following formula:

$$\mathbf{BSCR} = \sqrt{\sum_{ij} Corr_{ij} \bullet SCR_i \bullet SCR_j} + SCR_{\text{int}\,angibles} \qquad \textbf{Eq. 2.2.4}$$

[26] Un modello stocastico per il calcolo del Fair Value della Riserva Sinistri R.C.Auto in presenza dell'Indennizzo Diretto e valutazione del requisito patrimoniale in ottica Solvency II, Paola Fersini, 2014.

In the formula above, SCR_i and SCR_j represent the capital requirement for the i-th and j-th risk category; $Corr_{ij}$ represents the correlation between the two risk categories taken into account and $SCR_{intangibles}$ represents the capital requirement for the risk relative to intangibles. Var is used to calculate the capital requirements for each risk category. We can divide the risks modules we see in Figure 1, into 4 macro-categories:

- *Market Risk:*
  In Market Risk, we have to recognize the following sub-categories, put in order of importance: Equity risk, Interest Rate risk, Real Estate risk, Currency risk, and Credit Spread risk;

- *Underwriting risk:*
  As for Market Risk, also in this case, we recognize different types of risks:
  Biometric risks (Mortality risk, Longevity risk, Morbidity risk etc.), Lapse risk, and Expense risk within the Life insurance industry and Reserve risk, Premium risk, Catastrophe risk, the risk of new risk through renewals and new businesses for the Non-Life insurance industry;

- *Credit Risk;* and

- *Operational Risk.*

Whenever a risk category is made up of some sub-risks, the SCR for that specific. In order to find the correlation between various categories of risk, CEIOPS' researchers considered a wide variety of possible solutions. In the end the result chosen was based on 21 data points starting from

KEY INSIGHT 2

Here are listed all risk categories included in the ESA standard model and respective notation

- Underwriting risk for non life insurances **(SCR_{nl})**;
- Underwriting risk for life insurances **(SCR_{life})**;
- Underwriting risk for health insurances **(SCR_{health})**;
- Market risk **(SCR_{mkt})**;
- Default risk of the counterpart **(SCR_{def})**;
- Risk on intangibiles **($SCR_{intangibles}$)**.

September 9 to 1998 to October 28, 1998. This period is described by CEIOPS as one of "very extreme stress". The specific series used were the Morgan Stanley Capital International (MSCI) world index for equities and "the spread to gilts on UK AA rated corporate bonds". [27]

The correlation between various types of risk are represented in the first table presented below. In the second table are presented the correlation factors between the sub-risks composing market risk[28]. These matrices of correlation are developed by the EIOPA:

i j	market	default	life	health	non-life
market	1				
default	0,25	1			
life	0,25	0,25	1		
health	0,25	0,25	0,25	1	
non-life	0,25	0,5	0	0	1

i j	interest rate	equity	property	spread	currency	concentration
interest rate	1	0,5/0	0,5/0	0,5/0	0,5	0,5
equity		1	0,75	0,75	0,5	0,5
property			1	0,5	0,5	0,5
spread				1	0,5	0,5
currency					1	0,5
concentration						1

A simplification could be useful to get a better picture of how to evaluate the risk and the related capital requirements. Assume a possible loss arising from a single business line, and assign a random variable Xi. The aggregate losses arising from all lines of business is the sum of the single losses:

$$\mathbf{X = X_1 + X_2 + \ldots + X_n}$$

The probability distribution of the total losses depend not only on the single losses, but as already said, it depends also on the correlation between each other.

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[27]Summary of CEIOPS Calibration Work on Standard Formula, T.Herzog, 2011

[28]CEIOPS is proposing a two-sided correlation between interest rate and each of three other risks. The choice of the correlation factor depends on whether a rise or fall in interest rates is the crucial factor. In the event the insurer would have adverse results if interest rates fell, then the 50% correlation factor should be used. In the event the insurer would have adverse results if interest rates rose, then the 0% correlation factor should be used.

The *q-quantile*, x_q , is the smallest value satisfying:

$$\mathbf{Pr}\{X > x\,q\} = 1 - q\,.$$

As a risk measure, x_q is the Value-at-Risk and is used extensively in financial risk management of trading risk over a fixed (usually relatively short, one year) time period.[29]

A peculiarity of Solvency II, provided by the ESA is that it takes into account all tools used by insurance companies to mitigate risk (e.g. reinsurance or different hedging strategies). Indeed in the basic formula for the calculus if SCR we have seen that the two variables *RPS* and *NL_PS* are the absorbing capacity of the insurance companies. In particular in the model explained by EIOPA, this absorbing capacities are grouped into a single variable named *Adj* and represent a specific type of absorbing capacity. *Adj* stands for the absorbing capacity of insurance companies deriving from a reduction of technical reserves or from a reduction in assets, or both. The total absorbing capacity is given by the sum of these two way used by insurance companies to absorb better a possible default:

$$\mathrm{Ad_j} = \mathrm{Ad_{jTP}} + \mathrm{Ad_{jDT}}$$

In the last section we will see better how this standard model is applied, in order to have a better understatement of this topic. In particular we will see in depth all steps for the calculation of requirements for market risk.

[29]Artzner, P. (1999), "Application of Coherent Risk Measures to Capital Requirements in Insurance," *North American Actuarial Journal.*

3. Main problems of the Standard formula and possible corrections

3.1 Problems and critiques to the European Standard Model

There are several debates related to the topic of capital requirements. Probably the most interesting are those on the efficient allocation of risk in multi-line business and those on risk measure. The allocation of capital among different lines of business is crucial. If we consider the question of how to allocate capital among the individual risks X_1 , X_2 , ..., X_n when the capital requirement $\rho(X)$ has been determined for the total risk X. Let $K = \rho(X)$ represent the risk measure for the total risk X . Let K_i denote the allocation of K to the i-th risk. There were several authors which tried to determine efficient allocation. Among them, Denault (2001) defined a *coherent allocation of capital.* A coherent allocation must respect the following properties:

1. *Full allocation*

   This means that all of the capital is allocated to the risks.

2. *No undercut*

   $K_a + K_b + ... + K_z \leq \rho(X_a + X_b + ... + X_z)$ for any subset $\{a, b, ..., z\}$ of $\{1, 2, ... , n\}$.

   This means that any decomposition of the total risk will not increase the capital from that if the risks stood alone.

3. *Symmetry*

   Within any decomposition, substitution of one risk X_i with an otherwise identical risk X_j will result in no change in the allocations.

4. *Riskless allocation*

   The capital allocation (in excess of the mean) to a risk that has no uncertainty is zero. [30]

Although these properties are quite intuitive, they are not sufficient to determine a single allocation method, but they are sufficient for some authors, to conclude that the allocation of capital using Var and more in general the standard method described by Solvency II is not enough to ensure an efficient allocation.

Another important criticism to ESA comes from the use of Var as statistical measure. Indeed VaR is an "all or nothing" risk measure, that is whenever an event causes the ruin of the company

[30] Denault M. (2001) Coherent Allocation of Risk Capital, Working Paper, ETH RiskLab, Zurich.

occurs, there is no capital to amortize losses. This is one of the reasons why many economists suggest the use of alternative risk measures. for instance Overbeck (2000) suggests to use only the *Tail Value at Risk* (*TailVaR*), that provides a definition of "bad times": those times in which losses exceed some threshold, but without using up all available capital.The TailVaR is defined as

$$\mathrm{E}\lfloor X | X > x_q \rfloor.$$

It can be seen that this will be larger than the VaR measure for the same value of q described above since it is the VaR, x_q, plus the expected excess loss. TailVaR is a coherent measure in the sense given by Artzner *et al* (1999). A risk measure in order to be considered as a coherent one, must have the following four properties:

1. *Sub-additivity:*

   $\boldsymbol{\rho(X + Y) \leq \rho(X) + \rho(Y)}$

   This means that the capital requirement for two risks combined will not be greater than for the risks treated separately. This is necessary, since otherwise companies would have an advantage to disaggregate into smaller companies.

2. *Monotonicity:*

   If $X \leq Y$ for all possible outcomes, then $\boldsymbol{\rho(X) \leq \rho(Y)}$.

   This means that if one risk always has greater losses than another risk, the capital requirement should be greater.

3. *Positive Homogeneity:*

   For any positive constant λ , $\boldsymbol{\rho(\lambda X) = \lambda\rho(X)}$.

   This means that the capital requirement is independent of the currency in which the risk is measured.

4. *Translation invariance:*

   For any positive constant α , $\boldsymbol{\rho(X + \alpha) = \rho(X) + \alpha}$.

   This means that there is no additional capital requirement for an additional risk for which there is no uncertainty. In particular, by making X identically zero, the total capital required for a certain outcome is exactly the value of that outcome.[31]

[31]Artzner, P. (1999), "Application of Coherent Risk Measures to Capital Requirements in Insurance," *North American Actuarial Journal.*

Using TailVaR also implies a different allocation of capital across the lines of business: in this case, the allocation of capital for each risk should depends on the contribution of the specific risk to the total capital, that is[32]:

$$K_j = E\lfloor X_j | X > x_q \rfloor.$$

This formula is really intuitive, indeed it simply says that the capital required for each risk is the expected contribution to the shortfall when a shortfall occurs; this method is called by Overbeck the "contribution to shortfall" method.

Another point which is debated is the fact that he standard formula uses a lognormal distribution which is parameterized with a mean of 1 and a standard deviation parameter. The latter can be set in two alternatives way: or corresponding to the market-wide estimations or corresponding to the data of the company. Many authors, and among them Hampel and Pfeifer (2011), believe that companies should base calculation for SCR on their individual risk situation and believe also that the restriction of a mean of 1 is not appropriate. We can see it from a rearrangement of SCR formula, presented also by QIS 5:

$$\mathbf{(F^{-1}(0.995)-1)V = F^{-1}(0.995)V - V, XX}$$

Where X is as usual a long normally distributed random variable, V is a volume measure and SCR is given by the difference of the 0.995-quantile of the (absolute) risk X · V and the volume measure V , which is usually equal to the amount of premiums; this means that X is the *loss ratio*. But in QIS 5 the expected loss ratio is assumed to be 1. Thus, the use of the standard formula generally leads to a systematic bias in the SCR calculation[33]. Some insurance companies may get an advantage from a reduction of the SCR. Indeed on the one hand problems like the underestimation of capital requirements arising from application ESA, can be tolerate from a political perspective in order to influence the behavior of market participants. On the other hand, we have to say that companies which may have greatest benefits from the application of the standard formula, are those more exposed to risk. This fact is disputable also under a political point of view because the consumer protection , one of the central issue of the reform, comes less. All things considered we can say that the fact the standard formula can systematically underestimates the SCR, can lead to irregularity of the directives in Solvency II and it is undesirable in terms of consistency.

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[32]Measurement of Risk, Solvency Requirements and Allocation of Capital whiten Financial Conglomerates, H. Panjer 2001.

[33]Proposal for correction of the SCR calculation bias in Solvency II, M. Hampel. D. Pfeifer, 2011.

3.2 Problems related to skewness of distribution and a possible solution

Given the modular approach prescribed by ESA, an aspect that assume particular importance, is the aggregation phase, in which all modules and sub-modules in which BSCR is divided, are brought together. Following the standard model, risk charges shall be aggregated using the so called *squared root formula* described previously in *Equation 2.3,* assuming a linear correlation among risk module.The equation 2.3 can be rearranged, by squaring both sides of the equation and beset aside for a while the SCR for intangibles, and written as:

$$C_{\mathrm{SCR}}^2 = \sum_{i=1}^{n} C_i^2 + 2\sum_{i \neq j}^{n}\sum^{n} \rho_{ij} C_j C_j. \qquad \textbf{Eq. 3.2.1}$$

The square root formula can be obviously considered correct, in the sense that the prescribed overall confidence level of 99.5% will stay within the world of normal risk distributions, independently from the use of VaR or TVaR as risk measures.

However this formula will provide fair results as long as probability distribution of all risk-modules are normally distributed or they all have the same skew. Whenever this is not the case, and in most cases this is not, the application of the square root formula for the aggregation of risk categories will produce flawed results with severe consequences for insurance companies by hugely underestimates the capital requirements. Indeed in most cases the distributions of risk modules are differently skewed to the right. This is why this is considered as one of the main problems of the ESA and explains also why calibration is so important.

CEIOPS provides some guidelines for calibration, but they have been proved to be too general, and consequently they gave rise to the problems highlighted above (CEIOPS guideline for calibration are examined in key insight 3).

The question now is how to solve the calibration problems?

Many economists tried to answer this question and here we are going to analyze one of the most interesting solution proposed.

This solution was proposed by Sandström (2007), it is interesting also for its relative simplicity. What Sandström proposed in its work is to use a Cornish-Fisher expansion to basically transform the the skewed distributions into a standard normal distribution.

For a univariate data set, the formula for skewness is:

$$\gamma_i = (E(Y_i - \mu_i)^3)/\sigma_i^3 \qquad \textbf{Eq. 3.2.2}$$

This formula is referred to as Fisher-Pearson coefficient of skewness. The skewness for a normal distribution is zero, while positive values indicate that data are skewed to the right. If risk charges have a skewed probability function, then also the probability distribution of the total risk charge will be skewed positively.

Through the Cornish-Fisher expansion is possible to express the (1- α) quantile of a skewed distribution, as the (1- α) quantile of a normal distribution, plus a correction for the skewness, and we get: [34]

$$\mu + k_{V,1-\alpha}(\gamma)\cdot\sigma \qquad \textbf{Eq. 3.2.3}$$

In this equation the second term is a function of the original distribution, the skewed one, describing the new probability distribution.

If we apply this finding to Equation 3.2.1, we obtain the following equation representing SCR in terms of the transformed risk measure, Var or the TailVar, for the *n* risk modules:

$$C^2_{\mathrm{SCR}} = k^2_{\cdot,1-\alpha}(\gamma_{\mathrm{SCR}})\sigma^2_{\mathrm{SCR}} = k^2_{\cdot,1-\alpha}(\gamma_{\mathrm{SCR}})\sum_{i=1}^{n}\sigma_i^2 + 2k^2_{\cdot,1-\alpha}(\gamma_{\mathrm{SCR}})\sum_{i\neq j}^{n}\sum^{n}\rho_{ij}\sigma_i\sigma_j \qquad \textbf{Eq. 3.2.4}$$

The k. in the equation may represents either Var or TailVar.

In order to adequate this equation to the consistency approach prescribed by CEIOPS, Sandström add a “correct” quantile, obtaining:

$$C^2_{\mathrm{SCR}} = \sum_{i=1}^{n}\left[\frac{k^2_{\cdot,1-\alpha}(\gamma_{\mathrm{SCR}})}{k^2_{\cdot,1-\alpha}(\gamma_i)}\right]k^2_{\cdot,1-\alpha}(\gamma_i)\sigma_i^2 + 2\sum_{i\neq j}^{n}\sum^{n}\rho_{ij}\left[\frac{k^2_{\cdot,1-\alpha}(\gamma_{\mathrm{SCR}})}{k_{\cdot,1-\alpha}(\gamma_i)k_{\cdot,1-\alpha}(\gamma_j)}\right]k_{\cdot,1-\alpha}(\gamma_i)\sigma_i k_{\cdot,1-\alpha}(\gamma_j)\sigma_j$$

KEY INSIGHT 3

CEIOPS requires consistency approach, it aims at calibrate each component of risk present in the standard formula in a consistent way. This means that the same assumption and objectives set for the overall SCR must be set also for its risk component.

Therefore also in all risk categories must be used same parameters, time horizon, confidence level and so forth.

It has some benefits, like easier the application of internal models and the analysis of single lines of business, but it also has many drawbacks mainly linked to its simplicity.

For instance it does not account for risk diversification (an aspect that will be analyzed in Chapter 4), and makes mathematical assumptions, one above all that all risk categories have the same probability distribution) which leads to misestimations of SCR.

[34] Arne Sandström (2007) Solvency II: Calibration for skewness, Scandinavian Actuarial Journal, 2007:2, 126-134, DOI: 10.1080/03461230701250481

This equation can be reduced to:

$$C^2_{\mathrm{SCR}} = \sum_{i=1}^{n} f^2_{\cdot,i} C^2_i + 2\sum_{i \neq j}^{n}\sum^{n} \rho_{ij} f_{\cdot,i}\, C_i f_{\cdot,j}\, C_j \qquad \textbf{Eq. 3.2.5}$$

We can easily note that this is the equation 3.2.1 multiplied by a factor ($f.$), called by Sandström calibration factor, which varies according to the risk measure used (Var or TailVar)[35].

The main finding of Sandström's work is that without calibration, insurance companies applying the standard model, will misestimate capital requirements whenever the risk categories' distributions are not homogenous. Specifically they will have lower requirements with respect to those obtained with a calibrated model. Moreover the author also find that misestimation are less significant using Var as risk measure, than with TailVar. As many economists assert, Sandström too affirm that using a modular approach for SCR computation requires calibration.

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[35] Calibration factors values are computed using the formulas provided by Sandström, that are:

$$f_{V,i} = \frac{k_{V,1-\alpha}(\gamma_{\mathrm{SCR}})}{k_{V,1-\alpha}(\gamma_i)} = \frac{z_{1-\alpha} + \frac{\gamma_{\mathrm{SCR}}}{6}(z^2_{1-\alpha} - 1)}{z_{1-\alpha} + \frac{\gamma_i}{6}(z^2_{1-\alpha} - 1)} = \frac{6z_{1-\alpha} + \gamma_{\mathrm{SCR}}(z^2_{1-\alpha} - 1)}{6z_{1-\alpha} + \gamma_i(z^2_{1-\alpha} - 1)}$$

$$f_{TV,i} = \frac{k_{TV,1-\alpha}(\gamma_{\mathrm{SCR}})}{k_{TV,1-\alpha}(\gamma_i)} = \frac{1 + \frac{\gamma_{\mathrm{SCR}}}{6} z^3_{1-\alpha}}{1 + \frac{\gamma_i}{6} z^3_{1-\alpha}} = \frac{6 + \gamma_{\mathrm{SCR}}\, z^3_{1-\alpha}}{6 + \gamma_i\, z^3_{1-\alpha}}.$$

for Var an TVar respectively.

4. Analysis of Internal Models

4.1 Internal model, an introduction

An internal model is one constructed by the insurer for its specific needs. Internal models are expected to result in more accurate analysis, control, and management of the insurer's financial situation than do the more generic standard models as reported by the European Commission[36]. The benefits steaming from the use of an internal model can be substantial in efficiency in terms of capital and costs. Moreover it can enhance the "dialogue" between insurers and their supervisors, as well as with their shareholders, analysts and rating agencies.

As we have already seen in the first chapter, there is a close relationship between Solvency II and Basel II; indeed the latter can be considered as a sort of reference for the former. This two regulations are similar also in the field of internal model. Indeed, the objectives and the precepts for the regulatory approval of internal models are analogous in banking and insurance. However as pointed out by the CEIOPS itself, there are too many important differences between these sectors, one above all the nature of some risks, and consequently the risk profiles, which results in different rules.

Insurance companies may adopt a partial or a full internal model. Full internal models will substitute the whole standard formula, while partial internal models will substitute the standard formula for the calculation of single categories or sub-categories. In particular, as specified in 2009 CEIOPS' Directive of Solvency II insurers can use a partial internal model to estimate:

KEY INSIGHT 4

To be eligible to use an internal risk model, an insurer must have its model certified by the supervisor, a process that requires detailed documentation of the selected model and its underlying assumptions. Periodic examination of the model is also required to ensure that the model is properly adjusted to the dynamic financial environment. Therefore, standards for the structure and validation of internal models are essential. If an internal model is used, the resulting target capital should not be lower than the minimum capital requirements provided under Solvency II rules. Furthermore, regulators can require the use of an internal model if the insurer's particular conditions differ widely from assumptions made in the standard model.[35]

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[36]European Commission, 2002a, Risk Models of Insurance Companies or Groups, Markt/2515/02, Working Paper, 2002-05-17, Brussels.

[37] The Solvency II Process: Overview and Critical Analysis, University of St. Gallen, M. Eling, H. Shmeiser, J.T. Shmit, May 2006.

1. one or more risk modules or sub-modules of BSCR;

2. capital requirements for operational risk (which shall take into account the volume of operations, in terms of earned premiums and technical provisions held in respect of their insurance obligations.

3. the adjustment for the loss-absorbing capacity of technical provisions and deferred taxes. These adjustments must reflect potential compensation of unexpected losses through a simultaneous decrease in technical provisions or deferred taxes or a combination of the two.

Modelling an internal model is not an easy task. Before going through the steps required by Solvency II for the validation of an internal model, it is necessary to have some insight on how these models are drawn up. Statistical modelling is essential in any internal model that try to determine the probability distribution of the profit and loss account and available own funds of an insurance company. Based on the work of Chatfield (1995)[38], it is possible to draft a general scheme of the actual modelling steps:

I. Setting the model objectives: whether we are going to develop a full or a partial calculation of the SCR;

II. Model data collection, scrutiny, processing, and initial analysis;

III. Model formulation (*specification*)

IV. Model fitting (*estimation*)

V. Model checking (*validation*)

VI. Model documentation and communication

VII. Approvals

VIII. Model application[39]

Steps three, four and five are repeated iteratively until a satisfactory model has been found, and that the whole process has to be regularly rerun. As Ronkainen et. al (2008) indicate, it is possible to

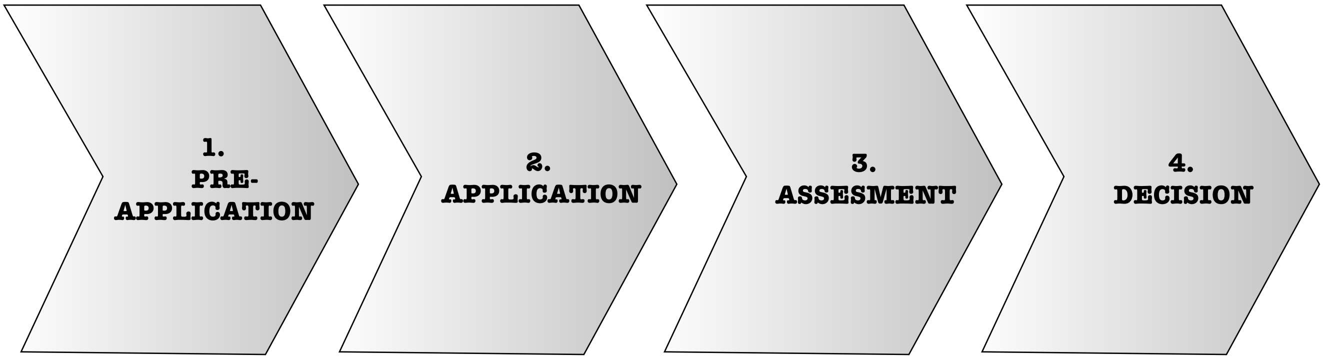
[38]Chatfield, C. (1995): Problem solving, A statistician's guide, Chapman & Hall.

[39] Rokainen et al., (2008),Challenges in developing internal models for Solvency II.

find some correspondences between model-building steps and approval steps, we will see it later in this chapter.

The procedure for the validation consists of four steps, as summarized in the following graph:

Graph 3.1 *Steps of validation process.*



The first step is denominated *pre-application*; it is not a mandatory step, nevertheless it is recommended by CEIOPS since it can give to the company substantial insight on the model they are developing. This phase is also really useful for the supervisor: in this phase he will be informed on how the company is organizing the formal application for the approval of its internal model and other secondary information. In this way, the company will also ease and speed the approval task to the authority, since he will start the application procedure with essential background on company's internal model.

The second step consist in the preparation of the *application pack,* that basically consists in the disclosure of the scope of the model, technical characteristics of the model, governance and dataset used. At this point the supervisor will have at most six months to analyze the model, and this is the third step of the procedure, the *assessment*. The supervisory authority has to establish whether the model can be used by the insurance company, and consequently validate it, or not. This decision is based on the guidelines supplied by Solvency II. In particular the general provisions for the approval of full or partial internal model are ruled by Article 120 and subsequents described in the Directive 2009/138/EC of the European Parliament.

Insurance companies are asked to provide, as a minimum, documentary evidence that the internal model respects the standards set out in Articles 120 to 125. CEIOPS sets the following standards and requirements with which the insurance company must comply:

- *Use Test:*

  This is not a straightforward requirement given the subjective nature of the regulations and involves companies having to provide evidence that risk and capital decision making is embedded into their business activities from an early stage. Indeed following Article 120 of CEIOPS, a company has to demonstrate that the internal model is widely used in and plays an important role in their system of governance. It requires the to demonstrate that:

a) the risk-management system implied by the internal model, which entails all "strategies, processes and reporting procedures necessary to identify, measure, monitor, manage and report, on a continuous basis the risks" (Article 44 CEIOPS-2009), shall be sound and well integrated into the organizational structure and in the decision-making processes of the insurance.

b) their economic and solvency capital assessment and allocation processes. This assessment must include, as pointed out by Article 45, the overall solvency needs; the compliance, on a continuous basis, with the capital requirements and with technical provisions; the significance with which the risk profile involved by the internal model deviates from the assumptions underlying the SCR calculated with the standard model.

- *Statistical Quality Standards:*

  The internal model, and in particular the calculation of the probability distribution forecast underlying it, must have some characteristics listed in Article121. We discuss here just the main features that the internal model has to respect. Firstly, the methods used to calculate the probability distribution must use adequate, applicable and relevant actuarial and statistical techniques based on current and realistic information and assumptions. The data sets used in the calculation must be revised by the company at least annually.

  Then the company must pay attention on how dependencies within and across risk categories, using the internal model. We also have to say hat by using an internal model, the company has the possibility to set new correlation coefficients among risk categories, provided that supervisory authorities are satisfied that the system used. Finally the model must take into consideration the effect of risk-mitigation techniques.

- *Calibration Standards:*

  Insurance companies are allowed to use in their internal model a different time period or risk measure from that of the Standard model as long as the outputs of the internal model can be used by the company to calculate the SCR with a level of protection at least equivalent to that set out in the Standard model. In calibrating the model, the role of data is crucial. The estimation results can be very sensitive to the chosen data set. Given that, supervisory authorities requires the calibration of the model, therefore may ask companies to run their internal model on some example portfolios using not their date but external dataset in order to verify it. Clearly there should not be too much freedom in respect to the dataset that could be used since CEIOPS tries to guarantee a level playing field for those companies that are using the standard formula with those that are using a partial or full internal model.

- *Profit and Loss Attribution:*

  As prescribed also by IASB, insurance companies has to disclose at least one time a year,the causes and sources of profits and losses for each major business unit. But CEIOPS also ask insurance companies to demonstrate how the internal model, and its related assumption on risks, explains the causes and sources of profits and losses.

- *Validation Standards:*

  "Insurance and reinsurance undertakings shall have a regular cycle of model validation which includes monitoring the performance of the internal model, reviewing the ongoing appropriateness of its specification, and testing its results against experience"(Art. 124- CEIOPS 2009)[40].

  Insurance companies have to demonstrate to supervisory authorities that the capital requirements provided by their internal model are appropriate by running statistical tests that we are going to see in detail later. Moreover, the model validation process includes an analysis of the stability of the internal model and in particular it tests how the results of the internal model change given a change in the model's key assumptions.

- *Documentation Standards:*

  Insurance companies are required to disclose the design and operational details of their internal model. The documentation have to prove that the model respects all the criteria that we have analyzed so far. The company must provide a detailed documentation about theory, assumptions, and mathematical and empirical bases characterizing the internal model. Furthermore must be documented also all cases in which the model can not be applied, and all relevant changes to it.

As we have previously said, there is a relation between the modelling and approval steps will be useful for both model-builders (e.g. insurance companies) and supervisors to clarify the processes and facilitate communication and co-operation. Indeed the calibration standard is closely related to

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[40] Official journal of the European Union, DIRECTIVE 2009/138/EC OF THE EUROPEAN PARLIAMENT AND OF THE COUNCIL of 25 November 2009

the estimation step, while the validation step is also part of the model building steps.
Finally the supervisor has to make a decision. He can decide to approve fully the model, this gives the company the possibility to immediately start using the model, or it can approve it partially, this implies that the company must revise the model and correct the problems found by the supervisor. Obviously the supervising authority can also decide to reject the model.

The use of an internal model brings not only benefits, but also some risks. One of the most important is the so called *Model Risk*: it represents the risk of an incorrect model that it is not applicable. The consequences of using such models, is that it can misestimate the probability of a significantly adverse event. Model risk shall be included under the category Operational Risk, since it can be considered as a consequence of inadequate internal process. Model risk arises especially in those situations where the outcome of the analysis are change significantly with the choice of a particular model, and there is uncertainty about what is the most suitable model.

But, to what extent capital requirements as compared of an internal model differs to those obtained using Solvency II standard model?
To answer this question, in the following paragraph we are going to analyze an internal model. This is particularly useful to better understand how an internal model is constructed, how it works and what are the effective benefits for insurance companies. We are going to analyze an internal model developed by Nadine Gatzert and Michael Martin, in order to try to solve a problem in computing capital requirements for credit and market risk by using Solvency II standard formula.

4.2 A Partial Internal Model to quantify SCR for credit and market risk

As we have seen in the first chapter, insurance companies invest a substantial part of their proceeds in government and corporate bonds. In recent years, after the crisis of European countries, insurance companies started to pay greater attention to credit risk associated with these types of financial instruments. Indeed the economic crises of countries like Greece, Spain, Portugal or Ireland demonstrate that it can not be taken for sure that highly indebted countries can obtain the needed financial support. Obviously this “new” consciousness calls for adequate credit risk models for insurers. However, even the last version of Solvency II provided bi QIS 5, does not require capital for credit risk inherent to riskier government bonds like Greece or Spain.
This is the reason why in last years was devoted always more attention in the development of new ways for an efficient capital requirements’ calculation for credit risk and market risk. However, even though this particular risk categories have been analyzed in depth, application to the insurance

sector have been scarcely inspected.

After the introduction of Solvency II the asset allocation of insurance companies changed. This new regulation has an impact which is someway similar to what happen in the banking sector after the introduction of Basel II, but with a different magnitude.

Many studies, conducted especially by rating agencies, found that insurers have indeed moved many of their proceeds to safer investments. In particular the investments in high-rated corporate and government bonds rose, and this is exactly what we have seen in the first chapter of this work. In addition to this a tendency to invest more in short-term bonds rather than long-term one was recorded along with a low level of equity holding. Another important factor to be treated is that Solvency II prescribes a special treatment for government bond issued or backed by States of the European Economic Area (EEA): Solvency II initially does not instruct any capital requirements for credit risk involved in investments financial instruments issued or guaranteed by any state of the EEA, including those more exposed to a default risk, like Greece and Spain.

Truthfully, this credit risk will not completely get rid off, since this risk associated with government bonds, will be considered in the process of ORSA (Own Risk and Solvency Assessment), one of the elements required by the Solvency directive (under article 45), for an effective governance model. Nadine Gatzert and Michael Martin (2012) address this problem by developing a partial internal risk model using a rating-based credit risk model that accounts for credit, equity, and interest rate risk inherent in a portfolio of both stocks and bonds[41]. Gatzer and Martin found a deep difference between the SCR obtain using the Solvency II standard formula and that obtained using their model. The explanation for such a result lies in the fact that an internal model allows many different assumptions and the possibility to fully reflect diversification strategy of the company; in addition to that the internal model of Gatzer and Martin has the important feature to emphasize the credit risk for low-rated bonds and EEA government bonds.

After this brief introduction we can start the comparison between how the standard formula calculate market and credit risks and how they are calculated using this partial internal model. We start from Solvency II standard model, to have a better understatement on how this process changes.

As already told in Chapter 2, in Solvency II standard model the BSCR is determine from six different risk modules and it takes into account also operational risk and adjustments for loss absorbency. Among the six risk categories included in the standard model we are interested in Market Risk. With no surprise, it is described as the biggest risk driver for life insurance companies,

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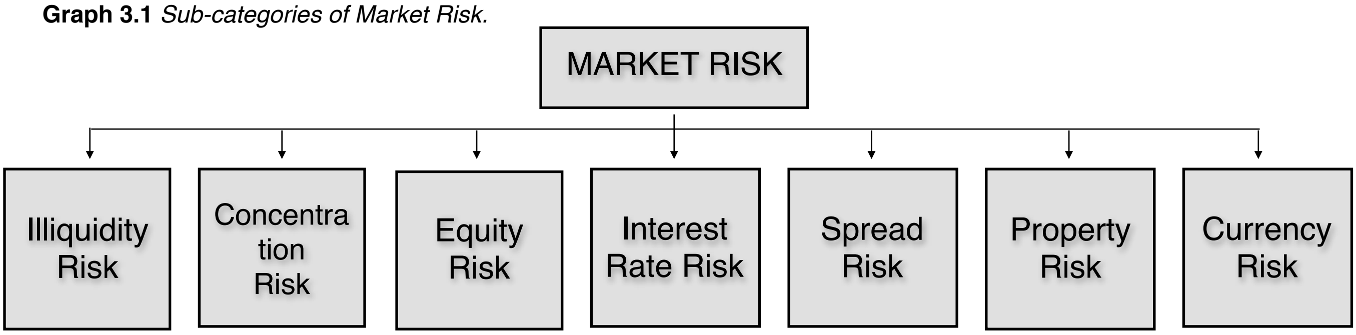
[41]Gatzert, N. Martin, M. (2012), "Quantifying credit and market risk under Solvency II: Standard approach versus internal model".

since they are the type of insurers who most invests in financial instruments. Market risk has seven sub-categories of risk, summarized in the graph below:

Graph 3.1 *Sub-categories of Market Risk.*



Solvency Capital Requirements for Market Risk should protect the company from variations in the market value of the financial instruments it owns. The basic calculation is based on the concept of NAV (Net Asset Value), introduced in the first part of this work, defined as the difference between market value of assets and best estimate of liabilities. ΔNAV represents the changes in NAV due to a shocked scenario for a given module or sub-module. It is computed as follows:

$$\Delta NAV = \max (NAV - (NAV \mid shock), 0)$$

In the construction of this model authors assume that liabilities will not be affected by changes in credit and market risks, therefore we can write $L = L \mid$ shock and consequently rearranging the previous equation we get:

$$\Delta NAV = \max (A - (A \mid shock), 0)$$ *with A stands for Assets*.

Credit risk is entailed in three risk modules, specifically spread risk module, market risk concentration module and default risk module. As we can see from the picture above, the first two modules are contained in market risk, while the third one is an extension of the spread risk sub module. The spread risk module contains all risks related to changes in term structure of interest rates and all risks responsible for variations in the value of assets and liabilities sensitive to credit risk changes. In default risk category includes risks related to government bonds of EAA and OECD states.

In this internal model are analyzed changes in Nav for the three most important sub-modules of Market Risk, that are *equity risk* (Mkt_{eq}), *interest rate risk* (Mkt_{int}) and *spread risk* (Mkt_{sp}), moreover Gatzer and Martin focused on just two classes of assets, stocks and bonds.

Firstly, we will calculate capital requirements for the equity risk sub-module. This means that we will find the part of SCR for market risk relative to fluctuations in equity prices. Notice that, as prescribed by Solvency II, the price of equity investments is computed using their market value. The approach used by Gatzer and Martin is the stressed scenario. This approach sets two shock scenario for the two classes included in equity sub-module:

I. *Global equity:* in this class we find all exposures transacted in EEA and OECD countries. Under the scenario testing approach, equity will decrease by 39% in stressed condition. Requirements are calculated as follows:

$$\begin{aligned} Mkt_{eq,Global} &= \max\left(0.3 \cdot MV_{eq,Global}(0), 0\right) \\ &= \max\left(0.3 \cdot \sum_{i \in Global} MV_{eq,i}(0), 0\right) \end{aligned}$$

MV stands for market value. **Eq. 4.2.1**

II. *Other equity:* this is the class of higher risks exposures, like investments in hedge funds or in non-listed equities. The stress scenario in this case prescribes a drop of 40% in their market value.

$$\begin{aligned} Mkt_{eq,Other} &= \max\left(0.4 \cdot MV_{eq,Other}(0), 0\right) \\ &= \max\left(0.4 \cdot \sum_{i \in Other} MV_{eq,i}(0), 0\right) \end{aligned}$$

Eq. 4.2.2

The two classes are correlated and this induce to the diversification effect, therefore the total SCR for the equity sub category is obtained by applying the standard square-root formula:

$$Mkt_{eq} = \max\left(\sqrt{\sum_{r_{eq},c_{eq}} CorrIndex_{r_{eq},c_{eq}} \cdot Mkt_{eq,r_{eq}} \cdot Mkt_{eq,c_{eq}}}, 0\right)$$

$$r_{eq}, c_{eq} \in \{Global, Other\}.$$

Eq. 4.2.3

Correlation coefficients are provided by CEIOPS.

As far as the SCR for interest rate risk sub-module, we start by saying that this sub category investigates the effects of changes in the term structure of interest rate on financial instruments. Therefore first if all we have to analyze the present value of all securities affected by changes in the term structure of interest rates:

$$PV_{int} = \sum_{t=1}^{T} \frac{CF(t)}{\left(1 + r_f(t)\right)^t}$$

Eq. 4.2.4

Now since the variation in term structure of interest rates can be either increasing or decreasing, we have two different stressed scenario, one for increased and one for decreased term structure of interest rate.

Consequently present value is computed two times, by adding an upward (s^{up} (t)) and a downward (s^{down} (t)) movement at time *t* of the risk-free interest term structure.

Therefore PV is computed as follows:

$$PV_{int}^{k} = \sum_{t=1}^{T} \frac{CF(t)}{\left(1 + r_f(t) \cdot \left(1 + s^k(t)\right)\right)^t}, \qquad \textbf{Eq. 4.2.5}$$
$$T = \max\left(t | CF(t) \neq 0\right), k \in \{up, down\}$$

Stressed parameters are given by CEIOPS and are summarized in the table below.

Table. 4.2.1

Maturity t (years)	Relative change $s^{up}(t)$	Relative change $s^{down}(t)$
1	0.70	−0.75
2	0.70	−0.65
3	0.64	−0.56
...	...	...
25	0.26	−0.30
>25	0.25	−0.30

Finally SCR for the whole interest risk sub-module is computed as we previously did for the equity sub-module:

$$Mkt_{int} = \max\left(Mkt_{int}^{up}, Mkt_{int}^{down}\right) \qquad where \quad Mkt_{int}^{k} = PV_{int} - PV_{int}^{k}, \; k \in \{up, down\} \quad .$$

Spread risk sub-module considers the impact of changes in credit spread to financial instruments. The solvency capital requirements for the sub-category (Mkt_{sp}) is obtained by summing uptake capital requirements for three different groups of asset, namely:

- SCR for bonds (Mkt_{sp}^{bonds});
- SCR for structure credit product (Mkt_{sp}^{struct});
- SCR for credit derivatives (Mkt_{sp}^{cd}).

$$\mathbf{Mkt_{sp} = Mkt_{sp}^{bonds} + Mkt_{sp}^{struct} + Mkt_{sp}^{cd}}.$$

Here we will focus only on the computation of capital requirements for bonds, the requirements for other two kind of assets can be computed analogously.

Just like for equity investments, also bonds are valued at their Market Value $MV_{sp,j}$ (0) of asset *j*. To evaluate requirements for bonds, Solvency II directives gives a rating based approach, this means

that stress parameters are asset specific and related to modified duration and to the rating of the bond. The modified duration has to be computed using the risk free rate provided by European Commission and the yield to maturity obtain with the following formula:

$$\mathbf{PV_{int,j} = CF_j(t) \cdot (1 + r_{YTM})^{-t}} \qquad \textbf{Eq. 4.2.6}$$

Once we compute the present value for all bonds, we have to combine them. In this case since the assets are uncorrelated, we will not have correlation coefficients. Consequently SCR for spread risk for bond *j,* is obtained solving this equation:

$$Mkt_{sp}^{bonds} = \max\left(\sum_j MV_{sp,j}(0) \cdot duration_j \cdot F^{up}\left(rating_j\right), 0\right) \qquad \textbf{Eq. 4.2.7}$$

It is important to notice that in this sub-category we should also have taken into account government bonds of EEA countries issued in their domestic currency, however Solvency II prescribes no allocation of requirements for them and also other types of financial instruments such as bonds of multilateral development banks and those of the European Central Bank.

Once we have computed all three risk sub-modules, we aggregate them to obtain the Standard Capital Requirements for Market Risk. We do this by the standard aggregation formula:

$$SCR_{mkt}^{SII} = \sqrt{\sum_{r,c} CorrMkt_{r,c} \cdot Mkt_r \cdot Mkt_c}, \qquad \textbf{Eq. 4.2.8}$$

$$\text{where } r, c \in \{eq, int, sp\}.$$

Correlation coefficients are provided by CEIOPS and are listed below.

Table 4.2.2

	Interest	Equity	Spread
Interest	1.00	*C*	*C*
Equity	*C*	1.00	0.75
Spread	*C*	0.75	1.00

Now we can begin the analysis of the partial internal model to quantify credit and market risks developed by Gatzer and Martin. Authors apply a Monte Carlo simulation to get the capital requirements based on a VaR at a 99,5%, as instructed by Solvency II. In this case the model can be substantially divided in two parts.

In the first part Gatzer and Martin quantify the market risk for stocks, they decide to described variations of prices of equity by a geometric Brownian motion. A stochastic process is said to follow a geometric Brownian motion, if it satisfies the stochastic differential equation below:

$$dS(t) = \mu_S \cdot S(t)dt + \sigma_S \cdot S(t)dW_S^{\mathbb{P}}(t) \qquad \textbf{Eq. 4.2.9}$$

where we have:

$W_s(t)$ which is a proper brownian motion process;

μ is a constant named percentage drift; and

σ represents volatility.

So we can consider the Brownian motion as a trajectory defined by the equation above, where the first term in the right side of the equation $\mu S_t dt$ controls the way in which trajectory evolve, its trend, while the second term $\sigma S_t dW_t$ controls for the random noise effect in this trajectory[42].

Solution of this differential equation is given by

$$S(t) = S(0) \cdot e^{\left(\mu_S - \frac{\sigma_S^2}{2}\right) \cdot t + \sigma_S \cdot \sqrt{t} \cdot Z_S(t)} \qquad \textbf{Eq. 4.2.10}$$

In this solution $Z_s(t)$ represents independent standard normally distributed random variables.This equation is widely used in finance , especially to model stock prices follows what is known as the Black-Scholes-Merton model.Indeed in this partial internal model, the geometric brownian motion is used exactly with this aim: modelling the development of equity prices.

Thanks to the analysis conducted above we are able to compute the market value of a portfolio composed by N_s assets:

$$MV_S(1) = \sum_{i=1}^{N_S} S_i(1). \qquad \textbf{Eq. 4.2.11}$$

Where $S_i(1)$ is the market value of stock i at time t=1, obtained with the formula described above.

The second part of the model concerns the modeling and the valuation of bonds. We can divide the process applied by Gatzer and Martin in two steps. In the first step is quantified the interest rate risk while in the second step credit risk is first calculated and then integrated to the valuation of market value for bonds.

[42] Karatzas and Shreve Brownian Motion Stochastic Calculus (1998).

1st step.

In the first step will be determined the risks arising from stochasticity of the term structure of interest rates. To do so, the authors start from the valuation of the market value for zero coupon bond (*ZCB*), with a price *p(t,h)* (h is the time to maturity and t represents the point in time considered). The market value for a ZCB is defined by the short term interest rate; we indicate it by *r(t)* and it is defined on the probability space $(\Omega_r, \mathcal{F}_r, \mathbb{Q})$,where $\mathbb{Q}$ represents a neutral probability measure.

Therefore the price for a ZCB is given by:

$$p(t,h) = E_t^{\mathbb{Q}}\left(e^{-\int_t^h r(s)ds}\right) \qquad \textbf{Eq. 4.2.12}$$

The short-term interest rate is valued through the CIR process. CIR process is derived from the model developed by Cox, Ingersoll and Ross; it is one of the most important one factor model describing the interest movements driven by only one source of market risk. It considers a mean reversing drift providing a strictly positive short-rate at any point in time *t*. By using a real world probability P, it can be written as:

$$\begin{aligned} dr(t) &= (\kappa_r \cdot \theta_r - (\kappa_r - \gamma_0 \cdot \sigma_r) \cdot r(t))\, dt + \sigma_r \cdot \sqrt{r(t)} dW_r^{\mathbb{P}}(t) \\ &= \hat{\kappa}_r \cdot \left(\hat{\theta}_r - r(t)\right) dt + \sigma_r \cdot \sqrt{r(t)} dW_r^{\mathbb{P}}(t), \end{aligned} \qquad \textbf{Eq. 4.2.13}$$

Price of a ZCB can also be written in a closed form, that is:

$$p(t,h) = e^{A(t,h) - B(t,h)\cdot r(t)} \qquad \textbf{Eq. 4.2.14}$$

Where we have: $A(t,h) = \frac{2 \cdot \kappa \cdot \theta}{\sigma_r^2} \cdot \ln\left(\frac{2 \cdot a \cdot e^{(\kappa + a)\cdot \frac{h-t}{2}}}{(\kappa + a)\cdot\left(e^{a\cdot(h-t)} - 1\right) + 2 \cdot a}\right)$

$$B(t,h) = \frac{2 \cdot \left(e^{a\cdot(h-t)} - 1\right)}{(\kappa + a)\cdot\left(e^{a\cdot(h-t)} - 1\right) + 2 \cdot a},$$

$$a = \sqrt{\kappa^2 + 2 \cdot \sigma_r^2}.$$

The last passage in the first step is described the correlation between stock and interest rates:

$$dW_r^{\mathbb{P}} dW_S^{\mathbb{P}} = \rho_{r,s} dt \qquad \textbf{Eq. 4.2.15}$$

2nd step:

Credit risk is integrated to calculate the market value of a bond portfolio at the end of a given period using the JLT model. The Jarrow and Turnbull (JLT) model is a rating-based credit risk model developed on the basis of Merton credit models. Merton developed a model for pricing default able bonds modeling bankruptcy as a statistical process. JLT model extends the model developed by Merton to take into account random interest rates. This model describes the state of a bond by default or non-default state and through credit ratings associated with it and the probability that a change in it will occur.

In this internal model credit transaction is assumed to follow a Markov process X, which is described in the formula below. This rating transaction chain uses data about transaction rates published by rating agencies.

$$X = (x(t),\ t \in \mathbb{N}_0) \qquad \textbf{Eq. 4.2.16}$$

The JLT model accounts also for the possibility of default for ZCB. Whenever bond issuer become insolvent, only a small fraction of the value of a non defaultable ZCB will be paid to creditor, in our case to an insurance company, this fraction is called the recovery rate, and will be paid at maturity.

The probability of a default is obviously related to the credit state associated with the bond, in the table below are described the credit rate used by most important rating agencies.

Moody's	S&P	Fitch	Definition
Aaa	AAA	AAA	Prime Maximum Safety
Aa1	AA+	AA+	High Grade High Quality
Aa2	AA	AA	
Aa3	AA-	AA-	
A1	A+	A+	Upper Medium Grade
A2	A	A	
A3	A-	A-	
Baa1	BBB+	BBB+	Lower Medium Grade
Baa2	BBB	BBB	
Baa3	BBB-	BBB-	
Ba1	BB+	BB+	Non Investment Grade
Ba2	BB	BB	Speculative
Ba3	BB-	BB-	
B1	B+	B+	Highly Speculative
B2	B	B	

Table 4.2.1 *Credit Rates adopted by most important rating agencies.*

B3	B-	B-	
Caa1	CCC+	CCC	Substantial Risk
Caa2	CCC	—	
Caa3	CCC-	—	
Ca	—	—	Extremely Speculative
C	—	—	
—	—	DDD	Default
—	—	DD	
—	D	D	

This relation between credit rating and default probability is described by equation 4.2.16.

The price for a defaultable coupon bond can be estimated with the following equation:

$$\begin{aligned}\hat{p}(t,h)_{x(t)=i} &= E_t^{\mathbb{Q}}\left(\mathbb{I}_{\{\tau>h\}}\cdot e^{-\int_t^h r(s)ds} + \mathbb{I}_{\{\tau\le h\}}\cdot\delta\cdot e^{-\int_t^h r(s)ds}\right)\\ &= E_t^{\mathbb{Q}}\left(e^{-\int_t^h r(s)ds}\cdot\left(\mathbb{I}_{\{\tau>h\}}+\mathbb{I}_{\{\tau\le h\}}\cdot\delta\right)\right)\\ &= p(t,h)\cdot\left(\delta+(1-\delta)\cdot\left(1-\lambda_{i,k}(t,h)\right)\right),\end{aligned}$$ **Eq. 4.2.17**

The indicator function $\mathbb{I}_{\{\tau>h\}}$ can assume two different values depending on whether the bond defaults or not. It assume a value equals to one if default occurs and zero otherwise; $1-\lambda_{i,k}(t,h)$ instead represents the probability that the default occurs between time t and maturity date h.

Now that we know how to value a ZCB, we can see how price a defaultable fixed income bond. The price of specific bond j, $B_j(t)$ is obtained by multiplying the price of a ZCB, obtained previously, by the sum of all future cash flows $CF_j(h)$ related to that bond. Therefore we have:

$$B_j(t) = \sum_{h=t+1}^{T_j} CF_j(h)\cdot\hat{p}_{x(t)=i}(t,h)$$ **Eq. 4.2.18**

Finally the stochastic market value of a portfolio with N_B bonds is:

$$MV_B(1) = \sum_{j=1}^{N_B}\left(\mathbb{I}_{\{\tau_j>1\}}\cdot\left(B_j(1)+CF_j(1)\right)+\mathbb{I}_{\{\tau_j\le 1\}}\cdot\delta\cdot FV_j\cdot n_j\right)$$ **Eq. 4.2.19**

Completed these two steps, we can finally compute the overall SCR, making use of the coming formula:

$$MV_B(1) = \sum_{j=1}^{N_B}\left(\mathbb{I}_{\{\tau_j>1\}}\cdot\left(B_j(1)+CF_j(1)\right)+\mathbb{I}_{\{\tau_j\le 1\}}\cdot\delta\cdot FV_j\cdot n_j\right)$$ **Eq. 4.2.20**

Another important step at this point is asses the model risk that stems from the use of an internal model, however its computation is beyond the scope of this work.
Gatzer and Martin continues their work by simulating their internal model again the ESA model, and the results obtained by the authors highlight severe flaws in the ESA model.
Results of their work shows very important differences between the capital requirements for market risk obtained using the standard formula and those obtained through their internal model. They also found out that the standard model underestimate the risk related to low-rated bonds, while overestimate capital requirements for high-rated bonds; additionally it does not give sufficient consideration to diversification, which also leads to misestimation of requirements.

Internal model may provide substantial benefits to insurance companies: a better understanding of risks and consequently a better risk management, moreover they can give rise to an improvement in decision-making. Insurers can therefore achieve a competitive advantage respect to similar companies using the ESA model. But as we have seen in the first part of this chapter developing a sound internal model model is not an easy tasks, and costs associated with the development, validation and of an internal model and fulfillment in time of the precepts required by regulator authorities have a really high costs.

Conclusions

In this paper we have analyzed Solvency II framework developed by the European Commission. The aim of the European Commission, is to harmonize supervision regulations in the European Union, in order to decrease information asymmetries in this sector and align incentives of policyholders and insurers.

This new supervision framework not only aims at limiting the number of insolvencies in the European market but, as studied by Doff (2008) have minimized costs of insurance insolvencies to the economy.

But Solvency II framework does not seem to be flawless. Problems of Solvency II are chiefly related to difficulties in developing a "one fits all" model, which inevitably will does not takes into account specific problems of particular types of insurance companies. This is especially true for the calculation of capital requirements. In chapter three we have widely discussed the most important problems related to the standard model for the calculation of capital requirements. Although for some companies, the standard formula can adequately reflect the capital requirements and also the interactions between lines of business[43], we have seen what the effects of the application of standard model could be on the majority of companies: it may severe misestimates capital requirements for many lines of business. In the last chapter we have seen how credit and market risks requirements are underestimated by the standard model, as demonstrated by Gatzner and Martin (2012). So it is not just by chance that many insurance companies requires internal model.

CEIOPS is obviously aware of this problems, and this is the reason why in Solvency II the use of internal models is encouraged. Well developed internal model provides a consistent view of the interaction between risk, capital and value across different lines of businesses. It can be really important in getting a better understatement of the risks faced by the company, and therefore in implementing a strategy aimed at increasing profitability while reducing risks, for instance thanks to reinsurance or diversifications. Understand the better way to diversify risk, means, for an insurance company, reduce the cost of holding capital.

But internal models have drawbacks too. First of all implementing an internal model, both full or partial, is really costly for companies. Costs in terms of preparation, development, and education go far beyond just getting the numbers right. This is because the development of internal models has to tackle very complex questions, and modeling well-embedded statistical models necessary in the

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[43]Benefits and challenges of using an internal model for Solvency II Oliver Gillespie, Dominic Clark, Henny Verheugen and Gary Wells.

development phase is really complex. Then, as we have extensively seen in chapter 4 the company's internal model must meet strict requirements set by supervisor authorities in order to be validated. Satisfying supervisors turns out to be very difficult, especially if we consider that controls by supervision authorities must be repeatedly passed during years.

Solvency II is undoubtedly a step forward toward a better regulatory framework in the European Union. It will benefits both insurance companies, in developing a better risk management and reducing their insolvency probability, and policyholders, since thanks to the harmonization introduced they will have a more easy access to material information. Furthermore it will promote stability in the economy as a whole. However it still has some important problems; problems for which European Commission, particularly CEIOPS, is already working on.

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