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The Shapley Value In The Newsvendor Game

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Abstract

In this paper the newsvendor model and the newsvendor game will be introduced, followed by a brief review of the literature on the topic. Some basic concepts of cooperative game theory and allocation methods are going to be explained and after the newsvendor game model will be explored, MATLAB will be used to compute the Shapley value in the game. The results will then be analyzed, with different levels of correlations and cost that will help to draw conclusions.

1 Introduction

1.1 Introduction to the newsvendor problem

At some point any commercial activity needs to face a tough dilemma, namely, what is the optimal quantity to order? To answer this problem, experts have always tried to find a framework that would give a solution to this issue. That framework is the news vendor boy, or better, the newsvendor model. The story is the following. Let's suppose there is a newsvendor boy that each day sells newspapers at a street corner. Each morning he has to buy a certain amount of paper, knowing that if he buys too little, he will disappoint customers and will face the opportunity cost of a missed profit. On the other hand, if he buys too many newspapers he will not be able to sell them, and those will lose their value, generating a cost. At this point the boy has to take a decision regarding the optimal quantity he needs to buy, knowing their cost, the cost of not buying enough of them (that is just the sell price minus the cost) and knowing the cost of buying too many (that is just the cost - the salvage value). What he does not know, of course, is the amount of newspapers he will sell, but for that statistics will come in handy.

The framework described above is a one-time business decision that occurs in many business context, among them the most important [9] are:

- Buying seasonal good. Retailers may have to buy seasonal goods once per season. A good example would be constituted by sun protection. Many retailers have to make an on-time order decision for all the summer season, in this way they run the risk of ordering too much or too little and incurring in a cost, the model can be really helpful for them.
- Making the last buy. If a product is going out of production, or it is the last time a retailer buys a certain product, the model will apply helping reducing the costs.
- Setting safety stock levels. A distributor has to set the safety stock. If the stock is too low, the probability of a stock out will be high. On the other hand if the safety stock is too high, the carrying cost will be high too.

1.2 The game

Now let us imagine that we have the same boy mentioned above, and let us imagine that he has an infinite number of friends and all of them sell newspapers at a different corner of a square with infinite corners; they may come together and decide to buy the product together if this will lower their cost. The situation we have just described is known as the newsvendor game.

In recent years, many companies have started to come together just like these boys, since they have been realizing that it could be helpful in reducing costs and increasing their profits [13]. Thanks to the new technology available and the improvement of logistics in recent years, these partnerships are now easier. The main driver of this reduction in cost are economies of scale and risk pooling. This means that, the players can have a gain sharing the risk due to the "compensation effect" that causes a consumption closer to the mean.

In this context allocating this extra profit among the players becomes non-trivial. One solution concept would be the set of all stable and rational allocations, the core, together with another interesting concept that will be analyzed in this thesis: The Shapley value.

1.3 Literature review

The news vendor model is the most celebrated in all of operations research and has been appearing in the literature for over 100 years [6] [3]. Regarding the game, it is much more recent, but, lately it has been studied even more intensively. Some have tried to frame the motivation behind this kind of industrial behavior [1] [2] [5]. Experts, although, had already started to examine the problem before. Eppen [7] was one of the first to analyze the problem, but he did not use game theory. Parlar [15] on the other hand, analyzed the problem using non-cooperative game theory. Cost allocation rules in centralized inventory systems has also been treated [8] and, in response to that, Robinson [16] suggested the use of the Shapley value in allocating cost. That was the turning point of the study, in the last decades the problem has been considered using a cooperative game theory toolbox. In particular, a remarkable result has been obtained proving that for the newsvendor game the core always exists and it is non-empty [14]. In recent years, the possibility of inventory-pooling coalition with a supplier as a per-se entity has been explored [10].

The paper now shifts to the introduction of few basic concepts, in section 1.4 there will be a brief review of the concept of cooperative games. The following section will present the concept of the Shapley value. Following, comes the introduction of the model that will be used throughout the thesis and in section 2.2 we are going to expand this to our newsvendor game. In section 2.3 the value of the coalitions will be computed with MATLAB, showing and explaining the code that has been created. It is going to be done using different levels of correlation coefficients, to try and understand what effect it will have on the results. Finally, there will be a computation of the Shapley value in section 2.4, briefly showing the script used. In the subsequent section the results obtained will be commented.

1.4 Cooperative games

Cooperative games represent situations in which the player may choose to cooperate in order to achieve a certain result [11]. The value we are going to analyze belongs to TU games, which means that utility can be transferred among players. Given a pair $(N; v)$ we can define these kind of games as follows:

- $N = 1, 2, ..., n$ is a finite set of players. A subset of N is called a coalition. The collection of all the coalition is denoted by 2^N
- $v : 2^N \longrightarrow \mathbf{R}$ is a function associating every coalition S with a real number v(S), satisfying $v(\varnothing = 0)$. This function is called the coalition function of the game.

The features of cooperative games make them very interesting. In fact, being players able to cooperate poses several problems, one being about stability. As already said the player can form a coalition to achieve better results, but what coalition should they setup is a non-trivial problem and it is not easy to understand whether this coalition will be stable. For example, a player could advocate to join an other coalition to get even more, but in this process he could make an other player worse off. Here another interesting issue comes up, players get together to achieve a certain result, in this case to lower their expected cost and, in this process, they may create a surplus, which is unclear how should be divided. About the multitude of studies about allocation of extra resources, two will be analyzed below: the core and the Shapley value.

1.5 The core

The concept of the core will be now briefly reviewed. First of all, the core is probably the most important solution concept for cooperative games. It is the set of all stable and rational possible allocations, that means players cannot make themselves better off arranging in a new way. It is important to notice that it could be empty, even though this is not the case as Scarsini et al. proved the newsvendor game always has a core and it is non-empty. Below are shown the property it satisfies.

$$\text{Efficiency:} \quad \sum_{i \in N} x_i = v(N)$$

$$\text{Coalition Rationality:} \quad \sum_{i \in S} x_i \geq v(S) \quad \forall S \subseteq N$$

The first states that the sum of all the singular allocations should be equal to the value of the grand coalition N (the grand coalition is just the coalition of all players). The second states that every player should get at least the same allocations he would get in a smaller coalition. The core has been criticized for being so extremely sensitive to oversupply of one type of player [11]. For example in the game of the right and left shoes, where we have 3 players, with the first having a right shoe, while the others having left shoe each. A shoe alone is worth noting, but a pair of different type can sell for 1. The core solution would allocate all the worth to the player with the right shoe, so as it can be observable there is a problem of sensitivity to oversupply. Moreover the core is not considered to be "fair". The Shapley value addresses some of these issues.

1.6 The Shapley value

The Shapley value is perhaps the most important single-valued solutions for cooperative games. It is considered to be a "fair" approach when it comes to allocation of resources and it is much used particularly in the field of politics. The Shapley value can be described as the participation a player would gain participating in that kind of game [20], precisely as the marginal mean contribution it brings. It is important to notice that the Shapley value may not be in the core, that means that the allocations are not stable and players may be better off deviating to an other feasible allocation. The Shapley value has the following characteristic function [17] [18]:

$$\phi_i(v) = \sum_{S \subseteq N \setminus [i]} \frac{|S|!(N - |S| - 1)!}{n!}(v(S \cup i) - v(S))$$

It is also possible to provide an axiomatic approach, in fact it can be defined as the only value that satisfies simultaneously these following properties:

- Efficiency: $\sum_{i \in N} x_i = v(N)$

- Simmetry $\quad v(S \cup i) = v(S \cup j)$

If i and J are two players and the rule above holds, then they should be rewarded equally, namely $\phi_i(v) = \phi_j(v)$

- Additivity [20]: if two coalition games described by gain functions v and w are combined, then the distributed gains should correspond to the gains derived from v and the gains derived from w, namely $\phi_i(v + w) = \phi_i(v) + \phi_i(w)$
- Null: If $v(S \cup i) = v(S)$, meaning that the player does not provide any contribution to the coalition, his reward $\phi_i(v)$ should be 0.

2 The Model and The MATLAB Script

2.1 The newsvendor model

In this chapter the newsvendor model is introduced. As already said previously this model is widespread and universally accepted, having been circulating for a long time. Let c_0 and c_u respectively denote the overage and the underage cost, that is the economic cost of ordering too much or too little. Let D denote the uncertain demand that each economic actor faces and Q a variable that represents the optimal order quantity. The cost function of Q and D will be the following:

$$Cost(Q, D) = \begin{cases} c_0(Q - D) & if D < Q \\ c_u(D - Q) & if D \geq Q \end{cases}$$

Then it is assumed that the demand D is a continuous random variable with density function f(D). So, the expected cost is computed as follows :

$$\mathbb{E}[Cost(Q)] = \int_0^\infty Cost(D, Q)f(D)dD =$$

$$= c_0 \int_0^Q (Q - D)f(D)dD \quad + \quad c_u \int_Q^\infty (D - Q)f(D)dD \tag{1}$$

In order to find the optimal Q, the derivative of the expected cost function is taken and set to zero to find:

$$Q^* = F^{-1}\left(\frac{c_u}{c_u + c_0}\right)$$

Where Q^* is the optimal order quantity, F^{-1} is the inverse of the cumulative distribution function (i.e. the quantile function).

2.2 Expanding the model to n players

The model described so far can be easily extended to more than one player. In fact, it is sufficient combine the singular D, the random demands each actor faces. So, the players will now consider to form coalitions trying to reduce their cost. In order to evaluate this, the coalition overall demand should be considered, in a way that will be explained in detail in the next chapters. Since the core always exists and it is non-empty, it will exist a stable solution of the game.

2.3 Computing coalition values with MATLAB

MATLAB will compute the actual value of the coalition.First of all it will find the optimal order quantity and then its respective expected cost, that will be considered as the value of the coalition.

In order to ease calculation and automatize it, the script has been divided in 4 parts for the following reasons. The text in the middle is the same (shown at the end), while the headings and the bottom changes.

This is the code created in Matlab :

```
%This is the heading to calculate the single player
   coalition
for 1=1:4
a=[0;0;0;0]
a(i)=1
end
```

```
%This is the heading to calculate all the 2 players
   coalition
for 1=1:3
        for j=i+i:4
        a=[0;0;0;0]
        a(i)=1
        a(j)=1
end

%This is the heading to calculate all the three player
   coalition
for 1=1:4
a=[1;1;1;1]
a(i)=0
end

%This is the heading to calculate the grand coalition, N
   .B. you do not need a loop
a = [1;1;1;1]

c0 = 100 ;
cu = 1000 ;
syms Q
syms D
mu_sing = [16, 30, 50, 42] ;
S = [1, 2, 5, 3] ;
p = 0 ;

sigma_v =
[S(1,1).^2, S(1,1).*S(1,2).*p , S(1,1).*S(1,3).*p,
S(1,1).*S(1,4).*p;S(1,1).*S(1,2).*p , S(1,2).^2,
S(1,2).*S(1,3).*p , S(1,2).*S(1,4).*p;S(1,1).*S(1,3).*p,
S(1,2).*S(1,3).*p , S(1,3).^2, S(1,3).*S(1,4).*p; S(1,1)
.*S(1,4).*p , S(1,2).*S(1,4).*p , S(1,3).*S(1,4).*p , S
   (1,4).^2];
```

```
chol(sigma_v)  ;
mu = mu_sing * a ;
sigma = a'*sigma_v*a ;
pd = makedist('normal',mu,sigma) ;
f = (Q-D).*pdf(pd,D) ;
z = c0.*int(f,D,0,Q) ;
anon_z = matlabFunction(z) ;
g = (D-Q).*pdf(pd,D) ;
t = cu.*int(g,D,Q,inf) ;
anon_t = matlabFunction(t) ;
tot = @(Q) anon_z(Q) + anon_t(Q) ;
[x fx] = fminsearch(tot,0)
```

Let's explain the code.

In the first rows there is the declaration of some variables. The overage and the underage cost are arbitrarily set respectively to 100 and 1000. After creating the symbolic variables Q and D, that will be used later for calculations it is possible, then, to shift to the definition of players's mean and standard deviation, shown as row vectors. Before turning to the next issue it is important to make some remarks. Throughout the whole paper, the main assumption will be that all random demands follow a normal distribution $D \sim N(\mu, \sigma^2)$. Means and standard deviations are listed respectively in mu sing and S. To compute the expected cost of a coalition it is necessary to sum the individual random variables, which represents the "aggregate demand" faced by the coalition.

$$\underset{\sim}{D} = (D_1, \ldots, D_n), \quad D_s = \sum_{i \in s} D_i \quad \forall S \subseteq N$$

The D_s are also distributed following a normal distribution and, as a mean, they have the sum of the individual means. For what regards the variance the procedure is slightly more complicated.

$$Given, \quad \Sigma = \begin{bmatrix} \sigma_1^2 & \sigma_1\sigma_2\rho & \sigma_1\sigma_3\rho & \dots & \sigma_1\sigma_n\rho \\ \sigma_2\sigma_1\rho & \sigma_2^2 & \sigma_1\sigma_3\rho & \dots & \sigma_2\sigma_n\rho \\ \dots & \dots & \dots & \dots & \dots \\ \sigma_n\sigma_1\rho & \sigma_n\sigma_2\rho & \sigma_n\sigma_3\rho & \dots & \sigma_n^2 \end{bmatrix}$$

with Σ being the covariance matrix and ρ the correlation coefficient, that is assumed to be constant across all the possible pairings. The variance for the D_s will then just be the sum of every single element in the covariance matrix. Sigma v represents the matrix and ρ, that will vary to analyze the effect on the shapley value. It is important to notice that Σ must be symmetric and positive definite, meaning that the scalar $A^T \Sigma A$ is positive for every non-zero column vector a of n real numbers.

It is now important to specify that, for the sake of representation the analysis will stick to a game with 4 players, however, the computational time is low as long as the number of players is below 15. Afterwards computing the Shapley value becomes quickly very complicated.

An important and useful property of the normal distribution that was essential for the entire procedure, is that a linear transformation of a normal random variable is itself a normal random variable. In particular, the following holds:

$$if \quad X \sim N(\mu_x, \sigma_x^2), \quad and \quad Y = aX + b, \quad where \quad a, b \in \mathbf{R}$$

$$then \quad Y \sim N(\mu_y, \sigma_y^2), \quad where \quad \mu_y = a\mu_x + b \quad ; \quad \sigma_y^2 = a^2\sigma_x^2$$

This made it possible to use the column vector 'a' in the code. It is composed of either 0 or 1 and it functions like a "switch", allowing to select which players to include, setting the respective column value to 1, and which players to exclude picking 0. For example, to compute the grand coalition value the vector must be filled with ones. If instead, only player 1 is considered, the vector will be the following: a (1 ; 0 ; 0 ; 0).

In this way, it easily possible to define the coalition random demand distribution using vector 'a'. It is important to notice that there are $2^n - 1$ possible coalitions, not considering permutations and the empty coalition. We will then have $mu = musing * a$, $sigma = a^t * sigma * a$. Note that, even if in theory the mean vector is treated as a column vector, Matlab prefers it to be a row vector.

In order to get the result in an automatic and fast way, it was necessary to create a loop in MATLAB considering all the possible coalitions. To do this the code had to be split in 4, one part for each of the coalition's possible number of player. The first part starts from the vector of zeros and looping add a one to a different position, for a total of 4 different outcomes. The second is actually a loop inside a loop: one mimics the behavior of the former loop, but only for the first 3 values while the other adds the remaining player, for a total of 6 different outcomes(all the 2 players coalitions). The third part is symmetric to the first, but this time there is the appearance of a vector of ones and the loop systematically adds a 0, creating all the 4 three-players coalitions(Note that the outcome will be in anti-lexicographical order). Last, for the grand coalition we do not need a loop since it is just 1 value calculated with the vector of ones.

From now on is is possible to start with the calculations. At the beginning the density function considered can be constructed, using the mean and the variance set up earlier and specifying that a normal distribution is taken into account. In the next line of codes, lies equation 1. For simplicity of use it has been split up in 2 parts (the 2 integrals), and reconstructed gradually. After defining the function to be integrated, the procedure continues doing so. Note that besides the three exceptions listed below, the two approaches are completely symmetric .The first exception deals with the extremes of integration: from 0 to Q the former, from Q to infinity the latter. The second one is the simply (D-Q) instead of (Q-D). Finally, the last one is the multiplying factor, c_0 instead of c_u. Both of these functions are then transformed into anonymous functions through a MATLAB handler. An anonymous function is a function definition that is not bounded to an identifier. It is important to notice that anonymous function is usually not accessible after its initial creation [12]. Many MATLAB functions accept function handles as inputs so that it is possible to evaluate functions over a range of values [19], like in the purpose of this paper. In fact, after reconstructing the function (now anonymous) needed, the introduction of the command [x fx] on fminsearch helps the computation, obtaining the optimal order quantity Q, starting from 0 and analyzing throughout the function, then it will return the expected cost linked to that value. Now, all the coalition values are set and the calculation of the Shapley value for each player will be the next step.

2.4 Computing the Shapley value

In order to calculate the Shapley value a package [4] born for this intent is needed and adapted for this purpose. Below, it is briefly explained the content and use of the package, showing a shorter version of it.

```
%v=[1 3 0 0 9 8 3 8 4 7 6 2 0 1 2];
%v=[a b c d ab ac ad bc bd cd abc abd acd bcd abcd]
v=[1 3 0 0 9 8 3 8 4 7 6 2 0 1 2]
n=log2(length(v)+1);
A=[matr(n) v'];
Shapleyvalue=zeros(1,n);
for i=1:n
G=sortrows(A,[n+1 -i]);
Shapleyvalue(i)=Shappie(G(:,n+2)');
end
Shapleyvalue
sum(Shapleyvalue);
```

The code takes in a row vector, that contains all the coalitions value in a lexicographical order. It then calculates the players' mean marginal contribution in all the permutations. It then return the Shapley value for every player. From a practical point of view, the main concern is about insertion of the value of the coalitions found previously in the right order, to get back the Shapley values of the players that will be crucial in our analysis.

2.5 The Approach

As stated earlier, the first goal of this paper is to find a way to compute the Shapley value in the newsvendor game, given that all players have a random demand that is distributed normally. Since it has not been assumed these demands to be identically independently distributed, they will have a correlation coefficient ρ. Here comes the second goal of the paper, ρ has to vary to observe and analyze the results.

3 Results and Analysis

3.1 The results

Given the vectors of the means:

$$\mu = \begin{bmatrix} 16 & 30 & 50 & 42 \end{bmatrix}$$

and the vectors of the singular standard deviation:

$$\Sigma = \begin{bmatrix} 1 & 2 & 5 & 3 \end{bmatrix}$$

The table below shows the results for the different levels of ρ arbitrary chosen:

Table 1: Shapley values

Player / p	-0.3	0	0.3	0.6	0.9
A	–4.6205	60.7729	106.8803	163.7018	254.9386
B	49.4529	157.5408	231.2327	315.8354	315.9332
C	485.8335	620.5754	719.5829	827.1511	848.8933
D	152.2633	285.0109	377.5041	383.5118	491.6350

Table 2: Expected costs

Coalitions / p	-0.3	0	0.3	0.6	0.9
A	179.9677	179.9677	179.9677	179.9677	179.9677
B	359.9353	359.9353	359.9353	359.9353	359.9353
C	899.8383	899.8383	899.8383	899.8383	899.8383
D	539.9030	539.9030	539.9030	539.9030	539.9030
AB	350.8215	402.4199	448.1158	489.5649	527.7688
AC	863.0945	917.6586	969.1555	1,018.1	1,064.7
AD	515.3489	569.1077	618.2092	663.6879	706.2441
BC	863.0945	969.1555	1,064.7	1,152.4	1,233.8
BD	551.7704	648.8826	733.2440	808.8540	877.9766
CD	899.8383	1,049.4	1,180.1	1,297.8	1,405.6
ABC	800.8055	985.7234	1,141.1	1,560.6	1,401
ABD	489.5649	673.3773	816	938.5964	1,046.3
ACD	828.6333	1,064.7	1,257.2	1,423.9	1,573
BCD	792.6753	1,109.4	1,345	1,560.6	1,405.6
ABCD	682.9292	1,123.9	1,435.2	1,690.2	1,911.4

3.2 Shapley value's stability

The Shapley value may or may not be in the core, this is fundamental to understand in order to know if this is a stable solution or if players want to deviate to make themselves better off. To do this, Matlab will come in handy again.

```
A = [ 1 0 0 0; 0 1 0 0; 0 0 1 0;0 0 0 1;
1 1 0 0; 1 0 1 0;1 0 0 1;0 1 1 0;
0 1 0 1;0 0 1 1;1 1 1 0; 1 1 0 1;
1 0 1 1;0 1 1 1 ];
x = [106.88 ; 231.23 ; 719.58 ; 377.5] ;
b = [ 179.96;359.93;899.83;539.9;448.11;969.15;
618.2;1064;733.24;1180;1141;816;1257.2;1345];
```

```
Ax<=b
```

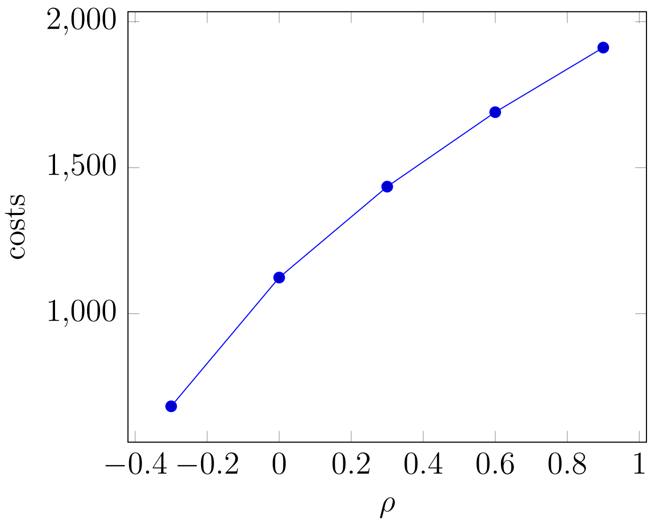
The code written above is created to check whether the Shapley satisfies one of the properties of the core, coalition rationality. Since the program return a vector of all ones it means the inequalities defined above through the linear system Ax = b are satisfied. It is easy to check that the second property, namely efficiency, it is satisfied: summing all the Shapley values we obtain the grand coalition cost. We can conclude then that the results we got are stable solutions to which players will stick.

3.3 Commenting on the results

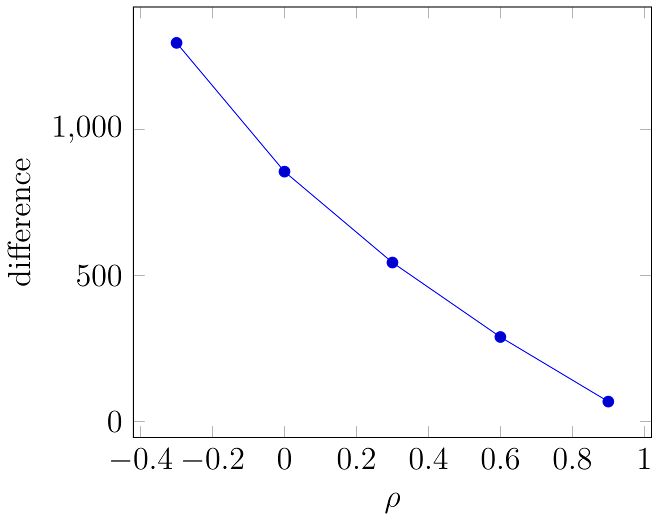
It is important to remember that since The Shapley value represents a cost in this case, it is preferred to be as lower as possible.

The table shown above shows many meaningful insights that here are going to be analyzed. The first, and probably the most straight-forward, arises from the differences of individual players' expected cost and variance. In fact, it is noticeable that the players with highest variance (like pl. C), tend to have much higher expected cost and in turn, Shapley value. On the opposite, players with small variance tend to have much lower expected cost and in turn Shapley values. It is important to notice that the variance it is much more important than the mean in this case. This happens because the cost is meant to be in some way arising from the difference between the actual demand faced and the demand we forecast. The bigger the variance, the more difficult is to actually determine a plausible number of customers we will have.

Another clear result stems from the analysis of outcomes. Through the table appears how a negative correlation coefficients among the random demands clearly constitutes an advantage for the entire coalition, also important is how the cost grows in correspondence of the increasing of the correlation coefficients.



The graph above can better help understanding the relation between the correlation coefficient and the expected costs. On the x-axis are labeled the values of ρ at which the observations are taken, while on the y-axis are plotted the expected cost. The data are considering only the grand coalition. The relation between the two variables it is almost linear, as it emerges from the blue line fitting the scatterplot. From this it si possible to obtain another useful relationships, that can be graphically represented.



Plotted there is the difference between the grand caolition cost and the sums of all singular player costs, varying with the correlation coefficient. As it is noticeable there is an almost linear relations here too, but this time the curve is downward sloping. As the correlation coefficient approaches to 1, the difference goes to 0. This implies that the positive effect brought by joining forces wears off as the random demands comes to behave too similarly. This arises from the fact that the "offsetting" effect is much more lenient. The situation gets particularly interesting when ρ is -0.3. In fact, the Shapley value of Player A is negative, pretty odd since it represents a cost. this is justified by the fact that whenever A joins a two player coalitions, the out coming result is even lower than single players cost. (i.e B = $359 \rightarrow AB = 350$).

Furthermore, it is interesting to notice that coalition BCD's value is lower than ACD's, even though player A brings a smaller variance than B. This mechanism resemble that of an insurance, with the negative correlation between players that effectively manage to reduce risk and in turns reducing cost helping bringing the consumption much closer to the mean.

3.4 Overage and underage Costs

It can be interesting to analyze the result when varying the overage and underage costs. In fact, these two variables influence the expected costs in ways that will be shortly observed. In the table below are shown the coalitions' cost and respective order quantity, keeping ρ fixed at 0.3 and letting c_u and c_o vary.
For readability: 1 = [100/1000] 2 = [1000/1000] 3 = [1000/100]

Table 3: My caption

Coalitions / Q & C	Q1	C1	Q2	C2	Q3	Cost3
A	17.33	179.96	16	797.88	14.66	179.96
B	32.67	359.93	30	1,595.8	27.32	359.93
C	56.67	899.83	50	3,989.4	27.32	359.93
D	46	539.9	42	2,393.7	37.99	539.9
AB	49.32	448.1	46	1,986.7	42.67	448.1
AC	73.19	969.15	66	4,296.7	58.8	969.15
AD	62.59	618.2	58	2,740.3	72.1	618.2
BC	87.89	1,064.7	80	4,720.3	72.1	1,064.7
BD	77.4	733.2	72	3,250.8	66.56	733.2
CD	100.75	1,180.1	92	5,232.1	83,24	1,180.1
ABC	104.46	1,141.1	96	5,058.9	87.53	1,141.1
ABD	94	816	88	3,621.4	81.94	816
ACD	117.3	1,257.2	108	5,573.8	98.67	1,257.2
BCD	132	1,354	122	6,002.7	111.95	1,354
ABCD	148.64	1,435.2	138	6,363.1	127.35	1,435.2

Many powerful insights emerges from this results that can enlighten on the effect of the underage and overage cost on expected costs. The first that comes into mind is the difference in the optimal order quantity, in fact increasing c_o means also decreasing optimal order quantity. This phenomena has a clear explanation because increasing the cost overage, then the risk of acquiring one more unit becomes more expensive since in case it remains unsold, a much higher cost will be borne. On the contrary, increasing c_u has the opposite effect, it makes much more expensive to acquire 1 unit less that might be sold causing a tendency to acquire more. This can be also seen from the costs, since in the two "lateral" instances remain the same even though quantity are different, while in the central situation are much higher, not being able to "adjust" and being every mistake relatively expensive from both sides. It is curios to notice that the quantity for "2" are all integers.

3.5 Conclusion

The purpose of this thesis was to provide a computational method for the newsvendor game and to understand how the parameters in the models affect the result. There are many ways in which this project could be brought forward and many interesting developments, for example other distributions beside the normal could be analyzed. The model could be improved considering for the cost of individual economic actors of giving up data, a topic very important lately since they are acquiring much relevance. I hope my future studies will allow me to gain a further understanding on these topics and to work again on this project to proceed in such directions.

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