



Department of Economics and Finance

Chair of Theory of Finance

Investment strategies: comparable portfolios constructed using Return on Equity and Market-to-Book factors.

SUPERVISOR

Prof. Nicola Borri

CANDIDATE

Davide Silvio Loreti

Student Reg. No. 682941

CO-SUPERVISOR

Prof. Pierpaolo Benigno

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Introduction

Empirical researches tried to assess whether it is possible to use fundamental factors in order to derive portfolios, trying to understand if they may be related to eventual common risk factors in asset pricing. The following empirical study seeks to further analyze this topic, using the market-to-book and the return on equity as two distinct tools in portfolio construction. The aim is to compare the characteristics of the derived portfolios and assess their impact in asset pricing, looking to similarities and differences between the two approaches.

The first chapter reviews the literature behind investment strategies and the common risk factors that have been derived from those portfolios. At first, the difference between value and growth investing and their principal screens for stocks selection is properly described, showing the different profitability of the constructed portfolios. Once the performance assessment has been concluded, the focus is shifted on deeply understanding how long/short portfolios based on value and growth investing may explain stock returns in asset pricing. The chapter briefly describes the value and size risk factors, with particular focus on understanding how they have been derived and their economic interpretation as additional sources of risk with respect the capital asset pricing model (CAPM).

The second chapter seeks to describe the data and the methodologies of the following research. Indeed, it explains how returns, market-to-book ratios and return on equity indices have been retrieved and computed. It is described the theoretical and statistical relation between the price to book value ratio and the return on equity of each stock, using correlation functions and regressions. Successively, it is explained how the two factors, singularly, have been used for portfolio construction and how their performances have been assessed. Finally, for each factor it is derived a long/short portfolio which is the one considered as risk factor in asset pricing.

The third and last chapter analyzes all the characteristics of the derived portfolios, assessing eventual similarities and differences between the two different single sorting approaches. The four principal moments (mean, standard deviation, skewness and kurtosis) of each portfolio are examined, looking to the presence of similar patterns between portfolios constructed with the market-to-book ratio and those with the return on equity. Considering the third and fourth moments, it is tested if returns are normal distributed or not, through the implementation of the Jarque-Bera test. Successively, it is reported and analyzed the value at risk, assessing the portfolios riskiness, focusing on understanding if the two factors generate portfolios with similar level of risk. Once the principal statistical characteristics have been scrutinized, the focus is shifted on an exercise of asset pricing. Indeed, the long/short portfolios linked to the two different

investment strategies, one based on PBV and the other one on ROE, have been used in order to assess if they explain stock returns and if they can be considered as similar risk factors linked to the same economic meaning. Indeed, two single factor model are applied, focusing on understanding the firm sensitivity to changes in the two common factors. The last analysis uses correlation functions and regressions in order to assess the relation between the portfolios created with the price to book value ratio and the respective portfolios created with the return on equity, trying to understand if they tend to move towards the same directions.

Chapter 1: Literature review

This first chapter is based on a quick review of the literature behind different investment strategies and the potential risk factors linked to that portfolios.

It is possible to observe two main sections within this chapter.

The first one seeks to describe the two main philosophies behind the investment strategies based on fundamental analysis. Specifically, the focus is on explaining the differences between value and growth investing with particular attention on describing the principle screens implemented for stock selection.

The second section provides empirical evidences of how some of the described screens may be used in asset pricing. Focusing on the famous three factors model. The value and size risks are described and illustrated with emphasis on their economic meanings.

1.1. Investment strategies

Investment strategies can be mainly divided into two main categories, fundamental and technical analysis.

The two approaches are implemented in order to select stocks that must be included in the portfolios.

Technical analysis relies on the assumptions that markets are not informationally efficient. More precisely, it is possible to exploit past transactions and historical prices in order to generate abnormal returns.

The investment strategies, based on technical analysis, try to spot eventual trends that can be used without investigating about the reason behind that trend.

Fundamental analysis, on the other hand, relies on the assumptions that markets are efficient, but they may fail in the pricing process for some stocks. For this reason, fundamental analysis tries to assess and derive the intrinsic value of a firm, analysing the fundamentals of individual stocks, and comparing it with its market value.

This approach is based on the idea of trying to select those firms that may be undervalued or overvalued by the market, therefore, if the spread between the intrinsic value and the market value is wide enough, that stock will be included in the portfolio.

Fundamental analysis can be divided into two main philosophies, value and growth investing.

The following sections will briefly introduce the two philosophies, with a particular focus on describing which are the principle screens used in order to identify and select value or growth stocks.

1.1.1. Value investing

Value investing is based on the identification of stocks which seem to be undervalued taking a long position with respect them.

This investment philosophy relies on the implementation of some quantitative and qualitative screens in order to identify undervalued stocks which can be included in the portfolio. This approach uses evaluation model in order to define the firm's intrinsic value and then compare it with its market value. If they differ by a wide margin, investors would include that stock in the portfolio waiting for an eventual reduction of the gap.

Fama and French (1998) defined value stocks those firms characterized by high book-to-market, earning-to-price or cash flow-to-price ratios. These indicators are the principle quantitative screens that can be implemented in order to identify undervalued firms and, therefore, value stocks.

Fama and French (1992, 1996) and Lakonishok, Shleifer and Vishny (1994) conducted empirical analysis testing the profitability of value stocks in the American market. Specifically, it has been proved that those stocks characterized by high book-to-market, earning-to-price or cash flow-to-price have higher average returns than those with low book-to-market, earning-to-price or cash flow-to price.

However, value stocks tend to be characterized by low earnings, in such a way the premium paid by those firms is associated with distress.¹ This premium is related to the market that is not able to correctly price distressed companies. For this reason, they are undervalued, but as long as the market is able to correct its pricing process and, moreover, those firms are capable to recover from the distress, above average returns are achieved by investing in those stocks.

In the next section of this chapter will be discussed the concept that this value premium is a compensation for bearing a risk that is not included in the capital asset pricing model (CAPM).²

So far it has been illustrated which are the principal screens that can be implemented in order to identify value stocks. Among all the cited ratios, the more important and the more frequent factor used by investment managers in portfolio construction is the book-to-market.

This ratio is used in order to divide stocks in different categories for which it is possible to identify different portfolios. Taking into consideration the portfolio containing stocks with high book-to-market ratios, it is associated with a value investing approach. It has a superior performance which can be explained by the

1 Fama, E. F., & French, K. R. (1995). Size and book-to-market factors in earnings and returns. *The journal of finance*, 50(1), 131-155.

2 Fama, E. F., & French, K. R. (1993). Common risk factors in the returns on stocks and bonds. *Journal of financial economics*, 33(1), 3-56

fact that it contains stocks with an earnings growth which is higher than expected, since they can be defined as distressed companies.³

Contrarian investors can be defined as value investors which seek to identify undervalued or overvalued stocks with a different screen with respect those previously named.

They rely on the concept that markets are inefficient and tend to overreact to good and bad news, affecting one or more firms. For this reason, contrarian investors are related to a simple investment strategy, purchase recent losers and sell recent winners. They believe that recent losers suffered a loss greater than expected and recent winners had a gain that exceed their intrinsic value. Therefore, it is reasonable to expect that the market will adjust over time reducing the profit of winners and reducing the loss of losers.

Thaler (1987) proved that investing in a portfolio containing US stocks with worst past performance strongly outperforms a portfolio containing US stocks with best past performance from 1933 and 1978.

Further evidence about the profitability of contrarian investment strategy is provided by Clayman(1994).

He proved that un-excellent stocks outperform excellent stocks in the period between 1976 and 1980.

1.1.2. Growth investing

Growth investors follow an investment strategy which is exactly the opposite of value investors. Indeed, they seek to identify those stocks that are characterized by higher earnings growth with respect other firms. These companies can be defined as young with large investments in innovation that are traded at multiple times their fundamentals since they have elevated growth prospects.

This section is focused on understanding the screens that can be implemented in order to identify growth stocks and the empirical evidences of their performance.

Fama and French (1998) defined growth stocks those firms characterized by low book-to-market, earning-to-price or cash flow-to-price ratios. They are characterized by persistently high earnings.⁴

However, several empirical evidences, illustrated in the value investing section, show that stocks with high book-to-market, earning-to-price or cash flow-to-price outperform those with low book-to-market, earning-to-price or cash flow-to-price ratios.⁵

Therefore, it is established that, as long as growth investors use the above three ratios as screens, they will end up with a portfolio which is less profitable than the one constructed with a value investing approach.

3 Fama, E. F., & French, K. R. (1995). Size and book-to-market factors in earnings and returns. *The journal of finance*, 50(1), 131-155.

4 Fama, E. F., & French, K. R. (1995). Size and book-to-market factors in earnings and returns. *The journal of finance*, 50(1), 131-155.

5 Fama, E. F., & French, K. R. (1998). Value versus growth: The international evidence. *The journal of finance*, 53(6), 1975-1999.

This less profitability is mainly due to the fact that stocks with low book-to-market, earning-to-price or cash flow-to-price ratios are linked to future earnings growth which is smaller than expected.⁶

Another screen that can be used in order to define and select growth stocks, is the size. Stocks are divided into different categories in terms of market capitalization with a particular focus on the first quantile which contains stocks with low market-cap.

Therefore, those firms can be defined as growth stocks. Indeed, young companies with high innovation and high potential growth, most of the times are small and, therefore, they can be identified within the small cap category.

Even if it is reasonable to expect a correct pricing process from the market in the assessment of small firms, empirical evidences demonstrated that small cap stocks tend to outperform big cap stocks over time.

Fama and French (1992) analysed the profitability of portfolios constructed with respect their size in the period from 1963 to 1990. They formed 12 portfolios sorted from the one containing stocks with the smallest market capitalization to the one containing firms with the biggest size. Considering the difference in average returns between the bottom and the top 2 portfolios, it is proved that small-cap stocks systematically outperform big-cap stocks.

The reason behind such result may be related to the fact that stocks characterized by a small market capitalization are subject to higher transaction costs, less disclosure obligation and moreover they can be defined as less liquid investments.

A further explanation of the difference in returns between these two categories of stocks can be due to the distinction in their beta exposures. However, even considering risk-adjusted returns, small-cap stocks still outperform big-cap stocks.

In the next sections will be deeply analysed the size risk factor. Specifically, it consists in a portfolio long stocks with small market capitalization and short stocks with big market capitalization, which is not included in the capital asset pricing model (CAPM) as common factor.⁷

Growth investors may implement other strategies in order to select growth stocks. In particular, initial public offerings (IPO), primary IPO, consists in private firms which decide to become publicly traded. It is possible to define such companies as potential growth investments.

Firms that are issuing new shares usually are young with high potential growth. Therefore, including such stocks in a portfolio can be considered as value investing.

⁶ Fama, E. F., & French, K. R. (1995). Size and book-to-market factors in earnings and returns. *The journal of finance*, 50(1), 131-155.

⁷ Fama, E. F., & French, K. R. (1993). Common risk factors in the returns on stocks and bonds. *Journal of financial economics*, 33(1), 3-56.

However, it is clear that the above average returns generated by investing in an IPO is mainly due to failure in the pricing process. Indeed, empirical evidence shows that the average under-pricing in US, from 1990 to 1994, was 16% for small issuing and 7% for large issuing.⁸

Even if this growth investment strategy seems to be profitable, it is necessary to assess whether such profitability is for the long or the short run.

Loughran and Ritter (1995) studied several IPOs comparing the average returns with respect different time horizon. They showed that investing in an IPO for more than one year the average return is less than 4% which is significantly lower than investing in non-issues.

1.2. Common risk factors

Previously the focus was on understanding which are the differences between two different investment strategy philosophies, value and growth investing, understanding the main screens used in order identify value or growth stocks.

In particular, the previous sections reported two main factors, value and size, that are linked to sources of risk. For this reason, empirical researches tried to understand the impact that those risk factors have on asset pricing in addition to the value premium derived from the market risk.

The capital asset pricing model (CAPM) is constructed in such a way the only source of risk that has been considered is the market risk. Indeed, alongside problems linked to the identification of a proxy for the market portfolio, the model suffers of another drawback. It is not able to consider through the market portfolio all the potential risk that explains the value premium in stocks returns.

For this reason, arbitrage pricing theory (APT) is focused on implementing single factor model or multifactor models in such a way it is possible to identify potential risk factors that may have an effect on asset pricing.⁹

Obviously, among all the tested multifactor model, particular importance must be given to the famous three factors model described by Fama and French (1996).

They identified three common factors for stocks. One is obviously related to the overall market risk linked to the market portfolio. The other two, on the other hand, are linked to the firm size and book-to-market.

8 Lee, I., Lochhead, S., Ritter, J., & Zhao, Q. (1996). The costs of raising capital. *Journal of Financial Research*, 19(1), 59-74.

9 Roll, R., & Ross, S. A. (1980). An empirical investigation of the arbitrage pricing theory. *The Journal of Finance*, 35(5), 1073-1103.

These two additional risk factors are derived from the two investment strategies previously described. Indeed, size is linked to growth investing since investors select stocks with respect their market capitalization, the other one is linked to value investing where stocks are picked through their book-to-market ratios.

The factors, that will be properly described in the successive sections, are able to explain the average stocks return and it has been shown their strong connection with economic fundamentals.¹⁰

Adding size and book-to market factors to the market risk it has been demonstrated that it is obtained a model which is able to explain a large portion of stock returns variation proved by its elevated r-squared values.¹¹

1.2.1. Value risk factor

As announced previously, the value risk factor is derived through the book-to-market ratio and it is considered in asset pricing since empirical evidences showed that portfolios containing stocks with high book-to-market ratios systematically outperform those containing stocks with low book-to-market ratios.

The risk factor has been created taking a long and a short position with respect portfolios created with a single sorting approach. Indeed, Fama and French (1993) divided the stocks in different groups, based on book-to-market, for which it is possible to observe two main portfolios. The one containing stocks with high book-to-market ratios and the one containing stocks with low book-to-market ratios.

The difference in monthly returns between the two portfolios consists in a long/short portfolio which is considered as common risk factor. It is called high-minus-low (HML).

They showed that the HML factor significantly captures common variation in stock returns that is not contemplated by the other two sources of risk, market and size.

This source of risk is linked to the fact that high book-to-market stocks tend to outperform low book-to-market stocks. It has been shown that those firms characterized by high book-to-market ratios have a strong negative impact on their prices due to financial distress. However, this above average returns are given by their ability to recover from the distressed scenario.¹²

The risk factor is not linked to the firm-specific risk, that can be diversified away, but is linked to market scenarios such as credit or liquidity crunch that systematically affect the market.

10 Fama, E. F., & French, K. R. (1992). The cross-section of expected stock returns. *the Journal of Finance*, 47(2), 427-465.

11 Fama, E. F., & French, K. R. (1993). Common risk factors in the returns on stocks and bonds. *Journal of financial economics*, 33(1), 3-56.

12 Fama, E. F., & French, K. R. (1995). Size and book-to-market factors in earnings and returns. *The journal of finance*, 50(1), 131-155.

Liew and Vassalou (1999) showed that there exists an economic connection between the LMH risk factor and the Gross Domestic Product (GDP) growth. They proved, through the implementation of a univariate regression between GDP growth and LMH, that there is a positive and statistically significant relation between the future economic growth and the HML performance, proved by a slope coefficient statistically greater than zero.

This result means that the book-to-market factor foretells the future GDP growth. This has been verified for ten developed markets, such as Australia, Italy, Germany and US.

Taking into consideration the Italian market, dividing the economic states in good or bad periods, they prove that the performance of the HML portfolio is 17.05 percentage points higher in the good state scenario rather than in bad state in the period between 1987 and 1996.

1.2.2. Size risk factor

As it was introduced, other risk factors have been considered in the asset pricing theory. The previous section explained how the high-minus-low (HML) common factor has been derived and which are the economical explanations of such source of risk. However, particular importance must be given to the firm size.

In the growth investing section, empirical evidences have been reported showing that small-cap stocks tend to systematically outperform big-cap stock.¹³

It has been exposed how stocks characterized by low market capitalization are subject to higher costs and different risks that may explain the higher premium paid by them. In particular, it must be considered that those firms have higher transaction costs, they suffer from liquidity risk and they are subject to less disclosure obligations. However, those are not the only reason of the higher premium paid by the market. Further analysis must be implemented in order to find out whether such performance is linked to an economic meaning and, therefore, if it is possible to create an addition risk factor for multifactor models in asset pricing.

Fama and French (1993) constructed portfolios containing stocks with different market capitalization. Specifically, these portfolios have been sorted from the one with the lowest market-cap stocks until the one with the highest market-cap stocks. As said, the one with low market-cap stocks systematically outperform those with high market-cap stock.

¹³ Fama, E. F., & French, K. R. (1992). The cross-section of expected stock returns. *the Journal of Finance*, 47(2), 427-465.

They decided to construct an additional risk factor. It consists in a portfolio based on a long position with respect the portfolio containing small-cap stocks and a short position on big-cap stock. This common factor is named small-minus-big (SMB).¹⁴

It has been proved that, implementing the three factors model, SMB factor significantly captures common variation in stock returns that is not considered by the other two sources of risk, market and value (HML). The regression model implemented by Fama and French (1996) showed that the estimated slope coefficients linked to the small-minus-big risk factor are positive, large and significative.

Therefore, it has been proved that SMB, alongside market risk and HML, can be properly used in the three factors model. Indeed, the three common factors are able to explain a large portion of total stock returns variation since the high r-squared values obtained by the model.

Liew and Vassalou (1999) proved, alongside for the HML risk factor, that there is a strong economic relation between SMB and the GDP growth.

In particular, implementing regression analysis they were able to demonstrate that there is positive relation between the returns of the small-minus-big portfolio and the future economic growth. Indeed, the risk factor is able to foretell future business cycle.

The analysis has been implemented in such a way the economic status has been divided between good and bad states and portfolio returns have been computed in relation to the two business cycles. It has been showed that, for the 10 developed markets included in the study, there is a statistical positive relation between the SMB returns in the good states and the bad states.

Looking to the Italian market, in the period from 1987 to 1996, they proved that the performance of the SMB portfolio is 29,01 percentage points higher for the good states rather than the bad states which presents negative returns.

¹⁴ Fama, E. F., & French, K. R. (1996). Multifactor explanations of asset pricing anomalies. *The journal of finance*, 51(1), 55-84.

Chapter 2: Data explanation and methodologies

The principal aim of this chapter is to define and explain all the data necessary to conduct the empirical study of interest. In particular, focusing on how returns are computed and how the ratios taken into consideration are derived.

For each data collected and presented there is, between brackets, the function that must be typed on Bloomberg in order to find that specific value that has been considered in the research.

The idea is to explain the reason why simple returns are considered and how the price to book value ratio (PBV) and the Return on Equity (ROE) have been evaluated.

After the description of each specific data the aim of the successive section is to define the relation between the price book-value ratio and the return on equity. It is explained first from a theoretical point of view and then applying statistical methods. In particular, at first, it is implemented a simple linear regression where the dependent variable (Y) is the price to book value ratio and the independent one is the return on equity. The second way in which such relation is tested is through the use of the simple correlation function present on MATLAB.

The last section of this chapter aims to describe the way in which the portfolios have been constructed and how their performances have been evaluated, using the market-to-book ratio and the return on equity as two different tools in portfolio construction.

2.1. Data

Data have been retrieved from Bloomberg and the period taken into consideration is from January 2001 to January 2018.

Thirty Italian stocks, included within the FTSE MIB 40 index, are considered in the research.

The Financial Times Stock Exchange Milano Indice di Borsa (FTSE MIB) is the benchmark stock market index for the Borsa Italiana, for which it is possible to identify the London Stock Exchange Group as the parent company of Borsa Italiana.

The Index, which has replaced the MIB-30 in 2004, consists on the forty most traded stocks on the exchange.

Those stocks for which a long past history is not available have not been considered. Indeed, for example, data about Ferrari S.P.A. has not been retrieved, since the IPO was really recent, January 2016.

The Data necessary for this research are the prices, the price-book value ratio and the Return-on-Equity of each stock.

It has been considered a monthly frequency in such a way the portfolios are rebalanced every month based on a single sorting approach implemented once with PBV and once with ROE.

The following table shows the stocks considered for the portfolio construction from the FTSE MIB 40 market index.

A2a	Atlantia	Azimut	Banca Generali	Banco Bpm
Bper Banca	Brembo	Buzzi Unicem	Campari	Enel
Eni	Exor	Generali	Intesa Sanpaolo	Leonardo
Luxottica	Mediaset	Mediobanca	Prysmian	Recordati
Saipem	Snam	Stmicroelectronics	Telecom Italia	Tenaris
Terna rete elettrica nazionale	Ubi Banca	Unicredit	Unipol	Unipolsai

Table 1: Stocks selected from the FTSE MIB index that are considered for the empirical study

2.1.1. Asset Returns

For the computation of assets returns, stock prices have been considered for each asset, in particular it has been retrieved, from Bloomberg, the monthly last price (PX_LAST) from January 2001 to January 2018 when the whole sample was available.

Within the sample there are some stocks for which the number of observations is smaller, therefore, it implies fewer stocks in the portfolios in early years.

The empirical study takes into consideration the creation of portfolios, for this reason it is necessary to aggregate returns across assets and the use of simple returns instead of log-returns is more suitable.

Log-return makes easier the computation of returns across time since its property of additivity over time.

$$r_{i,t+1} = \log(P_{i,t+1}) - \log(P_{i,t})$$

Where: $\log$ is the natural logarithm;

$P_{i,t}$ is the price of the security i at time t ;

$r_{i,t+1}$ is the log return.

In this case we can compute the return from period t to the period $t + n$, with $n > 0$ as follows.

$$r_{i,t:t+n} = \sum_{i=0}^n r_{i,t+i} = \log(P_{i,t+n}) - \log(P_{i,t})$$

Even though this property seems to be really useful, log-return makes difficult the computation across assets.

Indeed, as said before, it has been considered the simple returns defined as follows.

$$R_{i,t+1} = \frac{P_{i,t+1}}{P_{i,t}} - 1$$

Given the use of simple returns it is possible to compute the return of a portfolio p at time t , composed by N stocks, taking the simple or weighted average of assets return.

$$R_{p,t} = \frac{1}{N} \sum_{i=1}^N R_{i,t}$$

if the portfolio is equally weighted.

$$r_{p,t} = \sum_{i=1}^N w_i r_{i,t}$$

if each asset has a different weight w_i

In the case of a portfolio constructed using an equally weighted approach, each stock has the same importance within the portfolio construction. On the other hand, looking to the weighted average it is reasonable to expect a criterion such that a certain kind of stocks have a greater relevance with respect all the other. A typical example can be the market capitalization-weighted portfolio. Indeed, using this approach, the stocks with the higher market capitalization are characterized by a greater relevance than small-cap stocks in the portfolio.

2.1.2. Price to Book value ratio

The price book value ratio is a really common ratio used in finance.

It is fundamental since it puts into relation the market value of a specific stock with its book value of equity. It is computed simply dividing the stock price taken into consideration with the book value of equity per share.

The use of such ratio is largely applied in Finance, in particular for the assessment and identification of undervalued or overvalued publicly traded firms.

Generally, an enterprise can be defined as undervalued if it is traded at a price to book value ratio smaller than one. However, this is not enough to guaranty that the price will adjust to its long-term intrinsic value, generating above average returns.

The concept of undervalued firms relies on the existence of some market inefficiencies for which it is not able to correctly price stocks. Applying evaluation methodology, such as discount cash flow approach (DCF), it is possible to determine the theoretical long-term intrinsic value of the firm. Arbitrageur expects that the price will adjust to its fair value in the long run.

In the following empirical study, the price-book value ratio has been collected with a monthly frequency, on the other hand, the book value per share is considered at a semi-annual frequency.

In particular, as long as it is necessary to have consistent data, the price used for the computation of the ratio is the last price (PX_LAST), that is the same considered for the asset returns computation.

At the denominator there is the book value per share (RR020), with semi-annual frequency, which is computed as the firm's book value (RR010) divided by the number of outstanding shares.

$$\frac{P}{BV} = \frac{\text{Last Price (PX_LAST)}}{\text{Book Value per share (RR020)}}$$

$$\text{Book Value per share (RR020)} = \frac{\text{Book Value (RR010)}}{\text{Number of outstanding shares}}$$

The Book Value (RR010) precisely is the average book value that it has been computed on a semi-annual basis. The book value, in a simplify manner, can be seen as the amount that shareholders would receive if all the liabilities of the company are subtracted from its assets.

In the following study, the described ratio is used as parameter for the construction of portfolios which are sorted with respect different market-to-book values.

2.1.3. Return on Equity

The last index that must be taken into consideration for the research is the Return on Equity.

It is a measure of firm's profitability for a certain period. Specifically, it indicates the rate of return earned on the capital provided by shareholders after paying for all the other capital used.

Indeed, in order to compute the Return on Equity, it is needed the Net Income.

It is computed by taking revenues and subtracting the overall business costs including depreciation, interest expenses or income, taxes and other expenses.

Therefore, the ROE is computed as the Net Income available for common shareholders (T0089) divided by the average total common Equity on a semi-annual frequency (average RR010).

In this way we are considering the same Equity value used for the Price to book value ratio.

However, particular focus was given to the frequency of such index. The choice was between annual, semi-annual and quarterly frequency, which are related to the disclosure periods of corporate informations.

However, it has been considered the semi-annual one since the disclosure of periodicals statements is more complete on an annual or semi-annual frequency, due to regulatory obligations.

The use of such index is the same of the price to book value ratio, indeed, it is necessary for the construction of single sorted portfolios. For this reason, a shorter frequency would have been more suitable for a more frequent portfolio rebalance.

Data about the ROE do not have monthly frequency, therefore, in order to overcome such problem, the Return on Equity collected on a specific month (December or June) is the same for the successive five months, in particular the ROE relatives to December is the same until May and the ROE of June is the same until November.

$$ROE = \frac{Net\ Income\ (T0089)}{Average\ total\ Common\ Equity\ (Average\ RR010)}$$

However, even if the portfolio is constructed on a monthly frequency, the rebalance occur every six months, as long as the new ROE is published by the company.

2.2. Relation between PBV ratio and ROE

Before looking to the statistical correlation between the price to book value ratio and the return on equity for the stocks included in the study, it is necessary to describe the theoretical relation among them.

First of all, it must be considered that it is implemented a discount cash flows (DCF) model in order to determine the stock price.

The DCF approach relies on the fact that the price, or the fair value, of a stock can be assessed deriving the present value of future dividends, more specifically, future dividends per share (DPS), discounted at the cost of equity (k_e).

$$P_0 = \sum_{t=1}^T \frac{DPS_t}{(1 + k_e)^t} + \frac{P_T}{(1 + k_e)^T}$$

Where: P_0 is the stock price today;

DPS_1 is the dividend per share the next period;

K_e is the cost of equity.

This model is characterized by the fact that, at a certain time in the future T , it is no longer possible to forecast future dividend payments and, therefore, it is considered the future depreciation or appreciation of the stock price (P_T). It can be seen as the terminal value of the DCF approach.

Assuming a perpetuity model and a constant growth rate for dividends, it is considered T going to infinity, meaning that the firm will pay dividends for its entire life and those dividends will growth proportionally to a specific rate. These assumptions are needed in order to rewrite the present value of future dividends in a more simplified way.

$$P_0 = \frac{DPS_1}{k_e - g} = \frac{DPS_0(1 + g)}{k_e - g}$$

Where: P_0 is the stock price today;

DPS_1 is the dividend per share the next period;

K_e is the cost of equity;

g is the growth rate.

The theoretical relation between the two factors, price to book value ratio and return on equity, is observable simply making some adjustment to the previous equation. Indeed, multiplying and dividing the right-hand side for the earnings per share (EPS) and dividing the left-hand side and the right-hand side by the book value of equity per share (BVPS), the following result is obtained.

$$\frac{P_0}{BVPS} = \frac{ROE * Payout\ ratio * (1 + g)}{k_e - g}$$

$$ROE = \frac{EPS}{BVPS}$$

$$Payout\ ratio = \frac{DPS_0}{EPS}$$

Where: EPS is the earnings per share;

BVPS is the book value of equity per share;

DPS_0 dividend per share at time 0;

g dividend growth rate.

The obtained equation shows that several different factors affect the price to book value ratio. Specifically, its value is determined by their combinations where each element influences the ratio in a different way.

In particular, the ROE, the Payout ratio and the growth rate (g) positively affect the ratio, on the other hand, the cost of equity (k_e) has a negative impact over its value.

The payout ratio, as shown in the above equation, is the ratio between the dividends and the net income. It is the proportion of the total earning that are paid out as dividends to common shareholders. For its computation it is the same consider the overall dividend paid out to all shareholder in relation to the overall net income or the dividend per share in relation with the amount of earning per share.

$$\frac{DPS}{EPS} = \frac{DIV}{net\ income}$$

Where: *DPS* is the dividend per share;

EPS is the earning per share;

DIV is the total dividend paid out to shareholders.

The dividend growth rate (*g*) is the percentage growth rate of dividends for a certain period of time. The computation of the growth rate can rely on some assumptions made by the analyst or it can be easily computed taking the simple or geometrical average of the past dividend growth rates.

The cost of equity (*K_e*) can be seen as the compensation required by the market, more specifically by shareholders, in exchange for owning shares and bearing the risk linked to the ownership. Normally, it is expected that the cost of equity is higher than the cost of debt, meaning that the firm promises a higher return to shareholders instead of debtholders due to their higher risk profile. Financial analysts usually implement the capital asset pricing model (CAPM) for the cost of equity's evaluation.

$$K_e = r_f + \beta(r_m - r_f)$$

Where: β is the measure of systematic risk;

r_m is the market rate of return;

r_f is the rate of return investing in a risk-free asset.

Finally, it is possible to derive that the return on equity is one of the fundamental factors which positively affect the entity of the price to book value ratio. However, it is expected, from the statistical analysis, that its impact should be only marginal due to the presence of other relevant ratios, as it has been shown from the theoretical explanation.

In order to carry out the statistical analysis two different approaches are implemented. The first one is a linear time-series regression, using the ROE as independent variable and the PBV as the dependent one. The other one is the correlation function included within the MATLAB tools.

The Return on Equity it is used in numerical terms instead of percentage ones, indeed, it has been divided by 100 since from Bloomberg the ratio was retrieved directly in percentage terms.

The time-series regression takes into consideration as independent variable (x) the Return on Equity and as dependent variable (y) the Price to Book value ratio.

$$y_{t,i} = \alpha_i + \beta_i x_{t,i} + \epsilon_{t,i}$$

$$y_{t,i} = PBV_{t,i}$$

$$x_{t,i} = ROE_{t,i}$$

$$PBV_{t,i} = \alpha_i + \beta_i ROE_{t,i} + \epsilon_{t,i}$$

Where: $\epsilon_{t,i}$ is the error term of the linear regression;

β_i is the factor loading of the linear regression for the stock i ;

α_i is the intercept of the linear regression for the stock i .

In the above model, the two parameters, α and β , of each stock, show respectively the intercept and the slope coefficient of the linear regression.

On the other hand, the vector ϵ contains the error terms of the model for the stock i .

The above linear regression has been implemented on MATLAB using the fitlm code.

The default assumption is to use the ordinary least square (OLS) for the estimation of parameters.

The following table shows the results of the linear regressions, putting in evidence the estimated coefficients alpha and beta and the relative standard errors, t-statistics and p-values of each one, moreover, in the last two columns, are displayed the r-square and the adjusted r-squared coefficients.

Results of the regression between ROE and PBV ratio										
	α	SE α	t-statistic α	p-value α	β	SE beta	t-statistic β	p-value β	R2	Adjusted R2
A2A	1,363	0,075	18,184	0,000	3,646	0,648	5,628	0,000	0,135	0,131
ATL	1,014	0,246	4,123	0,000	10,692	1,150	9,293	0,000	0,355	0,351
AZM	2,470	0,356	6,938	0,000	3,595	1,197	3,005	0,003	0,059	0,052
BGN	1,975	0,222	8,881	0,000	7,925	0,714	11,100	0,000	0,519	0,515
BAMI	0,325	0,016	19,971	0,000	-0,212	0,114	-1,853	0,067	0,033	0,023
BPE	0,490	0,042	11,761	0,000	7,524	0,463	16,246	0,000	0,565	0,563
BRE	0,648	0,196	3,309	0,001	10,512	0,915	11,484	0,000	0,394	0,391
BZU	2,063	2,317	0,890	0,374	45,803	17,620	2,600	0,010	0,032	0,027
CPR	3,606	0,256	14,073	0,000	-7,679	1,770	-4,339	0,000	0,089	0,085
ENEL	1,010	0,082	12,342	0,000	4,145	0,556	7,458	0,000	0,215	0,211
ENI	1,002	0,039	25,875	0,000	4,202	0,203	20,749	0,000	0,680	0,678
EXO	0,790	0,023	34,808	0,000	0,895	0,188	4,755	0,000	0,181	0,173
G	1,689	0,142	11,897	0,000	4,151	1,204	3,447	0,001	0,060	0,055
ISP	0,814	0,032	25,471	0,000	3,330	0,290	11,474	0,000	0,424	0,421
LDO	1,333	0,038	35,027	0,000	1,989	0,238	8,373	0,000	0,268	0,265
LUX	1,347	0,270	4,988	0,000	17,417	1,426	12,211	0,000	0,423	0,421
MS	1,907	0,086	22,101	0,000	7,146	0,515	13,862	0,000	0,489	0,486
MB	0,794	0,051	15,562	0,000	5,327	0,584	9,119	0,000	0,317	0,313
PRY	2,888	0,070	41,540	0,000	2,548	0,190	13,377	0,000	0,599	0,595
REC	-0,123	0,687	-0,179	0,858	17,830	3,153	5,654	0,000	0,136	0,132
SPM	2,218	0,095	23,396	0,000	5,025	0,392	12,823	0,000	0,447	0,445
SRG	0,664	0,077	8,614	0,000	7,296	0,554	13,180	0,000	0,475	0,472
STM	1,615	0,060	26,705	0,000	3,900	0,749	5,206	0,000	0,127	0,123
TIT	1,103	0,041	26,778	0,000	2,114	0,384	5,502	0,000	0,130	0,125
TEN	1,180	0,092	12,871	0,000	8,083	0,432	18,702	0,000	0,717	0,715
TRN	2,435	0,082	29,857	0,000	0,210	0,406	0,518	0,605	0,002	-0,005
UBI	0,642	0,023	28,505	0,000	3,667	0,255	14,399	0,000	0,544	0,541
UCG	0,970	0,043	22,669	0,000	3,741	0,301	12,437	0,000	0,439	0,436
UNI	0,665	0,029	23,050	0,000	2,472	0,341	7,249	0,000	0,210	0,206
US	0,947	0,020	46,313	0,000	1,328	0,105	12,667	0,000	0,471	0,468

Table 2: Results of the following time-series linear regression $PBV_{i,t} = \alpha_i + \beta_i ROE_{i,t} + \epsilon_{i,t}$.

The first important coefficient that must be taken into consideration is the beta (β), or slope coefficient. It is easy to see, despite of few exceptions, that the estimated β of each stock is largely positive and characterized by a really low level of standard deviation. Therefore, it is reasonable to obtain a significantly large t-statistic for each stock. The t-statistic is considered in relation to the null hypothesis, beta equals to zero. It means that the test is focused on understanding and assessing if the slope coefficient is statistically equal to zero and, therefore, the return on equity it is or not one of the fundamental factors which drive the determination of the price to book value ratio. In particular the t-statistics alongside with the p-values are

fundamental to determine if the estimated value of the beta coefficient it is given by chance or it is statistically positive.

Generally, the t-statistic is given by the difference between the estimated beta ($\hat{\beta}_i$) and the tested value of the slope coefficient in the null hypothesis divided by the standard error of the coefficient.

$$t_{\hat{\beta}_i} = \frac{\hat{\beta}_i - \beta_0}{SE_{\hat{\beta}_i}}$$

$$H_0: \hat{\beta}_i = \beta_0$$

$$H_1: \hat{\beta}_i \neq \beta_0$$

Where: $\hat{\beta}_i$ is the estimated slope coefficient of the asset i ;

β_0 is the tested value;

$SE_{\hat{\beta}_i}$ is the standard error of the estimated beta of the asset i ;

$t_{\hat{\beta}_i} \sim$ t-student with T-2 degree of freedom.

In this specific case, the null hypothesis is related to test whether or not the estimated beta is equal to zero, therefore $H_0: \hat{\beta}_i = 0$, and the t-statistic follows a t-student distribution with T-2 degree of freedom.

$$t_{\hat{\beta}_i} = \frac{\hat{\beta}_i}{SE_{\hat{\beta}_i}}$$

$$H_0: \hat{\beta}_i = 0$$

$$H_1: \hat{\beta}_i \neq 0$$

Where: $\hat{\beta}_i$ is the estimated slope coefficient of the asset i ;

$SE_{\hat{\beta}_i}$ is the standard error of the estimated beta of the asset i ;

$t_{\hat{\beta}_i} \sim$ t-student with T-2 degree of freedom.

From the table 2, it is possible to observe that the values of the t-statistics are significantly larger than the threshold level of 2.575 linked with the 1% significant level, therefore, it is possible to say that the null hypothesis is strongly rejected for almost all the stocks, meaning that there is a positive serial correlation between PBV and ROE factors.

A further way in which it is possible to see the rejection of the null hypothesis is the p-value. It takes values significantly lower even than the 1% threshold level mentioned previously, meaning that the test guarantees that the probability of the type I error is at most 1%, or in other words, the risk to reject the null hypothesis when it is true is smaller than 1%.

It is possible to conclude, that the return on equity is a factor which strongly affect, in a positive manner, the price to book value ratio. Indeed, the statistical analysis of the slope coefficient gives as result the rejection of the null hypothesis that the estimated value of the beta is equal to zero. The rejection can be observed, among almost all the stocks in the sample, with a significant level of at least 1%, it means that the beta coefficient is large and positive, and its value is not given by chance.

The alphas, or intercept coefficients, are significantly larger than zero for almost all the observed stocks. Even in this case we have low standard error values and it would be reasonable to reject the null hypothesis, alpha equals zero.

As for the slope coefficient, the t-statistic is defined as the difference between the estimated alpha and the tested value of the intercept in the null hypothesis divided by the standard error of the coefficient.

$$t_{\hat{\alpha}_i} = \frac{\hat{\alpha}_i - \alpha_0}{SE_{\hat{\alpha}_i}}$$

$$H_0: \hat{\alpha}_i = \alpha_0$$

$$H_1: \hat{\alpha}_i \neq \alpha_0$$

Where: $\hat{\alpha}_i$ is the estimated slope coefficient of the asset i ;

α_0 is the tested value;

$SE_{\hat{\alpha}_i}$ is the standard error of the estimated beta of the asset i ;

$t_{\hat{\alpha}_i} \sim$ t-student with T-2 degree of freedom.

The specific case of such regression is referred to testing whether or not the intercept is statistically equal to zero.

$$t_{\hat{\alpha}_i} = \frac{\hat{\alpha}_i}{SE_{\hat{\alpha}_i}}$$

$$H_0: \hat{\alpha}_i = 0$$

$$H_1: \hat{\alpha}_i \neq 0$$

Where: $\hat{\alpha}_i$ is the estimated slope coefficient of the asset i ;

$SE_{\hat{\alpha}_i}$ is the standard error of the estimated beta of the asset i ;

$t_{\hat{\alpha}_i} \sim$ t-student with T-2 degree of freedom.

From the table 2, it is possible to observe significant large values of the t-statistics and, therefore, the null hypothesis, alpha equals zero, is strongly rejected for almost all the stocks within the sample.

Obviously, it is necessary to take into consideration the p-value. It is significantly low, showing, as the beta coefficient, small probability of the type I error, reject the null hypothesis when it is true. Despite of few exceptions, it is possible to conclude that the rejection of the null hypothesis is achieved with at least a significant level of 1%.

Finally, particular importance must be given to the R-Squared coefficient, or coefficient of determination. It is the proportion of the variance in the dependent variable that is predictable from the independent one. It can be seen as a measure that explain if the model is properly working.

The simple concept that can be derived is that the higher the R-Squared value the more the model fit the data in the process of explaining the dependent variable.

The R-squared coefficient can be computed in two different ways. One takes into consideration the ratio between the explained sum of squared and the total sum of squared. The other one is given by one minus the ratio between the residual sum of squared and the total sum of squared.

$$R^2 = \frac{ESS}{TSS} = 1 - \frac{RSS}{TSS}$$

$$ESS = \beta^2 \sigma_X^2$$

$$RSS = \sigma_\epsilon^2$$

$$TSS = \beta^2 \sigma_X^2 + \sigma_\epsilon^2$$

Where: *ESS* (explained sum of squared) is the variation explained by the model;

TSS (total sum of squared) is a measure of total variation;

RSS (residual sum of squared) is a measure of variation that is not explained by the model.

The R-Squared coefficient of a linear regression can be seen as the proportion of variation explained by the model itself with respect the total variance.

Its value ranges between 0 and 1, where 0 means that there is no prediction of the variance of the dependent variable and, therefore, the model does not fit the data at all. On the other hand, 1 means that the total variance is entirely determined by the explained sum of squared.

From the table it is possible to derive that the model is properly fitting the observed data but without extreme values of the R-Squared.

Indeed, on average, the coefficient of determination is closed to 40% which is a good indicator of the model itself. However, even though the R-Squared is positive, it does not reach the ideal value of 80% because of some other factors that may be fundamental in the determination of the dependent variable.

Specifically, at the beginning of this chapter, it was explained, from a theoretical point of view, which are the factors that mostly affect the price to book value ratio. Among them, there was the Return on Equity, the payout ratio, the growth rate and the cost of equity. For this reason, it is reasonable to expect higher values of the R-Squared if those other factors are included in the regression

Even though the other factors are not included within the regression model, the R-Squared and the Adjusted R-Squared are positive.

The Adjusted R-Squared is a modified version of the coefficient of determination that has been adjusted for the number of predictors in the model.

$$\bar{R}^2 = 1 - (1 - R^2) \frac{n - 1}{n - k - 1}$$

Where: $\bar{R}^2$ is the adjusted r-squared;

R^2 is the coefficient of determination;

n is the number of observations;

k is the number of regressors.

The final conclusion that can be derived from the implementation of the describe linear time-series regression is that the relation between the return on equity and the price to book value ratio is significantly high and positive.

However, the R-Squared coefficient is not high enough to say that the return on equity is the only factor that determines the entity of the price to book value. This is mainly due, as said at the beginning of the chapter, to the fact that the price to book value ratio is affected by other several factors, such as the payout ratio, the growth rate and the cost of equity.

The above results seem to be inconsistent in particular for two stocks, Banco BPM (BAMI) and Terna Rete Elettrica Nazionale (TRN). Both are characterized by high p-values and low t-statistics.

In particular it is possible to observe that the beta coefficient of Banco BPM takes value negative with low t-statistic and a p-value close to 7%, for this reason it is possible to say that the results of the linear regression for that stock are not sufficient to reject the null hypothesis that the Return on Equity affects the Price to Book value ratio, at the significant level of 1% as all the other stocks.

Even worst seems to be the case of Terna Rete Elettrica Nazionale. Indeed, in that case the p-value exceeds the level of 60%. It means that the value of the slope coefficient is mainly given by chance, since an actual relation between the Return on Equity and Price-Book value ratio does not exist for that stock.

For both of the cited stocks the R-squared coefficient is significantly low and close to zero, in such a way that the model does not explain the dependent variable at all.

Even though two out of thirty stocks have inconsistent results, it is possible to generalized that the profitability index, ROE, does affect the price to book value ratio with a significant level of at least 1%.

This relation can be analyzed even using the simple correlation function implemented on MATLAB.

Such function gives as results the correlation coefficient and the relative p-value for each stock included within the sample.

The correlation function is based on the Pearson correlation coefficient (PCC). It is a measure of correlation between two variable X and Y that, in this specific case, are respectively the return on equity and the price to book value factors.

The rho coefficient (ρ) can take value between -1 and 1, where, obviously, in case of 0 it means that there is no serial correlation, on the other hand, if rho takes value -1 it means there is a perfectly negative serial correlation and if it takes value 1 it means there is a perfectly positive serial correlation.

In the case of a perfectly negative correlation, all the data points lie on the fitted line such that if X increases, Y decreases.

If the coefficient is perfectly positive, therefore equals to one, a linear equation perfectly describes the data in such a way if X increases, Y increases as well.

The Pearson correlation coefficient is defined as the covariance between the two factors divide by the standard deviation of each factor.

$$\rho_{x,y} = \frac{COV(X,Y)}{\sigma_X \sigma_Y}$$

Where in this specific case:

$$\rho_{ROE,PBV} = \frac{COV(ROE,PBV)}{\sigma_{ROE} \sigma_{PBV}}$$

From the theoretical point of view the correlation coefficient for the whole population can be expressed in terms of mean and expectation.

$$COV(X, Y) = E[(X - \mu_X)(Y - \mu_Y)] = E[XY] - E[X]E[Y]$$

$$\sigma_X^2 = E[(X - \mu_X)^2] = E[X^2] - E[X]^2$$

$$\sigma_Y^2 = E[(Y - \mu_Y)^2] = E[Y^2] - E[Y]^2$$

Where: the mean of X is expressed as its expected value, $\mu_X = E[X]$;

the mean of Y is expressed as its expected value, $\mu_Y = E[Y]$.

Hence:

$$\rho_{x,y} = \frac{E[XY] - E[X]E[Y]}{\sqrt{E[X^2] - E[X]^2} \sqrt{E[Y^2] - E[Y]^2}}$$

The idea of using such function is to test the sample correlation coefficient between the price to book value and the return on equity for each stock. The Pearson correlation coefficient is obtained, in the sample, substituting the estimates of the covariance and the variances in the above equation.

$$\rho_{X,Y} = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^n (y_i - \bar{y})^2} \sqrt{\sum_{i=1}^n (x_i - \bar{x})^2}}$$

Where: $\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$;

$\bar{y} = \frac{1}{n} \sum_{i=1}^n y_i$.

$$\rho_{ROE, PBV} = \frac{\sum_{i=1}^n (ROE_i - \overline{ROE})(PBV_i - \overline{PBV})}{\sqrt{\sum_{i=1}^n (ROE_i - \overline{ROE})^2} \sqrt{\sum_{i=1}^n (PBV_i - \overline{PBV})^2}}$$

The next table shows results in terms of rho (ρ), for the correlation coefficient, and it displays the p-values in order to assess if the coefficients are significantly different from zero, where the test is based on a t-student distribution with n-2 degree of freedom.

Correlation between market-to-book ratio and return on equity					
	ρ	p-value ρ		ρ	p-value ρ
A2A	0,367	0,000	LUX	0,651	0,000
ATL	0,596	0,000	MS	0,699	0,000
AZM	0,243	0,003	MB	0,563	0,000
BGN	0,721	0,000	PRY	0,774	0,000
BAMI	-0,180	0,067	REC	0,369	0,000
BPE	0,752	0,000	SPM	0,669	0,000
BRE	0,628	0,000	SRG	0,689	0,000
BZU	0,179	0,010	STM	0,357	0,000
CPR	-0,299	0,000	TIT	0,360	0,000
ENEL	0,464	0,000	TEN	0,847	0,000
ENI	0,824	0,000	TRN	0,042	0,605
EXO	0,426	0,000	UBI	0,737	0,000
G	0,245	0,001	UCG	0,662	0,000
ISP	0,651	0,000	UNI	0,458	0,000
LDO	0,518	0,000	US	0,687	0,000

Table 3: Results of the correlation function between the PBV and the ROE for each stock

The table 3, shows results that lead to the same conclusion of the regression analysis.

Despite of few exceptions, the price to book value ratio and the return on equity have a really positive and high correlation.

The rho coefficient on average takes value close to 0,5, on the other hand, for few stocks, it is observed a negative correlation with a p-value higher than 5%, like for Banco BPM.

It is possible to say that the correlation between the price to book value ratio and return on equity is high and positive, meaning that if the latter increases than the former should increase as well and vice versa.

These results are really consistent, since the p-value of each stock is significantly lower than 1%. Therefore, it is possible to say that the rho coefficient is different from zero with a significant level of at least 1%, showing that there exists an actual relation between the two factors that is not given by chance.

However, as in the regression analysis, the results are not consistent for the same two stocks, Banco BPM (BAMI) and Terna Rete Elettrica Nazionale (TRN). The correlation coefficients, for those two stocks, are significantly low. In the case of Banco BPM, the rho coefficient reaches negative value, showing an opposite relation with respect all the other stocks and, moreover, the p-value is again really high with its value close to 7%.

Terna Rete Elettrica Nazionale has a positive rho coefficient, which at first sight seems to be in line with all the other results, but its p-value is significantly high exceeding the level of 60% which is related to the fact that its estimated value is mainly given by chance. Therefore, it is possible to say that there is almost no correlation between the ROE and the PBV for this specific stock.

Despite of the above exceptions it is possible to conclude that the relation between the two factors is significantly positive and different from zero.

The rho coefficient never reaches the level of the perfect positive correlation ($\rho = 1$). The reason behind such result is obviously related to the fact that the price to book value ratio is affected by several other factors that somehow can be summarized in the payout ratio, the growth rate and the cost of equity.

$$\frac{P_0}{BV} = \frac{ROE * Payout\ ratio * (1 + g)}{k_e - g}$$

2.3. Portfolio Construction

The aim of this section is to explain how the portfolios have been constructed using a single sorting approach.

As described at the beginning of chapter 2, the Data taken into consideration are the price, the price to book-value ratio (PBV) and the return on equity (ROE) with monthly frequency.

Prices are necessary for the computation of asset returns, on the other hand, PBV and ROE are used as tools in order to derive portfolios based to different investment strategies.

The idea is to use the two factors, singularly, for the creation of portfolios that are rebalanced on a monthly frequency base.

Looking to the rebalance process there is a difference between the real rebalancing frequency of the two factors. Indeed, if it is considered the price to book-value ratio, the portfolio can be easily rebalanced on a monthly based since data about this factor are available at that frequency. On the other hand, the return on equity, due to its nature, has been retrieved only twice a year, on December and June. Therefore, even if the Data are adjusted such that the ROE is the same in the successive months after the disclosure of financial statements, the real rebalance of the portfolio occur every six months.

Those portfolios are created using different tolls on MATLAB, such as FOR loops and IF clauses.

Let's take into consideration the price to book-value ratio as factor used for the construction of five portfolios.

This method is called single sorting since the composition of the monthly portfolio is given by sorting only one factor. Each month, the PBV ratios of all the thirty stocks included in the sample are sorted from the lowest to the highest value and, successively, it is divided into five categories.

However, in the sample it is possible to observe some NaN (not-a-number) values. This is due mainly to the different length of time between the stocks considered in the analysis and the lack of some observations from the database itself.

In order to overcome such problem, the `prtile` function has been used in order to divide, each month, the sample in five categories. The code reports the market-to-book value relative to a specific percentile that must be defined. It is constructed in such a way all the NaN are automatically excluded from the sample.

The construction of the five portfolios is implemented simply dividing, each month, the stocks into five categories.

The portfolios are constructed in such a way, starting from the first until the fifth, they contain increasing price to book value ratios. Therefore, the one created using the first quintile is composed by those stocks having the lowest price to book-value ratio. On the other hand, the portfolio linked to the fifth quintile of the distribution contains those stocks characterized by the highest values of the PBV.

When the whole sample is available, meaning that there are not NaN values at time t , each of the five portfolios contains six stocks. However, if it is considered the period around 2001, the average number of stocks included in each portfolio is close to three since only for few securities the time-series is available from that date.

This single sorting process is implemented each month in such a way a new portfolio composition is derived. Adopting such procedure, it is obtained a monthly rebalance of the five portfolios in relation to the adopted criteria.

Each month the five portfolios contain stocks that better suit their characteristics. Indeed, the portfolios, from the first to fifth, contain, respectively, stocks with low PBV, medium-low PBV, medium PBV, medium-high PBV and high PBV.

The process described until now is necessary in order to identify to which portfolio the stocks belong, successively the portfolio returns must be computed.

Returns are computed individually for each stock included in that specific portfolio then, using the mean function, the monthly returns are derived relying on the assumption that each stock has the same weight.

Within the computations the simple returns are considered since they are easier to use when aggregation across asset is needed.

Once those operations are concluded, five vectors of returns, linked to the five portfolios, are derived. For simplicity, from now on, those portfolios will be named P1, P2, P3, P4, P5, where:

- P1 contains stocks with low values of the PBV;
- P2 contains stocks with medium-low values of the PBV;
- P3 contains stocks with medium values of the PBV;
- P4 contains stocks with medium-high values of the PBV;
- P5 contains stocks with high values of the PBV.

The same process has been implemented for the creation of portfolios based on the return on equity index. Indeed, the ROE vector has been sorted in such a way the five percentiles are identified and used in order to construct different portfolios.

Once again, it is assumed that each stock has the same weight within a specific portfolio.

The main difference between the two factors, used for the portfolio construction, is that the portfolios generated by the return on equity are actually rebalanced every six months, on the other hand portfolio based on price to book value ratio are rebalanced on a monthly frequency.

Indeed, the portfolios based on ROE have the same composition from December to May and from June to November. This is due to the fact that the financial statements, where it is possible to derive the return on equity, are published every six months, specifically on December and June.

Implementing all the computation five portfolios, P1, P2, P3, P4, P5, are derived where:

- P1 contains stocks with low values of the ROE;
- P2 contains stocks with medium-low values of the ROE;
- P3 contains stocks with medium values of the ROE;
- P4 contains stocks with medium-high values of the ROE;
- P5 contains stocks with high values of the ROE.

Once described such portfolios, a particular importance must be given to the construction of a long/short portfolio, which is used as risk factor in asset pricing.

The decision about to which portfolios the long and the short position must be taken, derived from the famous high-minus-low (HML) risk factor studied by Fama and French (1993).

HML factor consists in a long position with respect the portfolio containing stocks characterized by high book-to-market ratios and a short position with respect the portfolio containing stocks with low book-to-market ratios.

In this empirical study, the ratio retrieved from Bloomberg is exactly the inverse of the one used from Fama and French (1993). Indeed, it is the price to book value ratio, which is the market value divided the book value of equity.

Since this slight difference, in order to mimic the risk factor, the opposite long/short position must be taken. Indeed, the long position is taken with respect the portfolio P1, containing stocks with low PBV, and the short position is taken with respect the portfolio P5, containing stocks with high PBV.

Within this empirical study, the generated risk factor will be called low-minus-high (LMH).

From a computation point of view, this portfolio has been constructed simply taking the difference between the return time-series of the first portfolio and the return time-series of the fifth one.

This portfolio, or risk factor, contains twelve stocks when the whole sample is available.

Six stocks, in relation to the long position, are bought and they positively affect the LMH performance if they appreciate in value. The other six stocks, in relation to the short position, are sold and they positively affect the long/short portfolio performance if they depreciate in value.

The same operation is implemented with the ROE index.

Indeed, a similar risk factor has been created taking a long position with respect the portfolio containing stocks with the low ROE values (P1) and the short position with respect the portfolio including stocks with high ROE values (P5). For this reason, it can be called low-minus-high as in the price to book value ratio scenario.

Chapter 3: Quantitative analysis

This chapter analyzes the characteristics of the portfolios, that have been described previously. The aim is to assess whether or not there exist some similarities between the two single sorting approaches in portfolio construction.

The first section is focused on the moments' analysis of each portfolio. Specifically, it is taken into consideration the analysis of the four principal moments (mean, standard deviation, skewness and kurtosis) and successively it is reported and analyzed the value at risk. Once the moments are described, the skewness and the kurtosis are used in order to implement the so called Jarque-Bera test that verifies whether or not portfolio returns are normal distributed.

The second section is fundamental since an exercise of asset pricing is implemented. After an assessment of the capital asset pricing model (CAPM), it is applied a single factor model, where the long/ short portfolio is the common factor. The model provides the estimate beta coefficients for each stock, describing the sensitive of each asset to changes in the risk factor. Moreover, it is determined the price of risk linked to the two long/short portfolios. After an individual assessment of the two risk factors (PBV and ROE) a comparison between them is carried out in order to determine if the two common factors are similar or present some differences from a statistical and economic point of view.

Successively, the focus is shifted on the analysis, through the use of correlation and regression functions, of the relation between the portfolios constructed with the price to book value ratio and the respective portfolios constructed with the return on equity.

The aim is to prove if those portfolios tend to move towards the same direction meaning that the two portfolio construction approaches generate similar portfolios.

The last section is based on testing the robustness of this empirical research. Specifically, it contains different tests that have been applied in order to check if the results do not change drastically changing some variables.

3.1. Moments analysis

The focus of this chapter is to evaluate and report the moments of each portfolio. The various characteristics of the five portfolios, created with respect the two investment strategies one based on the return on equity and the other on price to book value, must be analyzed. As described in the second chapter five portfolios are monthly rebalanced in such a way they contain stocks with different values of the PBV and ROE. In particular, it is possible to observe portfolios from P1, composed by stocks with the lowest values of the

PBV or ROE, to P5, that, on the other hand, contains stocks which are characterized by the highest values of the PBV or ROE.

This section takes into consideration the return distributions analysis. In particular, the four principal moments (mean, standard deviation, skewness and kurtosis) are analyzed and successively used in order to implement the Jarque-Bera test, verifying if the returns are normal distributed or not. Moreover, particular importance is given to the value at risk.

The following graphs show the returns distributions of each portfolio.

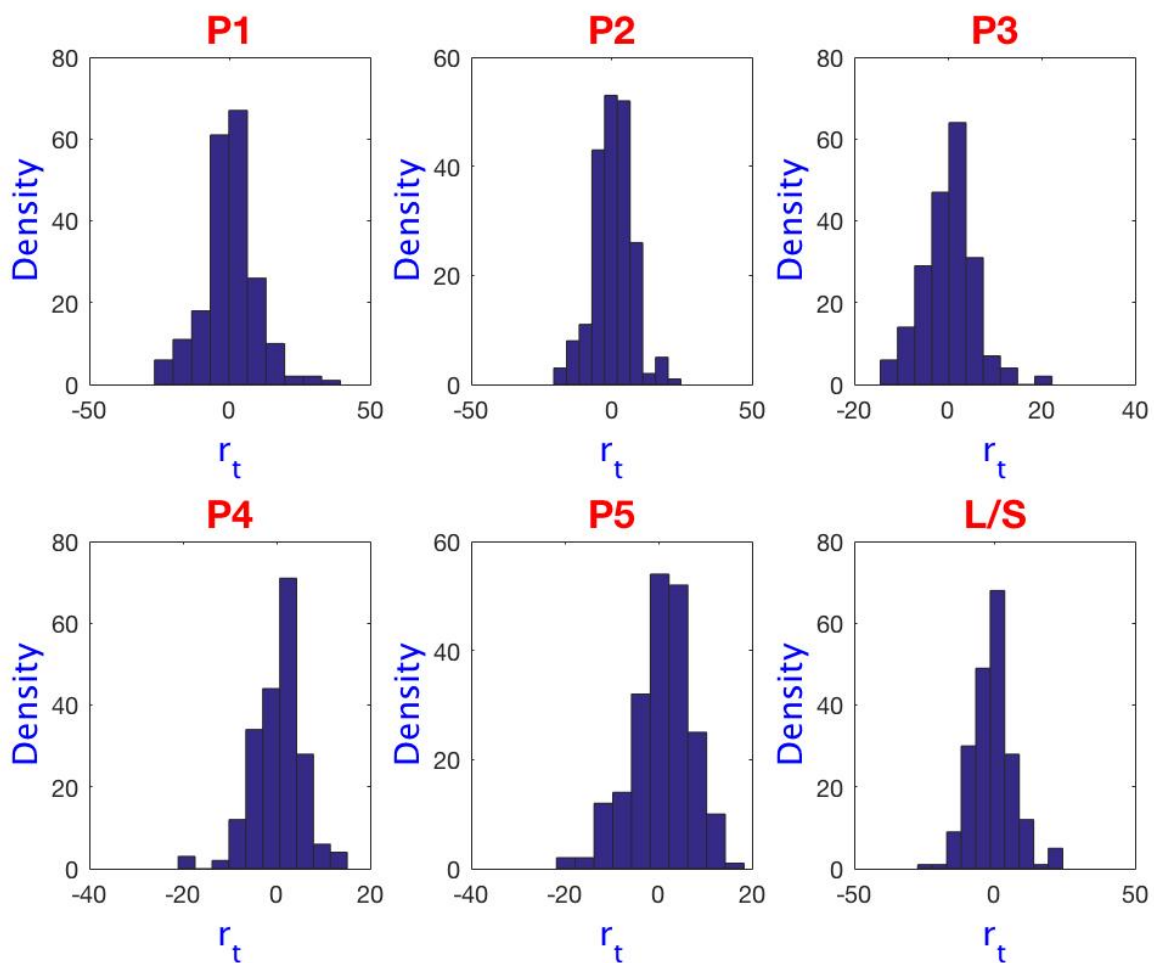


Figure 1: Return distributions of the five portfolios constructed using the price to book-value ratio approach.

Figure 1 is composed by six different histograms containing the return distributions of each portfolio created using the price to book value ratio. It is possible to observe the distributions of the five portfolios starting from P1, that is the one containing the stocks characterized by the lowest values of the price to book value factor, until P5 which contains the stocks with the highest values of the PBV.

Moreover, the last portfolio, which is denominated as L/S, is the one generated by taking a long position with respect P1 and the short position with respect P5. It is interesting to see how those graphs differ between each other.

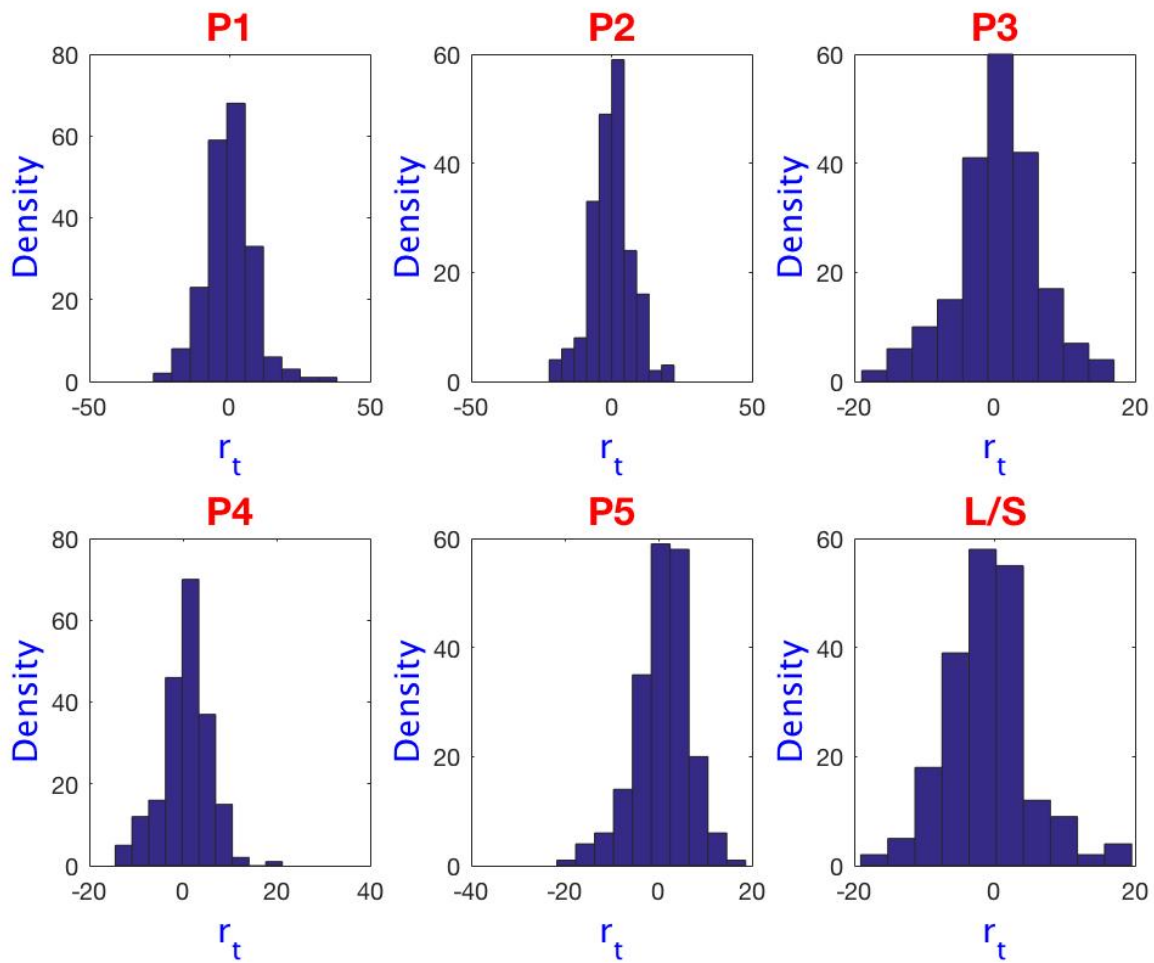


Figure 2: Return distributions of the five portfolios constructed using the return on equity index approach.

The figure 2, on the other hand, contains the six histograms of return distribution relative to the six portfolios constructed using a single sorting approach based on the return on equity. Starting from the first, which is the histogram related to the return distribution linked to the portfolio created using the stocks characterized by the lowest value of the return on equity, until the portfolio P5 which is the one generated using stocks with the highest values of ROE. The last histogram is the one containing the return distribution of the long/short portfolio, long P1 and short P5.

The figures provide a preliminary assessment of the differences, in terms of moments, that exist among the six portfolios. For example, it is possible to observe different mean values and standard deviations between them, simply looking to the charts.

3.1.1. Mean and Standard Deviation

Before start talking about the mean and the standard deviation, it must be understood the nature of the variables. The variables objective of the study are the portfolios monthly returns, which can be considered as discretional variables.

The first moment is the expected value or mean value. It is given by the mean of the returns for the whole sample, from January 2001 to January 2018. The aim of studying this moment is to understand, first from a statistical point of view, which is the central value of return distributions considering the frequency and likelihood of portfolio returns. The statistical explanation is not the only interpretation of the mean return. The financial interpretation of the first moment corresponds to the approximative expected monthly return that can be generated by investing in that specific asset. Indeed, the mean value shows what is the potential profit or loss derived from the investment. It considers, even if marginally, the impact of tail events, where those are the returns which are less likely to occur but somehow affect the central value. The mean return is derived simply taking the sample average of portfolio returns.

$$E[X] = \mu = \frac{1}{T} \sum_{t=1}^T x_t$$

Where: $E[X] = \mu$ is the expected value;

t is the vector of time starting from January 2001 until January 2018;

x_t is the vector of simple returns.

The showed formula is applied for the computation of monthly mean returns for each portfolio, starting from the five portfolios created using a single sorting based on the return on equity and the five portfolios constructed by the price to book value ratio, ending up with the analysis of the two long/short portfolios derived from the two different methodologies.

Alongside with the analysis of mean returns it is useful to assess and evaluate the standard deviation. It is the square root of the variance. Indeed, the variance is computed taking the square of the difference between the observed return and the mean value. It is the second moment of return distribution and it is the expected squared deviation of a random variable (X), that in this specific case is the vector of returns, from the central value. The variance is useful in order to assess and understand how far the returns are from the first moment. However, being more precisely, this role is carried out by the standard deviation, since it is adjusted in such

a way the volatility is expressed in the same units of the data. Low standard deviation shows that the returns are close the central value, on the other hand, high standard deviation shows that returns can be observed within a wide range.

The monthly returns are a discrete random variable and the variance can be expressed as follows.

$$Var(X) = \sigma^2 = \frac{1}{T-1} \sum_{t=1}^T (x_t - \mu)^2$$

$$SD = \sigma = \sqrt{\frac{1}{T-1} \sum_{t=1}^T (x_t - \mu)^2}$$

Where: $Var(X) = \sigma^2$ is the variance;

$SD = \sigma$ is the standard deviation;

μ is the mean value or first moment;

x_t is the vector of simple returns;

T is the number of observations.

The standard deviation, in finance, is useful in order to assess the volatility of portfolio returns. It is a measure of risk that needs to be taken into consideration when portfolio performance analysis is carried out.

In this specific case, the mean values and the standard deviations are necessary, at first, to evaluate the individual characteristics of the five portfolios and the long/short one created using the return on equity or the price to book value ratio. Specifically, it is assessed if there is a clear pattern from the portfolio P1 to P5.

Once such analysis has been implemented it is necessary to look to eventual similarities or differences between the portfolios created using the ROE and those constructed using the single sorting based on the PBV.

The following table shows the mean returns and the standard deviations relative to the five portfolios and the long/short portfolio constructed using the price to book value ratio as factor.

Mean and Standard deviation of portfolios based on PBV (in percentage terms)		
	Mean return	Standard_Deviation
P1	-0,029	9,659
P2	0,430	6,975
P3	0,190	5,664
P4	0,499	5,042
P5	0,674	6,638
Long P1/ Short P2	-0,703	7,428

Table 4: Mean and standard deviation of the five portfolios and the long short portfolio constructed using the single sorting based on price to book value ratio. Both are shown in percentage terms.

Looking to the table 4, specifically to the mean values, it can be observed a pattern among the five portfolios (P1, P2, P3, P4 and P5). Indeed, generally, it is possible to say that the mean return, starting from the portfolio characterized by the lowest values of the price to book value ratio (P1), tends to increase reaching its greatest value in correspondence of the portfolio containing stocks with the highest values of the price to book value ratio (P5). Since the first moments are expressed in percentage terms it is easy to assess that the first portfolio has a monthly mean returns very small and close to zero. However, if it is taken into consideration the portfolio P2, where are included those stocks characterized by medium low values of the price to book value, the monthly mean return seems to be more relevant, reaching a level of 0,430%. However, even though the described pattern can be recognized, a deviation arises when the portfolio containing the medium values of the price to book value ratios is considered. Indeed, the portfolio P3 has a mean return of 0,190% which is significantly lower than the portfolio P2. The portfolios P4 and P5 are in line with the described pattern in such a way the one containing stocks with medium high values of the PBV has a mean return slightly greater than the portfolio P2. On the other hand, the last portfolio, the one containing stocks with the highest values of the price to book value ratio (P5), has a mean return that is significantly higher than all the other portfolios. Indeed, it reaches a level of 0,674% where, just to be more specific, it corresponds to an annualized return greater of 8%.

Looking to the two extreme portfolios, it is observed that the portfolio P5 outperforms, on average, the portfolio P1 in the period from January 2001 until January 2018.

However, it is interesting looking to different intervals. Indeed, as several empirical studies have shown, there is a strong relation between the GDP growth and the returns of the first and fifth portfolios.¹⁵

Specifically, as long as long it is observed a period of economic growth and, therefore, of positive GDP growth, the portfolio P1, the one containing stocks with low price to book value ratios, outperforms the portfolio P5, that contains stocks with the highest PBV.

Looking to the Italian GDP chart, it is possible to identify the period from 2001 to 2007 as characterized by a significative and positive economic growth.

It is easy to understand that the end of this positive business cycle is linked to the global financial crisis.

The following chart reports the portfolios average returns computed in the period from January 2001 to January 2007.

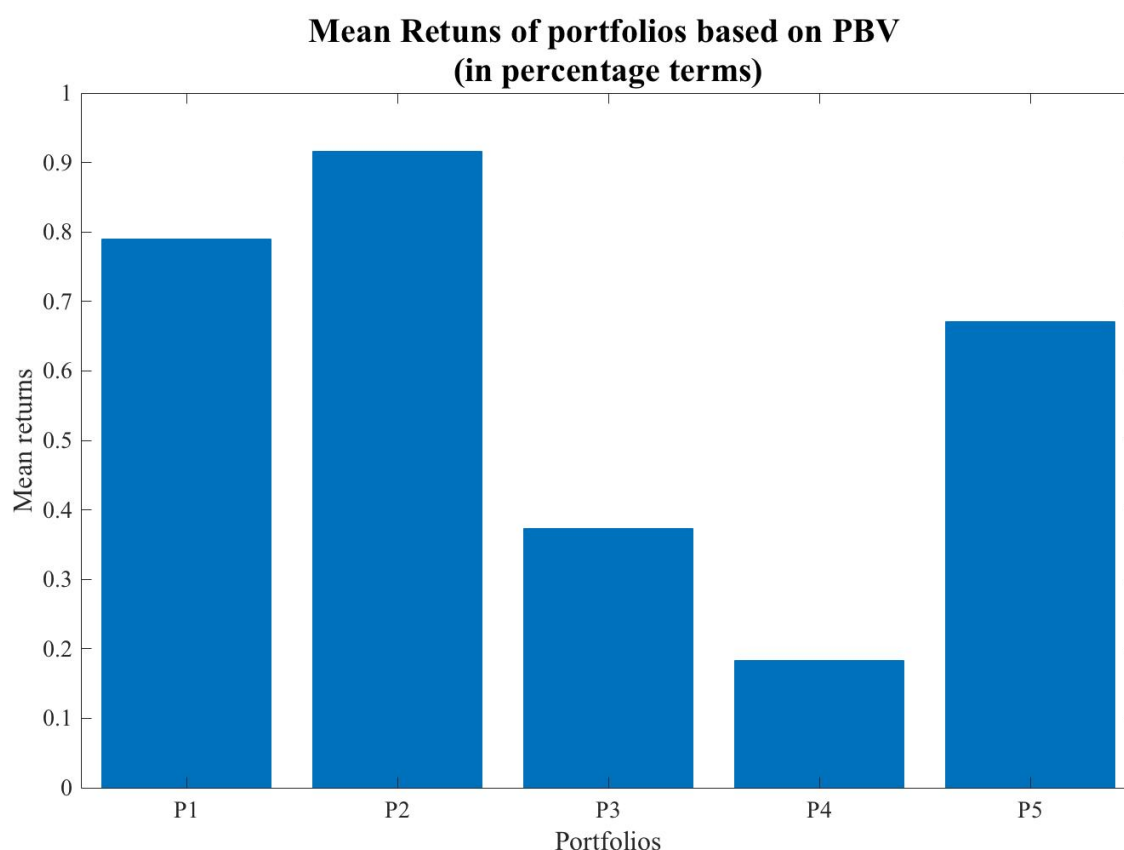


Figure 3: Mean returns of the five portfolios (P1, P2, P3, P4, P5) constructed using the single sorting approach based on PBV, in the period between January 2001 and January 2007.

¹⁵ Liew, J., & Vassalou, M. (2000). Can book-to-market, size and momentum be risk factors that predict economic growth?. *Journal of Financial Economics*, 57(2), 221-245.

It is clear that the portfolios containing stocks with low and medium-low PBV (P1 and P2) strongly outperform the fifth portfolio that contains stocks with high PBV.

Obviously, particular importance must be given to the two extreme portfolios, P1 and P5, since those are implemented for the risk factor construction.

The portfolio P1 has a monthly average return 0,12 greater than the portfolio containing stocks with the highest PBV.

Even if a clear pattern about the mean returns is observed, talking about the standard deviation a similar pattern does not arise.

Portfolios P3 and P4 are those with the lower standard deviations, meaning that their observed returns are closer to the central value than for all the other portfolios. It is possible to observe that, between the five portfolios, P1 is the one with the highest risk in terms of volatility, meaning it has the higher dispersion from the mean value. Indeed, that portfolio has a standard deviation of 9,659% that is 3 percentage points higher than the other extreme portfolio (P5).

This result is in line with the concept that those stocks characterized by low values of the price to book value ratio are riskier than those with high values. The higher riskiness can be related to the fact that sometimes a stock traded at a low price to book value ratio is a symptom of bad management or it is a firm in financial distress. This difference in term of risk, explained by the standard deviation, is even confirmed by other sources of risk that will be analyzed later on, such as the value at risk.

Now the focus is shifted to the portfolio composed by the long and the short position, taken respectively to portfolios P1 and P5. This portfolio is the one of particular relevance since it is used in order to implement an exercise of asset pricing. Specifically, the long/short portfolio is used as common risk factor. From the table 4 it is possible to observe that the mean return is significantly negative. Its value of $-0,703\%$ is mainly given by the fact that the short position is taken with respect the more profitable portfolio. The monthly performance of the long/short portfolio corresponds to an annualized return lower than -8% . The choice of creating the such portfolio, long P1 and short P5, is due to the aim of mimicking the famous risk factor, high-minus-low (HML), studied by Fama and French (1993). However, in this specific case, it can be called low-minus-high since Fama and French were using the book-to-market ratio instead of the market-to-book ratio as it is used in this study.

There are different empirical evidence showing that the cited risk factor anticipates the business cycle.¹⁶ In other word, it is a measure that anticipated the future GDP growth. Indeed, the portfolio yields larger returns in periods before good GDP growth than bad GDP growth.

This concept has been proved when the returns of the five portfolios have been computed in the period of GDP growth, from January 2001 and January 2007. Indeed, computing the mean return even for the long/short portfolio, it is possible to reports a monthly mean returns of 0,1197% which is significantly higher than the observed mean returns for the whole sample, from January 2001 to January 2018.

The long/short portfolio has a standard deviation of 7,428%. Its volatility stands between the standard deviations of P1 and P5, showing a lower dispersion from the central value than the first portfolio.

The next table contains the mean returns and the standard deviations of the five portfolios and the long/short portfolio constructed using a single sorting approach based on the return on equity.

Mean and Standard deviation of portfolios based on ROE (in percentage terms)		
	Mean return	Standard_Deviation
P1	-0,156	8,600
P2	-0,050	7,359
P3	0,270	6,106
P4	0,710	5,192
P5	0,798	5,869
Long P1/ Short P2	-0,954	6,160

Table 5: Mean and standard deviation of the five portfolios and the long short portfolio constructed using the single sorting based on return on equity. Both are shown in percentage terms.

From the table 5 it is possible to observe a similar pattern for the monthly mean returns with respect the previous scenario. Indeed, starting from the first portfolio (P1) the mean return increases reaching its highest value with the portfolio P5, which contains stocks with the highest return on equity. From a corporate point of view, it is easy to interpret since those firms characterized by the lowest values of return

¹⁶ Liew, J., & Vassalou, M. (2000). Can book-to-market, size and momentum be risk factors that predict economic growth?. Journal of Financial Economics, 57(2), 221-245.

on equity are companies with past poor performance. Hence, it is reasonable to expect a decrease in their market values.

Starting from the portfolio P3 positive mean returns are observed in such a way we find the most performing portfolio in the one containing those stocks with the highest return on equity values. The monthly mean return reaches a level close to 0,8% that corresponds to an annualized return of 10%.

Obviously, talking about the portfolio P5, its high mean return is linked to the good past performances of the companies included in the portfolio. Indeed, those firms are the one that are paying the most their shareholders in terms of profitability since their ROE are high.

Hence, in the period between January 2001 and January 2018 the portfolio P5 outperforms, on average, the portfolio P1.

In this empirical research, the aim is to assess eventual similarities and differences between the portfolios created using the market-to-book and ROE factors. For this reason, the same considerations implemented for the portfolios constructed with PBV must be carried out for the ROE.

The following analysis looks to eventual relation between the portfolio performance and the GDP growth. The monthly mean returns of the five portfolios have been computed from January 2001 to January 2007, period of positive economic growth in Italy.

The next chart reports the computed average returns of the portfolios constructed using the return on equity from January 2001 to January 2018.

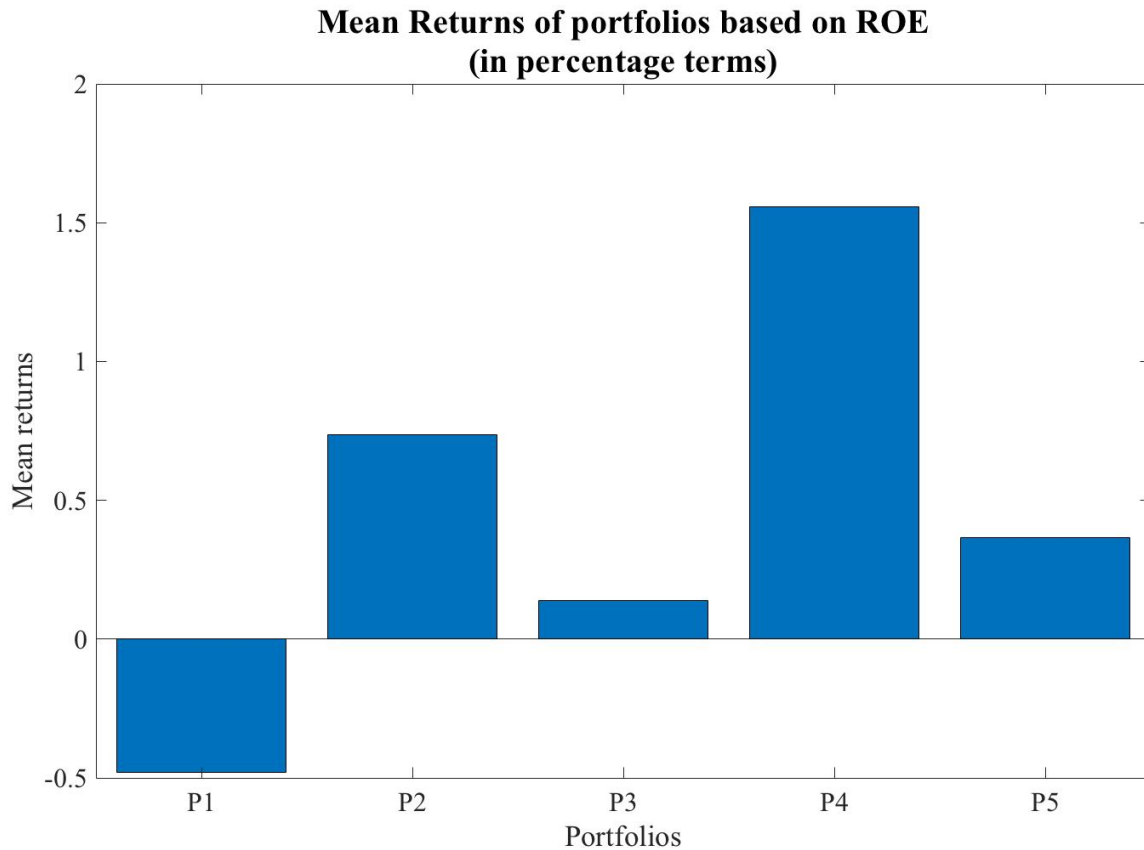


Figure 4: Mean returns of the five portfolios (P1, P2, P3, P4, P5) constructed using the single sorting approach based on ROE, in the period between January 2001 and January 2007.

As it is possible to observe, the relation between GDP and portfolio returns is not demonstrated. Indeed, the portfolio P5 still outperforms the portfolio P1 even in period of positive economic growth (positive GDP growth).

Such result has an implication regard the long/short portfolios, used as risk factor in asset pricing.

Taking into consideration the standard deviation it is easy to assess that, looking to the two extreme portfolios P1 and P5, the risk, in terms of volatility, is higher for the portfolio containing stocks with low values of ROE. Indeed, the first portfolio has a standard deviation of 8,6%, on the other hand, if it is considered the portfolio P5, its volatility decreases of almost 3 percentage points, reaching a level of 5,869%. It means that it is possible to observe returns closer to the central value for the portfolio P5 than for the portfolio P1. Such result is clearly in line with the fact that those stocks characterized by low return on equity are riskier than those with high values. This concept is confirmed by other source of risk that will be analyzed later on, such as the value at risk.

The portfolio composed by a long position on P1 and a short position on P5 is characterized by a negative mean return. Its value is slightly lower than $-0,95\%$ that in annualized terms corresponds almost to -11% . This result, as in the previous case, is due to the fact that the portfolio containing the highest ROE values outperforms the one with the lowest values. Its volatility has a value between P1 and P5, being closer to the standard deviation of the fifth portfolio. Such portfolio it is used for the implementation of an exercise of asset pricing in order to assess its role in the determination of asset returns as common risk factor.

Previously, it has been shown how there is no relation between the portfolios constructed with the return on equity and the GDP growth. This has a strong theoretical implication in asset pricing.

The value risk factor (LMH) has an economic meaning since it is able to predict future business cycle. On the other hand, it is not possible to say the same for the long/short portfolio constructed with respect the ROE.

The common factor has an average return of $-0,8462$ in the period from January 2001 to January 2007. Hence, the low-minus-high (LMH) risk factor based on ROE is not linked to the Italian economic growth.

Despite of few differences in the size of the first and second moments, the two factors, the ROE and the PBV, generate significantly similar portfolios in terms of mean return and standard deviation. In particular, there is the same pattern in terms of increasing monthly mean return. Moreover, the concept that the first portfolio has a higher volatility than the last one is observed for both methodologies of portfolio construction.

A slight difference that can be observed is linked to the portfolio P2 having a negative value in the case of portfolios constructed with a single sorting approach based on the return on equity instead of the one constructed using the price to book value ratio. It can be generalized the following concept. The first two portfolios P1 and P2 have higher monthly mean return when the price to book value ratio is used as factor. On the other hand, looking to the other two extreme portfolios, P4 and P5, the monthly mean returns are higher when a single sorting based on the return on equity is implemented.

These similarities are linked with the fact that the return on equity is one of the main factors considered in the price to book value ratio determination as it has been shown in the second chapter. However, those slight differences in the moments between the two strategies are due to the semi-annual rebalance frequency of portfolios based on the return on equity since the specific disclosure periods of financial statements, instead of a monthly rebalance typical of portfolios constructed using the price to book value ratio.

Looking to the long/short portfolios based on the two single sorting approach, it is possible to observe similarities between them. Indeed, both are characterized by negative monthly mean return and standard deviation close to the portfolio P5. It is possible to notice that the mean return and the standard deviation of the two long/short portfolios are lower when it is used the return on equity as factor for portfolio construction.

Even if the statistical moments of the portfolios constructed with the two approaches are similar, it is shown that the economic interpretations of the two LMH risk factors are different.

The PBV provides a common factor which is related to the business cycle. On the other hand, ROE generates a risk factor which is not related to GDP growth.

3.1.2. Skewness and Kurtosis

Once the first two moments have been considered, the focus is on the third and fourth, that are respectively skewness and kurtosis of return distributions.

The skewness, from a theoretical point of view, is a measure that shows the symmetry of a certain distribution. It can take value either positive or negative. If the skewness is negative it means that the left tail is longer or/and fatter than the right one. On the other hand, if it is positive, the opposite is true, therefore, the right tail is longer or fatter than the other one.

Skewness can take value equal to zero, and in that specific case it shows a perfect symmetry of the probability distribution, typical of a normal distribution. Skewness, alongside with kurtosis, are fundamental for the assessment of whether returns are normal distributed or not.

Despite the theory, from a financial point of view the skewness can be interpreted as follows. In case of negative skewness, therefore longer and fatter left tail, it is associated a high potential downside risk. In particular, comparing with a normal distribution, there is more chance of observing extreme negative returns.

If a positive skewness is considered than the difference with respect a normal distribution is that is more likely to observe high extreme positive returns.

Skewness can be seen as a measure of risk that shows which are the more likely extreme returns that can be observed for that specific portfolio and which are the more frequent observations. Indeed, a positive skewness shows more frequent small negative returns. On the other hand, a negative skewness shows more frequent small positive returns.

The sample skewness is computed as the third central moment divided by the standard deviation to the power of three.

$$\hat{S} = \frac{1}{T} \sum_{t=1}^T \left(\frac{x_t - \mu}{\sigma} \right)^3$$

Where: $\hat{S}$ is the skewness of the distribution;

μ is the mean value or first moment;

σ is the standard deviation or second moment;

x_t is the vector of simple returns;

T is the number of observations.

Kurtosis is the fourth moment of a probability distribution. It is a measure that is related to the tails in general. It shows how fat one or both tails are with respect a normal distribution.

When the fourth moment is considered particular importance must be given to the excess kurtosis. It is computed in such a way the estimated kurtosis is compared with the value related to a normal distribution. A normal distribution, with unknown mean and standard deviation, has a kurtosis equal to 3. Therefore, the excess kurtosis is given by the sample kurtosis computed minus the theoretical value of three linked with a normal distribution.

It is possible to observe three main situations of interest talking about the excess kurtosis. If it is large and positive it means that the distribution has fatter tails with respect the normal distribution, in this case it is called leptokurtic. On the other hand, if the excess kurtosis is large and negative, it means that the probability distribution has thinner tails than a normal distribution and it is called platykurtosis. The final situation is related to the case in which the excess kurtosis is close to zero, showing that the third moment is equal to the one of a normal distribution.

So far, it has been explained the kurtosis from a statistical point of view, but particular importance must be given to the financial meaning. Indeed, a high excess kurtosis is related to a higher likelihood to observe extreme returns that can be either positive or negative, in other words higher probability of observing tails events. It is clear that combining skewness and kurtosis is a useful method that can be applied in order to assess the riskiness linked to that specific portfolio.

$$\hat{K} = \frac{1}{T} \sum_{t=1}^T \left(\frac{x_t - \mu}{\sigma} \right)^4$$

$$excess\ kurtosis = \hat{K} - 3$$

Where: $\hat{K}$ is the kurtosis of the distribution;

μ is the mean value or first moment;

σ is the standard deviation or second moment;

x_t is the vector of simple returns;

T is the number of observations.

The following table reports the skewness and the kurtosis for all the portfolios, including the long/short portfolio, constructed using the price to book value ratio as factor.

Skewness and Kurtosis of portfolios based on PBV (in percentage terms)		
	Skewness	Kurtosis
P1	0,204	4,690
P2	-0,025	4,177
P3	0,165	4,499
P4	-0,465	4,477
P5	-0,525	3,694
Long P1/ Short P2	0,262	4,455

Table 6: Skewness and Kurtosis of the five portfolios and the long short portfolio constructed using the single sorting based on price to book value ratio.

From the Table 6 it is possible to observe that for all the portfolios the skewness is different from zero, only if it is considered the portfolio P2, it has a skewness small and close to zero.

Looking to the kurtosis all the stocks are characterized by higher values with respect the level of 3, typical of a normal distribution. Hence, the excess kurtosis is greater than zero for all the portfolios showing the fact that the distributions are all leptokurtic. Only in relation to the portfolio P5, the kurtosis can be seen closer to the value of a normal distribution, indeed the excess kurtosis has a value smaller than one.

Skewness and the kurtosis present a pattern looking to the five portfolios, from P1 to P5. In particular, the skewness and the kurtosis, starting from the portfolio P1, tend to decrease in value reaching their lowest level in relation to the portfolio P5.

Looking to the extreme portfolios, P1 and P5, it is possible to observe that those are characterized, respectively, by the highest and the lowest values of skewness and kurtosis. The portfolio P1 has a positive skewness meaning that there is more likelihood to observe positive extreme returns. More precisely, its distribution is characterized by a longer or/and fatter right tail, meaning that there are more extreme positive returns and more frequent small negative returns.

The excess kurtosis of the portfolio P1 is significantly greater than 1 showing the higher risk linked to that portfolio. This result can be seen from the figure 1 in relation with the return distribution of P5.

The portfolio P5 has a negative skewness, showing that the left tail is longer and fatter. However, it is characterized by frequent small positive returns, concept that is in line with its monthly mean return. The value of the skewness shows a higher likelihood of observing negative extreme returns. It means that the fifth portfolio is characterized by stocks that performs well on average. However, if a drop in the value of the portfolio occurs, it could be really significant and therefore entailing high negative extreme returns.

The kurtosis for the portfolio P5 is significantly low reaching an excess kurtosis smaller than 0,7. Therefore, it is the portfolio which has a kurtosis closer to the one of a normal distribution.

The last portfolio that must be considered is the one created taking a long position on P1 and a short position on P5.

It is possible to see that its skewness is even higher than the portfolio P1 and on the other hand, its kurtosis is slightly smaller than the first portfolio. This result shows that there is a positive likelihood to observe extreme positive returns, however, it is true that small negative returns are really frequent.

Once the skewness and kurtosis for the portfolios created using the price to book value ratio are analyzed the focus is shifted to those portfolios which are constructed using a single sorting based on the return on equity.

Skewness and Kurtosis of portfolios based on ROE (in percentage terms)		
	Skewness	Kurtosis
P1	0,428	5,129
P2	-0,056	3,819
P3	-0,269	3,829
P4	-0,098	4,731
P5	-0,711	4,537
Long P1/ Short P2	0,506	4,455

Table 7: Skewness and Kurtosis of the five portfolios and the long short portfolio constructed using the single sorting based on return on equity.

The table 7 shows similar results with respect the previous table. Starting from portfolio P1 until the portfolio P5 the skewness systematically tends to decrease and the kurtosis, despite of few exceptions, decreases as well.

The portfolio P1 has a skewness that reaches a value of 0,428 showing a right tail that is definitely longer or/and fatter than the left one. It means that, even though its mean return is negative, it is characterized by higher positive extreme returns instead of negative ones and more frequent small negative returns.

Moreover, it has a kurtosis greater than 5 and, therefore, it is characterized by an excess kurtosis greater than 2. It means that both tails are fatter than a normal distribution, showing a higher likelihood to observe positive and negative extreme returns. However, looking to skewness and kurtosis together, the right tail is the one where there are more extreme observations.

The portfolio P5 has characteristics which seem to be similar to the PBV scenario. Indeed, the skewness is large and negative, and the kurtosis is lower than the portfolio P1 but still in excess of 1,5 than a normal distribution. The distribution of the portfolio P5 has a left tail longer than the right one showing higher likelihood of observing negative extreme returns, but more frequent small positive returns.

Looking to the kurtosis of both portfolios, P1 and P5, the latter has both tails which are less fat than the portfolio P1 but still fatter than a normal distribution. Moreover, the portfolio P5 has more likely negative extreme returns instead of the first portfolio for which the opposite is true.

However, it must be considered that, even though P1 has a longer and/or fatter right tail, it is characterized by frequent small negative returns. On the other hand, the portfolio P5 has a longer and/or fatter left tail but with frequent small positive returns.

The long/short portfolio has a skewness higher than the portfolio P1 but a kurtosis smaller than it. This portfolio has its distribution characterized by frequent small negative return and a longer and fatter right tail. It means that it is possible to observe positive extreme returns with more likelihood than from a normal distribution.

Those results are in line with the concept that the portfolio P1 is riskier than P5, but it still has higher potential extreme returns that may be linked to eventual undervalued firms or to companies which are able to recover from the financial distress.

Even looking to skewness and kurtosis, it is possible to assess that there are similarities among the portfolios constructed with the two different methodologies, PBV and ROE.

The pattern is similar in both cases, showing decreasing values of the third and the fourth moments starting from the first portfolio to the portfolio P5. This shows the higher potential profitability of the portfolio containing stocks with the lowest return on equity or price to book value ratio values. Indeed, the higher skewness is linked to a greater likelihood of observing extreme positive returns. Those potential extreme returns may be linked to eventual undervalued firms which value is adjusted by the market, creating above average performance.

The portfolio P5, again in both cases, is characterized by a negative skewness and a lower kurtosis. These results are in line with the fact that those firms included in such portfolio are subject to the risk of being overvalued. For this reason, the risk of observing extreme negative returns is higher.

Similarities arise even for the long/short portfolios. Indeed, despite of few differences in terms of absolute values of such moments, the two portfolios are characterized by positive skewness and quite large kurtosis. More precisely, the third moment is slightly bigger than the portfolio P1 and, on the other hand, the fourth moment is smaller.

This difference in terms of absolute value is linked to the fact that there is no complete correlation between the two factors, ROE and PBV. Indeed, from a theoretical point of view, there are other factors that strongly affect the entity of the price to book value ratio. It means that the portfolios constructed with the two single sorting approaches are similar but not the same.

The two long/short portfolios are used as common risk factor in the asset pricing exercise. Therefore, since the characteristics of those portfolios are similar, it is reasonable to expect almost the same stocks sensitivity to changes in the common factor.

3.1.3. Jarque Bera test

In the previous section the analysis was focused on the third and the fourth moments of portfolio distributions. Skewness and kurtosis are reconsidered in order to run a test in such a way it is possible to verify whether or not the return distributions of such portfolios behave like a normal distribution with unknown mean and standard deviation. The implemented test is the Jarque-Bera test.

The JB-test is constructed in such a way the null hypothesis taken into consideration is the one to assess if the time series of returns is normal distributed, against the alternative hypothesis that it is not normal distributed.

The test is not focused on the assessment of the first two moments, mean and standard deviation. Indeed, both are defined as unknown but particular importance it is given to skewness and kurtosis for an assessment of symmetry of the distribution.

The normal distribution, as it is known, has a skewness equal to zero and a kurtosis of three. Therefore, the test looks to such moments in order to understand if those are statistical similar to their theoretical values linked to a normal distribution.

The decision whether accept or not the null hypothesis is related to the t-statistic value which is compared with a critical level linked to a chosen significance level. The JB-test is distributed as a chi-squared distribution with 2 degree of freedom ($JB \sim X^2(2)$) and, therefore, the threshold level is selected from the chi-square table.

The statistic is constructed considering the estimated skewness to the power of two and the excess kurtosis to the power of two.

$$JB = \frac{T}{6} \left[\hat{S}^2 + \frac{1}{4} (\hat{K} - 3)^2 \right]$$

H_0 : returns are normal distributed

H_1 : returns are not normal distributed

Where: $\hat{K}$ is the kurtosis of the distribution;

$\hat{S}$ is the skewness of the distribution;

T is the number of observations.

A chosen critical value is necessary in order to determine the probability of a Type I error, meaning the rejection of the null hypothesis when it is true.

Such test has been implemented on MATLAB using a simple jbtest code. The test reports the value zero or one when the null hypothesis is, respectively, accepted or rejected. Alongside with the hypothesis results, there are the p-values, the t-statistic values and the relative critical values linked to the significance level of 5%.

The following tables show the results linked to the implemented JB-test for the portfolios created with the price to book value ratio and the return on equity.

The critical level set on MATLAB, in order to decide whether reject or not the null hypothesis, is 5,6836 which is associated to a 5% significance level.

Jarque-Bera test (portfolios based on PBV)			
	Hypothesis	p-value	t-statistic
P1	1	0,0012	25,6967
P2	1	0,0101	11,7895
P3	1	0,0025	20,0337
P4	1	0,0012	25,8814
P5	1	0,0073	13,4482
Long P1 / Short P5	1	0,0024	20,3253

Table 8: Results of the Jarque-Bera test implemented to the five portfolios and the long/short portfolios created using the price to book value ratio.

As it is possible to observe from the table 8 the null hypothesis is rejected for each portfolio against the alternative hypothesis that the distribution does not behave as a normal distribution with unknown mean and standard deviation. Indeed, the column named hypothesis shows the value 1 which is related to a rejection of the null hypothesis at 5% significance level.

The p-values are really low and for most of the portfolios its value is even smaller than 1%. This shows that it is possible to reject the hypothesis of normality with a 1% probability to spot a Type I error.

Obviously, the t-statistic as well confirm the results since its value, for all the portfolios, is largely greater than 5,6836.

Particular importance must be given to the portfolio P2, which is characterized by a p-value slightly higher than 1%. Indeed, if it is considered the t-statistic of the second portfolio, its value is smaller, even if only slightly, than 11,8368 which is the critical value linked to a significance level of 1%. Therefore, it cannot be rejected the null hypothesis at a significance level of 1%, but it is still rejected at the 5% significance level. It means that for the portfolio P2 there is a higher likelihood of committing a Type I error.

Jarque-Bera test (portfolios based on ROE)			
	Hypothesis	p-value	t-statistic
P1	1	0,0010	44,7412
P2	1	0,0478	5,8147
P3	1	0,0225	8,3026
P4	1	0,0012	25,7987
P5	1	0,0010	37,2471
Long P1 / Short P5	1	0,0011	26,6977

Table 9: Results of the Jarque-Bera test implemented to the five portfolios and the long/short portfolios created using the return on equity.

The results showed in the table 9 are definitely similar to those of the previous scenario, even if slight differences are observable.

First of all, the rejection of the null hypothesis at a significance level of 5% is proved for all the portfolios, moreover, looking to the t-statistics and the p-values it is assessed that the null hypothesis is rejected at a significance level of 1% for 4 portfolios out of 6.

Looking to portfolios P2 and P3, it is possible to observe that the null hypothesis cannot be rejected at a significance level of 1%. Indeed, comparing the relative t-statistics with the thresholds level of 11,8368, those are significantly lower than it, therefore the probability of reject the null hypothesis when it is true is higher than for the other portfolios. The portfolio P2 has a t-statistic which is significantly close to the thresholds level linked to a 5% significance level.

The results of the two portfolios, P2 and P3, can be deducted even looking to the rough values of skewness and kurtosis from the previous section. Indeed, the third moment is really low and close to zero for the two portfolios rather than the other four. Moreover, the kurtosis is slightly higher than 3, reaching an excess kurtosis smaller than 1 for both portfolios.

Table 8 and table 9 show results which seem to be similar between them. Indeed, despite of differences in terms of p-values and t-statistics, it is possible to conclude that once again there are similarities between the portfolios created using the returns on equity and those constructed with the price to book value ratio. In particular, for all the 12 portfolios the null hypothesis, that the returns are normal distributed, is rejected with a significance level of 5%, meaning that the probability of observing a Type I error is of 5%.

Hence, the returns distributions of the portfolios constructed with the two single sorting approaches do not behave like a normal distribution with unknown mean and standard deviation.

3.1.4. Value at Risk

The value at risk (VaR) is the last measure of risk that must be considered in order to assess the several portfolios and in order to find eventual similarities and differences among those.

The VaR is a measure that shows the potential extreme losses in relation with a defined confidence interval of the return distribution. In particular, it reports the percentage (return-VaR) or monetary measure of downside risk. For example, taking into consideration a confidence interval of 95%, it means that there is 95% likelihood of observing a change in the portfolio that is not worse than the value at risk. In other words, it can be seen as the 5% probability of observing a loss greater than the VaR.

Alongside the expected shortfall (ES), the value at risk is a measure that looks to the tail events. It can be implemented for both the right and the left tail, but the principal aim is to assess the extreme risk linked a specific time horizon.

As long as the VaR is computed, some parameters must be defined. Particular relevance must be given to the time horizon since the value at risk shows the portfolio losses for a certain holding period. Therefore, it represents the change in portfolios value over a specific time horizon.

Once it has been defined, obviously, particular relevance must be given to the confidence interval. The more common confidence intervals considered are 95% and 99%.

There exist two main methods that can be implemented in order to assess the value at risk.

The first one, the delta-normal model, takes into consideration an assumption of normality. Assuming that the returns are normal distributed with mean μ and standard deviation σ , it is possible to use some conditional variance models in order to forecast the future standard deviation. The forecasted volatility, alongside with the inverse of the cumulative distribution function (CDF) of a standard normal distribution, are essential for the assessment of the one step-ahead VaR.

The formula used for the value at risk computation requires as inputs the confidence interval, the mean and the standard deviation at time $t + 1$.

$$VaR_{t+1}^P = -(\mu_P + F^{-1}(p)\sigma_{P,t+1})V_t$$

Where: VaR_{t+1}^P is the value at risk for the portfolio the next period;

$1 - p$ is the confidence level;

μ_P is the mean return of the portfolio p ;

$F^{-1}(p)$ is the inverse of the CDF;

$\sigma_{P,t+1}$ is the next period standard deviation;

V_t is the value of the portfolio.

The above equation gives as result the monetary amount of downside risk linked to the $1 - p$ confidence interval. If it is necessary to compute the return-VaR it can be used the same equation but simply without considering the value of the portfolio V_t .

The other method, named historical simulation (HS), uses all the historical observations, that can be either in terms of portfolio values or returns, in order to assess the potential loss linked to the defined percentile of the distribution, 5% or 1%.

The main strength of such method is that no assumption of normality is made. However, there is an important drawback linked to this approach. Specifically, the sample size has a significant role within the computation. The historical simulation is highly sensitive to eventual changes in the sample size. Specifically, the inclusion or exclusion of specific events, such as a financial crisis, strongly affect the value at risk computation. For example, if the market is bullish, for an eventual presence of a bubble, and a short sample is used, then the value at risk will be always smaller without actually reflect the true potential downside risk.

Within this empirical study the best method to be applied is the historical simulation. The reason is mainly related to the results given by the implemented JB-test, where it was proved that the returns of all the portfolios are not normal distributed with a significance level of at least 5%.

Before showing the results, few specifications must be made for a better comprehension of the value at risk. First of all, the time horizon. It is clear that, due to the monthly rebalance of the portfolio and, therefore, to the monthly computation of portfolios returns, it is considered a one-month time horizon. It means that the value at risk, computed with the historical simulation approach, is the loss that an investor may deal with in a month, with a specified probability.

The second parameter that must be set, obviously, is the confidence interval or in other words the probability that a potential loss can be higher than the VaR.

The interpretations of the results may be seen in two ways. First, the value at risk shows the loss that the portfolio suffered from the 2001 until the 2018. The other interpretation can be made if an assumption is defined. Indeed, if it is expected that the return distribution will be the same in the future, the value found by the historical simulation approach may foretell the one step-ahead value at risk.

Even if the historical simulation approach suits the best in this empirical study, the delta-normal method is considered as well in order to make a comparison between the two models.

The following tables report the VaR, computed with the two methodologies, of the portfolios constructed with the price to book value ratio and the return on equity.

The results are shown in absolute value, even though they must be interpreted as the loss linked to that specific confidence interval.

Value at Risk of Portfolios based on the PBV (in percentage terms)				
	Historical Simulation		Delta-Normal	
	VaR(5%)	VaR(1%)	VaR(5%)	VaR(1%)
P1	16,889	26,185	15,915	22,498
P2	13,291	17,552	11,043	15,796
P3	10,188	13,171	9,127	12,987
P4	7,524	16,190	7,794	11,230
P5	12,107	18,109	10,243	14,767
long P1/short P5	13,614	16,801	12,921	17,983

Table 10: Value at Risk (VaR) computed for the portfolios constructed using the price to book value ratio. The table reports the results of the historical simulation and delta-normal methods. Two confidence intervals are used, 95% and 99%.

Table 10 reports the results of the VaR computation for the portfolios created using a single sorting based on the price to book value ratio. First of all, let's consider the results derived from the application of the historical simulation approach.

The implementation of such method is carried out simply using the prctile code on MATLAB which provides the value of the time-series linked to the specified percentile of the distribution. It has been

implemented using the 5% and 1% percentiles. It reports the value at risk, specifically the return-VaR, linked respectively to the 95% and 99% confidence interval.

The first column shows the VaR values, for all the six portfolios, computed with a 95% confidence interval. Despite of the previous analysis a clear pattern does not appear. However, for almost all portfolios the value at risk is higher than 10%, an exception arise with respect the portfolio P4 characterized by a VaR(5%) of almost 7,5%.

Particular importance must be given to the two extreme portfolios, P1 and P5, and to the long/short portfolio. The main difference that can be noticed is the fact that the value at risk is significantly lower for the portfolio containing stocks with the highest values of the price to book value ratios (P5) with respect the one with the lowest values (P1).

The observed difference is more than 4%. This result, obviously, is in line with the fact that the portfolio P1 is riskier than P5. The riskiness has been assessed previously from the standard deviation, noticing that the fifth portfolio has a volatility smaller than the first one.

From the assessment of the value at risk it is noticed that, for the portfolio P1, there is 5% probability of losing more than 16%. On the other hand, with the same probability, the portfolio P5 may lose more than 12%.

The value at risk of the first portfolio is higher because of its greater volatility and major risk linked to several factor such as the case in which that particular firm is traded at a low price to book value ratio not because it is undervalued but for a bad management of the company or an eventual financial distress.

The portfolio P5 has lower risk, since for the major part of the time those firms are stable and mature with a good competitive position in the market. Therefore, it is reasonable to expect frequent small positive returns but a small likelihood of observing extreme values, that can be either positive or negative as the value at risk has shown.

Looking to the portfolio composed by a long position on P1 and a short position on P5, its VaR(5%) stands between those of the two extreme portfolios, being closer to P5.

It is expected to observe, in a month, a loss greater than 13% with a probability of 5%.

The lower value at risk relative to the portfolio P1 is mainly given by the less likelihood of observing extreme positive returns with respect the portfolio P5. Therefore, since the short position is taken with respect the fifth portfolio, the negative returns are given by the scenario in which P5 is gaining value, but it has been assessed that is really unlikely to observe significant positive returns.

Once the $\text{VaR}(5\%)$ has been assessed, the focus is shifted to the case in which a 99% confidence interval is considered. Obviously, in this case it is reasonable to expect higher values.

At first sight, the first portfolio is the one that display the higher variation between the value at risk at 5% and 1%. Indeed, for the portfolio P1, there is a probability of 1% of observing a loss greater than 26% that is 10% greater than the $\text{Var}(5\%)$.

Even though an actual pattern does not exist, as in the case of the value at risk with a confidence interval of 95%, the difference between P1 and P5 can be properly assessed. In particular, the value at risk is significantly higher when the portfolio containing stocks characterized by higher price to book value ratios is considered. The value at risk of the portfolio P5 is almost 8 percentage points smaller than P1, showing, again, that the first portfolio is riskier than the last one. The difference between the $\text{VaR}(1\%)$ of the two portfolios is bigger than the observed difference with a significance level of 95%.

The long/short portfolio is characterized by a value at risk lower even than the portfolio P5. This can be due to the properties of the two distributions. Indeed, the portfolio P5 has a really negative skewness meaning that there is a fatter and longer left tail instead of the right one. This result shows that the portfolio P5, despite of its positive mean return, has few and low positive extreme returns instead of the first portfolio. The reason of such result is given by the fact that the portfolio containing stocks with the lowest price to book value ratios has more likelihood of observing extreme positive returns due to presence of eventual undervalued firms or distressed companies able to recover from their situation.

On the other hand, the stocks within the portfolio P5 are correctly priced by the market, therefore, there are less extreme positive returns.

For this reason, the $\text{VaR}(1\%)$ of the long/short portfolio is significantly low.

A short statement must be made in relation to the results linked to the delta-normal approach.

The main feature that can be noticed is related to the smaller values linked to the use of such model. Indeed, for both value at risk, 5% and 1%, the results given by the delta-normal are smaller than those provided by the historical simulation for almost all portfolios. This is due to the assumption of normality for the portfolio distributions even if it has been proved the opposite through the Jarque-Bera test.

As showed in the previous section, the implemented JB-test displayed the rejection, at 5% significance level, of the null hypothesis of normal distribution. For this reason, implementing the delta-method, the results are obviously different from the other approach. However, the differences between the portfolios P1 and P5 is still sizable.

Value at Risk of Portfolios based on the ROE (in percentage terms)				
	Historical Simulation		Delta-Normal	
	VaR(5%)	VaR(1%)	VaR(5%)	VaR(1%)
P1	13,957	21,108	14,301	20,162
P2	13,208	19,629	12,154	17,169
P3	10,745	16,520	9,774	13,935
P4	8,641	13,497	7,829	11,367
P5	9,805	16,864	8,855	12,855
long P1/short P5	9,568	16,085	11,087	15,285

Table 11: Value at Risk (VaR) computed for the portfolios constructed using the return on equity. The table reports the results of the historical simulation and delta-normal methods. Two confidence intervals are used, 95% and 99%.

Table 11 shows the results of the value at risk, computed using the historical simulation and the delta-normal approach, linked to the portfolios created using the return on equity.

Once again, an actual pattern does not arise between the different portfolios.

Looking to the VaR(5%), it is observable that the value for each portfolio is around 10%, meaning that there is a 5% probability of observing a loss greater than 10%.

The portfolio P4, as in the PBV case, is the one with the lowest value at risk with respect all the other portfolios.

Particular importance must be given to the two extreme portfolios, P1 and P5, which are the one considered in the long/short portfolios construction. Specifically, the portfolio P5, containing stocks with the lowest return on equity values, has a VaR(5%) 4 percentage points smaller than the portfolio P1. Indeed, there is 5% probability of losing more than 9,8% investing in the fifth portfolio. On the other hand, the portfolio P1 has 5% likelihood of observing a loss greater than almost 14%.

Looking to the long/short portfolio (long P1, short P5) it is observable that its value at risk is even smaller than the portfolio P5. This is mainly due to the low extreme positive returns linked to the portfolio P5, this concept has been assessed in the previous section about the skewness which was negative for that specific portfolio.

Since the short position is taken with respect the fifth portfolio, the portfolio P5 affect negatively the long/short portfolio when an appreciation is observed.

The value at risk at 1% provides a further overview of the portfolio riskiness.

First of all, a pattern does not arise among all the portfolios, but, considering the portfolio P4, it is the one with the lowest value a risk.

On the other hand, the higher value at risk is linked with the portfolio P1. It has a $\text{VaR}(1\%)$ that reach a value around 21% which is significantly higher than the other extreme portfolio (P5). Indeed, the difference between the value at risk of the two portfolios, P1 and P5, is close to 5 percentage points.

The long/short portfolio has a value at risk lower than the portfolio P5, which is obviously linked to the characteristic of the fifth portfolio that has low extreme positive returns.

An explanation of such difference between the two extreme portfolios is linked to the intrinsic features of the firms included within the two portfolios. Considering the portfolio P1, that is the one containing those firms with the lowest return on equity, its high value at risk, both at 5% and 1%, is obviously due to the presence of firms that may be in financial distress or that may suffer a reduction in their prices due to bad management. Sometimes those firms are undervalued by the market or they may be able to recover from the distress. In those cases, the market will adjust their prices, and, for this reason, high positive extreme returns are observable for the portfolio P1.

On the other hand, the portfolio P5 contains stocks with the highest return on equity among all the securities in the sample. In this case, it is reasonable to expect a fair price by the market. Indeed, frequent small positive returns are evident for that portfolio, but there may be significant high negative returns due to the longer and fatter left tail.

Looking to the delta-normal approach, almost all the value at risk computed are lower than those derived from the historical simulation approach. This is mainly due to the fact that, testing those distributions for normality, the results show a rejection of normal distribution with unknown mean and standard deviation at least at 5% significance level.

The portfolios constructed with the two methodologies show some similarities and slight differences.

First of all, there is no pattern between the value at risk of the five portfolios constructed using both the price to book value ratio and the return on equity.

The portfolio P5, constructed with both single sorting approaches, is characterized by a VaR(5%) almost 4 percentage points lower than the portfolio P1. Therefore, in terms of relation among the different portfolios, there are significantly similarities that sustain the concept that the two factors, PBV and ROE, generate similar portfolios.

A reinforcement of such result is given even by the long/short portfolio which is considered as a risk factor in the asset pricing exercise. The common factor has a value at risk that is, in both cases, lower than the VaR of the portfolio P5. This result is mainly given by the low positive extreme return typical of the fifth portfolio.

It has been noticed that the relations between the five portfolios and the long/short portfolio are similar for both the applied methodologies.

Some differences arise when the absolute value of the VaR is considered. Such distinctions are more significant when the two extreme portfolios, P1 and P5, and the long/short portfolio are taken into analysis. Focusing on the VaR(5%), the portfolios P2, P3 and P4 are characterized by almost equal values between the two methodologies. On the other hand, the value at risk of the portfolios P1 and P5 are almost 3 percentage points lower when the single sorting approach based on the return on equity is considered. Moreover, the value at risk of the long/short portfolio based on the ROE is 4 percentage points lower than the one based on the price to book value ratio.

These results strengthen the higher riskiness that characterized the portfolios constructed with the price to book value ratio.

Looking to the VaR(1%), the first thing that is noticeable is that the long/short portfolios, constructed with the two methodologies, have almost equal value at risk. Indeed, the difference is lower than 1%.

It means that the two risk factors may suffer almost the same loss, or higher, with the probability of 1%.

On the other hand, the five portfolios show dissimilarities in their values. Specifically, the higher value at risk are observable in relation to those portfolios which are constructed with the price to book value ratio.

3.2. Asset pricing

Asset pricing theory tries to understand why some assets pay higher average return than others. The theory can be either used to comprehend the reason why prices or returns are what they are or assess if there are trading opportunities linked to some assets mispricing. Uncertainty and correction for risk factors are the main concept behind asset pricing.

The aim of this section is to analyze the effect generated by the risk factors, derived from the two long/short portfolios that have been investigated previously.

In particular, an exercise of asset pricing has been implemented in order to assess the relation between each stock with the common factor and successively the determination of the price of risk. Specifically, this exercise is implemented once for the portfolio constructed with the price to book value ratio and successively with the one derived using the return on equity.

The idea is to find similarities between the two risk factors in terms of betas and price of risk. In particular, the risk factor derived from the price to book value ratio is the one taken into consideration by Fama and French in the three factors model.¹⁷

The main important model implemented for asset pricing is the capital asset pricing model (CAPM). The CAPM is useful for the determination of expected asset returns using the excess returns of the market portfolio as common factor.

The model relies on the concept that the relation between the excess return of each stock is linked to the excess return of the market portfolio.

$$E(r_i) = r_f + \beta_i(E(r_m) - r_f)$$

Where: $E(r_i)$ is the expected return of asset i ;

r_f is the return of a risk-free asset;

$\beta_i = \frac{cov(r_i, r_m)}{\sigma_m^2}$ is the measure of systematic risk;

$E(r_m)$ is the expected market return.

¹⁷ Fama, E. F., & French, K. R. (1996). Multifactor explanations of asset pricing anomalies. *The journal of finance*, 51(1), 55-84.

Despite of the large use of this model, it suffers from some weaknesses.

The main problem is related to the identification of the market portfolio. Indeed, it is impossible to observe the true market portfolio and, therefore, a proxy must be used.

Typically, a value weighted market index is used such as S&P500 or FTSE MIB 40. However, even if a correct market index is used, the proxy may be inefficient and, therefore, it does not correctly represent the true market portfolio, leading to a rejection of the CAPM.

Despite of several empirical test that proves the rejection of the capital asset pricing model, it is still largely used in several financial fields.

In order to overcome the weakness linked to the CAPM, a further theory has been developed, the arbitrage pricing theory (APT).

APT was introduced by Ross, as an alternative to the capital asset pricing model.¹⁸

This theory is more generic, and it does not assume, as the CAPM does, that the markets are perfectly efficient and, therefore, there could be some failure in the pricing process. It is linked to the concept that arbitrageur would take advantage of such mispricing in order to make a profit.

APT does not imply several assumptions such as the identification of the market portfolio and the specification of the utility function. For this reason, it is obtained only an approximate relation between the expected asset returns and the considered risk factors.

Arbitrage pricing theory is developed in such a way a single factor or a multi-factor model can be implemented. Indeed, it can be analyzed the effect generated by a unique factor or it can be assessed how different risk factors affect, simultaneously, asset returns.

The most famous three factor model is the one described in the first chapter where, the market, the small-minus-big (SMB) and the high-minus-low (HML) factors are considered.¹⁹

The following section will report the results linked to the application of the capital asset pricing model for each stock. After that, the focus will be shift on the theoretical description, alongside the presentation of results, of a single factor model.

18 Roll, R., & Ross, S. A. (1980). An empirical investigation of the arbitrage pricing theory. *The Journal of Finance*, 35(5), 1073-1103.

19 Fama, E. F., & French, K. R. (1996). Multifactor explanations of asset pricing anomalies. *The journal of finance*, 51(1), 55-84.

3.2.1. Capital Asset Pricing Model (CAPM)

Before start talking about the derived risk factors and their implication in asset pricing, it must be taken into consideration a simply analysis and test of the capital asset pricing model (CAPM).

The model, as explained previously, is based on the relation between the expected excess return for the stock i and the expected excess return of the market portfolio.

In order to analyze the CAPM in the Italian market, it is necessary to run the following simple time-series regression.

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_i(r_{m,t} - r_{f,t}) + \epsilon_{i,t}$$

Where: $r_{i,t}$ is the return at time t of the stock i ;

$r_{f,t}$ is the return at time t of a risk-free asset;

$r_{m,t}$ is the return at time t of the market portfolio;

$\epsilon_{i,t}$ is the error term of the stock i ;

β_i is the slope coefficient of the stock i ;

α_i is the intercept coefficient of the stock i .

In order to run the described OLS regression it is needed to define some variables.

First of all, the market portfolio corresponds to the Italian stock index FTSE MIB 40, used as proxy.

Therefore, the monthly returns of it have been computed and stored.

Particular importance must be given to the risk-free return. It is considered the monthly yield of a three months Italian Treasury Bill issued by the Government, named Buoni Ordinari del Tesoro (BOT).

The following table reports the result of the time-series linear regression described above.

Capital Asset Pricing Model: results of the time-series regression										
	α	SE α	t-statistic α	p-value α	β	SE beta	t-statistic β	p-value β	R2	Adjusted R2
A2A	0,036	0,447	0,081	0,935	0,914	0,069	13,295	0,000	0,467	0,464
ATL	0,389	0,386	1,008	0,315	0,779	0,062	12,475	0,000	0,484	0,481
AZM	1,469	0,567	2,592	0,010	1,149	0,087	13,143	0,000	0,539	0,535
BGN	2,236	0,796	2,811	0,006	1,127	0,116	9,733	0,000	0,445	0,441
BAMI	-0,708	0,907	-0,781	0,436	1,627	0,132	12,331	0,000	0,563	0,559
BPE	0,115	0,570	0,201	0,841	0,975	0,088	11,024	0,000	0,377	0,374
BRE	1,475	0,551	2,680	0,008	0,937	0,085	11,079	0,000	0,378	0,375
BZU	1,003	0,549	1,829	0,069	1,028	0,085	12,067	0,000	0,420	0,417
CPR	0,385	0,431	0,893	0,373	0,341	0,067	5,085	0,000	0,117	0,112
ENEL	-0,207	0,291	-0,711	0,478	0,762	0,045	17,044	0,000	0,590	0,588
ENI	-0,311	0,302	-1,027	0,305	0,602	0,046	12,953	0,000	0,454	0,451
EXO	1,826	0,643	2,839	0,005	0,859	0,102	8,432	0,000	0,406	0,400
G	0,149	0,315	0,473	0,637	1,066	0,048	22,013	0,000	0,706	0,704
ISP	1,034	0,403	2,563	0,011	1,380	0,063	22,046	0,000	0,708	0,707
LDO	0,686	0,534	1,284	0,201	1,248	0,082	15,208	0,000	0,534	0,531
LUX	0,370	0,404	0,914	0,362	0,664	0,063	10,575	0,000	0,357	0,354
MS	0,363	0,617	0,588	0,557	1,236	0,095	13,040	0,000	0,457	0,454
MB	0,839	0,449	1,867	0,063	1,248	0,070	17,893	0,000	0,614	0,612
PRY	0,920	0,603	1,526	0,130	0,987	0,087	11,293	0,000	0,507	0,503
REC	0,914	0,487	1,879	0,062	0,551	0,076	7,293	0,000	0,209	0,205
SPM	0,172	0,679	0,253	0,800	0,867	0,104	8,313	0,000	0,255	0,251
SRG	-0,429	0,313	-1,370	0,172	0,324	0,049	6,571	0,000	0,184	0,180
STM	0,884	0,644	1,373	0,171	1,329	0,099	13,437	0,000	0,472	0,469
TIT	-0,530	0,468	-1,133	0,259	0,964	0,072	13,409	0,000	0,471	0,468
TEN	0,332	0,760	0,437	0,663	0,697	0,116	6,030	0,000	0,204	0,198
TRN	-0,003	0,323	-0,010	0,992	0,426	0,050	8,447	0,000	0,318	0,314
UBI	0,182	0,543	0,335	0,738	1,330	0,089	14,937	0,000	0,565	0,562
UCG	0,388	0,426	0,910	0,364	1,494	0,066	22,586	0,000	0,717	0,716
UNI	-0,380	0,660	-0,575	0,566	1,037	0,103	10,114	0,000	0,337	0,334
US	0,314	0,824	0,381	0,704	1,327	0,127	10,479	0,000	0,352	0,349

Table 12: Results of the Capital Asset Pricing Model (CAPM) running the following time series linear regression

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_i(r_{m,t} - r_{f,t}) + \epsilon_{i,t}.$$

From the table 12 it is possible to observe that the betas are all large and positive. The slope coefficient indicates the sensitivity of the stock i to changes in the market portfolios and, therefore, it can be seen as a measure of systematic risk. The estimated betas reflect the high sensitivity of all the stocks to movements in the FTSE MIB 40 index.

However, it is necessary to verify whether the estimated slope coefficients are statistically different from zero or if their values are mainly given by chance.

In order to run the test, it must be considered the following t-statistic.

$$t_{\hat{\beta}_i} = \frac{\hat{\beta}_i}{SE_{\hat{\beta}_i}}$$

$$H_0: \hat{\beta}_i = 0$$

$$H_1: \hat{\beta}_i \neq 0$$

Where: $\hat{\beta}_i$ is the estimated slope coefficient of the asset i ;

$SE_{\hat{\beta}_i}$ is the beta standard error of asset i ;

$t_{\hat{\beta}_i} \sim$ t-student with T-2 degree of freedom.

Looking to the table, more precisely looking to the t-statistic of beta, it is possible to observe that, for all the stocks, it is largely greater than the threshold level of 3,291 linked to the 0,001% significance level. Therefore, the slope coefficients are all statistically different from zero, with only a 0,001% probability of rejecting the null hypothesis when it is true.

From the CAPM, the intercept coefficient should be equal to zero. Looking to the table the estimated alphas are almost all positive and large, however, it is necessary to consider the test in order to verify if the estimated values are statistically different from zero.

$$t_{\hat{\alpha}_i} = \frac{\hat{\alpha}_i}{SE_{\hat{\alpha}_i}}$$

$$H_0: \hat{\alpha}_i = 0$$

$$H_1: \hat{\alpha}_i \neq 0$$

Where: $\hat{\alpha}_i$ is the estimated intercept coefficient of the stock i ;

$SE_{\hat{\alpha}_i}$ is the alpha standard error of the stock i ;

$t_{\hat{\alpha}_i} \sim$ t-student with T-2 degree of freedom.

Looking to the reported t-statistics of the intercept coefficients, it is possible to observe that they are smaller than the threshold level of 1,960 linked to 5% significance level for almost all the stocks. Indeed, 25 stocks out of 30 are characterized by an intercept coefficient with a t-statistic not enough in order to reject the null hypothesis. It means that they are not statistically different from zero and that their values can be mainly given by chance instead of a true estimated intercept.

The last coefficients that must be considered, in order to see how much the model is able to explain the variation of the excess return of each stock, are the R-squared and the adjusted R-squared.

From the table it is possible to observe that, for almost all the assets, the R-squared is large and close to 0,5. It means that almost the 50% of the total variation of stocks excess returns is explained by the model.

Once the first-pass regression has been implemented, the focus is shifted on the assessment of the market price of risk through a cross-sectional regression. The second-pass regression is implemented in such a way the mean excess returns of each stock and the estimated betas are used as variables.

$$\bar{r}_i - \bar{r}_f = \gamma_0 + \gamma_1 \hat{\beta}_i + \mu_i$$

Where: $\bar{r}_i$ is the average return of the asset i ;

$\bar{r}_f$ is the average return of the risk-free asset;

$\hat{\beta}_i$ is the estimated beta for the asset i ;

γ_0 is the constant term;

γ_1 is the price of risk of the common factor F ;

μ_i is the error term.

Looking to the gamma zero, the estimated intercept coefficient is equal to 0,112. However, its p-value is so high that it is not possible to say that the null hypothesis, gamma zero equals to zero, is rejected. Indeed, there is 83% probability of rejecting the null hypothesis when it is true, therefore, its value is mainly given by chance.

The slope coefficient, gamma one, is negative and large. Indeed, it takes value equal to -0,9950. However, looking to its statistics it is possible to conclude that, the estimated price of risk is not different from zero with a significance level of 5%. However, since its p-value is equal to 0,065 it is possible to say that the

null hypothesis, gamma one equals to zero, can be rejected at a significance level of 10%. It means that there is a 10% probability that the null hypothesis is rejected when it is true.

Particular importance must be given to the following t-test.

$$t_{\hat{\gamma}_1} = \frac{\hat{\gamma}_1 - (\bar{r}_m - \bar{r}_f)}{SE_{\hat{\gamma}_1}}$$

$$H_0: \hat{\gamma}_1 = \bar{r}_m - \bar{r}_f$$

$$H_1: \hat{\gamma}_1 \neq \bar{r}_m - \bar{r}_f$$

Where: $\hat{\gamma}_1$ is the estimated price of risk of the asset i ;

$SE_{\hat{\gamma}_1}$ is the standard error of the price of risk;

$\bar{r}_m$ is the mean return of the market portfolio;

$\bar{r}_f$ is the mean return of the risk-free asset;

$t_{\hat{\gamma}_1} \sim$ t-student with N-2 degree of freedom.

The above t-statistic can be implemented in order to verify whether the estimated price of risk is statistically equal to the average excess return of the market portfolio with respect the risk-free asset.

The results showed that the market price of risk is statistically equal to the excess market return. Indeed, it is obtained a t-statistic equals to 1,227 which is significantly lower than the threshold value of 2,0484 linked to 5% significance level and 28 degrees of freedom. Therefore, it is possible to say that the null hypothesis (H_0) cannot be rejected.

3.2.2. Single factor model

The single factor model in arbitrage pricing theory is called so since a unique common factor is considered for the assessment of asset returns.

The model has been developed by Ross and the idea is that asset returns are determined by two main components.²⁰ The first one is the expected return of the stock i and the other one is the unexpected component or surprise element. The latter can be divided in two further parts, one related to the common factor and the other one is related to the firm specific risk or idiosyncratic risk.

$$r_i = E(r_i) + \beta_i F + \epsilon_i$$

Where: r_i is the return of i ;

$E(r_i)$ is the expected component of i ;

$\beta_i F + \epsilon_i$ is the unexpected component of i ;

F is the common factor;

β_i is the factor loading of i ;

ϵ_i is the non-systematic risk of i .

The model requires some assumptions about the common risk factor F and the firm specific risk. Particularly, since $\beta_i F + \epsilon_i$ is the unexpected component of the asset i , it is reasonable to assume a zero expected value for F and ϵ_i due to their unpredictability nature, as follows.

$$E(F) = 0 \text{ and } E(\epsilon_i) = 0$$

Alongside with the zero expectation it must be considered that F and epsilon are characterized by their specific volatility.

$$VAR(F) = \sigma_F^2 \text{ and } VAR(\epsilon_i) = \sigma_{\epsilon_i}^2$$

²⁰ Roll, R., & Ross, S. A. (1980). An empirical investigation of the arbitrage pricing theory. *The Journal of Finance*, 35(5), 1073-1103.

The last assumptions are related to orthogonality conditions.

It is assumed that there is no covariance between the idiosyncratic risk of different firms.

$$E(\epsilon_i \epsilon_j) = 0 \quad \forall i \neq j$$

The last orthogonality condition is related to a zero covariance between the common factor F and the firm-specific risk of the asset i .

$$E(F \epsilon_i) = 0 \quad \forall i$$

The unexpected component of the single factor model is necessary for the computation of returns variance.

So far, it has been seen how the factor and the idiosyncratic risk have their own volatility.

This alongside with the orthogonality condition, $E(F \epsilon_i) = 0$, are necessary in order to divide the total variance in two different parts.

One is related to the common factor variation which is explained by the model and the other one is related to the volatility of the epsilon term.

In other word, the total variance is divided in the systematic risk, linked to the common factor, and the firm specific risk.

$$\sigma_i^2 = \beta_i^2 \sigma_F^2 + \sigma_{\epsilon_i}^2$$

Where: σ_i^2 is the total asset returns variance;

$\beta_i^2 \sigma_F^2$ is the systematic risk;

$\sigma_{\epsilon_i}^2$ is the firm-specific risk.

It is clear that the factor loading, β_i , strongly affects the factor volatility weight in the total variance evaluation for the asset i . Indeed, the higher the beta the greater is the impact of the factor variance in the total volatility computation.

As long as it is considered a well-diversified portfolio, through the diversification benefit, it can be reduced the portfolio specific risk in such a way it is really close to zero.

It means that the total variance of the portfolio is only given by the systematic risk and therefore unaffected by the idiosyncratic risk.

$$\sigma_P^2 = \beta_P^2 \sigma_F^2$$

Where: σ_P^2 is the total portfolio variance;

$\beta_P^2 \sigma_F^2$ is the systematic risk.

Previously, it has been described the single factor model. However, it is necessary to use time-series linear regression model in order to estimate the parameters that are not observable.

The OLS regression is useful for the estimation of the intercept (α_i), that corresponds to the expected component of the model, the factor loading (β_i), which is fundamental for the assessment of sensitivity to the common factor changes, and the firm-specific risk (ϵ_i).

The idea is to implement a simple linear regression with a unique regressor that corresponds, in this specific case, to the long/short portfolio constructed with a single sorting approach based once on the price to book value ratio and once on the return on equity.

The following regression model is the one applied for the exercise of asset pricing considering a single factor.

$$r_{i,t} = \alpha_i + \beta_i F_t + \epsilon_{i,t}$$

Where: $r_{i,t}$ is the return time series of the stock i ;

F_t is the time series of the factor;

$\epsilon_{i,t}$ is the error term time-series of the stock i ;

α_i is the constant term of the stock i ;

β_i is the factor loading of the stock i .

As explained previously, the above terms of the linear regression assume a specific meaning within this exercise.

Indeed, the estimate constant term corresponds to expected return, or the expected component of the model, for the stock i . Therefore, it reports the predictable returns for each stock, or the expected return when the common factor is zero.

On the other hand, since the common factor is not predictable, $\beta_i F_t + \epsilon_{i,t}$ corresponds to the unexpected component of the model. Specifically, it can be divided into two parts.

The first one, $(\beta_i F_t)$ is the amount of returns explained by the common factor, which is unique for each stock since the factor loading measure the sensitivity of asset returns to changes in the common factor. The other element of the unexpected component of the model is the error term $\epsilon_{i,t}$. It is the unpredictable return which is specific for each stock and it is not affected by other assets. In particular, epsilon reports the time series of unexpected returns which are, obviously, linked to the idiosyncratic risk, or in other words, to the firm specific risk.

Within this empirical study the factor F is substituted by the two long/short portfolios.

In the first scenario, it is considered the returns time-series of the long P1 short P5 portfolio based on the price to book value ratio, then it is implemented the model with the long/short portfolio constructed through a single sorting approach based on the return on equity.

Once the constant term, the factor loading, and the error terms are estimated a further step in asset pricing must be implemented. It is necessary to estimate the price of risk through a cross-sectional linear regression. It can be interpreted as the reward linked to that specific factor F over a defined time horizon. Specifically, it is the reward obtained for bearing that source of systematic risk within a specific period.

The price of risk can be either positive or negative.

If it is positive than it means that an investor is paid for a certain exposure to that specific risk. On the other hand, if the price of risk is negative than it means an investor pays in order to get that exposure.

Hence, the risk premium estimation is implemented through a cross-sectional regression using as regressors the estimated beta coefficients from the previous regression.

$$\bar{r}_i = \gamma_0 + \gamma_1 \hat{\beta}_i + \mu_i$$

Where: $\bar{r}_i$ is the average returns of the asset i ;

$\hat{\beta}_i$ is the estimated beta for the asset i ;

γ_0 is the constant term;

γ_1 is the price of risk of the common factor F ;

μ_i is the error term.

The estimated betas from the time-series regressions and the respective monthly average returns of the whole sample are the inputs of the cross-sectional regression described above

3.2.2.1. PBV risk factor

The following table reports the results of the time-series linear regression. Specifically, it displays alpha and beta, with the respective statistics, of each stock i .

Successively, the slope coefficients will be stored and used in the second pass regression.

The dependent variable is the returns time series of each stock and, on the other hand, the independent one corresponds to the long/short portfolio. The common factor (LMH) is the one created using a single sorting approach based on the price to book value ratio.

Alongside the alpha and beta coefficients, the table reports the R-squared and adjusted R-squared in order to assess the portion of variation that is explained by the model.

The regression is implemented using the fitlm code on MATLAB.

Asset Pricing: First pass regression, risk factor based on PBV										
	α	SE α	t-statistic α	p-value α	β	SE beta	t-statistic β	p-value β	R2	Adjusted R2
A2A	0,277	0,544	0,509	0,611	0,349	0,073	4,771	0,000	0,101	0,097
ATL	0,794	0,507	1,566	0,119	0,109	0,066	1,656	0,100	0,016	0,010
AZM	1,592	0,771	2,064	0,041	0,361	0,095	3,807	0,000	0,089	0,083
BGN	2,587	0,996	2,597	0,011	0,432	0,113	3,835	0,000	0,111	0,103
BAMI	0,271	0,926	0,293	0,770	1,230	0,105	11,738	0,000	0,539	0,535
BPE	0,691	0,526	1,315	0,190	0,855	0,071	12,125	0,000	0,422	0,420
BRE	1,529	0,652	2,344	0,020	0,134	0,088	1,527	0,128	0,011	0,007
BZU	0,963	0,685	1,405	0,161	0,115	0,092	1,256	0,210	0,008	0,003
CPR	1,191	0,425	2,802	0,006	-0,090	0,057	-1,579	0,116	0,013	0,008
ENEL	0,228	0,392	0,583	0,561	0,292	0,053	5,544	0,000	0,132	0,128
ENI	0,256	0,382	0,671	0,503	0,127	0,051	2,470	0,014	0,029	0,025
EXO	2,773	0,793	3,497	0,001	0,316	0,089	3,552	0,001	0,108	0,100
G	0,192	0,509	0,378	0,706	0,402	0,068	5,882	0,000	0,146	0,142
ISP	0,872	0,629	1,388	0,167	0,648	0,084	7,697	0,000	0,229	0,225
LDO	0,447	0,713	0,627	0,532	0,400	0,096	4,180	0,000	0,080	0,075
LUX	0,803	0,475	1,690	0,093	0,026	0,064	0,415	0,678	0,001	-0,004
MS	0,172	0,763	0,225	0,822	0,442	0,102	4,310	0,000	0,084	0,080
MB	0,892	0,584	1,528	0,128	0,690	0,078	8,810	0,000	0,279	0,275
PRY	0,835	0,826	1,011	0,314	0,110	0,095	1,157	0,249	0,011	0,003
REC	1,469	0,514	2,858	0,005	-0,036	0,069	-0,522	0,602	0,001	-0,004
SPM	0,307	0,768	0,400	0,690	0,095	0,103	0,923	0,357	0,004	-0,001
SRG	0,502	0,322	1,562	0,120	0,083	0,043	1,940	0,054	0,019	0,014
STM	0,390	0,835	0,467	0,641	0,216	0,112	1,927	0,055	0,018	0,013
TIT	-0,364	0,578	-0,630	0,530	0,353	0,078	4,545	0,000	0,093	0,088
TEN	0,413	0,831	0,497	0,620	-0,158	0,100	-1,574	0,118	0,017	0,010
TRN	0,725	0,359	2,019	0,045	0,103	0,045	2,287	0,024	0,033	0,027
UBI	0,755	0,601	1,256	0,211	0,910	0,078	11,715	0,000	0,444	0,441
UCG	0,142	0,649	0,219	0,827	0,779	0,087	8,951	0,000	0,285	0,281
UNI	0,136	0,620	0,220	0,826	0,896	0,083	10,782	0,000	0,366	0,363
US	0,404	0,818	0,494	0,622	1,042	0,110	9,477	0,000	0,308	0,304

Table 13: Results of the following time series linear regression $r_{i,t} = \alpha_i + \beta_i LMH_t + \epsilon_{i,t}$. Where LMH is the returns time series of the long/short portfolio. Long the portfolio containing stocks with low PBV (P1) and short the portfolio containing stocks with high PBV (P5).

First of all, the main important result that must be analyzed is the slope or beta coefficient.

As explained previously, it can be seen as the sensitivity of the stock i to changes in the long/short portfolio. Despite of few exceptions, the beta coefficients are largely positive with an average value higher of 0,4. Positive betas reflect the fact that as long as the long/short portfolio has positive returns then the stocks have a good performance as well. For some stocks the slope is really close to 1 showing a high and positive correlation between stock returns and the LMH risk factor.

In particular, looking to Banco BPM (BAMI) and UnipolSai Assicurazioni (US), it is possible to observe a beta that is statistically greater than 1. It means that those two stocks tend to overreact to eventual movement of the common factor.

Almost all the stocks considered in the study present a positive and high sensitivity to the created risk factor. Indeed, as several papers have shown, the risk factor created using the price to book value ratio is a predictor of future GDP growth.²¹ Therefore, it is reasonable to observe positive betas since positive returns of the long/short portfolio occur in growth period and stocks will positive react to such market conditions. Obviously, some stocks may be more or less affected by this condition. This is explained by differences in the slope coefficient values that reflect the individual stocks sensitivity to the common risk factor and therefore, indirectly, to the GDP movement.

Particular importance must be given to the assessment of the significance level of the beta coefficients. The null hypothesis related to the test is whether the slope coefficient is equal to zero, against the alternative hypothesis, alpha different from zero.

$$t_{\hat{\beta}_i} = \frac{\hat{\beta}_i}{SE_{\hat{\beta}_i}}$$

$$H_0: \hat{\beta}_i = 0$$

$$H_1: \hat{\beta}_i \neq 0$$

Where: $\hat{\beta}_i$ is the estimated slope coefficient of the asset i ;

$SE_{\hat{\beta}_i}$ is the beta standard error of asset i ;

$t_{\hat{\beta}_i} \sim t$ -student with $T-2$ degree of freedom.

Observing the standard errors, for the majority of the stocks, the values are significantly small, therefore, it is likely to observe high t-statistic values.

The minimum threshold level that must be considered, linked to a 5% significance, is of 1,960.

²¹ Liew, J., & Vassalou, M. (2000). Can book-to-market, size and momentum be risk factors that predict economic growth?. *Journal of Financial Economics*, 57(2), 221-245.

From the table 13, the results show that, for almost all the stocks, in particular for those with high and positive betas, the t-statistics are largely greater than the defined threshold level.

Hence, it is possible to say that the betas are significant at least with a 95% confidence interval.

However, the assessment of the slope coefficient quality can be even endorsed looking to the p-values.

A 5% significance level can be interpreted as a 5% likelihood to reject the null hypothesis, beta equal to zero, when it is true. It is possible to conclude that the majority of the stocks are characterized by betas which are significantly different from zero.

Even though, the results seem to be homogeneous among the sample, as introduced previously, some exceptions are displayed in the table. There are some stocks characterized by low beta values, in particular, three of them (TEN, REC and CPR) have negative coefficients.

However, those cases are not linked with a sufficient significance level in such a way the null hypothesis, beta different from zero, can be rejected. It means that there is a high likelihood that the results are given by chance instead of a true relation.

Those stocks characterized by betas lower than 0,1 or even with negative slope coefficient are characterized, as well, by a significance level that is not enough to explain the results. Indeed, the standard errors, the p-values and the t-statistics do not reach sufficient level in such a way it can be rejected the null hypothesis that the long/short portfolio, or risk factor, does not affect that specific stock.

Now let's have a look to the intercept, or alpha, coefficient.

So far it has been explained the single factor model and the meaning of the intercept. Alpha corresponds to the predictable part of stock returns described by the model, or from a statistical point of view is the return of the stock i when the common factor F is equal to zero.

From the table 13 the intercepts seem to be all large and positive. However, an assessment of whether the results are significant must be implemented.

The relevant t-statistic is constructed in such a way it is tested whether the alpha is statistically equal to zero or not.

$$t_{\hat{\alpha}_i} = \frac{\hat{\alpha}_i}{SE_{\hat{\alpha}_i}}$$

$$H_0: \hat{\alpha}_i = 0$$

$$H_1: \hat{\alpha}_i \neq 0$$

Where: $\hat{\alpha}_i$ is the estimated intercept coefficient of the stock i ;

$SE_{\hat{\alpha}_i}$ is the alpha standard error of the stock i ;

$t_{\hat{\alpha}_i} \sim$ t-student with $T-2$ degree of freedom.

Once again, the minimum threshold value that must be considered in order to ensure at least a 5% significance level is 1,960.

This time the results are exactly the opposite of the betas. Indeed, there are few exceptions for which the null hypothesis, alpha equals to zero, is rejected.

Standard errors are almost all large and greater than the estimated alphas, therefore the t-statistics are really low. This is obviously confirmed by the p-values that, for the majority of the stocks, are larger than 0,5, meaning that there is a 50% likelihood that the estimated intercept values are given by chance.

Summing up, the alpha coefficients are not statistically different from zero but, on the other hand, the betas are different from zero with a significance level of at least 5%.

However, the model presents an issue linked to the R-squared.

The R-squared measures the proportion of variation of the dependent variable explained by the model. It shows how much the variation of asset returns is explained by the model, specifically by the beta and the common risk factor.

$$R^2 = \frac{ESS}{TSS}$$

Where: R^2 is the r-squared measure;

ESS is the explained sum of squares ($\hat{\beta}_i^2 \sigma_{LMH}^2$);

TSS is the total sum of squares ($\hat{\beta}_i^2 \sigma_{LMH}^2 + \sigma_{\epsilon_i}^2$).

Table 13 shows that the r-squared values, despite few exceptions, are really low with an average value of 0,133. It means that only a relatively small portion of the total return variance of each asset is explained by the model and, therefore, by the common risk factor.

It means that there could be other risk factors that can be included in the regression in such way a multifactor model would be more suitable.

Looking to the famous three factor model of Fama and French (1996) it includes the market risk, the small minus big (SMB) and high minus low (HML) common risk factors. In this way, they were able to reach an r-squared which was close to 0,90, meaning that almost 90% of the total variation of returns was explained from the model.

Observing the results from the table 13, it is possible to notice that the r-squared values are significantly low, even smaller than 0,10, in correspondence with the non-statistical significance beta coefficients.

Once the first pass regression has been analyzed, the focus is shifted towards the second pass regression. It is necessary a cross-sectional regression for the price of risk estimation. Indeed, before implementing such analysis the inputs of the regression must be properly selected.

First of all, it needs to be computed the sample average of stocks returns for each asset and store them in a vector, which corresponds to the dependent variable.

Then, the estimated betas from the first pass regression must be stored into another vector that corresponds to the independent variable.

Within this analysis, the “slope” coefficient corresponds to the estimated price of risk linked to the common factor F , which in this case is the LMH risk factor based on the market-to-book ratio.

From the implementation of the second pass regression, or cross-sectional regression, the following results have been obtained.

The intercept coefficient (γ_0) is equal to 0,93163 and it is different from zero with a significance level of at least 0,001%.

However, particular importance must be given to the slope coefficient (γ_1) that, in this specific regression, provides an estimation of the price of risk. From MATLAB it is obtained a price of risk equal to -1.2689. Its negative value shows that potential investors are not receiving a positive risk, but they are paying for getting that risk exposure.

The estimated price of risk has significance level of at least 1% for which it is possible to reject the null hypothesis, γ_1 equals zero.

A further analysis that can be implemented, in relation to the slope coefficient, is to test whether the price of risk is statistically equal to the mean return of the long/short portfolio.

The following t-test has been implanted.

$$t_{\hat{\gamma}_1} = \frac{\hat{\gamma}_1 - \bar{r}_{LMH_{pbv}}}{SE_{\hat{\gamma}_1}}$$

$$H_0: \hat{\gamma}_1 = \bar{r}_{LMH_{pbv}}$$

$$H_1: \hat{\gamma}_1 \neq \bar{r}_{LMH_{pbv}}$$

Where: $\hat{\gamma}_1$ is the estimated price of risk;

$\bar{r}_{LMH_{pbv}}$ is the average return of the long/short portfolio based on PBV;

$t_{\hat{\gamma}_1} \sim$ t-student with N-2 degree of freedom.

The above t-statistic has a value of -1,4838 which is not sufficient in order to ensure a significance level of 5% for the rejection of the null hypothesis. For this reason, it is possible to say that the estimated price of risk is statistically equal to the mean return of the long/short portfolio

3.2.2.2. ROE risk factor

So far it has been analyzed the first and the second pass regression in relation to the risk factor created using the price to book value ratio.

This time, the results are linked to the time-series and the cross-sectional regressions implemented for the risk factor generated by a single sorting based on the return on equity.

The common factor (F) corresponds to the long/short portfolio, which moments have been analyzed in the previous sections. Specifically, the long position is taken with respect the portfolio composed by those stocks with the lowest return on equity values (P1) and the short one is taken with respect the portfolio with the highest ROE values (P5). For this reason, the common factor is called low minus high (LMH) as in the price to book value ratio case.

For the estimation of parameters and successively the price of risk, the same methodology, described in the PBV case, is implemented.

The next table reports the intercepts and the betas with the relative statistics for each stock. Alongside them, there are r-squared and adjusted r-squared coefficients linked to the time series regression.

Asset Pricing: First pass regression, risk factor based on ROE										
	α	SE α	t-statistic α	p-value α	β	SE beta	t-statistic β	p-value β	R2	Adjusted R2
A2A	0,514	0,534	0,962	0,337	0,505	0,086	5,885	0,000	0,146	0,142
ATL	0,825	0,512	1,612	0,109	0,124	0,085	1,467	0,144	0,013	0,007
AZM	1,530	0,795	1,925	0,056	0,315	0,126	2,498	0,014	0,040	0,034
BGN	2,525	1,013	2,491	0,014	0,483	0,150	3,223	0,002	0,081	0,073
BAMI	-0,133	1,163	-0,114	0,909	1,145	0,172	6,660	0,000	0,273	0,267
BPE	0,693	0,631	1,099	0,273	0,675	0,101	6,654	0,000	0,180	0,176
BRE	1,755	0,645	2,722	0,007	0,335	0,104	3,235	0,001	0,049	0,045
BZU	1,134	0,682	1,663	0,098	0,275	0,110	2,510	0,013	0,030	0,026
CPR	1,212	0,430	2,820	0,005	-0,043	0,069	-0,624	0,533	0,002	-0,003
ENEL	0,337	0,398	0,846	0,398	0,329	0,064	5,138	0,000	0,116	0,111
ENI	0,335	0,383	0,873	0,384	0,175	0,062	2,846	0,005	0,039	0,034
EXO	2,678	0,800	3,348	0,001	0,389	0,123	3,156	0,002	0,087	0,079
G	0,447	0,497	0,899	0,370	0,563	0,080	7,045	0,000	0,197	0,193
ISP	1,076	0,648	1,662	0,098	0,715	0,104	6,886	0,000	0,192	0,188
LDO	0,862	0,677	1,274	0,204	0,730	0,109	6,713	0,000	0,182	0,178
LUX	0,939	0,473	1,988	0,048	0,168	0,076	2,210	0,028	0,024	0,019
MS	0,636	0,720	0,884	0,378	0,813	0,116	7,021	0,000	0,196	0,192
MB	1,098	0,605	1,816	0,071	0,765	0,097	7,862	0,000	0,235	0,231
PRY	0,845	0,825	1,023	0,308	0,151	0,123	1,227	0,222	0,012	0,004
REC	1,485	0,518	2,870	0,005	-0,011	0,083	-0,127	0,899	0,000	-0,005
SPM	0,386	0,772	0,500	0,617	0,153	0,124	1,235	0,218	0,007	0,003
SRG	0,486	0,327	1,488	0,139	0,039	0,053	0,747	0,456	0,003	-0,002
STM	1,013	0,771	1,312	0,191	0,812	0,124	6,545	0,000	0,175	0,171
TIT	-0,108	0,566	-0,190	0,850	0,529	0,091	5,815	0,000	0,143	0,139
TEN	0,453	0,838	0,541	0,589	-0,124	0,131	-0,949	0,344	0,006	-0,001
TRN	0,668	0,366	1,825	0,070	0,044	0,059	0,748	0,456	0,004	-0,003
UBI	0,738	0,730	1,011	0,313	0,785	0,123	6,404	0,000	0,193	0,188
UCG	0,252	0,703	0,358	0,720	0,731	0,113	6,468	0,000	0,172	0,168
UNI	0,238	0,698	0,341	0,734	0,814	0,112	7,257	0,000	0,208	0,204
US	0,762	0,855	0,891	0,374	1,143	0,138	8,312	0,000	0,255	0,251

Table 14: Results of the following time series linear regression $r_{i,t} = \alpha_i + \beta_i LMH_t + \epsilon_{i,t}$. Where LMH is the returns time series of the long/short portfolio. Long the portfolio containing stocks with low ROE (P1) and short the portfolio containing stocks with high ROE (P5).

Table 14 shows the results of the first pass regression.

Starting from the slope coefficient, it is demonstrated a positive correlation between stock returns and the LMH risk factor. Indeed, despite few exceptions the average beta is greater than 0,5.

It is possible to observe some stocks which are particularly sensitive to changes in the long/short portfolio. Specifically, looking to Banco BPM (BAMI) and UnipolSai Assicurazioni (US), they tend to overreact to the common factor since their betas are greater than one. It means that if F increases in value than those stocks will increase more with respect the long/short portfolio.

There are some exceptions, such as CPR, REC and TEN, that are characterized by negative betas. Therefore, they tend to move towards the opposite direction of the common factor, meaning they can be used as hedging tools with respect that specific risk factor. However, it is necessary, looking to the different statistics, to assess whether betas are statistically different from zero.

The t-statistic is constructed as the ratio between the estimated value and the respective standard errors. The null hypothesis is beta equals to zero against the alternative one that is beta different from zero.

$$t_{\hat{\beta}_i} = \frac{\hat{\beta}_i}{SE_{\hat{\beta}_i}}$$

$$H_0: \hat{\beta}_i = 0$$

$$H_1: \hat{\beta}_i \neq 0$$

Where: $\hat{\beta}_i$ is the estimated slope coefficient of the asset i ;

$SE_{\hat{\beta}_i}$ is the beta standard error of asset i ;

$t_{\hat{\beta}_i} \sim$ t-student with T-2 degree of freedom.

For the majority of the stocks, it could be even sufficient looking to the standard errors in order to say that the null hypothesis is rejected. However, it is necessary to compare the t-statistics of each stock i with the relative threshold level 1,960 linked to the 5% significance level. Therefore, if the t-statistic is greater than 1,960 then the null hypothesis is rejected, at a 5% significance level, otherwise it cannot be rejected.

The t-statistics are significantly greater than the threshold level, strengthening the positive relation between stocks returns and the common factor.

This result is endorsed by the p-values which are largely smaller than 5%. Moreover, most of them are even smaller than 1%, meaning that there is 1% probability of rejecting the null hypothesis when it is true.

Particular importance must be given to those stocks having negative betas that have been named previously. Despite of the particular results linked to a negative slope coefficient, their t-statistics and p-values are not sufficient in order to ensure a rejection of the null hypothesis, beta equals zero.

Their p-values are all greater than 30% and therefore the estimated values are mainly given by chance instead of an actual relation with the risk factor.

Without considering the three cases with negative betas, there are other few stocks having high p-values. Most of them have their beta coefficients close to zero and due to their statistics, it is not possible to reject the null hypothesis at 5% significance level. Meaning that they are not significantly affected by the common risk factor.

Once the main coefficient has been analyzed, a quick sight to the constant term (α) is needed.

It corresponds, from the described model, to the predictable part of stock returns. All the stocks are characterized by positive and large constant term. However, it is not enough looking to the estimated values since the statistical properties must be assessed in order to decide if the expected part of the model is significantly different from zero.

The implemented test, linked to the null hypothesis alpha equals zero, is given by the ratio between the estimated coefficient and its standard error.

$$t_{\hat{\alpha}_i} = \frac{\hat{\alpha}_i}{SE_{\hat{\alpha}_i}}$$

$$H_0: \hat{\alpha}_i = 0$$

$$H_1: \hat{\alpha}_i \neq 0$$

Where: $\hat{\alpha}_i$ is the estimated intercept coefficient of the stock i ;

$SE_{\hat{\alpha}_i}$ is the alpha standard error of the stock i ;

$t_{\hat{\alpha}_i} \sim$ t-student with T-2 degree of freedom.

From table 14, the reported standard errors of the alpha terms are almost all really large or more precisely they are bigger than the estimated values (α). Therefore, the null hypothesis may be not rejected. Indeed, looking to the t-statistics, they are smaller than the 1,960 threshold level. Meaning, that, despite of few exceptions, the null hypothesis is not rejected with a significance level of 5%.

The estimated alpha coefficients are not statistically different from zero. It means that the estimated values from the regression are mainly given by chance. The p-values are really high reaching, for some stocks, a value greater than 0,8.

The single factor model implemented provide really clear and almost homogeneous results. Indeed, for almost all the stocks, the beta coefficient is high, positive and related to a significance level of at least 5%. On the other hand, the alpha coefficients, even if the estimated values are high and positive, are not statistically different from zero, meaning the null hypothesis is not rejected.

Last coefficient that must be considered is the r-squared. As explained several times, it is a measure that reports the proportion of stock returns variation that is explained by the model.

The r-squared values are not significantly high, meaning that only a little portion of the variance is explained by the model. Indeed, it reaches a maximum value of 0,25.

Looking to the lowest value of the r-squared it is possible to notice a coincidence. When it is reported a r-squared significantly low and close to zero in relation to a specific stock i , that firm has a low beta that is not statistically different from zero. Therefore, if the beta is not statistically significant even the model does not work properly having a low r-squared.

As in the previous case, this result is mainly due to the existence of several other risk factors that can be included within the model, such as the size and the value factors used in the famous three factors model by Fama and French (1996).

Once the estimated betas and the mean returns of each stock are stored, the second pass regression is implemented in such a way it is possible to derive the price of risk linked to the common risk factor.

Since the estimated betas seem to be very close to the results of the market-to-book factor, it is reasonable to expect similar results even in the second pass regression.

The cross-sectional regression provided as results an intercept coefficient (γ_0) equals to 1,0026 with a low p-value which can be associated with a significance level of at least 0,001%. It means that the constant term is statistically different from zero.

However, particular importance must be given to the slope coefficient (γ_1), that, as explained previously, corresponds to the price of risk. Its estimated value is negative with a considerable significance level. It is equal to -1,3247 with its statistics that ensure a significance level of at least 0,001%.

It is obtained that the estimated price of risk shows that investors are not receiving a positive risk premium for bearing that exposure.

It is possible to test, through the following t-statistic, whether the estimated price of risk is statistically equal to the average return of the long/short portfolio based on the ROE.

$$t_{\hat{\gamma}_1} = \frac{\hat{\gamma}_1 - \bar{r}_{LMH_{roe}}}{SE_{\hat{\gamma}_1}}$$

$$H_0: \hat{\gamma}_1 = \bar{r}_{LMH_{roe}}$$

$$H_1: \hat{\gamma}_1 \neq \bar{r}_{LMH_{roe}}$$

Where: $\hat{\gamma}_1$ is the estimated price of risk;

$\bar{r}_{LMH_{pbv}}$ is the average return of the long/short portfolio based on ROE;

$t_{\hat{\gamma}_1} \sim$ t-student with N-2 degree of freedom.

The t-statistic is equal to -1,0619 showing a significance level not sufficient to reject the null hypothesis when compared with the threshold level of 2,048. Therefore, it is possible to conclude that the price of risk is statistically equal to the average return of the long/short portfolio based on ROE.

3.2.2.3. Comparison between the two risk factors

So far, the first and second pass regression have been analyzed individually for both risk factors, the one based on the price to book value ratio and the one based on the return on equity.

Now, the focus is shifted on the assessment of eventual similarities and differences among the two approaches.

First of all, let's look to the first pass regression, where the beta coefficients have been estimated.

Looking to table 13 and 14 it is possible to observe similar results. In both cases the betas are high positive and characterized by a significance level of at least 5%. The difference between the betas of each stock, in terms of absolute value, are almost undetectable in the two scenarios.

These similarities show that the two common factors may be linked to the same risk and, therefore, each stock has almost the same sensitivity to changes in both long/short portfolios.

The same three stocks (CPR, REC and TEN) are characterized by negative and non-significant slope coefficients.

The only slight difference that arise is related to those stocks having non-significant betas. Few stocks have significant betas in relation to the risk factor based on the price to book value ratio but non-significant slope coefficients in relation to the risk factor based on the return on equity and vice versa.

Despite of this imperceptible difference, it is possible to conclude that the stocks react almost in the same manner with respect to changes in the two long/short portfolios.

Looking to the intercept coefficient, it is possible to observe other similarities. Indeed, despite few exceptions, each stock has similar alphas when the two risk factors are considered.

The alpha values seem to be non-significant in both scenarios.

The last concept that must be analyzed, in order to assess if the two risk factors or even better the two first-pass regression models are similar, is the r-squared coefficient. The results, again, show similarities between the two approaches for the determination of the long/short portfolios. R-squared values are really low in both cases, showing the need of a multifactor model for asset return explanation.

The only difference is that, when the risk factor based on the price to book value ratio is considered, the model is able to explain a higher proportion of returns variation for some stocks. On the other hand, the model implemented using the return on equity risk factor has a r-squared that ranges between 0,1 and almost 0,3. Despite of this slight difference, considerably the lowest r-squared values are linked to non-significant betas in the two models.

From the moments analysis a relevant result arises in relation to the economic interpretation of the two long/short portfolios. In particular, it has been shown that the long/short portfolio based on the market-to-book ratio has a relation with the business cycles. Indeed, in period of positive GDP growth from 2001 to 2007, it has been proved that stocks with low PBV values strongly outperform stocks with high PBV ratios. However, such relation has not been demonstrated in the ROE scenario. Therefore, the risk factor based on PBV is related to the Italian economic growth, on the other hand, such relation does not arise for the common risk factor based on ROE.

Looking to the two implemented second-pass regressions, it is possible to observe similarities between the two risk factors, the one based on PBV and the one based on ROE.

The estimated price of risk is negative and significantly different from zero in both cases, showing that potential investors are not receiving a premium for bearing that sources of risk.

Moreover, it has been tested whether the estimate risk premium is statistically equal to the average return of the respective long/short portfolios. The results showed that the null hypothesis have not been rejected, meaning that the price of risk, linked to the PBV or ROE factors, is statistically equal to the respective mean return of the long/short portfolio based on PBV or ROE.

3.3. Correlation between PBV and ROE portfolios

The aim of this last section is to study the correlation between the portfolios created with the price to book value ratio and the return on equity.

The relation is analyzed with two different methodologies. One using the sample correlation function present within the MATLAB tools. The other one running an OLS linear regression through the fitlm code. The idea is to study the correlation for all the six portfolios. The five portfolios, from the one containing stocks with the lowest PBV (P1) to the one containing stocks with the highest PBV (P5), will be compared with the respective five created with the return on equity approach. Once those have been analyzed the correlation between the two long/short portfolios, or risk factors, will be implemented.

It is reasonable to expect high, positive and significative correlation coefficients since the previous analyses showed many similarities among the different portfolios. The examination of moments reported similar results for both single sorting methodologies, presenting similar patterns and relations from the portfolio P1 to P5.

The long/short portfolios seem to be reasonably similar risk factors when the moments analysis and the asset pricing exercise have been implemented.

The correlation analysis is the last step necessary for a final assessment that portfolios created with the PBV and those with the ROE are similar and that they may be implemented in order to create similar risk factors in asset pricing.

The first approach implemented for the correlation assessment is through the correlation function.

It has been used, on MATLAB, the corr code that reports the correlation coefficients between two vectors with the respective p-values. More precisely, it is displayed the Pearson correlation coefficient (PCC) or rho coefficient (ρ). Its value ranges between -1 and 1 which correspond respectively to a perfectly negative correlation and a perfectly positive correlation.

The PCC is derived from the covariance between two vectors divided by the product of the two standard deviations. The null hypothesis, linked to the correlation coefficient, is rho equals to zero against the alternative hypothesis rho different from zero.

$$\rho_{X,Y} = \frac{COV(X,Y)}{\sigma_X \sigma_Y}$$

$$H_0: \rho = 0$$

$$H_1: \rho \neq 0$$

Where: $\rho_{x,y}$ is the correlation coefficient;

$COV(X, Y)$ is the covariance between X and Y;

σ_X is the standard deviation of X;

σ_Y is the standard deviation of Y.

Now the focus is on the correlation assessment between the five portfolios.

The next table reports five correlation coefficients with the respective p-values. The first column shows the results linked to the two P1 portfolios (those containing stocks with the lowest PBV or ROE) until the fifth column that reports the correlation between the two P5 portfolios (those containing stocks with the highest PBV or ROE).

Correlation between PBV and ROE portfolios					
	P1	P2	P3	P4	P5
ρ	0,8876	0,9010	0,8531	0,7984	0,8802
p-value ρ	0,0000	0,0000	0,0000	0,0000	0,0000

Table 15: Pearson correlation coefficients between the portfolios created with price to book value ratio and the respective portfolios based on the return on equity, with associated p-values.

Table 15 shows strong and significative results.

The rho coefficients, or the Pearson correlation coefficients, are all significantly large. Indeed, among the five portfolios, the correlation ranges between 0,79 and 0,90. This result shows that the portfolios created

with the single sorting based on the price to book value ratio and the return on equity comove towards the same direction, being almost perfectly correlated.

It is reasonable to expect that as long as the portfolio based on ROE has positive returns than this is observed even in the one based on PBV and vice versa.

The fact that a correlation is not exactly equals to one is due to several factors. First of all, the portfolio rebalance frequency may affect the returns entity. Indeed, as explained in the second chapter, the portfolios based on the price to book value ratio are rebalanced on a monthly frequency, on the other hand, those based on the return on equity are rebalanced every six months due to disclosure periods of financial statements. Another reason that explains the non-perfect portfolio correlation can be even related to a further analysis that has been implemented in the second chapter. Specifically, looking to each stock individually, the correlation between the price to book value ratio and the return on equity has positive but not extreme rho values, since other factors may be considered for the determination of the price to book value ratio, such as the growth rate or the payout ratio.

As always, it is not sufficient looking to the estimated values since it must be implemented a test in such a way the coefficients can be tested if they are statistically different from zero. Indeed, as explained previously, the null hypothesis linked to the Pearson correlation coefficient is whether rho is equal to zero, against the alternative hypothesis, rho different from zero.

The corr function on MATLAB, alongside the estimated parameters, reports the respective p-values. Hence, looking to the table, it is possible to observe p-values so small and close to zero that can be considered as zero. The null hypothesis is strongly rejected and therefore the estimated rho values are significantly different from zero. It means that there exists an actual relation between the several portfolios that it is not given by chance.

Another method for which such relation can be analyzed is through a simple linear regression.

The time-series regression models are created in such a way the dependent variables are the five portfolios constructed using the price to book value ratio and the regressors are the respective portfolios based on the return on equity.

Using the fitlm code on MATLAB five OLS regressions have been implemented.

The next table reports result in terms of estimated parameters with the relative statistics and the r-squared coefficient for each model.

The implemented t-statistic, in order to test whether the slope coefficient is statistically different from zero, is given by the ratio between the estimated beta and its standard deviation.

$$t_{\hat{\beta}_i} = \frac{\hat{\beta}_i}{SE_{\hat{\beta}_i}}$$

$$H_0: \hat{\beta}_i = 0$$

$$H_1: \hat{\beta}_i \neq 0$$

Where: $\hat{\beta}_i$ is the estimated slope coefficient of the asset i ;

$SE_{\hat{\beta}_i}$ is the beta standard error of asset i ;

$t_{\hat{\beta}_i} \sim$ t-student with T-2 degree of freedom.

Regression between PBV and ROE portfolios										
	α	SE α	t-statistic α	p-value α	β	SE beta	t-statistic β	p-value β	R2	Adjusted R2
P1	0,127	0,312	0,407	0,685	0,997	0,036	27,385	0,000	0,788	0,787
P2	0,472	0,212	2,224	0,027	0,854	0,029	29,510	0,000	0,812	0,811
P3	-0,024	0,208	-0,114	0,910	0,791	0,034	23,234	0,000	0,728	0,726
P4	-0,052	0,215	-0,243	0,809	0,775	0,041	18,848	0,000	0,638	0,636
P5	-0,120	0,223	-0,537	0,592	0,996	0,038	26,360	0,000	0,775	0,774

Table 16: Results of the following time series linear regression $r_{PBV,i,t} = \alpha_i + \beta_i r_{ROE,i,t} + \epsilon_{i,t}$ with $i = P1, P2, P3, P4, P5$.
Where $r_{PBV,i,t}$ is the time series returns of the portfolio i constructed with the PBV and $r_{ROE,i,t}$ is the returns time series of the portfolio i constructed with the ROE.

Table 16 shows results in such a way the correlation, previously analyzed, is strengthened.

The focus of the implemented regressions is mainly on the beta analysis because the constant term (α) does not have a relevant meaning. Moreover, looking to the estimated alpha coefficients, those are all small and the null hypothesis, alpha equals to zero, is not rejected except for the portfolio P2.

The slope coefficient, on the other hand, provides a confirm of the correlation that exist among the portfolios. The betas are all high with their values that range between 0,77 and almost 1. The standard errors of the betas are really low involving high t-statistics that ensure a significance level of at least 0,001%. It means that there is almost no probability of reject the null hypothesis when it is true. Therefore,

through the implementation of OLS regressions, it is proved that the couple of portfolios created with the price to book value ratio and the return on equity comove towards the same direction.

The results are even reinforced by the r-squared coefficients. Indeed, they range between 0,63 and 0,81 showing that at least more than the 60% of the total variation is explained by the model.

The correlation analysis now is shifted towards the long/short portfolio.

Its relevant importance is mainly related to its application as risk factor in asset pricing models.

The common factor is constructed taking a long position on the portfolio P1 and a short position on the portfolio P5. This risk factor can be even identified as low-minus-high.

It is reasonable to expect high correlation between the two long/short portfolios.

Using the correlation function on MATLAB, it is derived a rho coefficient of 0,633 with a p-value close to zero. This shows that the two long/short portfolios have a high and statistically positive correlation, entailing the fact that the two risk factors comove towards the same direction.

The following time series regression is applied for the beta estimation between the two long/short portfolios.

$$L/S_{PBV,t} = \alpha + \beta L/S_{ROE,t} + \epsilon_t$$

Where: $L/S_{PBV,t}$ is the returns time series of the long/short portfolio based on the PBV;

$L/S_{ROE,t}$ is the returns time series of the long/short portfolio based on the ROE;

The regression reports a beta equals to 0,763 with a t-statistic significantly high that ensures a rejection of the null hypothesis, beta equals zero, at least at a 0,001% significance level. The r-squared of the model is higher than 0,4, meaning that it works properly and that a great portion of variation is explained by it.

As for the five portfolios analysis, the alpha coefficient is not relevant and additionally it has a low value which is not statistically different from zero.

3.4. Robustness

This section aims to test the robustness of the applied model. Specifically, it verifies if replacing the type of Data needed for the research, the results change drastically.

The reason of such analysis lies on the fact that the long/short portfolio based on the price book value ratio has some characteristics which are against past empirical evidences.²²

The low-minus-high (LMH) risk factor should have a positive mean return since the stocks with low market-to-book ratios tend to outperform those with high market-to-book ratios. However, this is not the result obtained by this empirical study, on the contrary, it is obtained that the long/short portfolio has a negative mean return in the period between January 2001 and January 2018.

This section looks to the implementation of four different tests in order to verify the robustness of the study.

The first test consists on changing the way in which the market-to-book time series is derived.

The price to book value ratio, used in the study, has been directly retrieved from Bloomberg. In order to check if it is correctly computed, it has been stored the book value per share of each stock and used, alongside the prices, to derive the PBV. It was easy to assess that the computed market-to-book ratio and the one retrieved by Bloomberg were exactly the same. For this reason, it is possible to conclude that the abnormal behave of the long/short portfolio is not related to the market-to-book input.

The second test takes into consideration the application of the total return in order to compute the portfolios performance.

The original study is based on the use of stock prices in order to compute simple returns. However, computing the portfolios returns with this approach it is considered only the capital gain in the performance assessment, without contemplating other factors such as the dividend yield. For this reason, it has been retrieved from Bloomberg the total return time-series which take into consideration all the elements that may affect the real stocks performance.

Once such data have been retrieved, the portfolios have been redefined and their returns have been computed using the total return from Bloomberg.

The following table reports the four principle moments of the 5 portfolios based on the price to book value ratio and the long/short one.

²² Fama, E. F., & French, K. R. (1998). Value versus growth: The international evidence. *The journal of finance*, 53(6), 1975-1999.

Moments of the portfolios based on the PBV (in percentage terms)				
	Mean	Standard_Deviation	Kurtosis	Skewness
P1	0,204	9,654	4,651	0,174
P2	0,719	6,918	4,380	-0,017
P3	0,509	5,549	4,418	0,111
P4	0,655	5,220	5,372	-0,599
P5	0,978	6,428	3,704	-0,455
Long P1/ Short P5	-0,774	7,330	4,488	0,257

Table 17: Mean return, standard deviation, kurtosis and skewness of each portfolio constructed using the price to book value ratio. The returns computation is implemented using the total return time-series, therefore, it considers even the dividend yield.

Table 17 reports results which seem to be significantly similar to the one derived in the original analysis. Looking to the standard deviations, kurtosis and skewness they are almost unchanged between the two different approaches in performance evaluation. However, the first moment presents slight changes. It is possible to observe that, for all the five portfolios, the monthly mean returns are higher by few percent points. Moreover, it is noticeable that the portfolio P1, the one containing stocks with the lowest market-to-book values, has a positive mean return instead of a low and negative one from the original data. Despite of these variations, the portfolio P5 still strongly outperforms the portfolio P1 and, therefore, the long/short one has a negative average performance. Specifically, the constructed risk factor has a monthly mean return of -0,774 which is higher, in absolute value, that the one derived by the original study. Hence, this second test provided slight changes in the mean values but not sufficient in order to obtain the long/short portfolio with a positive performance, on the contrary, it has been reinforced the previous result.

The third test takes into consideration the reduction of the overall sample based on the following criteria. All the banks have been excluded from the analysis. This choice is based on the fact that the drop in the FTSE MIB 40 index is linked to the 2007/2008 financial crisis which had a strong negative effect on the banking system. Therefore, without including banks it is reasonable to expect that the portfolios mean returns will be higher.

The original sample consists in 30 stocks but, once all the banks have been removed, it contains only 22 firms. For this reason, 3 portfolios instead of 5 have been constructed using this dataset. Specifically, the first portfolio (P1) contains stocks with the low 30 percentile of the market-to-book, the second one (P2) contains stocks with the middle 40 percentile of the market-to-book and the last one (P3) contains stock with the high 30 percentile of the market-to-book.

The following table reports the four principle moments of the 3 portfolios and the long/short one (long P1 and short P3) using the original prices as tool for returns computation.

Moments of the portfolios based on PBV (in percentage terms)				
	Mean	Standard_Deviation	Kurtosis	Skewness
P1	0,267	7,450	4,803	0,244
P2	0,412	4,861	4,067	-0,275
P3	0,627	6,009	3,837	-0,620
Long P1/ Short P3	-0,359	5,496	4,351	0,096

Table 18: Mean return, standard deviation, kurtosis and skewness of each portfolio constructed using the price to book value ratio. The portfolio construction is implemented using a reduced sample of 22 stocks without including banks.

Table 18 shows results for which the same pattern of the original analysis is observable. Indeed, standard deviation, kurtosis and skewness tend to decrease passing from the first portfolio to the third one.

The only difference is related to the fact that the portfolio P1 has a positive mean return. However, it is not still sufficient in order to obtain a long/short portfolio with positive performance. Indeed, the portfolio containing stocks with the highest price to book value ratios (P3) still strongly outperforms the portfolio containing stocks with the lowest PBV (P1). As it is noticeable the risk factor (LMH) has a mean return that is higher than the one obtained by the original study but not enough to reach a positive performance. Hence, removing all the banks from the sample, it has been shown the robustness of the research confirming the results obtained using the original dataset.

The last test that has been implemented takes into consideration, simultaneously, two of the three changes. Indeed, it is considered the total return time-series for the computation of portfolios performance and, at the same time, it is used the reduced dataset for which all the banks have been removed.

Obviously, as for the previous test, 3 portfolios have been containing, from P1 to P3, stocks with low 30, medium 40 and high 30 values of the market-to-book ratio.

The following table reports the principle four moments of the three portfolios constructed using the single sorting approach and of the long/short portfolio.

Moments of the portfolios based on PBV (in percentage terms)				
	Mean	Standard_Deviation	Kurtosis	Skewness
P1	0,533	7,444	4,919	0,230
P2	0,738	4,792	4,173	-0,319
P3	0,913	6,077	4,025	-0,613
Long P1/ Short P3	-0,380	5,476	4,372	0,079

Table 19: Mean return, standard deviation, kurtosis and skewness of each portfolio constructed using the price to book value ratio. The returns computation is implemented using the total return time-series, therefore it considers even the dividend yield. The portfolio construction is implemented using a reduced sample of 22 stocks without including banks.

Table 19 reports the results of the last test that has been implemented in order to check the robustness of the study.

Once again, standard deviation, kurtosis and skewness are almost unchanged, in particular if they are compared with the table 15.

The only difference arises in relation to the monthly mean returns. Indeed, as it has been proved previously, using the total return in order to measure portfolios performance, it is observable that all the mean values are higher than the one reported from the table 18. However, the increase in performance of the third portfolio, the one containing stocks with the highest PBV, is slightly greater than the first portfolio. Therefore, it is still proved that P3 outperforms P1 on average. Once again, it is obtained a risk factor, the long/short portfolio, with a negative mean return.

Finally, it is possible to conclude that, even if small changes in the mean returns have been observed from the implemented tests, the stocks with high price to book value ratios still outperform those with low price to book value ratios in the period from January 2001 and January 2018. Therefore, it has been checked and confirmed the robustness of the study and of the considered dataset.

Conclusion

The research was focused on the analysis of the portfolios constructed using two single sorting approaches, one based on the market-to-book ratio and the other one on the return on equity. The study showed that there are several similarities and slight differences between the portfolios constructed with the two different methodologies. First of all, the moments of the portfolios based on PBV are similar to those based on ROE, deviating only by few percentage points. In particular, it was shown that the portfolios containing stocks with the highest market-to-book or return on equity values (P5) outperform the respective portfolios containing stocks with the lowest PBV or ROE (P1) in the period from January 2001 to January 2018. Interesting result is related to the performance assessment of portfolios from January 2001 to January 2007, period of positive GDP growth in Italy. It was demonstrated that, in the PBV scenario, the first portfolio strongly outperforms the fifth, however, such result was not obtained for the portfolios based on ROE. Therefore, the market-to-book ratio generates a risk factor which is linked to the GDP growth, on the other hand, it is not possible to say the same for the return on equity case.

Other similarities arise with respect the Jarque-Bera test, since it was proved that all the portfolios are not normal distributed with unknown mean and standard deviation. Moreover, the value at risk shows that portfolios based on PBV are slightly riskier with respect those based on ROE, specifically, they have higher losses linked to tail events.

Particular importance must be given to the exercise of asset pricing. It was shown that the long/short portfolios, long P1 and short P5, based on PBV and ROE explain stock returns since the estimated betas from a single factor model were high, positive and significative for almost all the stocks. Therefore, even if it was proved that only the risk factor based on PBV has a strong relation with business cycles, stocks tend to react in the same manner to changes in the two long/short portfolios. The estimated price of risk, in both cases, is negative reflecting the observed poor performance of FTSE MIB 40 index from January 2001 to January 2018, however, it would be reasonable, for the PBV risk factor, to observe positive risk premium in period of good GDP growth.

Finally, it was analyzed the correlation between portfolios based on PBV and the respective portfolios based on ROE. Results showed that they tend to move towards the same direction.

The slight differences and not perfect correlations between the portfolios based on the two single sorting approaches can be explained by the distinct portfolios rebalance frequencies and by the fact that other factors, such as payout ratio and dividend growth, influence the determination of the market-to-book ratio.

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Summary

Empirical researches tried to assess whether it is possible to use fundamental factors in order to derive portfolios, trying to understand if they are related to eventual common risk factors in asset pricing. The following empirical study seeks to further analyze this topic, using the market-to-book and the return on equity as two distinct tools in portfolio construction. The aim is to compare the characteristics of the derived portfolios and assess their impact in asset pricing, looking to eventual similarities and differences between the two approaches.

There are several different investment strategies for which it is possible to recognize two main philosophies related to fundamental analysis, **value and growth investing**.

Value investing is based on the implementation of some quantitative and qualitative screens in order to identify undervalued stocks which can be included in the portfolio. Fama and French (1998) defined value stocks those firms characterized by high book-to-market, earning-to-price or cash flow-to-price ratios. Specifically, it has been proved that those stocks with high book-to-market ratios have higher average returns than those with low book-to-market. The premium paid is related to the fact that the market is not able to correctly price distressed companies which are characterized by higher earnings growth than expected. For this reason, they are undervalued, but as long as the market is able to correct the pricing process, above average returns are achieved by investing in distressed firms. Contrarian investors, which can be defined as value investors, believe that markets are inefficient and tend to overreact to good and bad news. For this reason, they purchase recent losers and sell recent winners which are respectively undervalued and overvalued stocks.

The book-to-market ratio is fundamental in order to derive the value risk factor (HML). It is obtained simply taking the difference between the returns of the portfolio containing stocks with high book-to-market and the portfolio containing stocks with low book-to-market. Fama and French (1993) showed that the HML risk factor significantly captures common variation in stock returns that is not considered by the capital asset pricing model (CAPM). Such risk factor is linked to market situation such as credit or liquidity crunch that systematically affect the market.

Liew and Vassalou (1999) showed that there exists a positive economic relation between the LMH risk factor and the Gross Domestic Product (GDP) growth.

Growth investing is based on the implementation of some quantitative and qualitative screens in order to identify those stocks characterized by higher earnings growth with respect other firms. Fama and French (1998) defined growth stocks those firms characterized by low book-to-market, earning-to-price or cash

flow-to-price ratios. However, several empirical evidences show that stocks with high values of those ratios outperform those with low values.²³ Meaning that value stocks tend to outperform growth stocks.

Size is one of the main screens that can be implemented in order to identify and select potential growth stocks. Indeed, young companies characterized by high innovation and high potential growth most of the times are small and, therefore, they can be identified within the small cap category. Fama and French (1992) proved that a portfolio containing small cap stocks tends to outperform the one containing big cap stocks. The reason behind such result may be related to the fact that stocks characterized by a small market capitalization are subject to higher transaction costs, less disclosure obligation and, moreover, they are less liquid investments. Fama and French (1993) decided to construct an additional risk factor (SMB). It consists in a portfolio based on a long position with respect the portfolio containing small-cap stocks and a short position on the portfolio containing big-cap stocks. SMB factor significantly captures common variation in stock returns that is not considered by the CAPM and the HML risk factor. The regression model implemented by Fama and French (1996) showed that the estimated slope coefficients, linked to the small-minus-big risk factor, are positive, large and significative.

Liew and Vassalou (1999) showed that there exists a positive economic relation between the SMB risk factor and the Gross Domestic Product (GDP) growth.

In this research **data** have been retrieved from Bloomberg and the period taken into consideration is from January 2001 to January 2018. Thirty Italian stocks, included within the FTSE MIB 40 index, are used in the research. The Data necessary for this study are the prices, the price-book value ratio and the Return-on-Equity of each stock. It has been considered a monthly frequency in such a way the portfolios are rebalanced every month based on a single sorting approach implemented once with PBV and once with ROE.

Since this empirical study seeks to create portfolios, it is necessary to aggregate returns across assets and, therefore, the use of simple returns instead of log-returns is more suitable. They are computed using the historical prices of each stock in the period of interest. Moreover, firms have the same weight when included in the portfolio.

The price book value ratio is a really common ratio used in finance. It is fundamental since it puts into relation the market value of a specific stock with its book value of equity. It is computed simply dividing the price of the stock taken into consideration with its book value of equity per share. The use of such ratio

23 Fama, E. F., & French, K. R. (1998). Value versus growth: The international evidence. *The journal of finance*, 53(6), 1975-1999.

is largely applied in Finance, in particular for the assessment of undervalued or overvalued publicly traded firms. The price-book value ratio has been collected with a monthly frequency but the book value per share is considered at a semi-annual frequency. The described ratio is used as parameter for the construction of portfolios which are sorted with respect different market-to-book ratio values.

The last index that must be taken into consideration for the research is the Return on Equity.

It is a measure of firm's profitability for a certain period. Specifically, it indicates the rate of return earned on the capital provided by shareholders after paying for all the other capital used. Therefore, the ROE is computed as the Net Income available for common shareholders divided by the average total common Equity on a semi-annual frequency. Since the ROE is presented only into financial statements, which are disclosed less frequently, the index is retrieved every 6 months. For this reason, even if the portfolio composition is redefined each month, the true rebalance occurs twice a year when new performance indices are available.

Before looking to the statistical correlation between the price to book value ratio and the return on equity for the stocks included in the study, it is necessary to describe the theoretical relation among them. First of all, it must be considered that it is implemented a discount cash flows (DCF) model to determine the stock price, that can be seen as the present value of future dividends payment. It is considered a perpetuity model with a constant dividend growth rate. The theoretical relation between the two factors, PBV and ROE, is observable simply making some adjustment to the present value equation.

$$\frac{P_0}{BVPS} = \frac{ROE * Payout\ ratio * (1 + g)}{k_e - g}$$

Where: EPS is the earnings per share;

BVPS is the book value of equity per share;

DPS_0 dividend per share at time 0;

g dividend growth rate.

The obtained equation shows that several different factors affect the price to book value ratio. The ROE, the Payout ratio and the growth rate (g) positively influence the ratio, on the other hand, the cost of equity (k_e) has a negative impact over its value. It is expected, from the statistical analysis, that the impact of the ROE over the PBV value is only marginal due to the presence of other relevant ratios.

In order to carry out the statistical analysis two different approaches are implemented. The first one is a linear time-series regression, using the ROE as independent variable and the PBV as the dependent one. The other one is the correlation function included within the MATLAB tools.

The following time-series regression is the one implemented in order to analyze the relation between the market-to-book ratio and the return on equity of each stock.

$$PBV_{t,i} = \alpha_i + \beta_i ROE_{t,i} + \epsilon_{t,i}$$

Where: $\epsilon_{t,i}$ is the error term of the linear regression;

β_i is the factor loading of the linear regression for the stock i ;

α_i is the intercept of the linear regression for the stock i ;

$ROE_{t,i}$ is the return on equity time-series for the stock i ;

$PBV_{t,i}$ is the market-to-book ratio time-series for the stock i .

The results show that the slope coefficient is statistically different from zero for almost all the stocks. Specifically, the estimated betas are large and positive with t-statistics significantly greater than the threshold level of 2.575 linked with the 1% significant level. It is possible to conclude that the return on equity is a factor which strongly affect, in a positive manner, the price to book value ratio. However, the r-squared values, which are not significantly high and close to 1, show that the market-to-book ratio is determined by the ROE and other factors, as it has been proved from the theoretical point of view.

This relation can be analyzed even using the simple correlation function implemented on MATLAB.

Such function gives as results the correlation coefficient and the relative p-value for each stock included within the sample. The rho coefficient (ρ) can take value between -1 and 1, where -1 means that there is a perfectly negative serial correlation and 1 means that there is a perfectly positive serial correlation.

The results of the correlation function show that the price to book value ratio and the return on equity have a really positive and high correlation for almost all the stocks. The rho coefficient on average takes value 0,5 with a p-value smaller than 1%, meaning that the estimated correlations are significantly different from zero. The rho coefficient never reaches the level of the perfect positive correlation ($\rho = 1$). The reason behind such result is obviously related to the influence of several other factors in the price to book value ratio determination.

Portfolio construction has been implemented through a single sorting approach. Indeed, the two factors, singularly, are used in order to create five different portfolios each month. Considering the price to book value ratio, it is sorted from the lowest to the highest values in every period in which the portfolio compositions should change. The prtile function has been used in order to divide the sample in five categories each month. The portfolios are constructed in such a way, starting from the first until the fifth, they contain increasing market-to-book ratios. Indeed, each month the five portfolios contain stocks that better suit their characteristics. The portfolios, from the first to fifth, include, respectively, stocks with low PBV, medium-low PBV, medium PBV, medium-high PBV and high PBV. Successively, returns are computed for each stock and then, using the mean function across assets, the monthly portfolio returns are derived relying on the assumption that each stock has the same weight. Once those operations are concluded, five vectors of returns, linked to the different five portfolios (P1, P2, P3, P4, P5), are derived.

- P1 contains stocks with low values of the PBV;
- P2 contains stocks with medium-low values of the PBV;
- P3 contains stocks with medium values of the PBV;
- P4 contains stocks with medium-high values of the PBV;
- P5 contains stocks with high values of the PBV.

The same process has been implemented for the creation of portfolios based on the return on equity index. Indeed, the ROE vector has been sorted in such a way the five percentiles are identified and used in order to construct different portfolios. The main difference between the two factors, used for the portfolio construction, is that the portfolios generated by the return on equity are actually rebalanced every six months, on the other hand, portfolios based on price to book value ratio are rebalanced on a monthly frequency. The same operations of returns computation are implemented, obtaining five vectors of returns linked to the five different portfolios (P1, P2, P3, P4, P5).

- P1 contains stocks with low values of the ROE;
- P2 contains stocks with medium-low values of the ROE;
- P3 contains stocks with medium values of the ROE;
- P4 contains stocks with medium-high values of the ROE;
- P5 contains stocks with high values of the ROE.

Particular importance must be given to the construction of the two long/short portfolios, which are used as risk factors in asset pricing. The long position is taken with respect the portfolio P1, containing stocks with

low PBV or ROE, and the short position, respectively, on the portfolio P5, containing stocks with high PBV or ROE.

Once it has been described how the portfolios have been constructed, the focus is shifted on the **quantitative analysis** of their characteristics in order to assess whether or not there are some similarities between the two single sorting approaches. At first it is considered the analysis of moments, including mean returns, standard deviations, kurtosis, skewness and value at risk. Successively, a Jarque Bera test has been implemented in order to check whether or not the portfolios returns are normal distributed. Once, the main characteristics have been analyzed an exercise of asset pricing is carried out in order to study the long/short portfolios as potential risk factors. The last analysis includes the assessment of the correlations between the 6 portfolios based on PBV and the respective 6 portfolios based on ROE.

Before reporting results, it is necessary to briefly describe how moments are computed and their meanings. The first moment is the expected value or mean value. It is given by the mean of the returns for the whole sample, from January 2018 to January 2001. The financial interpretation of the first moment corresponds to the approximative expected monthly return that can be generated by investing in that specific asset.

$$E[X] = \mu = \frac{1}{T} \sum_{t=1}^T x_t$$

Where: $E[X]$ is the expected value;

t is the vector of time starting from January 2001 until January 2018;

x_t is the vector of simple returns.

The second moment is the variance. It is the expectation of squared deviation of a random variable (X), that, in this specific case, is the vector of returns. The variance is useful in order to assess the volatility of portfolio returns and, therefore, it can be considered as a measure of risk. For a better comprehension of the variance, it is necessary to consider the standard deviation since it is expressed in the same units of returns.

$$Var(X) = \sigma^2 = \frac{1}{T-1} \sum_{t=1}^T (x_t - \mu)^2$$

$$SD = \sigma = \sqrt{\frac{1}{T-1} \sum_{t=1}^T (x_t - \mu)^2}$$

Where: σ^2 is the variance;

σ is the standard deviation;

μ is the mean value or first moment;

x_t is the vector of simple returns;

T is the number of observations.

The third moment is the skewness, it is a measure that shows the symmetry of a certain distribution. It can take value either positive or negative. If the skewness is negative it means that the left tail is longer or/and fatter than the right one. On the other hand, if it is positive, the opposite is true, therefore, the right tail is longer or fatter than the other one. Skewness can be seen as a measure of risk that shows which are the more likely extreme returns that can be observed for that specific portfolio and which are the more frequent observations. Indeed, a positive skewness shows more frequent small negative returns. On the other hand, a negative skewness shows more frequent small positive returns.

$$\hat{S} = \frac{1}{T} \sum_{t=1}^T \left(\frac{x_t - \mu}{\sigma} \right)^3$$

Where: $\hat{S}$ is the skewness of the distribution;

μ is the mean value or first moment;

σ is the standard deviation or second moment;

x_t is the vector of simple returns;

T is the number of observations.

The fourth moment is the kurtosis, it shows how fat one or both tails are with respect a normal distribution. It must be considered the excess kurtosis, which is the difference between the estimated value and 3, the kurtosis of a normal distribution. A positive excess kurtosis is related to a higher likelihood to observe

extreme returns that can be either positive or negative, in other words there is higher probability of observing tails events.

$$\hat{K} = \frac{1}{T} \sum_{t=1}^T \left(\frac{x_t - \mu}{\sigma} \right)^4$$

$$\text{excess kurtosis} = \hat{K} - 3$$

Where: $\hat{K}$ is the kurtosis of the distribution;

μ is the mean value or first moment;

σ is the standard deviation or second moment;

x_t is the vector of simple returns;

T is the number of observations.

Once the main moments have been computed, it is necessary to implement the Jarque Bera test in order to check whether or not returns are normal distributed.

$$JB = \frac{T}{6} \left[\hat{S}^2 + \frac{1}{4} (\hat{K} - 3)^2 \right]$$

$$H_0 = \text{returns are normal distributed}$$

$$H_1 = \text{returns are not normal distributed}$$

Where: $\hat{K}$ is the kurtosis of the distribution;

$\hat{S}$ is the skewness of the distribution;

T is the number of observations.

An important measure of risk that must be considered is the value at risk (VaR). It is a measure that shows the potential extreme losses in relation with a defined confidence interval of the return distribution. In particular, it is the percentage (return-VaR) or monetary measure of downside risk. The confidence interval is necessary in order to determine which is the likelihood of observing a loss greater than the VaR.

		Moments (in percentage terms)				Jarque Bera test		Value at Risk	
		Mean	Standard deviation	Skewness	Kurtosis	Hypothesis	P-value	VaR(5%)	VaR(1%)
Single sorting based on PBV	P1	-0,029	9,659	0,204	4,690	1	0,001	16,889	26,185
	P2	0,430	6,975	-0,025	4,177	1	0,010	13,291	17,552
	P3	0,190	5,664	0,165	4,499	1	0,003	10,188	13,171
	P4	0,499	5,042	-0,465	4,477	1	0,001	7,524	16,190
	P5	0,674	6,638	-0,525	3,694	1	0,007	12,107	18,109
	Long P1/Short P5	-0,703	7,428	0,262	4,455	1	0,002	13,614	16,801
		Mean	Standard deviation	Skewness	Kurtosis	Hypothesis	P-value	VaR(5%)	VaR(1%)
Single sorting based on ROE	P1	-0,156	8,600	0,428	5,129	1	0,001	13,957	21,108
	P2	-0,050	7,359	-0,056	3,819	1	0,048	13,208	19,629
	P3	0,270	6,106	-0,269	3,829	1	0,023	10,745	16,520
	P4	0,710	5,192	-0,098	4,731	1	0,001	8,641	13,497
	P5	0,798	5,869	-0,711	4,537	1	0,001	9,805	16,864
	Long P1/Short P5	-0,954	6,160	0,506	4,455	1	0,001	9,568	16,085
		Mean	Standard deviation	Skewness	Kurtosis	Hypothesis	P-value	VaR(5%)	VaR(1%)

Table 20: Moments, JB-test, and Value at Risk of all the portfolios constructed with the two single-sorting approaches, one based on the return-on-equity and the other based on the price-to-book value ratio.

From the table it is easy to assess that there are significant similarities between the portfolios constructed with the price book value ratio and those constructed with the return on equity.

Let's deeply analyze the principle **four moments**. It is possible to observe that the estimated values follow the same pattern in both scenarios. Indeed, despite of few differences in the size of the first and second moments, it is observable an increasing mean return from P1 to P5. In both single sorting approaches, the fifth portfolio, containing stocks with the highest PBV or ROE, outperforms the first portfolio, containing stocks with the lowest PBV or ROE, by at least 0,70%. It is possible to observe almost the same pattern for the standard deviation. Indeed, it tends to decrease from P1 to P5, showing that, in both scenarios, the first portfolio is riskier than the fifth with a volatility at least 3 percentage points greater. However, the portfolios P3 and P4 are those with the lower standard deviations in the PBV scenario and P4 has the lowest standard deviation in the ROE case. The two long/short portfolios, that can be identified as LMH risk factors, have both negative mean return and a standard deviation significantly lower than the first portfolio. A slight difference that can be observed is that the second portfolio has a negative mean return in relation with the ROE factor instead of the positive one linked to the PBV. The main distinction between the two single sorting methodologies is their correlation with the GDP growth. Indeed, as Liew and Vassalou (1999) showed, the LMH risk factor based on PBV is positively correlated with the GDP growth. Indeed, the portfolio P1 outperforms on average the portfolio P5 by 0,12 in the period of economic growth from 2001 to 2007. However, this result does not arise for the portfolios based on return on equity, meaning that the two risk factors have different economic interpretations.

Once the first two moments have been analyzed, the focus is shifted on the skewness and kurtosis. The pattern is similar in both single sorting approaches, showing decreasing values of the third and the fourth moments from the first portfolio to the portfolio P5. This fact shows the higher potential profitability of the portfolio containing stocks with the lowest return on equity or price to book value ratio values. Indeed, the higher and positive skewness is linked to the greater likelihood of observing extreme positive returns. Those potential extreme returns may be linked to eventual undervalued firms which value is adjusted by the market, generating above average performances. The portfolio P5, again in both cases, is characterized by a negative skewness and a lower kurtosis. For this reason, the risk of observing extreme negative returns is higher, however, small positive returns are more frequent. Similarities arise even when the long/short portfolios are considered. Indeed, despite of few differences in percentage terms of such moments, the two portfolios are characterized by positive skewness and quite large kurtosis, showing higher positive extreme returns and more frequent small negative returns.

Once skewness and kurtosis have been analyzed, it is possible to implement the **JB-test** checking if portfolios are normal distributed. Looking to the table, specifically to the column hypothesis, it is possible to observe that it contains all values equal to 1. It means that all the null hypothesis, returns are normal distributed, are rejected with a significance level of at least 5%, as it is shown by the p-value column. Therefore, all the portfolios constructed with the return on equity or the price to book value ratio are not normal distributed and, therefore, not symmetric as it has been shown from the skewness and kurtosis analysis.

The last source of risk that must be analyzed is the **value at risk**. Once again, it is possible to notice some similarities and differences between the two single sorting approaches in portfolio construction. First of all, an actual pattern is not observable in both methodologies. Moreover, the portfolio P4, based on ROE and PBV, is the one with the lowest VaR(5%). However, particular importance must be given to the first and the fifth portfolios. The portfolio P5, constructed with both single sorting approaches, is characterized by a VaR(5%) almost 4 percentage points lower than the portfolio P1. It means that the latter is riskier than the former as it has been shown from the volatility assessment. Moreover, the VaR(5%) and VaR(1%) of the long/short portfolios are lower than the estimated value at risk of the fifth portfolio except for the VaR(5%) in the PBV scenario. These results are due to the portfolios P5, which are the one shorted in the LMH risk factors, are characterized by low positive extreme returns.

Looking to the value at risk of the two extreme portfolios and the long/short one, some differences arise between the two single sorting approaches. Specifically, the VaR(5%) of the portfolios P1 and P5 are almost

3 percentage points lower when the single sorting approach based on the return on equity is considered. Moreover, the value at risk of the long/short portfolio based on the ROE is 4 percentage points lower than the one based on the price to book value ratio.

Looking to the VaR(1%), the first thing that is noticeable is that the long/short portfolios, constructed with the two methodologies, have almost equal value at risk. On the other hand, the five portfolios show dissimilarities in their values, indeed, the higher value at risk are observable in relation to those portfolios which are constructed with the price to book value ratio.

Once the characteristics and specifics of all the portfolios constructed with the PBV and those with the ROE have been analyzed, an exercise of **asset pricing** is needed in order to understand the impact of the two low-minus-high risk factors in the determination of stock returns.

The **capital asset pricing model (CAPM)** seeks to identify the correlation between stock excess return and the market excess return.

The market portfolio is identified as the FTSE MIB 40 index and the risk-free asset is the monthly yield of a three months Italian treasury bill issued by the government (BOT).

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_i(r_{m,t} - r_{f,t}) + \epsilon_{i,t}$$

Where: $r_{i,t}$ is the return at time t of the stock i ;

$r_{f,t}$ is the return at time t of a risk-free asset;

$r_{m,t}$ is the return at time t of the market portfolio;

$\epsilon_{i,t}$ is the error term of the stock i ;

β_i is the slope coefficient of the stock i ;

α_i is the intercept coefficient of the stock i .

The results of the time-series linear regression, described above, show a significative positive relation between the excess returns of each stock and the market excess return. Indeed, the beta coefficients are all large and positive showing a high sensitivity of the stock i to changes in the market portfolio. Beta can be seen as a measure of systematic risk, specifically, the market risk. Particular importance must be given to the statistical properties of the slope coefficients. It is proved that, for all the 30 stocks included in the sample, the t-statistics are all significantly greater than the threshold level of 3,291 linked to the 0,001% significance level. It means that the null hypothesis, beta equals zero, is rejected for all the stocks. On the

other hand, the intercept coefficients are statistically equal to zero since almost all the firms have really high p-values. The last parameter that must be analyzed is the r-squared, which measures the proportion of variability explained by the model. It is on average equal to 0,5, meaning that the market portfolio explains the 50% of the overall return volatility. Once the betas have been estimated it is necessary to run a cross-sectional regression in order to derive the market price of risk.

$$\bar{r}_i - \bar{r}_f = \gamma_0 + \gamma_1 \hat{\beta}_i + \mu_i$$

Where: $\bar{r}_i$ is the average return of the asset i ;

$\bar{r}_f$ is the average return of the risk-free asset;

$\hat{\beta}_i$ is the estimated beta for the asset i ;

γ_0 is the constant term;

γ_1 is the price of risk of the common factor F ;

μ_i is the error term.

The slope coefficient, gamma one, is negative and large. Indeed, it takes value equal to -0,9950. However, looking to its statistics it is possible to conclude that, the estimated price of risk is not different from zero with a significance level of 5%. It could be useful to assess whether this estimated market price of risk is statistically equal to the mean excess return of the market portfolio. The implemented t-statistic is equal to 1,227 which is significantly lower than the threshold level of 1,960 linked to a 5% significance level. Therefore, the null hypothesis (H_0), market price of risk equals the mean excess return of the market portfolio, cannot be rejected.

Despite of the large use of the CAPM, it suffers from some weaknesses. In particular the main problem is related to the identification of the market portfolio. Indeed, it is impossible to observe the true market portfolio and, therefore, a proxy must be used such as a stock index. In order to overcome the weakness linked to the CAPM, a further theory has been developed, the arbitrage pricing theory (APT).²⁴

This theory is more generic, and it does not assume, as the CAPM does, that the markets are perfectly efficient. Moreover, it does not imply the identification of a market portfolio or the specification of the utility function.

24 Roll, R., & Ross, S. A. (1980). An empirical investigation of the arbitrage pricing theory. *The Journal of Finance*, 35(5), 1073-1103.

A **single factor model** is necessary in order to assess the effect of the derived common risk factors in asset pricing and the following time-series regression is used in order to estimate the beta coefficients of each stock.

$$r_{i,t} = \alpha_i + \beta_i F_t + \epsilon_{i,t}$$

Where: $r_{i,t}$ is the return time series of the stock i ;

F_t is the time series of the factor;

$\epsilon_{i,t}$ is the error term time-series of the stock i ;

α_i is the constant term of the stock i ;

β_i is the factor loading of the stock i .

Within this empirical study the factor F is substituted with the two long/short portfolios.

In the first scenario, it is considered the returns time-series of the long P1 short P5 portfolio based on the price to book value ratio, then it is implemented the model with the long/short portfolio constructed through a single sorting approach based on the return on equity.

Looking to the results of both first-pass regressions that have been implemented, the beta coefficients are almost all high, positive and with a significance level of at least 5%. The estimated slope coefficients are essentially the same between the two LMH risk factors, the one based on PBV and the one based on ROE. Three stocks (CPR, REC and TEN) are characterized by negative and non-significant slope coefficients for both common factors. Therefore, it is possible to conclude that almost all the stocks have the same sensitivity to changes in the two long/short portfolios. Looking to the intercept coefficient, it is possible to observe other similarities. In particular, the estimated alpha values are non-significant in both scenarios, showing that the predictable part of stock returns is not statistically different from zero. These results show that the two long/short portfolios are similar in asset pricing even if it has been proved that only the LMH based on PBV is linked to the future GDP growth.

R-squared values are really low in both cases, showing the need of a multifactor model for asset returns explanation, since the two LMH risk factors, singularly, are not able to explain a large proportion of the total variation. However, the coefficient of determination is slightly higher for the PBV risk factor. Once the first-pass regression has been implemented, a cross-sectional one is needed in order to estimate the price of risk.

$$\bar{r}_i = \gamma_0 + \gamma_1 \hat{\beta}_i + \mu_i$$

Where: $\bar{r}_i$ is the average return of the asset i ;

$\hat{\beta}_i$ is the estimated beta for the asset i ;

γ_0 is the constant term;

γ_1 is the price of risk of the common factor F ;

μ_i is the error term.

The estimated price of risk is negative and significantly different from zero in both cases, showing that potential investors are not receiving a premium for bearing that sources of risk from January 2001 to January 2018. Moreover, it has been tested whether the estimate risk premium is statistically equal to the average return of the respective long/short portfolios. The results showed that the null hypothesis have not been rejected, meaning that the price of risk, linked to the PBV or ROE factors, are statistically equal to the respective mean return of the long/short portfolios based on PBV or ROE.

The last analysis aims to study the **correlation** between the portfolios created with the price to book value ratio and the respective portfolios created with the return on equity. The relation is examined with two different methodologies. One using the sample correlation function included within the MATLAB tools. The other one running an OLS linear regression through the fitlm code.

The following table reports the Pearson correlation coefficients and the relative p-values of all the 6 portfolios.

Correlation between PBV and ROE portfolios						
	P1	P2	P3	P4	P5	Long/Short
ρ	0,8876	0,9010	0,8531	0,7984	0,8802	0,6327
p-value ρ	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000

Table 21: Pearson correlation coefficients between the portfolios created with the price to book value ratio and the return on equity, with associated p-values.

The Pearson correlation coefficients are all significantly large with values between 0,63 and 0,90. Moreover, their p-values are all small and close to zero, showing that the coefficients are significantly different from zero. This result shows that the portfolios created with the two single sorting approach based on price to book value ratio and return on equity comove towards the same direction, being almost perfectly

correlated. The fact that the correlation is not exactly equal to one is due to the different portfolio rebalance frequencies and, moreover, as it has been shown previously, the price to book value ratio is determined by other factors such as dividend growth rate and payout ratio. However, the correlation between the long/short portfolios is the lowest since it has been proved how they tend to react differently to period of GDP growth. Running a simple linear regression using the portfolios based on PBV as dependent variable and as independent one the respective portfolios based on ROE. The results report high and positive beta coefficients with values between 0,76 and 0,99. The relative statistics ensure a significance level of at least 0,001%. However, even in this case the long/short portfolio is the one with the lowest beta. The regressions results are reinforced by the r-squared coefficients. Indeed, they range between 0,63 and 0,81, showing that at least more than the 60% of the total variation is explained by the model.

Several tests have been implemented in order to verify the **robustness** of the research. The reason of such analysis lies on the fact that, in this empirical study, stocks with high market-to-book values outperform those with low market-to-book values, concept which is against past empirical studies.²⁵

Four main changes have been applied to the dataset, in order to check the robustness of the research.

- It has been checked if the PBV time-series retrieved from Bloomberg is the same than the one computed dividing the price by the book value per share.
- It has been used the total return time-series instead of simple returns computed using prices in order to evaluate portfolio performances, including dividend yields in the computations.
- Banks have been removed from the sample in order to avoid biases due to the recent financial crisis of 2007/2008.
- Banks have been removed from the sample and the total returns have been considered in order to evaluate portfolio performances.

The results of the above tests confirmed the robustness of the study. Indeed, in all the scenarios, stocks with high market-to-book values still outperform those with low market-to-book values for the Italian market in the period from January 2001 and January 2018.

²⁵ Fama, E. F., & French, K. R. (1992). The cross-section of expected stock returns. *the Journal of Finance*, 47(2), 427-465.