LIBERA UNIVERSITÀ INTERNAZIONALE DEGLI STUDI SOCIALI

A Quantitative Analysis of Sovereign Bond Spreads in the Eurozone

Master's degree in Economics and Finance

Master's thesis

Chair: **Theory of Finance**

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July 2017

Academic year 2016/2017

Department of Economics and Finance

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Table of Contents

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Acknowledgments

"Dubium sapientiae initium"

(René Descartes, *Meditationes de prima philosophia*)

I would like to thank for this work my advisor Prof. Nicola Borri, who guided me all during the writing of my dissertation and my Co-Advisor Prof. Pierpaolo Benigno for the useful suggestions he made. I have to thank also all my LUISS professors who taught me a lot during these two years of masters' degree. In particular Giuseppe Brandi who helped me by reviewing some parts of my dissertation and Alberto Cybo-Ottone for giving helpful suggestions.

I would also like to thank Larry Goldstein (University of Southern California), Iskander Karibzhanov (Bank of Canada), Harold Cole (University of Pennsylvania), Zachary Stangebye (University of Notre Dame) for the insights they gave me; even if I do not know all of them personally they were very helpful in delineating some operational aspects of my thesis.

I have to of course to thank my family, in particular my parents Massimo and Tiziana who made this possible, Sergio for the patient review and Eloisa for the infinite support and to whom this dissertation is dedicated.

Chapter 1

Foreword

1.1 Challenges of the European Union

There have been a lot of transformations in the geopolitical links of influence and forces in the Eurozone. According to Eurobarometer, up to 2015 the main worries of Europeans concerned the economic and public finance situation, together with unemployment; in 2015 economic-related issues have been slightly overtaken by immigration and terrorism, even though remaining key topics [1]. In the last ten years the old continent was shocked by two major crisis and some Europeans, especially the ones from countries that were hit the most and who saw their quality of life[2] decrease (Eurofound, 2012), found in the Union a threat rather than a hope. Presenting some data at an aggregate level, results can be seen as positive; citizens who consider themselves as European have grown from 45% in 2002 to 56% in 2015, while the

[1] the precise question was "What do you think are the two most important issues facing the EU at the moment?" (2 Answers; data from 5/2011 to 5/2016);

[2] for an explanation of these themes you could read a work I have written "*Felicità e soddisfazione della vita: un'analisi empirica della situazione europea tra indicatori economici e progresso sociale*" in Italian;

overall declared attachment to Europe passed from 45% of 2002 to 49% in end of 2015. However, it seems that there has been a different trend in some countries of the periphery: on one hand, in Portugal the percentage of people who declared themselves attached to the European Union remained constant around 55%, while in Spain there was a remarkable increase of 10 percentage points from 2002 to 2015. On the other hand, in Italy this percentage sharply dropped from 59% to 40% while in Greece it remained stable around 35%, highly negative. With the financial crisis - considering the restricted time horizon - results were even worse and it appears clear that the economic cycle affects how the European Union is seen. For what concerns the UK, the overall satisfaction was around 27% in 2002 and ended up to 36% in 2015. Nonetheless, Brits decided to leave the European Union on July 26th 2016.
The percentage of optimistic citizens about the Union declined from 69% before crisis (2007) to 50% ten years after. Another aspect that might be interesting to analyse is how Europeans have answered to the question “Taking everything into consideration, would you say that (your country) has on balance benefited or not from being a member of the EU?". In Greece, from 78% of people that answered positively in 2007, the portion declined to 48% in 2011, losing 30% in half of a decade. Similar negative trends were observed for Italy (47% to 43%), Spain (70% to 59%), UK (40% to 35%) and Portugal (68% to 51%). Moreover, it shall be pointed out that data availability ends in 2011; it is opinion of the writer that the trend kept going downwards. Also the opinion of Euro as unique currency showed a strong negative correlation with the crisis too: 67% of Italians in 2007 looked at it in a positive way while only 56% in 2014; also Spain (68% - 63%) and France (72% - 65%) showed a similar trend. Moreover, it is remarkable the difference in numbers for Greece during the sovereign crisis. Indeed, in that period only 45% of Greeks looked at the Euro in a non negative way, having registered a -25% drop in

consensus, with respect to few years before the crisis (European Commission, 2017). Hence, it seems that - especially in some areas - the Union is facing a reduction in consensus that could lead to its end if there will not be a strong response and if more inclusive social policies will not be pursued. This work will try to give some answers to one of the most relevant aspects in the life of every European citizen: economics. Its aim is to give some lines to policy makers in understanding to which extent they have to focus on different countries and how much attention they have to put on the entire picture.

1.2 A diachronic union

The juridical and historical beginning of the European Union shall be seek in the Treaty of Rome (25th March 1957), when the agreement on the European Economic Community (EEC) was signed; this first piece of primary legislation was later renewed with the Maastricht Treaty (7th Febraury 1992). Finally, the European Union (EU) as intended today was made official on 13th December 2007, in Lisbon. One of the key results that all these agreements tried to implement in the Community and that was finally achieved in the Treaty of Lisbon was the realisation of the internal market (Art. 3, Treaty on the European Union). The internal market allows for the free movements of goods, persons, services and capitals (Art. 49 *et seq.* Treaty on the functioning of the European Union), enlarging the scope of trades and increasing efficiency.

However, it might be suggested that if the aim of the Union is to achieve positive results shared among its members, behind a unique market there should also be a unique policy. Aiming at an internal market with different fiscal, monetary or prudential policies and regulations would advantage only determined countries: indeed, competing in a unique market with different rules for the players shall be something to avoid. Actually, in Europe there is a crucial issue in terms of regulation because of the different regimes, that create unbalances and that sometimes lead to the phenomenon of regulatory arbitrage. In particular, the EU - from a financial perspective - was partially built on mutual restrictions; some renowned constraints are, for instance, the ones outlined in the stability and growth pact, like the 60% maximum debt as percentage of GDP or the 3% limit on public deficit [3]. They have been studied and criticized (*e.g.* Magnifico, 2008 or Lane, 2012) because they were considered by some as the result of non solid/scientific studies, often labelled

[3] the limits where in the Maastricht Treaty of 1992 and in the Stability and Growth pact of 1997;

as ineffective (if not even harmful). In particular, these constraints had different meanings for different Euro countries; for some states it was relatively natural to respect them, while for others it was a relevant, if not impossible, effort and some never managed to achieve such results. Portugal, Ireland, Spain and Greece, that were respecting the 60% limit up to the first years of 1990s, considerably crossed it in the subsequent years. Similarly, also the 3% limit on public deficit was not respected by many countries, and this number seemed meaningless in some years during the crisis. 2009, for instance, was a devastating year for all Europe and in particular for the periphery countries: in fact, concerning deficit Portugal had a -9,25%, Spain a -11,16%, Italy a -5,24%, Greece a -13,58% and Ireland a -14,61% (IMF, 2010 and Lane, 2012), far away from the -3%.

With the creation of the European Central Bank (ECB) - one of the seven institutions of the EU - monetary policy was harmonized and coordinated. However, it has not been the same for fiscal policy, which remains a responsibility of single nations. This clearly creates unbalances: a unique market, with different taxations for both individuals and corporations, creates distortions, also in competition. Some authors regard the 2003-2007 period as a “missed opportunity" to tighten the gap between fiscal policies (Lane, 2012): that should be a main point of the regulatory agenda. On the other hand, for what concerns financial regulation, there is a convergence going on. In particular, from the first banking directive (77/780/EEC) which aimed to create a EU passport mechanism for banking activities [4], passing through the Lamfaloussy report of 2001 and, after the crisis of 2007-2008, the De Larosière report (25th February 2009). The latter, proposed to the European Commission by Jacques de Larosière and its group of work, tried to delineate the framework of reforms that were needed to update, in the light of the recent events, the previous system originally developed in 2001. The De Larosière report proposed the

[4] as defined in Annex I of Credit Requirement Directive IV (2013/36/EU);

so-called European Supervisory Authorities (ESAs): European Banking Authority (EBA), European Securities and Market Authority (ESMA) and the European Insurance and Occupational Pensions Authority (EIOPA). The European Systemic Risk Board (ESRB) was also created, with the role of assessing and contributing to macro-prudential analysis and with the main objective of helping to reduce systemic risk avoiding contagion effects. Financial stability was a key point in the regulatory agenda. On this wave, in 2012 the Banking Union was proposed, founded on a *corpus* of laws called single rulebook, with the objective of creating a safer environment. To this purpose, with Reg. 1024/2013 the Single Supervisory Mechanism (SSM) and with Reg. 806/2014 the Single Resolution Mechanism (SRM) were introduced, with the aim of conducting a coordinated and harmonic supervision (ECB, 2016)[5]. The joined regulation for monetary, fiscal and financial topics is crucial. Currently, the financial environment is a *unicum* in its essence while it has different speeds of regulation. Having different rules is a fragility of the system; the sovereign crisis was strongly connected with the banking crisis and a strong, coordinated European answer could have been far more effective. A taste of this has been seen with the political turbulences of 2015, when the Greek government - lead by the prime minister, Alexis Tsipras - announced a referendum to get the popular opinion on the economic conditions that were asked by creditors regarding the Greek public debt and if these should have been accepted or rejected. In this occasion, the possibility of Greece exiting from the Euro and of shocks in the European system had been real. However, even because of protection mechanism and ECB policy, it was possible to keep the situation under control avoiding serious effects as in the previous years.

[5]for a synthesis of these themes refer to a short paper I have written: *'A quick review of European financial stability institutions and the role of stress tests in the current juridical system* ' available at https://arxiv.org/pdf/1612.05227

1.3 Crisis and spreads: a metrix under the microscope

Since the adoption of the euro (1999-2001), according to the studies conducted by the Centre for Economic Policy Research (CEPR), there have been two relevant complete cyclical crisis episodes: the 2008-2009 subprime crisis and the 2011-2013 credit spread crisis. The first one started in the US house market, that was affected by a serious bubble that, in the end of 2006, started to deflate. In fact, house prices had an extraordinary growth, caused by the demand that was artificially high because of the sub-prime market: mortgages were granted without proper collaterals. Property value, in a world where real estate had striking growth rates, was considered to be enough as protection and often mortgages were granted up to the 100% of property value; risk was then re-packaged and sold. Later, also due to movements in interest rates, debtors started to not be able to repay the debt and they started to abandon their houses, making real estate prices - and the value of their mortgages - drop to the floor. In 2008, with the failure of Lehman Brothers the climax of the crisis was reached. Banks had high exposures in major Asset Backed Securities (ABS). The Federal Reserve (FED) and European Central Bank (ECB), who put all their efforts in trying to avoid bank run and situations of panic as in 1929, managed to keep a discrete level of stability. However, the crisis spread all over the world and from the US house market affected the vast majority of economic sectors in every corner of the world. One of the biggest crisis in history had just took place and its aftermaths would have lasted for many years.

In Europe, the subsequent crisis overflowed in 2011 and lasted up to 2013, even if its time borders are not perfectly shaped. It was caused by the fact that, in advanced economies, GDP was decreasing while Deficit was increasing and it was amplified by the fragility of the financial sector, in particular of the banking sector, that had emerged few years before with the subprime crisis. Greece contributed to instability when, in 2009, declared that budget deficit was around -13%, dramatically higher than previous estimates. Moreover, debt was revaluated from around €170 billions of 2004 to €260 billions of 2009[6] and by the end of 2011 Greek spread[7] reached quota of 35%; numbers never seen before in developed economies. In this period of crisis, many countries had to ask for help and bailouts occurred: Greece was helped for around €200 billions, 70% of which granted by the European Stability Mechanism (ESM), that played a crucial role in sovereign debt crisis. The ESM with its capacity of more than €500 billions and currently has [8], together with the European System of Financial Supervisors (ESFS), covered more than 50% of Greek public debt. The role played by these two institutions was essential and they managed to help countries in difficulty by applying very low interest rates, around 1% (ESM, 2017). In addition, they helped Portugal for €26 billions out of the €80 billions of bailouts, Ireland by €17.7 billions out of the €70 total billions. Furthermore, they offered the entire help that Spain needed of €40 billions. Some unpopular structural reforms and a period of austerity were required in these countries, making confidence in European Union decline. The strong linkage between the banking and the public sector was then clear (Consob, 2017) as it was clear that a GDP greater than 2%, often assumed for policy purposes was unrealistic for developed countries, especially in a crisis context. In this scenario, in fact, taxation increases and public investment decreases, with a subsequent deterioration of human capital due to unemployment creating a vicious circle (Lane, 2012). Nowadays, the situation for many of the countries that were subject to bailouts is recovering; Greece constitutes an exception and the situation is still critical. Its GDP growth is, in fact, minimal even if it is

[6]a related newspaper article is *Timeline of a crisis: how Greece's tragedy unfolded* of Telegraph;
[7]defined as the difference between Greek 10 year bond and German 10 year bund returns;
[8]further details can be found at https://www.esm.europa.eu/assistance/greece;

compared to other countries of the periphery (Figure 1.1). 36% of the population is beyond the poverty level[9] and unemployment rate is at 25%, making Greece one of the countries with the highest unemployment rate in the entire world (CIA, 2016). In Italy unemployment is around 12% and the portion of population in a poverty condition is at 30%, in Spain there are more unemployed (20%) and less people beyond the poverty level (21%), while in Portugal (poverty: 19%, unemployment 11%) and Ireland (both poverty and unemployment around 8%) the situation seems to be reversing in the last years (CIA, 2016)[10]. In 2016, GDP of Greece was 0.05% but, according to forecasts, it is supposed to recover and jump to 3% by 2018, for then declining to 1.8% in 2021, in Italy is stable around 0.8%. On the other hand, Spain had a significant increase in GDP growth, reaching 3% in 2016 and then it is expected to be around 2% and 1.5% for the next 5 years. In Portugal it is expected to be around 1%, while Ireland registered a 4.9% and for the next five years the GDP is forecast to grow around 3%. At an aggregate level, European Union GDP growth is believed to be around 1.8% for the next years (IMF, 2017). In these years, the different speeds of European countries were evident. However, projects for reducing this gap, like the creation of eurobonds, were rejected. In the meanwhile, spreads were considered as the main variables to check, synthesis of safety and stability of countries. If the economic situation will recover, it depends on a significant number of factors: from Brexit to the US passing through the oriental markets, but spreads will always be under the microscope on a daily basis and it is hence important to understand how and why they move.

[9] it shall be underlined that these numbers might be affected by tax evasion;
[10] some data referred to poverty level (CIA, 2016) might be of previous years;

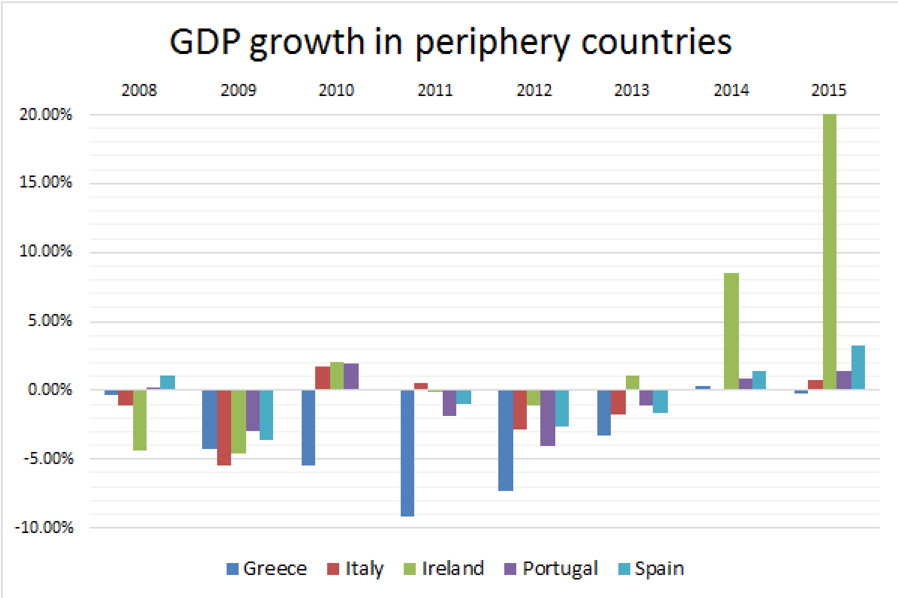

Figure 1.1: GDP growth in Periphery countries

1.4 Past literature

During periods of crisis both academics and public opinion focused one spreads as measure of risk. Spread is usually expressed in basis points[11] and it can be defined as the difference between the returns on a given product (*e.g.* sovereign bond) and a benchmark (*e.g.* the German Bund). At this point it shall be clear how interconnected the financial world is and that policy-maker shall have a good understand of this kind of measure reported in everyday news and of great public relevance. In this sense some studies, especially for emerging countries, have been conducted but still the mechanics of spread movements remain unfamiliar.

Ang and Piazzesi, in 2003, studied the relationships between the bond yields and macroeconomic variables by using, together with no arbitrage techniques, vector auto-regressive (VAR) models with latent factors. Longstaff, Pan, Pedersen and Singleton, in their successful paper of 2011 "How sovereign is sovereign credit risk?" tried to decompose the spread of Credit Default Swap (CDS) between global and

[11] 1 bps: $1/10\ 000 = 0.01\%$

local components for 26 countries (few of which Europeans). They showed high correlations between spreads: around 0.39 on average and 0.73 during the subprime crisis, as far as high commonality. On this aspect, it will be shown that in Europe these values are higher, suggesting that linkages are stronger in the old continent. They also found significant correlations with global variables. Moreover, the same Longstaff working with Ang (2011) tried to explain the CDS spreads in Europe and in the US, discovering that in Europe there is a higher portion of systemic risk, that they estimated to be around 35% of total risk. However, as significant explanatory variables, they only identify the market return, the change in Chinese CDS and the change in ITraxx, with an explanatory R^2 of around 40% regressing on the systemic/shared part and around 20% if regressing on the entire spread. Nevertheless, the use of ITraxx and CDS of China could be a critical assumption because it may be argued that they are similar variables to CDS and the analysis does not go to the basis of the relationship.

Finally, the methodology that will be applied in this work is taken by the work of Aguiar *et al.* (2016) who - in their first part of their paper - used the Kalman filter to construct common factors and explain the spread in emerging countries, showing that a high percentage of volatility can be explained with these common factors.

This work will try to apply a latent factors approach focusing on the Eurozone with a fully multidimensional and time varying model, trying to improve the one applied by Aguiar *et al.*.

Chapter 2

Data

2.1 The dataset

Countries subject to the analysis are Italy, France, Spain, Portugal, Belgium, Ireland, Netherlands, Austria and Finland; Germany has been taken as benchmark. These are roughly the countries that experimentally adopted the Euro in 1999; Greece - that joined the Euro in 2001 - was excluded because data are partially interrupted and not of high quality; in addition, there have been serious issues for what concerns liquidity and prices might not be informative. Minor countries have also been excluded because of data quality requirements. All the data has been collected in quarters starting from Quarter 2 of 2002 up to Quarter 4 of 2016. Where returns are used data has been gathered from Quarter 1 of year 2002. [1].

Variables obtained are, for what concerns country-specific:

[1]in particular, stock prices data has been updated on 10th January 2017, data from DataStream on 23rd February 2017, gdp-growth rates on 20th March and deficit related data, that were the latest to be updated, on the 1st of May. It shall be pointed out that some of the latest observations might be not definitive and henceforth could have been further changed by small amounts;

(i) returns on benchmark bond indices realized by DataStream - Eikon Thomson Reuters [2]; from these spreads are obtained by computing:

$$s_{it} := r_{it} - r_{BDt} \tag{2.1}$$

for $i = 1, ..., I$ representing the country, for $t = 1, ..., T$ representing the time period and being r_{BDt} the return on the German index at time t.
(ii) levels of deficit as percentage of GDP taken from EuroStat;
(iii) percentual GDP growth in real terms taken from OECD;
for what concerns global variables:
(iv) the Price-Earnings (PE) ratio for the US gathered from DataStream;
(v) Libor 3 month, UK interbank rate from DataStream;
(vi) VIX (CBOE) from DataStream;
(vii) Stock returns on S&P500 taken from Yahoo Finance.
In addition, CDS from the end of 2007, in euro and dollar terms have been taken from Datastream in order to conduct an incidental analysis on *Quanto* spreads. Furthermore, a variable that could be been used as measure of euro break-up risk has been created. This variable is constructed from Google Trends data (2017) by taking the number of researches on Google of the following keywords: *euro breakup, euro break, euro break-up, abandon euro, leave euro, euro exit, out euro, euro breakdown, euro referendum, euro collapse.* An average of these values has then been taken and defined as `trendOnline` that will later be inserted in the analysis.

At global level factors regarding the US (or UK for what concerns Libor) market have been taken; one of the objectives of this research will be to check to which extend spreads depend on non-local variables. By using non European elements, it

[2]for the methodological note on how these indices are constructed *cfr.* Markit iBoxx EUR Benchmark Index Guide, Jan. 2017 version

will be assured that they truly are of global dimension and not euro-related.

Furthermore, it could be argued that the use of Credit Default Swaps (CDS) - as some academics did - would better fit for the analysis. However, it shall be pointed out that bond indices could present the same liquidity and that CDS might be as well not fully informative for given countries in very peculiar time periods (*e.g.* Greek crisis). In addition, and most relevant, datasets for CDS were not available from 2002 but started some years after. Some authors (as the above quoted Ang and Longstaff, 2011) used CDS; nevertheless, one of their key argument for using CDS was that more observations - at a higher frequency - were available; this is reasonable in that context where only financial variables (that typically have with relatively higher frequency) are used, but it would not fit here the same. In this study, macroeconomic variables are likewise included and the vast majority of them has at most quarterly frequency. The advantage to which the authors refer would then disappear and using CDS, for this work, would cut by one third the sample size. Thence, it appears logic the choice to use bond indices.

All data has been made stationary and residuals have been checked in order to ensure that OLS techniques could be used. All analysis use explanatory lagged variables; in other words to explain variables at time t at most factors at $t-1$ have been used. The standard OLS model which tries to explain sovereign spreads registered an average adjusted-R^2 of around 37%.

2.2 Descriptive Statistics

GDP had a significant drop during the subprime crisis of 2008-2009 as can be seen from Figure 2.1. Crisis periods as reported by CEPR (2016) are highlighted. Debt as a percentage of GDP and the change in the real GDP. Debt levels have increased from 2008 to 2013 stabilizing from there onwards. In Figure 2.2 the evolution of public deficit can be seen. It shall be underlined *in primis* that these are macroeconomic variables, more stable and less volatile than financial ones; *in secundis* that Irish volatility that can be appreciated in both the figures.

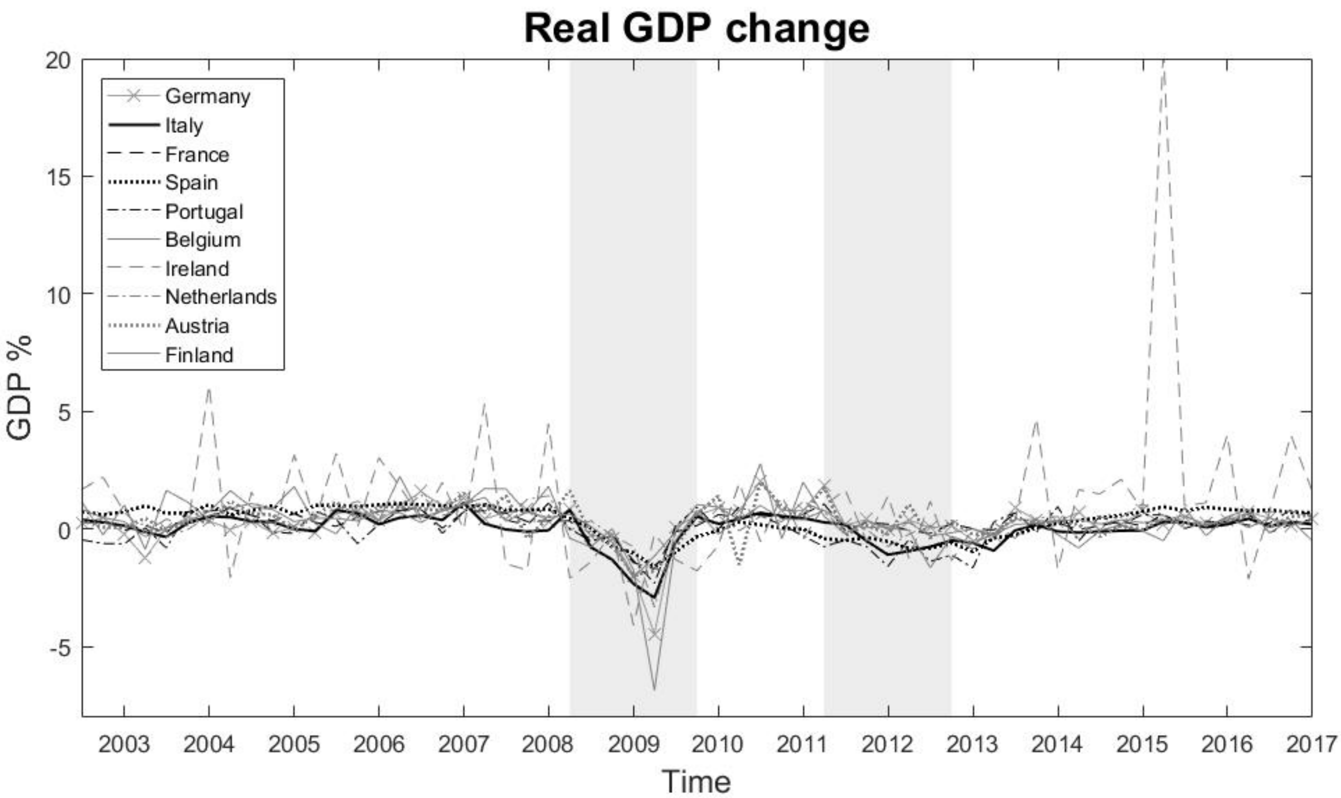

Figure 2.1: Change in real GDP growth; computation on OECD data

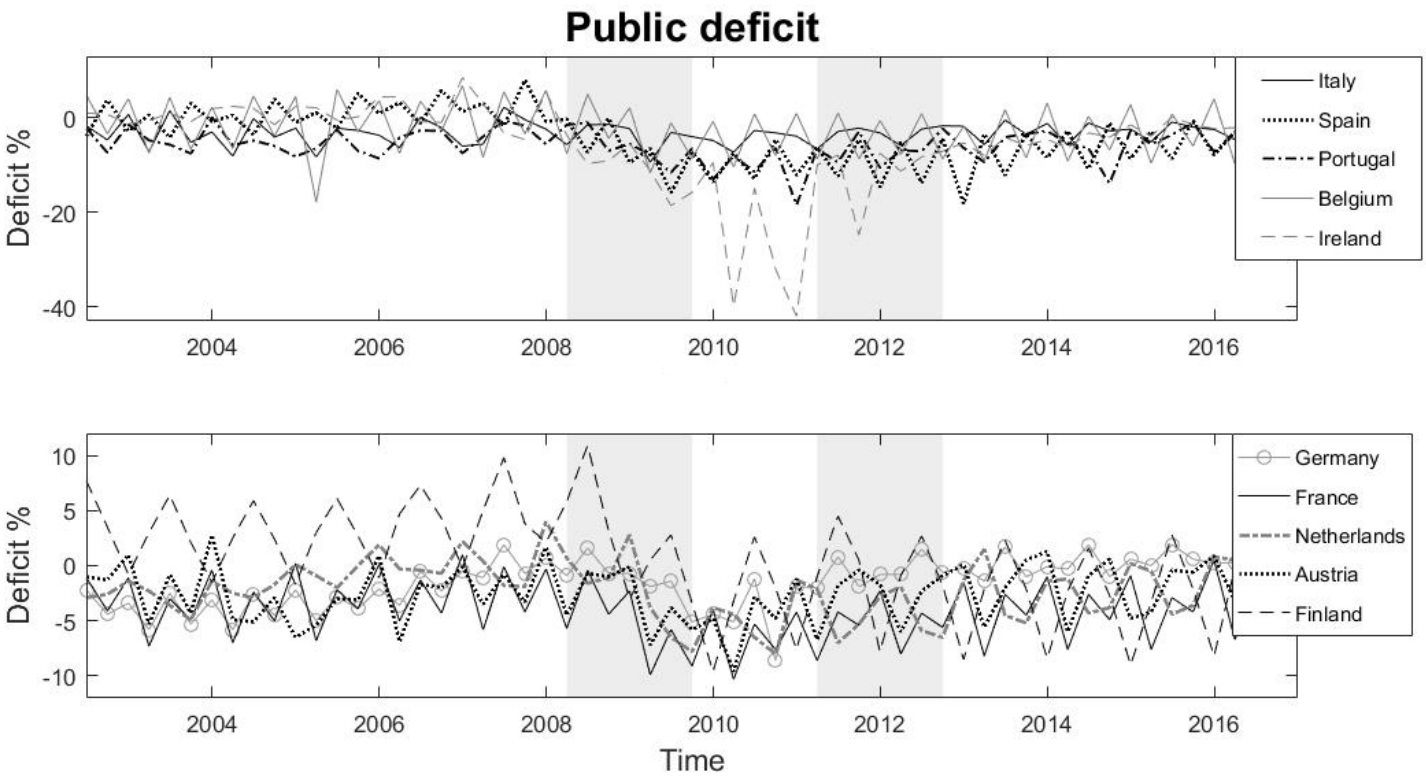

Figure 2.2: Public deficit; computation on Eurostat data

In Figure 2.3 the behaviour of researches for euro break-up is reported; two peaks are in 2012 and 2016. It could be argued that there was not fear (or wish) of abandoning euro in 2008 while there was in 2012. 2016 spike is clear and due to Brexit.

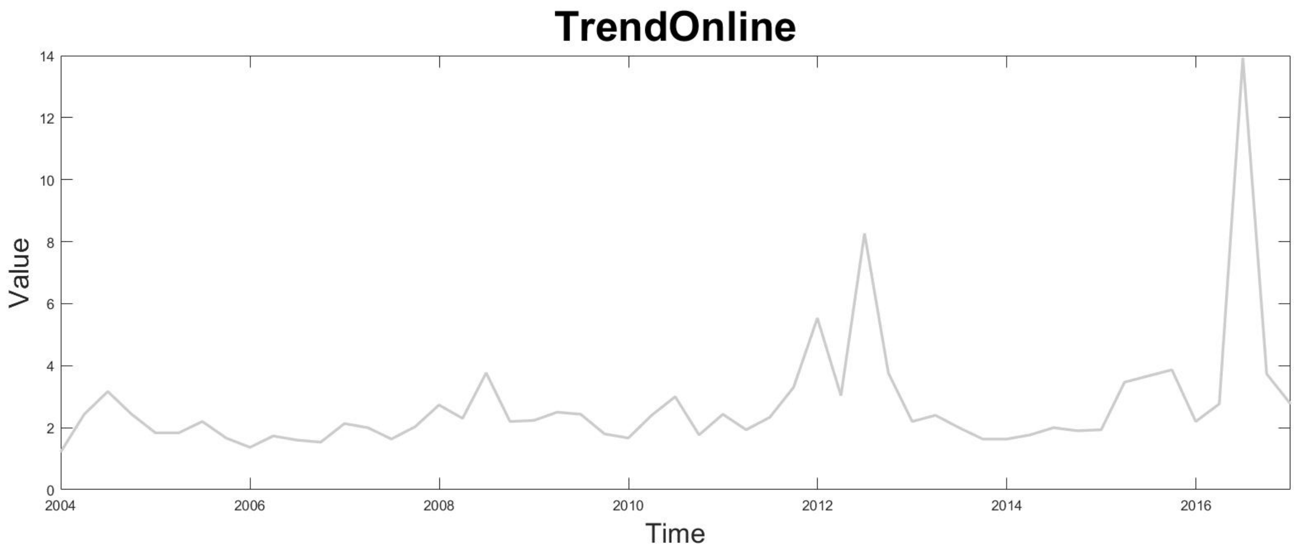

Figure 2.3: Google research index for euro break-up keywords; computation on Google Trends data

In Figure 2.4 spreads, as defined in formula (2.1), expressed in bps from 2002 onwards, are reported. In particular, during the 2008-2009, with the subprime crisis, that involved the entire world, spreads more than doubled while remaining at reasonable levels because sovereign stability was not under discussion. Differently, during the sovereign debt crisis of 2012-2013, the difference between returns of some countries - mainly the peripheral ones - and Germany spiked. Core countries like France, Netherlands, Austria and Finland, as reported in the graph, showed relatively low spreads. Portugal trespassed 1000 bps, followed by Ireland, Spain and Italy.

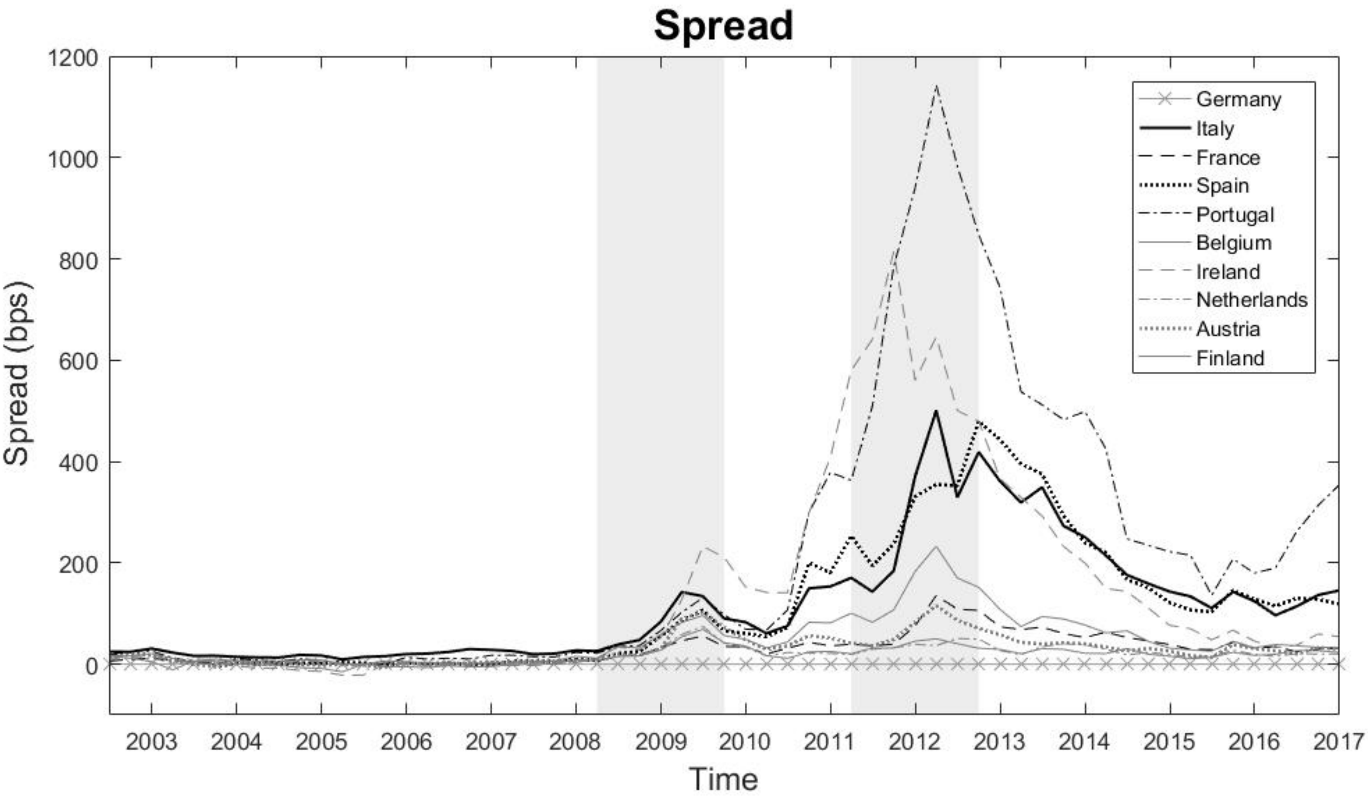

Figure 2.4: Spread for considered European countries

In the next table the countries that will be included in the analysis with their spread mean and standard deviation are reported:

Table 2.1: Statistical summary of countries in the sample

Variable	**Label**	**Mean Spread**	**Standard Deviation**
Italy	IT	117.33	118.63
France	FR	30.14	30.20
Spain	ES	112.26	130.55
Portugal	PG	220.40	285.16
Belgium	BE	47.14	49.76
Ireland	IR	141.17	203.10
Netherlands	NL	18.98	16.37
Austria	AU	28.47	28.46
Finland	FN	16.22	17.70

2.3 Correlations

A remarkable result is the correlation between returns and spreads in the given 14-year time horizon. In Table (2.2) the correlation matrix between returns is reported. There are strong correlations between France, Germany, Netherlands, Austria and Finland. Some correlation coefficients are even higher than 0.99; for instance between Finland, Netherlands and Germany or France and Austria. There it seems to be two clusters of data; one, that is going to be labelled as core, represented by Germany, France, Netherlands, Austria and Finland; the other, referred as periphery, composed by Portugal, Italy, Spain. Behaviours of Belgium (more orientated towards the first group) and Ireland (more orientated to the second group) are blurred. Again, it is interesting to point out the behaviour of the latter, the so-called "Celtic tiger", that registers the most volatile data in the entire Eurozone.

On the other hand, correlations between spreads, among with histograms of spreads themselves on the main diagonal, as reported in figure (2.5), seem to confirm the guess of two clusters. In any case, it is striking how high correlation coefficients are inside the same cluster. During the sub-prime crisis of 2008-2009, correlations

Table 2.2: Correlations of returns

Country	**BD**	**IT**	**FR**	**ES**	**PG**	**BE**	**IR**	**NL**	**AU**	**FN**
BD	**1.000**	0.6070	0.9853	0.4834	-0.0527	0.9266	0.3922	0.9946	0.9813	0.9928
IT	0.6071	**1.000**	0.7260	0.9619	0.6865	0.8271	0.8224	0.6570	0.7107	0.6526
FR	0.9853	0.7260	**1.000**	0.6086	0.0981	0.9718	0.5122	0.9940	0.9955	0.9927
ES	0.4834	0.9619	0.6086	**1.000**	0.7350	0.7157	0.8526	0.5329	0.5864	0.5249
PG	-0.0527	0.6865	0.0981	0.7350	**1.000**	0.2913	0.7789	0.0012	0.0876	0.0071
BE	0.9266	0.8271	0.9718	0.7157	0.2913	**1.000**	0.6762	0.9502	0.9730	0.9532
IR	0.3922	0.8224	0.5122	0.8526	0.7789	0.6762	**1.000**	0.4478	0.5170	0.4573
NL	0.9946	0.6570	0.9940	0.5329	0.0012	0.9502	0.4478	**1.000**	0.9936	0.9985
AU	0.9813	0.7107	0.9955	0.5864	0.0876	0.9730	0.5170	0.9936	**1.000**	0.9946
FN	0.9928	0.6526	0.9927	0.5249	0.0071	0.9532	0.4573	0.9985	0.9946	**1.000**

increased a bit, while during sovereign crisis of 2012-2013 correlations on average decreased and behaviours in some cases diverged. This result, obtained from the relatively few observations during the crisis period, could be explained by arguing that - after the 2008 crisis - probabilities of default (or their expectations) changed significantly between states and by recalling that while in 2008 Europe was considered to be solid and not in danger, during the sovereign debt crisis some doubts arose. Indeed, euro break-up was a possible scenario and there was a "flight to quality". It is not hard to think that Portugal and Germany, for instance, were considered in a different way by investors and henceforth moved in opposite directions.

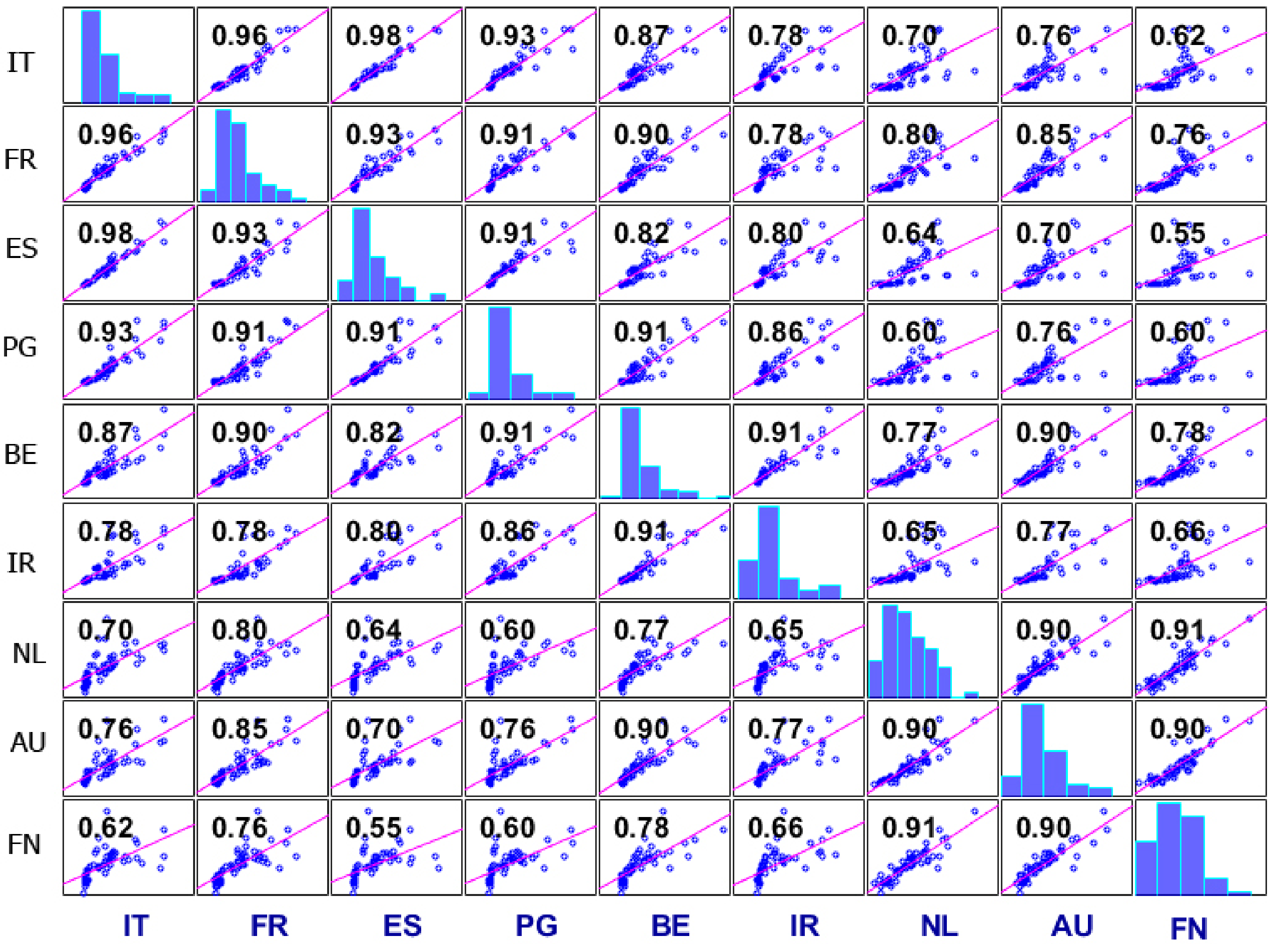

Figure 2.5: Correlations of spreads between different countries

2.4 Commonality: principal component

Given a time series by computing the variance-covariance matrix, it is possible to get through spectral decomposition the principal components (PC). It consists in obtaining from a series of observations linearly uncorrelated components that explain in a decreasing way the variance in the given sample. The fastest way of getting the principal components is by computing the variance-covariance matrix (Σ) and taking the largest eigenvalues (§ Appendix). Projecting the eigenvectors associated with the highest eigenvalues allows gathering the principal components. If we let $\lambda_i \in \sigma(\Sigma)$ the i-th eigenvalue, the corresponding eigenvector will explain $\frac{\lambda_i}{\sum_k \lambda_k}$ of the total variance. This kind of analysis is often used to explain how much commonality there is between observations.
In the specific case a Principal Component Analysis (PCA) has been run. Analysing the numbers reported in Table 5.2 it is possible to notice the great commonality present in spreads. On the entire sample, the first principal component explains more than 90% of the spread and the first three, summed together, reach 99%. It is important to highlight that, similarly for correlations, during the subprime crisis [3] the first PC increased and it explained around 96%. On the other hand, similarly to correlations, during the sovereign credit spread there was a significant decrease in PC1 and a strong increase in PC2 that reached 32.49%. This, as suggested above, could depend on the fact that during the 2011-2013 crisis, countries in the Eurozone started to move in opposite directions and not as an *unicum*, probably because of different levels of vulnerabilities and investors' reasoning related to a possible euro break-up.

From picture (2.6) the factor loadings for the different euro-countries can be observed. It appears from the first principal component that there are two groups.

[3] again, on a restricted sample

Table 2.3: Principal Components in Eurozone spreads

Time Horizon / PC	PC1	PC2	PC3
2002-2016	91.81%	5.70%	1.63%
2008-2009	96.04%	3.44%	0.45%
2011-2013	63.32%	32.49%	2.74%

The first one is composed by Italy, Spain, Portugal and Ireland. The second one from Netherlands, Austria, Finland and - even if more blurred - France and Belgium. The second principal component reflects the volatile behaviour of Ireland.

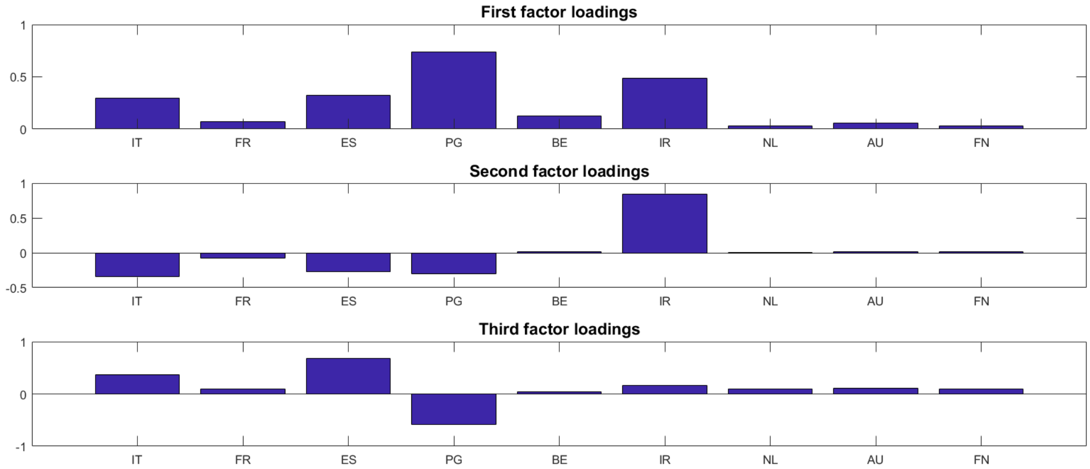

Figure 2.6: Factor loadings

2.5 Data manipulation: stationarity

In the application of standard econometric techniques to time series, it would be worth to check whether observations are stationary and deepen analysis if necessary (cfr. *infra*). In particular, to check whether variables were trend-stationary the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test was used, while Augmented Dickey-Fuller (ADF) was applied in order to check unit roots. It shall be pointed out that these tests are sometimes criticized and considered not to be fully informative, since they could often give back Type I errors. However, these tests have been run bearing in mind the limitations they might have. MATLAB has the commands `kpsstest` and `adftest`, that perform these tests. For KPSS, as suggested by Kwiatkowski et. al (1992), the test was run with a lag equal to $\sqrt{T}$ and so as lags considered have been 7 and 8. In this case, if the test gave back 0, then it was not possible to reject the null hypothesis that the series is stationary. For the ADF test the max lag is chosen as $[12 \cdot (T/100)^{1/4}]$ so was tested for lags from 1 to 11. In particular the most serious issue was observed for the stock level as it could have been imagined; both KPSS and ADF rejected the hypothesis of stationarity for different lags. In addition, also for the Libor tests the hypothesis of stationarity has been rejected. This is also intuitive looking at their autocorrelation functions in Figure 2.5. Even if autocorrelation functions are not a proper way to assess stationarity, a persistence can give the idea and the suspect that the variables are not stationary, as in this case. Therefore, for Stock, Libor and PE returns have been considered:

$$r_t = \frac{S_t - S_{t-1}}{S_{t-1}}$$

$$\Delta\%\text{Libor}_t = \frac{\text{Libor}_t - \text{Libor}_{t-1}}{\text{Libor}_{t-1}}$$

$$\Delta\%\mathrm{PE}_t = \frac{\mathrm{PE}_t - \mathrm{PE}_{t-1}}{\mathrm{PE}_{t-1}}$$

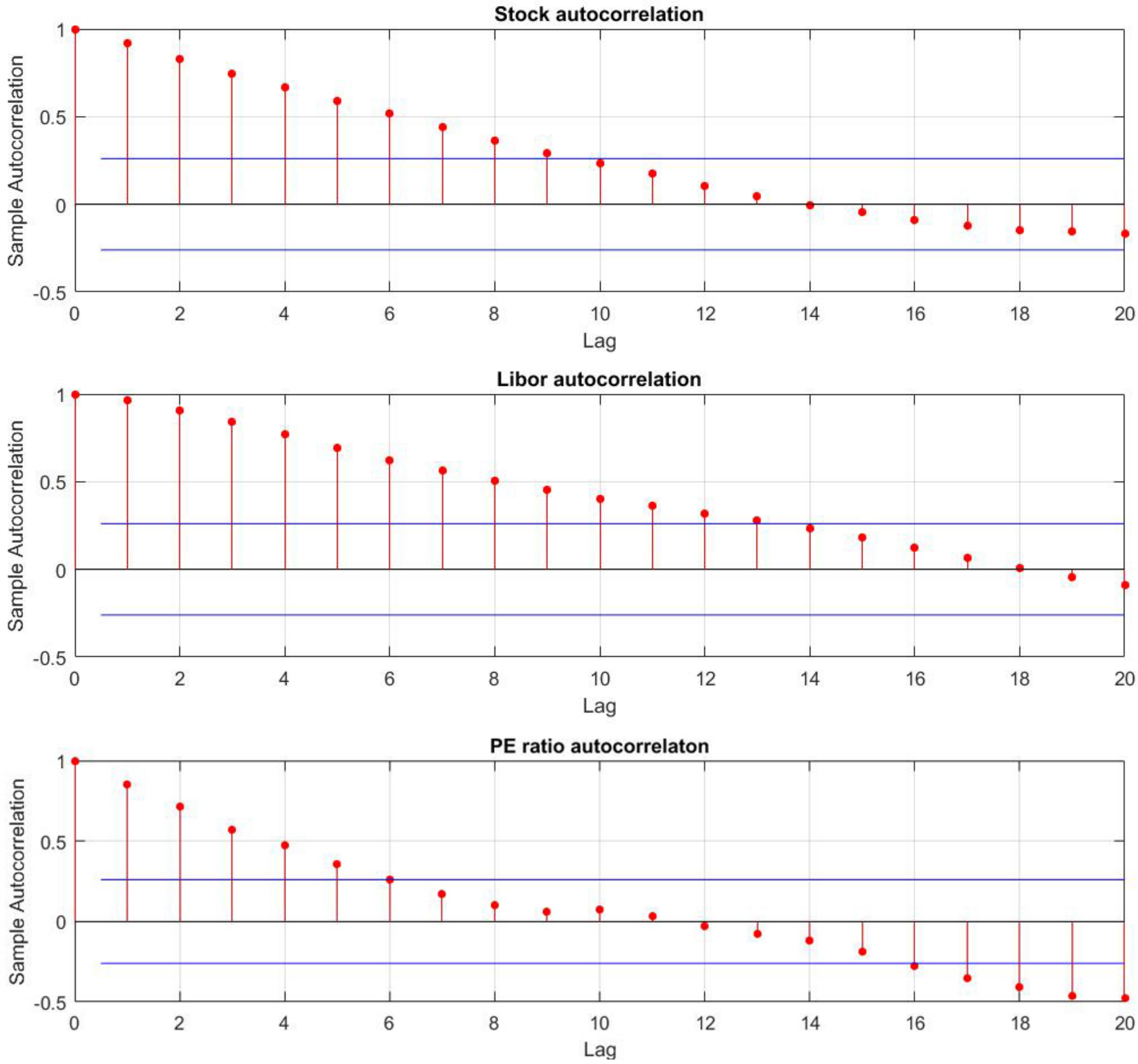

Figure 2.7: Autocorrelation of Stock and Libor

For the other variables, even if - in some cases - not with strong statistical certainty (10%), it was not possible to reject the null hypothesis of stationarity for the KPSS test. While for ADF tests VIX did not pass the test. Apart from Stock, Libor and PE that have been adjusted as described before, the other variables have not been modified, because no severe issues appeared.

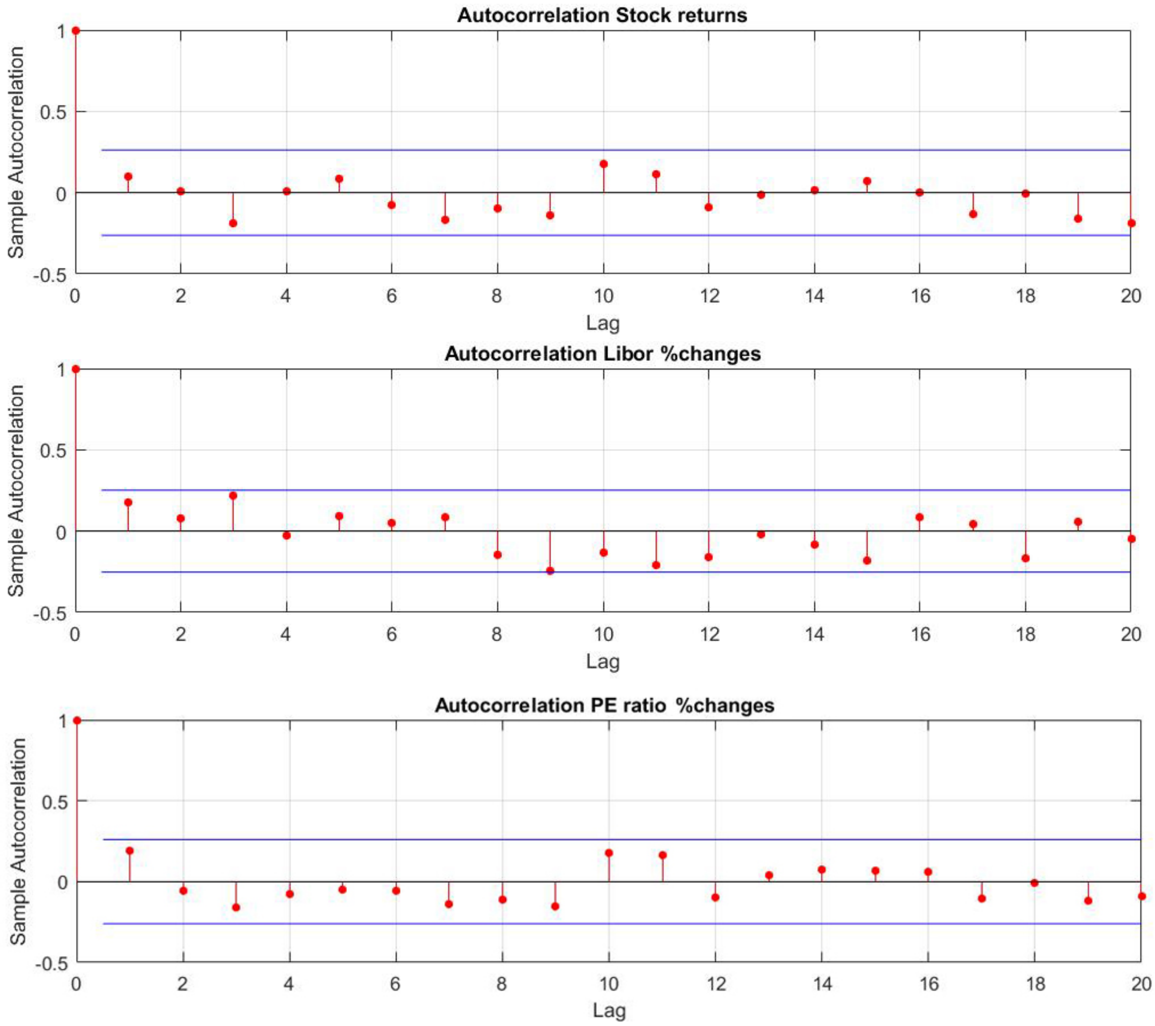

Figure 2.8: Autocorrelation of stock and libor with %change

Chapter 3

The Model

3.1 Filtering

Origins of modern filtering techniques are to be researched in the works conducted in the 40s by Kolmogorov (1941) and later by Wiener (1949). In 1960, Kalman started to build what he called "A new approach to linear filtering and prediction problems" and formalized the algorithm that then took his name. The Kalman Filter is a recursive method for estimation of measures in an uncertain dynamic system with constant update, in a system where the measured value contains random or unpredicted errors.

The Kalman filter is an optimal way among linear systems and it minimizes the mean squared errors. It reaches this purpose by giving relative weights to previous estimates and previous data. In fact, the Kalman gain represents how much importance has to be given to the new observation and how much to the previous estimate; in a simple dimensional case, it can be expressed as the error in the estimate divided by the sum of errors in the estimate and in the measurement. If we indicate the

Kalman gain as K the new estimate will weight K the measurement and $(1 - K)$ the previous estimate and - in general - we may assess that the smallest the K is the more stable the model will be. At each iteration of the process, an updated estimate is produced and it is used to track the variable we are monitoring.

Numerous applications have been done in tracking systems: the filter is often used in positioning system (like GPS) and it is included in space programs - for instance it was used for the Apollo program[1]. The filter is also used in the economic and financial field to track the evolution of (latent) variables in presence of noise.

In this work the Kalman Filter will be used to construct the two common latent factors that affect spreads in the different European countries. It is not said *ex ante* that results will increase explanatory power with respect standard procedures; for this reason, results will be compared with the outcomes from Ordinary Least Square techniques and it will be checked if an approach with use of Kalman Filter increases the power of the analysis in this specific context.

In the next section a formalization of the Kalman Filter applied to the specific case will be presented.

[1]for an extensive list of application in tracking system *cfr.* "Applications of Kalman Filtering in Aerospace 1960 to the Present" (M. Grewal and A. Andrrews, 2010)

3.2 Specification of the model: Kalman filter

It is possible to implement a Kalman filter approach and represent the system of spreads in a state-space form.

Let $\{\alpha^{(j)}\}_{j=1,...,J}$ the unobserved factors; in the specific case the model will be run with 2 factors, which is $J = 2$.

The specification of the model is the following:

$$s_{it} = \beta_{it}d_{it} + \gamma_{it}g_{it} + \delta_i^{(1)}\alpha_t^{(1)} + \delta_i^{(2)}\alpha_t^{(2)} + \varepsilon_{it} \tag{3.1}$$

$$\boldsymbol{\alpha}_t = \mathbf{F}\boldsymbol{\alpha}_{t-1} + \boldsymbol{\eta_t} \tag{3.2}$$

Being s the spread, d the deficit, g the GDP growth and αs the latent factors, for countries $i = 1, ..., I$ and times $t = 1, ..., T$.

Equation (3.1) is called *observation* (or measurement) equation while Equation (3.2) is called *transition* equation; for $i = 1, ..., I$ and $t = 1, ..., T$ representing the i-th country and t-th period (measured in quarters). In addition $\delta_i^j > 0$ is the response of different countries to the same common factors. It is required that δ is positive, in line with Aguiar M. *et al.* (2016), in order to have that all countries respond in the same way (with the same sign) to the factors.

$$\mathbf{F} = \begin{bmatrix} f_1 & 0 \\ 0 & f_2 \end{bmatrix}$$

is a $J \times J$ (in the specific case since there are two latent factors $J = 2$ is a 2×2) diagonal matrix. $\mathbf{F}$ is required to have the eigenvalues (§Appendix) inside the unit circle. But since $\mathbf{F}$ is diagonal the spectrum $\sigma(\mathbf{F})$ coincides with the elements of the diagonal that hence need to be less than one in absolute value, *i.e.* $|f_i| < 1$ for $i =$

$1,2$; so that stationarity - which is a feature that is desirable - is imposed, avoiding exploding trajectories. Moreover, $\varepsilon_{it} \sim N(0, \sigma_i^2)$ and $\eta_t \sim \mathrm{MVN}_J(0, \mathrm{diag}\ \sigma_i^2)$.

In time t it is possible to rewrite the Kalman filter that will be implemented in MATLAB. The two basic equations are:

$$\begin{cases} \mathbf{s}_t = \mathbf{B}_t + \mathbf{H}\boldsymbol{\alpha}_t + \boldsymbol{\varepsilon}_t \\ \boldsymbol{\alpha}_t = \mathbf{F}\boldsymbol{\alpha}_{t-1} + \boldsymbol{\eta}_t \end{cases} \tag{3.3}$$

Clearly, the equation is equivalent to the one previously reported, if we let:

$$\mathbf{s}_t = \begin{pmatrix} s_{1t} \\ s_{2t} \\ \vdots \\ s_{It} \end{pmatrix} \quad \mathbf{B}_t = \begin{pmatrix} \beta_{1t} d_{1t} + \gamma_{1t} g_{1t} \\ \beta_{2t} d_{2t} + \gamma_{2t} g_{2t} \\ \vdots \\ \beta_{It} d_{It} + \gamma_{It} g_{It} \end{pmatrix} \quad \mathbf{H} = \begin{pmatrix} \delta_1^{(1)} & \delta_1^{(2)} \\ \delta_1^{(1)} & \delta_1^{(2)} \\ \vdots & \vdots \\ \delta_I^{(1)} & \delta_I^{(2)} \end{pmatrix} \quad \boldsymbol{\alpha}_t = \begin{pmatrix} \alpha_t^{(1)} \\ \alpha_t^{(2)} \end{pmatrix}$$

$$\boldsymbol{\varepsilon}_t = \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \vdots \\ \varepsilon_{It} \end{pmatrix} \quad \boldsymbol{\eta}_t = \begin{pmatrix} \eta_t^{(1)} \\ \eta_t^{(2)} \end{pmatrix}$$

3.3 Derivation

In addition, to express the model in matrix form helps in checking the dimensions: $\mathbf{s_t}$ is a $I \times 1$ column vector, $\mathbf{B}_t$ is a $I \times 1$, $\mathbf{H}$ a $I \times 2$, $\boldsymbol{\alpha}_t$ a 2×1 (hence $\mathbf{H}\boldsymbol{\alpha}_t$ is a $I \times 1$) and $\boldsymbol{\varepsilon}_t$ a $I \times 1$. So the observation equation is dimensionally consistent. Also the transition equation is consistent since $\boldsymbol{\alpha}_t$ and $\boldsymbol{\alpha}_{t-1}$ are 2×1, $\mathbf{F}$ is a 2×2 matrix and $\boldsymbol{\eta}_t$ a 2×1 vector.

Regarding equation (3.3) some assumptions must be made:

$$\mathbb{E}(\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}'_\tau) = \begin{cases} \mathbf{S}_1 & \text{for } t = \tau \\ 0 & \text{otherwise} \end{cases}$$

and $\mathbf{S}_1$ is clearly a $I \times I$ matrix.

$$\mathbb{E}(\boldsymbol{\eta}_t \boldsymbol{\eta}'_\tau) = \begin{cases} \mathbf{S}_2 & \text{for } t = \tau \\ 0 & \text{otherwise} \end{cases}$$

and $\mathbf{S}_2$ is a 2×2. At this point in time is useful to define

$$\sum := \begin{pmatrix} \mathrm{VAR}(\boldsymbol{\eta}) \\ \mathrm{VAR}(\boldsymbol{\varepsilon}) \end{pmatrix} = \begin{pmatrix} \sigma_1^2 & & & \\ & \sigma_2^2 & & \\ & & \ddots & \\ & & & \sigma_{(I+2)}^2 \end{pmatrix} \tag{3.4}$$

With conditions:

$$\mathbb{E}(\boldsymbol{\eta}_t \boldsymbol{\varepsilon}'_\tau) = 0 \tag{3.5}$$

$$\mathbb{E}(\boldsymbol{\alpha}_t \boldsymbol{\varepsilon}'_\tau) = 0 \tag{3.6}$$

$$\mathbb{E}(\boldsymbol{\alpha}_t \boldsymbol{\eta}'_\tau) = 0 \tag{3.7}$$

for all t and τ.

To start the recursion it can be noted that [2]:

$$\hat{\boldsymbol{\alpha}}_{1|0} = \mathbb{E}(\boldsymbol{\alpha}_1)$$

with associated error variance:

$$\mathbf{P}_{1|0} = \mathbb{E}\{[\boldsymbol{\alpha}_1 - \mathbb{E}(\boldsymbol{\alpha}_1)][\boldsymbol{\alpha}_1 - \mathbb{E}(\boldsymbol{\alpha}_1)]'\}$$

And for a generic time t:

$$\hat{\boldsymbol{\alpha}}_{t+1|t} = \mathbb{E}(\boldsymbol{\alpha}_{t+1}|I_t) \tag{3.8}$$

$$\mathbf{P}_{t+1|t} = \mathbb{E}[(\boldsymbol{\alpha}_{t+1} - \hat{\boldsymbol{\alpha}}_{t+1|t})(\boldsymbol{\alpha}_{t+1} - \hat{\boldsymbol{\alpha}}_{t+1|t})'] \tag{3.9}$$

$$\mathbb{E}(\boldsymbol{\alpha}_t|\mathbf{I}_{t-1}) = \hat{\boldsymbol{\alpha}}_{t|t-1}$$

Where $\mathbf{I}_{t-1}$ is the Information Set at time $t-1$. It follows that the forecast can be expressed as:

$$\hat{\mathbf{s}}_{t|t-1} = \mathbb{E}(\mathbf{s}_t|\mathbf{B}_t, \boldsymbol{\alpha}_{t-1}) = \mathbf{B}_t + \mathbf{H}\hat{\boldsymbol{\alpha}}_{t|t-1}$$

With error:

$$\begin{aligned} \mathbf{s}_t - \mathbf{s}_{t-1} &= \mathbf{B}_t + \mathbf{H}\boldsymbol{\alpha}_t + \boldsymbol{\varepsilon}_t - \mathbf{B}_t - \mathbf{H}\hat{\boldsymbol{\alpha}}_{t|t-1} \\ &= \mathbf{H}(\boldsymbol{\alpha}_t - \hat{\boldsymbol{\alpha}}_{t|t-1}) + \boldsymbol{\varepsilon}_t \end{aligned} \tag{3.10}$$

[2] *cfr.* Hamilton, 1994

Using (3.10) it is possible to write the MSE as:

$$
\begin{aligned}
\mathbf{C}_t &= \mathbb{E}[(\mathbf{s}_t - \hat{\mathbf{s}}_{t|t-1})(\mathbf{s}_t - \hat{\mathbf{s}}_{t|t-1})'] \\
&= \mathbb{E}\{[\mathbf{H}(\boldsymbol{\alpha}_t - \hat{\boldsymbol{\alpha}}_{t|t-1}) + \boldsymbol{\varepsilon}_t][\mathbf{H}(\boldsymbol{\alpha}_t - \hat{\boldsymbol{\alpha}}_{t|t-1}) + \boldsymbol{\varepsilon}_t]'\} \\
&= \mathbb{E}\{[\mathbf{H}(\boldsymbol{\alpha}_t - \hat{\boldsymbol{\alpha}}_{t|t-1}) + \boldsymbol{\varepsilon}_t][(\boldsymbol{\alpha}_t - \hat{\boldsymbol{\alpha}}_{t|t-1})'\mathbf{H}' + \boldsymbol{\varepsilon}_t']\} \\
&= \mathbb{E}[\mathbf{H}(\boldsymbol{\alpha}_t - \hat{\boldsymbol{\alpha}}_{t|t-1})(\boldsymbol{\alpha}_t - \hat{\boldsymbol{\alpha}}_{t|t-1})'\mathbf{H}'] + \mathbb{E}[\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t']
\end{aligned}
\tag{3.11}
$$

Where it has been used (3.6) to conclude: $\mathbb{E}[(\boldsymbol{\alpha}_t - \hat{\boldsymbol{\alpha}}_{t|t-1})\boldsymbol{\varepsilon}_t'] = 0$.

In addtion, clearly is a $I \times I$ since we have a $(I \times 1) \times (1 \times I)$ for the first line and for the last one $\{(I\times2)\times(2\times1)\}\times\{(1\times2)\times(2\times I)\}+(I\times I) \to (I\times1)\times(1\times I)+(I\times I) \to (I \times I)$.

Recalling (3.9) it is possible to rewrite as:

$$
\mathbf{C}_t = \mathbb{E}[(\mathbf{s}_t - \hat{\mathbf{s}}_{t-1})(\mathbf{s}_t - \hat{\mathbf{s}}_{t-1})'] = \mathbf{H}\mathbf{P}_{t|t-1}\mathbf{H}' + \mathbf{S}_1 \tag{3.12}
$$

that is a compact form for the MSE of the error. It is possible to update this with new information on $\mathbf{s}_t$ using linear projections:

$$
\hat{\boldsymbol{\alpha}}_{t|t} = \hat{\boldsymbol{\alpha}}_{t|t-1} + \mathbb{E}\left[(\boldsymbol{\alpha}_t - \hat{\boldsymbol{\alpha}}_{t|t-1})(\mathbf{s}_t - \hat{\mathbf{s}}_{t|t-1})'\right]\left[\mathbb{E}(\mathbf{s}_t - \hat{\mathbf{s}}_{t|t-1})(\mathbf{s}_t - \hat{\mathbf{s}}_{t|t-1})'\right]^{-1}(\mathbf{s}_t - \hat{\mathbf{s}}_{t|t-1}) \tag{3.13}
$$

that is a $(2 \times 1) + \{(2 \times 1) \times (1 \times I)\} \times (I \times I) \times (I \times 1) \to (2 \times 1)$ as we would expect.

Using (3.10) it is possible to rewrite:

$$
\begin{aligned}
\mathbb{E}[(\boldsymbol{\alpha}_t - \hat{\boldsymbol{\alpha}}_{t|t-1})(\mathbf{s}_t - \hat{\mathbf{s}}_{t|t-1})'] &= \mathbb{E}[(\boldsymbol{\alpha}_t - \hat{\boldsymbol{\alpha}}_{t|t-1})(\mathbf{H}(\boldsymbol{\alpha}_t - \hat{\boldsymbol{\alpha}}_{t|t-1}) + \boldsymbol{\varepsilon}_t)'] \\
&= \mathbb{E}[(\boldsymbol{\alpha}_t - \hat{\boldsymbol{\alpha}}_{t|t-1})(\boldsymbol{\alpha}_t - \hat{\boldsymbol{\alpha}}_{t|t-1})'\mathbf{H}'] \\
&= \mathbf{P}_{t|t-1}\mathbf{H}'
\end{aligned}
\tag{3.14}
$$

that is $(2\times 1)\times(1\times I)\rightarrow(2\times I)$ both from first and last line. Then equation (3.13) becomes:

$$\hat{\boldsymbol{\alpha}}_{t|t}=\hat{\boldsymbol{\alpha}}_{t-1}+\mathbf{P}_{t|t-1}\mathbf{H}'(\mathbf{H}\mathbf{P}_{t|t-1}\mathbf{H}'+\mathbf{S}_1)^{-1}(\mathbf{s}_t-\mathbf{B}_t-\mathbf{H}\hat{\boldsymbol{\alpha}}_{t|t-1}) \tag{3.15}$$

which clearly is $(2\times 1)+\{(2\times 2)\times(2\times I)\}\times\{(I\times 2)\times(2\times 2)\times(2\times I)+(I\times I)\}\times\{(I\times 1)-(I\times 1)-(I\times 2)\times(2\times 1)\}\rightarrow(2\times 1)$ as expected.
For the forecast given (3.3):

$$\hat{\boldsymbol{\alpha}}_{t+1|t}=\mathbb{E}(\boldsymbol{\alpha}_{t+1}|I_t)=\mathbb{E}(\mathbf{F}\boldsymbol{\alpha}_t+\boldsymbol{\eta}_{t+1}|I_t)=\mathbf{F}\hat{\boldsymbol{\alpha}}_{t|t} \tag{3.16}$$

substituting (3.15) into (3.16) leads to:

$$\hat{\boldsymbol{\alpha}}_{t+1|t}=\mathbf{F}\hat{\boldsymbol{\alpha}}_{t|t-1}+\mathbf{F}\mathbf{P}_{t|t-1}\mathbf{H}'(\mathbf{H}\mathbf{P}_{t|t-1}\mathbf{H}'+\mathbf{S}_1)^{-1}(\mathbf{s}_t-\mathbf{B}_t-\mathbf{H}\hat{\boldsymbol{\alpha}}_{t|t-1})$$

Where $\mathbf{K}_t$ is referred as Kalman gain and defined as:

$$\mathbf{K}_t=\mathbf{F}\mathbf{P}_{t|t-1}\mathbf{H}'(\mathbf{H}\mathbf{P}_{t|t-1}\mathbf{H}'+\mathbf{S}_1)^{-1}$$

So that the forecast ends to be:

$$\mathbf{F}\hat{\boldsymbol{\alpha}}_{t|t-1}+\mathbf{K}_t(\mathbf{s}_t-\mathbf{B}_t-\mathbf{H}\hat{\boldsymbol{\alpha}}_{t|t-1})$$

with MSE:

$$\mathbf{P}_{t+1|t}=\mathbf{F}\mathbf{P}_{t|t}\mathbf{F}'+\mathbf{S}_2$$

3.4 Parameters estimation

In order to perform parameter estimation, a Maximum Likelihood Estimation (MLE) approach is used. The likelihood function is the function that indicates the probability of observing the data that have actually been observed.

The likelihood function is:

$$\mathcal{L}(\boldsymbol{\theta}, x_1, x_2, ..., x_n) = f(x_1, x_2, ..., x_n|\boldsymbol{\theta}) \tag{3.17}$$

That under hypothesis of independence leads to:

$$\mathcal{L}(\boldsymbol{\theta}, x_1, x_2, ..., x_n) = f(x_1|\boldsymbol{\theta}) \cdot f(x_2|\boldsymbol{\theta}) \cdot ... \cdot f(x_n|\boldsymbol{\theta}) = \prod_{i=1}^{n} f(x_i|\boldsymbol{\theta}) \tag{3.18}$$

According to MLE this quantity shall be maximized. However, often it is easier to find the maximum for the log transformation, since the result does not change because of the monotone transformation, and it simplifies the problem. Hence:

$$\ell(\boldsymbol{\theta}, x_1, x_2, ..., x_n) := \log\left[\mathcal{L}(\boldsymbol{\theta}, x_1, x_2, ..., x_n)\right] = \sum_{i=1}^{n} \log f(x_i|\boldsymbol{\theta}) \tag{3.19}$$

In the specific case as log likelihood to maximize will be taken the average over times:

$$\bar{\ell} := \frac{1}{T}\sum_{t=1}^{T} \ell_t \tag{3.20}$$

Where ℓ_t is the log-likelihood function at general time t.

Recalling that the density function of a multivariate normal is:

$$f(\boldsymbol{\mu}, \boldsymbol{\Sigma}) = \frac{1}{2\pi^{n/2}|\boldsymbol{\Sigma}|^{1/2}} \cdot e^{-1/2\left[(\mathbf{x}-\boldsymbol{\mu})'\boldsymbol{\Sigma}^{-1}(\mathbf{x}-\boldsymbol{\mu})\right]}$$

it is possible to apply the maximization to our case; where ε is the error (equivalent of $\mathbf{x}-\boldsymbol{\mu}$) and $\mathbf{C}$ the variance covariance matrix. Hence, reformulating the expression ends to be:

$$p(s_t|s_{t-1}) = \frac{1}{(2\pi)^{n/2}|\mathbf{C}_t|^{1/2}} \cdot e^{-1/2\left[\boldsymbol{\varepsilon}_t'\mathbf{C}_t^{-1}\boldsymbol{\varepsilon}_t\right]}$$

Taking logs:

$$\begin{aligned} \ell_t = \log p(\mathbf{s}_t|\mathbf{s}_{t-1}) &= \log\left((2\pi)^{-n/2} \cdot |\mathbf{C}_t|^{-1/2} \cdot e^{-1/2 \cdot \boldsymbol{\varepsilon}_t'\mathbf{C}_t^{-1}\boldsymbol{\varepsilon}_t}\right) \\ &= \log\left((2\pi)^{-n/2}\right) + \log\left(|\mathbf{C}_t|^{-1/2}\right) + \log\left(e^{-1/2 \cdot \boldsymbol{\varepsilon}_t'\mathbf{C}_t^{-1}\boldsymbol{\varepsilon}_t}\right) \\ &= -\frac{n}{2}\log(2\pi) - \frac{1}{2}\log(|\mathbf{C}_t|) - \frac{1}{2}\boldsymbol{\varepsilon}_t'\mathbf{C}_t^{-1}\boldsymbol{\varepsilon}_t \end{aligned} \tag{3.21}$$

By maximizing this quantity, in the end, what is obtained are the predicted time series of the two latent common factors $\boldsymbol{\alpha}$.

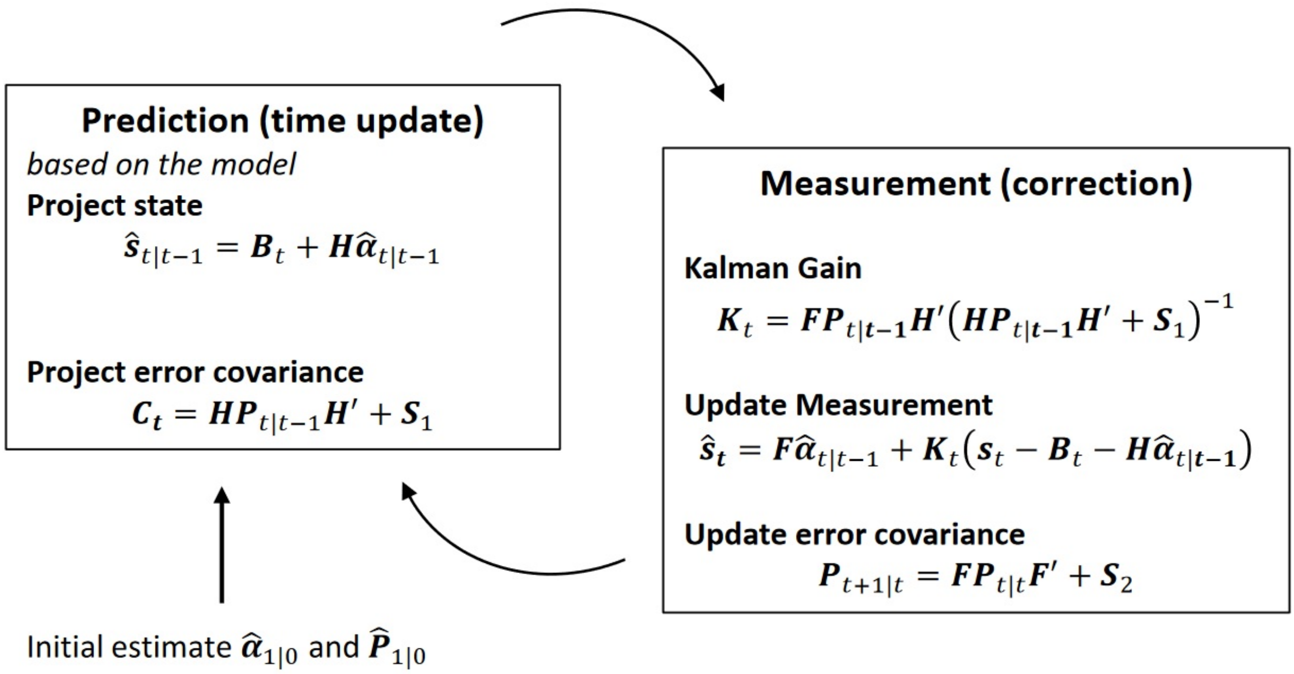

Figure 3.1: Functioning of the Kalman Filter

Chapter 4

The MATLAB implementation of the model

4.1 Preliminary steps

In order to apply the model of Kalman, some variables have to be properly defined. In particular, after having imported the data downloaded with the command `xlsread`, and having stored the returns on the indices in a variable called `IndexMatrix`, the spread matrix in basis points is obtained subtracting the German benchmark, with:

```
spreadMatrix=100*bsxfun(@minus,indexMatrix,indexMatrix(:,1))
```

At this point, it shall be discussed the $\mathbf{B}_t$ as defined in (3.3). Some authors (like Aguiar *et al.*, 2016) prefer to put a static vector $\mathbf{B}_t = \mathbf{B}$. In this analysis it has been preferred to consider a time varying parameter, because what would be optimal is to run analysis without having information on the future: analysis shall be conducted

- but this is a personal opinion - like if forecasts were run; this means that in trying to explain a variable at time t at most, factors at $t-1$ shall be used. This, clearly, is a subjective evaluation but it could increase the validity of results; for this reason a OLS with rolling window has been run to estimate $\mathbf{B}_t$. Window size has been chosen to be 12 months and coefficient are obtained running a regression of spreads on all the variables and picking the coefficients for GDP growth and deficit. The following portion of the code aims to get the time varying coefficient:

```
%Rolling regressions to get time varying B_t
%Create global empty variables that will register the values
COEFF=[];B_t=[];RSQ=[];
for i=2:10 %for the different countries (from 2 because 1 is
    Germany)
        %Create specific empty variables
        rsqi=[];rsqit=[];betai=[];betait=[];
        display('===Other country===')
        for t=13:size(spreadMatrix,1) %for the different
            times (from 13 because of the rolling window)
                DatatRegr=table(spreadMatrix(t-11:t,i),
                    debtMatrix(t-12:t-1,i),realGDPchange(t
                    -12:t-1,i),VIX(t-12:t-1,:),PE_ratio(t-12:
                    t-1,:),Libor(t-12:t-1,:),Stock_r(t-12:t
                    -1,:),trendOnline1(t-12:t-1,:),'
                    VariableNames',{'spread','debt','realgdp'
                    ,'vix','pe','libor','stock','trendonline'
                    });
                Mdl=fitlm(DataRegr,'spread~debt+realgdp+vix+
                    pe+libor+stock+trendonline+trendonline^2'
                    )
                betait=[betait; [Mdl.Coefficients.Estimate
                    (2) Mdl.Coefficients.Estimate(3)]]
                rsqi=Mdl.Rsquared.Adjusted;
                rsqit=[rsqit;rsqi];
        end

        B_ti=bsxfun(@times,betait(:,1)',debtMatrix(size(
            debtMatrix,1)-size(betait,1):size(debtMatrix,1)
            -1,i)')+bsxfun(@times,betait(:,2)',realGDPchange(
            size(realGDPchange,1)-size(betait,1):size(
            realGDPchange,1)-1,i)')
        B_t=[B_t; B_ti]; %This is the vector of the specific
             component refered as B_t
        COEFF=[COEFF betait]; %this is the time-varying
            vector of coefficients

        RSQ=[RSQ rsqit]; %this will be the vector of R^2
```

```
end
```

It is worth underlying that the `for` cycle starts from 13 and when $t = 13$ the `spreadMatrix` is taken from 2 to 13 while the explanatory variables X are taken from 1 to 12: there is a lag of one period, because -as stated- variables in $t-1$ shall be used to predict/explain spread at time t. The coefficients of the regressions are obtained with standard OLS (*cfr.* §Appendix) and finally, the vector $\mathbf{B}_t$ as in (3.3) is obtained by applying the equation of the regression to the values of the regressors and stored in `COEFF`. Furthermore, adjusted-R^2 [*cfr.* (5.1)] are gathered using the function `fitlm` and stored in the variable `RSQ`. This will be the starting brick of the model which should be able to outperform standard OLS. In this latter case, including all the variables that will after be used to explain the common factors, the average obtained adjusted-R^2 is around 37%. This is the benchmark which the model that will be presented (*cfr. infra*), using Kalman filtering, will try to improve. To get the coefficients the regression runned has been:

$$Spread_{it} = \beta_{0,it} + \sum_j \beta_{j,it} \cdot Variable_{j,it}$$

for $j = 2, ..., 10$ representing the country. The functional form has been chosen by a stepwise mechanism of maximization of the adjusted-R^2 by inserting the variables, their squares and their cross products of maximum order 2. Talking at an aggregate level, with regression run on the entire sample coefficients for debt[1] resulted positive and for GDP resulted negative - as it could have been imagined. A higher deficit is associated with a higher spread and a higher real GDP growth is associated with a lower spread. This holds for all countries with the only exception of Ireland that has a negative, even if not statistically significant, coefficient for the deficit; this could be due to the high volatility of Irish economy. At the end of this process the vector

[1] which is the public deficit;

$\mathbf{B}_t$ of dimension (47×9) is obtained. Of course 47 is 59-12 and 9 is the number of effective countries.

In table 4.1 the results from basic regressions are reported. It can be seen that the average explained variance is around 37%.

Table 4.1: Explained variance with basic OLS model

Country	IT	FR	ES	PG	BEL	IR	NET	AU	FN
Adjusted R^2	37.23%	44.50%	54.46%	29.85%	43.72%	14.52%	37.03%	32.32%	38.72%

4.2 Analysis for checking heteroskedasticity

As previously argued, some variables have been made stationary by taking returns. However, there might be the problem of residual heteroskedasticity. For this reason two checks will be made. Residuals will be plotted and analysed graphically and a robust regression will be run to see whether results are different. Mean of residuals is smaller than 10^{-13}, consistently with the intercept term; in addition, no deterministic patterns seem to emerge in the residuals. Residuals have been fitted with a polynomial of order 4 and coefficients, among with 95% confidence interval, are reported in Table 4.2. No case appears where confidence interval boundaries have the same sign; 0 is always in the interval, suggesting that there are nor positive nor negative significant coefficients. This suggests that there are not particular issues in applying OLS techniques to time series.

Table 4.2: Coefficients and confidence intervals of grade 4 polynomial fitting

Country/Coefficient	**Coefficient**	**Coefficient Value**	**Lower 95%**	**Upper 95%**
	c1	1.96	-17.3	21.22
Italy	c2	-0.47	-1.79	0.84
	c3	0.02	-0.01	0.05
	c4	-0.00	-0.00	0.00
	c1	1.78	-3.18	6.74
France	c2	-0.18	-0.53	0.15
	c3	0.01	-0.00	0.02
	c4	-0.00	-0.00	0.00
	c1	-0.42	-18.05	17.23
Spain	c2	-0.36	-1.56	0.83
	c3	0.02	-0.01	0.04
	c4	-0.00	-0.00	0.00
	c1	-8.19	-63.27	46.89
Portugal	c2	0.27	-3.48	4.02
	c3	0.01	-0.09	0.10
	c4	-0.00	-0.00	0.00
	c1	-0.91	-8.32	10.14
Belgium	c2	-0.14	-0.77	0.49
	c3	0.01	-0.01	0.02
	c4	-0.00	-0.00	0.00
	c1	-11.22	-54.01	31.57
Ireland	c2	0.39	-2.52	3.30
	c3	0.00	-0.07	0.07
	c4	-0.00	-0.00	0.00
	c1	-0.16	-2.75	2.43
Netherlands	c2	0.03	-0.15	0.20
	c3	-0.00	-0.01	0.00
	c4	0.00	-0.00	0.00
	c1	1.74	-2.35	5.79
Austria	c2	-0.08	-0.36	0.20
	c3	0.00	-0.01	0.01
	c4	-0.00	-0.00	0.00
	c1	-0.05	-2.72	2.64
Finland	c2	0.06	-0.12	0.25
	c3	-0.00	-0.01	0.00
	c4	0.00	-0.00	0.00

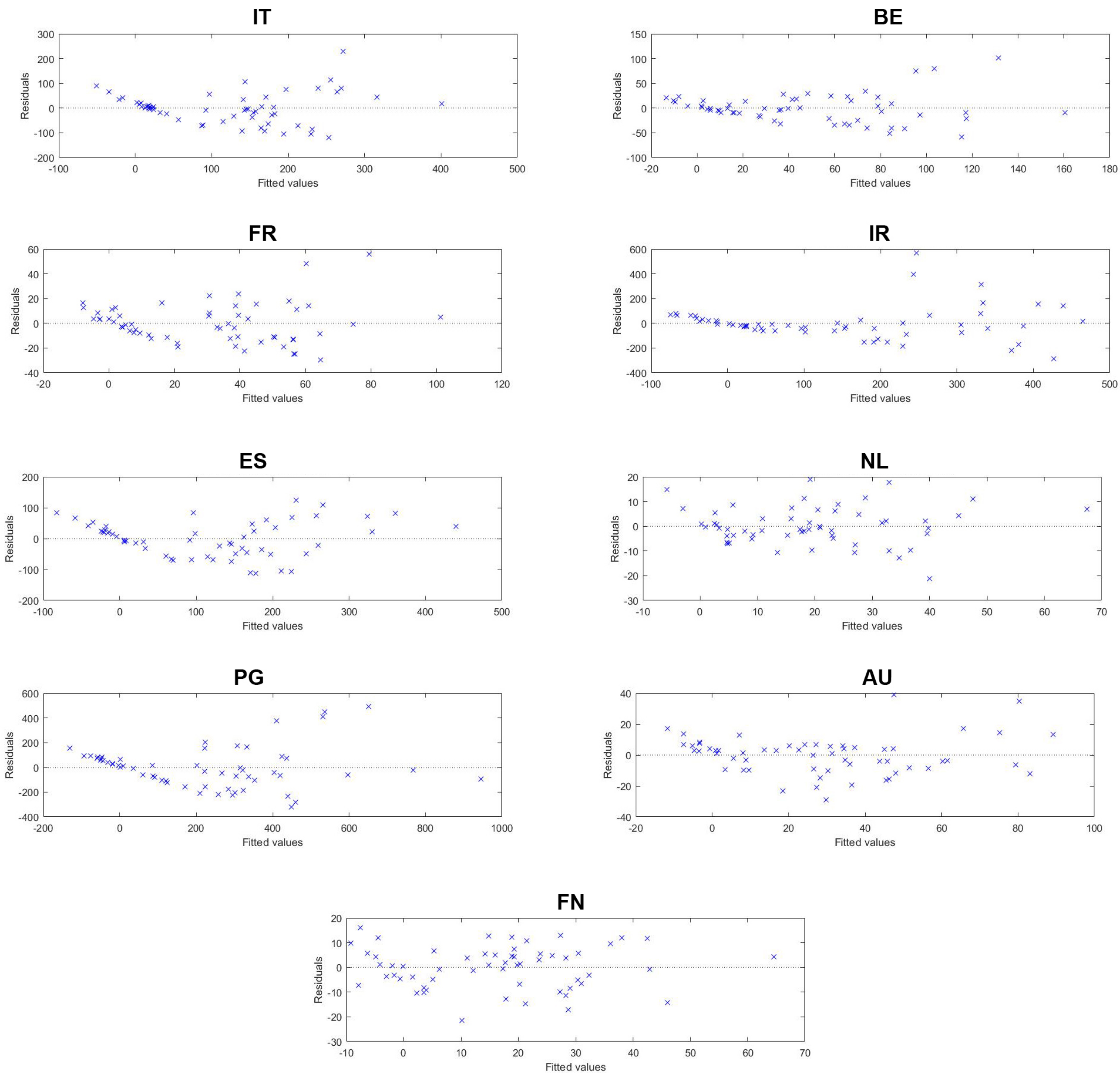

Figure 4.1: OLS regression residuals

4.3 The Kalman Function

To apply the Kalman Filter - as formalized before - in MATLAB the *KALCVF* function (Karibzhanov, 2003) has been used and extended in order to allow for a time-varying vector $\mathbf{B}_t$ [2] In particular the code that implements the Kalman filter is:

```
%This code has been edited from the original version by Karibzhanov, 2003
for t=1:T %create a for cycle from 1 to T
        D = H*P*H'+R; %mean square error matrix of the prediction error
        dy = data(:,t)-H*z-b(:,t); %estimation of the error accounting for a time varying B vector
        ddy = D\dy; %for the log likelihood
        L = L+log(det(D))+dy'*ddy; %log likelihood
        if nout==5
                PH = P*H';
                filt(:,t) = z+PH*ddy;
                vfilt(:,:,t) = P-PH/D*PH';
        end
        if t<T || lead>0
                FP = F*P;
                FPHG = FP*H'+G;
                z = F*z+FPHG*ddy+a; %prediction
                P = FP*F'-FPHG/D*FPHG'+V; %variance matrix of prediction
                P = (P+P')/2;
                if nout>1
                        pred(:,t+1) = z;
                        vpred(:,:,t+1) = P;
                end
        end
end

if lead>1 && nout>1
        for t=T+2:T+lead
                z = F*z+a;
                P = F*P*F'+V;
                pred(:,t) = z;
                vpred(:,:,t) = P;
        end
end

        L = -(Ny*log(2*pi)+L/T)/2; %update Likelihood
```

[2] I thank the author of the package Dr. Karibzhanov for some helpful suggestions;

4.4 Core Model

At this point to solve for the maximum likelihood a Monte Carlo approach, with different starting points, is used with the objective function `lolgl`:

```
%Randomize the starting point
J=45000;
paramNum=31;
magnitude=[10^-10*ones(paramNum,round(J/15)) 10^-9*ones(paramNum,round(J/15)) 10^-8*ones(paramNum,round(J/15)) 10^-7*ones(paramNum,round(J/15)) 10^-6*ones(paramNum,round(J/15)) 10^-5*ones(paramNum,round(J/15)) rand*10^-5*ones(paramNum,round(J/15)) 10^-4*ones(paramNum,round(J/15)) rand*10^-4*ones(paramNum,round(J/15)) 10^-3*ones(paramNum,round(J/15)) 10^-2*ones(paramNum,round(J/15)) 10^-1*ones(paramNum,round(J/15)) 10^-abs(randn/2)*ones(paramNum,round(J/15))];
magnitude=bsxfun(@times,magnitude,rand(paramNum,J*13/15));
randMatrix=[];
for h=1:J/15*2
        randList=[];
        for j=1:paramNum
                index = randsample(1:size(magnitude,1), 1, true);
                index2 = randsample(1:size(magnitude,2), 1, true);
                randList=[randList magnitude(index,index2)];
        end
        randMatrix=[randMatrix; randList];
end
startM=[magnitude randMatrix']; %starting random Matrix


%Kalman begins
data_fk=spreadMatrix(14:end-1,2:end)';
a_fk=zeros(2,1);
b_fk=B_t
lead_fk=0
logl_e=0;Sigma_e=[];

%Restrictions
%All parameters positive
A_c=-eye(33,31);
b_c=-10^-2*ones(1,33);

%Element of F matrix less than one
A_c(32,1)=1;A_c(33,2)=1;
b_c(1,33)=0.99;b_c(1,32)=0.99; %less than 1
b_c(1,1)=1;b_c(1,2)=1; %greater than -1, x has neg. coeff.
```

```
for j=1:J %Here we do MCMC
        start=startM(:,j); %starting Vector
        %Let's do the minimization start will change from time to time
        %Try to change alghoritm and see if liklihood increases
        options=optimoptions('fmincon','MaxFunctionEvaluations',5000);

        params=fmincon(@(params) -kalcvf(data_fk, lead_fk, a_fk, [params(1) 0; 0 params(2)], b_fk, reshape(params(3:20),9,2), diag(params(21:31))),start',A_c,b_c,[],[],[],[],[],options); %do the maximization of the likelihood

        params1=params(1);
        params2=params(2);
        .
        .
        .
        params29=params(29);
        params30=params(30);
        params31=params(31);

        F_es=[params1 0; 0 params2];

        H_es=[params3 params4;params5 params6; params7 params8; params9 params10; params11 params12; params13 params14; params15 params16; params17 params18; params19,params20];

        Sigma_es=diag([params21 params22 params23 params24 params25 params26 params27 params28 params29 params30 params31]);

        [logl, pred, vpred, filt , vfilt]=kalcvf(data_fk, lead_fk, a_fk, F_es, b_fk, H_es, Sigma_es);
        pred=pred';

        if logl<logl_e & isreal(logl) %If the log-likelihood is smaller (it is a negative quantity) than the previous one save the results
                display('==========================')
                logl_e=logl %Save effective logl
                F_e=F_es %Save effective F matrix
                H_e=H_es %Save effective H matrix
                Sigma_e=Sigma_es %Save effective var-cov matrix
                predd=pred %Save the obtained prediction
                filtt=filt' %Save obtained filt

        end

end
```

This is an application of the theoretical model as seen before. Vectorizing code is used and first of all a matrix `startM` is randomly constructed and it will contain the starting vector for every simulation. The starting point shall be a vector where the function is well defined and not particularly large, for instance we would like the $\mathbf{F}$ matrix to be stationary, so it would not make sense to start with values greater than 1 in absolute value. Many simulations have been done and it seems that the starting random matrix given as stated in line 5 is a good compromise. It is a manual way to do what also some packages do, but highlighting the idea behind it[3]. Each simulation from 1 to J will pick a row of the matrix and will try to maximize the `logl` parameter given that specific starting point. The maximization is done using the function `fmincon`. In MATLAB, there is not a buildin function for maximization, hence it is done minimizing the function with the negative sign: `fmincon -kalcvf` maximizes the output of the function *i.e.* `logl`. If the obtained value is greater [4] than the previous registered, the `if` statement starting in line 63 registers the new value and the associated parameters. Constraints require that every element is positive; `fmincon` admits as linear constraints $\mathbf{Ax} \leq \mathbf{b}$ by giving $\mathbf{A} = -\mathbf{I}$ where $\mathbf{I}$ is the identity matrix we obtain $\mathbf{x} \geq -\mathbf{b}$ by giving b very small negative numbers (excluding zero), like -0.01 we end up to have:

$$\begin{pmatrix} x_1 & & & & \\ & x_2 & & & \\ & & \ddots & & \\ & & & x_{n-1} & \\ & & & & x_n \end{pmatrix} \geq \begin{pmatrix} 10^{-2} \\ 10^{-2} \\ \vdots \\ 10^{-2} \\ 10^{-2} \end{pmatrix} \tag{4.1}$$

and this constraint ensures that all variables are positive. At this point it is required

[3] it could have also been done by using Sobol numbers, for instance, but for the purposes here described it seems reasonable the compromise applied;

[4] I refer to maximization and greater because I reason in terms of the theoretical model; it has been shown that in reality from a MATLAB point of view it is a minimization and hence, instead of greater it should be said smaller, since we are minimizing a negative quantity;

that the eigenvalues of the **F** matrix shall be in absolute value less than one as seen in the theoretical discussion. Even if this is not crucial because - in any case - the maximization autonomously finds values less than one in absolute value, constraints are set requiring the first two elements of the diagonal constraint matrix (1,1) and (2,2) to be the same but (1,1) (2,1) of the **b** vector to be 0.99 so that they are greater, when the sign is changed, than -0.99. Furthermore, two other lines are added to **A** and **b** to ensure that the elements of **F** are smaller than 1. So the last two elements of **A** are positive ones and also the two last elements of **b** are positive 0.99. So that $\mathbf{Ax} \leq \mathbf{b}$ ensures $x_1, x_2 \leq 0.99$ [5].

[5]recall that x_1 and x_2 are the first two parameters which are f_1 and f_2;

Chapter 5

Results

5.1 Model estimation

The model has been estimated by performing the Monte Carlo on 45 000 iterations. The obtained final value of `logl` is $-5.32 \cdot 10^7$. The procedure, as outlined before, estimated $\mathbf{F}$, $\mathbf{H}$, $\sum$ and the two common latent factors $\boldsymbol{\alpha}_1$ and $\boldsymbol{\alpha}_2$. Below estimates are reported:

$$\hat{\mathbf{F}} = \begin{bmatrix} 0.9876 & 0 \\ 0 & 0.9335 \end{bmatrix}$$

$$\hat{\mathbf{H}} = \begin{pmatrix} 0.0037 & 0.0384 \\ 0.0072 & 0.1636 \\ 0.0169 & 4.8158 \\ 0.0948 & 0.0079 \\ 0.0356 & 0.0008 \\ 0.0576 & 1.3016 \\ 0.1451 & 0.2576 \\ 0.1550 & 0.6095 \\ 3.4291 & 0.0400 \\ 0.1272 & 0.0346 \end{pmatrix}$$

$$\hat{\sum} = \begin{pmatrix} 3.9517 &&&&&&&&&& \\ & 3.4755 &&&&&&&&& \\ && 5.2376 &&&&&&&& \\ &&& 2.2340 &&&&&&& \\ &&&& 5.9671 &&&&&& \\ &&&&& 0.0011 &&&&& \\ &&&&&& 3.4959 &&&& \\ &&&&&&& 0.0020 &&& \\ &&&&&&&& 1.8182 && \\ &&&&&&&&& 3.0010 & \\ &&&&&&&&&& 1.6919 \end{pmatrix}$$

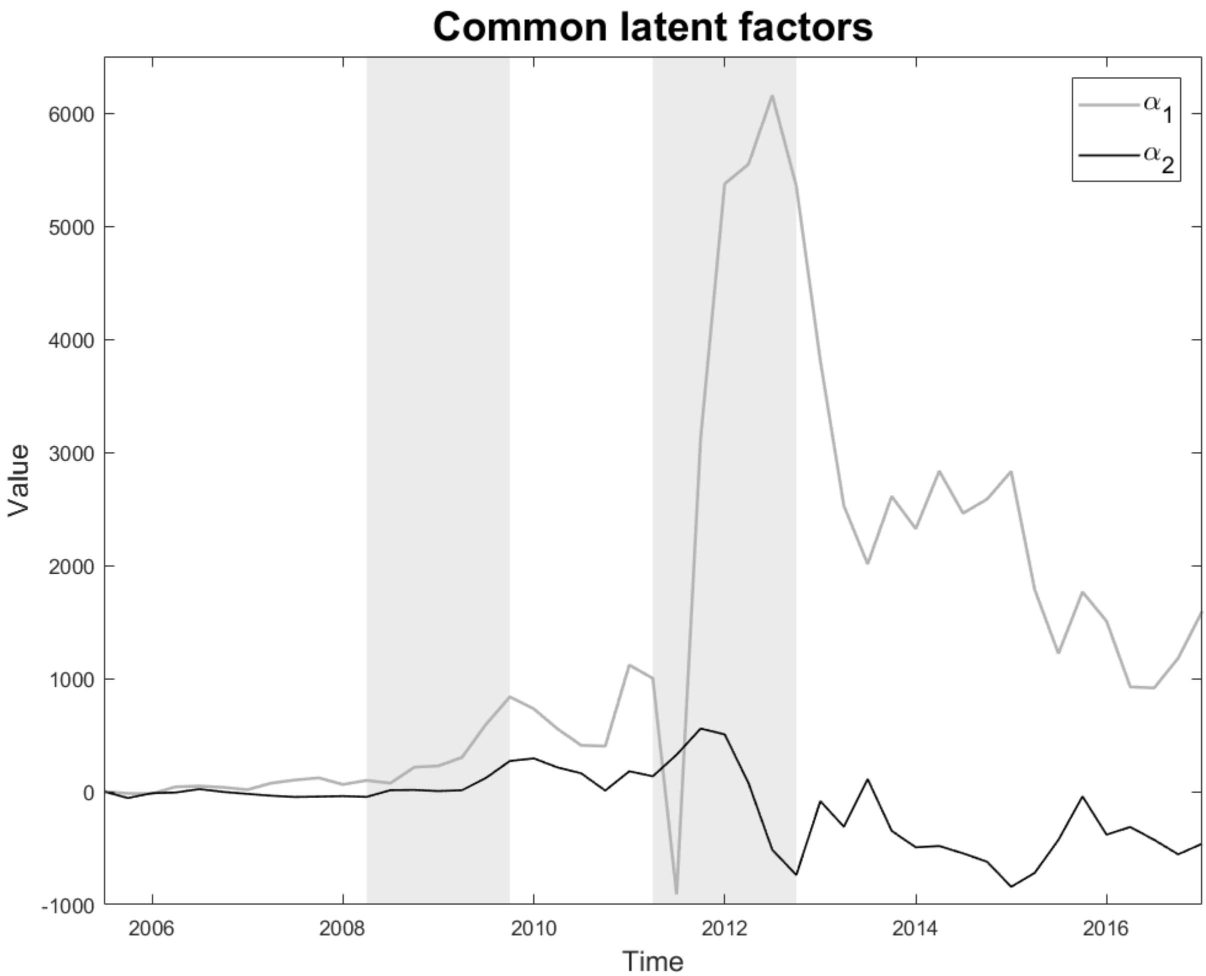

Figure 5.1: Latent factors

From Figure (5.1) it can be seen that the two factors remained relatively similar up to the sovereign crisis of 2012 when they significantly diverge with an important spike of $\boldsymbol{\alpha}_1$.

5.2 The R^2 as goodness of fit

In data analysis, it could be useful to understand how much each independent variable affects the dependent variable that shall be explained. From a linear regression it can be obtained the R^2 coefficient (sometimes also called coefficient of determination) that is defined, for a dependent variable y, as:

$$R^2 := 1 - \frac{\sum_i e_i^2}{\sum_i (y_i - \bar{y})^2} \tag{5.1}$$

Where $e_i := y_i - \hat{y}_i$ letting $\hat{y}_i$ the prediction. It measures the portion of variance that is explained by the model. In fact, when the errors are zero, their squared sum is zero and hence the negative term in the formula disappears, which implies that the R^2 is 1 and that 100% of the variance is explained by the model, that means that the model perfectly explains the data. In general, it measures the goodness of fit of a given model. One of the main issues of R^2 is that it increases as one increases the number of variables in the model. For this reason, often, it is used the adjusted-R^2 or other tests like F-statistic. [1]

[1] Another way in which the R^2 could be seen, for the bidimensional case, is as the square of the correlation coefficient, also called Pearson's coefficient, the fraction of the covariance over the product of standard deviations:

$$\rho_{x,y} := \frac{\sigma_{xy}}{(\sigma_x \cdot \sigma_y)}$$

This can be proved, recalling some OLS properties: $y = \hat{y} + e$ and $cov(\hat{y}, e) = 0$,

$$\begin{aligned} \rho^2_{y,\hat{y}} = \frac{\sigma^2_{y\hat{y}}}{(\sigma^2_{\hat{y}} \cdot \sigma^2_y)} &= \frac{\mathrm{COV}(\hat{y}, \hat{y}+e) \cdot \mathrm{COV}(\hat{y}, \hat{y}+e)}{\mathrm{VAR}(y) \cdot \mathrm{VAR}(\hat{y})} \\ &= \frac{\mathrm{COV}(\hat{y}, \hat{y}) \cdot \mathrm{COV}(\hat{y}, \hat{y})}{\mathrm{VAR}(y) \cdot \mathrm{VAR}(\hat{y})} \\ &= \frac{\mathrm{VAR}(\hat{y})^2}{\mathrm{VAR}(y) \cdot \mathrm{VAR}(\hat{y})} \\ &= \frac{\mathrm{VAR}(\hat{y})}{\mathrm{VAR}(y)} = R^2 \end{aligned} \tag{5.2}$$

where it was used: $\mathrm{COV}(A, B+C) = \mathbb{E}[A(B+C)] - \mathbb{E}[A] \cdot \mathbb{E}[B+C] = \mathbb{E}[AB] + \mathbb{E}[AC] - \mathbb{E}[A]\mathbb{E}[B] - \mathbb{E}[A]\mathbb{E}[C] = \mathrm{COV}(A, B) + \mathrm{COV}(A, C)$.

5.3 Variance decomposition

Once the R^2 has been defined and interpreted as the portion of variance that is explained by the model, it would be interesting to understand how much each regressor contributes to this measure (Lee and Lee, 2017) and which is the contribution of the single variables in explaining the regressand. This process is often called variance decomposition. Lindeman, Merenda, and Gold in 1980 proposed an algorithm based on sequentially add regressors to get sequences of explained variances and inferring the portion that a single regressor can explain (Gromping, 2007), this process is often called as LMG, from the authors' initials.

5.4 An R package: `relaimpo`

The variance decomposition problem is usually a complex one both from a theoretical and computational point of view. Grömping (2015) implemented the LMG approach in an R package called `relaimpo` with the approach LMG. The following code dynamically generates variance decomposition for the different countries in scope:

```r
#Application of relaimpo package, thesis SManduchi 01/2017

#install packages
install.packages(relaimpo)
install.packages(readxl)
install.packages (stringr)
#run packages
library(relaimpo)
library(readxl)
library(stringr)

dataset <- read_excel("Data.xlsx") #import database as a table


#define countries and their label
countries=c("ITA","FR","ES","PG","BEL","IR","NET","AU","FN")

#run a for cycle that generates the commands
for (i in countries){


cmdLinMod= paste("linmod_",i," <- lm(Spread_",i," ~ Debt_",i,"+ GDP_",i," + alpha1 + alpha2, data = dataset)") #variables are saved in Data.xlsx as Variable_CountryCode
cmdSummaryLm=paste("summary(linmod_",i,")")
cmdVarDec=paste("varDecomposed_",i," <- calc.relimp(linmod_",i,", type = c(\"lmg\", \"first\", \"last\", \"betasq\", \"pratt\"))")

#clean commands
cmdLinMod=str_replace_all(string=cmdLinMod, pattern=" ", repl="")
cmdSummaryLm=str_replace_all(string=cmdSummaryLm, pattern=" ", repl="")
cmdVarDec=str_replace_all(string=cmdVarDec, pattern=" ", repl="")

#evaluate/print commands
eval(parse(text=cmdLinMod))
eval(parse(text=cmdSummaryLm))
}
```

For instance, the generated command that runs the linear model for Italy turns out to be:

```
linmod_ITA <- lm(Spread_ITA ~ Debt_ITA + GDP_ITA +
    alpha1 + alpha2, data = dataset)
```

After, by using the `calc.relimp` command it is possible to obtain the variance decomposition of the previous estimated linear model:

```
varDecomposed_ITA<-calc.relimp(linmod_ITA,type=c("
    lmg","first","last","betasq","pratt"))
```

The previous line of code does the variance decomposition for different algorithms. Only LMG will be considered.

5.5 Spreads variance decomposition results

The variance decomposition run for spreads (Table 5.1) shows that the first factor $\boldsymbol{\alpha}_1$ explains a high portion of spread movements. The portion of explained variance by $\boldsymbol{\alpha}_1$ is high for Italy, Portugal and Ireland, while it is a bit lower for Spain (0.37). It is high for France and Belgium too that, in this sense, show a behaviour comparable to the periphery countries rather than the core ones. $\boldsymbol{\alpha}_2$ is of lower importance and cross-country it accounts for an average 10% of the variance. It can be seen that the goodness of the model significantly increases with respect to the standard OLS approach. The adjusted-R^2 increases from the 37% of the standard model to the 70% by using latent factors estimated by the Kalman model.

Table 5.1: Variance decomposition of explained spread

Country / Variable	**GDP**	**Deficit**	$\boldsymbol{\alpha}_1$	$\boldsymbol{\alpha}_2$	**Adj-$\boldsymbol{R}^2$**
ITA	0.12	0.02	0.58	0.11	0.83
FR	0.07	0.05	0.57	0.12	0.81
SPA	0.16	0.08	0.37	0.07	0.68
PG	0.12	0.02	0.69	0.06	0.89
BEL	0.07	0.02	0.61	0.08	0.78
IR	0.02	0.14	0.57	0.13	0.86
NET	0.16	0.10	0.16	0.06	0.48
AU	0.07	0.04	0.39	0.07	0.57
FN	0.19	0.08	0.26	0.06	0.59

An interesting aspect would be to study the evolution, over time, of the explained variance by the four regressors (GDP, Deficit, $\boldsymbol{\alpha}_1$ and $\boldsymbol{\alpha}_2$) on the regressand (the spreads). To this purpose, regressions have been run with rolling windows of one year and registering the variance decomposition for each country and for every time starting from middle 2008[2].

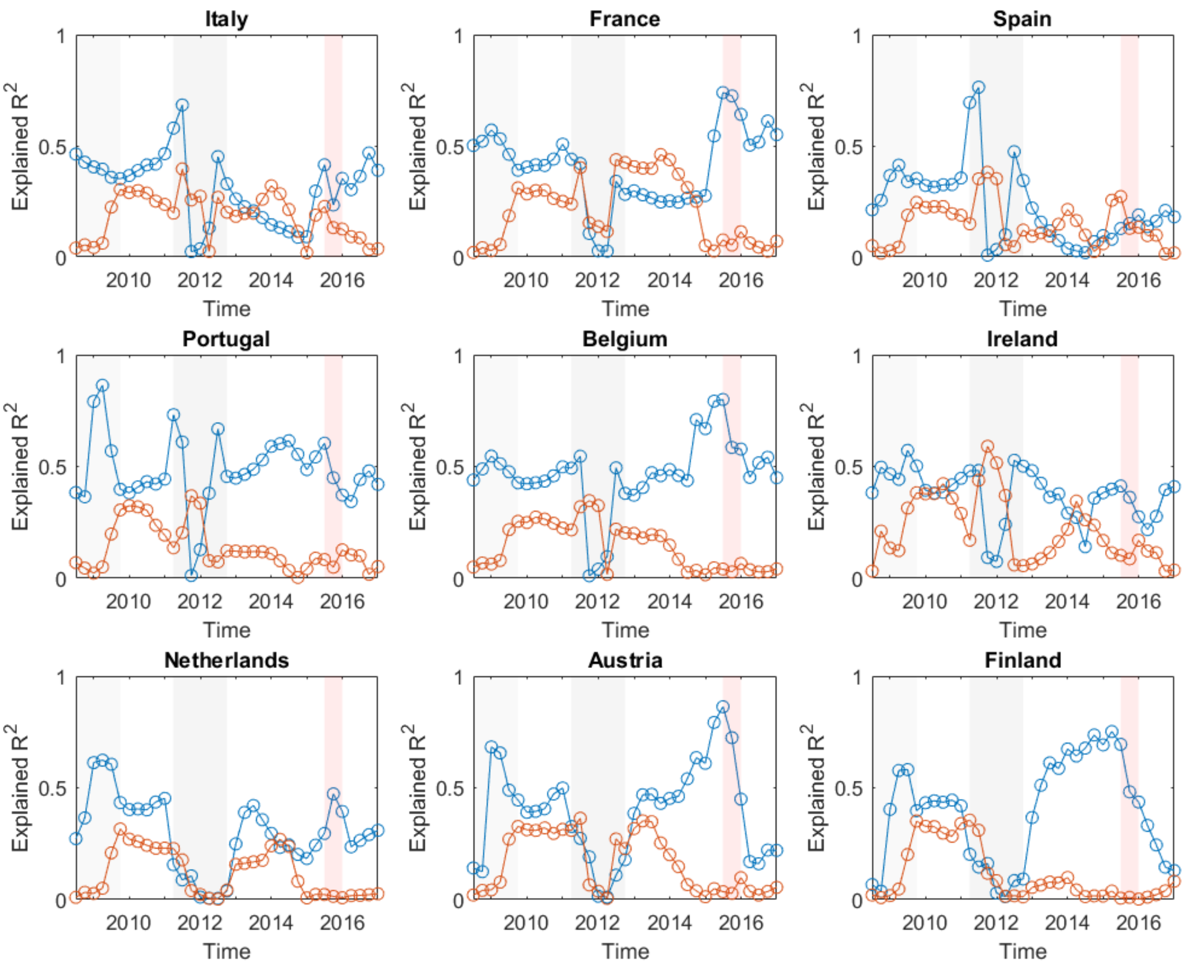

Figure 5.2: Explained variance of $\boldsymbol{\alpha}_1$ (blue) and $\boldsymbol{\alpha}_2$ (orange)

[2]in fact, the sample is reduced because some initial data are used to estimate the latent factors and some other data are used to create the first rolling window;

It might be seen from the graphs that the power of explanation is quite high for $\boldsymbol{\alpha}_1$ and that also GDP, as local variable, has a significant impact. The latent factors lose a lot of power during crisis. Indeed, in 2012 there is a drop that then recovers in the following years.

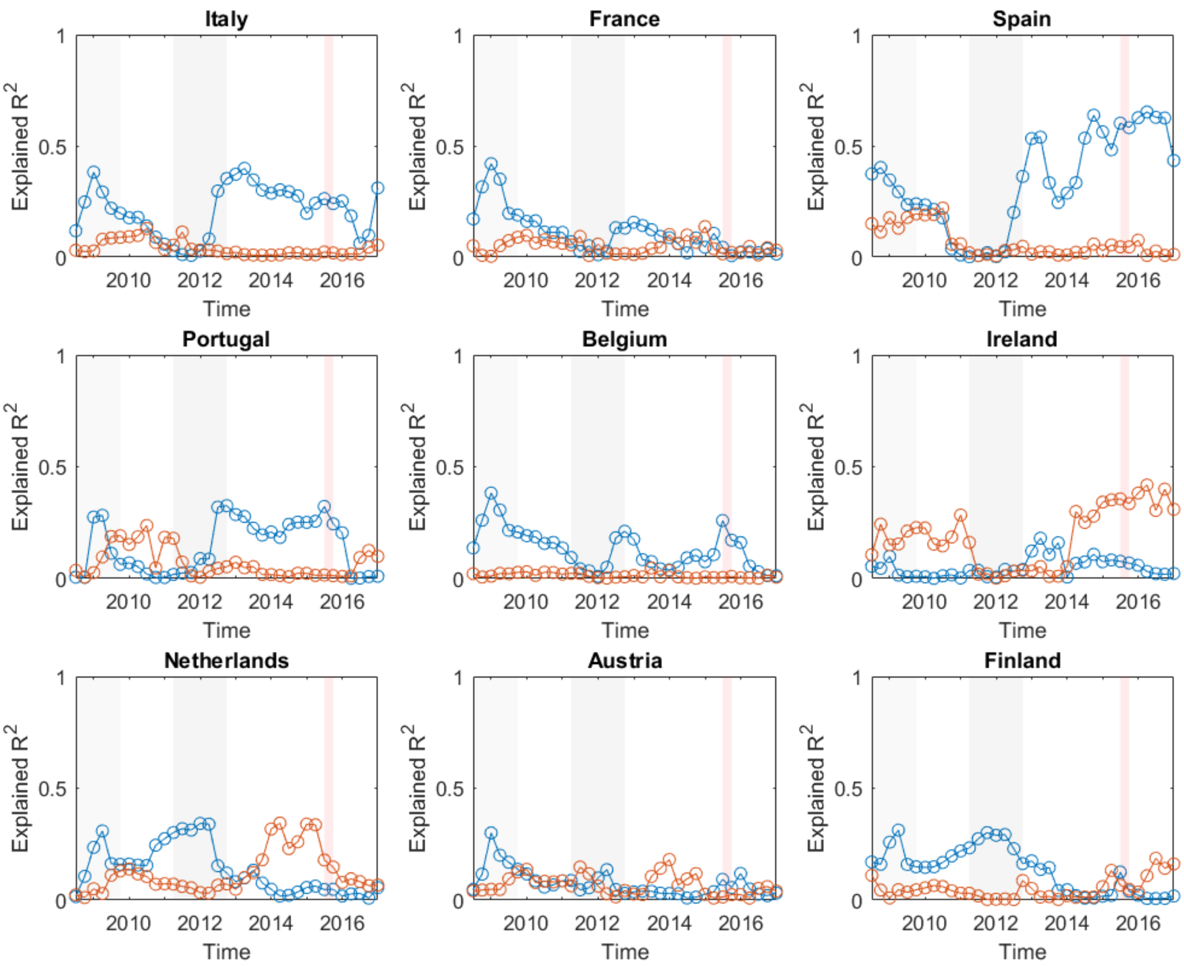

Figure 5.3: Explained variance of GDP (blue) and Debt (orange)

5.6 Decomposing $\boldsymbol{\alpha}_1$ and $\boldsymbol{\alpha}_2$

Once it has been seen that the explicative power of the latent factors is quite high and that they could be useful in modelling spreads in the Eurozone; a subsequent interesting question could be in which way these two latent factors may be explained. It appears that $\boldsymbol{\alpha}_1$ is linked to the fear of euro break-up (as measured by Google researches). A non-linear relation (in variables, not in coefficients) emerges for the online trend variable, since also the second power has a good impact on spreads. The TrendOnline variable itself (considering first and second power) explains more than the 30% of the first latent factor. In addition, the stock variable explains the 5% of the factor. On the other hand, $\boldsymbol{\alpha}_2$ is strongly dependent on the VIX and this latter explains the 16% of the first. In general, the two latent factors seem to reflect the fear of a failure of the common currency and the global uncertainty of the economy.

Table 5.2: Variance decomposition of latent factors

Variable / Factor	$\boldsymbol{\alpha}_1$	$\boldsymbol{\alpha}_2$
Price Earning	0.02	0.01
Libor	0.01	0.01
VIX	0.01	0.16
Stock	0.05	0.01
TrendOnline	0.19	0.03
TrendOnline^2	0.13	0.02
Total Adj-R^2	0.41	0.24

5.7 *Quanto* spread

Usually CDS on European countries trade in dollar terms, but they are also available in euros. The latter are cheaper than the first ones because they give a smaller protection. In fact, if one of the euro countries fails, there will likely be also troubles for the currency. *Quanto* spread is the difference between CDS traded in dollars and CDS traded in euros. It shall be underlined that some components might depend on the liquidity. However, in general, *Quanto* might be seen as a proxy of expected euro depreciation in the scenario where euro should break-up [3]. It was decided to study the relationship between latent factors and the Italian *Quanto* since it is one of the countries that might be linked the most to the probability of a euro break-up and its behaviour will affect the entire Eurozone.
Below a plot of the Italian *Quanto* is reported:

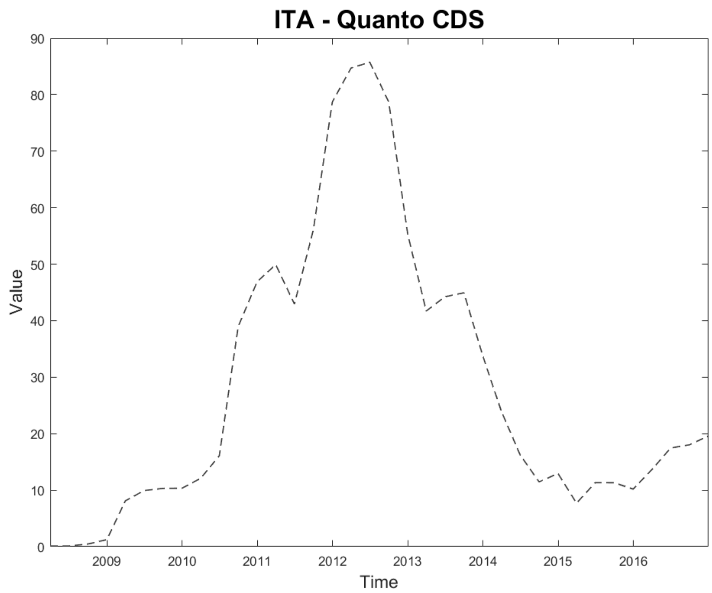

Figure 5.4: *Quanto* CDS for Italy

[3]this view it makes similar to choose the *Quanto* of a country rather than another, since they bring similar information;

Data on CDS only start from the end of 2007; it has been decided - as incidental analysis - to compute the variance decomposition also by including the *Quanto* spread [4].

The *Quanto* has a significant impact on the main latent factor $\boldsymbol{\alpha}_1$, as from Table 5.3. In fact, it explains the 46% of the variance of the first latent factor. This, again, seems to justify the thesis according to which spreads evolve and are strongly linked to the fear of a euro break-up.

Table 5.3: Variance decomposition of latent factors

Variable / Factor	$\boldsymbol{\alpha}_1$	$\boldsymbol{\alpha}_2$
Price Earning	0.02	0.01
Libor	0.01	0.01
VIX	0.02	0.27
Stock	0.04	0.03
TrendOnline	0.06	0.02
TrendOnline^2	0.05	0.02
Quanto ITA	0.46	0.05
Total Adj-R^2	0.66	0.41

[4] remembering that the sample is different with respect to the study reported in the previous chapter;

5.8 Granger test

The study ends with another incidental analysis on Granger causality. This is a method that allows to investigate causality between two variables. The core model has been presented and discussed; Granger will only be used to confirm its conclusions. Hence, it will not be given to this approach an extensive treatise. In simple terms, it is said that a time series $x_t^{(1)}$ Granger-causes $x_t^{(2)}$ if conditioning $x_t^{(2)}$ on $x_t^{(1)}$ the MSE is reduced.
Granger tests have been run on the different variables and only the significant ones are reported in Table 5.4. The conclusion of the Kalman model seem to be upheld.

Table 5.4: Significant Granger relations.

Relation	F-stat	p-value
$\boldsymbol{\alpha}_1$ ~*Quanto*-ITA	4.51	0.01
$\boldsymbol{\alpha}_1$ ~TrendOnline	3.47	0.02
$\boldsymbol{\alpha}_2$ ~VIX	2.29	0.09

a ~b means a Granger-causes b.
Only significant relationships have been reported

Once more, spreads appear to strongly depend on global variables. In particular, global uncertainty (VIX) and sentiment of euro break-up (TrendOnline variable and *Quanto* CDS).

Chapter 6

Conclusions

The work introduced the complex issue of stability in the Eurozone and it underlined how economic crisis might affect Europeans' lives and be related with the strength of EU institutions.

It has been showed how spreads are linked and that there is a high level of commonality. After having made data stationary it has been run a standard OLS technique to explain spreads with financial (VIX, S&P, Libor, PE), economic (GDP and Deficit) and social (researches on Google of euro break-up related keywords) variables. The average portion of explained variance (adjusted-R^2) was around 37%. Then the model with Kalman filtering was presented and implemented in Matlab. Its estimation allowed to construct the latent factor; a regression with GDP and Deficit as idiosyncratic variables and the two latent factors was run. The explanatory power of the model significantly grew showing an average adjusted-R^2 of around 70%. Hence, given the goodness of this approach the modelling of latent global factors might be used and work well also in forecasting. However, during periods of crisis some power of explanation is lost.

The last part of the analysis focused on how latent factors could be explained. It was shown that just a portion of them might be explained and they were mainly related to the fear of euro break-up, as measured by Google with specific keywords and to the VIX index. An incidental analysis on *Quanto* spread showed a strong correlation between this latter and the first latent factor.
In conclusion, the approach seems to be rewarding and it seems that spread dynamics are more dependent on Euro strength confidence rather than idiosyncratic/macroeconomic factors. Further researches might focus, when more data will be available, on running the model with CDS - eventually even considering only financial variables, removing the macroeconomic factors.

Another potential, interesting analysis could regard the construction of a dataset also before euro adoption, by using national currency and exchange rates. To this purpose, it might be worthy to check whether there is a structural change in behaviour before and after the euro adoption.
The study could therefore enforce EU institutions to increase efforts for what concerns the implementation of coordinated response and of a single mechanism of crisis management. It has been shown that a strong component of sovereign spreads depend on global variables and a global response should be in place for Europe.
E pluribus unum[1]: the Latin, immortal, maxim can be a target also for a stronger Europe.

[1] *Moretum*, Virgil (?);

Mathematical appendix

The following appendix defines the basic concepts that are going to be used in the thesis. It does not want to be a full treatise, but just to define and give a notation. This appendix is freely taken from the lectures in LUISS and from Spence *et al.* (2008), Hamilton (1994), Hansen (2017).

MATHEMATICS

Definition 1. ***(Vector)***

A vector $\mathbf{x}$ is a collection of numbers $x_1, ..., x_n$:

$$\mathbf{x}_1 = (x_1, x_2, ..., x_n)$$

is a row vector and

$$\mathbf{x}_2 = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$$

is a column vector.

$$\mathbf{x}_1 = \mathbf{x}_2^T = \mathbf{x}_2'$$

where ^{T} and $'$ represent the transpose operator.

Definition 2. ***(Dot product)***

Given two vectors $\mathbf{a} = (a_1, a_2, ..., a_n)$ and $\mathbf{b} = (b_1, b_2, ..., b_n)$ their dot product is:

$$\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + ... + a_n b_n$$

Definition 3. ***(Orthogonality)***

Two vectors $\mathbf{a}$ and $\mathbf{b}$ are orthogonal if their dot product is zero, *i.e.*

$$\mathbf{a} \cdot \mathbf{b} = 0$$

Definition 4. ***(Matrix)***

A rectangular collection of numbers is called a matrix. A $(n \times m)$ matrix $\mathbf{M}$ is represented with $\mathbf{M} \in \mathcal{M}^{n \times m}$ or $\mathbf{M} \in \mathbb{R}^{n \times m}$ and it has the following form:

$$\mathbf{M} = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1m} \\ a_{21} & a_{22} & \dots & a_{2m} \\ \vdots & & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nm} \end{pmatrix}$$

A vector can be thought as a matrix where either the number of rows or the number of columns is one.

a_{ij} is the element of the matrix that lies on the i-th row and j-th column.

Definition 5. ***(Diagonal of a Matrix)***

The diagonal of a matrix $\mathbf{M}$ is the collection of elements a_{ij} where $i = j$ for $i = 1, ..., \min(m, n)$.

Definition 6. ***(Square Matrix)***

A square matrix $\mathbf{M}$ is a matrix where $n = m$.

Definition 7. ***(Identity Matrix)***

An identity matrix $\mathbf{I}$ is a square matrix which has diagonal elements equal to one and the others are zeros. Namely: $a_{ij} = 1 \quad \text{if} \quad i = j \quad \text{and} \quad 0 \quad \text{otherwise}$.

Definition 8. ***(Transpose of a Matrix)***

The transpose of a matrix $\mathbf{A}$ $(n \times m)$ with generic elements a_{ij} is a matrix $(m \times n)$ with elements a_{ji}.

Definition 9. ***(Inverse of a Matrix)***

A square matrix $\mathbf{A}$ is said to be invertible if there exists a matrix $\mathbf{A}^{-1}$ such that

$$AA^{-1} = I$$

Definition 10. ***(Sum of Matrices)***

Let $\mathbf{A}$ and $\mathbf{B}$ two matrices whose elements respectively are a_{ij} and b_{ij} for $i = 1, ..n$ and $j = 1, ..., m$ then:

$$\mathbf{A} + \mathbf{B} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nm} \end{pmatrix} + \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1m} \\ b_{21} & b_{22} & \cdots & b_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nm} \end{pmatrix} \tag{1}$$

$$= \begin{pmatrix} a_{11} + b_{11} & a_{12} + b_{12} & \cdots & a_{1m} + b_{1m} \\ a_{21} + b_{21} & a_{22} + b_{22} & \cdots & a_{2m} + b_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} + b_{n1} & a_{n2} + b_{n2} & \cdots & a_{nm} + b_{nm} \end{pmatrix} \tag{2}$$

Definition 11. ***(Product of Matrices)***

Let $\mathbf{A}$ and $\mathbf{B}$ two matrices $(n \times m)$ and $(m \times k)$ whose elements respectively are a_{ij} for $i = 1, ..., n$ and $j = 1, ..., m$ and b_{ij} for $i = 1, ..., m$ and $j = 1, ..., k$ then the new matrix $\mathbf{C} = \mathbf{AB}$ will be a $(n \times k)$ where the elements of $\mathbf{C}$ are obtained in the following way:

$$c_{ij} = \sum_{h=1}^{m} a_{ih} b_{hj}$$

Definition 12. ***(Eigenvalue)***

Let W a vectorial space over a field F. Let $S : W \to W$ (esomorphysm) let $\mathbf{w} \in W \neq \mathbf{0}$ and λ a scalar $S(\mathbf{w}) = \lambda\mathbf{w}$, λ is called eigenvalue associated to the eigenvector $\mathbf{w}$.

Let W of finite-dimensional, then it can be represented by a matrix $\mathbf{A} \in \mathcal{M}^{n\times n}$. Then if $\mathbf{Ax} = \lambda\mathbf{x}$, $\mathbf{x}$ is eigenvector and λ is eigenvalue. The set of distinct eigenvalues is called **spectrum** of $\mathbf{A}$ and is denoted by $\sigma(\mathbf{A})$.

Theorem 1. ***(Eigenvalue and determinant)***

$\lambda \in \sigma(\mathbf{A}) \Leftrightarrow det(\mathbf{A} - \lambda\mathbf{I}) = 0.$

Theorem 2. ***(Eigenvalue and diagonal matrix)***

If a matrix $\mathbf{A}$ is diagonal then its eigenvalues are the diagonal elements.

$$\sigma(\mathbf{A}) = \mathrm{diag}(\mathbf{A})$$

PROBABILITY and ECONOMETRICS

A foundational approach will be adopted, hence no formal definition of set will be given. It can be thought in a simple way as a collection of objects. In general, the following properties hold:

Properties of sets.

(Commutative) $A \cup B = B \cup A$ and $A \cap B = B \cap A$

(Associative) $A \cup (B \cup C) = (A \cup B) \cup C$ and $A \cap (B \cap C) = (A \cap B) \cap C$

(Distributive) $A\,(B\, \cup\, C) = AB \cup AC$ and $A \cup (BC) = (A \cup B)(A \cup C)$

(De Morgan) $\overline{A \cap B} = \overline{A} \cup \overline{B}$ and $\overline{A \cup B} = \overline{A} \cap \overline{B}$.

Where $\overline{A} = A^c$ and is defined in Definition 14.

Definition 13. ***(Sample space)***

The set of all possible results of an experiment is called sample space; it is labeled as Ω.

Definition 14. ***(Complementary set)***

Let $A \in \Omega$. $A^c := \Omega \setminus A$ equivalent to $A^c = \{\omega\ s.t.\ \omega \notin A\}$ is called the complementary of A .

Definition 15. ***(Union)***

Let A and B two sets then $A \cup B := \Big\{\omega\ s.t.\ \omega \in A\ or\ \omega \in B\Big\}$

Definition 16. ***(Intersection)***

Let A and B two sets then $A \cap B := \Big\{\omega\ s.t.\ \omega \in A\ and\ \omega \in B\Big\}$

Definition 17. ***(Sigma field)***

A σ-field (also refered as σ-algebra) $\mathcal{F}$ is a collection of subsets of Ω that satisfies:

a) $\Omega \in \mathcal{F}$

b) if $A \in \mathcal{F} \Rightarrow A^c \in \mathcal{F}$

c) if $A_1, A_2, ... \in \mathcal{F} \Rightarrow \bigcup_{i=1}^{\infty} A_i \in \mathcal{F}$

Remark: from b) it follows that $\varnothing \in \Omega$; in fact, the empty set that has no elements in it, is the complementary of the sample space and must be in the field from property a).

Definition 18. ***(Disjoint set)***
A and B are disjoint if $A \cap B = \varnothing$.
A collection of sets $A_1, A_2, ...$ are disjoint if $A_i \cap A_j = \varnothing$ for all $i \neq j$.

Definition 19. ***(Measure)***
Let A a set and $\mathcal{F}$ a σ-field. A function $\mu : \mathcal{F} \to \bar{\mathbb{R}}$ is a called measure if:
a) $\mu(\varnothing) = 0$
b) if $\{A_i \in A\}$ is countable disjoint collection of sets then $\mu(\bigcup_{i=1}^{\infty} A_i) = \sum_{i=1}^{\infty} \mu(A_i)$
c) $\mu(S) \geq 0$ for all $S \in \mathcal{F}$

Definition 20. ***(Probability measure)***

$\mathbb{P} : \Omega \to [0; 1]$ is a probability measure if:

a) $\mathbb{P}(\varnothing) = 0$ and $\mathbb{P}(\Omega) = 1$

b) if $A_1, A_2, ...$ is a disjoint collection of members of $\mathcal{F}$ then

$$\mathbb{P}(\bigcup_{i=1}^{\infty} A_i) = \sum_{i=1}^{\infty} \mathbb{P}(A_i)$$

Definition 21. ***(Probability space)***

$(\Omega, \mathcal{F}, \mathbb{P})$ is called probability space.

Definition 22. ***(Independence)***

Two events A, B are said to be independent if $\mathbb{P}(A \cap B) = \mathbb{P}(A)\mathbb{P}(B)$.

Definition 23. ***(Conditional Probabilities)***

Let A, B two events in Ω and $\mathbb{P}(B) > 0$ then the conditional probability is:

$$\mathbb{P}(A|B) = \frac{P(A \cap B)}{\mathbb{P}(B)}$$

Theorem 3. ***(Bayes' rule)***

Let $\{A_i\}_i$ a partition of Ω and let E an event for which $\mathbb{P}(E) > 0$ then for any event A_j in the partition with non zero probability, the following holds:

$$\mathbb{P}(A_j|E) = \frac{\mathbb{P}(E|A_j)\mathbb{P}(A_j)}{\sum_i \mathbb{P}(E|A_i)\mathbb{P}(A_i)}$$

Definition 24. ***(Random variable)***

A random variable X is a function $X : \Omega \to A$ where A is a measurable space (*e.g.* $A = \mathbb{R}$) .

A random variable is discrete if A is finite and countable.

Definition 25. ***(Probability mass function, p.m.f.)***

Let X a discrete random variable, the probability mass function $f : \Omega \to A$ is:

$$f(x) := \mathbb{P}(X = x), \quad x \in A$$

Definition 26. ***(Probability density function, p.d.f.)***

Let X a continuous random variable, with a continuous c.d.f. F as in Definition 27 then the p.d.f. is defined as

$$f(x) := \frac{dF(x)}{dx}$$

if F is differentiable at x.

The definition may be extended as a non negative map Lebesgue integrable such that:

$$\mathbb{P}(X \in A) = \int_A f(x) d\mu$$

for all $A \subseteq \Omega$ and measure μ.

Definition 27. ***(Cumulative distribution function, c.d.f.)***

Let X a random variable, the probability distribution function is $F : A \to [0; 1]$ defined as:

$$F(x) := \mathbb{P}(X \leq x), \quad x \in A$$

Definition 28. ***(Expectation of a random variable)***

Let X a random variable defined on a probability space then the expected value of X referred as $\mathbb{E}[X]$ is defined, if the following exists, as:

$$\mathbb{E}[X] := \int_{\Omega} X dP$$

Definition 29. ***(Expectation of discrete and continuos random variable)***

Definition 28 is general and it makes use of Lebesgue integration, expectation can - however - be defined for discrete random variables as:

$$\mathbb{E}[X] := \sum_{x} x f(x)$$

and for continuous random variables:

$$\mathbb{E}[X] := \int_{x} x f(x)$$

Theorem 4. ***(Properties of expectation)***

(Expectation of a function) For discrete and continuos random variables:

$$\mathbb{E}[g(X)] = \sum_x g(x)f(x) \quad \text{and} \quad \mathbb{E}[g(X)] = \int_x g(x)f(x)$$

(Linearity) Let X_1 and X_2 random variables, $a, b \in \mathbb{R}$ and f, g functions, then it is said that the expectation is a linear operator. Namely:

$$\mathbb{E}[aX_1 + b] = a\mathbb{E}[X_1] + b$$

$$\mathbb{E}[f(X_1) + g(X_2)] = \mathbb{E}[f(X_1)] + \mathbb{E}[g(X_2)]$$

(Iterated expectations)

$$\mathbb{E}[X_1] = \mathbb{E}[\mathbb{E}[X_1|X_2]]$$

Definition 30. ***(Variance of a random variable)***

The variance of a random variable is defined as:

$$\mathrm{Var}(X) = \mathbb{E}[(X - \mathbb{E}[X])^2]$$

Theorem 5. ***(Properties of variance)***

$$\mathrm{Var}(X) = \mathbb{E}[X^2] - \mathbb{E}[X]^2$$

$$\mathrm{Var}(aX + b) = a^2 \mathrm{Var}(X)$$

Definition 31. ***(Random vector)***

A vector of random variables $X_1, X_2, ..., X_n$ is called a random vector.

Definition 32. ***(Independence of random variables)***

Discrete random variables $X_1, ...X_n$ are independent if and only if

$$\mathbb{P}(X_1 = x_1, ..., X_n = x_n) = \prod_{j=1}^{n} \mathbb{P}(X_j = x_j)$$

for all x_j.

Continous random variables $X_1, ..., X_n$ are independent if and only if

$$\mathbb{P}(X_1 \in A_1, ..., X_n \in A_n) = \prod_{j=1}^{n} \mathbb{P}(X_j \in A_j)$$

for all x_j. Which is equivalent to say that the joint p.d.f. is the product of the single p.d.f.

Definition 33. ***(Covariance)***

Let X and Y two random variables, the covariance is defined as:

$$\mathrm{Cov}(X,Y) = \mathbb{E}[(X - \mathbb{E}[X])(Y - \mathbb{E}[Y])]$$

Theorem 6. ***(Weak law of large numbers)***

Let $X_1, ...X_n$ a sequence of independently and identically distributed (i.i.d) random variables with mean μ and finite variance. Define $\overline{X}_n = \sum_{j=1}^{n} X_i$, then:

$$\lim_{n\to\infty} \mathbb{P}(|\overline{X}_n - \mu| > \epsilon) = 0$$

The sample mean converges in probability to its true mean, *i.e.* $\overline{X}_n \xrightarrow{p} \mu$ as $n \to \infty$

Definition 34. ***(Convergence in distribution)***

$X_1, ..., X_n$ are said to converge in distribution if

$$\lim_{n\to\infty} F_n(x) = F$$

for every $x \in \mathbb{R}$ where F is continuous. It is labelled as $X_n \xrightarrow{d} X$

Definition 35. ***(Convergence in probability)***

$X_1, ..., X_n$ are said to converge in probability if

$$\text{lim}_{n\to\infty}\mathbb{P}(|X_n - X| < \epsilon) = 0$$

for all $\epsilon > 0$. It is labelled as $X_n \xrightarrow{p} X$

Theorem 7. ***(Central limit theorem)***

Let $X_1, ...X_n$ a sequence of independently and identically distributed (i.i.d) random variables with mean μ and finite variance.

$$\sqrt{n}[(\frac{1}{n}\sum_i X_i) - \mu] \xrightarrow{d} N(0;\sigma^2)$$

Where $N(\mu;\sigma)$ represents the normal distribution:

$$f(x|\mu,\sigma) = \frac{1}{\sqrt{2\pi}\sigma} \cdot e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Definition 36. ***(Estimator)***

An estimator is a function of the data which is used to gather information on an unknown parameter. If we let the parameter to be θ its estimator is referred with $\hat{\theta}$.

Definition 37. ***(Unbiasedness)***

An estimator θ is unbiased if:

$$\mathbb{E}[\hat{\theta}] = \theta$$

Definition 38. ***(Consistency)***

An estimator θ is consistent if:

$$\hat{\theta}_n \xrightarrow{p} \theta$$

Definition 39. ***(Mean Squared Errors)***

The mean squared error of an estimator $\hat{\theta}$ is defined as:

$$\text{MSE} = \mathbb{E}[(\hat{\theta} - \theta)^2]$$

Definition 40. ***(Linear model)***

Let $\mathbf{y}$ a $(n \times 1)$ vector of dependent observations that is to be linearly explained by a set of m regressors $(\mathbf{x}_1, ..., \mathbf{x}_m) = \mathbf{X}$ where $\mathbf{x}_i$ is a $(n \times 1)$ vector for $i = 1, ..., m$. $\mathbf{X}$ is an $(n \times m)$ matrix of regressors, $\boldsymbol{\beta}$ an $(m \times 1)$ vector of coefficients and $\boldsymbol{\epsilon}$ a $(n \times 1)$ vector of errors.

The linear can be expressed as:

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon}$$

Which, in the bidimensional case, simplifies to:

$$y_i = \beta \cdot x_i + e_i, \qquad for \quad i = 1, ..., n$$

For the linear model there are different sets of assumption. In general, apart from the correct specification of the model, the following hypothesis are made:

- residuals are linearly independent;
- residuals have conditional 0-mean $\mathbb{E}[\boldsymbol{\epsilon}|\mathbf{X}] = 0$ and are normally distributed;
- residuals are homoskedastic.

The notation of the linear model is not unique and many times it can be found with or without pedices and transpose.

Definition 41. ***(Homoskedasticity and heteroskedasticity)***
The error is said to be homoskedastic if it does not depend on the regressors.
In the bidimensional case, if $\mathrm{Var}(e|x)$ $(=\mathbb{E}[e^2|x])$ is constant and does not depend on x.
It is heteroskedastic if it is not homoskedastic, namely depends on the x.

Theorem 8. ***(Ordinary Least Square)***

Let the sum of squared residuals:

$$s(\boldsymbol{\beta}) = \boldsymbol{\epsilon}'\boldsymbol{\epsilon} = (\mathbf{y} - \mathbf{X}\boldsymbol{\beta})'(\mathbf{y} - \mathbf{X}\boldsymbol{\beta})$$

The idea of OLS is that a way to get a good estimator is by minimizing this quantity for it $\boldsymbol{\beta}$.

Using relations of matrices and vectors, it is derived:

$$\frac{ds(\hat{\boldsymbol{\beta}})}{d\boldsymbol{\beta}} = -2\mathbf{X}'\mathbf{y} + 2\mathbf{X}'\mathbf{X}\hat{\boldsymbol{\beta}} = 0 \quad \Leftrightarrow \quad \hat{\boldsymbol{\beta}} = (\mathbf{X}'\mathbf{X})^{-1}(\mathbf{X}'\mathbf{y})$$

When residuals are assumed normal the same result can be obtained with an MLE approach.

Definition 42. ***(Time series)***

A time series is a collection of random variables which are ordered by time. It is labelled as $\{Z_t\}_t$.

Definition 43. ***(Autocovariance)***

Given a time series $\{Z_t\}_t$ (with $t \in \mathbb{N}$ or $\mathbb{Z}$) with finite second moment the autocovariance function is defined as:

$$\gamma(s,t) = \mathrm{Cov}(Z_s, Z_t)$$

Definition 44. ***(Weak stationarity)***

Given a time series $\{Z_t\}_{t\in\mathbb{Z}}$ this is said to be weakly stationary if:

$\mathbb{E}[Z_t] = \mu \qquad \text{for all} \quad t \in \mathbb{Z}$

$\gamma(s,t) = \gamma(s+j, t+j) \qquad \text{for all} \quad s,t,z \in \mathbb{Z}$

Definition 45. ***(Strict stationarity)***

Given a time series $\{Z_t\}_{t\in\mathbb{Z}}$ this is said to be weakly stationary if:

$$F(z_{t_1+j}, z_{t_2+j}, ..., z_{t_n+j}) = F(z_{t_1} + z_{t_2}, ..., z_{t_n}) \qquad \text{for all } n, j \text{ and } t_1, ..., t_n$$

Definition 46. ***(Ergodicity)***

A sequence $\{Z_t\}_{t\in\mathbb{Z}}$ is ergodic if for any two function f, g:

$$\lim_{n\to\infty} |\mathbb{E}[f(z_t, z_{t+1}, ..., z_{t+k})g(z_{t+n}, z_{t+n+1}, ..., z_{t+n+h})|$$

$$= |\mathbb{E}[f(z_t, z_{t+1}, ..., z_{t+k})|\mathbb{E}|g(z_{t+n}, z_{t+n+1}, ..., z_{t+n+h})|$$

for any f, g bounded

Definition 47. ***(Assumption of linear model for time series)***

The OLS technique, reported in Theorem 8, might be used also for time series data and under certain specific assumptions the estimator keeps its properties. In particular, if:

- $(\mathbf{y}_t, \mathbf{X}_t)$ are stationary and ergodic
- Errors and regressors are orthogonal

then the estimator is unbiased and consistent.

Acronyms

ADF: Augmented Dickey-Fuller test

Bps: Basis Points

CDS: Credit Default Swap

GDP: Gross Domestic Product

KPSS: Kwiatkowski Phillips Schmidt Shin test

LMG: Lindeman, Merenda, and Gold

MLE: Maximum Likelihood Estimator

MSE: Mean Squared Error

MVN_J: Multi-Variate Normal with dimension J

OLS: Ordinary Least Squares

PCA: Principal Component Analysis

PCi: i-th Principal Component

PE: Price Earnings

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LIBERA UNIVERSITÀ INTERNAZIONALE DEGLI STUDI SOCIALI

A Quantitative Analysis of Sovereign Bond Spreads in the Eurozone

Master's degree in Economics and Finance

Summary of the Master's thesis

Chair: **Theory of Finance**

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July 2017

Academic year 2016/2017

Department of Economics and Finance

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Foreword

The prolonged financial crisis has stressed the real and perceived impact of economic variables on the lives of EU citizens with consequent effects on their opinions regarding EU institutions. This correlation between economic crisis and the strength (in terms of public opinion) of EU institutions can be highlighted analyzing surveys conducted by the European Commission (2017). Spreads are important key-indicators, expecially during economic crisis, both representing a synthetic “judgment” of a Country-risk and having a direct impact on the cost of public debt.

This dissertation comes to this point and it tries to study whether spreads in the Eurozone might be modelled with latent factors and, eventually, how these latent factors might be explained.

Regarding the past literature, the use of latent factors is not new. In fact, Ang and Piazzesi, in 2003, studied the relationships between the bond yields and macroeconomic variables by using, together with no arbitrage techniques, vector auto-regressive (VAR) models with latent factors. Longstaff, Pan, Pedersen and Singleton, in their paper “How sovereign is sovereign credit risk?”, tried to decompose the spread of Credit Default Swap (CDS) between global and local components, showing a high commonality high correlations. This thesis will show that in Europe these values are even higher, suggesting that linkages are stronger.

Finally, the methodology here applied is inspired by the work of Aguiar *et. al.* (2016) who - in their first part of their paper - used the Kalman filter to construct common factors and explain spreads in emerging countries, showing that a high percentage of volatility can be explained with these common factors.

It will be checked whether the construction of latent factors with a Kalman-based model outperforms the standard OLS techniques.

Data

Countries in the scope of the analysis are Italy, France, Spain, Portugal, Belgium, Ireland, Netherlands, Austria and Finland; Germany has been taken as benchmark. These are roughly the countries that experimentally adopted the Euro in 1999; Greece - that joined the Euro in 2001 - was excluded because data is partially interrupted and not of high quality; in addition, there have been serious issues for what concerns liquidity and prices might not be informative. Minor countries have also been excluded because of data quality requirements. All data has been collected in quarters, starting from Quarter 2 of 2002 up to Quarter 4 of 2016. Where returns are used, data has been gathered from Quarter 1 of year 2002. [1].
Variables obtained are, for what concerns country-specific:
(i) returns on benchmark bond indices realized by DataStream - Eikon Thomson Reuters;
(ii) levels of deficit as percentage of GDP taken from EuroStat;
(iii) percentual GDP growth in real terms taken from OECD;
for what concerns global variables:
(iv) the Price-Earnings (PE) ratio for the US gathered from DataStream;
(v) Libor 3 month, UK interbank rate from DataStream;
(vi) VIX (CBOE) from DataStream;
(vii) Stock returns on S&P500 taken from Yahoo Finance.

In addition, CDS, from the end of 2007, in euro and dollar terms have been taken from Datastream in order to conduct an incidental analysis on *Quanto* spreads. Also a variable that it could be used as a measure of euro break-up risk has been created. This variable

[1] in particular, stock prices data has been updated on 10th January 2017, data from DataStream on 23rd February 2017, gdp-growth rates on 20th March and deficit related data, that were the latest to be updated, on the 1st of May. It shall be pointed out that some of the latest observations might be not definitive and henceforth could have been further changed by small amounts;

has been constructed from Google Trends (2017) by taking the number of researches on Google of the following keywords: *euro breakup, euro break, euro break-up, abandon euro, leave euro, euro exit, out euro, euro breakdown, euro referendum, euro collapse.* An average of this values has then been taken and defined as TrendOnline that will later be inserted in the analysis.

At global level factors regarding the US (or UK for what concerns Libor) market have been taken; one of the objectives of this research will be to check how much non-local variables have an impact on spreads; using non European elements, it will be assured that they truly are of global dimension and not euro-related.
Furthermore, it could be argued that the use of Credit Default Swaps (CDS) - as some academics did - would better fit for the analysis. However, it shall be pointed out that bond indices [2] could present the same liquidity and that CDS might be as well not fully informative for given countries in very peculiar time periods (*e.g.* Greek crisis). In addition, and most relevant, datasets for CDS were not available from 2002, but started some years after. Some authors (as the above quoted Ang and Longstaff, 2011) used CDS; nevertheless, one of their key argument for using CDS was that more observations - at a higher frequency - were available; this is reasonable in that context, where only financial variables (that typically have higher frequency) are used, but it would not fit here the same. In this study, macroeconomic variables are likewise included and the vast majority of them has at most quarterly frequency. The advantage to which the authors refer would then disappear and using CDS, for this work, would cut by one third the sample size. Thence, it appears logic the use of bond indices.

All data has been made stationary and residuals have been checked in order to ensure that OLS techniques could be used. All analysis are lagged by one period; in other words, to explain variables at time t factors at $t-1$ have been used. The standard OLS model, which tries to explain sovereign spreads, registered an average adjusted-R^2 of around 37%.

[2]for a methodological note of how they are constructed *cfr.*
Markit iBoxx EUR Benchmark Index Guide (Markit, 2017). Available at:
http://www.markit.com/Company/Files/DownloadFiles?CMSID=910be37be7154e13bbb18aa81e801e90

The Model

In 1960, Kalman started to build what he called "A new approach to linear filtering and prediction problems" and formalized the algorithm that then took his name. The Kalman Filter is a recursive method for estimation of measures in an uncertain dynamic system with constant update, in a system where the measured value contains random or unpredicted errors.

The Kalman filter is optimal among linear systems and it minimizes the mean squared errors. It reaches this purpose by giving relative weights to previous estimates and previous data. In fact, the Kalman gain represents how much importance has to be given to the new observation and how much to the previous estimate; in a simple dimensional case, it can be expressed as the error in the estimate divided by the sum of errors in the estimate and in the measurement. If we indicate the Kalman gain as K the new estimate will weight K, the measurement and $(1 - K)$ the previous estimate. In general, the smallest the K is the more stable the model will be. At each iteration of the process, an updated estimate is produced and it is used to track the variable we are monitoring.

Numerous applications have been done in tracking systems: the filter is often used in positioning system (like GPS) and it is included in space programs - for instance it was used for the Apollo program[3]. The filter is also used in the economic and financial field to track the evolution of (latent) variables in presence of noise.

In this work the Kalman Filter will be used to construct the two common latent factors that affect spreads in the different European countries. It is not said *ex ante* that the model will increase explanatory power with respect standard procedures; for this reason, results will be compared with the outcomes from OLS.

[3] for an extensive list of application in tracking system *cfr.* "Applications of Kalman Filtering in Aerospace 1960 to the Present" (M. Grewal and A. Andrrews, 2010)

It is possible to implement a Kalman filter approach and represent the system of spreads in a state-space form.

Let $\{\alpha^{(j)}\}_{j=1,...,J}$ the unobserved factors; in the specific case the model will be run with 2 factors, which is $J = 2$.

The specification of the model is the following:

$$s_{it} = \beta_{it}d_{it} + \gamma_{it}g_{it} + \delta_i^{(1)}\alpha_t^{(1)} + \delta_i^{(2)}\alpha_t^{(2)} + \varepsilon_{it} \tag{1}$$

$$\boldsymbol{\alpha}_t = \mathbf{F}\boldsymbol{\alpha}_{t-1} + \boldsymbol{\eta_t} \tag{2}$$

Being s the spread, d the deficit, g the GDP growth and αs the latent factors, for countries $i = 1, ..., I$ and times $t = 1, ..., T$.

Equation (1) is called *observation* (or measurement) equation, while Equation (2) is called *transition* equation; for $i = 1, ..., I$ and $t = 1, ..., T$ representing the i-th country and t-th period (measured in quarters). In addition $\delta_i^j > 0$ is the response of different countries to the same common factors. It is required that δ is positive, in line with Aguiar M. *et al.* (2016), in order to have that all countries respond in the same way (with the same sign) to the factors.

$$\mathbf{F} = \begin{bmatrix} f_1 & 0 \\ 0 & f_2 \end{bmatrix}$$

is a $J \times J$ (in the specific case since there are two latent factors $J = 2$ is a 2×2) diagonal matrix. $\mathbf{F}$ is required to have the eigenvalues (§Appendix) inside the unit circle. But since $\mathbf{F}$ is diagonal the spectrum $\sigma(\mathbf{F})$ coincides with the elements of the diagonal that hence need to be less than one in absolute value, *i.e.* $|f_i| < 1$ for $i = 1, 2$; stationarity improves quality of previsions and is a feature that it would be desirable.

In time t it is possible to rewrite the Kalman filter, that will be implemented in MATLAB. The two basic equations are, in matrix notation:

$$\begin{cases} \mathbf{s}_t = \mathbf{B}_t + \mathbf{H}\boldsymbol{\alpha}_t + \boldsymbol{\varepsilon}_t \\ \boldsymbol{\alpha}_t = \mathbf{F}\boldsymbol{\alpha}_{t-1} + \boldsymbol{\eta}_t \end{cases} \tag{3}$$

In order to perform parameter estimation, a Maximum Likelihood Estimation (MLE) approach is used. The likelihood function is the function that indicates the probability of observing the data that have actually been observed.

The likelihood function is

$$\mathcal{L}(\boldsymbol{\theta}, x_1, x_2, ..., x_n) = f(x_1, x_2, ..., x_n|\boldsymbol{\theta}) \tag{4}$$

That under independence leads to

$$\mathcal{L}(\boldsymbol{\theta}, x_1, x_2, ..., x_n) = f(x_1|\boldsymbol{\theta}) \cdot f(x_2|\boldsymbol{\theta}) \cdot ... \cdot f(x_n|\boldsymbol{\theta}) = \prod_{i=1}^{n} f(x_i|\boldsymbol{\theta}) \tag{5}$$

According to MLE this quantity shall be maximized. However, often it is easier to find the maximum for the log transformation, since the result does not change because of the monotone transformation, and it simplifies the problem. Hence,

$$\ell(\boldsymbol{\theta}, x_1, x_2, ..., x_n) := \log\left[\mathcal{L}(\boldsymbol{\theta}, x_1, x_2, ..., x_n)\right] = \sum_{i=1}^{n} \log f(x_i|\boldsymbol{\theta}) \tag{6}$$

In the specific case as log likelihood to maximize will be taken the average over time:

$$\bar{\ell} := \frac{1}{T}\sum_{t=1}^{T} \ell_t \tag{7}$$

Where ℓ_t is the log-likelihood function at general time t.

Results

The model has been estimated in MATLAB by performing the Monte Carlo on 45 000 iterations. The obtained final value of the likelihood is $-5.32 \cdot 10^7$. The procedure, as outlined before, estimated $\mathbf{F}$, $\mathbf{H}$, $\sum$ and the two common latent factors $\boldsymbol{\alpha}_1$ and $\boldsymbol{\alpha}_2$. The two latent factors turned out to have the following form:

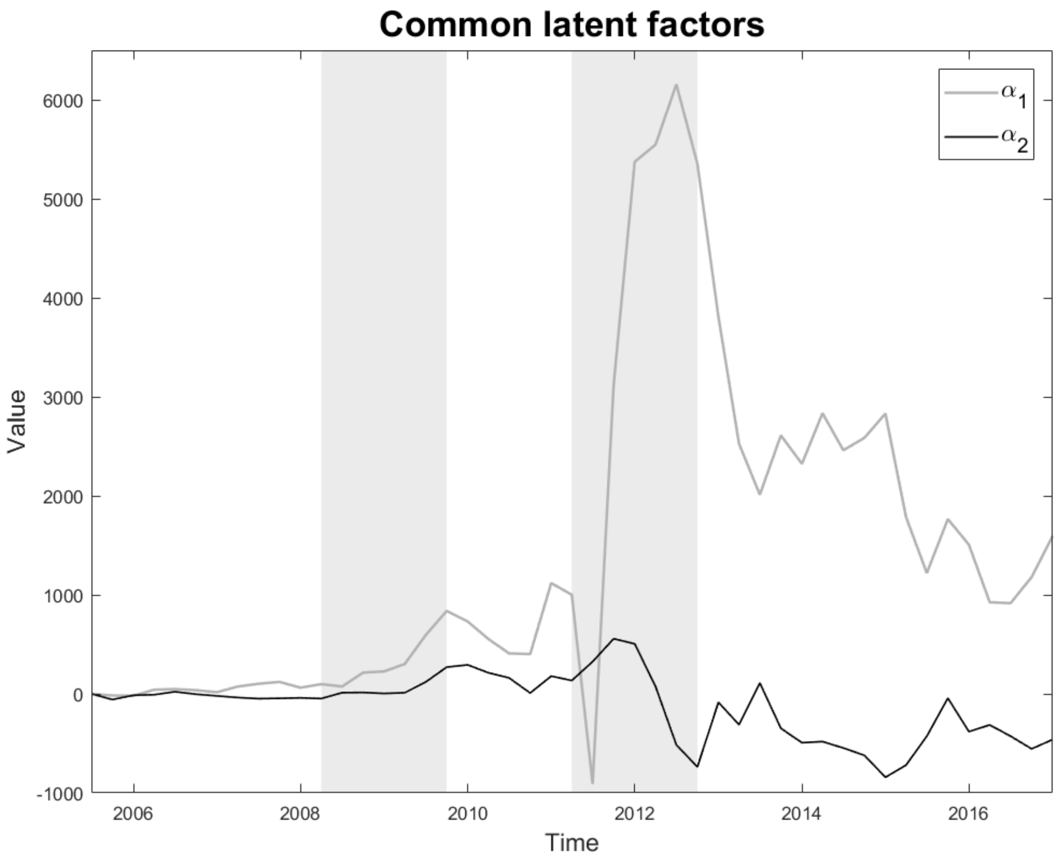

Figure 1: Latent factors

From Figure (1) it can be seen that the two factors remained relatively similar up to the sovereign crisis of 2012, when they significantly diverge with an important spike of $\boldsymbol{\alpha}_1$. The variance decomposition run for spreads (Table 1) shows that the first factor $\boldsymbol{\alpha}_1$ explains a high portion of spread movements. The explained variance by $\boldsymbol{\alpha}_1$ is high for Italy, Portugal and Ireland, while it is a bit lower for Spain (0.37). It is high for France and Belgium too that, in this sense, show a behaviour comparable to the periphery countries rather than the core ones. $\boldsymbol{\alpha}_2$ is of lower importance and cross-country; it accounts for an average 10% of the variance. It can be seen that the goodness of the model significantly increases with respect to the standard OLS approach. The adjusted-R^2 increased, from the 37% of the standard model to the 70% by using latent factors as estimated by the Kalman model.

Table 1: Variance decomposition of explained spread

Country / Variable	**GDP**	**Deficit**	$\boldsymbol{\alpha}_1$	$\boldsymbol{\alpha}_2$
ITA	0.12	0.02	0.58	0.11
FR	0.07	0.05	0.57	0.12
SPA	0.16	0.08	0.37	0.07
PG	0.12	0.02	0.69	0.06
BEL	0.07	0.02	0.61	0.08
IR	0.02	0.14	0.57	0.13
NET	0.16	0.10	0.16	0.06
AU	0.07	0.04	0.39	0.07
FN	0.19	0.08	0.26	0.06

An interesting aspect would be to study the evolution, over time, of the explained variance by the four variables (the regressors: GDP, Deficit, $\boldsymbol{\alpha}_1$ and $\boldsymbol{\alpha}_2$) on spreads (the regressand). To this purpose, regressions have been run with rolling windows of one year and it has been registered the variance decomposition for each country and for every time starting from middle 2008[4].

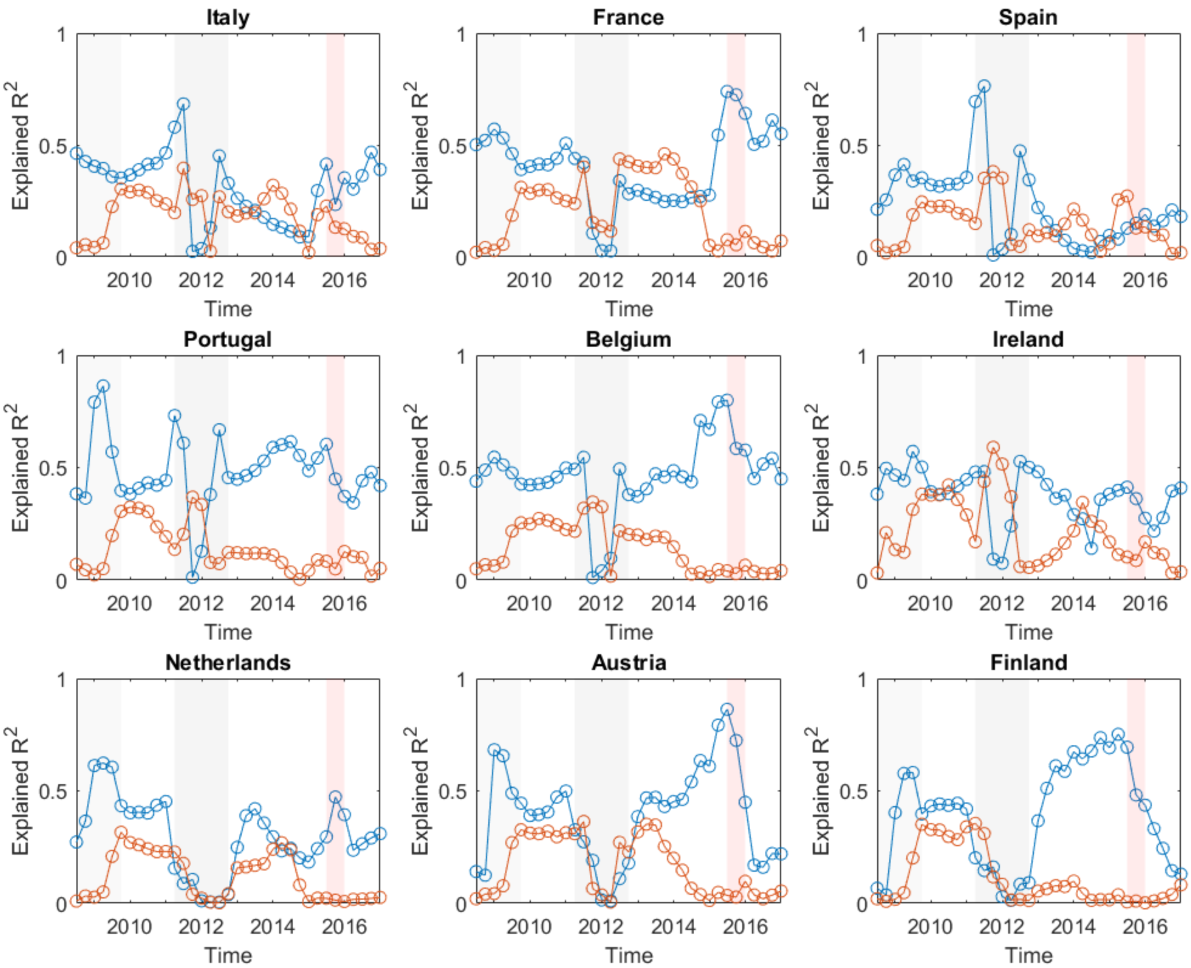

Figure 2: Explained variance of $\boldsymbol{\alpha}_1$ (blue) and $\boldsymbol{\alpha}_2$ (orange)

[4]in fact, the sample is reduced because some initial data are used to estimate the latent factors and some other data are used to create the first rolling window;

It might be seen from the graphs that the power of explanation is quite high for $\boldsymbol{\alpha}_1$ and that also $\boldsymbol{\alpha}_2$ and GDP have a significant impact. The latent factors lose a lot of power during crisis. Indeed, in 2012 there is a drop that then recovers in the following years.

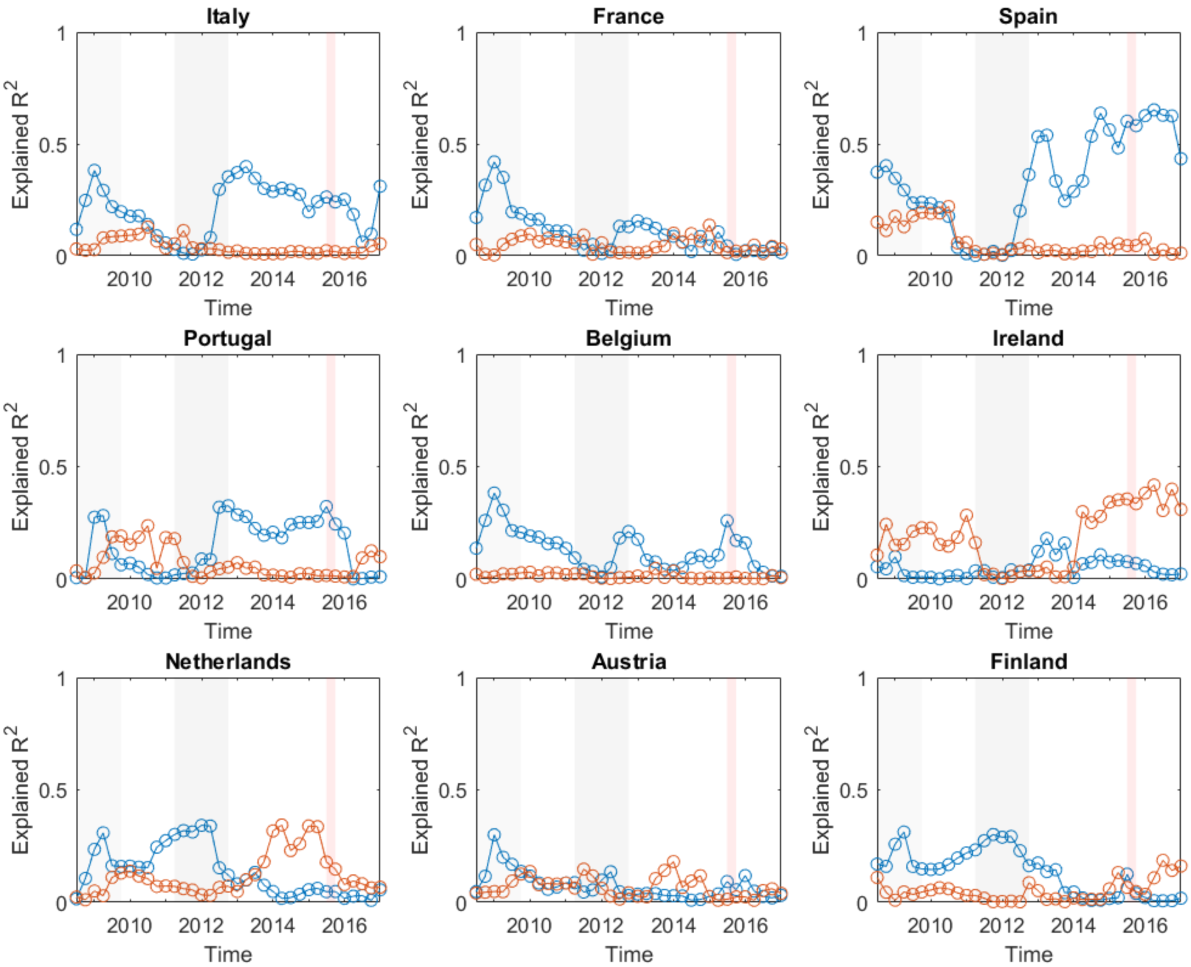

Figure 3: Explained variance of GDP (blue) and Debt (orange)

Once it has been seen that the explicative power of the latent factors is quite high and that they could be useful in modelling spreads in the Eurozone, a subsequent interesting question could be in which way these two latent factors may be explained. It appears that $\boldsymbol{\alpha}_1$ is linked to the fear of euro break-up (as measured by Google researches). A non-linear relation (in variables, not in coefficients) emerges for the online trend variable, since also the second power has a good impact. The TrendOnline variable itself (considering first and second power) explains more than the 30% of the first latent factor. In addition, the stock variable explains the 5% of the factor. On the other hand, $\boldsymbol{\alpha}_2$ is strongly dependent on the VIX and this latter explains the 16% of the first. In general, the two latent factors seem to reflect the fear of a failure of the common currency and the global uncertainty of the economy.

Table 2: Variance decomposition of latent factors

Variable / Factor	$\boldsymbol{\alpha}_1$	$\boldsymbol{\alpha}_2$
Price Earning	0.02	0.01
Libor	0.01	0.01
VIX	0.01	0.16
Stock	0.05	0.01
TrendOnline	0.19	0.03
TrendOnline^2	0.13	0.02
Total R^2	0.41	0.24

The *Quanto* spread is the difference between CDS traded in dollars and CDS traded in euros. It might be influenced by liquidity; however, in general, *Quanto* might be seen as a proxy of expected euro depreciation in the scenario where euro should break-up.
Below a plot of the Italian[5] *Quanto* is reported:

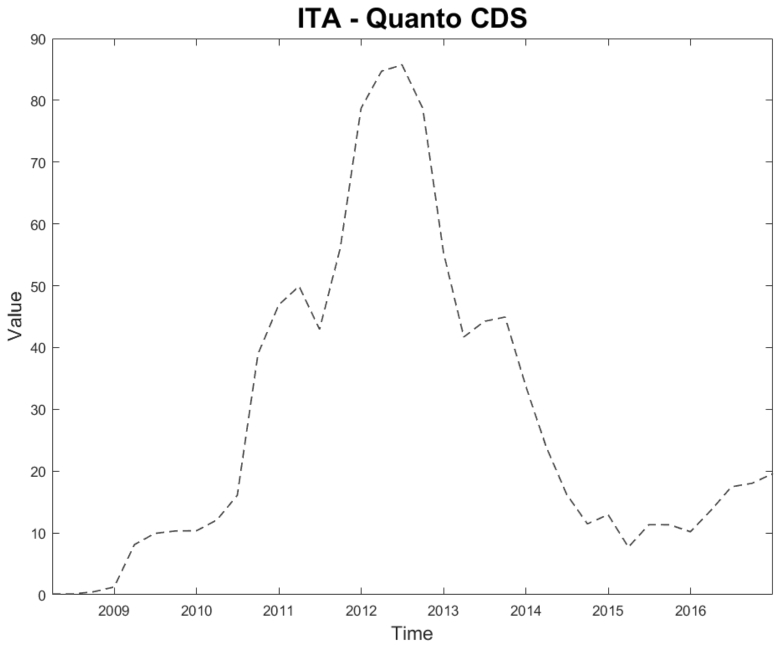

Figure 4: *Quanto* CDS for Italy

Data on CDS only starts from the end of 2007; it has been decided to compute the variance decomposition also by including the *Quanto*.

The *Quanto* has a significant impact on the main latent factor $\boldsymbol{\alpha}_1$, as from Table 3. In fact, it explains the 46% of the variance of the first latent factor. This, again, seems to justify the hypothesis according to which spreads evolve and are correlated to the fear of a euro break-up.

[5]it is similar to choose the *Quanto* of a country rather than another, since they bring similar information. It was decided to study the relationship between latent factors and the Italian *Quanto* since it is one of the countries that might be linked the most to the probability of a euro break-up and its behaviour will affect the entire Eurozone.

Table 3: Variance decomposition of latent factors

Variable / Factor	$\boldsymbol{\alpha}_1$	$\boldsymbol{\alpha}_2$
Price Earning	0.02	0.01
Libor	0.01	0.01
VIX	0.02	0.27
Stock	0.04	0.03
TrendOnline	0.06	0.02
TrendOnline^2	0.05	0.02
Quanto ITA	0.46	0.05
Total R^2	0.66	0.41

Conclusions

The work [6] introduced the complex issue of stability in the Eurozone and it underlined how economic crisis might affect Europeans' lives and be related with the strength of EU institutions.

It has been showed how spreads are linked and the high level of commonality. After having made data stationary it has been run a standard OLS technique to explain spreads with financial (VIX, S&P, Libor, PE), economic (GDP and Deficit) and social (researches on Google of euro break-up related keywords) variables. The average portion of explained variance (adjusted-R^2) was around 37%. Then the model with Kalman filtering was presented and implemented in MATLAB. Its estimation allowed to construct the latent factor; a regression with GDP and Deficit as idiosyncratic variables and the two latent factors was run. The explanatory power of the model significantly grew showing an average adjusted-R^2 of around 70%. Hence, given the goodness of this approach the modelling of latent global factors might be suggested and work well also in forecasting - always bearing in mind that during periods of crisis some power of explanation is lost.

The last part of the analysis focused on how latent factors could be explained. It was shown that just a portion of them might be explained and they were mainly related to the fear of euro break-up, as measured by Google with specific keywords and to the VIX index. An incidental analysis on *Quanto* spread showed a strong correlation between the latter and the first latent factor.

In conclusion, the approach seems to be rewarding and it seems that spread dynamics are more dependent on Euro strength confidence rather than idiosyncratic/macroeconomic factors. Further researches might focus, when more data will be available, on running the model with CDS - eventually even considering only financial variables, removing the macroe-

[6] refering to the complete version;

conomic factors.

Another potential interesting analysis could regard the construction of a dataset also before euro adoption, by using national currency and exchange rates. To this purpose, it might be worthy to check whether there is a structural change in behaviour, before and after the euro adoption.

The study could therefore enforce EU institutions to increase efforts for what concerns the implementation of coordinated response and of a single mechanism of crisis management. It has been shown that a strong component of sovereign spreads depends on global variables and a global response should be in place for Europe.

For this reason, it is still valid the Latin, immortal, maxim: *E pluribus unum*[7], for a stronger Europe.

[7] *Moretum*, Virgil (?);

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