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An Ambiguity Analysis under Heterogeneity with a Bayesian Spin

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Agli attimi rosso fuoco.

Alle scelte fuori binario.

Alle parole macigni di libertà.

A queste emozioni dilaganti,

e a tutti coloro che ancora ne respirano la Vita.

τα πάντα ῥεῖ.

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Now I would like to switch to Italian, in order to express some feelings that cannot easily be translated.

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Prost!

Abstract

We often have to deal with uncertainty regarding multiple aspects of the decision problems we face. This uncertainty may concern, for instance, our earnings, the likelihood to receive them in a given moment and in a given amount. The aim of this thesis is to contribute to the growing body of literature around “multi-dimensional uncertainty”, which enlarges the scope of ambiguity outside the frame of uncertainty about probabilities. It does so by analysing, both theoretically and empirically the evidence stemming from a multi-stage experiment in which subjects have to choose between lotteries whereby amounts of monetary prizes are not always known, whereas probabilities are always public knowledge. In the experiment, three different levels of information over some monetary prizes are randomized between subjects. The experimental evidence undergoes structural estimation exercises: these elicit the individuals' degree of risk aversion within the frame of a standard constant relative risk aversion (CRRA) utility function. Furthermore, we investigate whether a change of information, such as the one we reproduce through the different treatments conditions, translates into a change in behavior and, in turn, whether and how much this change translates into a significant change in their measured (CRRA) attitude toward risk. As to the behavioral content of the structural model for the uncertain payoffs, we propose two alternative specifications, labelled “naive” and “sophisticated”. The empirical evidence shows a moderate but significant degree of *love for ambiguity*, since less information given to subjects results in a lower estimate of their risk aversion, and, as a consequence, in a stronger attraction toward risk and uncertainty. A mixture model is implemented to identify the probability of individuals mirroring one behavioral model or the other, or, saying it differently, the percentage of observations compatible with either model. We conclude that our subjects have a strong tendency to behave as naive.

Keywords: heterogeneity; risk aversion; ambiguity.

Contents

CHAPTER 1

Introduction

Uncertainty regarding multiple aspects of the decision problems is an issue that often arises. This uncertainty may concern the amount of monetary earnings, the likelihood of these earnings, the actual date at which these earning are received, etc. In this respect, the way in which individuals behave in uncertain situations may as well varies for different dimensions of uncertainty. However, the theoretical and empirical economic discussion on these issues has been mostly focused on a specific kind of uncertainty, the *uncertainty about probabilities*. The aim of this dissertation is to contribute to the scarce but increasing body of research which deals with "multi-dimensional uncertainty", that enlarges the scope of ambiguity outside the frame of uncertainty about probabilities. We contribute on this by analyzing (both theoretically and empirically) existing evidence from a multi-stage experiment in which subjects have to choose between lotteries where probabilities were publicly known at all times but some of the monetary prizes were not.

It should be noted at this point that, from a pure bayesian perspective, all the different dimensions of ambiguity can be reduced to a single one by appropriately defining the "states of the world" as multidimensional objects defined over all uncertain dimensions. Within this augmented frame, a "bayesian" decision maker would simply form some subjective prior beliefs over this augmented set of states of the world and maximize an objective function, that represents her preferences based on them. If we accept this bayesian interpretation, every decision problem under multi-dimensional ambiguity can be appropriately reduced to a standard problem of uncertainty over probabilities. This can only stand if we show that people are able to build such complex and multi-dimensional spaces and behave accordingly. Conversely, if this was not be the case, it would matter which objects of uncertainty are domains and whether and how these domains might be correlated.

This thesis reports evidence form a multi-stage experiment conducted at the "Laboratory of Theoretical and Experimental Economics" of the University of Alicante by Albarrán et al [1]. In the experiment, three different levels of information over some monetary prizes are randomized between subjects. Specifically, in the full information treatment, TR2, subjects observe all the prizes of the lotteries they are asked to select; in the partial information treatment, TR1, they are not informed about their actual values, but they know that they are i.i.d. draws from a uniform distribution; finally in the no information treatment, TR0, they are just informed of the prize rankings.

The experiment develops along two ordered balance phases built upon two classic risk-elicitation protocols, the Holt and Laury [2] and the Hey and Orme [3], respectively. Hey and Orme [3] experiment is built around binary choices between lotteries over 4 fixed monetary prizes, such as $\{0, \frac{1}{3}, \frac{2}{3}, 1\}$. In the treatments with ambiguity, TR0 and TR1, the intermediate payoffs, Y and X, are communicated to the subjects, being between 0 and 1, with $Y < X$. In phase 1, instead, the subjects elicit, by the way of a Multiple Prize List (MPL), the certainty equivalent of the same lotteries used in phase 2.

The experimental evidence is read by the way of some structural estimation exercises in which the individuals' degree of risk aversion is elicited within the frame of a standard constant relative risk aversion (CRRA) utility function. Furthermore, it is analyzed whether a change of information, such as the one reproduced through the different treatments conditions, correlates to a change in behavior and, in turn, whether and, how this change trasforms into a significant change in their measured (CRRA) attitude toward risk.

The uncertain payoffs Y and X are identified as the first and the second order statistics from a uniform distribution in $[\,0, 1\,]$, where the order statistics of a random sample $\{\chi_1, \chi_2\}$ are defined as the sample values placed in ascending order.

While this is totally correct for TR1 subjects -given that they know the characteristics of the random generation process that yields the uncertain payoffs- the same statistical model was imposed for subjects in TR0, considering that they already had this information. This is purely an identification assumption, as there is no possibility to test whether this is truth for the expectations in TR0 about the X and Y distributions, or whether subjects in TR0 consider another distribution. On the other hand, it is highly probable that TR0 subjects will heuristically and automatically come up with such a distribution of the payoffs, as it occurs in Laplace's well known "principle of insufficient reason". In any case, the important here is that -based on this assumption- our structural model is able to estimate treatment effects, to such an extent that we are able test a null hypothesis in which CRRA in both TR0 and TR1 is the same. Since subjects are randomized within treatments, a significant change in the CRRA coefficient between TR0 and TR1 has to be interpreted as a genuine treatment effect due to a change in information.

Regarding the behavioral content of the structural model for the uncertain payoffs, two alternative specifications were considered, labelled as "naive" and "sophisticated", respectively. A naive decision maker figures out a point estimation of the unknown payoffs X and Y, starting from the information that these are draws of a uniform distribution in $[\ 0, 1\ ]$. This means that $E[X]$ and the $E[Y]$ are computed and then plugged inside the CRRA utility function to be maximized. On the other hand, a sophisticated decision maker will proceed with a true bayesian updating. In particular, she formulates a prior distribution over the X and the Y, and then calculate the expected utility from these densities.

We shall now here summarize our main findings. Our empirical evidence shows a certain degree of *love for ambiguity*, given that the less the information given to subjects, the lower their estimated risk aversion, and, consequently, the bigger their attraction toward risk and uncertainty. Moreover, the risk aversion coefficient estimated for TR0 is significantly lower from that estimated in TR2, although no statistically significant difference was found between estimated CRRA coefficients in TR0 and TR1. These findings are in a way in contradiction to the common wisdom of the literature, although they are consistent with other experimental literature that applies similar elicitation techniques as Andersen et al [4].

When comparing the two behavioral models, that is the "bayesian" against the "naive", the estimated likelihood of the naive approach is higher than the one of the bayesian. This suggests that the naive approach closely approximates subjects' decision rules, given the data. In this regard, a mixture model is implemented to identify the probability of individuals using each model, which corresponds to the percentage of observations compatible to each model. This mixture model aims to achieve a statistical reconciliation of these two dominant theories of choices under risk. It avoids any extreme declaration of "winners" and "losers", providing a more balanced metric to decide which theory performs better in a given domain given the experimental data. Due to the fact that the likelihood of our models are very close, this probability was estimated numerically, using a grid loop.

Specifically, a probability π_{BAY}, i.e. the probability of the subjects acting as bayesian in each of their decisions was estimated. Subsequently we let this π_{BAY} moving inside a grid $(0, 1)$, to finally choose the value that maximizes the likelihood function.

This numerical computation demonstrates that the subjects have a strong tendency to behave as naive, given the estimation result, which was $\pi_{BAY} = 0.2$.

The structure of this thesis is arranged as follows. In Chapter 2 the previous results on the long lasting debate of decision making under risk and uncertainty are presented. In Chapter 3 the experimental design is described in details. In Chapter 4 it is shown how the individuals' heterogeneity is treated under structural modelling, providing some examples applied to the data. In Chapter 5 our structural estimations are reported as a function of the two alternative behavioral specifications, the naive and the bayesian, along with the estimation of our mixture model. Finally, Chapter 6 summarizes our results and highlights possible future developments and more complex experimental investigations. Two appendices follow, containing the experimental instructions and the Stata codes used for the statistical analysis.

CHAPTER 2

Literature Review

2.1 Risk vs Uncertainty: previous approaches to a long-lasting debate

In Economic theory, an already established difference exists between the concept of risk and the concept of uncertainty, also defined as ambiguity. Indeed, a risky situation exists when both the outcome values and their distribution are known. On the other hand, an uncertain, or ambiguous, situation occurs when the outcomes but not their distribution are known.
Reporting Knight [5] "Even though the business man could not know in advance the results of individual ventures, he could operate and base his competitive offers upon accurate foreknowledge of the future if quantitative knowledge of the probability of every possible outcome can be had..."
Risk can be quantitatively measured by any form, while uncertainty cannot. Therefore, there are certain risks that can be fully covered by taking insurance policies such as fire, flood, theft, robbery etc.. whereas in uncertainty an insurance coverage is not possible.
Knowing the distribution of the outcomes can be considered as a hyperbolic situation in the daily decisions agents face. However, incomplete information, or uncertainty, is common in multiple aspect of the decision problem.
For example, the monetary earnings value may be uncertain, the likelihood of positive earn-

ings may be uncertain, the payment date may be uncertain etc... The proliferation and frequent introductions of new brands are involved in the marketplace uncertain processes such as the time of purchase and brand selection. When buying a used car, people cannot know beforehand that it will not break down in the near future. When considering a loan for a new home, one can never be certain whether a fixed or a variable plan will be more effective.
Consumers have learned eventually how to deal with uncertainty or ambiguity. Indeed, in real settings, exact probabilities are impossible to be assigned to specific events. For example, the probability the car breaking down in the next moths could be estimated between 10% and 40%. In these situations, the probabilities are ambiguous, which means there is uncertainty about the uncertainty, as the odds of an uncertain event aren't exactly known. For istance, whereas there is no ambiguity associated to a coin toss, on the other hand a new product technology, a computer hardware obsolescence or the purchase time relative to the next price cut are quite ambiguous situations.
In conclusion, as Heath and Tversky [6] point, "the potential significance of ambiguity stems from its relevance to the evaluation of evidence in the real word."
According to Savage [7], uncertainty does not exist, since individuals always have been deciding for a subjective probability distribution to be associated to all decisions. This reflects all the information players have on the likelihood of a specific event and is a Bayesian approach, where subjects create a subjective probability evaluation that represents their specific state of knowledge in an uncertain situation. The subjects make their choices by maximizing the subjective expected values they compute.
By contrast, Ellsberg [8] states that these subjective probabilities cannot be always established by the players. Indeed, Savage approach is true only when some uncertain choice events become risky, after individuals had assigned subjective probabilities. In many instances though the subjective probability assessment is impossible or meaningless and people do not maximize the mathematical utility expectation on the basis of numerical probability for these events. It is not also possible to derive from their choices in these gamble events numerical von Neumann-Morgenstern utilities. Thus, their choices are not ascribable to any usual criteria for predicting or prescribing decision-making under uncertainty. Still, these choices do not appear to be careless or random.

In the empirical evidence proposed by Ellsberg, individuals seem to stay away from lotteries which are ambiguous, that is, lotteries whose associated probabilities cannot be precisely stated. Attending to his experiments, people choices depend also on the level of ambiguity the decisions contain, namely the reliability of probability assessment. To sum up, people dislike ambiguity and this yields inconsistency with the Bayesian theory under uncertain decisions. Following Kahn and Sarin [9], ambiguity can be operationally defined as the second-order uncertainty, particularly as the probability distribution for the perceived frequencies.

On the other hand, Lauriola and Levin [10] state that ambiguity cannot be reduced to second-order probability distributions. Their empirical results show that subjects strongly dislike ambiguous lotteries compered to the ones with explicit probabilities ones. In addition, the same proportion of preferences for the first- and second-order lotteries are observed, when one is compared to the another. Thus it seems that behavioural science does not support the hypothesis that ambiguity could be reduced to explicit second order probabilities.

Klibanoff, Marinacci and Mukerji [11] achieve a separation between ambiguity, as a characteristic of the decision maker subjective beliefs, and ambiguity attitude, as a characteristic of the decision maker's tastes. Ambiguity is defined by them as the uncertainty about the priors relative to the decision. The value function is:

$$V(f) = \int_{\Delta} \Phi \int_{S} u(f) d\pi = E_{\mu}[\Phi(E_{\pi}[u \cdot f])]$$

where f is the action chosen, Φ the ambiguity aversion function over Δ, π the probability measure over S and μ the subjective prior.

Halevy [12] tests different theories' performances in a controlled experimental environment, extension of the original Ellsberg experiment. As a result of this study, derives the lack of the ability of a comprehensive model to universally capture ambiguity preferences. This work confirms the approach of Epstein [13], by defining ambiguity aversion as a non-probabilistic sophisticated behaviour model, with no commitment to a specific functional model. This suggests that the failure in reducing compound objective lotteries is the underlying factor of the Ellsberg paradox itself. Finally, it lies upon decision theory to uncover the theoretical relationship between the reduction and ambiguity aversion.

In Harrison and Rutstrom's work [14] is introduced for the first time the possibility of a "wedding" between utility theory and prospect theory, i.e. between the two front runners for choices under uncertainty. In the aforementioned literature, is to allow for heterogeneity is usually allowed in responses through standard methods, such as random or fixed effects models. Indeed, this is not helpful to identify which people and when behave according to which theory, as heterogeneity is only recognized within a given theory. This automatically excludes heterogeneous theories from co-existing in the same sample. Harrison and Rutstrom specify and estimate a grand likelihood function, namely a mixture model, allowing for different theories to coexist and each having a specific weight. Specifically, the different theories are the expected utility theory (EUT) and prospect theory (PT) by Kahneman and Tversky [15]. The EUT assumes constant relative risk aversion utility function defined over the final monetary prize the subject will receive if the lotteries are played out. On the other hand, PT has a utility function defined over gains and losses separately, with a probability weighting function converting the underlying probabilities of the lottery into subjective ones. The key idea behind this mixture model is to achieve a statistical reconciliation of these two dominant theories of choices under risk. The model avoids any extreme declaration of winners and losers, providing metric to decide which theory performs better in a given domain. It also provides insight regarding when one theory behaves better than the other, using individual characteristics effects and treatments on estimated probability of support.

The already mentioned Kahn and Sarin [9] develop a modified version of the subjective expected utility model which assigns a decision weight, $w(E)$, to each event E. Therefore, the value function of a lottery L is obtained as:

$$V(L) = w(E)u(x).$$

Specifically, the weight is a function of the individual attitude toward ambiguity, namely λ, of the mean value of the probability of the event E, $\overline{p}$ and of the standard deviation of p, σ. It is expressed as:

$$w(E) = \overline{p} - \lambda\sigma.$$

This is clearly a variance of the common mean variance model, that is degraded to the SEU classical model when there is no ambiguity effect.

As a consequence, an ambiguity adverse individual, with a $\lambda > 0$, dislikes higher variance

of p. On the other hand, if there is no ambiguity or the subject does not care about it, which means that $\lambda = 0$, the weight will reduce to the mean value, $w(E) = \overline{p}$.
The findings point a significant context effect, which means that individuals attitude toward ambiguity changes depending on the context. As a result, λ is individual and context specific. Also the payoffs involved, namely if there is a win or loss, the amount of ambiguity, i.e. the range effect, and risk, that is the mean effects, do influence the overall attitudes toward ambiguity.
Based on Andersen et all [4] people formulate some priors on both certain and uncertain events and subsequently calculate the weighted average subjective expected value, given the probability of the priors. As the authors stated in their horse race metaphor, *"the decision-maker might have one prior over the performance of horses if it rains and the track is heavy, and another prior over the performance of the horses if the track is dry"*.
Sarin and Winkler [16] model ambiguous decision-making situations through a modification in the utility functions rather than in the decision maker's probability. Indeed, utilities depend not only on the received payoff but also on the one that might have been received but was not. This preference based model is composed by a simple bet B which yields a monetary payoff of X if the event E does occur, otherwise a payoff Y, where we assume $X > Y$. The decision maker expected utility is :

$$U(B) = pu(x) + (1 - p)u(y).$$

If the decision maker feels ambiguous about p, some new modified utility functions will appear, which depend on the degree of ambiguity in the event probability:

$$v[u(x)|u(y)]$$

and

$$v[u(y)|u(x)].$$

The new expectation of the modified utility functions will be:

$$V(B) = pv(x|y) + (1 - p)v(y|x)$$

The ambiguity concern is related to both the payoffs, and, when they are very close, the ambiguity about E probability does not really matter. Specifically, as $x - y$ approaches to

0, $v(x|y)$ approaches to $u(x)$ and $v(y|x)$ to $u(y)$. So the greater the degree of ambiguity, the higher the effect on $v()$.

Gilboa and Schmeidler [17] propose a new decision making model under uncertainty. Firstly, they adopt the neobayesian paradigm which leads to a set of multiple priors instead of a unique one. Following this way of reasoning, the ambiguity adverse decision maker takes into account the minimal expected utility, over all priors in the set, while evaluating a bet. Then, he proceeds on maximizing this expected utility. This results in the maximization of the minimum expected value of the decision maker's utility.

Herrero and Villar [18] suggest a quantitative assessment on the relative desirability of some lotteries distribution in term of the likelihood of getting better results. They provide a consistent application of the willingness to pay principle which is coherent with the first order stochastic dominance criterion. In particular, if distribution A stochastically dominates distribution B, then the worth of A, that's the willingness to pay for it, will be larger than of B. However, this evaluation procedure does not compute the difference in the magnitude of the prizes, but only their rankings. For this reason, this assessment may not be exhaustive in case the size of the prize differences constitutes a relevant part of the evaluation problem. On the other hand, this suggests an immediate application for monetary lotteries when utilities are ordinal.

Eliaz and Ortoleva [19] inject in the decision making task different dimensions of uncertainty, for example in the winning probability, in the prize amount, in the payment date and in all their subsequent combination. This is called a "multidimensional" uncertainty, a situation that does influence the willingness to accept uncertainty. Namely, they questioned whether the presence of uncertainty in some dimensions influences the willingness to accept it in a specific dimension and whether decision makers are still uncertainty adverse when facing multidimensional uncertainty. Their findings suggest that in many circumstances decision makers are more likely to opt for uncertainty options, having more and correlated uncertain dimensions. Specifically, when they have the possibility to remove all uncertainty, people opt for the sure option. On the other hand, when uncertainty cannot be removed at all, the majority prefers the options with perfectly correlated uncertainty on several dimensions with respect to the ones with a unique uncertainty dimension.

Eichberger, Oechssler and Schnedler [20] arrive to a similar conclusion. Indeed, they conduct an experiment to examine the effect of introducing an additional source of ambiguity to the standard two-color experiment by Ellsberg. Their results showed that fewer subjects opt for betting on known proportions events, once a second source of ambiguity appears. This behaviour, which is in line with Eliaz and Ortoleva's findings, contrasts to the prediction of numerous decision making theories, such as the maximized expected utility.

Moore and Eckel [21] consider multiple levels of ambiguity, while analyzing the difference between the gain domain and the loss domain. Greater ambiguity aversion will be exhibited in the gain domain, if there is ambiguity in the probability of the event and also if there is ambiguity both in the probability and in the amount of the outcome. In the loss domain instead, the ambiguity aversion is mainly driven by the size of the ambiguity rather then by its specific location. This finding is particularly interesting, because it may explain why consumers are used to pay large insurance premiums, even if the expected loss is relatively low. In conclusion, they also note that decision makers do have a preference for gambles framed as investment opportunities, rather than as lotteries.

List et al [22] analyze how ambiguity aversion estimates change as function of the risk aversion assumed. In particularly, estimates of ambiguity aversion are greater when risk neutrality is assumed than when risk and ambiguity aversion parameters are jointly estimated. Thus, the methods based on risk neutral individuals, while qualitatively capture their ambiguity attitudes, do not jointly measure the risk attitude and may fall on the trap of overstating the true degree of ambiguity aversion.

2.2 Elicitation Procedures

To ascertain the individuals' risk attitudes in the experimental laboratory, there are 5 general elicitation procedures, as described by Harrison and Rutstrom [23].
The first is the Multiple Price List (MPL), where the subject faces an ordered array of binary lottery choices, to be made all at once. Then the subject has to pick the one that will be played. The experiment described in Holt and Laury [2] is an example of a MPL. Indeed, there are two binary lotteries, A and B, and the subject has to choose to play one of them. The difference between payoffs of the events of the lottery A is small (for example $2 and $1.6) while the difference between the payoffs of the events of the lottery B is big (for example $3.85 and $0.1). Namely, the highest difference in between the lotteries is their payoffs' standard deviation. The lottery A starts with a higher expected value than lottery B, giving in both cases a high probability to the lower pay-off. When we advance in stages, probabilities change, and the expected value of both lotteries increase. However, expected value of B do it faster, being in the final stage higher than the expected value of lottery A.
Figure 2.1 reports a MPL experiment example[1].

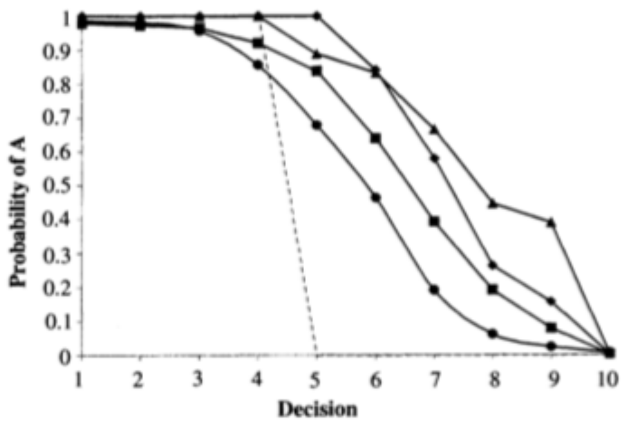
Figure 2.1: Holt and Laury (2002) lotteries

Option A	Option B
1/10 of $2.00, 9/10 of $1.60	1/10 of $3.85, 9/10 of $0.10
2/10 of $2.00, 8/10 of $1.60	2/10 of $3.85, 8/10 of $0.10
3/10 of $2.00, 7/10 of $1.60	3/10 of $3.85, 7/10 of $0.10
4/10 of $2.00, 6/10 of $1.60	4/10 of $3.85, 6/10 of $0.10
5/10 of $2.00, 5/10 of $1.60	5/10 of $3.85, 5/10 of $0.10
6/10 of $2.00, 4/10 of $1.60	6/10 of $3.85, 4/10 of $0.10
7/10 of $2.00, 3/10 of $1.60	7/10 of $3.85, 3/10 of $0.10
8/10 of $2.00, 2/10 of $1.60	8/10 of $3.85, 2/10 of $0.10
9/10 of $2.00, 1/10 of $1.60	9/10 of $3.85, 1/10 of $0.10
10/10 of $2.00, 0/10 of $1.60	10/10 of $3.85, 0/10 of $0.10

[1] Holt and Laury [2]

Figure 2.2 reports the proportion of safe choices in each decision[2].

Figure 2.2: Holt and Laury (2002) safe choices



Every lottery can be represented in this form:

$$L_i^k()x = \{x_i^H, x_i^L; \frac{k}{10}, \frac{10-k}{10}\},\ i = 0, 1;\ k = 1, ..., 10;\ x_i^H > x_i^L$$

If the subject is risk lover, he would be prone to choose B even when its expected value is lower than the expected value of A. On the other hand, if the subject is risk averse, he would be prone to choose A even when the expected value of A is lower than the B one. The expected behavior of a risk neutral subject would be to start choosing A and then subsequently B, according to the amount of the expected value.

However, it could exist a frame that encourages subjects to choose middle rows, win which case we do not have enough information to see the attitude to risk of the subject. Therefore, we could solve the problem by adding some parameters in these middle choices to achieve more information about the risk attitude.

[2]Holt and Laury [2]

Figure 2.3 reports the risk aversion classifications based on lottery choices[3].

Figure 2.3: Holt and Laury (2002) risk aversion classification

Number of safe choices	Range of relative risk aversion for $U(x) = x^{1-r}/(1-r)$	Risk preference classification	Proportion of choices		
			Low real[a]	20x hypothetical	20x real
0–1	$r < -0.95$	highly risk loving	0.01	0.03	0.01
2	$-0.95 < r < -0.49$	very risk loving	0.01	0.04	0.01
3	$-0.49 < r < -0.15$	risk loving	0.06	0.08	0.04
4	$-0.15 < r < 0.15$	risk neutral	0.26	0.29	0.13
5	$0.15 < r < 0.41$	slightly risk averse	0.26	0.16	0.19
6	$0.41 < r < 0.68$	risk averse	0.23	0.25	0.23
7	$0.68 < r < 0.97$	very risk averse	0.13	0.09	0.22
8	$0.97 < r < 1.37$	highly risk averse	0.03	0.03	0.11
9–10	$1.37 < r$	stay in bed	0.01	0.03	0.06

The second elicitation procedure is the series of Random Lottery Pairs (RLP), proposed by Hey and Orme [3], where the subject faces a sequence of multiple pairs and he has to pick one in each pair.

Individuals have to choose repeatedly between two lotteries over four fixed monetary prizes. They are also allowed to state indifference between the two lotteries. In all the lotteries, the support of each probability distributions is not full, in the sense that all lotteries in the experiments assign probability zero to at least one monetary outcome. Therefore, there are at most three prizes with positive probability in each lottery. At the end, one of the pairs is randomly selected for the payoff, and the preferred lottery of the two is the reward.

Figure 2.4 reports the original Hey and Orme's user interface for the experimental implementation of a RLP[4] and Figure 2.5 reports the 25 pairs of questions[5].

[3]Holt and Laury [2]
[4]Hey and Orme [3]
[5]Hey and Orme [3]

Figure 2.4: Hey and Orme (1994) user interface

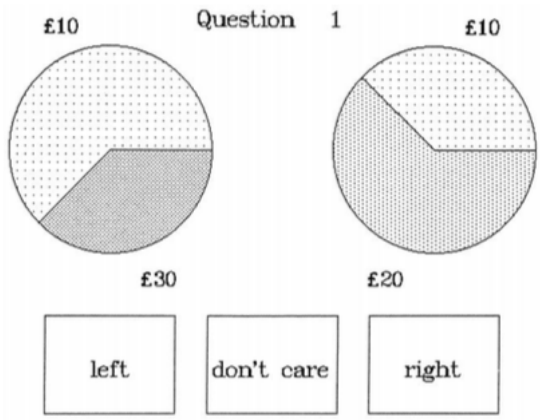


Figure 2.5: Hey and Orme (1994) pairs of questions

DEC	p1	p2	p3	q1	q2	q3	SET
1	0.625	0	0.375	0.375	0.625	0	A
2	0.375	0.625	0	0.5	0.25	0.25	D
3	0	1	0	0.125	0.5	0.375	C
4	0.125	0.75	0.125	0.25	0.5	0.25	D
5	0.5	0.375	0.125	0.625	0.125	0.25	D
6	0.25	0.75	0	0.375	0	0.625	A
7	0.25	0.625	0.125	0.375	0.25	0.375	D
8	0.25	0.25	0.5	0.125	0.625	0.25	A
9	0.125	0.375	0.5	0	1	0	B
10	0.125	0.25	0.625	0	0.5	0.5	A
11	0.125	0.875	0	0.25	0.625	0.125	D
12	0.25	0.75	0	0.5	0	0.5	A
13	0.625	0.375	0	0.75	0.125	0.125	D
14	0.125	0.5	0.375	0.25	0	0.75	B
15	0.125	0.75	0.125	0.375	0.125	0.5	B
16	0.375	0.375	0.25	0.5	0.125	0.375	D
17	0	0.75	0.25	0.125	0.375	0.5	D
18	0.5	0.125	0.375	0.375	0.5	0.125	A
19	0.75	0	0.25	0.625	0.375	0	A
20	0.25	0.375	0.375	0.375	0	0.625	C
21	0	0.875	0.125	0.125	0.625	0.25	A
22	0	0.625	0.375	0.125	0.25	0.625	A
23	0.25	0.5	0.25	0.125	0.875	0	D
24	0.5	0.5	0	0.625	0.125	0.25	D
25	0.25	0.5	0.25	0.375	0.25	0.375	A

The third is the Ordered Lottery Selection (OLS), where the subject selects a lottery from an ordered set. This method was developed by Binswanger [24]. There is a group of 8 lotteries. The first one is the secure option: with a probability of $\frac{1}{2}$ you win \$50 and with a probability of $\frac{1}{2}$ you win \$50. The second lottery is riskier but with a higher expected value:

with $\frac{1}{2}$ you get \$45 and with $\frac{1}{2}$ you get \$90. This pattern goes on, increasing at the same time both the expected value and the risk associated to the lottery.The degree of risk aversion can be estimated depending on the lottery chosen by the subject.

The fourth method is the Becker-DeGroot-Marschak auction where the minimum certain equivalent selling price to give up the endowed lottery is stated by the subject. Namely, it is an auction procedure where a number of lotteries are offered to the subject and for each of them he has to choose a "selling price". Then a "buying price" is given randomly. If the selling price is lower than the buying price, the subject will earn the difference. Depending on the price picked by the subject, his degree of certainty can be inferred. Anyway, this procedure can bring some errors if not well defined and performed.

Finally, the last one is the Trade-off design, where the lotteries' prizes are real-time endogenously defined by the subject's prior responses and certain equivalent elicited. It was first proposed by Wakker and Deneffe [25].

In this method we have four prizes: x_0, x_1, r and R. R is higher than r, and x_0 is a fixed low prize, for example \$0. Then, there are the two probabilities p and $1 - p$. In this case we have two lotteries in the first stage: $\{x_1, p; r, 1 - p\}$ and $\{x_0, p; R, 1 - p\}$. The subject has to choose the x_1 that makes herself indifferent between the two lotteries. In the second stage the two lotteries are: $\{x_2, p; r, 1 - p\}$ and $\{x_1, p; R, 1 - p\}$. Again, the subject has to choose the x_2 that makes him indifferent between the two lotteries. The experiment continues for more stages with this same procedure. The difference between the utility of x_2 and x_1 has to be the same as the one between x_1 and x_0. The utility of a prize equal to zero is zero, so the utility of x_2 is two times the utility of x_1, the utility of x_3 is three times the utility of x_1 and so on and so forth for the rest of x_n. The common problem of this elicitation is the overestimation tendency of the x_1. Furthermore, this first stage error might propagate in all the following ones, as all the subsequent lotteries depend on the x first established by the subject.

To conclude, a wide discussion regarding a theoretical model able to capture the ambiguity preferences is still open.

CHAPTER 3

Experimental Design

As stated earlier, the economic discussion among decision theorists has been often confined to a very specific kind of uncertainty, that is, uncertainty over probabilities, i.e. subjects know the outcomes but not the probabilities associated with them.
In contrast to the previous literature, the novelty of this work is that the uncertainty lies on the outcomes, namely on the payoffs, and not on their distributions.
The experimental layout of Albarrán et al [1] consists of two Phases, which are described in Section 3.1. Both of them are built around binary choices having the probability distributions over 4 monetary prizes according to the rule $0 < Y < X < 15$.
The treatment conditions, as clarified in Section 3.2, differentiate each other on the amount of information given to the subjects over the actual values of the intermediate payoffs, namely X and Y, which are randomized between-subjects. In all sessions, the amount X and Y are i.i.d. integers $\in [1, 14]$, drawn from a uniform distribution without replacement.
Furthermore, a strict bayesian interpretation may deal with this form of ambiguity as uncertainty on the probabilities and not on the outcome. Indeed, individuals can figure out some prior distributions of X and Y, and calculate their expected utility. In conclusion, the ambiguity conditions characterizing this work may be traced back to the ones traditionally embraced in economic literature, where uncertainty lies only on the prizes probabilities.

3.1 Sessions

The data were collected during an experiment run place at University of Alicante "Laboratory of Theoretical and Experimental Economics". The sessions conducted were 6, with 24 students per session, so for a total of 144 subjects that were recruited among the undergraduate students via the Orsee platform [26].

During all the phases, each subject was working in front of an individual screen with a user interface. Before the start of the experiment, the subjects were provided with a printed copy of the instructions, which were also read aloud by the experimenter in all sessions. The experimenters answered any questions raised by the subjects by using only the information provided in the instructions. A debriefing questionnaire was submitted at the end of the experiment, to provide us with data regarding social attitudes and individual characteristics. At the end of each session the subjects were compensated with real money. A random round was selected at the end of each of the 3 phases, and individuals were paid in cash according to their winnings in the 3 randomly selected rounds. The payment procedure just mentioned was described in the experiment instructions, and can be found in the Appendix A.

3.2 Treatments

In this multi-stage experiment the between subjects level of information over the lotteries monetary prizes is manipulated through three different treatments. In the FULL INFO treatment (TR2), prizes are precisely communicated to the subjects throughout the entire experiment; in the PARTIAL INFO treatment (TR1), the actual prizes are not communicated, but individuals know that $X > Y$ are drawn from a uniform distribution; in the NO INFO treatment (TR0) neither the x and y values nor the statistical process generating them are revealed to the subjects, they only know the values are i.i.d. $\in [1, 14]$ and that $X > Y$.

In Figure 3.1 the sessions and treatments are displayed.

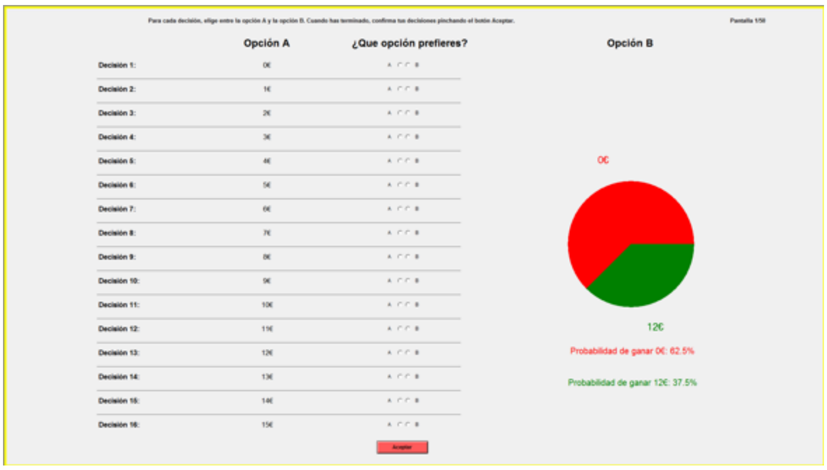
Figure 3.1: Sessions and treatments

Sessions	Level of information	Order of Phases
1 session with 24 subjects	TR=2 FULL INFO	123
1 session with 24 subjects	TR=2 FULL INFO	213
1 session with 24 subjects	TR=1 PARTIAL INFO	123
1 session with 24 subjects	TR=1 PARTIAL INFO	213
1 session with 24 subjects	TR=0 NO INFO	123
1 session with 24 subjects	TR=0 NO INFO	213

3.3 Phases

Phase 1 is a multiple price list certain equivalence elicitation [2] composed of 50 rounds. Subjects are asked about their certain equivalence for each of the 50 lotteries, as shown in Figure 2.5. The standard multiple price list format is then played, namely the subjects are asked to choose between a fixed amount $\in [0, 15]$ and the lottery in question, as we can see from the Figure 3.1, from Albarrán et al [1].

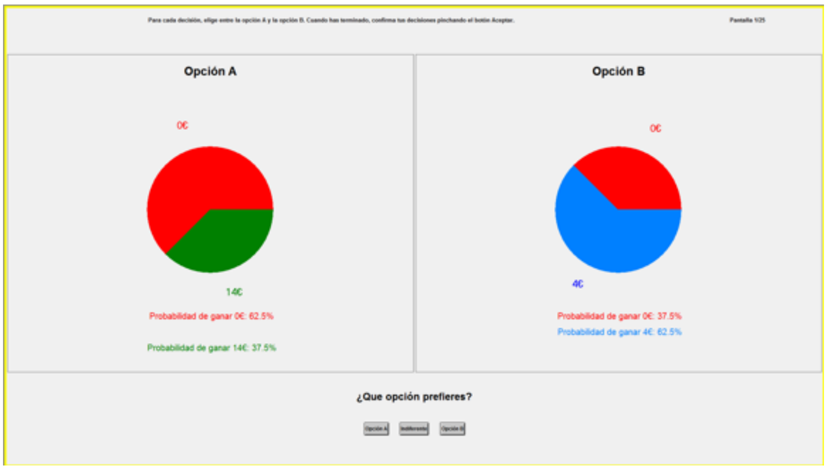
Figure 3.2: Phase 1 User Interface, Certain Equivalence



Phase 2 is a random lottery pair elicitation protocol [3], where subjects, throughout the 25 rounds, are asked to make a binary choice of the same lotteries of Phase 1, which are randomly ordered between subject. The 25 probability distributions pairs are the same as in Hey and Orme's [3], and randomized over these four combinations of fixed amounts $(0, y, x), (0, y, 15), (0, x, 15)$,
$(y, x, 15)$.

The uses interface, in fig 3.3, displays the 2 lotteries. Each color represents a prize probability and it remains the same during all rounds. Subjects are asked to select their preferred lottery using the corresponding button. Individuals can also express indifference, so no strict lottery preference is required, following Hey and Orme layout. In Figure 3.1 the user interface of Phase 2 is shown, from Albarrán et al. [1]

Figure 3.3: Phase 2 User Interface, Random Lottery Pair



3.4 Debriefing Questionnaire

At the end of the experiment, a debriefing questionnaire was given to the subjects. It contained socio-demographics questions, such as on gender, parents' education and wealth, personal education and the field of degree chosen. There were also some additional questions regarding abilities of the subjects, such as the GPA or the Cognitive Reflection Test [27]. Finally, there were questions on the subjects' personality, calculated using a reduced version of the Big Five test [28].

CHAPTER **4**

Structural Modelling Under Heterogeneity

4.1 Maximum Likelihood Estimation

Customized likelihood functions corresponding to specific models of decision making under risk and uncertainty are more and more popular among economists dealing with a wide range of fields, as suggested by Harrison [29]. This demand for customization is due to the numerous parametric functional forms experimental economists use to account for behaviour under risk and uncertainty. These functions also permit to represents "handwritten" models, used to explain decision rules, which may be different from the traditional ones. In behavioural econometrics it's becoming even more common to see user-written maximum likelihood estimations rather than pre-packaged model specifications.

Specifically, what a maximum likelihood estimation does is, conditional on the structural model under scrutiny, to select the value of the estimator which maximizes the probability of observing the collected data, i.e. the probability density function. It is given by a model such as $P(y|\theta)$, where θ represents the set of unknown parameters we want to estimate and Y the vector containing the observed decisions. The maximum likelihood estimator, θ^{*}, maximizes the the likelihood function $P(y|\theta)$ with respect to θ; this means that we maximize the probability of observing the data we actually observed as function of the parameters of the model.

With a big sample size the likelihood function, being the product of the probability density functions of all the subjects' outcomes, is close to 0. For this reason, locating its maximum may be difficult. The logarithmic function is usually employed to solve this problem in order to stretch the function vertically and making it easier to locate its maximum. Furthermore, the logarithmic transformation is strictly monotone, preserving the same local maxima.

In our specific model, so under the Expected Utility Model, the probabilities of each outcome k, namely p_k, are the ones induced by the experimenter.[1] This means that the expected utility is calculated as the sum of utilities of monetary prizes, each of which is multiplied by the corresponding probability.

Being i the subject, $k \in \{0, 1\}$ the index of the lottery equal to 0 for the right one and 1 for the left one, and $h \in \{1, 2, 3, 4\}$ the index of the prizes:

$$\mathrm{U}_i(L_k) = \sum_{h=1}^{H} u(\chi_i)\, p_h^h \tag{4.1}$$

where $u_i : R \rightarrow R, \quad L_k = \{\chi; p_k\}, \quad \chi \in \{0, y, x, 1\}$.

In this chapter we use the mean-variance utility function (4.2), given its simplicity and intuitiveness in calculations. The mean-variance (MV) utility function applied to a lottery i is as follow:

$$\mathrm{U}_i(L_k) = E[L_k] - \beta_i Var[L_k] \tag{4.2}$$

The utility of each individual is a function of the expected value of the lottery μ_k, and of its variance, $Var[L_k]$.

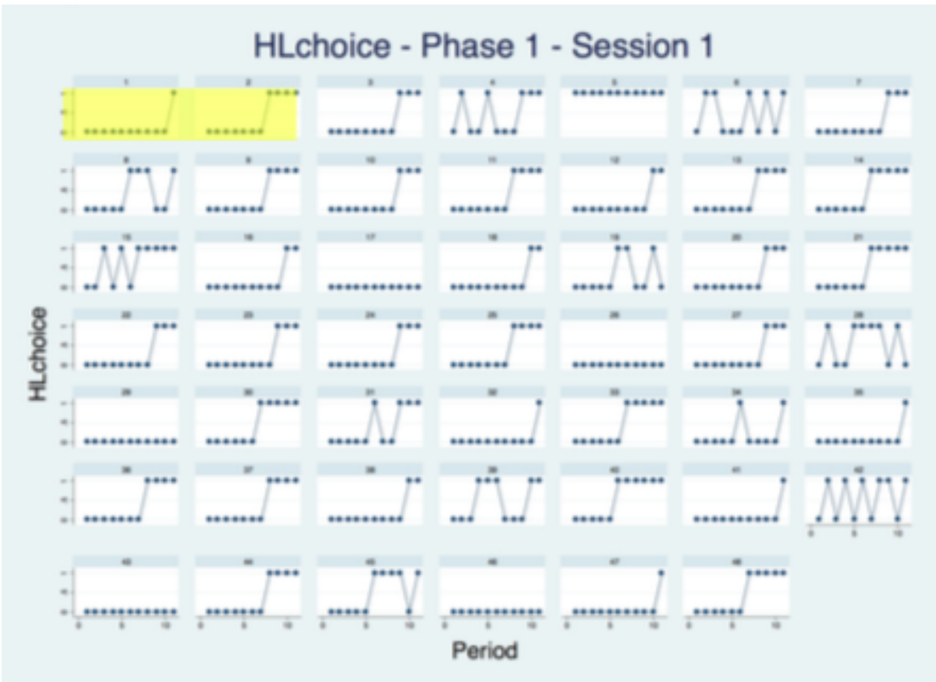
It can be shown that MV utility is equivalent to a VNM utility function (4.1) with a quadratic utility function $u(x) = x - \beta x^2$, where β is the only unknown parameter to be estimated. It represents the level of risk aversion. Indeed, the variance $Var[L_k]$ is used as a proxy of the risk of the lottery. An individual is considered to be risk-averse if β has a positive value, namely if, ceteris paribus, a higher value of the lottery variance decreases his utility. By the

[1] All maximum likelihood routines have been programmed and run with STATA v. 14, by STATA Corporation. The interested reader can find all codes being used in Appendix B.

same token, a negative β value is associated with a risk-seeking decision maker, hence $\beta = 0$ indicates risk neutrality.

In this section we shall estimate β using data from the full information treatment, TR2, where there is no ambiguity and subjects know the true value of both X and Y. In Phase 1, subjects make 16 choices per period. According to the usual Holt and Laury framework, the threshold value of β, such that the the decision maker switches to the certain amount rather then the risky lottery, is extracted. According to Moffatt [30], only this piece of information should be extracted from each individual facing the Holt and Laury lottery. Indeed, he states the list of people choices cannot be analyzed as an independent sequence. However, this does not seem to be the case for our data, as we can see from Figure 4.1 from Rodriguez and Ponti [31], where subjects choices are shown as a function of the periods (i.e., the individual choice between the lottery and a specific monetary prize ranging from 0 to 15).

Figure 4.1: Phase 1 Switching Point Graph



Here the highlighted subjects, subject 1 and 2, are a perfect example of a rational behavior, as indicated by the presence of a single switching point. Looking at subject 4 or 6 instead, a clearly irrational behavioral pattern emerges, with the presence of several switching points. Furthermore, these two cases are not unique. This demonstrates that we may not rely on a single switching point information per subject. Indeed, the emergent absence of a clear path individuals follow throughout the whole Phase suggests that we may also treat people's decisions as independent.

For Phase 2 we directly take the 25 decisions subjects make between the two lotteries.

For the lotteries of Phase 1 and Phase 2, the expected value is computed as:

$$E[L_k] = \sum_{h=1}^{H} p_k^h \times \chi_h$$

where π_h is the prize the subjects will receive with probability p_k.

The variance, as the expectation of the squared deviation of the prize random variable from its mean, is computed as:

$$\sigma_{L_i}^2 = \sum_{h=1}^{H} p_k^h \times (\chi_h - E[L_k])^2.$$

Then, using these values, for every left and right lottery we compute their utility function and the difference of these utility values:

$$U_i(L_k) = E[L_k] - \beta_i \sigma_{L_k}^2, \quad k = 0, 1.$$

This difference will prescribe the optimal behavior, namely the lottery to be chosen according to the utility maximization principle.

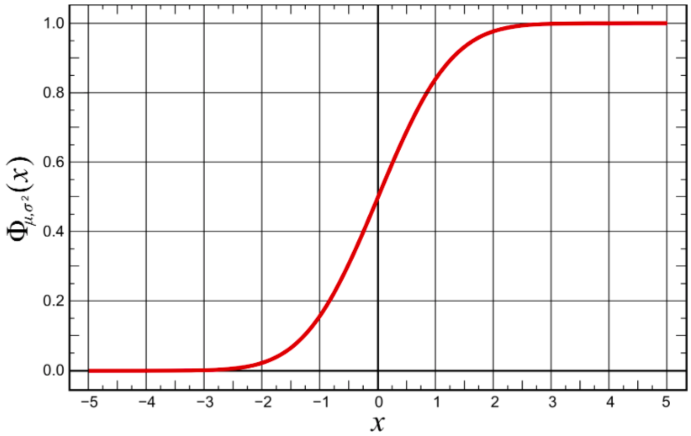
In Phase 1, the left lottery L_0 is actually a certain prize. This means that for all the left lotteries we have $\sigma_0^2 = 0$ and $E[L_0] =$ to the actual number displayed. Namely, the β never appears in the left lotteries of Phase 1.

Subsequently, we proceed with the calculation of the difference of the expected utility, as follows:

$$\Delta U = U(L_1) - U(L_0) = (E[L_1] - \beta \sigma_{L_1}^2) - (E[L_0] - \beta \sigma_{L_0}^2) = [...]$$

This latent index, based on latent preferences, is then brought back to the observed choices using a standard cumulative normal distribution function $\Phi(\Delta E[U])$. This probit function has a domain in $[-\infty, +\infty]$ and has a codomain in $[0, 1]$ as shown in Figure 4.2[2].

Figure 4.2: Normal c.d.f. Function



4.2 Dealing with Heterogeneity

Here we introduce the distinction between the various and competing approaches to stochastic modelling, in which the choices made by individuals express their heterogeneity.

The Random Preference Model (Loomes and Sugden [32]) explains heterogenity in decisions as heterogeneity in the structural component (β, in our case). This is an example of an heterogeneous agent approach, which attributes a variation in behavior of the population to variation in the parameter representing preferences.

The Fechner Model (Fechner [33]), in which the stochastic component in the decision making process is done applying an additive idiosyncratic error $\epsilon \sim N(0, \sigma_\epsilon^2)$.

The Tremble Approach (Loomes et al. [34]) assumes that there is a small positive probability

[2]http://data.princeton.edu

that the individual, at any point in time, loses concentration and adopt any possible behavior randomly with probability ω.
The Random Effect Model, whereby the unobserved heterogeneity is expected to be explained by a random effect parameter, η_i, which captures the between subject differences, and its variance, σ_η^2, is a measure of subject heterogeneity.

Random Preference Model

In this model, the structural parameter $\beta \sim N(\mu_\beta, \sigma_\beta^2)$ accounts for all the heterogeneity in the model.
For the maximum likelihood routine to work, we need to create some local variables, which are temporarily used only inside the program.
The parameters we want to estimate are μ_β and σ_β, which describe the distribution of $\beta \sim N(\mu_\beta, \sigma_\beta^2)$. This means that we maximize the log likelihood function with respect to μ_β and σ_β, where:
- μ_β is estimated through the choices made by subjects, the experimental data represents then the basis to construct likelihood functions, such that the selected optimal value of is directly dependent on the subjects' binary decisions between the two lotteries.
- σ_β is imposed to be strictly positive, given that it is the standard deviation of β, using the strictly monotone exponential transformation, and is estimated as a constant.
Equating the expected utilities and solving for β we get:

$$\beta = \frac{E[L_1] - E[L_0]}{\sigma_{L_1}^2 - \sigma_{L_0}^2}$$

This general formula holds for both Phase 1 and Phase 2. Furthermore, for Phase 1, it can be simplified, given the left lottery is always a fixed price, γ, and $E[L_0] = \gamma$ and $\sigma_{L_0}^2 = 0$.
Specifically, $\gamma_i = \frac{\delta_i - 1}{15}$, with δ_i being the prize of decision i, is scaled back by 1 position and then normalized in $[0, 1]$ dividing by 15.

$$\beta_{Phase1} = \frac{E[L_1] - \gamma}{\sigma_{L_1}^2}.$$

Therefore, we are left with this two values for β:

$$\beta_{Phase1} = \frac{E[L_1] - \gamma}{\sigma_{L_1}^2}$$

and

$$\beta_{Phase2} = \frac{E[L_1] - E[L_0]}{\sigma^2_{L_1} - \sigma^2_{L_0}}$$

The probability that the right lottery is chosen, conditional on the average risk aversion coefficient, namely the mean of β, μ_β, is equal to the probability that ΔU is positive:

$$P(k = 1|\, \mu_\beta) = P(\Delta U > 0 \mid \mu_\beta) = P(U(L_1) > U(L_0) \mid \mu_\beta)$$
$$= P(\Delta E[L_k] - \beta \Delta E[\sigma_k] > 0 \mid \mu_\beta) = P(\beta < \frac{\Delta E[L_k]}{\Delta E[\sigma_k]} \mid \mu_\beta).$$

Calling $\beta^* = \frac{\beta - \mu_\beta}{\sigma_\beta}$, we use the normal transformation and we get:

$$P_{i,t}(\beta < \frac{\Delta E[L_{k,t}]}{\Delta E[\sigma_{k,t}]} \mid \mu_\beta) = P(\frac{\beta - \mu_\beta}{\sigma_\beta} < \frac{\frac{\Delta E[L_{k,t}]}{\Delta E[\sigma_{k,t}]} - \mu_\beta}{\sigma_\beta} \mid \mu_\beta) = P(\beta < \beta^* \mid \mu_\beta) = \Phi(\beta^*_{i,t})$$

where Φ is the standard normal c.d.f.

In conclusion, the likelihood function of observing the right lottery chosen, given μ_β, is:

$$\mathcal{L} = \sum_{i,t} ln(P_{i,t}(\beta < \frac{\Delta E[L_{k,t}]}{\Delta E[\sigma_{k,t}]} \mid \mu_\beta))$$

An important property of the c.d.f. of both the probit and the logit model is symmetry.

By this,

$$\Phi(-\beta^*) = 1 - \Phi(\beta^*)$$

Following the same way of reasoning, the symmetric probability that the right lottery is chosen, is just:

$$P(k = 0|\, \mu_\beta) = P(\Delta U < 0 \mid \mu_\beta) = P(U(L_1) < U(L_0) \mid \mu_\beta) = P(\Delta E[L_k] - \beta \Delta E[\sigma_k] < 0 \mid \mu_\beta)$$
$$= P(\beta > \frac{\Delta E[L_k]}{\Delta E[\sigma_k]} \mid \mu_\beta)$$

Calling $\beta^* = \frac{\beta - \mu_\beta}{\sigma_\beta}$, we use the normal transformation and we get:

$$P_{i,t}(\frac{\beta - \mu_\beta}{\sigma_\beta} > \frac{\frac{\Delta E[L_{k,t}]}{\Delta E[\sigma_{k,t}]} - \mu_\beta}{\sigma_\beta} \mid \mu_\beta) = P_{i,t}(\beta > \beta^* \mid \mu_\beta) = \Phi(-\beta^*)$$

In conclusion, the likelihood function of observing the left lottery chosen, given μ_β, is:

$$\mathcal{L} = \sum_{i,t} ln(1 - P_{i,t}(\beta < \frac{\Delta E[L_{k,t}]}{\Delta E[\sigma_{k,t}]} \mid \mu_\beta))$$

Fechner Model

As hinted above, the Fechner Model introduces a stochastic component to take into account subjects' heterogeneity in the decision making process. This is included by adding an idiosyncratic error to ΔU, that is $\epsilon \sim N(0, \sigma_\epsilon^2)$. Then, rather than to parameters, attention is here posed to payoffs differences.

As soon as this error terms appears, the behavior is no longer deterministic and it is described in term of probabilities as follows:

$$P(k = 1) = P(U(L_1) + \xi_1 > U(L_0) + \xi_0) = P(\Delta U + \epsilon > 0) = P(\epsilon > -\Delta U)$$

$$= P(\frac{\epsilon}{\sigma_\epsilon} > \frac{-\Delta U}{\sigma_\epsilon}) = \Phi(\frac{\Delta U}{\sigma_\epsilon})$$

where ϵ is the difference between the two lotteries' errors in valuation ξ_1 and ξ_0, Φ is the standard normal c.d.f. and σ_ϵ represents the noisiness of the choice. This means $\sigma_\epsilon = 0$ fully explains a deterministic choice, while if $\sigma_\epsilon \to \infty$, the choice is entirely driven by noise, namely both right and left lotteries are chosen with 0.5 probability.

The parameter we want to estimate is σ_ϵ, namely we maximize the log likelihood function with respect to it. It is imposed to be strictly positive, being a standard deviation, trough the usual strictly monotone exponential transformation.

Again, the $U(L_i)$ and the ΔU are used as temporary variables.

Table 4.1 reports an application of the Fechner model using the Mean Variance utility function. Only TR2 data are used here, so β estimated is a proxy of individuals' aversion to risk (as represented by the variance).

Table 4.1: Fechner Model with Mean Variance Utility Function

VARIABLES	
Beta	0.458***
	(0.0827)
Sigma epsilon	2.033***
	(0.0644)
Obs.	61,339

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Tremble Parameter

According to the Random Preference model just described above, if the lottery L_i first-order stochastically dominates lottery L_k, the first one will always be chosen, no matter the risk attitudes of subjects. Indeed, any observed choice of a dominated lottery cannot be explained by the RP model.

For this reason the tremble parameter ω is introduced, and it represents the probability a subject loses concentration at any task and randomly chooses, with equal probability, between the two alternative lotteries. This does not necessarily imply he makes the incorrect choice, as under such a condition the correct and incorrect choices are equally likely.

This is not a model *per se*, rather it's an extension to be applied either to the Random Preference model or to the Fechner model.

In the RP model the

$$P(k = 1)(1 - \omega)\Phi(\frac{\beta^* - \mu_\beta}{\sigma_\beta}) + \frac{\omega}{2}$$

In Fechner,

$$P(k = 1) = (1 - \omega)\Phi(\frac{\Delta U}{\psi}) + \frac{\omega}{2}$$

This means that with $(1 - \omega)$ probability the correct choice prescribed by the model is done, and with (ω) probability the random choice equal to $\frac{1}{2}$ is made.

The Random Effect Model

In this model, the unobserved heterogeneity is explained by a random effect parameter, η_i, which is perpendicular to the other covariates. Specifically, η_i captures the between-subject differences, and its variance, σ_η^2, is a measure of subject heterogeneity. The probability of observing the right lottery chosen by subject i in period τ is now:

$$P(k=1) = f(\chi_{i,\tau}\,\beta) + \eta_i + \epsilon_{i\tau}$$

where $\eta_i \sim (0, \sigma_\eta^2)$ is the individual error, or heterogeneity, and $\epsilon_{i\tau} \sim N(0,1)$ is the idiosyncratic error. While $\epsilon_{i\tau}$ varies across subjects and periods, η_i has a unique value for every individual.

In our model, this probability can be explained as:

$$P(k=1) = \alpha E[\Delta\mu_{i,\tau}] + \beta E[\Delta\sigma_{i,\tau}^2] + \eta_i + \epsilon_i, \tau$$

where

$$E[\Delta\mu_{i,\tau}] = E[L_1] - E[L_0]$$

and

$$E[\Delta\sigma_{i,\tau}^2] = \sigma_{L_1}^2 - \sigma_{L_0}^2.$$

If the $\alpha = 1$ constraint is imposed, we obtain the usual mean-variance utility function, where a negative value for β is expected.

Only the data from TR2 individual is used, since we want to extract the risk aversion coefficient, while leaving aside any form of ambiguity which might arise from the missing payoffs' information of the TR1 and TR0.

We implement both probit and logit regressions, with the xtprobit and xtlogit functions, namely declairing our data structure to be a longitudinal panel. The Fechner approach is implemented by default when using these functions in Stata.

The results are shown in table 4.2.

The results of the probit and the logit are closely comparable. As expected, β is negative, and statistically significant at the 99% confidence level. Namely, the probability of choosing the right lottery is a negative function of the difference between the right and the left lottery variance.

Also the standard deviation of the random effect coefficient is statistically significant at the 99% confidence level, and it gives us information about the subjects heterogeneity. Specifically, the random effect coefficient $\eta_i \sim (0, 0.238)$ according to the probit model, and $\eta_i \sim (0, 0.363)$ according to the logit.

Table 4.2: Random Effect Probit and Logit Models

VARIABLES	Probit	Logit
Delta_VAR	-0.464***	-0.355***
	(0.0525)	(0.0811)
Random Effect SD	0.238***	0.363***
	(0.011)	(0.0169)
Obs.	195,459	195,459
Sbj.	279	279

Standard errors in parentheses *** p<0.01, ** p<0.05, * p<0.1

CHAPTER 5

Dealing with Ambiguity

5.1 Introduction

In conditions of uncertain outcomes, the Savage approach [7] has been traditionally used. In particular, individuals have been assumed to behave according to a unique subjective prior belief over all states of the world, and, given this, they would maximize their expected utility. This decision process clearly neglects the existence of any form of ambiguity, and it prescribes the way decision makers should deal with uncertain situations.

However, Ellsberg [8] claims that most individuals treat ambiguity differently than objective risk. In specific, he argues that people exhibit a significant degree of *ambiguity aversion*, placing a premium on outcomes for which probabilities are known. This general stylized fact has been replicated broadly and has important implications for the economics of optimal contracting, investment choices, and mechanism design.

One possible way to structurally identify ambiguity aversion is to assume that the latter influences people's degree of risk aversion (more precisely, the curvature of the utility function), an approach followed, among others, by Klibanoff et al. [11] and Andersen et al. [4].

As described in Chapter 3, in the experiment of Albarrán et al [1], prizes in the lotteries are distributed according to the rule $0 < y < x < 15$. In what follows, this prize domain is normalized, for the sake of simplicity, to lay within the unit interval $[\,0, 1\,]$, where \$0 is

0 and $\$15$ is 1. The treatment conditions -randomized between subjects- regard the amount of information given to them about X and Y. Furthermore, while in the full information treatment, TR2, people face a normal risky situation and there is no ambiguity influencing their decision, this is not the case for the partial information and no information treatments TR1 and TR0, respectively. As we shall see, some ambiguity preference appears from subjects' choices which is higher the less information is received.

5.2 Econometric strategy

In what follows we shall layout the identification assumptions underlying our structural estimations. Specifically, we need to define our identification strategy with respect to $i)$ subjects' risk attitudes and how the uncertain payoffs, X and Y, enter in subjects' calculations together with $ii)$ the behavioral model underlying subjects' optimization program. Regarding the former, as it will be explained in Section 5.2.1, we shall impose that subjects maximize a VNM CRRA utility function in all treatments and that, consistently with the TR1 experimental instructions, Y and X are calculated as first and second order statistics of a uniform distribution defined over the unit interval. Regarding the latter, that is explained in Section 5.2.2, we shall consider two alternative behavioral models, defined as *naive* and *sophisticated*. In the former, subjects are assumed to estimate first the uncertain payoffs and then use these expected payoffs in the expected maximization program; in the latter -consistently with a genuine bayesian approach- the order of integration is reversed.

5.2.1 Uncertain Payoffs and Risk Aversion

We read the experimental evidence by the way of some structural estimation exercises in which we elicit the individuals' degree of risk aversion within the frame of a standard constant relative risk aversion (CRRA) utility function, which generally performs better in more complex structural estimations.

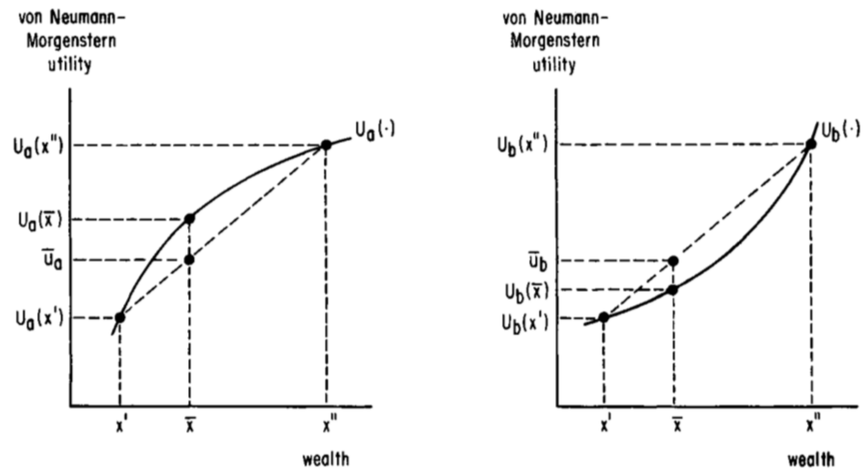
The utility function is given below:

$$u(\chi) = \begin{cases} \frac{\chi^{1-\rho}}{1-\rho} \text{ if } \rho \neq 1 \\ ln(\chi) \text{ if } \rho = 1 \end{cases} \tag{5.1}$$

where ρ is the (CRRA) coefficient which does not depend on χ, as formalized by Pratt [35]. As for its economic interpretation, $\rho > 0$ represents risk aversion, $\rho = 0$ risk neutrality and $\rho < 0$ risk loving.

In Figure 5.1[1], examples of $u(\chi)$ are presented for different values of ρ: concave in case of risk aversion (left) and convex in case of risk loving (right).

Figure 5.1: Risk Aversion Coefficient for both the Naive and Bayesian approaches



In Section 5.3.1 we check whether the change of information implemented by our treatments conditions generates a change in behavior and, in turn, (whether and) how this change is converted into a significant change in the measured (CRRA) attitude toward risk.

The uncertain payoffs Y and X are identified as the first and the second order statistics from a uniform distribution in $[\,0, 1\,]$, where the order statistics of a random sample $\chi_1, ..., \chi_n$ are

[1]Machina [36]

defined as the sample values placed in ascending order.
Specifically, let $f_k(n,\ z)$ denote the k_{th} order statistics of n draws, where $n = 2$ in our case, of a density function $f(\cdot)$. Let

$$X \sim f_2(2,\ z), \quad Y \sim f_1(2,\ z)$$

where

$$f_2(2,\ z) = 2\ z\ f(z)\ F(z), \quad f_1(2,\ z) = 2\ z\ f(z)\ (1 - F(z))$$

come from the general formula for the the k_{th} order statistics of n draws

$$n \binom{n-1}{k-1} f(z)\ (F(z))^{n-1}(1 - F(z))^{n-k}$$

being z a random draw from a uniform distribution and being

$$f(z) = 1 \text{ is the p.d.f. of } z, \quad F(z) = z \text{ is the c.d.f of } z.$$

While this is certainly true for TR1 subjects -since they know the characteristics of the random generation process that yields the uncertain payoffs- we impose the same statistical model for subjects in TR0, assuming they had this information. As we said, this is purely an identification assumption, as there is no possibility to test whether this is the true for expectations in TR0 about X and Y distribution, or whether subjects in TR0 consider a different type of distribution. On the other hand, it is highly probable that TR0 subjects will heuristically and automatically assume such a distribution of the payoffs, as it occurs in Laplace's well known "principle of insufficient reason". In any case, what is important here is that -thanks to this assumption- our structural model is able to estimate treatment effects, to such an extent that we are able test a null hypothesis in which CRRA in both TR0 and TR1 is the same. Since subjects have been randomized within treatments, a significant change in the CRRA coefficient between TR0 and TR1 has to be interpreted as a genuine treatment effect due to a change in information.
In the maximum likelihood function routine, ρ is analyzed through the individual choices subjects make, which are expressed in function of the treatments, to identify how a different level of information influences people's risk attitude.
Phase 1 observations are treated as a series of individual and independent choices between a

certain outcome and a risky lottery, whose expected value is computed and compared to the sure prize.

Instead, phase 2 data are used as a sequence of binary choices between lotteries. TR2 players know the true X and Y, so their ρ derived from a situation with no ambiguity. On the other hand, TR0 and TR1 players compute the lotteries expected values and variances, as function of the X and Y they figure out, and then the U_i and the ΔU.

A logit function is used to solve the usual binary choice model, explaining the $P(k=1) = P(\Delta U > 0)$ which is :

$$P(k=1) = \frac{e^{\Delta U}}{1+e^{\Delta U}} \quad \text{if } L_1 \text{ is chosen}$$

$$P(k=1) = \frac{e^{-\Delta U}}{1+e^{-\Delta U}} \quad \text{if } L_0 \text{ is chosen.}$$

The Fechner model is used, where people heterogeneity is expressed as function of a random error in the CRRA utility computation, i.e. $\epsilon \sim N(0, \sigma^2)$. In the whole of estimates we cluster all the observations made by the decisions of the same individual.

5.2.2 Identification of the Behavioral Model

Regarding the behavioral content of the structural model for the uncertain payoffs, we consider two alternative specifications, labelled as "naive" and "sophisticated", respectively. A naive decision maker figures out a point estimation of the unknown payoffs X and Y, starting with the information that they are draws from a uniform distribution in $[\,0, 1\,]$. This means that the $E[X]$ and the $E[Y]$ are computed first and then plugged into the CRRA expected utility function to be maximized. Specifically:

$$E[\,X\,] = \int_0^1 f_2(2,\ z)\, dz = \frac{2}{3}$$

$$E[\,Y\,] = \int_0^1 f_1(2,\ z)\, dz = \frac{1}{3}$$

where f_k is the k-th order statistics of a uniform distribution in $[\,0, 1\,]$.

Finally, the utility of a lottery k is:

$$U(L_k) = u(E[\,Y\,])\, p_y^k + u(E[\,X\,])\, p_x^k + p_1^k$$

given that the first price is 0, so $u(0) = 0$, and the last price is 1, so $u(1) = 1$.
A "sophisticated" decision maker, instead, will proceed based on a true bayesian updating, forming a prior distribution over the X and the Y, and then calculate the expected utility from these densities. Specifically:

$$U(X) = \int_0^1 u(z) f_2(2,\ z)\, dz;$$

$$U(Y) = \int_0^1 u(z) f_1(2,\ z)\, dz,$$

where $f_1(\cdot)$ and $f_2(\cdot)$ are the first and second order statistics of a uniform distribution in $[\,0, 1\,]$.
Finally, the utility of L_k, $U(L_k)$ equals to

$$U(L_k) = U(X)\, p_x^k + U(Y)\, p_y^k + p_1^k$$

given the first price is 0, so $u(0) = 0$ and the last price is 1, so $u(1) = 1$.
In conclusion, the two models differ due to the order of integration.

5.3 Results

The "atom" of our analysis is the decision made by subjects and our research question is how their ρ varies as function of the amount of information they receive, depending on their treatments, and how this process differs in the two distinct approaches, the naive and the bayesian one. We also query whether one model is more used than the other.

5.3.1 Treatment effects

Figure 5.2 reports the result of the structural estimation of the ρ as function of the different treatments, for both the two approaches.
Our empirical evidence shows a certain degree of *love for ambiguity*, as the less information given to the subjects, the lower their risk aversion, and, consequently, the bigger their attraction toward risk and uncertainty. Moreover, the risk aversion coefficient estimated for TR0 is significantly lower than that estimated in TR2, although there is no statistically significant

difference between estimated CRRA coefficients in TR0 and TR1. These findings are -somewhat- in contradiction with the common wisdom of the literature, although they are consistent with other experimental literature that applies similar elicitation techniques as ours, such as Andersen et al [4].

When comparing our two behavioral models, as shown in Table 5.1, the estimated likelihood of the naive approach is higher than that of the bayesian. This suggests that, based on our data, the naive approach approximates better subjects' decision rules.

Afterwards, we would like to identify the percentage of the subjects using each of the two models, i.e. the probability of them behaving either in a naive or a bayesian way.

Table 5.1: Risk Aversion Coefficient for both the Naive and Bayesian approaches

VARIABLES	Naive	Bayesian
TR_0	-0.0360***	-0.0452***
	(0.00953)	(0.0102)
TR_1	-0.0187*	-0.0254**
	(0.00973)	(0.0102)
Constant	0.774***	0.774***
	(0.00557)	(0.00557)
TR_1 – TR_0	-0.01723	-0.0198
	(.01111)	(0.0120)
Likelihood	-92261.238	-92604.987
Obs.	195,459	195,459

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

5.3.2 Naive or Sophisticated?

Up to now, we identified two different approaches individuals may follow to make their choices. The following step is to implement a mixture model to identify the probability of each observation being compatible with either model.

We use a binary mixture model, since a finite number of types, the naive and the bayesian, are assumed.[2]

The main advantage of this approach that the assumption of different subjects operating according to a single model is avoided. The behavior of a typical subject is often traced back to the average behavior, but it is quite possible this is not an accurate representation of every subject under study.

A possible answer to this issue could be the Average Treatment Effect, ATE, where a specific treatment effect is recognized to each individual. All subjects specific treatment effects are then assumed to vary randomly around an average, the ATE, i.e. the parameter being estimated.

If the distribution is bell-shaped and symmetric, the ATE will provide a sensible measure of the affect of the treatment. In other words, the ATE measure is relevant when the treatment has universal applicability so that it is reasonable to consider the hypothetical gain from treatment to a randomly selected member of the population.

However, this is not always the case, and this ATE can end up being far away form the actual treatment effect of any single subject.

The approach adopted by a finite mixture model is presented below. A total number of types in the population is decided, and a specific behavioral model is assigned to each of them. The parameters of these various models are estimated altogether, along with the mixing proportions.

In particular, we generate the probability π_{BAY}, namely the probability of our subjects acting as bayesian in each of their decisions.

We tried to estimate the ρ_{NAI} and ρ_{BAY}, i.e. the risk aversion coefficients for both the approaches, and π_{BAY} altogheter, but the likelihood function did not converge. Indeed,

[2] In case of an "infinite" mixture model, a continuous variation in some parameters indexing individual type is assumed, as happens for random coefficient models or random effect models

the likelihood functions of our models are very close. For this reason, we estimated this probability numerically, using a grid loop.

Subsequently we let π_{BAY} moving inside a grid $(0, 1)$, to finally choose the value that maximizes the likelihood function.

A possible drawback of this numerical procedure is the fact that the π_{BAY} standard error cannot be estimated, as it is shown in Figure 5.3. On the other hand, we can justify this statement by saying that our likelihood function is not function of it, given that it is just a product of probability.

This numerical computation demonstrates that our subjects have a strong tendency to behave as naive, given the estimation result of $\pi_{BAY} = 0.2$.

Figure 5.2: Mixture Model with $\pi_{BAY} = 0.2$

VARIABLES	
Rho_NAI	0.764***
	(0.00398)
Rho_BAY	0.637***
	(0.0109)
Pi_BAY	0.2
	-
Obs.	195,459

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

CHAPTER 6

Conclusions

The thesis aims to explore how subjects approach ambiguous decision making when uncertainty is on a different level than the one usually investigated. In fact, although most decision problems in daily life involve different dimensions of uncertainty, the majority of models discussed in the economic literature deal with a specific form of uncertainty, the one on probability of the payoffs. In this investigation, uncertainty lies on the prizes of the lotteries people have to choose, rather than on their probabilities.
We explore the question of whether there is some systematic different behavior that people exhibit while dealing with ambiguity. Specifically, we investigate how a change in the information given to decision makers influences their risk aversion and, in order to find an answer to this question, we analyze the effects of some between subjects treatment.
Our results suggest that increasing the amount of ambiguity, people modify their predisposition toward it, showing some degree of love in favour of it. Namely, the lower the information given, the lower their risk aversion.
When the decision problem is faced under uncertainty, two different specifications are presented, the naive and the bayesian. Indeed, according to the former, a pointwise estimation of the uncertain parameters is plugged inside the utility function. On the other hand, according to a more sophisticated paradigm, the bayesian, the prior distributions of these unknown parameters are used in order to compute the expected utility function, for each of them.
A mixture model demonstrates a strong majority of people, almost 80%, adopting the naive approach.

In this thesis, we measured the treatment effects in the variation of ρ parameter in two models, namely $\rho_N AI$ and $\rho_B AY$. Albarrán et al [1] adopt a different indentification strategy, where the ρ is extracted from the TR2 subjects and the treatment effects are measured as the X and Y estimations. Although ours is a different approach, the results are more than compatible, that further confirm our main findings.
It is my intention, in the future, to apply more complex models of decision making under uncertainty to these data, like the models suggested by Klibanoff et al. [11]. This project, which constitutes the core of the future research advisable in this thesis, shall also incorporate the possibility to extend uncertainty to more and different levels, like already been done in Eliaz et al. [19]. Furthermore, we wonder whether individuals have a higher predisposition toward the bayesian approach while facing multiple levels of ambiguity.
This thesis, therefore, can be considered not only as a partial and exploratory analysis, but also as a good starting point for numerous and extended future investigations.

APPENDIX **A**

Experiment Instructions

Here there are the instructions of the three treatments, respectively the Information treatment TR2, the Partial Information treatment TR1 and the No Information treatment TR0.
The text is written in Spanish, given the experiment was conducted at University of Alicante "Laboratory of Theoretical and Experimental Economics".

¡BIENVENIDO AL EXPERIMENTO!

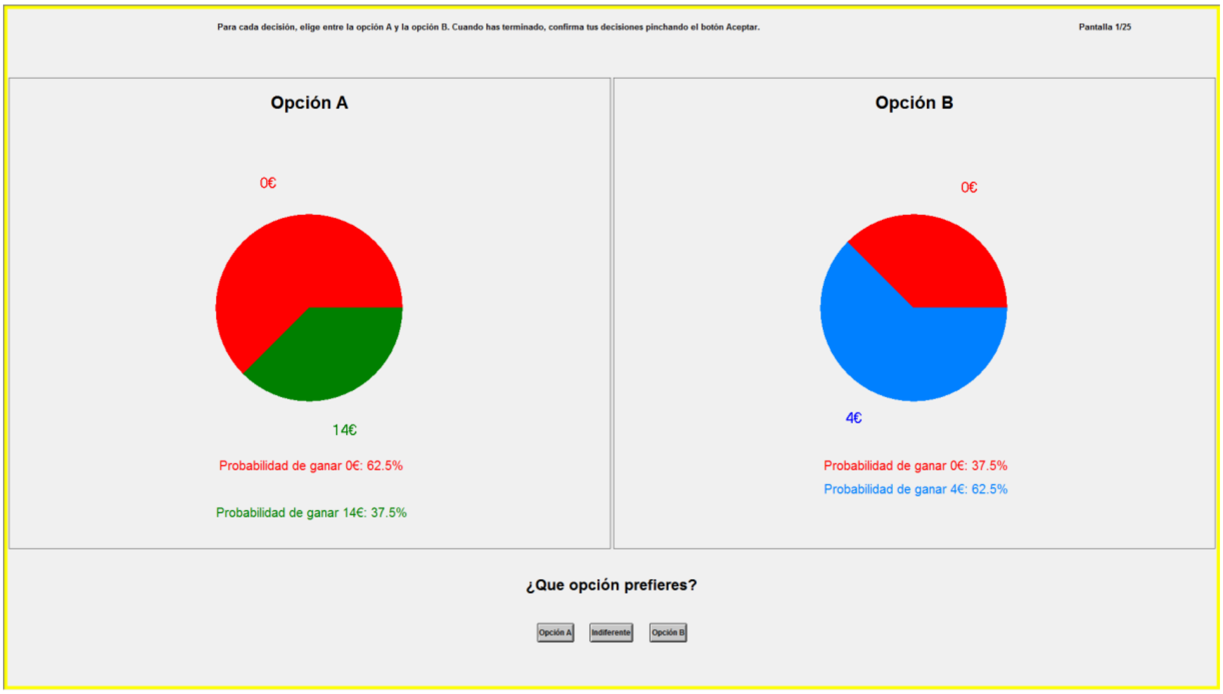
- Este es un experimento para estudiar el comportamiento individual al tomar decisiones. Sólo estamos interesados en lo que hacen los individuos en media.
- No pienses que se espera ningún comportamiento particular de ti. Sin embargo, ten en cuenta que tu actuación a lo largo del experimento influirá en la cantidad de dinero que puedes ganar.
- A continuación encontrarás una serie de instrucciones explicando cómo funciona el experimento y cómo usar el ordenador a lo largo del mismo.
- Por favor, es importante que no hables ni molestes a los otros participantes durante el experimento. Si necesitas ayuda, levanta la mano y espera en silencio. Serás atendido lo antes posible.

EL EXPERIMENTO

- El experimento consta de 81 rondas divididas en tres FASES.
- La cantidad de dinero recibida durante el experimento se determinará al concluir las 2 fases del mismo.

FASE 1

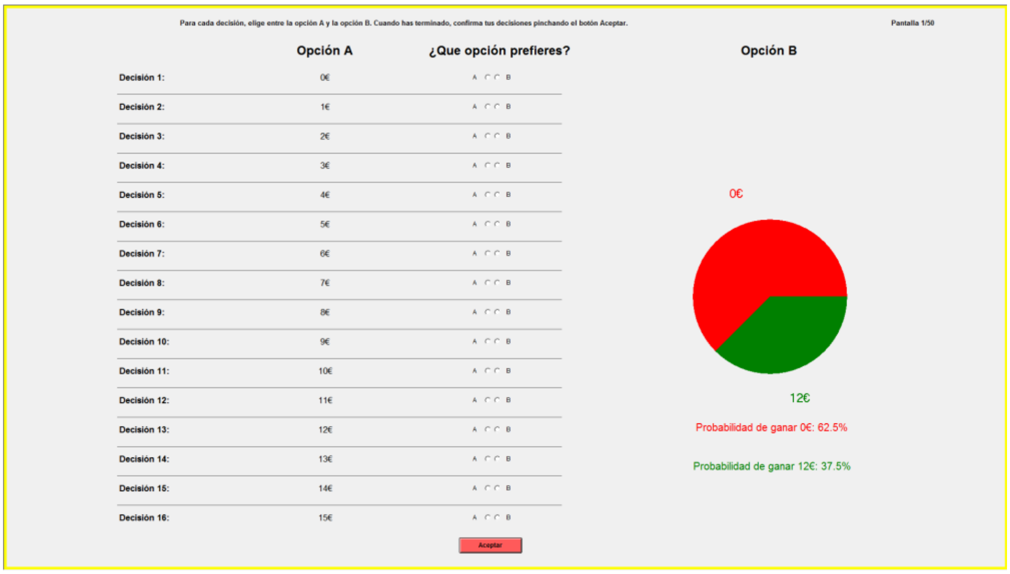
- En cada una de las 25 rondas de la FASE 1 te presentaremos una pareja de loterías entre las cuales tendrás que elegir tu preferida. Si estás indiferente entre las dos loterías, puedes señalarlo pinchando el botón “INDIFERENTE”. Al final del experimento el servidor determinará de forma aleatoria una de las 25 rondas, y se te pagará la cantidad de dinero que resulta de la lotería que has escogido. Si has elegido la opción “INDIFERENTE”, el ordenador sacará al azar una de las dos loterías y se te pagará la cantidad de dinero de la lotería escogida al azar por el ordenador.
- En cada ronda, en tu pantalla aparecerán las dos loterías entre las cuales tienes que escoger.



- En la figura que aparece sobre estas líneas tenemos un ejemplo de elección entre loterías. Si eliges la OPCIÓN A, puedes ganar € 0 con una probabilidad del 62.5% o € 14 con una probabilidad del 62.5%. Si, por el contrario, eliges la lotería de la DERECHA, puedes ganar € 0 con una probabilidad del 37.5 % y €4 con una probabilidad del 62.5%.
- En cada ronda, simplemente tendrás que elegir tu lotería preferida pinchando con el ratón en el botón correspondiente. Si estás indiferente entre las dos loterías, puedes señalarlo pinchando el botón “INDIFERENTE”.
- Es importante que juegues cada una de las 25 rondas de elección entre loterías como si fuera la que se va a jugar de verdad. Esto es debido a que al final del experimento, el ordenador escogerá una de las 25 rondas de manera aleatoria y jugará la lotería escogida por ti (o por el mismo ordenador, en el caso de indiferencia). La cantidad de dinero seleccionada se corresponderá a tu pago monetario asociado a la FASE 2.
- En resumen, la cantidad de dinero que recibirás para la FASE 2 dependerá de la ronda escogida aleatoriamente por el ordenador y del resultado de la lotería escogida por ti (o por el ordenador, en caso de indiferencia) en dicha ronda.

FASE 2

- En las 50 rondas de la FASE 2 participarás en un juego parecido al anterior pero con algunas variaciones.
- Así como en la FASE 1, también en la FASE 2 tendrás que elegir entre loterías sobre cuatro cantidades de dinero.
- La única diferencia con la FASE 1 es que, en la FASE 2, la OPCIÓN A es una determinada cantidad de dinero, que crece de 0€ a 15€ en cada una de las decisiones que tienes que tomar en cada ronda de la FASE 2.
- En cada ronda, en tu pantalla aparecerán las 16 cantidades de dinero (OPCIÓN A), y una lotería (OPCIÓN B), que tendrá exactamente el mismo formato de las loterías de la FASE 1.



- Cada ronda de la FASE 1 consta de 16 decisiones independientes, que se corresponden a cada fila.
- En la figura que aparece sobre estas líneas tenemos un ejemplo. La OPCIÓN A se corresponde una determinada cantidad de dinero, que crece de 0€ a 15€ en cada una de las decisiones que tienes que tomar en las sucesivas rondas de la FASE 1. En cada ronda, en tu pantalla aparecerán las 16 cantidades de dinero (OPCIÓN A), y una lotería (OPCIÓN B).
- Si eliges la OPCIÓN B en el ejemplo, podrías ganar 0€ con una probabilidad del 62.5% o 12€ con una probabilidad del 62.5%.
- En cada ronda de la FASE 1, la OPCION A seguirá siendo la misma (16 cantidades de dinero que suben de €0 hasta €15), mientras la OPCION B se corresponde a una lotería que irá cambiando de una ronda a otra.
- En cada una de las 16 decisiones, simplemente tendrás que elegir si prefieres la cantidad cierta de dinero asociada a cada decisión (OPCIÓN A) o jugar la lotería de la derecha (OPCIÓN B). Para tomar tu decisión, tendrás que pinchar con el ratón en el botón correspondiente.
- Es importante que juegues cada una de las 16 decisiones de cada ronda como si fuera la que se va a jugar de verdad. Esto es debido a que el ordenador -tras terminar el experimento- elegirá una de las decisiones de la FASE 1, y te pagará el dinero correspondiente a dicha decisión: la cantidad cierta si has escogido la OPCIÓN A o la resultante de jugar la lotería si has escogido la OPCION B.
- En resumen, la cantidad de dinero que recibirás en la FASE 1 dependerá de la ronda y decisión escogida aleatoriamente por el ordenador. Es decir, el pago monetario puede ser la cantidad cierta que elegiste o la cantidad resultante de la lotería escogida por ti.

¡BIENVENIDO AL EXPERIMENTO!

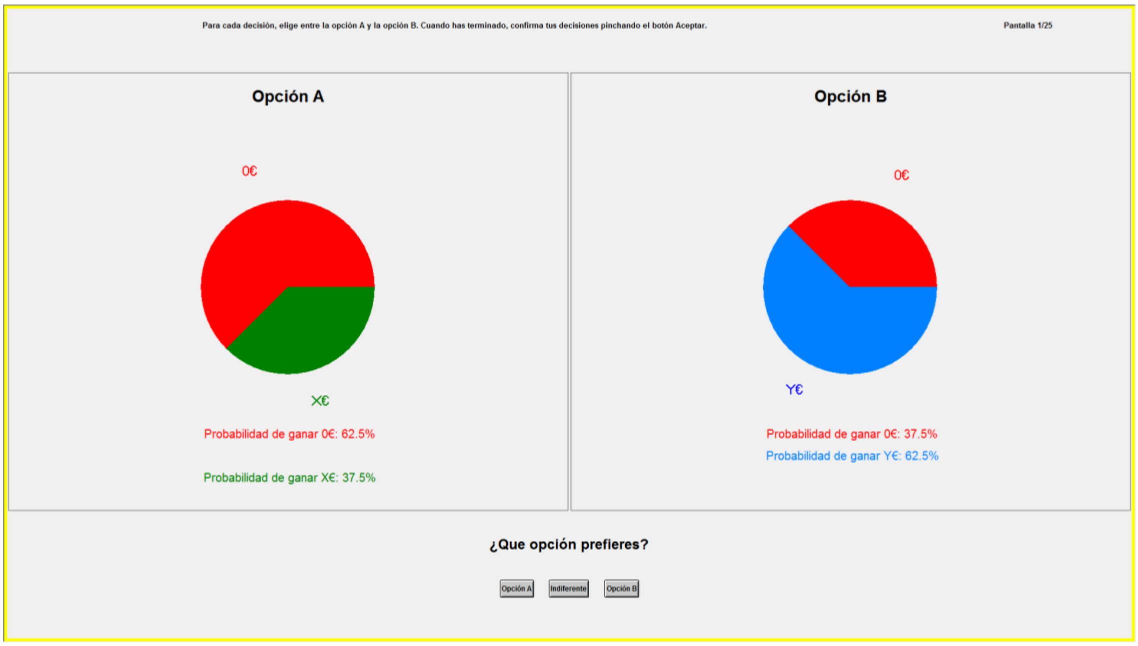
- Este es un experimento para estudiar el comportamiento individual al tomar decisiones. Sólo estamos interesados en lo que hacen los individuos en media.
- No pienses que se espera ningún comportamiento particular de ti. Sin embargo, ten en cuenta que tu actuación a lo largo del experimento influirá en la cantidad de dinero que puedes ganar.
- A continuación encontrarás una serie de instrucciones explicando cómo funciona el experimento y cómo usar el ordenador a lo largo del mismo.
- Por favor, es importante que no hables ni molestes a los otros participantes durante el experimento. Si necesitas ayuda, levanta la mano y espera en silencio. Serás atendido lo antes posible.

EL EXPERIMENTO

- El experimento consta de 81 rondas divididas en tres FASES.
- La cantidad de dinero recibida durante el experimento se determinará al concluir las 2 fases del mismo.

FASE 1

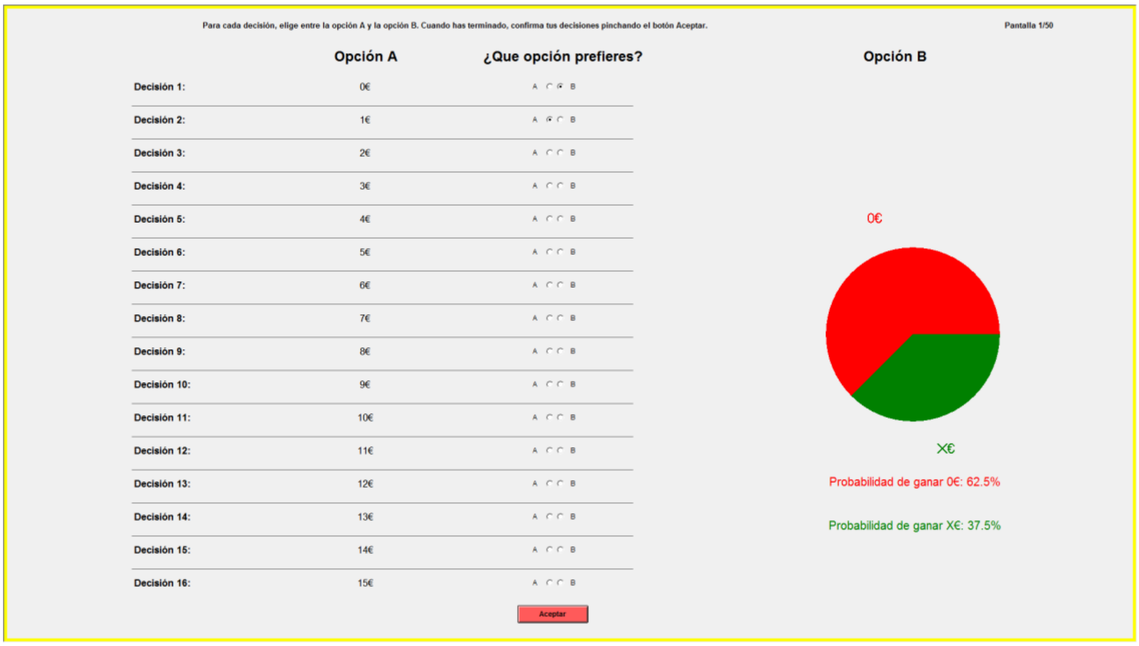
- En las 25 rondas de la FASE 1 te presentaremos una pareja de loterías entre las cuales tendrás que elegir tu preferida. Cada lotería asigna diferentes probabilidades de ganar cuatro premios de 0€, Y€, X€ y 15€, respectivamente. **El valor de Y y X** nunca se te comunicará y **cambiará en cada una de las 25 rondas de las que consta esta fase**. **Lo único que sabes es que 0€ < Y€ < X€ < 15€** (es decir, X es mayor de Y, y ambas cantidades están entre 0€ y 15€) y que X y Y pueden tomar cualquier valor entero entre 1€ y 14€ con la misma probabilidad.
- A cada premio está asociado un color. Esta asociación ente premios y colores se va a mantener a lo largo de todas las 50 rondas de esta fase.



- En la figura que aparece sobre estas líneas tenemos un ejemplo de elección entre loterías. Si eliges la OPCIÓN A, podrías ganar 0€ con una probabilidad del 62.5% o X€ con una probabilidad del 37.5%. Si, por el contrario, eliges la OPCIÓN B, podrías ganar 0€ con una probabilidad del 37.5 % y Y€ con una probabilidad del 62.5%.
- En cada ronda, simplemente tendrás que elegir tu lotería preferida pinchando con el ratón en el botón correspondiente. Si estás indiferente entre las dos loterías, puedes señalarlo pinchando el botón "INDIFERENTE". En este último caso, el ordenador escogerá una lotería por ti, seleccionando una de las dos loterías con la misma probabilidad.
- Es importante que juegues cada una de las 25 rondas de elección entre loterías como si fuera la que se va a jugar de verdad. Esto es debido a que al final del experimento, el ordenador escogerá una de las 25 rondas de manera aleatoria y jugará la lotería escogida por ti (o por el mismo ordenador, en el caso de indiferencia). La cantidad de dinero seleccionada se corresponderá a tu pago monetario asociado a la FASE 1.
- En resumen, la cantidad de dinero que recibirás en la FASE 1 dependerá de la ronda escogida aleatoriamente por el ordenador y del resultado de la lotería escogida por ti (o por el ordenador, en caso de indiferencia) en dicha ronda.

FASE 2

- En las 50 rondas de la FASE 2 participarás en un juego parecido al anterior pero con algunas variaciones.
- Así como en la FASE 1, también en la FASE 2 tendrás que elegir entre loterías sobre cuatro cantidades de dinero: **0€ < Y€ < X€ < 15€. Recuerda que Y y X pueden tomar cualquier valor entero entre 1€ y 14€ con la misma probabilidad, y que ese valor cambiará a lo largo de las 50 rondas de esta segunda fase**
- La única diferencia con la FASE 1 es que, en la FASE 2, la OPCIÓN A es una determinada cantidad de dinero, que crece de 0€ a 15€ en cada una de las decisiones que tienes que tomar en cada ronda de la FASE 2.
- En cada ronda, en tu pantalla aparecerán las 16 cantidades de dinero (OPCIÓN A), y una lotería (OPCIÓN B), que tendrá exactamente el mismo formato de las loterías de la FASE 1.



- Cada ronda de la FASE 2 consta de 16 decisiones independientes, que se corresponden a cada fila.
- En la figura aquí arriba tenemos un ejemplo. La OPCIÓN A se corresponde una determinada cantidad de dinero, que crece de 0€ a 15€ en cada una de las 16 decisiones que tienes que tomar (OPCIÓN A).
- La opción B se corresponde a una lotería, que se mantiene para todas las 16 decisiones. Si eliges la OPCIÓN B en el ejemplo, podrías ganar 0€ con una probabilidad del 62.5% o X€ con una probabilidad del 37.5%.
- En cada una de las 16 decisiones, simplemente tendrás que elegir si prefieres la cantidad cierta de dinero asociada a cada decisión (OPCIÓN A) o jugar la lotería de la derecha (OPCIÓN B). Para tomar tu decisión, tendrás que pinchar con el ratón en el botón correspondiente.
- Es importante que juegues cada una de las 16 decisiones de cada ronda como si fuera la que se va a jugar de verdad. Esto es debido a que el ordenador -tras terminar el experimento- elegirá una de las decisiones de la FASE 2, y te pagará el dinero correspondiente a dicha decisión: la cantidad cierta si has escogido la OPCIÓN A o la resultante de jugar la lotería si has escogido la OPCION B.
- En resumen, la cantidad de dinero que recibirás en la FASE 2 dependerá de la ronda y decisión escogida aleatoriamente por el ordenador. Es decir, el pago monetario puede ser la cantidad cierta que elegiste o la cantidad resultante de la lotería escogida por ti.

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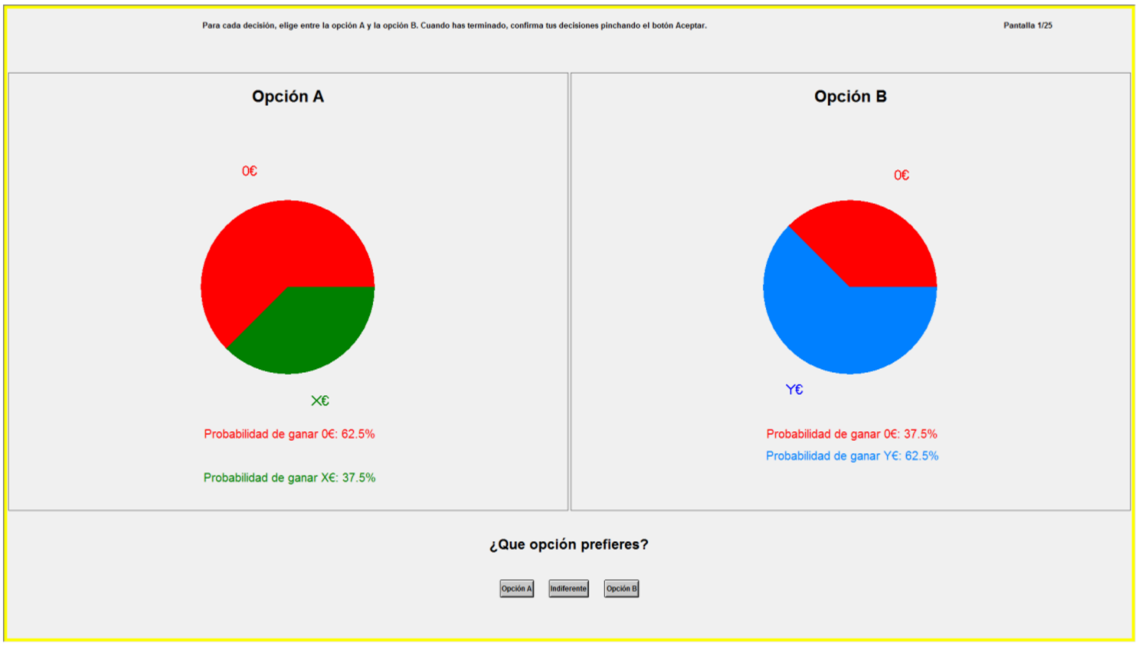
- La cantidad de dinero recibida durante el experimento se determinará al concluir las 2 fases del mismo.

FASE 1

- En las 25 rondas de la FASE 1 te presentaremos una pareja de loterías entre las cuales tendrás que elegir tu preferida. Cada lotería asigna diferentes probabilidades de ganar cuatro premios de 0€, Y€, X€ y 15€, respectivamente. **El valor de Y y X** nunca se te comunicará y **cambiará en cada una de las 25 rondas de las que consta esta fase**. **Lo único que sabes es que 0€ < Y€ < X€ < 15€** (es decir, X es mayor que Y, y ambas cantidades están entre 0€ y 15€). A cada premio está asociado un color. Esta asociación ente premios y colores se va a mantener a lo largo de todas las 50 rondas de esta fase.



- En la figura que aparece sobre estas líneas tenemos un ejemplo de elección entre loterías. Si eliges la OPCIÓN A, podrías ganar 0€ con una probabilidad del 62.5% o X€ con una probabilidad del 37.5%. Si, por el contrario, eliges la OPCIÓN B, podrías ganar 0€ con una probabilidad del 37.5 % y Y€ con una probabilidad del 62.5%.

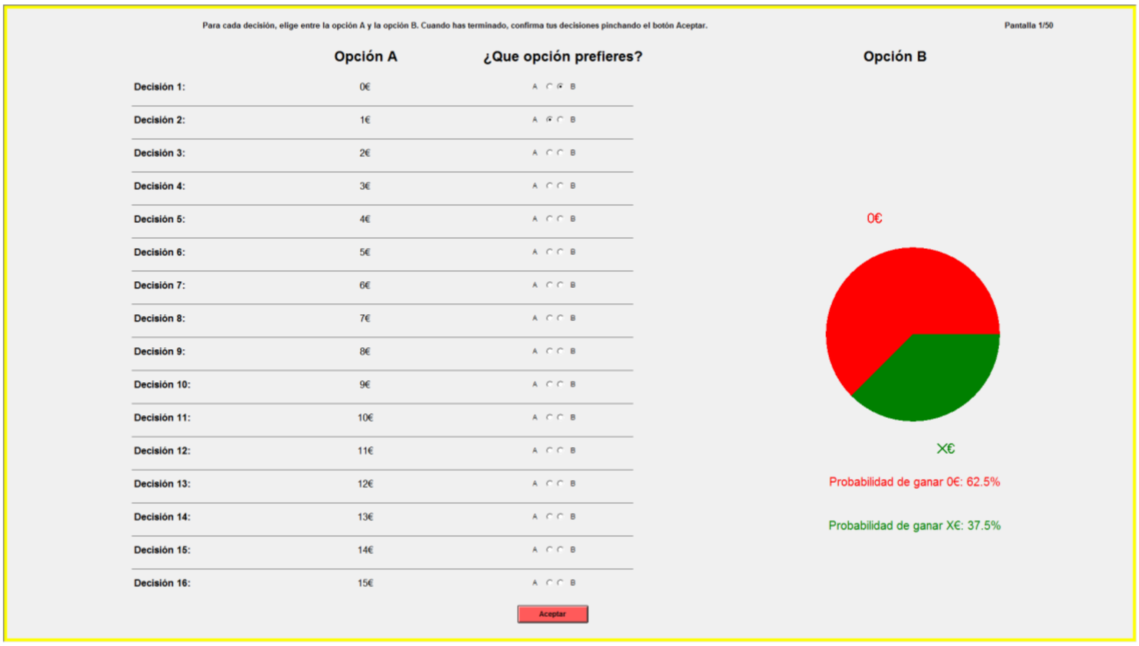
- En cada ronda, simplemente tendrás que elegir tu lotería preferida pinchando con el ratón en el botón correspondiente. Si estás indiferente entre las dos loterías, puedes señalarlo pinchando el botón "INDIFERENTE". En este último caso, el ordenador escogerá una lotería por ti, seleccionando una de las dos loterías con la misma probabilidad.

- Es importante que juegues cada una de las 25 rondas de elección entre loterías como si fuera la que se va a jugar de verdad. Esto es debido a que al final del experimento, el ordenador escogerá una de las 25 rondas de manera aleatoria y jugará la lotería escogida por ti (o por el mismo ordenador, en el caso de indiferencia). La cantidad de dinero seleccionada se corresponderá a tu pago monetario asociado a la FASE 1.

- En resumen, la cantidad de dinero que recibirás en la FASE 1 dependerá de la ronda escogida aleatoriamente por el ordenador y del resultado de la lotería escogida por ti (o por el ordenador, en caso de indiferencia) en dicha ronda.

FASE 2

- En las 50 rondas de la FASE 2 participarás en un juego parecido al anterior pero con algunas variaciones.
- Así como en la FASE 1, también en la FASE 2 tendrás que elegir entre loterías sobre cuatro cantidades de dinero**: 0€ < Y€ < X€ < 15€.** Recuerda que los valores de Y y X cambiarán en cada una de las 50 rondas de esta segunda fase.
- La única diferencia con la FASE 1 es que, en la FASE 2, la OPCIÓN A es una determinada cantidad de dinero, que crece de 0€ a 15€ en cada una de las decisiones que tienes que tomar en cada ronda de la FASE 2.
- En cada ronda, en tu pantalla aparecerán las 16 cantidades de dinero (OPCIÓN A), y una lotería (OPCIÓN B), que tendrá exactamente el mismo formato de las loterías de la FASE 1.



- Cada ronda de la FASE 2 consta de 16 decisiones independientes, que se corresponden a cada fila.
- En la figura aquí arriba tenemos un ejemplo. La OPCIÓN A se corresponde una determinada cantidad de dinero, que crece de 0€ a 15€ en cada una de las 16 decisiones que tienes que tomar (OPCIÓN A).
- La opción B corresponde a una lotería, que se mantiene para todas las 16 decisiones. Si eliges la OPCIÓN B en el ejemplo, podrías ganar 0€ con una probabilidad del 62.5% o X€ con una probabilidad del 37.5%.
- En cada una de las 16 decisiones, simplemente tendrás que elegir si prefieres la cantidad cierta de dinero asociada a cada decisión (OPCIÓN A) o jugar la lotería de la derecha (OPCIÓN B). Para tomar tu decisión, tendrás que pinchar con el ratón en el botón correspondiente.
- Es importante que juegues cada una de las 16 decisiones de cada ronda como si fuera la que se va a jugar de verdad. Esto es debido a que el ordenador -tras terminar el experimento- elegirá una de las decisiones de la FASE 1, y te pagará el dinero correspondiente a dicha decisión: la cantidad cierta si has escogido la OPCIÓN A o la resultante de jugar la lotería si has escogido la OPCION B.
- En resumen, la cantidad de dinero que recibirás en la FASE 2 dependerá de la ronda y decisión escogida aleatoriamente por el ordenador. Es decir, el pago monetario puede ser la cantidad cierta que elegiste o la cantidad resultante de la lotería escogida por ti.

APPENDIX B

STATA Codes

```
****FECHNER WITH MEAN VARIANCE ORDERED PROBIT



cap program drop    probit_MV_P123
program define      probit_MV_P123

args lnf kappa_beta kappa_sigma
tempvar beta sigma num0_MV num1_MV EV_0 EV_1 VAR_0 VAR_1 euDiff_MV
tempvar fMV1

    quietly {

gen double `beta'   =`kappa_beta'
gen double `sigma'=exp(`kappa_sigma')


*Defining EV and VAR:

    g `EV_0'=.
    g `EV_1'=.
    g `VAR_0'=.
    g `VAR_1'=.


    *Phase 1 (we have sure payoff on the left):

    replace `EV_0' = (Round-1)/15   if Phase==1
    replace `VAR_0'= 0              if Phase==1

    replace `EV_1'= prize_HO_1 * prob_HO_11 + prize_HO_2 *
prob_HO_12 + prize_HO_3 * prob_HO_13 + prize_HO_4 * prob_HO_14
if Phase==1


    replace `VAR_1'= ((prize_HO_1-`EV_1')^2) * prob_HO_11 + ((
prize_HO_2-`EV_1')^2) * prob_HO_12 + ((prize_HO_3-`EV_1')^2) *
prob_HO_13 + ((prize_HO_4-`EV_1')^2) * prob_HO_14 if Phase==1

    *Phase 2 and 3:
    replace `EV_0'= prize_HO_1 * prob_HO_01 + prize_HO_2 *
prob_HO_02 + prize_HO_3 * prob_HO_03 + prize_HO_4 * prob_HO_04 if
 Phase>1

    replace `EV_1'= prize_HO_1 * prob_HO_11 + prize_HO_2 *
prob_HO_12 + prize_HO_3 * prob_HO_13 + prize_HO_4 * prob_HO_14 if
 Phase>1

    replace `VAR_0'= ((prize_HO_1-`EV_0')^2) * prob_HO_01 + ((
prize_HO_2-`EV_0')^2) * prob_HO_02 + ((prize_HO_3-`EV_0')^2) *
prob_HO_03 + ((prize_HO_4-`EV_0')^2) * prob_HO_04 if Phase>1
```

```
    replace `VAR_1'= ((prize_HO_1-`EV_1')^2) * prob_HO_11 + ((
prize_HO_2-`EV_1')^2) * prob_HO_12 + ((prize_HO_3-`EV_1')^2) *
prob_HO_13 + ((prize_HO_4-`EV_1')^2) * prob_HO_14 if Phase>1



*The model:

gen double `num0_MV' = `EV_0' - (`beta'*`VAR_0')
gen double `num1_MV' = `EV_1' - (`beta'*`VAR_1')

generate double `euDiff_MV' = (`num1_MV' - `num0_MV')/`sigma'


generate double `fMV1' = 0


*FOR PROBIT

replace `fMV1' = normal( `euDiff_MV') if $ML_y1==1
replace `fMV1' = normal(-`euDiff_MV') if $ML_y1==0


replace `lnf' = ln(`fMV1')

}

end



xi: ml model lf probit_MV_P123 (beta: NEWchoice=) (sigma:=) if
NEWchoice!=. & Phase<3 & TR==2, robust technique(dfp) cluster(
idNEW) init(.5 .5, copy)
set more 1
ml maximize, dif
outreg2 using FechnerEx.doc, append
nlcom (beta: [beta]_cons) (sigma: exp([sigma]_cons))
```

```
****RANDOM EFFECT



keep if TR==2

*RANDOM EFFECT PROBIT MODEL


xtprobit NEWchoice newDelta_VAR if Phase<3, nocons offset(
newDelta_EV)


*RANDOM EFFECT LOGIT MODEL


xtlogit NEWchoice newDelta_VAR if Phase<3, nocons offset(
newDelta_EV)
```

```
****NAIVE



cap program drop logit_CRRA_P12_NAIVE2_TR_012
program define   logit_CRRA_P12_NAIVE2_TR_012

    args lnf kappa_rho
    tempvar rho X Y numP1_0 numP1_1 numP2_0 numP2_1  euDiff_P1
euDiff_P2 alpha
    tempvar fP1 fP2

    quietly {

    *** VAR TRANSFORMATIONS

    gen double `rho'=`kappa_rho'
    gen double `X'=(2/3)
    gen double `Y'=(1/3)

    ***Phase 1

    gen double `numP1_0'    = 0
    replace    `numP1_0'    = (((Round-1)/15)^(1-`rho'))/(1-`rho'
) if Round>1
    gen double `numP1_1'    = ((prize_HO_2^(1-`rho'))/(1-`rho'))*
prob_HO_12+((prize_HO_3^(1-`rho'))/(1-`rho'))*prob_HO_13+(1/(1-
`rho'))*prob_HO_14
    replace    `numP1_1'    = ((`Y'^       (1-`rho'))/(1-`rho'))*
prob_HO_12+((`X'^       (1-`rho'))/(1-`rho'))*prob_HO_13+(1/(1-
`rho'))*prob_HO_14 if TR<2


* get the Fechner index

generate double `euDiff_P1' = `numP1_1' - `numP1_0'
generate double `fP1' = 1
replace `fP1' = invlogit( `euDiff_P1') if $ML_y1==1 & Phase==1
replace `fP1' = invlogit(-`euDiff_P1') if $ML_y1==0 & Phase==1


***PHASE II

    gen double `numP2_0' = ((prize_HO_2^(1-`rho'))/(1-`rho'))*
prob_HO_02 + ///
                           ((prize_HO_3^(1-`rho'))/(1-`rho'))*
prob_HO_03+ (1/(1-`rho')) * prob_HO_04


    gen double `numP2_1' = ((prize_HO_2^(1-`rho'))/(1-`rho'))*
prob_HO_12 + ///
                         ((prize_HO_3^(1-`rho'))/(1-`rho'))*
```

```
prob_HO_13+ (1/(1-`rho')) * prob_HO_14


    replace  `numP2_0' = ((`Y'^(1-`rho'))/(1-`rho'))*prob_HO_02 +
 ///
                             ((`X'^(1-`rho'))/(1-`rho'))*prob_HO_03+
(1/(1-`rho')) * prob_HO_04 if TR<2


    replace `numP2_1' =  ((`Y'^(1-`rho'))/(1-`rho'))*prob_HO_12 +
 ///
                             ((`X'^(1-`rho'))/(1-`rho'))*prob_HO_13+(
1/(1-`rho'))*prob_HO_14 if TR<2


generate double `euDiff_P2' = `numP2_1' - `numP2_0'

generate double `fP2' = 1
replace `fP2' = invlogit(`euDiff_P2')  if $ML_y1==1 & Phase==2
replace `fP2' = invlogit(-`euDiff_P2') if $ML_y1==0 & Phase==2


  * now collect all the pieces

    replace `lnf'=ln(`fP1'*`fP2')


    }

end


xi: ml model lf logit_CRRA_P12_NAIVE2_TR_012 (rho:NEWchoice=i.TR)
 if NEWchoice!=., robust technique(dfp) cluster(idNEW) init(.5 .5
 .5, copy)
set more 1
ml maximize, dif

lincom TR_0 - TR_1
nlcom (delta_TR: [rho]TR_0 - [rho]TR_1)
```

```
****BAYESIAN



cap program drop logit_CRRA_P12_BAYES2_TR_012
program define   logit_CRRA_P12_BAYES2_TR_012

    args lnf kappa_rho
    tempvar rho X Y numP1_0 numP1_1 numP2_0 numP2_1  euDiff_P1
euDiff_P2 alpha
    tempvar fP1 fP2

    quietly {

    *** VAR TRANSFORMATIONS

    gen double `rho'=`kappa_rho'
    gen double `X'=  (2 * 1^(1 - `rho'))/((-3 + `rho') * (-1 +
`rho'))
    gen double `Y'=-(2 * 1^(1 - `rho'))/(-6 + 11 * `rho' - 6 *
`rho'^2 + `rho'^3)


***Phase 1

    gen double `numP1_0'    = 0
    replace    `numP1_0'    = (((Round-1)/15)^(1-`rho'))/(1-`rho'
) if Round>1
    gen double `numP1_1'    = ((prize_HO_2^(1-`rho'))/(1-`rho'))*
prob_HO_12+((prize_HO_3^(1-`rho'))/(1-`rho'))*prob_HO_13+(1/(1-
`rho'))*prob_HO_14
    replace    `numP1_1'    =       `Y'                          *
prob_HO_12+       `X'                     *prob_HO_13+(1/(1-
`rho'))*prob_HO_14 if TR<2

* get the Fechner index

generate double `euDiff_P1' = `numP1_1' - `numP1_0'
generate double `fP1' = 1
replace `fP1' = invlogit( `euDiff_P1') if $ML_y1==1 & Phase==1
replace `fP1' = invlogit(-`euDiff_P1') if $ML_y1==0 & Phase==1


***PHASE II

    gen double `numP2_0' = ((prize_HO_2^(1-`rho'))/(1-`rho'))*
prob_HO_02 + ///
                           ((prize_HO_3^(1-`rho'))/(1-`rho'))*
prob_HO_03+ (1/(1-`rho')) * prob_HO_04


    gen double `numP2_1' = ((prize_HO_2^(1-`rho'))/(1-`rho'))*
```

```
prob_HO_12 + ///
                              ((prize_HO_3^(1-`rho'))/(1-`rho'))*
prob_HO_13+ (1/(1-`rho')) * prob_HO_14


replace `numP2_0' = `Y' * prob_HO_02 + `X' * prob_HO_03 + (1/(1-
`rho')) * prob_HO_04 if TR<2
replace `numP2_1' = `Y' * prob_HO_12 + `X' * prob_HO_13 + (1/(1-
`rho')) * prob_HO_14 if TR<2


generate double `euDiff_P2' = `numP2_1' - `numP2_0'

generate double `fP2' = 1
replace `fP2' = invlogit(`euDiff_P2')  if $ML_y1==1 & Phase==2
replace `fP2' = invlogit(-`euDiff_P2') if $ML_y1==0 & Phase==2


  * now collect all the pieces

    replace `lnf'=ln(`fP1'*`fP2')


    }

end


xi: ml model lf logit_CRRA_P12_BAYES2_TR_012 (rho:NEWchoice=TR_0
TR_1) if NEWchoice!=., robust technique(dfp) cluster(idNEW) init(
.5 .5 .5, copy)
set more 1
ml maximize, dif

lincom TR_0 - TR_1
nlcom (delta_TR: [rho]TR_0 - [rho]TR_1)
```

```
***MIXTURE



cap program drop logit_CRRA_P12_MIX2_TR_012
program define   logit_CRRA_P12_MIX2_TR_012

    args lnf kappa_rho_NAI kappa_rho_BAY
    tempvar rho X_NAI Y_NAI X_BAY Y_BAY numP1_0 numP1_1 numP2_0
numP2_1  euDiff_P1 euDiff_P2
    tempvar rho_NAI rho_BAY pi_BAY alpha_NAI alpha_BAY
    tempvar fP1_NAI fP2_NAI f_NAI fP1_BAY fP2_BAY f_BAY

    quietly {

    *** VAR TRANSFORMATIONS

    gen double `rho_NAI'=`kappa_rho_NAI'
    gen double `rho_BAY'=`kappa_rho_BAY'
    gen double `pi_BAY'=pi_GRID
    gen double `X_NAI'=2/3
    gen double `Y_NAI'=1/3
    gen double `X_BAY'= (2 * 1^(1 - `rho_BAY'))/((-3 + `rho_BAY')
 * (-1 + `rho_BAY'))
    gen double `Y_BAY'=-(2 * 1^(1 - `rho_BAY'))/(-6 + 11 *
`rho_BAY' - 6 * `rho_BAY'^2 + `rho_BAY'^3)<2


***NAIVE

***PHASE I NAIVE


    gen double `numP1_0'    = 0
    replace    `numP1_0'    = (((Round-1)/15)^(1-`rho_NAI'))/(1-
`rho_NAI') if Round>1
    gen double `numP1_1'    = ((prize_HO_2^(1-`rho_NAI'))/(1-
`rho_NAI'))*prob_HO_12+((prize_HO_3^(1-`rho_NAI'))/(1-`rho_NAI'
))*prob_HO_13+(1/(1-`rho_NAI'))*prob_HO_14
    replace    `numP1_1'    = ((`Y_NAI'^       (1-`rho_NAI'))/(1-
`rho_NAI'))*prob_HO_12+((`X_NAI'^       (1-`rho_NAI'))/(1-
`rho_NAI'))*prob_HO_13+(1/(1-`rho_NAI'))*prob_HO_14 if TR<2


* get the Fechner index

generate double `euDiff_P1' = `numP1_1' - `numP1_0'
generate double `fP1_NAI' = 1
replace `fP1_NAI' = invlogit( `euDiff_P1') if $ML_y1==1 & Phase==1
replace `fP1_NAI' = invlogit(-`euDiff_P1') if $ML_y1==0 & Phase==1
```

```
***PHASE II NAIVE

    gen double `numP2_0' = ((prize_HO_2^(1-`rho_NAI'))/(1-
`rho_NAI'))*prob_HO_02 + ///
                           ((prize_HO_3^(1-`rho_NAI'))/(1-
`rho_NAI'))*prob_HO_03+ (1/(1-`rho_NAI')) * prob_HO_04


    gen double `numP2_1' = ((prize_HO_2^(1-`rho_NAI'))/(1-
`rho_NAI'))*prob_HO_12 + ///
                         ((prize_HO_3^(1-`rho_NAI'))/(1-`rho_NAI'
))*prob_HO_13+ (1/(1-`rho_NAI')) * prob_HO_14


    replace  `numP2_0' = ((`Y_NAI'^(1-`rho_NAI'))/(1-`rho_NAI'))*
prob_HO_02 + ///
                         ((`X_NAI'^(1-`rho_NAI'))/(1-`rho_NAI'))*
prob_HO_03+ (1/(1-`rho_NAI')) * prob_HO_04 if TR<2


    replace `numP2_1' =  ((`Y_NAI'^(1-`rho_NAI'))/(1-`rho_NAI'))*
prob_HO_12 + ///
                         ((`X_NAI'^(1-`rho_NAI'))/(1-`rho_NAI'))*
prob_HO_13+(1/(1-`rho_NAI'))*prob_HO_14 if TR<2


generate double `euDiff_P2' = `numP2_1' - `numP2_0'

generate double `fP2_NAI' = 1

replace `fP2_NAI' = invlogit( `euDiff_P2')  if $ML_y1==1 & Phase
==2
replace `fP2_NAI' = invlogit(-`euDiff_P2')  if $ML_y1==0 & Phase
==2


  * now collect all the pieces

gen double `f_NAI'=(`fP1_NAI' * `fP2_NAI')

***BAYES

***PHASE I BAYES


    replace    `numP1_0'    = (((Round-1)/15)^(1-`rho_BAY'))/(1-
`rho_BAY') if Round>1
    replace    `numP1_1'    = ((prize_HO_2^(1-`rho_BAY'))/(1-
`rho_BAY'))*prob_HO_12+((prize_HO_3^(1-`rho_BAY'))/(1-`rho_BAY'
))*prob_HO_13+(1/(1-`rho_BAY'))*prob_HO_14
    replace    `numP1_1'    =        `Y_BAY'
                         *prob_HO_12+        `X_BAY'
```

```
                              *prob_HO_13+(1/(1-`rho_BAY'))*prob_HO_14
if TR<2

* get the Fechner index

replace `euDiff_P1' = `numP1_1' - `numP1_0'
generate double `fP1_BAY' = 1
replace `fP1_BAY' = invlogit( `euDiff_P1') if $ML_y1==1 & Phase==1
replace `fP1_BAY' = invlogit(-`euDiff_P1') if $ML_y1==0 & Phase==1


***PHASE II BAYES


    replace `numP2_0' = ((prize_HO_2^(1-`rho_BAY'))/(1-`rho_BAY'
))*prob_HO_02 + ///
                                ((prize_HO_3^(1-`rho_BAY'))/(1-
`rho_BAY'))*prob_HO_03+ (1/(1-`rho_BAY')) * prob_HO_04


    replace `numP2_1' = ((prize_HO_2^(1-`rho_BAY'))/(1-`rho_BAY'
))*prob_HO_12 + ///
                             ((prize_HO_3^(1-`rho_BAY'))/(1-`rho_BAY'
))*prob_HO_13+ (1/(1-`rho_BAY')) * prob_HO_14


    replace `numP2_0' = `Y_BAY' * prob_HO_02 + `X_BAY' *
prob_HO_03 + (1/(1-`rho_BAY')) * prob_HO_04 if TR<2

    replace `numP2_1' = `Y_BAY' * prob_HO_12 + `X_BAY' *
prob_HO_13 + (1/(1-`rho_BAY')) * prob_HO_14 if TR<2


replace `euDiff_P2' = `numP2_1' - `numP2_0'

generate double `fP2_BAY' = 1
replace `fP2_BAY' = invlogit(`euDiff_P2')  if $ML_y1==1 & Phase==2
replace `fP2_BAY' = invlogit(-`euDiff_P2') if $ML_y1==0 & Phase==2


  * now collect all the pieces

gen double `f_BAY'=`fP1_BAY' * `fP2_BAY'


*HERE THE MIXTURE

    replace `lnf'=ln((1-`pi_BAY') * `f_NAI' + (`pi_BAY') *
`f_BAY' )


    }
```

```
    }

end



g pi_GRID=.5

replace pi_GRID=.5
xi: ml model lf logit_CRRA_P12_MIX2_TR_012 (rho_NAI:NEWchoice=
TR_0 TR_1) (rho_BAY:NEWchoice=TR_0 TR_1) if NEWchoice!=., robust
technique(dfp) cluster(idNEW) init(.5 .5 .5 .5 .5 .5, copy)
set more 1
ml maximize, dif



**************LOOP

log using Dile_2.log, replace


set more off
forvalues i=.181(.001).182{

replace pi_GRID=`i'

di "************** pi=`i' ***************"

xi: ml model lf logit_CRRA_P12_MIX_TR_012 (rho_NAI:NEWchoice=) (
rho_BAY:=) if NEWchoice!=., robust technique() cluster(idNEW)
init(.5 .5 .5, copy)
set more 1
ml maximize, dif
nlcom (rho_NAI: [rho_NAI]_cons) (rho_BAY: [rho_BAY]_cons)

}

log close
```

Bibliography

[1] Giovanni Ponti, Carmen Herrero, and Judit Albarràn, Pedro aan Alonso. An experiment on outcome uncertainty. *Departamento de Fundamentos del Analisis Economico - Univesidad de Alicante, Working Paper*, 2018.

[2] Charles A Holt and Susan K Laury. Risk aversion and incentive effects. *American economic review*, 92(5):1644–1655, 2002.

[3] John D Hey and Chris Orme. Investigating generalizations of expected utility theory using experimental data. *Econometrica: Journal of the Econometric Society*, pages 1291–1326, 1994.

[4] Steffen Andersen, John Fountain, Glenn Harrison, E Elisabet Rutström, et al. Estimating aversion to uncertainty. *Report.[233]*, 2009.

[5] Frank H Knight. Risk, uncertainty and profit, reprints of economic classics, augustus m. *Kelley, Bookseller, New York I*, 964, 1964.

[6] Chip Heath and Amos Tversky. Preference and belief: Ambiguity and competence in choice under uncertainty. *Journal of risk and uncertainty*, 4(1):5–28, 1991.

[7] Leonard J Savage. *The foundations of statistics*. Courier Corporation, 1972.

[8] Daniel Ellsberg. Risk, ambiguity, and the savage axioms. *The quarterly journal of economics*, pages 643–669, 1961.

[9] Barbara E Kahn and Rakesh K Sarin. Modeling ambiguity in decisions under uncertainty. *Journal of Consumer Research*, 15(2):265–272, 1988.

[10] Marco Lauriola and Irwin P Levin. Relating individual differences in attitude toward ambiguity to risky choices. *Journal of Behavioral Decision Making*, 14(2):107–122, 2001.

[11] Peter Klibanoff, Massimo Marinacci, and Sujoy Mukerji. A smooth model of decision making under ambiguity. *Econometrica*, 73(6):1849–1892, 2005.

[12] Yoram Halevy. Ellsberg revisited: An experimental study. *Econometrica*, 75(2):503–536, 2007.

[13] Larry G Epstein. A definition of uncertainty aversion. *The Review of Economic Studies*, 66(3):579–608, 1999.

[14] Glenn W Harrison and E Elisabet Rutström. Expected utility theory and prospect theory: One wedding and a decent funeral. *Experimental Economics*, 12(2):133, 2009.

[15] Daniel Kahneman. Prospect theory: An analysis of decisions under risk. *Econometrica*, 47:278, 1979.

[16] Rakesh K Sarin and Robert L Winkler. Ambiguity and decision modeling: A preference-based approach. *Journal of Risk and Uncertainty*, 5(4):389–407, 1992.

[17] Itzhak Gilboa and David Schmeidler. Maxmin expected utility with non-unique prior. *Journal of mathematical economics*, 18(2):141–153, 1989.

[18] Carmen Herrero and Antonio Villar. Ranking distributions of monotone attributes. 2014.

[19] Kfir Eliaz and Pietro Ortoleva. Multidimensional ellsberg. *Management Science*, 62(8):2179–2197, 2015.

[20] Jürgen Eichberger, Jörg Oechssler, and Wendelin Schnedler. How do subjects view multiple sources of ambiguity? *Theory and Decision*, 78(3):339–356, 2015.

[21] Evan Moore and Catherine Eckel. Measuring ambiguity aversion. *Unpublished manuscript, Department of Economics, Virginia Tech*, 2003.

[22] Uri Gneezy, Alex Imas, and John List. Estimating individual ambiguity aversion: A simple approach. Technical report, National Bureau of Economic Research, 2015.

[23] Glenn W Harrison and E Elisabet Rutström. Risk aversion in the laboratory. In *Risk aversion in experiments*, pages 41–196. Emerald Group Publishing Limited, 2008.

[24] Hans P Binswanger. Attitudes toward risk: Experimental measurement in rural india. *American journal of agricultural economics*, 62(3):395–407, 1980.

[25] Peter Wakker and Daniel Deneffe. Eliciting von neumann-morgenstern utilities when probabilities are distorted or unknown. *Management science*, 42(8):1131–1150, 1996.

[26] Ben Greiner. An online recruitment system for economic experiments. 2004.

[27] Shane Frederick. Cognitive reflection and decision making. *Journal of Economic perspectives*, 19(4):25–42, 2005.

[28] Oliver P John, Eileen M Donahue, and Robert L Kentle. The big five inventory—versions 4a and 54, 1991.

[29] Glenn W Harrison. Maximum likelihood estimation of utility functions using stata. *University of Central Florida, Working Paper*, pages 06–12, 2008.

[30] Peter G Moffatt. *Experimetrics: Econometrics for experimental economics*. Palgrave Macmillan, 2015.

[31] Ismael Rodriguez-Lara and Giovanni Ponti. Social motives vs social influence: An experiment on interdependent time preferences. *Games and Economic Behavior*, 105:177–194, 2017.

[32] Graham Loomes and Robert Sugden. Testing different stochastic specificationsof risky choice. *Economica*, 65(260):581–598, 1998.

[33] G Fechner. 1966. *Elements of psychophysics*, 1, 1860.

[34] Graham Loomes, Peter G Moffatt, and Robert Sugden. A microeconometric test of alternative stochastic theories of risky choice. *Journal of risk and Uncertainty*, 24(2):103–130, 2002.

[35] John W Pratt, Howard Raiffa, and Robert Schlaifer. The foundations of decision under uncertainty: An elementary exposition. *Journal of the American Statistical Association*, 59(306):353–375, 1964.

[36] Mark J Machina. Choice under uncertainty: Problems solved and unsolved. *Journal of Economic Perspectives*, 1(1):121–154, 1987.

[37] Marco R Steenbergen. Maximum likelihood programming in stata. *Department of Political Science. University of North Carolina, Chapel Hill*, 2003.

[38] Judit Alonso. Risk aversion and uncertain outcomes in the laboratory. *Departamento de Fundamentos del Analisis Economico - Univesidad de Alicante, Master Thesis*, 2016.

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An Ambiguity Analysis under Heterogeneity with a Bayesian Spin

-Summary-

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Abstract

We often have to deal with uncertainty regarding multiple aspects of the decision problems we face. This uncertainty may concern, for instance, our earnings, the likelihood to receive them in a given moment and in a given amount. The aim of this thesis is to contribute to the growing body of literature around “multi-dimensional uncertainty”, which enlarges the scope of ambiguity outside the frame of uncertainty about probabilities. It does so by analysing, both theoretically and empirically. The evidence stemming from a multi-stage experiment in which subjects have to choose between lotteries whereby amounts of monetary prizes are not always known, whereas probabilities are always public knowledge. In the experiment, three different levels of information over some monetary prizes are randomized between subjects. The experimental evidence undergoes structural estimation exercises: these elicit the individuals’ degree of risk aversion within the frame of a standard constant relative risk aversion (CRRA) utility function. Furthermore, we investigate whether a change of information, such as the one we reproduce through the different treatments conditions, translates into a change in behavior and, in turn, whether and how much this change translates into a significant change in their measured (CRRA) attitude toward risk. As to the behavioral content of the structural model for the uncertain payoffs, we propose two alternative specifications, labelled “naive” and “sophisticated”. The empirical evidence shows a moderate but significant degree of *love for ambiguity*, since less information given to subjects results in a lower estimate of their risk aversion, and, as a consequence, in a stronger attraction toward risk and uncertainty. A mixture model is implemented to identify the probability of individuals mirroring one behavioral model or the other, or, saying it differently, the percentage of observations compatible with either model. We conclude that our subjects have a strong tendency to behave as naive.

Keywords: heterogeneity; risk aversion; ambiguity.

Contents

CHAPTER 1

Dealing with Ambiguity

1.1 Introduction

In conditions of uncertain outcomes, the Savage approach [1] has been traditionally used. In particular, individuals have been assumed to behave according to a unique subjective prior belief over all states of the world, and, given this, they would maximize their expected utility. This decision process clearly neglects the existence of any form of ambiguity, and it prescribes the way decision makers should deal with uncertain situations.

However, Ellsberg [2] claims that most individuals treat ambiguity differently than objective risk. In specific, he argues that people exhibit a significant degree of *ambiguity aversion*, placing a premium on outcomes for which probabilities are known. This general stylized fact has been replicated broadly and has important implications for the economics of optimal contracting, investment choices, and mechanism design.

One possible way to structurally identify ambiguity aversion is to assume that the latter influences people's degree of risk aversion (more precisely, the curvature of the utility function), an approach followed, among others, by Klibanoff et al. [3] and Andersen et al. [4].

in the experiment of Albarrán et al [5], from which our data are from, prizes in the lotteries are distributed according to the rule $0 < y < x < 15$. In what follows, this prize domain is normalized, for the sake of simplicity, to lay within the unit interval $[\,0, 1\,]$, where \$0 is

0 and $\$15$ is 1. The treatment conditions -randomized between subjects- regard the amount of information given to them about X and Y. Furthermore, while in the full information treatment, TR2, people face a normal risky situation and there is no ambiguity influencing their decision, this is not the case for the partial information and no information treatments TR1 and TR0, respectively. As we shall see, some ambiguity preference appears from subjects' choices which is higher the less information is received.

1.2 Econometric strategy

In what follows we shall layout the identification assumptions underlying our structural estimations. Specifically, we need to define our identification strategy with respect to $i)$ subjects' risk attitudes and how the uncertain payoffs, X and Y, enter in subjects' calculations together with $ii)$ the behavioral model underlying subjects' optimization program. Regarding the former, as it will be explained in Section 1.2.1, we shall impose that subjects maximize a VNM CRRA utility function in all treatments and that, consistently with the TR1 experimental instructions, Y and X are calculated as first and second order statistics of a uniform distribution defined over the unit interval. Regarding the latter, that is explained in Section 1.2.2, we shall consider two alternative behavioral models, defined as *naive* and *sophisticated*. In the former, subjects are assumed to estimate first the uncertain payoffs and then use these expected payoffs in the expected maximization program; in the latter -consistently with a genuine bayesian approach- the order of integration is reversed.

1.2.1 Uncertain Payoffs and Risk Aversion

We read the experimental evidence by the way of some structural estimation exercises in which we elicit the individuals' degree of risk aversion within the frame of a standard constant relative risk aversion (CRRA) utility function, which generally performs better in more complex structural estimations.

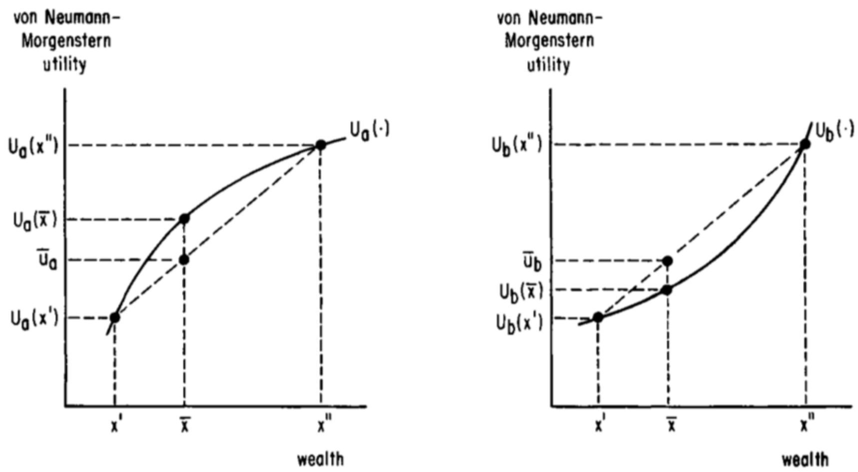
The utility function is given below:

$$u(\chi) = \begin{cases} \frac{\chi^{1-\rho}}{1-\rho} \text{ if } \rho \neq 1 \\ ln(\chi) \text{ if } \rho = 1 \end{cases} \tag{1.1}$$

where ρ is the (CRRA) coefficient which does not depend on χ, as formalized by Pratt [**?**]. As for its economic interpretation, $\rho > 0$ represents risk aversion, $\rho = 0$ risk neutrality and $\rho < 0$ risk loving.

In Figure 1.1[1], examples of $u(\chi)$ are presented for different values of ρ: concave in case of risk aversion (left) and convex in case of risk loving (right).

Figure 1.1: Risk Aversion Coefficient for both the Naive and Bayesian approaches



In Section 1.3.1 we check whether the change of information implemented by our treatments conditions generates a change in behavior and, in turn, (whether and) how this change is converted into a significant change in the measured (CRRA) attitude toward risk.

The uncertain payoffs Y and X are identified as the first and the second order statistics from a uniform distribution in $[\,0, 1\,]$, where the order statistics of a random sample $\chi_1, ..., \chi_n$ are

[1]Machina [6]

defined as the sample values placed in ascending order.

Specifically, let $f_k(n,\ z)$ denote the k_{th} order statistics of n draws, where $n = 2$ in our case, of a density function $f(\cdot)$. Let

$$X \sim f_2(2,\ z), \quad Y \sim f_1(2,\ z)$$

where

$$f_2(2,\ z) = 2\ z\ f(z)\ F(z), \quad f_1(2,\ z) = 2\ z\ f(z)\ (1 - F(z))$$

come from the general formula for the the k_{th} order statistics of n draws

$$n \binom{n-1}{k-1} f(z)\ (F(z))^{n-1}(1 - F(z))^{n-k}$$

being z a random draw from a uniform distribution and being

$$f(z) = 1 \text{ is the p.d.f. of } z, \quad F(z) = z \text{ is the c.d.f of } z.$$

While this is certainly true for TR1 subjects -since they know the characteristics of the random generation process that yields the uncertain payoffs- we impose the same statistical model for subjects in TR0, assuming they had this information. As we said, this is purely an identification assumption, as there is no possibility to test whether this is the true for expectations in TR0 about X and Y distribution, or whether subjects in TR0 consider a different type of distribution. On the other hand, it is highly probable that TR0 subjects will heuristically and automatically assume such a distribution of the payoffs, as it occurs in Laplace's well known "principle of insufficient reason". In any case, what is important here is that -thanks to this assumption- our structural model is able to estimate treatment effects, to such an extent that we are able test a null hypothesis in which CRRA in both TR0 and TR1 is the same. Since subjects have been randomized within treatments, a significant change in the CRRA coefficient between TR0 and TR1 has to be interpreted as a genuine treatment effect due to a change in information.

In the maximum likelihood function routine, ρ is analyzed through the individual choices subjects make, which are expressed in function of the treatments, to identify how a different level of information influences people's risk attitude.

Phase 1 observations are treated as a series of individual and independent choices between a certain outcome and a risky lottery, whose expected value is computed and compared to the sure prize.

Instead, phase 2 data are used as a sequence of binary choices between lotteries. TR2 players know the true X and Y, so their ρ derived from a situation with no ambiguity. On the other hand, TR0 and TR1 players compute the lotteries expected values and variances, as function of the X and Y they figure out, and then the U_i and the ΔU.

A logit function is used to solve the usual binary choice model, explaining the $P(k = 1) = P(\Delta U > 0)$ which is :

$$P(k = 1) = \frac{e^{\Delta U}}{1 + e^{\Delta U}} \quad \text{if } L_1 \text{ is chosen}$$

$$P(k = 1) = \frac{e^{-\Delta U}}{1 + e^{-\Delta U}} \quad \text{if } L_0 \text{ is chosen.}$$

The Fechner model is used, where people heterogeneity is expressed as function of a random error in the CRRA utility computation, i.e. $\epsilon \sim N(0, \sigma^2)$. In the whole of estimates we cluster all the observations made by the decisions of the same individual.

1.2.2 Identification of the Behavioral Model

Regarding the behavioral content of the structural model for the uncertain payoffs, we consider two alternative specifications, labelled as "naive" and "sophisticated", respectively. A naive decision maker figures out a point estimation of the unknown payoffs X and Y, starting with the information that they are draws from a uniform distribution in $[\,0, 1\,]$. This means that the $E[X]$ and the $E[Y]$ are computed first and then plugged into the CRRA expected utility function to be maximized. Specifically:

$$E[\,X\,] = \int_0^1 f_2(2,\ z)\, dz = \frac{2}{3}$$

$$E[\,Y\,] = \int_0^1 f_1(2,\ z)\, dz = \frac{1}{3}$$

where f_k is the k-th order statistics of a uniform distribution in $[\,0, 1\,]$.

Finally, the utility of a lottery k is:

$$U(L_k) = u(E[\,Y\,])\, p_y^k + u(E[\,X\,])\, p_x^k + p_1^k$$

given that the first price is 0, so $u(0) = 0$, and the last price is 1, so $u(1) = 1$.
A "sophisticated" decision maker, instead, will proceed based on a true bayesian updating, forming a prior distribution over the X and the Y, and then calculate the expected utility from these densities. Specifically:

$$U(X) = \int_0^1 u(z) f_2(2,\ z)\, dz;$$

$$U(Y) = \int_0^1 u(z) f_1(2,\ z)\, dz,$$

where $f_1(\cdot)$ and $f_2(\cdot)$ are the first and second order statistics of a uniform distribution in $[\,0, 1\,]$.
Finally, the utility of L_k, $U(L_k)$ equals to

$$U(L_k) = U(X)\, p_x^k + U(Y)\, p_y^k + p_1^k$$

given the first price is 0, so $u(0) = 0$ and the last price is 1, so $u(1) = 1$.
In conclusion, the two models differ due to the order of integration.

1.3 Results

The "atom" of our analysis is the decision made by subjects and our research question is how their ρ varies as function of the amount of information they receive, depending on their treatments, and how this process differs in the two distinct approaches, the naive and the bayesian one. We also query whether one model is more used than the other.

1.3.1 Treatment effects

Figure 1.2 reports the result of the structural estimation of the ρ as function of the different treatments, for both the two approaches.
Our empirical evidence shows a certain degree of *love for ambiguity*, as the less information given to the subjects, the lower their risk aversion, and, consequently, the bigger their attraction toward risk and uncertainty. Moreover, the risk aversion coefficient estimated for TR0 is significantly lower than that estimated in TR2, although there is no statistically significant

difference between estimated CRRA coefficients in TR0 and TR1. These findings are -somewhat- in contradiction with the common wisdom of the literature, although they are consistent with other experimental literature that applies similar elicitation techniques as ours, such as Andersen et al [4].

When comparing our two behavioral models, as shown in Table 1.1, the estimated likelihood of the naive approach is higher than that of the bayesian. This suggests that, based on our data, the naive approach approximates better subjects' decision rules.

Afterwards, we would like to identify the percentage of the subjects using each of the two models, i.e. the probability of them behaving either in a naive or a bayesian way.

Table 1.1: Risk Aversion Coefficient for both the Naive and Bayesian approaches

VARIABLES	Naive	Bayesian
TR_0	-0.0360***	-0.0452***
	(0.00953)	(0.0102)
TR_1	-0.0187*	-0.0254**
	(0.00973)	(0.0102)
Constant	0.774***	0.774***
	(0.00557)	(0.00557)
TR_1 – TR_0	-0.01723	-0.0198
	(.01111)	(0.0120)
Likelihood	-92261.238	-92604.987
Obs.	195,459	195,459

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

1.3.2 Naive or Sophisticated?

Up to now, we identified two different approaches individuals may follow to make their choices. The following step is to implement a mixture model to identify the probability of each observation being compatible with either model.

We use a binary mixture model, since a finite number of types, the naive and the bayesian, are assumed.[2]

The main advantage of this approach that the assumption of different subjects operating according to a single model is avoided. The behavior of a typical subject is often traced back to the average behavior, but it is quite possible this is not an accurate representation of every subject under study.

A possible answer to this issue could be the Average Treatment Effect, ATE, where a specific treatment effect is recognized to each individual. All subjects specific treatment effects are then assumed to vary randomly around an average, the ATE, i.e. the parameter being estimated.

If the distribution is bell-shaped and symmetric, the ATE will provide a sensible measure of the affect of the treatment. In other words, the ATE measure is relevant when the treatment has universal applicability so that it is reasonable to consider the hypothetical gain from treatment to a randomly selected member of the population.

However, this is not always the case, and this ATE can end up being far away form the actual treatment effect of any single subject.

The approach adopted by a finite mixture model is presented below. A total number of types in the population is decided, and a specific behavioral model is assigned to each of them. The parameters of these various models are estimated altogether, along with the mixing proportions.

In particular, we generate the probability π_{BAY}, namely the probability of our subjects acting as bayesian in each of their decisions.

We tried to estimate the ρ_{NAI} and ρ_{BAY}, i.e. the risk aversion coefficients for both the approaches, and π_{BAY} altogheter, but the likelihood function did not converge. Indeed,

[2]In case of an "infinite" mixture model, a continuous variation in some parameters indexing individual type is assumed, as happens for random coefficient models or random effect models

the likelihood functions of our models are very close. For this reason, we estimated this probability numerically, using a grid loop.
Subsequently we let π_{BAY} moving inside a grid $(0, 1)$, to finally choose the value that maximizes the likelihood function.
A possible drawback of this numerical procedure is the fact that the π_{BAY} standard error cannot be estimated, as it is shown in Figure 1.3. On the other hand, we can justify this statement by saying that our likelihood function is not function of it, given that it is just a product of probability.
This numerical computation demonstrates that our subjects have a strong tendency to behave as naive, given the estimation result of $\pi_{BAY} = 0.2$.

Figure 1.2: Mixture Model with $\pi_{BAY} = 0.2$

VARIABLES	
Rho_NAI	0.764***
	(0.00398)
Rho_BAY	0.637***
	(0.0109)
Pi_BAY	0.2
	-
Obs.	195,459

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Bibliography

[1] Leonard J Savage. *The foundations of statistics*. Courier Corporation, 1972.

[2] Daniel Ellsberg. Risk, ambiguity, and the savage axioms. *The quarterly journal of economics*, pages 643–669, 1961.

[3] Peter Klibanoff, Massimo Marinacci, and Sujoy Mukerji. A smooth model of decision making under ambiguity. *Econometrica*, 73(6):1849–1892, 2005.

[4] Steffen Andersen, John Fountain, Glenn Harrison, E Elisabet Rutström, et al. Estimating aversion to uncertainty. *Report.[233]*, 2009.

[5] Giovanni Ponti, Carmen Herrero, and Judit Albarràn, Pedro aan Alonso. An experiment on outcome uncertainty. *Departamento de Fundamentos del Analisis Economico - Univesidad de Alicante, Working Paper*, 2018.

[6] Mark J Machina. Choice under uncertainty: Problems solved and unsolved. *Journal of Economic Perspectives*, 1(1):121–154, 1987.