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## **The Right Momentum**

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# Abstract

In this final thesis we analyze momentum investing focusing on time series application. We first introduce momentum risk premium, behavioral theories and then we discuss the profitability of momentum trading. We prove the momentum effect through Sharpe analysis, multiple regression and backtesting procedures. We finally elaborate and present an automated momentum strategy that uses both relative and absolute momentum to generate abnormal returns.

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# Introduction

Whether we decide to invest our savings or our time into a project, a new business opportunity or into something that seems to be appealing to us, we strongly believe that our choice will be promising. When we do that we are betting, and every bet implies some risk. What we gather at the end it's the willingness to accept the risk, a premium.

In Finance the risk premium it is defined as *"the return in excess of the risk-free rate of return that an investment is expected to yield"* <sup>1</sup>. Maximizing this premium for a given level of risk it is always been the principal objective of modern portfolio theory. It's also well-known that Economics it's not an exact science, but a social science and the word social means human, which in turn implies irrationality, which again is the expression of our behavior. In recent years we filled this gap and we start to move away from the traditional view of risk premium into a new one, the so-called alternative risk premium.

Traditional risk premia are returns that can be harvested passively from directional long exposures in asset classes such as stocks, bonds, commodities and so forth. By contrast, ARP <sup>2</sup> is a dynamic and systematic source of return that behaves differently from those in traditional markets <sup>3</sup>. While traditional risk premium includes factor investing <sup>4</sup> or smart beta strategies <sup>5</sup> that are related to systematic risk, the latter is structured as a long-short investment and it may be independent of traditional risk premia. Moreover, ARP should not be confused with alpha strategies, which are believed to be driven by a manager's security selection and market-timing skills. The ARP on traditional financial assets such as commodities, equity or currencies often results from market behaviors and structured conditions of the market itself. If we start selling losers

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<sup>1</sup>Risk Premium, Investopedia [Internet]. Available from: <https://www.investopedia.com/terms/d/drawdown.asp> [Accessed 20 September 2018]. Alternative Risk Premia.

<sup>2</sup>Alternative Risk Premia.

<sup>3</sup>Communication department (2017), Alternative Risk Premia: What Do We Know? Viewpoint Amundi Asset Management [Internet]. Available from: <http://research-center.amundi.com/> [Accessed 20 September 2018].

<sup>4</sup>Factor investing is an investment approach that involves targeting quantifiable firm characteristics or "factors" that can explain differences in stock returns. Over the last 50 years, academic research has identified hundreds of factors that impact stock returns, Wikipedia [Internet]. Available from: [https://en.wikipedia.org/wiki/Factor\\_investing](https://en.wikipedia.org/wiki/Factor_investing) [Accessed 20 September 2018].

<sup>5</sup>Smart beta investing combines the benefits of passive investing and the advantages of active investing strategies Investopedia [Internet]. Available from: <https://www.investopedia.com/terms/d/smartbeta.asp> [Accessed 20 September 2018].

and buying winners, we are creating momentum. If we are betting on a cheap asset that has potential growth and a higher fair price, we are considering value investing. If we think that an asset yield is mispriced, we believe in carry opportunities.

These are the most common sources of ARP and are also well-known for many years among investors, the key difference is how they are implemented. In fact, if ARP strategies are included in portfolios alongside traditional investments, they enhance diversification and provide benefits (e.g. reduce drawdown exposure <sup>6</sup>) since they exhibit heterogeneous statistical properties.

After the 2008 global financial crisis, the diversification has become the primary objective of institutional investors and wealth funds who needed to restructure their portfolio in a low correlated environment. To complicate the situation, it was also the uncertainty of future behavior of the stock-bond correlation that continues to be negative since the 90's and should revert to positive over the long run <sup>7</sup>. As a reaction wealth funds and sophisticated investors have switched from common diversification procedures to a deleveraging equity solution using sovereign bond as hedging assets and corporate bond as lower beta assets (sovereign bonds are felt more as assets for hedging than for performance).

ARP are expanding the universe of traditional risk premia and are becoming the building blocks of multi-asset management. In 2017, a survey from bFinance <sup>8</sup> suggests that ARP has been the area of the greatest interest among investors on a rolling 12-month basis. In the same year, Deutsche Bank reported some survey results <sup>9</sup> showing the growth of the percentage of investors who allocate to ARP strategies, with an increase of 20% from 2015 the level has reached the 26 % in 2017. Furthermore, a recent Prime Brokerage survey <sup>10</sup> from Morgan Stanley reports that 79% of investors with more than \$5 billion of assets under management in hedge fund investments currently rely on ARP strategies or are considering to allocating on ARP.

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<sup>6</sup>Drawdown, Investopedia [Internet]. Available from: <https://www.investopedia.com/terms/d/drawdown.asp> [Accessed 20 September 2018].

<sup>7</sup>If economic conditions and monetary policy permit this change.

<sup>8</sup>"Manager Intelligence and Market Trends", bFinance (February 2017).

<sup>9</sup>15th Annual Alternative Investment Survey," Deutsche Bank (March 2017).

<sup>10</sup>"Recent Hedge Fund Trends," Morgan Stanley Prime Brokerage-Strategic Content Group (July 2017).

# Chapter 1

## Momentum Risk Premium

### 1.1 Introduction

*“A popular view is that individuals tend to overreact to information” and “if stock prices either overreact or underreact to information, profitable trading strategies that select stocks based on their past returns will exist.” Titman and Jegadeesh 1993.<sup>1</sup>*

In 1993 Titman and Jegadeesh first introduced the concept of Relative Momentum focusing on positive returns of stocks generated by buying recent winners and selling recent losers. Nowadays Momentum is considered among investors one of the oldest and most popular trading strategies over the entire industry. Often in contrast with contrarian investors that consider overreaction and underreaction of assets as something temporary, that will mean revert in the future. Contrarian’s strategy is based on the fact that some stocks are mispriced by the market and will revert to their fair value in the future. Specifically, by buying recent losers and selling recent winners they try to anticipate this process of mean-reversion. Decisions are made estimating the fundamental value of an asset and comparing it to his market value. Therefore, the contrarian approach is considered a form of value investing. Differently, momentum investors follow the trend and the market by betting on the under-reaction or on the over-reaction itself. This explains why they are considered as “lazy” investors and their strategies as trend following approaches. It may appear that *“value and contrarian investing are gratifying while momentum investing is shameful”*<sup>2</sup> but if we look at the composition of the quarterly portfolio holdings of 155 equity mutual from 1974 to 1984 we discover that the 77% of them were momentum investors (Grinblatt and Wermers, 1995).

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<sup>1</sup>Jegadeesh and Titman, 1993

<sup>2</sup>Philippe Ithurbide et al. (2017), Keep Up The Momentum, Discussion Paper - cross asset Investment Strategy.



## 1.2 Relative and Absolute Momentum

In order to fill the gap between momentum strategies and momentum risk premia, we need to distinguish momentum time series from cross-sectional momentum. These strategies both assume that the past trend is a predictor of the future trend and that the initial underreaction (anchoring<sup>3</sup>) and subsequent overreaction (herding<sup>4</sup>) may lead to price move continuation. What differs in terms of risk premia is the net exposure.

In time series momentum the net exposure is not equal to zero whereas in cross-sectional momentum it is equal to zero. In fact, time series momentum strategies are implemented considering only single assets and using their past history as a benchmark, e.g. buying an asset with recent positive performance or sell it if the recent performance is negative. These strategies create a positive or negative net exposure depending on the direction of the trend that has been followed.

Differently, cross-sectional momentum is not implemented on a standalone basis, but by considering a set of assets and by assuming that current winners will continue to outperform current losers, e.g. buying assets that have the best recent returns and selling the same number of assets that exhibit the worst performances. These strategies create portfolios that go long on outperformers and go short on underperformers ensuing a zero-net exposure. We may state in a very simple manner that absolute momentum allows comparing performance with himself, while relative momentum allows comparing performance with his peers.

## 1.3 The Effect of Correlation

Another distinction among momentum applications is made in terms of correlation. In particular, Roncalli et al. starting from the model of Brouder and Gaussel<sup>5</sup> showed how the impact of correlation varies between relative and absolute momentum. Their results are derived analytically from the assumption that an asset price  $s_t$  follows a geometric Brownian motion with constant volatility and with a time-varying trend.

Absolute momentum does not benefit from correlation<sup>6</sup>: if we look at Figure 1.1 we notice that the diversification gain is limited when adding more than three assets confirming the fact that this kind of strategy is indifferent from a positive or a negative correlation. Furthermore, Jusselin et al (2017) starting from the analytical model of Brouder and Gaussel went deeper inside the question of how the correlation of asset affects absolute momentum. The main result is that the sign of correlation does not influence the performance of a trend following strategy, specifically they demonstrated that the profit & loss scheme of such strategy does not depend on the sign of the

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<sup>3</sup>It refers to a behavioral finance phenomenon called anchoring behavior.

<sup>4</sup>It refers to a behavioral finance phenomenon called herding behavior.

<sup>5</sup>Brouder and Gaussel, 2011

<sup>6</sup>The tests on correlation are derived analytically.

correlation: a correlation of 80% has the same impact of a correlation of -80% (Figure 1.2 shows this effect).

Differently, relative momentum is a “friend of correlation” since his Sharpe ratio increases simultaneously with the correlation (Figure 1.3). The performance of a cross-sectional strategy depends more on the relative differences between the trends of the selected assets, i.e. the spread between only positive or negative trends as well as the spread between the trends of the winners and the trends of the losers<sup>7</sup>. Moreover, a high correlation among assets<sup>8</sup> will reduce the uncertainty of the outcome and the subsequent dispersion of the profit & loss of the strategy capturing the spread-risk between trends<sup>9</sup> and leaving the directional risk<sup>10</sup> of trend aside. In this last case its useful to compare the relative strategy with pairs trading (considering each pair as one investment bet). This trading approach looks similar to the one used in cross-sectional momentum since the performance of the strategy is related to a long/short matching of assets.

Since absolute momentum follows the market or the trend direction creating positive or negative net exposure, in case of big trends the time-series strategy should concentrate its exposure entirely on the asset that exhibits the trend (single bet) rather than diversifies the exposure on different assets (multiple bets). This is not the case of relative momentum that should be exposed to many relative bets in order to capture their spread risk. The choice of the universe is then important when considering a trend-following strategy. In a portfolio construction process, time series momentum should be used over a multi-asset universe (i.e. equity, currency, and commodity), whereas cross-sectional momentum strategies should be applied in a universe of homogeneous securities (i.e. geographical or sectorial divisions). A possible approach would follow an implementation of absolute momentum at a global level and relative momentum at a categorical level. From now on we will focus only on absolute momentum and on his applications (Roncalli et al. 2017).

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<sup>7</sup>The fact that relative momentum captures two opposite trends at the same time is quite difficult in reality (Roncalli, 2017a).

<sup>8</sup>With high correlation we mean that all the assets within the dataset need to be positively or negatively correlated in order to better capture the spread risk between trends. Best results are found when the correlation is positive and high (Roncalli, 2017b).

<sup>9</sup>Relative difference between trends.

<sup>10</sup>It is the risk of taking the “wrong” direction of the market.

## 1.4 Behavioral Theories and Market Efficiency

Momentum is based on the autocorrelation between asset returns which in turns assumes that there will be a trend continuation in the future (positive or negative). This concept goes against the two biggest assumptions of modern finance: the efficient market hypothesis and the random walk hypothesis. The efficient market hypothesis states that markets are efficient and that all private and public information is incorporated in the asset price. This excludes the possibility among investors to obtain abnormal rates of returns from the market<sup>11</sup>. EMH<sup>12</sup> was often equated to RWH<sup>13</sup> since they share common principles. The random walk hypothesis implies that aggressive competition among market participants to exploit any predictable patterns makes price changes fully random and unpredictable<sup>14</sup>. However, the EMH allows time variation in rational risk premia whereas the RWH assumes the case of constant expected returns. Thus, according to the evidence of return predictability only RMH is rejected (Anti Ilmanen 2011). Since financial markets are not exact and predictable we may encounter in some errors which are often described as anomalies or inefficiencies of the market.

Anomalies are distortions associated with the impossibility of an asset price to reflect his fair value. These errors suggested a more dynamic view of the market, formalized in the Adaptive market hypothesis. AMH<sup>15</sup> applies evolutionary and ecological principles to financial markets showing that human behavioral interactions “stimulate” the market and may reduce his degree of efficiency. Following EMH, anomalies are the reason for their own elimination and the ensuing finite process of exploitation self-stabilizes the market.

However, boom events such as the Tech bubble and the 2008 Global financial crisis caused a “lack of fundamentalism” among market operators carrying on the idea of persistence of these inefficiencies. An interesting point of view is expressed by Atti Ilmanen in his book (Expected Returns 2011):

*“To me, a fair conclusion is that recent events have undermined the validity of the EMH’s main idea (that market prices are always “right”, near the fair value), but have underlined the validity of its main implication for most investors (that beating the markets is extremely difficult, no free lunches). It is ironic that while the EMH is seen as one scapegoat for the 2008 crisis, behavioral finance has survived unscathed, with its reputation even enhanced. Although the EMH paradigm has faced a vigorous challenge from behavioral finance, there is no doubt that the EMH has been a powerful organizing principle for theoretical and empirical work in finance that has improved our understanding of asset returns.”*

Obviously, Behavioral finance shakes things up and turned the inefficiencies of the market into

<sup>11</sup>Strong form of Efficient market hypothesis.

<sup>12</sup>Efficient market hypothesis.

<sup>13</sup>Random walk hypothesis.

<sup>14</sup>Ilmanen, A. (2011), Expected returns: An investor’s guide to harvesting market rewards. John Wiley & Sons.

<sup>15</sup>Adaptive market hypothesis.

social and psychological behaviors. Anomalies such as Momentum are explained by BF<sup>16</sup> as an initial underreaction (anchoring) and a subsequent over-reaction (herding) of the value of an asset that may cause price move continuation. These anchoring and herding behaviors are fueled by the market itself. In particular, herd behavior is the tendency for individuals to mimic the actions (rational or irrational) of a larger group. In the financial market, this occurs when market operators “follow the trend” and copy the behavior of other investors by buying and selling the same asset. Similarly, anchoring demonstrates the tendency to attach or “anchor” our thoughts to a reference point, even if it may have no logical relevance. It is a cognitive bias where individuals place too much weight on the first piece of information that they get to make investment decisions<sup>17</sup>. In financial markets, the investor puts more weight on the latest or the most outstanding stock price, which becomes the anchor in making future stock predictions and leads to an initial under-reaction. Furthermore, the disposition effect that relates to the tendency of investors to sell shares whose price has increased while keeping assets that have dropped in value, may fuel the momentum anomaly by exacerbating the under-reaction since investors are more reluctant to sell and are more likely to realize large losses.

Other cognitive biases such as representativeness and misperception of regression to the mean contribute to generating momentum. The former is where an investor assumes the probability of the stock going in a certain direction by judging it relative to its performance in the past, whereas the latter describes how individuals do not acknowledge fully the degree to which there is a regression to the mean. Despite there is no general consensus, behavioral biases may be the principal cause of the momentum effect<sup>18</sup>

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<sup>16</sup>Behavioral Finance.

<sup>17</sup>Hazel Kadzikano (2017), Behavioral theories behind momentum. Quantsportal [Internet]. Available from: <http://www.quantsportal.com/behavioural-theories-behind-momentum/> [Accessed 20 September 2018].

<sup>18</sup>Breaking Down Fiance [Internet]. Available from: <http://breakingdownfinance.com/finance-topics/behavioral-finance/anchoring/> [Accessed 20 September 2018].

### The effect of correlation on absolute applications

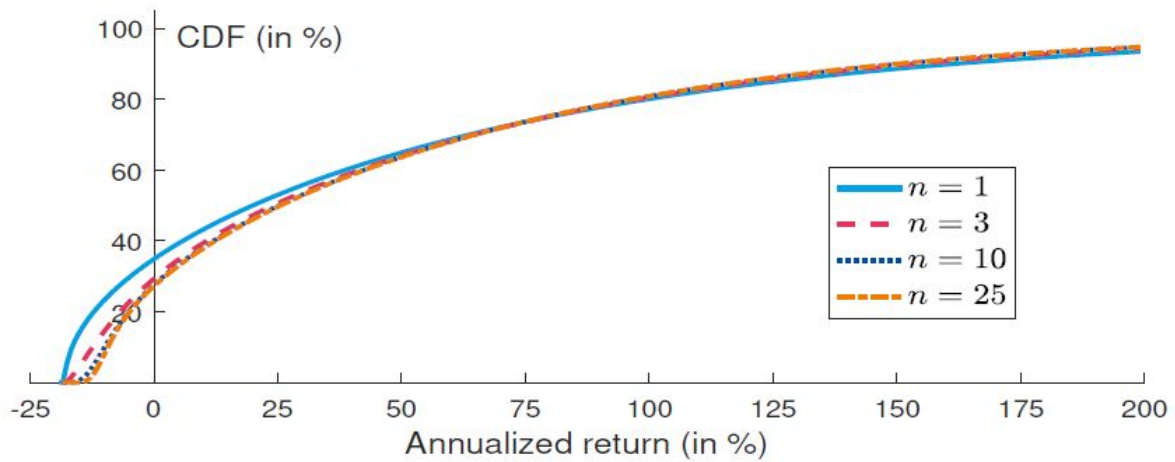


FIGURE 1.1: displays the Cumulative Distribution Function of a trend following strategy showing limited diversification gain when adding more than three assets. These results are derived analytically from the model of Brouder and Gaussel by Roncalli et al. 2017.

### The correlation symmetry on absolute momentum

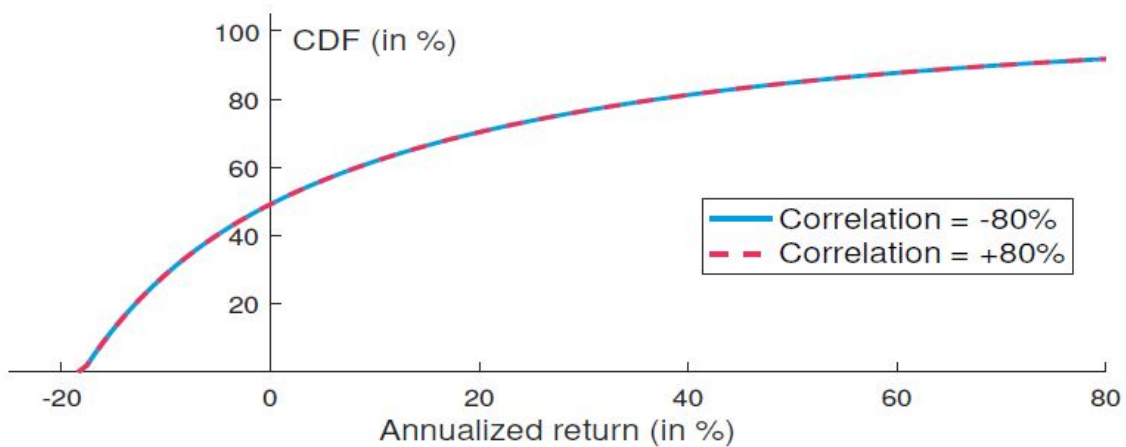


FIGURE 1.2: displays the cumulative distribution function of a trend following application zero showing that this kind of strategy is indifferent from a positive or a negative correlation. These results are derived analytically from the model of Brouder and Gaussel by Jusselin et al. 2017.

### The effect of correlation on relative applications

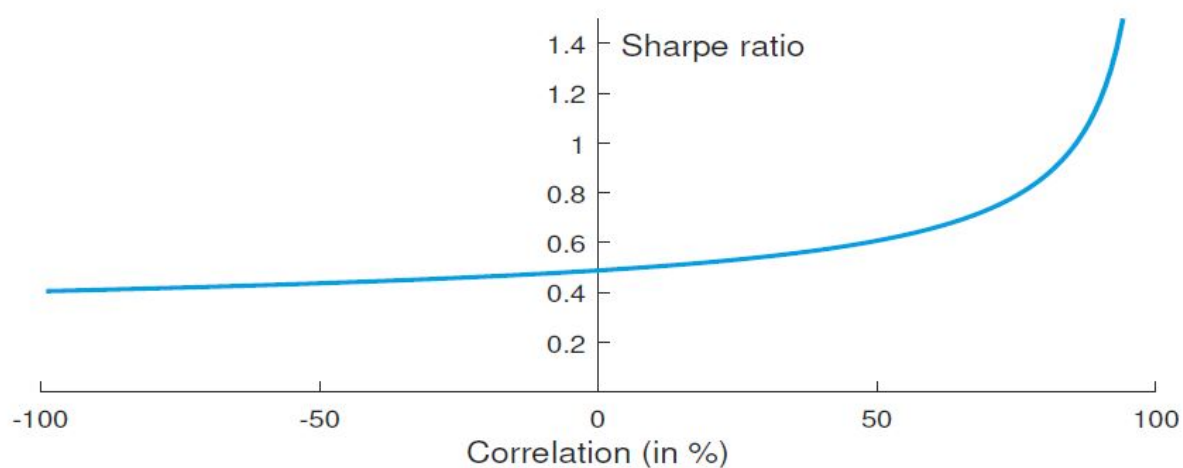


FIGURE 1.3: shows that relative momentum is a “friend of correlation” since the Sharpe ratio increases simultaneously with the correlation. These are the highest results and have been obtained when the correlation across assets was high and positive. Cross-sectional momentum worked well in capturing the spread-risk of the asset trends (Roncalli et al 2017).

## Chapter 2

# The Momentum Effect

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## 2.1 Introduction

Momentum trading strategies have always been historically successful in almost every asset classes demonstrating an excellent power of diversification and heterogeneous statistical properties.

Statistical evidence suggests that many financial assets show positive short-term autocorrelation (momentum bias) and negative long-term autocorrelation (mean reversion bias). Momentum bias<sup>2</sup> is generally confirmed except for very short (less than a week) and very long (greater than two years) windows where reversal biases<sup>3</sup> dominate. Most of these studies use monthly data without adjusting momentum signals and position sizes for volatility.

## 2.2 Momentum Trading

Academic studies often use past average returns as trading signals, but we know that there are various methodologies to create signals that do not outperform each other. In particular, the most common trend following rule is based on moving average (where the current market price of the stock it is compared with the average of his historical prices): if the current price is above (below) the moving average, we have a buy (sell) signal. Some momentum rules use a different kind of

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<sup>1</sup>We refer to absolute momentum effect.

<sup>2</sup>Using the expression of momentum biases, we refer to the momentum effect that can be interpreted as an anomaly or as a bias of the market.

<sup>3</sup>When the market suddenly revers: it changes trend direction (from positive to negative or vice-versa)Potters and Bouchaud, 2006.

lagging indicators such as a shorter moving average or an exponential moving average in order to improve the entire fit of the model<sup>4</sup>. In contrast, the use of technical momentum indicators that measures the magnitude of recent price changes to analyze overbought or oversold conditions such as the relative strength indicators<sup>5</sup>, underweights the magnitude of market moves emphasizing the frequency of up moves and down moves. Hence, this kind of measures is less effective to create momentum signals.

Another entry rule concerns acceleration or breakout signals filtering the recent price moves and the preceding market action. When a stock price “breaks” the so-called psychological level or resistance it could indicate a possible start of a trend and when this phenomenon is carried with sharp market moves it often represents a momentum signal. In addition to entry rules, exit rules (stop losses or take profit) are fundamental in order to minimize the loss when reversals occur. Trend followers typically diversify not just across a large number of assets but also across different historical window length and trading frequencies. The use of look-back periods varies depending on the underlying asset and the applied strategy. They could range from months to intraday trading in case of high-frequency data. The main rule is that optimizations and trend following models need to be dynamic and constantly rebalanced considering the state of the market at every time, usually, this kind of models are implemented and reviewed at least once a month.

One of the most important decisions regarding trend following applications is the volatility weighting procedure for positions and return signals. Since volatility varies across assets and over time (especially using slightly different asset classes like commodities and currencies) targeting the position and the return signal with respect to the volatility of the underlying asset could be a source of benefits for the entire strategy.

There is some evidence<sup>6</sup> that volatility targeting reduces risk and enhances the results of commodities strategies.

Firstly, if return signals are not adjusted to volatility, a less volatile asset will be preferred to a higher one. Specifically, the use of momentum strategies over a set of assets could penalize the instruments that present higher volatility excluding them from the active portfolio.

An example could be made considering again natural gas and gold. The former (which is highly volatile) will be more likely to be at either end of the rankings than the latter (which is three to four times less volatile). This bias of exclusion reduces the breadth of the entire asset universe. Secondly, if position sizes are not adjusted for differences in volatility across assets there is likely to be a long directional bias: the long or short positions produced by the ranking model can be market directional. Every long/short matching that does not include volatility weighting procedures will suffer from directional bias. This is true especially if we are considering heterogeneous

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<sup>4</sup>The use of different kind of moving averages highly depends on the type of stock that is under scrutiny.

<sup>5</sup>Are calculated considering the average gain of up periods during the specified time frame / average loss of down periods during the specified time frame, Investopedia [Internet]. Available from: <https://www.investopedia.com/terms/d/RSI.asp> [Accessed 20 September 2018].

<sup>6</sup>Ilmanen's study on commodities, Ilmanen, A. (2011), Expected returns: An investor's guide to harvesting market rewards. John Wiley & Sons.



universes such as commodities. In these asset classes, the directional bias will be stronger rather than bonds since in general commodities present greatest volatility differences among each other. (e.g. a dollar neutral long natural gas versus a short gold position is not empirically neutral to generally changing commodity prices; it effectively has a long directional bias).

Volatility weighting of positions also helps to reduce the nominal position sizes when recent volatility is higher creating volatility stabilization and enhancing risk-adjusted returns. A dynamic form of volatility weighting is to stop a trend following strategy after high recent volatility or better to create relative weights (fast and slow signals) as a function of the volatility level in the historical window to favor slow signals during bull markets and fast signals during bear markets (Ilmanen 2011<sup>7</sup>).

## 2.3 Empirical Evidence

There is evidence<sup>8</sup> that comes from the spurious use of momentum trading in different asset classes that suggest some recommendations during the applications of these strategies.

First of all, the attention needs to be focused on liquid assets since it is true that less liquid assets are more likely to trend, but their exploitation may be a difficult process to grasp (also considering the high level of transaction costs), that's why most of the strategies use highly liquid asset like futures and currencies. Secondly, theoretically poorly anchored asset prices allow grater trending because they give more scope for sentiment-driven changes, the lack of fair value of an asset may lead it to trend sharply. The explanation of this phenomenon is based on Behavioral finance biases such as the disposition effect and the anchoring behavior that may cause abnormal sentiment.

In particular, the investor's reluctance to realize small losses (disposition effect) exacerbates momentum effects. It fuels the underreaction or the moment which correspond to the difference between the current price and the price at which most investors bought.

Other conditions of the market may be helpful and can enhance the performance of absolute momentum applications. For example, it is well known that momentum works better between trends rather than range-bound markets, then macro trends and conditions should be constantly monitored. Another element to take under consideration is seasonality, it has been demonstrated that trend following strategies tend to perform better in December rather than January. The main explication is given by year-end tax-loss selling<sup>9</sup> and by the phenomenon of window dressing where institutions buy (sell) assets that have outperformed (underperformed) in order to make

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<sup>7</sup>Ilmanen, 2011

<sup>8</sup>Ilmanen, A. (2011), *Expected returns: An investor's guide to harvesting market rewards*. John Wiley & Sons.

<sup>9</sup>Tax selling refers to a type of sale in which an investor sells an asset with a capital loss in order to lower or eliminate the capital gain realized by other investments, for income tax purposes. Tax selling allows the investor to avoid paying capital gains tax on recently sold or appreciated assets, Investopedia [Internet]. Available from: <https://www.investopedia.com/terms/d/drawdown.asp> [Accessed 20 September 2018].

their portfolio better looking at the eyes of investors. Furthermore, momentum strategies tend to perform better when volatility starts to rise after a period of contained volatility. Finally, momentum time series tend to perform well when big events or announcements affect the assets, abnormal SR are documented for gov-bond and for commodities at the beginning of the event window (Ilmanen 2011).

## 2.4 Published Studies

Momentum strategies perform well in almost all asset classes. Moskowitz and Pedersen et al, as well as Gary Antonacci, studied the trend following applications on equities, currencies, commodities, and bonds finding substantial abnormal returns, especially during extreme markets. The empirical evidence suggests that trend following strategies using equity index futures are profitable in many countries, it also works in commodities, fixed income, foreign exchange, and alternative assets, as well as across asset classes.

### 2.4.1 Time Series Momentum

Pedersen and Moskowitz et al<sup>10</sup>. (2011) focused on absolute momentum strategies confirming the validity and effectiveness of the application regarding every asset classes. They further demonstrated the momentum effect and the subsequent reversals for each asset within the database. Their data begins in 1965 through 2009 and includes 58 instruments (24 commodities futures, 12 cross-currencies pairs forwards, 9 equity indices futures, and 13 gov-bond futures). In particular, they showed that in absolute momentum there is significant positive auto-covariance between an asset's return in the following month and his past one-year excess return. Following this statement, looking at an asset's excess return over the entire look-back period is the main task for applying absolute rules<sup>11</sup>.

#### *Volatility Model*

In order to analyze momentum strategies, volatility needs to be taken under scrutiny since it largely varies across contracts and among asset classes. It is well known that some asset classes such as commodities and equities present higher volatility than bond futures or currency forwards, but what is often slipping away among investors is that even among the same asset class

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<sup>10</sup>Moskowitz and Pedersen, 2012

<sup>11</sup>When this excess return is positive, we have a positive absolute momentum effect.

(e.g. commodities) there is substantial cross-sectional variation in terms of volatility. For example, the volatility of natural gas futures is about 50 times larger than 2-year US bond futures.

To make meaningful comparison across instruments with different volatilities and combine various assets into a diversified portfolio, Moskowitz et al. elaborated a volatility model and a filtering procedure.

In particular, they scaled the returns by their volatilities estimating each instrument's ex-ante or conditional variance  $\sigma_t^2$  at each point in time using a model called the exponentially-weighted lagged squared daily returns.

Specifically, they calculated the conditional annualized variance for each instrument as follows:

$$\sigma_t^2 = 261 \sum_{i=0}^{\infty} (1 - \delta) \delta^i (r_{t-1-i} - \bar{r}_t)^2 \quad (2.1)$$

where the scalar 261 scales the variance to be annual, the weights  $(1 - \delta) \delta^i$  add up to 1,  $\bar{r}_t$  is the exponentially weighted average return computed similarly and  $r_{t-1-i}$  is the return at time t-1-i. The parameter  $\delta$  is chosen so that the center of mass of the weights is:

$$\sum_{i=0}^{\infty} (1 - \delta) \delta^i = \frac{\delta}{1 - \delta} = 60 \quad (2.2)$$

To ensure that no look-ahead bias contaminates the results, they used the volatility estimates at time t-1 applied to time t returns throughout the analysis. This model is applied to all the assets within the database.

### ***Regression Analysis***

In order to show that in absolute momentum there is significant positive auto-covariance between an asset's return in the following month and his past one-year excess return, they applied regression analysis. They found that the past 12-month excess return of each instrument is a positive predictor of its future return. This time series momentum effect persists for about a year and then partially reverses over the long horizons.

After having estimated the conditional volatility for each asset, they divided all the returns for their estimated volatility. To show the time series predictability of asset returns, they run a pooled

panel regression (similarly to a Generalized Least Square) and compute t-statistics that account for group-wise clustering by time (at the monthly level). They used two different procedures. The first regression is run using lags of  $h$ , where  $h$  goes from 1 to 60 months and it is described by the following formula:

$$\frac{r_t^s}{\sigma_{t-1}^s} = \alpha + \beta \frac{r_{t-h}^s}{\sigma_{t-h-1}^s} + \varepsilon_t^s \quad (2.3)$$

Where the dependent variable  $r_t^s$  is the excess-return of the asset  $s$  on the month  $t$  scaled by the estimated volatility, regressors are lagged excess returns of the asset  $s$  at time  $t-h$ . The second procedure looks similar, but it focuses on the sign of the past excess return. In this framework, they again made the regression independent to volatility, so that all the estimates are comparable across instruments and among different asset classes. The regression is run using lags of  $h$ , where  $h$  goes from 1 to 60 months and it is described by the formula:

$$\frac{r_t^s}{\sigma_{t-1}^s} = \alpha + \beta_h \text{sing}(r_{t-h}^s) + \varepsilon_t^s \quad (2.4)$$

The results of the two regression procedures are similar and both show strong evidence of return continuation for the first year and reversals for the next four years. In both cases, the data exhibit a clear pattern, with most recent 12-month lag returns positive (and nine statistically significant) and most of the remaining lags negative. These findings slightly vary between asset classes and are robust across different lock-back periods and holding periods. Moreover, these results are positive not just on average, but for each asset class they have examined.

The findings of positive trends that partially reverse over the long-term may be consistent with initial under-reaction and delayed over-reaction, which behavioral theories suggest and that can produce these return patterns (anchoring, herding behavior, and the disposition effect). The results of the two pooled panel regressions are described in Figure 2.1.

### ***Momentum Strategy***

After showing the time series predictability they focused on the profitability of some trading strategies based on absolute momentum. In order to test the strategy, they varied both the “look-back period” and the “holding period” on every instrument within the dataset. Moreover, for

each asset  $s$  and month  $t$ , they considered the excess return over the past  $k$  months and depending on the sign of this excess return they hold the position for  $h$  months. Specifically, the two fundamental periods are described as follow:

- **Look-Back Period:** it is the time considered for creating a signal. It is constructed creating average functions of returns for each asset during  $k$  months. The signals are generated whether the excess return of these averages is positive or negative, in case of positive signal we go long, while in case of negative signal we go short.
- **Holding Period:** it represents how long the position remains open and it varies from the parameter  $h$ .

They sized each position with respect to estimated volatility to create an easier aggregation between strategies and across instruments with very different levels of volatility. This choice is helpful econometrically since having a time series with stable volatility ensure that the strategy it is not dominated by a few volatile periods and reduces the risk of being market directional<sup>12</sup>. For each trading strategy  $(k, h)$  they derived a single time series of monthly returns even if the holding period  $h$  is more than one month. Moreover regarding equity returns, some research papers<sup>13</sup> and studies often choose to skip the most recent month of the selected look-back period in order to create a separation between the momentum effect and the subsequent reversal. This method is useful in cases of liquidity issues since there may be more complex to close and open positions. In this analysis Moskowitz et al. choose to rebalance positions without skipping the last month of observation (this choice highly depends on the type of asset considered during the test).

In particular, they focus on the properties of the 12-month time series momentum strategy with a 1-month holding period (i.e.  $k = 12$  and  $h = 1$ ) for all the assets within the database. Figure 2.2 plots the cumulative excess return of the strategy for all futures contracts they have examined (on a log scale) scaled by the same conditional volatility. In order to have comparable metrics, they plotted also the cumulative excess returns of a diversified passive long position in all instruments with equal risk on every asset, the instruments are scaled by the same conditional volatility.

## 2.4.2 The Dual Approach

Gary Antonacci (2014) questioned the diversifier power of relative momentum enhancing the properties of absolute applications. He first discussed how the procedure of choosing winners and losers may create a segmentation of the set of assets reducing the benefits that come from a multi-asset diversification. This phenomenon may lead to opportunity loss by excluding lagging

<sup>12</sup>Market directional bias.

<sup>13</sup>Antonacci, 2013.

assets that may suddenly start outperforming. Secondly, he showed the superior ability of time series momentum with respect to relative applications to reduce the downside volatility by identifying a regime change. Finally, by applying absolute momentum to a 60/40 stock-bond portfolio and to a simple risk parity portfolio, he demonstrated that absolute momentum can enhance the expected returns and reduces the expected drawdown. He proposed the time series strategy as a powerful tactical overlay for improving the risk-adjusted performance of portfolios or as an attractive standalone strategy for absolute performances. Another relevant question is whether the combined use of cross-sectional and time series momentum can be profitable for the entire portfolio. Evidence suggests that using both relative and absolute momentum could have positive or negative effects on the performance of the associated portfolio. Specifically, cross-sectional momentum could neutralize the directional risk and time series momentum could be tailor-made to different assets, but their net benefits need to be determined empirically. An interesting study is proposed by Gary Antonacci in his book “Dual momentum” who studied the application of both strategies.

### *Look-back Periods and Sharpe Analysis*

The dataset begins in 1973 through 2012 and it includes 8 kinds of instruments (2 Equity indices futures, 6 gov-bond futures, 1 commodity index futures, and 1 real estate index).

In order to formulate the look-back period for the strategy, Antonacci used a series of clustered Sharpe ratios to measure the strength of the periods tested. The formation periods range from 2 to 18 months (18 months is the maximum formation period he considers). These Sharpe ratios are obtained in the following way: The first step is to cluster the observations with respect to the duration of the formation period. Secondly, calculate the SR over these clusters and finally compute the arithmetic mean of the founded SR. For example, if we are considering evaluating the formation period of 2 months over the asset  $s$ , we calculate the SR corresponding to the clusters of 2 months for the entire time series, then we average all the obtained SR values together to get the result. All these values are reported and compared with each other highlighting the formation periods associated with the higher SR. He showed that the higher occurrence across all assets is founded considering SR related to a formation period of 12 months (Figure 2.3). This result is confirmed also by Pedersen and Moskowitz (2011) that selected a 12 months formation period as the best look-back period in their applications.

### *Absolute Momentum on a 60/40 Portfolio*

Once the optimal look-back period is founded he applied the 12 months absolute momentum to improve the performance of a 60/40 portfolio. In this application, he focused only on long momentum strategies without considering short exposures. This portfolio is composed of 60% of the US MSCI and of 40% of the US Treasury indexes. To show the differences he compared the 60/40 portfolio with and without absolute momentum. With respect to investments in US stocks, the 60/40 portfolio without momentum proves some reduction in volatility and drawdown risk. In any case, the solid 0.92 correlation founded of the 60/40 portfolio with the S&P 500 demonstrates that the 60/40 portfolio is bearing the market risk of US stocks. Moreover since stocks are more volatile, they dominate the risk in this kind of portfolios that behaves almost as an equity portfolio. When 12 months absolute momentum is added, the MSCI US showed a 0.67 correlation to the S&P 500 (lower than 0.92) and presented superior ability to reduce drawdown by more than 70% enhancing the returns of the overall strategy. Figure 2.4 and Figure 2.5 presents the results in terms of drawdown and abnormal returns. Figure 2.4 by Antonacci and Figure 2.2 by Moskowitz follow different methodology and instruments, but they share common features. In particular, if we look at these figures analyzing the impact of the most recent global financial crisis, we see that time series momentum profits are larger in October, November, and December of 2008 during the peak of the Global Financial Crisis. In the third quarter of 2008, the moves of the associated instruments caused the strategy to be short in many contracts and absolute momentum suffered losses. In the fourth quarter of 2008 when assets fell further, time series momentum realized large profits. Moreover, at the end of the crisis (March and May 2009) absolute momentum suffered losses from the reversal of the market (the strategy was operating in the opposite direction since the third quarter).

### *Dual Momentum*

Gary Antonacci analyzed also the combined use of cross-sectional and time series momentum studying the application of both strategies at the same time. The dataset is composed of monthly observations of S&P 500 and MSCI ACWI ex-US indices for equities. For bonds, he used the Barclays US Aggregate Bond index. The time horizon is of nearly 40 years (from 1971 to 2016). In this application, he again focused only on long momentum strategies without considering short exposures. He first used relative momentum to switch between the S&P 500 and the ACWI ex-US and then absolute momentum to switch between stocks and bonds<sup>14</sup>. Specifically, regarding the use of relative momentum, he had positive relative signals<sup>15</sup> 55% of the time for S&P 500 and 45% of the

<sup>14</sup>In his paper "Dual momentum" he does not show the methodology used to switch between absolute and relative momentum.

<sup>15</sup>This means that he selected for a potential investment S&P500 55% of the time, while ACWI ex US 45% of the time.

time for ACWI ex-US. Since momentum studies use either a six or a 12 months formation period, he selected the 12 months timeframe as the preferred look-back period for testing the strategy. Again in this analysis, Antonacci choose to rebalance positions monthly without skipping the last month of observation.

Declared results are supportive for the strategy since dual momentum shows the lowest draw-down (-17%) and the highest SR (0.92) with to respect to absolute and relative strategies in a stand-alone application. Figure 2.7 shows the cumulative returns of the strategy and Figure 2.6 displays a summary of the statistics. The use of dual momentum strategy makes diversification more efficient investing in assets only when they show both positive relative and absolute momentum. Moreover, findings demonstrated that long momentum strategies worked extremely well when considering a combination of absolute momentum and relative momentum and that trend determination with absolute momentum can help mitigate downside risk and take advantage of regime persistence of the market, while both relative and absolute momentum can enhance expected returns.



## Time Series Predictability Across All Asset Classes

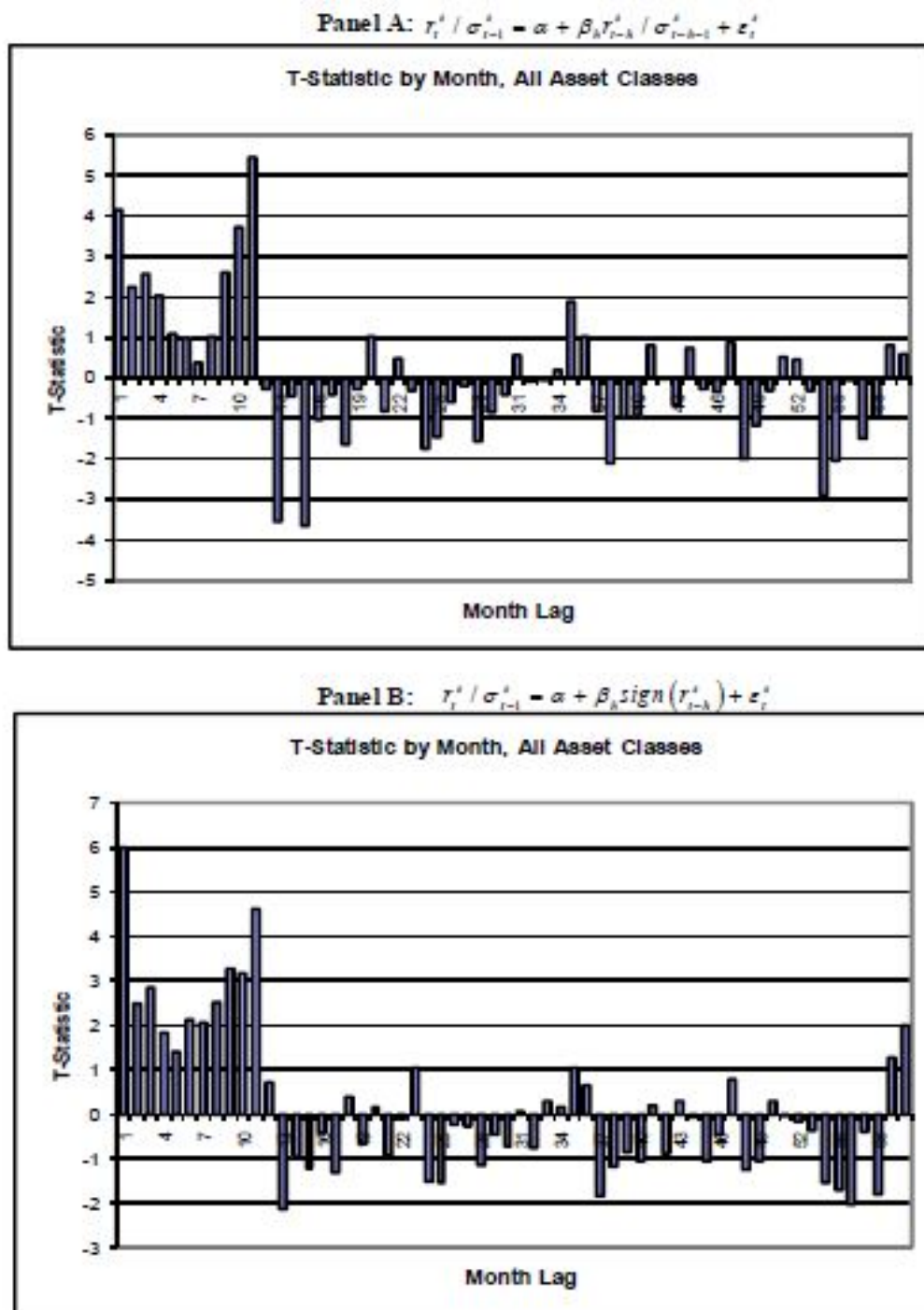


FIGURE 2.1: Panel A and Panel B of display the results of the two-regression analysis done by Pedersen & Moskowitz. They found that the past 12-month excess return of each instrument is a positive predictor of its future return, this time series momentum effect persists for about a year and then partially reverses over the long horizons. Figure 2.1 plots the t-statistics from the pooled regressions by month lag  $h$ . The positive t-statistics for the first 12 months (nine statistically significant) indicate strong return continuation or trend. The negative signs for the longer horizons indicate reversals, the most significant of which occur in the year immediately following the positive trend. (Pedersen and Moskowitz 2011).

**Cumulative Excess Return of Time Series Momentum and Diversified Passive Long Strategy,  
January 1985 to December 2009**

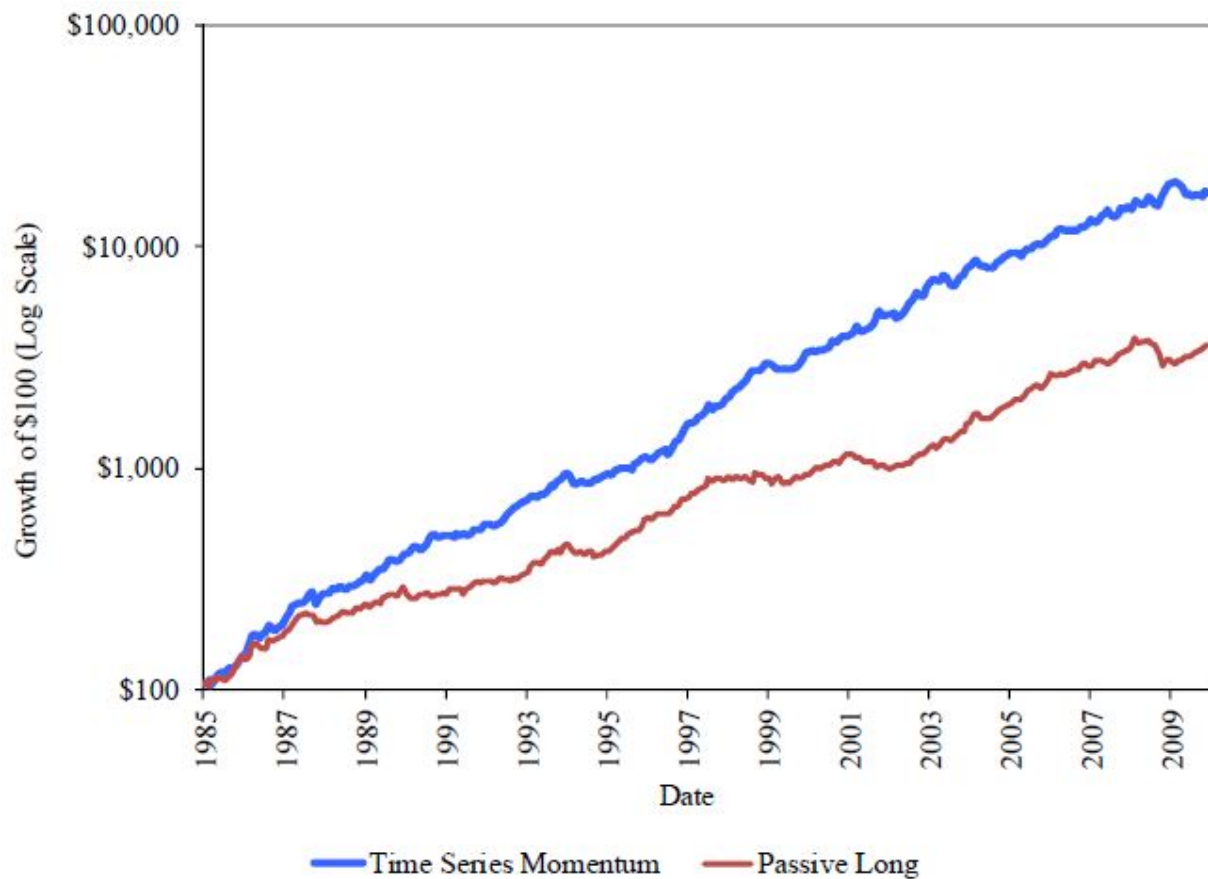


FIGURE 2.2: plots the cumulative excess return of the time series strategy for all futures contracts they have examined (on a log scale) compared to a passive long exposure for all instruments. To improve comparability, all the contracts are scaled by the same conditional volatility. (Pedersen and Moskowitz 2011).

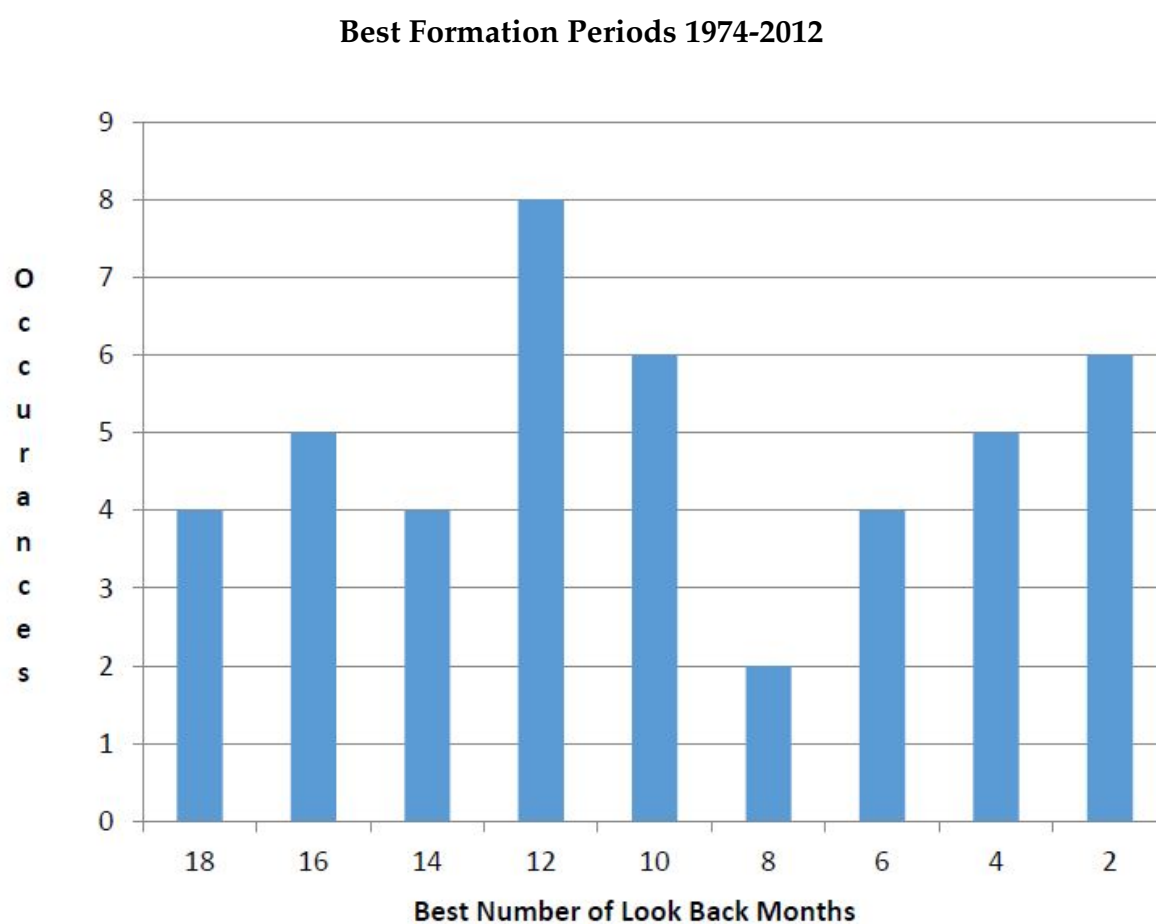


FIGURE 2.3: displays the results of the clustering process of formation periods for different assets. The highest Sharpe ratios for each asset is the one associated to the cluster of 12 months, then highest occurrences (8) are associated with a 12 months formation period (Antonacci 2011).

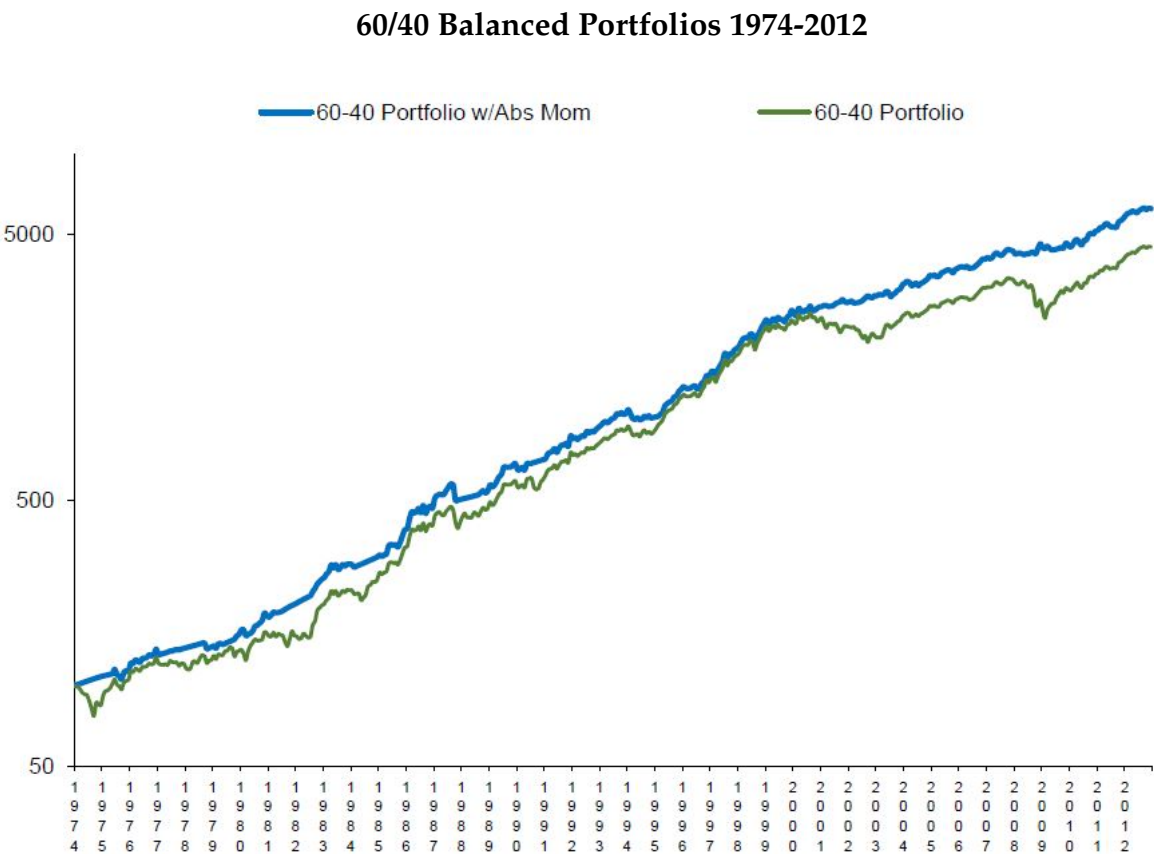


FIGURE 2.4: displays the cumulative results of the long-only 12-month absolute momentum strategy applied to a 60/40 portfolio compared with a 60/40 portfolio without momentum (Antonacci 2011).

### Drawdown Analysis 1985-2009

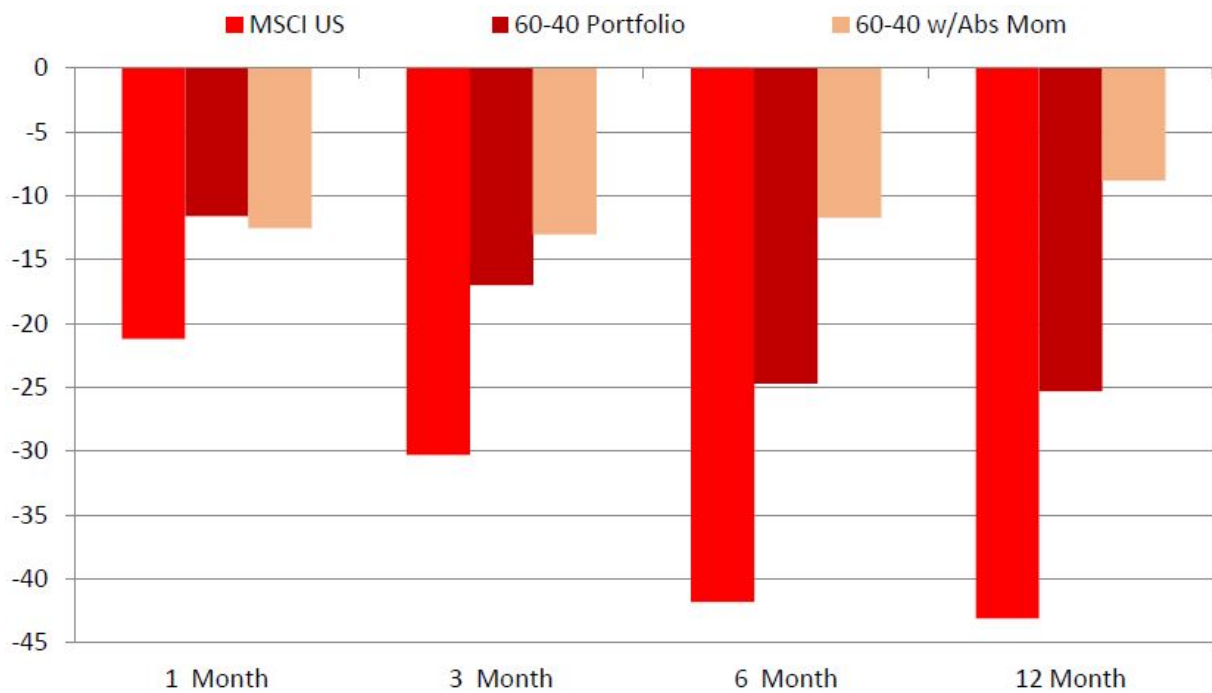


FIGURE 2.5: displays the volatility drawdown for MSCI US, 60/40 portfolio without momentum and 60/40 portfolio with absolute momentum highlighting the benefits of applying time series momentum to reduce drawdown risk. (Antonacci 2011).

### Dual Momentum Results 1971-2016

Jan 1971 - Dec 2016	CAGR	Standard Deviation	Sharpe Ratio	Worst Drawdown
Dual Momentum	17.0%	12.5%	0.92	-17.8%
Absolute Momentum	12.9%	11.9%	0.66	-29.6%
Relative Momentum	13.5%	15.9%	0.56	-54.6%
S&P 500 Index	10.7%	15.1%	0.42	-51.0%
MSCI ACWI ex-U.S.	10.2%	17.2%	0.37	-57.4%

FIGURE 2.6: displays the results of the long-only dual momentum strategy with a look-back period of 12 months. CAGR refers to Compound annual growth rate. (Antonacci 2016).

Cumulative Returns of the Dual Momentum strategy

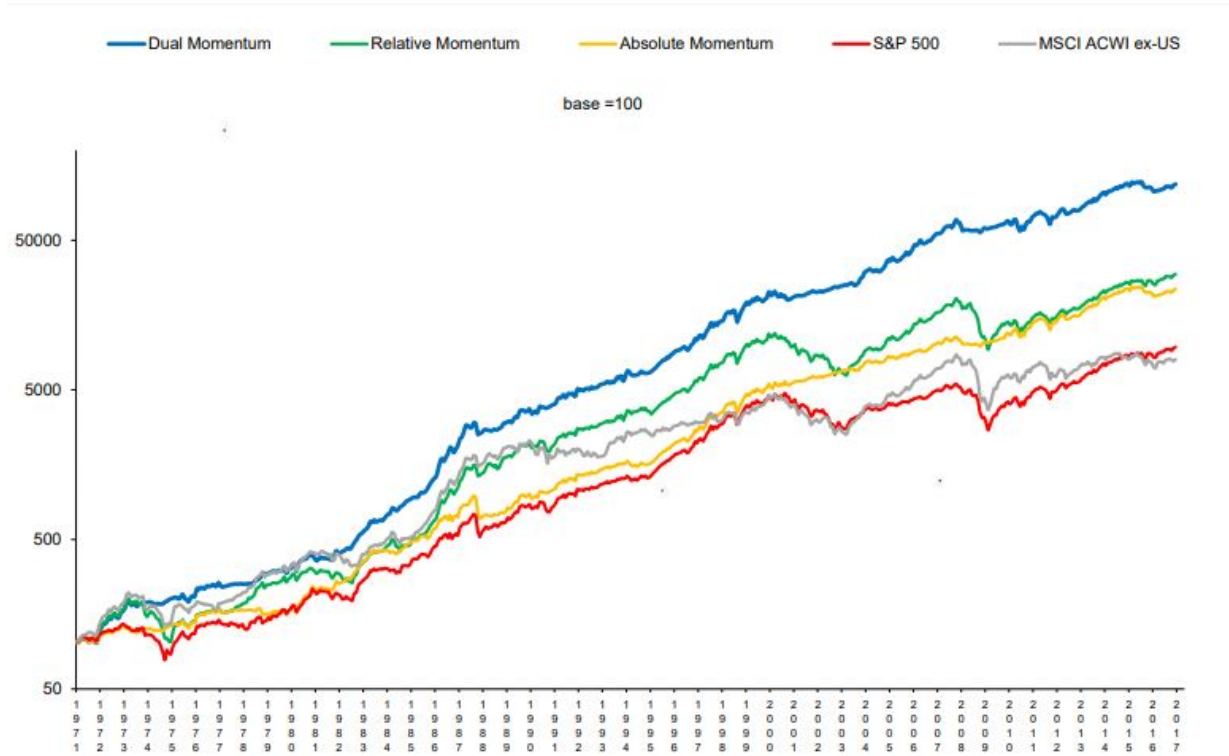


FIGURE 2.7: displays the cumulative returns on a log scale of the strategy compared with the singular use of absolute and relative momentum and with long passive exposures on S&P 500 and MSCI ex-US. (Antonacci 2016).

## 2.5 The Momentum Effect: A practical application

In this section, we want to prove the Momentum Effect through time series analysis leaving the cross-sectional application aside for the moment. In particular, we have replicated some ideas of applications and methodologies described by Pedersen & Moskowitz and Gary Antonacci in their studies.

### 2.5.1 Data and Methodology

Our dataset is composed of equity monthly returns that include dividends and interest with a formation period of approximately 20 years (from December 1998 to July 2018). We have selected the most representative equity indexes regarding the Eurozone: DAX, IBEX, CAC, AEX, FTSE, and UK. All the data are taken using a Bloomberg terminal post. The results are obtained using MATLAB.

### 2.5.2 Sharpe Evaluation on Formation Periods

The first step is to use a measure of performance such as the Sharpe ratio to find the best formation period (look-back period) within the dataset. Specifically, we divide the time series of returns into different periods of time and we calculate the respective annualized SR for each period, then we classify each formation period using the SR as a unit of measure (highest SR for the best formation period).

Following Gary Antonacci and his paper, the formation periods range from 2 to 18 months with an interval of 2 months. Results based on these settings show that the best cluster is at 12 months followed by 2, 14 and 16 months (Figure 2.8 and Figure 2.9 illustrate these findings). Many momentum research papers use a 12-months formation period as a benchmark strategy for research purposes reducing transaction costs and data snooping.

Figure 2.8 shows the same result in a more detailed way (it describes the best and the worst performer), while Figure 2.9 is made by averaging all the indexes. Comparing to the analysis produced by Gary Antonacci<sup>16</sup> (he used an extended and different dataset), in our results there are no significant levels<sup>17</sup> in the cases of 18, 10, 8, and 6 months. This suggests using formation periods that are close to 12-months or 2-months for a better performing application.

Since our findings suggested to focus on a 2-months and on a 12 months look-back period, we went deeper into the question producing another analysis and looking to the neighborhood of

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<sup>16</sup>Antonacci, 2012

<sup>17</sup>The Sharpe ratio calculated over these formation periods (6, 8, 10, 18 months) is relatively low comparing to the one of 12 and 2 months.

such critical windows. Results were interesting: we found that the highest level of SR is the one associated with formation periods of only one month (Figure 2.10). This strongly rejects the idea that a 12 months look-back period needs to be preferred among all windows of observations and reminds to the fact that preferences need to be determined empirically in every case since they may vary among instruments and across time.

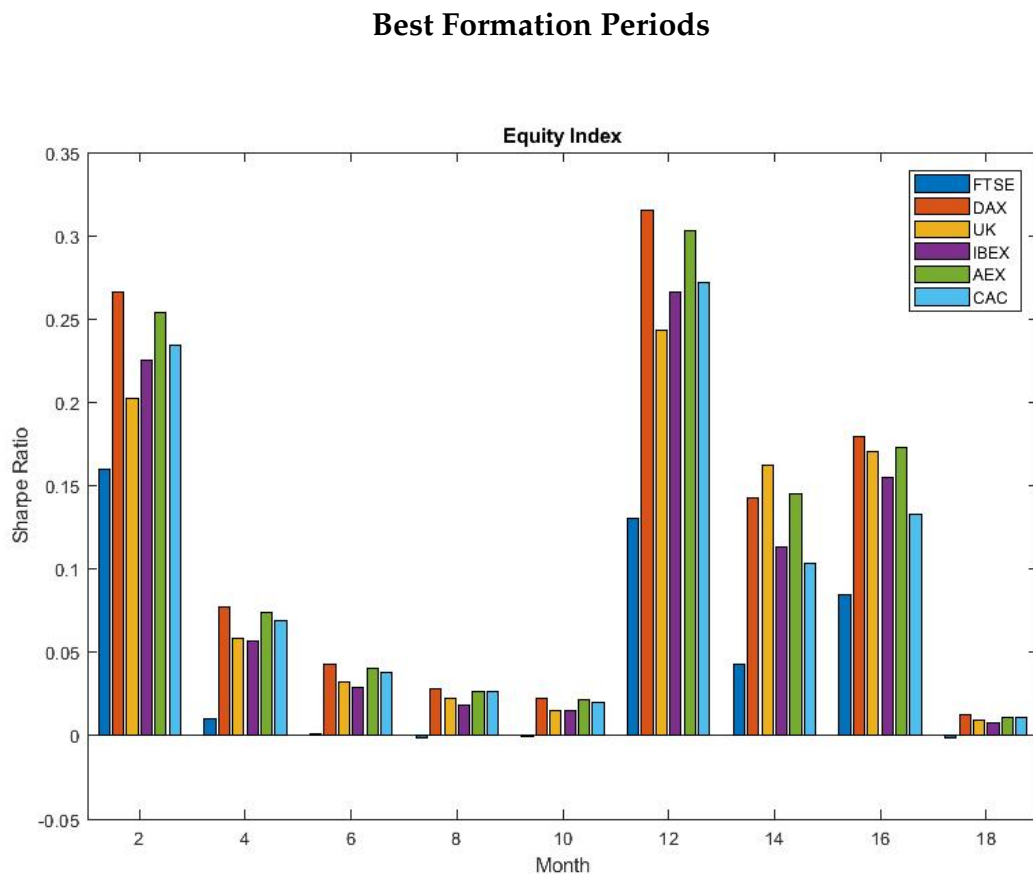


FIGURE 2.8: displays the results of the clustering process of formation periods for different assets. The highest Sharpe ratios for each asset is the one associated to the cluster of 12 months followed by 2, 16 and 14 months.



### Best Formation Periods

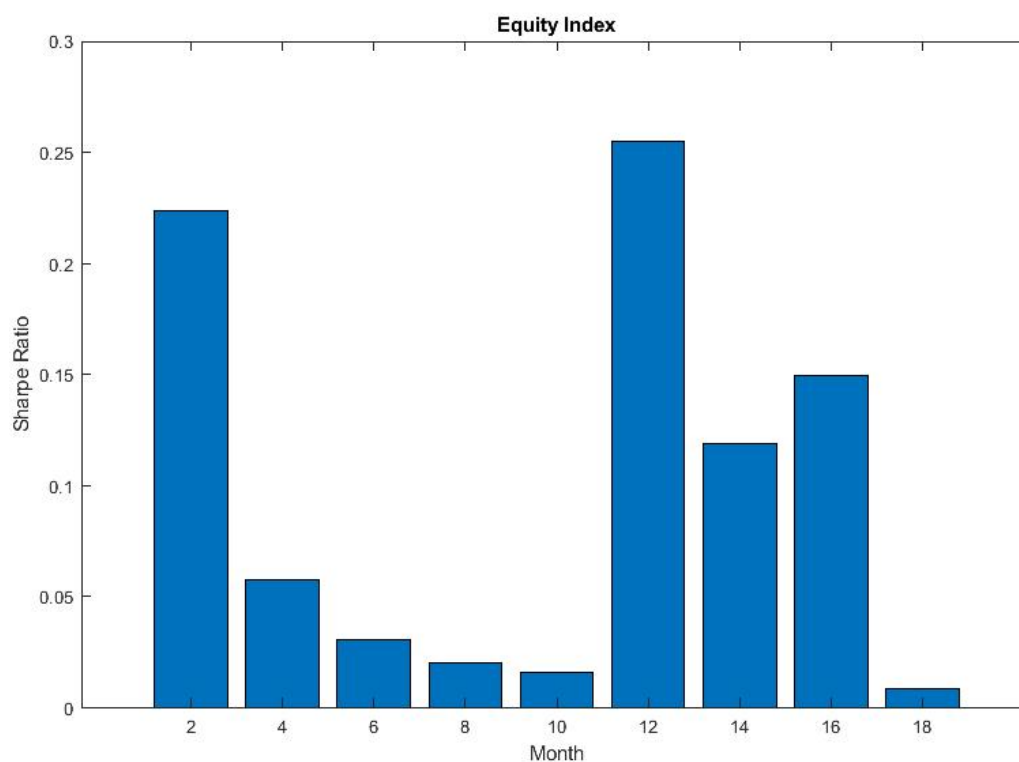


FIGURE 2.9: displays the results of the clustering process of formation periods for different assets. The highest Sharpe ratios for each asset is the one associated to the cluster of 12 months. The equity index is the arithmetic mean of all the index within the dataset.

### The Best Formation Period

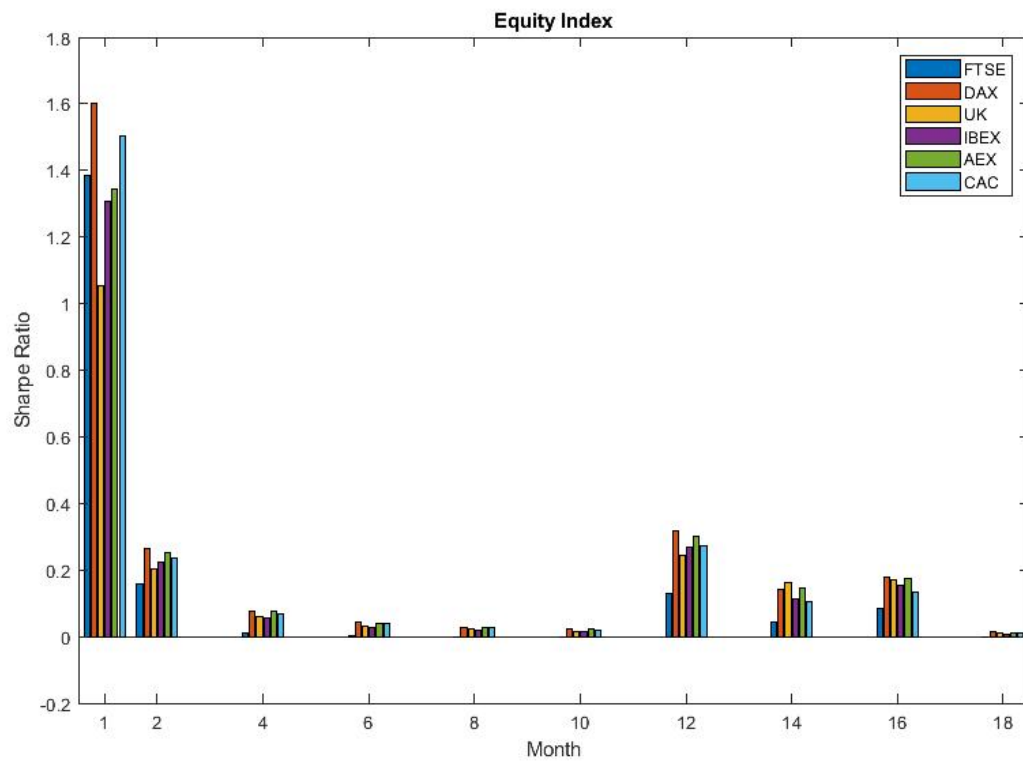


FIGURE 2.10: displays the results of the clustering process of formation periods for different assets. Expanding the window of the analysis the highest Sharpe ratio for each asset is the one associated to the cluster of one month. This strongly rejects the idea that a 12 months look-back period needs to be preferred among all windows of observations.

### 2.5.3 Volatility Model

The second step is to examine the time series predictability of returns across different time horizons using regression and volatility weighting. Practically, we proved the momentum effect by looking at the time series of excess returns and at the auto-covariance between the security's excess return next month and its lagged 1-year return. If this autocovariance is positive, it means that the asset future excess returns are explained by the past and then profitable trading strategies could be applied.

In order to make meaningful comparisons across assets, all returns are scaled by their ex-ante or conditional volatility which it is calculated over the selected period of analysis<sup>18</sup>. This stage is fundamental since volatility varies across assets and across time. The volatility model is applied to all the dataset and to ensure no look-ahead bias, the volatility estimates are at time  $t-1$  applied to time  $t$  returns. Differently from Pedersen and Moskowitz, we have estimated the ex-ante volatility for each asset with a simple univariate Garch (1,1). This choice has been made for simplicity reasons since the authors in their paper elaborate a more complex model of estimation. In order to formulate good estimates for the conditional variances and find the best possible Garch-fit for the data, we needed to test first the fit of the model and then the autocorrelation function of standardized residuals for each asset within the database. In doing so we estimate<sup>19</sup> the model using the MLE<sup>20</sup> on the return data and we test the fit looking at the conditional volatility (the standard deviation of conditional variance) and at the serial correlation of standardized residuals. In other words, we are calculating standardized residuals and checking if they respect the typical assumption of being independent and identically distributed, if they are IID<sup>21</sup> it means that there is no serial correlation and that the autocorrelation function is bounded on zero<sup>22</sup>. We look at the standardized residuals and at the squared standardized residuals<sup>23</sup> analyzing their autocorrelation functions. We confirmed the good fit of the model for each asset within the database (Figure 2.12 and 2.11 show the autocorrelation test and the qualitative fit of the conditional variance for the first asset returns).

We also tested the distribution of the standardized residuals with a Jarque-Bera Test<sup>24</sup>. If the volatility model (used to estimate conditional volatility) holds, then the standardized residuals should have approximately a normal distribution and nearly 95% of them will be between  $-2$  and  $+2$ <sup>25</sup> (where sigma is the standard deviation of the conditional volatility). Findings rejected the

<sup>18</sup>The conditional volatility is calculated over the selected time horizon, if we modify the period of analysis the volatility needs to be recalculated.

<sup>19</sup>In the estimation function we chose t-distribution with 5DOF instead of normality in order to have an approximately normal distribution for the standardized residuals and a better fit for the model

<sup>20</sup>The estimation function used Maximum likelihood estimator by default.

<sup>21</sup>Independent and Identically Distributed.

<sup>22</sup>This is very useful to show that we capture volatility in a correct way.

<sup>23</sup>Are calculated dividing square returns over the conditional volatility.

<sup>24</sup>In statistics, the Jarque-Bera test is a goodness-of-fit test of whether sample data have the skewness and kurtosis matching a normal distribution, Wikipedia [Internet]. Available from: [https://en.wikipedia.org/wiki/Jarque\\_Bera](https://en.wikipedia.org/wiki/Jarque_Bera) [Accessed 20 September 2018].

<sup>25</sup>Following the 68 95 99.7 rule.

null hypothesis when we tested normality of the standardized residuals using a Gaussian distribution as the distribution parameter of the volatility model. Differently, when we choose a T-distribution with 5 degrees of freedom as the distribution parameter of the volatility model we got better statistics. In this last case, results were supportive and confirmed that the distribution of the standardized residuals may be approximated to a normal distribution. The positive outcome of the Jarque-Bera test confirmed the good fit of the model to the data.

### The Qualitative Fit of The Conditional Volatility

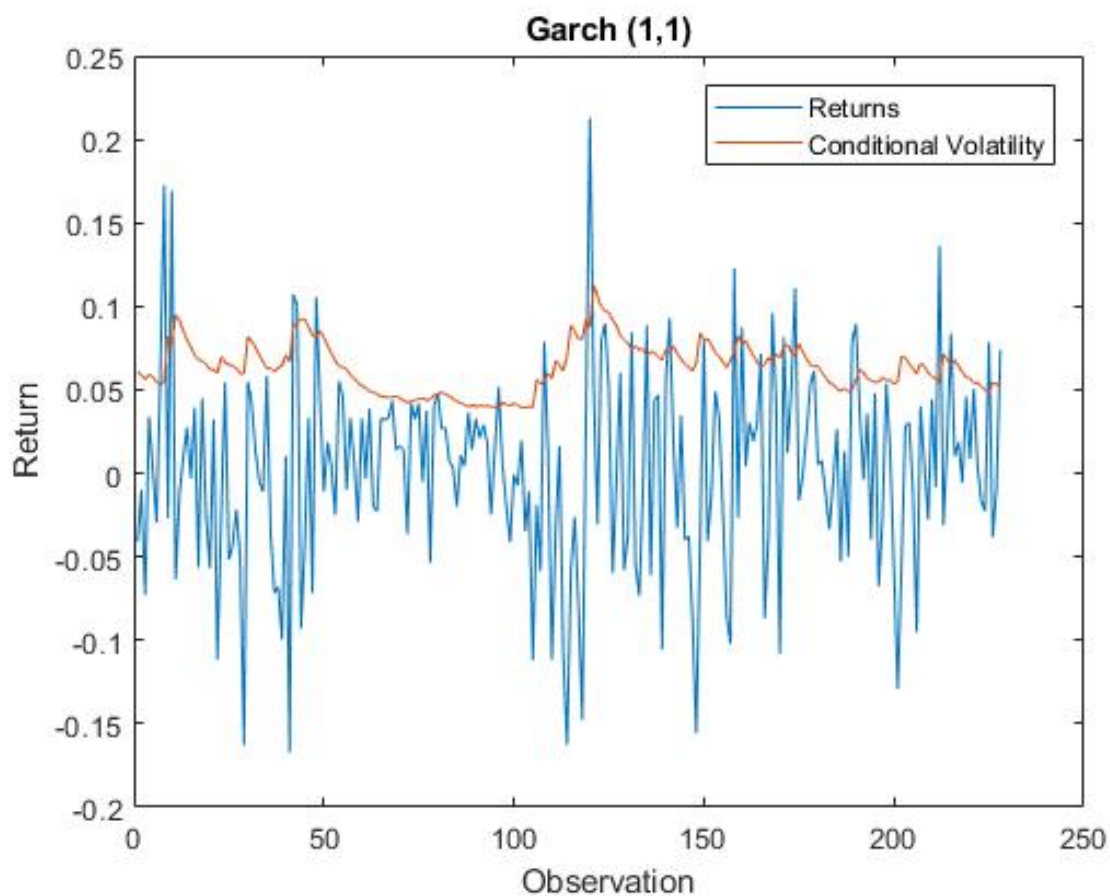


FIGURE 2.11: displays the qualitative fit of the estimated model to the data using Garch (1,1). This is a visual fit of the conditional volatility on the volatility of the index. This result refers only to the FTSE data but has been done successfully for every asset.

## Autocorrelation Fit

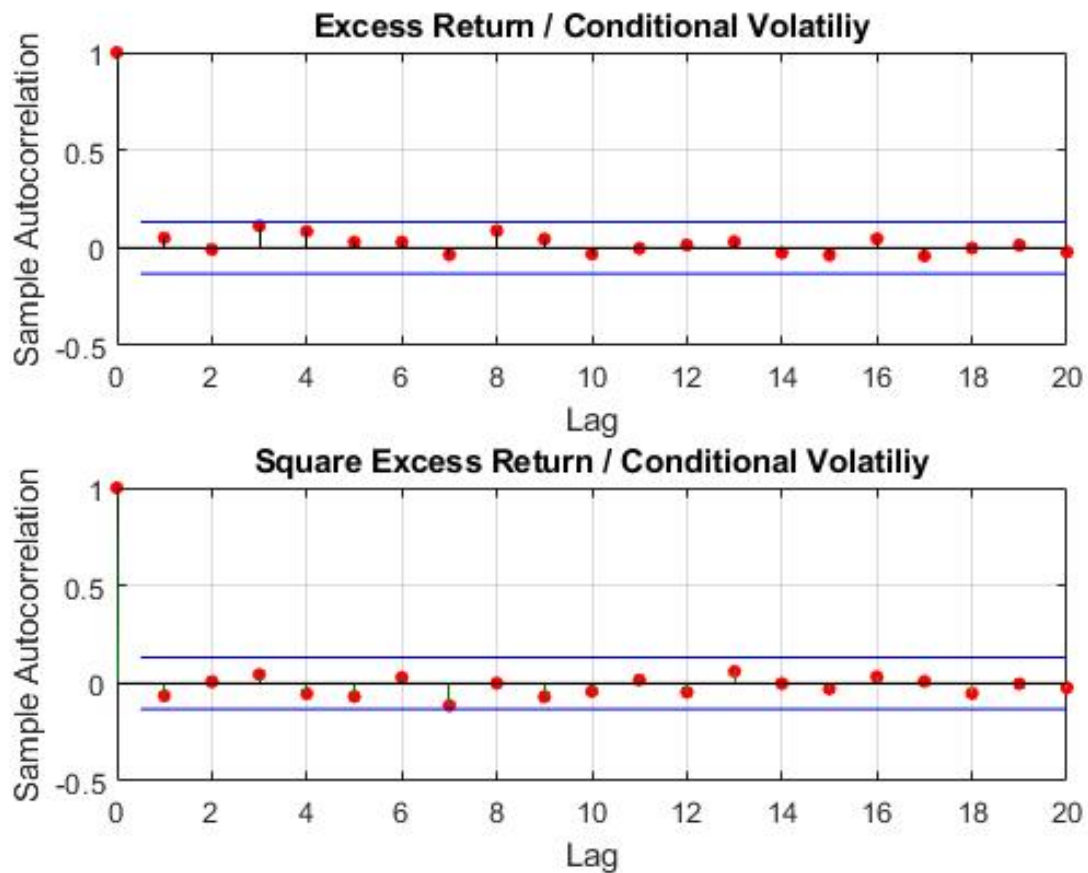


FIGURE 2.12: displays the autocorrelation functions of the standardized residuals (Panel A) and of the squared standardized residuals (Panel B) showing in both cases zero serial correlation. The former is calculated dividing the excess return for the conditional volatility, while the latter is obtained using squared excess returns over the conditional volatility. This result refers only to the FTSE data but has been done successfully for every asset.

## 2.5.4 Regression Analysis

Now that we have estimated the conditional volatility, we use regression analysis to useful predict the price continuation and subsequent reversal for every asset within the database showing the existence of the so-called momentum effect.

Following Pedersen & Moskowitz methodology, we regress the excess return  $r_t^s$  for the asset  $s$  in the month  $t$  on its return lagged  $h$  months, where both returns are scaled by their ex-ante<sup>26</sup> conditional volatilities<sup>27</sup>, we recall the formula:

$$\frac{r_t^s}{\sigma_{t-1}^s} = \alpha + \beta \frac{r_{t-h}^s}{\sigma_{t-h-1}^s} + \varepsilon_t^s \quad (2.5)$$

We conclude the test running a pooled panel regression and computing the t-statistics for each asset. Our regression analysis is done using the MATLAB function `fitlm` that choose OLS<sup>28</sup> by default. Specifically, we run a t-test on the betas of all the regressions and then we average obtaining a unique series of t-statistics that represent the t-test for all the equity indexes within the database. Figure 2.13 plots the average of t-statistics from the regressions by month lag  $h$ , where  $h$  goes from 1 to 60 months. Differently from Moskowitz & Pedersen et al., we found no positive trend of t-statistics for the first 12 months and a return continuation of only one month followed by negative signs when reversal occurs.

We found that in average the past one-month excess return of each instrument is a positive predictor of its future return. This time series momentum effect persists for each asset within the dataset (nearly 20 years).

Empirical evidence confirms the positive auto-covariance between an asset's return in the following month and his past one-month excess return. The results are less significant than the one of Pedersen & Moskowitz (t-statistics are not positive in the first 12 months) since we show a shorter momentum effect and a higher dispersion. Some explanations may be related also to the fact that we use a different dataset<sup>29</sup>, a smaller time frame and a simpler methodology for the estimation of the ex-ante volatility<sup>30</sup> and for the estimator<sup>31</sup> of the regression model.

<sup>26</sup>To ensure no look-ahead bias we scaled returns by their ex-ante (t-1). volatility

<sup>27</sup>Returns are divided by their estimated conditional volatility.

<sup>28</sup>Ordinary least square estimator.

<sup>29</sup>Our data-base differs since there are few instruments: we cover only the euro-equity sector and no futures are included.

<sup>30</sup>We choose a simple univariate Garch (1,1).

<sup>31</sup>We used the ordinary least square estimator (OLS) for the regression analysis.

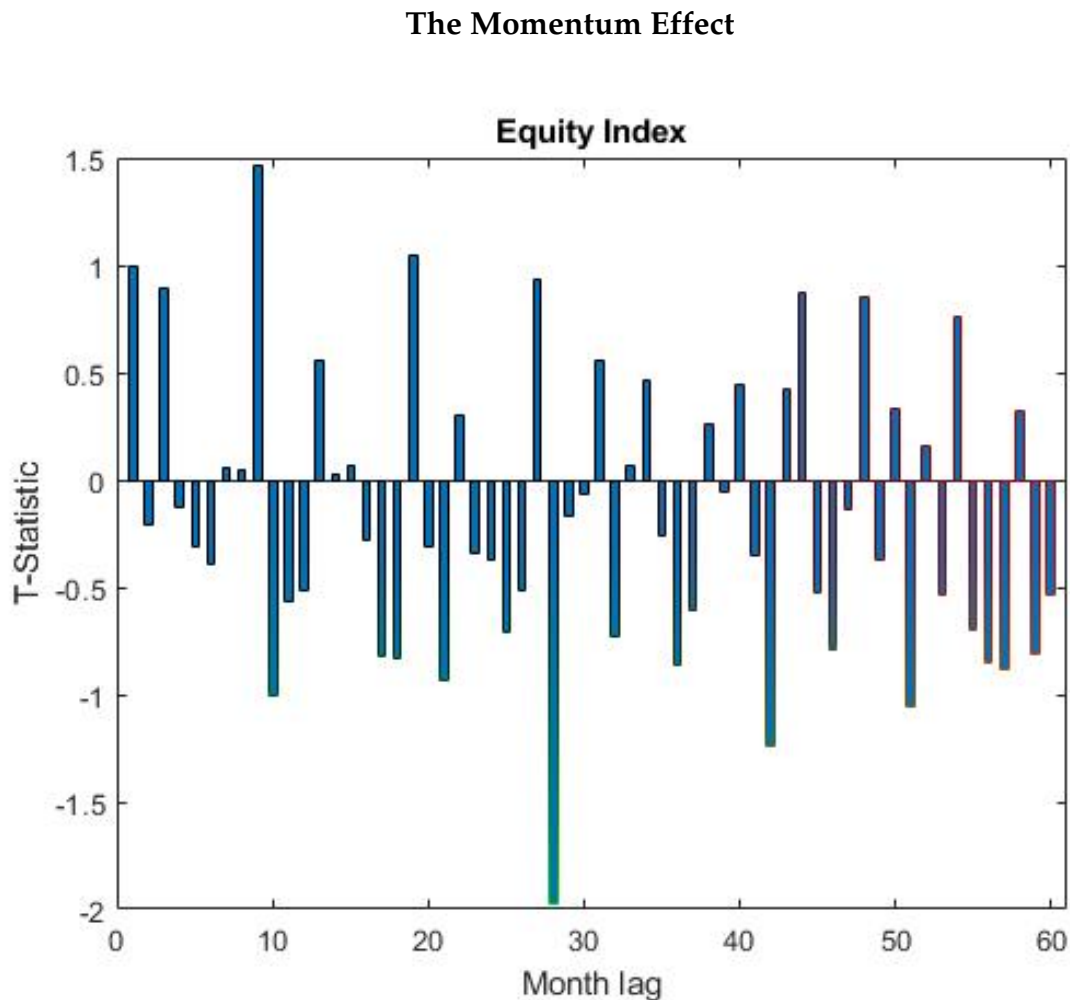


FIGURE 2.13: displays the results of our regression analysis plotting the average of the t-statistics of all the equity indexes. The positive significant trend of the t-statistics for the first month indicate a short return continuation. We found that in average the past one-month excess return of each instrument is a positive predictor of its future return. This time series momentum effect persists for all the dataset and the time horizon (nearly 20 years) considered during the analysis. Finally, this result confirms that in our analysis a one-month look-back period needs to be preferred among all windows of observations and reminds to the fact that these kinds of preferences need to be determined empirically.

## Chapter 3

# Momentum Strategy

### 3.1 Data and Methodology

In this section, we want to test momentum trading creating a strategy that uses both absolute and relative momentum at the same time<sup>1</sup>.

Our dataset is composed of equity monthly returns that include dividends and interest with a formation period of approximately 20 years (from December 1998 to July 2018). We have selected the most representative equity indexes regarding the Eurozone: DAX, IBEX, CAC, AEX, FTSE, and UK. All the data are taken using a Bloomberg terminal post. The results are obtained using MATLAB and Python.

### 3.2 Backtesting

Since our analysis in chapter two strongly rejects the idea that a 12 months look-back period needs to be preferred among all windows of observations and reminds to the fact that preferences need to be determined empirically, to discuss the profitability of a strategy based on time series momentum, we varied both the “look-back period” and the “holding period” on every instrument within the dataset. Moreover, for each asset  $s$  and month  $t$ , we considered the excess return over the past  $l$  months and depending on the sign of this excess return we hold the position for  $h$  months. Specifically, we recall the concept of formation and holding period periods as follows:

- **Look-Back Period:** it is the time considered for creating a signal. It is constructed creating average functions of returns for each asset during  $l$  months. The signals are generated

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<sup>1</sup>The effectiveness of a combined use of absolute and relative momentum has been studied by Antonacci in his paper, Gary Antonacci (2012), Risk Premia Harvesting Through Dual Momentum. Portfolio Management Consultants.



whether the excess return of these averages is positive or negative, in case of positive signal we go long, while in case of negative signal we go short.

- Holding Period: it represents how long the position remains open and it varies from the parameter  $h$ .

Table 1, Table 2, Table 3, Table 4, Table 5 and Table 6<sup>2</sup> show the results of time series<sup>3</sup> momentum backtested for each asset in case of positive signals<sup>4</sup> (long positions). Table 7, Table 8, Table 9, Table 10, Table 11 and Table 12<sup>5</sup> show the results of time series momentum backtested for each asset in case of negative signals<sup>6</sup> (short positions). These tables represent all the combinations of look-back periods and holding periods for every asset within the database over nearly 20 years (1999-2018). Specifically defining  $l_i$  = look-back period (with  $i = 1, \dots, 12$ ) and respectively  $h_j$  = holding period (with  $j = 1, \dots, 17$ ) we create 17 time series<sup>7</sup> of monthly returns for each asset moving from  $h_j$  to  $l_i$ . Each table corresponds to a different instrument and indicates cumulative monthly returns for every combination of  $(l, h)$ .

Moreover, we guarantee no overlapping of returns since we cumulate only returns that respect the duration of the selected holding period. For example, if we consider FTSE (1,6) and we have a positive signal in the month of May, we will invest 1 euro in June and we will be able to reinvest our capital only in December (6 months after) respecting the duration of the holding period, every other signal produced in the period between June and December will be treated as a new or separated investment (this process is automated for every asset and for every combination of  $l, h$ ). Our findings differ from Pedersen & Moskowitz study suggesting most of the times the use of a look-back period of one month. This is confirmed also by the results obtained with the Sharpe analysis (in Chapter II) of the formation periods where the look-back period of one month present the highest value of SR. Finally, we compute the mean for all the tables of positive signals as well as for all the tables of negative signals. These averages are represented in Figure 3.1.

<sup>2</sup>In the appendix.

<sup>3</sup>Time series or absolute momentum is applied by buying or selling the asset when positive or negative signals occur.

<sup>4</sup>When the excess return calculated over the formation period is positive.

<sup>5</sup>In the appendix.

<sup>6</sup>When the excess return calculated over the formation period is negative.

<sup>7</sup>This value depends on the max value of  $j$ .

## Average of Backtested Results

## Panel A

Equity Momentum: AVERAGE (1999-2018) CUMULATIVE

(l,h)	Hold 1	Hold 2	Hold 3	Hold 4	Hold 5	Hold 6	Hold 7	Hold 8	Hold 9	Hold 10	Hold 11	Hold 12	Hold 13	Hold 14	Hold 15	Hold 16	Hold 17
Look 1	58.7%	102.2%	222.6%	309.0%	357.8%	432.2%	489.3%	612.8%	597.2%	765.7%	970.4%	2585.0%	2456.1%	3134.8%	2826.6%	2662.9%	1481.1%
Look 2	25.0%	189.0%	133.7%	197.3%	209.6%	245.9%	272.4%	332.3%	292.8%	346.2%	351.7%	318.8%	337.1%	340.1%	331.8%	329.0%	376.0%
Look 3	40.4%	102.4%	128.7%	133.2%	138.3%	160.6%	197.5%	172.9%	211.6%	204.7%	188.1%	212.1%	190.3%	159.9%	197.5%	189.3%	139.1%
Look 4	-1.0%	31.3%	88.0%	108.9%	97.0%	158.6%	219.8%	171.3%	147.6%	164.0%	167.0%	204.2%	220.8%	235.1%	244.1%	219.2%	249.3%
Look 5	8.3%	31.2%	43.6%	32.5%	59.1%	66.0%	107.2%	169.3%	100.3%	78.8%	79.6%	84.2%	95.1%	99.0%	85.5%	109.6%	160.1%
Look 6	7.4%	25.6%	25.7%	20.0%	13.3%	54.7%	94.0%	71.7%	58.0%	43.4%	32.1%	85.3%	81.7%	90.2%	99.9%	83.0%	52.9%
Look 7	7.9%	19.4%	31.0%	159.6%	206.3%	222.9%	79.3%	55.4%	51.1%	76.7%	101.7%	86.9%	105.8%	81.3%	41.9%	41.8%	78.9%
Look 8	-3.3%	4.9%	23.6%	11.5%	10.6%	36.8%	45.7%	84.8%	51.1%	55.6%	48.7%	53.0%	55.7%	63.1%	58.7%	77.5%	87.2%
Look 9	13.1%	37.6%	57.2%	23.5%	40.6%	57.4%	79.4%	56.4%	84.1%	82.9%	104.7%	127.9%	84.4%	92.1%	97.1%	110.2%	105.0%
Look 10	28.0%	45.8%	36.0%	44.3%	55.8%	38.7%	46.2%	81.1%	65.0%	58.6%	77.6%	79.4%	49.0%	51.9%	30.7%	45.2%	46.9%
Look 12	0.5%	-7.9%	2.0%	-11.7%	-11.7%	19.2%	11.5%	20.1%	26.3%	37.5%	32.0%	32.0%	35.0%	29.2%	20.2%	13.0%	-3.6%

## Panel B

Equity Momentum: AVERAGE (1999-2018) CUMULATIVE SHORT

(l,h)	Hold 1	Hold 2	Hold 3	Hold 4	Hold 5	Hold 6	Hold 7	Hold 8	Hold 9	Hold 10	Hold 11	Hold 12	Hold 13	Hold 14	Hold 15	Hold 16	Hold 17
Look 1	-31.6%	-27.4%	-27.2%	-7.9%	-29.5%	10.4%	-13.7%	4.9%	24.0%	23.3%	66.7%	27.1%	13.1%	78.7%	70.8%	88.7%	106.9%
Look 2	20.9%	-90.8%	27.3%	27.4%	35.5%	30.8%	60.4%	46.0%	79.3%	41.0%	63.6%	61.7%	83.1%	60.5%	72.2%	57.3%	60.9%
Look 3	11.6%	37.8%	17.0%	20.7%	28.5%	15.2%	8.6%	27.5%	19.2%	9.4%	27.3%	-12.2%	-19.4%	-9.4%	17.5%	11.5%	27.0%
Look 4	32.7%	38.1%	39.8%	17.4%	45.4%	49.0%	55.9%	59.7%	71.6%	75.5%	86.3%	24.8%	54.0%	67.3%	62.5%	106.9%	106.1%
Look 5	13.0%	10.3%	6.1%	17.1%	10.0%	24.5%	30.3%	34.5%	50.0%	31.1%	24.6%	14.3%	25.8%	45.8%	68.4%	58.2%	67.5%
Look 6	1.0%	15.0%	21.8%	17.2%	29.8%	27.7%	25.1%	25.1%	17.6%	12.4%	25.2%	21.6%	19.3%	32.6%	33.9%	34.0%	34.7%
Look 7	8.9%	15.0%	17.6%	-48.4%	-38.9%	-31.4%	16.6%	23.8%	35.3%	45.5%	42.5%	38.7%	44.1%	55.6%	52.6%	43.4%	50.6%
Look 8	20.4%	24.8%	29.3%	24.2%	33.9%	41.8%	45.4%	35.8%	33.1%	36.9%	33.2%	37.1%	49.5%	57.2%	59.8%	53.5%	57.0%
Look 9	11.3%	28.3%	22.9%	21.1%	27.0%	20.9%	14.6%	22.3%	25.4%	30.8%	9.6%	9.7%	8.2%	17.8%	6.8%	17.3%	23.7%
Look 10	4.3%	-7.4%	-4.7%	-4.9%	-5.6%	13.6%	23.1%	22.6%	35.3%	22.1%	21.4%	10.8%	22.7%	23.8%	20.7%	18.6%	22.0%
Look 12	3.2%	13.5%	18.0%	9.7%	29.0%	26.9%	23.9%	24.5%	34.8%	38.4%	38.9%	16.0%	16.6%	19.9%	25.1%	38.3%	43.5%

FIGURE 3.1: shows the averages of the backtested cumulative returns of all the equity indexes (DAX CAC IBEX AEX and FTSE). Panel A illustrates the results in case of long positions while panel B displays the results in case of short positions. Also in this case, the look-back period of one month is the one associated with higher returns confirming that such time length window needs to be preferred in our analysis. Concerning the holding period or the amount of time an asset is held, we pass from 14 in panel A to 17 in panel B. In general, the backtested results of long positions (panel A) are more significative and present greatest differences in terms of returns than results obtained from short positions (panel B). Following our results, the best absolute strategy considering our equity indexes would be (1, 14) in case of long positions and (1, 17) in case of short positions. Conditional formatting has been made to highlight negative and positive results: red color indicates negative or lower returns, while green color indicates positive returns. In our calculations, there is no overlapping of observation since every holding period is respected before considering the impact of the successive investment.

### 3.3 Best Index Best Strategy Algorithm

The strategy can be summarized and automated with an algorithm<sup>8</sup> that uses both cross-sectional momentum<sup>9</sup> and times series momentum<sup>10</sup>. The former is used to switch between equity indexes and choose among them the “best index”. While the latter is used to switch between the previously founded best index and bonds applying the “best strategy” in every possible case (Figure 3.2). Since our results in the previous section confirmed the validity of formation periods of one month, we select this time horizon as the look-back period for the entire strategy.

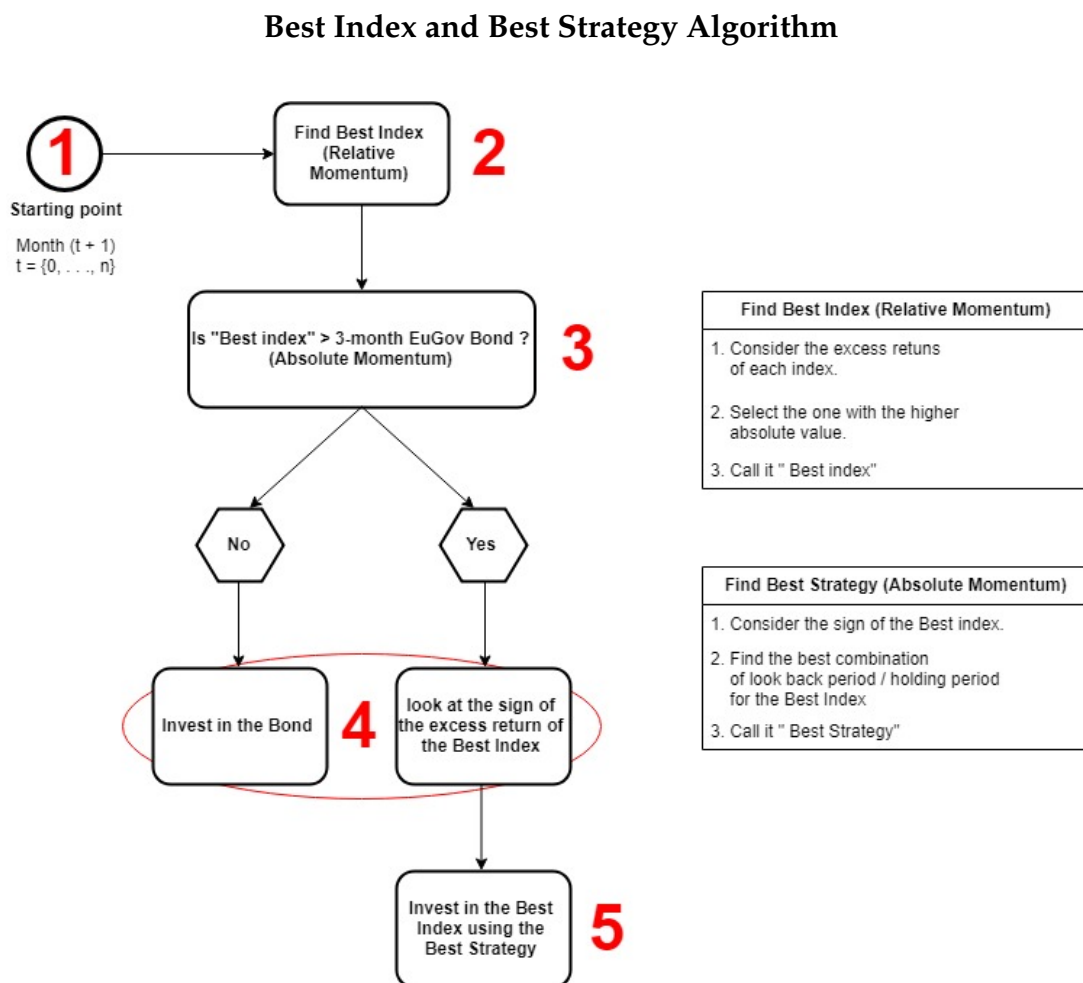


FIGURE 3.2: shows the Momentum Strategy and Illustrates with a graphical description each action of the Best Index Best Strategy Algorithm. The numbers in red are the steps of operation.

<sup>8</sup>The implementation is provisional, and it works only for  $t = 1$ .

<sup>9</sup>We use cross sectional momentum to switch from equity index.

<sup>10</sup>We use absolute momentum to switch from “Best Index” to bond and to find the “best period (l, h)”.

In order to better explain how the algorithm operates, we divide the procedure into different steps or moments:

- First step: Starting from the month  $t$  the algorithm collects the returns of every indexes  $s$  for that month of observations and calculates their excess return<sup>11</sup>  $r_t^s$ .
- Second step: it looks at the absolute value of these excess returns " $|r_t^s|$ " and it selects the one that has the higher absolute value " $\max |r_t^s|$ ". The index associated with the highest excess return (in absolute value) is called the "**Best index**".
- Third step: it compares<sup>12</sup> the excess return<sup>13</sup> associated with the best index to the return of a 3-month euro-government bond. If the excess return of the index is higher than the return of the bond, it generates a positive signal otherwise a negative signal.
- Fourth step: If the signal is negative it invests in the bond if the signal is positive it looks at the sign of the excess return and the bond is rejected.

Now in case the bond is rejected with a positive signal, the algorithm chooses to invest in the best index, but it needs to determine first the holding period of the investment. In order to find out how long the asset needs to be held by the strategy, the algorithm simply looks at a table of our backtested long/short results<sup>14</sup>. Then a new step must be followed:

- Fifth step: if the excess return associated with the best index  $s$  is positive, following the previous step it must take a long position on  $s$ . In doing so it looks at the backtested-long<sup>15</sup> results of the index  $s$  and selects the combination of  $(l, h)$  associated with the highest return. Since we set  $l = 1$ , it has to select only  $h$  respecting the value of  $l$ . If the excess return associated with the best index  $s$  is negative, this time must take a short position on  $s$ . Then it looks at the backtested-short<sup>16</sup> results of the index  $s$  and selects the combination of  $(l, h)$  associated with the highest return. Since we set  $l = 1$ , it has to select only  $h$  respecting the value of  $l$ .

The process described in the last step is called "**Best strategy**".

We repeat this procedure at the beginning of every month. These steps are recapped in Figure 3.2 with a graphical description.

<sup>11</sup>These excess returns are obtained subtracting from returns the arithmetic mean of the past returns calculated over the entire time series.

<sup>12</sup>It compares both negative and positive returns.

<sup>13</sup>It is always considered the absolute value.

<sup>14</sup>Grouped in the appendix.

<sup>15</sup>Are the result backtested using long positions and presented in forms of tables in section 3.2.

<sup>16</sup>Are the result backtested using short positions and presented in form of tables in section 3.2.

### 3.3.1 Test for $t = 1$

<sup>17</sup> We use this test as an example since we retrace the steps discussed previously:

- First step: Starting from the month of June 2015, the algorithm collects the returns of the indexes within our dataset (DAX, CAC, AEX, FTSE, UK, and IBEX) for that month of observations and calculates their excess return<sup>18</sup>  $r_t^s$ .
- Second step: the higher excess return in absolute value is the one associated with FTSE. Then it calls FTSE the “**Best index**”.
- Third step: it compares<sup>19</sup> the excess return<sup>20</sup> of FTSE to the return of a 3-month euro-government bond. FTSE has a 2% excess return while the bond has a return of 1% in June 2018. Since  $|2\%|$  is bigger than  $|1\%|$ , it generates a positive signal.
- Fourth step: Since the signal is positive it looks at the sign of  $|2\%|$ . 2% is positive so it moves to the fifth step.
- Fifth step: it invests in FTSE taking a long position. In doing so it looks at Table 1 (FTSE for long positions) and selects the combination of (1, 14). Then, it holds the position for 14 months and call (1, 14) the “**Best strategy**”.

Thanks to the Best index Best strategy Algorithm we know that for June 2015, **Best index** is FTSE and the **Best strategy** is (1, 14): taking a long position for 14 months on FTSE. We repeat this procedure at the beginning of every month.

The fifth step or the best strategy procedure is described in Figure 3.3 considering a long position on the index FTSE.

<sup>17</sup>The test is made using a dataset of approximately 20 years (from 1999 to 2018).

<sup>18</sup>These excess returns are obtained subtracting from returns the arithmetic mean of the past returns calculated over the entire time series.

<sup>19</sup>It compares both negative and positive returns.

<sup>20</sup>It is always considered the absolute value.

## Best Strategy of FTSE (1, 14)

Equity Momentum: FTSE (1999-2018) CUMULATIVE



(l,h)	Hold 1	Hold 2	Hold 3	Hold 4	Hold 5	Hold 6	Hold 7	Hold 8	Hold 9	Hold 10	Hold 11	Hold 12	Hold 13	Hold 14	Hold 15	Hold 16	Hold 17
Look 1													0.42%	3148.9%	14.4%		
Look 2	10.3%	126.7%	60.1%	104.3%	99.8%	129.5%	203.1%	172.7%	88.1%	84.7%	108.3%	137.9%	157.2%	186.8%	170.1%	157.1%	197.4%
Look 3	15.4%	77.1%	157.6%	112.6%	75.1%	160.5%	147.7%	67.6%	108.4%	65.5%	-6.2%	21.4%	-51.5%	-96.9%	-42.6%	-72.1%	-66.2%
Look 4	64.3%	73.9%	118.0%	133.0%	177.7%	173.1%	184.6%	209.6%	187.4%	172.1%	175.4%	108.1%	127.1%	143.9%	146.0%	131.6%	108.1%
Look 5	-18.9%	-19.5%	-6.4%	-22.4%	3.5%	-2.2%	9.2%	6.0%	34.6%	59.6%	34.5%	7.4%	14.4%	22.8%	33.6%	44.1%	67.0%
Look 6	4.9%	1.0%	30.8%	-8.4%	-16.5%	42.1%	56.8%	9.9%	13.8%	-8.1%	-15.7%	9.9%	1.8%	0.1%	6.7%	13.9%	-7.3%
Look 7	5.8%	24.8%	35.8%	163.6%	188.0%	180.6%	39.1%	6.6%	13.8%	35.3%	62.1%	50.7%	49.0%	42.7%	3.1%	3.6%	12.2%
Look 8	20.9%	28.4%	54.2%	27.2%	38.6%	57.8%	54.3%	28.9%	44.0%	38.0%	44.5%	23.4%	23.8%	26.9%	23.6%	95.7%	49.3%
Look 9	25.8%	22.3%	39.6%	-19.3%	-4.5%	4.8%	7.2%	-21.9%	6.7%	14.9%	-4.9%	0.2%	-35.4%	-11.0%	-16.7%	-7.7%	-3.3%
Look 10	26.8%	27.6%	24.8%	28.9%	51.8%	34.4%	35.1%	81.2%	70.0%	61.3%	67.3%	79.8%	45.7%	34.6%	10.7%	-1.5%	-19.0%
Look 12	1.8%	-10.5%	0.0%	-21.5%	-11.9%	19.3%	5.2%	0.6%	16.2%	13.2%	14.1%	22.4%	7.4%	-16.2%	-32.2%	-45.8%	-67.2%

FIGURE 3.3: displays the fifth step of the previous example using the results of FTSE backtested for long positions (Table 1). This figure shows how the Algorithm given a look-back period of one month selects the holding period with the highest return and finds the best strategy (1, 14). Using the Best index Best strategy algorithm we are able to apply the best strategy for each asset in every possible case. No matter in what kind of frequency of operation we choose (months, days, minutes), the algorithm will go long if momentum is positive, will go short when momentum is negative and will invest in bonds in case of range bounded markets enhancing returns and ensuring the best possible outcome for the entire strategy.

### 3.3.2 Modified Relative Momentum

In this application, we relax the concept of relative momentum using a light or modified version of cross-sectional application where we don't need to respect the condition of long/short matching or zero-net exposure. If we do not guarantee a zero-net exposure the relative strategy will suffer from directional bias because we are creating negative or positive exposure. Then, not adjusting positions for volatility will not be the primary cause of directional risk, since we already bearing that risk without guaranteeing the zero-net exposure. This allows us to simplify the procedures and to not worry about adjusting positions and signals for volatility. Thus, to reduce the complexity of the overall strategy we believe that it is more convenient to apply modified relative momentum and face directly the directional risk of the market.

The modified cross-sectional momentum is seen as a comparison of performance among assets that belong to the same category (comparing between peers). In other words, to apply modified relative momentum we don't need to buy some assets (winners) and sell the same number of assets (losers) but we only need to compare the performance of the instruments relative to their peers and choose the one that is outperforming (we create positive or negative net exposure). For example, considering two assets: X and Y. If X has outperformed Y by 2%, we say that X has a positive relative momentum effect with respect to Y. In this application, we only consider this modified version of cross-sectional momentum.

## 3.4 Results and Implementations

The algorithm has been tested only for the month of June 2015 and the results indicate the FTSE as the "best index" (with a positive signal) and the combination of look-back (1) and holding (14) as the "best strategy". There is not complete backtesting of our strategy since the algorithm has not been fully completed.

An interesting implementation of this algorithm would be to use this strategy in a pure high-frequency trading mode. Specifically, generating a risk premium algo-strategy by reducing the time horizon of the estimation and shifting the time of the operations from months to minutes. It would be interesting to analyze the results (the differences in terms of "best index" and "best strategy") and the risk factor associated with. Another implementation would be the creation of modules to separate investments and instruments that belong to different asset class (i.e. equities-module, currencies-module, ... etc.). The modules will protect the assets enhancing the diversification of the portfolio. In fact, the use of relative momentum over a big set of assets could cause a constant drop out of some instruments that are less likely to be part of the winner-loser category (i.e. lagging assets that may suddenly start outperforming), this will be a possible cause of exclusion from the active portfolio. The use of relative momentum on modules could reduce this bias of exclusion and guarantee a higher level of diversification over the entire strategy. Finally, in order

to protect the portfolio to large losses, it is necessary to set stop-losses or take-profit constraints over the entire strategy<sup>21</sup>.

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<sup>21</sup>There are many other useful and interesting implementations or studies such as drawdown exposure, expected shortfall and payoff analysis that we do not cover in this dissertation.



# Conclusion

Since 1993 momentum strategies were considered one of the most valuable techniques over the entire investment industry and have been subject of research of a great number of academic and professional studies. Alternative risk premia have changed the portfolio construction process shifting from a correlation diversification approach to a payoff diversification approach and becoming the building blocks of multi-asset management. Statistical evidence suggested that many financial assets show positive short-term autocorrelation or momentum bias<sup>22</sup> and that trend following strategies are profitable in almost every asset class. Moskowitz and Pedersen, as well as Gary Antonacci, studied the trend following applications on equities, currencies, commodities, and bonds finding substantial abnormal returns, especially during extreme markets. Nowadays every informed investor should know the main benefits associated with the use of momentum investing and alternative risk premia. Nevertheless, it is highly important to continue to study such interesting properties elucidating the role of a wide use of ARP into a pre-crisis environment. The primary objective of this final thesis was not only to discuss ARP and to collect the existing literature of momentum investing but also to prove the momentum effect and his profitability presenting a simple strategy that uses eurozone data and applies automated absolute rules. In the second and third chapter, we replicate some methodologies and applications described by Gary Antonacci and Moskowitz & Pedersen in their studies to prove the momentum effect<sup>23</sup>.

The first step was to use a measure of performance such as the Sharpe ratio to find the best formation period (look-back period) within the dataset. Results indicated that the highest level of SR was associated with formation periods of one month. Hence, we rejected the idea that a 12 months look-back period needs to be preferred among all windows of observations and we reminded to the fact that preferences need to be determined empirically in every case since they may vary among instruments and across time.

The second step was to examine the time series predictability of returns across different time horizons using regression and volatility weighting. Following Pedersen & Moskowitz methodology, we proved the momentum effect by looking at the time series of excess returns and at the autocovariance between the security's excess return next month and its lagged excess returns. In order

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<sup>22</sup>Using the expression of momentum biases, we refer to the momentum effect that can be interpreted as an anomaly or as a bias of the market, behavioral theories.

<sup>23</sup>We refer to absolute momentum effect.

to make meaningful comparisons across assets, we scaled all returns by their ex-ante volatility. Once we tested the validity of the volatility model, we used regression analysis to useful predict the price continuation and subsequent reversal for every asset within the database showing the existence of the so-called momentum effect. Once again findings were supportive, and results confirmed that in our analysis the look-back period of one-month needs to be preferred among all windows of observations.

Finally, in the last chapter, we tested momentum trading running a backtest and creating a strategy that uses both absolute and relative momentum at the same time<sup>24</sup>. The strategy has been automated with an algorithm<sup>25</sup> that uses cross-sectional momentum<sup>26</sup> to choose the “best index” and times series momentum<sup>27</sup> to select and apply the “best strategy” in every possible case. The backtest proves that returns associated with formation periods of one month are the highest among all assets within the database supporting results founded in the previous chapter. The strategy confirmed the validity of momentum investing showing the main benefits of mixing cross-sectional and time series momentum at an absolute level. In particular, such methods are interesting to understand past performances and best asset combinations as well as further potential uses in high-frequency trading and portfolio construction.

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<sup>24</sup>The effectiveness of a combined use of absolute and relative momentum has been studied by Antonacci in his paper, Gary Antonacci (2012), Risk Premia Harvesting Through Dual Momentum. Portfolio Management Consultants.

<sup>25</sup>The implementation is provisional, and it works only for  $t = 1$ .

<sup>26</sup>We use cross sectional momentum to switch from equity index.

<sup>27</sup>We use absolute momentum to switch from “Best Index” to bond a and to find the “best period (l, h)”.

# Appendix

## Results Backtested

In this appendix, we show the results of Long and Short positions for the assets within the database.

The followings tables are the cumulative monthly returns of all the backtesting we using a time horizon of nearly 20 years (1999-2018).

### FTSE backtested for long positions

Equity Momentum: FTSE (1999-2018) CUMULATIVE

(l,h)	Hold 1	Hold 2	Hold 3	Hold 4	Hold 5	Hold 6	Hold 7	Hold 8	Hold 9	Hold 10	Hold 11	Hold 12	Hold 13	Hold 14	Hold 15	Hold 16	Hold 17
Look 1	-6.8%	23.9%	133.3%	201.2%	182.2%	196.4%	150.8%	167.0%	99.0%	217.8%	536.5%	289.2%	450.4%	3119.1%	1408.7%	1359.2%	539.4%
Look 2	10.3%	120.9%	60.1%	104.3%	99.8%	115.3%	166.5%	154.6%	75.7%	69.7%	96.2%	127.4%	137.7%	153.9%	140.8%	157.1%	197.4%
Look 3	15.4%	77.1%	157.6%	112.6%	75.1%	160.5%	147.7%	67.6%	91.7%	56.4%	-14.0%	-35.9%	-94.0%	-126.1%	-66.3%	-72.1%	-66.2%
Look 4	64.3%	73.9%	118.0%	121.2%	177.7%	173.1%	184.6%	209.6%	167.5%	157.5%	161.5%	75.2%	88.7%	112.9%	112.6%	131.6%	108.1%
Look 5	-18.9%	-19.5%	-6.4%	-22.4%	3.5%	-2.2%	9.2%	6.0%	24.6%	52.7%	28.5%	39.3%	37.6%	42.7%	63.3%	44.1%	67.0%
Look 6	4.9%	1.0%	30.8%	-8.4%	-16.5%	42.1%	56.8%	9.9%	-1.8%	-16.0%	-22.7%	28.9%	22.6%	32.0%	34.1%	13.9%	-7.3%
Look 7	5.8%	24.8%	35.8%	163.6%	188.0%	180.6%	39.1%	6.6%	3.5%	20.2%	53.4%	80.6%	83.0%	64.8%	22.2%	3.6%	12.2%
Look 8	20.9%	28.4%	54.2%	27.2%	38.6%	57.8%	54.3%	28.9%	25.4%	23.6%	23.3%	44.1%	48.3%	72.5%	60.6%	95.7%	49.3%
Look 9	25.8%	22.3%	39.6%	-19.3%	-4.5%	4.8%	7.2%	-21.9%	-21.0%	-14.2%	-26.9%	33.5%	-11.1%	3.0%	2.5%	-7.7%	-3.3%
Look 10	26.8%	27.6%	24.8%	28.9%	51.8%	34.4%	35.1%	81.2%	62.8%	63.3%	88.9%	38.1%	12.6%	4.8%	-18.9%	-1.5%	-19.0%
Look 12	1.8%	-10.5%	0.0%	-21.5%	-11.9%	19.3%	5.2%	0.6%	-0.5%	0.7%	2.4%	53.4%	38.5%	25.3%	-0.6%	-45.8%	-67.2%

### DAX backtested for long positions

Equity Momentum: DAX (1999-2018) CUMULATIVE

(l,h)	Hold 1	Hold 2	Hold 3	Hold 4	Hold 5	Hold 6	Hold 7	Hold 8	Hold 9	Hold 10	Hold 11	Hold 12	Hold 13	Hold 14	Hold 15	Hold 16	Hold 17
Look 1	82.0%	114.7%	257.0%	449.0%	427.5%	562.5%	711.2%	826.1%	728.5%	664.8%	726.4%	4504.7%	4226.8%	6485.6%	7705.0%	10911.7%	1864.4%
Look 2	85.4%	276.0%	280.8%	301.6%	325.9%	352.6%	418.2%	564.1%	510.0%	599.1%	646.2%	448.9%	478.1%	519.5%	561.6%	463.5%	500.8%
Look 3	79.0%	278.5%	254.3%	227.4%	281.9%	320.7%	361.8%	367.4%	404.6%	423.3%	444.5%	380.5%	382.2%	381.0%	347.5%	324.6%	328.2%
Look 4	0.4%	52.5%	154.9%	171.2%	158.4%	267.1%	388.3%	179.4%	166.8%	189.6%	193.3%	272.6%	320.7%	275.9%	298.5%	276.9%	273.4%
Look 5	16.4%	31.2%	24.2%	26.0%	65.4%	73.7%	107.4%	173.9%	90.1%	96.6%	106.5%	80.2%	117.5%	142.7%	92.2%	169.8%	165.5%
Look 6	11.2%	41.0%	16.1%	-10.6%	-15.1%	46.8%	111.4%	90.7%	61.2%	41.9%	44.0%	116.4%	142.3%	150.0%	184.4%	106.1%	74.2%
Look 7	22.9%	43.2%	39.3%	170.9%	222.7%	221.1%	110.9%	75.1%	37.5%	68.8%	88.3%	83.7%	149.8%	117.1%	41.1%	31.1%	67.9%
Look 8	-3.6%	15.2%	13.6%	-5.3%	1.8%	7.7%	17.1%	108.9%	22.1%	22.6%	14.7%	16.7%	-2.6%	-14.9%	-0.7%	51.2%	38.1%
Look 9	25.9%	88.0%	109.9%	66.8%	99.0%	125.1%	169.1%	122.4%	178.4%	185.3%	246.3%	239.7%	167.3%	173.2%	161.3%	221.6%	201.9%
Look 10	48.8%	55.0%	51.4%	65.4%	75.8%	55.0%	60.6%	95.5%	101.8%	79.1%	98.7%	111.7%	87.8%	109.3%	78.7%	110.7%	91.8%
Look 12	-2.3%	-9.2%	-11.4%	-37.5%	-36.2%	-1.1%	7.6%	13.2%	32.7%	46.3%	39.0%	18.3%	24.0%	24.6%	21.5%	2.0%	-21.1%



## UK backtested for long positions

Equity Momentum: UK (1999-2018) CUMULATIVE

(l,h)	Hold 1	Hold 2	Hold 3	Hold 4	Hold 5	Hold 6	Hold 7	Hold 8	Hold 9	Hold 10	Hold 11	Hold 12	Hold 13	Hold 14	Hold 15	Hold 16	Hold 17
Look 1	20.6%	60.5%	133.7%	215.5%	328.4%	426.1%	569.8%	665.0%	725.2%	951.5%	1328.1%	6572.7%	1661.0%	1232.9%	2060.3%	1937.9%	3037.1%
Look 2	6.7%	178.4%	106.7%	171.6%	152.3%	192.4%	225.7%	281.7%	271.1%	322.7%	299.2%	385.0%	345.0%	361.5%	338.8%	407.2%	412.0%
Look 3	31.4%	86.3%	135.8%	129.9%	119.2%	146.5%	222.7%	204.0%	312.1%	304.9%	236.9%	311.5%	280.8%	218.1%	282.9%	255.9%	196.3%
Look 4	8.0%	39.0%	98.9%	123.9%	142.5%	200.1%	267.7%	301.5%	271.9%	322.5%	313.7%	275.9%	295.0%	335.3%	349.2%	387.9%	356.6%
Look 5	30.4%	62.7%	75.9%	70.8%	90.6%	105.9%	155.2%	243.8%	169.7%	109.7%	101.3%	144.0%	137.4%	146.6%	200.5%	234.6%	270.2%
Look 6	-16.9%	-16.6%	-4.7%	-4.0%	-18.0%	47.0%	58.0%	30.7%	23.3%	16.8%	-18.4%	62.5%	61.0%	53.2%	64.4%	45.8%	1.1%
Look 7	4.0%	8.5%	-3.4%	142.0%	167.3%	172.9%	79.4%	65.8%	73.5%	91.4%	127.2%	101.9%	131.6%	131.0%	90.9%	93.7%	128.0%
Look 8	9.1%	4.5%	26.4%	31.6%	36.6%	93.0%	109.1%	115.0%	88.9%	101.1%	86.5%	139.2%	151.8%	192.1%	178.2%	168.2%	147.9%
Look 9	5.3%	17.8%	41.6%	12.4%	32.8%	63.8%	92.1%	50.7%	113.6%	119.0%	111.1%	168.1%	132.5%	135.2%	143.6%	157.1%	104.3%
Look 10	26.2%	48.8%	34.1%	39.6%	44.2%	34.0%	37.6%	80.6%	69.7%	47.5%	46.4%	84.3%	41.0%	46.0%	27.9%	39.0%	32.6%
Look 12	-3.1%	-12.4%	8.5%	2.1%	4.6%	43.7%	33.5%	46.3%	39.4%	56.2%	40.0%	69.8%	77.0%	62.7%	66.5%	72.4%	46.6%

## IBEX backtested for long positions

Equity Momentum: IBEX (1999-2018) CUMULATIVE

(l,h)	Hold 1	Hold 2	Hold 3	Hold 4	Hold 5	Hold 6	Hold 7	Hold 8	Hold 9	Hold 10	Hold 11	Hold 12	Hold 13	Hold 14	Hold 15	Hold 16	Hold 17
Look 1	78.8%	153.0%	357.9%	439.1%	485.4%	648.6%	556.2%	900.6%	902.6%	1734.4%	2263.3%	1670.7%	6237.2%	9392.4%	3340.8%	1314.8%	1811.8%
Look 2	5.2%	144.9%	107.7%	155.3%	231.5%	295.8%	361.1%	369.2%	394.7%	355.6%	403.4%	384.4%	478.1%	438.6%	468.1%	353.3%	283.0%
Look 3	26.6%	65.7%	142.2%	138.7%	120.3%	121.3%	160.1%	139.5%	183.6%	152.3%	177.3%	180.6%	139.2%	146.6%	254.8%	212.2%	195.4%
Look 4	4.8%	43.5%	110.0%	68.6%	43.7%	74.4%	130.9%	140.1%	102.5%	80.9%	100.8%	170.3%	211.6%	231.6%	216.1%	169.1%	151.2%
Look 5	-6.4%	24.7%	14.0%	-10.1%	43.0%	69.6%	156.0%	216.5%	105.3%	118.7%	147.3%	159.0%	135.3%	111.4%	112.0%	114.1%	175.9%
Look 6	-5.8%	16.7%	42.8%	41.3%	57.7%	85.4%	121.2%	87.1%	98.9%	70.2%	104.7%	149.4%	108.2%	132.0%	150.8%	156.9%	143.5%
Look 7	11.5%	24.0%	60.6%	192.7%	305.8%	351.5%	120.2%	108.3%	138.2%	172.1%	213.5%	199.6%	196.2%	147.5%	128.2%	149.0%	176.6%
Look 8	-2.1%	20.4%	53.8%	24.1%	17.1%	61.4%	89.0%	103.5%	98.4%	102.0%	69.1%	63.4%	83.1%	80.6%	74.2%	130.6%	129.8%
Look 9	35.0%	66.7%	89.6%	54.1%	78.1%	57.7%	65.2%	75.0%	58.9%	53.4%	45.2%	102.8%	61.4%	102.5%	96.9%	53.8%	56.7%
Look 10	26.1%	39.3%	25.3%	33.2%	55.7%	47.3%	85.9%	122.5%	102.2%	100.7%	153.0%	103.7%	80.6%	66.9%	44.4%	95.5%	104.9%
Look 12	-10.6%	-16.7%	6.2%	-12.8%	9.8%	52.9%	22.5%	22.2%	72.6%	88.3%	84.7%	25.6%	20.3%	15.5%	6.0%	35.5%	31.4%

## AEX backtested for long positions

Equity Momentum: AEX (1999-2018) CUMULATIVE

(l,h)	Hold 1	Hold 2	Hold 3	Hold 4	Hold 5	Hold 6	Hold 7	Hold 8	Hold 9	Hold 10	Hold 11	Hold 12	Hold 13	Hold 14	Hold 15	Hold 16	Hold 17
Look 1	-29.6%	29.3%	135.1%	230.5%	245.6%	328.6%	364.6%	503.0%	444.2%	382.5%	735.2%	1732.9%	1670.7%	625.4%	2724.1%	1129.9%	957.7%
Look 2	43.1%	219.6%	183.3%	326.0%	296.2%	252.1%	275.0%	355.5%	279.3%	350.1%	356.2%	352.3%	404.9%	427.6%	411.8%	421.4%	464.3%
Look 3	22.2%	72.9%	58.7%	61.0%	31.0%	50.2%	60.2%	26.4%	54.4%	15.1%	-25.6%	58.0%	38.9%	-21.9%	-33.7%	3.2%	-31.9%
Look 4	6.8%	30.2%	73.8%	92.5%	79.3%	170.5%	235.1%	210.2%	171.0%	208.1%	227.5%	221.4%	203.5%	278.7%	298.7%	310.6%	286.3%
Look 5	-0.8%	42.4%	60.2%	88.0%	94.3%	83.9%	160.3%	248.2%	173.0%	132.9%	113.8%	107.7%	138.2%	171.4%	130.9%	162.6%	182.0%
Look 6	3.0%	17.3%	6.3%	-6.0%	-6.2%	52.8%	74.2%	51.1%	18.3%	16.0%	-16.3%	12.3%	9.9%	28.4%	27.6%	35.3%	-7.6%
Look 7	22.6%	36.6%	47.8%	179.1%	212.2%	198.9%	91.1%	71.1%	51.1%	93.8%	108.0%	50.4%	64.4%	62.7%	-1.7%	-15.6%	24.8%
Look 8	7.8%	9.9%	34.1%	25.0%	25.2%	69.1%	75.6%	140.7%	72.2%	76.4%	90.2%	84.6%	97.5%	124.3%	123.9%	112.9%	75.8%
Look 9	15.5%	45.9%	77.6%	22.0%	49.5%	72.8%	97.7%	87.4%	133.5%	133.9%	145.3%	153.0%	97.7%	107.8%	117.6%	168.2%	162.6%
Look 10	32.0%	61.0%	47.1%	71.0%	82.7%	61.8%	71.8%	118.9%	102.6%	94.6%	103.2%	79.1%	62.3%	66.7%	37.2%	45.5%	37.1%
Look 12	3.9%	-0.1%	14.7%	-5.6%	5.2%	41.6%	34.7%	62.2%	45.3%	66.7%	70.6%	58.3%	61.9%	59.8%	49.3%	38.1%	4.7%

## CAC backtested for long positions

Equity Momentum: CAC (1999-2018) CUMULATIVE

(l,h)	Hold 1	Hold 2	Hold 3	Hold 4	Hold 5	Hold 6	Hold 7	Hold 8	Hold 9	Hold 10	Hold 11	Hold 12	Hold 13	Hold 14	Hold 15	Hold 16	Hold 17
Look 1	63.7%	133.9%	264.8%	359.0%	428.8%	507.3%	514.2%	626.9%	567.5%	592.7%	634.8%	687.0%	708.6%	1011.3%	953.1%	542.1%	676.0%
Look 2	56.7%	196.9%	189.2%	299.6%	331.0%	387.7%	419.7%	444.1%	378.2%	450.4%	469.3%	417.2%	433.8%	441.7%	369.7%	393.7%	398.6%
Look 3	84.4%	160.5%	169.8%	225.9%	257.6%	291.3%	293.3%	259.5%	270.1%	248.4%	213.1%	240.5%	201.5%	162.3%	243.5%	228.9%	212.7%
Look 4	26.7%	82.8%	146.7%	204.4%	221.5%	329.4%	386.7%	272.8%	276.3%	308.3%	302.8%	294.2%	369.9%	392.9%	407.3%	331.0%	320.4%
Look 5	23.3%	33.2%	90.0%	54.9%	84.5%	92.5%	102.6%	194.1%	137.5%	97.0%	89.6%	89.4%	134.9%	131.4%	98.8%	104.3%	100.1%
Look 6	30.5%	73.9%	66.8%	64.2%	71.1%	131.5%	187.2%	153.4%	116.0%	93.4%	97.7%	179.9%	162.1%	177.9%	183.1%	160.0%	113.2%
Look 7	17.7%	46.5%	74.2%	209.1%	264.8%	326.3%	95.0%	52.2%	54.5%	90.8%	130.8%	121.5%	148.5%	109.1%	61.9%	50.3%	63.6%
Look 8	7.0%	18.6%	36.1%	12.0%	20.6%	41.3%	43.9%	52.0%	67.2%	74.9%	65.8%	57.1%	64.0%	72.2%	64.0%	98.3%	82.3%
Look 9	19.2%	57.7%	74.5%	25.7%	39.9%	70.0%	89.5%	47.0%	97.4%	94.7%	113.2%	141.7%	81.3%	88.7%	99.5%	118.5%	107.7%
Look 10	43.2%	57.8%	61.4%	70.9%	90.5%	67.7%	61.1%	104.9%	74.6%	60.5%	78.6%	83.0%	58.4%	65.5%	40.3%	51.3%	34.2%
Look 12	13.3%	5.0%	11.5%	-11.6%	-4.7%	26.4%	13.1%	18.0%	19.6%	30.0%	31.3%	52.2%	63.3%	54.9%	34.8%	9.1%	-16.2%



## FTSE backtested for short positions

Equity Momentum: FTSE (1999-2018) CUMULATIVE SHORT

(l,h)	Hold 1	Hold 2	Hold 3	Hold 4	Hold 5	Hold 6	Hold 7	Hold 8	Hold 9	Hold 10	Hold 11	Hold 12	Hold 13	Hold 14	Hold 15	Hold 16	Hold 17
Look 1	-23.9%	-7.4%	66.3%	79.4%	86.3%	88.9%	81.5%	64.9%	144.9%	109.5%	288.7%	347.2%	341.5%	463.4%	410.6%	398.7%	475.9%
Look 2	42.5%	-142.4%	50.3%	55.9%	75.1%	68.4%	118.0%	75.1%	137.4%	116.2%	131.6%	129.3%	165.4%	176.4%	191.6%	200.3%	236.5%
Look 3	31.3%	71.5%	64.5%	69.7%	83.6%	23.9%	71.2%	99.9%	66.7%	54.1%	73.7%	51.7%	49.2%	69.4%	72.4%	98.6%	135.6%
Look 4	61.2%	70.9%	72.1%	67.0%	105.0%	109.4%	109.4%	99.2%	118.9%	134.8%	133.4%	109.4%	145.2%	167.9%	158.7%	189.4%	200.0%
Look 5	28.1%	42.8%	27.9%	37.2%	9.6%	47.2%	78.4%	119.0%	87.0%	52.7%	63.9%	73.3%	83.0%	109.2%	122.4%	156.1%	178.6%
Look 6	9.3%	30.6%	41.7%	33.1%	49.9%	30.4%	51.7%	58.3%	36.4%	35.3%	59.6%	61.2%	64.3%	90.1%	87.1%	112.7%	119.2%
Look 7	7.7%	26.9%	32.3%	-71.6%	-50.0%	-39.0%	45.0%	45.9%	65.5%	85.5%	85.4%	83.0%	90.3%	116.9%	117.9%	122.7%	130.5%
Look 8	36.6%	44.2%	44.8%	40.4%	64.0%	71.5%	74.6%	49.0%	51.6%	58.5%	56.0%	74.8%	89.0%	85.9%	93.3%	101.2%	100.3%
Look 9	16.6%	42.7%	29.9%	32.8%	44.8%	37.6%	27.7%	34.5%	67.9%	77.5%	88.4%	80.0%	81.2%	92.5%	80.5%	66.7%	75.7%
Look 10	15.7%	7.4%	4.5%	-0.3%	-7.9%	31.5%	54.1%	57.6%	69.4%	55.8%	64.9%	54.4%	52.0%	57.2%	43.1%	38.9%	46.2%
Look 12	12.2%	25.1%	31.8%	21.0%	49.4%	49.5%	42.0%	43.9%	58.5%	65.0%	69.5%	33.7%	37.4%	41.0%	49.3%	70.4%	77.3%

## DAX backtested for short positions

Equity Momentum: DAX (1999-2018) CUMULATIVE SHORT

(l,h)	Hold 1	Hold 2	Hold 3	Hold 4	Hold 5	Hold 6	Hold 7	Hold 8	Hold 9	Hold 10	Hold 11	Hold 12	Hold 13	Hold 14	Hold 15	Hold 16	Hold 17
Look 1	56.1%	94.7%	121.9%	168.9%	219.6%	215.0%	294.6%	219.0%	388.6%	487.7%	422.4%	559.6%	299.6%	256.4%	448.3%	459.5%	725.4%
Look 2	-44.7%	71.3%	-66.1%	-22.0%	-31.7%	73.0%	42.5%	67.9%	73.1%	32.1%	-7.8%	-64.9%	-63.6%	-43.5%	-45.1%	30.2%	31.1%
Look 3	-48.7%	-79.3%	-53.6%	-60.4%	-82.4%	2.5%	-24.6%	-61.4%	25.2%	3.9%	-12.0%	-63.1%	-83.7%	-113.8%	77.8%	92.7%	82.0%
Look 4	-46.3%	-48.5%	-39.5%	-13.5%	-53.8%	-77.3%	-59.9%	36.8%	-30.1%	-34.9%	24.1%	-66.2%	-114.3%	-86.3%	-66.4%	52.1%	-9.8%
Look 5	-11.9%	-20.0%	17.4%	-8.3%	40.4%	43.2%	2.6%	-44.6%	-1.8%	50.6%	45.1%	22.9%	4.7%	8.0%	99.1%	72.5%	85.7%
Look 6	28.8%	17.1%	40.5%	20.1%	-1.6%	32.0%	17.8%	22.0%	37.4%	5.6%	-31.8%	-15.4%	-2.7%	3.4%	-2.1%	-16.0%	-27.1%
Look 7	-30.6%	-42.4%	-37.0%	73.3%	63.2%	75.5%	-14.2%	-15.6%	-16.1%	-33.1%	-41.2%	-21.3%	-46.3%	-55.4%	-43.5%	-31.8%	-44.1%
Look 8	-24.7%	-21.4%	12.4%	-1.3%	-29.2%	-33.4%	-21.1%	-0.2%	-5.2%	10.1%	41.5%	-12.1%	-29.8%	-14.1%	-20.9%	15.7%	17.8%
Look 9	-27.1%	-51.8%	-41.3%	-46.7%	-60.4%	-47.6%	-46.3%	-51.6%	-53.9%	-64.4%	-74.9%	-81.0%	-77.7%	-101.5%	-83.7%	-69.1%	-77.6%
Look 10	-11.9%	-7.5%	-1.6%	-1.9%	4.7%	-13.4%	-26.7%	-31.8%	-56.9%	-29.9%	-48.5%	-31.7%	-54.1%	-59.8%	-52.3%	-42.3%	-40.8%
Look 12	1.2%	-14.1%	2.2%	-3.9%	-43.8%	-38.1%	-23.7%	-8.3%	-29.8%	-42.8%	-41.2%	-34.2%	-41.7%	-45.5%	-49.3%	-30.2%	-48.3%

## UK backtested for short positions

Equity Momentum: UK (1999-2018) CUMULATIVE SHORT

(l,h)	Hold 1	Hold 2	Hold 3	Hold 4	Hold 5	Hold 6	Hold 7	Hold 8	Hold 9	Hold 10	Hold 11	Hold 12	Hold 13	Hold 14	Hold 15	Hold 16	Hold 17
Look 1	-31.1%	-58.5%	-53.9%	-68.6%	-66.3%	-38.5%	-60.3%	21.0%	99.7%	78.0%	24.8%	-85.8%	-135.5%	-75.8%	-86.8%	30.2%	53.8%
Look 2	25.5%	-96.5%	45.4%	35.4%	39.9%	22.9%	59.7%	8.7%	65.3%	28.6%	56.7%	57.0%	63.2%	35.6%	59.1%	35.8%	38.3%
Look 3	17.8%	65.7%	46.3%	43.8%	76.8%	37.3%	61.6%	94.7%	43.0%	60.6%	79.0%	62.7%	53.0%	77.8%	33.3%	-1.3%	10.7%
Look 4	42.9%	47.5%	51.5%	27.5%	77.2%	84.1%	89.6%	97.6%	124.6%	125.8%	108.2%	82.7%	119.5%	122.8%	118.7%	143.0%	153.8%
Look 5	25.9%	18.1%	7.8%	22.7%	33.9%	39.1%	39.4%	91.9%	52.8%	37.4%	21.9%	19.9%	11.9%	23.5%	41.7%	2.5%	-2.6%
Look 6	-22.6%	3.7%	13.2%	16.9%	27.0%	15.2%	16.0%	21.9%	0.3%	-10.3%	-0.7%	6.0%	6.9%	20.8%	22.2%	8.5%	11.0%
Look 7	13.1%	8.0%	6.7%	-84.9%	-76.6%	-70.5%	22.4%	34.6%	24.2%	39.7%	18.5%	7.0%	25.0%	52.9%	51.5%	58.6%	76.7%
Look 8	28.0%	31.3%	34.5%	38.1%	54.6%	63.6%	64.9%	33.8%	43.0%	42.3%	47.0%	61.9%	77.4%	82.7%	84.8%	87.0%	90.4%
Look 9	16.1%	36.5%	29.5%	20.4%	34.0%	28.8%	28.9%	39.7%	21.1%	27.3%	1.3%	5.8%	2.1%	14.3%	5.9%	28.3%	36.8%
Look 10	5.2%	-8.8%	-5.5%	-12.1%	-4.1%	13.9%	20.3%	20.8%	35.6%	9.7%	18.6%	5.6%	37.8%	38.8%	43.2%	30.1%	27.9%
Look 12	-3.9%	14.1%	16.0%	14.7%	38.4%	33.7%	34.0%	38.4%	56.9%	59.3%	28.5%	-8.9%	-4.5%	5.3%	10.3%	36.4%	47.2%

## IBEX backtested for short positions

Equity Momentum: IBEX (1999-2018) CUMULATIVE SHORT

(l,h)	Hold 1	Hold 2	Hold 3	Hold 4	Hold 5	Hold 6	Hold 7	Hold 8	Hold 9	Hold 10	Hold 11	Hold 12	Hold 13	Hold 14	Hold 15	Hold 16	Hold 17
Look 1	-19.0%	-23.7%	-31.6%	15.7%	-44.7%	-16.5%	-39.3%	-6.9%	-44.2%	-33.9%	-50.2%	-122.7%	-96.9%	-58.0%	-72.6%	-29.0%	-114.5%
Look 2	15.5%	-138.1%	20.5%	-23.8%	9.8%	-28.4%	0.4%	34.2%	36.9%	21.1%	48.1%	57.6%	83.5%	1.4%	21.8%	-70.0%	-130.2%
Look 3	32.9%	55.6%	16.1%	32.0%	37.9%	16.1%	-12.7%	9.3%	-36.2%	-30.1%	33.1%	-40.7%	-40.3%	-47.1%	9.4%	-4.0%	21.5%
Look 4	37.4%	38.9%	41.5%	-34.5%	-4.2%	-5.7%	19.0%	-23.8%	6.4%	-14.6%	-0.7%	-78.1%	-35.5%	-39.9%	-69.7%	-48.9%	-34.4%
Look 5	-1.9%	-6.3%	-43.4%	-12.6%	-41.8%	-23.5%	8.1%	-67.0%	35.9%	-51.2%	-99.8%	-145.8%	-96.9%	-39.5%	-22.5%	-55.1%	-32.0%
Look 6	14.2%	34.7%	34.2%	32.0%	42.7%	28.1%	43.6%	27.7%	22.9%	17.7%	25.2%	-8.3%	-14.8%	-2.6%	-1.7%	-0.2%	-9.0%
Look 7	7.7%	17.8%	29.3%	-82.2%	-51.3%	-39.7%	-7.8%	7.3%	43.4%	59.4%	54.6%	52.9%	58.1%	60.9%	47.7%	-3.5%	5.9%
Look 8	17.0%	27.7%	40.1%	35.6%	41.5%	50.8%	48.7%	100.8%	25.8%	27.0%	17.3%	29.3%	53.9%	57.3%	52.6%	-22.3%	0.4%
Look 9	21.7%	49.3%	33.9%	48.8%	43.9%	23.7%	11.3%	32.8%	-29.0%	-25.0%	-32.8%	-29.6%	-33.0%	-10.8%	-41.6%	-34.9%	-15.5%
Look 10	-3.1%	-22.9%	-35.0%	-27.8%	-33.8%	-1.3%	24.7%	26.1%	51.5%	36.0%	39.3%	24.3%	17.5%	16.4%	4.1%	-21.6%	-11.3%
Look 12	11.6%	25.6%	23.8%	9.8%	35.8%	28.3%	16.3%	3.9%	21.9%	30.1%	39.6%	42.5%	40.8%	42.2%	38.8%	12.4%	18.7%

## AEX backtested for short positions

Equity Momentum: AEX (1999-2018) CUMULATIVE SHORT

(l,h)	Hold 1	Hold 2	Hold 3	Hold 4	Hold 5	Hold 6	Hold 7	Hold 8	Hold 9	Hold 10	Hold 11	Hold 12	Hold 13	Hold 14	Hold 15	Hold 16	Hold 17
Look 1	-136.7%	-121.9%	-187.2%	-161.0%	-231.2%	-119.8%	-220.0%	-155.1%	-215.4%	-268.2%	-134.6%	-341.9%	-232.1%	-60.9%	-176.3%	-141.2%	-241.3%
Look 2	47.1%	-118.1%	65.8%	70.6%	79.1%	5.1%	65.4%	21.0%	76.5%	0.5%	64.1%	74.8%	117.3%	148.1%	159.2%	65.1%	116.3%
Look 3	1.3%	49.3%	-11.5%	-16.5%	-19.7%	-33.4%	-86.9%	-40.8%	-44.7%	-84.0%	-82.3%	-101.8%	-99.5%	-73.5%	-89.9%	-111.1%	-98.4%
Look 4	52.5%	60.0%	55.9%	7.4%	63.1%	90.1%	89.7%	76.1%	102.8%	125.6%	136.1%	9.0%	75.9%	104.1%	105.6%	160.5%	171.6%
Look 5	12.8%	7.2%	2.7%	34.7%	23.2%	29.6%	38.3%	60.6%	73.8%	81.9%	80.9%	74.9%	92.5%	109.3%	121.2%	128.0%	118.9%
Look 6	-22.3%	-21.3%	-27.2%	-35.0%	9.9%	35.5%	-12.3%	-17.0%	-30.4%	-22.1%	19.1%	8.5%	-6.5%	0.6%	10.9%	6.0%	20.6%
Look 7	31.6%	42.4%	32.6%	-63.9%	-65.1%	-66.5%	20.5%	40.0%	48.0%	56.6%	57.4%	35.6%	55.8%	79.4%	69.1%	57.6%	71.0%
Look 8	38.0%	39.0%	22.5%	18.5%	37.4%	51.9%	60.5%	11.2%	42.2%	43.3%	34.3%	42.8%	59.6%	75.9%	87.5%	96.3%	96.2%
Look 9	22.1%	50.2%	50.0%	39.9%	55.8%	45.2%	37.1%	44.3%	77.2%	89.1%	32.7%	37.8%	33.7%	54.8%	36.1%	58.4%	63.0%
Look 10	8.7%	-12.7%	3.6%	14.5%	14.3%	33.5%	39.6%	35.9%	61.2%	30.6%	14.2%	-14.5%	36.1%	42.9%	44.1%	70.7%	72.9%
Look 12	-1.8%	14.0%	17.4%	4.9%	49.2%	48.2%	42.1%	41.2%	51.7%	62.5%	73.6%	32.1%	36.8%	42.1%	57.0%	79.3%	92.3%

## CAC backtested for short positions

Equity Momentum: CAC (1999-2018) CUMULATIVE SHORT

(l,h)	Hold 1	Hold 2	Hold 3	Hold 4	Hold 5	Hold 6	Hold 7	Hold 8	Hold 9	Hold 10	Hold 11	Hold 12	Hold 13	Hold 14	Hold 15	Hold 16	Hold 17
Look 1	-35.0%	-47.6%	-78.7%	-81.8%	-141.0%	-66.5%	-138.6%	-113.2%	-229.7%	-233.4%	-150.7%	-193.6%	-97.8%	-53.0%	-98.5%	-185.8%	-257.8%
Look 2	39.6%	-121.1%	47.8%	48.3%	41.1%	43.6%	76.1%	68.9%	86.5%	47.6%	89.1%	116.4%	132.9%	45.3%	46.6%	82.3%	73.2%
Look 3	35.3%	64.2%	40.4%	55.4%	74.6%	45.2%	43.4%	63.6%	61.3%	51.9%	72.1%	18.1%	4.7%	30.5%	1.9%	-5.7%	10.9%
Look 4	48.7%	59.7%	57.4%	50.3%	85.4%	93.4%	87.7%	72.0%	107.0%	116.4%	116.9%	92.0%	133.2%	135.4%	128.0%	145.3%	155.6%
Look 5	25.2%	20.2%	24.0%	28.8%	-5.3%	11.4%	15.1%	47.1%	52.2%	15.5%	35.8%	40.5%	59.9%	64.4%	48.6%	45.1%	56.3%
Look 6	-1.3%	25.2%	28.6%	36.0%	50.8%	25.2%	34.0%	38.0%	39.0%	48.2%	79.7%	77.9%	68.7%	83.3%	86.8%	92.6%	93.5%
Look 7	23.9%	37.4%	41.7%	-61.4%	-53.7%	-48.0%	33.9%	30.4%	46.5%	64.6%	80.1%	75.2%	81.4%	79.0%	73.2%	56.6%	63.3%
Look 8	27.4%	27.8%	21.8%	13.9%	35.0%	46.4%	44.8%	20.2%	41.0%	40.3%	2.9%	25.7%	46.8%	55.5%	61.8%	43.0%	36.9%
Look 9	18.3%	43.0%	35.4%	31.4%	43.8%	37.6%	29.1%	34.1%	69.1%	80.1%	43.3%	45.3%	42.8%	57.4%	43.8%	54.3%	59.9%
Look 10	11.1%	0.4%	6.1%	-1.8%	-7.0%	17.3%	26.5%	27.1%	51.0%	30.2%	39.9%	26.7%	46.9%	46.9%	42.1%	35.7%	37.0%
Look 12	-0.1%	16.3%	16.9%	11.9%	44.7%	39.6%	33.1%	28.2%	49.3%	56.1%	63.5%	30.9%	31.0%	34.4%	44.8%	61.4%	73.6%

# Summary

Whether we decide to invest our savings or our time into a project, a new business opportunity or into something that seems to be appealing to us, we strongly believe that our choice will be promising. When we do that we are betting, and every bet implies some risk. What we gather at the end it's the willingness to accept the risk, a premium.

In Finance the risk premium it is defined as *"the return in excess of the risk-free rate of return that an investment is expected to yield"*<sup>28</sup>. Maximizing this premium for a given level of risk it is always been the principal objective of modern portfolio theory. It's also well-known that Economics it's not an exact science, but a social science and the word social means human, which in turn implies irrationality, which again is the expression of our behavior. In recent years we filled this gap and we start to move away from the traditional view of risk premium into a new one, the so-called alternative risk premium.

Traditional risk premia are returns that can be harvested passively from directional long exposures in asset classes such as stocks, bonds, commodities and so forth. By contrast, ARP<sup>29</sup> is a dynamic and systematic source of return that behaves differently from those in traditional markets<sup>30</sup>. While traditional risk premium includes factor investing<sup>31</sup> or smart beta strategies<sup>32</sup> that are related to systematic risk, the latter is structured as a long-short investment and it may be independent of traditional risk premia. Moreover, ARP should not be confused with alpha strategies, which are believed to be driven by a manager's security selection and market-timing skills. The ARP on traditional financial assets such as commodities, equity or currencies often results from market behaviors and structured conditions of the market itself. If we start selling losers and buying winners, we are creating momentum. If we are betting on a cheap asset that has potential growth and a higher fair price, we are considering value investing. If we think that an asset yield is mispriced, we believe in carry opportunities.

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<sup>28</sup>Risk Premium, Investopedia [Internet]. Available from: <https://www.investopedia.com/terms/d/drawdown.asp> [Accessed 20 September 2018]. Alternative Risk Premia.

<sup>29</sup>Alternative Risk Premia.

<sup>30</sup>Communication department (2017), Alternative Risk Premia: What Do We Know? Viewpoint Amundi Asset Management [Internet]. Available from: <http://research-center.amundi.com/> [Accessed 20 September 2018].

<sup>31</sup>Factor investing is an investment approach that involves targeting quantifiable firm characteristics or "factors" that can explain differences in stock returns. Over the last 50 years, academic research has identified hundreds of factors that impact stock returns, Wikipedia [Internet]. Available from: [https://en.wikipedia.org/wiki/Factor\\_investing](https://en.wikipedia.org/wiki/Factor_investing) [Accessed 20 September 2018].

<sup>32</sup>Smart beta investing combines the benefits of passive investing and the advantages of active investing strategies Investopedia [Internet]. Available from: <https://www.investopedia.com/terms/d/smartbeta.asp> [Accessed 20 September 2018].



These are the most common sources of ARP and are also well-known for many years among investors, the key difference is how they are implemented. In fact, if ARP strategies are included in portfolios alongside traditional investments, they enhance diversification and provide benefits (e.g. reduce drawdown exposure<sup>33</sup>) since they exhibit heterogeneous statistical properties.

After the 2008 global financial crisis, the diversification has become the primary objective of institutional investors and wealth funds who needed to restructure their portfolio in a low correlated environment. To complicate the situation, it was also the uncertainty of future behavior of the stock-bond correlation that continues to be negative since the 90's and should revert to positive over the long run<sup>34</sup>. As a reaction wealth funds and sophisticated investors have switched from common diversification procedures to a deleveraging equity solution using sovereign bond as hedging assets and corporate bond as lower beta assets (sovereign bonds are felt more as assets for hedging than for performance).

ARP are expanding the universe of traditional risk premia and are becoming the building blocks of multi-asset management. In 2017, a survey from bFinance<sup>35</sup> suggests that ARP has been the area of the greatest interest among investors on a rolling 12-month basis. In the same year, Deutsche Bank reported some survey results<sup>36</sup> showing the growth of the percentage of investors who allocate to ARP strategies, with an increase of 20% from 2015 the level has reached the 26 % in 2017. Furthermore, a recent Prime Brokerage survey<sup>37</sup> from Morgan Stanley reports that 79% of investors with more than \$5 billion of assets under management in hedge fund investments currently rely on ARP strategies or are considering to allocating on ARP.

Since 1993 momentum strategies were considered one of the most valuable techniques over the entire investment industry and have been subject of research of a great number of academic and professional studies. Alternative risk premia have changed the portfolio construction process shifting from a correlation diversification approach to a payoff diversification approach and becoming the building blocks of multi-asset management. Statistical evidence suggested that many financial assets show positive short-term autocorrelation or momentum bias<sup>38</sup> and that trend following strategies are profitable in almost every asset class. Moskowitz and Pedersen, as well as Gary Antonacci, studied the trend following applications on equities, currencies, commodities, and bonds finding substantial abnormal returns, especially during extreme markets. Nowadays every informed investor should know the main benefits associated with the use of momentum investing and alternative risk premia. Nevertheless, it is highly important to continue to study such interesting properties elucidating the role of a wide use of ARP into a pre-crisis environment. The primary objective of this final thesis was not only to discuss ARP and to collect the existing

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<sup>33</sup>Drawdown, Investopedia [Internet]. Available from: <https://www.investopedia.com/terms/d/drawdown.asp> [Accessed 20 September 2018].

<sup>34</sup>If economic conditions and monetary policy permit this change.

<sup>35</sup>"Manager Intelligence and Market Trends", bFinance (February 2017).

<sup>36</sup>15th Annual Alternative Investment Survey," Deutsche Bank (March 2017).

<sup>37</sup>"Recent Hedge Fund Trends," Morgan Stanley Prime Brokerage-Strategic Content Group (July 2017).

<sup>38</sup>Using the expression of momentum biases, we refer to the momentum effect that can be interpreted as an anomaly or as a bias of the market, behavioral theories.



literature of momentum investing but also to prove the momentum effect and his profitability presenting a simple strategy that uses eurozone data and applies automated absolute rules. In the second and third chapter, we replicate some methodologies and applications described by Gary Antonacci and Moskowitz & Pedersen in their studies to prove the momentum effect<sup>39</sup>.

The first step was to use a measure of performance such as the Sharpe ratio to find the best formation period (look-back period) within the dataset. Results indicated that the highest level of SR was associated with formation periods of one month. Hence, we rejected the idea that a 12 months look-back period needs to be preferred among all windows of observations and we reminded to the fact that preferences need to be determined empirically in every case since they may vary among instruments and across time.

The second step was to examine the time series predictability of returns across different time horizons using regression and volatility weighting. Following Pedersen & Moskowitz methodology, we proved the momentum effect by looking at the time series of excess returns and at the autocovariance between the security's excess return next month and its lagged excess returns. In order to make meaningful comparisons across assets, we scaled all returns by their ex-ante volatility. Once we tested the validity of the volatility model, we used regression analysis to useful predict the price continuation and subsequent reversal for every asset within the database showing the existence of the so-called momentum effect. Once again findings were supportive, and results confirmed that in our analysis the look-back period of one-month needs to be preferred among all windows of observations.

Finally, in the last chapter, we tested momentum trading running a backtest and creating a strategy that uses both absolute and relative momentum at the same time<sup>40</sup>. The strategy has been automated with an algorithm<sup>41</sup> that uses cross-sectional momentum<sup>42</sup> to choose the "best index" and times series momentum<sup>43</sup> to select and apply the "best strategy" in every possible case. The backtest proves that returns associated with formation periods of one month are the highest among all assets within the database supporting results founded in the previous chapter. The strategy confirmed the validity of momentum investing showing the main benefits of mixing cross-sectional and time series momentum at an absolute level. In particular, such methods are interesting to understand past performances and best asset combinations as well as further potential uses in high-frequency trading and portfolio construction.

<sup>39</sup>We refer to absolute momentum effect.

<sup>40</sup>The effectiveness of a combined use of absolute and relative momentum has been studied by Antonacci in his paper, Gary Antonacci (2012), Risk Premia Harvesting Through Dual Momentum. Portfolio Management Consultants.

<sup>41</sup>The implementation is provisional, and it works only for  $t = 1$ .

<sup>42</sup>We use cross sectional momentum to switch from equity index.

<sup>43</sup>We use absolute momentum to switch from "Best Index" to bond a and to find the "best period (l, h)".

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