

**LUISS – Libera Università Internazionale degli Studi Sociali**

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DIPARTIMENTO DI ECONOMIA E FINANZA  
Corso di Laurea Magistrale in Financial Economics

TESI DI LAUREA MAGISTRALE

**BIOECONOMIC MODELING**

An optimal control approach

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# 1 Thesis summary

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An important and common problem in economics is how to manage assets, meaning anything that produces economic value and can be owned or controlled in some way. A first categorization of assets that has important implications from an accounting perspective is, as usual, between tangible and intangible ones since intangible assets other than financial assets are very hard to define and evaluate. From an economic perspective, however, a perhaps more meaningful division is between *renewable* and *non-renewable* assets. It is fairly difficult to characterize what a renewable asset truly is: since it is an asset, it must provide economic value, meaning that it cannot be in infinite supply, and it must be possible to exclude, at least under some circumstances, others from its use; since it is renewable, it must be *naturally* replaced.

The key elements in this definition are (i) the natural occurring process that replaces the resource and (ii) the economic nature of the resource. For example, sunlight and wind are indeed renewable, but they are also in infinite supply: what is in finite supply is the equipment necessary to produce, store and channel electricity from solar and wind power. Oxygen and fresh water are also renewable, and it can no longer be safely assumed that they are in infinite supply or that the access to water cannot be restricted, and they should therefore be considered as renewable assets. However, it looks foolish as well as criminal to put a price on water or air and limit access to these vital resources.

Finally, the last sizable category that could and does, in fact, fit the characteristics of renewable assets are living biological organisms (other than humans), since all natural species are indeed renewable and some plants and animals have a long history of economic exploitation. The discipline that studies how to optimally manage these assets is called “Bioeconomics” and is the subject matter of this thesis. For clarity, I collect the remarks from the above discussion in the following two tentative definitions.

**Definition 1** (Renewable asset). *A renewable asset is an asset that can be naturally replaced. Examples of renewable assets are biological systems such as forests, marine and freshwater resources, grasslands and deserts, and wildlife populations.*

**Definition 2** (Bioeconomics). *Bioeconomics, a field at the interplay of economics and biology, is the discipline that aims at developing a theory of optimal management of renewable assets.*

While they are fairly restrictive and can certainly be improved, these working definitions manage to exemplify the *economic* nature of the problem: at least on a first reading, renewable resources are just another class, although fairly peculiar, of assets, and their management is subject to similar economic incentives and constraints. The definitions, however, give no information on whether the issue of managing renewable resources is also a *relevant* one, meaning that even serious mismanagement of the assets does not severely affect society as a whole.

To give an idea of the magnitude of the problem, just considering one type of renewable assets, namely marine and freshwater resources, in the European Union alone about 85 000 and 116 000 employees work, respectively, in the fishing and fish processing industry; moreover, these two sectors produced economic value for about EUR 3.5 billion and EUR 30 billion in 2013. Equally importantly, in some European coastal communities fisheries are the main employer accounting for more than half of local jobs (European Commission 2014a). These few figures about fisheries may seem minor compared to EU level employment or GDP, but they do not convey the complete story: fish as a source of protein are an important component of a healthy diet, and as part of the marine ecosystem are one of the drivers for the “blue economy” that generates about 5.6 million jobs with an economic value of about EUR 495 billion per year (European Commission 2014b).

If renewable assets are indeed highly valuable to society, then it remains the question of how to optimally manage them. Continuing with the fishery example, the European Union, recognizing the importance of the fishing industry, tackles this issue with “Regulation (EU) No 1380/2013” and “Regulation (EU) 2015/812” of the European Parliament and of the Council that lay down the legal principles behind the “Common Fisheries Policy (CFP).” Quoting from these regulations, the CFP

should ensure that fishing and aquaculture activities contribute to long-term environmental, economic, and social sustainability [...] should contribute to increased productivity, to a fair standard of living for the fisheries sector including small-scale fisheries, and to stable markets, and it should ensure the availability of food supplies and that they reach consumers at reasonable prices

In the above quotation, the emphasis is on the *economic* and *environmental sustainability* of the fishery policy, which is a fundamental and widely shared principle by managers of renewable resources. Hence, the bioeconomic task of managing renewable assets such as fish stocks, forests, or grasslands is

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surely becoming a necessity rather than a “luxury affordable only by rich nations” if humanity is to learn from its past and finally look for a sustainable growth path (Clark 2010). The problem is economic in nature, renewable assets being just another form of capital capable of generating consumption flows, and economic theory should suggest possible solutions.

Unfortunately, the prevalent economic incentives and constraints push firms towards unsustainable harvest levels that risk depleting and ultimately extinguishing the exploited populations. In the literature, such a dynamic is called bionomic equilibrium, and, especially for fisheries, it is a far too common state of affairs: attracted by the initial high profitability of a natural resource, harvesters continue entering the industry until economic profits for entrant firms are driven down to zero, and the population close to depletion. It is widespread because an open access unregulated resource, which is a necessary condition for bionomic equilibrium to occur, has traditionally been the case for fisheries and other renewable assets.

Resource managers can and should introduce regulations to avoid the bionomic equilibrium outcome, but what objective should they strive for? They could, for example, wish to implement the “Maximum sustained yield (MSY),” meaning the highest harvest rate which can be sustained over an indefinite period and which does not lead to depletion of the population. But the MSY has at least one major deficiency because it ignores the economic rationale behind harvesting such as prices and cost of harvesting completely.

Alternatively to the MSY, managers could look at the “Maximum economic yield (MEY),” meaning the highest sustainable rate that maximizes economic profits. Compared to the MSY, the MEY does explicitly consider economic incentives, but, like the MSY, it neglects the fundamental dynamic nature of the problem. When deciding harvest levels, economic agents select an entire harvest path that stretches into the future and that may or may not be optimal depending on the response of the exploited population. Hence, rather than the MSY or MEY, managers should adopt a dynamic bioeconomic model of harvesting as benchmark for their decisions.

For the above reasons, following Clark (2010) in this thesis I focus mainly on a linear dynamic bioeconomic model of a single price taker firm. Through the tools of optimal control theory, I first derive an existence result for an optimal harvest level, and henceforth, thanks to the Pontryagin’s maximum principle, necessary conditions for the optimal harvest and population level. The first important result is the so called modified golden rule which asserts that the marginal productivity of a renewable asset is exactly equal to its opportunity cost minus a “stock effect,” and is the best population equilibrium that a single profit maximizer firm wants to achieve.

The second important result is that the golden rule solution always has a higher population stock than the bionomic equilibrium, implying that the latter is suboptimal with important consequences for policy. In fact, if managers want to escape from the trap of bionomic equilibrium and enact re-

source preservation coupled with economic efficiency, then the golden rule seems a good compromise, and it is indeed what I define as a social optimum.

To achieve the golden rule solution I consider different types of regulations: input and output controls, which mean respectively restrictions on effort and total catch, taxes and quotas. What I find is that, in the model, classical input and output controls alone cannot prevent a bioeconomic system from reaching bionomic equilibrium, since they do not manage to significantly modify harvesters' economic incentives.

Taxes seem a good instrument, because they succeed in implementing the golden rule solution even if managers do not limit access to the resource. However, to be effective they rely on the precise estimation of a large number of potentially time-varying parameters, where even a minor mistake could determine under- or over-harvesting situations that could cause, respectively, large economic losses to private firms or depletion of the resource stock.

Like taxes, quotas, or more specifically "Individual fishing quotas (IFQs)" and "Individual transferable quotas (ITQs)" manage to steer harvesting towards the golden rule solution, but, unlike taxes, they rely only on the right biological modeling of the population to determine the total allowable catch, thus reducing the magnitude of the estimation problem. However, quotas are controversial because there is no right way to implement them, since, unlike taxes, quotas restrict access to the resource, and, especially in fisheries, firms are strongly opposed to buying the right to harvest when access to the resource had traditionally been unrestricted. Furthermore, if quotas are freely granted, there is no way to compensate, at least partially, losers from the policy. A possible solution is to implement a double regime of taxes and quotas, where the revenue from taxes could in part offset the losses of ineligible participants to the quota system.

When writing this thesis, I used like a handbook for the modeling part Clark's book: "*Mathematical Bioeconomics*" as the choice and order of the topics treated in this thesis show. As Clark does, the models presented are, while simple, general enough in scope to include many sorts of renewable assets such as fisheries, forestry, and other wildlife populations. However, the focus is always on fish stocks for many reasons: they are one of the renewable assets with the longest history of commercial harvesting and are a widespread and economically important resource. They are also a challenging problem since their distribution in space is not fixed as it is for trees, and at least some species are mobile enough that their sustainable exploitation requires international agreements. Notwithstanding the last point, this additional complexity of the fishery problem is not present in this thesis, because as I mentioned before, this work aims at presenting a few results that should hold qualitatively true for a general renewable asset, and because the subject matter is already mathematically complicated in its basic form. In this vein, I also do not consider more realistic models such as multi-species models, growth and aging models, predator-prey models, etc.

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To conclude, in this work my main contribution is to develop a dynamic bioeconomic model in a more rigorous optimal control formulation, and to analyze different forms of regulations in the setting of the model.

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