Dipartimento di Economia e Finanza

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Asset Pricing Models, Arbitrage Pricing Theory and Fundamental Analysis: Main Applications and the European Market Case

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Table of contents

Introduction

The practice of asset pricing has grown throughout the years thanks to the adoption of several new instruments and procedures. Financial economists all over the world were thusly inspired in creating new theories to explain the fluctuations of the market and in testing if these theories could be applied in real life. These tests were often successful to the point that an asset pricing model could become a landmark of the science, stand the test of time and inspire both great lines of work with numerous contributors and, of course, a healthy dose of criticism. This is the case of the Fama-French 3-Factor Model, which can be included in the inner circle of the works that changed the discipline forever, along with the Markowitz Model and the Capital Asset Pricing Model. After more than 20 years, this model is still widely regarded as the most efficient asset pricing model and inspired numerous expansions and empirical tests.

This model can be seen as the most prominent case of overlapping between two of the most widespread practices in financial economics: asset pricing and fundamental analysis. My intent is to offer a substantial survey of the literature that dealt with these issues throughout the years and to test the Fama-French 3-Factor and 5-Factor models in the stock markets of the European Union. I will try to check if the patterns found by Fama and French are able to explain the cross section of stock returns in this market over the last 20 years.

In the first section I will present the main theories and the relative empirical tests developed on asset pricing in the last 60 years. I will start from the Efficient Market Hypothesis, proceed with the Markowitz Model and then I will analyze the evolutions that led to the Capital Asset Pricing Model and eventually the Arbitrage Pricing Theory.

In the second section I will talk about the works that paved the way for the Fama-French 3-Factor Model and I will analyze this model thoroughly. Then I will present data, methodology and results of my own test of the 3-Factor on the European market.

In the third and last section I will examine the most recent progress in Arbitrage Pricing Theory, most notably the various expansions to the 3-Factor Model, including the Fama-French 5-Factor Model. Finally, I will present data, methodology and results of my empirical test on this last model as well, including further analysis for the Euro Area and UK stock markets.

Section 1

1.1 – Asset Pricing and the Efficient Market Hypothesis

The financial markets are an ever-changing and increasingly complex entity and everyone involved with them is eager to know the causes of their seemingly unpredictable fluctuations. Nowadays, with the enormous amount and variety of financial instruments available in the market, it is as difficult as ever to predict the outcomes of an investment. According to Bodie, Kane & Marcus (2011), experience and common sense suggest that investors are generally averse to risk, which means that they are willing to accept to take on more risky endeavors only if they are rewarded with higher returns. Unfortunately, it is impossible to observe risk or expected returns attached to financial products in real life because they are constantly influenced by an indefinitely large amount of events, which may relate to the single entity that issues the instrument, to its sector, to its country's economy or to the whole world. It is however possible to estimate them by using statistical processes and the historical record of the security's performance, even though these estimates will never be exactly precise and new unpredictable events will most certainly happen eventually.

According to Fama (2014), the research on the possibility of predicting or at least explaining the tendencies of the capital markets began between the 1950s and the 1960s, as soon as computers started to come out and econometricians could start building their models in a more efficient way, without having to waste lots of time in mechanic calculations. This research became prominent especially at the Massachusetts Institute of Technology and at the University of Chicago, thanks to the work of legendary researchers like Franco Modigliani, Merton Miller, Harry Roberts and Paul Samuelson. The first problem that presented itself to these pioneers of asset pricing was whether the financial markets reflected all the available information and, if not, what kind of information could be exploited to predict their future behavior.

This is what led Fama (1970) to elaborate his Efficient Market Hypothesis, which came in three very different forms:

- The Weak Form of the Efficient Market Hypothesis requires that current prices reflect all the information contained in past prices, so future prices cannot be predicted by analyzing past prices.
- The Semi-Strong Form of the Efficient Market Hypothesis requires that current prices reflect all publicly available information and adjust to it quickly and without bias, so no one can profit by operating on the market according to that information.
- The Strong Form of the Efficient Market Hypothesis requires that current prices reflect all information, both publicly available news and inside information. If markets satisfy this hypothesis, no one can ever gain excess returns consistently, and the market portfolio is the efficient one.

Testing these hypotheses is particularly difficult, as they are purely theoretical and it is not clear what effects of these hypotheses are directly observable.

Fama (1970) makes it clearer with some notation. We indicate as $E[R_{i,t+1} \mid \theta_t]$ the returns on asset i at time $t+1$ given the information set θ available at time t. According to the EMH, the expected returns should not depend on the available information set, and therefore these expectations should be constant:

$$E\left[R_{i,t+1}|\theta_t\right] = E\left[R_{i,t+1}\right] \quad (1)$$

This hypothesis becomes even stricter in another very popular theory of the 60s and 70s: the Random Walk hypothesis. In this case the random variable of asset returns is unpredictable not only in its expectation, but also in its distribution:

$$f\left(\tilde{R}_{i,t+1}|\theta_t\right) = f\left(\tilde{R}_{i,t+1}\right) \quad (2)$$

According to Cochrane (1999a), this is still a popular theory, but it is also ignored by the majority of financial economists, who are always researching new hypotheses to explain stock returns.

Asset pricing models are therefore not only an attempt at predicting returns in order to pick the right stock and make money: they are also a way to test if these hypotheses are verified in the market, it prices already reflect all the available information and therefore the market is efficient. For example, the "momentum" effect by Jegadeesh and Titman (1993) (with which I will deal more in depth in later sections of this thesis) is a particular form of short-term autocorrelation in stock returns and, if it actually exists, we can openly say that financial markets are not efficient, not even in the "weak" form. According to Fama (2014) these market inefficiencies might be a consequence of what financial economists indicate as "irrational behavior", which translates in financial market operators that, due to excessive fear, excessive confidence or mere conformity, behave differently from the return-maximizing and risk-averse "rational investor" that I discussed earlier.

There is a whole branch of economics called behavioral finance studying these kinds of anomalies, with interesting studies like the one by Benartzi and Thaler (1995), which managed to find and some fairly consistent patterns. For example, the stock market tends to overreact to information of bad earning prospects for the firm and, vice versa, tends to underreact to good news. However, Fama (2014) and financial economists in general tend to have little reliance on the ability of behavioral finance to be able to contribute with significant and consistent mathematical and statistic processes to the advancement of asset pricing and financial economics in general. This is simply because these are among the branches of economic science that are more based on solid evidence rather than theory and behavioral finance analyzes and tries to explain anomalies that are too big in number and too various in characteristics to be predicted in a mathematical model.

By this point of view, asset pricing focuses on whether these factors (and their relative premiums) derive from rational or irrational behavior: does the market require additional returns in order to take on more risk or does it react irrationally to new information?

Modigliani and Miller (1961) formulated the theory that stock price evolutions (and therefore stock returns) depend on the expectations of future dividends of the company, and this is a widely accepted theory in financial economics. However, Shiller (1981) finds that volatility in stock prices is too large to be explained by the change in expectation of future dividends, but there is no conclusive proof on whether a model can explain rationally this excessive variation. Moreover, if the model fails, is the model wrong or are returns moving irrationally and unpredictably? Are the patterns that researchers find through mathematical and statistic processes consistent or are they temporary and fortunate? Maybe the solution is a little bit of both: stock returns may be caused by irrational behavior, but irrational behavior itself may have some determinants. After all, Fama and French (1989) assert that predictability of stocks and bonds is rational, but the "animal spirits" of the market can influence it "in a way that is related to business conditions". This comes as no surprise, as many macroeconomic theories suggest that market operators (and people in general) tend to have particularly irrational behaviors in situations of economic distress.

It should be noted that the tendency to schematization is sometimes an obstacle for asset pricing researchers. As noted by Fama (1970), when testing market efficiency, many economists use models to calculate the theoretical long-term equilibrium returns on a stock, and assume that they are constant, which is unlikely to be true. This is because the equilibrium returns on a stock depend on various risk factors, and the company's exposure to these risk factors may change over time. The problem is to find the right variables that describe these risk factors, in order to track the premium that they add to the firm's stock returns. I will discuss later of what these factors may be, especially because in order to come to this particular view on financial markets there are several stepping-stones to consider. In the next sub-sections, I will talk about the most important asset pricing models, the theory behind them and the empirical evidence that supports them.

1.2 – Asset Pricing Models: History, Theory and Empirical Studies

1.2.1 – The Mean-Variance Approach and the Markowitz Model

Asset pricing is mainly concerned with practical applications; the ultimate aim (at least at first) is to find "the" efficient portfolio, with the optimal combination of risk and returns, ideal for the rational, risk averse investor.

The main measure of the total risk associated with a security is the standard deviation of its returns. The standard deviation is the square root of the variance, which, in turn, is the average squared deviation from the expected value of the returns, calculated historically. The expected return on a security is also generally calculated as the mean of the historical returns. Following these assumptions, the time series analysis of past returns came to be one of the main practices of econometricians involved with asset pricing (Bodie, Kane, Marcus, 2011). Generally, the main assumption on asset returns is that they have a normal distribution, which is symmetric and only needs two estimations (mean and standard deviation) for probabilities to be computed.

Unfortunately, the true distribution of returns is likely to be slightly different from normal, as samples often exhibit a certain level of asymmetry and higher probability of extreme results. The former effect is called skewness and is generally tilted towards the left (or negative) side, while the latter is called kurtosis. Nevertheless, to our purposes the assumption of normality seems to be a fair and effective approximation of reality.

The next step to build an efficient portfolio, always according to Bodie, Kane and Marcus, is to assess the investor's level of risk aversion. This is no easy task and I am not going to talk about the techniques associated to it. However, a good idea to control for risk when building a portfolio is to invest some of the wealth on riskless assets. What riskless assets are exactly and if any exists at all is a matter of debate, but they are typically associated with interbank loans and government bonds. These are securities with low returns and (theoretically) zero default risk and their returns can be thought of as the temporal cost of money.

The average historical returns of assets, their standard deviation and the risk-free rate of return (which I will call RFR for the remainder of this thesis) constitute the basis for the groundbreaking first asset pricing model, designed by Harry Max Markowitz.

The paper "Portfolio Selection" by Markowitz was published in 1952 and described the process of investment with a two-stage approach. The first stage involved the creation of expectations on future returns based on past experience and observation, while the second was about putting these expectations to work to build the portfolio. Markowitz starts from the hypothesis that investors intend to maximize the expected return from their investment, so given a discount rate *d*, invested wealth *X* and r_i as the expected return on a security *i*, with $i=1,\dots,N$ the investor maximizes the following function:

$$R = \sum_{t=1}^{\infty} \sum_{i=1}^{N} d_{it} r_{it} X = \sum_{i=1}^{N} R_i X_i \tag{3}$$

where R_i represents the total discounted returns on asset *i* and X_i is the wealth invested in it.

Normalizing the sum of all amounts invested in each security X_i to 1, the variables we need to find is the relative weights to be invested in each asset.

Of course, maximizing returns is not enough to find an efficient portfolio: there is a good chance that one security has the most returns, but investing all of the wealth on it would be a very risky endeavor. Markowitz suggests that, by the law of large numbers, there exists a portfolio that offers the best risk/return trade-off, thus allowing us maximize returns on the investment and minimize the standard deviation of these returns. Markowitz considers portfolios instead of single assets because diversification is needed to suppress all sources of risk that attain a single security; however, there is some level of risk (called systematic risk) that cannot be eliminated. Markowitz defines covariance as the degree of common variation between two or more securities:

$$\sigma_{ij} = E\left[(R_i - E[R_i])\left(R_j - E\left[R_j\right]\right)\right] \tag{4}$$

He also defines correlation as covariance relative to the product of the two securities' standard deviations:

$$\rho_{ij} = \frac{\sigma_{ij}}{\sigma_i \sigma_j} \tag{5}$$

So, for N securities in our portfolio, the variance of the returns on the investment will be:

$$\sigma_p = \sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j \sigma_{ij} \tag{6}$$

where w_i are the weights assigned to each security *i*.

Markowitz designs a plane in which we can represent all the different combinations of standard deviation and expected returns that we can obtain and postulates the existence of an "efficient frontier" that contains all the portfolios that have the maximum expected returns for each level of standard deviation.

Once the efficient frontier is found, we can introduce the risk-free asset, which can be found on the Y-axis of the Markowitz plane due to its standard deviation equal to zero. This will serve as the intercept of a set of lines that is tangent to the efficient frontier. Of these lines, the one that has the maximum slope, is called Capital Allocation Line and its point of tangency with the efficient frontier is the optimal risky portfolio, which is the portfolio that offers the best trade-off between standard deviation and expected returns. Investors will then choose the combination of risky and riskless assets (represented by a point on the CAL) that best suits their level of risk aversion. This process is called "separation property" and was acknowledged by James Tobin (1958).

According to Burmeister et al. (1994) The Mean-Variance-Covariance approach introduced by Markowitz has been object of long debate, leading to various opinions on the matter. While the MVC approach is still widely used among practitioners and the relationship between risk and return is widely recognized as the key point in asset pricing, the main criticism towards the model is due to its excessive reliance on the assumption of normality of returns (Bodie, Kane and Marcus, 2011). As I said before, this assumption, however "comfortable" is unlikely to be true, and therefore the standard deviation of past returns is probably not a reliable measure of risk. A more practical approach was needed and it came roughly ten years later with the Capital Asset Pricing Model.

1.2.2 – The Capital Asset Pricing Model

The CAPM was introduced independently by Sharpe (1964), Lintner (1965) and Mossin (1966), it was later expanded by Black (1972) and is therefore also known as Sharpe-Lintner-Black (SLB) Model. According to Bodie, Kane and Marcus, is a corner stone of modern finance, and arguably the model that inspired the most related studies, researches and tests. In order to understand this model, it is of paramount importance to keep in mind the assumptions upon which it is based:

- There are many investors, all equal in their wealth and all price-takers.
- Operators have the same holding period, which means their investments have the same time horizon.
- Only investment in publicly traded securities is allowed, but there is no restriction on short-selling and debt: everyone can lend and borrow money at the risk-free rate of interest.
- There are no taxes or accessory costs on transactions in the financial market.
- All investors are mean-variance optimizers, meaning that they all use the Markowitz model to select their portfolios
- Homogeneous Expectations: all investors have the same views on the evolutions of the market and they share the same information set.

The consequence of these assumptions is that investors choose the same risky portfolio and only change their exposure to it and to the risk-free asset according to their risk aversion (not all investors have the same risk aversion). Investors that are more risk averse will invest part of their wealth on the risky asset and lend the rest at the RFR for a secure profit; more risk-prone investors, on the other hand, will borrow at the RFR and invest all the money on the risky assets. Eventually loaned and borrowed money sums up to zero. This in turn means that the optimal risky portfolio is nothing but the market portfolio, which contains all the assets in the market, weighted by their value (or, alternatively, equally weighted). The Capital Allocation Line is now called Capital Market Line and all investors will choose their optimal portfolio along this line.

These assumptions leave very little to chance: the returns on the market portfolio can be calculated mathematically as a combination of the variance of the market returns and the average degree of risk aversion in the market, plus the RFR:

$$R_m = RFR + \sigma_m^2 \bar{A} \tag{7}$$

From now on, I will call *R_i-RFR* "excess returns" on security *i*: it is simply the compensation that the asset offers the investor for taking on more risk.

It follows that, according to their specific risk, every portfolio will lie on the CML, and the excess returns on every asset and portfolio will have a linear relationship with the market portfolio's excess returns:

$$R_i = RFR + \beta_i (R_m - RFR) \tag{8}$$

The coefficient of this linear relation is called β; it is one of the most important concepts in finance and is formally defined as:

$$\beta_i = \frac{Cov(R_i, R_m)}{\sigma_m^2} \tag{9}$$

According to this framework, the passive strategy (holding the market portfolio) is always efficient: stock picking and analysis are superfluous, as the market is completely efficient and there is no more information to exploit. Moreover, all the information about a security's risk is accounted for in its β, so there is no need to estimate enormous covariance matrices like in Markowitz: all we need is this simple coefficient that tells us how the stock reacts to market fluctuations.

The CAPM is just as simple as the β coefficient. According to Burmeister et al. (1994), it is of course a model with excessively restrictive assumptions, but for many years it was the most quick and efficient way to analyze performances in the stock markets and it was actually pretty close to reality, too. According to Brealey, Myers and Allen, the β coefficient is still the main measure of market risk, although it is often criticized for its volatility and the fact that is doesn't include new information, as past prices are not very good predictors of future prices. The β coefficient has the following interpretations:

- A negative β means that the security tends to move in opposite directions with respect to the market. A negative β is typically associated with funds including strong short positions.
- A β of 0 means that the security is not influenced by the movement market at all. It is typical of fixed income securities.
- A β between 0 and 1 means that the security moves in the same direction as the market, but in a less sharp fashion. It is generally associated with stocks of utilities companies, which sell food and essential goods.
- A β of 1 means that the security follows exactly the evolutions of the market. It is typical of passive funds that track the benchmark.
- A β bigger than 1 means that the security tends to accentuate the movements of the market. It is generally associated with stocks of companies that sell technology and luxury goods.

While the CAPM offers an interesting theoretical framework, its testability in real life proved to be a challenge.

The first empirical test on the CAPM was conducted by Black, Jensen and Scholes (1972), based on the version of the CAPM proposed by Black (1972):

$$R_{it} - RFR_t = \alpha_{it} + \beta_{it}(R_{mt} - RFR_t) + \epsilon_{it} \tag{10}$$

Their approach was to divide the stocks into portfolios with large spreads in their βs and to conduct a time series regressions of the returns of these portfolios against the market returns to obtain the portfolio βs. Since this division in portfolios could cause biases in the coefficient's estimations, they used the previous period's β as instrumental variable to build the portfolios and therefore correct possible systematic errors. The result of Black, Jensen and Scholes were promising: they proved the existence and positive influence of the β coefficient in stock returns, but of course, the assumptions of the model were rejected. Instead, Black, Jensen and Scholes found that each stock had a systematic element that that augmented or diminished returns according to factors that the β failed to capture. This element was called α and it served as the intercept in the standard model.

Fama and MacBeth (1973) came to similar conclusions using a different approach, which is worth analyzing more deeply, because I will use it to conduct my tests later. They build a second regression equation to test the model, along with the traditional one, which is used to estimate the coefficients. In this equation they use another measure (s_i) that stands for systematic risk that is not related to β_i for asset *i*. Their "test" equation is the following (the tilde indicates random variables):

$$\tilde{R}_{it} = \tilde{\gamma}_{0t} + \tilde{\gamma}_{1t}\beta_{it} + \tilde{\gamma}_{2t}\beta_{it}^2 + \tilde{\gamma}_{3t}s_{it} + \tilde{\epsilon}_{it} \tag{11}$$

The conditions to accept the CAPM are:

- Linearity: $E[\tilde{\gamma}_{2t}] = 0$.

- No systematic effect of non-β risk: $E[\tilde{\gamma}_{3t}] = 0$.
- Positive trade-off between risk and return: $E[\tilde{\gamma}_{1t}] = E[\tilde{R}_{mt} - \alpha_{it}] > 0$.
- The intercept coincides with the RFR (Sharpe-Lintner Hypothesis): $E[\tilde{\gamma}_{0t}] = RFR$.
- $\tilde{\gamma}_{2t}, \tilde{\gamma}_{3t}, \tilde{\gamma}_{1t} - E[\tilde{R}_{mt} - \alpha_{it}], \tilde{\gamma}_{0t} - RFR$ and $\tilde{\epsilon}_{it}$ are all "fair games", which means that although exceptions to the hypotheses may be observed ex-post, it is still convenient for the investor to act like the CAPM is valid.

The fundamental approach of Fama and MacBeth relies on this two-step regression: one to estimate the parameters (the "standard" model), the other to estimate their effects and to test their efficiency (the "test" regression). It should be noted that they use a similar method to Black, Jensen and Scholes to assign the βs to stocks. They estimate the βs based on data from the previous 7 years, use these estimations to rank the stocks and then divide them into 20 portfolios based on this ranking (they call this "Formation Period"). Then, they re-estimate the β for the portfolio using the standard model and data from the following 5 years and assign this estimation to all the stocks in the portfolio ("Initial Testing Period"). Eventually, they run the test regression using data from the following 4 years ("Testing Period"). They repeat this process for every year in the sample.

Fama and MacBeth use a cross-sectional approach, as opposed to the Black-Jensen-Scholes time-series approach. This means that they run the regressions each month for the various stocks, and then average the monthly estimations to obtain the results for each 5-year period.

The results they obtain seem to suggest that the first three hypotheses hold, or at least they are "fair games". This means that even if the model is not true, the behavior of the market indicates that investors tend to behave according to it. Serial autocorrelations are also low, meaning that the behavior of the market cannot be predicted by looking at previous prices. On the other hand, the S-L Hypothesis is rejected, so there is still a part of stock returns that remains unexplained because the intercept of the test regression does not coincide with the RFR.

Regardless of these flattering early results, Fama himself (2014) points out how contradictions to the CAPM would soon come out, as many researchers started finding more and more anomalies. Moreover, Roll (1977) moves a critique on the CAPM, stating the impossibility of a reliable test on the model. His critique comes from the fact that a true market portfolio is unobservable, and even if it were observable, a test of the CAPM would be a tautology, since any portfolio would be mean-variance efficient and the model would always be satisfied.

The hypothesis that stock returns were only affected by one catch-all systematic factor started to seem a bit restrictive: more and more new factors were tested to see if they had any influence on stock prices, even temporarily. This tendency gave way to the Arbitrage Pricing Theory.

1.2.3 – Arbitrage Pricing Theory and Factor Models

According to Bodie, Kane and Marcus, arbitrage is the act of trading securities according to temporary price inefficiencies in the market, leading to riskless profits. Namely, arbitrage is realized by selling an asset and buying another that has the same returns and, due to market failure, lower price. One of the implications of the CAPM is the perfect efficiency of the market, so opportunities for arbitrage should not exist, and this is the starting assumption of the Arbitrage Pricing Theory. During the 70s and 80s researchers in the financial markets started finding more and more anomalies to the SLB model and the opinion that they could be exploited to earn abnormal returns. These anomalies depend on various risk factor and there is no unique approach to this theory: during the last forty years, this approach kept re-inventing itself, thanks to the contribution of numerous authors, each with their personal take on the matter.

The APT was introduced by Stephen Ross in 1977, when he criticized the excessive restrictions imposed by Sharpe and Lintner, especially quadratic preferences and normal distribution of returns. He proposed that these irrealistic assumptions could be weakened, and an asset pricing model based on the concept of arbitrage could be built. The only assumptions that the author made were:

- Security-specific risk is diversifiable.
- Markets do not allow for persisting arbitrage, so if a security has more returns than another does, it must depend on its exposure to certain risk factors.
- These risk factors and their association with returns can be explained by a multi-factor model.

Therefore, Ross makes no assumption over the distribution of returns and the shape of investor's preferences. This is what makes this theory so versatile: Ross himself recognized this particular quality and predicted that his work could be the beginning of an extensive new line of literature. In fact, a great amount of studies on multi-factor models based on the APT started to come out.

A typical multi-factor model is not very different from the CAPM; the only difference is that there are various βs, one for each of the factors against which stock returns are tested. These factors may be macroeconomic or firm-specific: anything that, according to the author, has an influence on the returns on an asset. A typical linear multi-factor model looks roughly like the following equation:

$$R_{it} = \alpha_{it} + \beta_{1i}F_{1t} + \beta_{2i}F_{2t} + \beta_{3i}F_{3t} + \cdots + \epsilon_{it} \qquad (12)$$

The Arbitrage Pricing Theory has been tested by Roll and Ross (1980) and Cho, Elton and Gruber (1984) among others: both papers state that there is strong evidence pointing toward the existence of factors at influencing the market that are not accounted for in the CAPM. However, Dhrymes, Friend and Gultekin (1984) found some limitations in the techniques used to test the theory and their results indicate that the predicted number of factors at play depends on the size of the portfolio.

On the other hand, Shanken (1982) altogether doubts the testability of the theory, questioning the effectiveness of the techniques used to evaluate it. His criticism derives from the fact that Ross' theory does not imply a linear relationship between stock returns and risk factors, even if the number of securities in the market tends to infinite. Moreover, he states that the acceptance of such a theory has the impossible implication that all securities have the same expected returns and that new sets of returns can be manipulated arbitrarily and associated to any random variable to serve as factor models. According to Shanken, this makes the APT by itself inadequate to identify the drivers of stock returns. He even goes as far as to assert that the APT "is merely concerned with statistical correlations and is blind to aggregate economic considerations". He concludes that an equilibrium factor model based on the APT ultimately has the same problem as the CAPM: the absence of a true observable market portfolio, as noted by Roll (1977).

This point is reiterated by Black (1993), who warns financial economists from "data-snooping", which is the continuous scanning of returns databases to find always new risk factors to explain returns. Black states that many of these relationships may be temporary and merely due to chance.

It should be noted that advocates of the APT were quick to respond to Shanken's doubt: Dybvig and Ross refute his conclusions in their 1985 paper (eloquently titled "Yes, The APT Is Testable"). They say that there is no bias in APT tests if not towards rejection, and Shanken's examples of inadequate securities sets are misleading. They also reject the extension of the Roll critique to the APT, as factor models do not require a true market portfolio, and state that linearity is implied by the absence of arbitrage, and not by the infinite number of securities. The debate is particularly intense, but some criticism is to be expected on such a popular theory.

However, according to Cochrane (1999b), the popularity of factor models remains strong nowadays, and the majority of studies were able to find strong relations between stock returns and risk factors. Many of these tests involved macroeconomic drivers, as the stock market has been known to be correlated with the state of the economy in general, simply because in case of economic downturn basically every kind of investment measure goes down.

According to Burmeister et al. (1994) these macroeconomic risk factors may involve changes in investor confidence, interest rates, inflation, fluctuations in business activity and, of course, a market index is considered a relevant risk factor as well (just not the only one). Soon after elaborating the theory, Chen, Roll and Ross (1986) had also proposed the proxies same macro factors, proving their relevance. Their also tested for consumption indexes and oil prices, but were unsuccessful. Cochrane (1999a) noted how many attempts to find relationships between stock returns and various measures of consumptions were unsuccessful.

It is clear that when relevant factors are found, the model can be applied by selecting stocks according to their exposure to that factor. For example, if business activity is expected to increase, an investor can try to exploit this fluctuation by building a portfolio of stocks with high sensitivity to business cycles. These are the potential macroeconomic risk factors that, according to Burmeister et al., are most likely to influence the behavior of stock returns and serve as theoretical explanations for the effects of the proxies found by Chen, Roll and Ross:

- **Confidence Risk**: The risk that in the future there will be an unanticipated change in the willingness of investors to undertake risky endeavors. It is measured as the difference between the returns on relatively risky corporate bonds and government bonds with the same maturity. Generally every stock has some exposure to confidence risk, with smaller stocks typically being more exposed.
- **Time Horizon Risk**: The risk of unexpected changes in investor's desired time to receive their payoff. It is generally measured by the difference between returns on 20-year and one-month treasury bills. This spread defines the Term Structure of interest rates and its increase signals growing impatience among investors, who require more returns to hold a longer investment. Stocks tend to have a positive exposure to time horizon risk and growth stocks (which rarely pay dividends) are generally more exposed than income stocks.
- **Inflation Risk**: The risk of unexpected change in inflation. It is measured as the difference between the actual inflation rate and its expectation from a month before. Stocks generally have a negative exposure to inflation risk and this exposure tends to be greater for industries that sell luxury goods, as opposed to companies that sell necessities.
- **Business Cycle Risk**: The risk of unanticipated change in business activity. It is calculated as the difference in value of specific indexes that measure real business activity. Of course, stocks are positively exposed to such risk, more so for companies whose activity depends more on the state of the business cycle (for example, retail stores), while others tend to have more stable earnings and hence less exposure (for example, utility companies).
- **Market Timing Risk**: This is a measure of the variation in market returns that are not explained by the previous variables. Its coefficient can be compared to the CAPM's β, just like the APT can be thought of as a generalization of the SLB model.

This expanded CAPM proposed by Chen, Roll and Ross and by Burmeister et al. is proven to do a better job at explaining returns than the "regular" CAPM, but does not explain all the returns. This is why between the 80s and the 90s applications of the Arbitrage Pricing Theory started to contemplate both systematic and firm-specific risk. This kind of approach merged asset pricing and analysis of balance sheet fundamentals and gave way to the discovery of a number of market anomalies, eventually leading to what is arguably the most popular factor model of all times (Cochrane, 1999b): the Fama-French 3-Factor Model.

In the next sections, I will analyze the theory behind it and its applications and I will try to test the model in the stock markets of the European Union, trying to find out if it has some power in explaining stock returns in the last 20 years.

Section 2

2.1 – The Fama-French 3-Factor Model

In their seminal 1992 paper, Eugene Fama and Kenneth French proposed a new approach to asset pricing and to the arbitrage pricing theory. Their idea was to use the CAPM's β in addition to two variables that are specific to each firm in order to proxy for those risk factors that are not linked to the market as a whole, but attain a particular company and cause returns to deviate significantly from the values that the CAPM predicts. These two additional factors are the company's market capitalization (which they refer to as "size") and the ratio of book value of common equity over market capitalization (also called book-to-market).

Ever since their research came to prominence, industrial applications and empirical research on the matter has been particularly active, and many tests and proposed expansions to the model have been elaborated. I propose to test the model on the European stock market to see if some of the patterns discovered by Fama and French can still be found on this side of the world. First of all, I intend to examine the premises of this model.

The 1992 paper "The Cross-Section of Expected Stock Returns" is significantly different from 1993's "Common Risk Factor in the Returns on Stocks and Bonds": I will focus on the former and adopt its methodology and approach for my own analysis. The risk factors that Fama and French analyze and the relative proxies had already been studied in previous researches, and FF tested them together to see which could have some joint explanatory power and which were redundant or too heavily correlated with each other.

In fact, Banz (1981) was the first to acknowledge the explanatory power of size on stock returns in his paper "The Relationship between Return and Market Value of Common Stocks". Namely, he found that small-size stocks had unusually high returns compared with those one would expect by applying the regular CAPM. He did this by running a linear regression that included both the CAPM's β and the company's market capitalization compared to that of the whole market. His methodology is also similar to the one I am using, both deriving from Fama, MacBeth (1973). The so-called "size effect" was later acknowledged by various other researchers like Reinganum (1981), while Brown, Kleidon & Marsh (1983) found that the size effect may actually be reversed for lasting periods of time. This view was expanded by Bhardwaj and Brooks (1993), who find that relationships among β, size and returns vary according to economic conditions. Typically, the size effect is related to the risky nature of small firms: their securities are generally less liquid and more prone to systematic risk, and therefore investors have to be rewarded with higher returns. This is what studies by Amihud and Mendelson (1986) and Liu (2006) postulate, but there is also a line of thought that asserts that these abnormal returns are due to estimation risk and reporting inconsistencies from smaller firms (Banz, 1981).

Basu (1977) found another factor that defied the precision of the CAPM: he used the price/earnings ratio as a proxy for it, and it has been since known as the "value effect". According to Basu, this negative relationship between price/earnings ratio and returns was due to the market overreacting to news of a company's earnings:

when they are low, investors oversell their shares, while the opposite happens when earnings are high. This way, low-earnings companies tend to generate higher returns when the situation goes back to normal and their price increases quickly, while high-earnings stocks tend to perform badly.

Stattman (1983) was the first to call this anomaly "value effect", but he used the book-to-market ratio as a proxy for it instead of price/earnings. According to Petkova and Zhang (2003) this anomaly is not due to market overreaction, but rather it is caused by the fact that high book-to-market stocks tend to underperform in case of macroeconomic downturn and thus are rewarded with higher returns to compensate for this enhanced systematic risk. Chan and Chen (1991) seem to agree with this postulate, but do not rule out the possibility that book-to-market and the value factor are really due to the market's irrationality about a company's prospects. Chan, Hamao and Lakonishok (1991) found that the book-to-market helps explaining the cross-section of Japanese stock returns as well.

Financial leverage was also proved to be linked with the risk factors that drive stock returns by Bhandari (1988), who tested a regressive model which included a debt/equity ratio, size and β. Fama and French tested the role of leverage as well, but used two asset/equity ratios instead. The first ratio is book assets over book equity (described as a measure of book leverage), while the second one is book assets over market equity (market leverage). The interesting thing about financial leverage being a determinant for average returns is that while it is indeed a measure of risk, its role should be already captured by the β.

Until the Fama-French paper was published in 1992, the aforementioned variables were the most used to describe anomalies in the CAPM. They all seemed to be good proxies to explain the part of expected returns that remained unexplained under the Sharpe-Lintner-Black assumptions. However, as Fama and French pointed out these variables were composed by roughly the same fundamentals (price, shares outstanding, book equity and total assets) and some of them might have been redundant.

The conclusions of the Fama-French research is that size and book-to-market are the best proxies for these effects and the roles of the other variables seem to be absorbed by these two catch-all fundamentals. The CAPM's β is still included in the model, but is now almost completely dismissed, as the division in portfolios helps uncover the fact that β's role is almost nullified when its correlation with size is accounted for by dividing the sample in size portfolios.

They reach these conclusions in the two separate papers that I mentioned before. In the 1992 version (the one on which I am basing my thesis) they use a cross-sectional approach and build portfolios by ranking stocks based on their size, β and book-to-market. Then they analyze the performance of the portfolios to find some common patterns before running the cross-sectional regressions that legitimate their initial claims. As I said, this is the methodology I used, so I will go deeper in its description in the next sections. The results they obtained are the following:

- The CAPM β is positively correlated with average returns, but this correlation seems to be a consequence of its strong negative correlation with size. Once this correlation is accounted for, the role of the β seems to vanish almost completely, leaving little or no hope for the revival of the CAPM.
- As expected, size has a robust negative correlation with average returns, which also seems to absorb the role of the β.
- Book-to-market has a significant positive relationship with average monthly returns and its effect seems to be even stronger than the size effect.
- Market and book leverage both have a strong effect, but it is almost exactly opposite: positive for market leverage, negative for book leverage. Since ln(A/BE)-ln(A/ME)=ln(BE/ME), Fama and French conclude that their effect is absorbed by book-to-market, which may account for Chan and Chen's relative distress factor as well as for financial leverage.
- The earnings/price ratio shows a positive relationship with average returns when earnings are positive, while firms with negative earnings tend to have higher returns. However, the effect of earnings/price becomes irrelevant when size and book-to-market are added to the regression.

Due to these results, Fama and French introduced their 3-Factor model, which they developed further later that year.

The second Fama-French paper (published in 1993) is perhaps the most recognized and tested one, as it uses a time-series approach, which is similar to the one used by Black, Jensen and Scholes (1972) for the CAPM. Here, the risk factors are tested by building two portfolios based on size and then dividing each in three portfolios based on ranked book-to-market. The average returns of the three "big" (large size) portfolios are then subtracted from those of the "small" portfolio to mimic the size effect, which they call SMB (Small-Minus-Big). The same goes for the value effect, which is proxied by subtracting the average returns of the two low-book-to-market portfolios from those of the two high-book-to-market portfolios; they call this HML (High-Minus-Low). The regression equation they tested is:

$$R_{it} = a_t + b_{1t}(R_{mt} - RFR_t) + b_{2t}SMB_{it} + b_{3t}HML_{it} + \epsilon_{it} \tag{13}$$

This is done to eliminate any correlation that may exist between size and book-to-market to analyze the two effects separately. The conclusions of the 1992 paper are confirmed: SMB and HML seem to have a relevant effect on average returns, at least in the period 1963-1991. The paper also contains a similar test for bonds, using maturity (TERM) and default risk (DEF) as risk factors, but the results were not satisfying.

The Fama-French model shaped the way many approached asset pricing for years and is a cornerstone for the discipline as Cochrane (1999b) spectifies. However, as many models before it, it may not stand the test of time and, more importantly, the test of "place". The European Union is an extremely peculiar and diverse entity and as Modigliani, Pogue and Solnik found in their 1973 test of the CAPM on the European market, the results are very different across countries and the test suffered by the lack of data. This lack of data kept everyone from testing the CAPM before 1973, when the model had been thoroughly tested and expanded throughout the previous seven years in the U.S. market. Moreover, as Foye, Mramor and Pahor (2013) found, there are still many inefficiencies in the European market, with many Eastern European countries failing to reach the weak form of market efficiency defined by Fama (1970). We will get back on the works of Foye, Mramor and Pahor in the next chapters.

The issue of data availability has since improved due to the disclosure requirements for stock market quoted companies and the advancement of technology, but the level is not quite that of the United States yet. Europe was also struck by two devastating financial crises in the last 10 years and still has a hard time recovering: the consequence is a widespread mistrust in the financial markets, which is exacerbated by continuous political tension among EU countries, as reported by Bastasin (2015).

Anyway, my test of the model should be considered anything but conclusive: it is my humble contribution to the discussion on the validity of the Fama-French model and possibly to see if this leads to new evidence on the efficiency and integration of the European stock markets.

2.2 – Empirical Test

2.2.1 – Data Description

The data I used is relative to firms from the European Union, for a total of 3602 companies, recorded monthly. The data is relative to the period June 1996 - July 2015 due to the scarce availability of balance sheet data for previous periods. These are the countries that are part of the analysis, with the number of companies between parentheses:

- Austria (54)
- Belgium (77)
- Bulgaria (30)
- Croatia (25)
- Cyprus (20)
- Czech Rep. (8)
- Denmark (102)
- Estonia (11)
- Finland (107)
- France (426)
- Germany (454)
- Greece (164)
- Hungary (8)
- Ireland (51)
- Italy (146)
- Latvia (21)
- Lithuania (19)
- Luxembourg (17)
- Malta (10)
- Netherlands (74)
- Norway (100)
- Poland (146)

- Portugal (39)
- Romania (69)
- Slovakia (5)
- Slovenia (28)
- Spain (61)
- Sweden (305)
- Switzerland (174)
- UK (851)

The requirements for a stock to be selected and used in the study are that the instrument has to be the primary common stock for the company; it has to be active and must have data at least for the previous five years. As in the original paper, I excluded financial companies because they usually have a level of financial leverage that is higher than industrial firms and doesn't have the same meaning. All the data I gathered is from Bloomberg.

I am testing about the possible influence that some variables have on stock returns, so I have to make sure that these variables are known when the returns they are tested against come to fruition. This is why, like in the original paper, I coupled the returns with balance sheet data from six months prior and market data from the previous month. For example, the variables that contain balance sheet data relative to December of year *t-1* are associated with market variables relative to June of year *t* and used to explain the returns of July *t*. For variables that are composed by both balance sheet and market values (BE/ME, A/ME, A/BE, E/P), I obviously used values for the same month (six months prior to the returns).

Alford, Jones and Zmijewski (1993) show how balance sheet information is often untimely and inaccurate, citing European countries like Germany, Switzerland, Belgium, Denmark and Sweden, which constitute a relevant part of my dataset. On the other hand, the use of the Bloomberg Terminal makes sure that the value recorded is always the latest published and this should also make up for the inconsistencies that may arise for those companies that do not end their fiscal year in December. However, according to Fama and French, this makes little difference anyway.

The data I gathered for each firm are the following (descriptions are as provided by Bloomberg):

- **Current Shares Outstanding**: Total number of shares in circulation. This data may have been obtained from annual, semi-annual, and quarterly reports, Edgar filings, press releases, or stock exchanges from May 2000 to present. Prior to May 2000, daily shares outstanding data is populated from Interim and Annual Reports only for all single-share class companies and does not return data for Multiple Share companies. The value is in millions.

- **Last Price**: Last price of the security provided by the exchange. For securities that trade Monday through Friday, this field will be populated only if such information has been provided by the exchange in the past 30 trading days. The value is in Euros.
- **Dividend Yield**: The most recently announced net dividend, annualized based on the Dividend Frequency, then divided by the current market price. If the security is paying an interim/final dividend, then the indicated yield is calculated by adding the net amount from the most recently announced interim and the most recently announced final, and dividing the sum by the current market price. Abnormal Dividends are not included in this yield calculation. The value is in percentage points.
- **EBITDA**: Earnings Before Interests, Taxes, Depreciation and Amortization. Calculated by adding Depreciation and Amortization back to the Operating Income, it is an indicator of the profitability of the company. The value is in million Euros.
- **Total Common Equity**: The amount that all common shareholders have invested in a company according to the balance sheet. The value is in million Euros.
- **Total Assets**: The total of all short and long-term assets as reported on the Balance Sheet. The value is in million Euros.

In addition to these, I collected the time series of the 1-month LIBOR (London InterBank Offered Rate) and of the 1-month EURIBOR (EURo InterBank Offered Rate) and used them as risk-free rates.

2.2.2 – Methodology

Variable Construction

The variables I used for the study are constructed using the data I gathered in the following way:

- **Risk-Free Rate**: The RFR I used is a combination of the EURIBOR for observations from 1999 on and the LIBOR for the pre-1999 period.
- **Stock Returns**: Here I used data about price and dividend yield in this way:

$$R_{it} = [\ln(Price_{it}) - \ln(Price_{it-1}) + Dividend\ Yield_{it-1}] \times 100 \tag{14}$$

Thus, I obtained the historical returns for each stock in percentage points. It should be noted that the dividend yield was not always available when prices were; in that case, I only used price change as returns.

- **Market Capitalization (ln(ME))**: The total market value of a company's publicly traded shares. Here the straightforward procedure is:

$$\ln(ME_{it}) = \ln(Price_{it} \times Shares\ Outstanding_{it}) \tag{15}$$

- **Book-To-Market (ln(BE/ME))**: The ratio of the book value of common equity over the market capitalization of a company. I computed it like this:

$$\ln(BE/ME_{it}) = \ln[Common\ Equity_{it} \div (Price_{it} \times Shares\ Outstanding_{it})] \tag{16}$$

- **Market Leverage (ln(A/ME))**: The ratio of the book value of total assets over market capitalization. It is a measure of market leverage and according to Bhandari (1988), it should have a positive effect on returns. It is constructed this way:

$$\ln(A/ME_{it}) = \ln[Total\ Assets_{it} \div (Price_{it} \times Shares\ Outstanding_{it})] \tag{17}$$

- **Book Leverage (ln(A/BE))**: The ratio of the book value of total assets over book equity. It is a measure of book leverage and according to Fama and French (1992), it should have a negative effect on returns. It is constructed this way:

$$\ln(A/BE_{it}) = \ln(Total\ Assets_{it} \div Common\ Equity_{it}) \tag{18}$$

- **Earnings/Price Dummy (E/P Dummy)**: A variable that takes value 1 when the EBITDA for the company in that period is negative and value 0 when the EBITDA is positive. As in the original paper, I use this variable because according to Jaffe, Keim and Westerfield (1989), firms with negative earnings tend to have higher returns, and I intend to test this hypothesis as well.
- **Earnings/Price (E/P(+))**: The ratio of EBITDA over the price of the stock and, according to Ball (1978), a comprehensive proxy that should capture many of the drivers of stock returns that other variables don't manage to capture. It takes value 0 when earnings are negative. I built it like this:

$$E/P\ (+) = \frac{EBITDA}{Price} \tag{19}$$

- **Market Excess Returns**: As in Fama, French (1992), the market returns of my sample are calculated as the average returns of all 3602 stocks at each time, weighted by their market capitalization. Then I subtract the risk-free rate in order to obtain a proxy for the movements of the European stock market alone. Formally, the procedure to obtain them is:

$$Total\ Market\ Cap_t = \sum_{j=1}^{N} Market\ Cap_{jt} \quad (20)$$

$$R_{mt} = \sum_{i=1}^{N} \frac{Market\ Cap_{it}}{Total\ Market\ Cap_t} \times R_{it} \quad (21)$$

with N=3602.

β Estimation

As in Fama, MacBeth (1973), I estimated each stock's β as the risk of the asset *i* in the portfolio *m* (the market portfolio), measured relative to the variance of the returns of the market portfolio:

$$\beta_i = \frac{\text{cov}(R_i\ , R_m)}{\sigma^2(R_m)} \quad (22)$$

I decided to use the Fama-French approach, which is more comprehensive and consists in including in the beta the relationship of the stock returns with both the same-month and the previous-month market returns. The purpose of this is to obtain an estimate that is adjusted for the possible lag of information:

$$\beta_i = \frac{\text{cov}\left(R_i\ , R_{m,-1}\right)}{\sigma^2\left(R_{m,-1}\right)} + \frac{\text{cov}(R_i\ , R_m)}{\sigma^2(R_m)} = \beta_{i,-1} + \beta_{1,0} \quad (23)$$

According to Fowler and Rorke, if the market returns have a relevant auto-correlation, it is possible for these β estimations to be biased. In my case, market returns for the 1996-2015 period have a first-order autocorrelation of 0.3552, so I used a correction similar to the one proposed by Scholes and Williams (1977)[1]:

$$\beta_i(corrected) = \frac{\beta_{i,0} + \beta_{1,-1}}{1 + \rho_1(R_m)} \quad (23)$$

where $\varrho_1(R_m)$ is the first order autocorrelation of the market returns.

The single-stock β is re-calculated every year in June, and is used merely for ranking purposes. In fact, the βs I will use in the regressions are computed for each portfolio (with the same method), and every stock in the portfolio at that time is assigned the β of the portfolio for the whole year. I used this approach because as stated by Blume (1973) and demonstrated by Ang, Liu and Schwarz (2010), estimations of factor loadings are more precise for portfolios, as this helps reducing anomalies in the errors. The reasoning behind this is simple: the

[1] In the original paper, Scholes and Williams used: $\beta_i = \left(\beta_{i,0} + \beta_{i,-1} + \beta_{i+1}\right) / \left(1 + 2 \times \rho_1(R_m)\right)$

factor loadings of a stock contain all the idiosyncratic errors linked to that stock, while the factor loadings of a portfolio will merely contain the weighted average of the errors attached to each stock. The larger the portfolio, the smaller the errors.

The pre-ranking βs are calculated each year using return data up to 5, 4, 3 or 2 years prior, depending on data availability, while post-ranking βs for portfolios are computed using the whole sample. Of course, stocks change their post-ranking β by changing the portfolio to which they belong.

Portfolio Construction

Every year in June, the stocks were ranked by their current-period market capitalization. Subsequently, I calculated the 10 percentiles of ln(ME) and divided the stocks in 10 size groups. As this procedure was repeated each year, I eventually obtained the 10 size portfolios, with new securities each year, representing 10 segments of size.

Then I did the same for each of the 10 size portfolios that I had obtained, this time using the pre-ranking βs of the stocks to rank them. Again, I calculated the percentiles and divided them into 10 groups. At the end, I obtained 100 size/pre-ranking β portfolios, with stocks moving across them each year according to their varying ln(ME) and β.

Once I obtained these portfolios, I computed their returns as the equally-weighted average of the returns of every stock in the portfolio at a given time. I did the same for other variables as well, so I could check if the average values of these variables across portfolios show any regularities.

As in the original paper, I formed the portfolios based on the following variables, using the same procedure:

- 10 Size Portfolios (based on market capitalization)
- 10 β Portfolios (based on pre-ranking β)
- 100 Size/β Portfolios (based on market capitalization and then pre-ranking β)
- 10 Book-To-Market Portfolios (based on book-to-market)
- 10 E/P Portfolios (based on earnings/price)
- 100 Size/Book-To-Market Portfolios (based on size and then book-to-market)

Fama-MacBeth Regressions

I proceeded by testing the Fama-MacBeth linear regression models, in which the dependent variable (stock returns) is regressed against several sets of explanatory variables. The main characteristic of this method is its cross-sectional approach: each stock's returns at a given time are regressed against the independent variables relative to the same period, and then the results are averaged across periods. This way, each regression has a higher number of observations (N=3602) than it would have in a time series model (T=228), where each stock's time series of returns is regressed against the time series of the explanatory variables. I then proceeded to compute the t-Statistics for each intercept and slope that I estimated in order to test if they are significantly different from zero. The t-Statistics for a generic parameter b_i is:

$$t = \frac{b_i}{s.e.(b_i)} \tag{24}$$

where $s.e.(b_i)$ is the standard error of the estimated parameter, which is the standard deviation of the estimates over the square root of the number of observations:

$$s.e.(b_i) = \frac{\hat{\sigma}(b_i)}{\sqrt{T}} = \sqrt{\frac{\hat{\sigma}^2(b_i)}{T}} \tag{25}$$

I ran the following regressions:

1. $R_{it} = a_t + b_{1t}\beta_{it} + \epsilon_{it}$
2. $R_{it} = a_t + b_{1t}\ln(ME_{it}) + \epsilon_{it}$
3. $R_{it} = a_t + b_{1t}\beta_{it} + b_{2t}\ln(ME_{it}) + \epsilon_{it}$
4. $R_{it} = a_t + b_{1t}\ln(BE/ME_{it}) + \epsilon_{it}$
5. $R_{it} = a_t + b_{1t}\beta_{it} + b_{2t}\ln(ME_{it}) + b_{3t}\ln(BE/ME_{it}) + \epsilon_{it}$
6. $R_{it} = a_t + b_{1t}\ln(A/ME_{it}) + b_{2t}\ln(A/BE_{it}) + \epsilon_{it}$
7. $R_{it} = a_t + b_{1t}E/PDummy_{it} + b_{2t}E/P(+)_{it} + \epsilon_{it}$
8. $R_{it} = a_t + b_{1t}\ln(ME_{it}) + b_{2t}\ln(BE/ME_{it}) + \epsilon_{it}$
9. $R_{it} = a_t + b_{1t}\ln(ME_{it}) + b_{2t}\ln(A/ME_{it}) + b_{3t}\ln(A/BE_{it}) + \epsilon_{it}$
10. $R_{it} = a_t + b_{1t}\ln(ME_{it}) + b_{2t}E/PDummy_{it} + b_{3t}E/P(+)_{it} + \epsilon_{it}$
11. $R_{it} = a_t + b_{1t}\ln(ME_{it}) + b_{2t}\ln(BE/ME_{it}) + b_{3t}E/PDummy_{it} + b_{4t}E/P(+)_{it} + \epsilon_{it}$
12. $R_{it} = a_t + b_{1t}\ln(ME_{it}) + b_{2t}\ln(A/ME_{it}) + b_{3t}\ln(A/BE_{it}) + b_{4t}E/PDummy_{it} + b_{5t}E/P(+)_{it} + \epsilon_{it}$

I also ran regressions 5 and 8 for two subperiods: the pre-Crisis subperiod (from July 1996 to June 2007) and the post-Crisis subperiod (from July 2007 to June 2015).

2.3.2 – Results Summary

In Table 1 we can see the main characteristics of the 100 size/β portfolio, re-built each year according to the size and pre-ranking β percentiles. The sample is composed by 3602 stocks for the 1996-2015 period.

In Panel A we can see the average portfolio returns across the whole period. The first pattern that can be seen is that low-β portfolios tend to actually earn more returns than high-β portfolios, and the portfolios with the highest average βs actually have negative returns (except for the portfolios with the largest size). This pattern is not exactly precise, but from the data it is fairly clear that the bottom half of the β portfolios tends to have higher returns. This tendency is a reversal of the usual prediction, but it makes sense if we think about it: low-β stocks notoriously perform better in situations of economic distress, so it is no surprise that they were more popular in Europe during the last 20 years and performed better than the high-β stocks. Another tendency, which is even less pronounced but nonetheless clear, is that largest stocks tend to perform slightly better. This may again depend on the popularity of "conservative" stocks.

In Panel B we can see the post-ranking βs of the portfolios and they substantially confirm the results of Fama and French that pre-ranking βs predict the ranking of the portfolio βs with significant precision. The portfolio βs are seen to be somewhat positively correlated with size, but the tendency is fairly weak and we can see how βs go down for the two largest-size portfolios.

In Panel C we can see the average size (measured as ln(ME)) of the portfolios and it is fairly clear that there is no pattern in the relationship between size and post-ranking β, so in this case my research diverges from the Fama-French results.

In Table 2 I reported additional statistics of the size and β portfolios, like their average book-to-market, market leverage, book leverage, E/P Dummy, E/P and the average number of stocks in the portfolio.

Panel A includes the characteristics of the size portfolios. Returns show a weak increasing relationship with size: Small-ME has 1.6% monthly returns in average, while Large-ME has 2.7%. The portfolio with the lowest returns is ME-4 with 1.5%. Post-ranking βs also show a very weak increasing relationship with size: the highest portfolio β is 0.87 (ME-8), while the lowest is 0.61 (Small-ME). Large-ME has a β of 0.7, so there is little variation of post-ranking βs among size portfolios. Higher-size portfolios tend to have a lower book-to-market, thus confirming the Fama-French results: my results go from 0.08 for the smallest portfolio to the -1.05 for the largest. The negative relationship between size and market leverage is also confirmed (0.85 for Small-ME and 0.003 for Large-ME), as is the positive relationship between size and book leverage (0.82 for Small-ME and 1.06 for Large-ME). The E/P Dummy line is interesting because it describes the percentage of the companies in the portfolio that have negative earnings. Of course, the smaller the size, the higher the percentage: we go from a considerable 0.3 for Small-ME to 0.01 for Large-ME. We can also see how the earnings/price ratio is increasing with size and skyrockets from 0.3 to 5.2. The smallest half of the size portfolios actually seems to present no relationship between size and E/P, but we have to consider that negative-earnings companies are not included in the mix.

Panel B shows the same statistics for the pre-ranking β portfolios. We can see the weak negative relationship between β and returns (which is actually there only for the high-β half of the portfolios): I obtained an average 2.6% for Low-β and -0.14% for High-β, with a peak at 2.75% for β-2. The post-ranking βs follow the portfolio ranking: we go from 0.49 to 1.24. The size-β relationship is U-shaped: we have 3.92 for Low- β, 3.85 for High- β and a peak at β-5 with 5.06. The book-to-market ratio shows no clear relationship with β and neither do market leverage and book leverage, so the Fama-French results are more or less confirmed, while high-β portfolios tend to have slightly higher average earnings/price ratios and a lower percentage of firms with negative earnings.

Table 3 shows the results of the Fama-MacBeth Regressions. The role of the β seems to be reversed, as it tends to have a negative coefficient in all of the regressions in which it is included. The same reversal can be seen in the behavior of the size factor: its coefficients are all positive, while Fama and French found a negative relationship. As I already said, this reversal is possible and proven by previous studies and is probably due to the popularity of biggest and low-β stocks during economic distress. However, the size effect becomes negative when the E/P ratio is included in the regression. The positive effect of book-to-market is confirmed and so are the effects of market leverage (positive) and book leverage (negative). It should be noted that these last two have an effect that is very close in absolute value in the original paper, while here the effect of book leverage is much smaller, indicating that it is not as important in determining the performance of a company's stock. The role of the earnings/price ratio is positive as predicted, while the E/P Dummy, which has a mixed role in the Fama-French paper (it depends on the presence of the leverage and book-to-market factor), has negative coefficients in my research. This again may be a consequence of flight to quality: companies with negative earnings may offer positive excess returns in the U.S., but in a situation like last 20 years' Europe, companies with positive (and possibly high) earnings perform better. The big trouble with these regressions is that they all present very low t-Statistics, meaning that the coefficients are not significantly different from zero. The R^2 that I obtained for the various regressions do not go over 0.1, even without adjusting, meaning that these factors actually explain a low percentage of total returns variations. On the other hand, the coefficients are not lower than the original paper, so the problem is not in the estimations, but rather in the extremely high volatility of returns and therefore of the errors in the regressions.

Table 4 shows the statistics of book-to-market and earnings/price portfolios. Going from lowest to highest, book-to-market portfolios (shown in Panel A) seem to have increasing returns as predicted: they go from 0.07% to 3.71%. Post-ranking βs are slightly decreasing and so is average size; however, dispersion is low for both of them: we go from 0.83 to 0.56 for β and from 5.49 to 3.24 for size. We can also see how the majority of portfolios have negative book-to-market. As expected, market leverage is strongly increasing (from -1 to 1.93) and book leverage is decreasing (from 1.25 to 0.85). E/P is slightly increasing, but with some exceptions: we go from 0.66 to 1.22; the percentage of negative-earning companies in each portfolio has a somewhat U-shaped relationship with their book-to-market: Low-BE/ME has an average of 0.16% and High-BE/ME has 0.13%, while BE/ME-5 has 0.07%. The variation is too low to talk about a real relationship.

In Panel B we can see the same statistics of the E/P portfolios. It should be noted that there actually are 11 E/P portfolios instead of the usual 10: one of the portfolios is set aside for negative-earnings companies. We can see how, going from negative earnings to high-E/P, portfolio returns are strongly increasing: Neg-E/P has average monthly returns of -0.18%, Low-E/P has 0.88% and High-E/P has 4.59%. This may indicate that investors give a lot of importance to the earnings/price ratio when planning their decisions. Neg-E/P also has a higher post-ranking β than the other portfolios, but there is no regularity with βs across E/P portfolios; all of this is consistent with the results of Fama and French. A clear increasing pattern can be found with average sizes (from 1.44 for Neg-E/P to 7.66 for High-E/P), while no pattern can be found for book-to-market. Market and book leverage are all increasing: we go from 0.1 to 0.85 and from 0.32 to 1.18 respectively: this result agrees with Fama and French, but in my case Neg-E/P follows the tendency, while in the original paper it has higher values of leverage. We can also see that the negative-earnings portfolio includes an astonishing average of 754.5 stocks, while other portfolios generally do not have more than 150 each.

Table 5 presents the average returns of the 100 size/book-to-market portfolios and they show some peculiar tendencies. First of all, we acknowledge that the size effect is reversed for big stocks, while it does not show particular tendencies for small stocks. Besides, the size effect weakens for higher book-to-market portfolios: the High-BE/ME portfolio returns do not seem to have any relationship with size. Book-to-market generally has a positive influence on returns: this relationship is particularly clear for large and high-book-to-market portfolios, while small and low-book-to-market portfolio returns seem to be more dispersed. Low-BE/ME seems to be the only portfolio whose sub-divisions show negative returns. The lowest returns are attached to the portfolio that intersects Low-BE/ME and ME-2, with -1.44%, while the best performing of the 100 portfolios is the intersection between Small-ME and High-BE/ME, which yields 4.87% per month in average.

In Table 6 we can see the results for regressions 5 and 8 more in depth, which are the ones including β, size and book-to-market and size and book-to-market respectively. Here I reported the coefficients and t-Statistics for the whole periods and for the two sub-periods: pre-Crisis (1996-2007) and post-crisis (2007), as well as the average market returns. The first thing we can see is that market returns decrease in the post-Crisis period, as expected. The intercepts of the regressions are actually very high and surprisingly significant: they are important because they signal the existence of systematic factors that are not accounted for in the regression. In the whole time period, the intercept is 3.63 standard errors from zero for the regression that does not include the β and 10.82 for the one that does. This indicates that while many high-β stocks tend to underperform (the coefficient of the β is negative), they receive a premium from other systematic risk factors that have yet to be found. These t-Stats increase significantly in the pre-Crisis period: they are 5.26 and 11.19 respectively, but they decrease very sharply in the post-Crisis period to -0.34 and 3.24 respectively. This may indicate that risk factor became less systematic under the crisis and cannot explain returns anymore. The coefficients on the β are consistently negative (-0.21 in average), have a very high standard deviation (8.09 in average, lower in the post-Crisis period) and are not very significant (0.21 average t-Stat). The coefficients of the size factor are positive but have very low significance (0.02 t-Stat), while book-to-market has positive coefficients, but again, significance is low (0.07 standard errors from zero).

It is worth noting that the correlation between the risk-free rate and the alphas is 0.1757 for the 3-Factor model and -0.0297 for the 2-Factor. This can be seen as my version of the test of the Sharpe-Lintner Hypothesis. Of course, this correlation increases when the systematic factor β is included in the model, but it is not perfect, meaning that there are other systematic factors that are not accounted for.

The results of this test were not very conclusive, but in my opinion some degree of uncertainty in the European market of the last 20 years is to be expected. The test somewhat shows that book-to-market has a positive effect on returns and so does the E/P ratio, so the value effect is substantially confirmed. The only difference is that negative earnings stocks tend to perform badly in my case, while they have higher returns in the original paper. The size effect and the effect of β are reversed, which may be a consequence of flight to quality. The results show a generally low significance and this may be due to the excessive volatility of the market and the variance of market returns because the coefficient themselves are not very different in magnitude from the Fama-French coefficients (they actually are even higher). In the next section I will analyze more recent applications and expansions of this model and I will also run a similar test on the Fama-French 5-Factor model.

Section 3

3.1 – Expansions to the 3-Factor Model and the Fama-French 5-Factor Model

3.1.1 – The Momentum Effect

After the 3-Factor model was created, it became an instant classic and spawned numerous related studies and expansions, much like the CAPM did, and again numerous researchers started finding new anomalies.

By analyzing the 1965-1989 period, Jegadeesh and Titman (1993) found what was later to be known as the "momentum" effect. Their evidence suggested that there were significant abnormal returns to stocks that had a good performance in the previous 3 to 12 months and that a strategy of buying recent-past winners and selling losers led to good returns. These results seemed to dissipate in the following 2 years and to be mostly concentrated in the first year, especially around the time quarterly earnings are announced after the portfolio formation. After some months, however, this effects reverses and the loser portfolio starts to outperform the winner portfolio. According to the authors, this leads to questions about the common interpretations of investor behavior, where return reversal is seen as a symptom of overreaction to news and performance persistence is seen as underreaction. Instead, they propose the explanation that the market overreacts initially, but then prices go back to their long term values, or that investors underreact to news about short-term perspectives of the firm and overreact to long-term news. In 2000, Hong Lim and Stein found evidence of the three determinants of the momentum effect:

- It works mainly for small stocks and declines rapidly for big stocks;
- It works better for stocks that have low coverage by analysts;
- This last effect is greater for past losers than for past winners.

This leads to the conclusion that the momentum effect is caused by the gradual diffusion of information, possibly emphasized by the irrational behavior of the market.

However we put it, momentum became another big point of discussion in the 90s: it contradicted the Efficient Market Hypothesis and it represented and anomaly for both the CAPM and the Fama-French Model.

Carhart (1997) was the first one to include the momentum effect in the 3-Factor Model. His research started as a study on the persistence of performance by mutual funds, as he finds that funds that perform exceptionally and consistently better than others do not do so by putting superior stock-picking skills to work, but rather by cutting expenses and transaction costs (or, I should add, by running Ponzi schemes). Carhart does find that funds with high returns in the previous year tend to have higher average expected returns in the following year, but also have higher expenses. Moreover, the worst performing funds tend to keep having bad performances, probably due to their small dimensions, which can prevent them from performing certain operations: for example, many of those funds were not available for short positions. Carhart does acknowledge momentum as a relevant effect in the short term, and uses it to build the 4-Factor model that he uses to evaluate the performances of mutual funds. The model is built in this way:

$$R_{it} = a_{it} + b_{1t}(R_{mt} - RFR_t) + b_{2t}SMB_t + b_{3t}HML_t + b_{4t}PR1YR_t + \epsilon_{it} \quad (26)$$

where *PR1YR* denotes the returns on a portfolio that is long the best performing stocks of the last 12 months and short the worst performing, much like SMB for size and HML for book-to-market.

Fama and French came to conclusions similar to Carhart's in their own 2010 paper about the performances of mutual funds, while they found that the momentum factor is more or less present in most stock exchanges around the world, with the singular exception of Japan (2012).

However relevant, the momentum effect is something that in my opinion relates more to behavioral economics than to an approach to asset pricing that is based more on fundamental analysis. This is why I am not going to analyze momentum in the empirical part of this thesis; instead, I will focus on other expansions of the model that attain the firm-specific growth possibilities.

3.1.2 – The Fama-French 5-Factor Model

As Fama and French assert in their 2006 paper, a company's future stock returns can be estimated through its future discounted dividends, so if one firm has the same expected dividends as another, but lower price, it should have higher price growth in the future and hence higher returns. Modigliani and Miller (1961) show that the market value of a company is equal to the discounted equity earnings minus the change in book equity, and this creates the following equation, which should hold, even in the presence of information inefficiencies and irrational behavior:

$$\frac{M_t}{B_t} = \frac{\sum_{s=1}^{\infty} E[Y_{t+s} - (B_{t+s} - B_{t+s-1})]/(1+r)^s}{B_t} \quad (27)$$

where Y is earnings, B is book value, r is expected stock returns and M is market value. This leads to three statements:

- Holding everything else fixed except r, a higher B/M (book-to-market) leads to higher returns.

- Holding everything else fixed except *r*, a higher dB/B (change in book equity, or investment) leads to lower returns.
- Holding everything else fixed except r, a higher Y/B (earnings over book equity, or profitability) leads to higher returns.

The role of book-to-market according to Fama and French is clear enough by now, but here we see how new factors determine the returns on a stock, at least in theory. Expected earnings of a company are not directly observable of course, so a proxy is needed (assuming that the role of expected earning is not absorbed by other variables that are already included in the mix). Fama and French also test the validity of this equation and their results are substantially encouraging: the equation seems to represent a true relationship. However, one question that statistical procedures are unable to answer is whether the stock return anomalies associated with these variables are caused by rational or irrational circumstances. Are these abnormal returns caused by legitimate risk factor that are actually present in the market and have to be rewarded, or are they the result of under- or overreaction by irrational investors?

Another problem with this paper is in the choice of proxies: the authors use the percentage change in book equity as a proxy for investment and the ratio of earnings over book equity as a proxy for profitability, and they seem to cause problems of collinearity. Fama and French solve these problems by changing and correcting measures: they use asset change as a measure of investment, a lagged book-to-market and two different measures of firm stability and default probability (Piotroski, 2000[2] and Ohlson[3], 1980) as explanatory variables.

This seems to solve the technical problems, but the problem of the real causes of these effects remains unsolved and again, this may be a subject matter more related to behavioral economics.

In his 2013 paper, Novy-Marx proposes an alternative measure of profitability, which he calls "gross profitability" and measures as the ratio of gross profits to total assets. His results are significant: according to his paper, gross profitability not only has a relevant role in explaining returns, but it has as big an effect as book-to-market's and is also complementary to it, meaning they proxy for different risk factors. This is the "other side of value" that he talks about: while in conventional value strategies we finance the acquisition of inexpensive assets by selling expensive assets, here the investor sells unprofitable assets to buy profitable assets. In this way, a strategy based on profitability is seen as a growth strategy, which can be used to hedge against the risks of a value strategy. As in many other studies, small stocks represent an anomaly and are always very difficult to predict.

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[1] The Piotroski Score goes from 0 to 9 and the company is given points for satisfying 9 requirements in 3 categories: Profitability (requirements on ROA, ROA growth, OCF, and ROA/OCF), Leverage & Liquidity (growth of long term debt, Current Ratio, shares issued) and Operating Efficiency (Gross Margin, Asset Turnover)

[2] The O-Score by Ohlson is the result of a linear combination of 9 factors relative to the fundamentals of a company. These factors include current and total assets, current and total liabilities, capital, self-funding, net income and two dummies for liabilities>assets and income<0. Each of these factors has a pre-specified weight.

Aharoni, Grundy and Zeng (2013) also expand the 2006 paper by Fama and French by conducting their research at a firm level instead of the per-share level used by the original authors. They start from the Modigliani-Miller firm valuation equation as well, and then estimate a similar model as in the Fama-French paper. Their results are slightly better, as they manage to obtain a coefficient for investment that is negative and significant, which Fama and French could not. All the other results confirm the Fama-French research.

These expansions led Fama and French to the creation of a new model that included profitability and investment as risk factor together with the "historical" β, size and book-to-market. In order to do this, they proxy the two new risk factors by finding the returns on two portfolios: RMW and CMA. RMW (Robust Minus Weak) and is long on stocks with high profitability and short on stocks with low profitability, while CMA (Conservative Minus Aggressive) is long on stocks with low investment and short on stocks with high investment. The regression equation that describes the model is built as follows:

$$R_{it} = a_{it} + b_{1t}(R_{mt} - RFR) + b_{2t}SMB_{it} + b_{3t}HML_{it} + b_{4t}RMW_{it} + b_{5t}CMA_{it} \qquad (28)$$

As far as the "old" variables are concerned, The authors are able to find the same patterns that they did in 1993. It should be noted that according to Fama and French themselves, book-to-market, profitability and investment are correlated: firms with high book-to-market tend to have low profitability and investment, while the opposite is true for low-book-to-market firms. This is why it is suggested that HML may be redundant and RMW and CMA may account for the value effect, so they authors introduce another variable called HMLO (HML Orthogonal), which is constructed by summing up the intercepts and errors in the regression of HML on R_m-RFR, SMB, RMW and CMA. However, the results that Fama French obtain are significant and follow the expectations: small and profitable firms with non-aggressive investment policies and high book-to-market seem to have the highest returns. The test statistic that Fama and French use (developed by Gibbons, Ross and Shanken in 1989) rejects the hypothesis that the 5-Factor model captures all the patterns, but nonetheless the patterns are there, as the authors show, and their cross-sectional regressions still manage to explain the better part of the variation in average returns.

My intent of course is to see if any of these patterns can be found in the European market, but once again the idiosyncrasies of the European market may come out. This is why I decided to include another variable in my analysis, which is the earnings management indicator.

Foye Mramor and Pahor (2013a) tried to test the weak form of the Efficient Market Hypothesis of Fama (1970) in the stock markets of Eastern European countries that joined the EU in 2004. This hypothesis only requires that current prices fully reflect past prices and of course it is a necessary condition for the verification of the other two stronger forms of market efficiency. In the early 2000s, other studies tested this hypothesis in Easter European countries and were unsuccessful, but the general expectation was that inefficiencies would eventually be solved. Foye, Mramor and Pahor ran tests on serial independence, autocorrelation, heteroskedasticity and stationarity, and also added a control for liquidity. The tests were not successful for any of the countries in the region, and did not even show any sign of improvement over the situation of ten years prior.

According to Foye, Mramor and Pahor (2013b), these price inefficiencies are mainly due to problems with information, which in those countries is slowly (if ever) published, hard to come across and generally unreliable, especially when it comes to earnings. This is why they built a re-specified 3-Factor model that includes an earnings management indicator instead of the size factor. This indicator is built as a ratio of operating income over operating cash flow.

Another interesting factor model based on fundamental analysis and on the characteristics of the security was created at financial firm Barra, Inc. (now MSCI, Inc.). According to Bender and Nielsen (2010), it is based on 13 indicators:

- Size (market capitalization);
- Size Non-Linearity
- Currency Sensitivity (correlation with the reference currency's fluctuations);
- Leverage (debt/equity ratio);
- Volatility (standard deviation of stock price);
- Earnings Yield (earnings/price ratio);
- Trading Activity (average volume of trading on the stock);
- Momentum;
- Growth (book equity growth);
- Value (book-to-market);
- Dividend Yield
- Earnings Variation

I am not going to analyze or test this model in depth, but I thought it was worth mentioning to give a demonstration of how the Fama-French model's factors are still used by major investment companies, at least for applications in the United States.

In the next sections I am going to build this re-specified 5-Factor model to see if it works better than the 3-Factor and if investment and profitability really are the new fundamentals on which to keep an eye, and I will see if this earnings management indicator can really control for information inefficiencies in the European market. In the following analysis, I will keep the same methodology as before, I will use the same techniques as in the 1992 Fama-French paper and see if I obtain better results.

3.2 – Empirical Test

3.2.1 – Data Description

For this second analysis, the total of UE firms that satisfy the data requirements dropped to 3223 and the relevant period is now July 1997 – June 2015. For the Euro Area test the period is July 2000 – June 2015. The number of firms for each country is now:

- Austria (47)
- Belgium (70)
- Bulgaria (27)
- Croatia (25)
- Cyprus (18)
- Czech Rep. (7)
- Denmark (87)
- Estonia (11)
- Finland (101)
- France (380)
- Germany (396)
- Greece (155)
- Hungary (5)
- Ireland (48)
- Italy (140)
- Latvia (15)
- Lithuania (13)
- Luxembourg (17)
- Malta (8)
- Netherlands (66)
- Norway (89)
- Poland (146)
- Portugal (38)

- Romania (64)
- Slovakia (5)
- Slovenia (26)
- Spain (38)
- Sweden (224)
- Switzerland (157)
- UK (800)

The data requirements and the association of balance sheet data to returns relative to six months later work the same way as in the previous model. The only difference is that I gathered the following additional data for each company (the descriptions are once again as provided by Bloomberg):

- **Net Interest Expenses**: Total interest expenses for the company minus interest income. The value is as disclosed by the company if no other information is available. The value is in million Euros.
- **Cash Flow From Operations**: Total amount of money received by the company for ongoing regular business activities, as disclosed by quarterly reports. The value is in million Euros.

I conducted the analysis on a restricted sample: one relative to the Euro Area, the other relative to the UK. The number of firms in each sub-sample is 1617 for the Euro Area (which includes Austria, Belgium, Cyprus, Estonia, Finland, France, Germany, Greece, Ireland, Italy, Latvia, Lithuania, Luxembourg, Malta, Netherlands, Portugal, Slovakia, Slovenia and Spain) and 800 for the UK.

3.2.2 – Methodology

Variable Construction

For the expansion of the model, I discarded the leverage variables and added the following, using the new data:

- **Investment**: The percentage change in the company's total book assets, constructed this way:

$$Inv_{it} = \ln(Total\ Assets_{it}) - \ln(Total\ Assets_{it-1}) \tag{29}$$

- **Operating Profitability**: An indicator of the profitability of a company's regular business, expressed as a percentage of the company's total assets. I calculated it like this:

$$OP_{it} = \frac{EBITDA_{it} - Net\ Interest\ Expense_{it}}{Total\ Assets_{it}} \times 100 \qquad (30)$$

- **Earnings Management Indicator**: An indicator of the degree to which the company is likely to manipulate the results reported in the balance sheet, according to Foye, Mramor and Pahor (2013), obtained as a ratio of operating income over operating cash flow. I calculated it like this:

$$EMI_{it} = \frac{EBITDA_{it} - Net\ Interest\ Expense_{it}}{CFO\ _{it}} \qquad (31)$$

The Euro Area test includes only the EURIBOR as the RFR, while the UK test uses the LIBOR.

Portfolio Construction

The approach to portfolio construction here was the same as in the 3-factor model tests. I built the following portfolios to test the new explanatory variables:

- 10 Investment Portfolios (based on investment)
- 10 Profitability Portfolios (based on operating profitability)
- 100 Investment/Profitability Portfolios (based on investment and then operating profitability)

The pre-ranking and post-ranking βs are calculated in the same way as in the previous model.

For the Euro Area and UK analysis, I formed the following portfolios:

- 10 Size Portfolios
- 10 Book-To-Market Portfolios
- 100 Size/Book-To-Market Portfolios
- 100 Size/Pre-Ranking β Portfolios

Fama-MacBeth Regressions

Again, I used the same methodology as before to run the Fama-MacBeth cross-sectional regressions and to compute the t-Statistics. For this model, I ran the following regressions

1. $R_{it} = a_t + b_{4t} Inv_{it} + \epsilon_{it}$
2. $R_{it} = a_t + b_{5t} OP_{it} + \epsilon_{it}$

3. $R_{it} = a_t + b_{1t}\beta_{it} + b_{4t}Inv_{it} + \epsilon_{it}$
4. $R_{it} = a_t + b_{6t}EMI_{it} + \epsilon_{it}$
5. $R_{it} = a_t + b_{1t}\beta_{it} + b_{4t}Inv_{it} + b_{5t}OP_{it} + \epsilon_{it}$
6. $R_{it} = a_t + b_{2t}\ln(ME_{it}) + b_{4t}Inv_{it} + b_{5t}OP_{it} + \epsilon_{it}$
7. $R_{it} = a_t + b_{2t}\ln(BE/ME_{it}) + b_{4t}Inv_{it} + b_{5t}OP_{it} + \epsilon_{it}$
8. $R_{it} = a_t + b_{4t}Inv_{it} + b_{5t}OP_{it} + b_{6t}EMI_{it} + \epsilon_{it}$
9. $R_{it} = a_t + b_{7t}\,E/P\ Dummy_{it} + b_{8t}E(+)/P_{it} + b_{6t}EMI_{it} + \epsilon_{it}$
10. $R_{it} = a_t + b_{4t}Inv_{it} + b_{5t}OP_{it} + b_{7t}\,E/P\ Dummy_{it} + b_{8t}E(+)/P_{it} + \epsilon_{it}$
11. $R_{it} = a_t + b_{1t}\beta_{it} + b_{2t}\ln(ME_{it}) + b_{2t}\ln(BE/ME_{it}) + b_{4t}Inv_{it} + b_{5t}OP_{it} + \epsilon_{it}$
12. $R_{it} = a_t + b_{1t}\beta_{it} + b_{2t}\ln(ME_{it}) + b_{2t}\ln(BE/ME_{it}) + b_{4t}Inv_{it} + b_{5t}OP_{it} + b_{6t}EMI_{it} + \epsilon_{it}$
13. $R_{it} = a_t + b_{1t}\beta_{it} + b_{2t}\ln(BE/ME_{it}) + b_{5t}OP_{it} + b_{7t}\,E/P\ Dummy_{it} + b_{8t}E(+)/P_{it} + \epsilon_{it}$

Regressions 8, 11, 12 and 13 are also run for the pre-Crisis and post-Crisis subperiods. I ran the same regressions for the sub-samples relative to the Euro Area and the UK.

3.2.3 – Results Summary

In Table 7 we can see the statistics of the 100 portfolios ranked on investment and operating profitability. I reported their average returns, operating profitability, investment, β and size in order to look for regularities. The sample is composed by 3223 stocks for the 1997-2015 period.

Panel A shows the average returns of the portfolios. We can see that high-operating profitability portfolios tend to have higher returns as predicted by Fama and French, but the relationship is not perfectly increasing. On the other hand, investment has a U-shaped relationship with returns and this is consistent for every level of operating profitability. In the Fama-French 2014 paper, this relationship is predicted to be decreasing, but it actually depends on the level of book-to-market. The best performing portfolio is the intersection between OP-6 and Inv-8 with 5.13%, but this is clearly out of the pattern. In fact, the second best performing portfolio is High-OP/Inv-5 with 3.77%, which agrees with the pattern. The worst-performing portfolio is Low-OP/Low-Inv with -1.41%.

In Panel B I reported the average values of investment (percentage change in total assets) for the 100 OP/Inv portfolios. We can see that there seems to be a weak increasing relationship between the two. However, the portfolio with the highest investment is the intersection between Low-OP and High-Inv with 0.36, while Low-OP/Low-Inv has the lowest average investment with -0.11.

The average operating profitability for each of the 100 portfolios can be seen in Panel C: this relationship is U-shaped, as low and high investment portfolios seem to have the highest operating profitability. In particular, the portfolios that are obtained by dividing Low-Inv have the most extreme levels of operating profitability: we go from -39.25% for Low-OP to 36.86% for High-OP. They may be seen as outliers but the relationship is too regular and there are too many observations in these portfolios to dismiss this result.

Panel D shows no patterns in the portfolios as far as post-ranking βs are concerned. We can only notice a slight tendency for high- and low-investment portfolios to have higher βs.

Finally, Panel E shows no particular patterns for average size in the OP/Inv portfolios. Low-Inv and Low-OP portfolios seem to have the smallest size, but the variation is too low to reach any conclusion.

Table 8 shows more statistics for investment and operating profitability portfolios.

Panel A shows the statistics for the 10 investment portfolios: we can see the slight U-shape relationship with returns, with the best performing portfolios being Inv-5 and the worst being High-Inv with 0.93. This pattern is reversed for the post-ranking β: High-Inv has the highest with 0.91 and Inv-5 has the lowest with 0.64. There is no relationship with size that is worth noting and the same can be said about book-to-market: we have a slight tendency for the middle portfolios to have larger size and higher book-to-market, but again the dispersion is too low to reach conclusions. As I said, Low-Inv has a very high level of operating profitability at 15.45%, which is impressive considering that the next-highest value is 0.54%. The value of the E/P Dummy (which represents the percentage of companies in the portfolio with negative earnings) is decreasing in investment, but High-Inv has the second-highest percentage (0.15). Low-Inv has the highest value (0.25) and Inv-8 has the lowest (0.05), so, as expected, it seems that investment hurts a company's earnings, but it only does when it is very high. This U-shaped relationship can be noticed in most of the variables including the earnings/price ratio: again, the E/P increases until Inv-7, which has the highest value with 1.52, but then it drops and High-Inv has the minimum value with 0.55. The earnings management indicator is also higher for average-investment portfolios (0.23 for Inv-5) and lower for extremes (-0.08 for Low-Inv).

Panel B analyzes OP portfolios. Returns peak at OP-7 with 3.28%, post-ranking βs show no patterns, size peaks at OP-6 with 5.65 and book to market has a minimum in High-OP (-1.05) and a maximum in OP-3 (-0.17). All these variables show very low difference in value across portfolios and unclear patterns. As I said before, investment has an increasing relationship with operating profitability: we go from the 0.01 of Low-OP to the 0.09 of High-OP, while the percentage of negative-earnings companies goes from 0.34% for Low-OP to 0.02 for OP-8 before increasing slightly in OP-9 and High-OP. The earnings/price ratio and the earnings management indicator do not show any clear patterns.

Table 9 shows the results of the Fama-MacBeth regressions. We can see that the role of β, size and book-to-market are substantially the same as in the previous test. Investment shows consistently negative coefficients, while operating profitability seems to have a positive effect on returns. The EMI has negative coefficient as expected. Again, coefficients do show the expected sign and magnitude, but due to the excessive volatility, they are not significant, as they are always less than 0.1 standard errors from zero, with β again being the only exception.

We can see Table 10 for the results of regression 8 and 11, which are the Custom 3-Factor Model (OP, Inv and EMI) and the 5-Factor Model (β, ME, BE/ME, Inv and OP) respectively, including the sub-period analysis. The results are very similar to the 3-Factor model: the coefficients have the expected sign and magnitude, but have very low t-Statistics, which make them non-significant. The results are even worse for the post-crisis period: in the Custom 3-Factor the coefficient for OP is 0.0039 standard errors from zero and the coefficient for EMI is 0.0001 standard errors from zero. Things are somewhat better in the 5-Factor, indicating that size, book-to-market and especially β are necessary in the model. Once again, my results indicate strong positive intercepts with very high t-Statistics: I get 5.83 for the Custom 3-Factor and 9.87 for the 4-Factor. However, we witness a decrease of these t-Statistics for the post-Crisis period. This may indicate even more volatility and less explanatory power for the unknown systematic risk factors that are not included in the model.

Table 11 reports the full-period and sub-period statistics for regressions 12 and 13, which I will call 6-Factor (β, ME, BE/ME, Inv, OP and EMI) and Custom 4-Factor Models (β, BE/ME, OP, E/P Dummy and E(+)/P) respectively. In the first case, the results are basically the same as in the 5-Factor, indicating the scarce influence of the earnings management indicator. In the second case, I built this model taking into account the variables that have shown the best results (even though taking about "best results" is a bit of an overstatement). The results are of course very similar to previous regressions, but I noticed an interesting peculiarity here: the t-Statistic of the intercept actually increases in the post-Crisis period instead of going down, indicating that in this model the explanatory power of the unknown systematic factor increases. Moreover, the role of the earnings/price ratio becomes particularly non-significant in the second sub-period, as its coefficient is merely 0.004 standard errors from zero. Once again, the R^2 of the regressions barely reaches 0.1, even without adjusting.

The test of the S-L hypothesis for the four main models is indicated as the correlation between the risk-free rate and the intercepts of each model. It has the results of -0.2088 for the Custom 3-Factor, 0.1404 for the 5-Factor, 0.1367 for the 6-Factor and 0.0674 for the Custom 4-Factor, thus suggesting that an unknown systematic factor still has some explanatory power.

This test shows that operating profitability, investment and the earnings management indicator do not do much to explain the cross-section of stock returns and β, size and book-to-market are still better predictors, although their coefficients are not significant. In the next sub-sections I will see if the restriction of the model to the Euro Area only and the UK only leads to better results.

Euro Area

Table 12 shows the average monthly returns on the size/pre-ranking β and size/book-to-market portfolios for the Euro Area. The sample is composed by 1617 stocks for the period 1999-2015.

Panel A confirms the reversal patterns found in the full-sample model: the pre-ranking β portfolios seem to have decreasing returns, with negative returns for almost every portfolio in the High-β section. Size seems to have a positive effect on returns, again confirming the reversal of the size effect found in the previous test, but this time it is less pronounced as results often do not follow the pattern. The best performing portfolio is again the intersection between Low-β and ME-9 with 3.81%, while the worst performer is High-β/ME-2 with -0.82%.

Panel B leads to the same conclusions as the full-sample model as well: size and book-to-market portfolios both have increasing returns, but the effect is much stronger for book-to-market, while size has a less clear effect and more dispersed returns. The best performer is the intersection between High-BE/ME and Small-ME with 9.7% of average monthly returns, while the worst performer is the intersection between Low-BE/ME and ME-5 with -0.33%.

In Table 13 we can see more statistics for the 10 size and book-to-market portfolios.

Panel A shows the size portfolio statistics. We can see that returns are increasing in size except for the Small-ME portfolio, which has higher returns that the pattern would suggest. Nonetheless, Large-ME has the highest returns with 2.56%, while ME-3 has the lowest with 1.16%. Post-ranking βs seem to be increasing in size, but the relationship is not as clear as in the full sample, indicating that the reversal of the size effect on returns and βs is not as pronounced in the Euro Area as in the full sample. As noted in the previous test, book-to-market and size seem to have a clear negative relationship, which is to be expected by construction: from smallest to largest size, the results go from 0.22 to -0.91. Investment, operating profitability and the earnings management indicator show little or no patterns across the size portfolios (especially OP and EMI), while the earnings/price ratio and the percentage of negative-earning firms both confirm their patterns: increasing and decreasing in size respectively. E/P has a maximum value in Large-ME (5.77) and a minimum in ME-2 (0.11), while the E/P Dummy has a maximum in Small-ME (0.27) and a minimum in Large-ME (0.01).

Panel B shows the same statistics for the book-to-market portfolios. The positive relationship with returns and the weakly negative relationships with size and β are confirmed, even though the Euro Area shows a greater dispersion of results. Average investment seems to be decreasing across book-to-market portfolios: we go from a maximum of 0.08 for Small-ME to a -0.001 for Large-ME. The E/P Dummy seems to have a U-shaped relationship with book-to-market, but not very relevant, while the earnings/price ratio seems to be increasing (except for the High-BE/ME portfolio). OP and EMI do not seem to be related to book-to-market.

Table 14 shows the results of the Fama-MacBeth regressions of monthly returns on the explanatory variables. The results are consistent with the full-sample model, showing slightly smaller t-Statistics and hence even less significant coefficient. Nothing is particularly worth noting, besides the exceptionally bad performance of the earnings management indicator.

United Kingdom

In Table 15 we can see the average monthly returns for the size/pre-ranking β and for the size/book-to-market portfolios of United Kingdom stocks. The sample is composed by 800 stocks for the period 1997-2015.

Panel A shows the returns for the size/β portfolios and once again, the results of the full-sample model are confirmed. The intersection between High-β and ME-3 shows the lowest returns with -2.6% and the intersection of Small-β and Large-ME with 3.72% is the best performing portfolios. Here we can see again that the role of size is not clear and returns are quite dispersed, but the reversal pattern seems stronger than in the Euro Area.

In Panel B we can see the average returns on the 100 size/book-to-market portfolios. The results are the same as before and here too the reversal of the size effect seems more consistent. The best performer is the intersection of High-BE/ME and ME-9 with 3.66%, while the worst is the intersection of Low-BE/ME and ME-2 with -2.05.

Table 16 shows the characteristics of the 10 size portfolios and the 10 book-to-market portfolios.

As Panel A shows, the increasing relationship between size and returns is quite consistent: we go from 0.1% for Low-ME to 2.59% for High-ME. The size effect on post-ranking βs is virtually non-existent for the smaller half of the portfolio (small stocks are notoriously harder to predict) and decreasing for bigger portfolios. The highest value is 1.03 for ME-5 and the lowest is 0.62 for Large-ME. As in the full-sample test, we can see that the traditional negative relationship between β and size is actually more or less respected in the UK, at least for larger-size stocks. Book-to-market seems to be decreasing in size, thus confirming the results obtained until now: going from smallest to largest portfolio, we go from an average book-to-market of 0.01 to -1.18. As always, the earnings/price ratio seems to be increasing in size (from 0.13 to 10.69), while the percentage of firms with negative earnings is decreasing (from 0.33 to 0.01). Investment, operating profitability and the earnings management indicator do not show any pattern across size portfolios.

Panel B reports the same statistics for book-to-market portfolios in the UK. Returns are increasing, but the High-BE/ME portfolio had a worse performance than BE/ME-9: 2.3% average monthly returns against 2.56%. The worst performer is Low-BE/ME with 0.15%. Portfolios with high book-to-market seem to be generally composed of smaller stocks, but the relationship is not clear. Investment shows a slight tendency of decreasing as the book-to-market of the portfolio increases: from Low-BE/ME to High-BE/ME we go from 0.11 to -0.05. With the exception of a slight increasing tendency shown by the earnings/price ratio, all the other variables (β, OP, E/P Dummy, and EMI) do not show clear regularities across the book-to-market portfolios.

Table 17 shows the results of the Fama-MacBeth cross-sectional regressions of returns against the explanatory variables for the UK market. We can see that the previously obtained results are still there. The coefficients are slightly more significant in this case, with the notable exceptions of the EMI and even the earnings/price ratio, which apparently is not as relevant in the British market as in the rest of Europe.

This test substantially confirms the results of the 3-Factor Model tests: the coefficients have the sign and magnitude that the model predicts, except for a reversal of the size effect on β and returns. However, as the t-Statistics show, these coefficients are not significant and fail to explain the cross-section of expected returns.

Conclusion

In this thesis I researched the subject of asset pricing and its most important applications; namely, the Markowitz Model, the Capital Asset Pricing Model and the Fama-French Model. I presented a survey of the main literature on the matter to offer some background and then I focused on the interaction between asset pricing and fundamental analysis, with the work of Fama and French being the most prominent example. Then I tested the Fama-French 3-Facor and 5-Factor Models in the European stock market of the last 20 years, to see if, in the ever-changing and uncertain scenario of European finance, this model would shed some light.

My results were rather conflicting: in the last 20 years they indicate a reversal in the traditional negative effect of size on β and returns. In fact, stocks of larger firms now offer higher returns and high-β stocks tend to perform badly. This tendency is probably due to the concept of "flight to quality", according to which, in cases of economic downturn, investors tend to favor low-risk instruments instead of risky ones. Therefore, low-β and large stocks tend to have higher returns than they should according to economic theory. Even the traditionally negative relationship between β and size was found to be harder to observe, but it was somewhat more consistent in the UK, even though the reversal of the size effect seems to be slightly stronger in the British market. The positive effect of book-to-market is confirmed, as is the double role of leverage. With respect to the U.S. market and the Fama-French analysis, companies with negative earnings tend to have lower returns and this should be the consequence of flight to quality as well. In bull markets, firms with negative earnings are more risky, but are seen as an opportunity by investors who are confident that the stock will pick up. This is not the case in my results: European investors are possibly less incline to bet on their money on firms that are losing money. Apart from this, the value factor's influence on stock returns is substantially confirmed by my results.

On the other hand, the test of the 5-Factor model show how the new fundamental factors, proposed following the first Fama-French paper and recently included in the model by the authors themselves, are less easy to interpret and do not seem to add much to the 3-Factor Model. Their effects seem to be consistent with the prediction, but they do not significantly improve the explanatory power of the model on the cross-section of stock returns. The Earnings Management Indicator proposed by Foye, Mramor and Pahor has almost no effect at all.

The main problem with my tests is the extremely low significance of the coefficients and the low explanatory power of the regressions, as expressed by their R^2. Even though the situation slightly improves in the UK test, the coefficients never reach a t-Statistic high enough to be considered significantly different from zero. These issues may be explained by an excessive volatility of returns, which increases the standard errors and decreases the t-Statistics and R^2. As a matter of fact, the coefficients of the explanatory variables are very similar to if not higher in magnitude than those found by Fama and French, but their significance level is far lower.

Moreover, the consistence of the intercepts of the models and its low correlation with the risk-free rate suggest that there are more systematic factors influencing the stock returns aside from the market returns. Possible developments on this study may include searching for these macroeconomic factors and continuing the analysis of the European market, trying to see if its efficiency and predictability will improve in the future.

Tables

Table 1

3-Factor Model: Average Returns, Post-Ranking Betas and Average Size for Portfolios Formed on Size and Pre-Ranking Beta

In June of every year, I ranked the stocks in the sample by their Size, I calculated the percentiles and formed 10 portfolios according to them. Then I did the same for each of the 10 Size portfolios, this time based on their Pre-Ranking Beta, which is the Beta of each singular stock, calculated using data from 2 to 5 years prior, depending on availability. Then I calculated the average monthly returns for each of the 100 portfolios (equally weighted).
As specified in the Fama-MacBeth paper, I calculated the Betas by computing the Covariance of the stock or portfolio returns with the returns on a value-weighted portfolio of all the stocks in the sample and then dividing it by the Variance of these Market Returns. As in the Fama-French paper, I obtained the Betas of the stock returns related to both same-period and previous-period Market Returns, then I summed them up and corrected for autocorrelation. I computed the Post-Ranking Betas by using the whole sample and then assigned each portfolio's Beta to every stock that is in the portfolio year after year, so each stock is included in a Size/Pre-Ranking Beta portfolio in June and is assigned the Post-Ranking Beta of that portfolio for the next 12 months.
The average Size of the portfolio is the average logarithm of the Market Capitalization of each stock in the portfolio. The Market Capitalization is simply Price times Shares Outstanding.
All the data was obtained from Bloomberg. The period considered is July 1996 – June 2015, given the scarce availability of balance sheet data for previous years.

	All	**Low-β**	**β-2**	**β-3**	**β-4**	**β-5**	**β-6**	**β-7**	**β-8**	**β-9**	**High-β**
Panel A: Average Monthly Returns (in percentage points)											
All	1.9824	2.5169	2.7539	2.5883	2.6515	2.5856	2.1200	2.2075	1.5948	0.9415	-0.1360
Small-ME	1.5995	3.7312	1.7624	1.8257	1.9986	1.9475	1.7580	1.0155	1.5315	0.5678	0.2958
ME-2	1.5413	2.0158	1.4855	2.2517	2.0600	2.3239	1.3453	2.5952	1.5060	0.3220	-0.3058
ME-3	1.5043	2.0568	2.7418	2.0042	2.0822	1.7203	1.7121	1.8101	1.1675	0.0772	-0.2541
ME-4	1.4978	2.6181	2.2646	2.3046	2.4397	2.0061	1.2644	1.6534	0.8145	0.6343	-0.8188
ME-5	1.7342	2.0549	2.4514	2.9031	2.5569	1.9482	1.6121	1.7026	1.3780	0.8436	-0.1413
ME-6	1.7905	2.1353	2.9341	2.4674	2.5620	2.1997	1.9982	2.1427	1.4936	0.4960	-0.4232
ME-7	2.1797	2.6797	2.9111	2.7430	3.3010	2.6910	2.3866	2.5322	1.8733	1.3891	-0.6212
ME-8	2.4759	3.3827	3.0017	2.8027	3.0786	2.9977	2.2984	3.2812	2.2637	1.6655	0.1670
ME-9	2.6153	3.4906	2.9462	3.1023	2.8791	3.0119	2.8241	2.8446	2.0742	2.2335	0.7492
Large-ME	2.6699	3.4824	2.5684	3.1619	2.9394	3.1998	2.7400	2.7016	2.4301	2.2305	1.3335

	All	Low-β	β-2	β-3	β-4	β-5	β-6	β-7	β-8	β-9	High-β
Panel B: Post-Ranking Betas											
All		0.4908	0.4850	0.5250	0.6336	0.6659	0.7465	0.7945	0.9354	1.0483	1.2398
Small-ME	0.6145	0.3304	0.3522	0.3499	0.5311	0.5128	0.5851	0.7249	0.7987	0.8742	0.9665
ME-2	0.6620	0.3610	0.3971	0.4542	0.6046	0.5437	0.6330	0.5542	0.9713	0.9459	1.1214
ME-3	0.7178	0.4698	0.4369	0.4964	0.5187	0.6330	0.7878	0.7812	0.8488	1.0651	1.1092
ME-4	0.7732	0.5841	0.4772	0.5495	0.5998	0.7726	0.8152	0.7441	0.9152	1.0550	1.1575
ME-5	0.7680	0.4605	0.5551	0.5714	0.6604	0.6661	0.7776	0.7683	0.8559	1.0376	1.3170
ME-6	0.8372	0.6113	0.6534	0.5846	0.5145	0.7715	0.7219	0.9186	1.0145	1.2100	1.3460
ME-7	0.8561	0.5954	0.5630	0.6180	0.7294	0.7349	0.8029	0.8886	1.0313	1.1814	1.3904
ME-8	0.8651	0.5442	0.5483	0.7367	0.7019	0.7498	0.8774	0.8129	0.9529	1.2002	1.4787
ME-9	0.8320	0.5758	0.5340	0.6412	0.6703	0.6866	0.8260	0.8859	1.0011	1.1512	1.3520
Large-ME	0.6967	0.4024	0.5243	0.4655	0.5162	0.6610	0.7448	0.8040	0.7673	0.9178	1.1358

	All	Low-β	β-2	β-3	β-4	β-5	β-6	β-7	β-8	β-9	High-β
Panel C: Average Size (ln(ME))											
All	4.4388	3.9176	4.8991	5.0217	5.0280	5.0618	4.8843	4.7957	4.6088	4.3509	3.8474
Small-ME	1.1020	0.7583	0.8767	0.9608	0.9855	0.9824	1.0441	1.0653	1.0151	0.9177	0.9167
ME-2	2.2802	2.1914	2.1863	2.2459	2.1963	2.2305	2.2122	2.2224	2.2596	2.2095	2.2231
ME-3	2.9952	2.9552	2.9618	2.9309	2.9428	2.9590	2.9641	2.9696	2.9203	2.9033	2.9370
ME-4	3.5805	3.5541	3.5523	3.5401	3.5461	3.5734	3.4863	3.5670	3.5354	3.5067	3.4749
ME-5	4.1667	4.1266	4.1318	4.1589	4.1456	4.1140	4.1333	4.1004	4.1055	4.0988	4.0876
ME-6	4.7628	4.7040	4.7485	4.7396	4.7485	4.6911	4.7419	4.7191	4.6952	4.6814	4.6504
ME-7	5.4563	5.3873	5.4099	5.4008	5.4366	5.4029	5.3962	5.4374	5.3894	5.4026	5.3302
ME-8	6.2874	6.2640	6.2283	6.2363	6.2317	6.2172	6.2116	6.2552	6.2122	6.2044	6.0967
ME-9	7.3412	7.2885	7.2539	7.2839	7.2715	7.2935	7.2503	7.2310	7.2225	7.2102	7.1531
Large-ME	9.3793	9.4533	9.3257	9.3134	9.2763	9.0695	9.0448	9.0818	8.9300	8.8353	8.7649

Table 2

3-Factor Model: Portfolios Formed on Size and Pre-Ranking Beta: Main Properties

In June of every year, I ranked the stocks in the sample by their Size, I calculated the percentiles and formed 10 portfolios according to them. Then I did the same based on their Pre-Ranking Beta, which is the Beta of each singular stock, calculated using data from 2 to 5 years prior, depending on data availability.

For each portfolio, I calculated the average monthly Returns (equally weighted), which I obtained by adding the Dividend Yield (where available) to the percentage change in price.

The Post-Ranking Betas are calculated by using the portfolio returns for the whole sample and relating them to the Market Returns.

Then I calculated the average logarithm of the Market Capitalization of each stock in the portfolio.

The Book-To-Market value is obtained as the logarithm of Total Common Book Equity over Market Capitalization.

Two measures of Leverage are computed for each portfolio: the first one is Market Leverage, obtained as the ratio of Total Book Assets over Market Capitalization and the other is Book Leverage, which is Total Book Assets over Common Equity.

The Earnings/Price variable is obtained as the ratio of EBITDA over stock price. As in the original paper, I only used observations in which Earnings are positive, while I added a dummy variable that takes the value 1 when the EBITDA is negative and 0 when it is positive. By doing this, the E/P Dummy row in the table describes the proportion of companies in the portfolio that have negative Earnings.

All the variables are then averaged across all the stocks in the portfolio. The variables that contain balance sheet values (BE, EBITDA, A) are relative to 6 months prior to the Returns they are related to in order to allow some time for the market to process the new data, so when portfolios are formed in June each year, the balance sheet data refers to December of the previous year. I also added the average number of stocks that are in each portfolio every year.

All the data was obtained from Bloomberg. The period considered is July 1996 – June 2015.

	Small-ME	**ME-2**	**ME-3**	**ME-4**	**ME-5**	**ME-6**	**ME-7**	**ME-8**	**ME-9**	**Large-ME**
					Panel A: Size Portfolios					
Returns	1.5995	1.5413	1.5043	1.4978	1.7342	1.7905	2.1797	2.4759	2.6153	2.6699
β	0.6145	0.6620	0.7178	0.7732	0.7680	0.8372	0.8561	0.8651	0.8320	0.6967
ln(ME)	1.1020	2.2802	2.9952	3.5805	4.1667	4.7628	5.4563	6.2874	7.3412	9.3793
ln(BE/ME)	0.0774	-0.1730	-0.3023	-0.4450	-0.4198	-0.5061	-0.5106	-0.6976	-0.8842	-1.0490
ln(A/ME)	0.8543	0.7006	0.5532	0.4330	0.4387	0.3267	0.4123	0.2207	0.1058	0.0027
ln(A/BE)	0.8212	0.8755	0.8660	0.8843	0.8839	0.8332	0.9197	0.9219	0.9942	1.0591
E/P Dummy	0.2778	0.2019	0.1709	0.1435	0.1190	0.1020	0.0699	0.0400	0.0234	0.0093
E(+)/P	0.2931	0.1681	0.1927	0.1916	0.2138	0.3089	0.4757	0.7125	1.2468	5.8637
# of Stocks	232.7325	242.7368	246.8026	248.7895	249.5439	250.9211	251.8465	252.5921	252.9912	253.1711

	Low-β	β -2	β -3	β -4	β -5	β -6	β -7	β -8	β-9	High- β
Panel B: Pre-Ranking β Portfolios										
Returns	2.5169	2.7539	2.5883	2.6515	2.5856	2.1200	2.2075	1.5948	0.9415	-0.1360
β	0.4908	0.4850	0.5250	0.6336	0.6659	0.7465	0.7945	0.9354	1.0483	1.2398
ln(ME)	3.9176	4.8991	5.0217	5.0280	5.0618	4.8843	4.7957	4.6088	4.3509	3.8474
ln(BE/ME)	-0.5390	-0.6031	-0.5670	-0.5070	-0.5032	-0.4157	-0.5304	-0.5133	-0.5200	-0.5228
ln(A/ME)	0.3174	0.2999	0.3446	0.4521	0.4321	0.5854	0.4129	0.4015	0.4022	0.2563
ln(A/BE)	0.8650	0.8911	0.8862	0.9489	0.9380	0.9964	0.9447	0.9266	0.9234	0.7673
E/P Dummy	0.1327	0.0840	0.0739	0.0802	0.0815	0.0934	0.0973	0.1155	0.1508	0.2262
E(+)/P	1.1219	1.5455	1.3798	1.2643	0.9613	0.9171	1.0263	0.7619	0.7405	0.6190
# of Stocks	238.7895	251.2368	254.5921	255.5482	255.6886	256.4956	256.3070	255.8596	255.2763	253.6316

Table 3

3-Factor Model: Average Slopes and t-Statistics from the Fama-MacBeth Regressions

Each month the stock returns are regressed against the listed variables; the table shows the Time-Series Averages of the Slopes and t-Statistics obtained from the Cross-Sectional Regressions.

The t-Statistics test the hypothesis that the Slope is different from zero: they are calculated as the ratio of slope over Standard Error of the Regression (Standard Deviation of the estimates over square root of the number of observations) and is shown in the table under the Slopes, between parentheses.

The Post-Ranking Beta is assigned to each stock depending on the Size/Pre-Ranking Beta it is in at the beginning of each period.

The variable ln(ME) is logarithm of Market Capitalization (same month as the returns).

The variable ln(BE/ME) is logarithm of Book Equity over Market Capitalization (six months prior to the returns).

The variable ln(A/ME) is logarithm of Total Assets over Market Capitalization (six months prior to the returns).

The variable ln(A/BE) is logarithm of Total Assets over Common Equity (six months prior to the returns).

The variable E/P Dummy is a dummy variable that has value 1 when earnings are negative and 0 when earnings are positive (six months prior to the returns).

The variable E(+)/P is EBITDA over Price (six months prior to the returns); if Earnings are negative, its value is 0.

The number of observations in each Cross-Sectional Regression (which is also the total number of firms in the sample) is 3602.

All the data was obtained from Bloomberg. The period considered is July 1996 – June 2015.

β	ln(ME)	ln(BE/ME)	ln(A/ME)	ln(A/BE)	E/P Dummy	E(+)/P
-2.6553						
-0.1738						
	0.1198					
	0.0078					
-2.8792	0.1606					
-0.1893	0.0106					
		0.8370				
		0.0586				
-2.9177	0.3154	0.9294				
-0.2072	0.0224	0.0660				
			0.8814	-0.3497		
			0.0618	-0.0245		
					-2.7427	0.2204
					-0.1865	0.0150
	0.2877	1.0202				
	0.0202	0.0718				
	0.2544		0.9293	-0.0931		
	0.0179		0.0655	-0.0066		
	-0.1989				-2.7549	0.3226
	-0.0136				-0.1886	0.0221
	-0.0218	0.8283			-2.6581	0.2692
	-0.0015	0.0587			-0.1884	0.0191
	-0.0789		0.8594	-0.3832	-2.9487	0.2674
	-0.0056		0.0613	-0.0274	-0.2105	0.0191

Table 4

3-Factor Model: Portfolios Formed on Book-To-Market and Earnings/Price: Main Properties

In June of every year, I ranked the stocks in the sample by their Book-To-Market, I calculated the percentiles and formed 10 portfolios according to them, then I did the same based on their Earnings/Price ratio.

For each portfolio, I calculated the average monthly Returns (equally weighted), which I obtained by adding the Dividend Yield (where available) to the percentage change in Price.

The Post-Ranking Betas are calculated by using the portfolio Returns for the whole sample and relating them to the Market Returns.

I then calculated the average logarithm of the Market Capitalization of each stock in the portfolio.

The Book-To-Market value is obtained as the logarithm of Total Common Book Equity over Market Capitalization.

Market Leverage is computed as the ratio of Total Book Assets over Market Capitalization, while Book Leverage is Total Book Assets over Common Equity.

The Earnings/Price variable is obtained as the ratio EBITDA over stock price. When Earnings are positive, E(+)/P is the real E/P value and E/P Dummy is zero, while when Earnings are negative E(+)/P is 0 and E/P Dummy is 1.

The E/P Dummy row in the table also describes the proportion of companies in the portfolio that have negative Earnings.

All the variables are then averaged across all the stocks in the portfolio. The variables that contain balance sheet values (BE, EBITDA, A) are relative to 6 months prior to the Returns they are related to in order to allow some time for the market to process the new data.

All the data was obtained from Bloomberg. The period considered is July 1996 – June 2015.

	Low-BE/ME	BE/ME-2	BE/ME -3	BE/ME -4	BE/ME -5	BE/ME -6	BE/ME -7	BE/ME -8	BE/ME-9	High-BE/ME
Panel A: Book-To-Market Equity Portfolios										
Returns	0.0680	1.2767	1.8282	1.9212	2.4243	2.4168	2.2988	2.7010	2.4550	3.7107
β	0.8313	0.7814	0.7738	0.7693	0.7184	0.7030	0.7145	0.6227	0.6573	0.5641
ln(ME)	5.4868	5.4744	5.6352	5.3424	5.0954	5.0257	4.7519	4.2761	4.0826	3.2398
ln(BE/ME)	-2.2609	-1.3963	-1.0420	-0.7845	-0.5785	-0.3743	-0.1502	0.0861	0.3665	1.0769
ln(A/ME)	-0.9993	-0.4871	-0.1676	0.1643	0.3042	0.5402	0.7412	0.9635	1.2017	1.9328
ln(A/BE)	1.2537	0.9071	0.8722	0.9477	0.8817	0.9138	0.8901	0.8764	0.8300	0.8545
E/P Dummy	0.1592	0.0896	0.0791	0.0787	0.0752	0.0759	0.0841	0.0877	0.1057	0.1319
E(+)/P	0.6611	0.7342	0.8421	0.8228	0.7617	1.0163	1.1786	0.8756	1.1870	1.2211
# of Stocks	192.2544	193.5439	193.5526	193.5570	192.9781	193.5219	192.9605	191.7544	189.7807	184.8202

	Neg.-E/P	Low-E/P	E/P -2	E/P -3	E/P -4	E/P -5	E/P -6	E/P -7	E/P -8	E/P-9	High-E/P
Panel B: Earnings/Price Portfolios											
Returns	-0.1821	0.8835	1.3287	2.3557	1.4594	2.3189	2.2777	2.7991	2.7646	3.5388	4.5931
β	1.0333	0.6935	0.7467	0.7313	0.7592	0.7452	0.7196	0.6486	0.7143	0.6774	0.6418
ln(ME)	1.4438	3.3882	3.6538	4.2703	4.3385	4.7299	5.0039	5.6213	6.0243	6.6939	7.6563
ln(BE/ME)	-0.2202	-0.6176	-0.5021	-0.4990	-0.4561	-0.5420	-0.4855	-0.4147	-0.5832	-0.4283	-0.3295
ln(A/ME)	0.1039	0.1376	0.2757	0.3375	0.3810	0.3483	0.4904	0.5226	0.5663	0.7113	0.8509
ln(A/BE)	0.3275	0.7516	0.7700	0.8335	0.8920	0.8747	0.9755	0.9392	1.1285	1.1385	1.1819
E/P Dummy	1.0000	0.0436	0.0347	0.0237	0.0246	0.0215	0.0209	0.0167	0.0159	0.0110	0.0129
E(+)/P	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
# of Stocks	754.4825	143.8289	147.0658	147.7105	149.2115	148.9781	149.8377	149.4518	150.1278	149.6447	150.1798

Table 5

Average Returns for Portfolios Formed on Size and Book-To-Market Equity

In June of every year, I ranked the stocks in the sample by their Size and then I calculated the percentiles and formed 10 portfolios according to them. Then I did the same for each of the 10 Size portfolios, this time based on their Book-To-Market. Then I calculated the average monthly returns for each of the 100 portfolios (equally weighted). The Market Capitalization is simply price times shares outstanding, while the Book-To-Market is Total Common Equity over Market Capitalization.
All the data was obtained from Bloomberg. The period considered is July 1996 – June 2015.

	All	Low-BE/ME	BE/ME-2	BE/ME-3	BE/ME-4	BE/ME-5	BE/ME-6	BE/ME-7	BE/ME-8	BE/ME-9	High-BE/ME
Size/Book-To-Market Portfolios Average Monthly Returns											
All	2.1101	0.0680	1.2767	1.8282	1.9212	2.4243	2.4168	2.2988	2.7010	2.4550	3.7107
Small-ME	1.5995	0.0221	0.5876	1.2123	-0.0896	1.5874	1.7532	1.9272	2.0128	3.8311	4.8720
ME-2	1.5413	-1.4364	0.9949	-0.1072	0.9787	2.0445	2.2042	2.1543	2.6414	2.3770	3.9914
ME-3	1.5043	-0.4904	-0.8988	0.8868	1.7790	1.3373	2.2819	1.8719	1.4365	1.5172	2.5510
ME-4	1.4978	-0.3875	0.4559	1.1498	0.7345	1.7631	1.7542	1.9040	2.3044	1.7004	2.1427
ME-5	1.7342	-0.6714	0.6396	1.4699	1.3480	2.4609	2.2498	2.4409	2.3782	2.2839	3.4585
ME-6	1.7905	-0.6570	0.5333	2.1978	1.6973	2.3902	2.3183	2.6173	2.3208	2.6325	3.4013
ME-7	2.1797	-0.1979	1.1207	1.3286	2.0104	1.7422	2.7512	2.7286	2.8447	3.2070	3.4367
ME-8	2.4759	1.4751	1.5893	2.3845	1.4908	2.6381	2.3358	2.6785	2.9898	3.1724	3.0140
ME-9	2.6153	0.9983	1.6335	2.4638	3.0769	2.8181	2.6352	3.2916	2.7920	3.2377	3.7671
Large-ME	2.6699	1.5465	1.7073	2.1881	2.8848	3.1737	3.2925	3.2215	3.3326	3.3307	3.5072

Table 6

Subperiod Average Market Returns and Properties of the 2-Factor and 3-Factor Models

I calculated the Mean, Standard Deviation and t-Statistics for the Market Returns, the intercepts and the slopes of the 2-Factor and 3-Factor Models for the whole sample (June 1996 – July 2015), then I did the same for the pre-Crisis and post-Crisis periods (June 1996 – July 2007 and June 2008 – July 2015 respectively).
The 2-Factor Model has Size and Book-To-Market as explanatory variables, while the 3-Factor Model includes Post-Ranking Beta, Size and Book-To-Market.
I computed the equally-weighted (EW) Market Returns by simply averaging returns from all stocks in the sample at each month, while the value-weighted (VW) Market Returns are calculated on the basis of each firm's contribution to total Market Capitalization at each observation. Then I calculated their Mean and Standard Deviation.
The average slopes (b) and intercepts (a) derive from the Cross-Sectional Regressions of each stock's returns on the explanatory variables for each month; then I computed the Mean and Standard Deviation of the estimates.
The t-Statistics test the hypothesis that the Slopes, Intercepts and Average Returns are different from 0: they are calculated as the ratio of Mean over Standard Error (Standard Deviation of the estimates over square root of the number of observations).
The number of observations in each Cross-Sectional Regression (which is also the total number of firms in the sample) is 3602.
All the data was obtained from Bloomberg.

	07/96 – 06/15 (228 Obs.)			07/96 – 06/07 (132 Obs.)			07/07 – 06/15 (96 Obs.)		
Variable	**Mean**	**Std**	**t-Stat**	**Mean**	**Std**	**t-Stat**	**Mean**	**Std**	**t-Stat**
Value-Weighted and Equally-Weighted (VW) Market Portfolio Returns									
VW	2.0920	4.7596	6.6369	2.3887	4.5050	6.0918	1.6841	5.0846	3.2452
EW	2.1776	4.9939	6.5840	2.5334	5.6403	5.1605	2.3065	4.7433	4.7644
$R_{it} = a_t + b_{2t} \ln(ME_{it}) + b_{3t} \ln(BE/ME_{it}) + \epsilon_{it}$									
a	1.3123	5.4666	3.6249	2.4236	5.2987	5.2551	-0.1569	4.5563	-0.3373
b_2	0.2877	0.6994	0.0202	0.2129	0.8019	0.0166	0.3906	0.5134	0.0334
b_3	1.0202	1.3686	0.0718	1.3914	1.5263	0.1083	0.5097	0.8993	0.0436
$R_{it} = a_t + b_{1t}\beta_{it} + b_{2t} \ln(ME_{it}) + b_{3t} \ln(BE/ME_{it}) + \epsilon_{it}$									
a	3.3529	4.6810	10.8154	4.7725	4.9016	11.1865	1.0188	3.0844	3.2364
b_1	-2.9177	8.0885	-0.2072	-3.3353	9.4157	-0.2631	-2.3435	5.7882	-0.2012
b_2	0.3154	0.7288	0.0224	0.2368	0.8486	0.0187	0.4235	0.5058	0.0363
b_3	0.9294	1.2462	0.0660	1.2488	1.4005	0.0985	0.4903	0.8182	0.0421

Table 7

5-Factor Model: Average Returns, Average Investment, Average Operational Profitability, Post-Ranking Betas and Average Size for Portfolios Formed on Investment and Operational Profitability

In June of every year, I ranked the stocks in the sample by their Investments, I calculated the percentiles and formed 10 portfolios according to them. Then I did the same for each of the 10 Investment portfolios, this time based on their Operational Profitability. Then I calculated the average monthly returns for each of the 100 portfolios (equally weighted), and their Post-Ranking Betas.

Investment is the percentage change in the company's Total Assets.

The Operating Profitability is the company's EBITDA minus Net Interest Expenses, divided by the company's Total Assets.

The average Size of the portfolio is the average logarithm of the Market Capitalization of each stock in the portfolio.

All the data was obtained from Bloomberg. The period considered is July 1997 – June 2015.

	All	**Low-OP**	**OP-2**	**OP-3**	**OP-4**	**OP-5**	**OP-6**	**OP-7**	**OP-8**	**OP-9**	**High-OP**
Panel A: Average Monthly Returns (in percentage points)											
All	2.0244	-0.0018	0.9179	1.3456	1.9825	1.7789	2.8510	3.2795	2.9678	2.4164	2.7062
Low-Inv	1.5152	-1.4067	-0.2680	0.4268	0.9423	1.0056	1.5802	1.2327	2.3251	2.1115	2.5633
Inv-2	2.1511	0.1286	1.2345	2.2052	2.1168	2.2177	2.2231	2.1863	3.2374	3.4112	2.5053
Inv-3	2.3756	0.9022	0.8936	1.0343	2.0915	2.6114	2.8050	2.7821	2.6648	2.8217	3.4485
Inv-4	2.5317	0.6352	1.7164	2.0418	2.0068	2.7197	2.9225	2.6346	2.9492	3.1241	3.5202
Inv-5	2.8895	0.7052	2.1458	1.9964	2.8098	3.0383	2.7436	3.1179	3.3136	3.1103	3.7792
Inv-6	2.3008	1.1877	1.7945	2.2237	1.7645	2.9115	2.5475	3.1586	2.5811	3.3957	2.6969
Inv-7	2.6912	1.5173	2.6173	2.2179	2.2714	3.3932	2.8626	2.5318	2.8836	2.4401	3.0498
Inv-8	2.2275	1.2396	2.0545	1.8556	2.5033	2.9169	5.1292	2.6740	2.7393	2.6223	2.9045
Inv-9	2.0276	0.6622	1.9562	2.3041	2.0427	2.3447	2.4643	3.0678	1.9662	2.2475	2.1475
High-Inv	0.9256	-0.8809	0.0400	-0.1542	0.5043	1.5629	2.1733	1.1174	2.1114	0.7388	0.4325

	All	Low-OP	OP-2	OP-3	OP-4	OP-5	OP-6	OP-7	OP-8	OP-9	High-OP
Panel B: Average Investment (in percentage change)											
All	0.0304	-0.0047	0.0127	0.0196	0.0214	0.0246	0.0239	0.0287	0.0340	0.0496	0.0495
Low-Inv	-0.1143	-0.2330	-0.1897	-0.1500	-0.1113	-0.1253	-0.0807	-0.1162	-0.0769	-0.0989	-0.2519
Inv-2	-0.0220	-0.0395	-0.0316	-0.0238	-0.0272	-0.0255	-0.0274	-0.0227	-0.0147	-0.0173	-0.0232
Inv-3	-0.0048	-0.0099	-0.0134	-0.0221	-0.0094	-0.0060	-0.0056	-0.0058	0.0043	0.0015	0.0135
Inv-4	0.0052	0.0033	-0.0051	-0.0019	0.0017	-0.0018	0.0002	0.0070	0.0078	0.0121	0.0087
Inv-5	0.0124	-0.0021	0.0066	0.0056	0.0136	0.0099	0.0094	0.0104	0.0138	0.0206	0.0216
Inv-6	0.0196	0.0187	0.0071	0.0131	0.0100	0.0167	0.0218	0.0211	0.0232	0.0293	0.0272
Inv-7	0.0326	0.0233	0.0184	0.0280	0.0235	0.0288	0.0266	0.0389	0.0307	0.0408	0.0432
Inv-8	0.0470	0.0390	0.0330	0.0330	0.0443	0.0377	0.0383	0.0474	0.0428	0.0599	0.0700
Inv-9	0.0729	0.0533	0.0588	0.0622	0.0545	0.0592	0.0620	0.0734	0.0786	0.0808	0.1035
High-Inv	0.3182	0.3570	0.3006	0.2732	0.2291	0.1894	0.1907	0.1754	0.2022	0.2226	0.3192

	All	Low-OP	OP-2	OP-3	OP-4	OP-5	OP-6	OP-7	OP-8	OP-9	High-OP
Panel C: Average Operating Profitability (in percentage of Total Assets)											
All	5.1836	-5.5682	-0.5887	-0.1403	0.0695	0.1682	0.2617	0.3645	0.4956	0.7100	2.8813
Low-Inv	19.4204	-39.2549	-2.7853	-1.4259	-0.8205	-0.4365	-0.1070	0.0673	0.2479	0.5411	36.8618
Inv-2	-0.0247	-1.9126	-0.6453	-0.3078	-0.0836	0.0483	0.1546	0.2511	0.3800	0.5583	2.2386
Inv-3	0.1330	-1.4932	-1.2731	-0.1775	0.0538	0.1306	0.2301	0.2787	0.4045	0.5861	4.2309
Inv-4	0.1889	-1.1519	-0.2972	-0.0717	0.0589	0.1524	0.2279	0.3326	0.4120	0.5602	2.4083
Inv-5	0.0414	-1.5918	-0.2898	-0.0748	0.0954	0.1779	0.2360	0.3125	0.4477	0.5760	1.1953
Inv-6	0.3195	-0.9563	-0.2388	0.2632	0.0279	0.2055	0.2736	0.3793	0.4950	0.6466	2.4860
Inv-7	0.2586	-1.0822	-0.2338	0.0060	0.1422	0.2311	0.2991	0.4193	0.5445	0.7313	2.0126
Inv-8	0.1690	-1.7270	-0.1426	0.0962	0.1924	0.2773	0.3821	0.4824	0.6265	0.8206	1.7711
Inv-9	0.4745	-1.4164	-0.1350	0.0716	0.1893	0.2878	0.4170	0.5118	0.7242	0.9646	8.2859
High-Inv	-0.7791	-5.2974	-0.8476	-0.2979	-0.0617	0.1237	0.2928	0.4252	0.5892	0.9430	2.6948

	All	Low-OP	OP-2	OP-3	OP-4	OP-5	OP-6	OP-7	OP-8	OP-9	High-OP
Panel D: Post-Ranking Betas											
All		0.9269	0.8921	0.8167	0.7449	0.7376	0.6975	0.6712	0.7078	0.7268	0.6913
Low-Inv	0.7601	1.1665	1.0425	1.1284	0.6595	0.7430	0.8482	0.6324	0.7784	0.6593	0.8087
Inv-2	0.6827	1.0810	0.6305	0.7054	0.6801	0.5545	0.8645	0.7987	0.6399	0.6299	0.6481
Inv-3	0.7297	0.9180	0.8836	1.0248	0.7432	0.8483	0.7405	0.6455	0.7542	0.5547	0.6028
Inv-4	0.6607	0.6696	0.8398	0.6695	0.6637	0.7399	0.6692	0.7125	0.8422	0.6286	0.6743
Inv-5	0.6442	0.8165	0.7663	0.7222	0.5941	0.6657	0.7253	0.6165	0.6626	0.6521	0.5086
Inv-6	0.6933	0.9147	0.8276	0.8451	0.8106	0.5855	0.5910	0.7330	0.6930	0.6275	0.5508
Inv-7	0.6930	0.7093	0.7657	0.7397	0.7487	0.6128	0.6491	0.7529	0.6913	0.9139	0.6730
Inv-8	0.7511	0.7938	0.8600	0.7433	0.8550	0.7200	0.4802	0.6740	0.8497	0.7645	0.7067
Inv-9	0.8032	0.8122	0.8514	0.8036	0.7720	0.8089	0.7823	0.6991	0.8974	0.8765	0.8111
High-Inv	0.9090	0.9270	1.1380	0.8732	1.0515	0.9141	0.8214	1.0123	0.9251	1.1828	0.7753

	All	Low-OP	OP-2	OP-3	OP-4	OP-5	OP-6	OP-7	OP-8	OP-9	High-OP
Panel E: Average Size (ln(ME))											
All	4.5870	3.5093	4.2342	4.8308	5.0742	5.2736	5.4088	5.3995	5.3690	5.2600	4.6010
Low-Inv	3.5666	2.3773	2.8344	3.1306	3.1922	3.6248	3.6305	4.0172	4.1370	4.1002	3.2689
Inv-2	4.5641	3.5982	3.8306	4.3940	4.6341	4.5999	4.7652	4.6569	5.1340	5.2791	4.4125
Inv-3	4.9596	4.0523	4.2713	4.7595	4.9446	5.1226	5.3070	5.2297	5.3801	5.1824	4.9725
Inv-4	5.2211	3.9875	4.7198	5.1238	5.3454	5.5827	5.6817	5.4489	5.6479	5.4773	4.8254
Inv-5	5.3963	4.1615	4.8396	5.5535	5.4032	5.6354	5.7721	5.8291	5.7223	5.6009	5.0496
Inv-6	5.4669	4.6625	5.0728	5.6378	5.4882	5.6695	5.9980	5.9327	5.5581	5.6413	5.2507
Inv-7	5.4401	4.5259	5.3373	5.6544	5.8188	5.6100	5.6837	5.6495	5.6002	5.6926	5.3283
Inv-8	5.2472	4.4258	5.3463	5.1373	5.3755	5.5814	5.4266	5.5571	5.5017	5.3788	5.2054
Inv-9	4.9673	4.0712	4.8110	5.1467	5.2020	5.0922	5.1603	5.3400	5.2172	5.2330	4.7569
High-Inv	4.2474	3.1051	3.9116	4.1740	4.5663	4.4456	4.8306	4.7964	4.8180	4.4486	3.9963

Table 8

5-Factor Model: Portfolios Formed on Investment and Operating Profitability: Main Properties

In June of every year, I ranked the stocks in the sample by the company's Investment, I calculated the percentiles and formed 10 portfolios according to them. Then I did the same based on their Operating Profitability.

For each portfolio, I calculated the average monthly Returns (equally weighted), which I obtained by adding the Dividend Yield (where available) to the percentage change in price.

The Post-Ranking Betas are calculated by using the portfolio returns for the whole sample and relating them to the Market Returns.

Then I calculated the average logarithm of the Market Capitalization of each stock in the portfolio.

The Book-To-Market value is obtained as the logarithm of Total Common Book Equity over Market Capitalization.

Investment indicates the percentage change in Total Book Assets. Operating Profitability is the company's EBITDA minus Net Interest Expenses, divided by Total Assets.

The Earnings/Price variable is obtained as the ratio of EBITDA over stock price. As in the original paper, I only used observations in which Earnings are positive, while I added a dummy variable that takes the value 1 when the EBITDA is negative and 0 when it is positive. By doing this, the E/P Dummy row in the table describes the proportion of companies in the portfolio that have negative Earnings.

The Earnings Management Indicator is obtained as EBITDA minus Net Interest Expenses divided by Operating Cash Flows.

All the variables are then averaged across all the stocks in the portfolio. The variables that contain balance sheet values (BE, EBITDA, A) are relative to 6 months prior to the Returns they are related to in order to allow some time for the market to process the new data, so when portfolios are formed in June each year, the balance sheet data refers to December of the previous year. I also added the average number of stocks that are in each portfolio every year.

All the data was obtained from Bloomberg. The period considered is July 1997 – June 2015.

	Low-Inv	Inv-2	Inv-3	Inv-4	Inv-5	Inv-6	Inv-7	Inv-8	Inv-9	High-Inv
Panel A: Investment Portfolios										
Returns	1.5152	2.1511	2.3756	2.5317	2.8895	2.3008	2.6912	2.2275	2.0276	0.9256
β	0.7601	0.6827	0.7297	0.6607	0.6442	0.6933	0.6930	0.7511	0.8032	0.9090
ln(ME)	3.7781	4.7992	5.3084	5.1298	5.4037	5.6533	5.5587	5.2461	4.9347	4.7028
ln(BE/ME)	-0.4908	-0.3616	-0.3448	-0.1981	-0.3178	-0.5263	-0.4474	-0.6480	-0.8058	-0.7829
Inv	-0.1375	-0.0212	0.0014	0.0142	0.0239	0.0390	0.0479	0.0666	0.0926	0.3018
OP	15.4512	0.1835	0.2345	0.3310	0.2246	0.5280	0.4838	0.4241	0.5431	0.1517
E/P Dummy	0.2573	0.1327	0.0873	0.0620	0.0591	0.0510	0.0476	0.0465	0.0609	0.1483
E(+)/P	0.5550	0.8259	1.0483	1.1801	1.1570	1.3622	1.5161	0.8990	0.7184	0.5470
EMI	-0.0792	0.1720	0.1811	0.0456	0.2274	0.0141	0.1077	0.1129	0.1047	0.0349
# of Stocks	176.3465	178.3289	180.2061	180.4298	179.7807	179.8246	181.0088	180.9737	180.7124	178.3377

	Low-OP	OP -2	OP -3	OP -4	OP -5	OP -6	OP -7	OP -8	OP-9	High- OP
Panel B: Operating Profitability Portfolios										
Returns	-0.0018	0.9179	1.3456	1.9825	1.7789	2.8510	3.2795	2.9678	2.4164	2.7062
β	0.9269	0.8921	0.8167	0.7449	0.7376	0.6975	0.6712	0.7078	0.7268	0.6913
ln(ME)	3.9006	4.5126	4.9877	5.4163	5.4439	5.6504	5.4441	5.3852	5.4051	4.6488
ln(BE/ME)	-0.6044	-0.2633	-0.1654	-0.2160	-0.2495	-0.4231	-0.3848	-0.5661	-0.8504	-1.0533
Inv	0.0142	0.0250	0.0324	0.0397	0.0365	0.0377	0.0435	0.0595	0.0684	0.0886
OP	-5.7053	-0.3300	0.0431	0.2516	0.3304	0.4327	0.5490	0.7062	0.9206	2.0052
E/P Dummy	0.3448	0.2169	0.1543	0.0974	0.0367	0.0237	0.0207	0.0203	0.0222	0.0328
E(+)/P	0.5686	0.8933	1.0613	1.1906	1.1365	1.1805	0.9863	1.1388	0.8241	0.6189
EMI	-0.0680	-0.0232	0.0244	0.0232	0.0221	0.1465	-0.0535	0.3048	0.3663	0.2455
# of Stocks	166.5833	176.2182	170.1798	190.6667	172.3392	181.6204	171.9693	171.8816	170.8947	166.1228

Table 9

5-Factor Model: Average Slopes and t-Statistics from the Fama-MacBeth Regressions

Each month the stock returns are regressed against the listed variables; the table shows the Time-Series Averages of the Slopes and t-Statistics obtained from the Cross-Sectional Regressions.

The t-Statistics test the hypothesis that the Slope is different from zero: they are calculated as the ratio of slope over Standard Error of the Regression (Standard Deviation of the estimates over square root of the number of observations) and is shown in the table under the Slopes, between parentheses.

The variables Post-Ranking Beta, ln(ME), ln(BE/ME), E/P Dummy and E(+)/P are described in previous tables.

The variable Inv indicates the percentage change in Total Book Assets (six months prior to the returns).

The variable OP is the company's EBITDA minus Net Interest Expenses, divided by Total Assets (six months prior to the returns).

The variable EMI is an indicator of Earnings Management, and is calculated as EBITDA minus Net Interest Expenses over Cash Flow from Operations (six months prior to the returns).

The number of observations in each Cross-Sectional Regression (which is also the total number of firms in the sample) is 3223. All the data was obtained from Bloomberg. The period considered is July 1997 – June 2015.

β	ln(ME)	ln(BE/ME)	Inv	OP	E/P Dummy	E(+)/P	EMI
			-0.8090				
			-0.0553				
				0.1996			
				0.0132			
-4.0141			-0.7465				
-0.2794			-0.0520				
							-0.4415
							-0.0293
-4.4495			-0.4323	0.1417			
-0.3097			-0.0301	0.0099			
	0.1675		-0.5563	0.2116			
	0.0116		-0.0384	0.0146			
		0.7634	-0.3829	0.3895			
		0.0546	-0.0274	0.0279			
			-0.7092	0.2742			-0.3863
			-0.0479	0.0185			-0.0261
					-3.1575	0.0842	-0.5360
					-0.2116	0.0056	-0.0359
			-0.6746	0.2524	-2.6535	0.1647	
			-0.0463	0.0173	-0.1822	0.0113	
-4.2460	0.2903	0.9902	-0.3816	0.3692			
-0.3100	0.0212	0.0723	-0.0279	0.0270			
-4.1990	0.2798	0.9702	-0.2874	0.4294			-0.5304
-0.3054	0.0204	0.0706	-0.0209	0.0312			-0.0386
-3.5529		0.9904		0.5040	-1.9082	0.1415	
-0.2531		0.0706		0.0359	-0.1359	0.0101	

Table 10
Subperiod Average Market Returns and Properties of the Custom 3-Factor and 5-Factor Models

I calculated the intercepts and the slopes of the Custom 3-Factor and 5-Factor Models for the whole sample (June 1997 – July 2015), then I did the same for the pre-Crisis and post-Crisis periods (June 1997 – July 2007 and June 2008 – July 2015 respectively). The Custom 3-Factor Model has Investment, Operating Profitability and Earnings Management as explanatory variables, while the 5-Factor Model includes Post-Ranking Beta, Size, Book-To-Market, Investment and Operating Profitability.
The average slopes (b) and intercepts (a) derive from the Cross-Sectional Regressions of each stock's returns on the explanatory variables for each month; then I computed the Mean and Standard Deviation of the estimates.
The t-Statistics test the hypothesis that the Slopes, Intercepts and Average Returns are different from 0: they are calculated as the ratio of Mean over Standard Error (Standard Deviation of the estimates over square root of the number of observations).
The number of observations in each Cross-Sectional Regression (which is also the total number of firms in the sample) is 3223.
All the data was obtained from Bloomberg.

	07/97 – 06/15 (228 Obs.)			07/97 – 06/07 (132 Obs.)			07/07 – 06/15 (96 Obs.)		
Variable	**Mean**	**Std**	**t-Stat**	**Mean**	**Std**	**t-Stat**	**Mean**	**Std**	**t-Stat**
$R_{it} = a_t + b_{4t}Inv_{it} + b_{5t}OP_{it} + b_{6t}EMI_{it} + \epsilon_{it}$									
a	2.0232	5.2368	5.8335	2.4888	5.3388	7.0391	1.0481	4.3687	3.6225
b_4	-0.7092	5.1311	-0.0479	-0.5450	6.5974	-0.0408	-0.9145	2.2330	-0.0756
b_5	0.2742	1.4813	0.0185	0.4561	1.9700	0.0342	0.0467	0.1038	0.0039
b_6	-0.3863	2.9462	-0.0261	-0.6968	3.9318	-0.0522	0.0018	0.0759	0.0001
$R_{it} = a_t + b_{1t}\beta_{it} + b_{2t}\ln(ME_{it}) + b_{3t}\ln(BE/ME_{it}) + b_{4t}Inv_{it} + b_{5t}OP_{it} + \epsilon_{it}$									
a	4.1435	6.3408	9.8671	5.7146	7.4671	11.5559	1.5851	3.3462	7.1526
b_1	-4.2460	10.9288	-0.3100	-5.4941	13.5125	-0.4569	-2.6860	6.0965	-0.2340
b_2	0.2903	0.9511	0.0212	0.2650	1.1857	0.0220	0.3220	0.5332	0.0280
b_3	0.9902	2.1526	0.0723	1.4640	2.6889	0.1218	0.3979	0.8900	0.0347
b_4	-0.3816	6.6240	-0.0279	0.4239	8.5807	0.0353	-1.3885	2.2865	-0.1210
b_5	0.3692	1.4983	0.0270	0.4544	1.9767	0.0378	0.2628	0.4065	0.0229

Table 11

Subperiod Average Market Returns and Properties of the 6-Factor and Custom 4-Factor Models

I calculated the intercepts and the slopes of the 6-Factor and Custom 4-Factor Models for the whole sample (June 1997 – July 2015), then I did the same for the pre-Crisis and post-Crisis periods (June 1996 – July 2007 and June 2008 – July 2015 respectively). The 6-Factor Model has Post-Ranking Beta, Size, Book-To-Market, Investment, Operating Profitability and Earnings Management as explanatory variables, while the Custom 4-Factor Model includes Post-Ranking Beta, Book-To-Market, Operating Profitability, E/P Dummy and Earnings/Price.

The average slopes (b) and intercepts (a) derive from the Cross-Sectional Regressions of each stock's returns on the explanatory variables for each month; then I computed the Mean and Standard Deviation of the estimates.

The t-Statistics test the hypothesis that the Slopes, Intercepts and Average Returns are different from 0: they are calculated as the ratio of Mean over Standard Error (Standard Deviation of the estimates over square root of the number of observations).

The number of observations in each Cross-Sectional Regression (which is also the total number of firms in the sample) is 3223. All the data was obtained from Bloomberg.

	07/97 – 06/15 (228 Obs.)			07/97 – 06/07 (132 Obs.)			07/07 – 06/15 (96 Obs.)		
Variable	**Mean**	**Std**	**t-Stat**	**Mean**	**Std**	**t-Stat**	**Mean**	**Std**	**t-Stat**
$R_{it} = a_t + b_{1t}\beta_{it} + b_{2t}\ln(ME_{it}) + b_{3t}\ln(BE/ME_{it}) + b_{4t}Inv_{it} + b_{5t}OP_{it} + b_{6t}EMI_{it} + \epsilon_{it}$									
a	4.1650	6.4917	9.6876	5.7509	7.6604	12.7961	1.5873	3.4098	7.0288
b_1	-4.1990	11.1388	-0.3054	-5.4443	13.8319	-0.4505	-2.6424	6.0609	-0.2295
b_2	0.2798	1.0442	0.0204	0.2490	1.3155	0.0206	0.3184	0.5451	0.0277
b_3	0.9702	2.3538	0.0706	1.4236	2.9832	0.1178	0.4035	0.8983	0.0350
b_4	-0.2874	6.6933	-0.0209	0.6184	8.6521	0.0512	-1.4198	2.2990	-0.1233
b_5	0.4294	1.5305	0.0312	0.5592	2.0151	0.0463	0.2672	0.4080	0.0232
b_6	-0.5304	8.3891	-0.0386	-0.9625	11.2573	-0.0796	0.0097	0.0584	0.0008
$R_{it} = a_t + b_{1t}\beta_{it} + b_{3t}\ln(BE/ME_{it}) + b_{5t}OP_{it} + b_{7t}\,E/P\ Dummy_{it} + b_{8t}E(+)/P_{it} + \epsilon_{it}$									
a	4.9314	6.9446	10.7222	5.8365	8.9646	9.8309	2.7635	2.7107	15.3940
b_1	-3.5529	10.5536	-0.2531	-4.4885	13.0870	-0.3557	-2.3834	5.9195	-0.2072
b_3	0.9904	3.8507	0.0706	1.6604	5.0103	0.1316	0.1529	0.9147	0.0133
b_5	0.5040	3.4877	0.0359	0.7597	4.6613	0.0602	0.1844	0.3543	0.0160
b_7	-1.9082	7.5648	-0.1359	-1.2998	9.9281	-0.1030	-2.6687	2.2347	-0.2320
b_8	0.1415	0.6307	0.0101	0.2405	0.8342	0.0191	0.0177	0.0249	0.0015

Table 12

5-Factor Model (Euro Area): Average Returns for Portfolios Formed on Size and Book-To-Market and on Size and Pre-Ranking Beta

In June of every year, I ranked the stocks in the sample by their Size, I calculated the percentiles and formed 10 portfolios according to them. Then I did the same for each of the 10 Size portfolios, this time based on their Book-To-Market and then on their Pre-Ranking Beta, which is the Beta of each singular stock, calculated using data from 2 to 5 years prior, depending on availability. Then I calculated the average monthly returns for each of the 200 portfolios (equally weighted).
The method that I used to obtain the βs is specified in previous tables.
Returns are expressed in percentage points.
All the data was obtained from Bloomberg. The sample is restricted to Euro Area firms, for a total of 1617 stocks. The period considered is July 1997 – June 2015, given the scarce availability of balance sheet data for previous years.

	All	**Low-β**	**β-2**	**β-3**	**β-4**	**β-5**	**β-6**	**β-7**	**β-8**	**β-9**	**High-β**
Panel A: Size/Book-To-Market Portfolios											
All	1.9057	2.1597	2.7468	2.8056	2.8585	2.5009	2.3431	1.5984	1.4902	0.6760	-0.1218
Small-ME	1.9828	2.0676	2.0033	1.9360	2.6128	1.7904	1.9665	1.0533	4.7565	1.0347	1.7096
ME-2	1.4181	2.8345	1.3814	2.1338	2.9656	1.9267	1.4527	1.5725	1.1817	0.3036	-0.8243
ME-3	1.1624	1.9463	2.3939	1.6606	1.7974	2.5582	0.8820	1.3102	0.7782	-0.3537	-0.7211
ME-4	1.5174	1.6819	2.5800	3.1893	2.1201	1.8447	1.5870	0.4990	0.2382	0.8970	-0.5521
ME-5	1.4695	1.7221	2.2381	2.7886	2.6159	2.4183	3.0648	1.1401	1.0504	0.0788	-0.3206
ME-6	1.6672	1.9606	2.9717	2.3903	3.1446	2.5244	2.4752	1.1649	1.2578	0.5497	-0.7781
ME-7	1.8196	2.6256	3.1600	2.4715	3.0760	2.8244	1.6091	1.7510	1.2947	0.5532	-0.6371
ME-8	2.2011	3.2654	2.5736	3.3618	3.2182	3.0937	2.5512	2.0348	1.6460	1.0168	-0.0982
ME-9	2.5231	3.8078	3.1101	3.1198	3.0652	3.1396	2.9337	2.7003	2.1850	1.4823	0.4357
Large-ME	2.5646	2.6827	3.0478	3.2775	3.1049	2.7229	2.9756	2.3817	2.4649	2.0003	1.3086

	All	Low-BE/ME	BE/ME-2	BE/ME-3	BE/ME-4	BE/ME-5	BE/ME-6	BE/ME-7	BE/ME-8	BE/ME-9	High-BE/ME
Panel B: Size/Post-Ranking Beta Portfolios											
All	1.9897	0.3108	1.4126	1.6610	1.7878	2.2338	2.2989	2.3946	2.4853	2.3051	3.0071
Small-ME	1.9828	0.2234	0.6969	1.5921	1.2172	0.3023	1.3406	2.1897	1.4553	4.4953	9.6951
ME-2	1.4181	0.4227	-0.1122	0.6963	1.4226	2.6032	2.0951	1.4796	3.4851	1.4976	1.2553
ME-3	1.1624	-0.2254	0.0438	1.9964	1.0063	1.3651	2.6394	1.3166	2.0055	1.8693	2.1232
ME-4	1.5174	0.0509	1.0311	0.5030	1.4637	2.3410	1.9799	1.4971	1.8407	1.7020	2.1847
ME-5	1.4695	-0.3332	0.9094	1.2092	2.0098	1.3097	2.8584	2.8929	2.4551	1.2982	2.8638
ME-6	1.6672	0.6238	0.4536	1.4281	1.1854	3.1342	2.0901	2.2694	2.3741	2.6560	2.8266
ME-7	1.8196	0.5092	1.0288	1.2639	1.2750	1.7578	1.6356	2.8148	2.5053	2.5794	2.8691
ME-8	2.2011	1.3311	1.5823	1.8293	1.7337	2.2847	2.8931	2.7489	2.5986	2.5999	2.9624
ME-9	2.5231	0.9836	1.4450	1.9807	2.8417	3.1224	2.7906	3.0803	2.8993	3.4702	3.4399
Large-ME	2.5646	1.8058	1.5300	1.7922	2.5639	2.4416	3.0872	2.8413	3.4333	3.3695	3.2262

Table 13

5-Factor Model (Euro Area): Portfolios Formed on Size and Book-To-Market: Main Statistics

In June of every year, I ranked the stocks in the sample by the company's Size, I calculated the percentiles and formed 10 portfolios according to them. Then I did the same based on their Book-To-Market.

For each portfolio, I calculated the average monthly Returns (equally weighted), which I obtained by adding the Dividend Yield (where available) to the percentage change in price.

The Post-Ranking Betas are calculated by using the portfolio returns for the whole sample and relating them to the Market Returns.

Then I calculated the average logarithm of the Market Capitalization of each stock in the portfolio.

The Book-To-Market value is obtained as the logarithm of Total Common Book Equity over Market Capitalization.

Investment indicates the percentage change in Total Book Assets. Operating Profitability is the company's EBITDA minus Net Interest Expenses, divided by Total Assets.

The Earnings/Price variable is obtained as the ratio of EBITDA over stock price. As in the original paper, I only used observations in which Earnings are positive, while I added a dummy variable that takes the value 1 when the EBITDA is negative and 0 when it is positive. By doing this, the E/P Dummy row in the table describes the proportion of companies in the portfolio that have negative Earnings.

The Earnings Management Indicator is obtained as EBITDA minus Net Interest Expenses divided by Operating Cash Flows.

All the variables are then averaged across all the stocks in the portfolio. The variables that contain balance sheet values (BE, EBITDA, A) are relative to 6 months prior to the Returns they are related to. I also added the average number of stocks that are in each portfolio every year.

All the data was obtained from Bloomberg. The sample is restricted to Euro Area firms, for a total of 1617 stocks.

The period considered is July 1997 – June 2015.

	Small-ME	ME-2	ME-3	ME-4	ME-5	ME-6	ME-7	ME-8	ME-9	Large-ME
					Panel A: Size Portfolios					
Returns	1.9828	1.4181	1.1624	1.5174	1.4695	1.6672	1.8196	2.2011	2.5231	2.5646
β	0.5324	0.6434	0.7449	0.7584	0.7911	0.8421	0.8997	0.8783	0.8260	0.7110
ln(ME)	1.3995	2.6200	3.2966	3.8640	4.3959	4.9499	5.5862	6.3592	7.3975	9.2833
ln(BE/ME)	0.2160	-0.0597	-0.2405	-0.3305	-0.4444	-0.4885	-0.5413	-0.6096	-0.7648	-0.9067
Inv	-0.0091	0.0041	0.0307	0.0497	0.0474	0.0443	0.0518	0.0468	0.0470	0.0340
OP	-0.6661	0.3995	0.0483	0.5570	0.2302	0.3010	0.3075	0.3052	0.1182	0.2729
E/P Dummy	0.2660	0.1933	0.1541	0.1147	0.1151	0.0999	0.0664	0.0361	0.0267	0.0097
E(+)/P	0.1425	0.1106	0.1316	0.1492	0.1868	0.4455	0.3818	0.6461	1.1408	5.7667
EMI	0.1144	0.3378	0.0659	0.0628	-0.0690	-0.1485	-0.0028	0.5777	-0.0766	0.0588
# of Stocks	119.3073	128.8438	131.6771	132.7448	133.2969	134.0417	135.0365	135.3854	135.4167	135.7813

	Low-BE/ME	BE/ME-2	BE/ME -3	BE/ME -4	BE/ME -5	BE/ME -6	BE/ME -7	BE/ME -8	BE/ME-9	High-BE/ME
Panel B: Book-To-Market Portfolios										
Returns	0.3108	1.4126	1.6610	1.7878	2.2338	2.2989	2.3946	2.4853	2.3051	3.0071
β	0.9164	0.8564	0.7921	0.8572	0.7082	0.7149	0.6586	0.6747	0.7272	0.5434
ln(ME)	5.8776	5.8370	5.9364	5.7100	5.5160	5.3140	5.1075	4.7999	4.2917	3.4672
ln(BE/ME)	-2.0083	-1.2575	-0.9460	-0.7394	-0.5016	-0.3196	-0.1399	0.0923	0.3722	1.0887
Inv	0.0770	0.0555	0.0483	0.0419	0.0371	0.0341	0.0301	0.0165	0.0126	-0.0006
OP	0.1140	0.4410	0.4187	0.2166	0.2450	0.3828	0.2630	0.1662	0.1377	0.5851
E/P Dummy	0.1251	0.0741	0.0613	0.0607	0.0648	0.0666	0.0731	0.0808	0.0980	0.1491
E(+)/P	0.5608	0.6048	0.6877	0.8452	1.0031	1.0128	1.0071	1.0367	1.2028	0.7154
EMI	0.5100	0.3178	-0.2538	0.0691	0.1370	0.2405	0.0733	0.0013	0.0931	0.1740
# of Stocks	104.0000	104.9063	105.3281	105.1510	104.8698	105.1823	104.4635	103.8698	102.1719	99.0260

Table 14

5-Factor Model (Euro Area): Average Slopes and t-Statistics from the Fama-MacBeth Regressions

Each month the stock returns are regressed against the listed variables; the table shows the Time-Series Averages of the Slopes and t-Statistics obtained from the Cross-Sectional Regressions.

The t-Statistics test the hypothesis that the Slope is different from zero: they are calculated as the ratio of slope over Standard Error of the Regression (Standard Deviation of the estimates over square root of the number of observations) and is shown in the table under the Slopes, between parentheses.

The variables Post-Ranking Beta, ln(ME), ln(BE/ME), E/P Dummy and E(+)/P are described in previous tables.

The variable Inv indicates the percentage change in Total Book Assets (six months prior to the returns).

The variable OP is the company's EBITDA minus Net Interest Expenses, divided by Total Assets (six months prior to the returns).

The variable EMI is an indicator of Earnings Management, and is calculated as EBITDA minus Net Interest Expenses over Cash Flow from Operations (six months prior to the returns).

The number of observations in each Cross-Sectional Regression (which is also the total number of firms in the sample) is 1617.

All the data was obtained from Bloomberg. The period considered is July 1997 – June 2015.

β	ln(ME)	ln(BE/ME)	Inv	OP	E/P Dummy	E(+)/P	EMI
			-0.3989				
			-0.0260				
				0.1019			
				0.0065			
-3.6566			-0.5384				
-0.2434			-0.0358				
							-0.0067
							-0.0004
-3.4134			-0.3055	0.1155			
-0.2256			-0.0202	0.0076			
	0.1518		-0.4587	0.0967			
	0.0098		-0.0296	0.0062			
		0.6332	-1.0270	0.4545			
		0.0425	-0.0690	0.0305			
			-0.2136	0.0902			0.0200
			-0.0135	0.0057			0.0013
					-2.3392	0.1055	-0.0246
					-0.1482	0.0067	-0.0016
			-0.5453	-0.0292	-2.3668	0.0861	
			-0.0352	-0.0019	-0.1529	0.0056	
-3.1231	0.2041	0.6994	-1.2654	0.5145			
-0.2166	0.0142	0.0485	-0.0878	0.0357			
-2.0523	0.1686	0.4632	-1.6613	0.5645			0.0125
-0.1411	0.0116	0.0318	-0.1142	0.0388			0.0009
-1.8387		0.3219		0.4730	-1.3826	0.0316	
-0.1265		0.0221		0.0325	-0.0951	0.0022	

Table 15

5-Factor Model (UK): Average Returns for Portfolios Formed on Size and Book-To-Market and on Size and Pre-Ranking Beta

In June of every year, I ranked the stocks in the sample by their Size, I calculated the percentiles and formed 10 portfolios according to them. Then I did the same for each of the 10 Size portfolios, this time based on their Book-To-Market and then on their Pre-Ranking Beta, which is the Beta of each singular stock, calculated using data from 2 to 5 years prior, depending on availability. Then I calculated the average monthly returns for each of the 200 portfolios (equally weighted).
The method that I used to obtain the βs is specified in previous tables.
Returns are expressed in percentage points.
All the data was obtained from Bloomberg. The sample is restricted to UK firms, for a total of 800 stocks. The period considered is July 1997 – June 2015, given the scarce availability of balance sheet data for previous years.

	All	Low-β	β-2	β-3	β-4	β-5	β-6	β-7	β-8	β-9	High-β
Panel A: Size/Book-To-Market Portfolios											
All	1.6547	1.6882	2.8186	2.8101	2.6311	2.1342	2.3201	1.8160	1.4384	0.1786	-1.2880
Small-ME	0.1027	-0.4020	1.3370	0.3472	1.2370	0.5672	0.1356	0.1153	1.1951	-1.7102	-1.4359
ME-2	0.3331	0.1229	1.7148	1.7083	1.6526	0.8839	0.4486	0.3126	-0.0121	-1.8283	-1.7519
ME-3	0.8824	-0.2427	2.6997	2.6468	1.4753	1.7553	1.1177	1.2134	1.0576	-0.3017	-2.5985
ME-4	1.2081	1.5517	2.7780	2.5284	2.1524	1.8535	1.8649	1.9787	0.4875	-0.8658	-2.0303
ME-5	1.4501	2.9500	2.4338	2.6042	2.5218	2.3209	2.3907	0.9619	0.8719	-1.2665	-1.4518
ME-6	1.7649	1.9454	2.6198	2.4721	2.3011	3.1960	2.5241	2.0671	0.7903	1.4625	-1.5385
ME-7	2.2639	2.6934	2.9613	3.2329	2.3546	1.8849	3.5708	2.8957	2.0596	0.9204	0.0137
ME-8	2.9939	3.2624	3.5415	3.8148	3.4840	3.0619	3.0855	2.8746	2.8527	3.8375	0.4773
ME-9	2.7827	3.2785	3.4955	2.9710	2.7240	3.1743	3.6002	2.5297	3.2442	2.2316	0.5256
Large-ME	2.9456	3.7171	3.3560	3.6058	3.4006	3.4623	2.8671	3.0674	1.8087	2.3824	1.8758

	All	Low-BE/ME	BE/ME-2	BE/ME-3	BE/ME-4	BE/ME-5	BE/ME-6	BE/ME-7	BE/ME-8	BE/ME-9	High-BE/ME
Panel B: Size/Post-Ranking Beta Portfolios											
All	1.9501	0.1537	0.9378	1.9233	2.1485	2.1146	2.1655	2.6201	2.5733	2.5641	2.3002
Small-ME	0.1027	1.4123	0.9770	1.1488	-1.5048	1.2898	-0.8500	0.2567	1.1680	1.8158	0.8532
ME-2	0.3331	-2.0452	-1.3343	0.2899	-0.3076	2.2010	0.4653	1.8797	2.3460	0.8811	1.3855
ME-3	0.8824	0.0801	-1.2954	0.2269	1.2651	0.9705	2.7679	1.9727	2.1461	1.7256	2.3331
ME-4	1.2081	-1.3655	-1.7579	1.2485	1.0269	1.6435	2.0928	2.9322	3.1822	3.1861	1.8122
ME-5	1.4501	-1.0045	0.7329	1.3462	1.7476	1.7557	2.6727	2.5400	1.7787	2.7423	2.7958
ME-6	1.7649	-0.9768	0.8380	2.2334	1.8515	1.7539	2.0771	2.9616	2.9817	2.8808	2.4745
ME-7	2.2639	-1.3295	1.4325	1.9255	2.8384	2.2603	2.3037	2.9063	2.8337	3.4976	3.2765
ME-8	2.9939	1.6724	2.1685	2.3279	3.2652	3.3767	2.9879	3.8942	3.2435	3.6933	4.3739
ME-9	2.7827	1.8725	1.6039	2.6642	3.2712	2.7321	2.8504	3.1009	3.4532	3.3277	3.6594
Large-ME	2.9456	2.0436	2.2683	2.5385	3.3529	3.0797	3.2772	2.8153	3.1753	3.5030	3.6569

Table 16

5-Factor Model (UK): Portfolios Formed on Size and Book-To-Market: Main Statistics

In June of every year, I ranked the stocks in the sample by the company's Size, I calculated the percentiles and formed 10 portfolios according to them. Then I did the same based on their Book-To-Market.

For each portfolio, I calculated the average monthly Returns (equally weighted), which I obtained by adding the Dividend Yield (where available) to the percentage change in price.

The Post-Ranking Betas are calculated by using the portfolio returns for the whole sample and relating them to the Market Returns.

Then I calculated the average logarithm of the Market Capitalization of each stock in the portfolio.

The Book-To-Market value is obtained as the logarithm of Total Common Book Equity over Market Capitalization.

Investment indicates the percentage change in Total Book Assets. Operating Profitability is the company's EBITDA minus Net Interest Expenses, divided by Total Assets.

The Earnings/Price variable is obtained as the ratio of EBITDA over stock price. As in the original paper, I only used observations in which Earnings are positive, while I added a dummy variable that takes the value 1 when the EBITDA is negative and 0 when it is positive. By doing this, the E/P Dummy row in the table describes the proportion of companies in the portfolio that have negative Earnings.

The Earnings Management Indicator is obtained as EBITDA minus Net Interest Expenses divided by Operating Cash Flows.

All the variables are then averaged across all the stocks in the portfolio. The variables that contain balance sheet values (BE, EBITDA, A) are relative to 6 months prior to the Returns they are related to. I also added the average number of stocks that are in each portfolio every year.

All the data was obtained from Bloomberg. The sample is restricted to UK firms, for a total of 800 stocks.

The period considered is July 1997 – June 2015.

	Small-ME	ME-2	ME-3	ME-4	ME-5	ME-6	ME-7	ME-8	ME-9	Large-ME
Panel A: Size Portfolios										
Returns	0.1027	0.3331	0.8824	1.2081	1.4501	1.7649	2.2639	2.9939	2.7827	2.9456
β	0.9155	0.9238	0.9060	0.8974	1.0235	0.9564	0.9959	0.9419	0.8304	0.6167
ln(ME)	0.9421	2.0147	2.7158	3.3356	3.9403	4.6099	5.3626	6.2828	7.3295	9.2157
ln(BE/ME)	0.0104	-0.1900	-0.3818	-0.3841	-0.5331	-0.6807	-0.8237	-0.9819	-1.0892	-1.1787
Inv	-0.1453	0.0300	0.0030	0.0578	0.0717	0.0715	0.0739	0.0487	0.0476	0.0439
OP	85.5279	-1.0196	-1.2050	-0.2090	-8.1347	0.1428	0.3498	0.4553	0.3868	0.3550
E/P Dummy	0.3314	0.2770	0.2414	0.2118	0.1806	0.1517	0.0982	0.0369	0.0193	0.0084
E(+)/P	0.1277	0.2555	0.2632	0.3866	0.3116	0.5534	1.0428	1.5259	2.4906	10.6946
EMI	-0.0946	0.0328	0.0216	-0.8787	0.1776	0.5241	0.3775	0.0245	0.0830	-0.0051
# of Stocks	62.1354	63.9479	63.8073	64.1563	64.0573	64.7552	64.3177	64.2396	64.4375	64.3750

	Low-BE/ME	BE/ME-2	BE/ME -3	BE/ME -4	BE/ME -5	BE/ME -6	BE/ME -7	BE/ME -8	BE/ME-9	High-BE/ME
Panel B: Book-To-Market Portfolios										
Returns	0.1537	0.9378	1.9233	2.1485	2.1146	2.1655	2.6201	2.5733	2.5641	2.3002
β	0.9691	0.9234	0.8659	0.8507	0.9225	0.9575	0.9114	0.8535	0.8425	0.8519
ln(ME)	5.6731	5.6810	5.7744	5.6482	5.2361	4.9778	4.7929	4.3637	3.7986	3.1327
ln(BE/ME)	-2.3740	-1.5592	-1.2162	-0.9406	-0.7402	-0.5288	-0.3170	-0.0748	0.2178	0.8018
Inv	0.1071	0.0704	0.0596	0.0714	0.0518	0.0378	0.0167	0.0210	0.0018	-0.0510
OP	-0.5384	-0.0252	0.1980	0.0966	0.1081	0.0847	0.1067	0.0016	0.0041	-0.3743
E/P Dummy	0.1981	0.1296	0.1180	0.1076	0.1230	0.1184	0.1207	0.1198	0.1497	0.1714
E(+)/P	1.3317	1.3007	1.8330	1.6348	1.5355	1.3652	1.4354	1.7719	2.4554	2.6251
EMI	0.1453	0.0548	0.0620	0.1362	0.1082	0.4624	0.2119	-0.0150	-0.0929	0.2303
# of Stocks	56.3556	56.7222	56.6833	56.6500	56.3444	56.5944	56.6278	56.5722	56.3778	56.4556

Table 17

5-Factor Model (UK): Average Slopes and t-Statistics from the Fama-MacBeth Regressions

Each month the stock returns are regressed against the listed variables; the table shows the Time-Series Averages of the Slopes and t-Statistics obtained from the Cross-Sectional Regressions.

The t-Statistics test the hypothesis that the Slope is different from zero: they are calculated as the ratio of slope over Standard Error of the Regression (Standard Deviation of the estimates over square root of the number of observations) and is shown in the table under the Slopes, between parentheses.

The variables Post-Ranking Beta, ln(ME), ln(BE/ME), E/P Dummy and E(+)/P are described in previous tables.

The variable Inv indicates the percentage change in Total Book Assets (six months prior to the returns).

The variable OP is the company's EBITDA minus Net Interest Expenses, divided by Total Assets (six months prior to the returns).

The variable EMI is an indicator of Earnings Management, and is calculated as EBITDA minus Net Interest Expenses over Cash Flow from Operations (six months prior to the returns).

The number of observations in each Cross-Sectional Regression (which is also the total number of firms in the sample) is 800. All the data was obtained from Bloomberg. The period considered is July 1997 – June 2015.

β	ln(ME)	ln(BE/ME)	Inv	OP	E/P Dummy	E(+)/P	EMI
			-0.9280				
			-0.0650				
				0.0905			
				0.0062			
-5.2478			-0.8640				
-0.3712			-0.0611				
							0.0209
							0.0014
-5.0671			-0.9400	0.1158			
-0.3607			-0.0669	0.0082			
	0.2399		-0.9664	0.1567			
	0.0170		-0.0686	0.0111			
		0.5908	-1.0256	0.3837			
		0.0424	-0.0736	0.0275			
			-0.9370	0.1558			0.0486
			-0.0657	0.0109			0.0034
					-4.2729	0.0184	0.0391
					-0.2963	0.0013	0.0027
			-0.9176	-0.0089	-4.2012	0.0169	
			-0.0649	-0.0006	-0.2972	0.0012	
-4.5350	0.2857	0.7671	-1.1059	0.2401			
-0.3302	0.0208	0.0559	-0.0805	0.0175			
-4.5236	0.2899	0.7723	-1.1092	0.2396			0.0473
-0.3298	0.0211	0.0563	-0.0809	0.0175			0.0034
-3.7383		0.5723		-0.0612	-3.7993	0.0081	
-0.2679		0.0410		-0.0044	-0.2723	0.0006	

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Summary

In this thesis I analyzed the theory of asset pricing, the main asset pricing models and I particularly focused on the work of Fama and French. Then I ran some empirical tests on their 3-Factor and 5-Factor models on the European Union, Euro Area and United Kingdom stock markets.

I started by providing the definition of risk averse investor: this concept describes the market operators that is not willing to take on more risky investments unless they are rewarded with higher returns. According to Bodie, Kane and Marcus, most real people behave this way.

Since it is impossible to observe risk or expected returns of financial products in real life because they are constantly influenced by unpredictable events, we have to estimate them by using statistical processes and historical records.

According to Fama (2014), the research on the tendencies of the capital markets began between the 1950s and the 1960s, taking place especially at the MIT and at the University of Chicago. The first problem to address was whether the financial markets reflected all the available information and, if not, what kind of information could be exploited to predict their future behavior.

Fama (1970) elaborated the Efficient Market Hypothesis, to schematize the issue. There are three forms to the EMH: the Weak Form, which requires that current prices reflect all the information contained in past prices, the Semi-Strong Form, which requires that current prices reflect all publicly available information, and the Strong Form, which requires that current prices reflect all information. If financial markets satisfy the EMH, it means that expected returns on stocks are not predictable by using available information.

Another popular hypothesis at that time was the Random Walk hypothesis: according to it, returns are not only unpredictable in their expected value, but also in their distribution. Asset pricing models are therefore not only an attempt to predict the fluctuations of the market for practical reasons, but also means to test the efficiency of such markets.

According to Fama (2014) these market inefficiencies might be a consequence of financial market operators behaving differently from the rational risk-averse investor ("irrational behavior"). There is a whole branch of financial economics involved with the study of the behavior of investors called behavioral finance, but some (like Fama himself) consider it unlikely to be able to produce empirical contributions to asset pricing.

Modigliani and Miller (1961) formulated the theory that stock price evolutions depend on the expectations of future dividends and this is a widely accepted theory, but Shiller (1981) finds that volatility in stock prices is too large to be explained by the change in expectation of future dividends. Whether this excessive volatility is explained by rational or irrational behavior and whether it can be explained by a model is still a matter of debate. It is also possible, as Fama and French (1989) say, that irrational behavior can itself be explained by external variables, like economic conditions.

According to Bodie, Kane and Marcus (2011), from a practical point of view, asset pricing is involved with the elaboration of statistical and mathematical processes aimed at finding the "efficient portfolio", which offers the best trade-off between risk and return. Common practice is to measure the risk associated to a security by the historical standard deviation of its returns, while expected returns are calculated by averaging historical returns, maybe relative to a certain period. This is why time series analysis is such a big part of asset pricing.

Generally, the main assumption made about returns is that they are normally distributed. This is not true since they usually exhibit some degree of skewness (asymmetry, mostly towards the left side of the distribution) and kurtosis (fat tails, higher probability of extreme results), but most studies assume normality anyway for simplicity.

When planning an investment a good idea is to assess the investor's level of risk aversion. This is not easy, but a way to control the risk on a portfolio is to use a riskless asset, which is generally identified with an interbank loan or a government bond. The riskless asset has zero risk attached to it (at least in theory) and pays the risk-free rate (RFR), which can be interpreted as the intertemporal cost of money.

The first asset pricing model, developed by Harry Markowitz in his 1952 paper uses average returns, standard deviation of returns and the RFR as its basis. According to the author, the investment process has two stages: creation of expectations on future fluctuations (based on experience) and construction of the portfolio (according to these expectations). The variables that we need to calculate from a mathematical point of view are the weights to assign to each asset. We find them by maximizing the trade-off between expected returns and standard deviation. This implies that the portfolio must be heavily diversified in order to eliminate all firm-specific risk, thus leaving only systematic risk to be dealt with. This is done in the model by estimating the covariances of all assets with each other in order to find the weights that minimize the aggregate standard deviation of the portfolio. Markowitz designs a plane in which we can represent all the different combinations of standard deviation and expected returns that we can obtain and postulates the existence of an "efficient frontier" that contains all the portfolios that have the maximum expected returns for each level of standard deviation.

Tobin (1958) introduces the "separation property", which consists in introducing the risk-free asset: it lies on the Y-axis, as it has standard deviation equal to zero. Among the lines that pass through the risk-free asset and are tangent to the efficient frontier, we will choose the one with the maximum slope and we will call it Capital Allocation Line (CAL): its point of tangency with the efficient frontier indicates the efficient portfolio. Each investor will then choose the combination of efficient portfolio and risk-free asset that best suits their level of risk aversion.

This is called Mean-Variance-Covariance approach and is still widely used among practitioners. The Markowitz model has been criticized for its excessive reliance on the assumption of normality of returns, which has been disproven and therefore standard deviation may not be an adequate measure of risk (Bodie, Kane, Marcus, 2011). This criticism gave way to the Capital Asset Pricing Model.

The CAPM was introduced independently by Sharpe (1964), Lintner (1965) and Mossin (1966), it was later expanded by Black (1972) and is therefore also known as Sharpe-Lintner-Black (SLB) Model. According to Bodie, Kane and Marcus, it is a corner stone of modern finance, and arguably the model that inspired the most related studies, researches and tests. Its assumptions are

- There are many investors, all equal in their wealth and all price-takers.
- Same time horizon for all investors.
- All securities are publicly traded and there is no debt restriction.
- No taxes or transaction costs.
- All investors are mean-variance optimizers.
- Homogeneous Expectations.

The consequence of these assumptions is that investors choose the same risky portfolio and only change their exposure to it and to the risk-free asset according to their risk aversion. This in turn means that the optimal risky portfolio is nothing but the market portfolio, which contains all the assets in the market, weighted by their value (or, alternatively, equally weighted). The Capital Allocation Line is now called Capital Market Line, all portfolios will lie on it and all investors will choose their optimal portfolio along this line.

According to this model, all assets' excess returns have a linear relationship with the market excess returns (which are returns minus RFR). The slope of this relationship is called β, also defined as the covariance between asset and market returns over the variance of the market returns. In the model, all information about a stock is accounted for in its β, which tells us how a stock responds to market fluctuations, so long and complicate estimations of covariances between all assets are no longer needed. The CAPM may rely on excessively restrictive assumptions, but it is a simple and quick way to value a stock and is still popular: the β is still considered the main measure of market risk exposure.

Attempts to test the model empirically proved to be a challenge: the most popular papers on the matter were by Black, Jensen and Scholes (1972) and Fama and MacBeth (1973). The first research had a time-series approach: they regressed returns on portfolios built on ranked values of single-stock βs against market returns in order to obtain estimations of portfolio βs, which are more precise than single-stock estimations. They rejected the assumptions of the model but proved the existence of a positive effect of β on market returns, as well as the existence of another unrelated systematic factor that served as intercept of the model, called α.

Fama and MacBeth (1973) came to similar conclusions using a cross-sectional approach instead. They used the traditional model to estimate the coefficients though regressions, and then used another equation to run other regressions to test the validity of the model.

The conditions they tested are:

- The β has a linear effect.
- There is no other systematic risk factor.
- There is a positive trade-off between risk and return.
- The intercept of the test equation coincides with the RFR (Sharpe-Lintner Hypothesis).
- The hypotheses are "fair games", which means that although exceptions may be observed ex-post, it is still convenient for the investor to act like the CAPM is valid.

Fama and MacBeth, like Black, Jensen and Scholes, use a division in portfolios to estimate the βs, but they run the regressions for different stocks at the same time (cross-sectional approach) instead of running them for the different observations of the same stock (time-series approach). Then they average the monthly estimations to obtain the coefficients. The results they obtain seem to suggest that the first three hypotheses hold, or at least they are "fair games". Serial autocorrelations are also low, meaning that the behavior of the market cannot be predicted by looking at previous prices. On the other hand, the S-L Hypothesis is rejected, so there is still a part of stock returns that remains unexplained.

Fama (2014) points out how contradictions to the CAPM soon started to come out, while Roll (1977 moved a critique on the testability of the model, claiming that a market portfolio with the needed properties to test it is unobservable.

The Arbitrage Pricing Theory came to prominence after the CAPM, as a response to the excessive strictness of its assumptions. In fact, this is a very versatile and flexible theory and it has been applied in many different ways. It was developed by Ross (1977) and it assumes nothing with regards to investor preferences and distribution of returns; its only assumptions are:

- Security-specific risk is diversifiable.
- Markets do not allow for persisting arbitrage, so if a security has more returns than another does, it must depend on certain risk factors.
- These risk factors and their association with returns can be explained by a multi-factor model.

Therefore, we can say that a multi-factor model is a CAPM with more explanatory variables besides the market returns, each with its own β, which describes the asset's exposure to that particular factor.

The Arbitrage Pricing Theory has been tested by Roll and Ross (1980) and Cho, Elton and Gruber (1984) among others: both papers state that there is strong evidence pointing toward the existence of factors at influencing the market that are not accounted for in the CAPM. However, Dhrymes, Friend and Gultekin (1984) found some limitations in the techniques used to test the theory. Shanken (1982) altogether doubts the testability of the theory and says that it is only concerned with finding correlations and is ultimately inadequate to identify the drivers of stock returns from an economic point of view. He bases this critique on the fact that linear dependence of returns on factors is not implied and the tests are arbitrary in their choice of assets and factors. He also extends the Roll (1977) CAPM critique to the APT. Black (1993) also warns against the continuous search of variables that correlate with returns, even by chance and temporarily, which is called "data snooping". Dybvig and Ross (1985) among others refute Shanken's critique: they justify linear dependence with the no-arbitrage assumption, reject the extension of the Roll critique and dismiss the accusations of arbitrariness as "misleading".

Chen, Roll and Ross (1986) tested their own multi-factor model and found relevant relationships between some variables describing macroeconomic tendencies and the returns on stocks. These variables are of course proxies of risk factors that were described by Burmeister et al (1994):

- **Confidence Risk**: The risk of unanticipated changes in the investors' risk aversion. It is measured as the spread between risky corporate bonds and government bonds. Smaller stocks are typically more exposed.
- **Time Horizon Risk**: The risk of unanticipated changes in the investor's time horizon. It is measured as the spread between long-term and short-term government bonds. Growth stocks are typically more exposed.
- **Inflation Risk**: The risk of unexpected changes in inflation. It is measured as the spread between actual inflation and its previous month's expectation. Luxury stocks are typically more exposed and the exposure is usually negative.
- **Business Cycle Risk**: The risk of unanticipated changes in business activity. It is measured as the difference in value of specific indexes that measure real business activity. Stocks of companies whose activity depends more on the state of the business cycle are generally more exposed.
- **Market Timing Risk**: This is a measure of the variation in market returns that are not explained by the previous variables. Its coefficient can be compared to the CAPM's β, just like the APT can be thought of as a generalization of the SLB model.

This expanded CAPM is proven to work better than the original.

Between the 80s and the 90s applications of the Arbitrage Pricing Theory started to contemplate both systematic and firm-specific risk. This kind of approach merged asset pricing and fundamental analysis and many market anomalies were discovered, leading to the Fama-French 3-Factor Model, which, according to Cochrane (1999b), is the most popular multi-factor model ever created.

The purpose of Fama and French in their 1992 and 1993 papers is to build an asset pricing model that exploits the CAPM's β as well as proxies for two other risk factors to predict returns on stocks. These risk factors are the company's size (proxied by its market capitalization) and the value of the stock (proxied by the company's book-to-market ratio). This paper spawned numerous expansions, tests and related studies.

The patterns found and analyzed by the two authors were discovered in previous studies, soon after the CAPM became widely known and researchers started finding anomalies to its application. The so-called "size effect" was discovered by Banz (1981), who noticed how smaller stocks tended to have unusually high returns. It is worth noting that his methodology was similar to Fama and MacBeth (1973). While many researchers like Reinganum (1981) confirmed this effect, Brown, Kleidon and Marsh (1983) and Bhardwaj and Brooks (1993) found that there are periods in which relationships among β, size and returns are reversed. According to many academics like Amihud and Mendelson (1986) and Liu (2006) among others, the size effect should depend on the fact that smaller firms are generally more risky and their securities more illiquid, so their investors have to be rewarded with more returns. However, Banz himself suggests that it may depend on reporting inconsistencies.

Basu (1977) found a negative relationship between a company's price/earnings ratio and its stock returns, which, he suggests, may be due to market overreaction to news about earnings: the anomaly is due to the adjustment of prices after the frenzy. Stattman (1983) acknowledged this anomaly, called it "value effect" and used the book equity to market equity ratio as a proxy. Petkova and Zhang (2003) suggest that it is not due to overreaction, but rather to a rational increase in returns due to high book-to-market companies performing badly in case of economic distress. Chan and Chen (1991) conduct a similar analysis and agree with the rational explanation, but do not exclude that irrational behavior can amplify the effect. Chan, Hamao and Lakonishok (1991) find that this effect is relevant in the Japanese market as well.

Bhandari (1988) found also a leverage effect using a debt/equity ratio as proxy. Fama and French tested this effect but used the asset/market equity and asset/book equity ratios instead. This effect comes as a surprise, as the role of leverage should be included in the β.

These variables were used as proxies to explain anomalies in the CAPM, but since they were composed by roughly the same fundamentals, Fama and French suggested that some might have been redundant.

In the 1992 paper, the authors use a cross-sectional approach: they build portfolios based on size, β and book-to-market to find return patterns and then they run the cross-sectional regressions to find their explanatory power. Their conclusions are the following:

- The CAPM β's role is almost nullified: it has a strong correlation with returns, but also with size and its role vanishes when this correlation is accounted for.
- Size has a robust negative correlation with average returns, which also seems to absorb the role of the β.
- Book-to-market has a positive relationship with returns which is even stronger than the size effect.

- Market and book leverage both have a strong effect, but it is almost exactly opposite and it is absorbed by book-to-market.
- The earnings/price ratio has a positive effect (except when earnings are negative), but this effect too is absorbed by size and book-to-market.

In their 1993 paper, Fama and French use a more practical approach, based on Black, Jensen and Scholes (1972). Here, they build portfolios that are short big and low book-to-market companies and long large and high book-to-market companies. The returns on these portfolios are then used as proxies of the risk factors and are called SMB (Small-Minus-Big) and HML (High-Minus-Low). Then they tested the stock returns against market returns and these risk factors.

The results confirmed the previous paper's conclusions. They also tried a similar test for bond returns against default risk and maturity, but were unsuccessful.

This model however may have troubles in being tested in Europe due to the idiosyncrasies that the market presents, much like Modigliani, Pogue and Solnik's 1973 test of the CAPM on the European market had conflicting results and suffered from lack of reliable data. These inefficiencies are still active in the market according to Foye, Mramor and Pahor (2013). Even though information availability has of course been improved, the European market still presents inefficiencies and financial and political crises in the last 10 years are contributing to mistrust and lack of confidence, not to mention that according to Alford, Jones and Zmijewski (1993), even countries like Germany, Belgium and Switzerland present high inaccuracy of balance sheet information.

In order to test the model I gathered data from all 28 EU countries, for a total of 3602 firms, for the period June 1996 – July 2015. I only considered companies that had data for at least 5 years and I used their primary common stock. Only active stocks of non-financial firms were part of the analysis because of their high leverage. All the data is from Bloomberg. In order to correct for information lags, I associated returns with balance sheet data from six months prior. I used the 1-month LIBOR as risk-free rate up to December 1999 and the 1-month EURIBOR afterwards. I gathered the following data for each company:

- Current Shares Outstanding.
- Last Price.
- Dividend Yield.
- Earnings Before Interests, Taxes, Depreciation and Amortization.
- Total Common Equity.
- Total Assets.

Then I built the following variables:

- Stock Returns (percentage change in price plus dividend yield).

- Market Capitalization (price times shares outstanding).
- Book-To-Market (book equity over market capitalization).
- Market Leverage (total assets over market capitalization).
- Book Leverage (total assets over book equity).
- Earnings/Price Dummy (a dummy variable that takes value 1 when earnings are negative and 0 when they are positive)
- Earnings/Price Ratio (EBITDA over price).
- Market Returns (returns on a value-weighted portfolio of all assets in the sample).

I estimated the CAPM β as the covariance between stock and market returns over the variance of market returns. I used both same-period and previous-period market returns to correct for information lags, I summed the two βs up and then I corrected the estimate for the autocorrelation of market returns. Each year in June, I used data from to 2 to 5 years prior to calculate the single stock βs; then I ranked the stocks by their β and formed 10 portfolios accordingly; then I calculated the portfolio βs using the whole sample and assigned it to every stock in the portfolio each year. I did this because portfolio β estimations are more precise and stable.

I did similar portfolio divisions according to size, β, book-to-market and earnings/price. Besides, I also sub-divided each of the 10 portfolios in order to create 100 portfolios sorted by size/β and size/book-to-market. Then I calculated the equally-weighted average of returns and other variables for each portfolio in order to find regularities and clear patterns.

I proceeded by testing the Fama-MacBeth linear regression models, in which the dependent variable (stock returns) is regressed against several sets of explanatory variables. The approach is cross-sectional: each stock's returns at a given time are regressed against the independent variables relative to the same period, and then the results are averaged across periods. This way, each regression has a higher number of observations (N=3602) than it would have in a time series model (T=228). I then computed the t-Statistics for each intercept and slope that I estimated in order to test if they are significantly different from zero. The t-Statistic is calculated as the estimated parameter over the standard error of the regression. I then proceeded to run the Fama-MacBeth cross-sectional regressions, using various combinations of the explanatory variables. Regressions 5 (on ln(ME) and ln(BE/ME)) and 8 (on β, ln(ME) and ln(BE/ME)) are also run for two subperiods: the pre-Crisis subperiod (from July 1996 to June 2007) and the post-Crisis subperiod (from July 2007 to June 2015).

In Table 1 we can see the main characteristics of the 100 size/β portfolio, re-built each year according to the size and pre-ranking β percentiles. The sample is composed by 3602 stocks for the 1996-2015 period. In Panel A we can see the average portfolio returns across the whole period. Although this pattern is not exactly precise, we can see a tendency for low-β stocks to have higher returns, which is peculiar, since usually high-β stocks tend to perform better. Largest stocks also seem to perform better and this is another contradiction to the Fama-French results. This may be a consequence of "flight to quality" during the economic downturn. Panel B shows the post-ranking βs of these portfolios and they are fairly consistent with the pre-ranking βs as predicted, while the negative relationship between β and size is not observed, which is also clear in Panel C, where average portfolio sizes are reported.

In Table 2 I included additional statistics on size portfolios and pre-ranking β portfolios. The main regularities I could find are the weak increasing relationship between size and returns and between size and post-ranking β, but the variation is too little to say anything conclusive. Book-to-market shows a significant decreasing pattern with respect to size and so does market leverage, while book leverage is decreasing in size, so in these cases the Fama-French results are confirmed. The average E/P Dummy describes the percentage of companies in the portfolios that have negative earnings and its decreasing pattern (as well as the increasing pattern of earnings/price) confirms the expectations.

Panel B shows the same statistics for the pre-ranking β portfolios. We can see a weak negative relationship between β and returns (which is actually there only for the high-β half of the portfolios) and the post-ranking βs follow the portfolio ranking. The book-to-market ratio shows no clear relationship with β and neither do market leverage and book leverage, so the Fama-French results are more or less confirmed, while high-β portfolios tend to have slightly higher average earnings/price ratios and a lower percentage of firms with negative earnings.

Table 3 shows the results of the Fama-MacBeth Regressions and my conclusions are:

- The role of β seems to be reversed, as its coefficient is negative.
- The role of size is reversed as well: its coefficient is now positive. However, it turns negative when the earnings/price ratio is included in the regression.
- The coefficients of book-to-market, market leverage, and earnings/price are positive, while the one for book leverage is negative, all as predicted. The E/P Dummy has a negative role in my analysis, while its coefficients vary in the original paper (depending on the presence of leverage and book-to-market).
- All the regression coefficients have very weak t-Statistics, indicating that they are not significant, while the R^2 are all under 0.1. Since the coefficients are similar (and even higher) in magnitude to the Fama-French coefficients, this may be a consequence of excessive volatility in the market.

Table 4 shows the statistics of book-to-market and earnings/price portfolios. Going from lowest to highest, book-to-market portfolios (shown in Panel A) seem to have increasing returns as predicted. Post-ranking βs are slightly decreasing and so is average size; however, dispersion is low for both of them. We can also see how the majority of portfolios have negative book-to-market. As expected, market leverage is strongly increasing and book leverage is decreasing. E/P is slightly increasing, but with some exceptions, while the variation of the E/P Dummy is too low to talk about a real relationship. In Panel B we can see the same statistics of the E/P portfolios. It should be noted that there actually are 11 E/P portfolios instead of the usual 10: one of the portfolios is set aside for negative-earnings companies. Portfolio returns are strongly increasing in E/P and Neg-E/P does not have higher returns than other portfolios. Neg-E/P also has a higher post-ranking β than the other portfolios, which show no relationship with β: this is consistent with the results of Fama and French. A clear increasing pattern can be found with average size and both measures of leverage, while no pattern can be found for book-to-market. In my case, Neg-E/P follows the general tendencies, while in the original paper is had higher leverage and returns. This portfolio also contains five times as many stocks as the other portfolios in average.

Table 5 presents the average returns of the 100 size/book-to-market portfolios. We notice that the size effect is reversed for big stocks, but it does not show particular tendencies for small stocks and for higher book-to-market portfolios. Book-to-market generally has a positive influence on returns, but small and low-book-to-market portfolio returns seem to be more dispersed.

In Table 6 we can see the sub-period average market returns and results of regressions 5 and 8, including size and book-to-market and β, size and book-to-market respectively. Market returns decrease in the post-Crisis period, as expected. The intercepts of the regressions are actually very high and surprisingly significant and they may signal the existence of systematic factors that are not accounted for in the regression. They increase when β is included in the regression and this indicates that while many high-β stocks tend to underperform (the coefficient of the β is negative), they receive a premium from other systematic risk factors. These risk factors decrease in the post-Crisis periods as the t-Statistics of the intercepts go down. The coefficients on the β are consistently negative, have a very high standard deviation and are not very significant. The coefficients of the size factor are positive but have very low significance, while book-to-market has positive coefficients, but again, significance is low.

The intercepts of the model are not significantly correlated with the RFR, so the latter does not capture the systematic risk to which the stocks are exposed.

In the following section of the thesis, I analyzed the subsequent expansions to the 3-Factor Model and tested the Fama-French 5-Factor Model.

Jegadeesh and Titman (1993) found the first anomaly to the 3-Factor model: the momentum effect. This effect consists in abnormal excess returns for stocks that performed well in the last 3 to 12 months. The authors suggest that the market overreacts to news initially, but then prices go back to their long term values, hence the reversal. Hong Lim and Stein (2000) found evidence that the momentum effect works mainly for stocks that are small or are less covered by analysts, which is more true for past losers than for past winners, which indicates that the effect is due to gradual diffusion of information.

Carhart (1997) includes this effect in a model as well as the three Fama-French factors. He finds that mutual funds that performed well in the last year seem to have a better performance, but they also have higher expenses. He suggests that the performance of fund really depends on their ability to cut transaction costs and other expenses rather than on better stock-picking skills. Fama and French come to the same conclusions about mutual funds in their 2010 paper, while they confirm the momentum factor across the world (except Japan) in their 2012 work. However, as the momentum factor is clearly due to market inefficiency and does not relate to fundamental analysis, I did not test it.

Instead, I focused on the Fama-French 5-Factor Model, which includes investment and operating profitability as added explanatory variables. Fama and French started their work on the model in 2006, when they analyzed the Modigliani-Miller (1962) theory of discounted dividends, which postulates that stock prices are calculated as the discounted value of future dividends, at least in the long run. As a consequence, they found that the change in book equity (investment) should have a negative effect on stock returns, while the earnings/book equity ratio (profitability) should have a positive effect together with the book-to-market ratio. This paper presents a problem of collinearity of the selected proxies, but it is resolved by using change in assets instead of change in book equity as a proxy for investment, by lagging the book-to-market variable and by adding Piotrosky and Ohlson's measures of firm stability and default risk. The question about rationality or irrationality of these effects remains however unsolved. In 2013, Novy-Marx proposes an alternative measure of profitability called "gross profitability" which is gross profits over total assets. Aharoni, Grundy and Zeng (2013) conduct the Fama-French research at a firm level (instead of per-share level) and obtain slightly better results.

Fama and French elaborate their new 5-Factor model in 2014 by following the same procedure that they did in their 1993 analysis, except adding the two new factors (investment and profitability). Once again, the factors were proxied by returns on portfolios called RMW (Robust Minus Weak) for profitability and CMA (Conservative Minus Weak) for investment. They find that HML (the book-to-market proxy) is now redundant, so they substitute it with HMLO (High-Minus-Low Orthogonal), which is the sum of intercepts and errors of the regression of HML on the other explanatory variables. Their test statistic (elaborated by Gibbons, Ross and Shanken in 1989) rejects the model, but the authors find consistent patterns and find that the new model works better than the old one.

Foye, Mramor and Pahor (2013a) found that European markets are not efficient, so they introduced introduced a new variable called the Earnings Management Indicator. This should be an indicator of the balance sheet reporting inaccuracies of European firms and is proxied by the ratio earnings/operating cash flow. They test this variable together with the other Fama-French factors (2013b). Another interesting expansion to the Fama-French Model is the BARRA Model developed by MSCI, Inc., which includes 13 factors: size, size nonlinearity, currency sensitivity, leverage, volatility, earnings yield, trading activity, momentum, growth, value, dividend yield and earnings valuation. It is worth noting how the Fama-French factors are still in use by practitioners (at least for the U.S. market), but I am not going to analyze this model.

In testing this model, I used the same approach as before. Due to data availability, the number of firms in the sample dropped to 3223 and the time period is now 1997-2015. The dataset is the same; I only added the following data for each company:

- Net Interest Expense.
- Cash Flow From Operations.

I used the data to add the following variables to the mix:

- Investment (percentage change in total assets).
- Operating Profitability (EBITDA minus interest expenses over total assets).
- Earnings Management Indicator (EBITDA minus interest expenses over cash flow from operations).

I built 10 investment portfolios, 10 profitability portfolios and 100 investment/profitability portfolios to check for consistent patterns, then I ran the Fama-MacBeth Regressions considering various combinations of the explanatory variables. Regressions 8 (on Inv, OP and EMI), 11 (on β, ln(ME), ln(BE/ME), Inv and OP), 12 (on β, ln(ME), ln(BE/ME), Inv, OP and EMI) and 13 (on β, ln(BE/ME), OP, E/P Dummy and E(+)/P) are run for the two subperiods as well.

I also ran the test by restricting the sample to Euro Area countries and the UK. The sample size is down to 1617 firms for the Euro Area and 800 for the UK, while the relevant period is restricted to 1999-2015 for the Eurozone. In the monetary union I used only the EURIBOR as the RFR, while in the UK test I used the LIBOR. In the sub-sample tests I built 10 size portfolios, 10 book-to-market portfolios, 100 size/pre-ranking β portfolios and 100 size/book-to-market portfolios to check for regularities and ran the aforementioned 13 regressions.

In Table 7 we can see the statistics of the 100 portfolios ranked on investment and operating profitability. The sample is composed by 3223 stocks for the 1997-2015 period. Panel A shows that high-operating profitability portfolios tend to have higher returns as predicted by Fama and French, but the relationship is not perfectly increasing. On the other hand, investment has a U-shaped relationship with returns and this is consistent for every level of operating profitability. In the Fama-French 2014 paper, this relationship is predicted to be decreasing, but it actually depends on the level of book-to-market. In Panel B I reported the average values of investment and we can see that there seems to be a weak increasing relationship between the two variables. The average operating profitability for each of the 100 portfolios can be seen in Panel C: this relationship is U-shaped, as low and high investment portfolios seem to have the highest OP. In particular, the portfolios that are obtained by dividing Low-Inv have the most extreme levels of operating profitability. Panel D and Panel E show no patterns in the portfolios as far as post-ranking βs and size are concerned.

Table 8 shows more statistics for investment and operating profitability portfolios. Panel A shows the statistics for the 10 investment portfolios: we can see the slight U-shape relationship with returns, while this pattern is reversed for the post-ranking β. There is no relationship with size and book-to-market that is worth noting. As I said, Low-Inv has a very high level of operating profitability, while the value of the E/P Dummy is decreasing in investment, but High-Inv has the second-highest percentage so, as expected, it seems that investment hurts a company's earnings, but it only does when it is very high. This U-shaped relationship can be noticed in most of the variables including the earnings/price ratio and the EMI. Panel B analyzes OP portfolios and we can see that returns, size and bok-to-market show very low difference in value across portfolios and unclear patterns. As I said before, investment has an increasing relationship with operating profitability, while the percentage of negative-earnings companies is decreasing with the exception of the two highest OP portfolios. The earnings/price ratio and the earnings management indicator do not show any clear patterns.

Table 9 shows the results of the Fama-MacBeth regressions. We can see that the role of β, size and book-to-market are substantially the same as in the previous test. Investment shows consistently negative coefficients, while operating profitability seems to have a positive effect on returns. The EMI has negative coefficient as expected. Again, coefficients do show the expected sign and magnitude, but due to the excessive volatility, they are not significant, as they are always less than 0.1 standard errors from zero, with β again being the only exception.

We can see Table 10 for the results of regression 8 and 11, which are the Custom 3-Factor Model (OP, Inv and EMI) and the 5-Factor Model (β, ME, BE/ME, Inv and OP) respectively, including the sub-period analysis. The results are very similar to the 3-Factor model: the coefficients have the expected sign and magnitude, but have very low t-Statistics, which make them non-significant. The results are even worse for the post-crisis period. Things are somewhat better in the 5-Factor, indicating that size, book-to-market and especially β are necessary in the model. Once again, my results indicate strong positive intercepts with very high t-Statistics, but they decrease for the post-Crisis period. This may indicate even more volatility and less explanatory power for the unknown systematic risk factors that are not included in the model.

Table 11 reports the full-period and sub-period statistics for regressions 12 and 13, which I will call 6-Factor (β, ME, BE/ME, Inv, OP and EMI) and Custom 4-Factor Models (β, BE/ME, OP, E/P Dummy and E(+)/P) respectively. In the first case, the results are basically the same as in the 5-Factor, indicating the scarce influence of the earnings management indicator. In the second case, I built this model taking into account the variables that have shown the best results (even though taking about "best results" is a bit of an overstatement). The results are very similar to previous regressions. Again, the test for the S-L Hypothesis is rejected, so the systematic factor that we are missing is not captured by the RFR.

The conclusion of this test is that the new variables do not do much to improve the explanatory power of the model. I therefore conducted new tests on the Euro Area and the UK to see if I would find some differences in patterns.

Table 12 shows the average monthly returns on the size/pre-ranking β and size/book-to-market portfolios for the Euro Area. The sample is composed by 1617 stocks for the period 1999-2015. Panel A confirms the reversal patterns found in the full-sample model: the pre-ranking β portfolios seem to have decreasing returns, while size seems to have a positive effect on returns, again confirming the reversal of the size effect, but this time it is less pronounced as results often do not follow the pattern. Panel B leads to the same conclusions as the full-sample model as well: size and book-to-market portfolios both have increasing returns, but the effect is much stronger for book-to-market, while size has a less clear effect and more dispersed returns.

In Table 13 we can see more statistics for the 10 size and book-to-market portfolios. Panel A shows that returns are increasing in size except for the Small-ME portfolio, which has higher returns than the pattern would suggest. Post-ranking βs seem to be increasing in size, but the relationship is not as clear as in the full sample. Book-to-market and size seem to have a clear negative relationship like before, while investment, operating profitability and the earnings management indicator show little or no patterns. The earnings/price ratio and the percentage of negative-earning firms both confirm their patterns: increasing and decreasing in size respectively. Panel B shows the same statistics for the book-to-market portfolios. The positive relationship with returns and the weakly negative relationships with size and β are confirmed, while average investment seems to be decreasing. The earnings/price ratio seems to be increasing, while the E/P Dummy shows no relevant patterns. OP and EMI do not show regularities.

Table 14 shows the results of the Fama-MacBeth and they are consistent with the full-sample model, showing slightly smaller t-Statistics and hence even less significant coefficient. The earnings management indicator performed particularly badly.

In Table 15 we can see the average monthly returns for the size/pre-ranking β and for the size/book-to-market portfolios of United Kingdom stocks. The sample is composed by 800 stocks for the period 1997-2015. Panel A shows the returns for the size/β portfolios and once again, the results of the full-sample model are confirmed. The reversal pattern seems stronger than in the Euro Area and this seems clear from Panel B as well, from which we can also see the usual increasing relationship between book-to-market and returns.

Table 16 shows the characteristics of the 10 size portfolios and the 10 book-to-market portfolios. As Panel A shows, the increasing relationship between size and returns is quite consistent, while the traditional negative relationship between β and size is actually more or less respected in the UK, at least for larger-size stocks. Book-to-market seems to be decreasing in size, thus confirming the results obtained until now. As always, the earnings/price ratio seems to be increasing in size, while the percentage of firms with negative earnings is decreasing. Investment, operating profitability and the earnings management indicator do not show any pattern. Panel B reports the same statistics for book-to-market portfolios. Returns are increasing, but with some exceptions. High book-to-market seems to coincide with smaller size, but the relationship is not clear. Investment shows a slight tendency of decreasing as the book-to-market of the portfolio increases. None of the other variables show clear regularities.

Table 17 shows the results of the Fama-MacBeth cross-sectional regressions and the previously obtained results are still there. The coefficients are slightly more significant in this case, with the notable exceptions of the EMI and even the earnings/price ratio.

The results of my tests indicate that the original roles of β and size in determining returns are now reversed in the European stock markets. This may be a consequence of flight to quality, which may explain the exceptionally good performances of low-β and large-size stocks. The negative relationship between β and size is also weakened, but it is still there for larger stocks, especially in the British market. The value effect and the leverage effect are both confirmed, while stocks of negative-earnings firms perform significantly worse than predicted. This too may be a consequence of flight to quality. The effects of the new factors in the 5-Factor Model also confirm the expectations, but do not do much to improve the explanatory power of the model. The main issue with my tests is the extremely low significance of the regression coefficients, indicating that the factors predicted by Fama and French are active, but they do not do much to explain the cross section of monthly stock returns due to an excessive volatility of such returns. Moreover, the significant intercepts in the regressions may indicate the presence of relevant systematic risk factors that are not accounted for.