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Option Pricing for the Electricity Market

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ABSTRACT

The electricity industry has tended to be viewed as a natural monopoly in mostly every country after the Second World War, because of its importance for the national economy and society as a whole. The deregulation process, which started in Europe thanks to the ‘’European Electricity Directive’’ (1997) and in other OECD countries, was the inevitable result of the high demand for a restructuring of the electricity market in order to remove its inefficiencies. The physical characteristics of electricity and the liberalization process, which brought drastic changes to the ownership, competitiveness and regulation of the electricity industry, made this market jump into the financial one. In order to deal with risk exposures, volatility of prices and the instability of demand, the creation and trading of financial derivatives naturally started.

The aim of this work is to understand the complex process behind the pricing of these derivatives, which mostly depends on the unique nature and complexity of electricity.

INTRODUCTION

HISTORY

In 1989 the UK became the pioneer in privatizing the vertically integrated electricity industry. The English government was then followed by Norway and California in 1990 and 1996 respectively, and soon after by many other countries. The process of deregulation started in order to introduce competition in a previously monopolistic industry: the electric utility service was disaggregated into its basic components, which were then offered for sale at different rates. Generation, transmission and distribution were therefore unbundled and the creation of a central independent body was the natural following step in almost every country. This body, usually called independent system operator or Power Exchange, is concerned with the matching of supply and demand, the maintenance of system security and reliability and the matching of generators’ bids with demand bids. Thanks to the unbundling of these different components, the market for electricity enlarged and spot electricity prices started to be traded on worldwide exchanges. Power Exchanges, for risk management and speculation purposes, established the trade of futures and forwards contracts, which take into consideration the unique nature of electricity. First of all, electricity cannot be stored except at very high cost in batteries, thus, demand and supply must be continuously and instantaneously in balance. Moreover, the stages of electricity production are still considered to be natural monopolies because transmission and distribution involve large sunk capital costs and capital equipment, which leave no arbitrage opportunities. Lastly, electricity is subject to large seasonal fluctuations in demand and seasonal fluctuations in physical requirements for distributors. All these characteristics contribute to the volatility and to the exposure to risks of the whole electricity market.

INCREASED RISK WITH COMPETITION

The risks associated with the electricity market increased because of the introduction of competition. Before the liberalization, a vertically integrated state monopoly used to deal with fluctuations in demand by carrying excess capacity. With the new competitive setting the market had to face at least two new sources of risks: a more complex pricing structure and loop flows problems, which arise when independent producers introduce electricity within the transmission network. Therefore, if decentralized market provokes increased risks, it also has to provide ways to deal with it: the electricity market evolved so to give methods to reduce risks and to price risk so that it can be spread optimally among market participants. First of all, selling on the spot market offers normal returns because prices regress to the mean. Secondly, producers can sell long-term contracts with specified prices and adjustment clauses so that price fluctuations risks are reduced. Thirdly, producers can hedge spot market sales in futures market. Therefore, the application of financial derivatives to the electricity market can be a useful tool in order to reduce risks and achieve the Pareto efficiency among market players.

WORK STRUCTURE

The work is structured as follows:

1) Chapter 1 deals with the deregulation process of the electricity market, its general elements and it provides an overview of the Italian case. Afterwards, the issue of risk management and some financial instruments to hedge it are taken into account.

2) Chapter 2 deals with the Black-Scholes evaluation model for options and the assumptions on which it relies. Afterwards, it focuses its attention on the reliability of two of the assumptions: normal returns and constant volatility. The analysis is provided through the Matlab environment (R15).
3) Chapter 3 deals with the dynamic delta-hedging strategy of a hydroelectric power plant.

CHAPTER 1

1.1 THE ELECTRICITY MARKET

Many countries around the world have been restructuring their electricity markets with the purpose of liberalizing the electricity sector, but few have reached what could be called a competitive market. In fact, public ownership remains common in restructured electricity markets: in several OECD countries the state maintains its share both in networks and in power production. In order to reach a deep understanding it is important to take into consideration the set of constraints related to the electricity market and therefore highlight its deregulation process and development.

1.1.2 THE SET OF CONSTRAINTS

The constraints the restructuring process has to deal with have three different origins. First of all, the physical situation of a country sets the most important hurdle to overcome while deregulating the market. Every single state has to take into consideration the existence of indigenous power sources such as oil, gas, coal and therefore its dependence upon other countries on energy supplies. Moreover, the geographic distribution of demand, the market size and the degree of isolation of the country play an important role in the creation of competition. The second essential constraints to be considered are the macro-economic characteristics of each country. The availability of capital, the rate of demand grows and the level of economic development influence the financing options, the investment rate and the results that change in prices might have on final users. The last category of constraints is related to the socio-political environment of the energy system. As a matter of fact, the restructuring process success is the result of the interaction between formal and informal institution. Culture, property rights, sector legislation, the role of the state in the economy, the power of the central government all play fundamental roles in the shaping and in the feasibility of the restructuring process.

1.1.3 THE HISTORY OF THE DEREGULATION PROCESS

The earliest introduction of the concepts of privatization and electricity market occurred in Chile. Thanks to the 1982 Electricity Act, amended during the dictatorship of Augusto Pinochet, the previous state owned electricity companies were acquired by huge private investors, who were able to gain improving rates of return on capital. The transparency and rationality on electricity pricing were the main factors contributing to the success of this revolutionary reform. Anyway a further improvement to this model was masterminded by the Argentinian president Carlos Menem and his Minister of Energy, Carlos Bastos. In fact, the massive Argentinian privatization program, done in order to reduce the huge losses of state owned companies, also nurtured the electricity sector. The attraction of massive private investments was the result of two main reforms: the regulation of market concentration and the improvement of payments structure that guaranteed system reliability. The result was that the decrepit generation assets were rehabilitated and the overall system expanded exponentially. These two forward-looking countries were then followed by other Latin America countries, which assisted by the World Bank, tried to create some deregulated hybrid market structures. The decisive event for the European and more developed countries was pursued in 1990 by Margaret Thatcher government, which privatized the UK electricity industry. After the appearance of the UK model, the deregulation process took place in Scandinavian countries, the Netherlands and other Commonwealth countries. As a consequence of the overall success, the EU drove the reform by amending the ‘’European Electricity Directive’’ (1997), with the intention of creating a European market and of reducing end-user prices. Also the USA, with the fundamental role played by the Federal Energy Regulatory Commission (FERC), implemented the deregulation process by introducing independent system operators (ISOs) and regional transmission organizations (RTOs).

Even if the deregulation process had an overall success, due to the immaturity of the market design certain countries suffered by demand-supply gaps. The most notorious example is the Californian crisis of 2000 and 2001. The crisis occurred because of the market manipulation and illegal pipelines’ shutdown operated by the Texas energy company Enron. Energy traders, during days of peak demand, decreased supply by voluntarily shutting down power plants, in order to artificially create a shortage. From April 2000 up to December 2000, wholesale prices rose by 800%. Californian generators (Pacific Gas and Electric Company (PG&E) and Southern California Edison (SCE)) were forced to purchase electricity on the ‘’spot market’’, paying astronomical amounts, and were unable to raise the retail price. The government had in fact imposed price caps which in the end caused PG&E bankruptcy and SCE financial crisis.

1.1.4 CONSIDERATIONS

The Californian example shows how the inefficiency of the wholesale electricity market, the one allowing trade among generators, financial intermediaries and retailers both for current and future delivery of electricity, played a fundamental role. As a matter of fact, on the one hand the deregulation process caused several performance improvements in many countries around the world, by achieving cost and price reductions without reducing service quality. On the other hand, the creation of efficient wholesale and retail markets has encountered many difficulties also because of unexpected fluctuations in prices. Anyway, many problems that have emerged are now better understood by policymakers, who, with the help of financial markets, will eventually obtain an efficient market.

1.1.5 MARKET ELEMENTS

Electricity markets are run by Independent System Operators (ISOs) who are in charge of controlling the auction system. The auction market system consists of day-ahead negotiations, which establish daily the price for each hour of the following day, and real-time negotiations, which are computed every five minutes. Moreover, the ISOs is in charge of collecting the generators’ offers, which consist of generation levels and energy prices. Therefore, its final objective is to collect these offers and match them with energy bids, thus constructing the market supply and demand curves. Market operators are constantly aware of trades taking place within the market in order to maintain load and generation balance. The electricity commodities traded on the market are of two types: Power, which is the net electrical transfer rate calculated at a given moment and that is measured in megawatts (MW); and Energy, which is electricity that flows through a point for a given period of time and is measured in megawatt hours (MWh). In addition to these two commodities, because of the restructuring of the electricity market, it is possible to trade electricity derivatives such as electricity options and futures. The main objective of these derivatives is to mitigate market risks and to let generators and load service entities build hedging strategies in order to decrease uncertainty and increase market efficiency.

1.2 THE ITALIAN CASE

Before the deregulation took place, the Italian electricity market was organized as a legal public monopoly, with a vertically integrated structure. The "Ente Nazionale Energia Elettrica" (ENEL), through the State concession, had the right and duty to pursue all electricity activities. In order to increase market efficiency, the necessary step was to introduce competition in: generation, regulatory mechanisms, supply to liberalized and captive customers and transmission. Therefore, thanks to Bersani's law dated 16th March 1999, the historical operator ENEL was vertically separated. Barriers to entry in production and distribution were removed, generation, transmission and distribution were separated and a new market was created. During the first decade of 21st century, the Electricity Power Exchange (IPEX) was created and the main market players arose:

- The National Transmission Network Manager "Gestore della Rete di Trasmissione Nazionale" (GRTN)
- The Electricity Exchange Manager "Gestore del Mercato Elettrico" (GME)
- Unique Buyer "Acquirente Unico" (AU)
- The Network Owner "Terna S.p.A."
- Electricity Producers

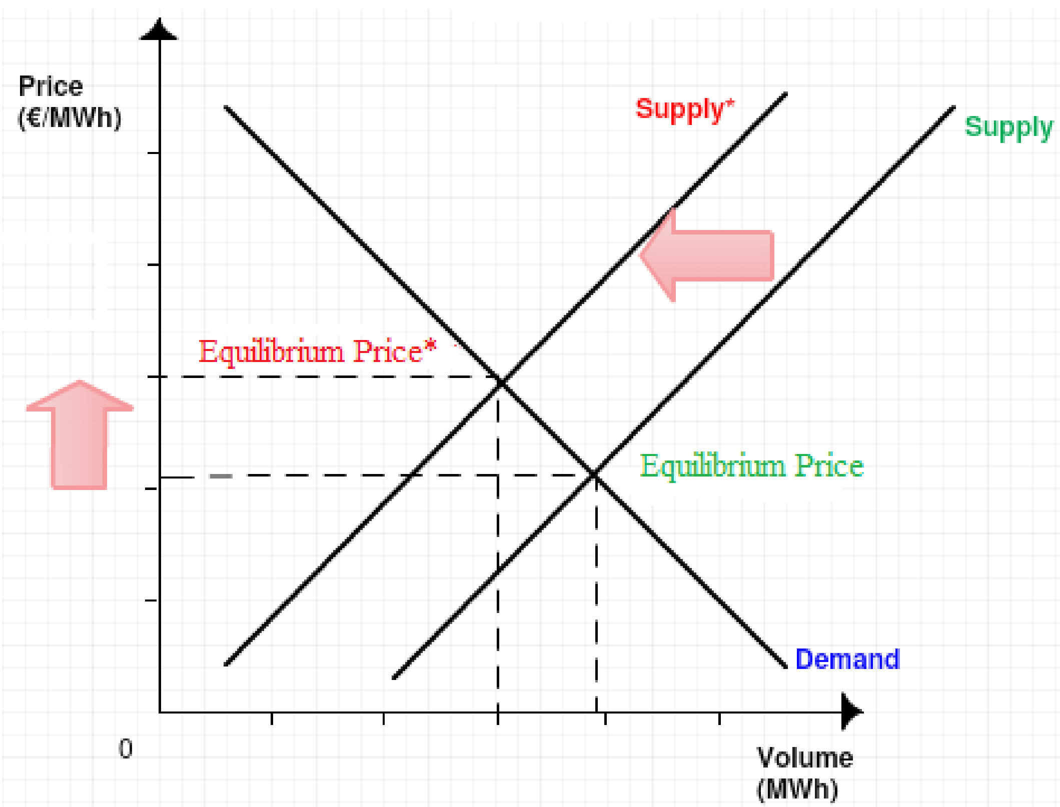
In addition to the numerous market participants it is also important to highlight the different kind of components of the Spot Electricity Market within the Italian Power Exchange. As a matter of fact, the Spot Electricity Market consists of:

- Day-Ahead Market (MGP)

- Intra-Day Market (MI)

- Ancillary Services Market (MSD)

Managed by the GME, the MGP is the “first” component and it represents the main arena for electricity trading. Contracts for the delivery of energy are made between sellers and buyers. The equilibrium price is calculated according to the standard supply-demand model. In fact, each buyer evaluates the volume of energy he will need to meet demand the following day, and how much he is willing to pay for this volume, hour by hour. On the other hand, each seller evaluates how much he can deliver and at what price, hour by hour. Therefore, the price is continuously adjusting to the equilibrium determined by supply and demand, which in turn depend on several factors such as: weather conditions, season, time and transmission capacity. The market hourly electricity price reflects the relentless change of these factors and can hence be described as a dynamic equilibrium price. For example, if transmission capacity gets constrained in order to react to bottlenecks occurring when large volumes of electricity have to be delivered, supply decreases and price increases thus reducing quantity demanded.



1. Shift of supply curve and new equilibrium price

This dynamic equilibrium price and the various adjustment processes, which occur hour by hour in order to adjust the price to the competitive one, make the Italian market look like a perfectly competitive one. Anyway several country-specific factors have not been taken into account in the model, which only represents a simplified view of electricity pricing.

In addition to this, it is important to highlight in the MGP the differentiation among three kinds of daily prices, which are dependent upon the level of electricity demanded hour by hour and during the whole day: baseload, peak load and off-peak.

The ‘’second’’ component managed by the GME, is the Intra-Day market (MI). The MI supplements the day-ahead market and helps secure the necessary balance between supply and demand in the power market. Because of unpredictable event the electricity supplied to the market could be lower or higher. Therefore, at the intra-day market sellers and buyers can bring the market back to balance by real-time trading.

Managed by Terna S.p.A., the ‘’third’’ component is the MSD. It represents the venue where Terna S.p.A. procures the resources that it requires for managing and monitoring the system relief of intra-zonal congestions, creation of energy reserve, real-time balancing. In the MSD, accepted offers are remunerated at the price offered by Terna, which acts as a central counterparty. On the other hand, GME is in charge of “organizing and economically managing the Electricity Market, under principles of neutrality, transparency, objectivity and competition between or among producers, as well as of economically managing an adequate availability of reserve capacity.” (GME)

1.2.2 THE ITALIAN WHOLESALE AND RETAIL MARKET

Since the beginning of deregulation, the Italian wholesale market has struggled to become a competitive market. Anyway in the last years there has been a significant improvement; the market share of the five largest operators has in fact decreased by 5%. Nowadays the market shares are divided as follows:

- ENEL (25%)
- ENI (9%)
- Edison (7.2%)
- E.On (4.4%)
- Edipower (4.6%)
- Small-sized operators (32.3%)

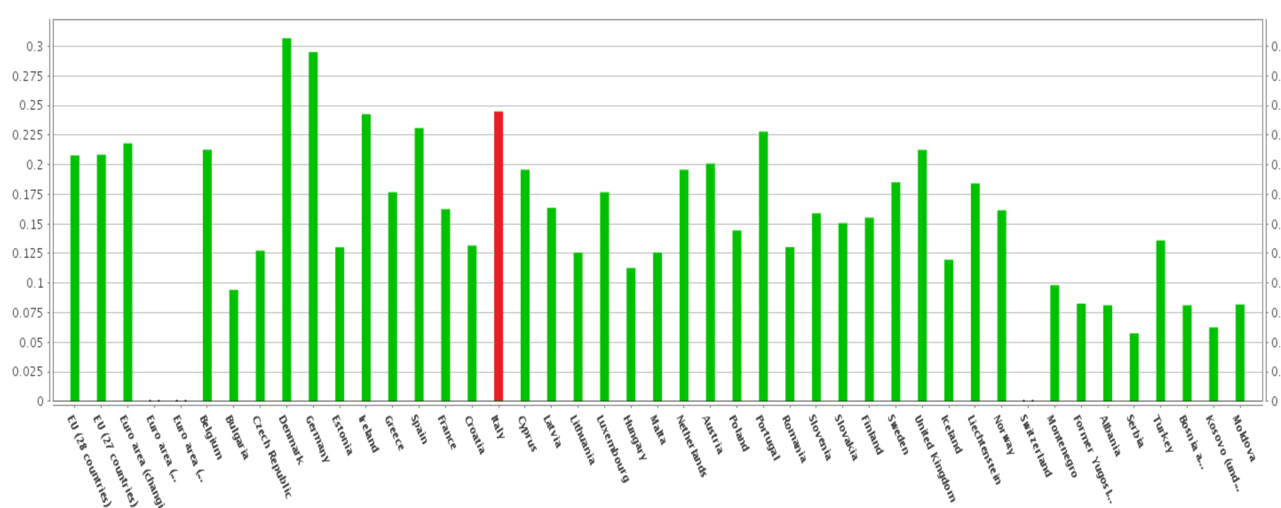
Comparing these data with the ones of 2011 it is possible to notice how all the five largest operators market shares decreased to the advantage of other small-sized producers, thus increasing competition within the market. Moreover, market integration with neighbour markets is improving, congestion management rules have been enhanced and Italy now represents one of the best interconnected systems in Europe. Nonetheless, the Italian electricity wholesale price is considerably higher than in other European countries, mainly due to the excessive reliance on gas fired plants, whose fundamental raw material wholesale price is far above the European average. Not only the wholesale market, but also the retail one was fostered by the deregulation process. Even if the standard offer market concentration has remained high, as ENEL provided 85% of the total supply, the free market has seen an increasing competition. As a matter of fact, the three main operators (ENEL, ENI and Edison) had a combined market share of 34.3%, with ENEL playing the leading position (20.3%). With only two companies having a market share greater than 5%, the overall retail market competition is at a medium level, which represents a serious improvement with respect to 2011, where the joint market share of these three companies accounted for 49.6%. Since January 2007 consumers are able to choose their own supplier. Those who do not make any choice are by law assigned a default supplier, the local DSO (Distribution System Operator), which provides electricity according to a "standard offer''. Nowadays the 80% of households and small-medium enterprises are served on the base of this ''standard offer'' while the others have to find an alternative supplier. If they are not able to find any suitable offer, the selection through an open auction of a Last Resort Supplier occurs. In the last years an independent data hub to support the switching process has been appointed. As a result of this deregulation measures, the switching rate of suppliers has been constantly increasing, thus benefiting competition.

1.2.3 THE OVERALL LEVEL OF EFFICIENCY IN ITALY

Having taken into account the general features of the Italian wholesale and retail market it is now essential to understand the problems threatening their efficiency. The main problem within the Italian electricity market is that production is highly inefficient with respect to other European countries. Production in the last two decades has switched from the massive use of oil, which accounted for the 64% of Italian electricity production in 1994, to the use of natural gas, which accounted for 59,5% in 2015. This switch assured the electricity market not to suffer from the volatile and high prices of oil, while instead to rely on a more “secure” primary production factor, natural gas. Nonetheless, there still are some fundamental anomalies in the Italian electricity sector, which do not allow it to be an efficient one:

- Lack of competition among producers
- Italy is one of the largest electricity importer in the world (14% of total demand)
- Natural gas has to be imported from above
- Taxes, system and network costs accounts for about 50% of the electricity final price

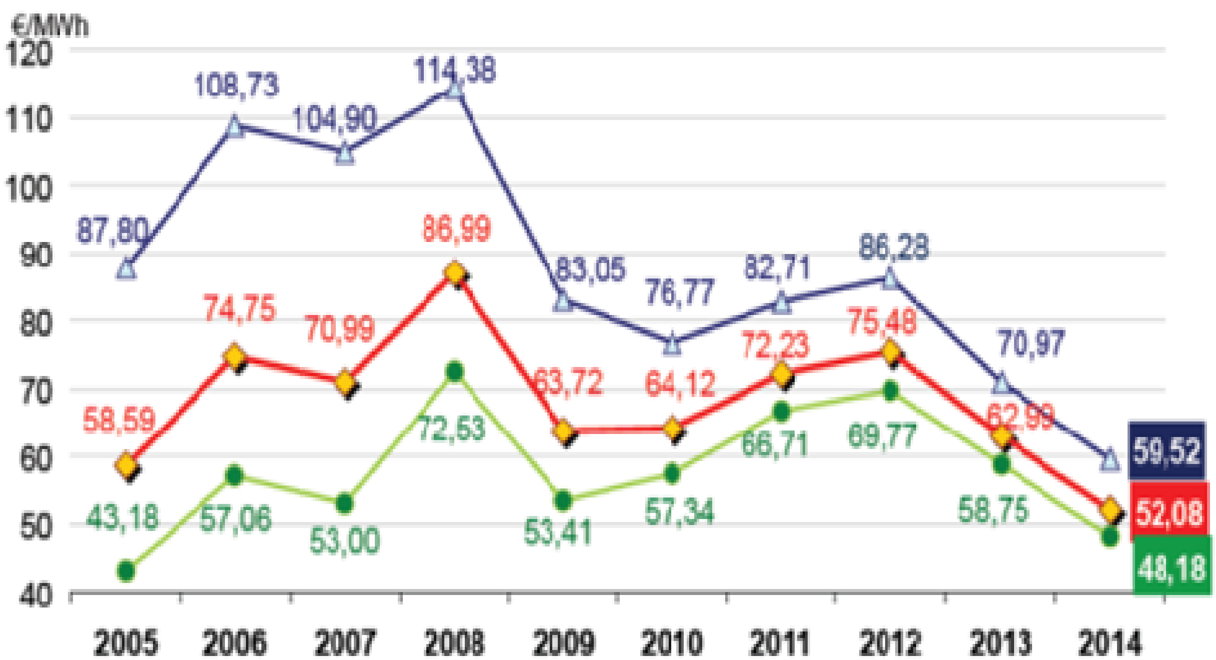
All these factors do not allow the Italian electricity market to be efficient and working as in other European countries. In fact, it is straightforward that the Italian electricity prices for household are among the highest in the Euro zone.



2. **Comparison of European electricity prices for medium size households**

In any case, Italy has made some important steps to improve the electricity market efficiency. For example, it has nearly eliminated the oil reliance of electricity production. Moreover, in 2014 the 44.5% of total domestic electricity production was generated by renewables. In addition, even if not all installed capacity was available for production, in 2014 the efficient net power capacity was 128 GW against an observed peak demand of 51.5 GW, thus assuring reliability and flexibility in generating capacity.

In the last six years, the switch to natural gas, the significant increase in renewables and the enduring overcapacity conditions had a beneficial effect on prices, which declined by approximately 40% as the fig shows.



3. Italian yearly average price for peak time electricity (blue), baseload (red) and off peak (green) set on the official exchange in the last decade. Source: GME

1.2.4 THE FORWARD ELECTRICITY MARKET

In addition to the spot electricity market that was described before, the forward electricity market (MTE) is the venue where forward electricity contracts with delivery and withdrawal obligation are traded (GME).

The forward electricity market is a medium term market in which demanders and suppliers can lock in energy prices and quantities for medium and longer periods. These contracts are settled with terms that are much longer than the hourly spot market and are therefore useful to reduce the risk of the volatility of the spot market prices. All the electricity market participants can trade in the MTE contracts of various types: peak load and base load contracts with different delivery periods, such as monthly, quarterly and yearly. The two main roles of the forward electricity market are first, to reduce problems in the bilateral contracting market, and second to improve the performance of the spot energy market, thus helping an efficient price formation.

1.2.5 THE OTC MARKET

The OTC (Over The Counter) contracts are the last mean for trading electricity securities and derivatives that have to be taken into account. These kind of contracts are stipulated outside the formal exchange market and consist of bilateral agreements negotiated through financial intermediaries. Generally, in the over the counter market dealers act as market makers by quoting prices and agreeing on the price, without other players being aware of the transaction. Nonetheless, these negotiations are subject to compatibility checks with transport and dispatch constraints, thus monitoring this less regulated market.

The final purpose of the dealer networks trading on the OTC market is to first of all manage the financial risk related to these securities. Banks buy and sell derivatives in order to limit price volatility risk exposure of their clients, who generally are electricity producers that want to immunize against unexpected market prices. In addition to this, financial intermediaries gain on each occurred transaction on the electricity derivatives. As a matter of fact, banks have often been criticized for conflict of interests, since they have an incentive to disclose unreliable information in order to stipulate as much contracts as possible.

1.3 ELECTRICITY PRICE VOLATILITY AND RISK MANAGEMENT

As it has already been highlighted spot electricity is a non-storable asset, thus it cannot be traded. Its price is calculated through a demand-offer equilibrium algorithm, which establishes the hourly clearing price for each of the six Italian market zones and then determines the Unique National Price (PUN), which is the average of the zonal prices weighted on the zones' volumes.

Spot prices are deeply influenced by volatility of demand and supply, seasonality, weather changes and many other factors, thus determining an hour by hour transformation. The electricity market does not have a precise model able to account for all these peculiarities and to exactly predict price movements. Therefore, market participants have to bear strong uncertainties and risks, which can only be overcome thanks to the shaping of electricity derivatives instruments.

Producers are exposed to two main risks: not obtaining a sufficient remuneration of the capital invested because of the marginal spot prices and to bear losses in the fuels procurement. Sellers on the other hand, are exposed to two different kind of risks: the price could be unacceptable for final consumers, who are unwilling to accept prices related to the stock exchange movements, and to volume uncertainties caused by clients' withdrawals. These are the reasons for which a liquid and transparent market for electricity financial derivatives could contribute to the efficiency enhancing of the Italian electricity market.

Anyway the creation of an efficient derivatives market is dependent upon the overcoming of some important structural problems. First of all, the use of market power by some operators, who are able to significantly alter the value of futures contracts can undermine the derivatives utility. Secondly, the multiplicative effect of the positions taken on the financial market can alter the behavioural incentives on the spot market. Hence, strict regulation and monitoring of operators' market power, and the duty of information diffusion imposed on market players with information advantages, should be established. Thus, liquidity, transparency and no information asymmetries could be guaranteed, and an efficient market for electricity derivatives could be assembled.

1.3.2 HEDGING IMPORTANCE

In the last years the electricity retail market has experienced increasing competition and has moved away from administratively determined, cost based rates towards market driven prices. In the new setting of a competitive electricity market, price volatility has become the main problem to be faced by market players. In fact, the birth of electricity derivatives was the necessary step to create an efficient market. Generators, marketers, consumers can now hedge price risks through the purchase of derivatives, which in most cases do not imply the physical delivery and are used to manage seasonal price fluctuations and not daily ones. The risks arise because volatile inputs prices (gas, coal, oil) are coupled with relatively stable output price and firms are vulnerable to these fluctuations while entering into commercial operations. For example, if a marketer sells electricity on the base of fixed price contracts but at the same time he buys it on the spot market, which is subject to the inputs prices fluctuations, he could suffer from huge losses. Even if new risks arise because of speculators and naïve investors, the possibility of hedging through the use of derivatives remain of the utmost importance and also fundamental for the creation of an efficient market.

1.3.3 THE DIFFERENT TYPES OF ELECTRICITY DERIVATIVES

The New York Mercantile Exchange (NYMEX) introduced electricity derivatives in 1996. The trading of these derivatives is done both in formal exchanges and in over the counter markets. However an important distinction has to be stressed: while on the NYMEX and other formal exchanges only futures and options on futures are traded, on the OTC market also other kind of derivatives are sold and purchased, such as forward contracts, swaps, plain, vanilla and exotic options.

- ELECTRICITY FORWARD CONTRACTS

Electricity forward contracts consist of the obligation to purchase or sell a prearranged volume of electricity. In addition to the amount of electricity, the contract states the price (forward price) that has been established in advance, and the expiration time, which represents the time the obligation will have to be satisfied. These kind of contracts are the primary instruments that both buyers and sellers use in order to neutralize price uncertainties, which are the main elements jeopardizing market efficiency and transparency. The party agreeing to buy electricity on a certain specified future time for a certain specified price assumes a long position; on the other hand the party agreeing to sell electricity at the same time and price assumes a short position. If the case of long position is considered, the payoff of a forward contract, which establishes the delivery of a unit of electricity at a future time T, is:

$$Payoff\ of\ Forward\ Contract = (S_T - F)$$

where F is the delivery price and S_T (settlement price) is the spot price of the asset at contract maturity. Anyway it is important to stress a distinction that arises because of the peculiar nature of electricity. Electricity value changes according to the time of delivery within a day. Electricity industry considers 16 hours, from 6:00 to 22:00, to be the ‘’peak-period’’. On the other hand 22:00-6:00 is considered to be the “off-peak” period. Therefore, there are various kind of electricity forward contracts: ‘’peak-period’’ contracts, ‘’off-peak’’ contracts and “around-the-clock” contracts, which include the 24 daily hours. Hence, the spot prices for electricity derivatives are calculated as an average of the hourly spot prices included in the different periods.

For example, if an ‘’off-peak’’ contract has been stipulated, the long position party pays the agreed price K at T_0 (initial time) and waits for the short position party to deliver the established MWh of electricity at time T (maturity time). The payoff of the forward contract can be evaluated by calculating the average of 22:00-6:00 spot prices of day T (S_T).

- ELECTRICITY FUTURE CONTRACTS

On March, 1996 the NYMEX launched the first electricity derivatives. Other energy related derivatives, such as oil, gas, gasoline had already been introduced in 1980s. Therefore, the birth of electricity derivatives was the consistent step after the electricity market deregulation.

A future contract is a standardized contract where all terms have been defined in advance: the delivery date, location, quality and quantity are defined by the market exchange and the only variable that has to be negotiated is the price. These types of contract are very similar to the forward ones with the most notable difference being the delivery quantity specified in the contracts. As a matter of fact, the quantity of electricity to be delivered in futures is generally significantly smaller. In addition to this, futures are exclusively traded on organized commodity exchanges, thus conferring price transparency, lower monitoring and transaction costs (financial payments instead of physical delivery) and reduced credit risks (gains and losses of electricity futures are paid out daily, instead of being cumulated and paid out in a lump sum at maturity time).

- ELECTRICITY SWAP CONTRACTS

Electricity swaps are contracts that allow holders to purchase a given amount of electricity at a fixed price, irrespective of the floating electricity price over a specified period of time. On the other side, the holder could also receive a fixed price against paying a floating price. These kind of contracts are equivalent to a strip of forward contracts with multiple maturity dates and identical forward price for each date. Generally, they are established for a fixed amount of electricity pertained to a floating spot price at either a consumer's or a generator's location. These swaps provide hedging possibilities against price uncertainty for short to medium terms.

- ELECTRICITY CALL AND PUT OPTIONS

In the last two decades, with the emergence of the deregulated electricity market and the relentless arising of new techniques for risk management, electricity options have gained fundamental importance. These options are nowadays not only based on price attributes, but also on timing, location and volume. Options contracts cover different and various time maturities, but the most important ones are those, which cover time periods up to two years.

A call option gives the option holder the right to buy a given amount of electricity by a certain date at a specified price. On the other hand a put option gives the option holder the right to sell a given amount of electricity by a certain date at a specified price. The date specified in the contract is called expiration date or maturity date, while the price is known as the strike price or exercise price. There are two types of options: American and European. The difference stems from the fact that the American options allow the holder to exercise the option at any time, while the European ones allow the holder to exercise the option only on the expiration date. The American ones account for the majority of traded options on exchanges.

The payoff of plain call and put options is:

$$Payoff\ of\ Electricity\ call\ option = max(S_T - K, 0)$$
$$Payoff\ of\ Electricity\ put\ option = max(K - S_T, 0)$$

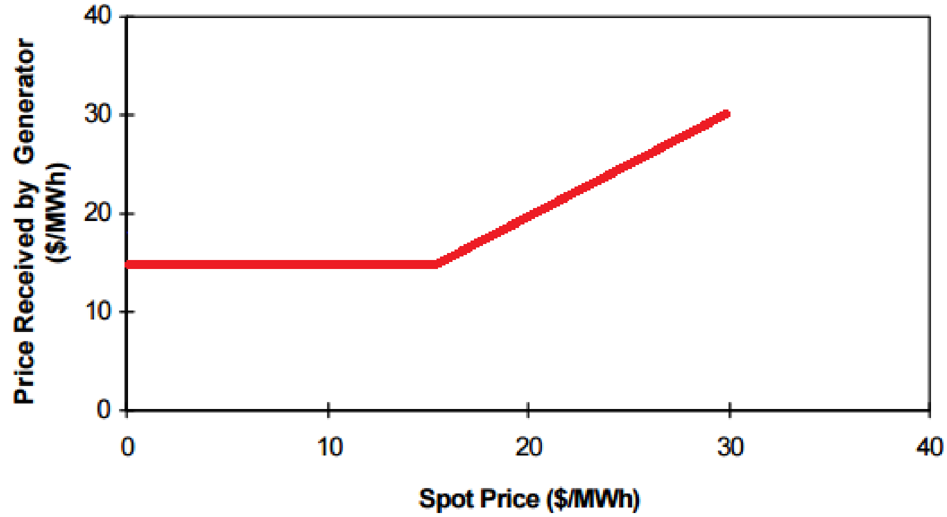
where *K* is the strike price and S_t is the electricity spot price at time T.
In general the underlying of electricity call and put options can be electricity futures, which are the most useful tool for power plants merchants and power marketers in order to hedge against price volatility and risk. Being call and put options such effective tools, they represent the most commonly traded derivatives over the counter. The evaluation model, which is used to provide the correct price of these derivatives is the Black-Scholes evaluation model. According to it, the general formula for a put option is:

$$P(S,t) = Ke^{-r(T-t)}N(-d_2) - S_tN(-d_1)$$

where *K* is the strike price, S_t is the underlying asset price, *N* represents the cumulative standard normal distribution and *r* is the interest rate. Alternatively the formula for a call option is:

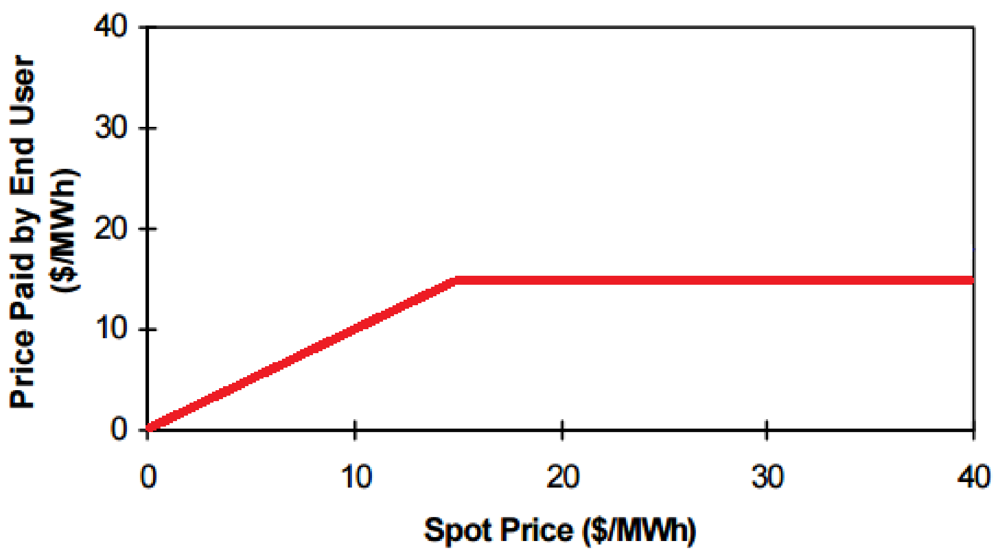
$$C(S,t) = S_tN(d_1) - Ke^{-r(T-t)}N(d_2).$$

The in deep meaning of the various components will be analysed in the second chapter.
Call and put options are generally used by generators, end users and marketers. Generators use put options in order to secure a minimum price for their electricity production: they not only avoid the risk of lower prices, but they can still benefit from increasing ones. For example, if the generator would like to receive at least $15/MWh he can purchase a put option paid up front. Therefore, if the electricity price goes above $15/MWh the generator would sell electricity on the spot market, otherwise he would exercise his put option, thus gaining $15/MWh for sure.



4. Payoff of a generator purchasing a put option

On the other side, end users can use call options in order to avoid the risk of higher prices. As a matter of fact, paying for a call option ensures the end user the possibility to exploit lower prices and a maximum ceiling price, thus reducing uncertainty. For example, if the end user would like to pay a maximum of $15/MWh he can purchase a call option paid up front. Therefore, if the electricity price goes below $15/MWh the end user would buy electricity on the spot market, otherwise he would exercise his call option, thus paying 15/MWh and no more.



5. Payoff of an end user purchasing a call option

Finally, marketers can exercise call and put options on exchanges or with other parties on behalf of end users or generators, or offer put and call options to generators and end users.

- CALLABLE AND PUTABLE FORWARDS

The last kind of derivatives with a single underlying are callable and puttable forwards. The callable forward contract implies that the contract holder longs one forward contract and shorts one call option agreeing upon the strike price with the purchaser. On the other side, the seller of the forward contract has the possibility to exercise the call option whenever the electricity price exceeds the strike price, thus annulling the forward contract at delivery time. The most notable example of this kind of contract is the interruptible supply contract thanks to which the purchaser gets an interruptible discount on the forward price.
On the other hand, a puttable forward contract implies that the contract holder longs one forward contract and longs one put option agreeing upon the strike price with the seller that is the one holding the short positions. In this type of contract the purchaser can exercise the put option whenever the electricity price goes below the strike price, thus annulling the forward contract at delivery time. The most notable example of this kind of contract is the dispatchable independent power producer (IPP) supply contract, according to which the purchaser pays a capacity availability premium over the forward price.

- OTHER DERIVATIVES

There are several other electricity derivatives traded on exchanges and the OTC market in order to ensure efficiency and transparency. Some examples include path dependent options, derivatives with multiple underlyings, options with variable volume and options on options. Anyway these are behind the scope of this paper and therefore will not be analysed.

In order to understand the pricing of electricity derivatives and in particular electricity options, it is important to discuss about the Black-Scholes model, which is the widely used one, and to verify whether more consistent pricing models are available.

CHAPTER 2

2.1 THE BLACK-SCHOLES EVALUATION MODEL

The Black-Scholes model is largely responsible for the birth of options market and options trading becoming increasingly popular in the last decades. As a matter of fact, with the development of this model, a standard method for pricing options and putting a fair value on them became available. Hence, options and other derivatives started to be seen as suitable financial instruments to be traded and thus became common on exchanges and OTC markets.

2.1.2 HISTORY OF THE BLACK-SCHOLES MODEL

The Black-Scholes formula was born in 1970, when Fischer Black, a mathematical physicist, and Myron Scholes, a professor of finance at Stanford University, wrote a paper titled “The Pricing of Options and Corporate Liabilities.” At the beginning, their paper was repeatedly rejected by economics journals, until in 1973 the Journal of Political Economy of Chicago University decided to publish it. According to Black and Scholes an option had a precise price, which could be calculated thanks to the equation provided in the paper. This equation became then knows as the Black-Scholes formula. A few months later, Robert Merton published “Theory of Rational Option Pricing", paper in which he introduced the term ‘’Black-Scholes option pricing formula’’ and he further developed and expanded this mathematical approach for option pricing. These three economists, even if surrounded by a great deal of skepticism, demonstrated that with the help of differential equations the true value of European call and put options could be evaluated. The contribution to modern financial theory of Black, Scholes and Merton was one of the most significant; as a matter of fact in 1997, two years after the death of Fischer Black, Myron Scholes and Robert Merton were awarded the Nobel Prize in Economics.

After the recognition of the ground-breaking result they had achieved, the Black-Scholes model started to be widely used for the pricing of options. In fact, the main idea behind successful trading and investing is to find assets that are either overpriced or underpriced. With the introduction of a correct price for options, option trading was no more considered to be too risky. On the contrary, options started to be seen as a way to create perfect hedging strategies in combination with the underlying asset. Therefore, both buyers and sellers, by repeatedly buying and selling options at the price set by the model, could break even (excluding commissions charged).

2.1.3 THE BLACK–SCHOLES–MERTON DIFFERENTIAL EQUATION

The Black-Scholes-Merton differential equation is an equation that must be satisfied by the price of any derivative dependent on a non-dividend paying stock. The idea is to set up a riskless portfolio with both a position in the derivative and in the stock. With no arbitrage opportunities the return of the portfolio is the risk-free interest rate, r.

There are several underlying assumptions upon which the Black-Scholes model rely in order to be theoretically coherent:

1) The price of the underlying asset (typically a stock) follows a geometric Brownian motion
2) The μ and σ are constant over the lifetime of the underlying security
3) The short-selling of securities is allowed
4) There are no dividends during the life of the derivative
5) There are no transactions costs or taxes. All securities are perfectly divisible
6) There are no riskless arbitrage opportunities.
7) Security trading is continuous
8) The risk-free rate of interest, r, is constant and the same for all maturities.

In addition to the assumptions, numerous are the variables or inputs that are used in the model to calculate the fair value of an option:

1) The current price of the underlying security (S_t)
2) The strike price (K)
3) The length of time until expiry ($\tau = T - t$)

4) The risk free interest rate during the period of the contract (r)
5) The implied volatility of the underlying security (σ)
6) The cumulative distribution function of a standard normal variable ($N(x)$ or $\phi(x)$)

2.2 DERIVATION OF THE BLACK–SCHOLES–MERTON DIFFERENTIAL EQUATION

For the purpose of this work the derivation and some other important elements will be taken from: John C. Hull in "Options, Futures and other Derivatives", Evan Turner "The Black-Scholes model and extensions", Fabrice Douglas Rouah "Four Derivations of the Black-Scholes Formula", Claudio Pacati "A proof of the Black and Scholes Formula" and Malik Magdon-Ismail "Computational Finance – The Martingale Measure and Pricing of Derivatives". The analysis starts by stating four important definitions for the further derivation of the Black-Scholes formula.

1) A stochastic process, $W(t)$, for $t \geq 0$, is a Brownian Motion if $W_0 = 0$, and for all t and s, with $s < t$,

$$W(t) - W(s)$$

is continuous, has a normal distribution with variance $t - s$, and the distribution of $W(t) - W(s)$ is independent of the behavior $W(r)$ for $r \leq s$.

More generally, a variable z follows a Brownian motion (Wiener process) if it has two specific properties:

- The change Δz during a small period of time Δt is $\Delta z = \epsilon\sqrt{t}$ where ϵ has a standardized normal distribution $\phi(0, 1)$.
- The values of Δz for any two different short intervals of time, Δt, are independent.

According to the first property, Δz itself is characterized by a normal distribution with mean = 0, standard deviation = $\sqrt{\Delta t}$ and variance Δt.

According to the second property, z follows a Markow process. A Markov process is a particular type of stochastic process where only the current value of a variable is relevant for predicting the future. The past history of the variable and the way that the present has emerged from the past are irrelevant.

2) The family *X* of random variables *X(t)* satisfies the stochastic differential equation (SDE),

$$dX(t) = \mu\big(t, X(t)\big)dt + \sigma\big(t, X(t)\big)dW(t) \tag{2.2}$$

if for any t,

$$X(t+h) - X(t) - h\mu\big(t, X(t)\big) - \sigma\big(t, X(t)\big)\big(W(t+h) - W(t)\big) \tag{2.3}$$

is a random variable with mean and variance which are O*(h)* and W*(t)* is a Brownian motion.

3) A stochastic process *S(t)* is said to follow a Geometric Brownian Motion if it satisfies the stochastic differential equation

$$dS(t) = \mu S(t)\, dt + \sigma S(t)\, dW(t) \tag{2.4}$$

with μ and σ constants and *W(t)* a Brownian motion.

4) An Ito Process, *X(t)*, is a process that satisfies the stochastic differential equation

$$dX(t) = \mu(t)X(t)\, dt + \sigma(t)X(t)\, dW(t) \tag{2.5}$$

The parameters μ and σ are functions of the underlying variable *X* and time *t*. The expected drift rate and variance of an Ito process change over time. In a small time interval between *t* and $\Delta t + t$ the variable changes from *x* to $X + \Delta X$, where ΔX is dependent upon Δt and $\epsilon\sqrt{\Delta t}$.

5) A martingale is a zero-drift stochastic process. A variable θ follows a martingale if its process has the form

$$d\theta = \sigma dz \tag{2.6}$$

where *dz* is a Wiener process. The variable σ may itself be stochastic. It can depend on θ and other stochastic variables. A martingale has the convenient property that its expected value at any future time is equal to its value today. This means that

$$E\big(\theta(T)\big) = \theta \tag{2.7}$$

where θ_0 and θ_T denote the values of θ at times zero and T, respectively.

Following this reasoning an equivalent martingale measure (Q) is a probability vector according to which

$$S(0) = e^{-r\mathrm{T}}E^{Q}\left[\frac{Si(T)}{S1(T)}\right] \tag{2.8}$$

The equivalent martingale measure (Q) is often referred to as the risk-neutral measure under the no arbitrage assumption.

Having stated these five definitions it is possible to proceed with the Ito's Lemma theorem. Let $X(t)$ be an Ito process satisfying equation (2.2), and let $f(x, t)$ be a twice-differentiable function; then $f(X(t), t)$ is an Ito process, and

$$d\big(f(X(t),t)\big) = \frac{\partial f}{\partial t}(X(t),t)dt\ + \frac{\partial f}{\partial X(t)}dX(t) + \frac{1}{2}\frac{\partial^2 f}{\partial X^2(t)}dX^2(t) \tag{2.9}$$

where $dX^2(t)$ is defined by

$$dt^2 = 0 \tag{2.10}$$

$$dtdW(t) = 0 \tag{2.11}$$

$$dW^2(t) = dt \tag{2.12}$$

Based on the fact that dt is infinitesimal it seems reasonable that $dt^2 = 0$ and $dtdW(t) = 0$. In order to explain $dW^2(t) = dt$ it will be useful to examine a random walk on Z, giving an intuitive proof rather than a strict mathematical one. Imagine that a man takes a step of length 1 or -1 with equal probability at time t, where t is a natural number greater than 0. Let $W(t)$ be the sum of steps from time $t = 0$ to t, then $E[W(t)] = 0$. Since there are t steps of length 1, $W^2(t) = t$. Therefore, it seems reasonable that $W(t) = \sqrt{t}$ and that in Δt steps $W(t) = \sqrt{\Delta t}$. If $\Delta t \rightarrow 0$, $W^2(t) \approx \Delta t$, since the time increment and steps have become arbitrarily small. Equation (2.12) follows.

Having stated these four definitions, the time-t price $C(S_t; K; T)$ of a European call option with strike price K and maturity equal to τ = T - t on a non-dividend paying stock with spot price S_t and a constant volatility σ when the rate of interest (r) is constant, can be expressed as

$$C(S(t);\ K;\ T) =\ S(t)\phi(d1) - e^{-r\tau}K\phi(d2) \tag{2.13}$$

where

$$d1 = \frac{ln\,\frac{S(t)}{K} + (r + \frac{\sigma^2}{2})\tau}{(\sigma\,\sqrt{\tau})} \tag{2.14}$$

and

$$d2\ =\ d1 -\ \sigma\sqrt{\tau} \tag{2.15}$$

and

$$\phi(y) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{y} e^{-\frac{1}{2}t^2}\,dt \tag{2.16}$$

is the standard normal cumulative distribution function.

2.2.2 MATHEMATICAL DERIVATION BY STRAIGHTFORWARD INTEGRATION

A portfolio with two assets driven by stochastic differential equation is taken into account. It includes a risky stock S and a riskless bond B.

$$dSt\ =\ \mu S(t)dt + \sigma S(t)dW(t) \tag{2.17}$$

$$dBt\ =\ r(t)B(t)dt \tag{2.18}$$

The value of the bond at time 0 is B_0 = 1 and the one of the stock is S_0. This model is valid under the previous stated assumptions. By Ito's Lemma the value V_t of a derivative written on the stock follows the diffusion

$$dV(t)\ = \frac{\partial V}{\partial t}\,dt\ + \frac{\partial V}{\partial S}\,dS\ + \frac{1}{2}\,\frac{(\,\partial^2\,V)}{\partial S^2}\,dS^2$$

$$= \frac{\partial V}{\partial t}\, dt \; + \; \frac{\partial V}{\partial S}\, dS \; + \frac{1}{2}\, \frac{(\,\partial^2\, V)}{\partial S^2}\; \sigma^2 S^2 dt$$

$$= \left(\frac{\partial V}{\partial t} \; + \; \mu S(t) \frac{\partial V}{\partial S} \; + \frac{1}{2} \sigma^2 S^2(t) \frac{(\,\partial^2\, V)}{\partial S^2}\; dS^2 \right) dt \; + \left(\sigma S(t) \frac{\partial V}{\partial S} \right) dW(t) \quad (2.19)$$

The Ito's Lemma is also used to derive the course followed by ln(*S*) if *S* follows the process described in equation (2.5). If

$$G = \ln(S)$$

Is defined. Since

$$\frac{\partial G}{\partial S} = \frac{1}{S}\,, \qquad \frac{\partial^2 G}{\partial S^2} = -\frac{1}{S^2}\,, \qquad \frac{\partial G}{\partial t} = 0$$

It follows from equation (2.17) that the process followed by *G* is

$$dG = \left(\mu - \frac{\sigma^2}{2} \right) dt + \; \sigma dz \qquad (2.20)$$

Since μ and σ are constant the equation *G* = ln(*S*) follows a generalized Wiener process. It has a constant drift rate (μ - σ^2/2) and a constant variance rate σ^2. The change in ln(*S*) between time 0 and some future time T is normally distributed with mean (μ - σ^2/2)T and variance σ^2T. Therefore, the stock price *S* a time T (S_T) is defined as

$$\ln S(T) \sim \phi \left[lnS(0) + \left(\mu + \frac{\sigma^2}{2} \right) T,\; \sigma^2 T \right] \qquad (2.21)$$

where S_0 is the stock price at time 0. Equation (2.21) shows that ln(S_T) is normally distributed. If the natural logarithm of a variable is normally distributed, the variable has normal distribution. Therefore, the stock price at time T, given its price today, follows a lognormal distribution and applying Ito's Lemma it follows the process:

$$dlnS(t) = \left(\mu - \frac{\sigma^2}{2} \right) dt + \; \sigma dW(t) \qquad (2.22)$$

hence

$$S(t) = S(0)e^{\left(\mu - \frac{\sigma^2}{2}\right) dt + \sigma W(t)} \tag{2.23}$$

On the other hand, applying Ito's Lemma to the function ln(B_t) it can be seen that ln(B_t) follows the stochastic differential equation

$$d\ln B(t) = r(t)dt \tag{2.24}$$

Since $B_0 = 1$, integrating from 0 to t the solution of the stochastic differential equation is

$$B(t) = e^{\int_0^t r(u)du} \tag{2.25}$$

If interest rates are constant therefore $r_t = r$ and $B_t = e^{rt}$. Thus, integrating from t to T gives the solution

$$B(t,T) = e^{\int_t^T r(u)du} \tag{2.26}$$

and $B_{t,T} = e^{rm}$.

Having stated the distributions of the processes it is possible to proceed with the straightforward integration. Considering an European call option price $C(S_t, K, T)$ that is the discounted time-t expected value of (S_t – K) under the equivalent martingale measure Q, when interest rates are constant. The starting equation of the straightforward integration is the following

$$C(S(t), K, T) = e^{-r\tau} E^Q[\,(S(T) - K) \mid \mathcal{F}(t)\,] =$$

$$= e^{-r\tau} \int_K^\infty (S(T) - K) dF\big(S(T)\big) =$$

$$= e^{-r\tau} \int_K^\infty S(T) dF\big(S(T)\big) - e^{-r\tau} K \int_K^\infty dF\big(S(T)\big) \tag{2.27}$$

To evaluate the two integrals, the results derived before are taken into account: the terminal stock price S_T follows the lognormal distribution with mean $\ln S_t + (r - \sigma^2/2)\tau$ and variance $\sigma^2\tau$, where $\tau = T - t$ is the time to maturity. The first integral in the last line of equation (2.27) uses the conditional expectation of S_T given that $S_T > K$

$$\int_{k}^{\infty} S(T)dF\big(S(T)\big) = E^{Q}[\,S(T) \mid S(T) > K\,] = L\big(S(T)\big)(K) \tag{2.28}$$

where $L(S_T)K$ stands for the lognormal distribution and stems from the conditional expectation $L(S_T)K = E[S_T/S_T>K]$. It can be shown that the general conditional expectation is

$$L(x)(K) = e^{\left(\mu+\frac{\sigma^2}{2}\right)}\frac{1}{\sigma}\int_{lnK}^{\infty}\frac{1}{\sqrt{2\pi}}e^{\left(-\frac{1}{2}\left(\frac{y-(\mu+\sigma^2)}{\sigma}\right)^2\right)}dy \tag{2.29}$$

and therefore $L(S_T)K$ can be written as

$$L\big(S(T)\big)(K) = e^{\left(\ln(S(T))+\left(r-\frac{\sigma^2}{2}\right)\tau+\frac{\sigma^2\tau}{2}\right)}x\phi\left(\frac{-lnK + lnS(t) + \left(r - \frac{\sigma^2}{2}\right)\tau + \sigma^2\tau}{\sigma\sqrt{\tau}}\right)$$

$$= S(t)e^{r\tau}\phi(d1) \tag{2.30}$$

Hence, the first integral of equation (2.27) can be written as

$$S(t)\phi(d1) \tag{2.31}$$

On the other hand, using the cumulative distribution function that states that

$$Fx(x) = \phi\left(\frac{lnx - \mu}{\sigma}\right) \tag{2.32}$$

it can be shown that the second integral can be written

$$e^{-r\tau}K\int_{K}^{\infty} dF\big(S(T)\big) = e^{-r\tau}K[1 - F(K)]$$

$$= e^{-r\tau}K\left[1 - \phi\left(\frac{lnK - lnS(t) - \left(r - \frac{\sigma^2}{2}\right)\tau}{\sigma\sqrt{\tau}}\right)\right]$$

$$= e^{-r\tau}K\,[1 - \phi(-d2)]$$

$$= e^{-r\tau} K \phi(d2) \tag{2.33}$$

Combining the terms in equation (2.30) and equation (2.33) leads to the Black-Scholes formula for the European call price

$$C(S(t);\, K;\, T) = S(t)\phi(d1) - e^{-r\tau} K \phi(d2) \tag{2.34}$$

2.3 CRITICS TO THE BLACK-SCHOLES MODEL

The main critics that has been moved to the Black-Scholes model are concerned with the assumptions on which the model itself relies. It is then important to analyze these assumptions and to evaluate the magnitude by which they affect the difference between option prices calculated with the Black-Scholes formula and the ones observed on the market. In fact, it is rare that the calculated value of an option comes out exactly equal to the price at which it is traded on exchanges. Therefore, the assumptions introduced at the beginning of Chapter 2 will be taken into account in order to spot the main flaws within the model.

1) The volatility of stock prices, which is assumed to be constant over the lifetime of the underlying security, changes. Generally the volatility changes in unexplainable ways, but the change seems to be related with the changes in the price of the security. In fact, while an increase in the stock price is associated with a decrease in volatility, a decline in the stock price is associated with a significant increase in volatility. Moreover, since the volatility of a stock cannot be observed, it is estimated by making use of the movements in prices and the fact that they move in opposite directions. Considering that underlying securities prices change, the volatility also changes and cannot be assumed to constant over the lifetime of the security itself. These possible changes in volatility will generally increase the option values, therefore making the writing of such options less attractive.
2) Borrowing and short-selling penalties are present on the market. Borrowing penalties mean that the rate at which an investor can borrow, even with securities as collateral, is higher than the rate at which he can lend. This affect the value of options: since they can provide leverage that can substitute for borrowing their value will generally slightly increase. On the other hand, short-selling penalties have more severe effects. Since the

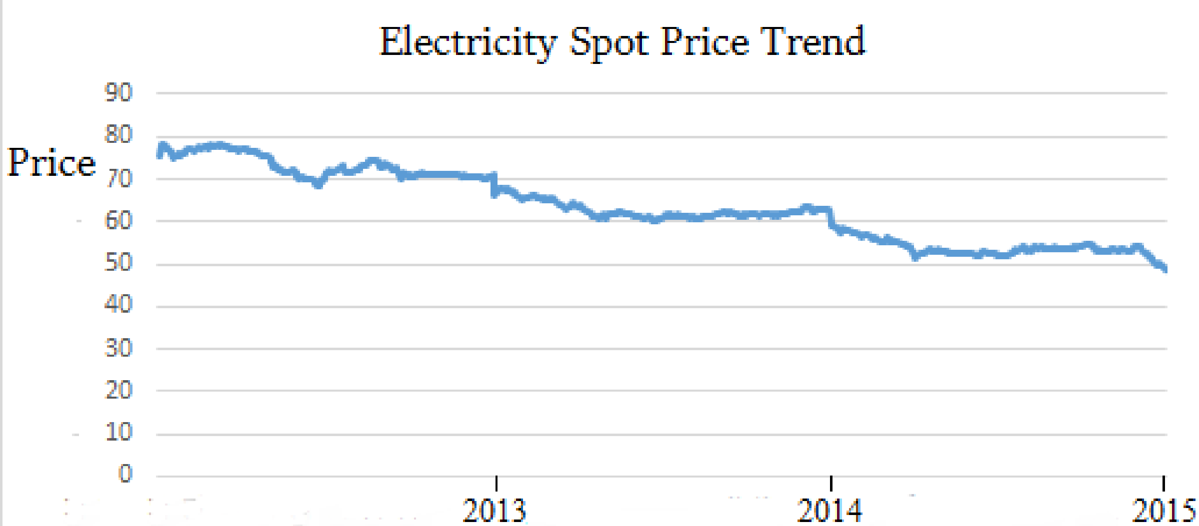
only way for investors to sell stocks short on a downtick is to undergo the expenses of borrowing them, such as the cash payment of the collateral, which gives interests far below market rates, to the stock lender or the cash due to the broker. These short-selling penalties on stocks generally may cause option values to be mispriced and more precisely, since put-options can be considered equivalent to selling stock short, short selling of stock tends to increase put options prices.

3) Transaction costs and taxes exist. First of all, brokerage charges on options or exchange memberships have to be generally paid and implicate a substantial burden on any potential profit on mispriced option. Nonetheless they represent a greater barrier for outside investors than for inside ones. Secondly, even if some investors do not pay taxes, the existence of taxes on dividends, capital gains, corporations and the presence of across countries different level of taxes, affects the option values.
4) Dividends are an important component on the market. Dividends lower the value of call options and increase the value of put options, if there are no offsetting adjustments in option terms. Therefore, if dividends are paid on the market the early exercise of a call option becomes more likely and the opposite holds for true for put options.
5) Security trading is not continuous. The existence of takeovers, not only settled ones but also possible ones, affects option values. If a takeover occurs the acquired firm options will become the acquiring firm options and the market value will therefore change. Moreover, if a takeover becomes possible the probability of the takeover occurrence itself will affect the option value.
6) Not only the stock volatility changes over time but also the interest rate does. While the volatility can only be estimated, the interest rate can also be observed; thus being easier to take into account. In fact, when both interest and volatility change, some complicated adjustments can be made to the formula to take these effects into account. Anyway the way interest changes affect option values cannot be nearly compared to the effects of volatility changes.
7) The price of the underlying asset does not follow a geometric Brownian motion and price returns are not normally distributed. This assumptions derives from the random walk

theory, which states that all price movements are random. Anyway, influences of various types, such as merger rumors and earnings surprises or sector, economic and political news, affect prices in a non-random manner.

2.3.2 TESTING FOR THE NORMAL DISTRIBUTION OF RETURNS

For the purpose of this work, the assumption that the underlying asset follows a geometric Brownian motion and more specifically that the price returns are normally distributed will be tested with data of the Italian electricity market, provided by Enel S.p.A. The data that are here taken into account represent the quarterly and yearly electricity prices of 2013, 2014 and 2015, which, as it has been described before, are traded starting from one year before the quarter or year of reference. The data consist of 253 observations for each of the three calendar years and of the twelve quarters. In order to give an insight of the trend of the price movements, the three calendar years prices are shown in the graph below.



6. Electricity spot price trend in 2013,2014 and 2015. Source: Enel S.p.A

Hence, the analysis of these price movements can start. First of all, the returns for the fifteen set of data are calculated. The example of the three calendar years is here provided:

```
rcal2013 = diff(log(Cal2013IT_Power_base))
rcal2014 = diff(log(Cal2014IT_Power_base))
rcal2015 = diff(log(Cal2015IT_Power_base))
```

Therefore, it is possible to provide an analysis of the returns through the most important statistical metrics: mean, standard deviation, skewness and kurtosis. The following table summarizes the results.

	Mean	Standard Deviation	Skewness	Kurtosis
Returns_Quarter1_2013	-4.45e-04	0.0055	1.0582	12.2921
Returns_Quarter2_2013	-4.94e-04	0.0054	0.0475	5.0830
Returns_Quarter3_2013	-2.20e-04	0.0045	0.3802	5.7941
Returns_Quarter4_2013	-4.33e-04	0.0040	0.2670	4.9028
Returns _2013	-2.39e-04	0.0050	1.3466	13.8493
Returns_Quarter1_2014	-2.13e-04	0.0042	0.3190	4.3301
Returns_Quarter2_2014	-0.0010	0.0054	-0.7833	4.3468
Returns_Quarter3_2014	-8.22e-04	0.0063	0.9041	13.4761
Returns_Quarter4_2014	-6.27e-05	0.0059	0.5407	7.5900
Returns _2014	-2.16e-04	0.0037	0.1600	3.9467
Returns_Quarter1_2015	-6.19e-04	0.0061	0.2083	4.3011
Returns_Quarter2_2015	-1.30e-04	0.0072	0.4362	5.6153
Returns_Quarter3_2015	6.99e-05	0.0075	0.2581	5.2538
Returns_Quarter4_2015	-5.60e-04	0.0072	-0.7570	8.5847
Returns _2015	-7.83e-04	0.0052	0.1783	6.3183

As it possible to infer from the data the mean returns is always very close to zero, which could leave the possibility for the returns to be normally distributed. On the other hand, at first glance, that hypothesis should be rejected since, while the standard deviation is generally very close to 0.5, except for some cases, the kurtosis is highly variable. Moreover, the returns are approximately always skewed on the right. Anyway, in order to assess whether the returns are normally distributed, a more precise analysis is needed.

The three tests that have been chosen to check for the normality of price returns are the most reliable and precise ones. A brief description of the Shapiro-Wilk, the Jarque-Bera and the Anderson-Darling tests is now provided thanks to “Statistics Explained”, Perry R. Hinton.

1) The Shapiro-Wilk test uses the null-hypothesis that a sample $x_1, \ldots, x_n$ comes from a normally distributed population. The SW test statistic is defined as

$$W = \frac{\left(\sum_{i=1}^{n} x(i)\alpha_i\right)}{\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

where

- x(i) is the ith order statistic
- $\bar{x} = (x_1 + \ldots + x_n)/n$ is the sample mean
- the constants α_i are given by

$$(\alpha_1, \ldots, \alpha_n) = \frac{m^T V^{-1}}{(m^T V^{-1} V^{-1} m)^{\frac{1}{2}}}$$

where $m = (m_1, \ldots, m_n)^T$

and $m_1, \ldots, m_n$ are the expected values of the order statistics of independent and identically distributed random variables sampled from the standard normal distribution, and V is the covariance matrix of these order statistics.

Therefore, if W is below a predetermined threshold the null hypothesis may be rejected. Moreover, the test has been empirically demonstrated to be the best test for detecting normality departures.

2) The Jarque-Bera test is the most frequently used by econometricians. It is a goodness of fit test of whether sample data have kurtosis and skewness matching a normal distribution. The JB test statistic is defined as

$$JB = \left(\frac{n-k-1}{6}\right)\left(S^2 + \frac{1}{4}(C-3)^2\right)$$

where

- n is the number of observations
- S is the sample skewness, defined as

$$S = \frac{\mu_3}{\sigma^3} = \frac{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^3}{\left(\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2\right)^{3/2}}$$

- C is the sample kurtosis, defined as

$$C = \frac{\mu_4}{\sigma^4} = \frac{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^4}{\left(\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2\right)^{2}}$$

- $\bar{x}$ is the sample mean
- σ^2 is the variance
- μ_3 is the estimate of the third moment
- μ_4 is the estimate of the fourth moment

The JB statistic asymptotically has a chi-squared distribution with two degrees of freedom, therefore it can be used to test the hypothesis that the data come from a normal distribution. It is based on comparing how far the asymmetry and kurtosis measures diverge from the values typical of the normal distribution, thus stressing their importance.

3) The Anderson-Darling test is used to test whether a sample of data comes from a population with a specific distribution. It is a modification of the Kolmogorov-Smirnov test and it is one of the most powerful statistical tools for detecting departures from normality since, differently from the KS test, it gives more weight to the tails. The AD test is defined as

$$A^2 = -n - S$$

where

- n is the number of observations
- S is defined as

$$S = \sum_{i=1}^{n} \frac{2i-1}{n}\left[\ln\big(\phi(Y_i)\big) + \ln(1-\phi(Y_{n+1-i}))\right]$$

- ϕ is the cumulative distribution function
- Y_i are the ordered data

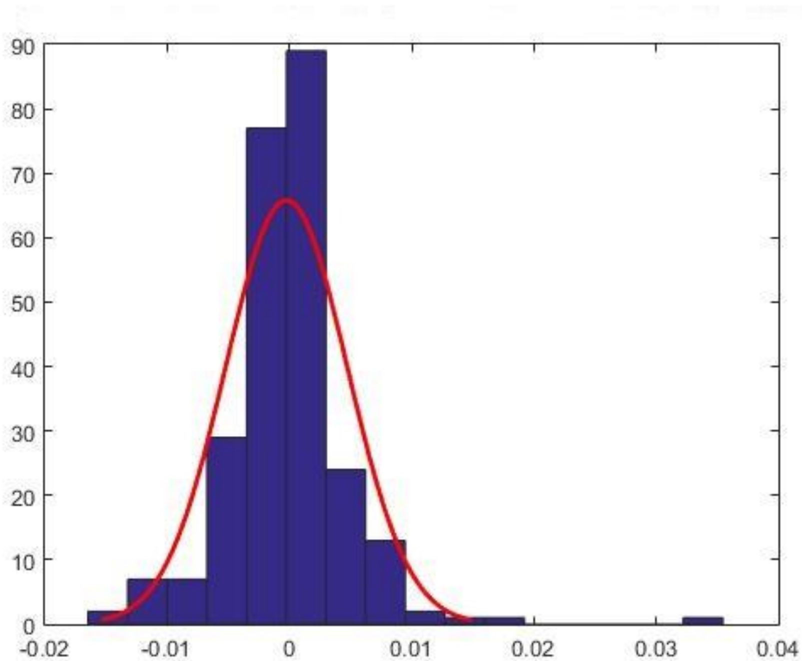
The critical values for the Anderson-Darling test are dependent on the specific distribution that is being tested. Even if the test is not as good as the Shapiro-Wilk, it represents a great improvement with respect to the Kolmogorov-Smirnov and other tests.

The three tests, being of the uttermost importance, are applied to the electricity price returns with the Matlab environment (R15). The null hypothesis that the returns are normally distributed is being tested at the 5% confidence level and the results are the following:

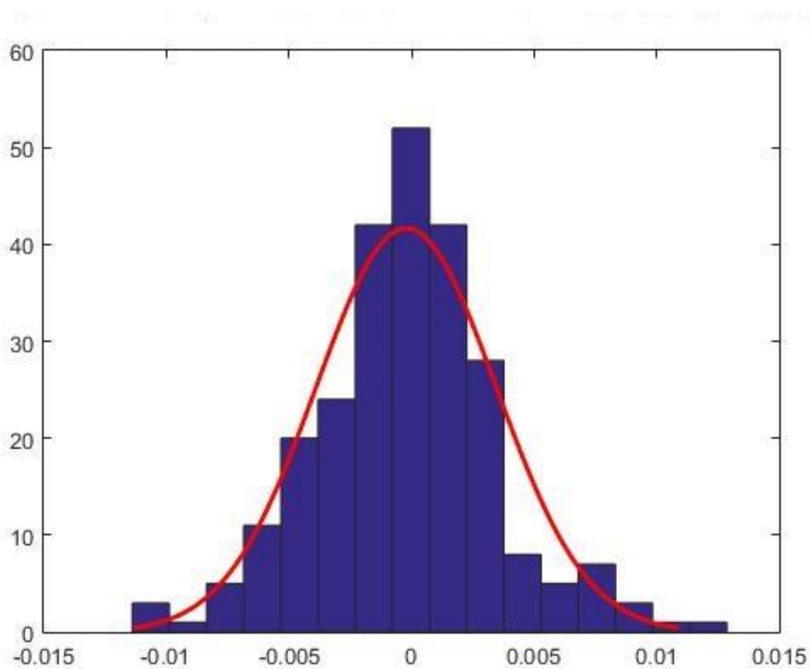
	Jarque-Bera	Shapiro-Wilk	Anderson-Darling
Returns_Quarter1_2013	Reject	Reject	Reject
Returns_Quarter2_2013	Reject	Reject	Reject
Returns_Quarter3_2013	Reject	Reject	Reject
Returns_Quarter4_2013	Reject	Reject	Reject
Returns_2013	Reject	Reject	Reject
Returns_Quarter1_2014	Reject	Reject	Reject
Returns_Quarter2_2014	Reject	Reject	Reject
Returns_Quarter3_2014	Reject	Reject	Reject
Returns_Quarter4_2014	Reject	Reject	Reject
Returns_2014	Reject	Reject	Reject
Returns_Quarter1_2015	Reject	Reject	Reject
Returns_Quarter2_2015	Reject	Reject	Reject
Returns_Quarter3_2015	Reject	Reject	Reject
Returns_Quarter4_2015	Reject	Reject	Reject
Returns_2015	Reject	Reject	Reject

The Matlab codes and calculations are provided in the appendix

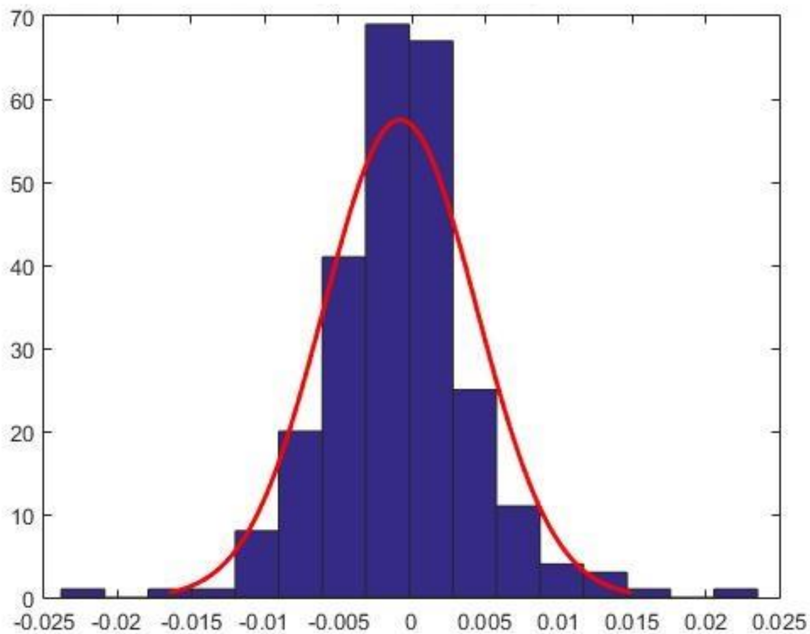
As it can be seen from the results the null hypothesis that the returns are normally distributed is rejected at the 5% significance level by the three tests, with no exception. A graphical insight can be given by comparing the returns of the three calendar years with a normal distribution:



7. Comparison between Price returns and Normal Distribution-2013



8. Comparison between Price returns and Normal Distribution-2014



9. Comparison between Price returns and Normal Distribution-2015

Hence, it is possible to infer that the Black-Scholes assumption does not find a confirmation when real data are taken from the market and analyzed: the prices do not follow a geometric Brownian motion and the returns are not normally distributed. This happens because the model neglects to account that prices do have high variability, which is detectable in real electricity prices and which is the main determinant of the risk associated with financial assets. Moreover, a careful analysis of returns, shows the presence of heavy tails, which detects a high probability of large price movements that the model does not take into consideration.

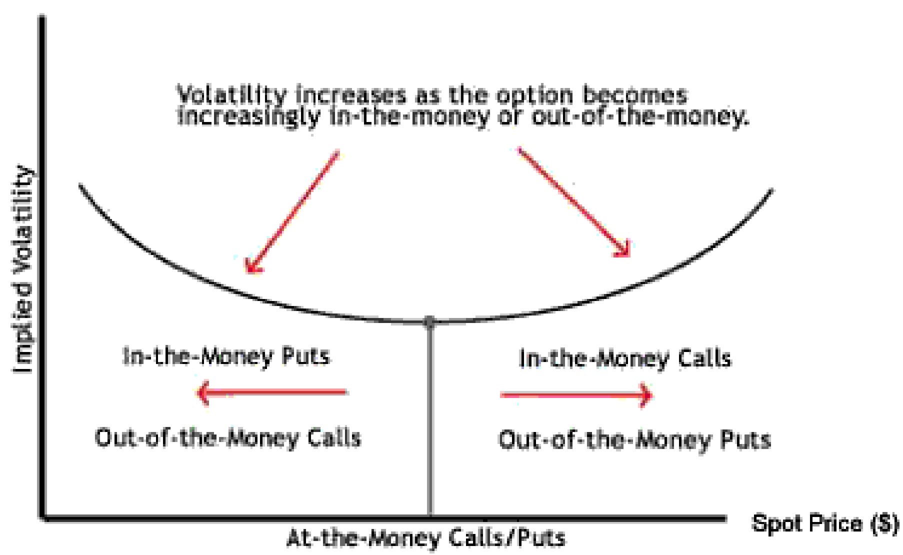
2.3.3 AN INSIGHT ON THE VOLATILITY SMILE

Being one of the main flaws of the Black-Scholes model, the assumption that the volatility is constant over the lifetime of the security is now analyzed carefully. If the model were correct it would not be possible to observe any change in volatility, hence a flat implied volatility surface (the 3D graph of implied volatility against strike and maturity) would be present. The volatility surface is a function of strike, K, time to maturity, T, and is defined as

$$C(S, K, T) = BS(S, T, r, K, \sigma(K, T))$$

Where $C(S, K, T)$ denotes the current market price of a call option with time to maturity T and strike K and BS is the Black-Scholes formula for pricing a call option. In other words, $\sigma(K, T)$ is the volatility that, when inserted into the formula, gives the current market price. Since the Black-Scholes formula is continuous and increasing in σ, there will always be a unique solution, $\sigma(K, T)$. Moreover, if the assumption were correct the volatility surface would be flat with $\sigma(K, T) = \sigma$ for all K and T. In reality however, not only is the volatility not flat but it actually varies, often significantly, with time.

The principal features of the volatility surface is that options with lower strikes tend to have higher implied volatilities. For a given maturity, T, this feature is often referred to as the volatility smile. The smile first appeared after the 1987 crash and was clearly connected in some way with the visceral shock of discovering, for the first time since 1929, that a giant market could drop by 20%. For a given strike, K, the implied volatility can be either decreasing or increasing with time to maturity. Nonetheless, $\sigma(K, T)$ tends to converge to a constant as $T \rightarrow \infty$, while for short-term options much higher volatilities are observed. The more an option is in-the-money or out-of-the-money, the greater its implied volatility becomes. The relationship between an option implied volatility and strike price can be seen in the graph below.



10. Relationship between an option implied volatility and strike price. Source: John C. Hull: *"Options, Futures and Other Derivatives"*

2.3.4 TESTING FOR THE VOLATILITY CONSTANCY

For the purpose of this work, the assumption that the volatility is constant over the lifetime of the underlying security will be tested with an extension of the data previously taken into account. The focus is now shifted on 113 call option prices related to years 2013, 2014 and 2015. The prices are based on a trading dataset provided by Enel S.p.A. The objective is to minimize the following functional:

$$\min_{\sigma>0} \sum_i \left[C^{BS}\left(S_{\tau_i}, \tau_i, r, K_i, \sigma\right) - C_i^{Obs}\right]^2$$

where

1) r is equal to 0 and constant over the three years
2) C^{BS} are the price estimated with the Black-Scholes formula
3) C^{Obs} are the call option prices observed on the market

The calibration procedure has been implemented in the Matlab environment (R15). Therefore, the following steps are carried out:

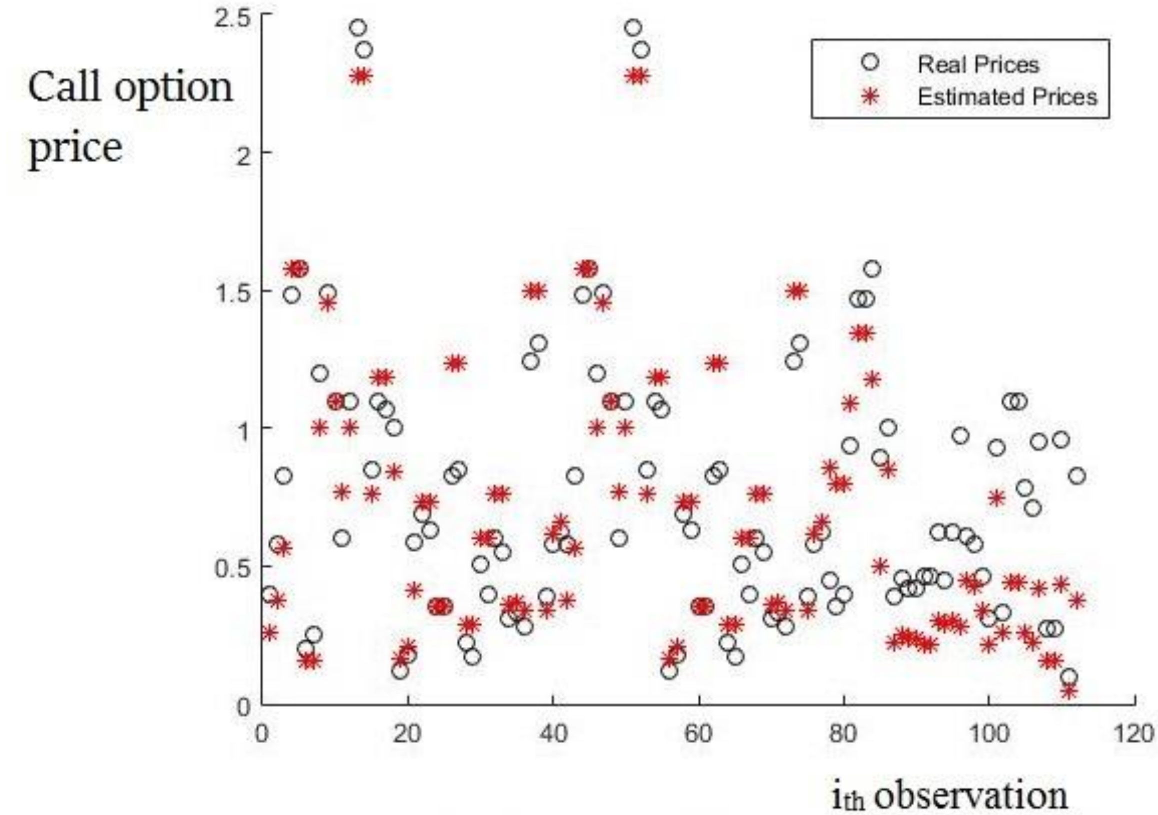
1) A function that describes the sum of the squared differences between prices estimated with a constant volatility *x* and real market prices is created.
2) The *x* minimizing this function is found
3) The call options prices are calculated with the estimated volatility *x*.

```
1)  fun = @(x) sum((blsprice(PowerMWh, Strike, 0, DeltaTOriginale, x)-Premio).^2)

2)  xx = fmincon(fun,0.09,[],[],[],[],0,1)                         x = 0.0780

3)  ys = blsprice(PowerMWh, Strike, 0, DeltaTOriginale, xx)
```

The prices calculated with the estimated constant volatility (σ=0.078) and the ones really observed in the market are then plotted to show if some differences are present.



11. Comparison between real prices and estimated prices

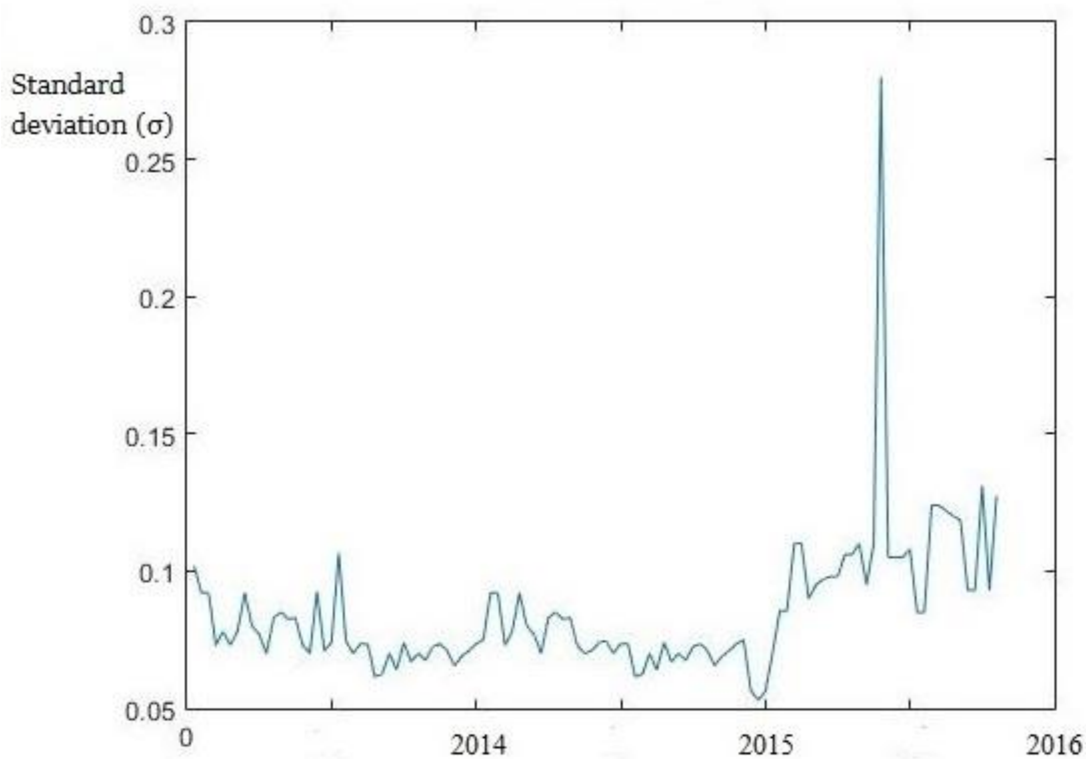
As can be seen from the graph, the prices estimated with the volatility minimizing the difference (σ=0.078) are most of the times different from the real ones. The magnitude of the differences is described by:

$$\sum\left[C^{BS}\left(S_{\mathrm{T}_i}, \mathrm{T}_i, r, K_i, 0.078\right) - \ C_i^{Obs}\right]^2$$

The following box describes the Matlab code for the calculation of the magnitude:

```
SumDiff = sum((blsprice(PowerMWh, Strike, 0, DeltaTOriginale, 0.078)-Premio).^2)
```

The result turns out to be equal to 5.9465 which, considering the 113 observations is not too high. Unfortunately, the constant volatility does not give the desired results (estimated prices equal to observed prices). Hence, it can be inferred that the assumption, according to which the volatility is constant over the lifetime of the underlying security, is an unreliable one. As a matter of fact, by plotting the implied volatility trend over the three years analyzed, it is possible to see that the volatility has not negligible variability.



12. Implied volatility trend over 2013, 2014 and 2015

2.4 ALTERNATIVES TO THE BLACK-SCHOLES MODEL

In order to address the differences arising between the Black-Scholes model predictions and the real market price movements, alternatives to the Black-Scholes option pricing model have been formulated. One of the first efforts that was made to overcome the problems related to the volatility of the underlying prices, was the creation of the stochastic volatility model. This model is a two dimensional diffusion process that, not only considers the evolution of S_t, but also allows volatility σ_t to fluctuate over time. As a consequence, the returns are not normally distributed, follow a discontinuous pattern and are supposed to have possible large movements. Another framework that has been developed is the jump-diffusion model. According to this model the normal evolution of prices is based upon a diffusion process, whose walk is interrupted by jumps that represent rare events (related to the financial market and happening at random time intervals). These jumps in the normal evolution of prices have also been integrated in a more general stochastic process called Lèvy processes (Cheang-Chiarella, 2011). The Merton model represent one of the extensions following the jump-diffusion model; according to it the jumps follow a Gaussian distribution. The development of the stochastic volatility model and the jump diffusion model provided useful insights for traders and the financial world in general. In fact, they revised some of the wrongful assumptions behind the Black-Scholes model, trying to give more precise estimates for option prices with the help of some more advanced mathematics and less relaxed assumptions. Nonetheless, the evolution of option pricing formulas has not arrived to an end: the continuous creation of derivatives driven by financial engineers, requires the elaboration of new pricing models, whose starting point should be the just mentioned models.

CHAPTER 3

3.1 THREE TYPES OF ELECTRICITY POWER PLANTS

The main components of the electricity market have already been introduced in Chapter 1, where the difference among generation, transmission and distribution in the market place has been analyzed. The work now focuses its attention on the generation sector of the electricity market, thus introducing the difference between spark and dark spread (U.S. Energy Information Administration). The dark spread is defined as the theoretical gross margin of a coal-fired power plant from selling a unit of electricity having made use of the coal required to produce this unit of electricity. Therefore, it is the difference between the price received by a generator for electricity produced with coal and the cost of coal needed to produce that electricity. Mathematically the dark spread is given by the equation:

$$\text{Dark spread}\left(\frac{€}{\text{MWh}}\right)=\text{Power price}\left(\frac{€}{\text{MWh}}\right)-\left[\text{Coal cost}\left(\frac{€}{\text{ton}}\right)+\text{Tranport cost}\left(\frac{€}{\text{ton}}\right)\right]*\frac{\text{Heat rate}\left(\frac{\text{MMbtu}}{\text{MWh}}\right)}{\text{Heat content}\left(\frac{\text{MMbtu}}{\text{ton}}\right)}$$

where:

1) Power price is the combination of off-peak and on-peak electricity prices, since the coal-fired power plants run both during day and night
2) Coal cost is the cost of purchasing coal through generally long-term contracts that are not publicly available
3) Transport cost is the cost of bringing the coal to the power plant
4) Heat rate is the measure of efficiency of the generating unit
5) Heat content is how much heating capacity a ton of coal has

On the other hand, the spark spread is defined as the theoretical gross margin of a gas-fired power plant from selling a unit of electricity, having made use of the gas required to produce this unit of electricity. Therefore, it is the difference between the price received by a generator for electricity produced with gas and the cost of gas needed to produce that electricity. Mathematically the spark spread is given by the equation:

$$\text{Spark spread}\left(\frac{€}{\text{MWh}}\right) = \text{Power price}\left(\frac{€}{\text{MWh}}\right) - \left[\text{Natural gas price}\left(\frac{€}{\text{MMbtu}}\right) * \text{Heat rate}\left(\frac{\text{MMbtu}}{\text{MWh}}\right)\right]$$

Where the variables are defined as for the dark spread, with the exception of coal price that is replaced with the natural gas price, publicly available on the spot market. These two different kind of power plants, coal-fired and gas-fired, will be turned on if the spark spread or the dark spread are positive, thus assuring profits for the generator.

The purpose of this chapter is to develop a hedging strategy for a hydroelectric power plant. Therefore, after having defined the coal and gas-fired plants, the hydropower one is taken into account. The hydroelectric power plants offer the lowest production cost with respect to all major fossil fuels and renewable energy sources. This cost advantage is possible thanks to the low maintenance, operations and fuel costs. The maintenance costs are spread over longer lifespans, since the power generating equipment used at these facilities can operate for long periods with no need of replacements or repairs. Moreover, the operational costs are the lowest compared to other energy sources. Finally the fuel costs are essentially negligible, since hydroelectric power derives from flowing water. This water stream comes from rivers or man-made installations, thanks to which water flows from high-level reservoirs down through a tunnel where turbines are placed. These turbines are able to extract the water kinetic energy and to convert it to mechanical energy, thus rotating at high speed. Finally, the generator converts the mechanical energy into electrical energy. The water flow and the vertical distance the water falls through, are the main determinants of the hydroelectric power generated. Hence, when these three type of plants are taken into account it can be said that the hydroelectric power plant is the best, if production costs only are considered. In fact, if a brief look is given to the equation for the dark and spark spreads, it is undeniable how the coal and natural gas costs play a determinant role in reducing profits from electricity production. On the other hand, water is essentially cost-free, so generators are free to decide to turn on the hydroelectric power plant and to produce just by having a look at the daily market potential gains. As a matter of fact, the market agents that are producers of electricity, have a natural long position with respect to the produced asset. Therefore, once they have decided to produce, since electricity is non storable, they will have to sell everything at every point in time, either on the spot market or at predetermined prices from earlier agreements (earlier forward contracts). Very important is the concept of hedging for these producers: they have strong incentives to make investments in order to reduce the risk of price movements in the future. For example, by stipulating forward contracts, according to which electricity has to be delivered at a future date at a predetermined price, they reduce price uncertainties.

3.2 WHAT IS DELTA-HEDGING

According to modern option pricing theory it is possible to create financial portfolios with exactly the same payoff structure as the underlying derivative. The optimal hedging strategy for a hydropower plant involves the use of a set of products that replicate the cash flows generated by hydro production. In fact, risk-averse behavior makes producers prefer to hedge production, in order to reduce the risk of price fluctuations and capital market imperfections. To achieve an optimal result the producer has to be able to plan and price production so that it is possible to get an estimation of how sensitive production value is with respect to changes in value of the available forward and future contracts (Wallace and Fleten, 2009).

Delta-hedging can be explained as the strategy that hedges the option that has already been sold, in order to create a riskless portfolio. The Greek letter delta (Δ) of a stock option is the ratio of the change in the price of the stock option to the change in the price of the underlying stock. It is the number of units of the stock we should hold for each option shorted in order to create a riskless portfolio (Options, Futures and other Derivatives, John C. Hull). In this case the underlying of reference is the electricity price. Therefore, the delta is simply the derivative of the option price with respect to the electricity price. As the electricity price changes, so does the delta and the hedger, in order to maintain a zero risk total position, must continuously sell or buy. By defining the price of the underlying (electricity) as *S*, and the value of the option to be hedged as *V*, the delta (Δ) is:

$$\frac{\partial V}{\partial S} = \Delta$$

Any change in the underlying price leads to a change in delta, if *V* does not depend linearly on *S*. The delta could also be seen as the slope of the curve that relates the option price to the electricity price. Hence, if the delta of a call option on the electricity price is 0.5, when the latter changes even by a small amount, the former price changes by 50% of that amount. The goal of delta hedging is to reach, thanks to a combination of transactions, a delta-neutral position (Δ=0), which means that the risk has been totally diversified.

For the purpose of this work the delta-hedging strategy for a call option will be analyzed. Therefore, an insight on call options delta is provided.

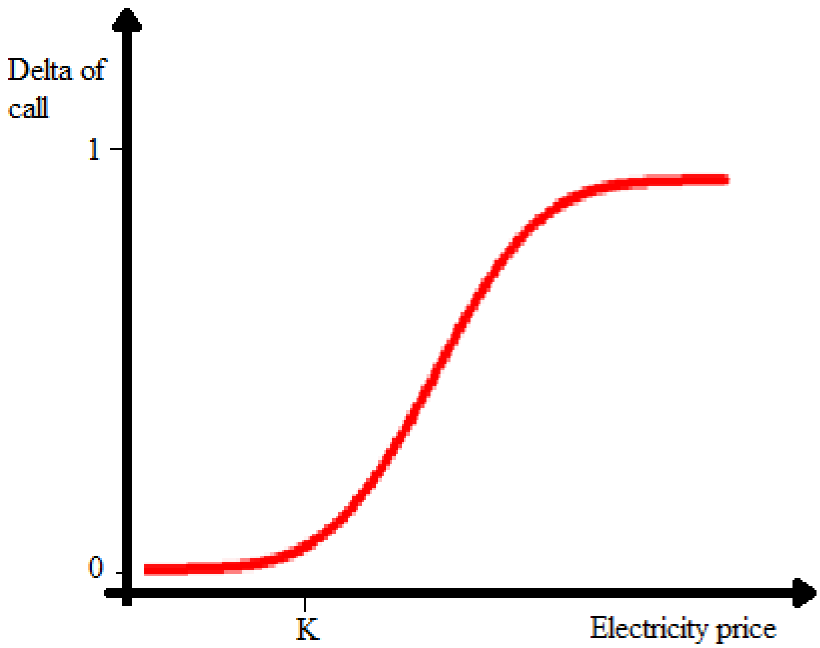
3.2.2 THE DELTA FOR A CALL OPTION

Considering the electricity properties, a portfolio including forward contracts only could be created. Such a portfolio, once hedged, is insensitive to changes in electricity prices. Unfortunately the same does not apply for a portfolio including options. As a matter of fact, contrary to the forward contracts, the delta of an option is directly related to the electricity price. Thus, since the delta of the option constantly variates, the hedger's position remains delta-neutral only for a short period of time. Hence, the delta position has to be rebalanced periodically.

The concept of delta is closely related to the Black-Scholes model previously introduced. The delta of a call option can in fact be defined as:

$$\Delta_C = \frac{\partial V}{\partial S} = e^{-r\tau} N(d_1)$$

where the parameters are the one previously defined in Chapter 2. Graphically the variation of delta with respect to price is defined as:



13. Variation of delta with electricity price

The formula gives the delta of a long position in one call option. The delta of a short position in one call option is $-e^{-rT}N(d_1)$. Using delta hedging for a short position in a European call option involves maintaining a long position of $e^{-rT}N(d_1)$ for each option sold (Options, Futures and other Derivatives, John C. Hull). Therefore, if an investor possesses a call option for electricity and the delta position of a call is positive, the strategic way to reach delta-neutrality is to bet on the underlying stock price to go up, selling part of the electricity that is not owned. Since, the electricity price and the call option value move in the same direction, the final purpose of the delta-hedging strategy is to determine the amount of electricity that has to be sold. The amount of electricity is defined in MWh and accordingly the delta-hedging strategy simply implies multiplying the delta position of the call option by the amount of MWh the contract is written upon.

3.3 THE SCENARIO OF A HYDROELECTRIC POWER PLANT

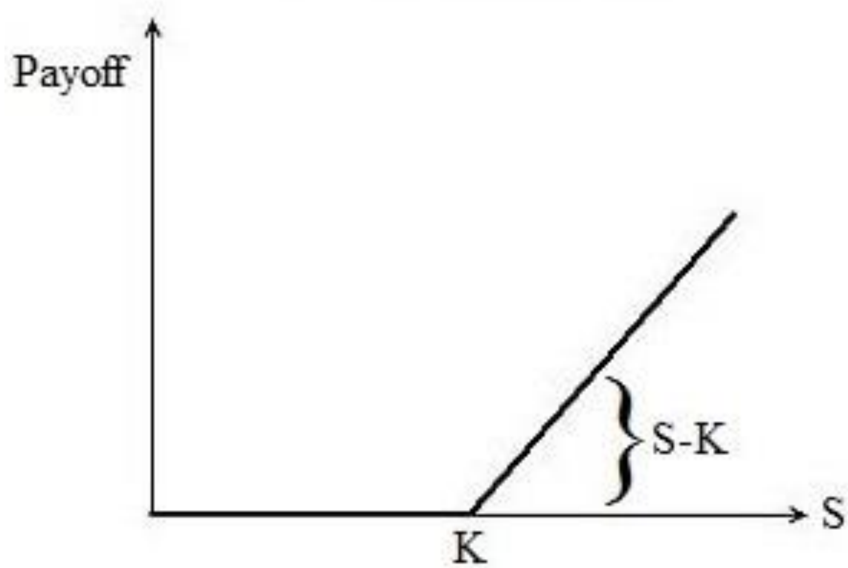
From now on, the analysis of this work is dedicated to the dynamic delta-hedging management of a hydroelectric power plant. Therefore, the scenario of this type of plant is introduced. As it was stated before, storing water to generate electricity is essentially cost-free. Without taking into consideration the maintenance costs and focusing on the mere production process, it could be stated that the gross margin received by a hydroelectric power generator is:

$$\text{Gross Margin}\left(\frac{€}{\text{MWh}}\right) \approx \text{Power price}\left(\frac{€}{\text{MWh}}\right) - \text{Operational costs}\left(\frac{€}{\text{MWh}}\right)$$

where:

1) Power price is the electricity spot price
2) Operational costs are the hydroelectric power plant expenses that are incurred while operating the plant. The plant will be activated as long as the electricity price is higher than the costs of electricity production.

Therefore, this type of plant could be seen as having a payoff similar to the one of a call option:



14. Payoff of a hydroelectric power plant

where:

1) *K* is the cost of producing electricity
2) *S* is the electricity price
3) Payoff is the gain the generator can extrapolate from the market

Therefore, there are two possible scenarios:

1) if $S > K,$ the generator sells today the electricity that he will produce in the future
2) whereas, if $S < K,$ the generator sold electricity on the market that he is no more sure to be able to buy (produce) at *K*. Hence, he will buy electricity on the market.

With these two kind of trading operations on the market, the generator is able to extract value. In fact, he continuously sells at a price $S > K$ and buys at a price $S < K$. Therefore, given the implied volatility of traded options, the dynamic management of a hydroelectric power plant, with a reference cost (strike) *K*, can be done. The generator has two possible outcomes:

1) he can either extract value with the "delta" dynamic management of the hydroelectric power plant
2) or he can sell volatility on the market

As a matter of fact, the option (hydroelectric power plant) implied volatility gives at t=0 a given value of the option. Nonetheless the realized volatility (volatility from t=0 up to T=Expiry), could give the generator real possibilities of extracting value. Therefore, after having introduced the general characteristics of delta-hedging in order to extract value, the delta-hedging strategy for volatility trading is explained.

3.3.2 DELTA-HEDGING STRATEGY FOR VOLATILITY TRADING

The delta-hedging strategy for volatility trading will now be considered. While continuous time hedging on the market is often assumed as possible, for the peculiarities of the electricity market, this work will base its analysis on the discrete time framework. More specifically, the hedging strategies applied will be daily based. The purpose of this strategy is to create a volatility arbitrage, which is defined as the profit to be made hedging options that are mispriced by the market, when your estimate of future actual volatility differs from that of the market as measured by the implied volatility (Natenberg, 1994). To be more precise, the volatility arbitrage is defined as:

$$\text{Volatility arbitrage} = \text{Implied volatility} - \text{Realized volatility}$$

The implied volatility is a forward looking estimate of volatility implied from options market prices. This volatility is how the market currently prices call options thanks to the Black-Scholes model. In Matlab the implied volatility is calculated as follows:

$$\text{Implied volatility} = \text{blsimpv}(\text{Price}, \text{Strike}, \text{Rate}, \text{Time}, \text{Value})$$

where:

1) Price is the current price of the underlying asset
2) Strike is the exercise price of the option
3) Rate is the annualized, continuously compounded risk-free rate
4) Time is the time to expiration of the option
5) Value is the price of a European option from which the implied volatility of the underlying asset is derived

On the other hand, the realized volatility is the observed volatility of price returns from the date of trade up to the expiry of the call option. This observed volatility can be found by daily applying the delta-hedging strategy to the plant and calculating the resulting volatility with “blsimpv”. In general, buying an option and selling the underlying asset results in a long volatility position, while selling an option and buying the underlying asset results in a short volatility position. A long volatility position will be profitable to the extent that the realized volatility on the underlying is ultimately higher than the implied volatility on the option at the time of the trade (Volatility arbitrage indices, Keith Loggie). Therefore, considering the dynamic delta-management of a hydroelectric power plant, the questions this chapter will try to answer are:

1) Did the implied volatility price correctly the call options (plant production)?
2) Does the dynamic delta-management of the hydroelectric power plant allow the generator to extract value from the market?
3) Was it more convenient to sell volatility on the market?

3.4 DYNAMIC MANAGEMENT OF A HYDROELECTRIC POWER PLANT

Since, as it has been stated before, the payoff of the hydroelectric power plant could be seen as the one of a call option, the data previously analyzed in Chapter 2 will be taken into account. Four different periods, extrapolated from Enel S.p.A. dataset, will be considered in order to set four different scenarios for a hydroelectric power plant. The data are the following:

DateTrade	Product	MW	Hours	Premium(€)	Expiry	Power (€/MWh)	Strike	Implied Volatility	Actual Volatility
27/02/2013	Cal-2014	100	8760	0,828	12/12/2013	65,3	69	9,21%	6,53%
22/01/2014	Cal-2015	50	8760	0,45	11/12/2014	57,85	60	5,65%	4,7%
28/04/2014	Cal-2015	200	8760	0,89	11/12/2014	53,7	56	11%	11,29%
24/07/2014	Cal-2015	50	8760	0,42	11/12/2014	53,45	56	9,8%	6,18%

The different variables are explained as follows:

1) DateTrade is the date at which the option has been traded
2) Product is the calendar year to which the option refers
3) MW is the amount of electricity subscribed in the call option
4) Hours are the number of hours subscribed in the call option
5) Expiry is the maturity of the call option
6) Power (€/MWh) is the spot electricity price
7) Strike is the exercise price of the call option
8) Implied volatility is a forward looking estimate of volatility
9) Actual volatility is the amount of randomness that "transpires" in the electricity price. For the purpose of this work, it is calculated as the observed volatility from fifty days before the date of trade up to the date itself.

Since the market has no perfect knowledge about the future, implied volatility and actual volatility will usually be different. If the generator thinks that his production (the premium of the call option) is underpriced, he should delta-hedge up to the expiry date in order to extract value. The choice now falls onto the volatility with which the dynamic management of the hydroelectric power plant has to be pursued. For the purpose of this work, delta-hedging has been done using the actual volatility.

- APPLICATION

- The dynamic delta-hedging strategy of the hydroelectric power plant is applied by calculating the delta with which the value of the portfolio is rebalanced daily. The Δ^a, where "a" stands for actual volatility, is the amount of underlying asset that has to be purchased on the spot market in order to rebalance the portfolio value. In Matlab it is calculated as follows:

$$\Delta^a = \ blsdelta(\ ElectricitySpotPrice, Strike, 0, TimeChange, ActualVolatility)$$

where:

1) ElectricitySpotPrice is a vector of changing electricity prices from the date of trade up to the expiry of the call option
2) Strike is a vector of the call option strike price, which is kept constant
3) 0 is the constant risk-free rate
4) TimeChange is a vector of the (decreasing) time up to expiry
5) ActualVolatility is a vector of the actual volatility, which is kept constant

- Afterwards, the value extracted from the market is calculated as the sum of the cross product of the Δ^a daily differences and the electricity spot prices. In Matlab it is given by:

$$Premium = sum(diff(\Delta^a).* \ ElectricitySpotPrice)$$

- The original premium, $V^{th}(t)$ defined as the theoretical production value of the hydroelectric power plant calculated with the implied volatility is then compared to the estimated premium, V^a, defined as the production value of the hydroelectric power plant, using the dynamic delta-hedging strategy. The total value that can be extracted from the market from t=0 up to expiry is therefore defined as:

$$e^{rt_0}\int_{t_0}^{T} d\left(e^{-rt}\left(V^{th}(t)-V^{a}(t)\right)\right) = V^{a}(T) - V^{th}(T)$$

- In conclusion, the realized volatility of the premium obtained through the delta-hedging strategy, can be calculated by the following:

$$\text{Realized Volatility} = \text{blsimpv}(\text{ElectricityPrice}, \text{Strike}, 0, \text{Time}, \text{Premium})$$

The results of the dynamic delta-management are shown in the following table:

Scenario	**Original Premium (V^{th})**	**Estimated Premium (V^a)**	**Value Extracted (V^a-V^{th})**	**Implied Volatility**	**Realized Volatility**	**Volatility Arbitrage**
1	0,828	0,6954	-0,1326	9,21%	8,54%	0,67%
2	0,45	0,3725	-0,0775	5,65%	5,15%	0,5%
3	0,89	0,3467	-0,5433	11%	6,66%	4,34%
4	0,42	0,2074	-0,2126	9,8%	7,46%	2,34%

RESULTS ANALYSIS

As it can be inferred from the table, a generator owning a hydroelectric power plant could not extract any value from the market. In fact, the value extracted using the dynamic delta-management of the plant in these four different periods, turned out to be always negative. Thus, it can be stated that rebalancing our options values by delta-hedging undervalues production. This might be either because only closing prices were analyzed, hence, the intra-day volatility was lost, or because the hedging strategy was options based. Moreover, in general, electricity option contracts, such as call options, are based on a forward looking estimate of volatility. Therefore, their value is generally overpriced because the market participants subscribing such options, want to manage the risk of future events, even unlikely ones. As a matter of fact, they are willing to pay a premium price in order to be hedged against the risk deriving from electricity price volatility. On the other hand, the generator could also use the delta-hedging strategy based on his view of future volatility. The purpose of this strategy is to create a volatility arbitrage, which is defined as the profit to be made hedging options that are mispriced by the market. As a matter of fact, the results show that a volatility arbitrage was possible in each of the four scenarios. In order to extract value by selling volatility, the generator could, for example, sell variance swaps contracts. These financial derivatives market has grown exponentially in the last decade and nowadays these are among the most liquid derivatives contracts in over-the-counter markets. Anyway these type of contracts are behind the purpose of this work and they are left to further research. In conclusion, the questions addressed at the beginning of this chapter are answered. First of all, implied volatility was overpricing call options, since the premium realized with delta-hedging turned out to be lower than the original one in each of the four periods. Secondly, the dynamic delta-management of the hydroelectric power plant did not allow the generator to extract any value from the market. Lastly, there were possibilities to extract value by using delta-hedging strategies for volatility trading and to profit from volatility arbitrage.

3.5 CONCLUSIONS

The analysis of this work began with the introduction of the deregulation process in the electricity market. The fundamental changes and the development phase that this market had to undergo literally struck market participants, who had to idealize and face problems never seen before. The history, the set of constraints and the general features of the electricity market were taken into account in order to give a better understanding of the changing nature of the market itself. The monopoly predominance was replaced by the continuous entrance and exit of competitors, who started to be identified as firms or ordinary people owning the necessary technology to enter the market. The increased number of market participants, fostered market efficiency also through the intervention of regulatory policies. Nonetheless, being electricity a non-storable asset and being spot prices deeply influenced by volatility of demand and supply, seasonality, weather changes and many other factors, the efficiency of the market encountered serious issues. Therefore, the necessary step was the creation of financial derivatives, which in most cases do not imply the physical delivery and are used to manage seasonal price fluctuations and not daily ones. The most important financial derivatives that have been mentioned are: forward, future, swap contracts, options and other more sophisticated instruments. Hence, to proceed with the analysis, an evaluation and pricing model was needed. As a matter of fact, the Black-Scholes option pricing model was taken into account even if two of its assumptions, the normal distribution of returns and the constancy of volatility over the lifetime of the underlying security, were proven not to be reflected on the real electricity market. Nonetheless, the Black-Scholes model is not only applied in real market pricing, but it is also useful to deal with hedging strategies for risk management. The delta-hedging strategy, which is one of the most important risk management hedging strategies, has been taken into account. This strategy, being a direct derivation of the Black-Scholes formula, has been applied to a hydroelectric power plant. The dynamic delta-management of the plant has been applied, in order to understand whether it is more likely for a generator to extract value through the hedging strategy or through the volatility selling. Four different periods have been considered and the dynamic delta-hedging strategy of the hydroelectric power plant has been applied by calculating the delta with which the value of the portfolio is rebalanced daily. Each of the four periods showed that a generator owning a hydroelectric power plant could not extract any value from the market by the dynamic delta-hedging strategy. Nonetheless the realized volatility was lower than the option implied one, thus leaving to the generator and market participants, volatility arbitrage possibilities.

This work has thus tried to provide a specific application of the Black-Scholes option pricing model. The flaws behind the model have been analyzed and an insight about the possible dynamic management of a hydroelectric power plant has been provided.

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MATLAB APPENDIX

Price returns

```
rcal2013 = diff(log(Cal2013IT_Power_base))
rcal2014 = diff(log(Cal2014IT_Power_base))
rcal2015 = diff(log(Cal2015IT_Power_base))
rq12013 = diff(log(Q12013IT_Power_base))
rq22013 = diff(log(Q22013IT_Power_base))
rq32013 = diff(log(Q32013IT_Power_base))
rq42013 = diff(log(Q42013IT_Power_base))
rq12014 = diff(log(Q12014IT_Power_base))
rq22014 = diff(log(Q22014IT_Power_base))
rq32014 = diff(log(Q32014IT_Power_base))
rq42014 = diff(log(Q42014IT_Power_base))
rq12015 = diff(log(Q12015IT_Power_base))
rq22015 = diff(log(Q22015IT_Power_base))
rq32015 = diff(log(Q32015IT_Power_base))
rq42015 = diff(log(Q42015IT_Power_base))
```

Mean of returns

```
murcal2013 = mean(rcal2013)
murcal2014 = mean(rcal2014)
murcal2015 = mean(rcal2015)
murq12013 = mean(rq12013)
murq22013 = mean(rq22013)
murq32013 = mean(rq32013)
murq42013 = mean(rq42013)
murq12014 = mean(rq12014)
murq22014 = mean(rq22014)
murq32014 = mean(rq32014)
murq42014 = mean(rq42014)
murq12015 = mean(rq12015)
murq22015 = mean(rq22015)
murq32015 = mean(rq32015)
murq42015 = mean(rq42015)
```

Standard deviation

```
sdrcal2013 = std(rcal2013)
sdrcal2014 = std(rcal2014)
sdrcal2015 = std(rcal2015)
sdrq12013 = std(rq12013)
sdrq22013 = std(rq22013)
sdrq32013 = std(rq32013)
sdrq42013 = std(rq42013)
sdrq12014 = std(rq12014)
sdrq22014 = std(rq22014)
sdrq32014 = std(rq32014)
sdrq42014 = std(rq42014)
sdrq12015 = std(rq12015)
sdrq22015 = std(rq22015)
sdrq32015 = std(rq32015)
```

```
sdrq42015 = std(rq42015)
```

Normalization of returns

```
rn_2013=(rcal2013-murcal2013)./sdrcal2013
rn_2014=(rcal2014-murcal2014)./sdrcal2014
rn_2015=(rcal2015-murcal2015)./sdrcal2015
rnq1_2013=(rq12013-murq12013)./sdrq12013
rnq2_2013=(rq22013-murq22013)./sdrq22013
rnq3_2013=(rq32013-murq32013)./sdrq32013
rnq4_2013=(rq42013-murq42013)./sdrq42013
rnq1_2014=(rq12014-murq12014)./sdrq12014
rnq2_2014=(rq22014-murq22014)./sdrq22014
rnq3_2014=(rq32014-murq32014)./sdrq32014
rnq4_2014=(rq42014-murq42014)./sdrq42014
rnq1_2015=(rq12015-murq12015)./sdrq12015
rnq2_2015=(rq22015-murq22015)./sdrq22015
rnq3_2015=(rq32015-murq32015)./sdrq32015
rnq4_2015=(rq42015-murq42015)./sdrq42015
```

Shapiro-Wilk test

```
Shrcal2013 = swtest(rcal2013)
Shrcal2014 = swtest(rcal2014)
Shrcal2015 = swtest(rcal2015)
Shrq12013 = swtest(rq12013)
Shrq22013 = swtest(rq22013)
Shrq32013 = swtest(rq32013)
Shrq42013 = swtest(rq42013)
Shrq12014 = swtest(rq12014)
Shrq22014 = swtest(rq22014)
Shrq32014 = swtest(rq32014)
Shrq42014 = swtest(rq42014)
Shrq12015 = swtest(rq12015)
Shrq22015 = swtest(rq22015)
Shrq32015 = swtest(rq32015)
Shrq42015 = swtest(rq42015)
```

Jarque-Bera test

```
JBrcal2013 = jbtest(rn_2013)
JBrcal2014 = jbtest(rn_2014)
JBrcal2015 = jbtest(rn_2015)
JBrq12013 = jbtest(rnq1_2013)
JBrq22013 = jbtest(rnq2_2013)
JBrq32013 = jbtest(rnq3_2013)
JBrq42013 = jbtest(rnq4_2013)
JBrq12014 = jbtest(rnq1_2014)
JBrq22014 = jbtest(rnq2_2014)
JBrq32014 = jbtest(rnq3_2014)
JBrq42014 = jbtest(rnq4_2014)
JBrq12015 = jbtest(rnq1_2015)
JBrq22015 = jbtest(rnq2_2015)
JBrq32015 = jbtest(rnq3_2015)
JBrq42015 = jbtest(rnq4_2015)
```

Anderson-Darling test

```
ADrcal2013 = adtest(rcal2013)
ADrcal2014 = adtest(rcal2014)
ADrcal2015 = adtest(rcal2015)
ADrq12013 = adtest(rq12013)
ADrq22013 = adtest(rq22013)
ADrq32013 = adtest(rq32013)
ADrq42013 = adtest(rq42013)
ADrq12014 = adtest(rq12014)
ADrq22014 = adtest(rq22014)
ADrq32014 = adtest(rq32014)
ADrq42014 = adtest(rq42014)
ADrq12015 = adtest(rq12015)
ADrq22015 = adtest(rq22015)
ADrq32015 = adtest(rq32015)
ADrq42015 = adtest(rq42015)
```

Constant volatility test

```
fun = @(x) sum((blsprice(PowerMWh, Strike, 0, DeltaTOriginale, x)-Premio).^2)
xx = fmincon(fun,0.09,[],[],[],[],0,1)
ys = blsprice(PowerMWh, Strike, 0, DeltaTOriginale, xx)
figure
hold on
plot(Premio,'ko')
plot(ys,'r*')
legend('Real Prices','Estimated Prices')
SumDiff = sum((blsprice(PowerMWh, Strike, 0, DeltaTOriginale, 0.078)-Premio).^2)
```

Dynamic delta-management of a hydroelectric power plant

```
RETURN2 = diff(log(PRICESTD2))
STD04 = std(RETURN2)
VOLATILITY2 = STD04*sqrt(252)
RETURN04 = diff(log(pricestd04))
STD04 = std(RETURN04)
VOLATILITY_04 = STD04*sqrt(252)
FFFF_1 = blsdelta(price1, strike1, 0, time1, VOLATILITY, 0)
Delta1 = diff(FFFF_1)
MONEY1 = DELTAS1.*price1
MONEYSUM1 = sum(MONEY1)
VOLCALC27 = blsimpv(65.3, 69, 0, 0.789041096, MONEYSUM1)
FFFF_2 = blsdelta(ASSETPRICE_2, STRIKEFISSO_2, 0, TIMECHANGE_2, VOLATILITY2, 0)
Delta2 = diff(FFFF_2)
MONEY2 = DELTAS2.*ASSETPRICE_2
MONEYSUM2 = sum(MONEY2)
VOLcalc1 = blsimpv(57.85, 60, 0, 0.884931507, MONEYSUM2)
RETURN28 = diff(log(pricestd28))
STD28 = std(RETURN28)
VOLATILITY28 = STD28*sqrt(252)
FFFF28 = blsdelta(ASSETPRICE28, STRIKEFISSO28, 0, TIMECHANGE28, VOLATILITY28, 0)
Delta28 = diff(FFFF28)
MONEY28 = DELTAS28.*ASSETPRICE28
```

```
MONEYSUM28 = sum(MONEY28)
VOLcalc28 = blsimpv(53.7, 56, 0, 0.621917808, MONEYSUM28)
RETURN24 = diff(log(PRICESTD24))
STD24 = std(RETURN24)
VOLATILITY24 = STD24*sqrt(252)
FFFF24 = blsdelta(ASSETPRICE24, STRIKEFISSO24, 0, TIMECHANGE24, VOLATILITY24, 0)
Delta24 = diff(FFFF24)
MONEY24 = DDELTAS24.*ASSETPRICE24
MONEYSUM24 = sum(MONEY24)
VOLcalc24 = blsimpv(53.45, 56, 0, 0.383561644, MONEYSUM24)
```