Department of Business and Management, Chair of Macroeconomics

Endogenous Growth and Business Cycles

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Academic year 2015/2016

Contents

Introduction

This work is focused on growth theory and particularly on what are the forces that allow an economy to grow in the long-run and which are the interrelations between growth in productivity and business cycle fluctuations.

First of all, we point out that growth can be measured as the percent rate of change in real, potential output produced in an economic system on a yearly basis. Since a larger amount of final goods per capita should extend the consumption possibilities in a country, growth in output has always been acknowledged by economic theorists, although this point has been sometimes disputed, as the main indicator of the evolution over time in the welfare of an economy. Because of that, analyses about why some countries are richer and grow faster than others are already showed in the classic works by Smith and Ricardo. Nevertheless, growth theory emerged as a stand-alone field of research only in the second half of the twentieth century, when the seminal works by Solow (1956) and Swan (1956) laid out the neoclassical model of growth. Another boost to growth theory as an independent sector of research in economics took place in the late 80's, when Romer (1986) and Lucas (1988) contributed to the birth of endogenous growth theory, according to which economic growth springs out by forces internal to the model.

Now growth theory is widely recognized as one of the main branches in macroeconomics and its interactions with other economic topics, such as industrial organization, labour economics, developmental economics and others, have been surveyed in recent years.

The objective of this thesis is to give a brief account of the development of growth theory and then to focalize on two topics. First of all, we describe the Schumpeterian approach to endogenous growth, according to which the innovation process has a dual effect: a positive one, since it allows growth in productivity and output, but also a negative one, given that it can destroy rents to previous innovators. Then, we try to explain the causal interrelations between growth and business cycle: namely, we discuss whether productivity growth is procyclical or countercyclical and finally if shifts in the amount of resources devoted to productivity enhancing activities can generate short-run fluctuations.

After this premise, follows a plan of the work.

In the **first chapter** we are going to present the main contributions to the development of growth theory through the years, from the very first model of growth built by Solow (1956) and Swan (1956), in which technological change is introduced only exogenously in order to allow for growth in the steady state, to the early attempts to endogenize technical progress, focusing in particular upon the *AK approach* and the models by Romer (1990b) and Grossman and Helpman (1991).

In the **second chapter** we are going to present a *Schumpeterian model of endogenous growth* following the formulation laid out by Aghion and Howitt (1992), in which vertical innovations make older vintages of products obsolete. In this way, the classical Schumpeterian notion of *creative destruction* can be introduced in the modeled economy and it ends up to have important implications, both positive and normative. First of all, it is described a basic, one-sector model, in which quality-improving innovations take place stochastically at a Poisson rate λ, thanks to research activities carried out by firms. Then, the model is extended to have more than one sector. Finally, it is presented an integrated model which reconciles in itself the neoclassical and the Schumpeterian approaches, since both capital accumulation and innovations are considered as forces drawing economic growth.

Finally, in the **third chapter** we are going to investigate the causal relationship between growth in productivity and business cycle fluctuations. We show that this relationship can work in two directions: on the one hand, long-run growth can be influenced by economic fluctuations: therefore, we discuss if growth could benefit or not from economic downturns and why, that is we investigate what are the forces that determine if growth is supposed to behave procyclically or countercyclically; on the other hand, shifts in the amount of resources devoted to research, and, hence, changes in the long-run technology growth may have massive effects on business cycle; therefore, we lay out a model of growth in which the arrival of a *general purpose technology* - i.e. a drastic technological innovation which affects the whole economy - by draining economic inputs from the final output to the R&D sector, causes an economic downturn in the short-run.

Chapter 1

The Development of Growth Theory

Introduction

In this first chapter we present a survey of the main contributions to the development of growth theory through the years, emphasizing particularly the dichotomy between exogenous and endogenous growth models: the former approach assumes that the rate of change in productivity is determined outside the model while according to the latter, technological change is essential in assuring growth in the long-run and depends on the individual decisions made by the economic agents.

The Neoclassical Framework

We start this review from the standard neoclassical model of growth, which was independently developed in the 50's by Solow and Swan[1] and for many years has established itself as the only benchmark in this field. According to this model, the technology of final output production is considered in the intensive form and it is assumed that output per capita is an increasing function of the intensity of capital. The dynamics of the stock of capital per person is influenced by two opposite forces: it depreciates over time but it can also be incremented by saving a fraction of output pro capite. If population growth and

[1]See Solow (1956) and Swan (1956).

technological progress are left aside, we observe that as capital increases, the rate of growth in output saved becomes smaller and smaller, because of the law of diminishing returns, while the effects of the depreciation become massive. In the steady state, these two opposites forces exactly balance each others, hence, in the long-run no growth in capital and output per capita is allowed. If we introduce population growth, this result continues to hold true[2], but this is not the case if we consider also technological progress. As matter of the fact, one of the main results of this model is that in order to allow for growth in output per person in the long-run technological advancement is needed. The steady state rate of growth would eventually converge to zero unless it is introduced in the model a rate of technological change capable of neutralizing the effects of the diminishing marginal product of capital. Although technological change is the only way to obtain growth in the steady state, it is taken as given and does not depend on the decisions of the agents: therefore, this approach goes also by the name of *exogenous growth theory.*

In the Solow-Swan model another operative assumption is made, namely that the rate of savings remains constant in the economy over time. Therefore, in the second half of the first section we analyze the Cass-Koopmans-Ramsey model[3], where this assumption is relaxed and the savings rate is endogenized. In this model the law of motion of capital remains pretty much the same as before, as well as the production function of final output; however, now the savings rate springs out from the individuals deciding their consumption plan over time in order to maximize their utility. If the model is solved for the steady state, we obtain the same results as in the Solow-Swan model; as matter of the fact, long-run growth in capital, output and consumption per capita is possible only if exogenous technological change is taken in consideration.

The Endogenous Growth Framework

We have already noticed that since innovation is the only force allowing long-run economic growth, it is unsatisfactory that the model cannot explain why technical change takes place in the economy. Therefore, another approach to growth theory has been developed through the years in order to endogenize technological advancement.

The first attempt to endogenize technological advancement consisted in defining the technology parameter as growing automatically in proportion to

[2]With the only difference that now output grows at the same rate of population.

[3]See Ramsey (1928); Cass (1965); Koopmans (1965).

capital: in this way, there are no diminishing returns of capital and growth is allowed also in the long-run. Because here the production function takes the form $Y = AK$, these models are also known as *AK models*[4]. According to the Frankel-Romer model, as capital is being accumulated, it is possible for output to grow proportionally since knowledge increases together with the stock of capital, offsetting the negative effects of diminishing returns: therefore, growth is made possible also in the long-run, by assuming a production function characterized by increasing returns to scale.

This family of models has two main shortcomings: first of all, they assume constant social returns to capital; however, if this hypothesis is relieved, in the long-run growth would converge either to 0 or to infinity and no balanced growth equilibrium would be supported. Then, the proportion according to which technology grows with respect to capital is taken as given and the innovation progress is external to firms' decisions and completely uncompensated. In order to address these two problems, two models were presented at the beginning of the 90's, namely those developed by Romer (1990b) and Grossman and Helpman (1991), where research activities are rewarded and technological progress springs out from firms engaging in R&D. In order to make balanced growth, increasing social returns to scale and endogenous technical change compatible, it is introduced market power in the intermediate sector, as suggested by Schumpeter (1942). Since both models assume non-perfect competition and operationalize the idea of "creative destruction", according to which technological innovations have a dual effect: on the one hand, they increase total factor productivity and generate rents to the new innovator; on the other hand, they destroy profits to previous innovators, they are also known as *Schumpeterian models of endogenous growth.*

The main difference between these two models is that Romer (1990b) assumes that horizontal innovations occur in the intermediate sector, while in Grossman and Helpman (1991) the innovation process takes place vertically[5]. However, they share the following distinguishing feature, i.e. technological change occurs because firms undertake research activities. As matter of the fact, each time a firm succeeds in discovering a new intermediate good which enhances productivity in final output sector, it earns a patent which grants monopoly profits. Therefore, enterprises are stimulated to carry out research

[4] See especially Frankel (1962); Romer (1986) but also Harrod (1939) and Domar (1946) can be regarded as early variants, where labour grows automatically in proportion to capital.

[5] However, in the appendix to this chapter it is shown that these two approaches lead to the same reduced form for the intensity of research.

and in this way technology grows over time. Moreover, in both these models it can be detected an *intertemporal spillover effect* generated by imperfect excludability of technological knowledge: in fact, new researchers can build on all innovations previously discovered in the economy and this positive externality cannot be captured by successful innovators. The *intertemporal spillover effect* in the end would make the balanced rate of growth lower than the socially optimal one[6].

1.1 The Neoclassical Growth Model with Exogenous Technological Change

1.1.1 Solow Model

The standard neoclassical model of growth was developed independently in the 50's by Solow and Swan [7] in order to overcome the shortcomings of the Harrod-Domar[8] model in which there was no labour as input to production and the capital-labour ratio was fixed. In this review we are going to follow the Solow model which has established itself as the benchmark in exogenous growth theory. In this model final output is produced by a production function defined as $Y = F(K, L)$, which is convenient to express in the intensive form:

$$y = f(k) \ , \tag{1.1}$$

where:

- y represents output per capita $\frac{Y}{L}$;
- k is the stock of capital per person $\frac{K}{L}$, or the *intensity of capital*;
- $f(\cdot)$ is a function exhibiting diminishing returns to capital.

Net investment per person is measured by the increase in capital pro capite over time in the following way:

$$\frac{dk}{dt} = \dot{k} = sy - \delta k \ , \tag{1.2}$$

[6]Actually, in Grossman and Helpman (1991) the equilibrium growth rate could be also more than optimal; an explanation is given in the welfare analysis of the model by Aghion and Howitt (1992) described in chapter two.

[7]See Solow (1956); Swan (1956).

[8]See Harrod (1939) and Domar (1946).

where s represents the marginal propensity to save, which is taken as given, and δ is the rate of depreciation of the existing physical capital. Substituting (1.1) for y in (1.2) we obtain the following difference equation:

$$\dot{k} = sf(k) - \delta k \ . \tag{1.3}$$

Because up to now we are considering neither population growth nor technological change, the accumulation of capital is the only force drawing output growth. The rate of increase of capital is the difference between the fraction of output saved and devoted to investment $sf(k)$ and the the fraction of capital that wears out δk. Because of the law of diminishing returns, as capital per capita increases, the rate of increase in output and, therefore, in output saved becomes smaller and smaller and approaches exactly the quantity needed to replace the fraction of capital per capita depreciated as k converges to the steady state intensity of capital k^* determined by the condition

$$sf(k) = \delta k \ .$$

Then, in steady state capital stock and capital per person remain fixed, hence, there is no growth in output and in output pro capite. An increase in the rate of savings will be able to raise growth only temporarily but the effect on the long-run rate of growth will be null.

The result of zero growth in the steady state persists also introducing population growth: in this case the condition for the steady state value k^* would be

$$sf(k) = (n + \delta)k \ ,$$

with n being the exponential rate of growth of population L. Again we have no growth in capital and output per person in the long-run[9].

This result did not fit well with the observations of long-run growth in output per person in many countries. In order to fix this shortcoming Solow introduced in his model a productivity parameter A whose rate of growth g were capable of offsetting the effects of diminishing marginal product of capital. The specification of production technology in this extension is similar as before, but now L represents effective labour force and is defined as $L = A \cdot N$, where A is a productivity parameter which measures the efficiency of each worker and

[9] In this second case, however, there is indeed growth in capital stock, and thereby in total output, in order to neutralize the diluting effect of population growth on capital pro capite.

N the units of labour. Now the rate of growth of L equals $n + g$[10]; therefore, following the previous reasoning, the condition for the steady state level of k^* can be rewritten as

$$sf(k) = (\delta + n + g)k \ .$$

If now we compute the rate of growth of capital per person $\frac{K}{N}$ in the steady state, we obtain that it is equal to the rate of growth in the productivity parameter g[11]. Hence, through the intoduction in the model of technological change, we are able to explain growth in capital and, hence, in output per capita also over the long-run.

1.1.2 The Cass-Koopmans-Ramsey Model

The Solow-Swann model of growth makes the assumption that the savings rate s remains fixed along the whole transition path to the steady state. However, in the model built initially by Ramsey (1928) and then extended by the work of Cass (1965) and Koopmans (1965) the savings rate is microfounded, since individuals choose independently their consumption profile over their lifetime, according to their intertemporal preferences for consumption.

Here the technology of production is the same as before in the Solow-Swan model and is represented by the equation $F(K, L)$, which is homogenous of degree one and exhibits constant returns to scale in capital and labour. It is useful to normalize the economy as having only one individual[12], i.e. $L = 1$. Then, we can rewrite the production function in the form $Y = F(K)$. In this economy now we introduce the idea of a representative individual who must allocate its income to consumption and saving in order to maximize the utility function

$$U = \int_{t=o}^{\infty} e^{-\rho t} u(c_t) dt \ , \tag{1.4}$$

where:

- c_t is its consumption at time t;
- $u(\cdot)$ is the instantaneous utility function which is defined as $u(c_t) = \frac{c_t^{1-\theta} - 1}{1-\theta}$;
- ρ is the intertemporal discount rate.

[10] $L = A \cdot N$. Since $\frac{\dot{A}}{A} = g$ and $\frac{\dot{N}}{N} = n$, we obtain that $\frac{\dot{L}}{L} \simeq n + g$.

[11] We know that $\frac{K_t}{N_t} = A_t k^*$, then, since k^* is steady over time by definition, we obtain finally that $\frac{d\frac{K_t}{N_t}}{dt} = g$.

[12] We are then not considering anymore population growth.

The law of motion of capital per person over time is:

$$\dot{K} = F(K) - c - \delta K \ . \tag{1.5}$$

In order to maximize his utility at any point in time, the representative individual must solve the maximization problem of the Hamiltonian:

$$H = u(c) + \lambda(F(K) - c - \delta K) \ , \tag{1.6}$$

where λ is value of investment calculated in current units of utility. The first-order condition for optimality is $u'(c) = \lambda$ and we can determine λ in terms of optimal control theory by deriving the Euler equation

$$\rho\lambda = \lambda(F'(K) - \delta) + \dot{\lambda} \tag{1.7}$$

and the transversality condition $\lim_{t\to\infty} e^{-\rho t}\lambda K = 0$, which ensures that capital will not be accumulated forever without never consuming it. As in the Solow-Swan model, now we search for a steady state, that is a state in which both the stock of capital and λ remain constant. By imposing $\dot{\lambda} = 0$ in (1.7) we obtain the condition for the steady state level of capital

$$F'(K) = \rho + \delta \ .$$

Therefore, we can conclude that even if we endogenize the consumption choices of individuals, there is no long-run growth in capital and output if no technological change is considered in the model, since the capital stock converges to his stationary state value K^*, which solves the previous condition, forcing output to converge to a stationary value too.

As before, now we introduce exogenous technological progress in the model through a productivity parameter A growing at the exponential rate g. We express the production function in two arguments, namely capital and effective labour, as $F(K, AL)$ and we assume again a fixed, unitary labour force, hence obtaining $F(K, A)$. The conditions for optimality in the model remain the same as in the model without growth in productivity[13]; however, now technical progress allows for growth in capital and output also in the long-run since the diminishing returns to capital are constantly offset by the growth in productivity. Substituting the condition $u'(c) = \lambda$ in the equation (1.7) op-

[13]Now the Euler equation looks like $\rho\lambda = \lambda\left[\frac{\partial F(K,A)}{\partial K} - \delta\right] + \dot{\lambda}$.

portunely rewritten and knowing from the specification of the instantaneous utility function that $u'(c) = c^{-\theta}$, it can be obtained the following equation for the constant growth of consumption over time:

$$\frac{\dot{c}}{c} = \frac{1}{\theta}\left[\frac{\partial F(K,A)}{\partial K} - \delta - \rho\right] . \tag{1.8}$$

The marginal product $\frac{\partial F(K,A)}{\partial K}$ depends only on the ratio $\frac{K}{A}$, then if both K and A grow at the same exogenous rate g, the marginal product of capital will not fall down over time and growth is allowed also in the long-run. According to (1.8), a steady state with output, capital and consumption all growing at a positive rate exists only if the ratio of capital to productivity satisfies the following condition:

$$g = \frac{1}{\theta}\left[\frac{\partial F(K,A)}{\partial K} - \delta - \rho\right] . \tag{1.9}$$

If this condition holds together with the transversality condition[14], the economy will have an optimal growth path in which consumption, capital and output all grow at the same exogenous rate of technological change g.

1.2 Attempts to endogenize technology

It has been showed before through the Solow-Swan and the Cass-Koopmans-Ramsey models that technology and innovation play a fundamental role to avoid zero growth in the steady state and allow for growth in capital and output in the long-run. However, they have been introduced in the previous models only exogenously, by a productivity parameter growing at a rate taken as given. However, it has always been believed by the most theorists that technology improvements depend on decisions of economic agents as much as capital accumulation. In order to fix the shortcoming of the previous exogenous models in explaining what are the forces that draw technological change, there have been many attempts to endogenize technology, such as the *AK approach*[15] or the pioneering works by Romer (1990b) and Grossman and Helpman (1991).

[14] A necessary and sufficient condition for the transversality condition to be satisfied in steady state is $\rho + (\theta - 1)g > 0$.

[15] See Harrod (1939); Domar (1946); Frankel (1962); Romer (1986).

1.2.1 The AK approach

The main cause of zero growth is the fact that the specification of production function as having constant returns to scale forces marginal product of capital to decrease over time if the other inputs, namely labour and technology, are taken as given. In the end, this decreasing behaviour of returns to capital excludes at all growth in output. In order to counterbalance the effects of diminishing returns, a family of models makes one of the other inputs grow automatically in proportion to capital. They usually go by the name of *AK models* since the production function takes the form $Y = AK$. An early variant of this kind of models is that built by Harrod and Domar[16], in which labour grows in proportion to capital. However, we are going to analyze the Frankel-Romer model, in which conversely it is the technology parameter A that increases proportionally to K [17]. In this model technical knowledge is considered itself as a capital good that can be accumulated over time by carrying out research activities. The aggregate production function, as defined by Frankel, takes the form:

$$Y = \bar{A}K^{\alpha}L^{1-\alpha} \ , \tag{1.10}$$

where the technological parameter $\bar{A}$ is an increasing function of the intensity of capital $\frac{K}{L}$:

$$\bar{A} = A \cdot \left(\frac{K}{L}\right)^{\beta} . \tag{1.11}$$

If the analysis is restricted to the case in which $\alpha + \beta = 1$, combining (1.10) and (1.11) together yields the equation

$$Y = AK \tag{1.12}$$

from which springs the name *AK approach* given to this typology of models. In this model as capital grows, final output increases proportionally, since knowledge increases too automatically with capital stock, offsetting the effects produced by the diminishing marginal returns to capital. In particular, the law of motion of capital is described by the differential equation

$$\dot{K} = sAK - \delta K$$

[16]It was developed independently by Harrod in 1939 and Domar in 1946 and had strong influences on the works of Solow and Swan.

[17]See Frankel (1962) and Romer (1986).

and therefore the growth rate of capital will be equal to:

$$g = \frac{\dot{K}}{K} = sA - \delta \ .$$

Since output increases in proportion to capital, its rate of growth will be g too. If we introduce in the economy population growth, the rate of growth of capital and output per person will be $g - n$.

More then thirty years after the publication of this paper by Frankel, Romer resumed it and re-expressed the previous analysis in terms of Ramsey model of intertemporal utility maximization. Assuming the same technology of production as in the model built by Frankel, and normalizing labour to be equal to 1, the representative firm must solve the following dynamic optimization problem:

$$\max \int_0^\infty u(c_t)e^{-\rho t}dt$$
$$\text{s.t.} \quad \dot{K} = \bar{A}K^\alpha - c \quad , \text{with } \dot{K} \geq 0 \ ,$$

where the dynamics of the productivity parameter are exogenously determined by the equation $\bar{A} = AK^\beta$ as in the Frankel model[18]. Assuming the same instantaneous utility function as in the Cass-Koopmans-Ramsey model [19] and using optimal control theory it can be derived the following Euler equation

$$-\theta\frac{\dot{c}}{c} = \rho - \alpha\bar{A}K^{\alpha-1}$$

and then, substituting in this condition the equation for $\bar{A}$ yields the equation:

$$-\theta\frac{\dot{c}}{c} = \rho - \alpha AK^{\alpha+\beta-1} \ . \tag{1.13}$$

If now we focus the attention to the case analyzed by Frankel in which $\alpha + \beta = 1$, it can be obtained the same result as in the previous model, with the economy growing at a finite, positive rate:

$$g = \frac{\alpha A - \rho}{\theta}.$$

[18]Remember that here labour force has been assumed to be equal to unity.
[19]That is assuming the isoelastic function $\frac{c^{1-\theta}-1}{1-\theta}$.

1.3 Two Schumpeterian Models of Endogenous Growth

So far two main approaches to growth theory have been analyzed: the *neoclassical growth framework* and the *AK approach*; the primary difference between these classes of models is the following: the technology of production assumed in the neoclassical models exhibits constant returns to scale: this means that as capital grows over time, its marginal product decreases up to exclude growth in output per capita in the long-run. The only way to avoid zero growth, then, is to introduce an exogenous parameter representing technological knowledge, whose growth is able to counterbalance the effects generated by the law of diminishing returns. However, in the AK models the technology of production is assumed to have increasing returns to scale, since as capital grows over time, one other input to production, namely labour or, especially, technology, increases automatically in proportion. In this way, marginal returns to capital do not fall down and growth is allowed also in stationary state and is determined by the rate of technological change which is no more taken as given, but endogenously generated in the model.

Although the AK approach seems to explain well how technology changes over time, it shows also some difficulties: first of all, if the assumption of constant social returns to capital[20] is relaxed, the economy presents two opposite asymptotic behaviours:

- if $\alpha + \beta < 1$, growth will converge to 0 as in the neoclassical model without progress in technology;
- if $\alpha + \beta > 0$, growth will diverge over time and then there is no *balanced growth equilibrium* in the economy.[21]

Another feature of the model that is not completely satisfying is the fact that albeit growth is endogenously explained, the proportion in which technology increases with respect to to capital is unexplained and taken as given; furthermore, the process of accumulation of knowledge is external at all to firms and completely uncompensated[22].

In the early 1990s, two papers were presented by Romer (1990b) and by Grossman and Helpman (1991) in which rewards to technological innovations

[20] That is the condition imposed by Frankel $\alpha + \beta = 1$.

[21] Such an equilibrium is defined to be one in which capital, output and consumption grow all at the same constant rate of technological change over time.

[22] As noticed by Romer (1990b) and Aghion and Howitt (1998b).

were taken in consideration as the main incentive for individuals to do research. In both models, these rewards take the form of monopoly rents which accrue to the successful innovator. As matter of the fact, the solution to reconcile in a single model balanced growth in the long-run, increasing social returns to capital due to technology spillovers and technological change not taken as given, but generated by maximizing individuals responding to market incentives is to introduce market power in the intermediate good sector, as suggested by Schumpeter (1942).[23] Since these models follow the suggestion made by Schumpeter of introducing in the economic system non-perfect competition and use his notion of "creative destruction", they are also called *Schumpeterian models of endogenous growth.* The main differences between these two models is that Romer based his work on horizontal product innovations - i.e. each innovation is a design for a new, different product - while Grossman and Helpman introduced in their paper vertical product innovations, i.e. research is aimed at enhancing the quality of existing products.[24]

1.3.1 Romer's Model of Growth through Technological Change

In his model Romer (1990b) claims that growth depends on technological development, which in turn comes from the decisions of individuals whose goal is to maximize their profits or utility. The peculiar feature of this model is that technology is neither a private nor a public good; instead, it is a non-rival, only partially excludable good. In this manner non-convexity is introduced and hence the equilibrium cannot be with perfect competition, but only monopolistic competition can be supported.

Non-Rivalry and Partial Excludability of Technology

The first peculiar point of the Romer's article is that technology as a production input cannot be considered neither as a conventional nor as a public good: in fact, on the one end, conventional economic goods are both rival and excludable and therefore they are privately produced in a competitive market; on the other hand, pure public goods are at the same time non-excludable and non-rivalrous, but they can be introduced in a price-taking model by allowing the existence of a government which can impose taxes. Here technology is non-rival, but, since

[23]As pointed out by Romer (1990b) in the discussion of the premises to his model.

[24]However, Grossman and Helpman (1991) show in their paper the similarity in the two approaches.

technological change arises from action of maximizing individuals, carrying out research activities must grant benefits that are at least partially excludable.

Romer detects two important implications introduced by the non-rivalry of technology:

- first of all, the stock of non-rival goods, unlike human capital, can be increased without bound on a pro capite basis: in fact, while human capital accumulated by an agent goes lost when he ceases to leave, any non-rival good produced by an individual - like a patent or a scientific principle - lives also after his death;
- then, the fact that technological knowledge is non-rivalrous allows for only partial excludability, that is, we can observe a knowledge spillover.

The next step is to take in consideration the strict link between non-rivalry and non-convexity: i.e. if one input of production is a non-rival good, then the technology function cannot give constant returns to scale, taking in account the inputs all together. Using mathematics, given that $F(A, X)$ is a production function where X is a vector of all the rival inputs and A is a vector of all the non-rival inputs, then we have that

$$F(A, \lambda X) = \lambda F(A, X)$$

but

$$F(\lambda A, \lambda X) > \lambda F(A, X) \ ,$$

that is the function is homogeneous of degree one only with respect to the argument X. Therefore, if A is a production input as well, the production function cannot be concave and then that kind of firm cannot survive as a price taker. In fact, since

$$F(A, X) = X \cdot \frac{\partial F(A, X)}{\partial X} \ ,$$

then

$$F(A, X) < A \cdot \frac{\partial F(A, X)}{\partial A} + X \cdot \frac{\partial F(A, X)}{\partial X} \ .$$

Therefore, if all inputs were remunerated at their real marginal product, the firm would have negative profits.

Romer points out that this difficulty emerged many times in previous growth models, however it has been always avoided by taking the technological input A as exogenous or as provided by the government: in both cases this factor of production receives no return and it is assumed that each firm can freely employ it. These models consider technology as the principal driver of growth and as non-rival , however, they are inconsistent with the Romer's premise of partial excludability of knowledge. In this way, the fact that it is the individual behaviour which generates technological change has been ignored.

In this model the way to keep together these three features - i.e. growth driven by technological change, non-rivalry of technology and individual decisions to invest in R&D responsive to market incentives - is to introduce an equilibrium with market power.

The Definition of the Model

Romer starts the description of the model by defining which are the inputs considered; he identifies four inputs:

- physical capital K;
- labour L;
- human capital H;
- an index A which represents the level of technology available to producers.

Here knowledge is divided in a rival component H and in a non-rival technological component A. As said before, A can be accumulated without bound and each new unit of knowledge can be considered as a new design for a producer durable good.

The economy modeled here is composed of three sectors:

1. the research sector, which employs human capital and the disposable knowledge to produce new knowledge, i.e. new designs;
2. an intermediate good sector, which uses designs and a fraction of output to produce a wide range of durables that are employed by firms in the final market;
3. a final good sector, in which firms make use of labour, human capital and producer durables to produce final output Y, which in turn can be consumed or devoted to investment in capital goods.

In order to simplify the analysis, Romer uses some assumptions:

- population and labour supply are kept constant;

- the stock of human capital and its fraction supplied to the market are assumed constant;

- assuming that final output can be saved to increase the stock of capital goods implies that the market for capital goods uses the same technology as the market for final goods;

- labour and physical capital do not enter at all in the production of new knowledge.

The production function in the final good market is described by a Cobb-Douglas technology :

$$Y(H_Y, L, x) = H_Y^{\alpha} L^{\beta} \sum_{i=1}^{\infty} x_i^{1-\alpha-\beta} \ ,$$

where H_Y^{α} is the fraction of human capital devoted to final good production and the physical capital is fractioned in a numerable infinity of producer durables $x = \{x_i\}_{i=1}^{\infty}$. We can also define an index A such that $x_i = 0$ for all $i \geq A$. We can notice that this technology function is homogeneous of degree one, hence in the final good market price-taking behaviour can be supported.

However, how anticipated in the previous section, the market for producer durables cannot be described by a representative, price-taking firm; in fact, each durable i is produced by a different firm which has to develop or buy a design for this durable before starting the production, whose inputs are η units of saved output to produce one unit of the producer durable i. The firm which has developed a new design obtains a patent and rents each durable at rate $p(i)$ and is in front of a downward-sloping demand curve, being the unique seller of good i. Then, assuming no depreciation of the capital good rented, the value of each unit $x(i)$ of the durable is equal to the present value of the infinite flow of rents that it generates. Now it is convenient to separate the sector in which research and development activities for new durables are carried out from that in which this durables are actually produced.

Now Romer specifies the process of creation of new designs, i.e. the dynamics of $A(t)$. First of all, to make the analysis simpler, the index i which identifies different types of durable goods is considered no more a discrete variable but a continuous one; hence, the new specification for the production technology is:

$$Y(H_Y, L, x) = H_Y^\alpha L^\beta \int_0^\infty x(i)^{1-\alpha-\beta} di \ . \tag{1.14}$$

Now the rate of production of new designs by developer j is described by $\delta H^j A^j$, where δ is a parameter of productivity and $H^j$and A^j are the fraction of human capital employed and the technological knowledge available to researcher j. Because each agent in the R&D sector has the access to all the knowledge, given its non-rivalry, the superscript j can be elided and then the aggregate rate of accumulation of designs becomes:

$$\dot{A} = \delta H_A A \ , \tag{1.15}$$

where H_A is the sum of all the amounts of human capital used by each researcher j. This formulation presents many interesting features:

- the more knowledge is available at a specific point in time, the higher will be the productivity of human capital devoted to research at that time;
- the rate of accumulation of new designs is linear in A, holding H_A constant: in this way unbounded growth is made possible because there is no incentive for human capital to shift from research to manufacturing sector as A grows.

Here Romer notices that knowledge enters the model in two ways: the invention of a new design allows for the production of a new durable good in turn used to produce output but also enlarges the stock of knowledge and consequently the productivity of human capital in research sector. The individual who has produced the design has property rights over its use only in the intermediate market but not in the research sector: in this sense technology is considered as only partially excludable. Since a researcher can use all the existing stock of knowledge in his activities, it follows the relation:

$$w_H = P_A \delta A \ ,$$

where w_H is the rental price of a unit of human capital and P_A the price of new designs.

Now Romer proceeds to analyze the maximization problem of the intermediate good producer: he takes as given P_A , the capital interest rate r and the spot price of capital goods (which is equal to 1, since the rate of conversion between goods and capital is one for one) and sets the price for the ith durable in order to maximize profits from a price list $p(i)$ whose range is $\mathbb{R}_+ \cup \{\infty\}$, where the price for durables not yet produced is $p(i) = \infty$. Observing this price list for durables, the final output firm in turn sets the quantity demanded $x(i)$ for each durable, by the following maximization problem:

$$\max_{\{x\}} \int_0^\infty [H_Y^\alpha L^\beta x(i)^{1-\alpha-\beta} - p(i)x(i)]di \ .$$

The resulting demand function, then, is

$$p(i) = (1 - \alpha - \beta) H_Y^\alpha L^\beta x(i)^{-\alpha-\beta} \ . \tag{1.16}$$

Facing this demand curve, each durable producer that has sustained a fixed cost for acquiring a design will chose the output $x(i)$ in order to maximize the profit:

$$\begin{aligned} &\max_{\{x\}} p(x)x - r\eta x \\ \Longleftrightarrow \ &\max_{\{x\}} (1 - \alpha - \beta) H_Y^\alpha L^\beta x^{1-\alpha-\beta} - r\eta x \ . \end{aligned}$$

Given that this monopoly maximization problem presents constant marginal costs and elasticity of demand, the price will be:

$$\bar{p} = \frac{r\eta}{(1 - \alpha - \beta)}$$

and the monopoly profit will be

$$\pi = (\alpha + \beta)\bar{p}\bar{x} \ .$$

Now Romer analyzes the decision of the intermediate good firm to buy a new design: it depends on the comparison between the cost of the design P_A and the present value of the flow of profits: since the research market is competitive, in equilibrium these two quantities must be equal:

$$\int_t^\infty e^{-\int_t^\tau r(s)ds} \pi(\tau) d\tau = P_A \ .$$

Differentiating with respect to t we obtain the equation:

$$\pi(t) - r(t) \int_t^\infty e^{-\int_t^\tau r(s)ds} \pi(\tau) d\tau = 0$$

and then substituting it in the expression for P_A from the research arbitrage equation, it is finally obtained the condition:

$$\pi(t) = r(t)P_A \ . \tag{1.17}$$

Romer concludes the model by introducing Ramsey consumers with discounted, constant elasticity preferences:

$$\int_0^\infty U(C)e^{-\rho t}dt, \qquad \text{with } U(C) = \frac{C^{1-\theta}-1}{1-\theta} \text{ for } \theta \in [0,\infty) \ .$$

The intertemporal solution if the consumer is facing an interest rate r is :

$$\frac{\dot{C}}{C} = \frac{(r-\rho)}{\theta} \ .$$

Consumers are endowed with fixed amounts of L and H whose supply is inelastic and at time 0 they own firms in the market for durables, whose profits are entirely distributed to consumers as dividends. Since the symmetry in the model, all the durables available are supplied at the same level. Final output, then, can be specified as:

$$\begin{aligned} Y(H_Y, L, x) &= H_Y^\alpha L^\beta \int_0^\infty (\bar{x})^{1-\alpha-\beta} di \\ &= (H_Y A)^\alpha (LA)^\beta K^{1-\alpha-\beta} \eta^{\alpha+\beta-1} \ . \end{aligned}$$

Balanced Growth Path

Now Romer solves the model for an equilibrium in which the level of technology A, the stock of physical capital and final output grow at a constant exponential rate. Since the equation for $\dot{A}$ is linear in A, the technology index will grow at a constant rate if the amount of human capital employed in research sector is held also constant. Along the balanced growth path the ratio K to A should be constant, than it is implied that $\bar{x}$ is constant too. Because of the accumulation of both capital and technology, the human capital productivity in the final market grows in proportion to A and, by the equation $\dot{A} = \delta H_A A$, the same does the productivity of human capital in the research sector; hence, if the price for new designs does not change, the repartition of H between H_Y and H_A will remain constant.

The final task which needs to be completed is checking if the balanced growth path described above follows the equilibrium conditions. We know that the present value of the stream of monopoly profits generated by any durable must be equal to the price of the design:

$$P_A = \frac{\pi}{r} = \frac{\alpha + \beta}{r}(1 - \alpha - \beta) H_Y^{\alpha} L^{\beta} \bar{x}^{1-\alpha-\beta} \ .$$

The condition which determines the distribution of H between final output and research sector is that the wage must be the same in both sectors: in the former, the wage is equal to marginal product of human capital, in the latter it is equal to $P_A \delta A$. Then H_Y and H_A must be chosen so that

$$w_H = P_A \delta A = \alpha H_Y^{\alpha-1} L^{\beta} A \bar{x}^{1-\alpha-\beta}$$

and then substituting for P_A yields:

$$H_Y = \frac{1}{\delta} \frac{\alpha}{(1 - \alpha - \beta)(\alpha + \beta)} r \ .$$

The exponential rate of growth for A given the fixed level of $H_A = H - H_Y$ corresponds to δH_A. Equation $Y = H_Y^{\alpha} L^{\beta} A \bar{x}^{1-\alpha-\beta}$ shows that final output grows at the same rate as A if H_Y, L and $\bar{x}$ are fixed and the same does the stock of capital since the relation $K = A \bar{x} \eta$. If the ratio of capital to output is constant, also the ratio of consumption to output must be constant

$$\frac{C}{Y} = 1 - \frac{\dot{K}}{K} \cdot \frac{K}{Y}$$

and therefore we have that the growth rate g for all these variables is the same:

$$g = \frac{\dot{C}}{C} = \frac{\dot{Y}}{Y} = \frac{\dot{K}}{K} = \frac{\dot{A}}{A} = \delta H_A$$

Combining it with the expression for H_Y and the constraint for H finally yields:

$$g = \delta H - \frac{\alpha}{(1 - \alpha - \beta)(\alpha + \beta)} r \ .$$

To close the model, the relation between the growth rate and the interest rate implied by the preferences, i.e. $g = \frac{\dot{C}}{C} = (\frac{r - \rho}{\theta})$ can be rewritten as

$$g = \frac{\delta H - \Lambda \rho}{\theta \Lambda + 1} \ , \tag{1.18}$$

where

$$\Lambda = \frac{\alpha}{(1-\alpha-\beta)(\alpha+\beta)} \ .$$

Moving to the welfare analysis of the equilibrium growth rate, it can be pointed out that in this model it results always less than the socially optimal one. First of all, we are going to find this optimal growth rate as the solution to the social planning problem:

$$\begin{aligned} \max \quad & \int_0^\infty \frac{C^{1-\theta}-1}{1-\theta} e^{-\rho t} dt \\ s.t. \quad & \dot{K} = \eta^{\alpha+\beta-1} A^{\alpha+\beta} H_Y^\alpha L^\beta K^{1-\alpha-\beta} - C \\ s.t. \quad & \dot{A} = \delta H_A A \\ s.t. \quad & H_Y + H_A = H \ . \end{aligned}$$

The balanced growth path derived as a solution to this problem has the following optimal growth rate:

$$g^* = \frac{\delta H - \Theta\rho}{\theta\Theta + (1-\Theta)} \ ,$$

with $\Theta = \frac{\alpha}{\alpha+\beta}$. By comparing this socially optimal growth rate with the equilibrium one (1.18), we can notice that the former is always greater than the latter:

$$g = \frac{\delta H - \Lambda\rho}{\theta\Lambda + 1} < g^* = \frac{\delta H - \Theta\rho}{\theta\Theta + (1-\Theta)} \ .$$

This is due to two reasons: first of all because the research sector produces an input which is used in a market that follows monopoly pricing: the researcher receives only a fraction $\frac{1}{1-\alpha-\beta}$ of the increase in output generated by the production of a new design (this is captured by the relation $\Lambda = \frac{1}{(1-\alpha-\beta)}\Theta$). The second reason is that research generates intertemporal spillovers by raising the productivity of all future researchers which, being not excludable, cannot be incorporated in the price for designs (this effect is captured by the equations for g and $g*$ having different denominators).

1.3.2 Grossman and Helpman's Model of Growth through Quality Improvements

Also Grossman and Helpman (1991) developed a model in which growth is driven by technological progress; however, here it takes a different form compared to Romer (1990b): innovations, rather than increase the spectrum of durable goods available to firms to produce final output, improve the quality of final goods. Hence, at an economy-wide level, there is a continuous set of final goods and each one can be produced in infinite qualities ordered along its own *ladder*[25].

In the economy modeled here, a fixed, uncountably infinite number of different final goods indexed by ω can be supplied. This set is represented by the unit interval $[0, 1]$. Each different good ω is available in a countably infinite number of qualities: at $t = 0$ each product has quality equal to 1; as time goes on, quality j of good ω is given by $q_j(\omega) = \gamma^j$: that is, each successful innovation in product ω raises its quality by the factor $\gamma > 1$. Technological progress is driven by R&D activities carried out by firms: if the entrepreneur i does research at intensity λ_i[26] for the interval dt, a successful innovation takes place with probability $\lambda_i dt$.

The consumers in the economy must solve the maximization problem

$$\max \quad U = \int_0^\infty e^{-\rho t} \log u(t) dt \tag{1.19}$$

$$\text{s.t.} \quad \int_0^\infty e^{-R(t)} C(t) dt \leq A(0) \; . \tag{1.20}$$

$R(t)$ is the cumulative interest over time, with $r_t = \frac{dR(t)}{dt}$. If we impose then r_t constant at some level r, we have $R(t) = rt$. The log of the instantaneous utility function is defined by:

$$\log u(t) = \int_0^1 \log \left[\sum_j q_j(\omega) x_{jt}(\omega) \right] d\omega \; , \tag{1.21}$$

with x_{jt} representing how much product ω of quality j is demanded at time t. In the intertemporal constraint, A_0 is the present value of the stream of incomes over time and $C(t)$ represents how much resources are allocated to consumption at time t:

$$C(t) = \int_0^1 \left[\sum_j p_{jt}(\omega) x_{jt}(\omega) \right] d\omega \; .$$

[25]We are referring to the expression *quality ladder* used by Grossman and Helpman in their work (1991).

[26]To do research at intensity λ_i, firms need to employ a units of labour per unit of time.

First of all, the consumer chooses $C(t)$ in order to maximize the instantaneous utility[27]and then chooses the time path to maximize the life-time utility. The solution to this problem is defined by the intertemporal constraint (1.20), the transversality condition and the following Euler equation:

$$\frac{\dot{C}}{C} = r - \rho \ . \tag{1.22}$$

In the economy modeled here, the only input to production is labour; one unit of whatsoever final good of any quality requires one unit of labour to be produced. In each sector ω there is only one firm which has a quality lead over the other competitors and this lead is exactly one step above the nearest follower. Then, each cutting-edge product is exchanged at price:

$$p = \gamma w \ . \tag{1.23}$$

The flow of profits which accrues to the leader in each sector is given by the equation:

$$\pi = \left(1 - \frac{1}{\gamma}\right) C \ . \tag{1.24}$$

To produce any kind of final good a firm needs a design, which is produced by firms engaging in R&D and which is permanently protected by a patent. Because, as for we have defined above research activity, cutting-edge firms have no cost advantages in carrying out R&D activities[28], only followers will invest resources to overcome the leader. Since each sector warrants the same flow of profits, and assuming that in each one research is undertaken at the same aggregate intensity λ[29], followers are indifferent to the industry in which to perform R&D. By employing $a\lambda_i$ units of labour per unit of time, with probability $\lambda_i dt$ a firm can be a successful innovator and, then, obtain the present value V of the flow of profits over the time it will remain leader in the sector. Therefore, its objective is to maximize the expression:

$$V\lambda_i dt - wa\lambda_i dt \ .$$

[27]It is evident that the consumer will choose for each product the quality $j = J_t(\omega)$ which warrants the lowest quality adjusted price $\frac{p_{jt}(\omega)}{q_{jt}(\omega)}$. Assuming, therefore, that he consumes the same quantity of each product ω, the demand function at time t takes the form $x_{jt}(\omega) = \frac{C(t)}{p_{jt}(\omega)}$ if $j = J_t(\omega)$ and 0 otherwise.

[28]In this feature of the research sector, it can be identified the *spillover effect*, since once a new technology has been discovered, all the firms can use it as starting point.

[29]In this way, firms expect that their leadership will last the same amount of time in each industry.

In order to allow for an equilibrium with positive but finite intensity of research, we impose the condition $V = wa$. In this case, however, the optimal level of individual intensity of research λ_i is indefinite. If we interpret V as the stock market value of the firm, we can write a research arbitrage equation by imposing the expected rate of return per unit of time to be equal to the interest rate:

$$\frac{\pi + \dot{V}}{V - \lambda} = r \ ,$$

where $\dot{V}$ is the appreciation of the stock market value per unit of time. Substituting in the condition $V = wa$, the research arbitrage equation becomes:

$$\frac{\pi}{wa} + \frac{\dot{w}}{w} = r + \lambda \ . \tag{1.25}$$

Combining together the Euler equation (1.22), (1.24) and the arbitrage equation (1.25), and, then, normalizing the wage rate w to be 1, it can be obtained the low of motion of consumption:

$$\frac{\dot{C}}{C} = \frac{(1 - 1/\gamma)C}{a} - \rho - \lambda \ . \tag{1.26}$$

Grossman and Helpman close the model by deriving the market clearing equation. The amount of labour employed in the manufacturing sector at time t is given by $l = \frac{C_t}{\gamma}$, while the amount engaged in the R&D sector is defined by the expression $n = a\lambda$. If L is the total labour force, the labour market clearing equation looks like the following:

$$a\lambda + \frac{C}{\gamma} = L \ . \tag{1.27}$$

Then, the evolution of the economy over time is fully described by the law of motion of consumption (1.26) and the resource constraint (1.27) for any initial value of C. The rate of growth g here is defined to be the rate of change in the instantaneous utility.[30] Substituting the demand functions derived in ([27]) and the expression for the limit price (1.23) in (1.21) yields:

$$\log u(t) = \log C - \log \gamma + \int_0^1 \log q_t(\omega) d\omega \ .$$

[30] In order to make the model more comparable to the Romer's one previously analyzed, we can regard each product ω as an intermediate good, the instantaneous utility function (1.21) as a constant return to scale production function and $u(t)$ as final output produced at time t. In this case, the rate g represents growth in output.

Knowing that if the aggregate intensity of research in each sector is λ, a fraction λ of the whole set of products increase its quality in the time interval dt, we can rewrite the previous equation as

$$\log u(t) = \log C - \log \gamma + t\lambda \log \gamma \tag{1.28}$$

and, then, differentiating (1.28) with respect to t, we obtain the equation for the growth rate:

$$g = \lambda \log \gamma. \tag{1.29}$$

It is evident from this equation that growth can be supported only if the aggregate intensity of research λ is different from 0, that is to say growth is allowed only if individuals, by engaging in research activities, make technological progress happen in the economy.

If we impose $\dot{C} = 0$ [31] in (1.26), it can be obtained the expression:

$$\lambda = \frac{(1 - 1/\gamma)C}{a} - \rho \tag{1.30}$$

which, combined with (1.27), yields the equilibrium level of aggregate intensity of research:

$$\lambda = \frac{(1 - 1/\gamma)L}{a} - \frac{\rho}{\gamma} \ . \tag{1.31}$$

From this equation we can see that the rate of growth increases if the size of the economy, i.e. L, becomes bigger.[32] Moreover, also a reduction in a, by making research less costly, determines a bigger rate of growth. Also an increase in the quality jump γ ends in an economy growing faster, since it raises both the size of steps in the quality ladder and the equilibrium aggregate intensity of research.[33]

We can now compare the equilibrium rate of growth, which is ultimately determined by (1.31), to the optimal one, which, given the value of γ, is defined by the expression:

$$\lambda^* = \frac{L}{a} - \frac{\rho}{\log \gamma} \ . \qquad (1.32)^{34}$$

Comparing (1.31) to (1.32), it is found out that for levels of the size of quality

[31] That is, in equilibrium the flow of consumption remains steady.

[32] This is the *scale effect* which can be found also in Romer (1990b) and Aghion and Howitt (1992).

[33] In fact, by increasing the expected flow of monopoly profits, firms have more incentives to engage in research activities.

[34] In order to find the optimal level of growth we maximize the expression for utility $U = \frac{\log E - \log \gamma + (\lambda/\rho) \log \gamma}{\rho}$, which is obtained by using (1.19), (1.28) and the fact that both C

improvements γ rather small or rather large, firms have too many incentives to do research, resulting in a growth rate exceeding the optimal one; however, for intermediate values of γ, the growth rate will result smaller than the optimal one.[35]

and λ are fixed in stationary state.

[35]This result in welfare analysis is quite different from that presented in Romer (1990b), since there the equilibrium growth rate must always fall short of the socially optimal one. We are going to explain this divergence in the next chapter, following the approach of Aghion and Howitt (1992).

Appendix 1 A

Reconciling the Quality Improvements and the Product Variety Approaches

As said above, the main difference between the economy modeled by Romer and that described by Grossman and Helpman is that in the former technological progress expands the variety of intermediate goods while in the latter it results in improvements in the quality of products. It will be proved [36] that the two approaches can be reconciled since in both of them the reduced form for the intensity of research is the same. However, it will be also shown that the two approaches present divergences as far as welfare analysis is concerned.

The preferences are the same as in (1.19); however, now the instantaneous utility (1.21) can be replaced by the production function:

$$y(t) = \left(\int_0^{A(t)} x_t(\omega)^{\alpha} d\omega \right)^{\frac{1}{\alpha}} , \tag{1.33}$$

where $x_t(\omega)$ is the amount of intermediate good of variety ω employed as input to production at time t [37] and the index $A(t)$ measures how many varieties are available to final output producer at time t. Then, y_t is final output produced at any point in time. A unit of any kind of intermediate good is assumed to be produced employing one unit of labour; hence, the marginal cost of the intermediate sector firms is the wage rate w and the mark-up pricing is used for any product:

$$p = \frac{1}{\alpha} w . \tag{1.34}$$

Then, combining this condition whit the static demand functions derived in note ([27]),the profit earned by the producer of each kind of intermediate good is:

$$\pi = (1 - \alpha) \frac{C}{A} .$$

In order to make innovations happen in the economy - that is, to expand the set of varieties of the intermediate producer good - firms must engage in research activities. A design for a new type of intermediate good requires a_A/A units of labour to be developed, where the number of varieties available at any point in time A can be regarded as the stock of technological knowledge which

[36]See Grossman and Helpman (1991), namely the fourth section "Quality versus Variety".
[37]We are using now the interpretation briefly laid out in note ([30]).

is available to any potential innovator.[38] The cost of development of a new variety is then:

$$c_A = \frac{wa_A}{A} \ .$$

Since each successful innovator becomes monopolist in the sub-market for the new kind of good discovered by himself for infinite time, the condition $\frac{\pi}{r} = c_n$ must hold. Taking the derivative with respect to time yields the following research arbitrage equation:

$$\frac{(1-\alpha)C}{wa_A} + \frac{\dot{w}}{w} - \frac{\dot{A}}{A} = r$$

which, normalizing the wage rate to be 1, using (1.22) and assuming the rate of increase in the amount of varieties to be equal to λ_A[39], can be rewritten as:

$$\frac{\dot{C}}{C} = \frac{(1-\alpha)C}{a_A} - \rho - \lambda_A \ . \tag{1.35}$$

Eventually, a stationary state is described by the labour market clearing equation

$$a_A\lambda_A + \alpha C = L \tag{1.36}$$

and by the condition obtained from (1.35) imposing stationarity in C:

$$\lambda_A = \frac{(1-\alpha)C}{a_A} - \rho \ . \tag{1.37}$$

Comparing these two conditions, i.e. (1.36) and (1.37) to the pair of conditions for the steady state in the Grossman-Helpman model (1.27) and (1.30), it can be noticed that the two reduced form systems are equivalent[40] and therefore the same goes for the whole comparative static analysis. Assuming that $A(0) = 1$ it can be derived the analogous for (1.28):

$$\log u(t) = \log C + \log \alpha + \lambda_A(\frac{1}{\alpha} - 1)t \ . \tag{1.38}$$

Taking the derivative with respect to time yields the rate of growth in the economy:

$$g = \lambda_A(\frac{1}{\alpha} - 1) \ .$$

[38] This is again the *spillover effect* detected by Romer (1990b) due to the fact that knowledge is a non-rival good.

[39] Which is nothing but the aggregate intensity of research.

[40] In the latter λ_A, a_A and α take the place of respectively λ, a and $\frac{1}{\gamma}$.

To compare the welfare properties of this second approach to those of the vertical innovation model, we substitute(1.38) in the utility function (1.19) to obtain the welfare function

$$U = \frac{1}{\rho}\left[\log C + \log \alpha + \frac{\lambda_A}{\rho}(\frac{1}{\alpha} - 1)\right]$$

whose *argmax* λ_A^* represents the socially optimal aggregate intensity of research:

$$\lambda_A^* = \frac{L}{a_A} - \rho\frac{\alpha}{1-\alpha} \ . \tag{1.39}$$

By substituting (1.36) in (1.37) for C, we can obtain the following expression for the equilibrium value of λ_A:

$$\lambda_A = (1-\alpha)\frac{L}{a_A} - \rho\alpha \ .$$

By comparing this equilibrium intensity of research to the socially optimal level, it can be noticed that whenever the equilibrium value of λ_A is positive, it always falls short of the optimal value, which diverges from the result obtained in the model with vertical innovations and corresponds to the welfare properties presented by Romer in his product variety model (1990b).

Chapter 2

The Schumpeterian Approach to Endogenous Growth

Introduction

In the first chapter we have surveyed the main contributions to growth theory: firstly, we have analyzed the standard neoclassical model developed by Solow (1956) and Swan (1956) and it has been shown that one of its main results is that long-run growth in output per capita is possible only if it is introduced in the economy technological progress. Originally, growth in productivity was introduced as exogenous in the models; however, this kind of approach seemed unsatisfactory to many economic theorists, who tried to endogenize the rate of technological change. Therefore, we have given a rapid account of the AK approach and, then, two models in which technological change springs out from individual decisions have been described, namely Romer (1990b) and Grossman and Helpman (1991). In this second chapter, we try to go deeper in the analysis of the Schumpeterian approach to endogenous growth. This field of endogenous growth theory goes by this name because it refers to the concept of *creative destruction* exposed by Schumpeter in his work "Capitalism, Socialism and Democracy"[1]. In fact, in this kind of models technological progress takes the form of a series of vertical innovations in intermediate good sectors: on the one hand, they enhance the efficiency in producing final output, having

[1]See Schumpeter (1942).

a positive effect on the economy as a whole, since they are the engine dragging long-run economic growth; on the other hand, they also make old types of intermediate goods obsolete, eventually driving them out of the market and destroying the flow of monopoly profits accruing to previous innovators. This is nothing but the *creative destruction* process theorized by Schumpeter, according to which the innovation process has two sides: one constructive and the other destructive.

The One-Sector Model

First of all, we analyze the model built by Aghion and Howitt [2] in which the economy is assumed to have only one intermediate sector. Final output is produced employing the available amount of the intermediate good which, in turn, is manufactured using labour as the only input. The technology of production of final output is made better off every time a new version of the intermediate good is discovered. Firms engage in research diverting a fraction of labour from production because the successful innovator is capable of driving out its competitors, becoming a monopolist. Hence, the stream of monopoly profits can be regarded as returns to research.

We can immediately notice two similarities between this model and the Romer's one presented in the first chapter:

- the intermediate sector is not perfectly competitive but is characterized by market power, since successful innovators are able to gain monopoly profits;
- we can detect a *spillover effect*, since research is carried out on the basis of the previous version of the intermediate good discovered by another firm which cannot exclude this positive externality.

Then, we solve the model for a stationary equilibrium with perfect foresight, where the fraction of workers devoted to research remains the same for each period, and we check how research and, thereby, the long-run growth rate are affected by the parameters in the model. We find out that they are decreased by an increment in the interest rate and increased by a raise in the size of the innovations and in the total stock of labour[3]. However, an increase in the rate at which new versions of the intermediate good are discovered has ambiguous effects: on the one hand, it raises the effectivity of researchers; on the other

[2]See Aghion and Howitt (1992).

[3]The latter effect is the usual scale effect.

hand, it increases also the rate of creative destruction, discouraging research. However, the first side of the causal relationship is found to be predominant.

After that, we move on to compute the balanced growth rate resulting in the economy and we compare it with the socially optimal growth rate, which would be chosen by a benevolent social planner. The result is that, unlike Romer's model but similarly to the Grossman and Helpman's one previously examined, here the rate of growth can be more or less than optimal, since, in addition to the *spillover* and the *appropriability effects* detected by Romer (1990b) which tend to make growth smaller, we have also to consider the textitbusiness-stealing and the *monopoly-distortion effects*, which, being not internalized by firms engaging in R&D, tend to make the rate of growth larger than the optimal one.

The Multi-Sectoral Framework

In the second section of this chapter, the basic model is extended to have many intermediate sectors, in order to stick more to the real word, where each consumer good is manufactured employing a large set of intermediate goods. The dynamics of the model are nearly the same as in the one-sector framework, however, another effect springs out from introducing a variety of intermediate goods: the *technology spillover*. If the spillover effect detected before works in the intertemporal dimension, the *technology spillover* works in the spatial one, since it captures the progressive diffusion of the cutting-edge technology throughout the economy, once it is discovered in one sector. In fact, once a new technology has been discovered, it becomes the basis on which firms in all the other sectors do their research activities.

In this extension, we notice also another facet of the *creative destruction* process: non-innovating sectors experience a decrease in employment and profits as the wage rate steadily grows due the continuous arrivals of innovations. Apart from this additional, negative effect on the long-run growth rate, all the previous results hold true also in the multi-sectoral framework.

Integrating Capital Accumulation and Technological Change as Drivers of Growth

After presenting a variant of the multi-sector framework where intermediate goods are the only input to research, we analyze an attempt to reconcile the neoclassical and the Schumpeterian approaches to growth theory: according to the former one, accumulation of capital is considered the main determinant of economic growth and therefore no long-run growth in output is allowed if it is not introduced an exogenous rate of evolution in the productivity parameter; according to the latter, it is emphasized the role of endogenous technological change, since it is the only force that eventually makes growth in final output possible in the long-run.

However, it is undoubtedly intuitive that both capital accumulation and technological progress play an essential role in determining the economic growth of a country, therefore in the third section it is showed a model in which this dichotomy is reconciled[4]. In this model, final output is produced employing labour and a continuous set of intermediate goods, according to a certain technology. Intermediate goods are manufactured using only capital as input and the more advanced they are, the more their production process becomes capital intensive. Firms carry out research activities by employing a fraction of final goods, therefore we have that output can be alternatively consumed, used in the R&D sector or saved in order to accumulate capital. If the model is solved for a balanced growth path, we notice that all the usual comparative statistic results holds true, but now also capital accumulation ends up to produce a positive effect on long-run growth. This finding contradicts one of the main results of the Solow-Swan model, namely that growth in output is not affected by the accumulation of capital in the long-run. Therefore, in this model it is proved that both capital accumulation and technological innovation are able to influence the dynamics of output in the long term, as suggested by the economic intuition.

2.1 The Basic One-Sector Framework

2.1.1 Initial Considerations

We start from a basic model where exists only one intermediate sector in which, as in Grossman and Helpman (1991), but unlike Romer (1990b), vertical innovations randomly take place, improving the quality of the unique intermediate good produced in the economy. Since innovations improve the quality of products, it is introduced in the model a factor of obsolescence, i.e. new, better products make old ones obsolete. In this factor of obsolescence is exemplified the Schumpeter's notion of *creative destruction*, according to which progress generates both gains and losses.

[4]See Aghion and Howitt (1998a).

Aghion and Howitt assume in this model that individual innovations affect the whole economy and that the length of each period between two successive innovations is stochastic. Although this randomness in the innovation process, the amount of research in a period is negatively related to the expected amount of research in the next period. This is due to two effects:

- **creative destruction**: the revenue to research in the current period is the stream of monopoly rents which accrues to the successful innovator along the next period; this stream will last until a new innovation is produced, when the immediately previous one which used to warrant the rent becomes obsolete. Hence, the expected present value of this stream depends negatively on the Poisson rate of arrival of a new innovation: more research expected in the next period will increase this rate and then discourage research in the current period;

- a **general equilibrium effect** spreading through the wage of skilled labour, which can be used either in research or in manufacturing sector: more expected research in the next period determines a higher expected demand for skilled labour and thereby a higher real wage. More expensive wages in the next period will reduce the monopoly rents paid to successful current research and this will eventually discourage research in the current period.

A feature of this model is that average growth in stationary equilibrium can be more or less than socially optimal because of two conflicting forces: on the one hand, as in the Romer's model previously analyzed, the *appropriability* and the *intertemporal spillover effects* cause a less than optimal growth rate; on the other hand, it can be identified a *business-stealing effect*[5], according to which researchers do not internalize the destruction of rents deriving from their innovations: this phenomenon leads to a more than optimal growth if the size of innovations is taken as given[6].

[5]This is the same effect found in the patent race literature, see Tirole (1988).

[6]We have noticed yet this difference in welfare analysis between the Romer's product variety model and the vertical innovation framework used by Grossman and Helpman (1991) and Aghion and Howitt (1992): in fact, in the former, the equilibrium growth rate always falls short of the socially optimal one, while in the latter it can be more or less than socially optimal.

2.1.2 Assumptions

The model takes in consideration two types of goods: intermediate and consumption goods. The individuals have additive preferences over lifetime consumption and the rate r of time preference is constant: hence, as in the Romer's model, r is also the interest rate.

Labour can be fractioned in three categories:

- **unskilled labour**, which can be used only in the production of consumption good;
- **specialized labour**, which can be employed only in research sector;
- **skilled labour**, which is an input both in the research and the intermediate good sectors.

Since the first two categories does not influence the following analysis, in order to keep the notation simple we normalize both unskilled and specialized labour to unity. The only category of workers whose allocation between manufacturing and research is endogenously determined and substantial to the model is skilled labour. Population growth is not considered in the model; hence, the stock of skilled labour is assumed to be fixed at the amount N. The consumption good is produced by using the intermediate good; hence, the production function for the final good sector can be formalized as:

$$Y = AF(x) \ , \tag{2.1}$$

where A represents the total factor productivity and x the quantity of intermediate good employed in the production.

The intermediate good production function has only skilled labour as one for one input:

$$x = L \ , \tag{2.2}$$

where L is the fraction of skilled labour N devoted to production in the intermediate sector.

Finally, in the research sector a new innovation is discovered at a Poisson arrival rate

$$\lambda\varphi(n) \ , \tag{2.3}$$

where n is the amount of skilled labour devoted to research and both λ and φ are determined by the technology of research; namely, λ is a constant parameter and φ is a constant-returns, concave function. Unlike the Romer's model, here the arrival rate of innovation does not depend on past knowledge, but only on the current flow of inputs to research.

Time τ is continuous and the index $t = 0, 1, 2, \ldots$ represents the period starting with the tth innovation and lasting until the next one is dicovered. The duration of each interval is a random variable distributed as an exponential with parameter $\lambda\varphi(n_t)$.

The innovation consists of the invention of a new intermediate good which allows for a more efficient production of the final good; namely, each innovation increases the total factor productivity A by the factor $\gamma > 1$ [7]; therefore the technology index at time t is $A_t = A_0\gamma^t$.

In this model research and final good sectors are perfectly competitive, but this is not the case for the intermediate sector, since a successful innovator earns a patent (which is assumed to last potentially perpetually) and thereby can monopolize the market until the arrival of a new innovation.

2.1.3 Description of the Model

First of all, Aghion and Howitt assume that innovations are drastic, that is, there is no competition from previous innovators, since a new innovator can always drive the previous one out of the market. The intermediate monopolist wants to maximize the present value of the stream of profits over the current interval, whose length is uncertain, taking the aggregate amount of research in each period as exogenously determined.

Let x_t be the amount of intermediate good produced by the monopolist over the period t, which is also equal to the amount of skilled labour employed at that time in that sector. Since the final good market is competitive, the inverse demand function facing the monopolist is the marginal product of a final good producer:

$$p_t = A_t F'(x_t) \tag{2.4}$$

and, then, he chooses to maximize the function

$$\pi = (A_t F'(x_t) - w_t)x_t \ ,$$

where w_t is the wage of skilled labour at time t.

[7]For now, we are not considering lags in the diffusion of the cutting-edge technology. This lag between the discovery of a new technology and its actual application in the final output sector will be central in our discussion of the effects produced by the arrival of a new *general purpose technology* on the business cycle.

Now let $\omega_t = \frac{w_t}{A_t}$ be the "productivity-adjusted wage" and let the marginal revenue function $\tilde{\omega}(x) = F'(x) + xF''(x)$ follow the conditions:

$$\tilde{\omega}'(x) < 0 \ , \forall x > 0 \ ; \quad \lim_{x \to 0} \tilde{\omega}(x) = \infty \ ; \quad \lim_{x \to \infty} \tilde{\omega}(x) = 0 \ .$$

Then, the output chosen by the monopolist is given by the first-order condition

$$\omega_t = \tilde{\omega}(x_t) \ , \tag{2.5}$$

which can be also expressed as

$$x_t = \tilde{x}(\omega_t) \ , \tag{2.6}$$

where $\tilde{x}$ is the inverse function $\tilde{\omega}^{-1}$.

Finally, the profits over period t are defined by:

$$\pi_t = A_t \tilde{\pi}(\omega_t) \ , \tag{2.7}$$

where $\tilde{\pi}(\omega) = -[\tilde{x}(\omega)]^2 \cdot F''[\tilde{x}(\omega)]$.

Then, Aghion and Howitt analyze the research sector: there is no contemporaneous spillover and then the objective of the firm is to maximize the expected profits, which are defined as

$$\lambda\varphi(n_t^i)V_{t+1} - w_t n_t^i \ ,$$

where n_t^i is the fraction of skilled labour employed by the firm i at time t and V_{t+1} the present value of the $t+1st$ innovation. Using the Kuhn-Tucker conditions for maximizing it is obtained

$$w_t \geq \varphi'(n_t)\lambda V_{t+1} \ , \ n_t \geq 0 \ , \quad \text{with at least one equality,} \tag{2.8}$$

where $\varphi'(n_t)$ is the derivative with respect to n_t of the function $\varphi(n_t)$ and n_t is the aggregate amount of skilled labour devoted to research in the whole economy at time t.

As for an outside research firm, the value V_{t+1} is the expected present value of the stream of monopoly profits π_{t+1} generated by the $t+1st$ innovation over a period whose length is a random variable distributed as an exponential with parameter $\lambda\varphi(n_{t+1})$ and is defined by the following expression:

$$V_{t+1} = \frac{\pi_{t+1}}{r + \lambda\varphi(n_{t+1})} \ .$$

In this model the current monopolist at time t chooses not to do research because his value of discovering the next innovation would be $V_{t+1} - V_t$, which is less than the value V_{t+1} to an outside firm[8].

In the model it can be detected an *intertemporal spillover*, which is the same spillover identified by Romer in the previous model: each subsequent innovation increases the total factor productivity by the same factor γ and with the same probability, but starting from a value higher by γ than the previous innovation. Then, each innovator captures the rents generated by his innovation only during one interval; after that - i.e. after new innovations are discovered - these rents are beneficial to successive innovators, which do their research on the basis of the present innovation without compensating the current innovator[9].

It can also be noticed that the concept of creative destruction enters the model through the rate $\lambda\varphi(n_{t+1})$; that is, each new innovation determines profits for the new innovator, but also destroys the monopoly rents generated by the previous innovation.

2.1.4 Solving the Model for a Balanced Growth Path

The only decision to be made at any point in time in this model is how to allocate the stock of skilled labour N between the intermediate good sector and research. Combining the previous maximizing conditions (2.5), (2.7) and (2.8), the expression for V_{t+1} and the equilibrium condition for the skilled labour market, $N = n_t + x_t$, we can obtain

$$\frac{\tilde{\omega}(N - n_t)}{\lambda\varphi'(n_t)} \geq \frac{\gamma\tilde{\pi}\{\tilde{\omega}(N - n_{t+1})\}}{r + \lambda\varphi(n_{t+1})} \ , \ n_t \geq 0 \ , \text{ with at least one equality.} \qquad (2.9)$$

This condition implies that the amount of skilled labour devoted to research during period t depends on the amount devoted to research at $t+1$, as described

[8] At furthest, if we introduce more than one sector in the model, the current monopolist is willing to engage into research activities in other sectors where he is not the incumbent.

[9] In this respect can be recognized the nature of technology knowledge as a non-rivalrous, only partially excludable good.

by the following expression:

$$n_t = \psi(n_{t+1}) \ , \tag{2.10}$$

where $\psi : [0, N) \to \mathbb{R}_+$ is a strictly decreasing function. This negative relationship is due to two reasons: on the one hand, an expected increase in research next period is likely to raise skilled labour wages and thereby to reduce the rents from the next innovation; on the other hand, more research in the next period would increase the rate $\lambda\varphi(n_{t+1})$ and hence shorten the period in which the next innovator would be monopolist.

We can now define $c(n_t)$ as the marginal cost of research and $b(n_{t+1})$ as the marginal benefit of research; they are defined by the following two equations:

$$c(n_t) = \frac{\tilde{\omega}(N - n_t)}{\lambda\varphi'(n_t)} \ , \tag{2.11}$$

which is strictly increasing in n_t, and

$$b(n_{t+1}) = \frac{\gamma\tilde{\pi}\{\tilde{\omega}(N - n_{t+1})\}}{r + \lambda\varphi(n_{t+1})} \ , \tag{2.12}$$

which is strictly decreasing in n_{t+1}. The negative relationship of current research on future research is evident looking at the expression for the marginal benefit of research: an expected increase in the fraction n_{t+1} of skilled labour devoted to R&D in the next period would both decrease the numerator of equation (2.12) by raising the productivity-adjusted wage[10] and augment its denominator by raising the rate of arrival $\lambda\varphi(n_{t+1})$.

An equilibrium with *perfect foresight* is defined as a sequence $\{n_t\}_0^\infty$ which satisfies equation (2.10) for all $t \geq 0$. A stationary equilibrium with perfect foresight is a sequence having n_t constant for all t and it can be found as a solution to $\hat{n} = \psi(\hat{n})$. If $c(0) < b(0)$, then there exists a unique perfect foresight stationary equilibrium with $\hat{n}$ positive, which is defined by:

$$\frac{\tilde{\omega}(N - \hat{n})}{\lambda\varphi'(\hat{n})} = \frac{\gamma\tilde{\pi}\{\tilde{\omega}(N - \hat{n})\}}{r + \lambda\varphi(\hat{n})} \ . \tag{2.13}$$

Growth is positive because innovations occur at a Poisson rate $\lambda\varphi(\hat{n}) > 0$. However, if $c(0) \geq b(0)$, then $\hat{n} = 0$, since it would be disadvantageous to start at all research activities. Since in this second situation there would be no growth because of $\lambda\varphi(0) = 0$, we assume that it is always convenient to devote

[10]Notice that $\frac{d\tilde{\omega}}{dn_{t+1}} > 0$ and that $\frac{d\tilde{\pi}}{d\tilde{\omega}} < 0$.

at least the first unit of skilled labour to research activities, that is $b(0) > c(0)$.

Analyzing the condition for stationary equilibrium (2.13), it can be noticed that the amount $\hat{n}$ of skilled labour devoted to research increases for the following reasons:

1. **a decrease in the interest rate r**, since it increases the present value of the monopoly profits;
2. **an increase in the factor γ** by which each innovation raises the total factor productivity, since it raises the monopoly profits;
3. **an increase in the total stock of skilled labour**, since it reduces the wage and thereby increases the flow of profits[11];
4. **an increase in the arrival rate of new innovations**: it has two opposite effects: on the one hand, it will reduce the marginal cost of research because of more effective researchers, on the other hand, it expands also the *creative destruction* rate in the next period. However, the former effect is always bigger than the second.

As previously done, now the model is solved for a *balanced growth path*. We know that during period t real output is given by

$$Y_t = A_t F(N - \hat{n}) \ .$$

This implies that $Y_{t+1} = \gamma Y_t$. Hence, the time path for the log of real output $\ln Y(\tau)$ is a random step function with $\ln Y_0 = \ln F(N - \hat{n}) + \ln A_0$ as starting point, the size of each step being equal to $\ln \gamma > 0$ and the time interval between each step being defined as a sequence of iid random variables distributed as exponentials with parameter $\lambda\varphi(\hat{n})$. As in the Romer's model, this specification implies that a discrete sequence of observations on the log of the output is a random walk with constant positive drift. Then, it follows that the average growth rate (AGR) of the economy is given by:

$$AGR = \lambda\varphi(\hat{n}) \ln \gamma \ . \qquad (2.14)$$

In conclusion, we can register how the average growth rate is affected by the parameters considered in the model by combining (2.14) and the comparative static analysis on condition (2.13) realized above:

[11] This is the familiar *scale effect* already detected in the two Schumpeterian models presented in the first chapter: Romer (1990b) and Grossman and Helpman (1991).

- AGR is raised by an increase in the arrival rate of new innovations and in their size and by an increase in the stock of skilled labour and in the degree of market power;
- AGR is lowered by an increase in the interest rate.

2.1.5 Results in Welfare Analysis

Now, as usual, we are going to analyze the welfare properties of the stationary equilibrium found above; namely, it is interesting to compare the average growth rate along the balanced growth path with the socially optimal one. In order to find this optimal growth rate, the following expression for the expected utility is supposed to be maximized:

$$U = \int_0^\infty e^{-r\tau} \sum_{t=0}^{\infty} \Pi(t,\tau) A_t F(N-n) d\tau \ ,$$

where, $\Pi(t,\tau)$ is the probability that t innovations have occurred up to time τ. This probability is described by the expression:

$$\Pi(t,\tau) = \frac{[\lambda\varphi(n)\tau]^t e^{-\lambda\varphi(n)\tau}}{t!} \ .$$

Substituting this probability into the expression for expected welfare yields:

$$U = \frac{A_0 F(N-n)}{r - \lambda\varphi(n)(\gamma-1)} \ . \tag{2.15}$$

Here the expected welfare is defined as the present value of a perpetual, where the first flow of output at time 0 $A_0F(N-n)$ is discounted at the *social discount rate* $r - \lambda\varphi(n)(\gamma-1)$, which is less than the private interest rate $r + \lambda\varphi(\hat{n})$ because of the growth of output over time[12]. By maximizing (2.15), we find the condition for the the optimal level of skilled labour devoted to research n^*:

$$\frac{F'(N-n^*)}{\lambda\varphi'(n^*)} = \frac{(\gamma-1)F(N-n^*)}{r - \lambda\varphi(n^*)(\gamma-1)} \ . \tag{2.16}$$

Once the optimal level of research n^* is found, it also defines the socially optimal average growth rate:

$$AGR^* = \lambda\varphi(n^*) ln\gamma \ .$$

[12]The general formula for the discounting of a perpetual rent is $\frac{F_0}{i-g}$, where F_0 is the first cash flow, i is the interest rate and g the rate of growth in the cash-flows.

We can easily notice that:

- $AGR > AGR^*$ iff $\hat{n} > n^*$;

- $AGR < AGR^*$ iff $\hat{n} < n^*$.

It can be checked which direction of the inequality is true by comparing the condition for balanced growth equilibrium under *laissez-faire* (2.13) with the condition a social planer would be supposed to fulfill in order to maximize social welfare (2.16). As in Grossman and Helpman (1991), but unlike the product-variety model by Romer (1990b)[13], here the average growth rate may be more or less than optimal. This is due to two, contrasting pairs of forces working at the same time in the economy:

- on the one hand, the **spillover effect**[14] and the **appropriability effect**[15] detected also by Romer, which make the AGR less than optimal;

- on the other hand, the **business-stealing effect**[16], which is caused by the researchers not internalizing the destruction of rents payed to the immediately preceding innovator, and the **monopoly-distortion effect**[17], which both induce the AGR to be greater than the optimal one.

Since these two pairs of forces operate in opposite directions, we cannot say a priori if the *laissez-faire* AGR would result more or less than socially optimal.

[13]In Romer (1990b) the growth rate always falls short of the optimal one.

[14]This effect is captured by the *social discount rate* $r - \lambda\varphi(n^*)(\gamma - 1)$ replacing in (2.16) the private one $r + \lambda\varphi(\hat{n})$. The former is less than the latter, since in the social planning problem it is considered the positive externality caused by an innovation on the infinite number of following ones.

[15]This is embodied in the flow of profits $\tilde{\pi}\{\tilde{\omega}(N - \hat{n})\}$ being replaced in (2.16) by total output $F(N - n^*)$.

[16]This effect is captured by the replacement in the right-hand side numerator of the factor γ in (2.13) by the factor $(\gamma - 1)$ in (2.16).

[17]This is represented by the substitution in the left-hand side numerator of marginal product $F'(N - n^*)$ in (2.16) for the productivity-adjusted wage $\tilde{\omega}(N - \hat{n})$ in (2.13) and it is due to the fact that the wage at which skilled labour is paid in the *laissez-faire* case is less than its marginal product, since it is also employed in an intermediate sector dominated by a monopolist.

2.2 The Multi-Sectoral Extension

2.2.1 A Multi-Sector Model with Skilled Labour as Input to Research

Up to now we have assumed that only one intermediate good is needed in the production process of final output; however, usually firms in order to produce final goods employ many, different typologies of intermediate goods[18]. In this extension to the model presented in the previous section, we are going to introduce many sectors for different intermediate goods; this plurality gives birth to a new effect: a **technology spillover**, which is the progressive diffusion throughout all the sectors of a new technology discovered in one of them.

There is still only one consumption good, but now it is produced by employing a continuous set of different intermediate goods, indexed on the unit interval:

$$Y_t = \int_0^1 A_{it} F(x_{it}) di \ , \tag{2.17}$$

where A_{it} is the productivity of the tth generation of good i and each intermediate good x_i is produced one for one using labour L. As before, each sector i is monopolized by the last successful innovator, who has in front of him the marginal product of the final output producer as his inverse demand function:

$$p_{it} = A_{it} F'(x_{it}) \ .$$

Hence, as before, the intermediate good monopolist's output will be:

$$x_{it} = \tilde{x}(\omega_{it}) \ , \tag{2.18}$$

where ω_{it} is the productivity-adjusted wage in sector i $\frac{w_t}{A_{it}}$, and the flow of profits is defined by the following expression:

$$\pi_{it} = A_{it} \tilde{\pi}(\omega_{it}) \ . \tag{2.19}$$

In the model, then, it is assumed a different research sector for each intermediate good. In each sector, firms engage in research activities in order to discover a new technology and monopolize that sector. Therefore, we have as many Poisson arrival rates $\lambda\varphi(n_{it})$, with n_{it} being the fraction of skilled labour devoted to research in sector i, as many are the sectors. We define A_t^{max} as

[18] In the description of this extension we are going to follow Aghion and Howitt (1998b).

the productivity of the leading-edge technology across all the sectors; since innovations in each sector contributes to augment this parameter, and since, being the expected profits the same in whatsoever sector, the amount of labour devoted to research is the same quantity n_t in each one of them, the motion law of this cutting-edge parameter can be written as:

$$\dot{A}_t^{max} = A_t^{max} \lambda \varphi(n_t) \ln \gamma \ . \tag{2.20}$$

When a firm discovers a new innovation, he can start to produce the intermediate good using the cutting-edge technology and driving out of the market the incumbent. In this way the productivity parameter in that sector will immediately switch from A_{it} to A_t^{max}. In this feature the **technology spillover** can be identified: each innovation is implemented only in the sector where it came up, but by contributing to the increase in A_t^{max}, it allows the next innovator in another sector to find a more productive technique. At each point in time, there is a distribution of the technology parameters across all the sectors, with values extending from 0 to A_t^{max}. In the long-run, the distribution of the relative productivity parameters, defined as $a_{it} = \frac{A_{it}}{A_t^{max}}$, is described by the function:

$$H(a) = a^{\frac{1}{\ln \gamma}} \ , \quad 0 \leq a \leq 1 \ .$$

However, the distribution of the absolute productivity parameters is continuously shifted to the right due to technological progress at the rate given by (2.20).

Since in the economy innovations come up continuously, the wage rate will increase over time, causing a reallocation between sectors and a decline of the profits in the non-innovating ones, as can be deduced by equation (2.18) and (2.19). This decline is another facet of the *creative destruction* process, called by Aghion and Howitt (1992) *crowding out.*

We define the economy-wide productivity-adjusted wage rate as: $\omega_t = \frac{w_t}{A_t^{max}}$ and from now on we index a sector by its relative productivity index a; in this way we can rewrite (2.18) as:

$$x_{it} = \tilde{x}(\frac{\omega_t}{a}) \ .$$

If $h(a) = H'(a)$ is the density of sectors, the labour market clearing condition can be written as:

$$n_t + \int_0^1 \tilde{x}(\frac{\omega_t}{a})h(a)da = N \ , \tag{2.21}$$

which provides the positive relationship between n_t and ω_t

$$\omega_t = \hat{\omega}(N - n_t)$$

due to the fact that as the fraction of labour devoted to research increases, its scarcity makes the wage increase too.

We can now rewrite final output as:

$$Y_t = A_t^{max} \int_0^1 aF[\tilde{x}(\frac{\omega_t}{a})]h(a)da \ .$$

It is easy to notice that the only force drawing growth in final output is the increase in the cutting-edge productivity parameter described by the law of motion (2.20); therefore, this rate of *technological spillover* is also the economic growth rate:

$$g_t = \lambda\varphi(n_t) \ln \gamma \ , \tag{2.22}$$

which is identical to the expression for the AGR (2.14) in the previous section and which holds at any point in time, also if the level of research is non-stationary.

As in the basic one-sector model, we focus our analysis to stationary equilibria, that is equilibria in which the amount of skilled labour devoted to research remains fixed for all t:

$$n_t = \hat{n} \qquad \forall t \ .$$

Since the allocation of skilled labour remains fixed, also the productivity-adjusted wage rate is constant at level ω and the growth rate keeps steady at level $g = \lambda\varphi(\hat{n}) \ln \gamma$.

We now turn our attention to the research sector: the value of an innovation at time t is the discounted flow of profits that the successful innovator receives during period s until a new technology is discovered. Since the wage rate w_t grows over time at rate g and the probability that in a sector a new innovation comes up, drawing out of the market the firm which innovated at time t, is $e^{\lambda\varphi(\hat{n})s}$, this present value can be written as:

$$V_t = A_t^{max} \int_0^{\infty} e^{-(r+\lambda\varphi(\hat{n}))s} \tilde{\pi}(\omega e^{gs})ds \ .$$

At this point, we know that the wage rate w_t must equal the expected marginal product of research; by dividing both sides of this equality by A_t^{max}, it is obtained the following research arbitrage condition:

$$\omega = \lambda \int_0^\infty e^{-(r+\lambda\varphi(\hat{n}))s} \tilde{\pi}(\omega e^{(\lambda\varphi(\hat{n}) \ln\gamma)s}) ds \ . \tag{2.23}$$

We can notice that the labour market clearing equation (2.21)[19] is an upward sloping function and that the research arbitrage equation (2.23) is a downward sloping function of the fraction of skilled labour devoted to research : therefore, also in this extension a unique stationary equilibrium level of research exists, which is found as the solution to the following condition:

$$\frac{\hat{\omega}(N-\hat{n})}{s\lambda\varphi'(\hat{n})} = \int_0^\infty e^{-(r+\lambda\varphi(\hat{n}))s} \tilde{\pi}(\omega e^{(\lambda\varphi(\hat{n}) \ln\gamma)s}) ds \ , \tag{2.24}$$

which is similar to the condition for the standard model (2.13). All the comparative statics analyzed in the previous section apply also in this multi-sectoral extension, but now we have a new effect: the mentioned **crowding out**, which is embodied in the exponential rate $e^{(\lambda\varphi(\hat{n}) \ln\gamma)s}$ at which wage increases over time. This effect reduces the flow of profits and, therefore, the present value of a new innovation, emphasizing the *creative destruction* effects of technological progress, which tends to destroy the rents of current monopolists who were successful innovators in the past.

2.2.2 A Variant of the Multi-sector Model: Intermediate Goods as Input to Research

Up to know, we have considered only skilled labour as input into the research sector; however, in this variant of the multi-sectoral model only intermediate goods are considered as input to research, according to the same technology as that used in the final output sector. Therefore, equation (2.17) can be rewritten as:

$$Y_t = C_t + Z_t = \int_0^1 A_{it} F(x_{it}) di \ ,$$

where Z_t is the fraction of final output saved to be devoted to research activities. Since it is likely that as technology becomes more and more complex, it is required a greater flow of inputs to research to keep technological progress steady, we define $z_t = \frac{Z_t}{A_t^{max}}$ as the productivity-adjusted level of research. If

[19] Re-expressed in the stationary form using the condition $n_t = \hat{n} \quad \forall t$.

the arrival rate of innovations is always defined by the Poisson rate $\lambda\varphi(z_t)$, the rate of technological growth remains the same as (2.20):

$$\frac{\dot{A}_t^{max}}{A_t^{max}} = \lambda\varphi(z_t)\ln\gamma \ .$$

Let we focus our attention only to stationary equilibrium, where both consumption C_t and the fraction of output devoted to research Z_t grow at the same rate as the technology parameter A_t^{max}: here we have that the productivity-adjusted level of research keeps steady over time at level $\hat{z}$ and the rest of the analysis remains pretty much the same as in the previous subsection. Now, since no skilled labour is employed in the research sector, the labour market clearing condition becomes:

$$\int_0^1 \tilde{x}(\frac{\omega_t}{a})h(a)da = N \ ,$$

and since now the marginal cost of research is no more w_t but A_t^{max}, the new research arbitrage equation takes the form:

$$1 = \lambda \int_0^\infty e^{-(r+\lambda\varphi(\hat{z}))s}\tilde{\pi}(\omega e^{(\lambda\varphi(\hat{z})\ln\gamma)s})ds \ .$$

It is straightforward to see that these two conditions are pretty much the same as (2.21) and (2.23); hence, assuming labour or intermediate goods as input to the research process approximately leads to identical results.

2.3 An Integrated Model of Growth through Innovation and Capital Accumulation

From our discussion in the first chapter and in the first two sections of the second one it can be inferred that the literature on growth theory is to be dichotomized in two main approaches: on the one hand, the neoclassical approach, in which accumulation of physical capital is emphasized as the main determinant of growth[20]; on the other hand, the Schumpeterian approach, which stresses a technological innovation process endogenously explained by the model as the main source of growth in output. Both these approaches

[20] And this would eventually lead to zero growth in the long-run due to the effects of diminishing returns in the production function, unless exogenous growth in technology is introduced.

tend to restrict the view to only one of these two fundamental features of an economy. However, it cannot be denied that both capital accumulation and technological change play an essential role in determining output growth in the long-run. In this third section, following Aghion and Howitt (1998a), we try to reconcile this dichotomy in one, comprehensive model in which innovation and capital accumulation are considered as two sides of the same process.

The model is as usual composed of three sectors: the final output, the intermediate and the research and development sectors. At any point in time, final output Y_t can be devoted to consumption C_t, to the production of physical capital K_t and to the R&D sector as input to research (Z_t). The technology of production of output is described by a concave function increasing in labour N, which is assumed to be constant over time, and in a flow of intermediate goods x_i which are as usual indexed on the unit interval:

$$Y_t = C_t + K_t + Z_t = \int_0^1 A_{it} F(x_{it}, N) di \ . \tag{2.25}$$

Each kind of intermediate good is produced by using capital as the only input:

$$x_{it} = \frac{K_t}{A_{it}} \ , \tag{2.26}$$

where A_{it} appears in the denominator to state that the more an intermediate good is technologically advanced, the more its production is capital intensive. As usual research is aimed at one specific type of intermediate good and the successful innovator becomes monopolist in that market until he is replaced by a new, successful innovator. Each monopolist faces the inverse demand function derived by the marginal product in final output production of the intermediate good:

$$p_{it} = A_{it} \frac{\partial F(x_{it}, N)}{\partial x_{it}}$$

and produces having a cost function defined as $\zeta_t K_{it} = \zeta_t A_{it} x_{it}$, where ζ_t is the cost of capital and is the summation of interest rate r_t, depreciation rate δ and the subsidy rate provided by government to stimulate firms to hold capital β_k:

$$\zeta_t = r_t + \delta - \beta_k \ .$$

Since marginal revenue and marginal cost functions differ between intermediate monopolists only in the parameter A_{it}, they all produce the same quantity x_t.

Substituting this quantity in (2.26) yields:

$$x_{it} = x_t = k_t N \tag{2.27}$$

where k_t is capital per worker in efficiency units $A_t N$ and A_t is an average of the productivity parameters distributed across the sectors. By substituting (2.27) in (2.25), it is obtained the familiar production function in the intensive form:

$$\frac{Y_t}{A_t N} = y_t = F(k_t, 1) = f(k_t) \ , \quad \text{with } f' > 0 \ , \ f'' < 0 \ .$$

Using the previous results, the equilibrium condition for the interest rate is defined by:

$$r_t = R(k_t) - \delta + \beta_k \ , \tag{2.28}$$

where $R(k_t)$ is the marginal revenue function, which is strictly decreasing in k_t.[21]. The flow of profits to monopolist in each sector is proportional to its productivity parameter and is increasing in the intensity of capital k_t[22]:

$$\pi_{it} = A_{it}\tilde{\pi}_t(k_t)N \ , \quad \text{with} \quad \frac{d\tilde{\pi}_t}{dk_t} > 0 \ .$$

The research sector works exactly as in the variant of the multi-sector model presented in subsection 2.2.2 and the arrival parameter of new innovations in each sector is always $\lambda\varphi(z_t)$. Furthermore, now we assume that research activities can be subsidized by the government at rate β_z. As usual we can write the research arbitrage condition as:

$$1 - \beta_z = \lambda \int_0^\infty e^{-(r_s + \lambda\varphi(z_s))s} \tilde{\pi}(k_s) N ds \ , \tag{2.29}$$

which can be rewritten as:

$$1 - \beta_z = \lambda \frac{\pi(k_t)N}{r_t + \lambda\varphi(z_t)} \ .$$

By plugging (2.28) in the arbitrage condition and by solving for the level of research it can be shown that z_t depends positively on capital intensity k_t:

$$z_t = \tilde{z}_t(k_t) \ , \quad \text{with} \quad \frac{d\tilde{z}_t}{dk_t} > 0 \ . \tag{2.30}$$

[21]It can be defined as: $R(k_t) = \frac{df(k_t)}{dk_t} + k_t \frac{d^2 f(k_t)}{dk_t^2}$.

[22]This feature embodies the *scale effect*: the more is the amount of capital per efficiency worker, the more is the flow of profits accruing to the monopolist and ,thereby, the more are the incentives to engage in research activities.

In this relationship we can recognize the positive effect of capital accumulation on growth in the long-run, since more capital per person implies a greater level of research activities.

As shown in the previous section, the cutting-edge technology parameter A_t^{max} grows at rate:

$$g_t = \frac{\dot{A}_t^{max}}{A_t^{max}} = \lambda\varphi(z_t)\ln\gamma \ .$$

Since the ratio A_t^{max}/A_t converges to $1 + \ln\gamma$, the law of motion of capital pro capite is described by the following differential equation:

$$\dot{k}_t = f(k_t) - c_t - z_t\frac{(1+\ln\gamma)}{N} - (\delta + g_t)k_t \ . \tag{2.31}$$

As far as consumer preferences are concerned, we assume as before the rate of time preferences and the intertemporal elasticity of substitution to be constant. Hence, we can find the Euler equation:

$$\dot{c}_t = c_t\left\{\frac{r_t - \rho}{\theta} - g_t\right\} \ . \tag{2.32}$$

Equations (2.31) and (2.32) are the same conditions for optimal growth as in the Cass-Koopmans-Ramsey model, but now the rate of technological change is no more taken as given but endogenously determined by the level of research z_t.

It is possible now to solve the model for a stationary equilibrium with balanced growth, where the rate of growth in final output per person is equal to the rate of change in the technology parameter $g = \lambda\varphi(\hat{z})\ln\gamma$, which is found as the solution to the system composed of the capital equation:

$$R(k) = \rho + \theta\lambda\varphi(\hat{z})\ln\gamma - \beta_k + \delta$$

and the research arbitrage equation:

$$1 - \beta_z = \lambda\frac{\pi(k)N}{\rho + (\theta\ln\gamma + 1)\lambda\varphi(\hat{z})} \ .$$

Performing comparative static analysis, all the usual observations hold, but now it is obtained a new, rather interesting result: a subsidy to capital will end up to raise permanently the growth rate in the long-run. This finding contradicts the conventional proposition that capital accumulation has no effects on long-run growth, as shown in the Solow-Swan model and in Romer (1990b) and Grossman and Helpman (1991) and it is due to the fact that capital here is identified to be an input also to the R&D process. This result is consistent with the empirical studies[23], which show that capital accumulation is as substantial to long-run growth as technological change, and with the intuitive sentiment that if technology and capital are the two main determinants of output, both of them should play a fundamental role in determining long-run economic growth.

[23] See for example De Long and Summers (1991) and Mankiw et al. (1992).

Chapter 3

Productivity Growth and Business Cycles

Introduction

That growth in productivity and short-run fluctuations are phenomena closely connected is a proposition largely supported by economists since the seminal work by Schumpeter (1934); however, growth and business cycle theory have had an independent development throughout the years. With the rise of the *real business cycle theory*, in which technology shocks are the principal causes of short-run economic fluctuations, the Schumpeterian formulation according to which productivity growth and cycles are two sides of the same phenomenon emerged again. The causal relation between productivity growth and cycles can be regarded as reciprocal:

- first of all, we focus on the direction of the causal relation from business cycles to growth: we analyze which are the main mechanisms through which economic fluctuations influence the long-run growth rate and we discuss if growth can be considered procyclical or countercyclical;

- then, we focus our attention on the opposite side of the direction, that is from productivity growth to business cycles: here we see that an increase in the amount of research in order to achieve a higher productivity growth rate, by draining resources away from final output production, would result in a short-run economic downturn, as shown by the literature on the *general purpose technologies* (GPTs)[1].

From Fluctuations to Growth

As written before, in the former section of this chapter we discuss the effects that short-run fluctuations may have on productivity, and, hence, economic growth. The main question we try to give an answer to is the following one:

> *"Does economic growth show a procyclical or a countercyclical behaviour?"*

Unfortunately, we are not able to find out an unambiguous answer to this question, since fluctuations have contrasting effects on technological change. First of all, we identify two mechanisms through which economic downturns affect negatively productivity growth:

- **financial constraints**[2]: firms, when engage in research activities, are constrained by the availability of financial resources: since the fall in output determined by a slump worsens their net cash flow, firms should obtain more borrowings in order to keep stable the intensity of research; however, due to information asymmetry in the markets for capitals, the phenomenon of credit rationing may take place, that is some firms are not able to obtain more loans from banks. If these firms are not big enough to obtain financial resources in other ways (e.g. in the equity market), they are forced to reduce expenditures on R&D causing a decrease in technological change. The whole mechanism is also exacerbated by the fact that during recessions, the scope of credit rationing is likely to be larger, since financial institutions are willing to reallocate their portfolio from loans, which have become riskier, to safe assets;

- **hindering in learning by doing**[3]: while employees are carrying out their tasks, they learn increasingly how to do them in the best way possible and acquire skills; in this way the production process becomes more and more efficient and the economy experiments a positive growth in productivity. Since an economic slump causes a fall in occupation, the effects of learning by doing are less conspicuous and therefore technological change is forced to experiment a slowdown.

[1] See Helpman and Trajtenberg (1998a,b), Howitt (1998) and Aghion and Howitt (1998c).
[2] See Stiglitz (1993).
[3] See Stadler (1990).

However, we should also recall the lesson by Schumpeter[4] that during recessions the competitive struggle between firms is fiercer, hence less efficient firms are driven out of the market and replaced by more efficient ones. In this way, after a recession the productivity should result to be higher than before. This is only one of the arguments that can be employed to support countercyclicality of growth, therefore in the second part of the first section we describe a model[5] which analyzes the main features of the *opportunity cost approach*. According to it, since in order to engage in research activities firms are supposed to give up a certain amount of production and during recessions the returns to output are lower, the opportunity cost of carrying out R&D and reorganizational activities is lower and, therefore, firms are stimulated to undertake productivity-enhancing activities, causing a higher rate of growth. Conversely, during economic expansions the opportunity cost of research would rise because of the high returns to production and the firms are likely to reduce the efficiency-enhancing activities, determining a fall in the rate of technical change. According to the *opportunity cost approach*, hence, growth in productivity should prove to have a countercyclical behaviour.

To sum up, business cycle fluctuations produce ambiguous effects on productivity and economic growth: whether it is to be considered procyclical or countercyclical depends on which one of the contrasting effects eventually prevails on the other.

From Growth to Fluctuations

In the latter section in this chapter we analyze the other direction of the causal relationship, that is if positive shifts in the amount of resources devoted to research aiming at speeding up the evolution of the technological parameter and, thereby, of the long-run economic growth rate are likely to produce short-run fluctuations in output. In order to assess this issues we introduce the notion of *general purpose technology* (GPT)[6]. A general purpose technology is a major technological innovation which affects almost all the sectors of an economy and which requires a number of complementary innovations in order to be implemented successfully.

General purpose technologies are introduced in the basic Schumpeterian model of endogenous growth presented in chapter two by allowing for a two-

[4]See Schumpeter (1934).
[5]Namely, that developed by Aghion and Saint-Paul (1998).
[6]See Bresnahan and Trajtenberg (1995).

steps innovation process[7]: firstly, a new GPT should come up in the economy, then firms engages in research in order to discover the new vintage of the intermediate good in order to implement it. Solving the model, we can observe that now the economy is characterized by a series of two-phases cycles: during phase 1, which starts with the arrival of a new GPT, the economy experiments a downturn, since a large fraction of labour force is diverted from productive activities to research activities; in phase 2, which begins as soon as the intermediate good implementing the new technology is discovered, the whole labour force is reallocated to final output and therefore we should observe a rise in final output to levels not attainable before.

However, two empirical questions are raised by the implications of this model: first of all, the framework predicts large slowdowns due to the reallocation of labour to research but this is empirically unlikely, since only a negligible percentage of workers is employed in R&D sector. Then, according to the model, as soon as a new GPT springs up, a sudden fall in final output should take place; however, empirical studies prove that it takes many decades for a disruptive innovation to have a tangible impact on macroeconomic variables and dynamics.

While there are many reasons to explain the wide fluctuations in output caused by the arrival of a GPT, such as the augmented obsolescence rate of physical and human capital or the job destruction due to employees unable to work with the new technology, it is more difficult to tackle the second empirical issue. In order to do that, the Schumpeterian model with GPTs is extended to have many sectors and a three-steps innovation process: first of all, a GPT should arrive, then each sector needs to design a template to start developing the new intermediate good, then it must be developed, finally implementing the cutting-edge technology. In order to generate in the model a lag in the response of final output to the discovery of a disruptive technology in the second stage is introduced a *social learning mechanism*, that is firms may design the template on their own or they may imitate it by observing other successful sectors. Since at the beginning only few sectors have implemented the new GPT, the *social learning mechanism* is not very effective and only a negligible fraction of labour is devoted to research. However, after some time more sectors have succeeded in implementing the cutting-edge technology, hence, this mechanism becomes way more effective and a large fraction of workers is diverted from production activities to R&D, causing a massive fall in output. In this way, the model is

[7]See Aghion and Howitt (1998c).

able to stick to the empirical evidence that there is a delay between the arrival of a GPT and the resulting decrease in output.

Hence, the introduction of GPTs in the Schumpeterian framework allows us to sustain that the innovation process is able to generate business cycle fluctuations around the growth path of the economy determined by the rate of technological change; in fact, phases of reduction in output due to the diversion of labour from productive activities to research (characterized by a lag with respect to the arrival time of the disruptive technology, because of the *social learning mechanism*) alternate with phases in which output raises to levels unattainable before, thanks to the increase in the productivity parameter.

3.1 The Effects of Business Cycles on Growth

Has a recession negative or positive effects on long-run growth?
This has always been one of the most debated question in economic theory. The idea that short-run slowdowns could have a positive effect on the economy in the long-run originates from Schumpeter (1934): his suggestion was that during periods of recession, since the struggle for survival becomes fiercer and fiercer, those firms which are less efficient are driven out of the market, being replaced by more efficient ones[8]. This process of augmented natural selection would end up in a rationalization of the economy, leading to a higher level of long-run growth.

However, on the other side of the field, Stiglitz (1993) pointed out that during economic slumps firms usually reduce their production of output, slowing down the learning by doing process, and that, due to the imperfections in the market for capitals, the problems of *credit rationing* and *equity rationing* are likely to force them to decrease their expenditures on R&D activities. These two effects combined together would eventually lead to a lower growth path than that which would have been attainable if the economic slowdown had not happened.

It is arguable that both the Schumpeterian efficiency-enhancing effect and the negative effects pointed out by Stiglitz are at work during an economic recession and it is not easy to say a priori which one would eventually prevail on the others. In the following subsections we analyze models which try to assess this issue and to detect the conditions under which a recession has

[8]This point has been questioned by many economists, since there in no mechanism assuring that the new entrant firms are actually more efficient than those which have been driven out of the market.

positive or negative effects on the growth path.

3.1.1 Negative Effects of Economic Slowdowns on Productivity Growth

Maybe the most natural way to think about the effects that business cycle fluctuations could produce on productivity and economic growth is that research activities and, thereby, the growth path are negatively affected by economic slumps. It is quite intuitive arguing that when firms are challenged by the many, sometimes insurmountable difficulties brought about by the adverse macroeconomic scenario and are forced to reduce their production of output, to lay off more and more workers and sell off large amounts of their productive structure, when firms are dramatically on the edge of the bankruptcy, they have few resources to invest in productivity-enhancing activities and even fewer incentives to engage in investments which will be remunerative only in the long term, taking on also the risk of the R&D process being unsuccessful.

Credit Rationing

The most straightforward thought is that firms are likely to cut down their expenditures on developmental activities because of the resources restraints they are experimenting in a much tighter way during economic slumps. This is the idea supported by Stiglitz (1993) in his analysis of the effects of short-run fluctuations on productivity growth: he underlines particularly the financial constraints under which all firms, especially those who are not big enough to have a significative contractual power, are forced to operate. As a matter of the facts, expenditures of firms on research projects are constrained by their financial resources, i.e. net cash flow and borrowings: a shock to the economy would have a massive, negative effect on the cash inflows, therefore reducing the first kind of funding for R&D. Firms may balance this reduction in net cash flow by increasing the amount of borrowings demanded: however, due to the existence of *adverse selection* in the capital markets, we can observe the phenomenon of *credit rationing*, that is not all the firms are capable of obtain more borrowings in order to keep steady the intensity of research activities[9]: especially those firms which are not big enough to collect resources in the equity markets and are perceived by financial institutions to be more risky would not

[9] We can also think of a situation where all firms obtain only a fraction of the funding demanded.

be able to offset the reduction in cash flows with more borrowings. On the other hand, economic downturns are also likely to exacerbate the rationing in the credit market: in fact, during a recession not only the probability of bankruptcy of a firm increases but also its implied warranty, namely, its present value, decreases; therefore, banks, which are assumed to be risk averse, are likely to reallocate their portfolio from loans to risk free assets: in the end, this should reduce the amount of loans provided to firms, increasing credit rationing. Hence, during an economic slump, thanks to imperfections in the capital markets, firms are forced to reduce their expenditures on R&D, causing the productivity growth rate to be smaller.

Learning by Doing

Another explanation for procyclicality of technological change can be now introduced: we should recall that one of the first methods through which productivity growth has been endogenized in growth models was the learning by doing mechanism: according to it, while employees are doing their job activities, they learn better and better how to carry them out, increasing the efficiency of the production process. If we assume the learning by doing process to exist in the economy, then the more labour is employed and the more output is produced, the larger should result the productivity growth rate.

This is the insight emphasized by Stadler (1990): according to his analysis, total factor productivity characterizing the economy at a certain time depends positively on output produced and upon the amount of labour force employed in the previous period. Therefore, if we introduce in the model a negative shock to the economy, the resulting fall in employment and production should compress the rate of growth of the technological parameter; this in turn should reduce also output produced in the next period and this fact would lower even more the productivity growth rate and so forth. Hence, economic downturns, by slowing down the learning by doing mechanism, would cause the entire growth path of output to shift down.

In conclusion, we have just discussed two possible explanation for productivity growth to be procyclical: on the one hand, imperfections in the market for capitals and the phenomenon of credit rationing should tighten firms financial constraints during recessions, leading to a cut back in R&D expenditures; on the other hand, negative shocks to the economy causing a fall in output and occupation should hinder the learning by doing mechanism. These two effects combined are likely to make the rate of technological change fall during economic downturns and, conversely, increase during economic upturns; hence, productivity growth should have a procyclical behaviour.

3.1.2 Positive Effects of Recessions on Productivity Growth: The Opportunity Cost Approach

Although in the previous subsection growth has been proved to be procyclical, there is also a big part of literature that, referring to the Schumpeterian idea that recessions may serve as a cleaning up mechanism to eliminate inefficiency and misallocation of resources, has stressed the fact that many are the factors which can make productivity growth countercyclical:

- the *cleaning up effect* firstly presented by Schumpeter (1934), according to which less efficient firms are driven out of the market during recessions, due to augmented competition, and are substituted by entrants which have a higher level of efficiency. This process would end up in raising the total factor productivity and therefore the whole growth path of the economy[10];

- the *disciplinary effect*, according to which during recessions the probability of failure for those firms which do not engage in reorganizational activities is bigger, thereby, pushing firms to carry them out in order to avoid this greater chance of bankruptcy[11];

- the *externality effect*, according to which, since the discrepancy in expected productivity between different kind of resources is bigger during slowdowns, it is easier for a firm to perform the selection process during recessions without making allocation mistakes[12];

- the *opportunity cost effect*, according to which during recessions the opportunity cost of carrying out productivity-increasing activities is lower because of smaller returns to production. Therefore, firms are stimulated during economic slowdowns to divert resources from manufacturing to R&D activities, because relatively less costly, and this in turn would raise the growth rate of the productivity parameter.

[10]See also Caballero and Hammour (1991).
[11]See Aghion and Saint-Paul (1998).
[12]See Dellas (1993).

Now we are going to present a model in which the basic features of the *opportunity cost approach* are analyzed, as presented by Aghion and Saint-Paul (1998) [13].

In this model we assume that in order to undertake productivity-enhancing activities, such as research, development of new products or trainings for workers, the firm must give up producing a certain amount of output. During recessions, since returns to production of final output are lower, the opportunity cost of these activities decreases and, thus, firms have more incentives to carry them out; because of that, productivity growth is bigger during slowdowns, showing, hence, a countercyclical behaviour along the business cycle.

In the model it is assumed an economy in which are produced several kinds of different final goods. The demand at time t for each one of them is defined by the following expression:

$$D_{it} = \frac{y_t}{p_t} \cdot \left[\frac{p_{it}}{p_t}\right]^{-\eta} \quad , \tag{3.1}$$

where y_t represents the scale of the aggregate demand in the economy, p_{it} is the price of good i and p_t is the current aggregate price index in the economy, resulting from the following formulation:

$$p_t = \left[\int_0^{N_t} p_{it}^{1-\eta} di\right]^{\frac{1}{1-\eta}} \quad ,$$

in which N_t is the number of varieties available to production in the world modeled here at time t.

Each good i is produced by a different monopolist having his own level of technology a_{it}. The capacity of production of each monopolist is fixed at $e^{a_{it}}$. Now we turn to consider the law of motion of each single technology parameter: let $g_{it} = \dot{a}_{it}$ be the rate of growth over time of technology characterizing firm i. This rate is to be chosen by each firm and springs from the productivity-enhancing activities undertaken by it. A monopolist would like to increase g_{it} in order to make the net present value of his firm bigger; however, in order to engage in the R&D activities required to widen technological growth, he must give up a certain amount of final good production, thus narrowing his current profits. It is assumed that the fraction of forgone output required to assure the technological growth target g_{it} is determined by the function $k(g_{it})$, having

[13] See also Hall (1991) and Aghion and Saint-Paul (1993).

the following features:

$$k' \geq 0 \ , \quad k'' > 0 \ , \quad k'(0) = 0 \ .$$

After the monopolist has chosen the amount of research, his net product is given by:

$$x_{it} = e^{a_{it}} \cdot \phi_{it} \ , \tag{3.2}$$

where $\phi_{it} = 1 - k(g_{it})$. Combining the equilibrium condition for each good i $x_{it} = D_{it}$, equation (3.1) and equation (3.2) we can obtain the following expression for the price of each good at time t:

$$p_{it} = y_t^{\frac{1}{\eta}} \cdot p_t^{\frac{\eta-1}{\eta}} \cdot e^{-\frac{a_{it}}{\eta}} \cdot \phi_{it}^{-\frac{1}{\eta}} \ . \tag{3.3}$$

The objective of each firm is to maximize its net present value, which depends on the state variables y_t and p_t and on the choice variable a_{it}:

$$V_t(a_{it}) = \pi_{it} dt + (1 - r dt) \cdot E_t \left[V_{t+dt}(a_{it} + g_{it} dt) \right] \ , \tag{3.4}$$

where $\pi_{it} = x_{it} p_{it}$ is the flow of profits at time t and r is the real interest rate. If we differentiate this expression with respect to g_{it} we can obtain the first order condition which, after having substituted in it (3.2) and (3.3), looks like:

$$\frac{\eta - 1}{\eta} \pi_{it} \frac{k'(g_{it})}{\phi_{it}} = E_t \left[\frac{\partial V_{t+dt}}{\partial a_{it}} \right] \ . \tag{3.5}$$

If we differentiate the expression for $V_t(a_{it})$ with respect to a_{it}, it is obtained the Euler condition:

$$\frac{\partial V_t}{\partial a_{it}} = \frac{\eta - 1}{\eta} p_{it} x_{it} + (1 - r dt) E_t \left[\frac{\partial V_{t+dt}}{\partial a_{it}} \right] \ . \tag{3.6}$$

The number N_t of monopolists producing different goods at any point in time is defined by the following entry and exit conditions: a firm which would like to enter the market is supposed to pay a fixed cost C. If the firm is to leave the market, it can recoup only a fraction $\theta C e^{\beta a_{it}} e^{-\beta a_t}$ of the entry cost[14], with $\theta \in [0, 1]$. In this formulation, then, technological growth rate influences not only the present value of a "going on" firm, but also the liquidation value of an undertaking which is going to exit from the market.

Since we are interested in symmetric equilibria, we assume that all the firms

[14]Note that a_t is the average technology parameter at time t.

in the market and all the new entrants share the same level of technology; therefore, we have $a_{it} = a_t \quad \forall i$. In this peculiar case we can observe that the liquidation value of a firm is constant at level θC. After having defined the entry cost, we observe that three situation may happen:

- if $V_t > C$, new firms are stimulated to enter the market until the condition $V_t = C$ holds; hence, in this case N_t would be increasing;

- if $V_t < \theta C$, firms which are currently within the market have incentives to exit the market until the point in which $V_t = \theta C$ is reached; therefore, in this case N_t would be decreasing;

- if $\theta C < V_t < C$, there is a stall and N_t would remain unchanged over time.

Now that we are focusing on symmetric equilibria, we can recalculate the expression for the aggregate price index as:

$$p_t = y_t e^{-a_t} N_t^{\frac{\eta}{1-\eta}} \phi_t^{-1} \ .$$

If we substitute this new expression in (3.3) it is obtained a new formulation for the individual price of the different goods:

$$p_{it} = \frac{y_t e^{-a_t}}{N_t \phi_t} \ , \tag{3.7}$$

which remains constant for each good i. Using these last two equations, we define the profit as:

$$\pi_t = x_t \cdot p_t = \frac{y_t}{N_t} \ ,$$

that is, the profit is nothing but the index for the world demand at time t divided by the number of firms in the economy in that period.

We now focus on the steady state of the economy, where profits, the number of firms and the marginal value to the monopolist due to an increase in a are all constant. If υ is the steady state level of $\frac{\partial V}{\partial a}$, by combining together (3.6) and (3.7) it can be obtained the following expression:

$$\upsilon = \left(\frac{\eta - 1}{\eta}\right) \cdot \frac{y}{rN} \ ,$$

which, substituted in (3.5) for $E_t \left[\frac{\partial V_{t+dt}}{\partial a_{it}}\right]$ yields the condition for the steady state value of g:

$$\frac{rk'(g)}{1-k(g)} = 1 \ .$$

From this equation it is straightforward to notice that a permanent shift in aggregate demand y has no effects on long-run growth.

Now aggregate demand fluctuations are introduced in the model in order to inquire what are their effects on long-run growth in productivity. The world may be in expansion (E) or in recessions (R); with probability γ the economy switches from E to R, with probability ϵ from R to E. We have that:

- if the economy is in expansion, $y_E > y$;
- if the economy is in recession, $y_R < y$.

We are interested in a stochastic steady state in which only p and a are allowed to adjust over time, while all the other variables are constant. Let us denote g_j and N_j as the constant values of g and N in state j, with $j \in \{E, R\}$.

If we go through the same steps as done before for the steady state without business fluctuations, we get the following two conditions for the steady state value of g_j, respectively during expansions and recessions:

$$r\frac{k'(g_E)}{1-k(g_E)} = \frac{(r+\epsilon)+\gamma\frac{d_R}{d_E}}{(r+\epsilon+\gamma)} \ , \tag{3.8a}$$

$$r\frac{k'(g_R)}{1-k(g_R)} = \frac{(r+\gamma)+\epsilon\frac{d_E}{d_R}}{(r+\epsilon+\gamma)} \ , \tag{3.8b}$$

in which $d_j = \frac{y_j}{N_j}$ is the individual demand facing each firm. We assume free entry during expansion, that is $V = C$, and that the downturns are capable of stimulating firms to exit the market, i.e. $V = \theta C$. Therefore, we should have $N_E > N_R$. Using these assumptions, (3.4) and the fact that $\pi_t = \frac{y_t}{N_t}$, the following two expressions for d_j can be obtained:

$$d_E = \{r + \gamma(1-\theta)\}C \ ,$$

$$d_R = \{r\theta + \epsilon(\theta - 1)\}C \ .$$

Now we have all the ingredients to check if the productivity growth rate is bigger during recessions or expansions. If θ is strictly less than 1, we obtain that $d_E > d_R$. Since the left-hand sides of (3.8a) and (3.8b) are identically specified and both increasing in g_j, $d_E > d_R$ implies that $g_R > g_E$, that is, the growth rate in productivity is larger during recessions. Hence, according to the *opportunity cost approach*, productivity growth is found to be countercyclical. This is mainly due to the fact that the opportunity cost of engaging in research activities, which can be identified as the foregone profits resulting from diverting resources from production activities, is larger in expansions than in recessions. Hence, undertaking productivity improving activities is less costly in recessions and this fact explains the countercyclicality of productivity growth.

3.1.3 Concluding Remarks

We have shown in this section that it is not clear a priori what is the effect produced by economic fluctuations on productivity growth; namely, it is difficult to claim that growth is unambiguously procyclical or countercyclical. On the one hand, economic slowdowns could provide incentives for firms to undertake reorganizational activities increasing their efficiency and make the opportunity costs of research less costly, thus stimulating productivity-enhancing activities. On the other hand, during slowdowns, since less output is produced and a smaller amount of inputs are employed, the learning by doing process would be less intense; moreover, it is likely that capital market imperfections and stricter liquidity constraints for firms would reduce expenditures on research, thereby resulting in a lower long-run growth path. The answer to the interrogative whether long-run growth is procyclical or countercyclical depends on which one of these two opposite effects prevails.

An absolute answer to this question cannot be given ex ante. Hence, it is reasonable to state that if growth benefits or not from slowdowns relies on how efficient are the markets in the economy: if they are characterized by a high degree of efficiency, productivity growth could prove to be countercyclical, since the cleaning up mechanism through which less efficient firms are eliminated and replaced by better one is likely to have a massive effect and the liquidity constraints reducing research expenditures would be less tight, due to the reduced imperfections in the market for capitals. In this case, therefore, a slowdown could end up to have a positive effect on the long-run growth rate of the economy, as predicted by Schumpeter. Conversely, if the market for capital is characterized by significant imperfections and there is no assurance that firms eliminated from the market are replaced by more efficient ones, the reduced magnitude of the rationalization and the exacerbated liquidity constraints would eventually compress productivity growth.

3.2 The Effects of Growth on Business Cycles: the *General Purpose Technologies Approach*

Up to know we have surveyed what are the influences that economic fluctuations have on the productivity growth rate and we have found that it is not easy to state if growth is to be considered procyclical or countercyclical. Now we turn to the other direction of the causal relationship: *does economic growth produce short-run fluctuations?*

A first answer to this question can be provided using the framework developed by Aghion and Howitt (1992) and presented here in the second chapter. Equation (2.10) provides a negative relationship between research in the current period and research in the following one. Besides the stationary equilibrium shown in chapter two, also a periodic solution can be sustained where periods with high amounts of labour devoted to research alternate with periods with low R&D activities. This oscillation in the fractions of skilled labour devoted to research and to production would eventually produce fluctuations in output.

However, we are going to focalize our attention on the new approach aiming at explaining the effects of growth on the business cycle through the distinction between drastic and incremental innovations[15].

3.2.1 First Contributions

An early model which manages to generate short-run economic fluctuations by assuming that when a drastic innovation, called breakthrough, takes place, it is followed by a cluster of improvements is Cheng and Dinopoulos (1996). In this paper, if the degree of diminishing returns to improvements is low enough, a steady state cannot be supported and there is a continuous succession of breakthroughs and improvements. These cyclical dynamics of the technological innovation process would end up to generate short-run fluctuations in output around its long-run growth path.

Another seminal work in this field is Helpman and Trajtenberg (1998a), in which the concept of **general purpose technology** (GPT)[16] is introduced: a general purpose technology is a drastic technological innovation which affects

[15] See the seminal work by Cheng and Dinopoulos (1996) and the developments by Helpman and Trajtenberg (1998a) and Aghion and Howitt (1998c).

[16] This term has been used for the first time by Bresnahan and Trajtenberg (1995).

most sectors in an economy and requires a wave of secondary, complementary innovations in order to be implemented in the production of final output. This structure of the innovation process generates two-phases cycles:

- the first phase starts after a new GPT has come up in the economy[17] : since, to be implemented, it requires a cluster of secondary innovations, resources are diverted from manufacturing to R&D, thereby causing a fall in real output;
- the second phase begins when enough complementary innovations has been discovered and the new GPT can be finally implemented in the production of final output. Since this new GPT assures a higher degree of productivity, final output increases during this phase.

Hence, in this model, although the discovery of a new GPT raises productivity growth in the long-run, it also causes economic slowdowns due to the subtraction of inputs from the manufacturing sector.
In the next subsection we are going to present another version of this model,[18] in which general purpose technologies are introduced in the Schumpeterian framework presented before in the second chapter.

3.2.2 GPTs as Source of Short-Run Fluctuations

In Aghion and Howitt (1998c) the basic idea introduced firstly by Helpman and Trajtenberg (1998a) that a new GPT cannot be implemented without the discovery of a certain amount of complementary innovations is introduced in the basic Schumpeterian framework presented in chapter two by partitioning the research process in two steps: first of all, a new GPT comes up in the economy at certain arrival times endogenously determined in the model; then, in order for the new GPT to be implemented, the development of a *critical mass* of intermediate goods needs to be carried out. During this period, final output is likely to fall down, as resources are diverted from manufacturing to development.

Introducing GPTs in the Basic Schumpeterian Framework

As in chapter two, the technology of production is described by the following equation:

$$y = AF(x) , \qquad \text{where:}$$

[17]GPTs are assumed to arrive at fixed intervals of time.
[18]See Aghion and Howitt (1998c).

- $F(\cdot)$ is a function increasing at a decreasing rate in the argument;
- x is the flow of the intermediate good employed in the production process. Since the production process of this intermediate good is characterized by a one for one technology, x is also the amount of skilled labour N devoted to the intermediate sector;
- A is a parameter which embodies the technology of the current GPT.

As anticipated, now the innovation process is composed of two phases:

1. a new GPT must be discovered in the economy;
2. to be implemented, the new GPT requires a certain amount of new intermediate goods to be developed. In order to keep the analysis simple, we pose this *critical mass* to be one.

The development process of the intermediate good implementing GPT_i can start only after its arrival and no one engages in research for GPT_{i+1} before the intermediate good i has been discovered.

The economy, hence, is characterized by a series of two-phases cycles, each one starting with the arrival of a new GPT and ending with the arrival of the next one, the transition from the first to the second phase being determined by the discovery of the implementing intermediate good. During phase 1, the fraction n of skilled labour is devoted to research, in order to discover the new version of the intermediate good[19]; in phase 2, the whole amount of skilled labour N is devoted to final output production[20]. It is straightforward to notice that in this model, each time a new GPT is discovered, the economy enters a period of recession, corresponding to the phase 1 of the cycle, in which final output falls down by the amount $A_{t-1}F(N) - A_{t-1}F(N-n)$.

The arrival of a new GPT is the random outcome of the continuous usage of the previous one and happens at the Poisson rate μ. The implementation of the cutting-edge GPT raises the current parameter A by the factor γ [21].

The discovery of the new version of intermediate good which allows to implement the modern GPT is the result of the research process carried out by firms, and is assumed to happen at the Poisson arrival rate λn.

Now, we move on to find the stationary equilibrium of the model, where the fraction of skilled labour devoted to research activities during phase 1 is

[19] Therefore, we have that final output in phase 1 is equal to $A_{t-1}F(N-n)$

[20] Hence, the amount of consumption good produced in phase 2 equals $A_tF(N)$.

[21] Therefore, we have that $A_t = \gamma A_{t-1}$, with $\gamma > 1$.

constant at level $\hat{n}$ for each cycle. We can now reiterate the solving process shown in chapter two in order to find the condition for $\hat{n}$. First of all, ω_j is defined to be the productivity-adjusted wage rate and v_j the productivity-adjusted present value of the flow of profits accruing to the successful innovator during phase j. The equilibrium condition in the research sector is that the wage rate must be equal to its marginal product, hence, the following condition must hold[22]:

$$\omega_1 = \lambda\gamma v_2 \ . \tag{3.9}$$

We know that v_j can be interpreted as the productivity-adjusted value for the firm of an innovation during phase j. It can be defined by the following system of Bellman equations:

$$r v_1 = \tilde{\pi}(\omega_1) - \lambda\hat{n} v_1 \ , \tag{3.10a}$$

$$r v_2 = \tilde{\pi}(\omega_2) + \mu(v_1 - v_2) \ . \tag{3.10b}$$

From (3.10a) we see that the value of an innovation during phase 1 of the following cycle is the flow of profits during that phase minus the capital loss deriving from the discovery of a new version of intermediate good, weighted by its probability. Equation (3.10b) shows that the value of an innovation during phase 2 is determined by the monopolist stream of profits to the successful innovator minus the net capital loss caused by the arrival of a new GPT, weighted by its likelihood. Combining system (3.10) and the equilibrium condition (3.9) yields the following research arbitrage equation:

$$\omega_1 = \frac{\lambda\gamma[\tilde{\pi}(\omega_2) + \mu\tilde{\pi}(\omega_1)(r + \lambda\hat{n})^{-1}]}{r + \mu} \ . \tag{3.11}$$

Since, as explained before, no one carries out research activities during phase 2, the labour market clearing condition in this phase is $N = \tilde{x}(\omega_2)$, which determines independently the productivity-adjusted wage rate ω_2. Once we have that value, we can notice that condition (3.11) describes ω_1 as a function of $\hat{n}$. The steady state value of $\hat{n}$, as usual, is determined by the research-arbitrage condition (3.11) and the labour market clearing condition in phase 1 of the cycle:

$$N = \hat{n} + \tilde{x}(\omega_1) \ .$$

[22]Notice that firms engages in development activities in phase 1, however, the successful one becomes monopolist only in phase 2.

As in chapter two, the stationary state level of research $\hat{n}$ is positively influenced by λ, γ, the scale of the economy N and negatively affected by r. Now, we have that also the arrival rate of the successive GPT μ tends to decrease the level of research, as it reduces the expected length of phase 2 of the cycle in which the monopolist has discovered the new intermediate good, reducing thus the period during which he is able to earn monopoly profits.

The size of the fall in final output can be calculated as

$$\ln[F(N)] - \ln[F(N - \hat{n})]$$

and is an increasing function in the level of research, thereby positively correlated with the long-run growth rate.

As in chapter two, the average growth rate will equals the size of each innovation $\ln \gamma$ weighted by its frequency. It takes a complete cycle for a GPT to be implemented and its expected length is nothing but the sum of the expected lengths of phase 1 and phase 2:

$$\frac{1}{\lambda \hat{n}} + \frac{1}{\mu} = \frac{\mu + \lambda \hat{n}}{\mu \lambda \hat{n}} .$$

As the frequency with which a new GPT is implemented is the inverse of this expected length, the long-run growth rate will be:

$$g = \frac{\mu \lambda \hat{n}}{\mu + \lambda \hat{n}} \ln \gamma .$$

We can se that the average growth rate is increasing in $\hat{n}$, γ and λ, while μ produces two opposite effects on g: a negative effect, since as μ grows, the steady state level of research falls down; a positive effect, since the bigger is the arrival rate of new GPTs, the greater will be the frequency of implementation.

In this model it has been shown that, even if technological progress is the only explanation for growth in the long-run, it also causes business cycle fluctuations, since each time a new GPT is discovered, inputs are diverted from production to research, causing slumps in final output, until the cutting-edge technology is implemented. An interesting result of the model is also that the wage rate ends up to be higher during recessions, since the supply of labour must be split between manufacturing and the R&D sector.

Two Empirical Questions about the Model

This model presents two main empirical problems which call into question its relevance:

- the *size of slowdowns*: in this model, the whole fall in output is due to the redirection of labour from production to research. Given that the fraction of the US labour force devoted to the R&D sector is estimated to be the 2.5 % of the whole amount, it is unlikely that this diversion could generate tangible fluctuations in output.

- the *timing of recessions*: according to this model, once a new GPT comes up in the economy, we should observe a sudden fall in output. However, as pointed out by David (1990), it takes decades for a disruptive technological innovation to have an impact on macroeconomic dynamics and to spread throughout the sectors. It is also highly unlikely that firms could divert a considerable amount of the labour force from production to research activities, which will be remunerative only in many years.

The first problem can be assessed by giving many reasons for why a major technological change should generate a short-run decrease in output:

1. in order to implement the cutting-edge GPT, firms may engage in highly risky experimentation projects; given that this projects absorb resources but are successful and, thereby, remunerative only on a sporadic base, the economy would experiment a slowdown since large amounts of capital would be no more employed in less risky, remunerative activities based on the old technology;

2. the high costs due to the required learning process for the employees in order to use new equipment in which the cutting-edge GPT is embodied could produce a slump in the production process;

3. each time a new GPT is implemented in a sector, the fraction of the labour force unable to work with the new technology becomes unemployed. Since it takes time for unemployed workers to find a new job, also if the fraction diverted from manufacturing to research is small, this destruction of jobs generated by the implementation of the new GPT could account for a tangible fall in output;

4. the slowdown could be worsened by the augmented obsolescence rate of both physical and human capital due to the wave of complementary

innovations, since each time an intermediate good is discovered which implements the modern GPT in a sector, a fraction of the capital previously utilized in that sector fades away because of the conversion process.

In order to address the second shortcoming, the innovation process is redefined as being composed of three phases, the new one accounting for the *technology spillover*, according to which firms learns how to implement the cutting-edge GPT also by observing the successful process of adoption completed by other firms.

Implementing New GPTs through Social Learning

In this extension to the previous model we have a continuous set of sectors on a unit interval; once a GPT comes up, each one of them needs to develop its own new intermediate good in order to implement it. As anticipated, the innovation process is composed of three phases: first of all, a new GPT arrives in the economy; then, each sector designs the template required to start developing the new version of its intermediate good; eventually, each sector discovers the new vintage of the intermediate good, thereby implementing the cutting-edge technology.

Aggregate output is produced employing intermediate goods according to the following technology:

$$Y = \left[\int_0^1 A(i)^\alpha x(i)^\alpha di \right]^{\frac{1}{\alpha}} .$$

It is assumed that $A(i) = 1$ in those sectors which have not implemented yet the new GPT, while $A(i) = \gamma$, $\gamma > 1$ in those industries which use the new GPT; $x(i)$ is the the amount of intermediate good produced by sector i, as usual according to a one for one technology.

Under the definition of the innovation process described above, each sector can be in one of three stages:

- **stage 1**: the cutting-edge technology has arrived, but the sector has not designed yet the template;
- **stage 2**: the sector has discovered the template and is doing research in order to find its new version of the intermediate good;
- **stage 3**: the sector has invented the modern intermediate good, thereby implementing the new GPT.

Let h_1, h_2 and h_3 be the fraction of sectors in stage 1, 2 and 3. At the beginning of the cycle, we have $h_1 = 1$ and $h_2 = h_3 = 0$. There are two ways to discover a new template: a sector can find it autonomously with probability $\lambda_1 < 1$ or it can imitate the template observing a certain number k of the m compatible sectors which are in stage 3. The latter is the technological spillover, or *social learning mechanism*, thanks to which the new GPT spreads throughout the economy. The probability that the social learning mechanism is successful for a sector is given by:

$$\phi(m, k, h_3) = \sum_{j=k}^{m} \binom{m}{j} h_3^j (1 - h_3)^{m-j} \ .$$

After having obtained the template, in order to reach stage 3 a sector needs to discover its new version of intermediate good by engaging in development activities. In order to keep the analysis simple, we assume that research is carried out by a fixed amount $\bar{n}$ of labour and it is successful according to the Poisson rate λ_2 . Now we can write down the laws of motion of the fractions of sectors in stage 2 and 3 (please see figure 3.1, where the parameters are assumed to take on the following values: $m = 10$, $k = 3$, $\lambda_1 = 0.005$ and $\lambda_2 = 0.3$):

$$\dot{h}_2 = \{\lambda_1 + \phi(m, k, h_3)\}(1 - h_2 - h_3) - \lambda_2 h_2 \ , \tag{3.12a}$$

$$\dot{h}_3 = \lambda_2 h_2 \ . \tag{3.12b}$$

Now we turn to the production side: we know that sectors in stage 1 and 2 produce under the old technology, while those in stage 3 under the new one. We can therefore rewrite the equation for output as:

$$Y = \left[\int_0^{1-h_3} x_{old}(i)^\alpha di + \gamma^\alpha \int_{1-h_3}^{1} x_{new}(i)^\alpha di \right]^{\frac{1}{\alpha}} \ , \tag{3.13}$$

in which x_{old} and x_{new} are the amounts of intermediate good produced and, thus, of labour employed respectively in sectors using the old and the cutting-

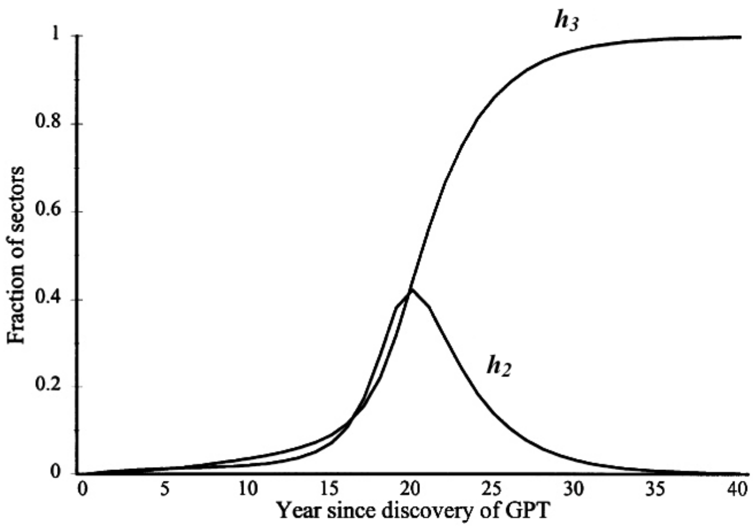


Figure 3.1: Dynamics of sectors

edge GPT. We can now derive these two labour demand equations as:

$$x_{old} = Y\left(\frac{w}{\alpha}\right)^{\frac{1}{\alpha-1}} , \tag{3.14a}$$

$$x_{new} = Y\left(\frac{w}{\alpha\gamma^{\alpha}}\right)^{\frac{1}{\alpha-1}} . \tag{3.14b}$$

Combining together (3.13), the system (3.14) and the following labour market clearing condition

$$(1 - h_3)x_{old} + h_3 x_{new} + h_2\bar{n} = N ,$$

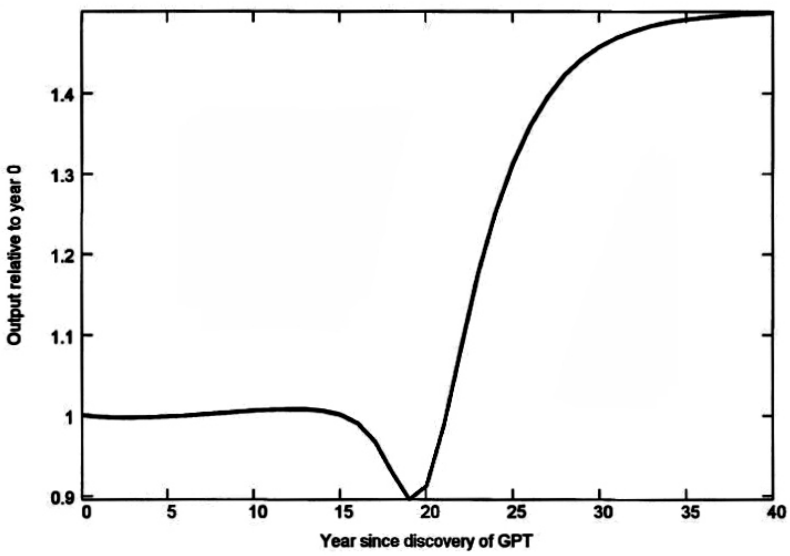
we can obtain the expression for aggregate final output as a function of h_2 and h_3:

$$Y = (N - h_2\bar{n})(1 - h_3 + h_3\gamma^{\frac{\alpha}{1-\alpha}})^{\frac{1-\alpha}{\alpha}} .$$

If a simulation is run on the time path of aggregate output[23] (as illustrated in figure 3.2), with the starting point being the moment in which a new GPT arrives in the economy, we can see that there is no slump in the first fifteen years, since, at the beginning, the transition of sectors from stage 1 to stage 2 is very slow and, therefore, a negligible amount of labour is subtracted from

[23] Aghion and Howitt (1998c) set the following parameter values in order to run the simulation: $\bar{n} = 6$, $N = 10$, $\gamma = 1.5$ and $\alpha = 0.5$.

Figure 3.2: Time path of aggregate output



manufacturing. However, in the next five years the decrease in output becomes very tangible and the economy experiments a massive slowdown: this is due to the fact that, as the fraction of sectors in stage 3 becomes larger, the social learning mechanism becomes more and more effective; hence, the fraction of industries in stage 2 reaches a peak and a wide number of workers are diverted from production to development, causing aggregate output to fall. However, after the nineteenth year, more and more sectors effectuate the transition from stage 2 to stage 3: in this way, workers start being reallocated to manufacturing, but also the fraction of sectors producing under the new, more efficient GPT becomes wider and wider; therefore, aggregate output starts to rise and in the long-run stabilizes at a level γ times larger than its value at time 0.

Therefore, introducing social learning in the Schumpeterian framework with GPTs allows the model to stick to the empirical evidence that it takes a long time for a major technological innovation to produce relevant effects on the economy.

3.2.3 Concluding Remarks

The question we have tried to answer throughout this second section of the third chapter is whether the innovation process aimed at determining a positive, long-run growth rate causes short-run oscillations in final output. By introducing GPTs in the Schumpeterian framework developed in chapter two we are able to support the idea that the continuous arrival of new, disruptive technologies is able to generate two-phases, business cycle fluctuations along the growth path of the economy: in fact, the diversion of labour force from production to research, following the discovery of a major technological innovation, produces a fall in final output, even if this effect is delayed by the social learning mechanism considered in the second part of the section, which makes the amount of final good produced remain stable in the first years after the arrival of the breakthrough, when only few sectors have successfully implemented it. However, once a GPT has been definitively implemented, the labour force is reallocated to the production sector, which is now characterized by a technology more efficient: therefore, final output raises to a level higher than that produced before the arrival of the cutting-edge GPT.

Conclusions

Along all the present work we tried firstly to explain how the main shortcomings of the neoclassical model of growth - namely, zero growth in the steady state and the need to introduce technological progress only taken as given in order to make long-run growth possible - have been assessed and solved by the literature on endogenous growth theory. Then, after having presented the Schumpeterian framework developed by Aghion and Howitt (1992), we showed how it could be reconciled with the neoclassical view that it is capital accumulation, not innovation process, the main determinant of potential output in the long-run. Finally, we described how productivity, and thereby economic growth are closely interrelated with business cycle.

In this conclusion we are going to discuss what are the main drawbacks of the models described and which facts from the empirical evidence still require to be explained.

The first critique to the Schumpeterian, endogenous growth approach comes from the analysis of empirical data on the evolution of total factor productivity and growth in output in Asian countries[24]. The analysis aims to identify how much advancements in total factor productivity contribute to explain economic growth with respect to other determinants: the finding is that growth in this countries is mainly due to other forces than technological progress, such as accumulation of human capital. Hence, this study suggests that technological change plays only a minor, nearly negligible role in explaining growth in output.

Another critique to the Schumpeterian approach is brought forward from those empirical papers questioning the existence of the *scale effect*[25]. As matter of the fact, according to endogenous growth literature, since the amount of

[24]See Young (1995).

[25]See Jones (1995).

workers devoted to research activities is an increasing function of the total endowment of labour force in a country and technological change is increasing in the number of researchers, increases in the scale of an economic system should result in raising the long-run productivity growth. However, observing the data it can be seen that the massive increments in population and in the number of people devoted to R&D occurred in the recents years did not manage to raise growth as expected. This finding suggests the hypothesis that also the research sector experiments the negative effects of diminishing returns in the production process of new knowledge. If this implication is true, we are brought back to the neoclassical model, which states that the long-run growth rate is unaffected by endogenous parameters and that in the steady state no growth is allowed unless we introduce exogenous forces.

Another criticized discrepancy between empirical evidence and the predictions made by the Schumpeterian approach is again due to the presence in these models of the *scale effect*: according to this effect, since each country has a certain endowment of labour force, dissimilar to that characterizing other ones, we should observe a different rate of economic growth for each economy. However, empirical evidence suggests that all the countries in the world are converging to the same long-run growth rate. Therefore, again it could be deduced that the contribution to growth provided by technological change is negligible.

As far as the effects of productivity growth on business cycle are concerned, we have already pointed out the two main empirical questions raised by critics, i.e. the timing and the size of the slowdown generated by the discovery of a GPT; in chapter three, following Aghion and Howitt (1998c), we have sketched some ways to address these problems, but also other extensions to the model of growth through GPTs can be developed in order to make it stick closer to the data.

Concluding, despite the empirical criticism endogenous growth literature has drawn on itself, the insights it provides in explaining why and how countries experiment economic growth in the long-run and what are the interrelations between productivity growth and business cycle are undeniable; therefore, we think that further research in this field of growth theory is likely to help us understand deeper how these phenomena affecting the whole world work.

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