LUISS Guido Carli
LIBERA UNIVERSITÀ INTERNAZIONALE DEGLI STUDI SOCIALI

An analysis of Arbitrage and Cointegration based Pairs Trading in the Cryptocurrency Market

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Advisor

Nicola Borri

Co-Advisor

Pierpaolo Benigno

Student

Alessandro Furlan

Student Number

685501

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Department of Economics and Finance

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Introduction

The scope of this paper mainly concerns the investigation and subsequently exploitation of absolute and relative price discrepancies in the new and attractive Cryptocurrency market, a volatile and fragmented space characterized by a multitude of exchanges and virtual issued currencies, the former represented by centralized and decentralized trading platforms, dislocated in several areas of the globe, that operate as market makers or matching systems, while the latter are digital electronic systems whose technological development relies on the academic works of modern Cryptography and Network security.

Chapter 1 covers an analysis of the cryptocurrency market from an historical, technological and statistical point of view; firstly, the Distributed ledger technology (DLT), the infrastructure upon which Cryptocurrencies rely, is introduced and followed by a classification of the digital cryptographic assets according to a set of parameters and metrics. Afterwards, basic statistics of the main cryptocurrencies are provided, with a major emphasis over Bitcoin network, the first digital and unregulated currency system to appear in 2009. Graphical methods and statistical tests are then outlined to assess the presence of Normality in daily returns distribution of a group of selected cryptocurrencies, chosen among the ones with most liquidity and historical data. The results possess fundamental implications for risk-management applications, as Value at risk (VaR) and Expected Shortfall (Es) computations.

Chapter 2 briefly reviews academic literature over arbitrage phenomena and market completeness, with a focus on the Law of One Price and No-arbitrage principles, applicable both to absolute and relative pricing theories. With regard to relative asset price misalignments, a popular type of "relative value" arbitrage and market neutral strategy is introduced, namely the pairs trading, a quantitative investment strategy extensively researched and experimented in a broad range of traditional markets since mid-1980s, when a group of mathematicians and computer scientists at Morgan Stanley were the first to formally theorize the underlying statistical property of mean reversion. Pairs trading seeks to exploit relative price deviations from an equilibrium level between components of a pair through the activation of matched long and short positions, and thus make a profit from market inefficiencies (relative value arbitrage), while hedging against market risk (market-neutral: absence of correlation between the strategy expected return and the market): the spread, measured as the price difference of the paired components, should possess the property of mean-reversion or stationarity; despite short-term deviations, where positions are opened in the pair (long the undervalued asset and short the overvalued one), its long-term behavior should converge to an average value or equilibrium term, where they are subsequently closed. Univariate pairs trading frameworks, used to identify the potential pairs, are properly investigated: Distance and Cointegration methodologies are exposed, along with the series of statistical tests and estimation procedures.

First section of Chapter 3 examines then absolute price discrepancies of digital coins between exchange platforms and the subsequent occurrence of simple arbitrage strategies. The fragmentation of the cryptocurrency space in more than two hundreds trading platforms, characterized by different trading volumes and buying pressure, encourages this kind of analysis. Hence, cryptocurrencies and exchanges have been ordered and selected on the basis of determined metrics, represented by trading volumes (liquidity) and extension of the historical data for the former, and a geographic order for the latter, with the intent of choosing the most representative platform for macro-region. The profitability of such risk-less strategies may be eroded by consistent transaction costs and hurdles.

Therefore, second section of the chapter shifts the focus to exploitation of relative price discrepancies inside the same exchange platform, in order to minimize many of the listed transaction costs and risks, more specifically, the execution time and the complex system of fees: Deposit, withdrawal and trading fees. Pairs trading strategy is subsequently investigated: cointegration approach is used to identify potential pairs. The analysis is restricted to few cryptocurrencies: Bitcoin, Litecoin, Dash, Monero and Ethereum. An explanation of the choice of such small sample relies on the lack of liquidity that interests the majority of other minor cryptocurrencies; moreover, Litecoin, Dash, Monero were all forks of the original Bitcoin code, with whom share some network features and technological developments. Hence, considering the strict connection with BTC, it is plausible to explore the evolution of relative price dynamics. Unit root rests are performed to check stationarity, and Engle-Granger two-step approach is adopted to form the pairs. Finally, once trading rules have been delineated, an automatic trading system is activated to capture deviations of the formed spread, and in-sample and out-of-sample performance metrics of the strategy are reported, along with final discussions.

Chapter 1

The Cryptocurrency Market

1.1 Intro

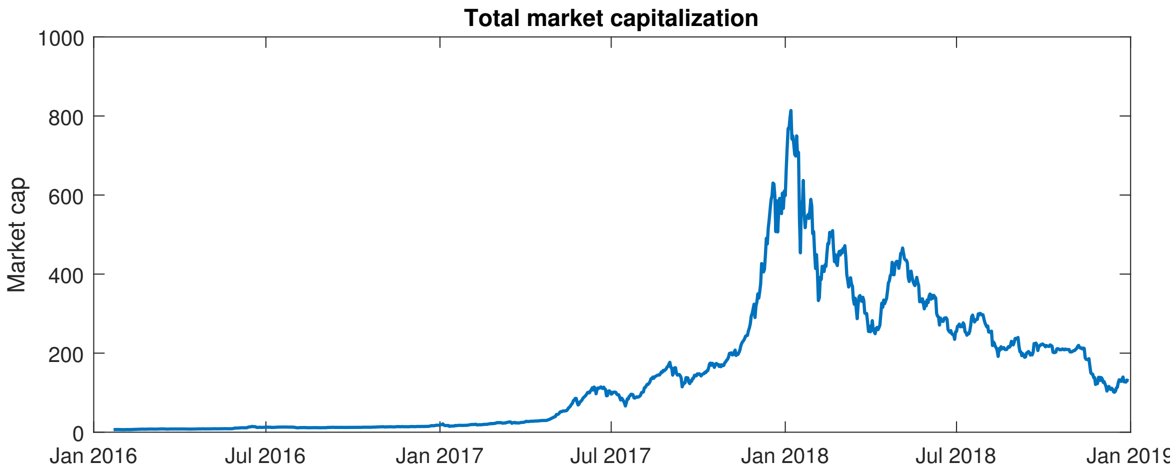
The year 2017 has experienced an exponential growth of the Cryptocurrency market, that reached a total capitalization of 800 billion dollars in the 4th quarter. With the introduction of the Bitcoin Futures market by the Chicago Board Options Exchange (CBOE) and the CME Group Inc (CME.O)[1], a consistent decline of the total market value has followed what has been defined in the academic world as one of the largest asset bubbles of all times [2] (**Figures 1.1**, **1.2**).
Since the release of Bitcoin protocol in 2008 [3], an increasing number of projects and initiatives have entered the new and attractive ecosystem, build upon mathematical and probabilistic models, mainly with regard to Cryptography and Network technologies.
At November 2018, more than 2000 cryptocurrencies have been traded on the several Crypto-exchanges located in all the areas of the Globe, setting an all time high record; a trading mania has hit several western and eastern inhabitants and enthusiasts, including third world countries; according to academic papers and financial analysts, this fear to be left out the market (FOMO) and the rise of trading bots have contributed to the rising prices of all the main cryptocurrencies.
Hileman and Rauchs (2017) have documented as more than 90% of all cryptocurrencies and tokens have copied the original code of Bitcoin, thus not providing any innovation or utility, hence raising questions about the real value that could justify their quotation.
Valuation remains a sensitive argument: the traditional valuation approaches have reveled not to be appropriate; despite a part of the academic world denies the pos-

[1] respectively on December 10 and 17, 2017
[2] Nouriel Roubini and Robert Shiller positions
[3] Satoshi Nakamoto, *Bitcoin: A peer-to-peer electronic cash system*, 2018

sibility of a valuation model for cryptocurrencies, suggesting the absence of every intrinsic value, some attempts have been made: **Hayes (2015)** has provided a cost of production model for the valuation of Bitcoin, while **Pagnotta and Buraschi (2018)** have addressed the valuation topic in a new type of production economy: a decentralized financial network.

The rising attention of the media and the public to the new sector has been accompanied by a series of negative and opaque events, including several hacks and fund losses, as the Mt Gox exchange hack, market manipulations, insider trading events, Crypto-exchanges disputable behaviors, that increased the climate of uncertainty and doubt around a sector not fully understood by the regulators and agencies yet.

Figure 1.1



Notes: Total Market capitalization; values on the vertical axis are in Usd Billion. Data extracted from Coinmarketcap

1.2 A glimpse of the Distributed Ledger Technology

The term ledger generally refers to the collection and classification of a series of accounts, usually financial and economic information related to business activities, stored in a double-entry bookkeeping system. Since their appearance in the Middle Age, a common factor of all ledgers have been the fundamental presence of a trusted party that acted as gatekeeper in order to protect the validity and originality of the data. Since then, all ledgers created and adopted in every field of the human society have been centralized; but centralization of a system presents different types of risk, the most important about the presence of a single point of failure, the record-keeper itself [4]. Conversely, a distributed ledger, or shared ledger, is a database spread and synchronized in a large network of participants, called Nodes, that possess a shared copy of the digital data, and, more importantly, no "central" server or administrator is required.

The network is peer-to-peer, to remark the equality status of each member in the execution of the tasks, and requires a consensus, a set of rules and agreements, to ensure the replication of the data. The distributed ledger technology (DLT) is both the sum of the protocols and supporting infrastructure that allow Nodes in different locations to propose and validate transactions and update records in a synchronized way across the network [5] (**Figure 1.3**).

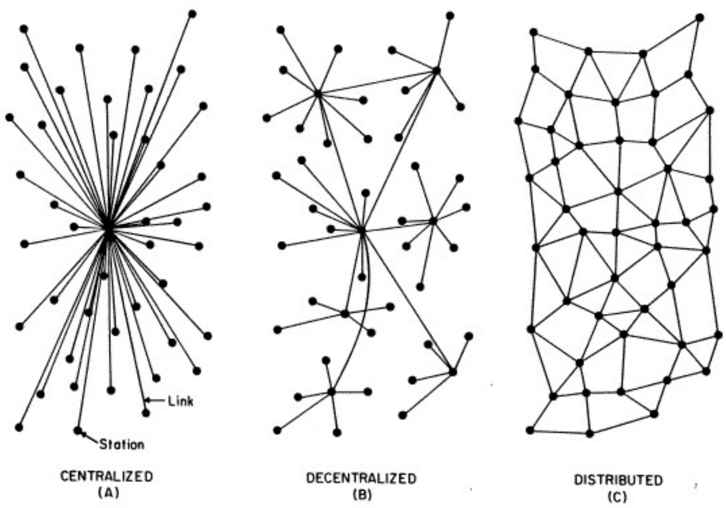


Figure 1.3. Comparison of Systems. Source Baran (1962).

[4] Loss and counterfeit of data due to carelessness of natural events

[5] Morten Bech, Rodney Garratt, *Central bank Cryptocurrencies*, International banking and financial market developments, BIS, Quarterly Review, September 2017, 55-67

As a result, the DLT provides an irrevocable and auditable transaction history [6], invulnerable to censorship and exclusion, counterfeit and loss of data .
The theorization of distributed computing systems in the 1980s opened a series of questions and doubts on their practical realization and ability to ensure trust and consensus between the participants of the network, a problem known as "Byzantine fault tolerance" [7].
Chaum (1984), **Chaum et al. (1990)**, **Okamoto et al. (1992)** and **Wei (1998)** were the first attempts at solving the mathematical problem and providing a concept of Cryptocurrency, a virtual currency, or digital asset, that heavily relied on the use of Cryptography to secure transactions.

On october 31, 2008, the release of a paper called "Bitcoin: A Peer-to-Peer Electronic Cash System" by an obscure author named Satoshi Nakamoto revealed to be the optimal solution so far, providing a version of the DLT based on a series of chained blocks with the use of cryptography, i.e., the blockchain.
A blockchain is a tamper-proof, shared digital ledger that records transactions in a decentralized peer-to-peer network [8] and reaches a decentralized consensus through a Proof-of-Work algorithm [9] (POW).
It's the core technology underlying Bitcoin, that makes use of pre-existing technologies and applications:

1. A P2P network
2. Public Key Cryptography (i.e. ECDSA – the Elliptic Curve Digital Signature Algorithm)
3. Cryptographic hash functions (i.e. SHA-256 and RIPEMD-160)

At the same time the blockchain is only a part of Bitcoin, the latter not just identifiable a currency or an asset but rather a collection of concepts and technologies that form the basis of a digital money ecosystem[10]. A summarized denition of BTC would make use of one word: Code.
Bitcoin is code, an open-source and programmable code, and it is decentralized, it does not rely on any Authority; this aspect is its truly advantage with respect

[6] Jan Loeys, Joyce Chang *Decrypting Cryptocurrencies: Technology, Applications and Challenges*, JP Morgan perspectives (2018)
[7] Leslie Lamport, Marshall Pease, Robert Shostak (1982)
[8] Andreas Antonoupolous, *Mastering Bitcoin*, (O'relly, 2017)
[9] Other versions use different algorithms as POS and POA
[10] Andreas Antonoupolous, *op. cit.*

to traditional systems, it can be upgraded, it can evolve in the time, it is a living organism nourished by the work of hundreds of developers and thinkers; this is also the reason why it has been proven to be difficult for regulators all over the world to categorize it under specific categories: currency, asset, or commodity ?
Probably it is neither of them, or will it become in the future.

The present work does not have as goal the research whether Bictoin could possess all of the traits of money or assets; different academic studies have dealt with the argument: **Lo and Wang (2014)**, **White (2014)**, **Mittal (2012)**, **Ametrano (2016)**, **Yermack (2015)**, **Baur et al. (2018)**.

The Blockchain is a particular realization of the DLT, but others have been theorized in the years and currently tested, as the Tangle, a directed acyclic graph (DAG) used to store transactions for the internet of things (IoT),i.e., the infrastructure of Iota, a new generation and distributed cryptocurrency. The tangle dismisses the need of mining activity, the latter necessary to confirm and validate transactions of the blockchain, that raised concerns about the consumption of electrical energy [11] and effective decentralization of the network after the creation of concentrated mining pools. [12]
The elimination of the mining process allows users to transfer digital assets without the necessary payment of transaction fees, a nice feature in the field of micropayments and internet of things.

LITERATURE. Since 2014 the academic literature on the DLT and cryptocurrencies has lived an exponential growth; several studies from universities, research centers and central banks have analyzed their properties and mathematical foundations, their regulatory collocation and social implications, and their potential applications for every work sector, from trade finance to digital identity and voting system (**Figure 1.4** in the appendix A):

Luther (2013) analyzes the network effects and switching costs of the adoption of alternative currencies and technologies, while **Catalini and Gans (2016)** prove the reduction of verification and networking costs improve innovation.

[11]Camilo Mora, Randi L. Rollins, Katie Taladay, Michael B Kantar, "Bitcoin emissions alone could push global warming above 2°C", *Nature Climate Change* volume 8 (2018), 931-933

[12]Adem Efe Gencer, Soumya Basu, Ittay Eyal, Robbert van Renesse, Emin Gün Sirer, *Decentralization in Bitcoin and Ethereum Networks*, Cornell university, 2018

Glaser and Bezzenberger (2015) focus on how Decentralized consensus systems and smart contracts provide the technological basis to establish predefined, incorruptible protocols to organize human behavior and interconnection. **Davidson et al. (2016)** discuss the implications of the DLT on the current Governance systems. **Hileman and Rauchs (2017)** present a systematic and comprehensive picture of the rapidly evolving cryptocurrency ecosystem, illustrating how cryptocurrencies are being used, stored, transacted and mined.

Rohr and Wright (2017) investigate the leverage the power of a blockchain and the Internet to facilitate capital formation and the Democratization of Public Capital Markets. **Malinova and Park (2017)** and **Khapko and Zoican (2017)** analyze the implementation of DLT in financial markets and its effects on settlement times, market makers' strategies, investor trading behavior and welfare, and trading costs. **Löber and Houben (2018)** focus the attention on potential integration of the DLT with central banking and introduction of Central bank digital currencies (CBDCs).

Kroll et al. (2013) examine the mining mechanism of public blockchains, linking it to game theory and the presence of Nash equilibrium.

1.3 Classification of Cryptocurrencies

As stated in the previous section, a first classification of cryptocurrencies can entail the type of distribute ledger technology; most of cryptocurrencies and tokens are linked with blockchains, each one with peculiar features regarding the dimension of the blocks, the number of transactions per block or the mining algorithm (Proof of work, Proof of stake and Proof of Authority algorithms).
Others, as Iota or Ripple, are based on different infrastructure, the Tangle the former, a common shared ledger the latter; technically, Ripple can not be described as cryptocurrency, but rather a protocol, a real-time settlement system and currency exchange that supports fiat currencies and the Xrp token, the native token of the network, to enable instant transactions between parties at insignificant fees.The token Xrp was issued at its creation and then distributed, it is not minable, a different paradigm with respect to Bitcoin and other cryptocurrencies.

A second subdivision can be applied to the nature of decentralization of these infrastructures and protocols : Decentralized, or permissionless blockchain, as the Bitcoin chain, or centralized and permissioned, as the Rypple one; the protocol could be open source, as most of the crypto code, or closed source (Hashgraph). This distinction resembles the ancient separation between the Internet and the Intranet, the former represented by permissionless protocols, while the latter by permissioned ones. This level of classification could reveal of particular interest in the complex and new field of valuation models, as decentralization, with the elimination of the single point of failure, could be considered a feature that brings utility while solving third party risks, failures, technical issues, that characterizes the modern and fragmented financial infrastructures.

A final subdivsion could entail the nature and purpose of cryptocurrencies, based on several criteria; different studies cite as possible principles the level of governance, the issuance and distribution mechanism, the transaction processing and the audit system [13]; different combinations of these features define specific types of digital assets. A digital asset is essentially any type of data in binary format; it is scarce as it admits a defined owner, or group of owner, in each time state.
A large part of cryptocurrencies is primarily used as medium of exchange with the use of its own dedicated blockchain.

[13] Pavel Kravchenko, "The periodic table of cryptocurrencies", *Coindesk*, January 28, 2018

Conversely, a token can be described a special type of virtual currency, an accounting unit that represents the owner's balance in a designated asset or utility [14]; tokens can be build on top of other cryptocurrencies' blockchains to create and execute smart transactions and contracts, or decentralized applications (Dapps), as happens on the Ethereum blockchain.
A naive codification would be of the following list:

- **Cryptocurrencies**, virtual fungible currencies not issued by a central authority that make use of cryptography to exchange value between users; their primary goal is to serve as mean of payment in a secure and decentralized system.
- **Platform currencies**, virtual currencies that allow the creation and execution of smart contracts ,Dapps tokens and collectibles on the blockchain to perform more complex and structured transactions, not necessarily financial transfers; The Ethereum network is the most known platform, but doubts and concerns have recently arose on the efficacy and security of its smart contracts [15]
- **Security tokens**, tradable tokens that represent assets or securities.
- **Utility tokens**, tokens that provide future access to goods & services launched by the project; they are not intended for investment.
- **Crypto-collectibles**, cryptographically unique and non-fungible digital assets
- **Crypto-fiat currencies**, i.e. stablecoins, are cryptocurrencies pegged 1 : 1 to Fiat currencies as the US dollar or the Euro; their rise has followed the exceptional volatility of crypto prices to meet investor's demand of a stable and fast instrument connected instantly with the market but that at the same time could preserve the properties of unit of account and store of value.

A more detailed classification has been elaborated by **Thomas Euler (2018)**, who considers five dimensions under which order the digital cryptographic assets: Purpose, Utility, Legal status, Underlying value and Technical layer.

[14] Pavel Kravchenko, "The periodic table of cryptocurrencies", *Coindesk*, January 28, 2018
[15] Lucianna Kiffer, Dave Levin, Alan Mislove *Analyzing Ethereum's Contract Topology*, 2018

However, the large part of cryptocurrencies revealed to be a clear copy of Bitcoin code, not providing any substantial innovation to the field but focusing on the different features of the network, as a different issuance and distribution scheme, or block time [16]. Most of the times these alternatives to the original conception of the blokchain proved to possess evident security flaws and not to be capable of preventing attacks to the network, a problem noted as "51% attack" that could enable the double spending of the digital currency. Actually, an article published by **Shanaev et al. (2018)** brings the evidence that the majority of such attacks are anticipated by the activation of Pump and Dump schemes with the final result of prices and volumes manipulation. The authors deploy an event study methodology to assess the influence of 51% attacks to cryptocurrency prices and report, among the various results, that the negative price response, in the order of 10-15% loss, is robust in various event windows. Moreover, prices of the attacked cryptocurrencies do not recover to pre-attack levels one week after the event, and evidence of insider trading prior the attacks is confirmed by the analysis of abnormal positive returns antecedent few days the event.

This analysis narrows the number of cryptocurrencies that could bring a real utility to traditional systems, as an effective and proved level of decentralization and security without the service of trusted third parts.

[16] Garrick Hileman, Michel Rauchs, *Global cryptocurrency benchmarking study*, University of Cambdridge, 2017

1.4 Main Statistics

Cryptocurrencies have the unique feature to be exchangeable at every day of the week, with no closing times as in the traditional markets.

Table 1.1 represents the list of most capitalized cryptocurrency at the date of January 16, 2019; the comparison with snapshots of the market at previous dates would demonstrate how new cryptocurrencies have emerged in the last years, or months, and fast can be the process to scale rankings.

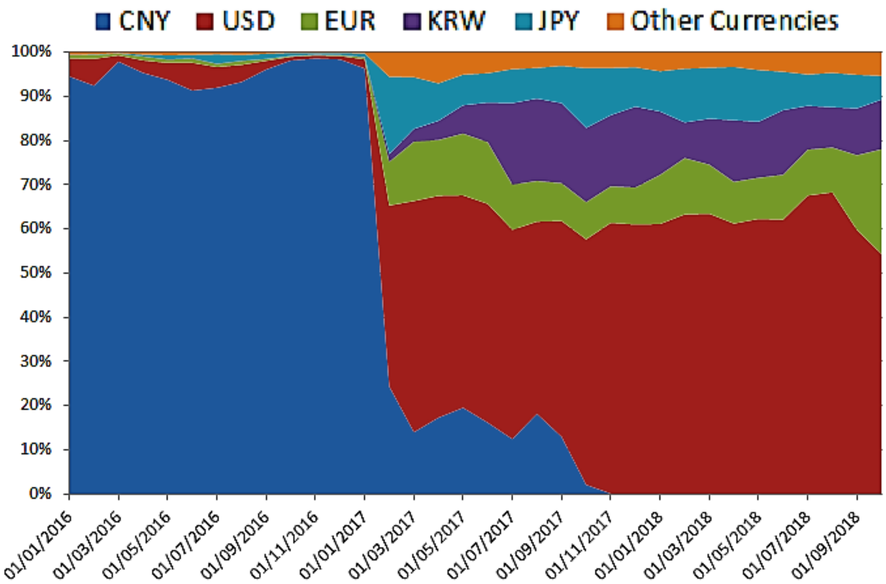
Bitcoin is the first cryptocurrency to appear in 2009, with the release of the first client on the 3rd of January; it is the most liquid and traded cryptocurrency in the entire market; in fact, it is possible to exchange it inside more than two hundred exchanges, located in all the areas of the globe, from the United States of America to South Korea, including Europe and third world countries.

Table 1.1: Top 15 cryptocurrencies by market capitalization

Rank	**Name**	**Symbol**	**Price**	**Marketcap**
1	Bitcoin	BTC	$3.664,10	$64.066.913.601
2	Ripple	XRP	$0,331296	$13.596.504.735
3	Ethereum	ETH	$124,31	$12.978.577.703
4	Bitcoin cash	BCH	$129,47	$2.274.707.084
5	EOS	EOS	$2,46	$2.226.360.399
6	Stellar	XLM	$0,107665	$2.059.314.945
7	Litecoin	LTC	$31,86	$1.913.065.27
8	Tron	TRX	$0,024993	$1.665.804.085
9	Bitcoin SV	BSV	$78,64	$1.381.638.153
10	Cardano	ADA	$0,044904	$1.164.232.549
11	IOTA	MIOTA	$0,308556	$857.641.044
12	Binance coin	BNB	$6,12	$790.630.750
13	Monero	XMR	$45,68	$763.933.011
14	Dash	DASH	$71,89	$616.199.372
15	Nem	XEM	$0,056754	$510.785.606

Figure 1.5 shows Bitcoin trading volumes per currency, starting from January 2016; it is clearly visible the huge decline of Bitcoin trading in the Chinese currency after the new regulations and restrictions imposed by the Chinese Authorities in 2017; at the same time, there has been registered a consistent increase of the Btc-to-stablecoin trading instead to fiat currencies, after the broader adoption by the exchanges of new issued covered , or partially covered, stable cryptocurrencies tied to the Us dollar.

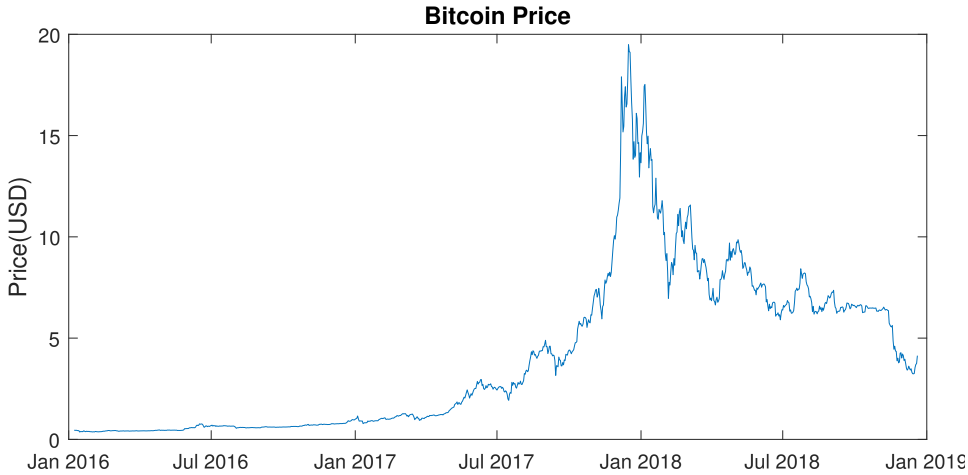
Figure 1.5



Notes: Bitcoin Trading volumes by currency. Data extracted from Bitcoinity with the adjunt of Korean exchanges data from Cryptocompare api

Figure 1.6 illustrates Bitcoin price evolution in the last three years; Btc price exponentially surged in 2017, when it registered an impressive annual growth rate equal to 1268% and reached an all time high of $20,000 in December, and then declined at the start of 2018, concurrently with the introduction of Bitcoin Futures, to mark an annual price fall around 73% and maximum drawdown of 81.53% . However, it has not been the first year that the cryptocurrency experienced this extreme level of volatility, as happened in 2013, when it passed from $13.28 to the all time high of the period of $807.78, or in 2011, when suffered a price loss of 90% after the hack of Mt Gox exchange.
Since its inception, BTC has been the cryptocurrency with the highest market capitalization, property that still holds at the date of writing, January 2019, as it is possible to see from **Figure 1.7** (Appendix A), that shows the evolution of its dominance, expressed as percentage of the total market capitalization.

Figure 1.6. Bitcoin Price



Notes: Bitcoin Price, values in thousands; data extracted from Coinmarketcap, Matlab representation

In the years a growing debate has considered the possibility for other cryptocurrencies to overtake its market cap and acquire the special status of symbol and brand of the market, situation, called the Flippening phenomenon, that has never occurred yet but was very close to occur on June 20, 2017, when Ethereum capitalization reached near the 30% of the total market cap and BTC one was declining to 37%. **Figures 1.8, 1.9, 1.10, 1.11, 1.12, 1.13** (Appendix A) display the main statistics of BTC network: the monetary emission, the evolution of average transaction fees, the growth of hash rate, the distribution of mining pools and the energy consumption index.

Bitcoin monetary emission is embedded in the protocol, that implies a maximum issuance of 21 million units of the currency, following a geometric distribution scheme; in fact the number of Bitcoins generated per block by users, i.e. the miners, halves every 210000 blocks, approximately 4 years. This algorithm makes BTC a currency with finite supply, hence with deflationary properties, an opposite paradigm with respect to the traditional inflationary fiat currencies.
However, it is not reasonable to exclude ex ante a possible review of the issuance scheme, as suggested by the same developers who follow the progress of the project. With the current scheme, once all the units will be mined, the protocol entails verifiers of the network will receive as only source of income the transaction fees payed by users, thus maintaining the reward incentive in order to validate the transactions.

BTC transactions are usually confirmed in 60 minutes, but in times of elevated traffic they can take longer times, even days. A higher payment of the fees speeds the confirmation process; during the exponential rise of its price at the end of 2017, fees reached the threshold of $80 per transaction, due to the increased demand of daily transactions.

This unsustainable level of fees began to decline with the adjunct of new features to the protocol, as the "batching", the combine of multiple transactions into a single operation to reduce the space and cost into the limited block space, and the "Segregated witness" update (Segwit), a change in the transaction format that reduced its cost of a factor of ten to one hundred times. Currently, Segwit is adopted by the majority of cryptocurrency exchanges.

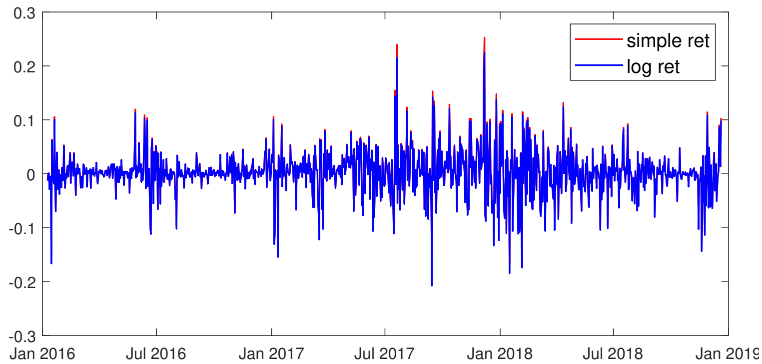
Moreover, a second reason of this decline can be traced back in the reduction of the total number of daily confirmed transactions, that passed from a daily average of 500000 to 250000.

Another interesting statistics of bitcoin network regards its hash rate, the measurement unit of computing power needed to solve the mathematical problems for security reasons. In the years it has showed an exponential growth, touching the all time high level of 60 trillion of hashes per second in August,2018. A higher number of miners and mining pools determines as primary effect the growth of the network hash rate and the adjustment of the difficulty mechanism, the latter a measure of the effort needed to find a new block, to maintain the extraction time of each block unchanged. The increase in the difficulty consequently complicated the computing costs and led to a progressive concentration of the hashing power, distributed to fewer and fewer mining pools, thus raising concerns about the effective decentralization of the entire network, highlighted by **Gencer et al. (2018)**. For all 2018, only 5 Bitcoin mining pools accounted for more than 60% of the total hashrate distribution, and the scenario has not evolved since.

The electricity consumption needed to perform the mathematical computations could have powered 6,7 millions of U.S households in 2018 or satisfied the electricity demand of a country as Austria, approximately 70 TWh per year, and, even though it has nearly halved at the end of the same year, it is estimated to grow more in the years. The creation of Bitcoin Energy Consumption index, that tracks the annual amount of electricity consumed by miners, has helped to raise the consciousness on the unsustainability of the Proof of Work algorithm, certified by scientific studies (**Mora et al. (2018)**).

Figures 1.14, 1.15, 1.16 display the main statistics of other principal cryptocurrencies, Ethereum, Litecoin, Ripple, Dash, Monero and Stellar Lumens, and their comparison with Bitcoin.
The analysis of daily returns for this group of cryptocurrencies exhibits the presence of extreme outliers; in fact, Cryptocurrency daily returns are very high, as their volatility, in comparison to traditional financial markets. Therefore, differences between simple and log daily returns are visible when the ratio between consecutive prices is far from one, a situation that occurred more than once for some cryptocurrencies, as Ripple, or Stellar Lumens, that presented, at a daily frequency, returns above the unity (100%). **Figure 1.17** plots the comparison of daily log and simple returns for Bitcoin for the period 01/2016-12/2018; the same computations have been executed for the other main cryptocurrencies with at least three years of historical data (**Figure 1.18**, Appendix A).

Figure 1.17. Bitcoin log vs simple returns



Normality assumption of returns distribution can be assessed with graphical methods and statistical tests. The qualitative approach, based on the comparison of the sample data histogram to a normal probability curve (Frequency distribution Histogram) or sample data quantiles to Normal ones (Quantile-Quantile plot), rejects Normality in favor of *Leptokurtosis*.
Cryptocurrency daily log returns are not normal but *Leptokurtic*, as they are more peaked towards the mean, i.e., higher Kurtosis than Normal distribution, and display evident fatter tails. In this sense, the Q-Q plot is an useful tool to highlight large deviations in the tails from the normal distribution (heavy tails). Moreover, some cryptocurrencies show evident signs of Skewness. **Figure 1.19, 1.20** report the Frequency distribution Histogram and Q-Q plot of the daily log returns for Bitcoin (**Figure 1.21, 1.22** for the other cryptocurrencies, Appendix A).

Figure 1.19. Frequency distribution Histogram of Bitcoin daily log returns

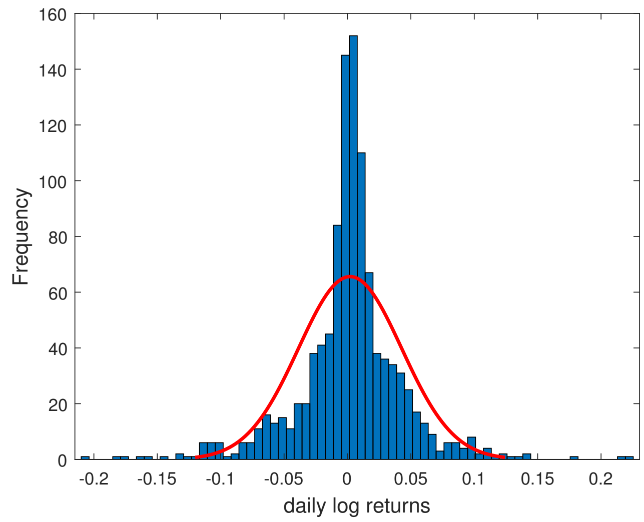
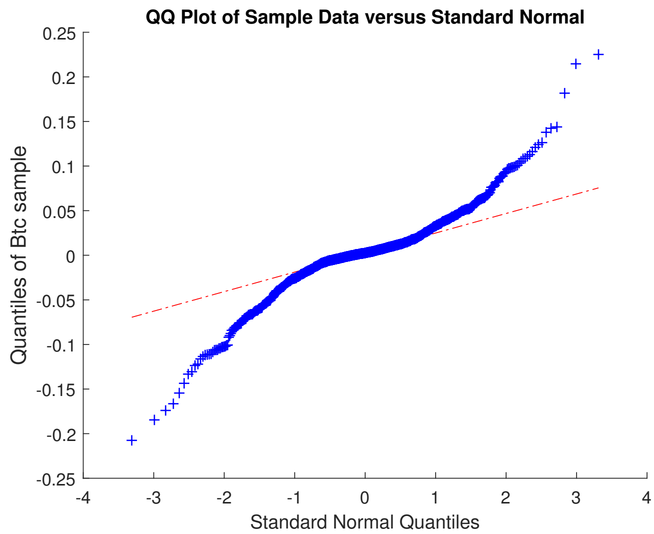


Figure 1.20. Quantile-Quantile plot of Bitcoin daily log returns



The presence of Leptokurtosis in a distribution of values has relevant effect on risk management activities, likewise Value at Risk (VaR) and Expected Shortfall (ES) computations. As a result, extreme levels of returns are more likely to occur than VaR estimates, based on the assumption of Normality, would indicate. Thus, there would be an underestimate of potential risk coming from extreme outliers. **Borri (2018)** uses the CoVaR risk-measure to estimate the conditional tail-risk for bitcoin, ether, ripple and litecoin and finds that these cryptocurrencies are highly exposed to tail-risk within cryptomarkets; as a consequence, single cryptocurrencies could be exposed to tail events that negatively impact portfolios with large negative returns. The empirical findings of non-normality in the distribution of daily log returns are confirmed by significance tests: Jarque-Bera and Kolmogorov-Smirnov Normality tests both reject the null hypothesis that the sample data follow a Normal distribution; Matlab version of the test returns in both cases the result $h = 1$, rejection of the null hypothesis. **Table 1.2** contains the results of the test.

Table 1.2: Normality tests

	BTC	**ETH**	**XRP**	**LTC**	**DASH**	**XMR**	**XLM**
J-B test	1	1	1	1	1	1	1
K-S test	1	1	1	1	1	1	1

Notes: Results of Jarque-Bera and Kolmogorov-Smirnov tests to assess Normality

Other distributions should be considered in the analysis to obtain the one that could best fit to the data, with resulting effects for investment and risk-management activities, as Normal distribution clearly do not characterize the sample data. According to **Chan et al. (2017)**, the generalized hyperbolic distribution gives the best fit for Bitcoin and Litecoin, while for the smaller cryptocurrencies the normal inverse Gaussian distribution, generalized t distribution, and Laplace distribution provide the best goodness of fit. To conclude, **Table 1.3** contains summary statistics of daily log returns: Mean, standard deviation, Skewness and Kurtosis.

Table 1.3: Summary statistics

	BTC	**ETH**	**XRP**	**LTC**	**DASH**	**XMR**	**XLM**
Mean	0.0021	0.0044	0.00389	0.0021	0.0032	0.0044	0,0040
Std.	0.0407	0.0655	0.0792	0.0595	0.0627	0.0733	0.0880
Skew.	-0.1704	0.2511	2.9711	1.2829	0.8635	1.0639	1.9803
Kurt.	7.4237	6.7155	39.8509	15.3698	8.7617	10.1430	17.1846
Obs.	1077	1077	1077	1077	1077	1077	1077

Notes: evident signs of positive Kurtosis; XRP, LTC, XLM are positively skewed

Chapter 2

Arbitrage and Pairs Trading: Literature Review

2.1 The Law of One Price and the No-arbitrage condition

An arbitrage strategy is unanimously defined, both in the academic and in the trading field, an investment strategy designed to take advantage of one or more assets' price discrepancies generated in different markets; thus, this type of strategy, likewise nominated riskless strategy as it does not bear risk, requires no capital commitment and guarantees a positive payoff for the investor.
A typical example of arbitrage opportunity involves the simultaneous purchase and sale of the same security, quoted in multiple markets at different price:

$$\Delta_{arb} = P_X^A - P_X^B$$

,with $P_X^A > P_X^B$, where P_X^A and P_X^B are the prices of asset X in the markets A,B and Δ_{arb} is the profit generated by the combined trades.

The existence of arbitrage opportunities in financial markets is in contrast with the Law of one price (LOP) and Fundamental theorem of equilibrium; the former, that can be considered a special case of the No-arbitrage theory and constitutes the basis of the purchasing power parity (PPP), holds that assets with identical payoffs, in every state of nature, must trade at the same price (**Ingersoll (1987)**). **Cochrane (2000)** links the existence of the LOP to the presence of a discount factor: "*there is a discount factor that prices all the payoffs by*

$$p_t = E_t(m_{t+1}x_{t+1})$$

if and only if the law of one price holds.", where the equation represents the basic asset pricing relation (with m_{t+1} the stochastic discount factor), that simply expresses the value of any stream of uncertain cash flows (see **Cochrane (2000)** for theorem proof, pag 64-68).

The Fundamental theorem of finance constitutes the basis of modern capital market theory and encompasses a more general version of No-arbitrage condition. The theorem states the principle according which security market prices are rational and in equilibrium, in the sense they do not allow for arbitrage opportunities; however, when market conditions ensure the exploitation of price deviations of homogeneous assets, then, the pressure reinforced by arbitrageurs will restore equilibrium levels. As a result, arbitrage activities would led to a convergence of price in different markets, in accordance with the efficient market hypothesis (EMH): asset prices fully reflect all past and current publicly available information and all private information (**Fama (1970)**). It is therefore the quickness of market response to arbitrage occurrences that defines it as efficient or not; if price discrepancies persist over a long period of time, as, for example, changes in demand and supply are not rapidly incorporated into current asset prices with the effect of deviations from their true intrinsic value, the market is then not able to restore equilibrium levels in the short-term, and arbitrage occurrences arise from market inefficiencies.
In mathematical terms, a formal definition of No-arbitrage (NA) condition can be defined:

$$\text{NA} \iff \{\eta | A\eta > 0\} = \emptyset$$

[1], i.e., there exist no arbitrage portfolios, assumed that:

(1)

$$\Omega = \{\theta_1, ..., \theta_m\}$$

is the state space with a finite number of states of nature

(2)

$$p = (p_1, ..., p_n)$$

is the price vector of n traded assets, and $(\eta_1, ..., \eta_n)$ is the vector of investments in asset $(1, ..., i, ..., n)$

(3)

$$-p\eta = \sum_i p_i \eta_i \leq 0$$

represents a portfolio with no positive cost

[1]Stephen Ross, *Neoclassical finance* (Princeton university press, 2005)

(4)
$$G\eta > 0$$

is the positive payoffs in some state of nature of the portfolio, where

$$G = [g_{ij}] = [\textit{payoff of security } i \textit{ if state } \theta_{ij} \textit{ occurs}]$$

represents the Arrow-Debreu tableau of possible security payoffs, as explained by **Ross (2005)**; The rows i of matrix G represent states of nature, while the columns j are the traded assets. "Each row of G contains the payoffs of n securities in the particular state of nature, and each column lists the payoffs of that particular security in the different states of nature"[2]

(5)
$$A\eta = \begin{bmatrix} -p \\ G \end{bmatrix} \eta$$

is the resulting arbitrage portfolio with no negative payoffs and a positive payoff in some state of nature[3];

This mathematical principle states that does not exist a portfolio that has a positive payoff and no cost at all; but if it exists and persists over time, while equilibrium level is not restored in the market, then the No-arbitrage condition is not satisfied, and the market in which the arbitrage portfolio is constructed is not complete and arbitrage free (See **Dybvig and Ross (1987)** for proof of the Fundamental theorem of Finance).

[2]Stephen Ross, *Neoclassical finance* (Princeton university press, 2005)
[3]Stephen Ross, *op. cit*

2.2 Relative Value Arbitrage

The LOP and EMH also apply to "relative value arbitrage", an investment strategy based on the concept of "relative pricing", the latter a methodology that infers the value of an asset or security in terms to another, i.e., through a comparison analysis. It is a completely different valuation approach to "absolute asset pricing", where assets are priced from fundamental factors. Then, if two assets are close substitutes (**Gatev et al. (2006)**) and present similar payoffs, they should trade at similar price (a variant of LOP, called "near-LOP"). In case of price deviation from an equilibrium level, for example due to a significant change in the relationship between two securities prices from its historical average, a relative value arbitrage strategy could be activated to profit from this temporary misalignment once it has been corrected. Hence, relative arbitrage strategies seek to exploit price discrepancies between similar financial assets, even under the circumstance of wrong price valuation (prices of the assets do not truly reflect their fair value), in contrast with the "near"-LOP and EMH. Market neutral strategies, as matched long/short strategies, and Convertible arbitrage strategies are considered examples of relative value arbitrage strategies that include multiple assets; they are not entirely risk-free, but based on the investor's perspective.

A typical expression of relative arbitrage and market neutral strategy is represented by pairs trading, a popular strategy that belongs to the category of statistical arbitrage, and seeks to exploit temporary price deviations between a couple of assets; however, it is not a risk-free strategy and in many circumstances neither market neutral.

2.3 Pairs Trading

2.3.1 History

The first appearance of pairs trading as investment strategy should be credited to the American investor and trader Jesse Livermore, who conceived the trading methodology called "sister stocks" in the early 1920s; his investment rules were simply based on the selection of stocks whose prices had moved together under normal market conditions, and subsequent opening of positions whenever their prices would have diverged. Therefore, positions would be held until price convergence was achieved or stop loss levels hit. Although Livermore was probably the first to experiment the methodology, the formal theorization of pairs trading, and its broad adoption as investment strategy, took place in the following decades. In the mid 1980s, a group of mathematicians, statisticians and computer scientists was assembled by Morgan Stanley quant-trader Nunzio Tartaglia[4], in order to develop statistical and quantitative methods able to identify the presence of arbitrage opportunities in the equity market. In particular, the group developed high-tech automated trading systems, one of the innovations of the period, that employed the use of consistent filter rules to execute trades[5]. The first results were astonishing: the group reportedly made a $50 million profit in 1987. They did not replicate the same level of performance in the following years, and the group disbanded in 1989; meanwhile, pairs trading was gaining attention from the market and press, as an innovative market neutral strategy that could be implemented both by institutional and retail traders.

2.3.2 Definition and approaches

The essence of a pairs trading strategy relies on the identification of some form of temporarily mispricing or anomaly between a pair of assets, the latter that could be represented by stocks, interest rates, currency rates or exchange rates. Whenever this divergence, called spread, is large enough to the investor perspective, the pair of assets could be traded with the idea that the price divergence would correct itself and return to an equilibrium level at some point in the future.

The success of the strategy depends on the approach chosen to identify potential profitable pairs[6]; in fact, pair identification remains the principle hurdle to the ac-

[4]Ganapathy Vidyamurthy, *Pairs Trading: Quantitative Methods and Analysis* (Wiley Finance, 2004)

[5]Evan Gatev, William N. Goetzmann, K. Geert Rouwenhorst, *Pairs Trading: Performance of a Relative-Value Arbitrage Rule* (2006)

[6]Francois-Serge Lhabitant, *Handbook of Hedge funds* (Wiley Finance, 2006)

tivation of a profitable pairs trading strategy. The first attempts were based on fundamental valuation, and comprised the analysis of financial and accounting data to perform the selection, usually stocks that belonged to the same industrial sector. Obviously, this approach had the limit to take into consideration only a limited number of assets due to the amount of time required to derive financial ratios and perform comparisons. More recently, with the proliferation of computer statistical software and tools, it is possible to deploy advanced algorithms and techniques to fathom the entire market of a given asset class and select among hundreds or thousands of assets the ones whose price satisfy prespecified metrics.
Among the statistical methods theorized to identify the pairs, two have emerged and subsequently tested in the years in a wide array of markets: the Distance method, introduced by **Gatev et al. (1999)**, and the Cointegration approach, a more sophisticated version that heavily relies on econometric techniques.

2.3.3 The Distance method

Gatev et al. (1999) use some sort of *distance function* to measure the co-movements of the pair components; a justification of the approach comes from the analysis of the main features pair traders of the period looked when forming the pairs, that is they were searching assets prices that "moved together". The authors define the *tracking variance (TV)*, a measure of distance between two normalized asset prices, for instance stock prices, computed as the sum of their squared differences over a formation period. Then, a minimum-distance criterion is used to match the assets; in other words, stocks that minimize this distance measure are selected to form the pairs and subsequently tested in the trading period.

If $\{P_1^A, P_2^A, ..., P_t^A, ..., P_T^A\}$ and $\{P_1^B, P_2^B, ..., P_t^B, ..., P_T^B\}$ are the price series of stocks A,B, the tracking variance can be estimated as the following[7]:

$$TV = \frac{1}{T}\sum_{t=1}^{T}(Q_t^A - Q_t^B)^2$$

, where $Q_t^A = P_t^A / P_1^A$ and $Q_t^B = P_t^B / P_1^B$ are the normalized prices of the two stocks, and $\delta_t = Q_t^A - Q_t^B$ is their difference, or spread.

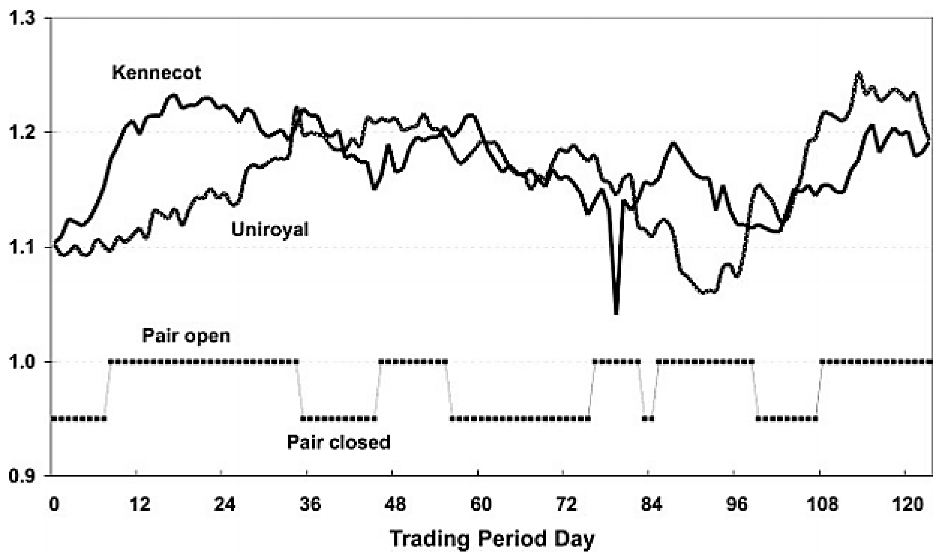
[7]Paolo Vitale, *Pairs trading and Statistical arbitrage*, lecture notes, Equity Markets and Alternative Investments, Luiss University, Academic year 2015-2016

Positions in a pair are activated whenever the asset prices distance has reached a certain threshold, defined in the formation period. In this sense, the authors use as trading rule the standard deviation metric: once "*prices diverge by more than two historical standard deviations*", $\delta_t > |2SD|$, a long position is assumed on the undervalued stock, and a short position on the overvalued one; positions are then closed when the spread cross back to another threshold or a stop loss level is hit. Standard deviation of the tracking variance can be defined as:

$$SD = \left(\frac{1}{T-1} \sum_{t=1}^{T} \left[\left(Q_t^A - Q_t^B \right)^2 - TV \right]^2 \right)^{1/2}$$

Figure 2.1 is the illustration of an example provided by the authors; a pair trading strategy applied to a couple of Us stocks, Kennecot and Uniroyal, in the trading period from August 1963 to January 1964. The graph also displays the positions of the strategy, opened and unwound whenever the pair spread is above or below the threshold defined in the formation period.

Figure 2.1. Daily normalized prices: Kennecott and Uniroyal, August 1963 - January 1964



Notes: The graph plots in the upper part the normalized prices of two Us stocks, Kennecot and Uniroyal, and in the bottom one the positions obtained by the pair trading rule

The position in a pair is constituted by a long position on a stock and a short position on the other one, so, the return over the holding period for this pair is simply the difference in returns between the two stocks. The pair is open 5 times, so the total return of the strategy in the interval of time is the result of the product of the corresponding 5 returns obtained with the activation of positions.

The main advantage of Distance methodology relies on the absence of parameters to be estimated; it is a parametric-free approach. Therefore, it is not subject to model mis-specifications and mis-estimations, as illustrated by **Krauss(2015)**.
On the other hand, this methodology presents several drawbacks with regard to the spread variance and mean reversion requirements; firstly, the choice of Euclidean squared distance as measure to select pairs is analytically suboptimal[8], as the "ideal pair", the one that minimizes the TV, would present null squared distance, but also a null spread and no profits; in fact, this methodology led to the formation of pairs with low spread variance and limited profits, a choice in contrast with the investment goals of a rational investor. Secondly, the methodology does not investigate on the nature of correlation between the pair components, as it does not make use of any statistical test to confirm some long-run equilibrium relationship. As a consequence, the high level of correlation could be spurious and the pair may not possess mean reverting properties, with implications on the strategy profitability. With regard to this last aspect, **Gatev et al. (2006)** confirm the profitability of pairs trading strategies, but at the same time record a small magnitude of the profit levels, justified by the exacerbation of arbitrage strategies by speculative funds. The declining profitability is documented by **Do and Faff (2010, 2012)** too, who replicated the methodology in a wider trading period, and found that approximately one third of the pairs, formed with the distance measure, did not converge or possess mean reversion properties at all.

2.3.4 Statistical Arbitrage: the Cointegration method

The application of cointegration approach to pairs trading has been introduced by **Vidyamurthy (2004)**; it exploits co-movement between pair components by cointegration testing, with the **Engle-Granger (1987)** two step procedure, or the **Johansen (1988)** method in the context of multiple cointegrating relations.

[8]Krauss (2015)

Cointegration is a statistical property that characterizes a set of non-stationary time series data X_t, Y_t, i.e, random processes ordered by time t. In time series analysis, non stationary variables X_t, Y_t possess time-varying moments: unconditional mean, variance and autocovariance are not constant over time (or just one of them):

$$E[Y_t] = \mu_t$$

$$Var(Y_t) = kf(\mu_t)$$

$$Cov(Y_t, Y_{t-h}) = \gamma_t(h)$$

, where k is a constant and f() a known function. An example of non-stationary variable is the one generated by an AR(1) model with a slope parameter of unity, $\phi = 1$, also called a *random walk* model.

However, non stationary variables can be made stationary by differencing; in this case they are said to be Integrated processes of order d, I(d) , where d is the degree of differencing required to make the variable stationary [9]. If X_t, Y_t are then integrated time series of order 1, I(1) , their first differences, $\{x_t - x_{t-1}\}$, $\{y_t - y_{t-1}\}$, are stationary time series with constant unconditional moments.

In general, as expressed by **Caldeira, Moura (2013)**, linear combinations of non-stationary variables are also non-stationary, but any linear combination that produces as result a stationary time series is said to be a cointegration relation.

In mathematical terms, if there exists a vector β such that the linear combination

$$\epsilon_t = Y_t - \beta X_t \sim I(0)$$

is stationary, the two variables X_t, $Y_t \sim I(1)$ are said to be cointegrated.

In this sense, Cointegration expresses the long-term relationship that ties two or multiple variables together, as asset prices, under a common stochastic trend, even though they might diverge in the short term; it is a measure of long-run comovements in the variables, not to be confused with the concept of correlation; in fact, correlation is a short-term measure, liable to great instability over time [10], while a cointegrating relation may even occur in periods of low static correlation. It does not indicate whether the two variables move in the same direction, but rather focuses on long term behaviour of their distance, or difference. Hence, it is possible to have cointegration jointly with correlation or not (cointegration without correlation).

The framefork introduced by Vidyamurthy relies on this statistical property: most financial price series are not stationary time series, but rather geometric random walks; however, if a linear combination of them is found to be stationary, then their

[9]Robert Sollis , *Empirical finance for finance and banking* (Wiley, 2012)

[10]Alexander (1999)

distance, or spread, possesses mean reversion traits. Consequently, a trading strategy could be constructed pairing non stationary but cointegrated asset prices ,as it is expected that their evolution will diverge in the short term but eventually retrace to an equilibrium level in the long-run.

2.3.4.1 Unit root tests and Stationarity

Antecedent the cointegration testing, the first essential stage of the analysis relies on the identifcation of non-stationarity in the asset price time series. If they are integrated processes, then there is the possibility of being cointegrated.
Statistical tests are utilized to verify the presence of unit roots, i.e., non-stationarity, in the price time series: the Augmented Dickey Fuller (ADF) test (**Said and Dickey (1984)**), the Phillips-Perron (PP) test (**Phillips and Perron (1988)**) and the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test (**Kwiatkowski et al. (1992)**).

The ADF test is a version of the initial Dickey-Fuller test for checking the presence of unit root in a time series sample, but it encompasses a larger set of time series models, not focusing exclusively on a simple first order autoregressive model AR(1) (Dickey-Fuller test, 1979) but including variables that may contain high order dynamics (ARIMA, ARMA models). Constants and deterministic trends may be added to the model:

$$\Delta Y_t = \mu + \gamma t + \phi_1^* Y_{t-1} + \sum_{i=1}^{k} \beta_i \Delta Y_{t-i} + \epsilon_t$$

,where:

$\Delta Y_t = Y_t - Y_{t-1}$

$\phi_1^* = (\phi_1 - 1)$ is the coefficient to assess unit root presence

μ is the constant term

γ is the trend component

ϵ_t is an IID random error term

The model tests the null hypothesis of $\phi_1^* = 0$, i.e., unit root in the sample, against the alternative hypothesis $\phi^* < 0$; the t-statistic is compared with the DF critical values and not the ones associated to the Student's t-distribution, as the authors proved that under the null hypothesis the t-statistic did not converged asymptotically to the Student's t-distribution but rather followed a non-standard distribution.

The model includes k lagged values of the dependant variable ΔY_{t-i}, as it allows for higher-order autoregressive processes; moreover, the lag length parameter could be optimized via an information criterion (AIC, SIC).

Phillips-Perron test for unit root builds on the DF test as well, but addresses the issue of higher order dynamics of the interested variables through a non parametric correction of the t-statistic. In fact, the authors *"adjust the statistics computed from a simple first order autoregression to account for serial correlation of the differenced data"* [11].

To conclude, in KPSS test for stationarity, the presence of unit root in the sample data is contained in the alternative hypothesis and tested against the null hypothesis of trend stationarity; hence it encompasses the possibility for a time series to reject the presence of unit root yet to be trend-stationary (time series data is stationary around a deterministic trend).

2.3.4.2 Engle-Granger approach

Engle and Granger (1987) propose a two-step approach to find a cointegrating relation between the components of a pair. Once it is proven via an unit root test that two variables, for instance X_t, Y_t, are integrated processes of order 1, I(1), then the first step of authors' procedure entails the performance of a linear regression of a variable (Y_t) on the other (X_t), using the *Ordinary least squares* (OLS) model for parameter estimation:

$$Y_t = \beta_0 + \beta_1 X_t + \epsilon_t$$

$$\hat{\epsilon}_t = Y_t - \hat{\beta}_0 - \hat{\beta}_1 X_t$$

where β_0 is a constant term, the intercept, β_1 is the hedge ratio, or cointegration coefficient, and $\hat{\epsilon}_t$ are the fitted errors, the latter subsequently tested for stationarity with the ADF test (Second step), excluding drift or deterministic trend in the model. In case of rejection of the null hypothesis of unit root in the residuals, the components of the pair are said to be cointegrated, while $\hat{\epsilon}_t$ is the estimated *cointegrating vector.*

The authors demonstrated as cointegrated time series can then be expressed in terms of *error-correction* (Engle–Granger representation theorem).

[11] James D. Hamilton, *Time Series Analysis* (Princeton university press, 1994) chapter 17

According to **Alexander (1999)**, *"the mechanism which ties cointegrated series together is a 'causality...'"*, known as "Granger casuality", *"...in the sense that turning points on one series precede turning points in the other"*.
In fact, the Error correction model (ECM) links the first differences of the two variables, which capture short-term dynamics, to a long-run equilibrium term, represented by the lagged fitted error, and estimates the speed of adjustment parameter, the rate at which the dependant variable adjusts to this equilibrium term after a change in the other variable. In other words, the model incorporates the tendency of cointegrated variables to converge to a common stochastic trend:

$$\Delta Y_t = \gamma \Delta X_t + \theta \hat{\epsilon}_t + \eta_t \qquad \text{(ECM)}$$

, θ is the speed-of adjustment parameter, which is negative for cointegrated variables. A more general version includes lags of the relevant variables $\Delta X_t, \Delta Y_t$ on the right-hand side of the equation. In the context of cointegration, all variables of the ECM are stationary processes, I(0), hence the standard inference techniques are therefore valid.

When it comes to modeling multivariate time series data, the Engle−Granger two-step procedure may produce biased results, as the order of time series data in dependant and independent variable assumes a crucial role, while **Johansen (1988)** methodology permits more than one cointegrating vector and is commonly regarded as the standard procedure. However, as expressed by **Alexander (1999)**, for many financial applications, the Engle-Granger approach can represent the optimal methodology, as "*it is very straight-forward to implement*"; secondly, its linear combinations present the minimum variance, a nice feature in the context of risk management applications, and, finally, it is quite natural the choice of dependant variable in the linear regression, while the bias of small-sample is not realistic as financial sample size are quite large most of the times.

2.3.4.3 Evidences of profitability

The profitability and superiority of the cointegration approach has extensively documented by a wide array of academic studies: **Perlin (2006)**, **Caldeira and Moura (2013)** document consistent excess returns on the Brazilian financial market; **Hong, Susmel (2003)** cointegrated pairs-trading results for 64 Asian shares display consistent positive profits, which are robust to different holding and estimation periods. **Rad et al. (2015)** find excess returns on a monthly basis on the entire US equity market from 1962 to 2014, and report that the cointegration method proves to be the superior strategy during turbulent market conditions. **Huck and Afawubo (2015)**, using the components of the SP 500 index, reveal that cointegration provides high, stable and robust returns. **Blázquez et al. (2018)** find that pairs trading strategy based on the distance and cointegration techniques generates residual series with better properties than the other techniques for a given pair of stocks within the US financial sector.

2.3.5 Other methods

Other pairs trading techniques, as time series approach **Elliott et al. (2005)**, or the the stochastic control approach **Jurek and Yang (2007)** , ignore the formation period over which estimate the pairs but mostly focus on the optimization of trading rules and signals, as highlighted by **Krauss (2015)**. In particular, **Elliott et al. (2005)** approach *"propose a mean-reverting Gaussian Markov chain model for the spread which is observed in Gaussian noise"*, a parametric framework that makes use of a state space model. It comprehends a set of states over the time, as the evolution of price series of a pair, and a set of observations on these states; however, the observable variable is a linear function of the hidden variable but contains statistical noise, with the implication that the "true" state is not ever directly observable.
Hence, the Kalman filter, an optimal linear algorithm, can be deployed to estimate the "true" state of a variable, as the dynamic hedge ratio of a pair spread; it updates the expected value of a hidden variable according to the latest value of an observable variable [12]. The measurement prediction error (or forecast error) estimated by the algorithm represents the deviation of the spread from its true state; thus, a trading strategy could be activated whenever this deviation is quite large, with negative or positive sign, depending on its predicted standard deviation[13].

[12] Ernie Chan, *Algorithmic Trading: Winning Strategies and Their Rationale* (Wiley, 2013)
[13] See Elliott et al (2005), and Chan (2013) for a detailed explanation of the Kalman filter

Chapter 3

Empirical Analysis

3.1 Arbitrage Strategies

Simple arbitrage strategies in the cryptocurrency market regard the possibility of buying and selling at the same time selected cryptocurrencies in different exchanges in order to retain the difference in price, defined as *premium* (when positive). One of the main reasons that suggests the analysis of price discrepancies relies on the fragmentation of the cryptocurrency space: there existed more than two hundred trading platforms in 2018 according to Coinmarketcap website, with sensible different trading volumes and buying pressure, certified by academic studies[1]. Moreover, the majority of them was open to foreign traders and investors, thus reinforcing the possibility for arbitrageurs to exploit temporary price misalignments.

3.1.1 Data

Historical Data of the exchange rate of cryptocurrencies versus the Us dollar have been collected with the use of Cryptocompare api on Python software. Cryptocompare is a global cryptocurrency market data provider that gives access to real-time pricing data on more than five thousand coins; the reliability of the data have been confirmed through the comparison with other data providers as Coinmarketcap and Investing.com. The data set contains the open, high, low and close pricing data (OHLC), trade volumes and the timestamp in Universal Coordinated time (UTC). Even if cryptocurrency markets have no closing times, closing prices for all the cryptocurrencies have been used to test the strategies, and they correspond to the midnight UTC ($00:00$ UTC). I have restricted the analysis to three of the most liquid cryptocurrencis, with the largest market capitalization and at least one year of historical price data: Bitcoin (BTC), Ether (ETH) and Litecoin (LTC).

[1] Borri and Shakhnov (2018), Igor Makarov and Antoinette Schoar (2018)

The exchanges covered by the analysis have been classified following a geographic order:

1. Asia: Bithumb (South Korea, fiat currency: Korean won), bitFlyer (Japan, fiat currency: Japanese Yen), Bitfinex (Hong Kong, fiat currency: Us dollar)
2. US: Coinbase (fiat currency: Us dollar), Kraken (fiat currency: Us dollar, Eur, Canadian dollar)
3. Europe: Bitstamp (Luxembourg, fiat currency: Eur, Us dollar)

The time frame of the analysis covers the period from May 22th, 2017 to December 20th, 2018 due to data unavailability of previous prices from the Korean exchange Bithumb; however, the analysis of daily volumes shows a lower market liquidity in the period prior to 2017, thus sustaining the hypothesis to focus the analysis on the selected time frame. Daily exchange rates versus the Korean won (KRW) and Japanese Yen (JPY) have been converted with the KRW/USD and JPY/USD pairs obtained from Investing.com.

3.1.2 Methodology and Results

The selected cryptocurrencies are strictly homogeneous assets: every unit possess the same properties, thus it should be traded at the same price in different markets, and any price differences should be eliminated by the market. However, cryptocurrencies prove not to satisfy the law of one price (LOP) as they can trade at sensible different prices in multiple exchanges.
Figure 3.1 shows the so-called "Kimchi Premium" [2], a large gap in cryptocurrencies prices recorded on the South Korean exchange compared to foreign ones; this difference appear quite evident between December 2017 and February 2018, when Bitcoin price was 40% higher than rates in the Us, Europe exchanges.

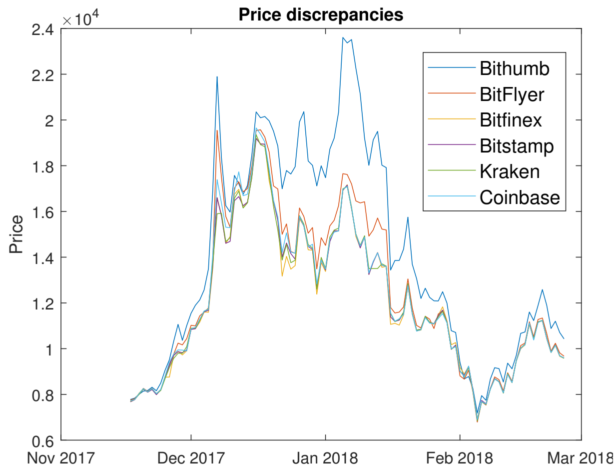
We can define such premium over USD as:

$$Pr = \frac{P_{BTC/KRW} * S^{KRW}}{P_{BTC/USD}} - 1$$

, where $P_{BTC/KRW}$ and $P_{BTC/USD}$ are respectively the Bitcoin price to the South Korean won and the US dollar, while S^{KRW} is the spot exchange rate between the South Korean won and US dollar.

[2] literally the country fermented cabbage dish

Figure 3.1



Notes: Bitcoin daily prices in US dollar on the selected exchanges; Korean and Japanese prices have been converted with the spot exchange rates from *Investing.com*. Data extracted with Cryptocompare api

More formally, Borri and Shakhnov (2018) define cryptocurrency discounts as:

$$D_{m,j} = \frac{P_{m,j}}{P_{1,1}} - 1$$

, where $D_{m,j}$ is the discount in market m in the currency j, $P_{m,j} = \frac{S^j}{P^*_{m,j}}$ is the units of coin obtained in market m with one U.S. dollar, expressed as the ratio between S^j, the spot exchange rate in unit of currency j per Us dollar, and $P^*_{m,j}$, the unit of currency $j = 1, ..., J$ required to buy one coin, i.e., Bitcoin, in market m. $P_{1,1}$ is the price in market $m = 1$ (Bitstamp) in the currency $j = 1$ (Us dollar).
Figure 3.2, 3.3 show the evolution of Bitcoin discounts in the period 05/2017-12/2018, selecting Bitstamp as reference exchange and expressing all the trading pairs in USD values. **Table** 3.1 reports the main statistics of Bitcoin discounts in percent over the USD price for the sample of international exchanges: mean, standard deviation, maximum and minimum values, first order autocorrelation and the total number of observations. The empirical findings confirm Borri and Shakhnov (2018) results: discounts are volatile, time-varying, as they can be positive or negative between the same pair of exchanges, and persistent. **Figure 3.4, 3.5** and **Table 3.2, 3.3** report the same results for the coins ETH, LTC.

Figure 3.2: BTC Kimchi premium to European exchange Bitstamp, 05/2017-12/2018

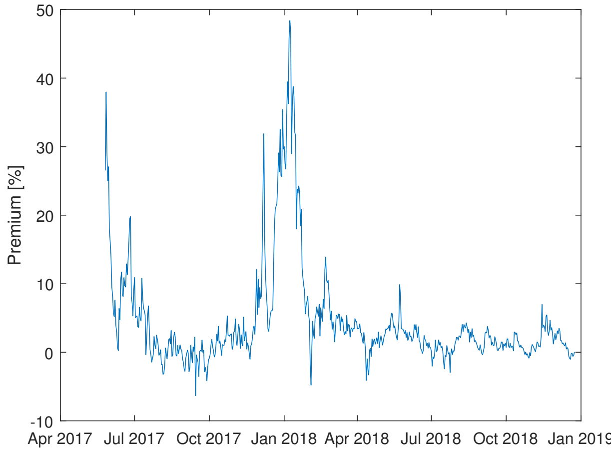


Table 3.1: Summary statistics

BTC Discount (in %)						
Exchange	**Mean**	**Std.**	**Max.**	**Min.**	**AC(1)**	**obs.**
Bithumb	4.5889	7.9131	48.4129	-6.3397	0.9409	578
BitFlyer	0.8783	2.3370	17.7331	-11.8885	0.7365	578
Bitfinex	0.1362	1.3235	5.8344	-6.7514	0.8029	578
Kraken	-0.0126	0.5842	2.0650	-4.8908	0.3919	578
Coinbase	0.1491	0.6743	6.4871	-1.2800	0.6109	578

Notes: main statistics of Bitcoin daily discounts in percent over Bitstamp exchange, period 05/2017-12/2018. Btc Discounts are volatile, time-varying and persistent

Price discrepancies are more observable during 2017 and across regions than exchanges within the same region; in fact, since March,2018 prices in Us and Europe exchanges seem to be aligned, while large deviations appeared and still appear between western exchanges and Asian ones (Bitfinex). A reduction of the magnitude of these discounts could be attributed to a major decrease of daily trading volumes and buying pressure, starting from the first quarter of 2018, and the deployment of advanced arbitrage strategies by new speculative funds that entered the market in 2017 and 2018 [3].

[3] According to *Crypto fund research*, crypto-funds constituted more than 16% of the total hedge fund launches in 2017 and more than 20% in 2018

However, as stated above, when these price discrepancies take form they tend to persist over time, in terms of days and weeks, and could allow traders to perform arbitrage strategies but the presence of several hurdles might influence their profitability, as documented by **Borri and Shakhnov (2018)**.

In fact investors may face considerable transaction costs and risks, represented by:

- Deposit, withdrawal and trading fees to exchanges to execute the orders
- Bid-ask spread
- Mining fees to move the assets between exchanges
- Execution risks due to settlement time for fiat deposit[4] and transaction confirmation times for cryptocurrencies[5]
- Counterparty risk due to disputable and opaque behaviours of the same exchanges: unexpected website maintenance, deposits and withdrawals suspended for selected coins, trade suspensions for hackings
- National and international interventions and restrictions of trading and international capital flows, as happened in South Korea at the beginning of 2018 [6]

The majority of such transaction costs could be minimized following some precautions:

1. Selection of transparent crypto-exchanges with large volumes, low bid-ask spread and trading fees, low level of hackings and technical failures, and in compliance with national or international rules.
2. Selection of coins that requires little or no time to move from an exchange to another, paying minimal mining/transaction fees
3. Execution of large trades in order to reduce or elude deposit, withdrawal and trading fees [7], transaction costs with the heaviest weight

[4]Between two and five business days

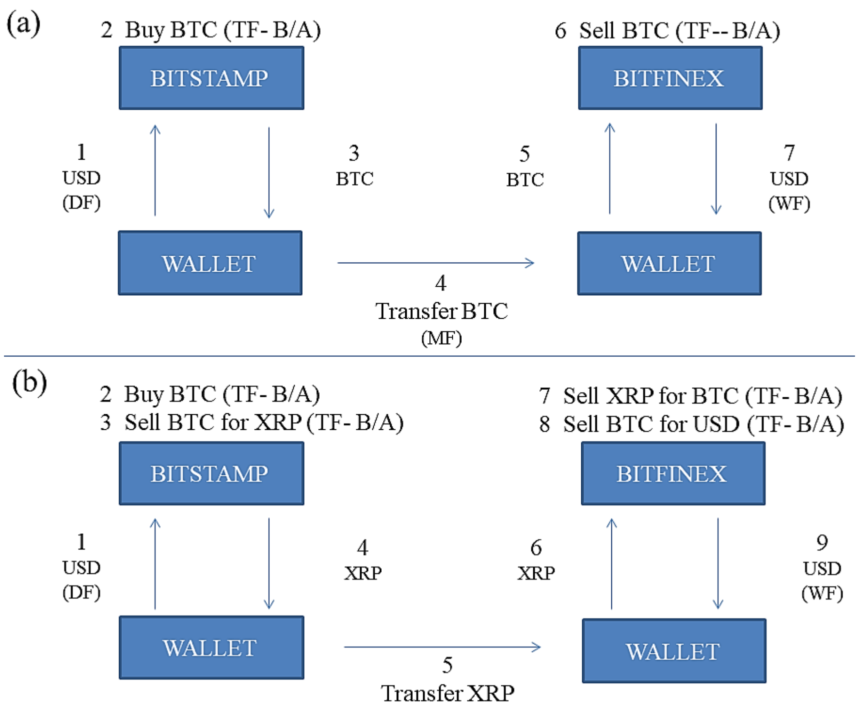
[5]Mining confirmation times can take between 5-10 to 60 minutes, depending on the type of cryptocurrency

[6]Since February 2018, crypto-trading is only allowable to citizens of the country

[7]Not applicable to every exchange

For example, we could construct a simple arbitrage strategy that involves a cryptocurrency with a significant discount between a pair of exchanges but includes a second currency for transfer; this second coin has to be traded in both exchanges, not present the same discount and take few time for the transfer. The main advantage of this strategy relies in the minimization of the execution time and transaction fees (tx) payment; in fact Bitcoin transaction confirmations take between 30 minutes and 1 hour to finalize, at an average tx fee of 0.8$, but with the possibility of reaching even $60-80$\$ in periods of heavy network traffic, while coins as Ripple (XRP), or Stellar Lumens (XLM) may take few seconds at infinitesimal tx fees (0.007$). A drawback is represented by the higher trading costs due to the execution of more trades. I select Bitcoin as the cryptocurrency with discount, Bitstamp and Bitfinex as the pair of exchanges, while Ripple is the selected currency to execute fast transfers. Hourly Btc, Xrp prices have been gathered via Cryptocompare api. Discounts computations are represented in **Figure 3.6**; Bitcoin discounts are expressed in Us dollar , while Ripple discounts take as reference Bitcoin. **Figure 3.7** illustrates a comparison of the two arbitrage strategies: simple and with a second currency.

Figure 3.7 Arbitrage strategies



Notes: (a) simple arbitrage, (b) arbitrage with the inclusion of a second currency; Legend: DF= deposit fee; MF= Mining fee; TF= Trading fee; WF= Withdrawal fee; B/A= Bid-ask spread. Author illustration

Table 3.4 reports summary statistics of the hourly discounts between Bitfinex and Bitstamp exchanges.

Table 3.4: Summary statistics of hourly discounts

		Discount (in %)						
Exch.	**Pair**	**Mean**	**Std.**	**Max.**	**Min.**	**AC(1)**	**obs.**	$D > 2$
Bitf.	**BTC/USD**	2.3435	1.1959	11.2796	0.4006	0.9608	2001	58.12
Bitf.	**XRP/BTC**	- 0.0053	0.2304	2.7100	-1.9740	0.0657	2001	0.0005

Notes: main statistics of BTC/USD and XRP/BTC hourly discounts in percent over Bitstamp exchange, period 08/2017-12/2018; every observation corresponds to one hour

Bitcoin presents a consistent discount, with an average value of 2.3435% and with 58.12% of probability of being larger than 2% (1163 over 2001 observations); the pair XRP/BTC presents no significant discount, as more of 97% of observations lie in the range $-0.5 - 0.5\%$; therefore, when BTC discount is consistent and assuming the Usd deposit has already been executed, a trading arbitrage strategy can be structured as the following:

1. Buy Bitcoin on the exchange with cheaper price, i.e., Bitstamp
2. Sell Bitcoin for Ripple
3. Transfer Ripple to the second exchange, Bitfinex
4. Sell Ripple for Bitcoin
5. Buy Us dollar with Bitcoin
6. Withdraw Usd from exchange

The above strategy minimizes the execution time and the payment of mining fees. However, it is reasonable to highlight as mining fees are very tiny and irrelevant with respect to other costs, for example the higher trading costs that have to be accounted; hence, the strategy can be considered profitable in cases of consistent price deviations, in the minimum order of $2 - 3\%$ with the actual system of fees[8].

[8] See table of fees, Bitstamp and Bitfinex exchanges

Table 3.5 reports the transaction costs. Usd deposit fee is 0.05% on Bitstamp (Eur deposit is free of charge), while Usd withdrawal fee is 0.10% on Bitfinex; Ripple deposits and withdrawals are free of charge on the two exchanges; trading fees vary between $0\% - 0.20\%$ on Bitfinex, and can be minimized with higher trade volumes, and between $0.10\% - 0.25\%$ on Bitstamp; mining fees are null with XRP transfers; remain to consider the bid-ask spread, represented in **Figure 3.8**.

In the case of reduced fees[9], the investor should only face the bid-ask spread, deposit and trading fees on Bitstamp, and Usd withdrawal fee on Bitfinex (no withdrawals fees of xrp from Bitstamp, no mining fees of xrp, no deposit and trading fees on Bitfinex). With a BTC price discrepancy of 1% and a null XRP/BTC discount the strategy generates an average outcome of -0.02% in the case of full fees payment, and a 0.485% profit in the case of reduced fees[10]. Hence, trading signals of entry positions may be placed at levels that equal 2% or 3% (**Figure 3.9**); whenever the discount is above such levels the arbitrage strategy can deliver a net profit.

3.1.3 Conclusions

Price discounts in cryptocurrency markets are in decline, mainly due to reduction of trading volumes, new strategies deployed by speculative funds and expert arbitrageurs and new measures adopted by exchanges[11]; however, in periods of uncertainty and volatility, during particular news or events, as Cryptocurrency forks[12], deviations may re-appear and in double-digit levels too; furthermore, with the introduction of liquid institutional platforms it is expected that the time period such that these arbitrage occurrences generate abnormal profits will shorten, in favor of high frequency trading strategies.

[9] For high monthly traded volumes

[10] Author computations, all transaction costs included except the bid-ask spread

[11] Introduction of new fees

[12] Bitcoin cash fork and subsequently hashing war on November, 14th raised uncertainty in the market and daily volatility, after months of relative price stability

3.2 Pairs Trading

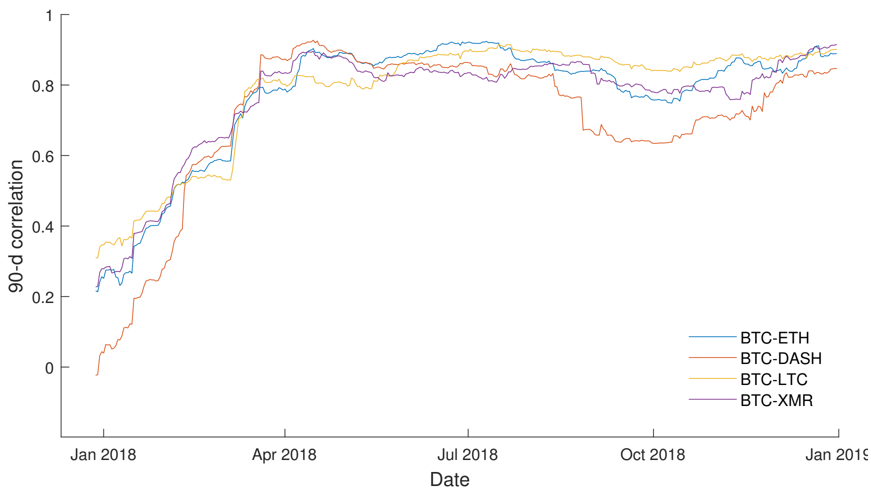
3.2.1 Intro

Cryptocurrencies proved to be extreme volatile assets, mostly correlated between them but largely uncorrelated with traditional financial markets (**Chuen et al. (2017)**, **Bianchi (2018)**). **Figure 3.10** displays the moving correlation of daily returns between Bitcoin and four altcoins; it is observable how, starting from January 2018, correlation between daily returns surged dramatically, passing from weak to a strong positive association. The simultaneous presence of volatility and correlation in the market is an opportunity to examine market neutral trading strategies, that do not take into account the market trend ; on the contrary, they allow investors to profit from any market condition.

Consequently, I analyze the process of constructing pair trading strategies in the cryptocurrency market; a set of cryptocurrencies is chosen to form the pairs and subject to a series of statistical tests.

The scope of the research is not to find the most profitable pair but rather demonstrate that the cryptocurrency market is not efficient, as it allows the construction of profitable arbitrage strategies, and, hence, reject the Efficient Market Hypothesis.

Figure 3.10: 90-day correlation for Bitcoin and four altcoins, Bitfinex



Notes: The graph plots the 90-day correlation between Bitcoin and four altcoins: Ethereum, Dash, Litecoin and Monero. Data extracted from Bitfinex exchange, author computations

3.2.2 Data

Historical data of cryptocurrencies have been gathered via Cryptocompare api for the period 08/2017-12/2018 from Bitfinex exchange. The dataset contains the OHLC data and the timestamp in UTC of five cryptocurrencies, selected among the most liquid and with at least one year of historical price data: Bitcoin (BTC), Ether (ETH), Litecoin (LTC), Dash (DASH) and Monero (XMR). All the pricing data are expressed in Us dollar. Litecoin, Dash, Monero were all forks of the original code Bitcoin, with whom share some network features and technological developments. Hence, considering the strict connection with BTC, it is plausible to explore the evolution of relative price dynamics. The Hong kong based exchange has been selected for the research as it displays significant trading volumes of the main cryptocurrencies and allows short selling practice; moreover, historically it has not suffered major technical failures or hacks, thus, the execution risk is of low order compared to other exchanges. The choice to use daily prices is explained by the persistence that characterizes prices deviations and movements in the market.

3.2.3 Training set and Test set

The historical data are divided in two parts in order to avoid the look-ahead bias[13]: the first part, the training set, comprehends the least recent observations; parameters of the model are optimized on this portion of data. The second part, the test set, is the set of observations where the resulting model is tested; the two portions should be equal in size, as expressed by **Chan (2013)**, but in the presence of insufficient training data, the size of the test set should at least be one-third of the training set. As a result, the first 365 observations of the dataset are contained in the training set, where parameters are optimized; the last 123 observations are contained in the testing set, where I will test the model optimized in the first portion of data. In the end performance measures of the two sets are compared; the performance of the second part should at least be reasonable, otherwise it could face the data-snooping bias. A more rigorous method of out-of-sample testing is to use moving optimization of the parameters[14]; this means that parameters are dynamically optimized in a moving look-back window, i.e., the parameters constantly adapt to the changing historical data.

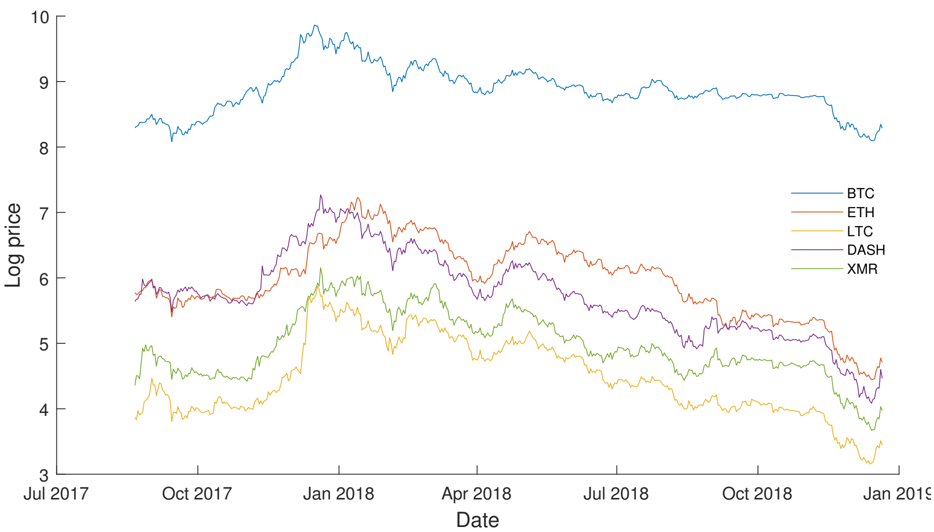
[13] When future information or data are used to construct and back-test a trading strategy for a time period antecedent to their availability

[14] Ernie Chan, *Quantitative Trading: How to Build Your Own Algorithmic Trading Business* (John Wiley & Sons, 2009)

3.2.4 Methodology: the Cointegration method

To construct the trading strategy at first I need to find the pairs of cryptocurrencies. The Cointegration method, proposed by **Vidyamurthy (2004)** and based on the works of **Engle and Granger (1987)**, is employed to identify the cointegrated pairs, as it has proved to generate more robust pairs and better risk-adjusted performance measures in traditional markets. In order to be cointegrated, price series of the selected cryptocurrencies have to be integrated of order one, i.e., they are non stationary processes but can be brought to stationary through differencing, and there exists a vector β such that their linear combination generates a time series whose residuals, or error terms, are stationary. Log prices of the five cryptocurrencies, represented in **Figure 3.11**, display evident signs of co-movement; hence prices of the selected cryptocurrencies could be interpreted as being in a long-run equilibrium relation, where their difference, denoted as *Spread*, is not constant in the short-run as they can deviate from the equilibrium level, but it is expected that they will retrace the path to this equilibrium in the long-run.

Figure 3.11 Log prices, Bitfinex



Notes: The graph plots the log price of five cryptocurrencies: Bitcoin, Litecoin, Ethereum, Dash, Monero

The analysis of autocorrelation and partial autocorrelation functions is an useful tool to decide if a variable is stationary or non-stationary; however, statistical tests provide more robust results.

The augmented Dickey-Fuller (ADF) test and Kwiatkowski, Phillips, Schmidt, and Shin (KPSS) test are then applied to absolute prices series of the five cryptocurrencies in the training set. Matlab version of the tests return a logical value with the rejection decision and the p-value: $h = 1$ indicates rejection of the unit-root hypothesis in favor of the alternative model (no root) for the ADF test, confirming the stationarity of log prices series; on the contrary, $h = 0$ indicates failure to reject the unit-root hypothesis. Opposite situation for the KPSS test: $h = 1$ indicates rejection of stationarity for the price series, while $h = 0$ confirms it. At a confidence level of 95% the null hypothesis is rejected for p-values smaller than the significance level $\alpha = 0.05$.
Results of the tests are contained in **table 3.6**, that shows p-values ; the null hypothesis of the presence of unit roots (ADF) in the prices time series is accepted, confirming the daily prices of the selected cryptocurrencies are not stationary; on the other hand, the ADF test rejects the null hypothesis for the first difference of prices; KPSS test confirms the findings (Prices are not stationary); as a result, prices are integrated process of order one, $I(1)$.

Table 3.6: Unit root test: Augmented Dickey Fuller test

	BTC_t	ETH_t	$DASH_t$	LTC_t	XMR_t
ADF-test	0.3713	0.5245	0.5760	0.4461	0.3397
KPSS-test	0.01	0.01	0.01	0.01	0.01
	ΔBTC_t	ΔETH_t	$\Delta DASH_t$	ΔLTC_t	ΔXMR_t
ADF-test	< 0.01	< 0.01	< 0.01	< 0.01	< 0.01
KPSS-test	0.1	0.1	0.1	0.1	0.1

Notes: P-values of the ADF and KPSS tests: Prices are integrated time series of order 1; their first difference is stationary

Thereupon, Engle-Granger two-step approach can be computed: at first, a regression is performed in order to estimate parameters of the linear relation between components of a pair. The *Ordinary Least Squares regression* (OLS) model is used to estimate parameters $\beta = [\beta_0, \beta_1]$, respectively the intercept and coefficient of the relation:

$$Y_t = \beta_0 + \beta_1 X_t + \epsilon_t$$

β_1 is the Hedge ratio, the coefficient that ensures the mean reversion of the spread. Bitcoin is assumed as the dependent variable, while Dash, Litecoin, Ethereum and

Monero are,in turn, the independent variable of the model; however, if we switch the roles of variables in every regression, for example, for the pair BTC/ETH, taking ETH as dependent variable and BTC as the independent one, we will not obtain the same estimated parameters. Afterwards, unit root tests are computed on the fitted errors $\epsilon_t = Y_t - \hat{\beta}_0 - \hat{\beta}_1 X_t$, the residuals of the model, to check stationarity. In case of rejection of the null hypothesis of unit root in the residuals (ADF), the two cryptocurrencies are then cointegrated; the vector containing the estimated parameters ϵ_t is called *Cointegrating vector*. **Table 3.7** illustrates results of the OLS regression for the pair BTC/DASH.

Table 3.7: OLS regression BTC-DASH

	Dependent variable BTC	
Variable	Coefficient	T-statistic
DASH	10.799*	34.362
	(0.314)	
Intercept	3,306.270*	19.841
	(166.635)	
R^2	0.765	
F statistic	1,182.314*	
obs.	365	
Note:	$*p < 0.001$	

The coefficients are statistically significant at the 5% significance level, showing p-values smaller than 0.1%. The large coefficient of determination, $R^2 = 76.50\%$, implies that 76.5% of the variability of the dependent variable is explained by the linear model, thus, a positive result. We can then describe their linear relation as:

$$\mathrm{BTC_t} = 3,306.27 + 10.79\mathrm{DASH_t} + \epsilon_\mathrm{t}$$

, where 10.79 is the hedge ratio.

Tables 3.8, 3.9, 3.10 show the OLS results for the other pairs.

To conclude, **Table 3.11** contains p-values of unit root test on the residuals. They appear to be stationary for all except the pair BTC/ETH, where p-value lies on the edge of the regions of acceptance and rejection; thus, while for the pairs BTC/DASH, BTC/LTC and BTC/XMR the spread can be defined using the coefficients obtained by the OLS model in the training set, for the pair BTC/ETH, the computation of a dynamic hedge ratio should be more appropriate to capture the changing levels of the pair components; in fact, although cointegration is not achieved for a pair of variables, a profitable mean reverting strategy can be constructed if their spread displays enough short-term mean reversion.

Table 3.11: Unit root test (ADF) on the residuals

	BTC/ETH	BTC/DASH	BTC/LTC	BTC/XMR
	$\mathrm{e_t}$	$\mathrm{e_t}$	$\mathrm{e_t}$	$\mathrm{e_t}$
ADF-test	0.0492	0.0026	0.0047	0.0032

Notes: P-values of the ADF test for the residuals of the OLS model: Residuals are stationary, hence the pairs are cointegrated, except for the pair BTC/ETH, where p-value is on the edge of the regions of acceptance and rejection

3.2.5 Half-line of mean reversion, trading rules and returns computation

Consider again the pair BTC/DASH; residuals of the OLS model, that represent the spread of the pair, are stationary, hence a mean reverting strategy could be constructed taking simultaneously a long position on one asset and short position on the other one; we can define the spread as:

$$\delta_t = \mathrm{BTC_t} - \beta_1 \mathrm{DASH_t}$$

, where $\beta_1 = 10.79$.

A long position on the spread denotes the opening of a long position of 1 unit on BTC and a short position of 10.79 units on DASH; a short position on the spread in exactly the opposite. However, the fact that a process shows signs of mean reversion does not automatically imply that it is profitable to construct a mean-reverting strategy; in fact, the half-line of mean reversion, the period of time a price series reverts to its mean, could be very long, in terms of several months or years. Hence, before setting trading rules, it is preferable to compute this measure of speed of mean reversion; the shorter the half-line of a process, the higher the chances of profitability for a mean reverting strategy. **Table 3.12** displays the half-line of the selected pairs; for the BTC/DASH pair, the spread mean reverts in about 14 days, which means that it will full revert to the mean in about 28 days, a remarkable result. Once demonstrated that the pairs are cointegrated and the half-line of mean reversion is of a reasonable order, we can then back-test the pairs in the trading period, or test-set, and compare performance metrics of the strategies with the ones for the training period, although it should be mentioned that performance statistics computed over the training period suffer of the look-ahead bias as they have used the same data for parameter optimization and back-test.

Table 3.12: half-line mean reversion

	BTC/ETH	BTC/DASH	BTC/LTC	BTC/XMR
λ	-0.0195	-0.0495	-0.0434	-0.0498
half-line	35.60	13.97	15.98	13.9262

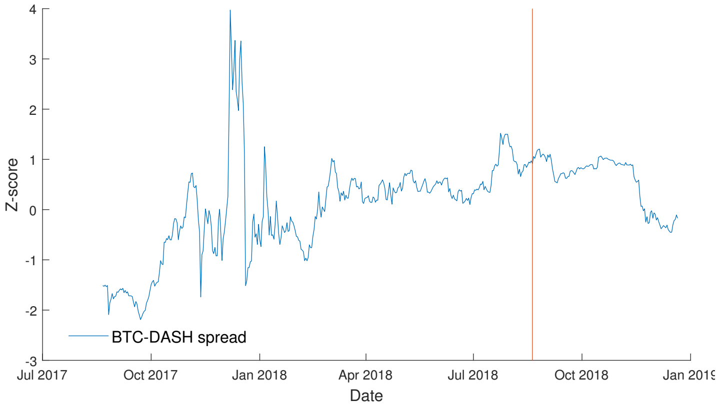
Notes: Half-line of mean reversion, expressed in days; the parameter λ measures the time it takes for a price or spread to mean revert. It is negative for mean reverting processes, a value very close to 0 means the half-line will be very long, hence even if the process is mean reverting the trading strategy could not be profitable.

Trading rules have to be defined to set the entry and exit thresholds and, for this reason, the Bollinger bands system is employed. At first, the pair spread has to be standardized in order to normalize the trading signals; mean (u_δ) and standard deviation (σ_δ) of the spread are computed within the training set, and the Zscore is subsequently defined by:

$$\mathrm{Z_t} = \frac{\delta_\mathrm{t} - \mathrm{u}_\delta}{\sigma_\delta}$$

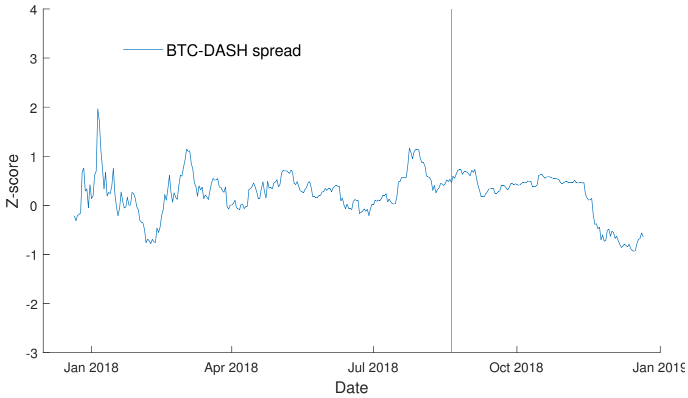
Figure 3.12 plots the Zscore for the pair BTC/DASH, **Figure 3.13, 3.14** for the other pairs ; a red vertical line divides the graph in two regions: the left-hand side represents the training set, where all computations and optimization of parameters have been executed, while the right-hand side the test set; it is quite evident as in the test set the precision of the model is less accurate, but it preserves the stationarity property. The mean reverting spread is even more visible if we consider a smaller sample of data for the training set, starting from January 2018 (240 days); it is represented in **Figure 3.15**; however, optimization of parameters has been conducted on the larger sample of data in order to have enough amount of data to validate the model; in fact for daily trading models there should be at least three years of data with daily prices, but since cryptocurrency relations proved to move much faster than in traditional markets and volumes of the previous years have been very low, the period of 1 year has been selected for parameter optimization.
Positions on the spread are opened whenever the Zscore diverges more than n historical standard deviations from the mean; more precisely, entry and exit signals are set as the follows: entry signals are generated whenever the Zscore falls below a pre-determined threshold, for example 1 standard deviation, $\mathrm{Z_t} < -1$ (long signal), or when it goes above it, $\mathrm{Z_t} > 1$ (short signal).

Figure 3.12 Z-score BTC-DASH



Notes: The red line divide the graph into two regions: the left part represents the Z-score over the training set, while the right-hand side over the test set

Figure 3.15 Z-score BTC-DASH



Notes: Z-score of the pair BTC-DASH, estimated using a small sample of data

Exit signals can be generated as the opposite of entry signals, which means that positions are closed when the Zscore moves beyond the opposite band, or an additional threshold can be set, for example when the Zscore reverts to the mean (exit signal = 0). We can formally define them:

- entry signal = n_1 and exit signal = n_2, with $n_1 > n_2$
- long entry signal = Zscore $< -n_1$
- long exit signal = Zscore $\geq -n_2$
- short entry signal= Zscore $> n_1$
- short exit signal = Zscore $\leq n_2$

Values n_1 and n_2 are free parameters that can be optimized in the training set; empirical evidences have demonstrated that the choice of smaller thresholds generates shorter holding periods and more round trip trades, with higher profits (**Chan (2013)**); however, in markets that present significant transaction costs, as the cryptocurrency market, where trading fees may vary between $0.10 - 0.30\%$, the choice of having fewer trades and longer holding periods might be the optimal one.
Returns are computed through a mark-to market system (MTM), that entails the division of the Profit and Loss (P&L), generated by the pair trading strategy, over the Gross market value of the portfolio.
For example, consider $P_t^A = \$76.30$ $P_t^B = \$27.12$ be the dollar price of two cointegrated cryptocurrencies at time t, while $\beta = 2.13$ is the Hedge ratio and the spread is obtained as

$$\delta_t = 76.30 - (2.13 * 27.12) = 76.30 - 57.77 = 18.53$$

; moreover, assume that the the Zscore has fallen a given threshold, as 1 standard deviation, and it is expected to mean revert at some point in the future; hence the strategy entails the opening of a long position on the undervalued cryptocurrency A ($-\$76.30$), and a short position of 2.13 units on the overvalued one B ($+57.51$) ; at time t, the P&L amounts to $\$-18.53$. At time t+1, prices of the two cryptocurrencies are changed: $P_{t+1}^A = \$84.57$ and $P_{t+1}^B = \$23.61$, with a spread $\delta_{t+1} = 34.28$. The Zscore has retraced to its mean, or a given threshold, hence the positions are closed: $+\$84.57$ from the sale of the undervalued cryptocurrency and $-\$50.29$ from the repurchase of the overvalued one, with a final P&L equal to $\$34.28 + (-\$18.53) = \$15.75$; the same result can simply be obtained subtracting the two values of the spread. The final return of the strategy is obtained dividing the P&L with the total traded notional, of Gross market value of portfolio, $M_p = \$76.30 + \$27.12 = \$103.42$:

$$r_{t+1} = \$15.75/\$103.42 = 15.23\%$$

3.2.6 Automated Trading System: In-sample vs Out-of sample results

An automated trading system is activated in order to capture the deviations of the spread for the cointegrated pairs, choosing as backtest platform the scripting language of MATLAB; however, this algorithm strategy could be implemented, for instance, using Bitfinex API, that allow users to recreate all the features of the platform and insert their algorithms to execute trades; positions are automatically opened and closed in the correspondence of the signals, while P&L are computed at a daily frequency. Existing positions of the previous day are carried forward whenever the following day's positions are indeterminate. In the end, daily returns are computed according to the procedure explained in the previous paragraph, and subsequently compounded. In the first instance, transaction costs are not considered; the corresponding trading strategies are shown to be protable under different entry/exit thresholds.

Figure 3.16 illustrates the positions in the pair BTC/DASH as the Zscore evolves over the time in the training set, and the profit and loss generated by the strategy with two different trading rules: the first, z_1, opens positions when the Zscore is above or below 1 standard deviation and close them when it reverts to the mean ($z_1 : n_1 = 1, n_2 = 0$), while the second, z_2, closes them when the Zscore moves beyond the opposite band ($z_2 : n_1 = 1, n_2 = -1$). Hence, a position of 1 in the pair indicates the purchase of 1 unit of the spread, in other words, a long position of 1 unit of BTC (+1), and a short position of 10.79 units of DASH (-10.79); converse situation for a position of -1. **Figure 3.17** displays the results for the test set (out of sample). **Figure 3.18** plots the cumulative compounded gross returns of the pair BTC/DASH generated in the training and test periods. The strategy performs better with the first rule either in the training set or in the test set; in fact, it presents a lower maximum drawdown and a higher Sharpe ratio. Out-of the sample results are consistent with the model. **Table 3.13** summarizes some of the performance metrics.

Return and risk measures of the strategy do not vary considerably with the inclusion of transaction costs, represented by the bid-ask spread, trading fees and margin funding rates (borrowing costs to short sell the cryptocurrencies). All the data that regard these costs have been gathered from Bitcoinity, Bitfinex, Polobot and Cryptolend websites, but historical data of the borrowing rates revealed to be difficult to be obtained; hence, annual average values of these rates have been used for the analysis (**Table 3.14** in the Appendix B). Gross profits of the strategy are preserved with the inclusion of all transaction costs, as displayed in **figure 3.19**.

Figure 3.16 In sample positions and P&L, BTC-DASH

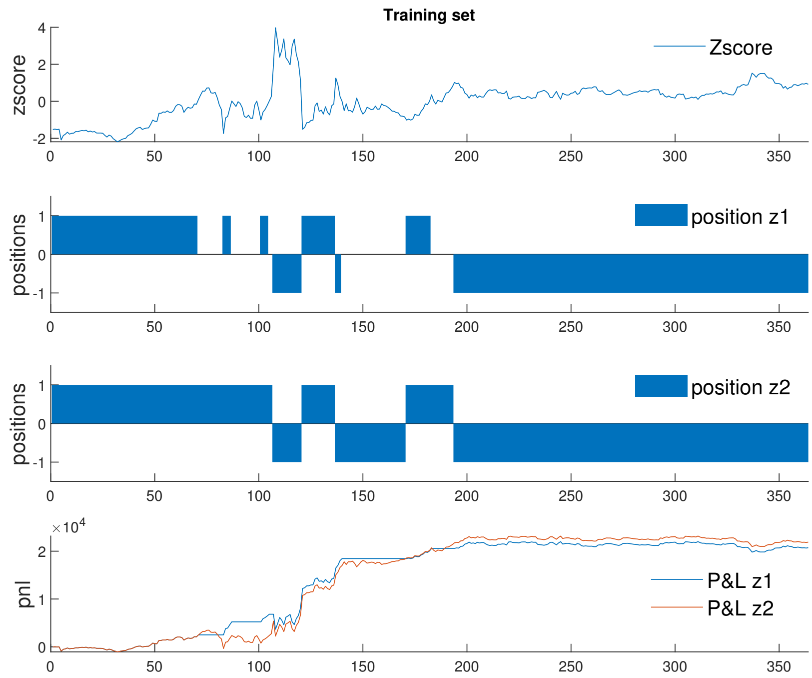


Figure 3.17 Out of sample positions and P&L, BTC-DASH pair

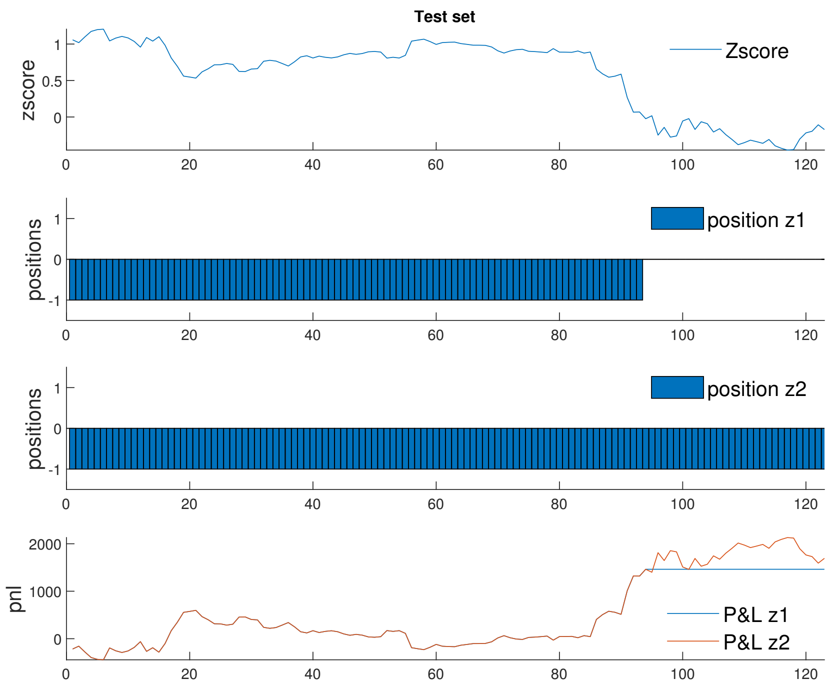
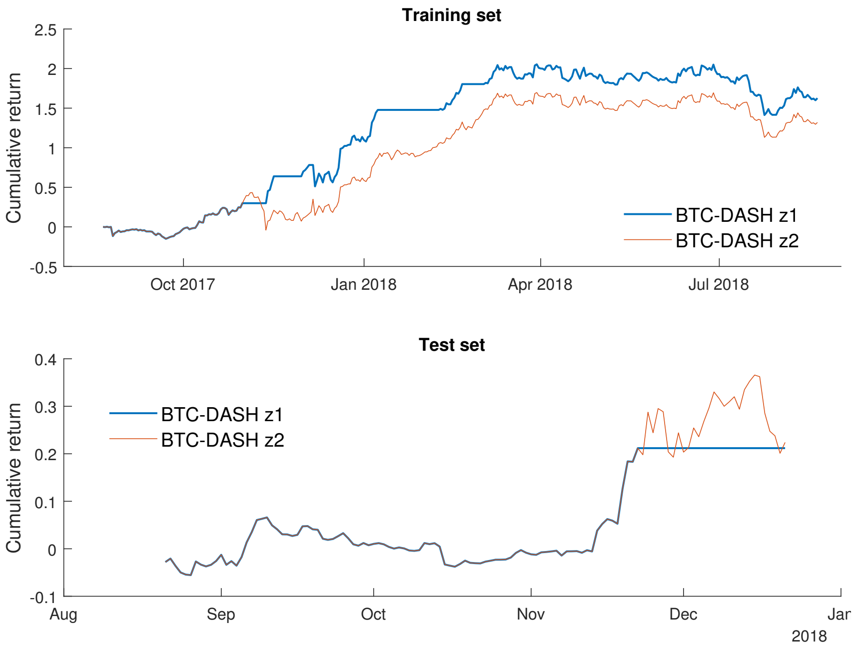


Figure 3.18 In sample and out of sample cumulative compounded returns, BTC/DASH



Notes: The graph plots the cumulative compounded return of the strategy for the training and test periods with different thresholds

Table 3.13: In sample and out of sample performance measures, BTC/DASH

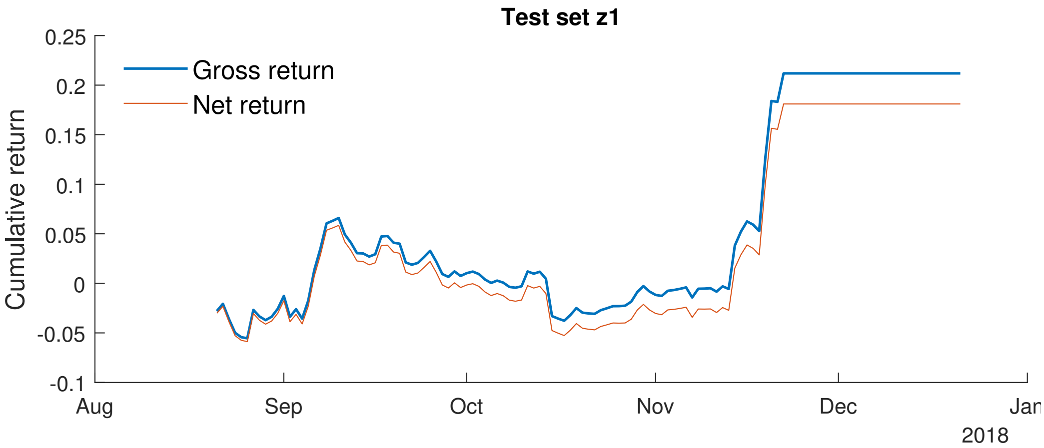
	Training set		**Test set**	
Threshold level	z_1	z_2	z_1	z_2
Gross profit %	162.59	131.92	21.19	22.40
Max.Drawdown %	-24.16	-26.42	-6.69	-13.05
Sharpe Ratio	2.45	1.85	1.44	1.05

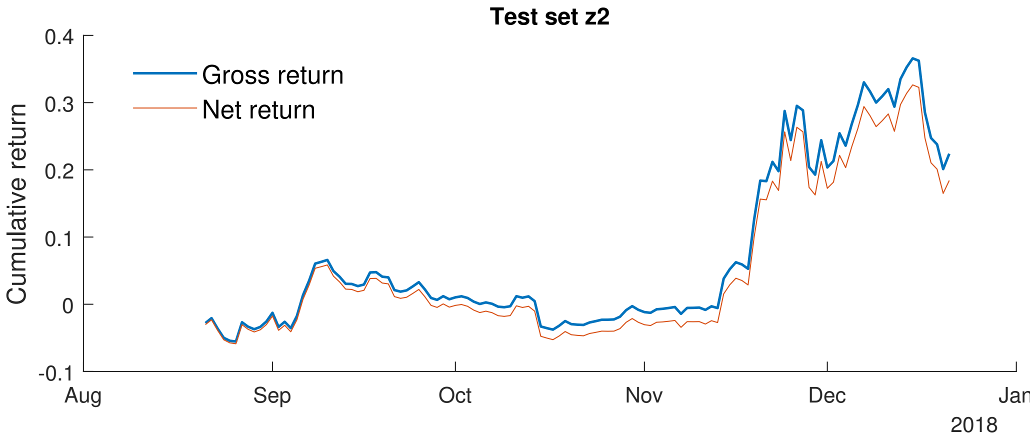
Notes: Performance of the automated trading system with Bollinger bands applied to the BTC-DASH pair

---

Table 3.15 reports performance metrics of the automated strategy with the inclusion of transaction costs, under the two trading rules. Graphs and tables of the other pairs are included in the Appendix, as a comparison of the profitability of the three cointegrated pairs. BTC/DASH pair presents the best risk-adjusted performance measures, even with the inclusion of transaction costs. (**Figures 3.20**, **3.21**, **3.22**, **3.23**, **3.24**, **3.25**, **3.26**, **3.27**, **3.28**, **3.29**; **Tables 3.16**, **3.17**).

Figure 3.19 Out of sample gross vs net returns of the strategy, BTC/DASH





Notes: The two graph plot a comparison of gross and net cumulative returns generated by the strategy in the test set under the two different rules z_1 and z_2

Table 3.15: Out of sample performance measures of the strategy includ. transaction costs, BTC/DASH

Threshold level	z_1		z_2	
	Gross	**Net***	**Gross**	**Net***
Cum. return%	21.19	18.10	22.40	18.42
Max.Drawdown %	-6.69	-6.69	-13.05	-13.06
Sharpe Ratio	1.44	1.25	1.05	0.89

Notes: * indicates the inclusion of Bid-ask spread, trading fees and margin funding rates (short selling costs) in the computation

An implementation of the strategy could be executed by a "scale in" process, in order to capture even the slight deviations of the spread; in fact, the scaling, or averaging, of positions lets the trader capture potential profits otherwise lost, as the spread deviates further and further from its mean and then reverse to it; hence, capital could be added to maximize the profit potential (scaling-in); on the other hand, the strategy also could scales out gradually. To implement scaling-in with the Bollinger bands system, multiple entry and exit thresholds have to be defined. As expressed in the previous section, empirical evidence demonstrates that the definition of thresholds with smaller magnitude determines shorter holding periods and more round trip trades, generally with higher profits in absence of transaction costs (**Chan (2013)**). However, in the presence of transaction costs, as in this context, the higher number of trades would consequently erode profits, due to payment of consistent trading fees; for this reason, a scale-in process is not activated.

3.2.7 Discussions

Cryptocurrencies prices of the arbitrarily selected pairs proved to be cointegrated processes in the training set, except for the pair BTC/ETH, whose residuals, obtained by the OLS model, barely passed the ADF test. Hence, a pair trading strategy that involved cointegrated pairs has been constructed using the Bollinger bands method as trading rules; an automated trading system has been defined to capture the deviations of the formed spread; positions have been automatically activated once thresholds parameters were satisfied. The system exhibited large gross profits both in the training and test sets and maintained the profitability even taking into account the transaction costs represented by the Bid-ask spread, trading fees and Margin funding rates. These empirical findings confirm the hypothesis that the Cryptocurrency market is not Efficient, as it allows profitable arbitrage trading strategies. However, due to the magnitude of transaction costs (short selling costs are extremely high for certain cryptocurrencies), the construction of a mean reverting portfolio, comprised of several cryptocurrencies, has not been performed. Further analysis should consider the selection of other cryptocurrencies to form the pairs and use hourly or minutely historical data to evaluate high-frequency trading strategies; an algorithmic system could be used to optimize the selection in the formation period. Secondly, for not truly cointegrated pairs but that display mean reversion, the computation of dynamic hedge ratio should be more opportune to adapt to the changing levels of the pair components. To conclude, the use of Kalman filter algorithm could be deployed to compute dynamic hedge ratios of the pairs.

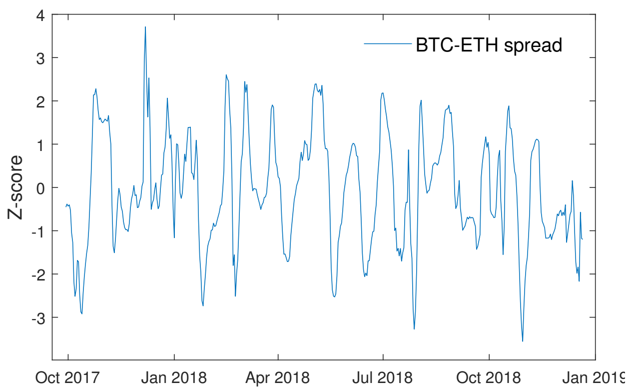
3.2.8 Dynamic hedge ratio: BTC/ETH pair

This section develops a pair trading strategy that considers the use of a dynamic hedge ratio to define the spread between the pair components. In fact, residuals of the BTC/ETH pair, obtained with OLS method, were the only one not to provide enough statistical certainty of being a stationary process; hence, the construction of a pair strategy that make use of the cointegration approach would have not provided the same level of profitability obtained by other pairs and, thus, has not been performed. However, the absence of a cointegration relationship between the components of a pair does not exclude the possibility that it could possess traits of mean reversion in the short term. In this sense, a pairs arbitrage strategy can be constructed taking into consideration the dynamic nature of the hedge ratio.

A rolling linear regression is then utilized to capture the changes over time in the hedge ratio; a lookback window is set, and a regression is performed on the observations contained in the window; afterwards, the window is moved forward in time of one observation and regression repeated, incorporating one new observation every time. As a final result, the process will generate a vector containing the hedge ratio, estimated with multiple regressions. Short size length of the lookback windows is found to provide better results; the use of a short window determines the execution of parameters estimations on a smaller sample of data; on the contrary, a longer window enhances the possibility for the data-generating process to change over the time period covered by the window with the result that oldest data does not represent any longer the current behavior of the model. This free parameter could be subject to an optimization process, for instance, through cross-validation technique. Sensitivity analysis is then performed by varying the lookback period over a range of values: results indicate that the use of a lookback window of 20 days in the training set provide the highest gross profits (**Figure 3.30** in the Appendix). Hence, the window length is set at 20 days.

The Zscore of the pair is computed considering the moving average and standard deviation of the spread; it is represented in **Figure 3.31** with the lookback window of 20 days. The strategy is back-tested using the same trading rules applied to the other pairs: z_1 and z_2. The main difference with the previous approach relies on the absence of parameter estimations exclusively in the training period; here, parameters are continuously estimated; however the subdivision of sample data in training and test sets is maintained and returns computed in both sets; for instance, the training set has been useful to find the optimal lookback period, subsequently implemented in the test set.

Figure 3.31 Z-score of BTC-ETH pair (lookback window of 20 days)



Figures 3.32, 3.33 illustrate the positions in the pair as the Zscore evolves over the time in the training and test sets, and the profit and loss generated by the strategy under the two different trading rules.

Figure 3.32 In sample positions and P&L generated by the strategy

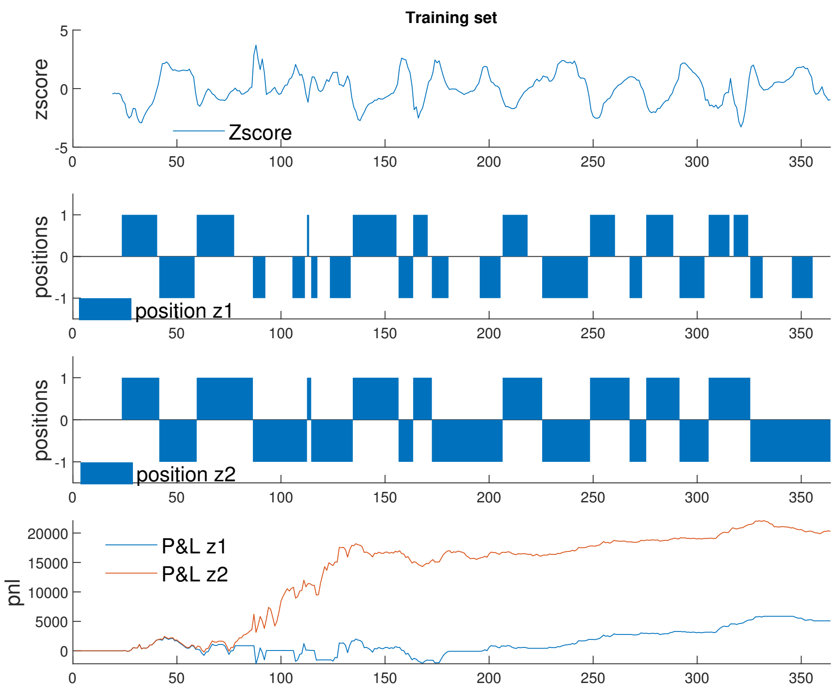


Figure 3.33 Out of sample positions and P&L generated by the strategy

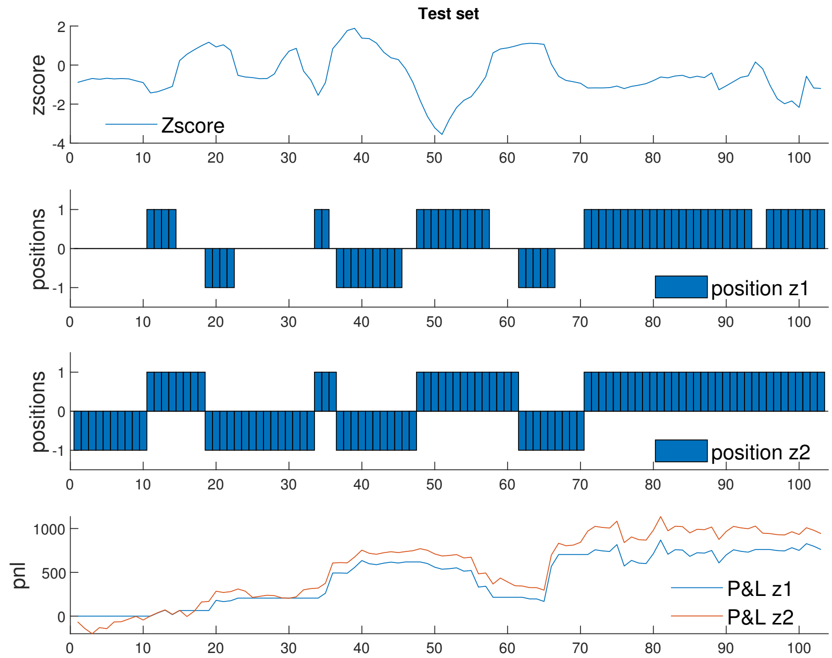
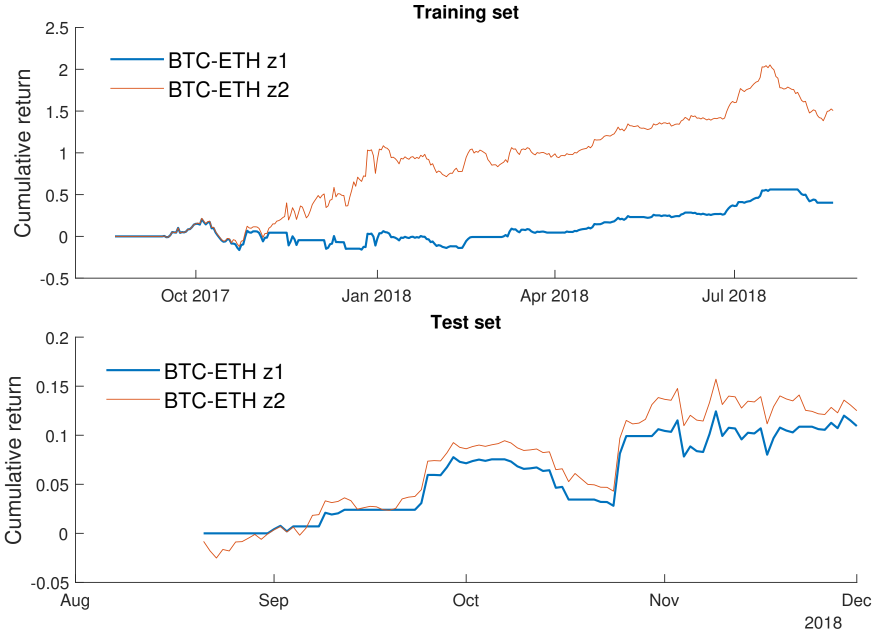


Figure 3.34 plots the cumulative compounded gross returns of the pair in the two sets, transaction costs excluded. The strategy performs better according to trading rule z_2 both in the training and test sets; a sensitivity analysis that considers different thresholds to form trading rules could be executed to find the best range of thresholds under which perform trades. **Table 3.18** contains performance metrics of the strategy in both sets, while **Figure 3.35** and **Table 3.19** include transaction costs in the out-of-sample analysis. Trading fees represent transaction costs with the heaviest weight and completely annihilate gross returns, due to the higher number of performed trades; in fact, aside from opening and closing positions when Zscore has reached certain threshold, a constant rebalancing of portfolio is needed to adapt to the dynamic hedge ratio; hence, positions in ETH are daily rebalanced to best match the synthetic spread formed with the dynamic hedge ratio, with the result of payment of higher trading costs. The reduction or avoidance of trading fees, for instance performing the strategy with large volumes[15], could enhance profitability, aligning net returns to gross ones. Moreover, the short selling costs of Ethereum proved to be much greater in comparison to Bitcoin; this aspect explains the divergence in performance between the first and second trading rule once margin funding rates have been accounted (with the second rule, the time over which ETH is short sold is longer, with the effect of higher margin funding costs).

[15] Maker fees on Bitfinex are erased for monthly traded volumes above $7.5 million

Figure 3.34 In sample and out of sample cumulative compounded gross returns



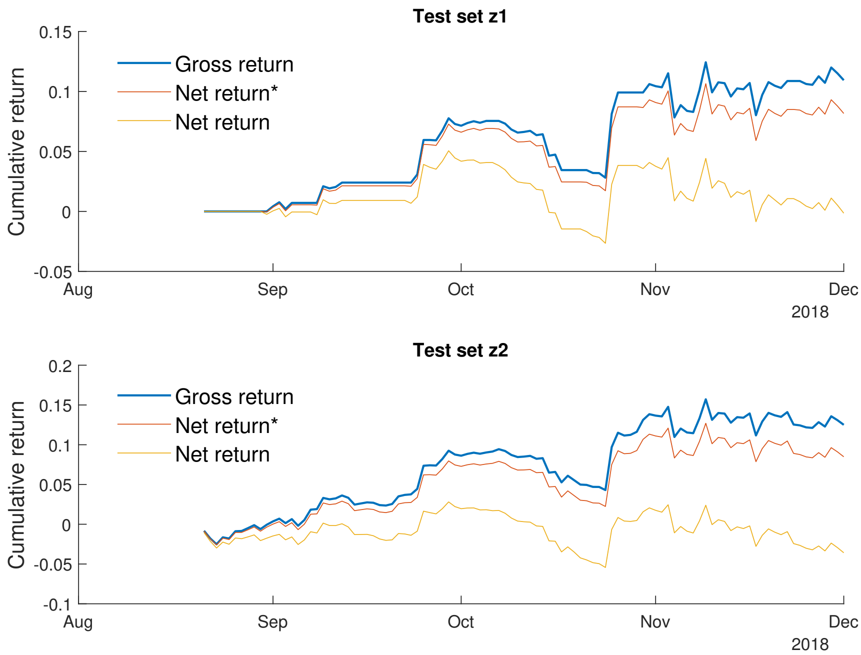
Notes: The graph plots the cumulative compounded gross returns of the strategy under the two trading rules; upper part training set, at the bottom test set (lookback window of 20 days)

Table 3.18: In sample and Out of sample performance measures, BTC-ETH

	Training set		**Test set**	
Threshold level	z_1	z_2	z_1	z_2
Gross profit %	40.32	150.60	10.94	12.50
Max.Drawdown %	-24.52	-24.52	-8.07	-8.07
Sharpe Ratio	0.93	1.93	1.16	1.22

Notes: In sample and Out of sample performance of the automated trading system with Bollinger bands applied to the BTC-ETH pair (lookback window of 20 days)

Figure 3.35 Out of sample gross vs net returns, BTC-ETH



Notes: The graph plots the cumulated compounded gross returns of the strategy under the two trading rules (lookback window of 20 days)

Table 3.19: Out of sample performance measures includ. transaction costs, BTC/ETH

Threshold level	z_1			z_2		
	Gross	**Net***	**Net**	**Gross**	**Net***	**Net**
Cum. return%	10.94	8.17	0.15	12.50	8.50	-3.59
Max.Drawdown %	-8.07	-8.08	-8.09	-8.07	-8.08	-8.09
Sharpe Ratio	1.16	0.88	0.03	1.22	0.86	-0.31

Notes: * indicates the exclusion of trading fees from the computation

Further analysis should consider the opportunity to add stop loss orders to the system to mitigate effects of large short term price swings, thus reducing maximum drawdown in the pair, a concept valid for the cointegrated pairs too (BTC/DASH, BTC/LTC, BTC/XMR); however, a drawback could be represented by the loss of potential profits, not captured anymore with the activation of a stop loss order.

3.2.9 Matlab Code

```
%% PAIRS TRADING: COINTEGRATION METHOD WITH ENGLE-GRANGER TESTING

%Load file with pricing data (obtained via API)

data = readtable('Pricbitf20172018.xlsx');
S=size(data,1)
dates=data(1:S,1);
dates=dates{:,1};
assetNames = data.Properties.VariableNames(2:end);
assetPrice = data(:,assetNames).Variables;
BTC=assetPrice(1:S,1);
ETH= assetPrice(1:S,2);
DASH=assetPrice(1:S,3);
LTC=assetPrice(1:S,4);
MONERO=assetPrice(1:S,5);
-------------------------------------------------------------------------
%% DIVIDE SAMPLE DATA IN TRAINING AND TEST SETS
trainset=1:365;
testset=trainset(end)+1:S;
-------------------------------------------------------------------------
%% UNIT ROOT TESTS: ADF, KPSS
% ADF: h=0, failure to reject the null hyp of STATIONARITY
% KPSS= h=1 rejects null hyp of STATIONARITY

%BTC
[h1,pVal1] = adftest(BTC(trainset),'model','ARD')
[h1D,pVal1D] = adftest(diff(BTC(trainset)),'model','ARD')

%ETH
[h12,pVal2] = adftest(ETH(trainset),'model','ARD')
[h1D2,pVal1D2] = adftest(diff(ETH(trainset)),'model','ARD')

%DASH
[h13,pVal3] = adftest(DASH(trainset),'model','ARD')
[h1D3,pVal1D3] = adftest(diff(DASH(trainset)),'model','ARD')

%LTC
[h14,pVal4] = adftest(LTC(trainset),'model','ARD')
[h1D4,pVal1D4] = adftest(diff(LTC(trainset)),'model','ARD')

%XMR
[h15,pVal5] = adftest(MONERO(trainset),'model','ARD')
[h1D5,pVal1D5] = adftest(diff(MONERO(trainset)),'model','ARD')
```

```

% KPSS test for stationarity
[h_kpss,pValue_kpss] = kpsstest(BTC(trainset))
[h_kpssd,pValue_kpssd] = kpsstest(diff(BTC(trainset)))

[h_kpss1,pValue_kpss1] = kpsstest(ETH(trainset))
[h_kpss1d,pValue_kpss1d] = kpsstest(diff(ETH(trainset)))

[h_kpss2,pValue_kpss2] = kpsstest(DASH(trainset))
[h_kpss2d,pValue_kpss2d] = kpsstest(diff(DASH(trainset)))

[h_kpss3,pValue_kpss3] = kpsstest(LTC(trainset))
[h_kpss3d,pValue_kpss3d] = kpsstest(diff(LTC(trainset)))

[h_kpss4,pValue_kpss4] = kpsstest(MONERO(trainset))
[h_kpss4d,pValue_kpss4d] = kpsstest(diff(MONERO(trainset)))


%% ENGLE GRANGER PROCEDURE: STEP 1

%Choose the pair and perfom Linear regression in the training set

%BTC-DASH

Y=BTC
X=DASH

Y_train=Y(trainset);
T=rows(Y_train);
X0_train=ones(T,1); %intercept
X1_train=X(trainset);
X_train=[X0_train,X1_train];

results=ols(Y_train,X_train);
beta_OLS=results.beta;
hedgeRatio=beta_OLS(2);
errors=results.resid;
Rsquared=results.rsqr


%%  ENGLE GRANGER PROCEDURE: STEP 2

%Perform Unit root tests on the fitted errors.
%Not include constant or deterministic trend when performing test

[h1a,pVal1a] = adftest(errors,'model','AR')

```

```
%% PAIR SPREAD AND ZSCORE

spread=Y-hedgeRatio*X;

% Zscore
spreadMean=mean(spread(trainset)); % In-sample mean of the spread
spreadStd=std(spread(trainset)); % In-sample standard deviation ... of the spread
zScore=(spread - spreadMean)./spreadStd; % z-score of the spread
------------------------------------------------------------------------


%% HALFLINE OF MEAN REVERSION (Credit to Ernie Chan)

%Find value of lambda and thus the halflife of mean reversion by ... linear regression fit

ylag=lag1(spread(trainset));  % lag1 is a function written by ... Ernest Chan.
∆Y=spread(trainset)-ylag;
∆Y(1)=[]; % To remove the NaN in the first observation of the ... time series.
ylag(1)=[];
regress_results=ols(∆Y, [ylag ones(size(ylag))]);
halflife=-log(2)/regress_results.beta(1);

fprintf(1, 'halflife=%f days\n', halflife);

------------------------------------------------------------------------
%% AUTOMATED TRADING SYSTEM
%(Rielaboration of Ernie Chan Code from "Algorithmic Trading: ... Winning Strategies and Their Rationale")

% Trading rules: Bollinger bands, Z1

entryZscore=1;
exitZscore=0;

longsEntry=zScore < -entryZscore;
longsExit=zScore > -exitZscore;

shortsEntry=zScore > entryZscore;
shortsExit=zScore < exitZscore;

numUnitsLong=NaN(length(spread), 1);
numUnitsShort=NaN(length(spread), 1);


```

```
numUnitsLong(1)=0;
numUnitsLong(longsEntry)=1;
numUnitsLong(longsExit)=0;
numUnitsLong=fillMissingData(numUnitsLong); % fillMissingData is ...
    a function created by Chan: it carries forward an existing ...
    position from previous day if today's position is ...
    indeterminate (NaN).

numUnitsShort(1)=0;
numUnitsShort(shortsEntry)=-1;
numUnitsShort(shortsExit)=0;
numUnitsShort=fillMissingData(numUnitsShort);

numUnits=numUnitsLong+numUnitsShort;
cl=[X Y];
positions=repmat(numUnits, [1 size(cl, 2)]).*[-hedgeRatio ...
    ones(size(hedgeRatio))].*cl; % [-hedgeRatio ...
    ones(size(hedgeRatio))] is the units allocation, ...
    [-hedgeRatio ones(size(hedgeRatio))].*cl is the dollar ...
    capital allocation, while positions is the dollar capital in ...
    each coin.

pnl=sum(lag1(positions).*(cl-lag1(cl))./lag1(cl), 2); % daily ...
    P&L of the strategy
ret=pnl./sum(abs(lag1(positions)), 2); % return is P&L divided ...
    by gross market value of portfolio
ret(isnan(ret))=0;
------------------------------------------------------------------------
%PERFORMANCE MEASURES

%Cumulative return
CumRet_trainset=cumprod(1+ret(trainset))-1;
CumRet_trainset=CumRet_trainset(end);
CumRet_testset=cumprod(1+ret(testset))-1;
CumRet_testset=CumRet_testset(end);

%Maximum drawdown
MaxDD_train = maxdrawdown(1+ret(trainset));
MaxDD_test = maxdrawdown(1+ret(testset));

%Sharpe ratio
Sh_train=sqrt(size(trainset,2))*mean(ret(trainset))/std(ret(trainset));
Sh_test=sqrt(size(testset,2))*mean(ret(testset))/std(ret(testset));

%Compare results with different trading rule Z2, and different pairs
%%NOTES: Functions used: rows, ols, lag1, fillMissingData
```

```
%%PAIRS TRADING: DYNAMIC HEDGE RATIO

% First and second sections equal to Cointegration method file

...

%% DYNAMIC HEDGE RATIO (Rielaboration of Chan code)

X=ETH
Y=BTC

lback=20; % Lookback set arbitrarily short
hedgeRatio=NaN(size(X, 1), 1);
for t=lback:size(hedgeRatio, 1)
    regression_result=ols(Y(t-lback+1:t), [X(t-lback+1:t) ...
        ones(lback, 1)]);
    hedgeRatio(t)=regression_result.beta(1);
end

cl=[X Y];
---------------------------------------------------------------------

%% SPREAD AND ZSCORE

spread=sum([-hedgeRatio ones(size(hedgeRatio))].*cl, 2); % ...
    Spread of the pair
hedgeRatio(1:lback)=[]; % Removed because hedge ratio is ...
    indeterminate
spread(1:lback)=[];
cl(1:lback, :)=[];

MA=movingAvg(spread, lback); %Credit to Ernie Chan
MSTD=movingStd(spread, lback);
zScore=(spread-MA)./MSTD
---------------------------------------------------------------------

%%AUTOMATED TRADING SYSTEM

%Same as in the cointegration file: try different trading rules

...

%%NOTES: functions used: fillMissingData, lag1, movingStd, ...
    movingAvg, ols, rows
```

Concluding Remarks

Cryptocurrency market proves not to be efficient as it allows profitable arbitrage and relative value arbitrage strategies, even with the inclusion of transaction costs. Price discrepancies of digital coins persist over a considerable amount of time, in terms of days and weeks. Although several constraints and obstacles may limit the exploitation of these temporary misalgninments, simple arbitrage strategies between exchanges were profitable for all 2017 and even 2018 but with a lower magnitude.

The main obstacles remain the execution time, needed to perform the combined trades, and the international capital restrictions, that may limit the choice of platforms where perform strategies, thus affecting the outcome. The reduction of buying pressure and global trading volumes, consequence of a minor interest of the public and perhaps several opaque and fraudulent events that interested the space, and the advanced strategies performed by speculative funds have probably eroded most of arbitrage opportunities between exchanges, thus upgrading the efficiency level of the market. However, in time of elevate volatility and uncertainty, due to relevant positive or negative news, price discrepancies are likely to re-occur due to the fragmented organization of the cryptocurrency space, characterized by multiple exchanges with sensible different volumes and local demands.

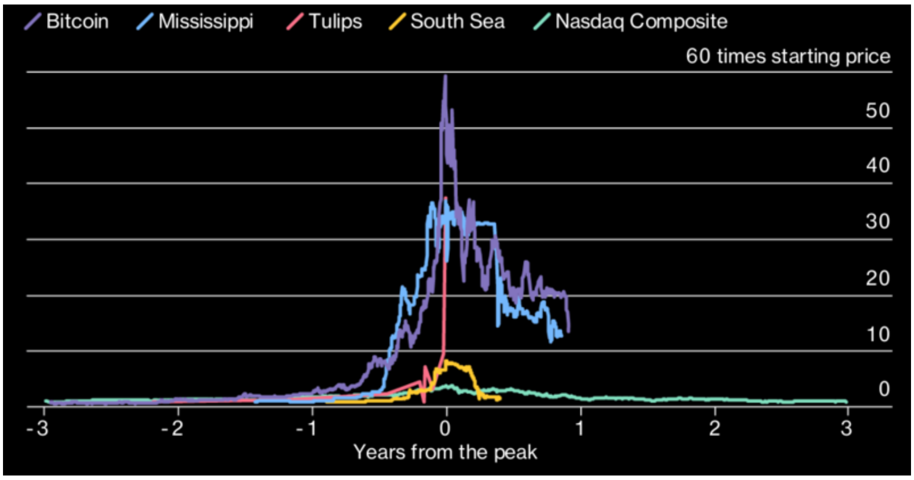
A solution to main obstacles to arbitrage exploitation has been represented by the activation of more sophisticated trading strategies that seek to find price discrepancies inside the same exchange, for example, investigating temporary price misalignments between a pair of assets, as implemented by pairs trading. Cointegration method applied to a group of 5 cryptocurrencies has identified 3 pairs with mean reversion property, and the automated trading system, based on the Bollinger bands rules, exhibited consistent gross and net cumulative returns under different thresholds. For non stationary pairs, the computation of a dynamic hedging ratio has been performed to adapt to the changing levels of the pair components.

Further research on the subject should explore the potential explanations of cointegrating relations in the cryptourrency space, while extending the dataset of possible pairs and adopting more sophisticated frameworks, as the stochastic spread approach ; Kalman filter could be used to correctly infer the exact entry and exit trading signals, or the true weights to assume in each pair component. In conclusion, high-frequency pairs trading could be tested with the use of minutely pricing data, but only in the context of trading fees minimization, as they represent transaction cost with the heaviest weight.

Appendix A

Figures and Tables Chapter 1

Figure 1.2. Comparison of greatest asset bubbles of all times



Source: Bloomberg, International center for finance

Figure 1.4. DLT applications

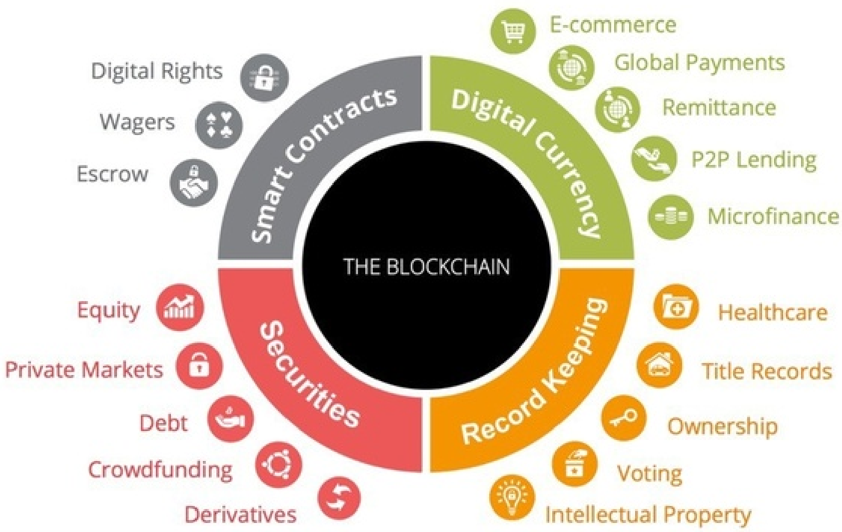
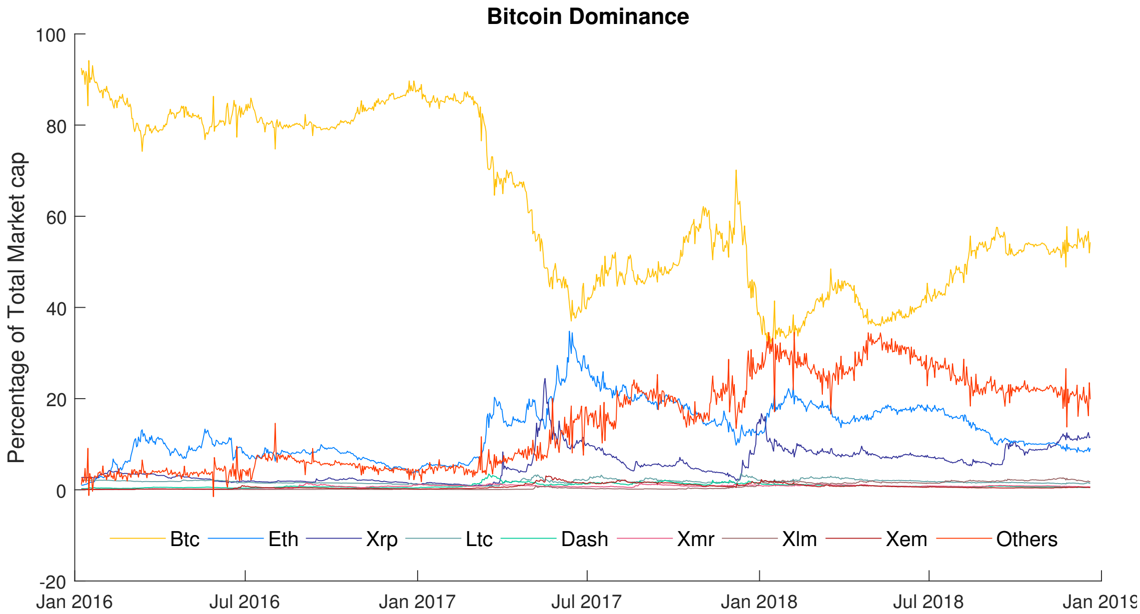
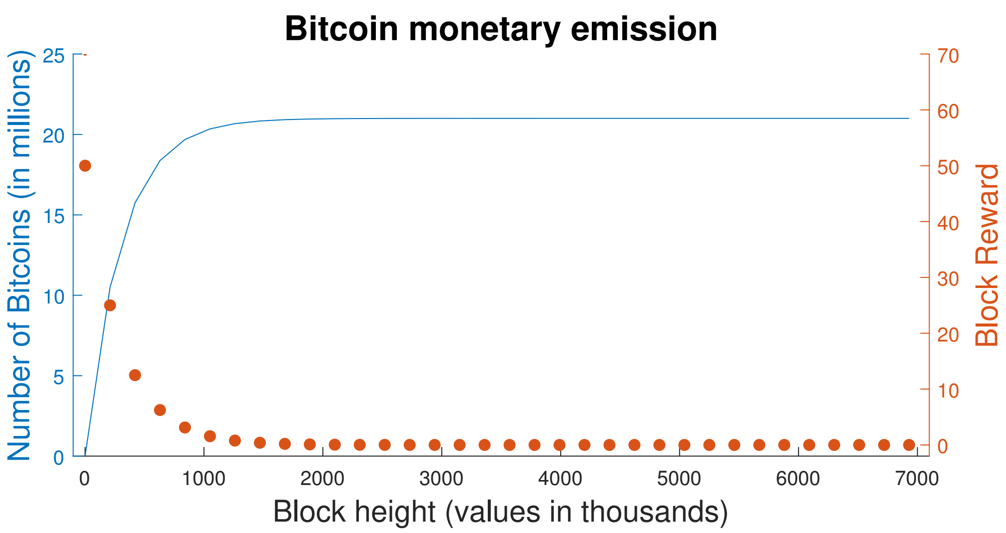


Figure 1.7. Bitcoin Dominance



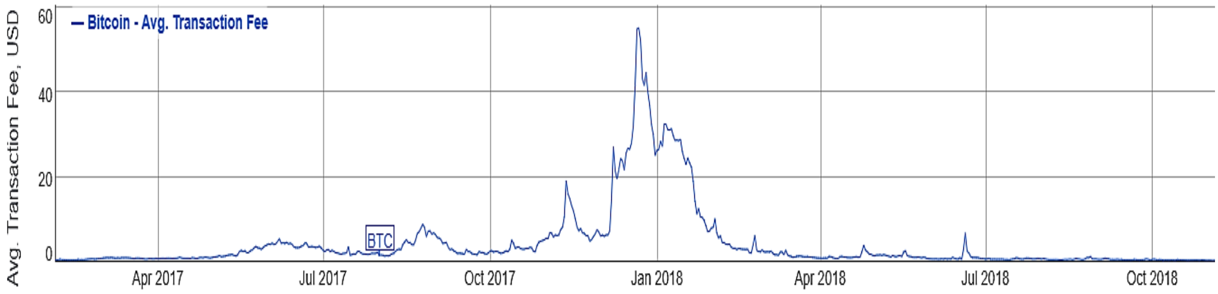
Notes: Bitcoin dominance (%), data extracted from Coinmarketcap, Matlab representation

Figure 1.8. Bitcoin Monetary Emission



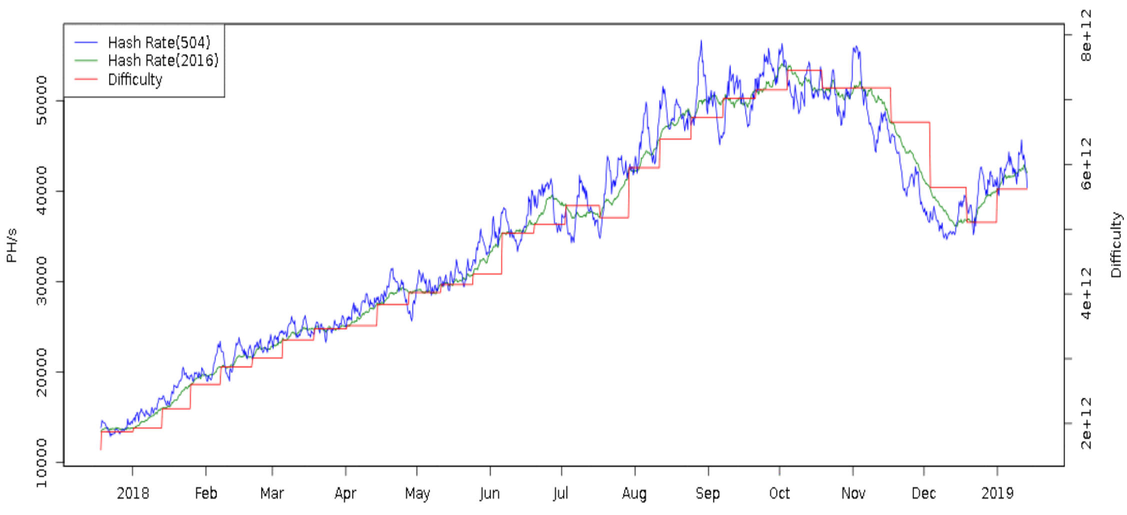
Notes: Bitcoin monetary emission, Author illustration with Matlab software

Figure 1.9. Bitcoin average transaction fees



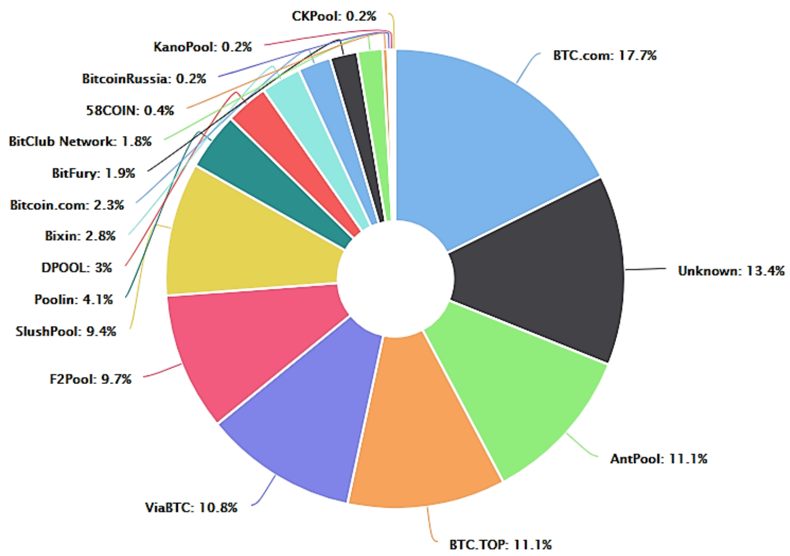
Source: Bitinfocharts.com

Figure 1.10. Bitcoin hash rate and difficulty



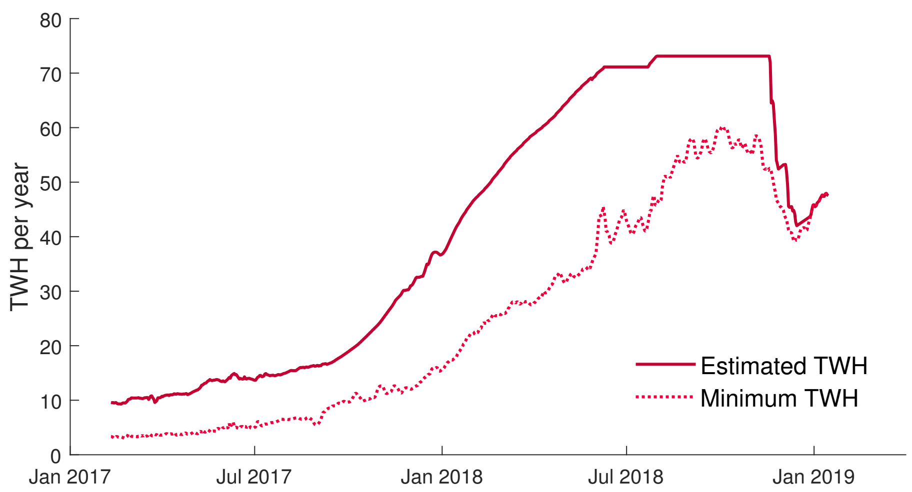
Source: Bitinfocharts

Figure 1.11. Bitcoin hash rate distribution



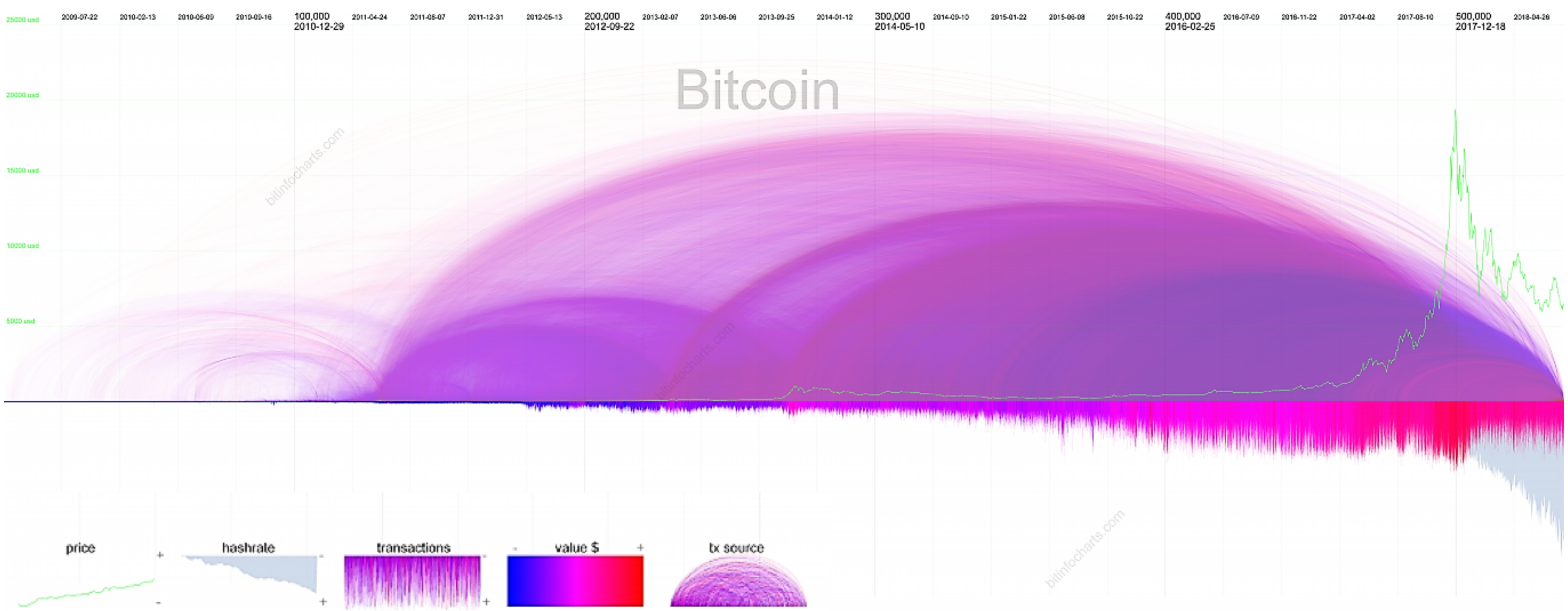
Notes: The pie chart shows Bitcoin hash rate distribution between Mining pools, November 2018, Source: Blockchain.com

Figure 1.12. Bitcoin energy consumption index



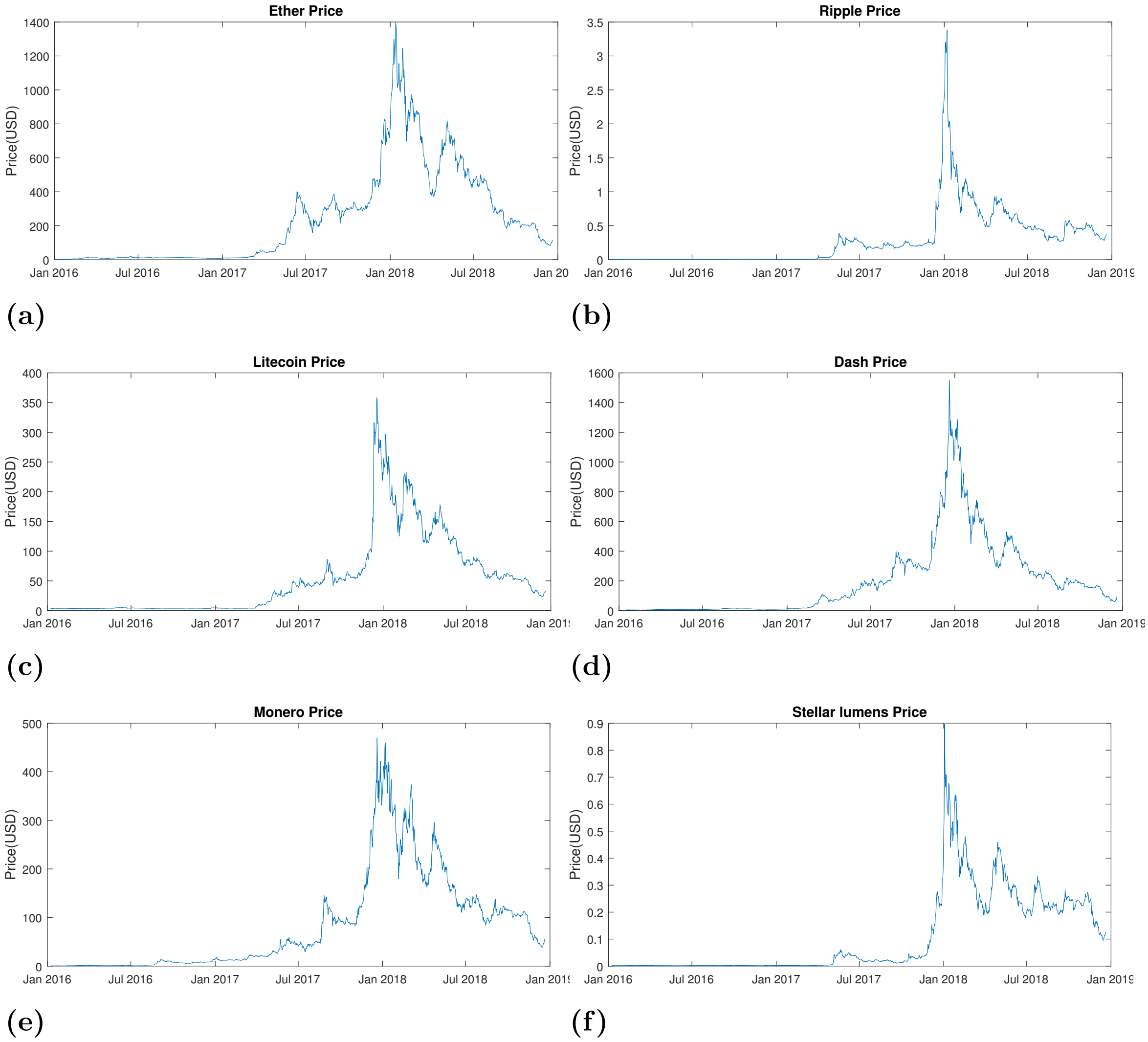
Notes: The graph plots the estimated and minimum amount of energy Bitcoin network needs to execute the mining process. Author illustration with Matlab software; data from digiconomist.net

Figure 1.13 Bitcoin visualization



Source: Bitinfocharts

Figure 1.14. Prices



Notes: Main cryptocurrencies price: (a) Ether, (b) Xrp, (c) Ltc, (d) Dash, (e) Monero, (f) Stellar Lumens. Source: Coinmarketcap

Figure 1.15. average tx fees for main cryptocurrencies

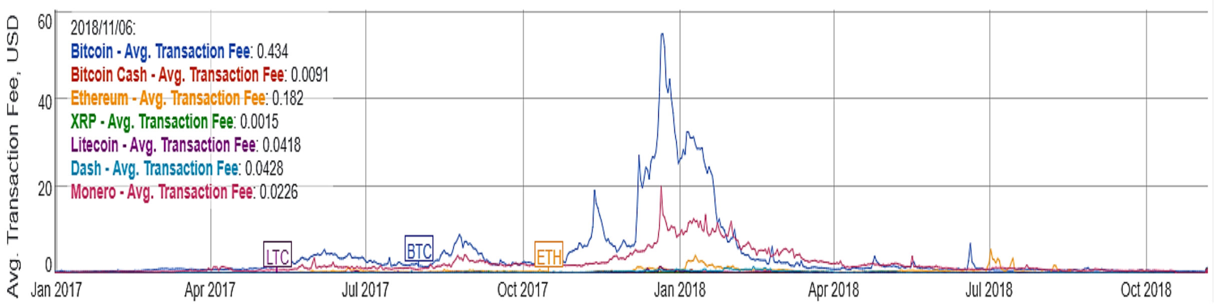


Figure 1.16. average tx fees in the period august-november 2018

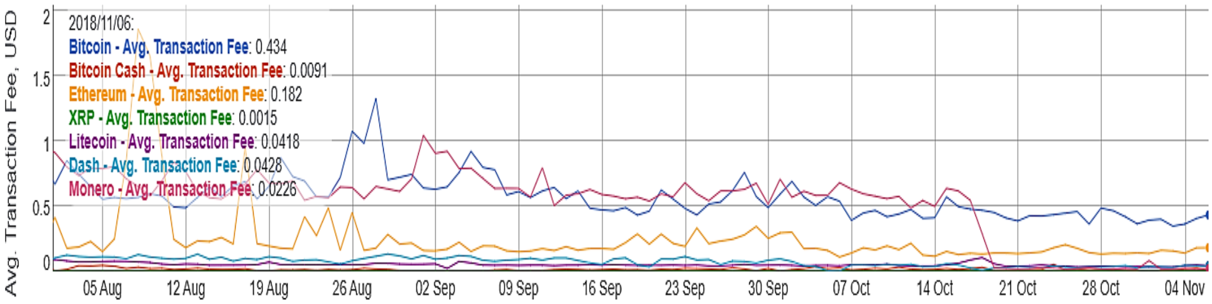
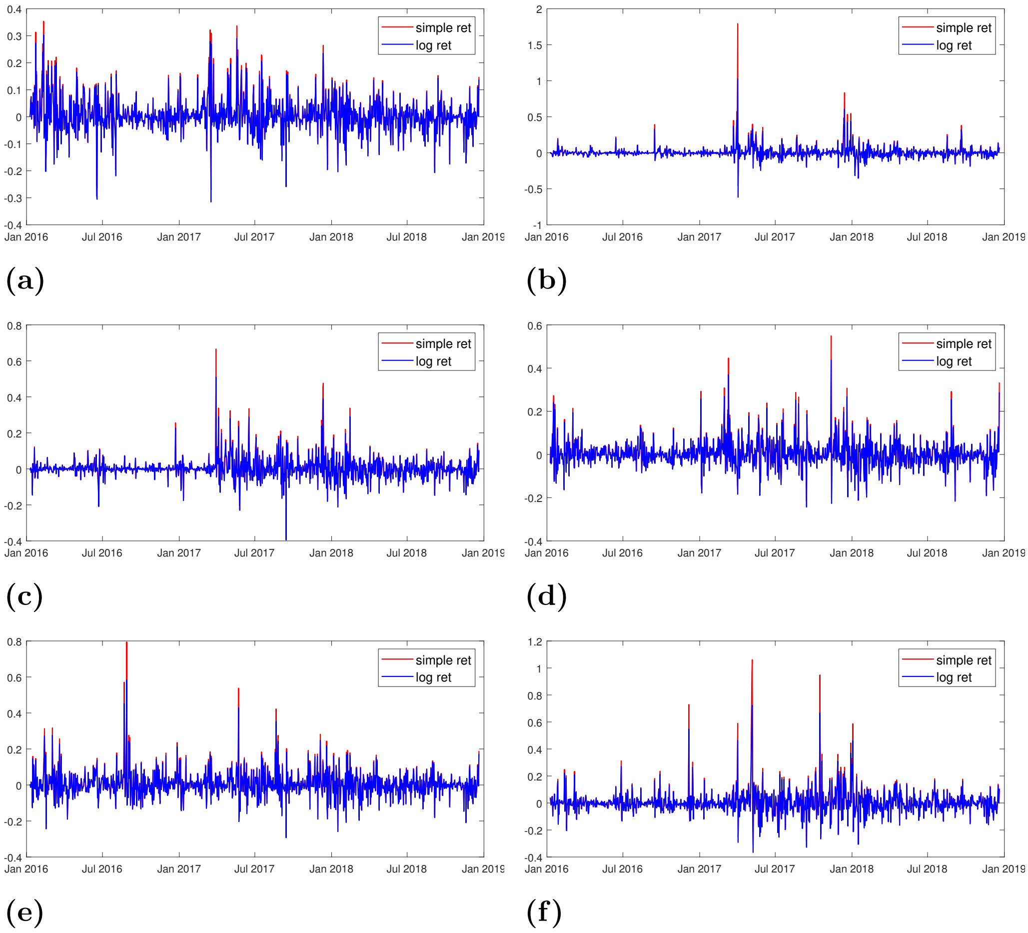
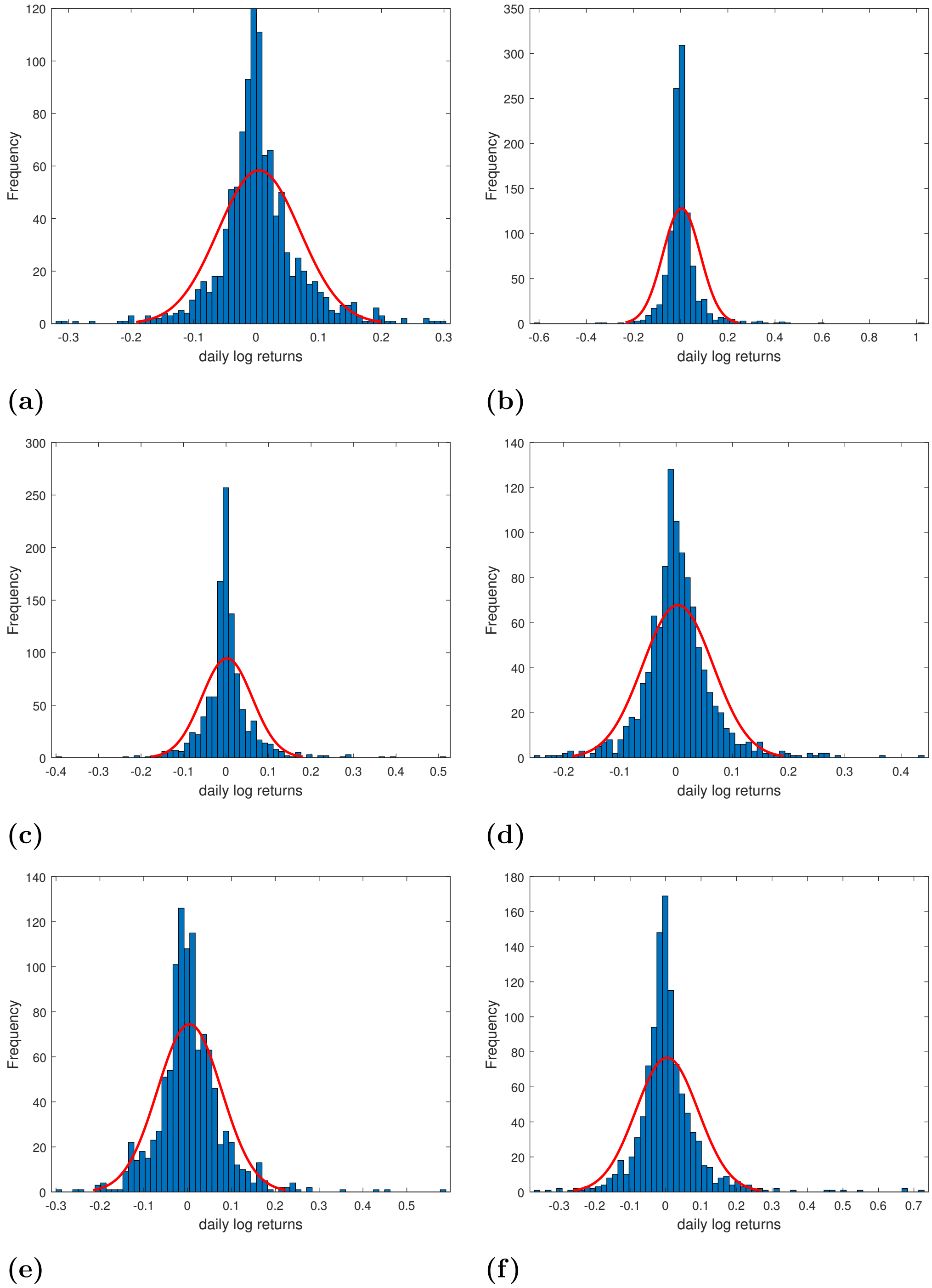


Figure 1.18. Log vs simple returns



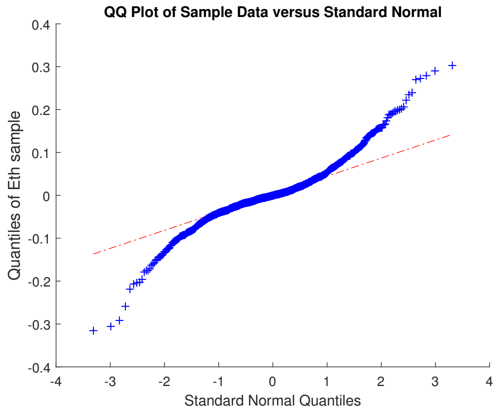
Notes: Log vs Simple daily returns: (a) Ether, (b) Xrp, (c) Ltc, (d) Dash, (e) Monero, (f) Stellar Lumens. Source: Coinmarketcap

Figure 1.21. Frequency distribution Histogram of daily log returns

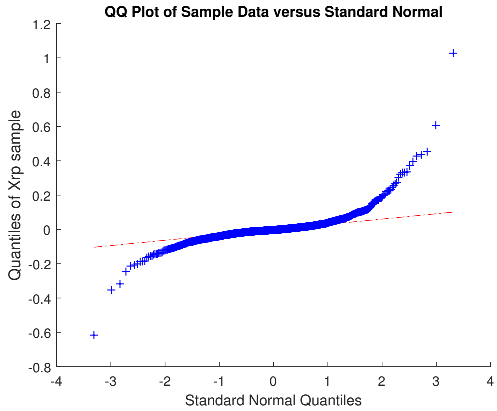


Notes:(a) Ether, (b) Xrp, (c) Ltc, (d) Dash, (e) Monero, (f) Stellar Lumens.

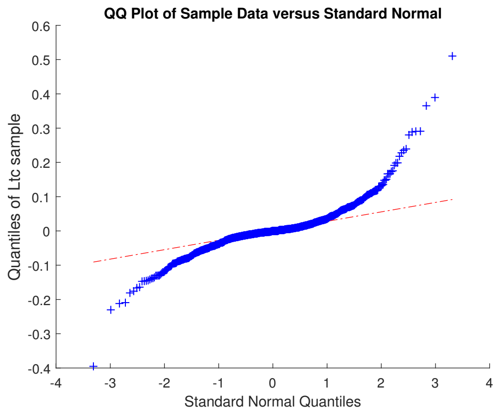
Figure 1.22. Quantile-Quantile plot of daily log returns



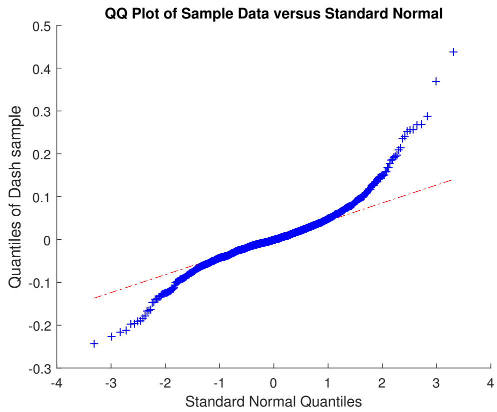
(a)



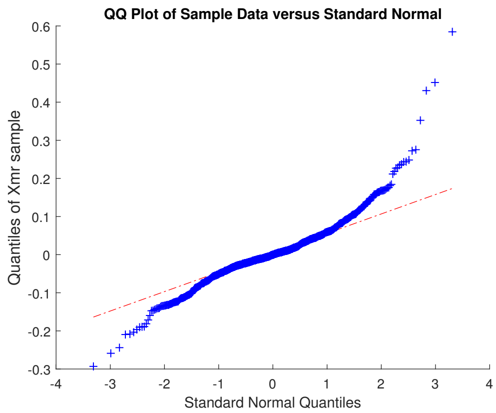
(b)



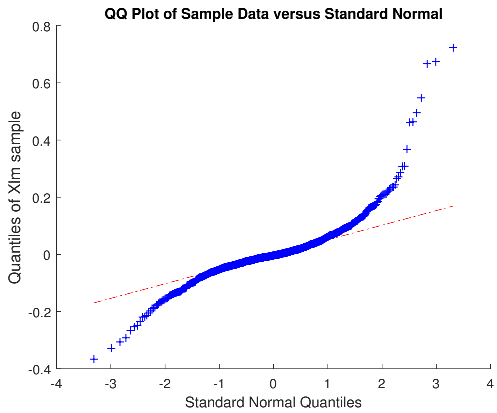
(c)



(d)



(e)



(f)

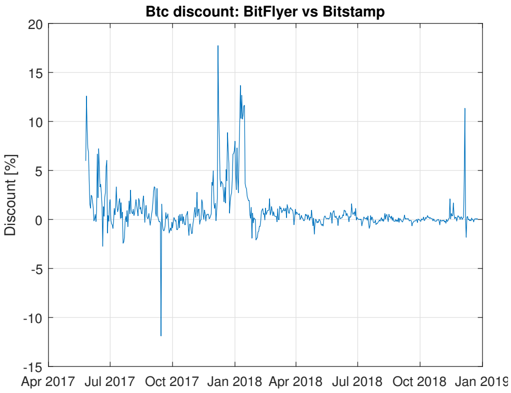
Notes:(a) Ether, (b) Xrp, (c) Ltc, (d) Dash, (e) Monero, (f) Stellar Lumens

Appendix B

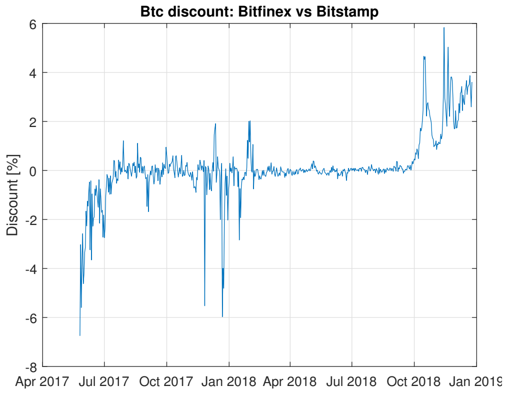
Figures and Tables Chapter 3

B.1 Arbitrage strategies

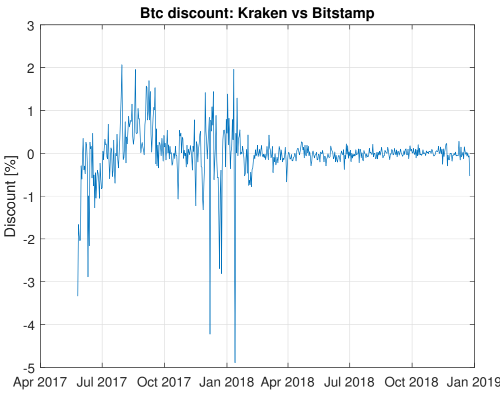
Figure 3.3 Bitcoin discounts



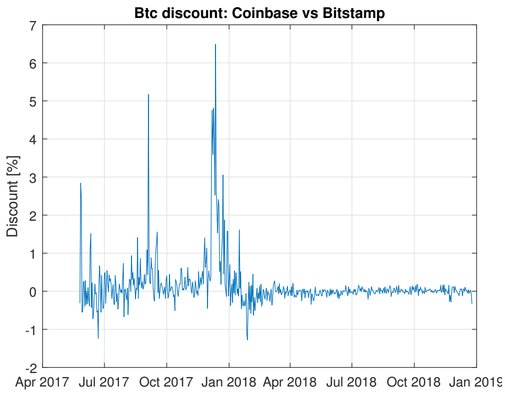
(a)



(b)



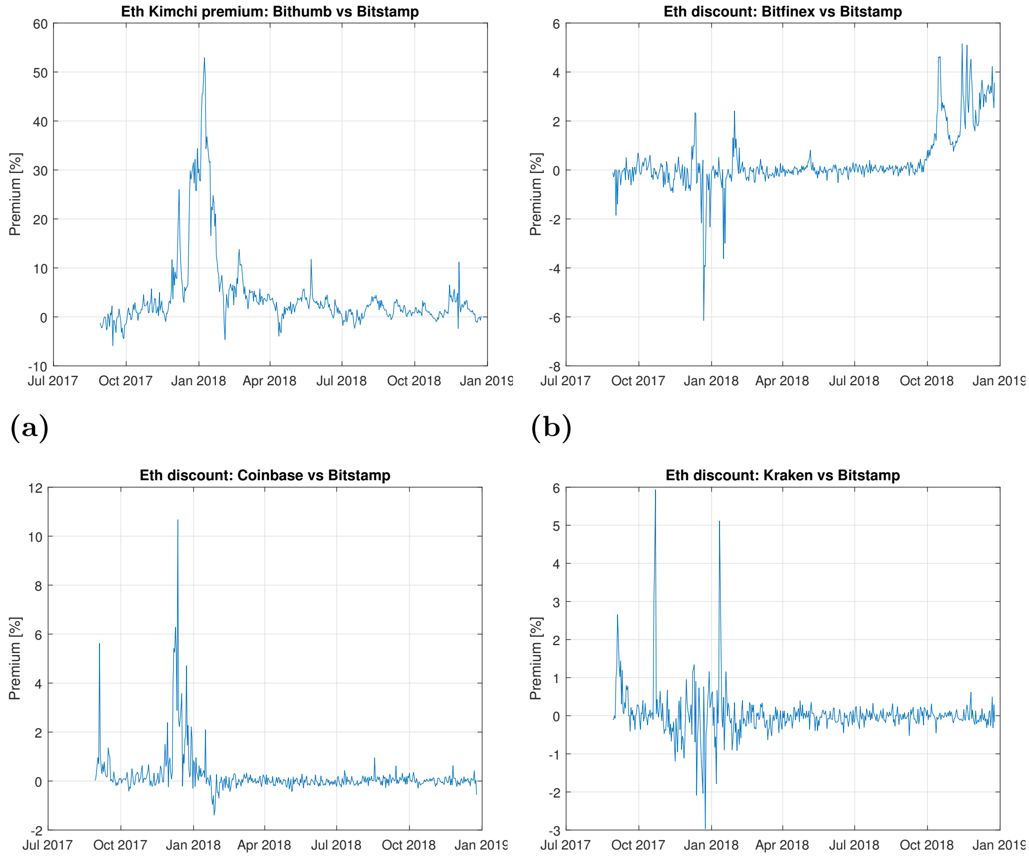
(c)



(d)

Notes: The four graphs plot Bitcoin discounts with respect to Bitstamp exchange: (a) BitFlyier, (b) Bitfinex, (c) Kraken, (d) Coinbase

Figure 3.4 Ethereum discounts



(c) **(d)**

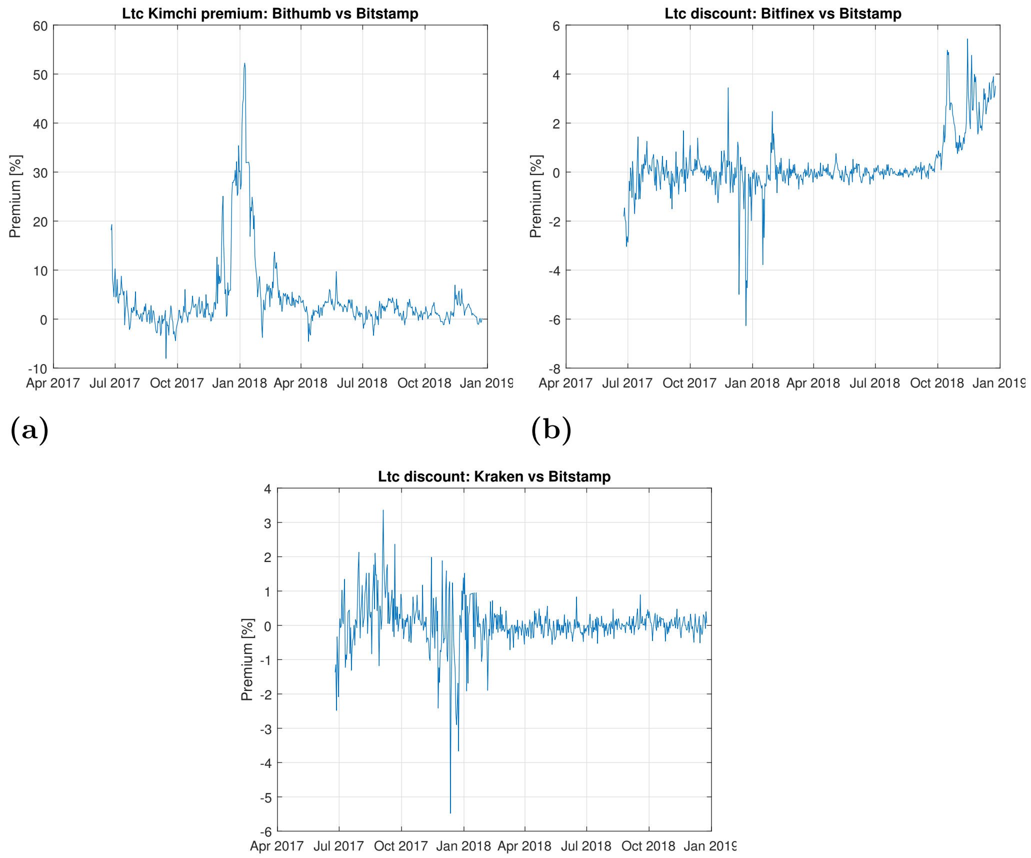
Notes: The four graphs plot Ethereum discounts with respect to Bitstamp exchange: (a) Bithumb, (b) Bitfinex, (c) Coinbase, (d) Kraken

Table 3.2: Summary statistics of ETH discounts

ETH Premium (in %)						
Exchange	**Mean**	**Std.**	**Max.**	**Min.**	**AC(1)**	**obs.**
Bithumb	4.3025	8.1939	52.8957	-5.8970	0.9533	474
Bitfinex	0.3578	1.1998	5.1505	-6.1507	0.8383	481
Coinbase	0.2174	0.9172	10.6721	-1.3904	0.6320	481
Kraken	0.0179	0.6054	5.9340	-2.9696	0.4130	481

Notes: main statistics of Etherum daily discounts in percent over Bitstamp exchange, period 08/2017-12/2018.

Figure 3.5 Litecoin discounts



(c)

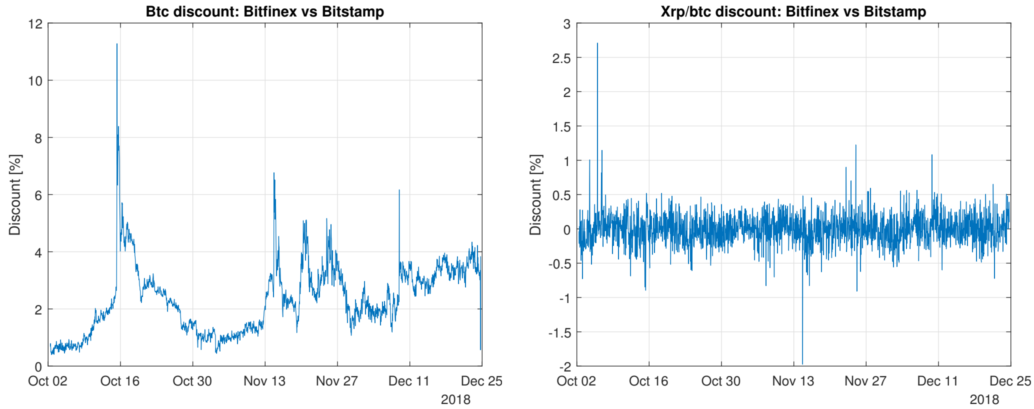
Notes: The graphs plot Litecoin discounts with respect to Bitstamp exchange: (a) Bithumb, (b) Bitfinex, (c) Kraken

Table 3.3: Summary statistics of LTC discounts

LTC Premium (in %)						
Exchange	**Mean**	**Std.**	**Max.**	**Min.**	**AC(1)**	**obs.**
Bithumb	4.0947	7.6621	52.2455	-5.8970	0.9426	542
Bitfinex	0.2118	1.1784	5.4358	-6.2684	0.7775	535
Kraken	0.0189	0.6967	3.3618	-5.4816	0.4149	535

Notes: main statistics of Litecoin daily discounts in percent over Bitstamp exchange, period 06/2017-12/2018.

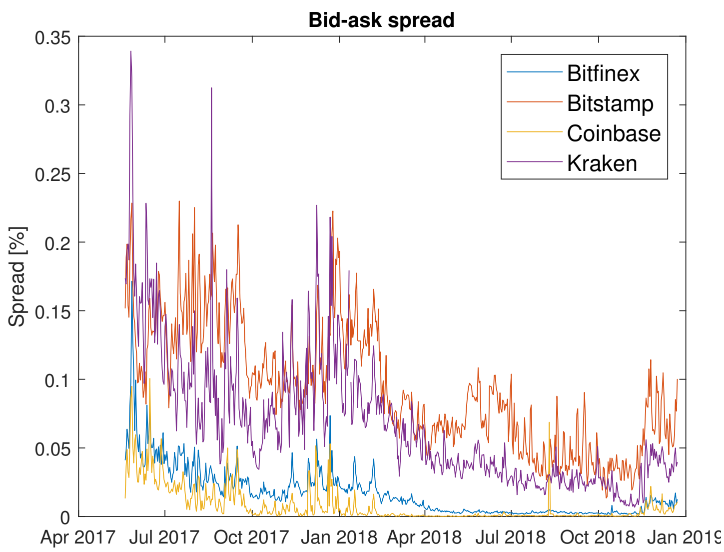
Figure 3.6 Hourly discounts in percent over Bitstamp exchange, period 10/2018-12/2018



(a) **(b)**

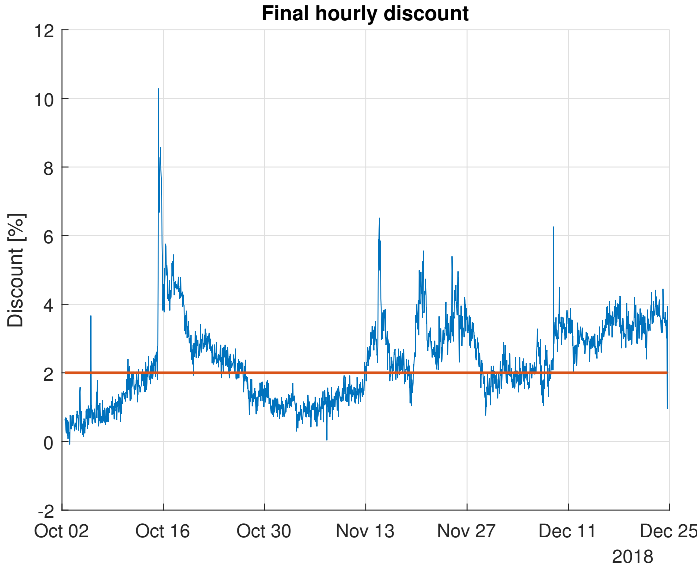
Notes: (a) BTC/USD discount, (b) XRP/BTC discount

Figure 3.8 Daily Bid-ask spread



Notes: The chart represents the daily bid-ask spread in Bitfinex, Bitstamp, Kraken and Coinbase exchanges

Figure 3.9 Final hourly discount



Notes: The graph plots the hourly discount of the strategy; the red line represents the threshold over which the arbitrage strategy is profitable considering transaction costs

Table 3.5: Fees

Exchange	**Coin**	**Deposit** (%)	**withdrawal** (%)	**trading** (%)
Bitstamp	USD	0.05	0.09	0.1-0.25
	BTC	free of charge	free of charge	0.1-0.25
	XRP	free of charge	free of charge	0.1-0.25
Bitfinex	USD	0.1	0.1	0-0.20
	BTC	free of charge	0.0004*	0-0.20
	XRP	free of charge	0.02*	0-0.20

Notes: Trading fees may vary based on the total monthly traded volumes; * entries represent absolute values

B.2 Pairs trading

Table 3.8: OLS regression BTC-ETH

	Dep. variable BTC	
Variable	Coefficient	T-stat.
ETH	9.177*	18.986
	(0.483)	
Intercept	3,205.918*	10.919
	(293.604)	
R^2	0.498	
F stat.	360*	
obs.	365	
Note:	$*p < 0.001$	

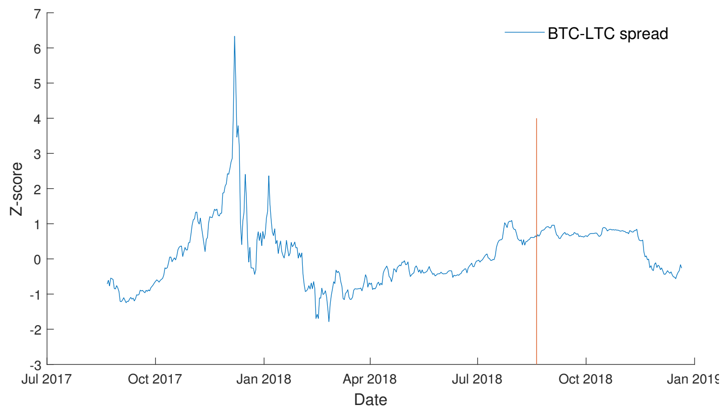
Table 3.9: OLS regression BTC-LTC

	Dep. variable BTC	
Variable	Coefficient	T-stat.
LTC	42.351*	35.489
	(1.193)	
Intercept	3,046.752*	18.131
	(168.035)	
R^2	0.776	
F stat.	1,263.721*	
obs.	365	
Note:	$*p < 0.001$	

Table 3.10: OLS regression BTC-XMR

	Dep. variable BTC	
Variable	Coefficient	T-stat.
XMR	30.992*	35.357
	(0.877)	
Intercept	2,411.819*	13.061
	(184.659)	
R^2	0.775	
F stat.	1,252.148*	
obs.	365	
Note:	$*p < 0.001$	

Figure 3.13 Z-score BTC-LTC



Notes: The graph plots the Z-score over the pair BTC-LTC

Figure 3.14 Z-score BTC-XMR

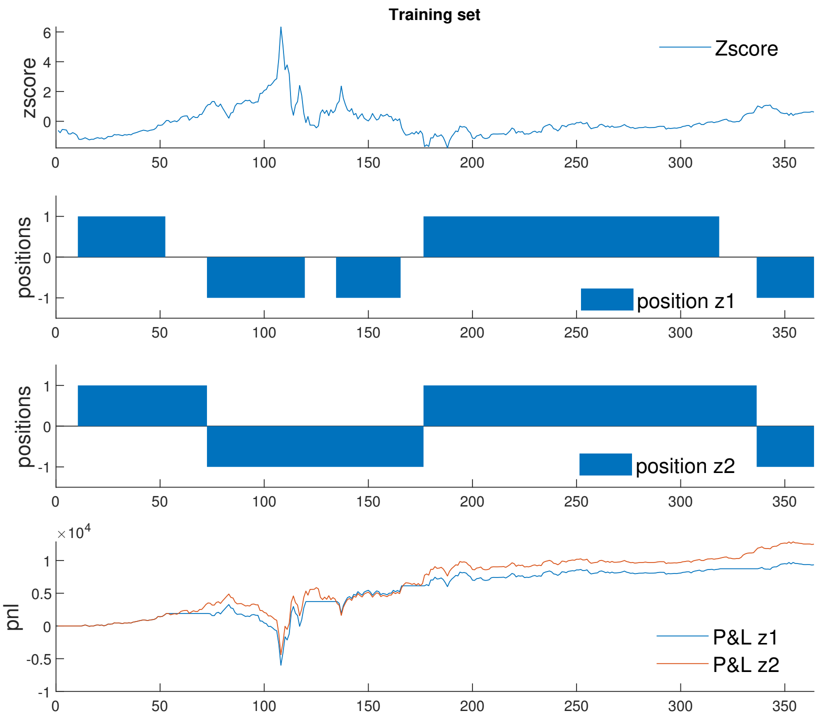
Notes: The graph plots the Z-score over the pair BTC-XMR

Table 3.14: Average daily margin funding rates, January 2019

	Polobot	**Cryptolend**
BTC	0.0233	0.0062
ETH	0.0346	0.0105
DASH	0.0477	0.0269
LTC	0.0365	0.0364
XMR	0.0035	0.0025

Notes: Values are in percentage

Figure 3.20 In sample positions and P&L, BTC/LTC pair



Notes: The graph plots positions and P&L of the pair BTC/LTC in the training set

Figure 3.21 Out of sample positions and P&L, BTC/LTC pair

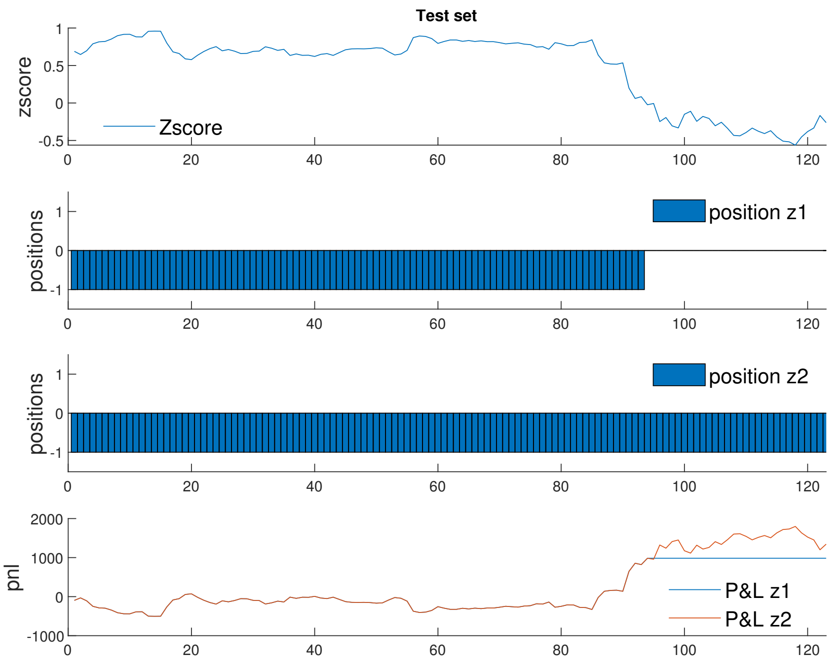
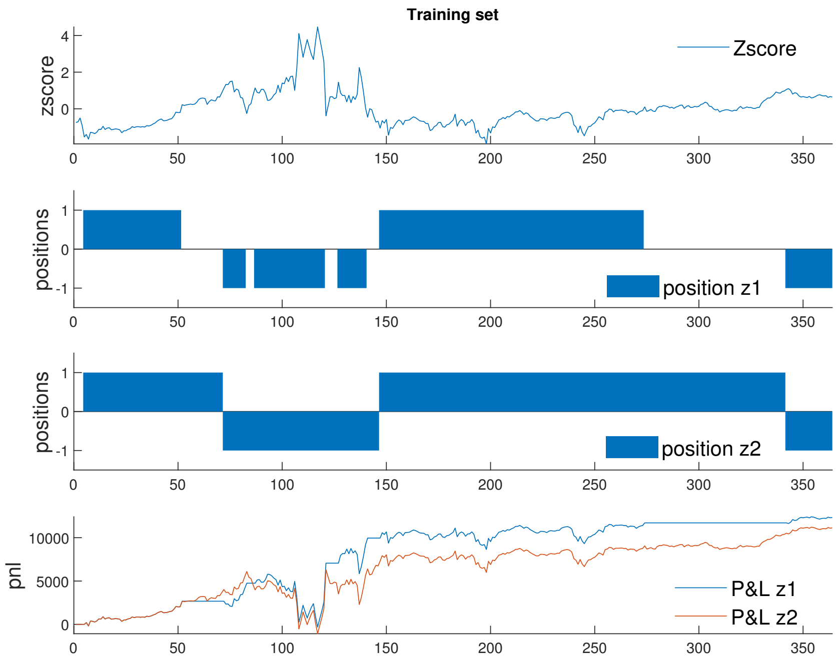
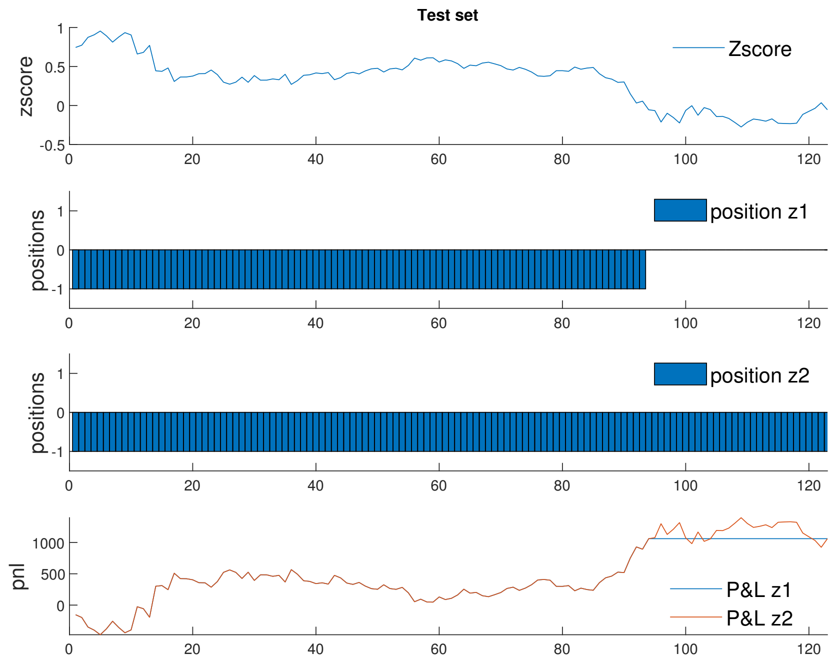


Figure 3.22 In sample positions and P&L, BTC/XMR pair



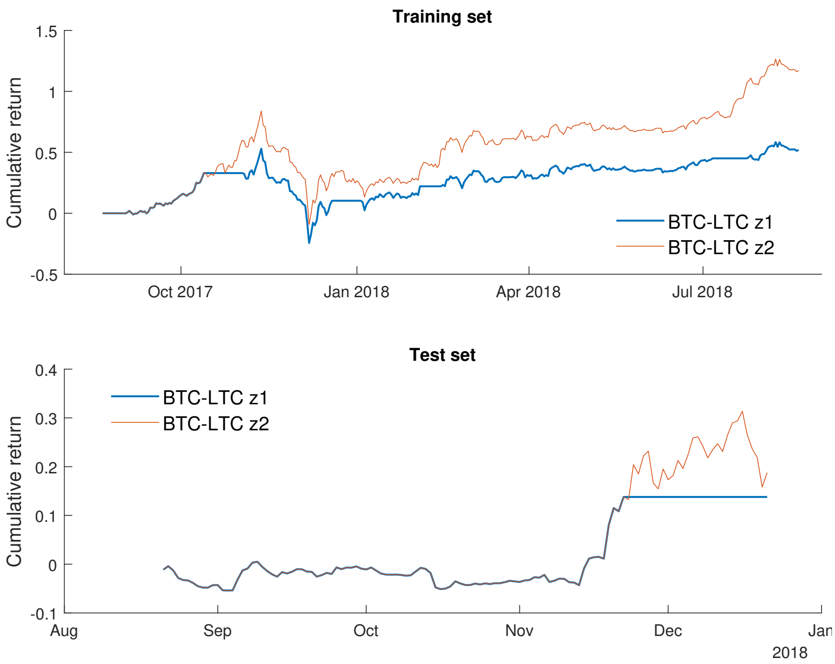
Notes: The graph plots positions and P&L of the pair BTC/XMR in the training set

Figure 3.23 Out of sample positions and P&L, BTC/XMR pair



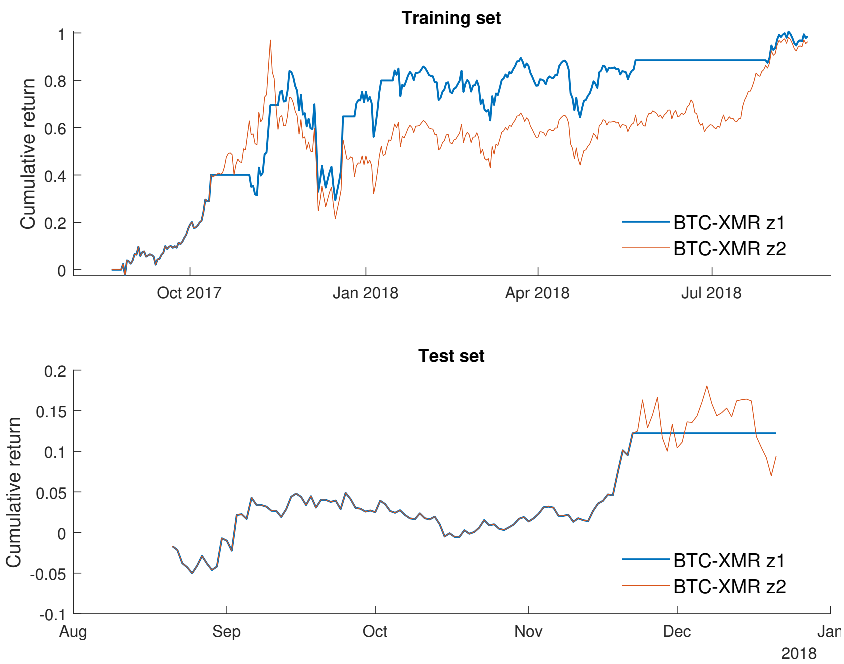
Notes: The graph plots positions and P&L of the pair BTC/XMR in the test set

Figure 3.24 In sample and out of sample cumulative gross returns, BTC/LTC pair



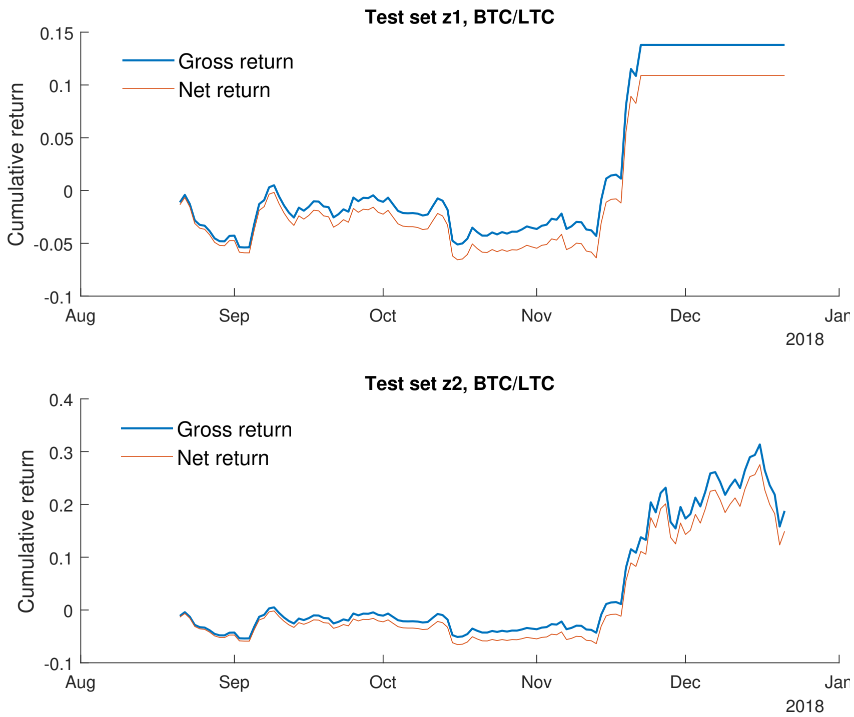
Notes: The two graphs plot cumulative gross returns of BTC/LTC under two different rules

Figure 3.25 In sample and out of sample cumulative gross returns, BTC/XMR pair



Notes: The two graphs plot cumulative gross returns of BTC/XMR under two different rules

Figure 3.26 Out of sample gross vs net returns, BTC/LTC pair



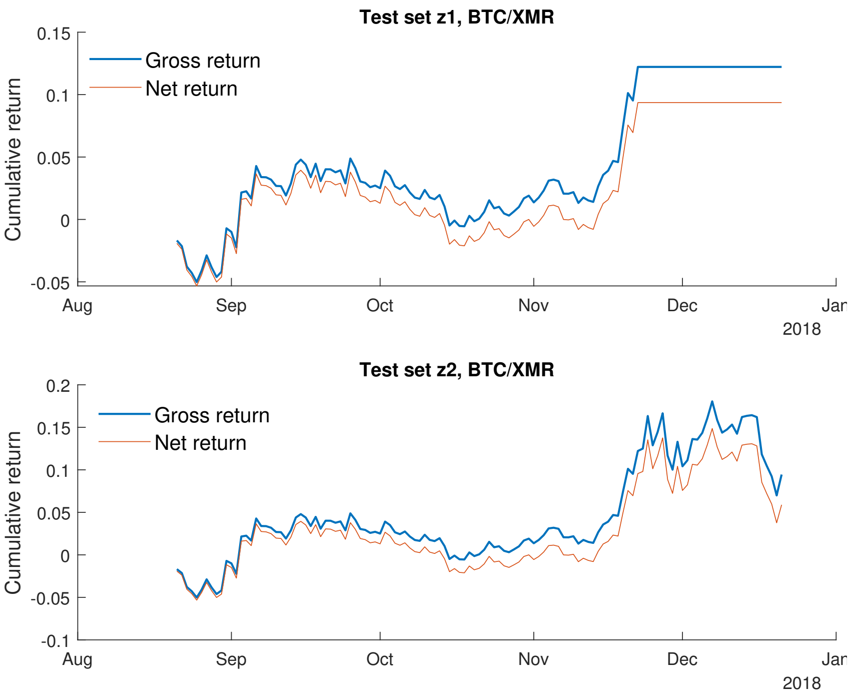
Notes: The two graphs plot a comparison of gross and net cumulative returns of the pair BTC/LTC generated by the strategy in the test set under the two different rules z1 and z2

Table 3.16: Out of sample performance measures of the strategy, BTC-LTC pair

Threshold level	z_1		z_2	
	Gross	**Net***	**Gross**	**Net***
Cum. return%	13.79	10.89	18.77	14.92
Max.Drawdown %	-6.95	-6.95	-11.33	-11.33
Sharpe Ratio	1.21	0.98	1.05	0.86

Notes: * indicates the inclusion of the bid-ask spread, trading fees and margin funding rates in the computation

Figure 3.27 Out of sample gross vs net returns, BTC/XMR pair



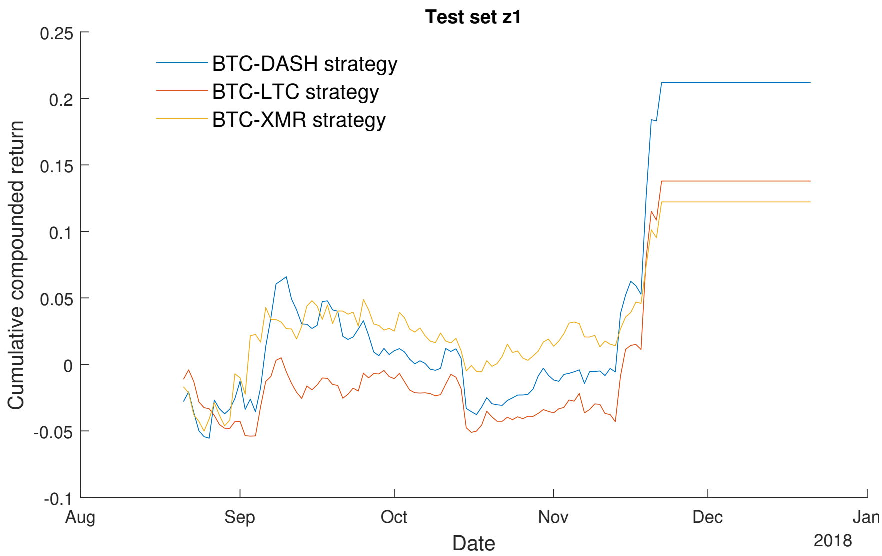
Notes: The two graphs plot a comparison of gross and net cumulative returns of the pair BTC/XMR generated by the strategy in the test set under the two different rules z1 and z2

Table 3.17: Out of sample performance measures of the strategy, BTC-XMR

Threshold level	z_1		z_2	
	Gross	**Net***	**Gross**	**Net***
Cum. return%	12.22	9.36	9.44	5.90
Max.Drawdown %	-5.73	-5.74	-8.39	-8.39
Sharpe Ratio	1.17	0.92	0.69	0.46

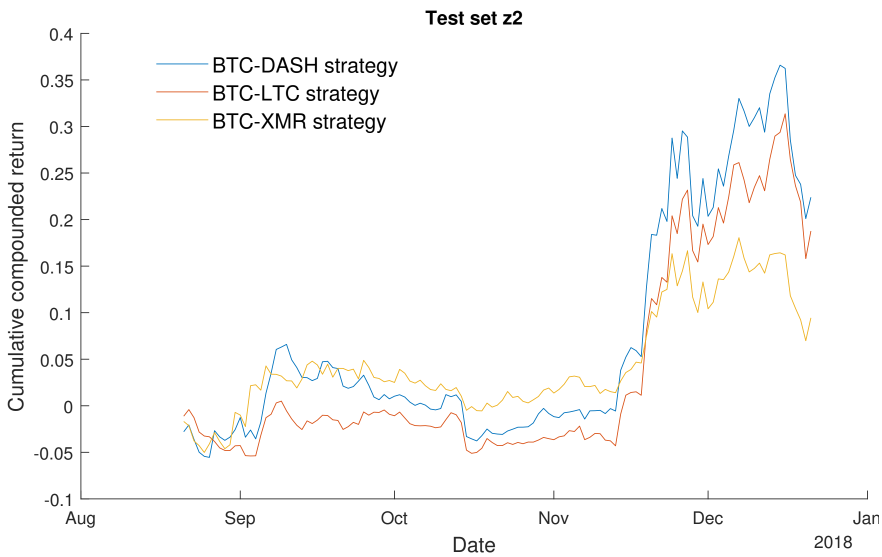
Notes: * indicates the inclusion of the bid-ask spread, trading fees and margin funding rates in the computation

Figure 3.28 Out of sample comparison of gross compounded returns under z1



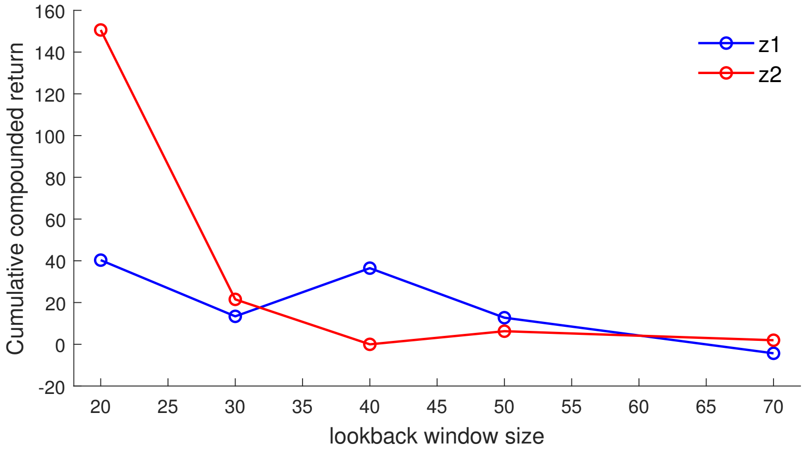
Notes: The graph plots cumulative gross returns of the three pairs under trading rule z_1

Figure 3.29 Out of sample comparison of gross compounded returns under z2



Notes: The graph plots cumulative gross returns of the three pairs under trading rule z_2

Figure 3.30 Sensitivity analysis of the lookback window



Notes: The graph plots cumulative compounded returns obtained in the training set based on the choice of lookback window size

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An analysis of Arbitrage and Cointegration based Pairs Trading in the Cryptocurrency Market

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Advisor

Nicola Borri

Co-Advisor

Pierpaolo Benigno

Student

Alessandro Furlan

Student Number

685501

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Department of Economics and Finance

Introduction

The scope of this paper mainly concerns the investigation and subsequently exploitation of absolute and relative price discrepancies in the new and attractive Cryptocurrency market, a volatile and fragmented space characterized by a multitude of exchanges and virtual issued currencies, the former represented by centralized and decentralized trading platforms, dislocated in several area of the globe, that operate as market makers or matching systems, while the latter are digital electronic systems whose technological development relies on the academic works of modern Cryptography and Network security. Firstly, an analysis of the cryptocurrency market from an historical, technological and statistical point of view is illustrated. Basic statistics of the main cryptocurrencies are provided, with a major emphasis over Bitcoin network, the first digital and unregulated currency system to appear in 2009. Graphical methods and statistical tests are then outlined to assess the presence of Normality in daily returns distribution of a group of selected cryptocurrencies. Follows a briefly review of academic literature, that covers arbitrage phenomena and market completeness, with a focus on the Law of One Price and No-arbitrage principles. Afterwards, a popular type of "relative value" arbitrage and market neutral strategy is introduced, namely the pairs trading, a quantitative investment strategy that seeks to exploit relative price deviations from an equilibrium level between components of a pair. Univariate pairs trading frameworks are investigated, along with the series of statistical tests and estimation procedures. The empirical analysis examines then absolute price discrepancies of digital coins between exchange platforms. The profitability of such risk-less simple arbitrage strategies may be eroded by consistent transaction costs and hurdles. As a result, second section of the analysis shift the focus to the exploitation of relative price discrepancies inside the same exchange, in order to minimize many of the listed transaction costs and risks: the execution time, and the complex system of fees. Pairs trading strategy is then investigated: Unit root rests are performed to check stationarity, and Engle-Granger two-step procedure is adopted to form the potential pairs. Finally, once trading rules have been delineated, an automatic trading system is activated to capture deviations of the formed spread, and in-sample and out-of-sample performance metrics of the strategy are reported, along with final discussions.

The Cryptocurrency Market

The year 2017 has experienced an exponential growth of the Cryptocurrency market, that reached a total capitalization of 800 billion dollars in the 4th quarter. With the introduction of the Bitcoin Futures market by the Chicago Board Options Exchange (CBOE) and the CME Group Inc (CME.O), a consistent decline of the total market value has followed what has been defined in the academic world as one of the largest asset bubbles of all times. Since the release of Bitcoin protocol in 2008 [1], an increasing number of projects and initiatives have entered the new and attractive ecosystem, build upon mathematical and probabilistic models, mainly with regard to the Cryptography and Network technologies. In particular, large part of digital cryptographic assets make use of cryptography to secure transactions through a series of concatenated blocks, i.e., the blockchain, a version of the more general Distributed ledger technology (DLT). At November 2018, more than 2000 cryptocurrencies have been traded on the several Crypto-exchanges located in all the areas of the Globe, setting an all time high record; a trading mania has hit several western and eastern inhabitants and enthusiasts, including third world countries; according to academic papers and financial analysts, this fear to be left out the market (FOMO) and the rise of trading bots have contributed to the rising prices of all the main cryptocurrencies. **Hileman and Rauchs (2017)** have documented as more than 90% of all cryptocurrencies and tokens have copied the original code of Bitcoin, thus not providing any innovation or utility, and raising questions about the real value that could justify their quotation. However, a naive codification of the digital cryptographic assets would consider six main categories: cryptocurrencies, platform currencies, security tokens, Utility tokens, Crypto-collectibles, and Stablecoins. More general subsdivisions can entail the type of distribute ledger technology, or the nature of decentralization of the network (permissionless or permissioned). The rising attention of the media and the public to the new sector has been accompanied by a series of opaque events, including hacks and fund losses, as the Mt Gox exchange hack, market manipulations, insider trading events, Crypto-exchanges disputable behaviors, that increased the climate of uncertainty and doubt around a sector not fully understood by the regulators and agencies yet.

[1] Satoshi Nakamoto, *Bitcoin: A peer-to-peer electronic cash system*, 2018

MAIN STATISTICS. Cryptocurrencies have the unique feature to be exchangeable at every day of the week, with no closing times as in the traditional markets. **Table 1** represents the list of most capitalized cryptocurrency at the date of January 16, 2019.

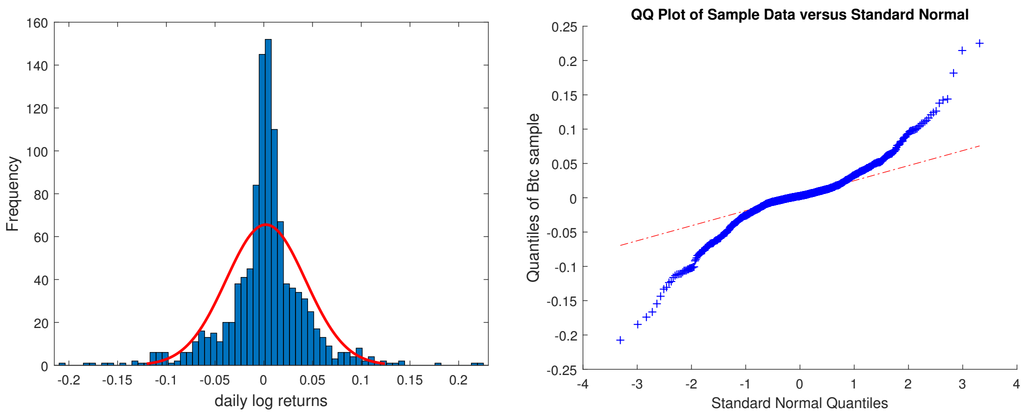
Table 1: Top 15 cryptocurrencies by market capitalization

Rank	Name	Symbol	Price	Marketcap
1	Bitcoin	BTC	$3.664,10	$64.066.913.601
2	Ripple	XRP	$0,331296	$13.596.504.735
3	Ethereum	ETH	$124,31	$12.978.577.703
4	Bitcoin cash	BCH	$129,47	$2.274.707.084
5	EOS	EOS	$2,46	$2.226.360.399
6	Stellar	XLM	$0,107665	$2.059.314.945
7	Litecoin	LTC	$31,86	$1.913.065.27
8	Tron	TRX	$0,024993	$1.665.804.085
9	Bitcoin SV	BSV	$78,64	$1.381.638.153
10	Cardano	ADA	$0,044904	$1.164.232.549
11	IOTA	MIOTA	$0,308556	$857.641.044
12	Binance coin	BNB	$6,12	$790.630.750
13	Monero	XMR	$45,68	$763.933.011
14	Dash	DASH	$71,89	$616.199.372
15	Nem	XEM	$0,056754	$510.785.606

Bitcoin is the first cryptocurrency to appear in 2009, with the release of the first client on the 3rd of January; it is the most liquid and traded cryptocurrency in the entire market, and with the highest market capitalization, property that still holds at the date of writing, January 2019. Main statistics of BTC network regards the monetary emission, the evolution of average transaction fees, the growth of hash rate, the distribution of mining pools and the energy consumption index. All of these network features have been extensively researched by academic and professional studies, with a major focus over the effective decentralization nature of the system and its potential long-term effects on the environment. The analysis of daily returns for a group of selected cryptocurrencies has evidenced the remarkable level of volatility that characterizes the cryptocurrency space; as a result, differences between simple and log daily returns are clearly visible for many coins; in fact, this situation materializes when the ratio between consecutive prices is far from one, i.e., when assets record consistent daily returns in the double-digit range ($30 - 50\%$).
Absence of Normality in the distribution of daily returns is assessed through graphical methods and statistical tests (Jarque-Bera and Kolmogorov-Smirnov Normality tests); both display the presence of *Leptokurtosis*: cryptocurrency returns are more peaked towards the mean with respect to a Normal distribution, and present fatter tails. The empirical findings reveal to be fundamental for many risk management

applications, as Value at Risk (VaR) and Expected Shortfall (ES) estimations. **Figure 1** reports the Frequency distribution Histogram and Q-Q plot of the daily log returns for Bitcoin (graphs of the other cryptocurrencies in Appendix A).

Figure 1 Graphical methods



(a) **(b)**

Notes: (a) Frequency distribution Histogram and (b) Q-Q plot of BTC daily log returns

Other distributions should be considered in the analysis to obtain the one that could best fit to the data, with resulting effects for investment and risk-management activities, as Normal distribution clearly do not characterize the sample data. According to **Chan et al. (2017)**, the generalized hyperbolic distribution gives the best fit for Bitcoin and Litecoin, while for the smaller cryptocurrencies the normal inverse Gaussian distribution, generalized t distribution, and Laplace distribution provide the best goodness of fit. To conclude, **Table 2** contains summary statistics of daily log returns: Mean, standard deviation, Skewness and Kurtosis.

Table 2: Summary statistics

	BTC	**ETH**	**XRP**	**LTC**	**DASH**	**XMR**	**XLM**
Mean	0.0021	0.0044	0.00389	0.0021	0.0032	0.0044	0,0040
Std.	0.0407	0.0655	0.0792	0.0595	0.0627	0.0733	0.0880
Skew.	-0.1704	0.2511	2.9711	1.2829	0.8635	1.0639	1.9803
Kurt.	7.4237	6.7155	39.8509	15.3698	8.7617	10.1430	17.1846
Obs.	1077	1077	1077	1077	1077	1077	1077

Notes: evident signs of positive Kurtosis; XRP, LTC, XLM are positively skewed

Arbitrage and Pairs Trading: Literature Review

An arbitrage strategy is an investment strategy designed to take advantage of one or more assets' price discrepancies generated in different markets; it does not require capital commitment and guarantees a positive payoff for the investor.
The existence of arbitrage opportunities in financial markets is in contrast with the Law of one price (LOP) and Fundamental theorem of equilibrium; the former holds that assets with identical payoffs, in every state of nature, must trade at the same price (**Ingersoll (1987)**), while the latter constitutes the basis of modern capital market theory and encompasses a more general version of No-arbitrage condition. The theorem states the principle according which security market prices are rational and in equilibrium; however, when market conditions ensure the exploitation of price deviations of homogeneous assets, then, the pressure reinforced by arbitrageurs will restore equilibrium levels. As a result, arbitrage activities would led to a convergence of price in different markets, in accordance with the efficient market hypothesis (EMH): asset prices fully reflect all past and current publicly available information and all private information (**Fama (1970)**). It is therefore the quickness of market response to arbitrage occurrences that defines it as efficient or not; if price discrepancies persist over a long period of time, the market is then not able to restore equilibrium levels in the short-term, and arbitrage occurrences arise from market inefficiencies.
The LOP and EMH also apply to "relative value arbitrage", an investment strategy based on the concept of "relative pricing". If two assets are close substitutes (**Gatev et al. (2006)**) and present similar payoffs, they should trade at similar price (a variant of LOP, called "near-LOP"). In case of price deviation from an equilibrium level, for example due to a significant change in the relationship between two securities prices from its historical average, a relative value arbitrage strategy could be activated to profit from this temporary misalignment once it has been corrected. Market neutral strategies and Convertible arbitrage strategies are considered examples of relative value arbitrage strategies that include multiple assets; they are not entirely risk-free, but based on the investor's perspective.

PAIRS TRADING. A typical expression of market neutral strategy is represented by pairs trading, a popular strategy that belongs to the category of statistical arbitrage. Its essence relies on the identification of some form of temporarily mispricing between a pair of assets, the latter that could be represented by stocks, interest rates, currency rates or exchange rates. Whenever this divergence, called spread, is large enough to the investor perspective, the pair of assets could be traded with the idea that the price divergence would correct itself and return to an equilibrium level at some point in the future. The success of the strategy depends on the approach chosen to identify potential profitable pairs; the first attempts were based on fundamental valuation. Among the statistical methods theorized to identify the pairs, two have emerged and subsequently tested in the years in a wide array of markets: the Distance method, introduced by **Gatev et al. (1999)**, and the Cointegration approach.

Distance method. **Gatev et al. (1999)** use some sort of *distance function* to measure the co-movements of the pair components. The authors define the *tracking variance (TV)*, a measure of distance between two normalized asset prices, computed as the sum of their squared differences over a formation period. Then, a minimum-distance criterion is used to match the assets. Stocks that minimize this distance measure are selected to form the pairs and subsequently tested in the trading period. The standard deviation metric is used as trading rule: once "*prices diverge by more than two historical standard deviations*", a long position is assumed on the undervalued stock, and a short position on the overvalued one; positions are then closed when the spread cross back to another threshold or a stop loss level is hit. The main advantage of Distance methodology relies on the absence of parameters to be estimated; it is a parametric-free approach. On the other hand, this methodology presents several drawbacks with regard to the spread variance and mean reversion requirements; firstly, the choice of Euclidean squared distance as measure to select pairs is analytically suboptimal. Secondly, the methodology does not investigate on the nature of correlation between the pair components, as it does not make use of any statistical test to confirm some long-run equilibrium relationship.

Cointegration method. A more sophisticated approach that heavily relies on econometric techniques; it has been introduced by **Vidyamurthy (2004)** and exploits co-movement between pair components by cointegration testing, with the **Engle-Granger (1987)** two step procedure, or the **Johansen (1988)** method in the context of multiple cointegrating relations. Cointegration is a statistical prop-

erty that characterizes a set of non-stationary time series data X_t, Y_t, i.e, random processes ordered by time t. Non stationary variables can be made stationary by differencing; in this case they are said to be Integrated processes of order d, I(d) , where d is the degree of differencing required to make the variable stationary. In general, linear combinations of non-stationary variables are also non-stationary, but any linear combination that produce as result a stationary time series is said to be a cointegration relation.
In mathematical terms, if there exists a vector β such that the linear combination

$$\epsilon_t = Y_t - \beta X_t \sim I(0)$$

is stationary, the two variables X_t, $Y_t \sim I(1)$ are said to be cointegrated, i.e., there is a long-term relationship that ties them under a common stochastic trend. Most financial price series are not stationary time series, but rather geometric random walks; however, if a linear combination of them is found to be stationary, then their distance, or spread, possess mean reversion traits. Antecedent the cointegration testing, the first essential stage of the analysis relies on the identifcation of non-stationarity in the asset price time series; this step can be performed via the the Augmented Dickey Fuller (ADF) test, the Phillips-Perron (PP) test, or the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test. Once it is proven that two variables, for instance X_t, Y_t, are integrated processes of order 1, I(1), then the first step of Engle-Granger procedure entails the performance of a linear regression of a variable (Y_t) on the other (X_t), using the *Ordinary least squares* (OLS) model for parameter estimation:

$$Y_t = \beta_0 + \beta_1 X_t + \epsilon_t$$
$$\hat{\epsilon}_t = Y_t - \hat{\beta}_0 - \hat{\beta}_1 X_t$$

where β_0 is a constant term, the interceipt, β_1 is the hedging ratio, and $\hat{\epsilon}_t$ are the fitted errors, the latter subsequently tested for stationarity with the ADF test (Second step), excluding drift or deterministic trend in the model. In case of rejection of the null hypothesis of unit root in the residuals, the components of the pair are said to be cointegrated, while $\hat{\epsilon}_t$ is the estimated *cointegrating vector*.
As expressed by **Alexander (1999)**, the Engle-Granger approach can represent the optimal methodology, as "it is very straight-forward to implement"; secondly, its linear combinations present the minimum variance, a nice feature in the context of risk management applications, and, finally, it is quite natural the choice of dependant variable in the linear regression. The profitability and superiority of the cointegration approach has extensively documented by a wide array of academic studies: **Perlin (2006)**, **Caldeira and Moura (2013)**, **Hong, Susmel (2003)**, **Rad et al. (2015)**, **Huck and Afawubo (2015)**, **Blázquez et al. (2018)**.

Empirical Analysis

SIMPLE ARBITRAGE STRATEGIES. They concern the possibility of buying and selling at the same time selected cryptocurrencies in different exchanges in order to retain the difference in price, defined as *premium* (when positive). One of the main reason that suggests the analysis of price discrepancies relies on the fragmentation of the cryptocurrency space: there existed more than two hundred trading platforms in 2018 according to Coinmarketcap website, with sensible different trading volumes and buying pressure, certified by academic studies[2]. Moreover, the majority of them was open to foreign traders and investors, thus reinforcing the possibility for arbitrageurs to exploit temporary price misalignments. Historical Data of the exchange rate of three cryptocurrencies versus the Us dollar have been collected with the auxiliary of Cryptocompare api on Python software (from May 22th, 2017 to December 20th, 2018). The exchanges covered by the analysis have been classified following a geographic order.

Methodology and results Cryptocurrencies prove not to satisfy the law of one price (LOP) as they can trade at sensible different prices in multiple exchanges. In particular, Korean exchanges recorded, especially in 2017, consistent gap in price, called the "Kimchi premium". **Table 3** reports the main statistics of Bitcoin discounts in percent over the USD price for the sample of international exchanges: mean, standard deviation, maximum and minimum values, first order autocorrelation and the total number of observations. The empirical findings confirm that price discounts are volatile, time-varying, as they can be positive or negative between the same pair of exchanges, and persistent.
Identical result for the other coins: Ether (ETH) and Litecoin (LTC). Price discrepancies are more observable during 2017 and across regions than exchanges within the same region. Investors may face considerable transaction costs and risks, represented by a complex system of fees, bid-ask spread, execution and counterparty risks, international capital restrictions. The majority of such transaction costs could be minimized following some precautions; for example, we could consider a simple arbitrage strategy that involves a cryptocurrency (BTC) with a significant discount

[2]Borri and Shakhnov (2018), Igor Makarov and Antoinette Schoar (2018)

Table 3: Summary statistics

BTC Discount (in %)						
Exchange	**Mean**	**Std.**	**Max.**	**Min.**	**AC(1)**	**obs.**
Bithumb	4.5889	7.9131	48.4129	-6.3397	0.9409	578
BitFlyer	0.8783	2.3370	17.7331	-11.8885	0.7365	578
Bitfinex	0.1362	1.3235	5.8344	-6.7514	0.8029	578
Kraken	-0.0126	0.5842	2.0650	-4.8908	0.3919	578
Coinbase	0.1491	0.6743	6.4871	-1.2800	0.6109	578

Notes: main statistics of Bitcoin daily discounts in percent over Bitstamp exchange, period 05/2017-12/2018. Btc Discounts are volatile, time-varying and persistent

between a pair of exchanges, Bitstamp and Bitfinex, but includes a second fast currency for transfer (XRP). The main advantage of this strategy relies in the minimization of the execution time and transaction fees (tx) payment; a drawback is represented by the higher trading costs due to the execution of more trades. Hourly Btc, Xrp prices have been gathered via Cryptocompare api, while **Table 4** reports summary statistics of the hourly discounts. Bitcoin presents a consistent discount, with an average value of 2.3435% and with 58.12% of probability of being larger than 2% (1163 over 2001 observations); the pair XRP/BTC presents no significant discount, as more of 97% of observations lie in the range $-0.5 - 0.5\%$; therefore, when BTC discount is consistent and assuming the Usd deposit has already been executed, a trading arbitrage strategy can be structured as the following:

1. Buy Bitcoin on the exchange with cheaper price, i.e. Bitstamp
2. Sell Bitcoin for Ripple
3. Transfer Ripple to the second exchange, Bitfinex
4. Sell Ripple for Bitcoin
5. Buy Us dollar with Bitcoin
6. Withdraw Usd from exchange

The above strategy minimizes the execution time and the payment of mining fees. However, it is reasonable to highlight as mining fees are very tiny and irrelevant with respect to other costs, for example the higher trading costs that have to be accounted; hence, the strategy can be considered profitable in cases of consistent price deviations, in the minimum order of $3 - 5\%$ with the actual system of fees[3]. With a BTC price discrepancy of 1% and a null XRP/BTC discount the strategy

[3]Deposit, withdrawal and trading fees

Table 4: Summary statistics of hourly discounts

		Discount (in %)						
Exch.	**Pair**	**Mean**	**Std.**	**Max.**	**Min.**	**AC(1)**	**obs.**	$D > 2$
Bitf.	**BTC/USD**	2.3435	1.1959	11.2796	0.4006	0.9608	2001	58.12
Bitf.	**XRP/BTC**	- 0.0053	0.2304	2.7100	-1.9740	0.0657	2001	0.0005

Notes: main statistics of BTC/USD and XRP/BTC hourly discounts in percent over Bitstamp exchange, period 08/2017-12/2018; every observation corresponds to one hour

generates an average outcome of -0.02% in the case of full fees payment, and a 0.485% profit in the case of reduced fees[4]. Hence, trading signals of entry positions may be placed at levels that equal 2% or 3%; whenever the discount is above such levels the arbitrage strategy can deliver a net profit.

Conclusions Price discounts in cryptocurrency markets are in decline, mainly due to reduction of trading volumes, new strategies deployed by speculative funds and expert arbitrageurs and new measures adopted by exchanges[5]; however, in periods of uncertainty and volatility, during particular news or events, as Cryptocurrency forks[6], deviations may re-appear and in double-digit levels too; furthermore, with the introduction of liquid institutional platforms it is expected that the time period such that these arbitrage occurrences generate abnormal profits will shorten, in favor of high frequency trading strategies.

PAIRS TRADING. Cryptocurrencies proved to be extreme volatile assets, mostly correlated between them but largely uncorrelated with traditional financial markets. The simultaneous presence of volatility and correlation in the market is an opportunity to examine market neutral trading strategies, that do not take into account the market trend.

Methodology. Cointegration-based pairs trading is then applied to a set of cryptocurrencies, selected among the most liquid and with at least one year of historical price data, traded on Bitfinex exchange: Bitcoin (BTC), Ether (ETH), Litecoin (LTC), Dash (DASH) and Monero (XMR). In particular, Litecoin, Dash, Monero were all forks of the original code Bitcoin, with whom share some network features

[4] Forh highly traded volumes, all transaction costs included except the bid-ask spread

[5] Introduction of new fees

[6] Bitcoin cash fork and subsequently hashing war on November, 14th raised uncertainty in the market and daily volatility, after months of relative price stability

and technological developments. Hence, considering the strict connection with BTC, it is plausible to explore the evolution of relative price dynamics. The historical data are divided in two parts in order to avoid the look-ahead bias[7]: the first part, the training set, comprehends the least recent observations; parameters of the model are optimized on this portion of data. The second part, the test set, is the set of observations where the resulting model is tested, i.e., the trading period. Prices of the selected cryptocurrencies confirm to be integrated process of order one, $I(1)$, via the augmented Dickey-Fuller (ADF) and Kwiatkowski, Phillips, Schmidt, and Shin (KPSS) tests. Afterwards, Engle-Granger procedure is used to form the pairs; a linear regression is computed to estimate model parameters: the hedging ratio and the intercept. Bitcoin is the dependant variable, while the other coins are in turn the independent ones. Afterwards, unit root tests are computed on the fitted errors. They appear to be stationary for all except the pair BTC/ETH, where lies on the edge of the regions of acceptance and rejection; thus, while for the pairs BTC/DASH, BTC/LTC and BTC/XMR the spread can be defined using the coefficients obtained by the OLS model in the training set, for the pair BTC/ETH, the computation of a dynamic hedging ratio should be more appropriate to capture the changing levels of the pair components. Consider, for example, the pair BTC/DASH; the spread can be defined as:

$$\delta_t = \mathrm{BTC_t} - \beta_1 \mathrm{DASH_t}$$

, where $\beta_1 = 10.79$.

A long position on the spread denotes the opening of a long position of 1 unit on BTC and a short position of 10.79 units on DASH; a short position on the spread in exactly the opposite. The Zscore is subsequently defined by:

$$\mathrm{Z_t} = \frac{\delta_\mathrm{t} - \mathrm{u}_\delta}{\sigma_\delta}$$

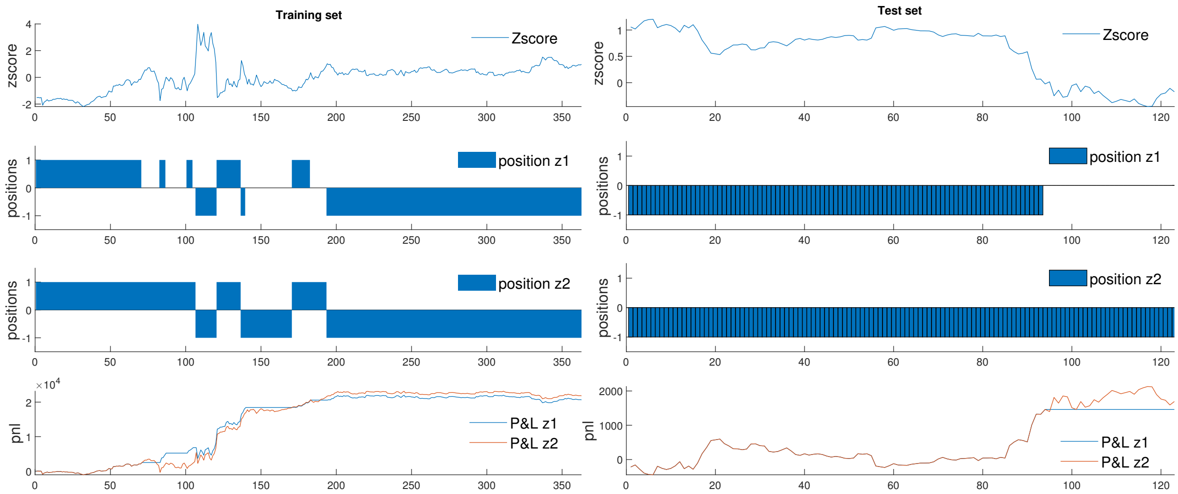
, where u_δ and σ_δ are the mean and standard deviation of the spread. Once trading rules are defined (Bollinger bands), an automated trading system can be activated; it automatically opens and closes positions in the pair in correspondence of predetermined signals; more specifically, entry and exit signals are determined on the basis of Zscore standard deviations from the mean.

Daily returns of the strategy are computed through a mark-to market system (MTM), that entails the division of the Profit and Loss (P&L) over the Gross market value of the portfolio. Existing positions of the previous day are carried forward whenever the following day's positions are indeterminate. **Figure 2** illustrates the positions in

[7] When future information or data are used to construct and back-test a trading strategy for a time period antecedent to their availability

the pair BTC/DASH as the Zscore evolves over the time in the training and test sets, and the profit and loss generated by the strategy with two different trading rules: the first, z_1, opens positions when the Zscore is above or below 1 standard deviation and close them when it reverts to the mean ($z_1 : n_1 = 1, n_2 = 0$), while the second, z_2, closes them when the Zscore moves beyond the opposite band ($z_2 : n_1 = 1, n_2 = -1$).

Figure 2. In-sample and out-of-sample positions and P&L, BTC-DASH



(a) (b)

Notes: (a) In-sample, (b) Out-of-sample

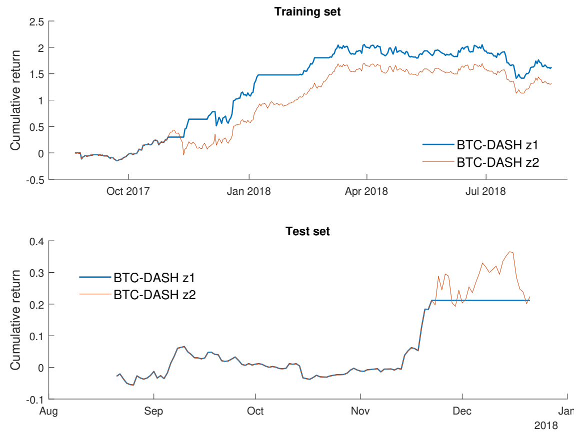
Figure 3 plots the cumulative compounded gross returns of the pair BTC/DASH generated in the training and test periods. The strategy performs better with the first rule either in the training set or in the test set; in fact, it presents a lower maximum drawdown and a higher Sharpe ratio. Out-of the sample results are consistent with the model. **Table 5** summarizes some of the performance metrics.

Table 5: In sample and out of sample performance measures, BTC/DASH

	Training set		Test set	
Threshold level	z_1	z_2	z_1	z_2
Gross profit %	162.59	131.92	21.19	22.40
Max.Drawdown %	-24.16	-26.42	-6.69	-13.05
Sharpe Ratio	2.45	1.85	1.44	1.05

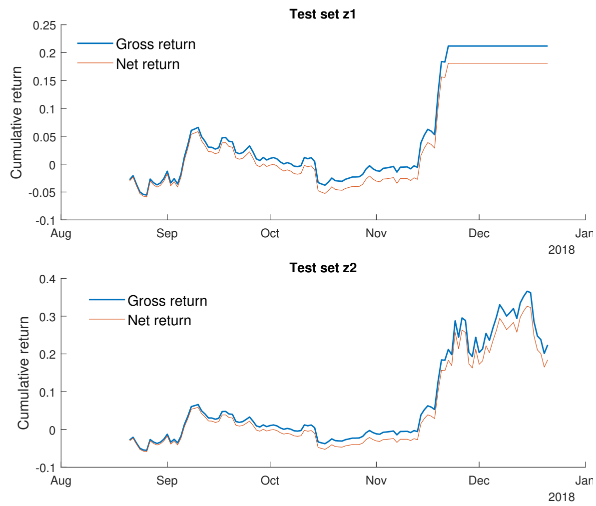
Return and risk measures of the strategy do not vary considerably with the inclusion of transaction costs, represented by the bid-ask spread, trading fees and margin funding rates (borrowing costs to short sell the cryptocurrencies). Gross profits of the strategy are preserved with the inclusion of all transaction costs, as displayed in **figure 4**.

Figure 3 In sample and out of sample cumulative compounded returns, BTC/DASH



Notes: Training set in the upper part; test set a the bottom

Figure 4 Out of sample gross vs net returns of the strategy, BTC/DASH



Notes: In the upper part test set under z_1; at the bottom z_2

Table 6 reports performance metrics of the automated strategy with the inclusion of transaction costs, under the two trading rules. The strategy presents similar performance metrics for the other pairs BTC/LTC, BTC/XMR.

Table 6: Out of sample performance measures of the strategy includ. transaction costs, BTC/DASH

Threshold level	z_1		z_2	
	Gross	**Net***	**Gross**	**Net***
Cum. return%	21.19	18.10	22.40	18.42
Max.Drawdown %	-6.69	-6.69	-13.05	-13.06
Sharpe Ratio	1.44	1.25	1.05	0.89

Notes: * indicates the inclusion of Bid-ask spread, trading fees and margin funding rates (short selling costs) in the computation

On the other hand, with regard to the pair BTC/ETH, a dynamic hedging ratio is computed to adapt to the changing levels of the pair components. A rolling linear regression is utilized to capture the changes over time in the hedging ratio. The Zscore of the pair is computed considering the moving average and standard deviation of the spread; Positions in ETH are rebalanced at a daily frequence to best match the synthetic spread formed with the dynamic hedging ratio, with the result of payment of higher trading costs. The reduction or avoidance of trading fees, for instance performing the strategy with large volumes[8], could enhance profitability, aligning net returns to gross ones. Moreover, the short selling costs of Ethereum proved to be much greater in comparison to Bitcoin; this aspect explains the divergence in performance between the first and second trading rule once margin funding rates have been accounted (with the second rule, the time over which ETH is short sold is longer, with the effect of higher margin funding costs). **Table 7** contains out-of-sample performance metrics of the strategy with the inclusion of transaction costs.

Table 7: Out of sample performance measures includ. transaction costs, BTC/ETH

Threshold level	z_1			z_2		
	Gross	**Net***	**Net**	**Gross**	**Net***	**Net**
Cum. return%	10.94	8.17	0.15	12.50	8.50	-3.59
Max.Drawdown %	-8.07	-8.08	-8.09	-8.07	-8.08	-8.09
Sharpe Ratio	1.16	0.88	0.03	1.22	0.86	-0.31

Notes: * indicates the exclusion of trading fees from the computation

[8] Maker fees on Bitfinex are erased for monthly traded volumes above $7.5 million

Concluding Remarks

Cryptocurrency market proves not to be efficient as it allows profitable arbitrage and relative value arbitrage strategies, even with the inclusion of transaction costs. Price discrepancies of digital coins persist over a considerable amount of time, in terms of days and weeks. Although several constraints and obstacles may limit the exploitation of these temporary misalgninments, simple arbitrage strategies between exchanges were profitable for all 2017 and even 2018 but with a lower magnitude.
The reduction of buying pressure and global trading volumes and the advanced strategies performed by speculative funds have probably eroded most of arbitrage opportunities between exchanges, thus upgrading the efficiency level of the market. However, in time of elevate volatility and uncertainty, due to positive or negative relevant news, price discrepancies are likely to re-occur due to the fragmented organization of the cryptocurrency space, characterized by multiple exchanges with sensible different volumes and local demands.
A solution of the main obstacles to arbitrage exploitation (execution time, capital restrictions) has been represented by the activation of more sophisticated trading strategies that seeks to find price discrepancies inside the same exchange, for example, investigating temporary price misalignments between a pair of assets, as implemented by pairs trading. Cointegration method applied to a group of 5 cryptocurrencies has identified 3 pairs with mean reversion property, and the automated trading system, based on the Bollinger bands rules, exhibited consistent gross and net cumulative returns under different thresholds. For non stationary pairs, the computation of a dynamic hedging ratio has been performed to adapt to the changing levels of the pair components.
Further research on the subject should explore the potential explanations of cointegrating relations in the cryptourrency space, while extending the dataset of possible pairs and adopting more sophisticated frameworks, as the stochastic spread approach ; Kalman filter could be used to correctly infer the exact entry and exit trading signals, or the true weights to assume in each pair component. In conclusion, high-frequency pairs trading could be tested with the use of minutely pricing data, but only in the context of trading fees minimization, as they represent transaction cost with the heaviest weight.

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