Department of Economics and Finance
Chair of Theory of Finance

Inside Investment Certificates: Structure and Application in the Developing Italian Market

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Summary

1. Introduction

After 12 years since the infamous subprime crysis took place in the Unites States, poisoning the real economy and financial markets all over the world, it is possible to say that every cloud has, at least, silver lining. The sense of omnipotence characterizing both “too big too fail” and small investors soon left space to uncertainty, bringing whichever actor inside the financial markets to the stabilisation of that risk/return balance they prematurely forgot about.

The growing risk aversion led, indeed, to investors’ need of security they could not find in no asset class anymore, except in perfectly liquid instruments.

In this contest, structured product found fertile ground to develop and among them, broadly, the Certificates, defined as securitized derivatives built through the combination of other derivatives instruments (mainly options and futures) and/or basic assets like bonds (Zero Coupon), thanks to which it is possible to generate original financial instruments with their own specific chatacteristics, different from the assets they come from.

Given that definition, it is immediate to understand how Certificates present all the characteristics of syntetic assets, despite the fact that they stand out from them due to the presence of one particular risk category, the counterparty credit risk, since they are structured to realize different payoffs.

Investment Certificates present themself as an alternative to products that can ensure an adequate saving plan for the investor (retirement plans, insurance policies and shares of mutual funds) giving them the possibility to invest in something that is taylor made on their needs, at the same time.

This dissertation aim to further explore a subject only skimmed by the scientific literature, mostly due to its brief history, trying to make a financial instrument more clear while it is still shrouded in mistery and from whom the most get away just for their denomination of derivative, term that should simply stand for “financial security with a value that is reliant upon an underlying asset or group of assets” and not for toxic product.

Background information will be given in the first part, before moving forward to study the structure of this financial instrument, analyzing its composition, studying the variables at stake at the beginning and then coming to its possible uses in various fields.

We will construct from scratch a Bonus Cap Certificate, then, studying deeply the options inside it by pricing them, its performance will be analyzed in order to understand how a simple certificate can be useful in many situation in the portfolio management environment.

Within the limit of my contribution, the final aim of this dissertation is to increase the financial culture of the average investor, which, translating Giovanna Boggio Robutti, head of the FEDUF (Fondazione per l'Educazione Finaziaria e al Risparmio), "compares thousands of websites in order to buy a new phone, hundreds of car dealerships to buy a car, but, when it comes to save money, believes that a friend's suggestion is more than enough".

2. Certificates

Certificates are financial instruments characterized by a large variety of risk/return profiles that allow to provide to the investors taylor-made financial solutions related to every different investment need.

Investment Certificates are securitized derivatives, in that a combination of financial contracts assembled in single securities which can be traded like stocks.

In Italy they are traded in two different markets and: Cert-X (EuroTLX) and SeDeX (Borsa Italiana).

These products are issued by financial institutions which assume the obligation to pay the due cash flows, following what stated in the related prospectus, that must be released for every tranche of products.

A single Certificate presents different alternative characteristics:

- Directionality of the underlying asset (exposition to positive and negative movements)
- Cash Flows during the life of the instrument (coupons)
- Early redemption with premium for the investor
- Full, partial or conditioned capital protection of the initial amount invested
- Redemption premium at maturity

This diversity of profiles can be achieved because the investor renounces, with the purchase of a certificate, to the dividends paid by the underlying asset during the life of the certificate.

Options contained in the certificates can belong to the standard category, the so-called "plain vanilla" options, or to the "exotic" one, which includes options characterized by special provisions. It is usual to find certificates incorportating the so-called "barrier options", in particular put ones: this kind of options provide the possibility to hedge one's position against a fall in the underlying's value as long as it stays above the level identified by the barrier.

There are four main categories of cetificates: Capital Protectection, Conditional Capital Protectection, No Capital Protection and Leverage.

Each one of these product can be structured in many ways thanks to the infinite combination of exotic option available Over The Counter, in order to create unique products that can fit almost every investor's need: as Mauro Camelia explains in his "Il Libro dei Certificati", "a certificate can be considered as the point of conjunction of investor's preferences and derivatives components".

2.1. History and Current Framework

Germany is for sure the ancestor of all the Certificates markets nowadays available. The first certificate was issued in the far 1989, anticipating by almost 9 years the not as succesful debut of such product at Piazza Affari.

As mentioned before, the trigger for the rebirth of this instrument was the 2007/2008 crysis and along with this, the insecurity that, from that time on, every investor could feel. Since then, this particular market knew a great expansion year by year, reaching the top in 2015.

As mentioned before, Certificates are currently traded in two different markets: SeDeX (Borsa Italiana) and EuroTLX.

The first one is born in 2004 as a Borsa Italian segment for Certificates and Covered Warrant exchange, which are denominated Securitised Derivatives, from which the name SeDeX, acronym of Securitised Derivatives Exchange.

The first Covered Warrant was admitted to trade in the stock market in 1998.

The success of such product led to the trading of thousands of them and to the birth of the Coverd Warrant Market (MCW).

Ever since the first years, next to the covered warrants, certificates begin to be exchanged.

Over the years the number of certificates and their volumes grew progressively just as their variety and for this reason in 2004 Borsa Italiana substituted the MCW with SeDeX.

From that moment on, this market is grown way further until it became one of the most important European Markets for volumes exchanged.

- 1998: The first Covered Warrants are traded in the stock market
- 1999: Benchmark-kind Certificates on indexes are traded, initially denominated as Covered Warrants Call “Strike 0”
- 2000: Birth of Covered Warrant Market (MCW), dedicated to the exchange of Covered Warrants and the first Certificates. The Market experiences a strong grow in Covered Warrants number and volume
- 2001: MCW sets the record for contracts exchaged and new issues
- 2002: First few Leverage Certificates enter the Market
- 2002: Introduction of the autonomous definition of Certificates inside Borsa Italiana regulation

- 2004: Birth of SeDeX, which substitutes the MCW. The new market reflects the growing weight of the certificates next to Plain Vanilla Covered Warrants from the point of view of both denomination and segmentation. Two out of four new segments are dedicated to Certificates. There are new types of exchanged Certificates and also the variety of underlying grows, opening to new asset classes such as commodities
- 2005: For the first time in the history of the market Certificates' volume exceeds Covered Warrants'
- 2006: New underlyings are available, such as emerging markets indexes, interest rates and new commodities; also, new certificates belonging to the Outperformance category begin to trade (Twin Win and Express Certificates)
- 2007: Borsa Italiana merges with London Stock Exchange
- 2009: SeDeX migrates to TradElect platform
- From this moment on the market stays basically the same as today, except being assisted by a new MTF[1], EuroTLX, of which we have meaningful data only starting from 2016.

Nevertheless, EuroTLX underwent through many changes in time before becoming the established market today is:

- 2000: TLX is launched as a Unicredit group SSO
- 2003: TLX S.p.A. becomes operator of the TLX market
- 2003: TLX S.p.A. runs TLX (regulated market) ed EuroTLX (Multilateral Trading Facility).
- 2006: Banca IMI becomes TLX's partner.
- 2007: Banca IMI starts operating as Market Maker.
- 2007: ECB recognizes EuroTLX as a "Non-regulated market acceptable by the ECB for collateral management purposes".
- 2007: with the application MiFID, according to art.19 subsection 12 of LD 164/07 EuroTLX becomes a MTF (Multilateral Trading Facility).
- 2008: The first Specialist (Banca Popolare di Vicenza) starts running operations on their own "branded" instrument (bonds and certificates, issued directly from the intermediary or intended for their network)
- 2009: Morgan Stanley becomes the first foreign Specialist

[1] 15th december 2009: CONSOB authorization for the management of multilateral trading systems and the simultaneous termination of the TLX regulated market.
1st January 2010: TLX Spa is renamed EuroTLX SIM Spa and closes the Regulated market, keeping only the MTF.

- 2009: BNL – Bnp Paribas group becomes the first non partner Market Maker on EuroTLX
- 2009: CONSOB gives authorization for the activity of management of Multilater Trading Facilities and at the same time for the termination of the regulated market, TLX
- 2010: TLX Spa is renamed EuroTLX SIM Spa and closes the regulated market, only maintaining the MTF
- 2011: UBS becomes the first foreign Broker
- 2011: Introduction of the domestic central counterpart
- 2011: On the BTP Day, in only one trading day 53.600 contracts are concluded for a countervalue of 1,2 billions euro
- 2012: For the first time the annual non partners third party trading activity overcomes the shareholders' one. In october a record for monthly trading countervalue exchanged is set (12.1 billions euro)
- 2013: 70% stake of EuroTLX is bought by Borsa Italiana. Banca IMI and Unicredit, which used to split the ownership before, keep maintaing 15% each
- 2014 EuroTLX migrates its platform on the new Millenium IT technology, already used for the markets run by Borsa Italiana and London Stock Exchange
- 2016: introduction of the feature "Request for Quote" (RFQ) for the trading activity on big size as an alternative to the negotiation model with the Central Limit Order Book (CLOB)
- 2017: TradeGate AG is admitted as Market Maker on the foreign stock segment and requests admission to trading for 200 new instruments that lead this particular segment to have 350 stocks trading
- 2017: EuroTLX Quote starts, the new EuroTLX segment meant for debt instruments admitted on market's initiative without listing obligation, only in Request for Quote mode

Numbers speak for themself: The new listings trend in the primary marker from 2006 until today is significantly increasing, with a peak reached in 2015 and already overcame by partial data of this year (primary market data are available up to 3rd quater) from the point of view of number of new listings (not yet in terms of countervalue).

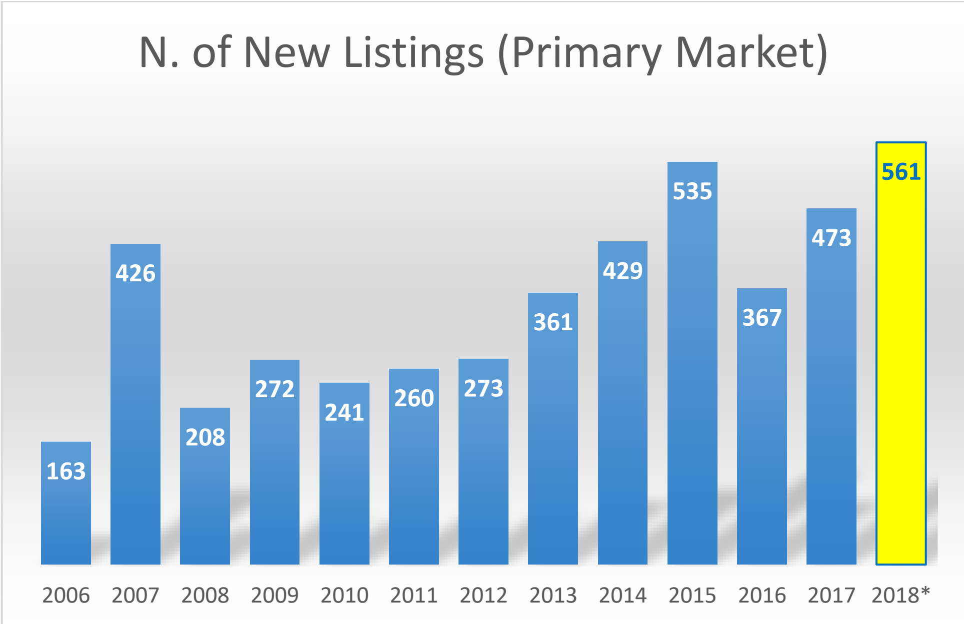


Figure 1 (Source: ACEPI)

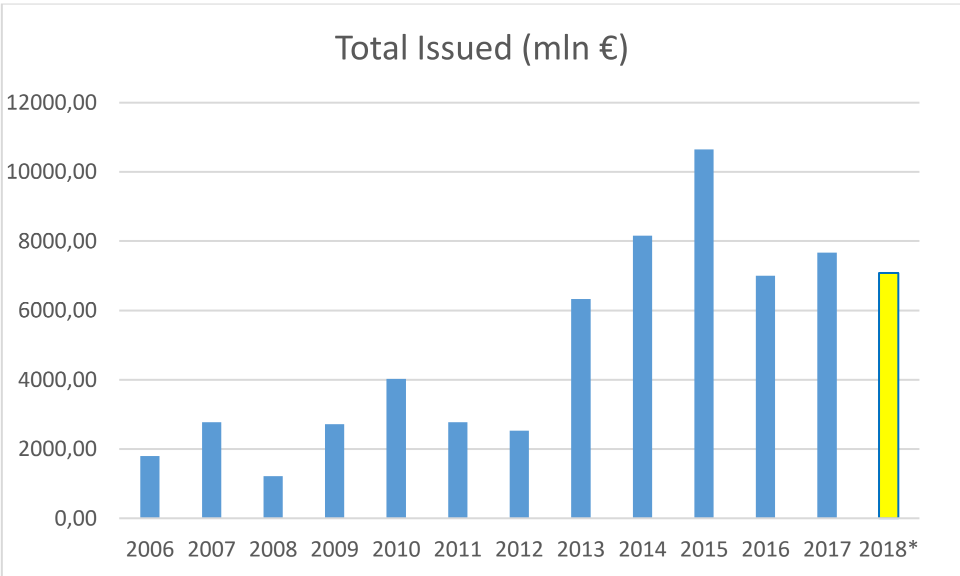


Figure 2 (Source: ACEPI)

As for secondary market, meaning among private subjects trades, we have consistent data only starting from 2010 that can be considered partial until 2016, when EuroTLX became an established market, yet it is still clear that also for this market the trend is upward looking, considering 2018 data are available up to the 1st quarter.

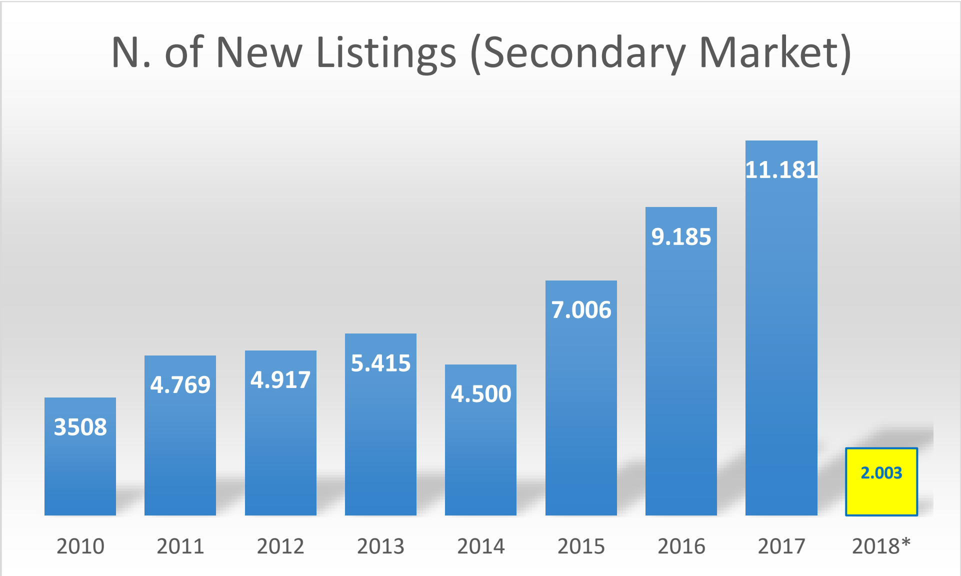


Figure 3 (Source: ACEPI)

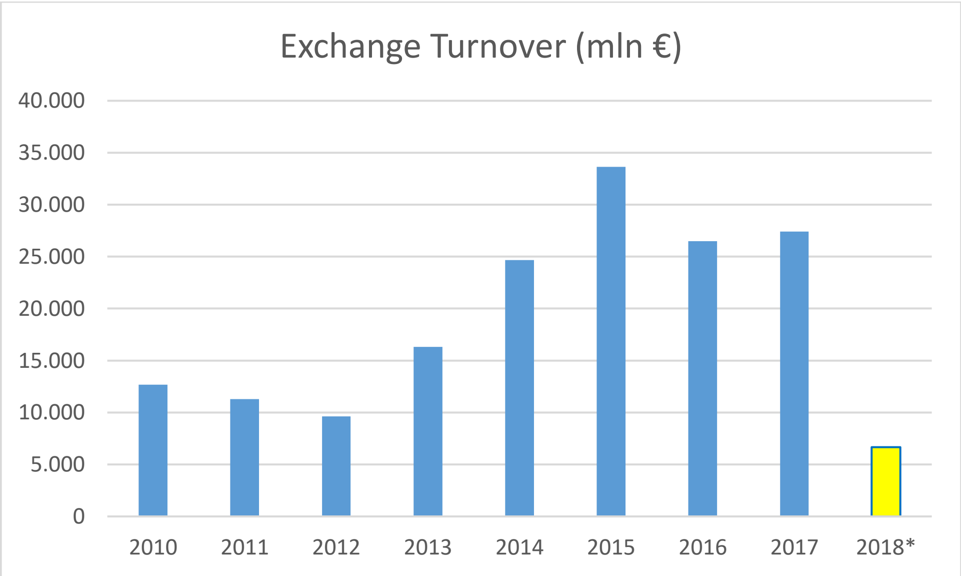


Figure 4 (Source: ACEPI)

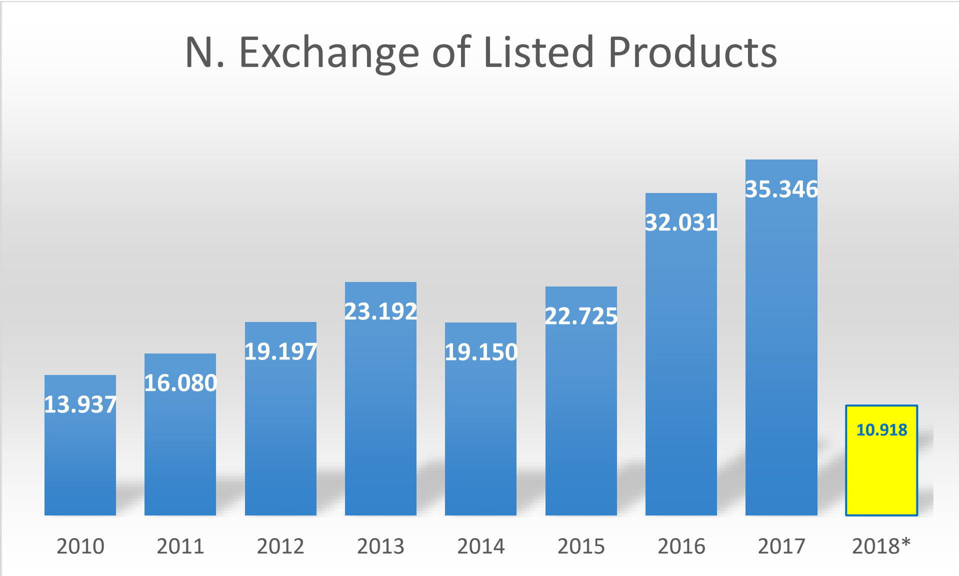
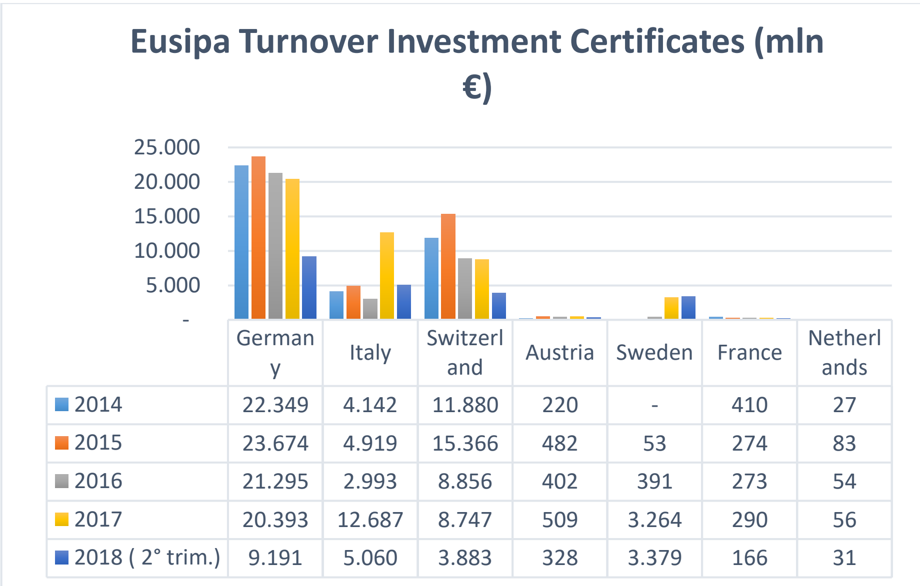


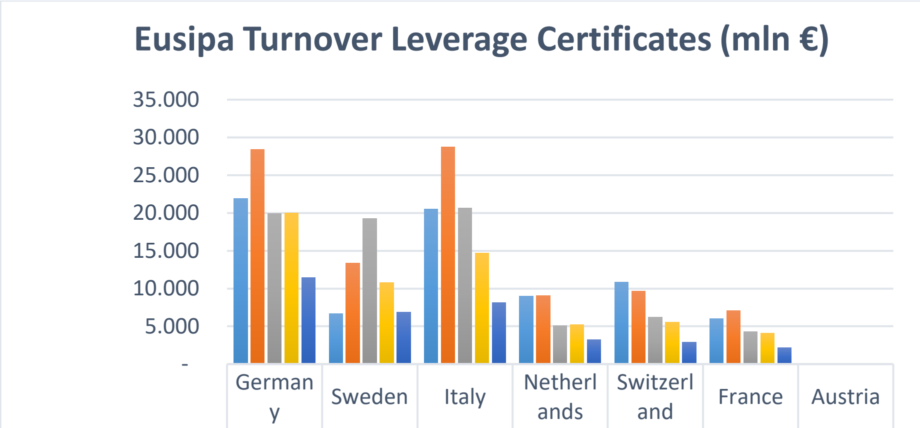
Figure 5 (Source: ACEPI)

This consistent growth led Italy to be one of the most important European Certificates Markets, as EUSIPA data show, especially for the leverage segment:



	Germany	Italy	Switzerland	Austria	Sweden	France	Netherlands
2014	22.349	4.142	11.880	220	-	410	27
2015	23.674	4.919	15.366	482	53	274	83
2016	21.295	2.993	8.856	402	391	273	54
2017	20.393	12.687	8.747	509	3.264	290	56
2018 (2° trim.)	9.191	5.060	3.883	328	3.379	166	31

Figure 6 (Souce: EUSIPA)



	Germany	Sweden	Italy	Netherlands	Switzerland	France	Austria
2014	21.904	6.697	20.520	8.993	10.876	6.018	18
2015	28.393	13.403	28.716	9.087	9.659	7.053	43
2016	19.917	19.260	20.654	5.085	6.231	4.267	24
2017	19.985	10.765	14.717	5.212	5.544	4.084	32
2018 (2° trim.)	11.446	6.875	8.126	3.238	2.894	2.205	20

Figure 7 (Source: EUSIPA)

2.2. Certificates Classification

Despite their brief history, Certificates underwent through bright changes and, in time, a extraordinary level of customization have been reach, in order to meet investor's need and market's dynamics.

Considering the wide range of certificates nowadays available in the market, The Italian Associaton for Certificates and Investment Products (ACEPI) created a map based on the European Structured and Investment Product Association (EUSIPA) one, in order to provide a guide for the investors not to get lost in such large variety of products.

With this map it is possible to lead back every certificate, no matter how singular its structure is, to a category, based on the level of risk (and therefore profit) the investor is willing to take, with clear explanation of profit and loss scenarios.

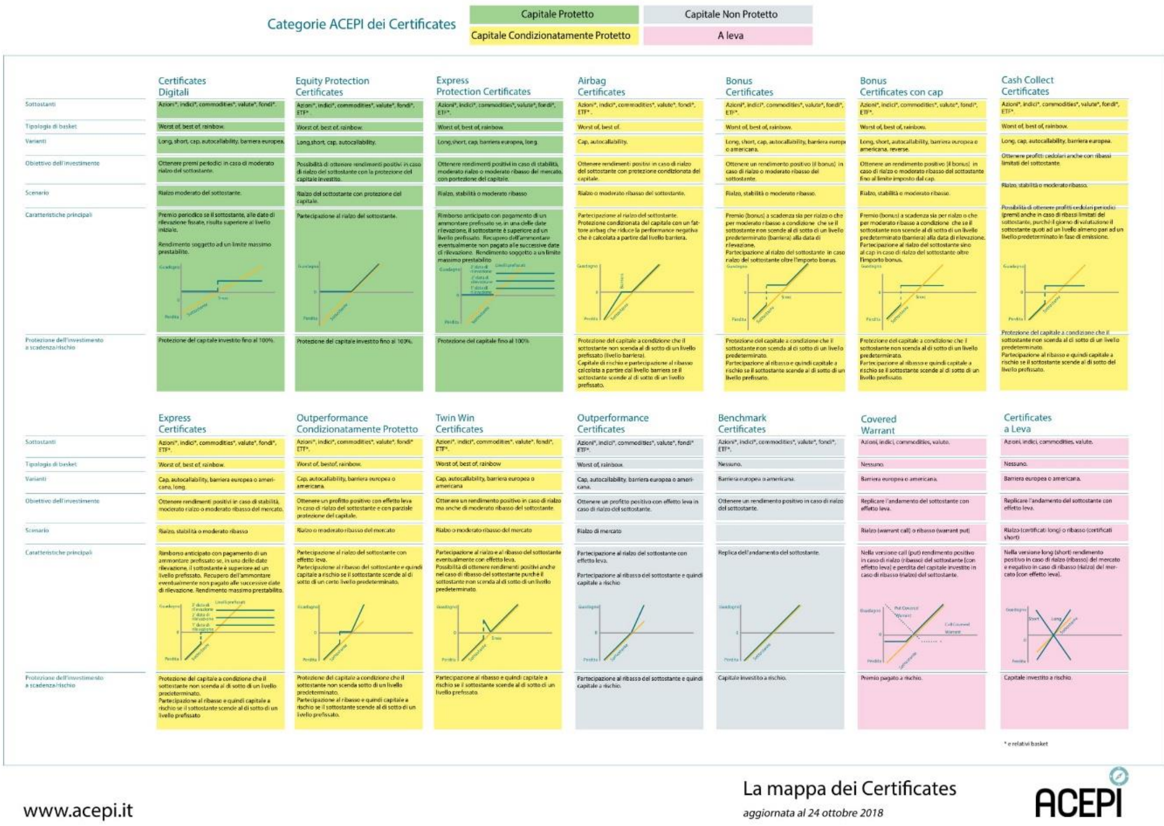


Figure 8 (Source: ACEPI)

Another thing that can create confusion is the wide range of product names available on the market, as each issuing bank calls a certificate in its own way[2]. For this reason ACEPI attached to the above map a list of all the product's names for each different issuer, bringing them back into their own specific macrocategory.

[2] For example Athena Certificates issued by BNP Paribas are exactly the same product of Bonus Certificates issued by Unicredit.

STRUMENTI A CAPITALE PROTETTO

Nome	Tipologia	Emittenti
Bonus Protected	Bonus Protected	Banca IMI
		Mediobanca
Cash Collect Protetto	Digitale	Mediobanca
		UniCredit
Digital	Digitale	Banca IMI
Protezione Premium	Digitale	Banca IMI
		Mediobanca
Double Win	Double Win	Deutsche Bank
Accelerator	Equity Protection	Société Générale
Equity Protection	Equity Protection	Banca Akros
		Banca IMI
		BNP Paribas
		Deutsche Bank
		Mediobanca
		Société Générale
Equity Protection Alpha	Equity Protection	Banca Akros
Equity Protection Autocallable	Equity Protection	Banca Akros
Equity Protection Cap	Equity Protection	Banca Akros
		Banca IMI
		Deutsche Bank
		Mediobanca
Pac Certificate	Equity Protection	BNP Paribas
Pac Crescendo Plus	Equity Protection	Société Générale
Premium Certificate	Equity Protection	BNP Paribas
Protection	Equity Protection	UniCredit
Protection Barrier	Equity Protection	Banca IMI
Protection con Cap	Equity Protection	UniCredit
Protection Double Barrier	Equity Protection	Banca IMI
Short Protection	Equity Protection	UniCredit
Short Protection con Cap	Equity Protection	UniCredit
Express Protected	Express Protection	Banca IMI
		Deutsche Bank
		BNP Parisbas
		Mediobanca
		UniCredit

STRUMENTI A CAPITALE CONDIZIONATAMENTE PROTETTO

Nome	Tipologia	Emittenti
Accelerator	Airbag	Banca IMI
Parachute	Airbag	Deutsche Bank
Bonus	Bonus	Banca Akros
		Banca IMI
		BNP Paribas
		Deutsche Bank
		Mediobanca
		Société Générale
		UniCredit
Bonus Autocallable	Bonus	Banca Akros
		Banca IMI
Bonus Autocallable Plus	Bonus	Banca IMI
Bonus Cliquet	Bonus	BNP Paribas
Bonus Plus	Bonus	Banca IMI
Bonus Pro	Bonus	BNP Paribas
Bonus Protection	Bonus	BNP Paribas
Double Bonus	Bonus	Banca IMI
PEPP	Bonus	Deutsche Bank
Reverse Bonus	Bonus	Banca Akros
		BNP Paribas
		Deutsche Bank
		Mediobanca
		UniCredit
Bonus Cap	Bonus Cap	Banca Akros
		Banca IMI
		BNP Paribas
		Deutsche Bank
		Mediobanca
		Société Générale
		UniCredit
Bonus Cap Doppia Barriera	Bonus Cap	UniCredit
Easy Express	Bonus Cap	BNP Paribas
		Société Générale
Recovery 80	Bonus Cap	BNP Paribas
Recovery Bonus Cap	Bonus Cap	Mediobanca
		Société Générale
Reverse Bonus Cap	Bonus Cap	BNP Paribas
		Deutsche Bank
		Mediobanca
		Société Générale
		UniCredit
Top Bonus	Bonus Cap	BNP Paribas
		UniCredit
Top Bonus Doppia Barriera	Bonus Cap	UniCredit
Bonus Plus	Cash Collect	BNP Paribas
Cash Collect	Cash Collect	Banca IMI
		BNP Paribas
		Deutsche Bank
		UniCredit
Cash Collect Autocallable	Cash Collect	Banca IMI
		UniCredit
Cash Collect Doppia Cedola	Cash Collect	UniCredit
Cash Collect Protection Plus+	Cash Collect	Société Générale
Equity Premium	Cash Collect	Banca Akros
Equity Premium Autocallable	Cash Collect	Banca Akros

ACEPI

La mappa dei Certificates

Figure 9 (Source: ACEPI)

STRUMENTI A CAPITALE CONDIZIONATAMENTE PROTETTO

Nome	Tipologia	Emittenti
Premium	Cash Collect	Banca IMI
Reverse Cash Collect	Cash Collect	BNP Paribas
Reverse Equity Premium	Cash Collect	Banca Akros
Reverse Phoenix	Cash Collect	Mediobanca
Short Cash Collect	Cash Collect	UniCredit
Digital	Digital	Banca IMI
Alpha Express	Express	Deutsche Bank
Athena	Express	BNP Paribas Mediobanca Société Générale
Athena Airbag	Express	BNP Paribas
Athena Buffer	Express	BNP Paribas
Athena Bull & Bear	Express	BNP Paribas
Athena Coupon Locker	Express	BNP Paribas
Athena Double / Chance / Win	Express	BNP Paribas
Athena Early Coupon	Express	BNP Paribas
Athena Duo	Express	BNP Paribas
Athena Fast Bonus	Express	BNP Paribas
Athena Fast Plus	Express	BNP Paribas
Athena Floating Plus	Express	BNP Paribas
Athena Jet / Cap / Relax	Express	BNP Paribas
Athena Jump	Express	BNP Paribas
Athena Lockback	Express	BNP Paribas
Athena Match Race	Express	BNP Paribas
Athena Plus	Express	BNP Paribas
Athena Premium Plus	Express	BNP Paribas
Athena Protection	Express	BNP Paribas
Athena Relax	Express	BNP Paribas
Athena Scudo	Express	BNP Paribas
Athena Step Down	Express	BNP Paribas
Athena Trio	Express	BNP Paribas
Athena Up	Express	BNP Paribas
Autocallable Preformance Plus	Express	Deutsche Bank
Double Express	Express	Banca IMI
Express	Express	Banca IMI Deutsche Bank Mediobanca UniCredit
Express Coupon	Express	Société Générale Mediobanca UniCredit
Express Doppio Centro	Express	Deutsche Bank
Express Plus	Express	Banca IMI Deutsche Bank Mediobanca
Express Premium	Express	Banca IMI
Express Protection Plus+	Express	Société Générale
Fixed Premium Athena	Express	BNP Paribas
Flored Footer	Express	BNP Paribas
Phoenix	Express	BNP Paribas Mediobanca Société Générale
Reverse Autocall	Express	Mediobanca
Reverse Express	Express	Deutsche Bank Mediobanca
Tris	Express	Deutsche Bank
Outperformance Protected / Cap	Outperformance Cond. Prot.	Mediobanca Société Générale
Reverse Twin Win	Twin Win	BNP Paribas

STRUMENTI A CAPITALE CONDIZIONATAMENTE PROTETTO

Nome	Tipologia	Emittenti
Twin Win	Twin Win	Banca Akros Banca IMI BNP Paribas Deutsche Bank Mediobanca Société Générale UniCredit
Twin Win Autocallable	Twin Win	Banca Akros Banca IMI BNP Paribas UniCredit
Twin Win con Cap	Twin Win	Banca Akros Mediobanca Société Générale UniCredit
Twin&Go	Twin Win	Deutsche Bank Société Générale

STRUMENTI A CAPITALE NON PROTETTO

Nome	Tipologia	Emittenti
Benchmark/Tracker	Benchmark/Tracker	Banca IMI Mediobanca Société Générale UniCredit Vontobel
Benchmark Open End	Benchmark/Tracker	UniCredit
Outperformance	Benchmark/Tracker	Banca IMI Mediobanca
Reflex	Benchmark/Tracker	Banca IMI
X-Pert	Benchmark/Tracker	Deutsche Bank
Discount	Discount	Deutsche Bank Mediobanca Société Générale UniCredit
Double Chance	Outperformance	Deutsche Bank
Jet	Outperformance	BNP Paribas
Outperformance	Outperformance	Deutsche Bank Mediobanca Société Générale

STRUMENTI A LEVA

Nome	Tipologia	Emittenti
Corridor	Corridor	Société Générale UniCredit
Covered Warrant	Covered Warrant	Banca IMI BNP Paribas Société Générale UniCredit Vontobel
Facktor	Leva Fissa	Deutsche Bank
Leva Fissa	Leva Fissa	BNP Paribas Société Générale UniCredit Vontobel
Mini Future	Leva variabile	BNP Paribas UniCredit
Turbo	Leva variabile	BNP Paribas UniCredit

Figure 10 (Source: ACEPI)

2.2.1. Capital Protection Certificates

Certificates with capital protection allow the investors to invest in financial asset eluding the risk of losing the invested capital, if purchased at issue and held till maturity. If such certificates are bought on the secondary market, there is no full capital protection (it is only possible to forecast the worst possible performance of the certificate if held till maturity), keeping out of the equation the CCR (Counterparty Credit Risk), which however characterize every single structured product.

This kind of product find its success in the growing need of safety by the investor, fitting their needs with the strong prudential predilection which characterize them. For this reason capital protection certiifcates turn out to be seen with a positive eye also from the supervisory bodies, especially because their payoffs are clear, a characteristic that explains why they are widespread among retail clients.

"Capital guaranteed products guarantee the redemption of the invested capital at maturity in addition to participating to a certain degree to the performance of an underlying risky asset"[3].

In this category we find: Equity Protection Certificates, Digital Certificates and Express Protection Certificates, but also other more particular instrument such as Butterfly Certificates and Twin Win Protection.

Equity Protection Certificates are the simplest product in this category and they are meant for those investors who want to participate, to a certain extent, to the performance of the underlying financial asset, without carrying the risk of losing all, or part of, the capital invested, since a level of protection below which the settlement amount can not drop is set, also in case of negative performance of the underlying.

The level of protection, expressed as a percentage of the initial evaluation value (the strike price) of the underlying asset is determined when the instrument is issued. Equity Protection Certificate may also have a maximum upper limit to participation in the movements of the underlying financial asset (Cap Level).

Depending on the characteristics established initially by the issuer, participation in the movements of the underlying may be less than, equal to or greater than 100%. The weight of

[3] Bluemke, 2009.

the tracking performance of the underlying is indicated by the participation rate. Depending on the type and characteristics of Equity Protection certificates, at maturity of the Certificate there are several scenarios that an investor may come across.

Assuming a Long Equity Protection certificate, there are two scenarios at maturity:

- The final value of the underlying financial asset is less than the Protection Level: the investor receives a Settlement Amount equivalent to the level of capital protection;
- The final value of the underlying financial asset is higher than the initial evaluation level: the investor receives the positive performance wich will be proportional to the participation factor provided.

In the case of Equity Protection Long Certificates with Cap, a further option can be added to the above mentioned scenarios, namely the final evaluation value of the underlying financial exceeds the Cap level at maturity: in this case, the investor will receive a settlement amount equal to the Cap itself, without benefitting from additional increases recorded by the underlying financial assets.

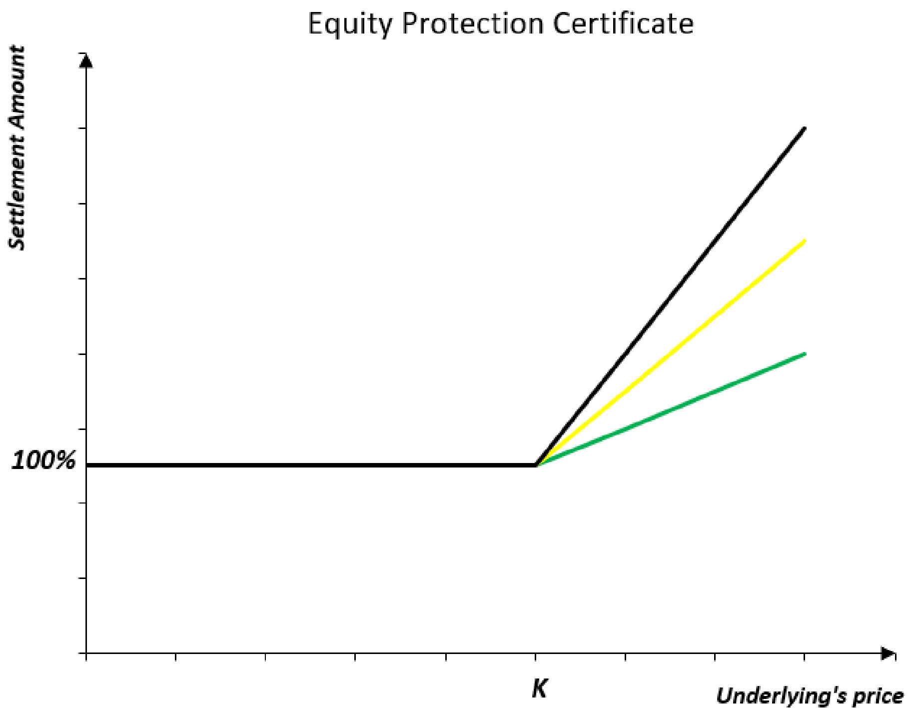


Figure 11 (Personal elaboration)

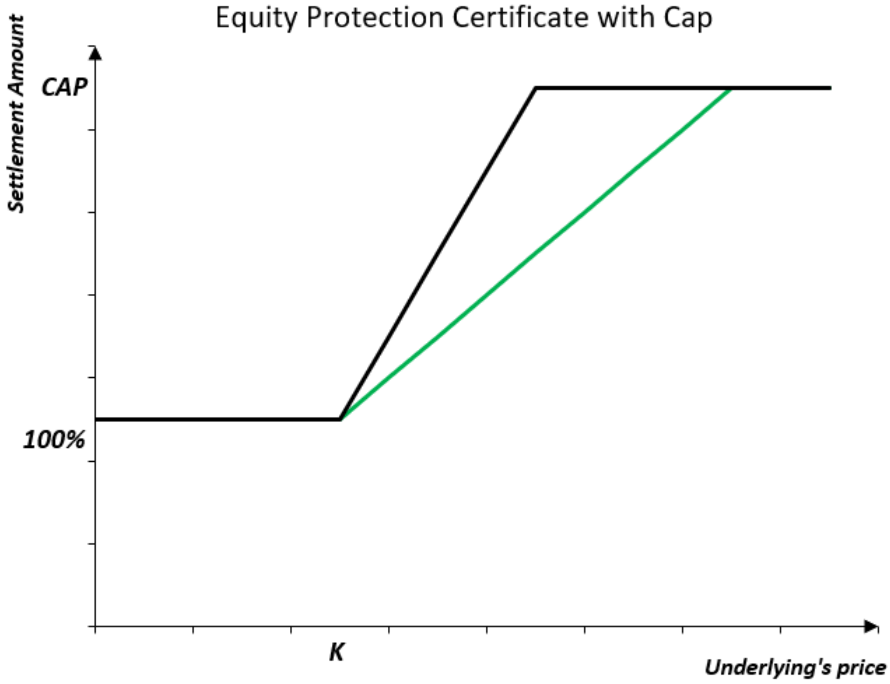


Figure 12 (Personal elaboration)

In the graphs provided 100% is the protection level with respect to the face value of the certificate, i.e. the percentage below which the final settlement can not go at maturity. Equity Protection Certificates may have a protection level that is less than 100%, staying in the capital protection category anyway.

K is the strike price, in that the level above which the settlement amount begins to grow commesured to a participation rate which is graphically explained by the slope of the curve for values greater than x.

In the second graph the particular case of an Equity Protection Certificate with Cap is drew, therefore the settlement amount can not exceed the Cap Level fixed by the issuer. Nevertheless, this particular kind of Equity protection Certificate can be endorsed with a participation rate, as highlighted by the slope of the curves in the graph.

Digital Certificates pay a coupon, usually annual, in case the underlying is above, on predetermined observation dates, a predefined level and a premium at maturity if the underlying is greater or equal to the initial level. If this is not the case, the product provides full capital protection (or partial as in the Equity Protection case).

They can be considered a capital protection variation of another certificate, the Cash Collect, which is why they are also known as Cash Collect Protection. Other commercial definitions are Target Cedola and Protection Premium.

The Long version of a Digital Capital Protection Certificate, equip the investor with the opportunity to receive a regular predetermined income. This remuneration is called a Digital Amount and is paid at the occurrence of a digital event. It occurs when the reference value of the underlying asset, during the respective digital evaluation periods, is at or above the Digital level or "trigger", which is usually, but not always, the Strike Price itself.

At maturity, there are two possible scenarios for an investor holding a Digital Capital Protection Certificate:

- If the value of the underlying financial asset is quoted at a level equal to or higher than the initial evaluation, he receives the return of the issue price of the Certificate plus the last Digital Amount determined at the time the instrument was issued;
- If the value of the underlying financial asset is quoted to a level below that of initial evaluation he receives only the issue price of the Certificate, or whichever protection percentage of it, expressed in the final prospectus.

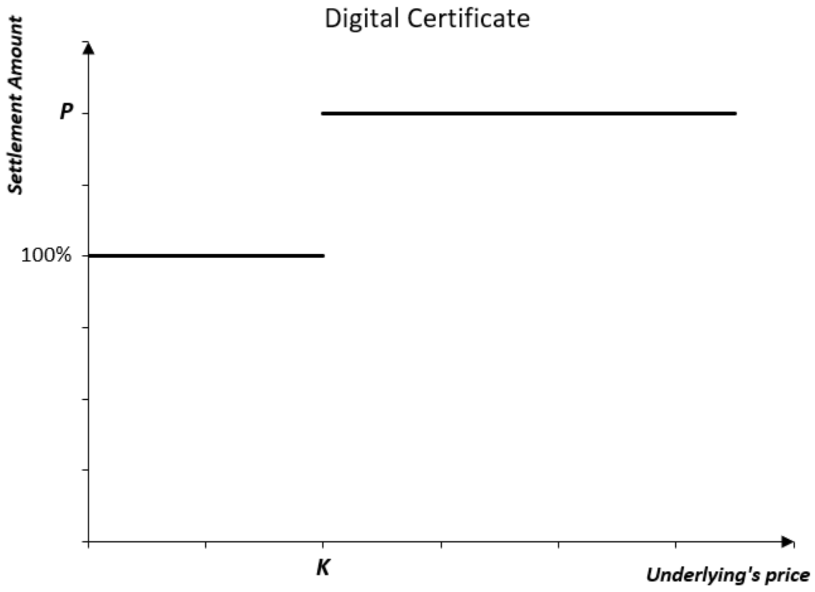


Figure 13 (Personal elaboration)

P is the premium paid via periodic coupons and not at once, and K is the Strike Price, or whichever level set during the issuing phase above which the investor is allowed to receive the coupons.
Also in this case 100% is the protection level, which could be also set at a lower percentage of the face value.

Express Protection Certificates represents a capital protected variation of the Express Certificate. This product allows to receive the payment of a premium, that increases over time (a feature known as memory effect), upon occurrence of a predetermined event (trigger) and in such case, the early redemption of the capital takes place (autocallability).
During the issuing phase, in correspondance to the initial fixing, the trigger level above (or below, in the case of a Short Express protection Certificate) which the underlying asset must be, in order for the early redemption of the face value together with a predetermined premium to take place, is set.
At maturity the following are the possible different scenarios an investor may encounter:

- During the life of the certificate, the underlying asset, in corrispondance to at least one of the predetermined observation dates, is above (or below for a Short investor) the trigger level: in this case the investor will receive an automatic early redemption amount which will be inclusive of the face value of the certificate and the premium correspondent to the observation date (the further the observation date, the bigger the premium, due to the so called "memory effect", thanks to which every premium not paid is sum up to the following, until conditions for an early redemption occur).
- At maturity the value of the underlying asset is below the Strike Price: the investor will receive an amount equal to the predetermined percentage of protection.
- At maturity the value of the underlying asset is above the Strike Price, but below the trigger level: in such case only the face value will be paid[4].

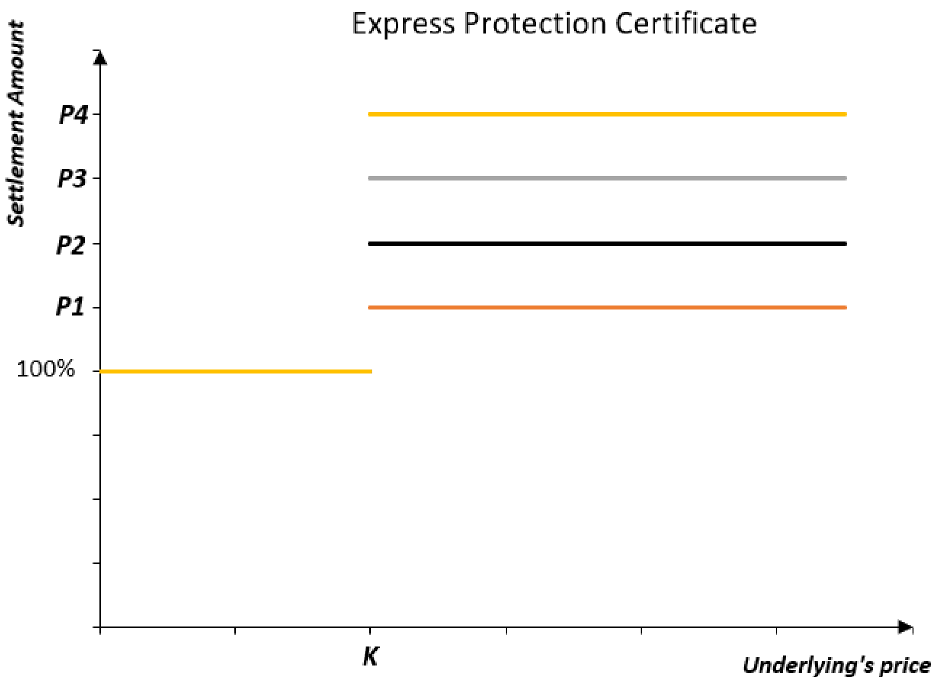


Figure 14 (Personal elaboration)

[4] For an Express Certificate whose protection level is set at 100%, the last two scenarios lead to the same result.

In the graph above 100% is the protection level below which the settlement amount can not go, P_i are the premium amounts an investor can receive, together with the face value, depending on the observation date i in which the underlying asset's value is above the strike price K (or any other predetermined trigger level).

2.2.2. Conditional Capital Protection Certificates

Certificates characterized by conditional capital protection are financial products that allow investors to indirectly invest in certain assets while enjoying a partial capital protection, which is activated if pre-arranged barrier levels are not reached during the life of the certificate.
The barrier for this kind of Certificates can be either of American type, for which the underlying asset can not reach the predetermined barrier level (or Knock-Out Level) for the whole life of the instrument, or European type, for which the value of the underlying asset with respect to the Knock Out Level is verified only at maturity.
The most important Certificate among the Conditional Capital protection category is the Bonus Certificate, thanks to one of the highest risk/return trade off available.
Other important certificates belonging to this category are: Cash Collect, Express, Outperformance and Twin Win.

Bonus Certificates can be used to invest in underlying financial assets such as shares, stock indexes, currencies, commodities or interest rates.
The singulairty of this type of product is the ability to provide the investor a return at maturity through the payment of a premium, the so called Bonus, even in event of a moderate fall of the underlying asset. For this type of Certificate, usually, the capital protection at maturity is conditional on the underlying asset reaching a specified price, called the Barrier Level[5].

[5] The barrier for Bonus Certificates is tipically of American type, even if there is no shortage of Bonus Certificates with European barriers.

The Bonus Certificate allows the investor to obtain a return on capital invested called Bonus even in the event of an eventual decline of the underlying asset. The ability to achieve the Bonus is guaranteed unless the value of the underlying asset reaches a predetermined level and is expressed in percentage terms and called the Barrier level. The evaluation of the Barrier, as mentioned before, may take place at maturity, or during the lifetime of the Certificate and if this level is reached, the investor receives a Settlement Amount at maturity related to the negative performance of the underlying asset.

At the maturity of a Bonus Certificate there are two possible scenarios the holder of the Certificate may incur:

- If the value of the underlying has not reached the Barrier Level, the investor will receive the higher of the value of the invested capital plus the performance of the underlying on the final observation date and the Bonus which represents a percentage greater than 100% of the issue price.
- If the value of the underlying has reached the Barrier, the investor will receive a settlement amount which corresponds to the performance of the underlying.

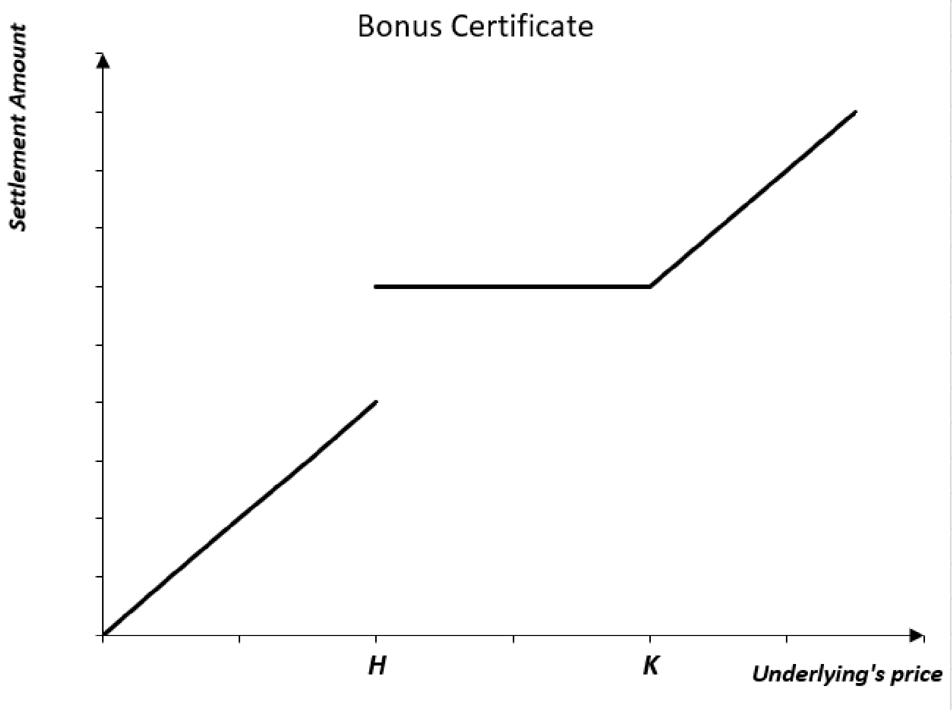


Figure 15 (Personal elaboration)

In the graph above H is the barrier level below which the settlement amount will be linearly reduced by the negative performance of the underlying asset with respect to the Strike Price and above which the investor is rewarded with the Bonus.
K is the Strike Price: if the underlying asset's value at maturity is above that level the settlement amount will follow its positive performance with respect to the strike price.

The main feature of ***Cash Collect Certificates*** is the payment of a periodic premium over the life of the instrument: they can be guaranteed premiums, depending on the performance of the underlying asset up until maturity or a combination between the two, i.e. assured premiums for a certain period of time and then conditional premiums for the rest of the certificate's life.
At maturity, the investor will receive the issue price plus the last premium if the underlying asset is equal to or above a specific fixed level ("barrier level") set at issuance. On the other hand, if the price of underlying asset is below the barrier level, the investor is basically long in the underlying asset. However, in case of a downside performance, the Cash Collect Certificate allows an investor to reduce the loss thanks to the premiums paid during the certificate's life.
Cash Collect Certificates can distribute premiums without any condition, regardless of the trend of the underlying financial asset in the first months of the life of the certificate. The payment of these premiums occurs at pre-established dates, which are determined at the time of the issuance of the certificate. In addition to the fixed premiums, Cash Collect Certificates also provide for the payment of conditional premiums if the value of the underlying financial asset is equal to or above the barrier level at the pre-established dates determined at the time of issuance. There are two possible scenarios at maturity:

- The final value of the underlying financial asset is equal to or above the barrier level: in this case, the certificate reimburses the issuance price plus the last premium;
- The final value of the underlying financial asset is below the barrier level: in this case, the investor receives a reimbursement correlated with the negative performance of the underlying asset, calculated with respect to the strike price.

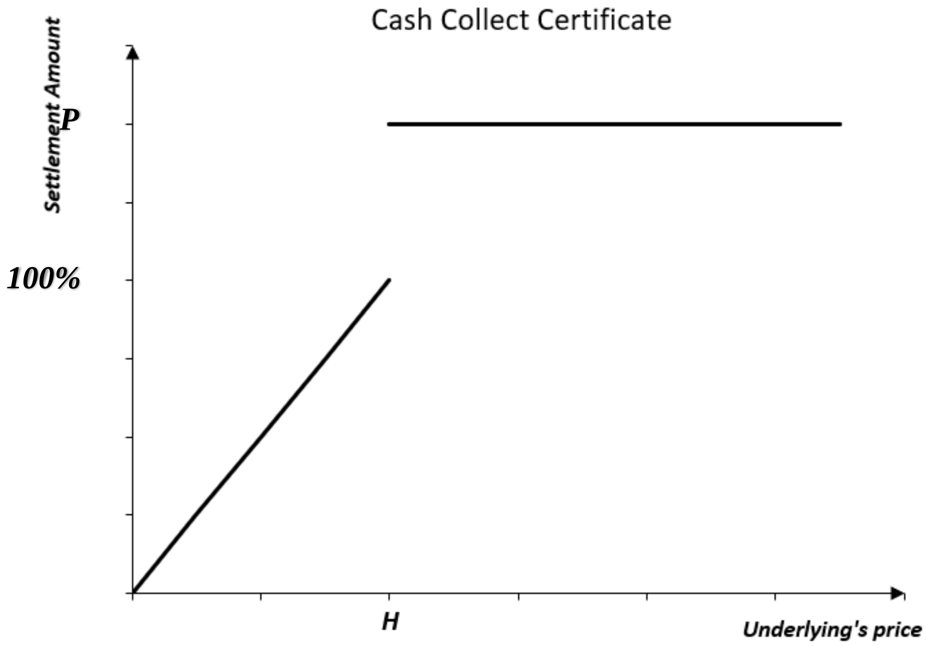


Figure 16 (Personal elaboration)

In figure 16. H is the barrier level, above which the investor will receive the premium P , which is paid periodically and not in one unique solution as it happens for Bonus Certificates[6], while 100% is the face value of the certificate.

The distinctive feature of ***Express Certificates*** is that they can expire in advance, upon the occurrence of predetermined conditions, established at the time the Certificate is issued, redeeming to the investor the issue price as well as an additional amount. The protection of the issue price of the Certificate is subject to the success of the underlying asset not to reach a certain threshold set at the time of issue: the Barrier level.

There are three different scenarios for such a certificate:

- During the life of the certificate, the underlying asset is above (or below, if Short) a predetermined trigger level in correspondance to at least one of the observation dates: in this case the investor will receive an automatic early redemption amount together with a coupon whose value depends on the observation date in which the early

[6] In particular this graph is identical to the Bonus Cap Certificate's one. Both instruments pay a premium to the investor if predetermined conditions occur, the difference stays in how that premium is paid: periodically via coupons in this case; at maturity and all at once for the Bonus Cap.

redemption conditions are met (as for the Capital Protection case, the further the observation date, the bigger the premium, thanks to the already discussed “memory effect”)

- At maturity the value of the underlying asset is lower than the initial fixing but higher than the barrier level: the investor will receive a settlement amount which will be exactly equal to the face value.
- At maturity the value of the underlying asset is above the Strike Price but below the predetermined trigger level: the investor will receive the face value exclusively
- At maturity the value of the undelying asset is below the barrier level: the settlement amount in this case will be reduced proportionally to the negative performance of the undelying aset itself with respect to the initial price.

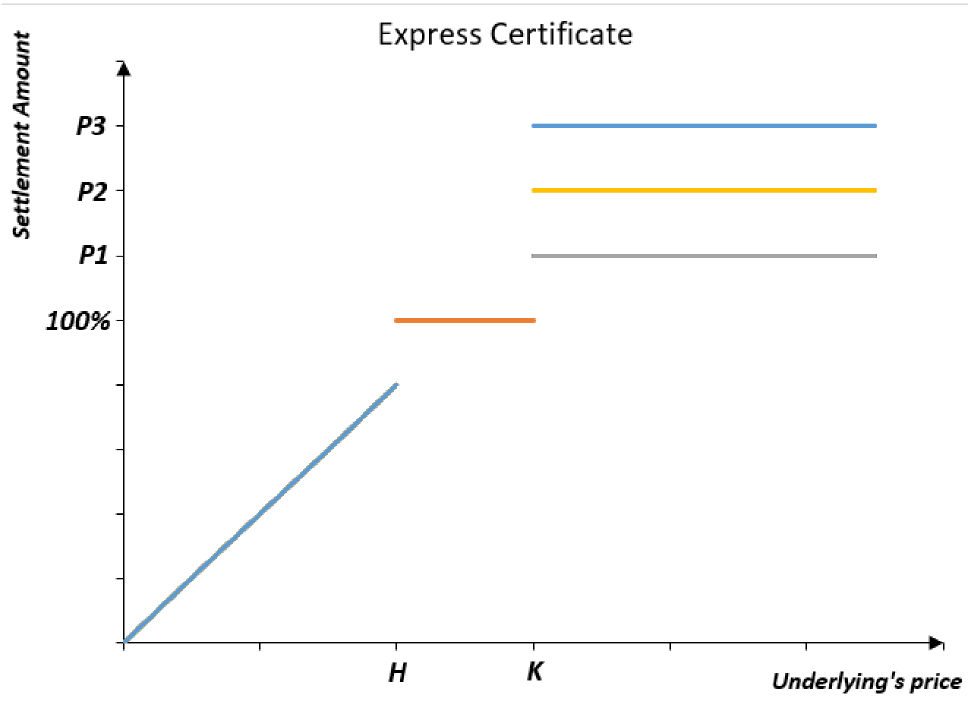


Figure 17 (Personal elaboration)

In the graph above P_i are the different premium that one could redeem together with the face value at maturity, depending on the observation date the condition for an early redemption event to happen occur; 100% is the face value, H is the barrier level below which the investor starts suffering losses and K is the Strike Price that, in this case, coincide with the trigger level[7].

The most important characteristic of an ***Outperformance Certificate*** is the ability to replicate proportionally the performance of the underlying asset, giving the investor a reasonable protection subordinated to a Barrier Event.

The Outperformance is based on a participation rate that can be either a downside participation rate, which allows the investor to mitigate the losses accrued in the underlying asset during the life of the Certificate thanks to the "Air Bag", a factor that is activated when the value of the underlying at maturity has violated the "Barrier Level" or upside participation rate, which, on the other hand, allows to enhance profits. Upside Participation in the movements of the underlying financial asset may be limited to a maximum value, called Cap.

The following scenarios may occur at maturity:

- The final evaluation value of the underlying financial asset is greater than the initial evaluation. In this case, the investor receives the full issue price of the Certificate plus the accrued performance from the underlying, multiplied by the upside participation rate, if any;
- The final evaluation value of the underlying financial asset is lower than the initial evaluation but a Barrier event has not occurred. In this case, the investor receives just the issue price of the Certificate;
- The final evaluation value of the underlying financial asset is lower than the Barrier Level. In this case the investor loses the conditional protection on the capital but the factor Airbag allows the investor to mitigate losses.

  If no downside participation rate is provided, the settlemet amount will be proportionally reduced by the negative performance of the underlying asset calculated with respect to the Strike Price.

[7] Trigger Levels can easily be set above the Strike Price.

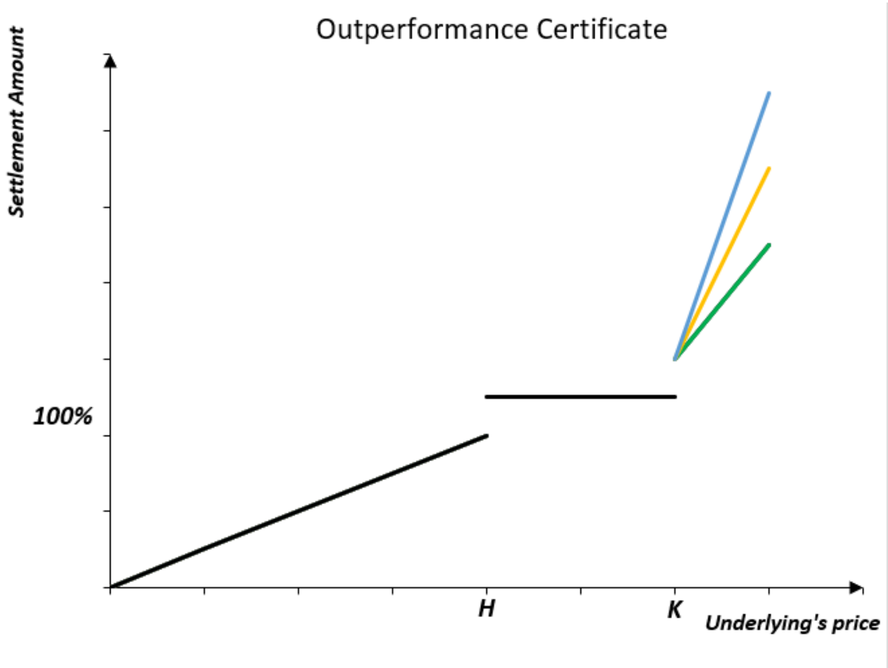


Figure 18 (Personal elaboration)

This first graph represents the Settlement Amount the investor is going to redeem at maturity with respect to movements of the underlying asset for an Outperformance Certificate that provides an upside participation rate, which is graphically represented by the different slopes of the lines from the Strike Price K on; H is the barrier level and 100% is the issue price of the Certificate.

If, instead, a downside participation rate is provided (Airbag), figure 19 graphically explains what the settlement amount evolution will be like:

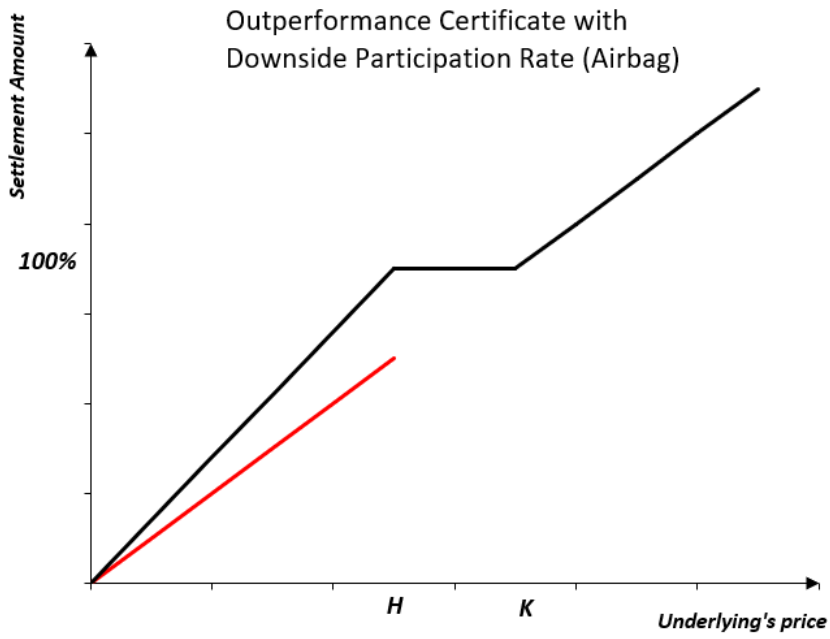


Figure 19 (Personal elaboration)

The red line is the price of the underlying asset, and it is clear that the Airbag allows to reduce losses with respect to it. On the other hand, there is no provision of profit enhancement participation rate, therefore, from K on, the settlement amount will linearly follow the underlying.

In order to structure Outperformance Certificates with Airbag a Cap may be needed or at least a reduced upside participation rate (below 100%), in that or the profit are stopped at the Cap level or the settlement amount will follow the underlying less than linearly[8].

Twin Win Certificates allow investors, at maturity and within certain limits set at the time the Certificate was issued, to participate in the performance, in absolute value, of the underlying asset, whether positive or negative and favourably for the investor. The extent of this participation in the performance of the underlying financial asset is fixed at the time the Certificate is issued. The condition is that the underlying at maturity is worth more than the level set when the Certificate was issued. This threshold level is called the Barrier level and

[8] Otherwise the certificate would be too expensive for the Issuer.

breaching this Barrier causes the loss of the capital protection, resulting in a decrease of the investment value for the holder of the Certificate.

Twin Win are Certificates suited to take advantage of the uncertainty in the financial markets. At maturity they may benefit from both positive performance by the underlying asset, participating in them with a predetermined participation rate and negative by converting it in gain, using a predetermined weighted participation rate, until the Barrier level.

In the case of a Barrier Event the Certificate loses capital protection and the investor receives at maturity a loss commensurate to that he would have had by investing directly in the underlying at the initial evaluation date of the Certificate. The Certificate's participation in the movement of the underlying asset may be less, equal to or greater depending on the participation factor set by the issuer at the time the Certificate was issued.

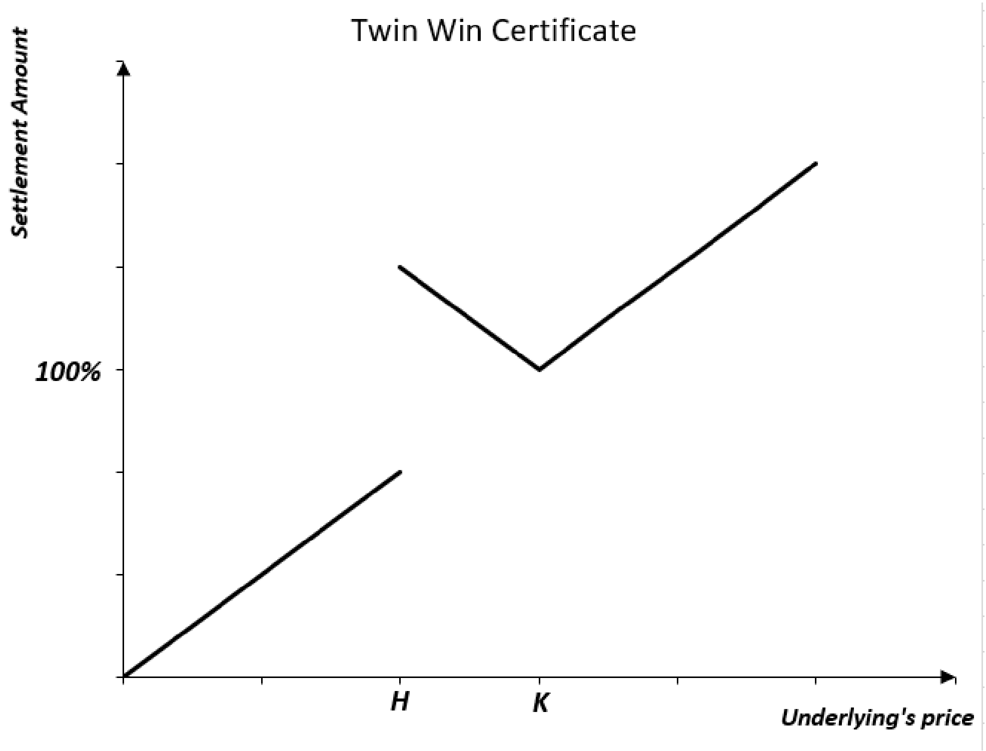


Figure 20 (Personal elaboration)

In the graph above is well explained how the settlement amount grows for absolute changes of the undelying asset's value, at least until it stays above the barrier level H[9].

2.2.3. No Capital Protection Certificates

Certificates with no capital protection are very simple products whose aim is to replicate the performance of their underlying assets and for this reason investors are exposed to the very same risk of investing in the underlying asset.
Some certificates belonging to this category may replicate the underlying asset's performance in proportions.
They are classified in Benchmark and Outperformance.

Benchmark certificates are the oldest certificates without any capital protection provision and what they do is to replicate the exact performance of the undelying financial asset, in fact they are also known as "delta 1 certificates"[10]. They can replicate upward or downward trends, depending whether they have a short or long exposition on the underlying asset.

[9] The slope of the lines from H on can vary depending on the participation rate set by the issuer.
[10] As will be later discussed, the greek letter delta is a measure of sensitivity of the options' price (and therefore certificates') to movements of the underlying financial asset. A delta equal to 1 means that for a 10% change in the underlying price, the certificate will have the very same change in its.

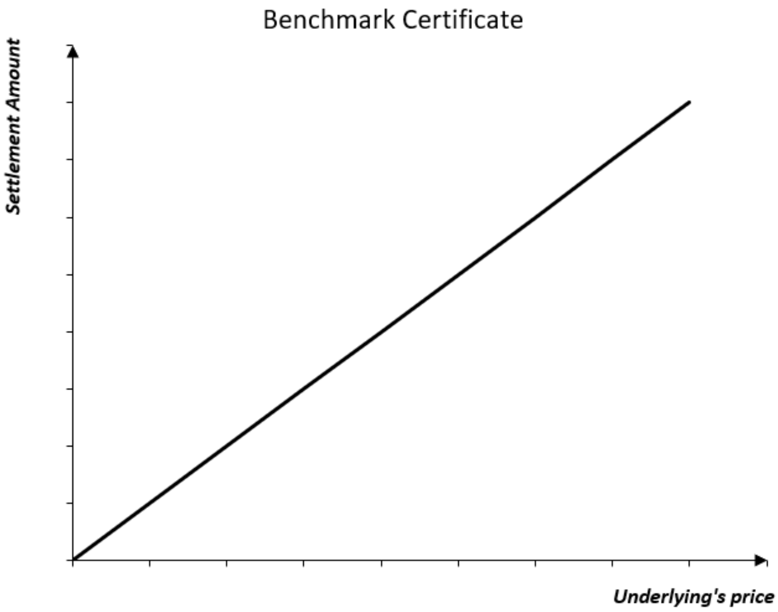


Figure 21 (Personal elaboration)

Outperformance Certificates are constructed exactly like a Benchmark, except they have an important feature added: they replicate the exact upward (or downward) perfomance of the underlying financial asset until a certain level (usually the strike price), from which they start amplificating it, in relation to a predetermined participation rate.

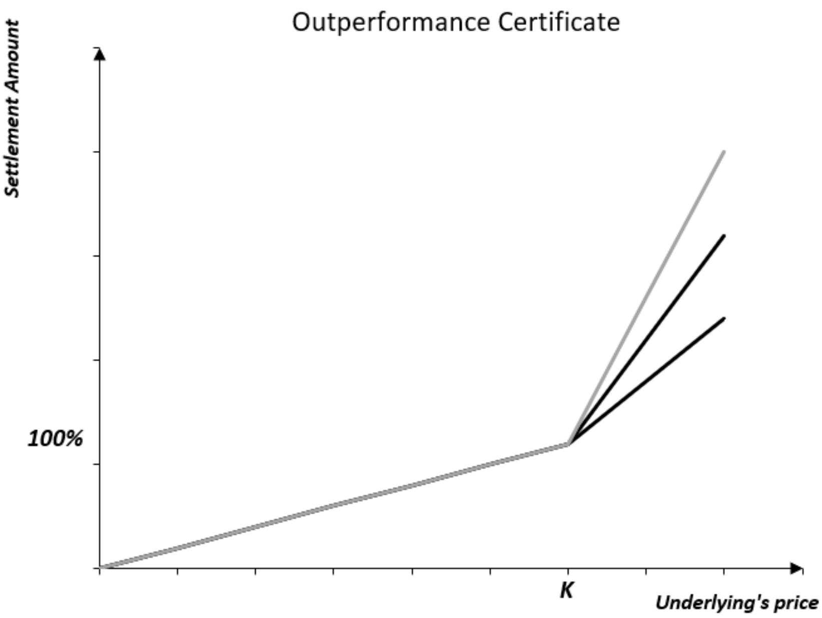


Figure 22 (Personal elaboration)

In the graph above the slope of the lines from K on is determined by the participation rate set by the Issuer.

2.2.4. Leverage Certificates

Leverage certificates are financial products providing investors with a more than proportional exposure to the performances of their underlying assets, making the benefitting from either positive or negative performances of the underlying asset possible, depending on the typology of the certificate.

They are classified in constant leverage and variable leverage. The latter have been created in order to avoid the so called Compounding Effect generated by the constant leverage certificates held in portfolio in the long time horizon.

The calculation basis for Fixed Leverage Certificates is set on a daily basis depending on the closing price of the underlying financial asset.

For this reason, for holding periods greater than one trading day, the so called "Compounding effect" is generated.

If the product is held in portfolio for more than one day its performance could dramatically differ from the underlying asset's in the same period, in addition it is multiplied by the leverage. This difference could even worsen for significant changes in the underlying value, as explained by the following graph illustrating an example of Compounding Effect for a Fixed Leverage 7x Certificate:

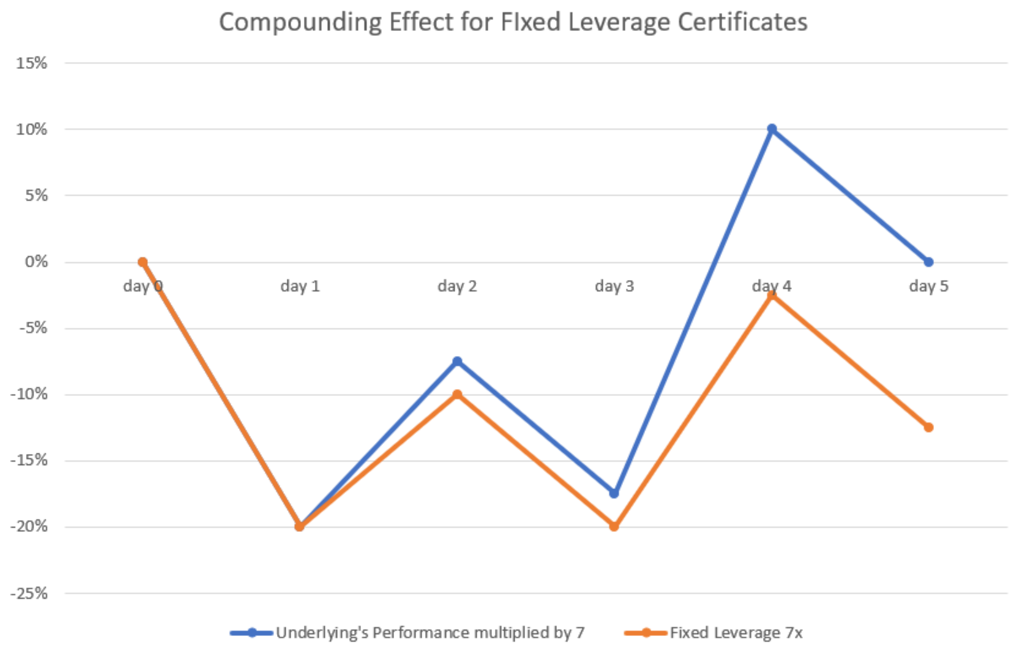


Figure 23 (Personal elaboration)

Above is shown the Compounding Effect that causes a meaningful underperformance of a Fixed Leverage 7x Certificate held in portfolio for 5 trading days with significant changes in the underlying asset’s price.

3. Options

Once it is clear how a certificate works it is possible to move forward to the analysis of their detailed composition, namely the options available for a Bank that wants to structure and issue such financial products.

In the following part we leave space to a focus on the financial options, that surely are the core of certificate's world, starting from Plain Vanilla Options before proceeding to Exotic ones.

Knowing deeply the characteristics of every single option inside these structured product allow to give technical and operational reason to the choices made by the Issuer Bank in terms of structures proposed and also to understand how and in which measure the price of a Certificate will be sensitive to the variations of the underlying.

This chapter will brings particular attention to the world of exotic options, which are the main reason for the asymmetrical settlement profile that characterize investment certificates. These particular kind of options, indeed, stands out for high sensitivity of their prices to market's variables, first of all the volatility.

While we talk about plain vanilla for derivatives such put and call option both american or european type, since they present standard properties, with well defined and clear payoffs and traded actively by brokers o regulated markets (the first options market to ever exist was the Chicago Board of Options Exchange in 1973), with the denomination of exotic options we mean all the non standard contracts, traded OTC (Over The Counter) and therefore outside of the regulated markets, which have been create in time by financial engineers.

Referring to the timeless classification made by John Hull in his "Options, Futures and Other Derivatives", inside the exotic options category we find: Non-standard american, gap, forward start, shout, compound, chooser, barrier, binary, lookback, cliquet, asian and basket (or raimbow) options.

The last six options are the most frequently used by ingeneers while structuring Investment Certificates and therefore we will mainly focus on their analysis.

3.1. Plain Vanilla Options

A ***call option*** is a derivative instrument that guarantees the purchaser the right, but not the obligation, to buy a stock (called underlying) at maturity (or by maturity) at a given price (strike price K).

In the case of exercise of index options, it is not possible to receive the underlying but only the cash consideration.

Obviously, the exercise will make sense (excluding the cost paid to purchase the option, the so-called "premium") only if the price of the underlying (S) will be higher than the strike (K) and the profit will be equal to the difference between the price of the market and the strike, it follows that the option payoff, upon expiry of the contract, is given by:

$$max(S - K\,, 0) = (S - K)^{+}$$

A put option is a derivative instrument that grants the holder the right to sell the underlying at a fixed price at maturity. In this case, the exercise will make sense (always excluding the cost paid to purchase the option, the so-called "premium") only if the price of the underlying will be lower than the strike; the realized profit will amount to the difference between the strike and the market price:

$$max(K - S\,, 0) = (K - S)^{+}$$

A put option (as well as a call option) can present itself in different styles. The most relevant types are the European put option, which can be exercised only on a certain date (maturity) and the American put option, which can be exercised at any time between the conclusion of the contract and the deadline.

The following chart summarizes the risk profile (profits and losses) related to the purchase of a call option:

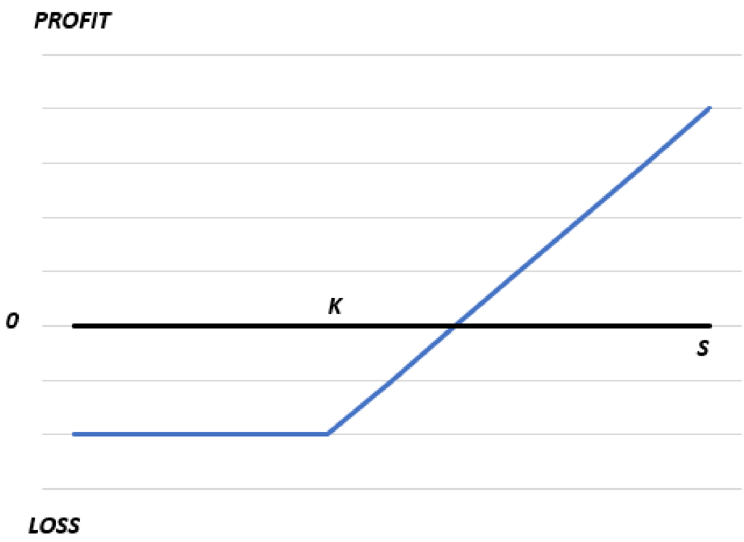


Figure 24. Profit and loss diagram for a long call option. (Personal elaboration)

The horizontal axis of the graph indicates the price of the underlying: to the right there is a price increase, to the left, ie to the origin of the axes, prices fall.
The vertical axis indicates instead the profits (or losses) of the purchaser of the option. As already mentioned, the exercise of the option will acquire meaning only if the market price of the underlying will be higher than the strike price. Since the purchase of the call has a cost (it is the premium that must be granted to those who grant us the option, to those who agree to guarantee the purchaser the right to buy the underlying at the set price) the payoff chart of the call has started in negative territory. In the event of a fall in prices, the value of the call will tend to zero and the maximum loss that the investor will bear will be the premium paid.

The **put option** allows you to earn if the market goes down. The buyer of put options wants to bet on the downside of the market without the costs associated with "short selling" (short selling, ie securities that do not own), and without exposing themselves to possible losses if the market goes in the opposite direction to that hoped.

Figure 25. Profit and loss diagram for a long put option. (Personal elaboration)

In general, the "long" positions on the options allow you to take a position by betting on the rise or fall of the market, with a possible maximum loss limited to the premium paid to purchase the option.

The option buyer has the right not to exercise his right to maturity (or within the deadline for American options), and hence the limit to the losses that may be incurred. On the contrary, the seller is always obliged to honor the commitment provided by the option he has granted to the buyer.

In the case of a call seller, the payoff will be as follows:

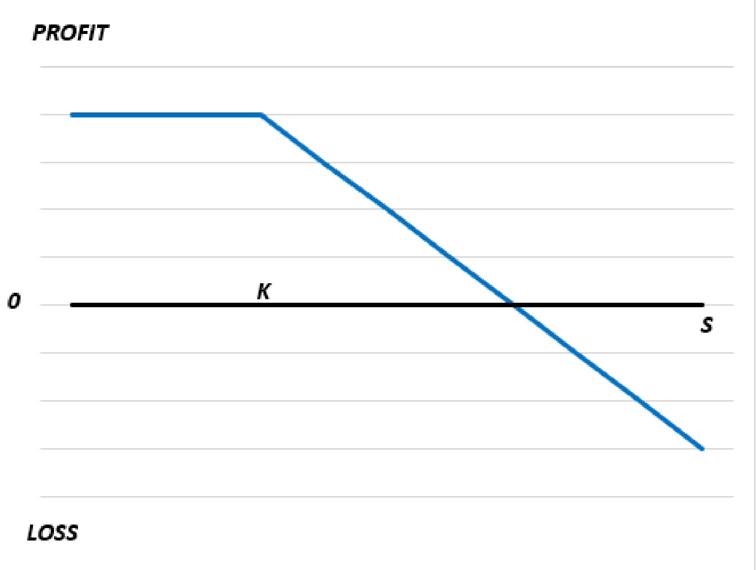


Figure 26. Profit and loss diagram for a short call option. (Personal elaboration)

The initial fixed profit (the premium received) decreases with the increase in the price of the underlying above the option strike: the seller of the option therefore hopes that the market will remain steady or fall. Against a limited immediate profit, the loss is potentially unlimited.

In the case of the put:

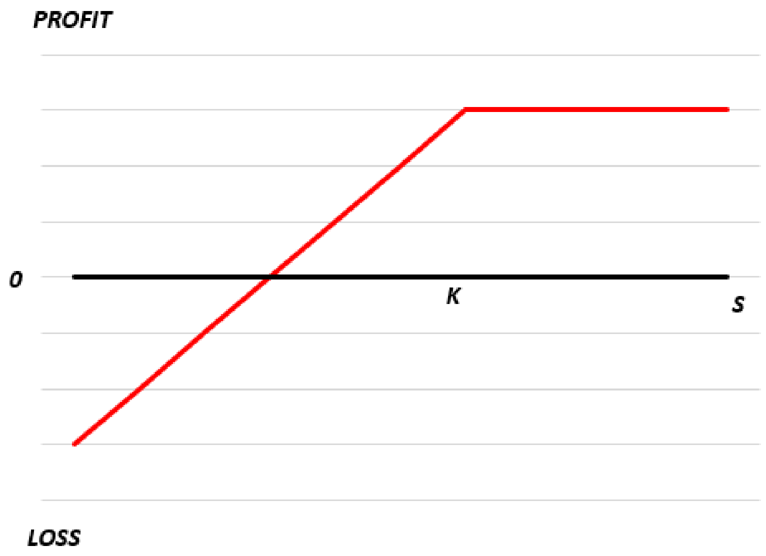


Figure 27. Profit and loss diagram for a short put option. (Personal elaboration)

In this case, however, the erosion of the initial premium received occurs if the price of the underlying decreases below the strike: the maximum profit will be obtained if the price remains constant or will rise. Even for the seller of the put the profit will be limited, while the loss "almost" unlimited (the underlying can not in fact take on a negative value, the maximum loss will be when this will have a value equal to zero).

Taking a short position in options has a risk profile certainly superior to the same transaction but in a long position. It is for this reason that very often the sale of these instruments is associated with the purchase and sale of other financial assets (for example, shares, ETFs, futures and options on indices and shares) to implement more complex investment strategies that can pursue different purposes such as hedging the position in shares or derivatives, increasing portfolio performance or pure speculation.

3.1.1. "Moneyness" of the Options

The profit or loss for the buyer of an option depends on the overrun (for the call) or the discount (for the put) of the exercise price by the prices of the underlying (net of the effect of the premium paid to buy the option).

If at maturity the price of the underlying will be lower than the strike, the holder of a call option will have more convenience to buy the underlying on the market and the option will not be exercised.

An option is know to be:

- "At the money" (ATM) when the exercise price is equal to the current price of the underlying asset.
- "In the money" (ITM) when the exercise price is lower (call) / higher (put) at the current price of the underlying asset.
- "Out of the money" (OTM) when the exercise price is higher (call) / lower (put) at the price of the underlying asset.

Let's consider the case of a put option with a strike at € 14 and illustrate it with a graphical example:

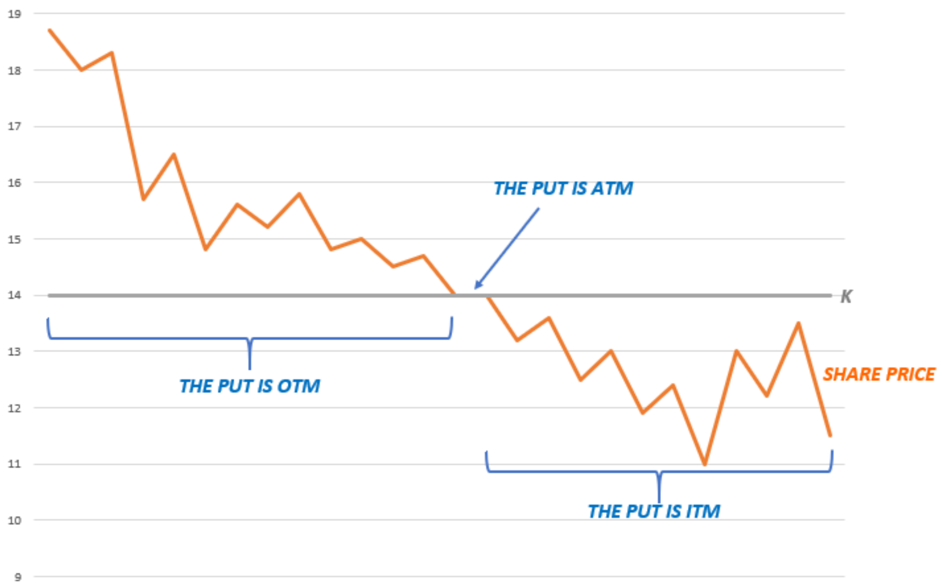


Figure 28. "Moneyness" of a put option. (Personal elaboration)

Values of the underlying below the exercise price (ITM put) will result in a profit and therefore the exercise of the right of the holder of the option to sell at a price higher than the market price; if the underlying will have a market price higher than the strike (OTM put), the option will not be exercised. In the case of the call the exact opposite occurs: if the price of the underlying is greater than the strike, the buyer will have a profit and will exercise the right to buy (call ITM), viceversa, for lower values, the call (OTM call) will not be exercised.

An ITM option at maturity is therefore the one that would give a profit if exercised: the value of this profit is the so-called intrinsic value of the option, which is the most important component of the price of an option close to maturity.

Once it is clear how the so-called plain vanilla options work, we can move on to discuss particular types of options, the exotic ones.

3.2. Exotic Options

The principal feature of a standard american option lays on the fact that it can be execised in whichever moment from the issue date till expiry. ***Non standard american options***, on the other hand, provide predetermined windows of time in which it is possible to be exercised, or the strike price could vary during the life of the option.
This kind of options can be, and usually are, valued using the binomial tree.

Following the above reasoning, ***Gap Options*** could easily be called non standard european options, since they are a modified version o plain vanilla options of european type.
Gap Call Option's Payoff is given by $S_t - K_1$, with $S_t > K_2$, meaning that the payoff of a regular european call option is increased (or decreased if $K_1 > K_2$) by the gap $K_2 - K_1$.
The payoff of a Gap Put Options, on the other hand is $K_1 - S_t$, with $S_t < K_2$.

As well explained by the name itself ***Forward Start Options*** are Options that will start sometimes in the future.
At initiation, a forward start option contract spells out all the defined characteristics relevant to any option, except for its strike price. The maturity date, underlying asset and size become set, as does the activation date. The only unknown for the contract itself is the strike. However, in terms of pricing the contract, the future price of the underlying asset is also, of course, unknown. Upon activation with its new strike price, the pricing of a forward start option becomes the same as any regular option.
For options on stocks, one benefit is that they enable investors to participate in the total value of the underlying company as the share price increases with dividend growth. In this case, the options holder does not take any market risk before the option becomes active.

A ***Shout Option*** is an option that allows the investor to stop a price of the underlying asset only once during the life of the option, in the case in which it considers the highest (or the lowest in the case of a shout put option) below, "declaring it" to the other party.

At maturity, the investor will be able to assert the price declared by him in the event that the final value of the underlying is lower (or higher), otherwise the final value will be used to calculate the payoff.

Compound Options are basically options on options and they can be of four types: a call on a call, a call on a put, a put on a call and a put on a put, and that's why they present two strike prices and two exercise dates. A call on a call option gives the investor the right but not the obligation to buy another option and clearly it will be exercised if the price at t_2 of the second option is lower than the exercise of the first.

A ***chooser option*** is an option in which the investor has the right but not the obligation to decide, after a specified period, whether his option is a call or a put, mathematically:

$$max\ (c\ ,p)$$

It may happen that the call and the put in question do not have same strike prices or maturity as in the standard case, in questo caso il loro comportamento si avvicina più ad una compound option che ad una chooser.

The exotic options which are most commonly used in the structuring of a certificate are certainly the ***barrier options*** (or digital options).
In this particular type of options, the payoff depends almost exclusively on whether the underlying reaches (or does not) a certain level called a barrier. One of the strengths of the barrier options is certainly the price, in fact they are much cheaper than the normal OTC options without barrier and it is through these tools that the issuers can build very interesting structures for the investor, given how much they can save in packaging.
The barrier options can be divided into Knock-Out and Knock-In: a KO stops existing if the predetermined barrier level is reached, while the KI begins to exist in this case.

Each of these types is in turn divided into various options: Down and Out, Up and Out (both call or put) in the case of KOs, Down and In and Up and In (call or put) in the case of KI.
Down-and-out and down-and-in provide that the barrier level is placed below the initial price (Strike Price) set at the time of issue; on the contrary, up and out and up and in have a higher barrier than the strike price.
The barrier options are so cheap OTC due to the fact that the volatility of the underlying greatly impacts their price as the increase in the price increases the probability of a Knock Out or In event.
For this reason, Financial Engineers have created the Parisian Options: barrier options in which the barrier event must last for a predefined time in order to trigger the mechanism.

Binary options are special options that provide a discontinuous payoff. The simplest binary option is the cash or nothing option (call or put), in which the payoff is 0 if the underlying ends up below (or above) the strike price, while it pays to premium if it stays above (or below) that threshold.
Another type of binary option is the asset or nothing option, but it will not be treated as it is not related to the structuring of certificates.

A ***Cliquet Option*** is simply a series of call and put options put together in order to grant rules to determine the strike price (that's why they are also called strike reset options).
In addition there may also be upper and lower limits on the total payoff over the whole period or sometimes cliquets terminate at the end of a period if the asset price is within a certain range.

A ***Lookback option*** is a contract that allows to structure the payoff based on the life of the option itself. That is to say that the trend of the underlying will be taken into account throughout the circulation phase and the difference between the final value of the same and its minimum period will be the payoff for the investor (in the case of a lookback call). On the contrary, by purchasing a put of this type, the investor will profit from the distance between the final value

of the underlying asset compared to the maximum reached in the period of circulation of the option.
A call option of this type allows the investor to buy the underlying asset at the lowest price in the life of the option, while a put ensures that the underlying is sold at the highest price.

Asian options give the possibility to structure the payoff on the basis of the arithmetic average of the prices of the underlying asset during the life of the option, in this case it is called an average price option call or put.
The payoff for this type of options is given by:

$$max\ (0, S_{ave} - K) \qquad \text{for a call asian option}$$
$$max(0, K - S_{ave}) \qquad \text{for a put asian option}$$

An Asian option can also base its final payoff on the arithmetic average of the strike and in this case it will be:

$$max\ (0, S_t - S_{ave}) \qquad \text{for a call asian option}$$
$$max\ (0, S_{ave} - S_t) \qquad \text{for a put asian option}$$

This mechanism guarantees that the final price will never be inferior than the strike price (or superior).

Finally, European ***basket options*** fall into the largest category of ***rainbow options***, i.e. options that include more than one underlying asset. The basket options have a payoff linked to the performance of a basket of underlying stocks or indices, which, being less volatile than the average volatility of individual securities, guarantees a premium reduction compared to a similar option on a single basket title, in line with the principle underlying the decorrelation of returns with a view to portfolio management.
Basketball options are often priced considering the value of the basket as a single underlying and applying the canonical formula. However, the key variable in the pricing of basket options

is precisely the correlation between the returns of the various assets included in the basket: the lower the correlation is the cost of the option.

It is therefore understandable that the correlation analysis is an element of absolute importance when it comes to basket options as it is determinant both on the cost in the structuring phase, in any case considerably lower than an identical written option on a single stock, both on the risk-return analysis.

The use of these options has become widespread recently, with the addition of some peculiarities that make them particularly attractive on the part of issuers, especially due to the remarkably low price.

These are the "worst of" basket options, in which the option of the worst underlying asset within the basket is taken into consideration by the option. There is also clearly the "best of" alternative, very little used because of the obviously higher cost, especially in periods of low volatility

3.3. The Black-Scholes Model

The need to estimate the price of options in an increasingly accurate manner has led operators to develop increasingly advanced pricing techniques that are suitable for considering the effects of sudden changes in market conditions.

Over the last few decades, numerous models for the determination of option prices have appeared in the literature, each of which is based on different assumptions with respect to market variables, such as the distribution of probability of the underlying or the type of volatility. to be used in the calculation.

Although today there is no uniformity of judgment on which the best performing model is, the totality of scholars and operators are in agreement in recognizing the influence on the methods of subsequent evaluation exercised by the Black & Scholes model (BS), elaborated at the beginning of the 70's by Fisher Black and Myron Scholes (1973).

This importance derives above all from its ability to provide correct pricing, even if based on hypotheses that are sometimes excessively simplistic, on the other, from its peculiarity as one of the few tools available to operators for analytical evaluation of options.

It was first developed in order to price European financial options (options that can not be exercised before the maturity) and from the first version it contributed e inspired creating all the following pricing models.

It is necessary to give the right credit to Merton for his important contribution towards the consecration of the so called Black and Scholes model, due to his adjustments and upgrades.

The basic assumption of the BS model concerns the distribution of share prices. In particular, the authors of the model hypothesize that it is possible to represent the logarithmic returns of the securities through a normal distribution. As a result, prices will be distributed according to a log-normal distribution.

This assumption is in fact tantamount to hypothesizing that, in every instant of time dt, the spot price of the stock follows a pattern of behavior, known as Brownian geometric motion, with expected rate of return (drift rate, μ) and variance (variance rate, σ_2) constants, mathematically:

$$dS = \mu S dt + \sigma S dz$$

This means that at any moment the price, on the one hand, moves according to its own trend, while on the other it makes oscillations around it, caused by the term dochastic disorder dz, or unpredictable, whose effects are amplified by volatility σ.
As it happen for the binomial model, another hypothesis is the possibility to create an option-equivalent portfolio, partly consisting of unities of the underlying, partly of risk-free bonds.
The principal difference with respect to the binomial model is that this hypothesis is built over the first fundamental assumption that the rates of return are distributed following a normal statiscal law, and also for this reason it is possible to consider the BS model as the limit in the continuous case of the binomial model which is by definition discrete.
Black & Scholes model allows to define and evaluate an option starting from six fundamental variables known, which are:

- S = value of the underlying asset
- K = strike price of the option
- T = time to maturity[11]
- r = annual continuously compounded risk-free rate
- σ = underlying's volatility[12]

Given this initial set of data, Black and Scholes show that, in the event of geometrical stochastic Brownian process, the value of a Plain Vanilla Call Option having as underlying a non-dividend paying stock is:

$$C = SN(d_1) - Ke^{-rT}N(d_2)$$

And equivalently, the value of a Plain Vanilla Put Option having as underlying a non-dividend paying stock:

$$P = Ke^{-rT}N(-d_2) - SN(-d_1)$$

[11] Time to maturity is normally measured as the number of trading days left in the life of the option divided by the number of trading days in 1 year.
[12] Usually 360 days volatility

Where:

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{\ln\left(\frac{S}{K}\right) + \left(r - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

And $N(x)$ represents a normal standard distribution, meaning a normal distribution with mean equal to 0 and standard deviation equal to 1: it is the probability that a variable with a standard normal distribution will be less than x.

Figure 29. N(x) is represented by the shaded area, from John C. Hull, Options, futures, and other derivatives (University of Toronto, 10th Edition)

The Black and Scholes Model is grounded on the idea of the replicant portfolio, in that, since in an efficient market there are no arbitrage opportunity, this portfolio must have the same value of an option given from the combination of a credit asset, a debt asset and the underlying. Taking into account the pricing formula for the Call Opton, in fact, it is possible to interpet the first term of the equation ($SN(d_1)$) as the value of the stock an investor should buy while the second term ($Ke^{-rT}N(d_2)$) as the value of the bond he should issue.

Therefore, the value of the option itself express the difference between a predetermined value of the underlying asset and the value of the debt.

$N(d_1)$, the number of shares of the underlying asset requested in order to create the replicating portfolio is known as delta of an option and it express the sensitivity of the option price to the movements of the underlying, i.e. the probability that the option at maturity will read a value of the underlying that is higher than the strike price [13].

Please note that, as it happen in the binomial model, these are "pseudoprobabilities" e they do not correspond to the effective ones, not taking into account the particular scenario in which all financial operators are risk-neutral.

BS model assumptions are:

- Underlying's prices follow a log-normal process
- The markets are perfectly efficient with no attrition[14]
- Markets' interest rate is the same for borrowing or lending, moreover it is constant for the whole life of the option
- Underlying's volatility is constant for the life of the option

If the markets behave following the above assumptions, then the model offers a rigorous base for calculating both value and risk-characteristics of a position in options.

The fundamental factor for the calculations is given by the variation in the stock price. The option value sensitivity to the price factor is measured by the delta coefficient which mesures the relationship between changes of C (or P) and S, ceteris paribus.

For example the delta of a Call option lays in an interval between 1 and 0 and its value is close to 0 when S is way below the strike price K and the option's expiration date is close. In such case the probability that the stock price rise in a way that allow the option to go back "in the money" is remote: the market believes that the option will expire without value and therefore the connection with the underlying stock's price is weak. On the other side delta tend to 1 for price S way above K, since the market expect the option to be exercised.

From the Black & Scholes formula it is possible to derive other interesting coefficients, namely Gamma, Theta, Vega and Rho (also known as Greeks).

[13] In the money

[14] No taxes nor costs of transaction

They measure sensitivity of delta to small changes in the stock price, of the value of the option to small changes of time to maturity, of the value of the option to small changes in volatility and of the value of the option to interest rates' changes, respectively.

3.3.1. Adjustments to the Black-Scholes Formula

Financial Theory have brought many refinements to the Black and Scholes formula. In 1973 Robert Merton abandoned the no dividend distribution assumption during the life of the option, providing us the new formulas:

$$C = Se^{-qT}N(d_1) - Ke^{-rT}N(d_2)$$

$$P = Ke^{-rT}N(-d_2) - Se^{-qT}N(-d_1)$$

Where q is the dividend yield of the underlying stock, and:

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r - q + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{\ln\left(\frac{S}{K}\right) + \left(r - q - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

Operationally, this means that the Black–Scholes–Merton formulas can be used
knowing that the stock price is reduced by the present value of all the dividends during the life of the option.
In 1976 Jonathan Ingerson abandoned the no taxes or transaction costs restriction and Robert Merton again the assumption of constant interest rates.

The original formula was then developed to be extended to currency options, bonds, futures and interest rates options (caps, floors, etc.).

More recently, empirical adjustments have been made in order to evaluate american options (options that can also be exercised anticipately with respect to their maturity).

4. Inside a Bonus Cap

In this chapter we will proceed by dismantling a certificate into all the options inside it, in order to better understand how the options described before work together, creating personalized payoffs.

The certificate taken into account is a Bonus Cap issued by one of the most important italian banks on the 7th of July 2017, expiring on the 21st of June 2019 and written on Intesa Sanpaolo stock.

A Bonus Cap certificate belongs to the conditional capital protection category and it allows investors to have a fixed premium paid at maturity (the bonus) while seeing their investment protected, until certain conditions for the underlying hold.

Getting into details this Bonus Cap offers a 113,3% premium at maturity (calculated from the nominal price) with a Cap at the same level, meaning that the investor can not receive more than the 113,3 € at maturiry, even if the underlying stock registers an above cap performance[15].

At the same time this certificate admit an american barrier (observed continuously, intraday[16]), put at 60% with respect to the strike price, for the equivalent of a price of the Unicredit Stock of 1,71 €, below which the Settlement amount will be linearly reduced by the negative performance of the underlying with respect to the strike price.

All Bonus Cap Certificates are built putting together three options (or legs of a certificate), two plain vanilla and one exotic option, here the details.

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[15] Meaning that Intesa Sanpaolo at maturity registers a performance higher than the 113,3% with respect to its Strike Price.

[16] If the share price of Intesa Sanpaolo goes below the barrier in any moment during a trading day a Knock Out event will occur and the certificates will then follow linearly the underlying, and it will be the equivalent of holding the stock.

+*Plain Vanilla Call* with strike price at 0, that allows to linearly replicate the performance of the underlying:

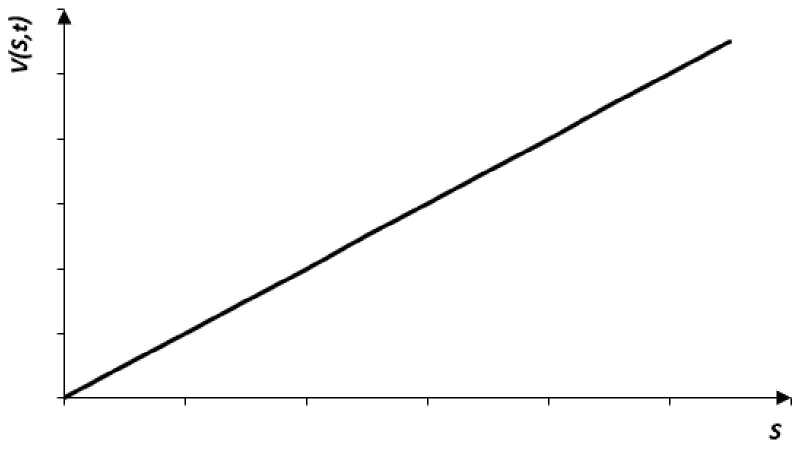


Figure 30. Payoff V(S,t) of a Long Call Option with Strike equal to 0. (Personal elaboration)

-*Plain Vanilla Call* with strike price K at the Cap level, which is, in our case, 3,22905 €. This leg is used to reduce the price of the certificate itself and put a limit to the premium that the investor can receive:

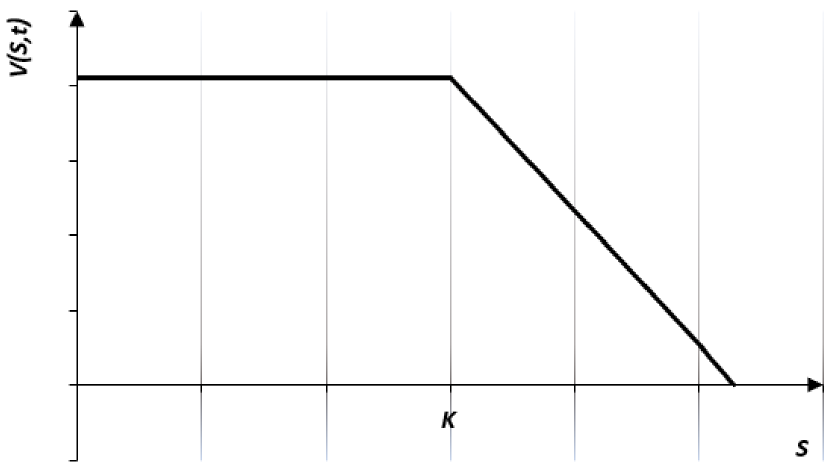


Figure 31. Payoff V(S,t) of a Short Call Option with Strike K. (Personal elaboration)

+*Put Down and Out Barrier Option* with strike price at the Bonus level (19,8708 €) and barrier at the barrier level (13,92 €). This last leg of the certificate is based on a Knock Out Event: it ceases to exist if the underlying goes below the barrier:

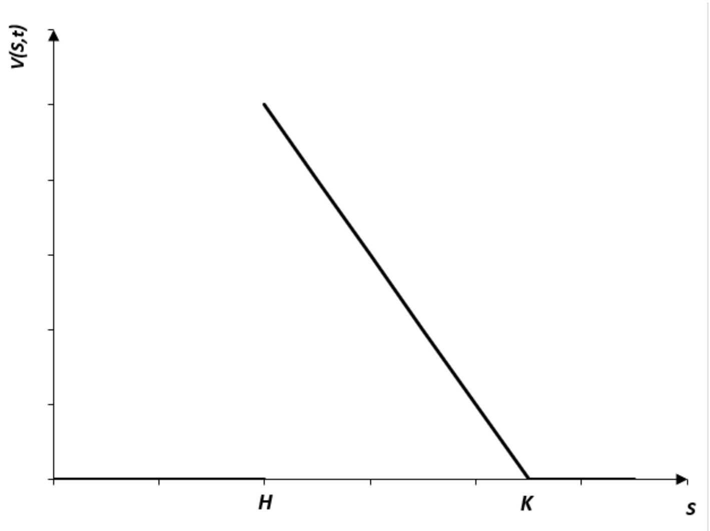


Figure 32. Payoff V(S,t) of a Put Down & Out barrier option with Barrier Level at B and Strike Price K. (Personal elaboration)

These three legs put together create the Bonus Cap Certificate, one of the most traded certificate with conditional capital protection, which present, consequently the following payoff:

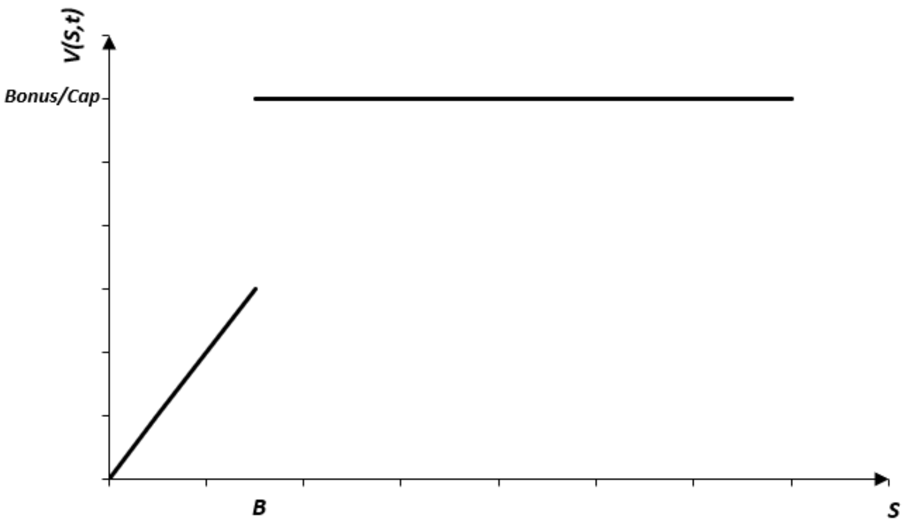


Figure 33. Payoff V(S,t) of a Bonus Cap Certificate with Barrier at B. (Personal elaboration)

It offers the opportunity to be rewarded with more than the performance of the underlying until it stays between the barrier and the bonus (and cap) level.
For Example, given a barrier at 10 €, a strike price of 12,5 € and both bonus and cap level set at 15 € (120% of the strike price), suppose that the underlying stock at maturity has a price of 13 €, for a certificate whose nominal price is 100 € the investor will receive 120 €, even if the underlying did not reach the cap level, for a 20% of profit, against the 4% that he would receive if instead of pursuing a certificate he decides to buy the underlying financial asset at the strike price (in correspondance to the certificate's issue date).
In addition, buying a certificate basically expose the investors to the very same risk of holding the stock in negative scenarios: using the same example, now suppose that the underlying at maturiy has a price of 8 €, which is below the barrier level, in this case he will receive a settlement amount of 64 €, for a 36% loss, that is exactly the same loss he would bear whether he decides to buy the stock: $\frac{8€}{12,5€} - 1 = -36\%$.
Let's go back to the Bonus Cap we are willing to investigate: from the chapter before we now know how to price both plain vanilla options, let's use those equation, together with specific formulas for the more difficult calculation of exotic options (barrier options in this particular case), to check the fair value of this certificate today.

The first leg of the certificate is built by buying the so called "***Strike 0" Call Option***:

$$C = Se^{-qT}N(d_1) - Ke^{-rT}N(d_2)$$

That easily becomes:

$$C = Se^{-qT}N(d_1)$$

Since the Strike Price K is 0[17].
Intesa Sanpaolo's shares are currently trading at 2,049 € (S) and the italian bank have a dividend yield (q) equal to 9,96%, moreover the option will expire on the 21th of June 2019, therefore

[17] It is impossible to put the Strike price exactly at 0, therefore in the calculation we will use an approximation close to it, i.e. $1e^{-20}$.

there are 148 days left till expiration, then T will be given from $\frac{148}{365} = 0{,}405479$. The last data needed are the annual volatility of Intesa Sanpaolo which is 26,645% and the italian risk free rate (r) that is equal to the yield of a 10 years italian government bond which is currently 2,394% that becomes 2,4429% since we need it to be continously compounded.
Therefore, the above equation becomes:

$$C = 2{,}049 \times e^{-0{,}0996 \times 0{,}405479} \times N(d_1)$$

Now we focus on $N(d_1)$, we know that d_1 is given by:

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r - q + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$d_1 = \frac{\ln\left(\frac{2{,}049}{1e^{-20}}\right) + \left(0{,}024429 - 0{,}0996 + \frac{0{,}26645^2}{2}\right) \times 0{,}405479}{0{,}26645 \times \sqrt{0{,}405479}}$$

$$d_1 = 248{,}4127558$$

Using the tables for the Cumulative Normal Distribution we obtain that:

$$N(248{,}413) = 1 = \delta$$

And, finally:

$$C = 2{,}049 \times e^{-0{,}0996 \times 0{,}405479} \times 1 = 1{,}967898291\ €$$

1,9679 € is the price of one single Strike 0 Call inside the Bonus Cap and it explain what mentioned at the beginning of the chapter, i.e. this leg is used to replicate the underlying, therefore the price of such options is always very close to the share price S.

Let’s move forward pricing the second Call Option that has to be sold.

It won't have strike 0 in this case, since it is used to fix the Cap of our certificate, and for this reason will have a K exactly at the Cap Level of 3,22905.

Using the same data set:

$$C = Se^{-qT}N(d_1) - Ke^{-rT}N(d_2)$$

$$C = 2{,}049 \times e^{-0{,}0996 \times 0{,}405479} N(d_1) - 3{,}22905 \times e^{-0{,}024429 \times 0{,}405479} N(d_2)$$

$$d_1 = \frac{\ln\left(\frac{2{,}049}{3{,}22905}\right) + \left(0{,}024429 - 0{,}0996 + \frac{0{,}26645^2}{2}\right)0{,}405479}{0{,}26645 \times \sqrt{0{,}405479}}$$

$$d_1 = -2{,}776031088$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

$$d_2 = -2{,}776031088 - 0{,}26645 \times \sqrt{0{,}405479} = -2{,}94569917$$

$$C = 2{,}049e^{-0{,}0996 \times 0{,}405479} N(-2{,}776) - 3{,}22905e^{-0{,}024429 \times 0{,}493151} N(-2{,}946)$$

$$C = 0{,}000262819\ €$$

0,000536 € is the unit price we will receive by selling a single Call with this characteristics today.

The final step in order to give a fair value of the Bonus Cap analyzed is to price the third leg of the certificate, namely the Put Down & Out Barrier Option.

Adjustments to Black & Scholes Model have brought us to the following parity equations for pricing barrier options:

$$C = C_{di} + C_{do}$$
$$P = P_{di} + P_{do}$$

Where C and P are the values of regular call and put options and C_{di}, C_{do}, P_{di} and P_{do} are the values of a Call Down & In, a Call Down & Out, a Put Down & In and a Put Down & Out Barrier Options, respectively.

The Option used to create the payoff of the Bonus Cap Certificate is the latter, namely P_{do}, let's move forward to price it.

From Hull (*Options, Futures and Other Derivatives*, 10th Edition, 2018) we know the Black and Scholes Formula adjusted for pricing correctly P_{di}:

$$P_{di} = -SN(-x_1)e^{-qT} + Ke^{-rT}N\left(-x_1 + \sigma\sqrt{T}\right) + Se^{-qT}\left({}^{H}/_{S}\right)^{2\lambda}[N(y) - N(y_1)]$$
$$- Ke^{-rT}\left({}^{H}/_{S}\right)^{2\lambda-2}[N\left(y - \sigma\sqrt{T}\right) - N\left(y_1 - \sigma\sqrt{T}\right)]$$

And from the parity shown above we can derive the value of the option we are looking for:

$$P_{do} = P - P_{di}$$

We first need to evaluate *P* with a Strike Price at the Bonus Level of 3,22905€ of Intesa Sanpaolo, using the same data set we used for pricing the two Call options:

$$P = 3{,}22905e^{-0{,}02442T}N(-d_2) - \ Se^{-qT}N(-d_1)$$

With

$$d_1 = \frac{\ln\left(\frac{2{,}049}{3{,}22905}\right) + \left(0{,}024429 - 0{,}0996 + \frac{0{,}26645^2}{2}\right)0{,}405479}{0{,}26645 \times \sqrt{0{,}405479}}$$

$$d_1 = \ -2{,}776031417$$

and

$$d_2 = = -2{,}776031417 - \ 0{,}26645 \times \sqrt{0{,}405479} = \ -2{,}945699499$$

Therefore:

$$P = 3{,}22905e^{-0{,}02442T}N(2{,}776) - \; Se^{-qT}N(2{,}946)$$

$$P = 1{,}229846673 \text{ €}$$

Now we need to find the value of the Put Down & In Barrier Option using the formula provided by Hull and to do this it is necessary to explain 4 parameters used in such equation.

$$\lambda = \frac{r - q + \frac{\sigma^2}{2}}{\sigma^2}$$

$$y = \frac{\ln(\frac{H}{S \times K})}{\sigma\sqrt{T}} + \; \lambda\sigma\sqrt{T}$$

$$y_1 = \frac{\ln(\frac{H}{S})}{\sigma\sqrt{T}} + \; \lambda\sigma\sqrt{T}$$

$$x_1 = \frac{\ln(\frac{S}{H})}{\sigma\sqrt{T}} + \; \lambda\sigma\sqrt{T}$$

Now we can proceed to P_{di} evaluation and consequently to P_{do}, which is our goal.

$$\lambda = \frac{0{,}024229 - 0{,}0996 + \frac{0{,}26645^2}{2}}{0{,}26645^2} = -0{,}561631073$$

$$y = \frac{\ln(\frac{1{,}71}{2{,}049 \times 3{,}22905})}{0{,}26645 \times \sqrt{0{,}405479}} + \; -0{,}0561631073 \times 0{,}26645 \times \sqrt{0{,}405479}$$

$$y = \; -4{,}907940929$$

$$y_1 = \frac{\ln(\frac{1,71}{2,049})}{0,26645 \times \sqrt{0,405479}} - 0,561631073 \times 0,26645\sqrt{0,405479}$$

$$y_1 = -1,161245623$$

$$x_1 = \frac{\ln(\frac{2,049}{1,71})}{0,26645\sqrt{0,405479}} - 0,561631073 \times 0,26645\sqrt{0,405479}$$

$$x_1 = 0,970663889$$

Now we can put these coefficients inside the formula for the evaluation of P_{di}.

$$\begin{aligned} P_{di} = & -2,049N(-0,97)e^{-0,0996\times 0,405479} \\ & + 3,22905e^{-0,024229\times 0,405479}N\left(-0,97 + 0,26645\sqrt{0,405479}\right) \\ & + 2,049e^{-0,0996\times 0,405479}\left({}^{1,71}/_{2,049}\right)^{[2\times(-0,56)]}[N(-4,9) - N(-1,16)] \\ & - 3,22905e^{-0,024229\times 0,405479}\left({}^{1,71}/_{2,049}\right)^{[2\times(-0,56)-2]}[N\left(-4,9\right. \\ & \left.- 0,26645\sqrt{0,405479}\right) - N\left(-1,16 - 0,26645\sqrt{0,405479}\right)] \end{aligned}$$

$$P_{di} = 0,569371824\ €$$

Remember

$$P_{do} = P - P_{di}$$

Therefore

$$P_{do} = 1,230119982\ € - 0,622337856\ € = 0,660474849€$$

We finally know what is the price of every option inside the Bonus Cap given by the Black-Scholes Model[18] and we can create a package composed by buying one Strike 0 Call and one Put Down & Out Barrier Option and selling a Call with strike price at the bonus level.

$$Package = +C_{K_0} + P_{do} - C_{K_B}$$

$$Package = +1{,}967898291€ + 0{,}660474849€ - 0{,}000262819€$$

$$\boldsymbol{Package = 2{,}6281103212}\ €$$

4.1. Face Value and Multiple

In order to get to a Fair Evaluation of the Certificate we need to explain the concept of Face Value and multiple.

Certificates are built using packages and then they are placed at a Face Value that is usually 100€[19].

In order to get to this unit value the Issuer has to buy many option packages, therefore a Certificate controls more than one share of the underlying and

the ratio between the Face Value and the Strike Price (which is the share price of the underlying at the issue date) returns the Multiple.

The Multiple express the countervalue that the Certificate move or equivalently the number of shares of the underlying.

Going back to our study, the Face Value of the Bonus Cap is 100€, the Strike Price of Intesa Sanpaolo is 2,85€, therefore:

[18] Other models can lead to slighlty different results.

[19] There are a lot of Certificates whose Face Value is 1.000 or 10.000 or more particular values, but 100 is certainly the most common, since it is in the nature of this financial instrument to be meant for retail investors. However the Face Value is always mentioned inside the Final Terms of the Certificate and it usually includes an implicit placement fee.

$$Multiple = \frac{100}{2,85} = 35,0877193$$

This particular Bonus Cap Certificate controls 35,0877 Intesa Sanpaolo shares, which means that the Issuer bought 35,0877 of the above priced packages, leading us the final price of the Certificate:

$$Bonus\ Cap\ Certificate = 35,0877 \times\ 2,628110321 =\ 92,21434652\ €$$

The Certificate today has a Black-Scholes Fair Value of 92,214 €, against a Market Value of 87,5€/88,15 (Bid/Ask), which means that, following our calculation, it is slightly underestimated by the Market.

4.2. Market Maker

One important issue about Certificates it's certainly its Liquidity.
It is guaranteed on both Cert-X and SeDex by the role of the Specialist Operator known as Market Maker, that provides trading offers both for buying and selling for a minimum number established during the admission phase and declared in the Final Prospectus.
The intermediary who wants to operate as a specialist is obliged to respect a number of listing rules made in order to allow a fair trading conduct.
Thanks to the Market Maker, indeed, a price at which is possible to buy or sell Certificates is always available.
The Specialist has to present a book in which there must be bid and ask price, meaning the prices at which is possible to sell or buy, resepctively.
This mechanism ensure the investors with the capability to find a continuously uptaded value of the instrument so that he is able to disinvest, to open or augment a position any time.
The Market Maker has also the obligation to show proposals to buy or sell for a minimum volume, that is set by Euro TLX and SeDeX.

Only for cases included in the market regulation, the Specialist has the right to ask for temporary exemptions from the above obligations, and the Supervision Authority has the faculty to accept or reject the request whether conditions for acceptance are not met.

The Specialist reserves the right to ask for exemption from the obligation to present ask offers whether the collection of the necessary amount for ensuring the market making activity turns out to be excessively expensive because of particular markets events which must be in the public domain.

Although, before starting an investment, one should match his own investment time horizon with the maturity of the instrument, to be sure to invest in products which can be reasonably held until maturity, it may happen that one has the need to sell, both for necessity or for opportunity.

In this case, it turns out to be fundamental to have an efficient secondary market, which allows the sell of financial produtcs with no difficulties.

It is also relevant that the liquidation takes place with no excessive costs, taking into account transaction costs applied by the intermediaries, and implicit costs stricktly linked to the so called bid-ask spread.

It can be useful to focus on the definition of bid-ask spread. For every instrument traded on the secondary market it exist a correspondant trading book, with bid offers and ask offers, meaning prices at which the Market Maker is willing to buy and sell, respectively.

Whenever a corrispondence between price offered and requested it's found, proposals are crossed and the trade contract materially takes place.

All the other offers, which can't find a suitable counterpart, are showed inside the book following a price order, from the highest to the lowest for bid offers and from the lowest to the highest for ask offers, according the so called best bid and offer (BBO) criterion.

The difference between the best bid and ask offers it's defined as bid-ask spread.

In an efficient market the theoretical price of the product is supposed to lay exactly between those two best proposals, at the half of the spread.

Consequently, he who buys at the best available price is buying at a price that is higher than the theretical one, and viceversa.

This difference represents the implicit cost deriving from the bis-ask spread.

Going back to the product which is object of the dissertation, almost all the investment certificates, right after the placement, are admitted for trading through SeDeX and EuroTLX platforms. In such markets, as already explained, liquidity is provided by the presence of a Market Maker, and/or Specialists, which commit to show continuously bid and ask offers, with differentials between the two that must be inside certain limits, established by regulation. Specifically, regulation from both the markets fix the maximum spread for most of the products exchanged at 3,5%. Nevertheless, markets operators commit to maintain differential levels which are inferior to the one fixed, in an interval between 1% and 1,5%.

4.3. Issue Date Price and Implicit Placement Fee

It is now interesting to investigate what was the Fair Value of the Bonus Cap in correspondance to the Issue Date, in order to better understand what gives back the unit face value of 100.

The dividend payed by Intesa Sanpaolo in 2017 was 0,178€ per share and the strike price on the 7th of July 2017 was 2,85€, therefore our q will be given by $\frac{0{,}178}{2{,}85} = 6{,}2456\%$. The risk-free rate r, given by the continuously compunded yield of a 10 years Italian BTP issued at the end of 2017 was 1,7166% and regarding the volatilty σ we notice that it was higher at that time, for a value of 27,957%.

In addition, we need to modify the parameter T, that will be 1,619178 ($\frac{591(days\ to\ expiry)}{365}$) and S, that will be exactly the Strike Price K=2,85€.

Plugging this data inside the previously used equations we have:

$$Bonus\ Cap\ Certificate_{issue\ date} = 96{,}85158706\ €$$

Even if the Certificate Fair Value at the issue date was 96,85€, the placement took place for 100€ and the difference between the Face Value and the amount actually paid for the packages is exactly the implicit placement fee for the Issuer, that earned 3,15€ (3,15%% fee) per certificate, against a 1,11% indicated in the Key Investor Document.

It is necessary to say that the price we found may also be significantly different from the actual one the issuer paid, moreover different methods of pricing can lead to different results.

From this observation it is possible to proceed trying to debunk the myth that the issuing bank benefits from constructing "bad" structured products, that is to say products that will hardly pay the investor at maturity.

The only source of income for a bank that issues certificates derives from the ability of structurers to find options Over The Counter at the lowest possible price, succeeding, at the same time, in creating interesting structures.

After that, at maturity, it is not the issuing bank that materially pays the investor, its outlay dates exclusively to the structuring phase, therefore there is no interest on the part of the issuer to ensure that the product does not pay the final amount or the premium to the investor.

Morevover, the issuing bank allows the investor to invest in packages that he would not be able to compose on his own, as there are options over the counter inside, that is to say strategies such as those offered by investment can not be replicated by the retail investor.

However, knowing the structure of a certificate, in that the options inside it, allows the investor to understand if the product is well structured or not, even without performing a fair price evaluation like the one provided.

An interesting doubt, instead, could arise with regard to the purchase of certificates in placement when they could be easily bought on the secondary market, once they have discounted the implicit placement fee.

The first and most important reason link us to what previously said while talking about the market maker.

In fact, a number of Issuer reserve the right to go in the market in “bid-only”, meaning that that there will always be, as regulation impose, an offer available for an investor to sell the certificate, but not to buy it.

Therefore, if one is interested in a particular structure, which he believes to be efficient and profitable, sometimes the only way to invest in it is to buy the certificate at placement.

In addition, it may happen that despite the implicit placement fee, the underlying’s peformance in the first days of trading brings the certificate to a price which can be either the equal to the face value value itself or even higher, in that an investor could also miss the right time to enter the investment.

4.4. Greek Letters

Every single derivative contract's value, $F(t)$ at a certain time t depends on:

- Underlying's value, $S(t)$;
- The time to expiration (calling T the maturity of the contract, time to expiration will be $T-t$);
- Interest rate, r;
- Underlying price's volatility, σ;

Therefore, it is clear that we must know how to calculate the value of the derivative contract as the above variables change. In more mathematicals terms, we want to calculate the option's derivatives with respect to such measures.
This measures are so in important in the financial context, and more specificly in the risk management context, that they have earned their own name, the Greeks.

The first greek letter is named Delta ($\varDelta$) and it explain how the value of the option change as the underlying's value change, namely it is the first derivative of $F(t)$ with respect to $S(t)$:

$$\Delta = \frac{\delta F(t)}{\delta S(t)}$$

Similarly, the Gamma (Γ) is the second derivative of the option's value with respect to the undelrlying's price:

$$\varGamma \ = \frac{\delta^2 F(t)}{\delta S(t)^2}$$

Teta (Θ) express the variation that the option's value suffers as time to maturity increase or decrease, i.e. the first derivative of $F(t)$ with respect to $T-t$:

$$\varTheta \ = \frac{\delta F(t)}{\delta (T-t)}$$

Rho (P) is the first derivative of $F(t)$ with respect to the interest rate r:

$$\mathrm{P} = \frac{\delta F(t)}{\delta(r)}$$

And Vega ($\mathcal{V}$), which is not properly a greek letter, is the first derivative of $F(t)$ with respect to volatility σ of the underlying's price:

$$\mathcal{V} = \frac{\delta F(t)}{\delta(\sigma)}$$

In the following Table the formulas of the above defined greek letters are sum up, according to Black-Scholes Model, both for the case of a call option and for a put:

Greek Letter	Call Option	Put Option
Δ	$N(d_1)$	$-N(-d_1)$
Γ	$\frac{1}{S(t)\sigma\sqrt{T-t}}N'(d_1)$	$\frac{1}{S(t)\sigma\sqrt{T-t}}N'(d_1)$
Θ	$Ke^{-r(T-t)}(rN(d_2) + \frac{\sigma N'(d_2)}{2\sqrt{(T-t)}}$	$Ke^{-r(T-t)}(-rN(-d_2) + \frac{\sigma N'(d_2)}{2\sqrt{(T-t)}}$
P	$(T-t)Ke^{-r(T-t)}N(d_2)$	$(T-t)Ke^{-r(T-t)}N(-d_2)$
$\mathcal{V}$	$Ke^{-r(T-t)}\sqrt{T-t}N'(d_2)$	$Ke^{-r(T-t)}\sqrt{T-t}N'(d_2)$

Table 1. Greek Letters Black-Scholes formulas for Call and Put Options

Black-Scholes strict assumptions do not guarantee this formulas can always work, in fact, if the BS hypothesis are not met, the greeks can always be calculated through numerical simulations, making the variables move and checking the correspondent change in the option price, or viceversa.

4.5. Stock vs Certificate

Now we finally have all the data in order to perform an analysis of the performances of both the certificate and the stock, since the time the certificates was admitted for trading. In this way we can understand how efficient such a instrument can be with respect to the stock itself.

Figure 34. Stock vs Certificate price evolution from issue date to 28/01/2019. (Data: Bloomberg)

As is evident, except for the period from January 2018 to May of the same year, the certificate yield (orange line) has always been above that of the share price (green line), why?

In the first part of the graph the temporal component heavily impacts on the certificate price (theta), ie the positive change of the underlying (delta) is held marginally in consideration in the pricing of the derivative, given that the expiration date is still far away. Going forward in time, the chances that the underlying asset, that is to say the Intesa Sanpaolo stock, will reach the price of 1.71 € (barrier level at which the barrier option within the certificate ceases to exist and consequently the product linearly follows the performance of the underlying) decrease, so the value of the certificate will increase and, above all, will overreact to the movements of the underlying, in both directions, as is evident from the end of December until today where there is a growth trend of Intesa Sanpaolo (please note that the expiration date is set for June 21 this year).

Likewise, a significant drop by the stock towards the barrier level will significantly increase the probability of a Knock Out event, so, approaching maturity (with the theta basically at zero) the delta will govern the price of the certificate, which could also go below the share price (in percentage), together with another important component of every Bonus Cap Certificate, more particularly of every barrier option, the Vega.

We already said that vega measures the sensitivity of the option price to the volatility of the underlying; this greek letter becomes fundamental analyzing barrier option, therefore Bonus Cap Certificates.

In fact, the more the underlying asset is volatile, the more probable will be for it to break the barrier and the option price will suffer from it, especially if the barrier is of american type, ie continuosly checked.

The situation described above is that relating to the purchase of the certificate during placement, but, as we all know, there is a secondary market of certificates that allows to structure interesting strategies.

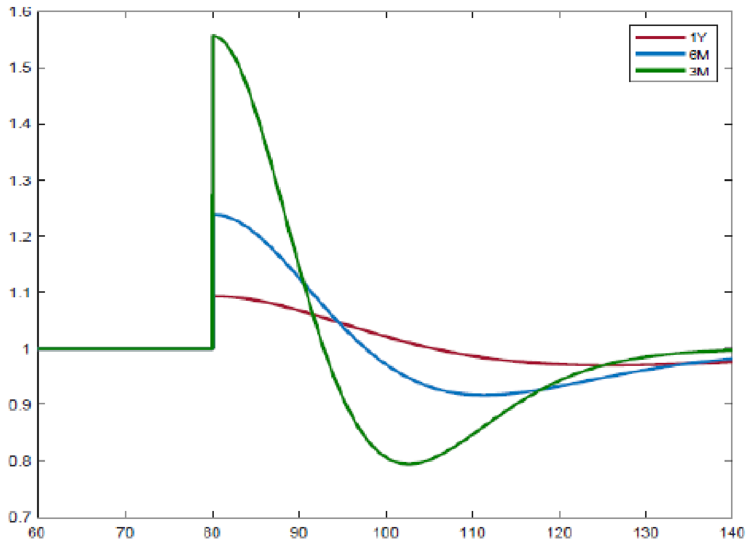


Figure 35. Delta of a Bonus Cap Certificate

The graph above helps understanding the evolution of the delta of a Bonus Certificate like the one analyzed. The delta becomes one and the Certificate linearly replicate the performance of the underlying financial asset if the barrier level is broken. Just above that level is where the delta is positively larger, then when the price moves higher the probability that a barrier event will occur is certainly reduced. At even higher price levels, the options doesn't "feel" the barrier, therefore the delta is lower than one (the delta of the put option is slightly negative and that of the call option is one).

4.6. Switch to Recovery

Let's assume an investor bought Intesa Sanpaolo shares at a price of € 2.5 and on January 2nd finds them at a value of € 1.96 and has therefore achieved a loss of 21.6%. In correspondence with the price of € 1.96 of the share, the certificate quoted € 72.34 and € 72,8 was the ask price, ie the actual price at which the investor would have bought.

An excellent recovery strategy on the stock would have been made by selling the shares at a unit price of € 1.96 and purchasing the certificate for € 72.8.

The price of the certificate today is € 87.5 / 88.15 for a value of the underlying security of € 2.12572.

Holding the stock in the portfolio until today would have result in a loss of almost 15%, while, accepting the capital loss resulting from the liquidation of the security (21.6%) and at the same time switching to the Bonus Cap Ceritficate, a yield of 20.2% would be obtained, not sufficient to offset the entire loss, but allowing to get close to the initial capital invested[20].

This is why Certificates are ideal tools for recovering situations of non-performing portfolio.

4.7. Scenario Analysis

As of today, the following is the scenario analysis for the Bonus Cap Certificate discussed in the dissertation:

Scenario	-50%	-40%	-30%	-20%	-10%	0%	10%	20%	30%	40%	50%
Share Price Intesa Sanpaolo	1,0	1,2	1,4	1,6	1,8	2,0	2,3	2,5	2,7	2,9	3,1
Certificate's Settlement Amount	35,9	43,1	50,3	57,5	113,3	113,3	113,3	113,3	113,3	113,3	113,3
P&L certificate	-59,2%	-51,1%	-42,9%	-34,8%	28,5%	28,5%	28,5%	28,5%	28,5%	28,5%	28,5%

Table 2. Scenario Analysis for Bonus Cap Certificate (ISIN: DE000HV40CG5). (Personal elaboration)

With the following graphical representation of profit & loss evolution:

[20] $1{,}96 \times (1 + 0{,}202) = 2{,}35592$ € vs the initial price of 2,5 €.

Figure 36. Profit and loss diagram of the Bonus Cap Certificate (ISIN: DE000HV40CG5) vs Stock

From the graph above it's easy to see the convenience of investing in such product instead of purchasing the stock.

The "profit area" is significantly larger than the "loss area" and the investor is willing to give up the gains that would derive from a growth of more than € 2,634, in order to gain in situations of sideways, and especially in situations where investing in the stock would only generate losses.

Moreover, the difference in the downward scenario prospected between the certificate and the stock is basically negligible.

5. Conclusions

The purpose of this thesis was to clarify a financial instrument still wrapped in fog, whose a priori exclusion from the range of investment products available to the investor, exclusively for the denomination of financial derivative, appears superficial.

Although their growth in the italian market is significant, it is believed, in fact, that, in the eyes of many, these structured products must be avoided at all costs, yet, for their nature and, above all, for the widespread belief that the only ones to gain from the circulation of the certificates are the issuers, who create bad products to make money at investors' expense.

The reality is different: investment certificates are products characterized by high flexibility designed to allow the investor to gain access to asymmetrical income profiles compared to the underlying asset, which he could not otherwise.

The family of investment certificates is now very large and the degree of customization of the payoffs reached is incredible, precisely to meet the needs of different types of investor.

Going back to the classic EUSIPA categorization, to which ACEPI refers, this thesis presents the main types of certificates, describing the characteristics and payoffs offered, thanks to a focus on plain vanilla and exotic options underlying, whose understanding, even basic, allows discerning good products the bad ones.

Subsequently, we moved on to the detailed analysis of a particular Certificate belonging to the family of Bonus Cap Certificates, in turn within the conditional capital protectection certificates category. After having introduced, in fact, basic notions of the most commonly used pricing model, the Black-Scholes model, we proceeded calculating the fair value of this Bonus Cap, both today, to check for any liquidity problems of the product, and back to the date of issue, to clarify the implicit fees that characterize the instrument.

Furthermore, a very interesting situation within the portfolio management environment is presented: retracing the certificate path compared with the underlying Intesa Sanpaolo stock from the date of issue to date, the certificate's efficiency has been verified in situations of recovery of losses deriving from from the holding in the portfolio of a non-performing security, thanks to a mention of the measures involved in the performance of a certificate, the Greeks.

The case studied in this thesis is an infinitesimal percentage of the total potential of investment certificates, which can be used as a long-term investment product, for short-term speculation, liquidity management with capital protection, loss recovery and, as highlighted, portfolio optimization.

The investment product at the center of the analysis is certainly not easy to understand, besides, the aim of the thesis was not to put an end to the investment certificates' argumentation, but rather to open one.

6. Appendix

Sottostante	ISP IM Equity	INTESA SANPAOLO		Trade	01/27/2019
Prz sott	Mid	2.0457	EUR	Rglmn	01/28/2019
Risultati					
Prezzo totale	91.67	Valuta	EUR	Vega	-0.53	Valore tempo
Price (Cert)	2.6127	Delta %	202.94	Theta	0.10	Gearing
		Gamma %	-6.9433	Rho	0.00	Parità - %

Capped Bonus Certifica	Leg 1	Leg 2	Leg 3
Stile	Vanilla	Knockout	Vanilla
Tipo barriera		Terminale	
Esercizio	Europeo		Europeo
Call/Put	Call	Put	Call
Buy/Sell	Buy	Buy	Sell
Strike	0.00	3.2291	3.2291
Strike % Money	100.00% ITM	57.85% ITM	57.85% OTM
Shares/Cert	35.087719		
Posizione	1.00		
Direzne barriera		Down & Out	
Livello barriera		1.71	
Spostamento barriera		0.00	
Status barriera		Attivo	
Scad	06/21/2019 17:50		
Tempo a scad	145 00:00		
Tipo prz	Assoluto		

Figure 37. Bloomberg Terminal pricing for Bonus Cap Certificate (ISIN:DE000HV40CG5) I

Sottostante	ISP IM Equity	INTESA SANPAOLO		Trade	01/27/2019	
Prz sott	Mid	2.0457	EUR	Rglmn	01/28/2019	
Risultati						
Prezzo totale	91.67	Valuta	EUR	Vega	-0.53	Valore tempo
Price (Cert)	2.6127	Delta %	202.94	Theta	0.10	Gearing
		Gamma %	-6.9433	Rho	0.00	Parità - %

Capped Bonus Certifica	Leg 1	Leg 2	Leg 3
Vol Personz	26.645%	26.645%	26.645%
Forward Carry	1.8438		
EUR Rate MM	-0.240%		
Rend dvd	27.488%		
Flusso div scontato	0.20		
Borrow Cost	0.000%		
Prz gamba tot	64.76	26.92	0.00
Leg Prc (Cert)	1.8455	0.7673	-0.0001
Delta gamba %	100.00	103.06	-0.12
Leg gamma	0.0000	-6.9188	-0.0245
Vega gamba	0.00	-0.53	0.00
Theta gamba	0.00	0.10	0.00
Rho gamba	0.00	0.00	0.00
Val tempo gamba	-7.02		0.00
Gearing gamba	1.11	2.67	-18,656.30
Parità gamba	-9.78		57.85

Figure 38. Bloomberg Terminal pricing for Bonus Cap Certificate (ISIN:DE000HV40CG5) II

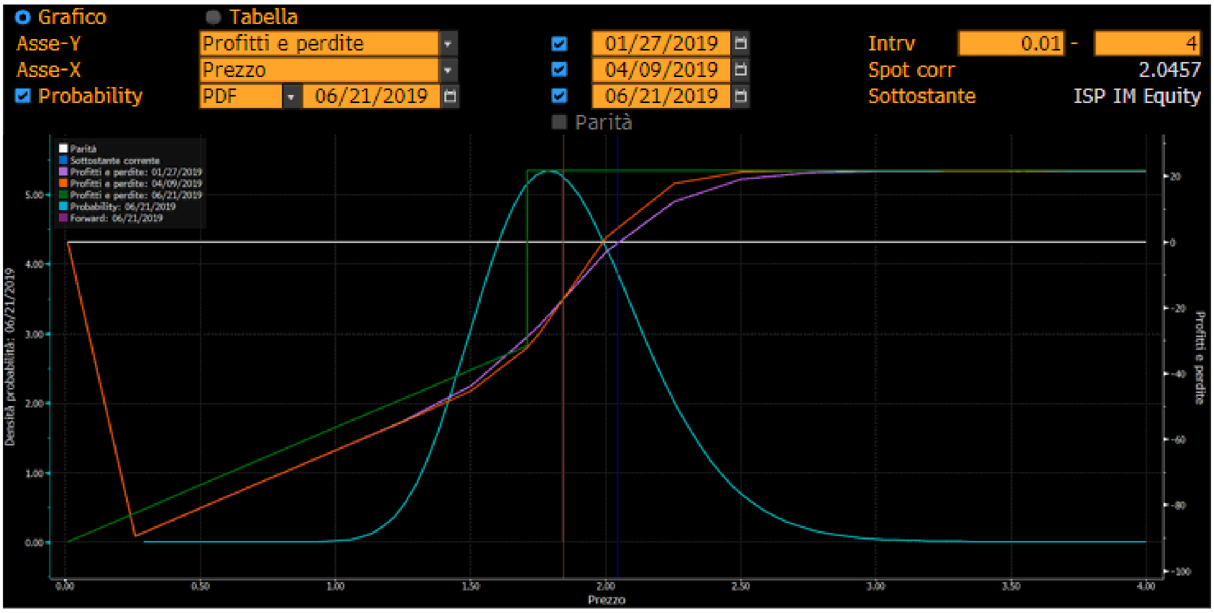
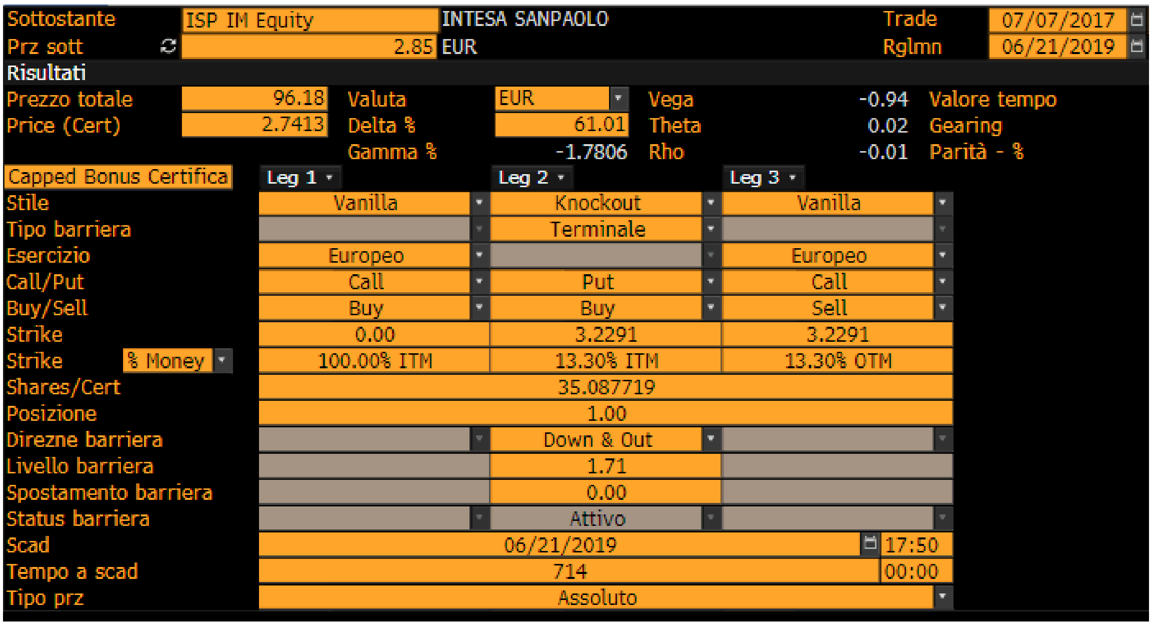


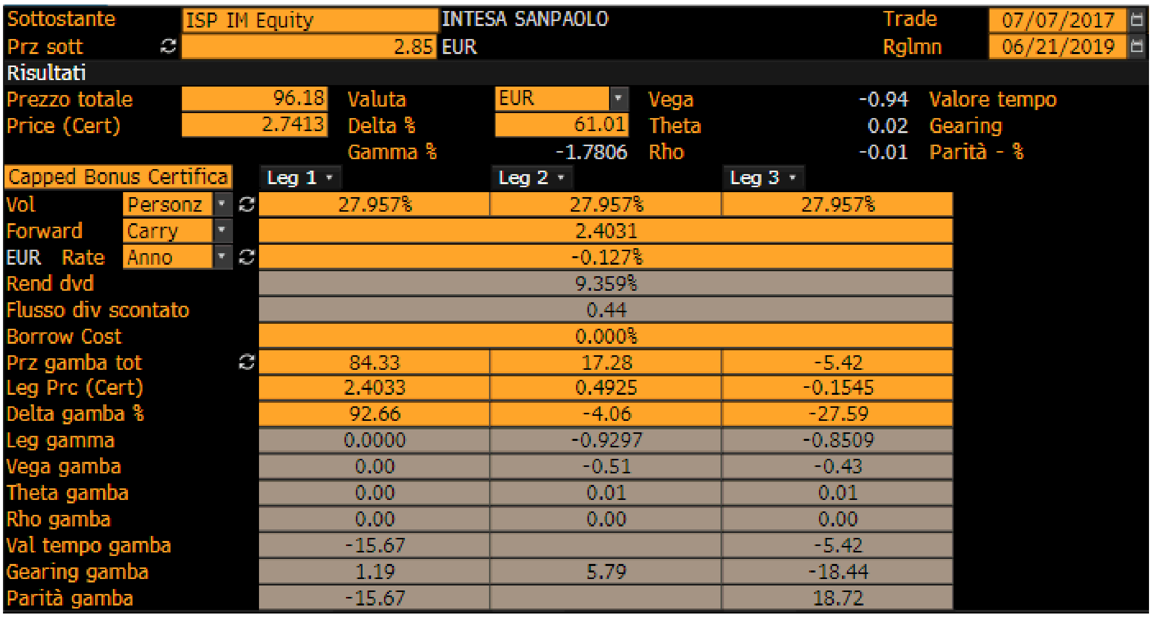
Figure 39. Bloomberg Terminal Profit and Loss diagram, with probability distribution



Sottostante	ISP IM Equity	INTESA SANPAOLO	Trade	07/07/2017
Prz sott	2.85	EUR	Rglmn	06/21/2019
Risultati				
Prezzo totale	96.18	Valuta	EUR	Vega -0.94 / Valore tempo
Price (Cert)	2.7413	Delta %	61.01	Theta 0.02 / Gearing
		Gamma %	-1.7806	Rho -0.01 / Parità - %

Capped Bonus Certifica	Leg 1	Leg 2	Leg 3
Stile	Vanilla	Knockout	Vanilla
Tipo barriera		Terminale	
Esercizio	Europeo		Europeo
Call/Put	Call	Put	Call
Buy/Sell	Buy	Buy	Sell
Strike	0.00	3.2291	3.2291
Strike % Money	100.00% ITM	13.30% ITM	13.30% OTM
Shares/Cert	35.087719		
Posizione	1.00		
Direzne barriera		Down & Out	
Livello barriera		1.71	
Spostamento barriera		0.00	
Status barriera		Attivo	
Scad	06/21/2019 17:50		
Tempo a scad	714 00:00		
Tipo prz	Assoluto		

Figure 40. Bloomberg Terminal pricing for Bonus Cap Certificate (ISIN:DE000HV40CG5) back at the issue date, I



Sottostante	ISP IM Equity	INTESA SANPAOLO		Trade	07/07/2017
Prz sott	2.85	EUR		Rglmn	06/21/2019

Risultati

Prezzo totale	96.18	Valuta	EUR	Vega	-0.94	Valore tempo	
Price (Cert)	2.7413	Delta %	61.01	Theta	0.02	Gearing	
		Gamma %	-1.7806	Rho	-0.01	Parità - %	

Capped Bonus Certifica		Leg 1	Leg 2	Leg 3
Vol	Personz	27.957%	27.957%	27.957%
Forward	Carry		2.4031	
EUR Rate	Anno		-0.127%	
Rend dvd			9.359%	
Flusso div scontato			0.44	
Borrow Cost			0.000%	
Prz gamba tot		84.33	17.28	-5.42
Leg Prc (Cert)		2.4033	0.4925	-0.1545
Delta gamba %		92.66	-4.06	-27.59
Leg gamma		0.0000	-0.9297	-0.8509
Vega gamba		0.00	-0.51	-0.43
Theta gamba		0.00	0.01	0.01
Rho gamba		0.00	0.00	0.00
Val tempo gamba		-15.67		-5.42
Gearing gamba		1.19	5.79	-18.44
Parità gamba		-15.67		18.72

Figure 41. Bloomberg Terminal pricing for Bonus Cap Certificate (ISIN:DE000HV40CG5) back at the issue date, II

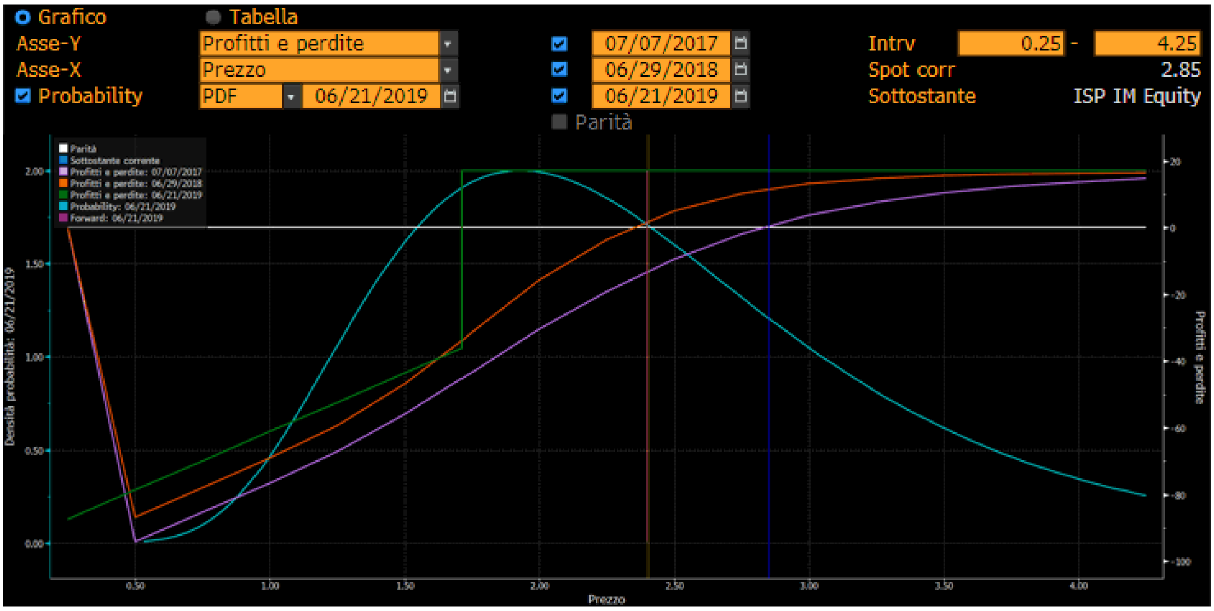


Figure 42. Bloomberg Terminal Profit and Loss diagram, with probability distribution, back at the issue date

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Inside Investment Certificates: Structure and Application in the Developing Italian Market

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ACADEMIC YEAR 2017/2018

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1. Summary

2. Introduction

After 12 years since the infamous subprime crysis took place in the Unites States, poisoning the real economy and financial markets all over the world, it is possible to say that every cloud has, at least, silver lining. The sense of omnipotence characterizing both “too big too fail” and small investors soon left space to uncertainty, bringing whichever actor inside the financial markets to the stabilisation of that risk/return balance they prematurely forgot about.

The growing risk aversion led, indeed, to investors’ need of security they could not find in no asset class anymore, except in perfectly liquid instruments.

In this contest, structured product found fertile ground to develop and among them, broadly, the Certificates, defined as securitized derivatives built through the combination of other derivatives instruments (mainly options and futures) and/or basic assets like bonds (Zero Coupon), thanks to which it is possible to generate original financial instruments with their own specific chatacteristics, different from the assets they come from. Given that definition, it is immediate to understand how Certificates present all the characteristics of syntetic assets, despite the fact that they stand out from them due to the presence of one particular risk category, the counterparty credit risk, since they are structured to realize different payoffs.

Investment Certificates present themself as an alternative to products that can ensure an adequate saving plan for the investor (retirement plans, insurance policies and shares of mutual funds) giving them the possibility to invest in something that is taylor made on their needs, at the same time.

This dissertation aim to further explore a subject only skimmed by the scientific literature, mostly due to its brief history, trying to make a financial instrument more clear while it is still shrouded in mistery and from whom the most get away just for their denomination of derivative, term that should stand for “financial security with a value that is reliant upon an underlying asset or group of assets” and not for toxic product.

Background information will be given in the first part, before moving forward to study the structure of this financial instrument, analyzing its composition, studying the variables at stake at the beginning and then coming to its possible uses in various fields.

We will construct from scratch a Bonus Cap Certificate, then, studying deeply the options inside it by pricing them, its performance will be analyzed in order to understand how a simple certificate can be useful in many situation in the portfolio management environment.

Within the limit of my contribution, the final aim of this dissertation is to increase the financial culture of the average investor, which, translating Giovanna Boggio Robutti, head of the FEDUF (Fondazione per l’educazione finaziaria e al risparmio), “compares thousands of websites in order to buy a new phone, hundreds of car dealerships to buy a car, but, when it comes to save money, believes that a friend’s suggestion is more than enough”.

3. Certificates

Certificates are financial instruments characterized by a large variety of risk/return profiles that allow to provide to the investors taylor-made financial solutions related to every different investment need.

Investment Certificates are securitized derivatives, in that a combination of financial contracts assembled in single securities which can be traded like stocks.

In Italy they are traded in two different markets and: Cert-X (EuroTLX) and SeDeX (Borsa Italiana).

These products are issued by financial institutions which assume the obligation to pay the due cash flows, following what stated in the related prospectus, that must be released for every tranche of products.

A single Certificate presents different alternative characteristics:

- Directionality of the underlying asset (exposition to positive and negative movements)
- Cash Flows during the life of the instrument (coupons)
- Early redemption with premium for the investor
- Full, partial or conditioned capital protection of the initial amount invested
- Redemption premium at maturity

There are four main categories of cetificates: Capital Protectection, Conditional Capital Protectection, No Capital Protection and Leverage.

Each one of these product can be structured in many ways thanks to the infinite combination of exotic option available Over The Counter, in order to create unique products that can fit almost every investor's need: as Mauro Camelia explains in his "Il Libro dei Certificati", "a certificate can be considered as the point of conjunction of investor's preferences and derivatives components".

3.1. History and Current Framework

Germany is for sure the ancestor of all the Certificates markets nowadays available. The first certificate was issued in the far 1989, anticipating by almost 9 years the not as succesful debut of such product at Piazza Affari.

As mentioned before, the trigger for the rebirth of this instrument was the 2007/2008 crysis and along with this, the insecurity that, from that time on, every investor could feel. Since then, this particular market knew a great expansion year by year, reaching the top in 2015.

Numbers speak for themself: The new lisitngs trend (primary market) from 2006 until today is significantly increasing, with a peak reached in 2015 and already overcame by partial data of this year (primary market data available up to 3rd quater) from the point of view of number of new listings (not yet in terms of countervalue).

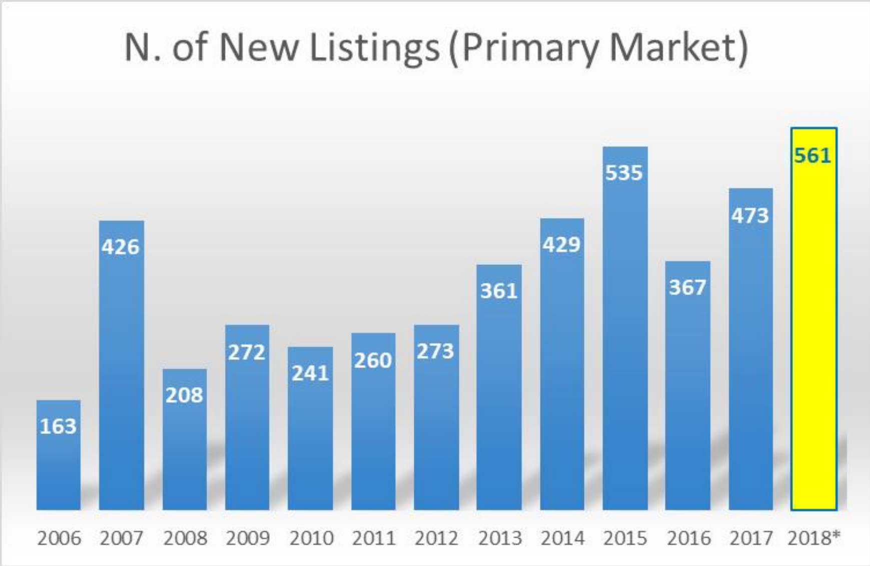


Figure 1. (Source: ACEPI)

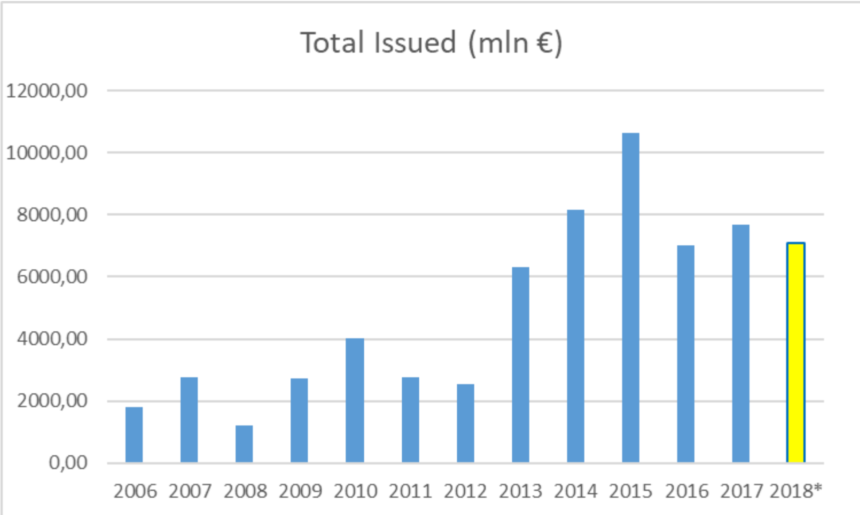


Figure 2. (Source: ACEPI)

As for secondary market, meaning among private subjects trades, we have consistent data only starting from 2010 that can be considered partial until 2016, when EuroTLX became an established market, yet it is still clear that also for this market the trend is upward looking, considering 2018 data are available up to 1st quarter.

Figure 3. (Source: ACEPI)

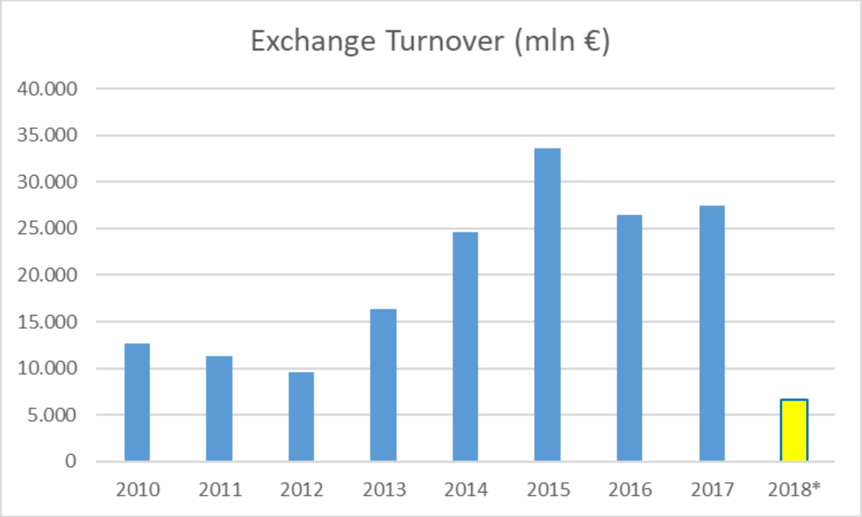


Figure 4. (Source: ACEPI)

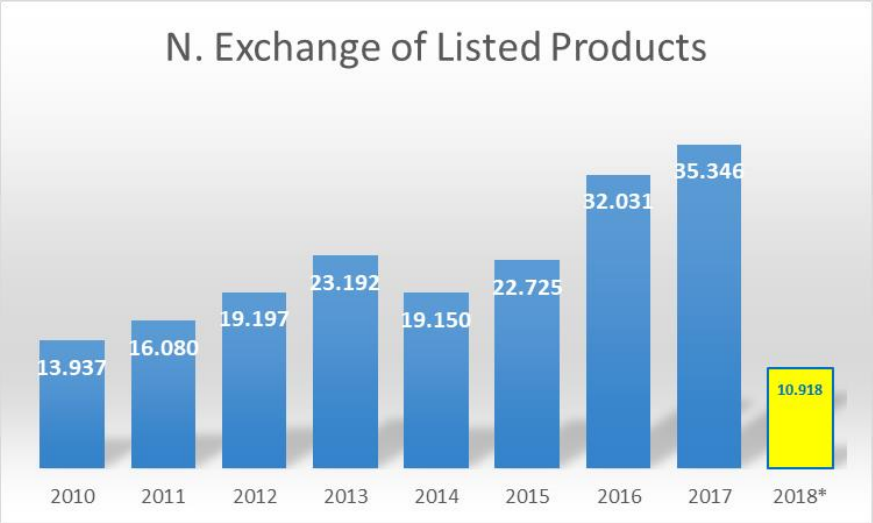
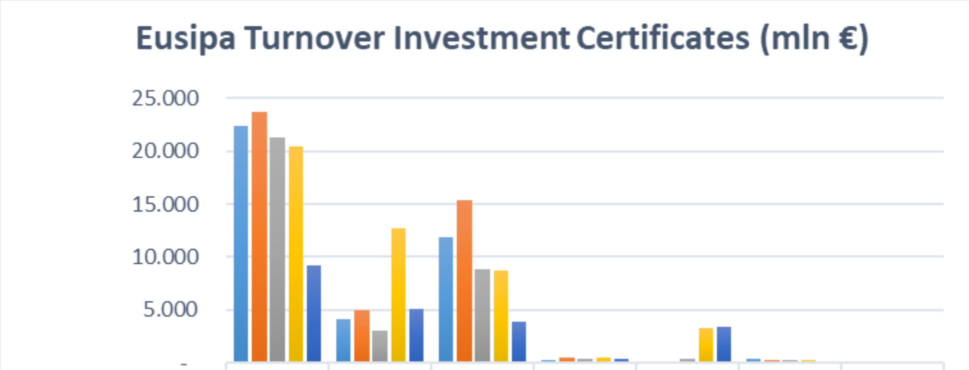


Figure 5. (Source: ACEPI)

This consistent growth led Italy to be one of the most important European Certificates Markets, as EUSIPA data show:



	Germany	Italy	Switzerland	Austria	Sweden	France	Netherlands
2014	22.349	4.142	11.880	220	-	410	27
2015	23.674	4.919	15.366	482	53	274	83
2016	21.295	2.993	8.856	402	391	273	54
2017	20.393	12.687	8.747	509	3.264	290	56
2018 (2° trim.)	9.191	5.060	3.883	328	3.379	166	31

Figure 6. (Source: ACEPI)

3.2. Certificates Classification

Despite their brief history, Certificates underwent through bright changes and, in time, a extraordinary level of customization have been reach, in order to meet investor's need and market's dynamics.

3.2.1. Capital Protection Certificates

Certificates with capital protection allow the investors to invest in financial asset eluding the risk of losing the invested capital, if purchased at issue and held till maturity. If such certificates are bought on the secondary market, there is no full capital protection (it is only possible to forecast the worst possible performance of the certificate if held till maturity), keeping out of the equation the CCR (Counterparty Credit Risk), which however characterize every single structured product.

This kind of product find its success in the growing need of safety by the investor, fitting their needs with the strong prudential predilection which characterize them. For this reason capital protection certiifcates turn out to be seen with a positive eye from the supervisory bodies, especially because their payoffs are clear and therefore they are widespread among retail clients.

"Capital guaranteed products guarantee the redemption of the invested capital at maturity in addition to participating to a certain degree in the performance of an underlying risky asset"[1].
In this category we find: Equity Protection Certificates, Digital Certificates and Express Protection Certificates, but also other more particular instrument such as Butterfly Certificates and Twin Win Protection.

3.2.2. *Conditional Capital Protection Certificates*

Certificates characterized by conditional capital protection are financial products that allow investors to indirectly invest in certain assets while enjoying a partial capital protection, which is activated if pre-arranged barrier levels are not reached during the life of the certificate.
The barrier for this kind of Certificates can be either of American type, for which the underlying asset can not reach the predetermined barrier level (or Knock-Out Level) for the whole life of the instrument, or European type, for which the value of the underlying asset with respect to the Knock Out Level is verified only at maturity.
The most important Certificate among the Conditional Capital protection category is Bonus Certificate, thanks to one of the highest risk/return trade off.
Other important certificates belonging to this category are: Cash Collect, Express, Outperformance and Twin Win.

3.2.3. *No Capital Protection Certificates*

Certificates with no capital protection are very simple products whose aim is to replicate the performance of their underlying assets and for this reason investors are exposed to the very same risk of investing in the underlying asset.
Some certificates belonging to this category may replicate the underlying asset's performance in proportions. They are classified in Benchmark and Outperformance.

3.2.4. *Leverage Certificates*

Leverage certificates are financial products providing investors with a more than proportional exposure to the performances of their underlying assets, making it possible to benefit from either positive or negative performances of the underlying asset, depending on the typology of the certificate.
They are classified in constant leverage and variable leverage. The latter have been created in order to avoid the so called Compounding Effect generated by the constant leverage certificates held in portfolio in the long time horizon.

[1] Bluemke, 2009.

The calculation basis for Fixed Leverage Certificates is set on a daily basis depending on the closing price of the underlying financial asset.

For this reason, for holding periods greater than one trading day, the so called “Compounding effect” is generated.

If the product is held in portfolio for more than one day its performance could dramatically differ from the underlying asset’s in the same period, in addition it is multiplied by the leverage.

4. Options

Once it is clear how a certificate works it is possible to move forward to the analysis of their detailed composition, namely the options available for a Bank that wants to structure and issue such financial products. Knowing deeply the characteristics of every single option inside these structured product allow to give technical and operational reason to the choices made by the Issuer Bank in terms of structures proposed and also to understand how and in which measure the price of a Certificate will be sensitive to the variations of the underlying.

This chapter will brings particular attention to the world of exotic options, which are the main reason for the asymmetrical settlement profile that characterize investment certificates. These particular kind of options, indeed, stands out for high sensitivity of their prices to market’s variables, first of all the volatility.

While we talk about plain vanilla for derivatives such put and call option both american or european type, since they present standard properties, with well defined and clear payoffs and traded actively by brokers o regulated markets (the first options market to ever exist was the Chicago Board of Options Exchange in 1973), with the denomination of exotic options we mean all the non standard contracts, traded OTC (Over The Counter) and therefore outside of the regulated markets, which have been create in time by financial engineers. Referring to the timeless classification made by John Hull in his “Options, Futures and Other Derivatives”, inside the exotic options category we find: Non-standard american, gap, forward start, shout, compound, chooser, barrier, binary, lookback, cliquet, asian and basket (or raimbow) options.

4.1. The Black-Scholes Model

The need to estimate the price of options in an increasingly accurate manner has led operators to develop increasingly advanced pricing techniques that are suitable for considering the effects of sudden changes in market conditions.

Over the last few decades, numerous models for the determination of option prices have appeared in the literature, each of which is based on different assumptions with respect to market variables, such as the distribution of probability of the underlying or the type of volatility. to be used in the calculation.

Although today there is no uniformity of judgment on which the best performing model is, the totality of scholars and operators are in agreement in recognizing the influence on the methods of subsequent evaluation exercised by the Black & Scholes model (BS), elaborated at the beginning of the 70's by Fisher Black and Myron Scholes (1973).

This importance derives above all from its ability to provide correct pricing, even if based on hypotheses that are sometimes excessively simplistic, on the other, from its peculiarity as one of the few tools available to operators for analytical evaluation of options.

It was first developed in order to price European financial options (options that can not be exercised before the maturity) and from the first version it contributed e inspired creating all the following pricing models.

It is necessary to give the right credit to Merton for his important contribution towards the consecration of the so called Black and Scholes model, due to his adjustments and upgrades.

4.1.1. Adjustments to the Black-Scholes Formula

Financial Theory have brought many refinements to the Black and Scholes formula. In 1973 Robert Merton abandoned the no dividend distribution assumption during the life of the option, providing us the new formulas:

$$C = Se^{-qT}N(d_1) - Ke^{-rT}N(d_2)$$

$$P = Ke^{-rT}N(-d_2) - Se^{-qT}N(-d_1)$$

Where q is the dividend yield of the underlying stock, and:

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r - q + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{\ln\left(\frac{S}{K}\right) + \left(r - q - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

Operationally, this means that the Black–Scholes–Merton formulas can be used
knowing that the stock price is reduced by the present value of all the dividends during the life of the option.

5. Inside a Bonus Cap

In this chapter we will proceed by dismantling a certificate into all the options inside it, in order to better understand how the options described before work together, creating personalized payoffs.

The certificate taken into account is a so called Bonus Cap issued by one of the most important Italian banks on the 7th of July 2017, expiring on the 21st of June 2019 and written on Intesa Sanpaolo stock.

A Bonus Cap certificate belongs to the conditional capital protection category and it allows investors to have a fixed premium paid at maturity (the bonus) while seeing their investment protected, until certain conditions for the underlying hold.

Getting into details this Bonus Cap offers a 113,3% premium at maturity (calculated from the nominal price of 100 €) with a Cap at the same level, meaning that the investor can not receive more than the 113,3 € at maturiry, even if the underlying stock registers an above cap performance[2].

At the same time this certificate admit an american barrier (observed continuously, intraday[3]), put at 60% with respect to the strike price, for the equivalent of a price of the Unicredit Stock of 1,71 €, below which the Settlement amount will be linearly reduced by the negative performance of the underlying with respect to the strike price.

All Bonus Cap Certificates are built putting together three options (or legs of a certificate), two plain vanilla and one exotic option, here the details.

[2] i.e. Intesa Sanpaolo at maturity registers a performance higher than the 113,3% with respect to its Strike Price.

[3] If the share price of Intesa Sanpaolo goes below the barrier in any moment during a trading day a Knock Out event will occur and the certificates will then follow the underlying asset linearly, and it will be equivalent to holding the stock.

+*Plain Vanilla Call* with strike price at 0, that allows to linearly replicate the performance of the underlying:

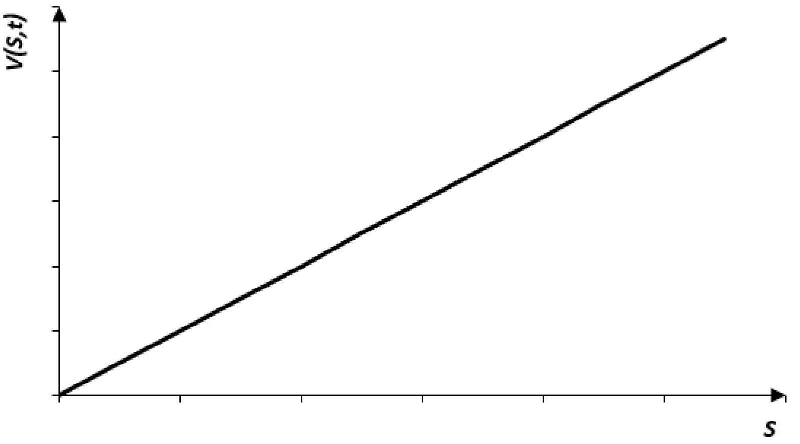


Figure 7. Payoff V(S,t) of a Long Call Option with Strike equal to 0. (Personal elaboration)

-*Plain Vanilla Call* with strike price K at the Cap level, which is, in our case, 3,22905 €. This leg is used to reduce the price of the certificate itself and put a limit to the premium that the investor can receive:

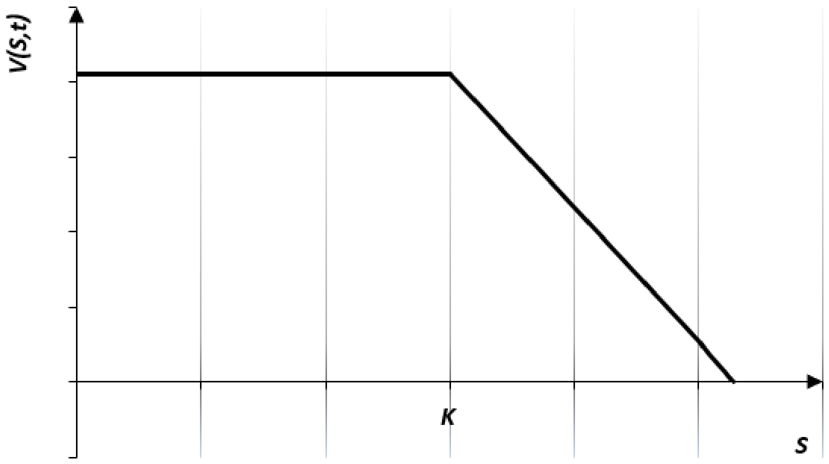


Figure 8. Payoff V(S,t) of a Short Call Option with Strike K. (Personal elaboration)

+*Put Down and Out Barrier Option* with strike price at the Bonus level (19,8708 €) and barrier at the barrier level (13,92 €). This last leg of the certificate is based on a Knock Out Event: it ceases to exist if the underlying goes below the barrier:

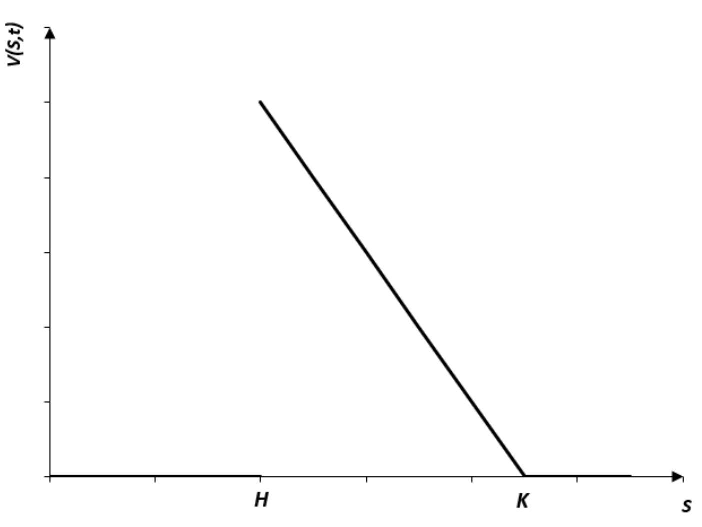


Figure 9. Payoff V(S,t) of a Put Down & Out barrier option with Barrier Level at B and Strike Price K. (Personal elaboration)

These three legs put together create the Bonus Cap Certificate, one of the most traded certificate with conditional capital protection, which present, consequently the following payoff:

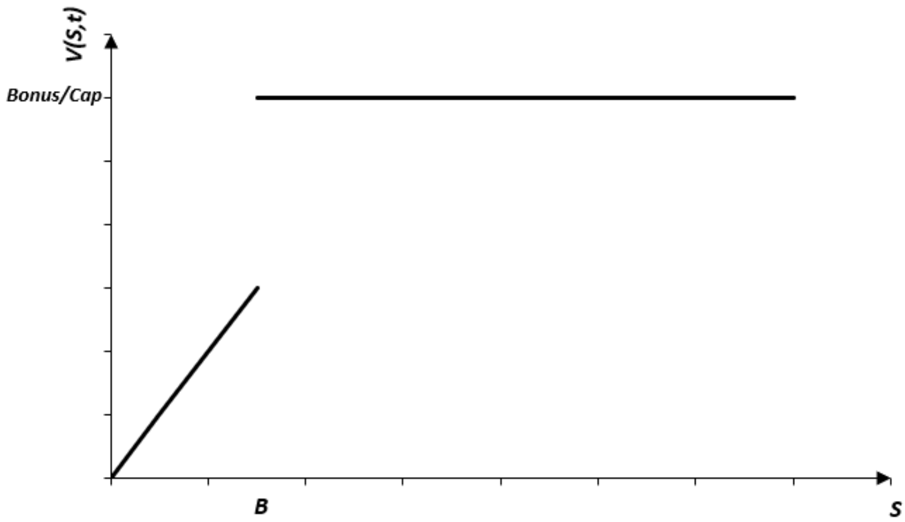


Figure 10. Payoff V(S,t) of a Bonus Cap Certificate with Barrier at B. (Personal elaboration)

It offers the opportunity to be rewarded with more than the performance of the underlying until it stays between the barrier and the bonus (and cap) level.

From the chapter before we now know how to price both plain vanilla options, let's use those equation, together with specific formulas for the more difficult calculation of exotic options (barrier options in this particular case), to check the fair value of this certificate today.

The first leg of the certificate is built by buying the so called "***Strike 0" Call Option***:

$$C = Se^{-qT}N(d_1) - Ke^{-rT}N(d_2)$$

That easily becomes:

$$C = Se^{-qT}N(d_1)$$

Since the Strike Price K is 0[4].

Intesa Sanpaolo's shares are currently trading at 2,049 € (S) and the italian bank have a dividend yield (q) equal to 9,96%, moreover the option will expire on the 21th of June 2019, therefore there are 148 days left till expiration, then T will be given from $\frac{148}{365} = 0{,}405479$. The last data needed are the annual volatility of Intesa Sanpaolo which is 26,645% and the italian risk free rate (r) that is equal to the yield of a 10 years italian government bond which is currently 2,394% that becomes 2,4429% since we need it to be continously compounded.

Therefore, the above equation becomes:

$$C = 2{,}049 \times e^{-0{,}0996 \times 0{,}405479} \times N(d_1)$$

Now we focus on $N(d_1)$, we know that d_1 is given by:

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r - q + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$d_1 = \frac{\ln\left(\frac{2{,}049}{1e^{-20}}\right) + \left(0{,}024429 - 0{,}0996 + \frac{0{,}26645^2}{2}\right) \times 0{,}405479}{0{,}26645 \times \sqrt{0{,}405479}}$$

$$d_1 = 248{,}4127558$$

Using the tables for the Cumulative Normal Distribution we obtain that:

$$N(248{,}413) = 1 = \delta$$

And, finally:

[4] It is impossible to put the Strike price exactly at 0, therefore in the calculation we will use an approximation close to it, i.e. $1e^{-20}$.

$$C = 2{,}049 \times e^{-0{,}0996 \times 0{,}405479} \times 1 = 1{,}967898291 \text{ €}$$

1,9679 € is the price of one single Strike 0 Call inside the Bonus Cap and it explain what mentioned at the beginning of the chapter, i.e. this leg is used to replicate the underlying, therefore the price of such options is always very close to the share price S.

Let's move forward pricing the second Call Option that has to be sold.

Since it is used to fix the Cap of our certificate, it will have a strike price K exactly at the Cap Level of 3,22905. Using the same data set:

$$C = Se^{-qT}N(d_1) - Ke^{-rT}N(d_2)$$

$$C = 2{,}049 \times e^{-0{,}0996 \times 0{,}405479} N(d_1) - 3{,}22905 \times e^{-0{,}024429 \times 0{,}405479} N(d_2)$$

$$d_1 = \frac{\ln\left(\frac{2{,}049}{3{,}22905}\right) + \left(0{,}024429 - 0{,}0996 + \frac{0{,}26645^2}{2}\right) 0{,}405479}{0{,}26645 \times \sqrt{0{,}405479}}$$

$$d_1 = -2{,}776031088$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

$$d_2 = -2{,}776031088 - 0{,}26645 \times \sqrt{0{,}405479} = -2{,}94569917$$

$$C = 2{,}049 e^{-0{,}0996 \times 0{,}405479} N(-2{,}776) - 3{,}22905 e^{-0{,}024429 \times 0{,}493151} N(-2{,}946)$$

$$C = 0{,}000262819 \text{ €}$$

0,000536 € is the unit price we will receive by selling a single Call with this characteristics today.

The final step in order to give a fair evaluation of the Bonus Cap analyzed is to price the third leg of the certificate, namely the Put Down & Out Barrier Option.

Adjustments to Black & Scholes Model have brought us to the following parity equations for pricing barrier options:

$$C = C_{di} + C_{do}$$
$$P = P_{di} + P_{do}$$

Where C and P are the values of regular call and put options and C_{di}, C_{do}, P_{di} and P_{do} are the values of a Call Down & In, a Call Down & Out, a Put Down & In and a Put Down & Out Barrier Options, respectively.

The Option used to create the payoff of the Bonus Cap Certificate is the latter, namely P_{do}, let's move forward to price it.

From Hull (*Options, Futures and Other Derivatives*, 10th Edition, 2018) we know the Black and Scholes adjusted formula for correctly pricing P_{di}:

$$P_{di} = -SN(-x_1)e^{-qT} + Ke^{-rT}N\left(-x_1 + \sigma\sqrt{T}\right) + Se^{-qT}\left({}^{H}/_{S}\right)^{2\lambda}[N(y) - N(y_1)] - Ke^{-rT}\left({}^{H}/_{S}\right)^{2\lambda-2}\left[N\left(y - \sigma\sqrt{T}\right) - N\left(y_1 - \sigma\sqrt{T}\right)\right]$$

And from the parity shown above we can derive the value of the option we are looking for:

$$P_{do} = P - P_{di}$$

We first need to evaluate P with a Strike Price at the Bonus Level of 3,22905€ of Intesa Sanpaolo, using the same data set we used for pricing the two Call options:

$$P = 3{,}22905e^{-0{,}02442T}N(-d_2) - \ Se^{-qT}N(-d_1)$$

With

$$d_1 = \frac{\ln\left(\frac{2{,}049}{3{,}22905}\right) + \left(0{,}024429 - 0{,}0996 + \frac{0{,}26645^2}{2}\right)0{,}405479}{0{,}26645 \times \sqrt{0{,}405479}}$$

$$d_1 = \ -2{,}776031417$$

and

$$d_2 == -2{,}776031417 - \ 0{,}26645 \times \sqrt{0{,}405479} = \ -2{,}945699499$$

Therefore:

$$P = 3{,}22905e^{-0{,}02442T}N(2{,}776) - \ Se^{-qT}N(2{,}946)$$

$$P = 1{,}229846673\ €$$

Now we need to find the value of the Put Down & In Barrier Option using the formula provided by Hull and to do this it is necessary to explain 4 parameters used in such equation.

$$\lambda = \frac{r - q + \frac{\sigma^2}{2}}{\sigma^2}$$

$$y = \frac{\ln(\frac{H}{S \times K})}{\sigma\sqrt{T}} + \lambda\sigma\sqrt{T}$$

$$y_1 = \frac{\ln(\frac{H}{S})}{\sigma\sqrt{T}} + \lambda\sigma\sqrt{T}$$

$$x_1 = \frac{\ln(\frac{S}{H})}{\sigma\sqrt{T}} + \lambda\sigma\sqrt{T}$$

Now we can proceed to P_{di} evaluation and consequently to P_{do}, which is our goal.

$$\lambda = \frac{0{,}024229 - 0{,}0996 + \frac{0{,}26645^2}{2}}{0{,}26645^2} = -0{,}561631073$$

$$y = \frac{\ln(\frac{1{,}71}{2{,}049 \times 3{,}22905})}{0{,}26645 \times \sqrt{0{,}405479}} + -0{,}0561631073 \times 0{,}26645 \times \sqrt{0{,}405479}$$

$$y = -4{,}907940929$$

$$y_1 = \frac{\ln(\frac{1{,}71}{2{,}049})}{0{,}26645 \times \sqrt{0{,}405479}} - 0{,}561631073 \times 0{,}26645\sqrt{0{,}405479}$$

$$y_1 = -1{,}161245623$$

$$x_1 = \frac{\ln(\frac{2{,}049}{1{,}71})}{0{,}26645\sqrt{0{,}405479}} - 0{,}561631073 \times 0{,}26645\sqrt{0{,}405479}$$

$$x_1 = 0{,}970663889$$

Now we can put these coefficients inside the formula for the evaluation of P_{di}.

$$P_{di} = -2{,}049N(-0{,}97)e^{-0{,}0996\times 0{,}405479} + 3{,}22905e^{-0{,}024229\times 0{,}405479}N\left(-0{,}97 + 0{,}26645\sqrt{0{,}405479}\right)$$
$$+ 2{,}049e^{-0{,}0996\times 0{,}405479}\left({}^{1{,}71}/_{2{,}049}\right)^{[2\times(-0{,}56)]}[N(-4{,}9) - N(-1{,}16)]$$
$$- 3{,}22905e^{-0{,}024229\times 0{,}405479}\left({}^{1{,}71}/_{2{,}049}\right)^{[2\times(-0{,}56)-2]}[N\left(-4{,}9 - 0{,}26645\sqrt{0{,}405479}\right)$$
$$- N\left(-1{,}16 - 0{,}26645\sqrt{0{,}405479}\right)]$$

$$P_{di} = 0{,}569371824 \text{ €}$$

Remember

$$P_{do} = P - P_{di}$$

Therefore

$$P_{do} = 1{,}230119982 \text{ €} - 0{,}622337856 \text{ €} = 0{,}660474849\text{€}$$

We finally know what is the price of every option inside the Bonus Cap given by the Black-Scholes Model[5] and we can create a package composed by buying one Strike 0 Call and one Put Down & Out Barrier Option and selling a Call with strike price at the bonus level.

$$Package = +C_{K_0} + P_{do} - C_{K_B}$$

$$Package = +1{,}967898291\text{€} + 0{,}660474849\text{€} - 0{,}000262819\text{€}$$

$$\boldsymbol{Package = 2{,}6281103212} \text{ €}$$

[5] Other models can lead to slighlty different results.

5.1. Face Value and Multiple

In order to get to a Fair Evaluation of the Certificate we need to explain the concept of Face Value and multiple. Certificates are built using packages and then they are placed at a Face Value that is usually 100€[6].

In order to get to this unit value the Issuer has to buy many option packages, therefore a Certificate controls more than one share of the underlying and the ratio between the Face Value and the Strike Price (which is the share price of the underlying at the issue date) returns the Multiple.

The Multiple express the countervalue that the Certificate move or equivalently the number of shares of the underlying.

Going back to our study, the Face Value of the Bonus Cap is 100€, the Strike Price of Intesa Sanpaolo is 2,85€, therefore:

$$Multiple = \frac{100}{2{,}85} = 35{,}0877193$$

This particular Bonus Cap Certificate controls 35,0877 Intesa Sanpaolo shares, which means that the Issuer bought 35,0877 of the above priced packages, leading us the final price of the Certificate:

$$Bonus\ Cap\ Certificate = 35{,}0877 \times\ 2{,}628110321 =\ 92{,}21434652\ €$$

The Certificate today has a Black-Scholes Fair Value of 92,214 €, against a Market Value of 87,5€/88,15 (Bid/Ask), which means that, following our calculation, it is slightly underestimated by the Market.

5.2. Issue Date Price and Implicit Placement Fee

It is now interesting to investigate what was the Fair Value of the Bonus Cap in correspondance to the Issue Date, in order to better understand what gives back the unit face value of 100.

The dividend payed by Intesa Sanpaolo in 2017 was 0,178€ per share and the strike price on the 7th of July 2017 was 2,85€, therefore our q will be given by $\frac{0{,}178}{2{,}85} = 6{,}2456\%$. The risk-free rate r, given by the continuously compunded yield of a 10 years Italian BTP issued at the end of 2017 was 1,7166% and regarding the volatilty σ we notice that it was higher at that time, for a value of 27,957%.

[6] There are a lot of Certificates whose Face Value is 1.000 or 10.000 or more particular values, but 100 is certainly the most common, since it is in the nature of this financial instrument to be meant for retail investors. However the Face Value is always mentioned inside the Final Terms of the Certificate and it usually includes an implicit placement fee.

In addition, we need to modify the parameter T, that will be 1,619178 ($\frac{591(days\ to\ expiry)}{365}$) and S, that will be exactly the Strike Price K=2,85€.
Plugging this data inside the previously used equations we have:

$$Bonus\ Cap\ Certificate_{issue\ date} = 96{,}85158706\ €$$

Even if the Certificate Fair Value at the issue date was 96,85€, the placement took place for 100€ and the difference between the Face Value and the amount actually paid for the packages is exactly the implicit placement fee for the Issuer, that earned 3,15€ (3,15% fee) per certificate, against the 1,11% indicated in the Key Investor Document.
It is necessary to say that the price we found may also be significantly different from the actual one the issuer paid, moreover different methods of pricing can lead to different results.
From this observation it is possible to proceed trying to debunk the myth that the issuing bank benefits from constructing "bad" structured products, that is to say products that will hardly pay the investor at maturity.
The only source of income for a bank that issues certificates derives from the ability of structurers to find options Over The Counter at the lowest possible price, succeeding, at the same time, in creating interesting structures. After that, at maturity, it is not the issuing bank that materially pays the investor, its outlay dates exclusively to the structuring phase, therefore there is no interest on the part of the issuer to ensure that the product does not pay the final amount or the premium to the investor.
Moreover, the issuing bank allows the investor to invest in packages that he would not be able to compose on his own, as there are options over the counter inside, that is to say strategies such as those offered by investment can not be replicated by the retail investor.
However, knowing the structure of a certificate, in that the options inside it, allows the investor to understand if the product is well structured or not, even without performing a fair price evaluation like the one provided.
An interesting doubt, instead, could arise with regard to the purchase of certificates in placement when they could be easily bought on the secondary market, once they have discounted the implicit placement fee.
The first and most important reason link us to what previously said while talking about the market maker.
In fact, a number of Issuer reserve the right to go in the market in “bid-only”, meaning that that there will always be, as regulation impose, an offer available for an investor to sell the certificate, but not to buy it.
Therefore, if one is interested in particular structure, which he believes to be efficient and profitable, sometimes the only way to invest in it is to buy the certificate at placement.
In addition, it may happen that despite the implicit placement fee, the underlying’s peformance in the first days of trading brings the certificate to a price which can be either the equal to the face value value itself or even higher, in that an investor could also miss the right time to enter the investment.

5.3. Stock vs Certificate

Now we finally have all the data in order to perform an analysis of the performances of both the certificate and the stock, since the time the certificates was admitted for trading. In this way we can understand how efficient such a instrument can be with respect to the stock itself.

Figure 11. Stock vs Certificate price evolution from issue date to 28/01/2019. (Data: Bloomberg)

As is evident, except for the period from January 2018 to May of the same year, the certificate yield (orange line) has always been above that of the share price (green line), why?

In the first part of the graph the temporal component heavily impacts on the certificate price (theta), ie the positive change of the underlying (delta) is held marginally in consideration in the pricing of the derivative, given that the expiration date is still far away. Going forward in time, the chances that the underlying asset, that is to say the Intesa Sanpaolo stock, will reach the price of 1.71 € (barrier level at which the barrier option within the certificate ceases to exist and consequently the product linearly follows the performance of the underlying) decrease, so the value of the certificate will increase and, above all, will react more amplified to the movements of the underlying, in both directions, as is evident from the end of December until today where there is a growth trend of Intesa Sanpaolo (please note that the expiration date is set for June 21 this year).

Likewise, a significant drop by the stock towards the barrier level will significantly increase the probability of a Knock Out event, so, approaching maturity (with the theta basically at zero) the delta will govern the price of the certificate, which could also go below the share price (in percentage), together with another important component of every Bonus Cap Certificate, more particularly of every barrier option, the Vega.

We already said that vega measures the sensitivity of the option price to the volatility of the underlying; this greek letter becomes fundamental analyzing barrier option, therefore Bonus Cap Certificates.
In fact, the more the underlying asset is volatile, the more probable will be for it to break the barrier and the option price will suffer from it, especially if the barrier is of american type, ie continuosly checked.
The situation described above is that relating to the purchase of the certificate during placement, but, as we all know, there is a secondary market of certificates that allows to structure interesting strategies.

5.4. Switch to Recovery

Let's assume an investor bought Intesa Sanpaolo shares at a price of € 2.5 and on January 2nd finds them at a value of € 1.96 and has therefore achieved a loss of 21.6%. In correspondence with the price of € 1.96 of the share, the certificate quoted € 72.34 and € 72,8 was the ask price, ie the actual price at which the investor would have bought.
An excellent recovery strategy on the stock would have been made by selling the shares at a unit price of € 1.96 and purchasing the certificate for € 72.8.
The price of the certificate today is € 87.5 / 88.15 for a value of the underlying security of € 2.12572.
Holding the stock in the portfolio until today would have result in a loss of almost 15%, while, accepting the capital loss resulting from the liquidation of the security (21.6%) and at the same time switching to the Bonus Cap Ceritficate, a yield of 20.2% would be obtained, not sufficient to offset the entire loss, but allowing to get close to the initial capital invested[7].
This is why Certificates are ideal tools for recovering situations of non-performing portfolio.

5.5. Scenario Analysis

As of today, the following is the scenario analysis for the Bonus Cap Certificate discussed in the dissertation:

Scenario	*-50%*	*-40%*	*-30%*	*-20%*	*-10%*	0%	10%	20%	30%	40%	50%
Share Price Intesa Sanpaolo	1,0	1,2	1,4	1,6	1,8	2,0	2,3	2,5	2,7	2,9	3,1
Certificate's Settlement Amount	35,9	43,1	50,3	57,5	113,3	113,3	113,3	113,3	113,3	113,3	113,3
P&L certificate	*-59,2%*	*-51,1%*	*-42,9%*	*-34,8%*	*28,5%*	*28,5%*	*28,5%*	*28,5%*	*28,5%*	*28,5%*	*28,5%*

Table 1. Scenario Analysis for Bonus Cap Certificate (ISIN: DE000HV40CG5). (Personal elaboration)

With the following graphical representation of profit & loss evolution:

[7] $1{,}96 \times (1 + 0{,}202) = 2{,}35592$ € vs the initial price of 2,5 €.

Figure 12. Profit and loss diagram of the Bonus Cap Certificate (ISIN: DE000HV40CG5) vs Stock

From the graph above it's easy to see the convenience of investing in such product instead of purchasing the stock.

The "profit area" is significantly larger than the "loss area" and the investor is willing to give up the gains that would derive from a growth of more than € 2,634, in order to gain in situations of sideways, and especially in situations where investing in the stock would only generate losses.

Moreover, the difference in the downward scenario prospected between the certificate and the stock is basically negligible.

6. Conclusions

The purpose of this thesis was to clarify a financial instrument still wrapped in fog, whose a priori exclusion from the range of investment products available to the investor, exclusively for the denomination of financial derivative, appears superficial.

Although their growth in the italian market is significant, it is believed, in fact, that, in the eyes of many, these structured products must be avoided at all costs, yet, for their nature and, above all, for the widespread belief that the only ones to gain from the circulation of the certificates are the issuers, who create bad products to make money at investors' expense.

The reality is different: investment certificates are products characterized by high flexibility designed to allow the investor to gain access to asymmetrical income profiles compared to the underlying asset, which he could not otherwise.

The family of investment certificates is now very large and the degree of customization of the payoffs reached is incredible, precisely to meet the needs of different types of investor.

Going back to the classic EUSIPA categorization, to which ACEPI refers, this thesis presents the main types of certificates, describing the characteristics and payoffs offered, thanks to a focus on plain vanilla and exotic options underlying, whose understanding, even basic, allows discerning good products the bad ones.

Subsequently, we moved on to the detailed analysis of a particular Certificate belonging to the family of Bonus Cap Certificates, in turn within the conditional capital protectection certificates category. After having introduced, in fact, basic notions of the most commonly used pricing model, the Black-Scholes model, we proceeded calculating the fair value of this Bonus Cap, both today, to check for any liquidity problems of the product, and back to the date of issue, to clarify the implicit fees that characterize the instrument.

Furthermore, a very interesting situation within the portfolio management environment is presented: retracing the certificate path compared with the underlying Intesa Sanpaolo stock from the date of issue to date, the certificate's efficiency has been verified in situations of recovery of losses deriving from from the holding in the portfolio of a non-performing security, thanks to a mention of the measures involved in the performance of a certificate, the Greeks.

The case studied in this thesis is an infinitesimal percentage of the total potential of investment certificates, which can be used as a long-term investment product, for short-term speculation, liquidity management with capital protection, loss recovery and, as highlighted, portfolio optimization.

The investment product at the center of the analysis is certainly not easy to understand, besides, the aim of the thesis was not to put an end to the investment certificates' argumentation, but rather to open one.

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