DEPARTMENT OF ECONOMICS AND FINANCE

Chair in Theory of Finance

Estimating the Stochastic Discount Factor: evaluation and historical approaches

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Financial markets have become more and more complex over the last 30 years and especially after the subprime crisis. For this reason, it is important to be able to understand the mechanisms that move the market variables by interpreting them properly.

My final dissertation aims at studying and analyzing different methodologies to estimate stochastic discount factors using the three-factor model developed by Fama-French and then testing their performance in pricing assets during the past 30 years.

Estimation techniques play a crucial role in this context because they may dramatically affect profits and losses of a portfolio. Therefore, having a good proxy of the real discount factor as a starting point could help to avoid mistakes in pricing an asset.

According the fundamental theorem of asset pricing, the price of any asset is equal to its expected discounted payoff. In asset pricing theory, the payoff is discounted by a factor, which depends on market parameters and on the data. Apart from asset pricing, another use of the stochastic discount factor is to evaluate the performance of actively managed portfolios.

Based on Cochrane (2001), risk neutral valuation implies the existence of a positive random variable, which is called the stochastic discount factor and is used to discount the payoffs of any asset. Modern asset pricing theory assumes the existence of this discount factor m so that the following equation holds:

$$p = E(mx)$$

Where:

- p is the price of any asset;
- m is the stochastic discount factor (SDF);
- x is the asset payoff(s).

The term *stochastic discount factor* refers to the way m generalizes the standard discount factor ideas: on one hand, it can incorporate all risk-corrections by defining a single stochastic discount factor – the same for each asset – and putting it into the expectation. On the other hand, m is defined stochastic or random because it is not known with certainty at the present time.

As far as the performance of actively managed portfolios is concerned, it is evaluated under the assumption that there are no arbitrage opportunities in financial markets. This assumption implies that exists at least one positive stochastic discount factor capable of price all assets. Under such condition, the price of any asset is given by the expected value of the future asset payoff adjusted by the stochastic discount factor so that the following equation is always satisfied:

$$1 = E_t(m_{t+1}R_{i,t+1})$$

Where

- m_{t+1} is the stochastic discount factor at time t +1;
- E_t is the expectation conditioned on the information available up to time t;
- $R_{i,t+1}$ is the gross return;
- $R^e_{i,t+1}$ is the excess return of any asset at time t+1 (defined as the difference between the return on the asset and the risk free rate).

In this framework, the objective of the analysis is to estimate the discount factor by using the linear specification according to which the discount factor is defined as follows:

$$m = a - f'b$$

Where *f* is a k x 1 vector of risk factors, *a* is a scalar constant and *b* is a k x 1 vector of parameters.

In the unconditional case (when the investors do not consider the information set available at time t) the weight of the parameters *a* and *b* are time-constant, so it is assumed that they does not change over time.

It is possible to assume that the payoffs and discount factors are independent and identically distributed (i.i.d.) over time, so that the conditional expectations would be the same as the unconditional expectations. In this framework, this assumption will not be made, because in practice is not feasible to know the investors' information set: for this reason, it will be consider only unconditional models.

It is necessary to establish a risk free rate in order to calculate the excess return: in this framework, the US one-month Treasury bill will be considered the “safe” asset (the instrument with certain future return).
The models are estimated by using monthly data for the period from July 1984 to February 2015.

In order to estimate the discount factor, the Fama-French (1993) three factor specification of the stochastic discount factor will be used. Since all the factors are excess returns, the stochastic discount factor in the unconditional version of the Fama-French three-factor model can be specified as:

$$m = 1 - b_1 YM - b_2 HML - b_3 SMB$$

To estimate the discount factor, it is possible to run the following regression:

$$1 = b_1 YM_t + b_2 HML_t + b_3 SMB_t + \eta_t \qquad (1)$$

Moreover, it is possible to show that solving the following moment condition is equivalent to the OLS regression above mentioned:

$$E(R^e m) = E(mf') = 0$$

Indeed, since the factors are excess returns, it is possible to write:

$$\begin{cases} E[mYM] = 0 \\ E[mSMB] = 0 \rightarrow E[mf] = 0 \\ E[mHML] = 0 \end{cases}$$

In addition, $m=1-b_j'f_{jt}$. If this equivalence is substituted into the assumption, the moment condition can be written as:

$$E\left[\left(1 - b_j f_j\right) f_j\right] = 0$$

Therefore, 1-b_j'f_j is exactly η_t (the error term in the regression), so that:

$$E[\eta_t f_j] = 0$$

which is specifically the strict exogeneity condition (the errors in the regression should have a zero conditional mean), a necessary assumption to calculate the OLS regression.

However, this computational process could lead to some problems: the estimation of the regressors could be biased due to the fact that the dependent variable is a constant.

Cochrane (2001) shows that, if the factors are excess returns, the predicted b_j of the linear model is equal to:

$$b_j = \frac{E[R^e]}{Cov(R^e, f_j)} = \frac{E[f_j]}{Cov(f_j, f_j)} = \frac{E[f_j]}{Var(f_j)} \quad (2)$$

Therefore, the following step is to calculate the betas from our data (in table 1, the first row is the predicted beta, calculated using equation 2, while the second row shows the estimated beta, derived from regression 1):

b_s	Mkt-RF	SMB	HML
Predicted	0,035122	0,007363	0,025439
Estimated	0,041265	0,007153	0,044815

Table 1 – Differences between predicted and estimated betas

The prediction of the theory is quite close to the actual estimation.
From the previous equation, it is possible to get the values of t; then, the p-values are calculated and shown in the next table:

Significance test	Mkt-RF	SMB	HML
t	0,5155	-0,0119	1,0473
p-value	0,3032	0,5047	0,1478

Table 2 – t and p-value of the test

From the values above follows that the null hypothesis can't be rejected at the predeterminated confidence level so that it is statistically correct to say that the theoritical coefficients and the estimated ones are equivalent.

These findings provide an evidence of the validity of the model.

2.3 Estimation results

At this point, it is possibile to compute m_t from the following equation:

$$m_t = 1 - b_{YM}YM_t - b_{HML}HML_t - b_{SMB}SMB_t$$

Using the betas estimated in the previous section.

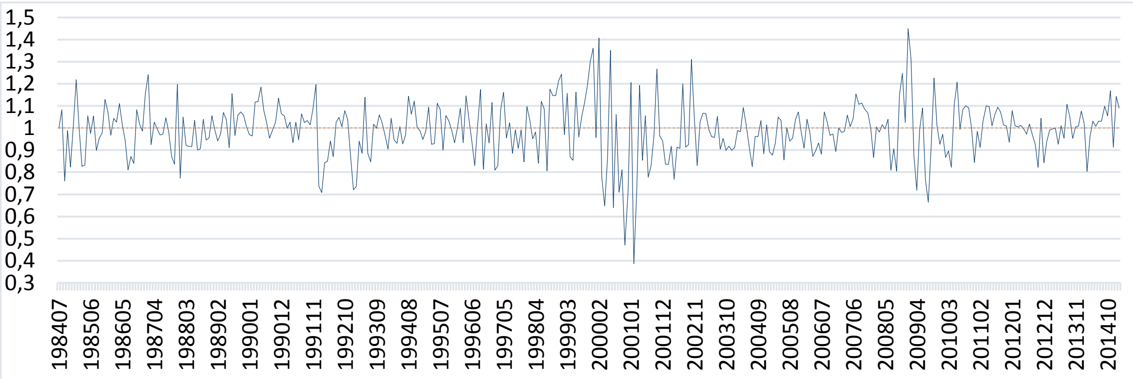

Figure 1 – Historical representation of the Discount Factor

The table below provides the summary statistics for the time-series of the SDF:

	MEAN	STD	MIN	MAX
Discount Factor (m)	0,98932	0,12860	0,38640	1,44889

The results are interesting because the SDF has no negative values and the mean is close to one, due to the mean-normalization assumption. The empirical discount factor seems to be quite stable with the exception of these two periods.

In order to check the performance of the discount factor, it is relevant to test if a set of SDF candidates satisfy the law of one price, such that:

$$E(mR_i) = 1$$

Thus, we say that a SDF correctly "prices" the assets if this equation is satisfied.
In fact, despite the assumption, it is possible that the empirical discount factor may not respect the moment condition and some assets could have significant pricing errors.
The pricing error can be written as:

$$\theta = E(mR_i) - 1$$

In practice, the thetas of all assets should be zero. In fact, the moment condition on which the estimation is based states that the discount factor assigns a price equal to one to any gross return, as the theory suggests.

At this point, it can be considered the set of different test portfolios and indices in order to establish if the discount factor estimated in this paper prices other assets. Let y_t be the returns of these other assets and let $p_t = m_t y_t$ be the price of the assets' residuals. Then, p_t -1 should not be significantly different from zero.
Figure 3 shows the summary of the effectiveness of the discount factor among the portfolios.

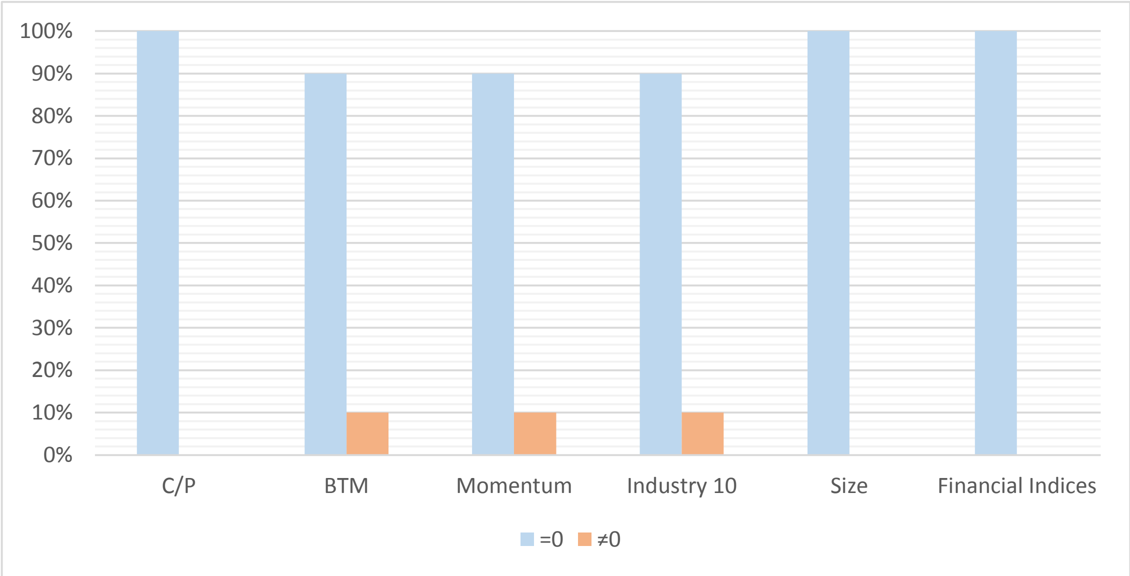

Figure 2 – Percentage of pricing errors significantly different from 0 by type of asset

Generally, the empirical discount factor model prices well, indeed it is consistent around the 94% of the time.

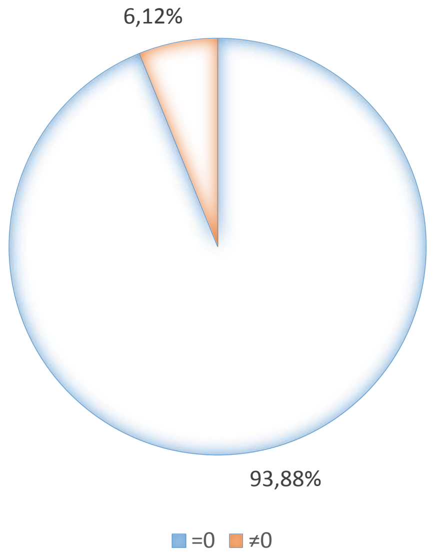

Figure 3 – General percentage of pricing errors significantly different form 0 among all assets

One way to understand the validity of the empirical discount factor is to compare the estimated *m* with the discount factor evaluated by other researchers.

Since the principal aim of the paper is to develop a simple model to estimate a discount factor that is a good proxy of the real one, if the difference between this m and the ones estimated using more theoretical and complicated methodologies is small, the central objective is satisfied.

Not only does this work aim at describing the approach of the previous models, but it also compares their results. Therefore, the next table provides the statistics concerning the above mentioned models in the first three rows, with the addition of the one described in this paper in the last row. In particular, the table shows the mean, the standard deviation, the minimum and the maximum value and also the first order autocorrelation of the estimated stochastic discount factor (ρ_1) , the number of m<0 in the sample and the unconditional HJ distance.

Unconditional models	E(m)	SD(m)	ρ_1(m)	min(m)	max(m)	n.(m<0)	HJ uncon
Farnsworth et all	0,9890	0,2250	0,2340	-0,2040	1,6800	2	0.235
Bessler et all	0,9905	0,4745	0,0550	-0,7183	2,1711	3	0,4909
Gutierrez & Gaglianone	0,9967	0,3346	N/A	-0,5184	2,1010	N/A	0,4207
Empirical Discount Factor[1]	0,9893	0,1286	0,1608	0,3864	1,4489	0	0,4318

Even if the discount factors are estimated using a different set of data, it is still possible to see how close the estimation conducted in this paper is to the other ones:

- the mean and the standard deviation are in line with the other estimations;
- the first order autocorrelation shows a small value;
- it is the only model to have a minimum value higher than zero, so that it does not have any negative discount factors;
- the HJ distance is in line with the other models.

Although the discount factors have been estimated with different sets of data, it is still possible to see some similarities between the estimates of this work and the others':

- The mean and the standard deviation are coherent with the other estimations;
- The first order autocorrelation shows a small value as well;
- The HJ distance is consistent with the other models.

[1] Discount Factor estimated in the paper

Even though the model applied in this work seems to compute an efficient discount factor easily, it is important to take into account its limits and features. In fact, it is based only on one assumption (the discount factor places a value of zero on the excess return) whereas the other methods explained involve more than one moment conditions.
Furthermore, the other papers, having more assumptions, have expanded the scope of the discount factor.

The methodology proposed to estimate the stochastic discount factor as well as the other models are all based on the Fama and French factors, these well represent the main characteristics of US firms.
The discount factor calculated here could not be generally applied and consistent for all the assets in the financial market due to the shortcomings concerning the computational approach.
In addition, the comparison between the models could be not completely reliable because of different data sets, which can negatively affect the results.
Nevertheless, it is undeniable that the m estimated could be a good proxy of the real discount factor: as stated in the introduction, it could be used as a first approximation for the real discount factor to test if the price of an asset is correct, when dealing with of new items first appearing in the financial market.

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