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Interbank Contagion in the Italian Banking Sector

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Abstract

The interbank market is extremely important, as it allows banks to manage liquidity shocks by lending to one another. However, it is also the most obvious channel through which shocks can propagate throughout the banking system. In this paper, we use data on 544 publicly owned banks obtained from Orbis Bank Focus to analyse the risk of contagion in the Italian interbank market. In order to do so, we use two different methodologies: (1) entropy maximization, and (2) a methodology not dependent on the specific topology of the network. We find that only the failure of a large bank can trigger a chain of defaults in the system, and this chain generally dissipates in a few rounds. Moreover, large banks are more likely to default due to outside shocks rather than due to the default of other large banks, but the failure of a large bank is likely to cause the failure of many smaller banks. This is consistent with the money centre structure of the Italian banking system.

Keywords: interbank contagion, systemic risk, maximum entropy, RAS

1. Introduction

Over the past decades, there has been a huge interest from researchers and policymakers alike in the functioning of the financial system, and the body of work investigating the effects of linkages among financial institutions has been growing consistently. This has been even more so since the financial crisis of

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2008. As the effects of the bankruptcy of Lehman Brothers reverberated all over the global financial system, regulators have increasingly focused on the supervision of systemic risk.

In this context, the interbank market is extremely important, as it allows banks to manage liquidity shocks by lending to one another. Nevertheless, bilateral exposures are sometimes very large, and they represent the most obvious medium through which shocks can propagate throughout the system. One of the fundamental questions in research is, in fact, the extent to which linkages among banks are helpful by allowing them to share risks, and when they become harmful by contributing to the propagation of contagion to other banks[1]. When banks are hit by liquidity shocks, and hence find themselves short of funds, they first try to withdraw their deposit at other banks. This may lead to direct contagion problems when the overall liquidity need of the interbank market is larger than the aggregate amount of liquidity available[2]. When this is the case, banks are unable to raise enough liquidity, and will turn to other methods, such as liquidating long-term assets, which will lead to indirect contagion[3] through other channels, such as asset prices[4].

A notable body of work has shown that risk of contagion depends strongly on the precise structure of the interbank relationships. Generally, a complete market structure enhances the resistance of the system to contagion, while incomplete structures, credit chains and money centre structures are more likely to suffer from contagion effects[5]. Usually, the structure of bilateral exposures is not known, since banks only disclose aggregated exposures on their balance sheet, and this is no exception for the Italian interbank market, as detailed information about exposures is only issued by individual banks to the Bank of Italy due to regulatory reasons.

Due to this lack of information, it is common to construct interbank markets by estimating an interbank-lending matrix using aggregate exposures from banks balance sheet data, assuming that interbank lending is as dispersed

[1]See Glasserman and Young (2016), p. 787.
[2]See Upper and Worms (2002).
[3]For more on direct and indirect contagion, see de Bandt and Hartmann (2000).
[4]See Cont and Schaanning (2017).
[5]See Allen and Gale (2000) and Freixas, Parigi and Rochet (2000).

as possible[6]. Then a sensitivity analysis is performed, in which an arbitrary exogenous shock is supposed to hit the market[7].

In this paper, we analyse the Italian interbank market by adopting this approach. We construct an interbank-lending matrix by running the Iterative Proportional Fitting algorithm on aggregate bank exposure data from 2015, then we assume the largest banks in the system to fail in turn and we investigate the effects on the rest of the system. However, since maximizing entropy has been shown to give rise to estimated effects of contagion that are different from those obtained from actual data in the Italian banking sector in the past[8], and since the topology of the network has been shown to substantially affect the extent of contagion, we also use node-level information to bound contagion effects in the network following Glasserman and Young (2015)[9], without assuming any detailed information on the topology.

The rest of the paper is organized as follows. The following section presents briefly the literature that we have drawn upon. The third section describes the data used in this paper. Section 4 contains the entropy maximization methodology used for constructing the interbank-lending matrix and the simulation and presents the results. Section 5 contains the methodology used for bounding contagion effects without assuming topology information and presents the results. Finally, the last section concludes and summarises the main results.

[6]See Upper and Worms (2002).
[7]See, for example, Mistrulli (2002) and van Lelyveld and Liedorp (2006).
[8]See Mistrulli (2002).
[9]See Glasserman and Young (2015).

2. Literature review

Financial contagion in interbank markets has been increasingly studied in the last two decades.
De Bandt and Hartmann (2000) distinguish between direct contagion, which occurs due to direct financial links, such as direct exposures, between banks, and indirect contagion, which occurs due to events affecting more than one bank at the same time, like changes in expectations or asset prices[10].
For a recent example on modelling and estimating effects from indirect contagion, see Cont and Schaanning (2017), who "model the phenomenon of fire sales in a network of financial institutions with common asset holdings"[11].
In this paper we focus on direct contagion in the interbank market. For a recent review on the overall state of the art of research on contagion in financial networks, see Glasserman and Young (2016).

One of the cornerstones on the subject of direct contagion, which is at the base of a large part of the subsequent research on contagion, is "Systemic Risk in Financial Networks" by Eisenberg and Noe (2011), in which the authors develop a model to analyse "complex financial systems featuring cyclical obligations between parties" passing through a clearing vector. The model is very elegant theoretically and succeeds to describe at a fundamental level how interbank markets work.

Allen and Gale (2000) and Freixas, Parigi and Rochet (2000) describe possible topological structures of interbank markets. At a broad level, they distinguish three types of structures: (1) a complete structure, where each bank is connected to all the others, (2) an incomplete structure, in which banks are only connected to some, "neighbour" banks, and (3) a money centre structure, in which minor banks are connected to a large bank (or banks) but not to one another. Generally, interbank markets that present a complete structure are the most resilient to shocks, while markets that present a chain structure are the least resilient to shocks.

A large strand of empirical research that tries to estimate the effects of shocks from a contagion point of view has been carried out in many countries.

[10] See De Bandt and Hartmann (2000) and van Lelyveld and Liedorp (2006).
[11] R. Cont, and E. Schaanning (2017), p.1.

A clear and compact overview was presented in 2006 by van Lelyveld and Liedorp. Examples are Upper and Worms (2004) on Germany, Degryse and Nguyen (2007) on Belgium, Mistrulli (2007) on Italy, Wells (2004) on the United Kingdom, and van Lelyveld and Liedorp (2006) on the Netherlands.

There is some common methodology in this strand of empirical research, which varies depending on the data at hand.
The best case scenario, clearly, is that data on actual (non-aggregate) bilateral exposure at the node level is available, such as in the case of Mistrulli (2007), who had data only available to the Bank of Italy due to regulatory reasons. This allows the creation of an interbank-lending matrix that reflects precisely the interlinkages of the interbank market of the country at hand, which gives room for a detailed analysis.
A second possibility is that data on large exposures at the node level for at least some banks is available, such as in the case of van Lelyveld and Liedorp (2006). This means that the amounts of the largest linkages are known, and one only needs to estimate the rest of the linkages. This task has generally been accomplished in the literature solving a cross-entropy minimization problem through the RAS algorithm, also known as Iterative Fitting Procedure outside of economics[12].
The last possibility, which is our case, is that only data about aggregate exposure is available at the node level. In this case, the methodology usually followed consists of assuming lending as dispersed as possible in the interbank market. Since complete interbank markets are the most resilient to shocks, this means that we are directing our estimates against the hypothesis of contagion[13]. Nevertheless, it has been shown by Mistrulli (2007) that, since the Italian interbank market is characterised by a multiple money centre structure, results obtained using the maximum entropy method may be "statistically different from the one obtained on the base of actual bilateral exposures". Moreover, he specifies that, in this particular case, maximum entropy may overvalue the severity of contagion, since "in the presence of large players, [...] complete markets may be even more conducive to contagion than incomplete ones"[14].

[12] See Wells (2004) and van Lelyveld and Liedorp (2006). For more on the RAS algorithm, see Blien and Graef (1997).

[13] See Upper and Worms (2004), p. 4.

[14] See Mistrulli (2007), p. 25-30.

Due to these problems with the maximum entropy method, we turn to another strand of literature in order to make our estimates more reliable and to compare the two approaches.
Glasserman and Young (2015) are able to derive results on the potential magnitude of network effects on contagion using only minimal node level information, such as asset size, leverage and a financial connectivity measure given by the fraction of a financial institution's liabilities held by other financial institutions. In particular, they are able to bound contagion and amplification effects without detailed knowledge of the network structure. Moreover, they find that "it is relatively difficult to generate contagion solely through spill-over losses in a network of payment obligations", and that the structure of the network matters more for the amplification effects[15].
We apply their model to our dataset on the Italian banking market, generating bounds to contagion and amplification effects, and see where they fall with respect to the entropy maximization method. This allows us to put the entropy maximization method in perspective, and to have a broader and better understanding of the current solidity of the Italian interbank market. It is important to note that the model developed by Glasserman and Young differs from the common methodology previously used in empirical research on interbank contagion. Instead of assuming ad-hock shocks to certain institutions, in fact, they assume a full-fledged shock distribution and analyse the probability of default cascades that are due to network connections. In order to do so, they analyse the difference between default probabilities in a network system compared to a similar system in which all connections have been severed. This is not only theoretically more elegant than what was done previously, but it also has the added benefit of requiring no sensitive or confidential information and generally more easily available data.

[15]See Glasserman and Young (2015).

3. Data

We obtain data on gross bilateral exposures for 544 Italian publicly owned financial institutions from the database Orbis Bank Focus, by Bureau Van Dijk (previously "Bankscope"). This represents a large enough share of the interbank market to conduct some meaningful analysis, even though non-publicly owned banks are missing. Moreover, only minor and smaller banks are missing from the dataset. Only one bank (Banca di Credito Cooperativo della Contea di Modica) has been dropped due to problematic data. All the major banks in terms of assets, equity and loans, which are the focus of our analysis, are present in the dataset. When we talk about the Italian banking sector, from now on, we talk about the set of banks on which we have data.

Moreover, for each institution included, we have data on total assets, total liabilities, total equity, tier 1 equity, and overall loans.

In 2015, interbank exposures accounted for about 10.6% of overall assets in the Italian banking sector. They were around 1.5 times the overall equity of the banking system, and almost 1.7 times its tier 1 capital. Loans were about 55% of assets, while the leverage ratio was about 14.

	Interbank loans	Interbank loans / assets	Interbank loans / equity
Total	410,620,612	0.105892022	1.518729346
Mean	753,432.3157	0.11273497	2.124716278
Standard deviation	4,973,577.34	0.137849668	16.98255004

	Interbank loans / Tier 1 equity	Assets / equity	Assets / Tier 1 equity
Total	1.666026015	14.34224524	15.73325343
Mean	1.618931458	15.30200525	12.6670272
Standard deviation	5.509549365	86.02055799	16.61960017

Table 1: Summary of the data used. Figures for Interbank loans are in thousands of euro.

4. Entropy maximization

4.1. The interbank lending matrix

In order to describe the interlinkages between the financial institutions in the Italian interbank network, we use a $N \times N$ square matrix of the following form[16]:

$$X = \begin{bmatrix} x_{11} & \dots & x_{1j} & \dots & x_{1N} \\ \vdots & \ddots & \vdots & \iddots & \vdots \\ x_{i1} & \dots & x_{ij} & \dots & x_{iN} \\ \vdots & \iddots & \vdots & \ddots & \vdots \\ x_{N1} & \dots & x_{Nj} & \dots & x_{NN} \end{bmatrix}$$

Each cell entry x_{ij} describes the amount that bank i lends to bank j . Moreover, let $a_i = \sum_{j=1}^{N} x_{ij}$ be the row marginals, representing the total amount that bank i is lending to other banks, and let $l_j = \sum_{i=1}^{N} x_{ij}$ be the column marginals, representing the total amount that bank j is borrowing from the system. Hence, the marginals represent aggregate exposures.

In our dataset, we have available aggregate exposures a_i and l_j, but we do not know the distribution of those exposures, i.e. the single cell entries x_{ij}. Therefore, we have a lack of information problem which cannot be solved easily. Since we have $N^2 - 2N$ unknowns to be estimated, and since we do not have additional information to add to our matrix, our network is under-identified.
By standardizing, a_i and l_j can be interpreted as realizations of the marginal distributions $f(a_i)$ and $f(l_j)$, and x_{ij} can be interpreted as their joint distribution. Therefore, if $f(a_i)$ and $f(l_j)$ are independent, then $x_{ij} = a_i l_j$[17].
This is the common approach in the empirical literature, which amounts to maximizing the entropy in the network[18] by letting each bank lend as dis-

[16]For a (somewhat lengthy) introduction to modelling networks, see Newman (2010). For use in this particular context, see Upper and Worms (2004).
[17]See Upper and Worms (2004), pp. 8-9.
[18]Here we are following information theory terminology.

persedly as possible to other banks. This is generally the safest assumption to make in case no additional information is available, and means that we are assuming that the network of interbank claims has a complete structure.

Mistrulli (2007), using actual (non-aggregate) node level data on exposures, points out that the Italian interbank market is characterised by a money centre structure, however he avoids to mention which banks represent the money centres in the network as well as to give any additional, more detailed, information on the specific topology. We could assume that the largest banks are the money centres, however there would be multiple consequent assumptions to make that may lead to inaccurate or completely wrong results, such as which banks are connected to which others, to what degree, and so on.
Therefore, we stick to the common approach and assume maximum entropy in the network to generate somewhat general results. Moreover, since Mistrulli (2007) mentions that entropy maximization tends to overestimate the effects of contagion in this case, we are also making a conservative assumption. Notice that our case is particular with respect to the general one: by assuming independence and maximizing entropy, it is generally believed that tests are biased against the hypothesis of contagion, since a complete market structure is considered, on average, the strongest. In our case, on the other hand, if the structure of the interbank market has not changed since the analysis done by Mistrulli, and therefore there is a small number of "large players raising funds from many relatively small counterparts"[19], the maximum entropy approach is probably overestimating contagion[20].

We want to estimate cell entries so that lending is as dispersed as possible and so that individual entries are proportional to the two marginals and add up to them.
In order to do so, we make use of the Iterative Proportional Fitting (IPF) procedure, also known as RAS in economics, which is commonly used to estimate cell entries in the literature.

[19]See Mistrulli (2007), p. 26
[20]For an in-depth explanation, see Mistrulli (2007), section 4.

The IPF procedure works in steps as follows[21]:

1. We set seed values in the matrix. Since we want lending as proportional and dispersed as possible given the marginals, we set each cell entry x_{ij} equal to 1. Moreover, since a bank cannot lend to itself, we set each entry the diagonal of the matrix equal to zero:

$$\begin{cases} x_{ij}^{I} = 1 & \forall\, i \neq j \\ x_{ij}^{I} = 0 & \forall\, i = j \end{cases}$$

2. We apply row constraints:

$$x_{ij}^{II} = x_{ij}^{I} \frac{a_i}{\sum_j^N x_{ij}^{I}}$$

3. We apply column constraints:

$$x_{ij}^{III} = x_{ij}^{II} \frac{l_j}{\sum_i^N x_{ij}^{II}}$$

4. We test for convergence. We stop here if:

$$|x_{ij}^{III} - x_{ij}^{II}| < \beta \quad \forall\, x_{ij}$$

   Where β is a pre-designated small parameter.
   If this condition is not reached at step 4, then x_{ij}^{III} become the new seed values and we start over from step 2. Notice that the procedure has the added benefit of retaining zeros in steps 2 and 3, so that the diagonal remains zero.

[21] For more information on the IPF (RAS) algorithm, see the appendix of van Lelyveld and Liedorp (2006) (where they apply it in cross-entropy minimization) or, for a detailed explanation, Blien and Graef (1997).
We run the IPF algorithm through the IPFN package for the Python programming language (https://pypi.python.org/pypi/ipfn/).

4.2. Simulation methodology

After we have obtained the interbank-lending matrix, we go on with a simulation to analyse the effects of contagion on the network. We assume each bank to fail in turn due to shocks that come from outside the banking system. Such idiosyncratic shocks are rare, but not impossible. Indeed, a situation of this kind occurred with the bankruptcy of Lehman Brothers, as the bank went from expecting to be bought by Korea Development Bank on September 9, 2008, to filing for Chapter 11 bankruptcy on September 15, 2008. Another example is that of Barings Bank[22].

Let B be the set of banks in our network. Let $d \subset B$ denote the first bank that defaults, let $D_d^n \subseteq B$ be the set of banks that have defaulted, and let $S_d^n \subseteq B$ be the set of banks that have survived in round n of the contagion initiated by the failure of bank d[23].

We check the effects of the failure of bank d on the rest of the banks by assuming that a bank fails if its exposure to the failed bank is larger than its tier 1 capital:

$$\theta x_{jd} > c_j$$

Where θ is the loss rate and c_j is the tier 1 capital of bank j. If bank j fails, we remove it from S_d and we add it to D_d.

If one or more banks fails after the failure of bank d, then we go on with another round and other banks fail if their exposure to all failed banks is larger than their tier 1 capital:

$$\theta \left(\sum_{i \in D} x_{ji} \right) > c_j$$

We continue the process until we reach a point where no additional default occurs.

[22]See van Lelyveld and Liedorp (2006), p. 114.

[23]Here we use a notation similar to the one used by Mistrulli (2007) to facilitate comparison.

Clearly, this methodology presents a series of shortcomings[24]: (1) loss rates are constant, while in reality they change depending on a multitude of factors, (2) we use only data on Italian banks, therefore we cannot model contagion effects with foreign countries, (3) we only focus on a specific channel for contagion, (4) We rule out various situations, such as the possibility of netting between debtor and creditor bank, bank runs, issuance of shares, and so on.

Some problems could be solved if more data were available, while others would need different modelling tools to be tackled.

4.3. Simulation results

Following the methodology just outlined, we estimate an interbank-lending matrix applying the Iterative Fitting Procedure. Subsequently, we run a simulation letting each bank fail in turn and checking whether other banks have an exposure that results in a loss larger than its tier 1 capital.

A practical issue is that the sum of all the aggregate exposures (i.e. the sum of the marginals) do not add up, since Italy is a net borrower from foreign banking systems. This means that the Italian banking system, overall, borrows more from foreign banking systems than it lends. To solve this problem, we add an additional item to our interbank lending matrix to represent borrowings from the rest of the world. We assume that the rest of the world never fails after an Italian bank fails, and we also assume that the rest of the world does not fail due to outside shocks. It seems unlikely that foreign banks fail for the whole amount of the Italian exposure together, and we do not have detailed information about this exposure, so this is the most reasonable assumption we can make.

We follow Furfine (1999) in that, since it is difficult to make assumptions on an actual loss rate that might be observed in reality, we run the simulation at four different loss rates: 0.25, 0.5, 0.75 and 1. Generally, due to netting of liabilities, contract covenants, collaterals and other measures that banks take to somehow cover their exposure, a loss rate of 1 seems difficult to be actually observed[25]. Nevertheless, it serves as an informative worst case scenario.

[24] For a more detailed explanation, see Mistrulli (2007), pp.8-10.

[25] James (1991) found loss rates to be around 30 percent without including bankruptcy

At a loss rate of 0.25, which seems fairly reasonable for a real-world situation, only the bankruptcy of four banks triggers a chain of defaults: Unicredit, Intesa Sanpaolo, Banca IMI and Monte dei Paschi di Siena. The chain of defaults extinguishes at round 4.

Results for the different loss rates on the number of banks that fail in each round are given in the figures in this section. For each round, we show the mean amount of banks that fail in the n^{th} round, and the mean of assets lost at each round. We also show the maximum amount of assets lost and the maximum number of banks that fail for each loss rate. We assume that when a bank fails all its assets are lost.

Even at a loss rate of 1, only the failures of fourteen banks due to exogenous shocks cause a chain of default larger than 2 rounds. Unicredit, the largest bank for assets, is the one that causes the largest number of defaults in the lowest number of rounds (4). It is also the bank that creates the absolute largest chain of defaults: 64 against the 50 caused by Intesa Sanpaolo.

Most default cascades only take 4 rounds to extinguish. The banks that cause a default cascade that takes the most number of rounds to extinguish are Banco BPM and BNL, which create a 6-rounds default cascade.

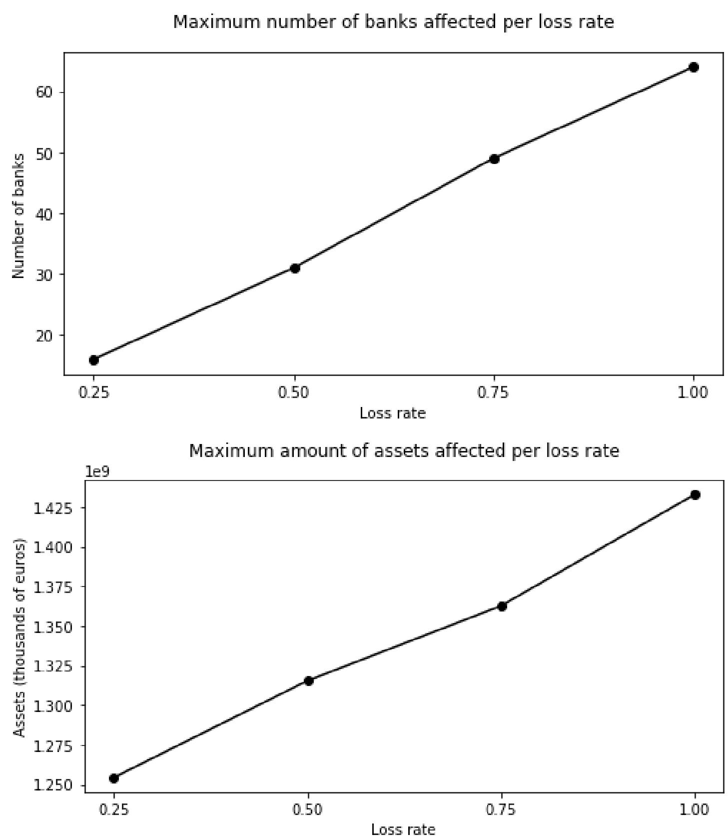
At lower loss rates, the chain of defaults starts to dissipate already after the first two rounds. At a loss rate of 0.75, it peaks on average at the fourth round, while at a loss rate of 1 it peaks earlier at the third round. The large amount of assets lost already at the first round is explained by the fact that only the failure of large banks in the first rounds is going to cause default cascades to the rest of the system.
Notice that, on average, a larger amount of assets is lost per round when the loss rate is lower than 1. This occurs because more smaller banks make other smaller banks fail at a loss rate of one, which brings down the average. Nevertheless, the total amount of assets lost per round is larger at a loss rate of 1 than for smaller loss rates, as well as the maximum.

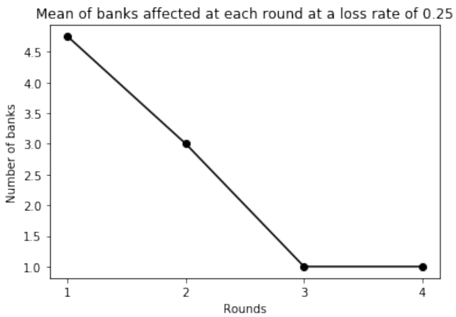
Our results are consistent with results from van Lelyveld and Liedorp (2006)

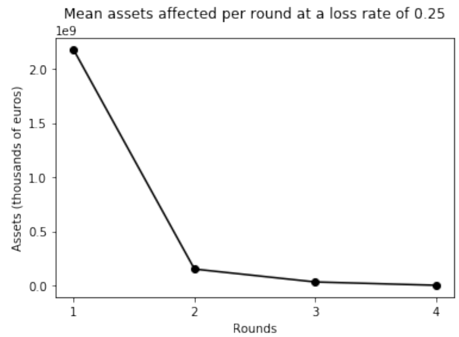
costs. Due to the different time and environment, these results probably do not translate to our situations, but point against the possibility of loss rates of 100 percent.

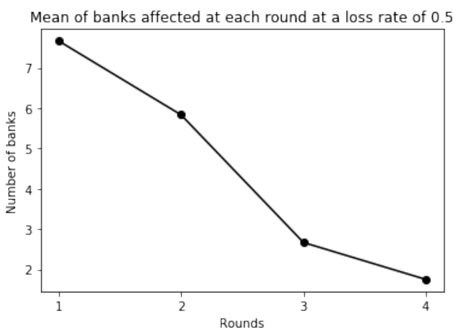
in that we find, especially for larger loss rates, that when the largest banks (such as Unicredit) fail, there are less total rounds with respect to when other, slightly smaller banks fail. This occurs because the failure of the largest banks makes more non-contagion-proof banks fail already in the first rounds. This also explains why defaults peak, on average, earlier at a loss rate of 1 than at a loss rate of 0.75.

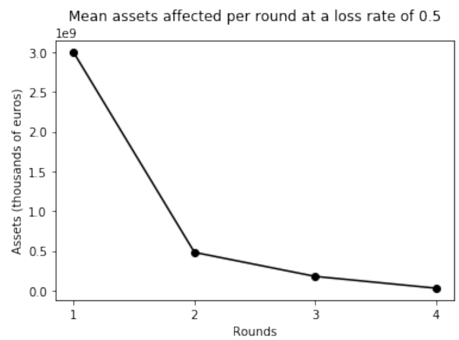
Overall, entropy maximization shows that only the failure of large banks causes other banks to fail. Moreover, a chain of defaults begins only if the failure of a large bank causes either another large bank to default or many smaller banks. The banks that create the largest systemic risks are Unicredit, Intesa Sanpaolo, Banco BPM, Monte dei Paschi di Siena, Banca IMI, BNL and UBI, which unsurprisingly are also some of the largest ones. This is consistent with what we expected to see in an interbank market that presents a money centre structure, as affirmed by Mistrulli (2007).

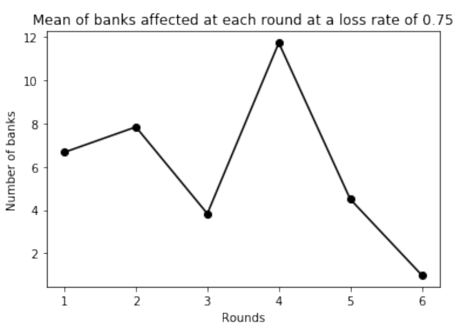


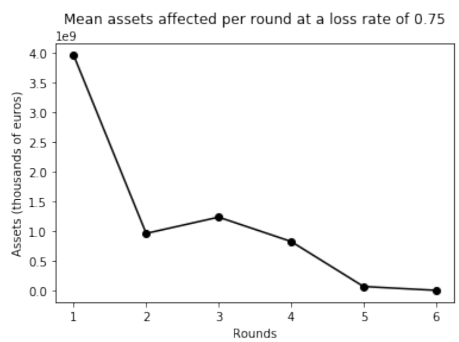

Mean of banks affected at each round at a loss rate of 0.25
Number of banks
Rounds

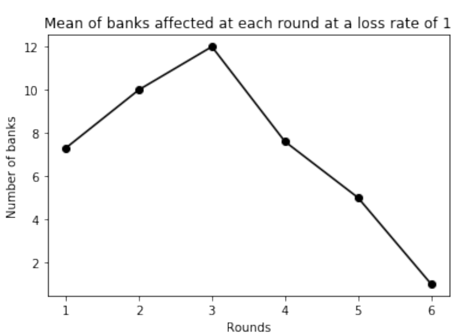

Mean assets affected per round at a loss rate of 0.25
1e9
Assets (thousands of euros)
Rounds

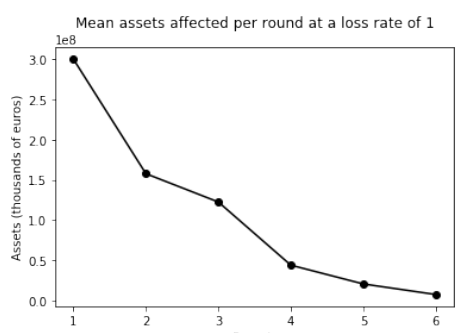

Mean of banks affected at each round at a loss rate of 0.5
Number of banks
Rounds


Mean assets affected per round at a loss rate of 0.5
1e9
Assets (thousands of euros)
Rounds


Mean of banks affected at each round at a loss rate of 0.75
Number of banks
Rounds


Mean assets affected per round at a loss rate of 0.75
1e9
Assets (thousands of euros)
Rounds


Mean of banks affected at each round at a loss rate of 1
Number of banks
Rounds


Mean assets affected per round at a loss rate of 1
1e8
Assets (thousands of euros)
Rounds

5. Topology-independent analysis

5.1. Methodology

Glasserman and Young (2015) are able to obtain bounds on contagion that do not depend on the specific network topology. Since Mistrulli (2007) found that results from entropy maximization may not be accurate for the specific topology of the Italian interbank market, we turn to this method to obtain results that are somewhat more general, but at the same time may be more representative[26].

Let $\beta_i > 0$ be the fraction of bank i's liabilities due to other banks. We assume this is larger than zero, otherwise these banks could be considered outside of the network. Let w_i be the tier 1 equity of bank i, and c_i the value of its assets that are outside of the network. Moreover, let $\lambda_i = c_i/w_i \geq 1$ be the leverage of its outside assets.

We let bank i be hit by a shock, and we let D be a set of banks, where $i \notin D$. The probability that the shock causes all banks in D to default is at most:

$$P\left(X_i \geq w_i + \frac{1}{\beta_i}\sum_{j\in D} w_j\right)$$

And it is impossible for i to affect the banks in set D if:

$$\sum_{j\in D} w_j > w_i\beta_i(\lambda_i - 1)$$

Where Glasserman and Young call $w_i\beta_i(\lambda_i - 1)$ the contagion index for i. They find that this "provides a measure of the relative likelihood that the

[26]Here we only provide a brief summary of the methodology developed by Glasserman and Young (2015) in order to make the rest of the section understandable. For a better grasp on the subject and for proofs, we suggest to see their paper directly.
We are going to use the same notation used by them in order to make comparison more straightforward.

nodes in D default due to direct shocks to their outside assets compared to the likelihood that they default due to contagion from i"[27].

Contagion is weak if banks in the default set default more probably due to outside shocks than due to contagion from i:

$$P\left(X_i \geq w_i + \frac{1}{\beta_i}\sum_{j\in D} w_j\right) \leq P(X_i > w_i)\prod_{j\in D} P(X_j > w_j)$$

Here, independent shocks are assumed on the right side of the equation.

Now, assume that shocks are independent and identically distributed with a beta density of the following form[28]:

$$h_{q,p}(y) = \frac{y^{p-1}(1-y)^{q-1}}{B(p,q)}, \quad y \in [0,1], \quad p,q \geq 1$$

Then, contagion is weak if:

$$\tilde{\lambda}_D \overline{w}_D \geq w_i \beta_i (\lambda_i - 1)$$

Where $\overline{w}_D$ is the average tier 1 equity in set D, and $\tilde{\lambda}_D$ is the harmonic mean of the leverage ratios.

[27]Glasserman and Young (2015), p. 5.

[28]Glasserman and Young note that legal capital requirements use a Gaussian copula model that can be approximated by beta distributions, see Glasserman and Young (2015), note 10, p. 6.

5.2. Results

We have taken all the banks whose bankruptcy made any other bank fail in the maximum entropy simulation, and we have examined their potential for contagion according to the methodology just outlined. For each bank we have available in our dataset, or we are able to compute, in-network assets, outside assets, tier 1 equity, the β, which is equal to the fraction of the bank's liabilities due to other banks[29], and λ. We drop observations for which β is equal to zero[30].

Moreover, for each bank we compute the "Weak Ratio" (WR):

$$\mathrm{WR} = \frac{\tilde{\lambda}_D \overline{w}_D}{w_i \beta_i (\lambda_i - 1)}$$

It follows that contagion is weak if WR > 1, and vice-versa.

Here, we deviate from the methodology outlined by Glasserman and Young of taking as the default set D two banks at a time for each bank we analyse the potential of contagion for. Since the Italian interbank market has a money centre structure, this means that, generally, large banks lend money either to a large number of smaller banks or to one another. Therefore, the bankruptcy of a systemically important bank should have repercussion either on few other money centre banks or on a large number of smaller banks.

Consequently, for each bank whose bankruptcy causes other banks to default with entropy maximization, we provide weak ratios for five different kinds default sets: (1) the two largest banks in the dataset (group 1)[31], (2) the 2 largest banks that are smaller than the bank analysed (group 2), (3) the 10 largest banks whose default does not cause other banks to default in entropy maximization (group 3), (4) 50 medium banks, (group 4) and (5) the 200 smallest banks in the dataset (group 5).

[29] Notice that in this case our data is precise, while Glasserman and Young (2015), using EBA 2011 stress test data, estimated this to be proportional to the bank?s in-network assets.

[30] Which are minor banks that probably present problematic data.

[31] We do not provide figures for Unicredit and Intesa Sanpaolo because they are themselves the two largest banks in the dataset.

We provide a summary of the results in table 2.

We find that contagion is always weak from each of the banks analysed when we take Unicredit and Intesa Sanpaolo (the two largest banks in the dataset) as the default set. This means that Unicredit and Intesa Sanpaolo are generally (much) more likely to default due to external shocks than due to the default of any of the banks analysed.
The only two banks that are more likely to make the next two largest banks default with their own default rather than due to an external shock are Mediocredito Italiano and Unicredit Leasing (respectively, their default sets are "Monte dei Paschi di Siena Capital Services" and "Banca Popolare di Vicenza" for Mediocredito Italiano, and "FinecoBank" and "Banca Aletti" for Unicredit Leasing).
Only Unicredit and Intesa Sanpaolo are more likely than an external shock to affect the 10 largest banks whose default does not cause other banks to go bankrupt in entropy maximization. These results point out that it is unlikely that the failure of a large bank makes other large banks go bankrupt, except for rare cases, and unless the banks defaulting are the largest in the system (namely Unicredit and Intesa Sanpaolo).

The situation, however, changes completely when we take into account medium and small banks. For all the banks we have analysed, when the default set is made up of either medium or small banks, we find that the weak ratio is always smaller than 1, and generally presents very small values. This implies that contagion in those cases is not weak, and that most medium and small banks are more likely to default from the default of a large bank rather than from an outside shock.

Overall, these results are in line with Glasserman and Young (2015) in that, when we compare large banks with other large banks, we generally find large weak ratios, while when we compare large banks to smaller banks, we find small weak ratios. Moreover, we are able to compute weak ratios of large banks taking as a default set a large number of banks of very small size, which they were not able to accomplish due to the limitations of the EBA 2011 stress test dataset, and in such case, we find very small weak ratios (all smaller than 0.005).

Finally, our results from this section are in line with the money centre structure of the interbank market found by Mistrulli (2007): just as we would expect in such a kind of network, the failure of the largest banks (the money centres) is much more likely to make a large number of smaller banks go bankrupt than another large bank.

Bank	**WR group 1**	**WR group 2**	**WR group 3**	**WR group 4**	**WR group 5**
Unicredit		3.99159	0.45637	0.03980	0.00025
Intesa Sanpaolo		2.87614	0.81326	0.07092	0.00044
Banco BPM	33.69689	6.57101	2.20110	0.19195	0.00120
Banca Monte dei Paschi di Siena	42.60316	6.46013	2.78286	0.24268	0.00152
Banca IMI	17.35396	2.29555	1.13357	0.09885	0.00062
UBI Banca	68.03917	6.55412	4.44435	0.38757	0.00242
BNL	33.03379	2.92628	2.15778	0.18817	0.00118
Iccrea Holding	92.61072	3.82696	6.04938	0.52754	0.00329
Mediocredito Italiano	18.35533	0.95490	1.19898	0.10456	0.00065
Monte dei Paschi di Siena Capital Services	42.60316	6.46013	2.78286	0.24268	0.00152
Banca Popolare di Vicenza	73.58238	3.82619	4.80644	0.41915	0.00262
Dexia CREDIOP	57.23598	1.54973	3.73868	0.32603	0.00204
Unicredit Leasing	42.71088	0.35605	2.78990	0.24329	0.00152
Findomestic Banca	67.27409	1.02649	4.39438	0.38321	0.00239

Table 2: Summary of weak ratios.

6. Conclusion

We have used data on 544 publicly owned banks taken from the database "Orbis Bank Focus" to analyse the risk of contagion in the Italian interbank market.
In order to do so, we have made use of two methodologies: (1) entropy maximization, which is the standard method used in the literature when no additional data apart from gross exposures are available, and (2) a methodology not dependent on the specific topology of the network that allowed us to compare probabilities of defaults from contagion rather than from outside shocks developed by Glasserman and Young (2015).

With entropy maximization, we found that only the failure of one of the largest banks triggers a chain of defaults in the systems, and that this chain of defaults dissipates in a few rounds even at the unrealistically high loss rate of 1. Moreover, chains of defaults occur only when the failure of a large bank causes at least another large bank or a large number of smaller banks to default.

Since Mistrulli (2007) using actual data on (non-aggregate) exposures at the node level reported to the Bank of Italy finds that entropy maximization may fail to approximate the true interlinkages occurring in the money centre Italian interbank market, we turn to a topology-independent analysis. For each of the banks that caused a chain of defaults in entropy maximization, we calculate weak ratios assuming different default sets. We find results that go hand-in-hand with the money centre structure of the interbank market affirmed by Mistrulli: generally, large banks are more likely to default due to outside shocks rather than due to the default of other large banks, but the failure of a large bank is likely to cause the failure of many small banks.

Overall, as long as the largest banks (the money centres) remain safe, it looks unlikely for the system to suffer systemic risk. Even when a large bank goes bankrupt, the system looks threatened only at large loss rates.

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