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RISK-AVERSION IN TRAFFIC NETWORKS:

AN EXPERIMENT ON BEHAVIORAL CHOICE UNDER RISK

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Abstract

Routing games are amongst the most studied and well-discussed domains of game theory. The modeling of such environments is key to understand the reality of everyday traffic routing. I specifically investigate the impact of risk-aversion and theories of choice to model route-choice in the context of traffic networks under stochastic environments. I interpret carefully all the postulates of the fundamental theory of choice of von Neumann and Morgenstern to discuss and analyze eventual paradoxes and violations of one of the core axioms. Subsequently, I conduct an analysis of the most successful alternative theories developed throughout the years that try to overcome such paradoxes in the former. The use of additive consistency as a plausible condition that allows to reduce risk-averse route choice to a standard shortest path problem, will be the subject of the experiment in this paper. Despite the classical theories of choice, many controversial behaviors persist, thus I also discuss such paradoxes that result from the adoption of other axioms, which try to substitute or avoid the crucial axiom of Independence.

Keywords: *Behavioral game theory, networks, traffic, additive consistency, Independence, paradoxes, Expected utility theory, experimental economics*

Chapter 1

Introduction

Congestion games are amongst the most historic, influential and well-studied classes of games. Their purpose is to capture settings in which the agents' payoffs depend upon the resources they choose and how congested each of them is. However, how really relevant are these calculations to understand the everyday reality of the agents and the traffic networks? At this point, theory alone does not suffice to provide a clear and most definitive answer, thus it makes sense to examine real world networks at a finer level of detail. The aim of this thesis is to discuss the impact of risk-aversion in the setting of traffic networks, and to examine the role of additive consistency in such context. Traffic networks are characterized by stochastic patterns, which can be approximated, although measuring the travel time to choose a route, is basically a matter of comparing random variables. The preferences in route-choice situations vary according to the individual, the trip purpose, and other personal traits that are unique to each subject. A key point is to address the mechanisms by which route choices are made, or simply, how a solution can be reached through the characterization of a rational route choice under risk. The risk factor is nowadays much present in economics, thus utility models must be able to explain such element, which truly differentiates itself from the classical setting of choice under uncertainty. Economists, in fact, tend to distinguish the two ways in which agents may not know for sure what will happen in the future on the basis of probabilities: in the first case, the probabilities of different outcomes are known, while in the latter case they are not. The ”risk versus uncertainty” debate is long-running and far from resolved at present. In this work, I will keep my focus on the former.

In order, to better motivate elements that are external to the rationality of agents, an introduction to behavioral game theory had seemed only fair for the final aim of my work. Previous research shows that behavioral game theory is a new field, though it does not lack analytical rigor. The theory does not aim at disproving the original model, but at empowering the tool for strategic situations analysis. After having discussed in Chapter 2 the framework for a network setting and having provided a brief introduction to the main traits and objectives of behavioral game theory, I will further articulate the role of the Expected Utility Theory (EUT) of von Neumann-Morgenstern in Chapter 3. Its extreme simplicity in economic contexts has attributed it with an enduring popularity to assess the choices of agents in stochastic environments. However, this well-known theory, is not so general that it lacks empirical content, and indeed, paradoxical behavior that is inconsistent with the theory has been documented for at least the past thirty years. The independence axiom results in the most controversial pattern in such context, though other axioms exist, all involving properties such as transitivity, completeness, and continuity. Only the former ensures a representation of the expected utility theory that is linear in the probabilities, and a further formulation of the EUT, which consists in the definition of additive consistency. Moreover, the impact of risk still stands to construct a controversial topic, due mainly to its subjectivity, hence, several alternatives have been proposed over the years, each one of them with their own advantages and their own limitations, trying to overcome or avoid the paradoxes.

As proof, with the only exception of the approach based on stochastic dominance, in all the others a somewhat paradoxical situation may occur. Roughly two decades of intensive research on the matter has generated a great deal of theoretical innovation, and additionally, a much richer body of evidence against which models can be judged. Regardless of the heaviness of the gathered research, empirical evidence has shown that the agents do not always conform to the postulates of expected utility, thus violating some fundamental axioms that best characterize it. A significant amount of theoretical effort has been devoted towards the development of alternatives to EUT, alongside with an ongoing experimental program aimed at testing those theories.

A citation from Chris Starmer seems to fit: *"There can be few areas in economics that could claim to have sustained such a rich interaction between theory and evidence in an ongoing effort to develop theories in closer conformity with the facts. Considered together, the accumulated theory and evidence present an opportunity to reflect on what* *has been achieved. Perhaps the most obvious question to address to this literature is this: has it generated, or does it show the prospect of generating, a serious contender for replacing EUT, at least for certain purposes? If the question seems disarmingly straightforward, providing a clear-cut answer will not be.”*

Chapter 4 tries to answer the question by providing a core overview of the most useful alternative theories to the standard model and a detailed description on their solutions to the controversies of the EUT and whether they offer a viable alternative to it. To this extent, it seems imperative to structure Chapter 5 on a more in depth description of all paradoxes and most controversial behaviors of the theories of choice, as further proof to their failure, thus the need for a solution to the problem, which can be found in the entropic risk measures and the characterization of the additive consistency concept. Such axiom is at the center of the experiment proposed in Chapter 6, a Pilot experiment useful to set the basis for a following field experiment to be conducted in the near future.

Final remarks and conclusions are drawn as results of the Pilot experiment and possible explanations for the behavior of agents, followed by the intentions of continuing the work and use this paper to structure a fine field experiment with the rights incentives for participants and the adequate measures and equipment.

Chapter 2

Survey of previous work

2.1 Framework: Networks setting

Traffic congestion is a major issue in every big or even mid-sized city. With this hindsight, the setting of choice for this thesis is indeed the one of traffic networks, in which are present the requisites necessary to study and understand the behavior of single agents in situations of route-choice. Moreover, I decided to focus on investigating the use of risk measures and theories of choice to model risk-averse routing, believing in the assumption that individuals studied in such environments tend to always be risk-averse agents.

An important, although trivial, observation to be made is that travel times can be very hard to predict reliably as they are subject to various random phenomena caused by many different reasons, e.g. congestion, weather, traffic jams or accidents, and so on and so forth. It could be taken any specific road segment at any point in time, and we could notice the stochastic pattern of travel times, meaning that when it comes to choosing a route to travel from a certain origin to reach a destination, we really are comparing combinations of random variables. Nevertheless, we should not forget the impact that personal preferences have on such choice; individuals are different from each other, thus they would result to prefer different routes, mainly because these preferences are subsequent to the nature of the trip purpose of the single users. Let's consider some examples: a person who must arrive on time for a certain meeting or to get on a flight, will definitely be considered risk-averse, so will care more about the travel time reliability rather than a tourist or someone who doesn't have any specific appointment, considered to be risk-neutral, thus care only about expected travel time. A question, amongst several others to be answered throughout this thesis, is how to model traffic and to design networks to perhaps control traffic, to understand the mechanisms behind route-choice. I will concentrate on the problem of modeling choices under risk, as opposed to the more general category of uncertainty.

Needless to say, we should first analyze the behavior of drivers, especially the role of risk-aversion in the network congestion and traffic equilibrium. In particular, I have decided to consider a scenario in which the risk factor is most present, therefore, economists in a similar setting, will rather prefer a general utility theory. Such traditional theory of choice, is considered a well-established field, a core component of game theory. Roughly two decades of intensive research on the matter has generated a great deal of theoretical innovation, and additionally, a much richer body of evidence against which models can be judged. Regardless of the robustness of the gathered research, empirical evidence has shown that the agents do not always conform to the postulates of expected utility, thus violating some fundamental axioms that best characterize it.

Although the *Expected Utility Theory* is considered a pillar of game theory itself, for it is the most used and respected theory of choice in stochastic environments, such studies and researches belong to a very young area, in fact, many new theories have just recently been proposed, as alternatives to the traditional von Neumann and Morgenstern expected utility theory. An interesting topic to discuss in this paper is whether any of the alternative theories developed over the years, aiming to substitute or, at least, get along with the traditional theory itself, are actually worthy.

The reason it is referred to as a new field of studies is due, in most part, to the behavioral aspect that has been introduced, namely the extent to which any individual in reality, doesn't really behave as their own self-interest tells them to. An acknowledgment which alters completely the equilibria dictated by the rationality of the agents, key to all the original utility theories.

2.2 Introduction of the Behavioral Factor in economics

"Game Theory is a probability calculation technique used to analyze situations with strategic interactions between different decision makers and predict the outcome of their decisions."[1]

Theorists tend to describe game theory as simply "analytical", a body of answers to mathematical questions about what players with various degrees of rationality will do. If people do not play according to what theory says, their behavior ha not proved the mathematics wrong. Models involving highly rational players result in their full and exhaustive understanding of the strategic situations, always aiming at maximizing their consistent preferences, given their rationally-formed beliefs about the behavior of their opponents. In practice, however, the tools of analytical game theory are used to predict, and also explain (or "postdict") and prescribe. Keeping this in mind, few assumptions must be introduced, which can be easily found in a standard equilibrium analysis:

1. Strategic thinking: belief based on the fact that each player carries out an analysis of what others may or may not do;

2. Optimization; all players always choose their best response, given those beliefs;

3. Adjust both Strategic Thinking and Optimization until they reach a common understanding (equilibrium).

In all the situations that may arise, a person (or firm, or any other agent) must anticipate what others will do and what others infer from the person's own actions. Moreover, a game is a mathematical "x-ray" of the crucial features of such situations. The introduction presented hereby is really helpful in introducing such basic ideas in game theory, which I will make more clear by providing some very intuitive notation;

Notation: Player i's strategy is denoted s_i. A vector of strategies, one for each player, is denoted

$$s = (s_1, s_2, s_3, \ldots., s_n).$$

[1] Citation from Mulder P., Game theory (2018).

The part of this vector which removes player i's strategy (i.e every other player's strategy) is denoted s_{-i}. The utility of player i's payoff from playing s_i is

$$u_i(s_i, s_{-i})$$

Game theory has a very clear paternity. Von Neumann and Morgenstern introduced many of its main features in 1944. A few years later, John Nash proposed a "solution" to the problem regarding how rational players would play, now named Nash equilibrium. Nash's intuition, based on the idea of equilibrium in a physical system, was that players would adjust their strategies until no player could benefit from changing. All players are then choosing strategies that are best (utility-maximizing) responses to all the other player's strategies.

In the real world, the whole ensemble of these assumptions is often untenable. Players most likely do not behave like ineffable optimizers in the sense of pure material self-interest, and it would be negligent and not accurate to think of the resulting consistent and structured deviations as inexplicable irrationalities.

"Behavioral game theory is the mathematical approach to modeling these issues as a consequence of the limited rationality of the human nature, by analyzing the strategic decisions made by interacting players."

Experimental results are clay for behavioral game theory. In the long-run, the primary goal is not to "disprove" game theory, which is a common reaction to think so of psychologists and sociologists, but rather to improve it by establishing regularity, which inspires new theories that most likely rely on plausible psychological foundations. Without some sort of observation, theoretical assumptions are grounded in a casual pseudo-empirical work—informal opinion polls in seminar and office discussions and using one's own intuitions (a one-respondent poll). Of course, experimental data are only one component of behavioral game theory. Detailed facts about cognitive mechanisms and field tests are crucial too .

Broadly speaking, behavioral game theory is separable into two branches of models:

- The first branch of models presumes that the mismatch between theory and real-world behaviors may be the result of capacity and/or informational constraints.

Hence, decisions are not best described by strictly maximizing behavior. Maximization fails when players do not have the complete set of information at their disposal about the structure of the game and/or about the payoffs consequences of different actions taken by themselves and others for the other players.

- The second branch explains behaviors that are contradicting between scenarios that involve self-interest and those which involve unbounded rationality players that are guided by alternative preferences. More precisely, material self-interest is flawed, hence replaced with higher-order motives.

Both strands are very appealing in a way, so that the model that best fits will be chosen accordingly to the context of the application. Although violations of this theory, rationality remains an excellent instrument for the approximation of consumer choices and other decisions. But game theory is different: the players' fates are indeed intertwined. The presence of players, even if only few, who do not optimize or think strategically, can result in changing what rational players should do. In response, what a population of players is likely to do when some are not thinking strategically nor optimizing can only be predicted by an analysis that uses the tools of (1)–(3) but accounts for bounded rationality as well, preferably in a precise way. An alternative way to define the equilibrium condition (3) is that players are never surprised when the strategies of other players are revealed.

At the opposite extreme, in evolutionary models, players have no cognition and therefore "no choice". By contrast, the approach of Behavioral Game Theory (BGT) is to seek empirical information about how human beings – as opposed to highly rational beings– behave in strategic situations. Thus, the true role of BGT is to serve as a middle ground in the contest between two extremes but at the same time, taking all the advances of formal Game Theory. Furthermore, BGT aims at making the classical game theory, a more powerful tool for the analysis of strategic situations.

Chapter 3

Theories of choice

3.1 An overview: the Expected Utility Model

The dominant theory of individual decision making in stochastic environments is von Neumann-Morgenstern utility theory. Its enduring popularity among economists and statisticians is due to its simplicity and its consistency with a wide range of attitudes toward risk. It should be no surprise, therefore, that developing a better understanding of the determinants of individual choice behavior seemed a natural research priority to many theorists.

However, this well-known theory is not so general that it lacks empirical content, and indeed, paradoxical behavior that is inconsistent with the theory has been documented for at least the past thirty years. As it will be discussed more specifically in the following sections, the most controversial assumption in such theory appears to be the *Independence axiom*; though other axioms exist, all involving properties such as transitivity, completeness, and continuity. The relevance of such properties lies in the guarantee they give to ensure the existence of a utility functional defined on a space of possible random variables, lotteries, or prospects, depending on what the specific theory wants to describe. Nevertheless, it is the Independence axiom that results in the representation of the expected utility theory that is linear in the probabilities, and it is this assumption, beyond all others, that is crucial to the further realization and actual proof of the expected utility function of von Neumann-Morgenstern.

The explanation of such a revolutionary theory, should lead back to its original roots, so that it is understood in its entirety. To be highlighted, current trends that point toward the generalization of this model, lead economists to notice that the expected utility hypothesis was itself first put forward as an alternative to an earlier, more restrictive theory of risk-taking. The theory, in fact, stems originally from Daniel Bernoulli's (1738) as a solution to the famous *St. Petersburg Paradox* proposed in 1713 by his cousin Nicholas Bernoulli.

It was the conventional wisdom at the time that lead economists to believe that it would be reasonable for any individual to pay any sum up to the expected value of a gamble. Despite this shared belief, Bernoulli presented a counterexample, dramatically illustrating that individuals usually consider more than the mere expected values, contrary to the beliefs generated during the 17th Century as a consequence to the rise of a more modern probability theory. The Paradox presents a situation of this type:

"A coin is flipped repeatedly until a head is shown; if you enter the game, you receive a payoff 2^n*, where* n *is the number of the throws producing the first head."*[1]

It is trivial to see that its expected monetary payoff is infinite, yet Bernoulli believed that most people would be prepared to pay only a relatively small amount to enter the game. He used this intuition as evidence that the "value" of a gamble in the eyes of an individual is not, generally, equal to its expected monetary value. Indeed, there exists a traditional critique of the plausibility of expected value theory, which is based on real experiments that propose hypothetical situations, in which people do not seem to be willing to pay a large amount of money in order to play a St. Petersburg game with infinite expected value. A traditional defense of the theory sets its ground on the observation that "no agent can credibly offer the St. Petersburg game for another to play in a real-payoff experiment; that is because it could result in a payout obligation exceeding any agent's wealth, and therefore, that this challenge to expected value theory has no bite." Nevertheless, he posed a theory for which individuals place subjective values, or "utilities," on monetary outcomes and the value of a gamble is the expectation of these utilities. Daniel Bernoulli's resolution revolutionized economics as it was: firstly because he confirmed that people's utility from wealth is not linearly related to wealth itself, but rather it increases at a decreasing rate, thus introducing the concept known

[1]Formal description of the game extracted from Advances in Behavioral Economics, by Colin F. Camerer, George Loewenstein, Matthew Rabin.

today as *diminishing marginal utility*; furthermore, the idea that a person's valuation of a risky prospect is not its expected return, but rather the expected utility they may receive from it.

While Bernoulli's theory, as the first statement of EUT, solved the St. Petersburg dilemma, it did not find much consensus among modern economists until the 1950s. Their skepticism is partly explained by the fact that Bernoulli's theory presupposes the existence of a cardinal utility scale; an assumption that did not merge well with the drive towards ordinalization of the first half of the twentieth century. It was only in 1947, when a great interest in the Theory was revived by John von Neumann and Oskar Morgenstern, who attempted to axiomatize this hypothesis in terms of agents' preferences. Since then, several alternative axiomatizations have been advanced, some of which seem highly interesting, some even compelling, from a normative point of view. The proposition that has been launched by Bernoulli, is still considered to be an inspiration underlying the hypothesis that individuals possess what is now termed a *von Neumann-Morgenstern utility function* [2]. The hypotheses posed by John von Neumann and Oskar Morgenstern were well developed and further discussed in their book *Theory of Games and Economic Behavior*. While formally identical, has nonetheless a somewhat different interpretation from Bernoulli's.

Remarkably, they viewed the progression of the expected utility model as something of a side note in the evolution of the theory of games. Although it shares the name "utility," $U(.)$ quite differs from the ordinal utility function of standard consumer theory. The latter can be subjected to any monotonic transformation, whereas the von Neumann-Morgenstern utility function is cardinal, for it can only be subjected to transformations of the form $a \cdot U(x) + b(a > 0)$, changing the origin and/or scale of the vertical axis, but not the "shape" of the function itself. For it is a theory of individual behavior, the model shares many of the elemental assumptions of traditional consumer theory. Thus, in all presentable cases, it is assumed that the objects of choice, which may be of diversified nature, can be unambiguously and objectively described, and that situations ultimately implying the same set of availabilities will lead to the same choice. Moreover, the implicit assumption that each individual is able to perform the mathematical operations necessary to truly determine the set of availabilities is also taken into consideration.

[2] Concepts and ideas extracted from the book "Theory of Games and Economic Behavior", by von Neumann J. and Morgenstern O.

Finally, each case also assumes that preferences are transitive, so that if an individual prefers one object (either a commodity bundle or a risky prospect) to a second, and prefers this second object to a third, he or she will prefer the first object to the third. A detailed description of such axioms will be provided in the next section.

3.2 Axiomatics that derive the Expected Utility Theory

Without risk, economists usually believe that individuals possess a certain utility function, as means to convert ordinal preferences into some real-valued function, the unambiguously known *utility function*. Although this appears to be a plausible scenario, when the factor risk enters the picture, economists seem to prefer the use of the expected utility theory. As already mentioned, such theory have functions of the functional form:

$$U = \sum_{i=1}^{n} p_i u(x_i),$$

where p is the probability of each agent of obtaining x. The implication of such formula is that a risky prospect will be (weakly) preferred over any other if its utility is higher than the utility provided by all other prospects. A behavioral interpretation usually lies underneath the function itself, describing *risk-neutral* agents as whomever has linear utility functions, whereas *risk-averse* individuals have concave utility functions $u" < 0$, and *risk-lovers* are visualized as convex functions $u" > 0$. An agent with a concave utility function will always prefer a certain amount x to any risky prospect with expected value equal to x.

The means necessary for the dominant theory of individual decision-making to work are three axioms, which have to hold. Whether these axioms are in fact true or not, will help answer one of several questions that happen to be fundamental for this thesis:

"Under which conditions can we find a cost function that represents the preference relationship between two random variables?"

Cost functions are considered here because the setting used is traffic games, in which agents will need to decide which segment among all roads they prefer to travel on, hence preferences are expressed in terms of minimization of the traffic times (the less time the better), analogous to expressions of costs. Moreover, I have decided to keep my attention on the study of random variables;

1. Let *RV* be the set of random variables;

2. Let $\preccurlyeq$ be the preference relation:

   - **Reflexivity**. $X \preccurlyeq X, \forall X \in RV$.
   - **Completeness**. This entails that for each pair $X, Y \in RV$, either $X \preccurlyeq Y$ or $Y \preccurlyeq X$ or both. If the answer is both, then the agent is indifferent between X and Y.
   - **Transitivity**. $\forall X, Y, Z \in RV$, if $X \preccurlyeq Y$ and $Y \preccurlyeq Z$, then $X \preccurlyeq Z$.

Let us consider a map

$$V : RV \to \mathbb{R},$$

assigning a real number $V(X)$ to each random variable $X \in RV$. The map $V(\cdot)$ is said to represent the preference $\preccurlyeq$, provided that

$$X \preccurlyeq Y \iff V(Y) \leq V(X).$$

Now, given these preliminaries aforementioned, the expected utility hypothesis can be derived from three axioms: ordering, continuity, and independence. Together, the axioms of ordering and continuity imply that preferences over random variables can be represented by a function $V(\cdot)$ which assigns a real-valued index to each variable. The function $V(\cdot)$ is a representation of preference in the sense that an individual will choose X over the other variable if, and only if, the value assigned to X by $V(\cdot)$ is no less than that assigned to Y [1].

1. The **Ordering axiom** requires both completeness and transitivity.

2. **Continuity** requires that for all random variables X , Y and Z, where $X \preccurlyeq Y$ and $Y \preccurlyeq Z$: *there exists some p such that* $(X, p; Z, 1 - p) \backsim Y$, where $\backsim$ represents the relation of indifference, and $(X, p; Z, 1 - p)$ represents a (compound) variable which results in X with probability p and Z with probability $1 - p$.

3.2.1 The Independence Axiom: its cruciality in the EUT

To assume the existence of such preference function has seemed to be for many economists the obvious starting line for any economic theory of choice; briefly, it sums up to assuming that agents have well-defined preferences, and in the meantime, also imposing a minimal restriction on the specific nature of those preferences. For those who wish to answer the question previously stated, the natural answer centers around what further restrictions can be placed on the function $V(\cdot)$. More precisely, the axioms of ordering and (weak) continuity impose some restrictions that better define the cost function we wish to characterize, but it is the independence axiom that places quite stronger restrictions on the precise form of preferences. In the end, the independence property requires that:

$$\forall(X, Y, Z)$$

if

$$X \preccurlyeq Y$$

then

$$(X, p; Z, 1-p) \preccurlyeq (Y, p; Z, 1-p), \forall p.$$

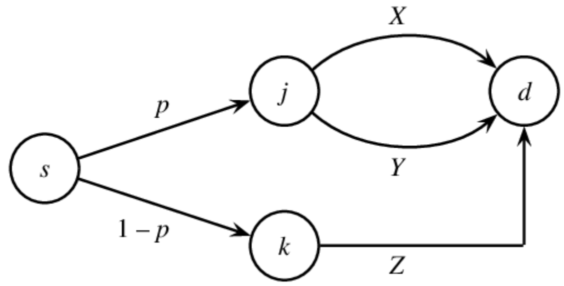
If all three axioms hold, preferences can be represented by:

$$V(X) = \sum_{i=1}^{n} p_i u(x_i)$$

where X is any variable, and $u(\cdot)$ is a "utility" function defined on the set of consequences.

It is this axiom which provides the standard theory with most of its empirical content, and it is this very axiom which most alternatives to EUT will relax or substitute with other weaker assumptions or hypotheses. The approach hereby used, as previously mentioned, is based on stochastic dominance, deriving from the nature of travel times; with its exception, in many other scenarios a paradoxical situation tends to occur. To interpret the Independence axiom, imagine to have a situation in which an agent must choose between two options of random

Figure 3.1: Variables representation



variables, the same used so far, X, Y, to travel from point j to d. A further alternative is proposed, going a step backwards, at point s. At this point, let us assume that there is a bridge to cross, and with a certain probability p that bridge will be open, and with probability $1 - p$ that bridge is closed, and in this case the agent will not be facing the same choice as before, between X and Y, but will have to take a detour and go through Z to reach the destination d. (See figure 3.1)

Thus, they're facing a choice between the lotteries $(X, p; Z, 1-p)$ and $(Y, p; Z, 1-p)$. Therefore, according to the aforementioned description of the Independence axiom, the first lottery should be the one preferred. Before entering more into details of the specific situation just introduced, let us consider some related literature, which adds some perspective, taken from different and more specific points of view.

3.2.2 Some related, practical literature

Despite the relative youth of the area regarding risk-sensitive choice in the settings of traffic networks, the literature which connects to this topic is quite wide. Going beyond the aim of this thesis, which will tend to focus more on the agents' behavior in such environments and the impact and implications of risk-aversion on their preferences, I shall make a reference to some more practical literature, keen on discussing transportation issues.

An early reference is Loui [8], who used the traditional approach of expected utility theory to formulate a model for route choice under stochastic weights, and which highlighted the crucial role played by utilities of exponential form. More recently, in Bates et al. [5], Noland and Polak [13], and Hollander [6] presented informal discussions on risk evaluation for route choice. A mean-risk model for network equilibrium, in which risk is quantified by the expected value plus the standard deviation, was studied by Nikolova and Stier-Moses [11], considering the case where only expected travel times depend on the traffic intensity as well as the case when the variances are also flow dependent. Algorithms for computing optimal routes for this mean-standard deviation objective were developed by Nikolova [10] and Nikolova et al. [12]. Route choice using α-percentiles was investigated by Ordóñez and Stier-Moses [9], considering also an approach using robust optimization and a third one in which risk aversion is treated by adding a variability index to the expected value. Since computing an α-percentile equilibrium is difficult, they proposed two approximation algorithms. Percentile equilibria in route choice were also considered by Nie [27] and Nie and Wu [28], who considered route choice with preferences based on the on-time arrival probability. An algorithm for the latter objective function was given by Nikolova et al. [12]. Finally, Nie et al. [29] and Wu and Nie [30] develop a different approach in which route choice is restricted by including a stochastic dominance constraint with respect to a referential path. For a detailed account of these and other relevant references, we refer to the literature review in Nikolova and Stier-Moses [11].

The Paradox introduced that sees individuals reversing their preferences when considering additional variables, can be eliminated through theories of choice and risk measures. The phenomenon is due to the characterization of the so-called *additive consistency preferences*, free from the paradox and dependent on *entropic risk measures*. *Additive consistency*, in a setting in which agents express their preferences over random variables, seems to be a relevant condition, necessary to translate the notion of *translation invariance*, resulting in a reduction of risk-averse route choice to a standard shortest path problem.

Additionally, it is also an obvious violation of the *Independence axiom*; such axiom appears to be the most controversial assumption in the basic von Neumann-Morgenstern theory. The independence axiom is an assumption that a lottery X

is weakly preferred to a lottery Y, if and only if a compound lottery that yields X with probability q and some other lottery Z with probability $1-q$ is weakly preferred to a compound lottery that yields Y with probability q and Z with probability $1-q$, for *any* lottery Z. Hence, the choice between two lotteries, X and Y, is independent of the possible existence of a common prospect Z [22].

Moreover, it seems now clear the immediate correlation with the *additive consistency* preferences, in a random variable environment.

3.2.3 Additive consistency

Measuring risk still stands to be a controversial topic, due mainly to its subjectivity, hence, several alternatives have been proposed over the years, each one of them with their own advantages and their own limitations. In this paper I intend to investigate the use of risk measures and theories of choice to model risk-averse routing. The additional axiom of additive consistency takes a step further, providing us with a much stronger result derived from its imposition onto the characterization of the other axioms that model the EUT. Economists believe that adding the element of additive consistency will have very precise and strong conclusions, which is what I am keen to test through the design of experiments and discussing their results. Moreover, having chosen the framework of network settings, trying to model the route choice under risk, it has been discussed that additive consistency preferences are free from all the paradoxes that contradict the independence axiom, hence the Expected Utility theory, and that this situation occurs only on the basis of *entropic risk measures* [3].

An axiomatic approach to risk quantification was initiated by Artzner *et al.* [4] by considering a *risk measure* as a map $\rho : \mathfrak{X} \to \mathbb{R}$ that attaches a scalar value to each random variable $\mathfrak{X} : \Omega \to \mathbb{R}$ on a given probability space $(\Omega, \mathfrak{F}, \mathbb{P})$.

The domain $\mathfrak{X}$ is called the *prospect space* and is a linear space of random variables containing the constants, usually a subspace of $L^{\infty}(\Omega, \mathfrak{F}, \mathbb{P})$.

[3]The characterization of additive consistency that follows is based on the work of Cominetti R., Torrico A., "Additive Consistency of Risk Measures and Its Application to Risk-averse Routing in Networks".

A risk measure ρ induces a preference $X \preccurlyeq Y \iff \rho(X) \leq \rho(Y)$, which defines a weak order, namely a binary relation that is reflexive, transitive and total (every two elements are comparable).

In this paper, prospects are interpreted as disutilities so that smaller values are preferred and, against common usage, $X \preccurlyeq Y$ is read as *"X is preferred to Y"*. As usual, $X \prec Y$ denotes strict preference and we write $X \sim Y$ when simultaneously $X \preccurlyeq Y$ and $Y \preccurlyeq X$.

Two natural properties that can be expected from a risk measure are:

- NORMALIZATION: $\rho(0) = 0$,
- MONOTONICITY: if $X \leq Y$ almost surely then $\rho(X) \leq \rho(Y)$.

Normalization is not restrictive, as one can always take $\rho(X) - \rho(0)$ instead of $\rho(X)$. Monotonicity has a clear intuitive meaning: larger costs convey higher risk. In the context of routing, paths with larger travel times are less preferred. Another basic property that holds for many risk measures is

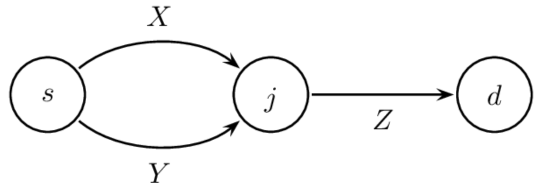
- TRANSLATION INVARIANCE: $\rho(X + m) = \rho(X) + m$ for all $m \in \mathbb{R}$.

This condition is equivalent (under normalization) to requiring simultaneously:

- NORMALIZATION ON CONSTANTS: $\rho(m) = m$ for all $m \in \mathbb{R}$.
- TRANSLATION CONSISTENCY: $\rho(X) \leq \rho(Y) \Rightarrow \rho(X + m) \leq \rho(Y + m)$ for all $m \in \mathbb{R}$.

Normalization on constants is thus not such a strict requirement: it suffices to have $m \mapsto \rho(m)$ strictly increasing and continuous, since then this function has an inverse σ and we may substitute ρ by $\sigma \circ \rho$. Also, translation consistency is a plausible condition stating that preferences between prospects are not altered when we add to them a constant term. While this postulate is not universally accepted in finance (attitudes towards risk might change after receiving a heritage), it seems very likely in the context of route choice: it states that when X is

Figure 3.2: Combinations of variables



preferred over Y, the same holds if we add a segment with constant travel time $Z \equiv m$.

A stronger property requires that preferences are preserved when adding an independent random variable, namely, if $X \preccurlyeq Y$ then $X + Z \preccurlyeq Y + Z$ for all $Z \perp (X, Y)$, where $\perp$ denotes independence. Intuitively, also referring to Figure 3.2, since the arc (j, d) is compulsory and one must inevitably pass through it, the decision at s should not depend on Z. This seems all the more plausible since, due to the independence, even if one observes Z, this reveals no information that could affect the choice between X and Y.

This explanation motivates the following definition for expected utilities.

Additive consistency: A risk measure $\rho : \mathfrak{X} \to \mathbb{R}$ is called *additive consistent* if for all $X, Y, Z \in \mathcal{X}$ with $Z \perp (X, Y)$ we have

$$\rho(X) \leq \rho(Y) \;\Rightarrow\; \rho(X + Z) \leq \rho(Y + Z).$$

Additive consistency is closely related to an apparently stronger condition of additivity for sums of independent risks, namely:

Let $\rho : \mathfrak{X} \to \mathbb{R}$ be normalized on constants. Then ρ is additive consistent if and only if $\rho(X + Y) = \rho(X) + \rho(Y)$ for all $X, Y \in \mathfrak{X}$ with $X \perp Y$.

Lastly, an important note to be highlighted is the fact that a risk measure that is monotone, translation invariant, sub-additive and positively homogeneous is called *coherent* [4]. Positive homogeneity translates the notion of scale invariance, while sub-additivity captures the idea that a merger of two risks cannot create additional risk. The validity of these latter axioms has been thoroughly debated in finance, while they seem less natural for route choice. A weaker property is *convexity* which still supports a useful dual representation for risk measures.

To conclude, in investigating the use of risk measures and theories of choice to model risk-averse routing, it can be stated that additive consistency is a relevant condition that extends the notion of translation invariance and reduces risk-averse route choice to a standard shortest path problem. As a matter of fact, as shown in [3], within various classes of risk functionals including EUT models as well as distorted risk measures and rank dependent utilities, the only maps that comply with the axiom of additive consistency are the entropic risk measures. For this reason, testing the validity of this axiom will be the object of our Pilot experiment.

In fact, this paper judges and evaluates the different alternatives to the traditional model of EUT that may satisfy the additive consistency axiom, and in the chapters to come, it is proven that within various classes of risk functionals, such as distorted risk measures as well as rank dependent utilities, the only maps that satisfy additive consistency stand out to be the entropic risk measures, and among expected utility maps these are the only ones that are translation invariant. These affirmations are relevant because, although the suitable axioms under which the preferences of an agent admit a scalar representation, the known representation results provide no type of information on the regularity of the functions involved. Translation invariance and additive consistency suffices to guarantee that regularity holds automatically and lead to the entropic risk measures.

3.3 Choice under risk

The concept of risk is inescapable in economics, thus economists instinctively need a theory of individual decision making under risk. EUT is much to be accounted for in this capacity. In fact, the theory presents a consistent degree of intuitive appeal, and it almost appears evident that any satisfactory theory of decision making under risk will necessarily account for both the consequences of choices and their corresponding probabilities. By definition, these are the dimensions relevant in the domain of risk. Indeed, the EUT manages to provide one elementary way of combining probabilities and consequences into a single "measure of value", which has various appealing properties. One such property is monotonicity, already briefly stated in the previous section, specifically to the extent of the additive consistency property. Now, to provide a broader representation of such monotonicity, it can be defined as follows:

Let $x_1, ..., x_n$ be consequences ordered from best (x_1) to worst (x_n). It may be stated that a random variable X with $\mathbb{P}(X = x_j) = p_j$ first-order stochastically dominates another variable Y with $\mathbb{P}(Y = x_j) = q_j$ if, for all $i = 1, ..., n$:

$$\sum_{j=1}^{i} p_j \geq \sum_{j=1}^{i} q_j$$

with a strict inequality for at least one i.

Monotonicity is the property that stochastically dominating variables are preferred to others which they dominate, and it is widely acknowledged that any satisfactory theory, either descriptive or normative, should entail monotonicity.

To model risk preferences this way means ending up into combining potentially distinct concepts into a single function: any attitude to chance and towards consequences should all be captured by the utility function. No more than a couple of decades ago, the theory of choice under risk was likely to be considered one of the few economic analyses that received the most success, and the reasons being that it rested on solid axiomatic foundations. Furthermore, it had seen important breakthroughs in risk aversion, the analytics of risk, and their applications to economic issues, and it stood ready to contribute the theoretical foundations for the newly emerging "information revolution" in economics [14].

Nowadays, choice in a stochastic scenario is a dynamic field: the traditional theory of choice as it was once known, is constantly changing and continuously evolving to stand up to the challenges on several grounds from both within and outside economics.

3.4 Further comments and hindsights on the EUT

My personal concern, however, lies in the descriptive merits of the Expected Utility Theory and, raising a crucial question of whether such theory provides a sufficiently accurate representation of actual choice behavior. The evidence from a substantial amount of empirical tests has incremented some real doubts on this score.

Expected utility theory states that the decision maker will choose the risky prospect with the highest expected utility. The assumptions behind this theory dominated economic analysis of individual decision-making under risk from the early 1950s to the 1990s. However, beginning in the late 1970s, the accumulation of robust experimental evidence in spite of the expected utility hypothesis prompted decision theorists to submit several alternative theories, such as prospect theory (Kahneman and Tversky 1979), Rank-dependent utility (Quiggin 1993), Choquet expected utility (Schmeidler 1989), maxmin expected utility (Gilboa and Schmeidler 1989) or the steady model of ambiguity aversion (Klibanoff, Marinacci, and Mukerji 2005). As of the present moment, though, none of these non-expected utility theories has reached the dominant status that the von Neumann-Morgenstern utility theory once enjoyed. As Gilboa and Marinacci (2013, p. 232) have argued in a recent survey of the decision-theoretic literature [4], *"it is not clear that a single theory of decision making under uncertainty will replace expected utility theory, and even if a single paradigm will eventually emerge, it is probably too soon to tell which one it will be."* Due to the absence of a univocal alternative, and to its simplicity and adaptability, expected utility theory remains the primary model in numerous areas of economics dealing with risky decisions, such as game theory, finance and the theory of asymmetric information. Therefore, the question of why economists came to accept it in the first place remains pertinent.

In *"Theory of Games (1944-1947)*, von Neumann and Morgenstern mention a series of axioms about the individual's preferences over indifference classes of lotteries, introducing proof that an individual obeying these axioms will then follow expected utility theory. The axiomatization includes the completeness, trans-

[4] Ambiguity and the Bayesan Paradigm, report (April 2011).

itivity and continuity of preferences, but does not feature an assumption corresponding to today well-known Independence Axiom. In the first three papers [5], completed in April 1950 and named the *"Japanese paper"* because of its later publication in a Japanese journal, Samuelson declared that he found von Neumann and Morgenstern's axioms "opaque". *"On the one hand*, he observed, *"they concern preference relations and seem therefore to be ordinal in nature; on the other hand, the axioms imply that the preference relations can be represented by the expected-utility formula, which features a cardinal utility function."* Samuelson was puzzled by this apparent contradiction and could not understand how von Neumann and Morgenstern's ordinal axioms imply expected utility theory.

Are these axioms realistic? In the next chapter, I will review some alternative theories in which the crucial Independence axiom is taken into different forms, or substituted with weaker-restrictions hypotheses; furthermore, subsequently to the explanation of such theories, I will list and further discuss in depth the paradoxical situations that lead to the violation of the Axiom. Without the substantial existence of the Axiom, no further application of the additive consistency property can be done.

[5] Samuelson, Paul A. 1950a. "Probability and the Attempts to Measure Utility." Economic Review 1(3): 167–73.

Chapter 4

Alternative Theories

The existence of some empirical studies and researches, dating from the early 1950s, have revealed a variety of patterns in choice behavior that appear inconsistent with the well-established Expected utility theory. The case being, a natural but not so trivial question arises regarding the sufficiently accurate representation that the standard theory provides for the actual behavior manifested by the agents when making their decision. Few examples of patterns that are inconsistent with the EUT will be illustrated briefly in the next sections, so that the nature of this evidence can be shown. Although some evidence that contradicts the traditional EUT exists, the latter can still be thought as stimulating in the development of all the other theories, nevertheless, due to the existing problems that the Independence axiom has brought up to the attention of the scholars, several other alternatives have emerged, trying to provide the means for the resolution of such dilemma, or at least, having the purpose to define the theory in a way that doesn't involve such axiom, or that provides alternative (weaker) hypotheses.

4.1 Dual Theory of choice, by Yaari

The new proposal of the theory by Yaari investigates the consequences of the following modification of Expected Utility theory: *"instead of requiring independence with respect to probability mixtures of risky prospects, it considers independ-*

ence with respect to direct mixing of payments of risky prospects." Therefore, such slight modification nonetheless brings a huge impact on the concepts behind the EUT, thus giving birth to a new theory of choice under risk, the so-called *Dual theory* is obtained, by Yaari [18].

Within this new theory, new questions arise:

- Numerical representation of preferences;
- properties of the utility function;
- the possibility for resolving the "paradoxes" of Expected Utility theory;
- the characterization of risk aversion; and
- comparative statics.

In practice, this implies that, whereas the traditional theory of choice introduces risk aversion through the amplification of the effects, or costs of bad outcomes through a non-linear transformation $c(X)$, the Dual theory states that risk-averse agents perceive the probabilities of bad outcomes as exaggerated, thus their choice is biased.

The main reasons behind the willingness of Yaari to generate a different model are purely methodological. In fact, the expected utility theory presents an agent's attitude towards risk which is forever bounded together with the one towards wealth. Stepping on a more fundamental level, we can consider risk aversion and diminishing marginal utility of wealth, both synonymous for expected utility theory, completely different, though in reality very similar; Yaari's own words in his paper: *"[...] they are horses of different colors [...]."*[1]

Risk aversion in fact, expresses an attitude towards risk (increased uncertainty is more harmful) while the latter expresses an attitude towards wealth (the loss of something hurts more when the agent is poor than otherwise). A question may now arise, that is whether these two notions can be kept separate from each other in a full-fledged theory of cardinal utility, or not. The dual theory discusses and analyzes this property.

[1] Citation extracted from the paper The dual theory of choice under risk. Econometrica, Vol. 55, No. 1 (1987), page 95.

On the other hand, some empirical reasons are also considered: behavior patterns which are systematic, yet inconsistent with expected utility theory, have often been observed. So deeply rooted is the economists' commitment to expected utility, that there is the tendency to regard such behavior patterns as ”paradoxical”, perhaps even as ”irrational.” The dual theory ends up rationalizing many of the ”paradoxes” of expected utility theory, even though it is obvious that the dual theory itself won't lack of its own ”paradoxes”, many of which turn out to become rationalized under expected utility.

A very natural way of using cardinal utility to deal with choice under risk consists of preferences represented by a measure which is defined on appropriate subsets of the payment-probability plane. Both expected utility and the dual theory are special cases of this approach, that have preferences represented by a product measure, and which is possible to factorize into two marginal measures.

The difference in the two theories is that the EUT utilizes a Lebesgue measure along the probability axis, while the alternative theory does so along the payment axis. Dropping the condition that one of the marginal measures be Lebesgue produces a theory which generalizes both expected utility and dual theory. Recently, Quiggin (1982) has proposed a special version of this generalized theory, throughout a paper which studies the perception of risk from a cognitive perspective, and which I will analyze more in depth in the following sections.

The dual theory has furthermore the property that utility is linear in wealth, meaning that by applying an affine transformation to the payment levels of two gambles, the result obtained is the direction between preferences unchanged. This phenomenon happens to be true only in case the agent is risk neutral, regarding the EUT. To this peculiar argument, Yaari firmly believes that linearity in payments is not an empirically viable proposition. Behavior which is inconsistent with such linearity is probably often observed, however, a proper perspective should be used in the view of such evidence. The dual theory of choice under risk is obtained by taking the independence axiom of expected utility theory and being postulated for convex combinations which are formed along the payment axis, instead of being postulated for convex combinations, formed along the probability axis.

Let V be the set of all random variables defined on some given probability space, with values in the unit interval $[0, 1]$ [2]. Let Γ be the set of functions, which are non-increasing and right continuous with $\Gamma(1) = 0$, so that G is the *decumulative distribution function* (DDF) of some $v \in V$. Now define a set-valued function, $\widehat{G}$, by writing, for

$$0 \leqslant t \leqslant 1,$$

$$\widehat{G}(t) = \{x \mid G(t) \leqslant x \leqslant G(t-)\},$$

where

$$G(t-) = \lim_{s \to t, s<t} G(s),$$

for

$$t > 0,$$

and

$$G(0-) = 1.$$

$\widehat{G}$ is simply the set-valued function which "fills up" the range of G, to make it coincide with the unit interval. The values of $\widehat{G}$ are closed and for each p, where

$$0 \leqslant p \leqslant 1,$$

there exists some t that $p \in \widehat{G}(t)$.

Using $\widehat{G}$, we may define the (generalized) inverse of G, to be denoted G^{-1}, by writing $G^{-1}(p) = min\{t \mid p \in \widehat{G}(t)\}$. Note that G^{-1}, like G, belongs to Γ and that, $\forall G \in \Gamma$. Furthermore, if G and H belong to Γ and $\|\|$ stands for L_{1-} norm, then $\|G - H\| = \|G^{-1} - H^{-1}\|$.

Of course, if G is invertible, then G^{-1} is just the inverse function of G. A mixture operation for $DDF's$ may now be defined as follows: If $G, H \in \Gamma$ and if $0 < a < 1$, then $\alpha G \boxplus (1 - \alpha)H$ is the member of Γ given by

$$\alpha G \boxplus (1 - \alpha)H = (\alpha G^{-1} + (1 - \alpha)H^{-1})^{-1}.$$

If $J = \alpha G \boxplus (1 - a)H$, for some $0 \leqslant \alpha \leqslant 1$, then it shall be said that J is

[2]Methodology and setup taken from "The dual theory of choice under risk. Econometrica" (1987).

a convex combination of G and H. With the operation $\boxplus$, the set Γ becomes a mixture space. Considering now the preference relation $\succeq$, the axiom that gives rise to the dual theory of choice under risk:

AXIOM-Dual Independence:

If $G, G', H \in \Gamma$ and α is a real number satisfying $0 \leqslant a \leqslant 1$, then $G \succeq G'$ implies $\alpha G \boxplus (1-\alpha)H \succeq \alpha G' \boxplus (1-\alpha)H$.

The utility U of the dual theory has two noteworthy properties: firstly, U assigns to each random variable its *certainty equivalent*. This means that if v belongs to V, then $U(v)$ is equal to that sum of money which, when received certainly, is considered by the agent equally as v. The second important property of U is *linearity in payments*: when the values of a random variable are subjected to some fixed positive affine transformation, the corresponding value of U undergoes the same transformation.

To notice that the dual theory of choice influenced by the risk factor needs to be discussed and interpreted in the light of other non-expected-utility theories that have been proposed recently. For my concern, I shall restrict my attention to specifically three prominent and representative contributions, namely those of Mark Machina, John Quiggin, and Peter Wakker.

4.2 Rank-dependent Expected Utility Theory, by Wakker

The first complementary theory to be discussed is the Rank-dependent utility, by Peter Wakker [17]; the latter amounts for the largest stream in non-expected utility, and in this case, risk aversion is separated into two parts: the first part is concerned with the 'probabilistic' side, i.e., convexity of a (cumulative) probability transformation, while the second part is diminishing marginal utility, though an axiomatic separation of these two sides has not been obtained in the literature so far. This theory serves to describe a simple way to elicit and characterize utility differences, entirely independent of probabilistic risk attitudes, and has been recently used to modify prospect theory, in Tversky and Kahneman (1992) and Wakker and Tversky (1991) [3]. Once marginal utility has been isolated, it can be filtered out, and probabilistic risk attitudes can also be characterized, independently of marginal utility. This section aims to show a new answer to the classical economic question provided by Rank-dependent utility (RDU), and how it does so.

In the early writings on EU, the difference between risky and riskless utility was not yet an issue. Bernoulli (1738) and Cramer (1728), the origins of expected utility, implicitly ascribed an intrinsic meaning to utility.

It began to be a problem to the eyes of many scholars, precisely ordinalists, at the beginning of this century. They started questioning the meaningfulness of (riskless) cardinal utility, and while their arguments were not primarily directed towards risky utilities, as a result they deprived the risk-less interpretation of risky utility of its basis. Such criticism, though strong and persistent, didn't make the idea of cardinal risk-less utility disappear.

The utility results of von Neumann and Morgenstern (1944), although remarkable, did not change the state of things. In fact, the testimony of few other experts suggest it; Arrow (1951, p. 425) wrote, about risky utility: *"[...] the utilities assigned are not in any sense to be interpreted as some intrinsic amount of good in the outcome [...],"* and, furthermore, about cardinal utility under certainty: *"which is a meaningless concept anyway."* The above citations express the representational viewpoint, which holds that utility, elicited from decisions under risk, has been proved to be applicable there, and only there. Up till now has argued for an intrinsic meaning of utility, prior to risk; but the picture changes completely if risk is taken into account, thus if risk attitude consists of more than marginal utility, a theory must be developed that incorporates more components of risk attitude.

RDU seems to be the most popular generalization of EU nowadays. As with all deviations from EU, it allows for the modeling of the well known paradoxes, such as the Allais paradox and the Ellsberg paradox. It serves the purpose of this section by allowing for one unified utility. To bear in mind that even the RDU is not without critics and weaknesses. The major weakness, according to the model

[3] Theories discussed later on through the research of Prospect Theory.

proposed by Peter Wakker is that a change of an outcome does not affect the decision weights, as long as it did not affect its ranking as compared to the other outcomes. Such change may suddenly and drastically change decision weights when it does affect the ranking. It is his clear belief that this sudden mutation of decision weights fits and can never pretend to be more than an approximation of natural processes or of a perfect normative approach. This seems to be the major weakness of RDU. Another point of concern is that no very convincing empirical support for RDU has yet been obtained.

The theory in the specific states the following properties and conditions [4]:

$\wp$ denotes the set of probability distributions, called *lotteries*, over an interval $(0, M)$, with M being an arbitrary fixed positive number; elements of $(0, M)$ are called outcomes. By $\succcurlyeq$, it is denoted the preference relation of a decision maker on $\wp$,with $\backsim$, $\succ$, $\preccurlyeq$,and $\prec$ as usual;

$\succcurlyeq$ is a weak order if it is complete $(P \succcurlyeq Q$ or $Q \succcurlyeq P \forall P, Q)$ and transitive.

A function $V : \wp \to \mathbb{R}$ represents $\backsim$ if $[(P \succcurlyeq Q] \iff [V(P) \geqslant V(Q)]$.

Cardinal, as it is explained by Wakker, is nothing more than an abbreviation of the intractable 'unique up to a positive affine transformation'. In fact, descriptive contexts treat preferences as dependent on the way lotteries are presented to subjects, but the difference in this theory is that it assumes that preferences depend only on probability distributions and not on the way of presentation.

DEFINITION 1. *Rank-dependent utility*, (RDU) holds if there exist a strictly increasing continuous utility function $U : [0, M] \to \mathbb{R}$ and a strictly increasing probability transformation $\varphi : [0, 1] \to [0, 1]$ with $\varphi(0) = 0$ and $\varphi(1) = 1$, such that $\backsim$ is represented by:

$$P \mapsto \int_{\mathbb{R}_+} \varphi \circ G_{U,P} d_\tau + \int_{\mathbb{R}_-} [\varphi \circ G_{U,P} - 1] d_\tau.$$

Here, $G_{U,P}$ is the decumulative distribution function of U under P and the integral is the Rank-dependent utility (RDU) of the distribution P.

[4]Definitions and design used in Peter Wakker's paper named "Separating marginal utility and probabilistic risk aversion." Theory and Decision.

The continuity of φ has not been incorporated in the definition of RDU, because there is interest in discontinuities, primarily at 0 and 1.

OBSERVATION 2. RDU is convex, concave, or linear (with respect to probability mixtures) if and only if φ is.

Proof. I only consider convexity. The decumulative distribution function of a (probability) mixture of probability distributions being the same mixture of decumulative distribution functions, convexity of φ implies the same for RDU by Definition (1). Conversely, if I set $U(M) = 1, U(0) = 0$, then the equality

$$q(p) = RDU(p, M; 1-p, 0)$$

shows that convexity of RDU implies convexity of φ .

Throughout the paper presented by Wakker, the following assumption is used:

ASSUMPTION 3.

- Weak ordering,
- (Strict) Stochastic Dominance $P \succ Q$ whenever $P \neq Q$ and P stochastically dominates Q.

4.3 Anticipated Utility Theory, by Quiggin

The model suggested by John Quiggin [16] is presented as an axiomatic approach to the problem that many other economists have been trying to solve for decades, the problem of the Independence axiom so much discussed earlier. In fact, this alternative theory suggests that the most intractable difficulties with the practical application of the EUT are related to the axiom of irrelevance of independent alternatives. In many different experiments, it has been proved that whenever arguments based upon this axiom were put before the eyes of the subjects, their answers clearly revealed that they did not accept its validity. A great portion of them continued to reject the axiom even after an intense and well-detailed discussion with the experimenter. The theory presents some weaker form of the independence axiom, for it is based on a weighted sum of utilities formed using 'decision weights', rather than a mathematical expectation, and for this reason it is termed Anticipated Utility. Despite its differences, it kept some similarities with the traditional theory, such as transitivity in pairwise choices and the fact that does not admit violations of dominance. Like all the other theories formulated, this one either was without criticisms; more specifically two points can be addressed: First of all, there is the consideration that the EUT is sufficient for all practical purposes, but that would be true only if the violations were merely random. It is instead clear that they are not, for people actually do not act rationally all the time like the EUT assumes. Also, the criticism that the anticipated utility theory may not be general enough, though it is still much more general than the standard theory of expected utility.

An advantage of this model is that it doesn't derive weights as a function of individual probabilities, but it avoids such difficulties embodied in this technique. To do so, it derives the weights directly from the entire probability distribution; typically, events at the far extremes of ranges would be overweighted, and to remedy some intermediate outcomes must be underweighted.

The theory deals with individual preferences over a set named X of outcomes, with the respective set Y of associated prospects. The set of preferences represents a part of the possible states of the world into mutually exclusive events. Precisely, each prospect $y \in Y$ consists of a pair of vectors $[(x_1, x_2, ..., x_n); (p_1, p_2, ..., p_n)]$, such that $x_1, x_2, ..., x_n \in X$ and $\sum_{i=1}^{n} p_i = 1$. Usually, it will happen that the number of different outcomes n will not be the same as the number of different prospects. Moreover, individual preferences are denoted by such a relationship P, assumed to be complete, reflexive and transitive, and also by the associated indifference relationship called I, that is, if an outcome that belongs to the set X is indifferent to a certain risky prospect of Y, it will be denoted as the certainty equivalent of y, described as $c = CE(y)$. If two outcomes are indifferent, there will be no distinction made between them. What this theory succeeds to do, unlike others, is to assign different decision weights to outcomes that have the same probabilities. That is an important note to point out because if this is not the case, then it fails to take into account the fact that people may distort the probabilities of extremes outcomes, but they don't need to treat the intermediate outcomes as having the same probability.

The anticipated utility function is defined as follows [5]:

$$V = h(p) \cdot U(x) = \sum_{i=1}^{n} h_i(p)U(x_i),$$

where U is a utility functions with properties and characteristics very similar to the function of EUT, while $h(p)$ is a vector of decision weights, that satisfies $\sum_{i=1}^{n} h_i(p) = 1$. The $p's$ represent the probabilities of obtaining an outcome x and they are not the same as the decision weights $h's$, as previously stated. The result of this procedure, after addressing some additional impositions, is that $h_i(p)$ depends on all probabilities up to n, and not merely on p_i, therefore, the relationship between the probability of an event x_i and its associated decision weight depends upon its position in the preference ranking of possible outcomes.

Ultimately, to be able to generate a theory which is far more general than the traditional model, it is mandatory to set some weaker axioms that substitute the problematic independence one. The axioms of independence and continuity are deeply weakened in this scenario, if applied only to $x \in X$. Furthermore, the completeness axiom is adopted;

Axiom 1. P is complete, reflexive and transitive (completeness).

Then, the dominance and continuity axioms are modified to apply to outcomes:

Axiom 2. $x_1, x_2 \in X, x_1 P x_2 \Rightarrow x_1 P[(x_1, p; x_2, 1-p)]$ (dominance).

Axiom 3. If $x_1, x_2, x_3 \in X, x_1 P x_2 P x_3 \exists p^*$ such that $x_2 I[(x_3, p^*; x_1, 1-p^*)]$ (continuity).

Finally, a much weaker independence axiom is used to ensure that $h(p)$ is independent of the outcomes:

[5]Model used also in the paper of Quiggin J., "A theory of anticipated utility", J. Econom. Behav. Organ.

Axiom 4. If $y_1 = (x, p)$ and $y_2 = (x', p)$ and for each $i, i = (1, 2, ..., n)$, there exists

$$c_i = CE[(x_i, 1/2; x'_i, 1/2)], i = (1, 2, ..., n)$$

and

$$x^*_1 = CE(y_1), x^*_2 = CE(y_2).$$

Then,

$$[c; p]I[(x^*_1, 1/2; x^*_2, 1/2)]$$

(independence).

It is important to point out that the sets X, Y must be rich to some extent, otherwise, if they do not have enough elements, utility functions may not be unique, thus the proof of existence theorems is made even more complicated. So two stronger assumptions must be made to derive uniqueness, for the utility class of functions in the Anticipated Utility theory is much larger:

R.1. If $x_1, x_2, ..., x_n \in X, x_n P x_{n-1} P, ..., P x_1$, and $\sum_{i=1}^{n} h_i(p) = 1$, then $(x; p) \in Y$.

R.2. If $y \varepsilon Y, \exists x = CE(y) \in X$.

To conclude, if X and Y satisfy all 4 axioms and these two further assumptions, the function aforementioned of the anticipated utility theory is confirmed.

Some additional considerations regarding the subjects can be highlighted, in terms of their attitudes towards prospects, which results in being due to both attitudes to the outcomes and the probabilities at once. Therefore, with respect to the first, risk-preferences are much relevant, but interpreted slightly differently, thus risk-aversion is perceived as a concave function. As for the probabilities, they distort the extremes events. Just as the expected utility model, Anticipated Utility theory permits the analysis of anticipations which are not mathematical expectations, thus not excluding behavior which would be considered irrational in other contexts. Although the framework of the theory is general, it manages to address the predictions and the outcomes in a specific manner.

4.4 Generalized Expected Utility Theory, by Machina

It is repeatedly highlighted the fact that what the expected utility model lacks, though its undoubted popularity, alternative theories have been trying to compensate. Research has extended to the areas of psychology literature, where experts have discussed and contextualized that preferences of subjects are not linear in probabilities. This section focuses on the proposal made by Mark Machina [15] in 1982. It is common knowledge that the EUT can be derived from three familiar axioms of *ordering, continuity and independence*. Machina's model keeps the first two axioms aforementioned, but drops the third axiom, substituting it with two alternative hypotheses (Hypothesis I and Hypotesis II), integral part of the theory itself. Both hypotheses concern the shape of a fixed non-linear preference functional over probability distributions and are consistent with most behaviors assumed in the original EUT. Hypothesis I describes the usual shape of a local utility function for a given distribution and, furthermore, Hypothesis II shows the ways in which such function changes as different initial distributions are evaluated. It can be further shown that Hypothesis II coincides with the respective violations of the independence axiom, in a formal characterization. More specifically, the first one represents the concept of *monotonicity*, while the second one is the so-called *fanning-out* hypothesis.

The model of GEUT [6] can be seen as an alternative approach to non-expected utility preferences, which links properties of attitudes toward risk directly to the probability derivatives of a general 'smooth' preference function $V(P) = V(x_1, p_1; ...; x_n, p_n)$. This approach is based on the observations that for the expected utility function $V_{EU}(x_1, p_1; ...; x_n, p_n) \equiv U(x_1) \cdot p_1 + ... + U(x_n) \cdot p_n$ the value $U(x_i)$ can be interpreted as the coefficient of p_i, and that many theorems involving a linear function's coefficients continue to hold when generalized to a nonlinear function's derivatives. By adopting the notation $U(x; P) \equiv \partial V(P)/\partial prob(x)$ and the term 'local utility function' for the function $U(\cdot; P)$, standard expected utility characteriza-

[6]Terminology and procedure extracted from "Expected utility" theory without the independence axiom. Econometrica", by Machina.

tions can be generalized to any smooth non-expected utility preference function $V(P)$ in the following pattern:

(a) $V(\cdot)$ exhibits global first order stochastic dominance preference if and only if at each lottery P, its local utility function $U(x;P)$ is an increasing function of x.

(b) $V(\cdot)$ exhibits global risk aversion (aversion to small or large mean-preserving increases in risk) if and only if at each lottery P, its local utility function $U(x;P)$ is a concave function of x.

(c) $V^*(\cdot)$ is globally at least as risk averse as $V(\cdot)$ if and only if at each lottery $P, V^*(\cdot)$'s local utility function $U^*(x;P)$ is a concave transformation of $V(\cdot)$'s local utility function $U(x;P)$.

Regarding the Allais Paradox and other remarked violations of the Independence Axiom, it can be shown that the indifference curves of a smooth preference function $V(\cdot)$ will fan out in the probability triangle if and only if $U(x;P^*)$ is a concave transformation of $U(x;P)$, whenever P^* first order stochastically dominates P.

It should be noticed that Hypothesis I doesn't imply that absolute risk-aversion decreases in wealth, as if the subjects were utility-maximizers.

Chapter 5

Failures and Paradoxes

The violations that the new-borne theories aforementioned have been trying to overcome can be divided into two primary, though broad, categories:

(a) those that have explanations in terms of "conventional" theory of preferences,

(b) and those which do not.

I decided to keep my focus on the former group, which in fact consists of a series of observed violations of the independence axiom in the expected utility theory.

Such violations tend to take the form of either one of two effects [1]:

- Common consequence effect,
- Common ratio effect.

As for the former effect, suppose there are four compound lotteries:

$$x_1 := (x, p; y^{**}, 1 - p)$$

$$x_2 := (y, p; y^{**}, 1 - p)$$

$$x_3 := (x, p; y^{*}, 1 - p)$$

$$x_4 := (y, p; y^{*}, 1 - p)$$

where all the lottery supports contain only non-negative outcomes, $p \in (0,1), x > 0$, the support of y contains outcomes both greater and less than x, and y^{**} first order stochastically dominates (FOSD) y^*.

Subjects are proved to often show the following behaviors:

$$x_1 \succ x_2, x_4 \succ x_3.$$

This specific pattern is not consistent with the independence axiom, since it implies that $x \succ y \iff x_1 \succ x_2 \iff x_3 \succ x_4$.

The most famous version of the common consequence effect is the Allais paradox [20].

Common ratio effect instead, can be presented as four lotteries:

$$y_1 := (x, p; 0, 1-p)$$

$$y_2 := (y, q; 0, 1-q)$$

$$y_3 := (x, \alpha p; 0, 1-\alpha p)$$

$$y_4 := (y, \alpha q; 0, 1-\alpha q)$$

where $1 > p > q > 0, 0 < x < y$ and $\alpha \in (0,1)$.

People often exhibit the following pattern of preferences, also inconsistent with the axiom:

$$y_1 \succ y_2, y_4 \succ y_3.$$

5.1 Allais and Ellsberg Paradoxes

The ideas and theorems formulated by Daniel Ellsberg [23] throughout his researches, dated 1961, can be viewed as strictly interconnected with the previous mentioned Allais Paradox [20], developed by Maurice Allais in his paper *"Le Comportement de l'homme rationnel devant le risque: critique des postulats et* *axiomes de l'école américaine"* in 1953 [1].

Various experiments empirically prove that individuals' decisions can be inconsistent with expected utility theory. Essentially, they describe two significantly different violations; the former refers to a controversy in the validity of the axiom regarding the subjective probability, whereas the latter regards the expected utility itself. Ellsberg's experiments have proved that more information is needed in situations, compared to other non-expected utility theories developed by other economists, and described in the previous chapter. To prove his believes, he used an anticipated utility theory approach, suggested by Quiggin, although to model the Ellsberg paradox as a two-stage lottery does not count for the model of Anticipated Utility theory, instead, it depends on a theory that doesn't necessarily satisfy the reduction of the compound lotteries axiom [2].

It is well known that in the expected utility framework, the condition that certain prizes are preferred to random variables is equivalent to the assumption that the utility function u is concave.

Consider the following four lotteries:

- $A_1 = (0, 0.89; 1000000, 0.11)$
- $A_2 = (0, 0.9; 5000000, 0.1)$
- $B_1 = (1000000, 1)$
- $B_2 = (0, 0.01; 1000000, 0.89; 5000000, 0.1)$

From his research, Allais understood that most people usually prefer A_2, to A_1, but then they choose B_1, to B_2, although by expected utility theory $A_2 \succcurlyeq A_1, \iff B_1 \succcurlyeq B_2$ [3].

The standard form of the Allais paradox involves the following lotteries:

- $A_1^* = (0, 1-p; x, p)$
- $A_2^* = (0, 1-q; y, q)$

[1] see the references in the Bibliography.

[2] Consult the paper "The Ellsberg paradox and risk aversion: an anticipated utility approach", by Segal U., as reference for more insights on the problems and theorems described in this section.

[3] Similar results were also reported by Kahneman and Tversky [1979].

- $B_1^* = (0, 1-p-r; x, p+r)$
- $B_2^* = (0, 1-q-r; x, r; y, q)$

where $0 < x < y, p > q$. According to the traditional model of utility function, $A_2^* \succcurlyeq A_1^*, \iff B_2^* \succcurlyeq B_1^*$, instead the evidence shows that if $A_2^* \sim A_1^*$, then $B_1^* \succ B_2^*$.

To informally conclude, the Allais Paradox relies on the fact that whenever people approach gambling situations in a slightly different way, they will result in preferring the uncertainty, which in any other case is to be rejected. Some further observations in fact can be made; to begin with, expected utility theory does not really apply in the real world , therefore we may observe its many critiques and alternative approaches. Moreover, individuals tend to give extra values to situations with total absence of risk, on the contrary to remote and highly uncertain risk.

5.2 The Preference Reversal Phenomenon

Among the most controversial patterns of behavior observed during the experiments is the Preference Reversal Phenomenon [22].

In the setting presented to the subjects, they are usually given a choice between *"a lottery with a high probability of winning a modest amount of money, called* **P bet***, and a lottery with a low probability of winning a large amount of money, a* **$ bet***."* [4]

Moreover, they elicit the lowest amount of money for which the subject would willingly sell either one of these lotteries.

What is considered most controversial is the fact that the subjects typically face a situation in which they would choose the P bet over the $ bet, but at the same time they would put a higher selling price on the $ bet. Preference reversals were much discussed by many scientists, even in the field of psychology, where David Grether and Charles Plott remark: *"[...] the inconsistency is deeper than mere*

[4] Description of a preference reversal experiment by Charles A. Holt. (See reference 22)

lack of transitivity or even stochastic transitivity. It suggests that no optimization principles of any sort lie behind even the simplest of human choices [...].”

Furthermore, human choices between lotteries with hypothetical monetary payoffs have repeatedly, but not overwhelmingly, violated the independence axiom.

I take the example of the experiments conducted by Grether and Plott [5]; their research, psychology-based, concludes that all earlier experiments were irrelevant from an economist’s perspective. The reason lies on the nature of the experiments themselves, which was not based on monetary or any other incentives to motivate subjects to behave rationally, and there was no control for income effects in all of the remaining experiments that did have real incentives. So they ran a series of preference reversal experiments first without then with both monetary incentives and a random lottery-selection device that guarantees the control over income effects. The most evident surprise was in finding out that the result was actually the opposite of their initial intuition; they found preference reversals to be even more pervasive in the series with monetary incentives. This result is especially noteworthy because it has been shown in other experiments that violations of the Bayes rule in information-processing tasks are common with inexperienced, financially unmotivated subjects, but that such violations are less frequent with experienced, financially motivated subjects (Grether, 1978).

To prove their ideas, they assumed that the axiom of independence implicit in the expected utility theory doesn’t hold, and they further developed the experiment around such assumption. So the lottery choices for some pairs were made by the subject after the elicitation of selling prices for those lotteries, and the procedure was reversed for other pairs of lotteries. For the analysis of a simple case of only one pair of lotteries, suppose first that the two elicitations precede the lottery choice. Let $\widehat{r_\$}$ and $\widehat{r_p}$ denote the selling prices that were selected in the elicitation stage. Then the choice between X_p and $X_\$$ in the final state is a choice between the compound lotteries

$$[X_p, 1/3; B(\widehat{r_\$}; X_\$), 1/3; B(\widehat{r_p}; X_p), 1/3]$$

[5]For more insights on the data and experiment, see the reference ”Preference reversals and the Independence Axiom”, by Holt C.

and

$$[X_\$, 1/3; B(\widehat{r_\$}; X_\$), 1/3; B(\widehat{r_p}; X_p), 1/3].$$

Let the lottery Z be defined:

$$Z \equiv [B(\widehat{r_\$}; X_\$), 1/2; B(\widehat{r_p}; X_p), 1/2].$$

It follows from these three equations, that the choice between X_p and $X_\$$ in the final stage is a choice between

$$(X_p, 1/3; Z, 2/3) \text{ and } (X_\$, 1/3; Z, 2/3).$$

The direct implication of the independence axiom is that the left-hand compound lottery present in the second to last equation, is weakly preferred to the right-hand compound lottery in the last equation, if and only if X_p is weakly preferred to $X_\$$.

The choice between the P bet and the \$ bet in the final stage of the preference reversal experiments is actually a choice between the compound lotteries between the last two equations, and if the independence axiom does not hold, the bet that the subject chooses in the final stage may not be the bet that is actually preferred by the subject, that is, the bet that would be chosen in a direct choice between X_p and $X_\$$. The bet that is actually preferred in such a direct choice may be given a higher selling price in the experiment, even though it is not selected in the pairwise comparison of the compound lotteries. If this occurs, a violation of the independence axiom may result in choices that appear to violate transitivity. The preceding discussion pertained to the case in which elicitation of the subject's selling prices is done before the subject makes the choice between the P bet and the \$ bet. If, instead, the choice between these two bets is to be made before the elicitation of selling prices, then the choice between X_p and $X_\$$ in the first stage is a choice between the compound lotteries, where Z is a lottery that in this case represents the subject's beliefs about the possible outcomes in the subsequent elicitation stage. As before, a violation of the independence axiom may result in preference reversals in which the bet in the compound lottery selected is the bet with the lowest selling price. Of course, it is possible to specify a utility functional that does not satisfy independence and for which the choices of the lottery and selling prices are independent of the order in which these choices are made. The point being, the lottery chosen (by the individual whose preferences are represented by this utility functional) may not be the lottery that would be preferred in the absence of the elicitation procedures.

It is well known that many individuals make choices that are direct violations of the independence axiom in other contexts. Therefore any theory of rational choice in such contexts must be derived from a set of axioms that does not include or imply the independence axiom, at least not in its usual ”strong” form.

Preference reversals could also be generated by intransitivities, but to abandon transitivity would be a drastic step that would make it difficult to construct a formal choice theory with empirical content. The transitivity assumption is needed for the existence of a utility functional that represents preferences over lotteries; independence is a strong assumption about the functional form of this utility functional (that pertains to the linearity with respect to the probabilities).

5.3 Violation phenomena discussed through Prospect Theory

As a critique to the traditional expected utility theory, a descriptive model of decision making under risk, Prospect Theory [21] aims at proposing an alternative model, like many others before did. It is known that choices among risky prospects exhibit various pervasive effects inconsistent with the basic tenets of utility theory. In expected utility theory, the utilities of outcomes are weighted by their probabilities, though it may happen that a series of choice problems result in people's preferences systematically violating this principle. Firstly, agents are inclined to overweight outcomes that are considered certain, relative to outcomes that are merely probable, classified as the certainty effect. Allais choice problem is the best known counter-example to EUT that exploits such effect. The Allais' example, refers mostly to moderate large gains rather than the extremes.

To name them:

(a) the so-called *certainty effect*; people underweight outcomes that are merely probable in comparison with outcomes that are obtained with certainty. This tendency most times contributes to risk-aversion in choices involving sure gains and to risk-seeking in choices involving sure losses.

(b) on the other hand, the *isolation effect*; subjects discard components shared by all prospects under consideration. This tendency, instead, leads to inconsistent preferences when the same choice is presented in different forms [6].

Therefore, this new theory of choice assigns each value to gains and losses rather than to final assets and the probabilities are replaced by decision weights. The value function is usually concave for gains, and convex for losses, moreover is normally steeper for losses. Also, decision weights are lower than the corresponding probabilities, except in the range of low probabilities. This theory helps to demonstrate several phenomena that are contradictory with the expected utility theory. Kahneman and Tversky replicate Allais' results by a choice experiment based on hypothetical situations, using moderate gains. In the choice problems presented to the agents, the decision maker is expected to evaluate each of the edited prospects, and finally choose the prospect of highest value. The overall value of the edited prospects is denoted by V, which in turn is expressed in terms of two scales, π and v.

The first scale, π, connects to each probability p a decision weight $\pi(p)$, which reflects the impact of the probability p on the overall value of the prospect. On the other hand, the second scale, v, assigns to each outcome x a number $v(x)$, which reflects the subjective value of that outcome. To note that the outcomes are defined in relation to a reference point, which functions as the zero point of the value scale. Hence, v measures the value of deviations from that reference point. In a plausible situation where winning is possible but not probable, most people choose the prospect that offers the larger gain. Such scenario proposed in the

[6] Definitions of the two effects, and the description of the theory to follow take inspiration from the work of Kahneman D., Tversky A .,Prospect theory: An analysis of decision under risk, Econometrica, Vol. 47, No. 2 (1979), pp. 263-291.

experiment conducted by Kahneman and Tversky, illustrates common attitudes toward risk that cannot be captured by the expected utility model. Furthermore, the experiment shows that a violation of the independence axiom is attributed in this case to sub-certainty, more specifically, such analysis shows that an Allais-type violation will occur whenever the v-ratio of the two outcomes presented in the experiment is bounded by the corresponding π-ratios [7]. To be highlighted, the alternative representations of choice problems that are instead induced by shifts of reference point, and sketch several extensions of the present treatment. The violation of the substitution axiom is attributed in this case to the sub-proportionality of π. Expected utility theory is violated therefore, whenever the v-ratio of the two outcomes is bounded by the respective π-ratios. The same analysis applies to other violations of the substitution axiom, both in the positive and in the negative domain.

The present theory states that attitudes toward risk are determined jointly by v and π, and not solely by the utility function. Moreover, in prospect theory, the overweighting of small probabilities favors both gambling and insurance, while the $S-shaped$ value function used tends to inhibit both behaviors. Although prospect theory predicts both insurance and gambling for small probabilities, economists feel that the hereby presented analysis falls far short of a fully adequate account of these complex phenomena. Indeed, there is evidence from both experimental studies and researches, and observations of economic behavior, that *"the purchase of insurance often extends to the medium range of probabilities, and that small probabilities of disaster are sometimes entirely ignored. Furthermore, the evidence suggests that minor changes in the formulation of the decision problem can have marked effects on the attractiveness of insurance."* [8]

A change of reference perspective, in fact, will result in the alteration of the preference order for prospects, and although the paper by Kahneman D., Tversky A. has been concerned mainly with monetary outcomes, the theory is readily applicable to choices involving other attributes.

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[7] For more insights, see the experimental setups and developments discussed in the paper reference by Kahneman D., Tversky A., "Prospect theory: An analysis of decision under risk. Econometrica."

[8] Citation from Prospect Theory: An Analysis of Decision under Risk. (See reference 21)

Chapter 6

Experimental Design

6.1 Brief introduction on Experimental Economics

Experimental economics is meant to study and address, through new methods, the many questions that economists have studied for ages. It is yet to be considered a solid theory, as the evidence gathered so far is not as strong and reliable as the one of economics itself, nevertheless the impact of the experimental methods to the economist's toolbox is remarkable, for several reasons.

Firstly, some respectable economists have doubted the possibility that economics is, to a considerable extent, an experimental science.

Second of all, the behavioral revolution in economics owes its existence to the path-breaking contributions of the experimental side of this area of knowledge. The same assumption applies for recent academic beliefs that the presumption of "representative agents", that is, the oversimplifying assumption that economic actors tend to be rational and act as guided by their own self-interest, is not necessarily true at all times. Indeed, experimental economics results have brought up much newer and more interesting theorizing.

Nowadays, it seems unreasonable to think of any theory that has not been studied, or at least been tested, experimentally. Amongst the most relevant characteristics of experimental economics is its methodology, for it distinguishes economics from other social sciences.

As the experiment conducted in this thesis will show, most of the individuals included in the experimental set up and development are usually provided with monetary incentives. And it is generally not accepted to use deception. Though, this doesn't mean that most of the experiments conducted in the history of economics have followed such rule.

Precise scripts are necessary to advocate replication, in fact, decision-making is not studied as "as-if" proposition, but rather in laboratories that represent the problem studied. Usually subjects are instructed to enact a precise role in such representations, such as buyer or seller, or that of a voter.

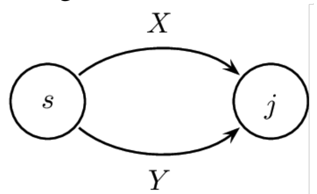
The various methodological practices are much more significant than they may first appear to be, due to the way an individual conducts an experiment. Distinctive choices of particular implementation details and design can crucially affect results and make the difference between accepting or rejecting a theory.

Experimental economists' methodological practices have also influenced emerging experimental versions of many other fields of expertise such as philosophy, finance, political science, and has led to useful reflections in many more social sciences. Increasingly, experimental economics had to share the spotlight with behavioral economics and neuroeconomics. In fact, many prominent experimental economists straddle these areas.

6.2 Experimental Environment

Starting in the mid-1980s, a number of researchers turned their attention towards testing the non-expected utility theories. The majority of this work involved experimental testing, part of it was designed to compare the predictive abilities of competing theories, whereas another part was designed to test novel implications of particular theories, and lastly, a part of which was designed to test the descriptive validity of particular axioms. A large volume of work has emerged in this arena, providing a much richer evidential base against which theories can be judged. The purpose of this section is to make the subjects of the experiment more familiar with the environment, so that the results of the experimental setting

Figure 6.1: Combinations of variables



are effective and reasonable, and more importantly, to explain the objective of the experiment, what results I would like to achieve.

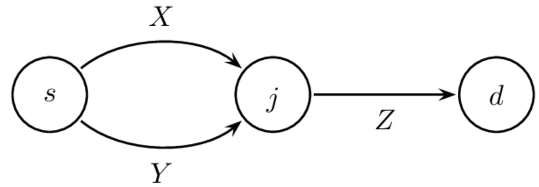
Specifically, the experiment in question may seem trivial in the setting of network games, because the assumptions made and discussed in previous chapters are very intuitive and straightforward in such environment, but it is actually not; in fact, in many other fields, the concept of additive consistency doesn't work at all. In fact, it is my intention to test whether such condition is meaningful and respected in this case. The additive consistency concept is without doubt crucial to the characterization of the Expected Utility Theory, being the last imposition, the one that forces on the theory a last axiom to obtain very much stronger and precise conclusions. Thus, the final purpose of the experiment is to ask agents repeatedly one basic question, and assess whether their answers are consistent:

"If you were to go from a certain point s to point j, and you were to choose between road X and road Y, which would be the one preferred?"

Hence, a really rough and informal idea of what the set-up will be, consists of a choice between two random variables, X and Y, to get from point s to j.

The intuition I would like to observe and prove is that if an individual, as their first choice, selects X, then the variable X should still be preferred to Y in case I add an additional variable, let us call it Z, to both original variables. Being that variable independent of the other two, then the second choice to be made by the agent should be consistent with the first choice of X, to get from point j, just reached, going a step further to point d.

Figure 6.2: Combinations of variables



I can develop different scenarios, one the inverse of the other, or simply diversified settings; hence, the agent could be first presented with a choice to make between X and Y, and only later Z is added, or I could set up the opposite structure, or in the end, I could also set the choice directly between $(X + Z)$ and $(Y + Z)$, so not dividing into two consequent steps the decision.

The experiment will also contribute to assembling the means to assess whether the context matters or not. To further explain, I would like to know whether the way in which I present the variables to the agents really makes a difference, and also finding the proper way to construct the experiment so that I will be ensured that the individuals are not biased towards a specific choice. Last but not least, the risk-aversion parameter will be calculated, or at least attempted to be derived. To this end, in fact, it would be crucial the prior assessment of the risk attitude of each agent, and after the experiment has been carried out, test whether they remained consistent. Finally, the assessment of risk attitudes will help to address some final questions to the participants in the experiment, trying to understand how they think, and how they make the decision.

6.3 Experimental Setup and Methodology

The current experiments are designed to examine the impact of risk attitudes on single agents' choices, and whether the additive consistency holds.

6.3.1 Assessment of Risk attitude

Despite the relevance of risk-aversion in all standard theories of choice, experimental research has provided shallow guidance as to how risk-aversion should be modeled. In fact, risk-aversion effects are often controversial or even ignored throughout the analysis of laboratory data, as a result, most theorists bypass its issues by assuming that the ”payoffs” for a game are already measured as utilities. The nature of risk aversion is ultimately an empirical issue, thus experiments run in laboratories should produce meaningful evidence to complement field observations by providing careful controls of payoffs and probabilities. While there might be many economists admitting that risk aversion may be important, it is instead commonly asserted that decision makers should be approximately risk-neutral for the low-payoff decisions involving money, typically encountered in the laboratory. One way to assess the situation is to provide subjects with high-payoff lottery choices. Firstly, for purpose of this paper, understanding the inclination of the agent to risk is a key factor in the evaluation and afterwards implementation of the experiment; depending on the degree of their attitudes towards risk, their choices will change. Furthermore, I decide to test such predisposition to risk through a very standard and indicative table. Such table is known as *”Multiple Price list”*, from an analysis conducted by Holt and Laury, the reference dates back to the year 2000. (See Table 6.1)[1]

The explanation behind this table is quite simple; each individual is shown this table, consisting of 10 different choices to be made, and for each choice, they are provided with two possible options, Option A and Option B. For each option, there are displayed the probabilities of two different payoffs that may occur, one with a higher probability of occurrence than the other. The task hereby presented

[1]Multiple Price List: a model to assess the risk-aversion of the agents, taken from the paper ”Risk aversion and incentive effects”, C. A. Holt and S. K. Laury, (2000).

Table 6.1: Risk Attitudes

	Option A				*Option B*			
Choices	*Prob*	*Payoff*	*Prob*	*Payoff*	*Prob*	*Payoff*	*Prob*	*Payoff*
1	0.1	2.00	0.9	1.60	0.1	3.85	0.9	0.10
2	0.2	2.00	0.8	1.60	0.2	3.85	0.8	0.10
3	0.3	2.00	0.7	1.60	0.3	3.85	0.7	0.10
4	0.4	2.00	0.6	1.60	0.4	3.85	0.6	0.10
5	0.5	2.00	0.5	1.60	0.5	3.85	0.5	0.10
6	0.6	2.00	0.4	1.60	0.6	3.85	0.4	0.10
7	0.7	2.00	0.3	1.60	0.7	3.85	0.3	0.10
8	0.8	2.00	0.2	1.60	0.8	3.85	0.2	0.10
9	0.9	2.00	0.1	1.60	0.9	3.85	0.1	0.10
10	1.0	2.00	0.0	1.60	1.0	3.85	0.0	0.10

is very immediate, for each of the 10 choices, they will have to choose either A or B. The only rule imposed is that once they switch their preference from A to B or vice versa, they cannot switch back.

Notice that the payoffs for Option A, 2.00 or 1.60, are less variable than the potential payoffs of 3.85 or 0.10 in the ”risky” option B. In the first choice, for instance, the probability of obtaining the high payoff for both options is $1/10$, therefore, only a true risk-seeker would prefer Option B.

Whenever the probability of obtaining the high payoff increases enough (moving down the table), a person should switch over to Option B. In fact, a risk-neutral person would normally prefer the option A about four or five times before switching to B, and even the most risk-averse person should cross over by the last few rows, since option B yields a sure payoff of 3.85 in the very last case.

To summarize:

- if a player switches options within the first 3 choices, they are assessed to be risk-loving,
- if they switch between choice number 4 and 5, they are risk-neutral,

- and finally, if they change their preference later on, within the last choices, they are concluded to be risk-averse.

Through the implementation of such chart, it may be possible to actually compute the risk-aversion parameter, let's call it β, for each and every individual, a result that is very much wanted as a conclusion to the experiment. Additionally, having the table as a benchmark, I will ask players their most favorite choice, and make them play it for real. This way, the outcome of the lottery played will be their final incentive.

6.3.2 The figurative description of random variables

I now begin explaining the actual steps of the experiment they will be conducting. Random variables in question will be described through histograms, expressed in real terms of traffic time (how many minutes will it take them to get from point s to j), and the frequency with which each outcome of time will occur. In doing so, players will not be frightened by a language they don't understand, because histograms provide a visual representation of variables that otherwise would be difficult to explain, and they are also a very intuitive tool to use.

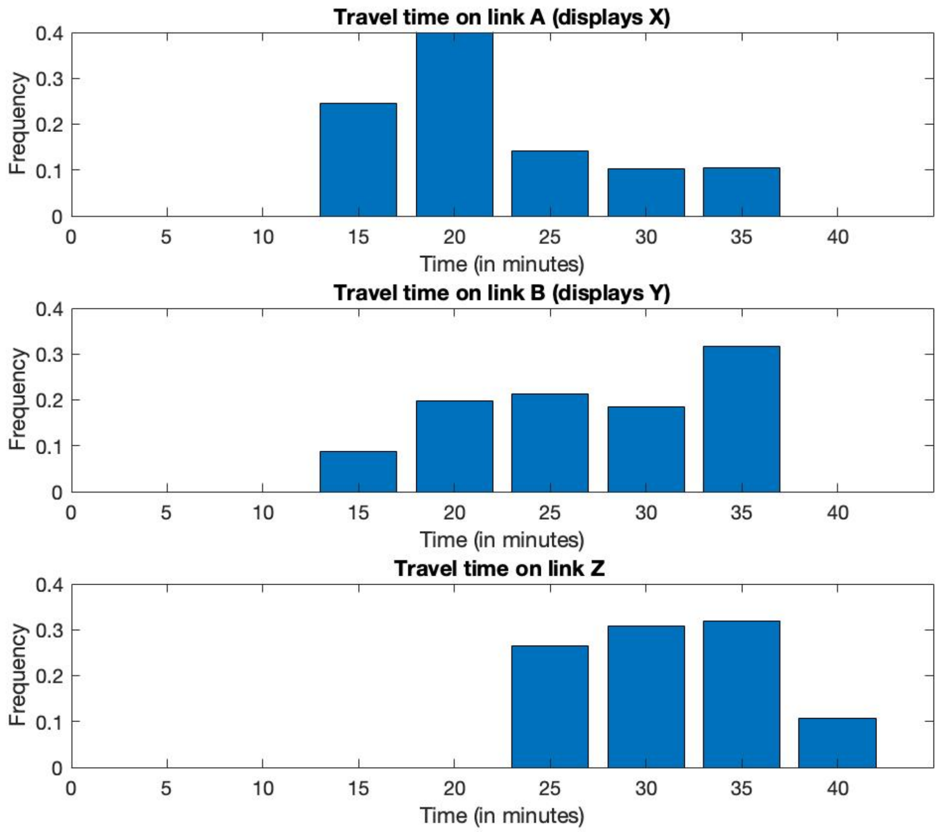
I will select six subjects, and I will assess the experiment for eleven times, so they don't get too much used to the procedure, enough to have their choices forced or biased in some way.

The scenario chosen consists into deciding which is best, in terms of time spent on the street, between two random variables presented as A and B.

Always to keep in mind that the higher the probabilities for the least-time spent outcomes, the better it is for the agents; the meaning behind this, in fact, is that a person always prefers to spend less time to travel.

Before presenting the variables with histograms, I discuss with the agents what they mean, providing them with the figure that visualizes these concepts. Firstly, I explain the intuition behind the scenario (See Figure 6.1), for the first set of graphs. Afterwards, once the individuals have made their choices for the first 11 proposals of graphs representing only a choice between A and B, I show them

Figure 6.3: Variables representation

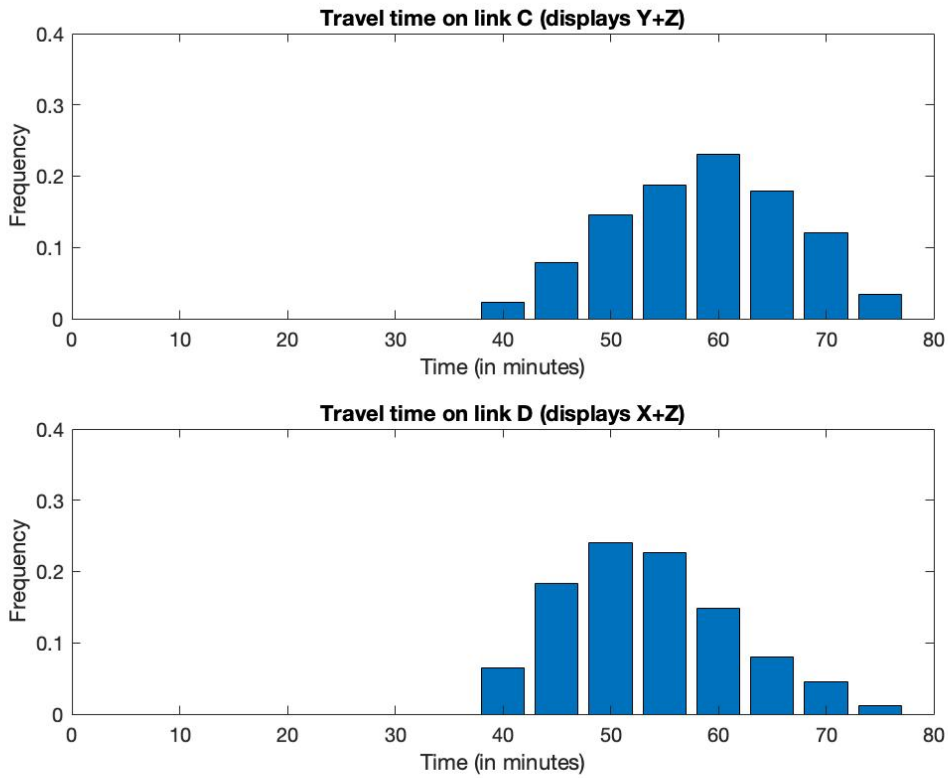


another figure (See Figure 6.2), that represents the choice which includes the common variable Z. Following such explanations, I present the figurative representations of the random variables. (See Figures 6.3 and 6.4)

The crucial step, is that the participants do not know that the second set of graphs is nothing more than the combinations of $(X + Z)$ and $(Y + Z)$, where X is the representation called A and Y is for B. In the Figure 6.3, the display of variable Z is actually not shown to the agents in this type of set up.

In the sample of Figure 6.4, there is a situation in which $(X + Z)$ is given as a representation of another variable called D and $(Y + Z)$ is given as C, hence the

Figure 6.4: Combinations of variables



order in which they are presented is inverted. But that isn't always the case, in fact, the way I programmed the Matlab Software to run the experiment, the order is random, thus sometimes will be the same, others won't.

Thus, without knowing which histogram represents which variable, the participants in the experiment won't be biased or induced into choosing one option or the other. This scenario will be played several times, to each player, and the alternatives proposed will be the same for all. This is possible because of the absence of interaction between the players, so they cannot copy what the others are doing, and by presenting the same objective alternatives to all, depending on their decision, it can be argued whether there is the action of a behavioral factor that conditions the final choice. In fact, when the experiment is over, I will ask each subject privately, why and how they made their decision, and in the very end, I will reveal the random variables for each graph, demonstrating for instance that C is nothing else than $(Y + Z)$, and finally ask them whether they would change their minds and choose differently.

To setup the experiments, I used the Matlab software to generate variables with the respective random probabilities, and the figurative representations will be presented in a random order, depending on what the software run. (See Appendix for the programming codes and the figurative representations)

6.4 Development and Results of Pilot experiment

The development of the pilot experiment proved to be very enlightening, from several different perspectives, but more importantly, I believe that the way in which it was set up was appropriate for my purpose, to demonstrate that the axiom of additive consistency really holds for such environment. The Pilot served as the basis to build up a future experiment, carried out more specifically and thought throughout, to better understand what to do and how to do it. The subjects were chosen to have diverse criteria and characteristics based on demographics, education, etc. so that the sample selected wouldn't be too specific and limited. Moreover, the subjects considered were students who belong to different areas of studies, some of them from the economics department, others from Law and others from Political Science studies. Additionally, some other subjects are workers, in their mid-thirties, to perhaps observe a new behavior.

The first 4 subjects are less than 30 years of age, mainly students, whereas the last two are adults. The patterns that can be observed from the graphs, and that I will further comment in this section appear to be clear; the risk-attitude of the subjects seems to have had a great impact on their choices, in fact, risk-averse players, given their pessimistic nature, demonstrated to act as driven by a diversification [2] of their portfolio to minimize the risk; namely, they were more inconsistent with themselves, through all of their own choices, because they wanted to reduce the

[2] In economics and investment planning there is a fundamental concept, which is the Portfolio Diversification; "it is the practice of spreading your investments around so that your exposure to any one type of asset is limited. This practice is designed to help reduce the volatility of your portfolio over time".

risk as much as possible, thus they adopted a diversification strategy by varying their choices.

The aforementioned pattern will result in a decrease in consistency of the subjects throughout their choices. In fact, they tend to variate perfectly their choices, firstly between A and B, and secondly between C and D. Considering the second treatment proposed to the subjects, the cognitive level of difficulty is much higher, namely the visual understanding and differentiation of the graphs is not that intuitive as it is in the first treatment. To this end, their diversification of choices is not optimal.

The other risk-attitude observed in the Pilot is risk-neutrality, in particular, two agents present it. Their results are quite interesting and clarifying. The reason for this is that they tend to be more consistent with themselves; being risk-neutral means that an individual is indifferent when facing a risky situation, namely in a stochastic environment, sometimes the subject would prefer an outcome, other times another.

In the second treatment, risk-neutral agents are still more consistent compared to risk-averse agents, but by a slighter percentage. I believe that the real motive behind it lies in the increased difficulty in implementing the action of diversifying the portfolio, since they do so on the basis of a visual assessment of the graphs. Thus, wherever they lack consistency, they lack visual understanding. Whereas risk- neutral agents don't really care about risk, so in their choices they vary perspectives.

The last 2 subjects interviewed are above 30 years of age, equally dividing themselves into risk-neutral and risk-averse. Regardless of their age, by tracking the effects of risk-aversion, they show the exact same pattern explained earlier in the case of younger students.

A further differentiation and analysis may be made according to the gender of the individuals. Males are more consistent compared to women, perhaps due to a different mental approach that is inherent in our biology.

To finally conclude my observation and interpretation of the results of the experiment, the assessment of the consistency presented by the subjects will reveal violations of the property of additive consistency in only 30% on average of the cases. Though 67% of the subjects demonstrated to be inconsistent only in 22% of the cases. The consistency hereby discussed refers to the actual coherence in choosing the graphs in the second treatment, which include the same variable of the graph previously preferred in the first treatment, with simply the addition of a third common variable Z. Thus the axiom seems to work in the overall context of traffic networks.

Find below the figures representing the Pilot experiment's results [3].

[3] The specific tables 6.5, 6.6, and 6.7, represent a summary of the results of the various aspects being assessed in the course of the Pilot. More specifically, the first two tables are to show the preferred graph (A or B, and C or D) in the two treatments of graphs presented, and the risk-attitude of each subject. The third table instead, discusses the consistency in the respective graphs throughout, that is the graphs representing the same values, with the addition of Z.

Figure 6.5: Summary of choices

Subjects	1	2	3	4	5	6
Risk Attitude	Risk-averse	Risk-averse	Risk-averse	Risk-neutral	Risk-averse	Risk-neutral
Choice 1	A	A	A	B	B	B
Choice 2	B	A	A	B	B	A
Choice 3	B	B	B	B	B	B
Choice 4	B	B	B	B	B	B
Choice 5	B	B	B	B	B	A
Choice 6	B	B	B	B	B	B
Choice 7	A	A	A	A	A	B
Choice 8	A	A	A	B	A	B
Choice 9	B	B	B	B	B	A
Choice 10	B	B	B	B	B	B
Choice 11	A	A	A	B	A	B

Figure 6.6: Summary of choices

Subjects	1	2	3	4	5	6
Choice 12	D	D	C	C	C	D
Choice 13	C	D	D	C	D	C
Choice 14	C	C	C	C	C	C
Choice 15	C	C	C	C	C	C
Choice 16	C	C	D	D	D	C
Choice 17	D	D	D	C	C	C
Choice 18	D	D	C	D	D	C
Choice 19	C	C	C	C	C	C
Choice 20	D	D	D	C	D	D
Choice 21	D	D	D	D	D	C
Choice 22	C	C	C	C	C	C

Figure 6.7: Consistency in additive consistency in the choices

Order	CONSISTENCY	Subject 1	Subject 2	Subject 3	Subject 4	Subject 5	Subject 6
Same	Sets 1-16	YES	YES	NO	YES	YES	NO
Same	Sets 2-18	YES	NO	YES	YES	YES	YES
Swap	Sets 3-12	NO	NO	YES	YES	NO	NO
Swap	Sets 4-17	NO	NO	NO	YES	YES	YES
Swap	Sets 5-19	YES	YES	YES	YES	YES	NO
Same	Sets 6-21	YES	YES	YES	YES	YES	NO
Swap	Sets 7-13	NO	YES	YES	NO	YES	YES
Same	Sets 8-14	YES	YES	YES	NO	NO	NO
Same	Sets 9-20	YES	YES	YES	NO	YES	NO
Swap	Sets 10-22	YES	YES	YES	YES	YES	YES
Same	Sets 11-15	YES	YES	YES	NO	YES	NO

Chapter 7

Conclusions and Future work

This work served to introduce the Expected Utility Theory, which in spite of its popularity amongst economists, doesn't lack presence of violations of some of the axioms that characterize it. Specifically, it reports the violation of the independence axiom, that further originates an additional notion, additive consistency, which has been the ultimate purpose of this paper, that is to test whether such additional axiom can be supported in the context of traffic networks. Moreover, the hidden framework present all throughout the work is the behavioral environment, which in fact is detected by assessing the risk-attitude and the consistency of the subjects within their choices.

There are few relevant insights that have emerged from the Pilot experiment, and subsequently interpreted. For instance, subjects behave differently, and the difference in their behaviors, thus following actions, lies in the chemistry of people. As addressed already in this paper, behavioral game theory aims exactly at understanding and capturing the deviations of players from their objective self-optimizing choices. Such chemistry may involve characteristics of the agents such as age, gender, and more importantly risk-attitude.

Nevertheless, experimental results are clay for such context, to simply track eventual similarities in behavior and their causes, combining therefore many diverse fields. Throughout my work, it has been shown how people react otherwise and how they decide, based on different evaluations or even, their own personal traits which are innate. Risk-attitude, as a matter of fact, is not something that develops or changes drastically during the life of a person, instead, it is intrinsic in our DNA, in our chemistry, that each and every human being is born with. Nonetheless, it can adjust as the individual experiences new patterns, or becomes more conscious about things, but it never truly changes in a drastic way. After having made data analysis and a posterior evaluation, there are cases in which the visual representation of the several variables are very much alike, especially in the second treatment, thus it made it very hard, for even the more conscious subjects, to decide in a consistent manner, and respect their previous decisions. This is the case of the 5th subject, very knowledgable, with a solid background and working experience in the financial side of a multinational. Since the beginning, he had a clear vision on what it was happening, though not too much to be biased toward the experiment itself. He understood the framework and the purpose of the process, and he knew that his mistakes were made due to a mere visual misevaluation. It can be seen from the summary tables of the results, that few sets are more controversial than others, for they present visual images harder to differentiate amongst. Such factor results in a spectrum of answers much more various compared to other more obvious representations.

Another potential, interesting feature, from an economics perspective, could regard the rationality itself, or more precisely, alternative motives that could deviate the agents from the obvious rational choice, and thus decide on the basis of their instincts.

To this purpose, it might be worthy to check whether there is a structural change in behavior before and after the revelation of the logic and purpose of the experiment. Additionally, to consider the absence of consciousness on their part regarding the benchmark used while making the choice; rationality in fact, may be present, but mostly, they don't know the intuition behind it. Better yet, they may think they do know, but then in the assessment of their actual behavior, they would prove to be inconsistent. This demonstrates that the controversy lies within the rationale and not the risk assessment; people do not know the motives behind their decisions, most of the times they are made out of pure instinct, or a logic that is so indirect, that is not even thought of.

Moving on, the first subject was decisive in making me realize other aspects; for instance, the timeframe for the amount of information given, to prove the relevance of the context. To this end, I would first show the individuals Figure 6.1, to depict the idea, and afterwards I would make them choose for the first 11 sets of graphs. Subsequently, I would proceed with showing Figure 6.2, introducing a more complex scenario, only then, I would finally sow the other sets of histograms. By doing so, agents know enough to make a valid decision and for me to test the concept of additive consistency in the setting of network games, without having biases. In the end, the combinations of different scenarios and ways to present the variables and assess the experiment are various, and all very important to draw conclusions. For lack of time and space, I only chose one setup as means to design the experiment, to gather meaningful comments and conclusions.

The study could therefore assess the trueness and adequateness of the *Additive consistency* concept in such scenario considered. It has been shown that a strong component of agents have understood the rationale and have acted in a coherent manner, though a certain level of complicatedness in the setup remains, that is a person should still have at least a percentage of knowledge on the tools utilized in the Pilot, or some sort of educational background. Nonetheless, a dependence on the intellectual abilities of the subject may be considered, although still remains an observation, additional researches and studies should be done on the matter. This specific interpretation has been extrapolated especially from the last subject, a woman with no education who, in fact, indirectly didn't fully understand the objective, or simply couldn't cope with it.

In conclusion, the approach seems to be rewarding and it seems that the structure of the setup is complex enough to not be directly detected and thus having biased agents. Further researches might focus, when more data will be available, on running the model in an even more adequate framework, with all possible setups, and formulating the basis for a real laboratory experiment, also considering a larger sample of subjects, removing the minor errors and adjustments hereby discussed.

A very useful approach, to take inspiration from, can be depicted from the Large-scale Field experiment that was conducted in Singapore in 2017 [2]. Their goal was to perform their first-to-our-knowledge game theoretic investigation and modeling of a real world traffic network of Singapore, based on a large scale field experiment with thousands of participants, specifically they focused only on students. They used a dataset that enabled them to inspect at any moment the concurrent decision making of the many commuters, as they adapted to the ever-changing traffic conditions. More specifically, they used the rich dataset from Singapore's National Science Experiment (NSE), an initiative led by researchers at the Singapore University of Technology and Design. They were able to track the decisions of the agents at any moment in time thanks to some custom-made sensors that they carried for up to 4 consecutive days, resulting in millions of measurements.

They stressed out an interesting interconnection between the Price of Anarchy effects and Tragedy of the Commons behaviors, to understand which players were lured in by the effects of a low decreasing Price of Anarchy [1]. In order to study the cost incurred when coordination is lost, few assumptions must be made to describe the selfish behavior of agents. Traditionally, the assumption has been that they will play Nash equilibrium strategies, and that the price of anarchy of a game is defined as *"the ratio of the value of the objective function in the worst Nash equilibrium to the social optimum value."* It does not seem realistic, however, to assume that all agents in a system will necessarily play strategies that form a Nash equilibrium. Even with centralized control, Nash equilibria can be computationally hard to find. Selfish players also tend to minimize their regrets, another plausible explanation to the behavior of the agents in the Pilot. Regret minimization seems a realistic assumption in this case because there exist a number of efficient algorithms for playing games that guarantee regret that tends to zero, because only localized information is required, and because in a game where the actions of any single player do not greatly affect the decisions of other players (as is often studied in the network settings), they can only improve their situation by switching from a strategy with high regret to a strategy with low regret.

Variations of course will have to be made, most importantly, the model investigated in this thesis is not a game theoretic model, but rather a setting in which

[1]The following assumptions and descriptions are extracted from Regret Minimization and the Price of Total Anarchy, by Avrim Blum, MohammadTaghi Hajiaghayi and Katrina Ligett.

individuals choose independently of other agents.

My intention is to continue the studies and proceed with a field experiment and analyze researches and methodologies to lay down a flawless design and set up an experiment which will result in the ultimate proof of the worthiness of additive consistency in such setting discussed. I will work directly under the supervision and perhaps, in collaboration with the Supervisor of my thesis, to pursue such intentions as final completion of the work.

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Chapter 8

Appendix

The following appendix defines the basic codes from MATLAB used to program the Pilot experiment of this thesis. It does not want to be a full discourse, though a mere scope for notation. This appendix is exclusively derived from my analysis, formulation and final implementation of the data.

```
>
>> n = 5;
m = 5;
k = 4;
```

Distribution of X

```
d=randi([-1,1],1,1);
d=0;
pX = [zeros(3+d,1); 0.2+rand(n,1)];
pX = pX/sum(pX);
```

Distribution of Y

```
pY = [zeros(3,1); 0.2+rand(m,1)];
pY = pY/sum(pY);
```

```
Distribution of Z
r=randi([2,6],1,1);
pZ = [zeros(r,1); 0.2+rand(k,1)];
pZ = pZ/sum(pZ);

Distribution of X+Z
pXZ = conv(pX,pZ);

Distribution of Y+Z
pYZ = conv(pY,pZ);

fig1=figure(1);set(fig1, 'Position', [46 282 560 420])

maxpqr=max([length(pX),length(pY),length(pZ)]);
W1 = 0:5:(5*(maxpqr-1));

if rand(1,1)<0.5
    iX=1;iY=2;
else
    iX=2;iY=1;
end
subplot(3,1,iX); bar(W1,[pX;zeros(maxpqr-length(pX),1)]);
axis([0 5*maxpqr 0 0.4])

subplot(3,1,iY); bar(W1,[pY;zeros(maxpqr-length(pY),1)]);
axis([0 5*maxpqr 0 0.4])

pause

fig2=figure(2);set(fig2, 'Position', [689 285 560 420])
maxconv=max(length(pYZ),length(pXZ));
W2 = 0:5:(5*(maxconv-1));
```

```
if rand(1,1)<0.5
    iXZ=1;iYZ=2;
else
    iXZ=2;iYZ=1;
end
subplot(2,1,iXZ); bar(W2,[pXZ;zeros(maxconv-length(pXZ),1)]);
axis([0 5*maxconv 0 0.4])
subplot(2,1,iYZ); bar(W2,[pYZ;zeros(maxconv-length(pYZ),1)]);
axis([0 5*maxconv 0 0.4])

pause

if iX==iXZ display ("Same order");
else display ("Swap order")};
end

close all
```

Find below the insights and reasoning processes of the subjects throughout the experiment.

- The first subject is a female, 21 years old, who studies economics; she proved to be consistent for most of the choices, except for three cases (sets 3-12, 4-17, 7-13). Nonetheless, those cases are the ones in which the values and the distributions are very similar, thus the choice between the two graphs was not so clear and obvious. When this is the scenario, people tend to vary their decisions, perhaps by looking at different factors to base their decision, or better yet, they reach a point in which they become indifferent. This hypothesis has been confirmed in the Pilot.

  Firstly, the subject had to proceed to the assessment of risk attitudes, where the final evaluation was as a risk-averse individual. Although the subject switched options within the 6th choice, that means that the individual was really close to being risk-neutral. This behavior was mostly present in the choice patterns she showed. The paradox detected was in the explanation of the rationale behind their choices. According to the subject explanation, when making a decision, she was concerned with the maximum value of the time values, and with the respective probability values. It is obvious that such answer is consistent with her attitude towards risk, given the pessimistic nature of risk-aversion.

- A second subject is also a female, 22 years old, who studies Political science as a master. Her results are very similar to the ones of the first female subject analyzed; she demonstrated to be risk-averse, by also switching from Option A to Option B in the 6th choice. She proved to be consistent in all choices except for three as well (sets 2-18, 3-12, 4-17). Interestingly that she used slightly different benchmarks to make her decisions, respectively she took particularly into account the extreme values in the first eleven sets of graphs, more intensely the minimum values and only afterwards the maximum values. For the other sets, namely the last 11 choices, she still looked at the extreme values, both of them, but since the distributions of the histograms in these cases are more similar to one another, she looked at the expected value as well. After explaining to her the intuition behind the experiment and I showed her the cases where she was not consistent with the additive consistency axiom, I asked her whether she would like to change her mind about the graph she picked out. Only in one case she would, which is the set 2-18. The reason why is the expected value, which she believes to be more concentrated towards to lowest values of time.

- The third subject is also a female, 21 years old, who studies economics. She is more risk-averse than the first two subjects tested, she in fact switched options at the 7th choice. She was more detailed in her explanation of the her rationale behind the decisions and actually even more true to her risk attitude. As a matter of fact, she would look at the extreme values, as the previous subjects, but more particularly at the maximum values, that is the worst outcome that could be. This really means that she is acting as a risk-averse person, being pessimistic and exaggerating the probability of the worst possible outcome. Furthermore, she was not consistent to the theory in two cases (sets 1-16, 4-17) but only decided to change the second case, after having learned the intuition of the experiment. The reason behind it is that the lowest values were actually more probable to occur in the first graph, and moreover the worst outcome, meaning the highest time value, had a higher probability in the second graph. Thus she would switch and choose C instead of her previous answer D.

- The fourth subject, male, 22 years old, who studies Law, is the only person so far assessed to be risk-neutral. To this end, in fact, his results are quite interesting and clarifying. The reason for this is that he switched options during the risk-assessment on the 4th choice, so almost being risk-loving. While providing me his answers to the second part of the experiment, meaning the choice between the various sets of graphs, he didn't prove to be as consistent with the axiom as much as the previous subjects. Nonetheless, my intuition sets its ground in his risk-attitude; being risk-neutral means that you are indifferent when facing a risky situation, namely in a stochastic environment, to this extent in fact, sometimes he would prefer on outcomes, other times another. Moreover, I strongly believe that his background of knowledge and current studies set his mind to think all angles of a situation, to argue all different aspects of a case, whichever it may be. As a matter of fact, when asked the reason behind his choices, he told me that he would look at the probabilities, more than at the time values, and compare each probability, of each graph, for each specific time vale, with the other graph. Finally, his decision would be the result of an overall evaluation of all probabilities, and he would choose the graph with the highest overall probabilities. Furthermore, he was not consistent in 4 cases (sets 7-13, 8-14, 9-20, 11-15).

- The fifth subject is a male, 60 years old, with a strong background in finance, and with a current position of a CEO in an Italian company. To nobody surprise, he performed best amongst all subjects, there were only two cases in which he wasn't consistent, i.e (sets 3-12, 8-14). For all the sets that he had to make a choice for, he clearly explained to me that the motive behind his choices was the overall average of the time values and the respective values. He always tried to find, through a rough mathematical calculation in his head, the real expected values, visually comparing then the worst and best outcome distributions to assess where they stood. All of this process seems consistent with a risk-neutral behavior, in fact, he switched on the 5th choice. Once the subject has understood the rationale and the purpose of the pilot experiment, and once I have showed him the two choices that are not consistent with the axiom, his reaction was a response that I find very interesting and stimulating for this thesis purpose: the agent would change his choice at this point, but only because he understands and believes that the mere reason for his mistakes was based on an erroneous visual assessment of the graphs, not because he was not consistent in his mind about the scenario. Moreover, to note that such two cases were in fact sets very much similar to one another.

- The sixth and last subject is a female, around 55 years old, she is a housewife, with no educational background. The risk-assessment resulted in her being risk-neutral, she in fact switched options within the 4th one. This specific individual proved to be particularly inconsistent, throughout the whole pilot assessment. Hence, I believe it is best to highlight the consistent choices (sets 2-18, 4-17, 7-13, 10-22); Her benchmark in making their decisions was again the expected values in the various sets. Though, her not being much consistent in the decisions may seem to be irrelevant for the experiment, whereas in fact, provides a very important insight for the future development of the real experiment.

Find below the histograms that represent the variables used for the Setup; such graphs were presented to the subjects in the precise order in which they are reported in the present Appendix.

Figure 8.1: Variables representation (set 1)

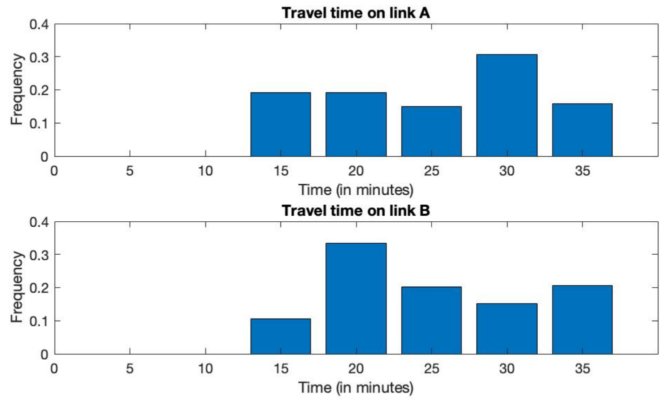

Figure 8.2: Variables representation (set 2)

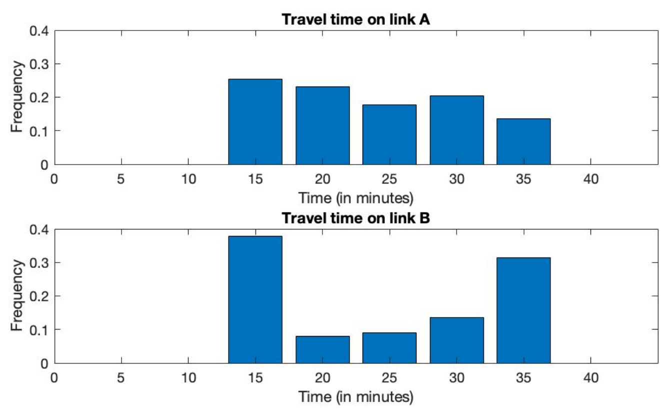

Figure 8.3: Variables representation (set 3)

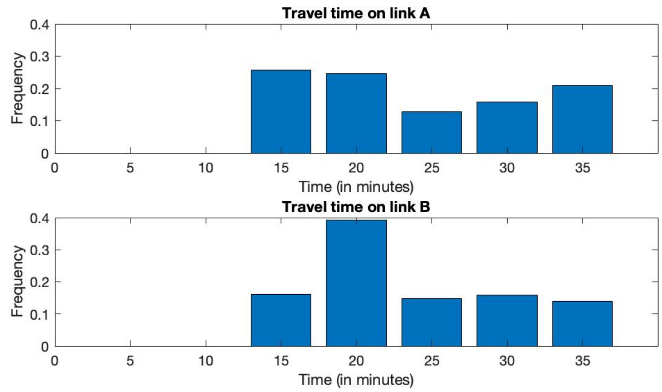

Figure 8.4: Variables representation (set 4)

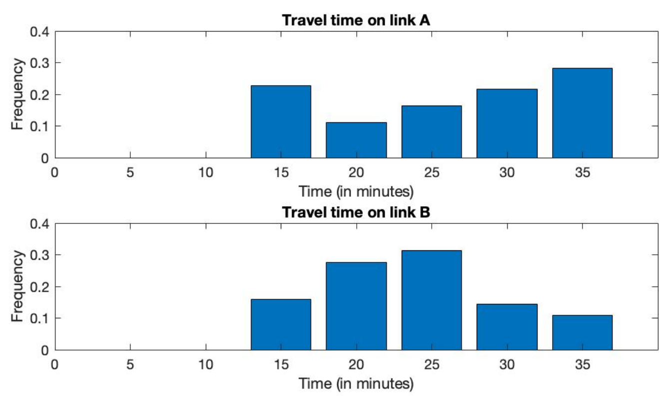

Figure 8.5: Variables representation (set 5)

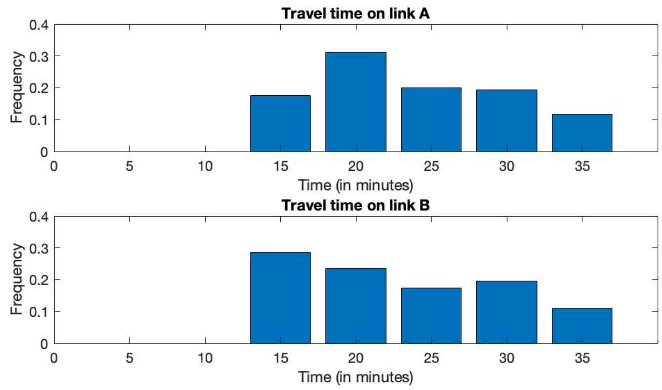

Figure 8.6: Variables representation (set 6)

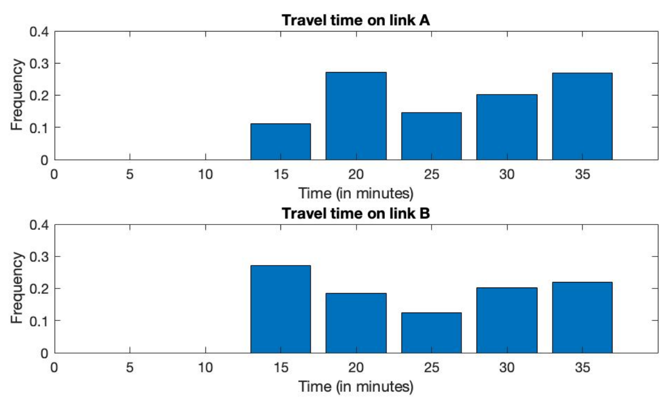

Figure 8.7: Variables representation (set 7)

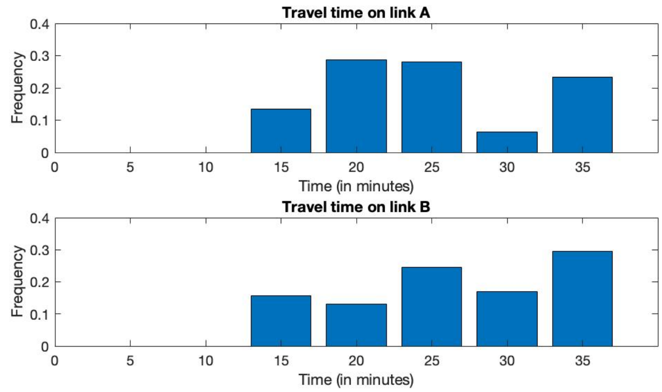

Figure 8.8: Variables representation (set 8)

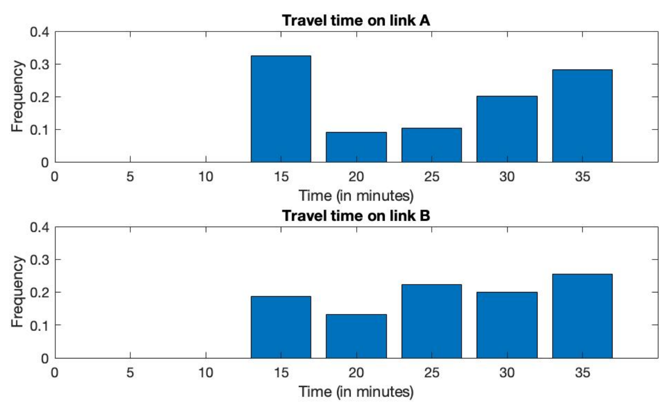

Figure 8.9: Variables representation (set 9)

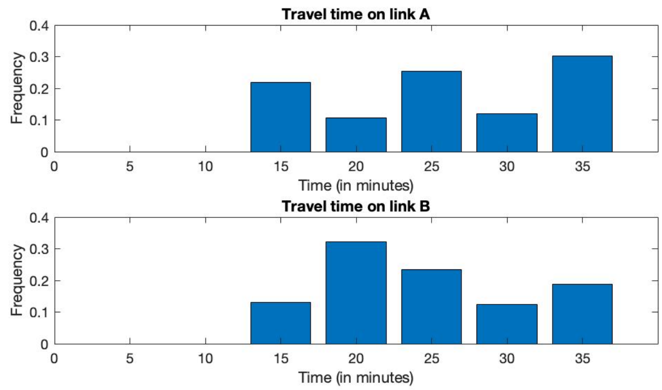

Figure 8.10: Variables representation (set 10)

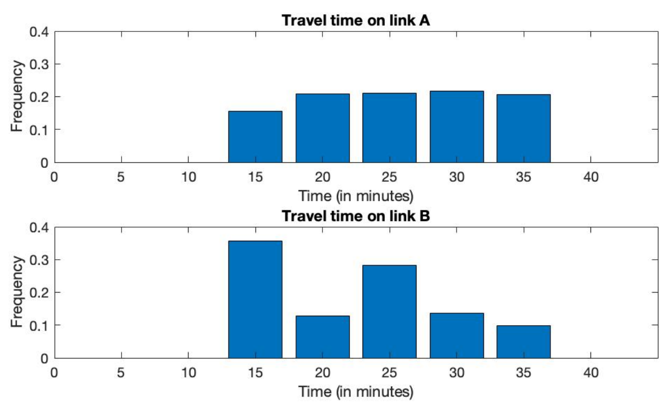

Figure 8.11: Variables representation (set 11)

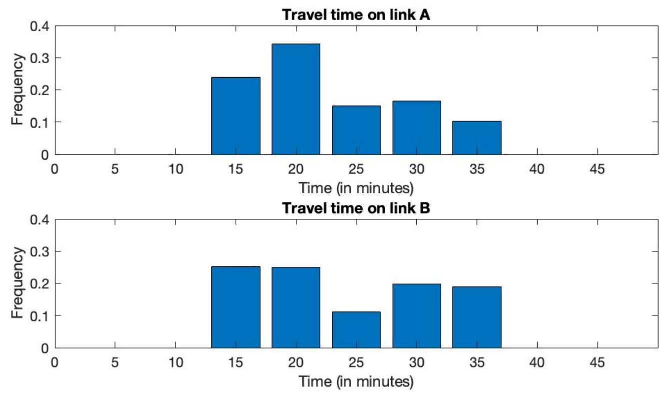

Figure 8.12: Combinations of variables (set 3)

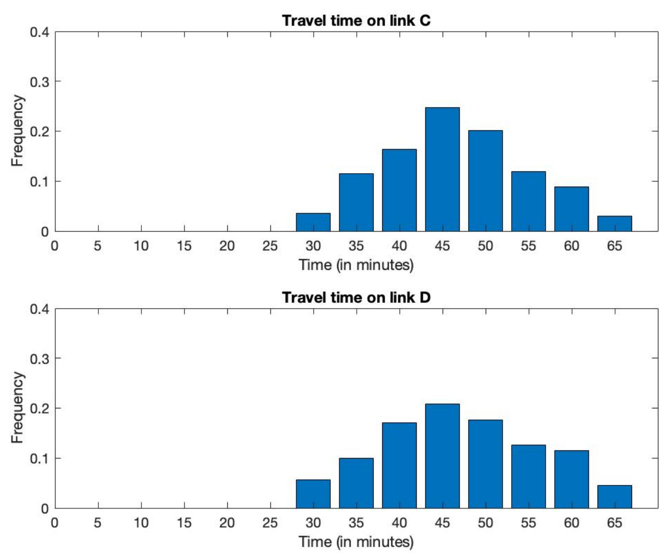

Figure 8.13: Combinations of variables (set 7)

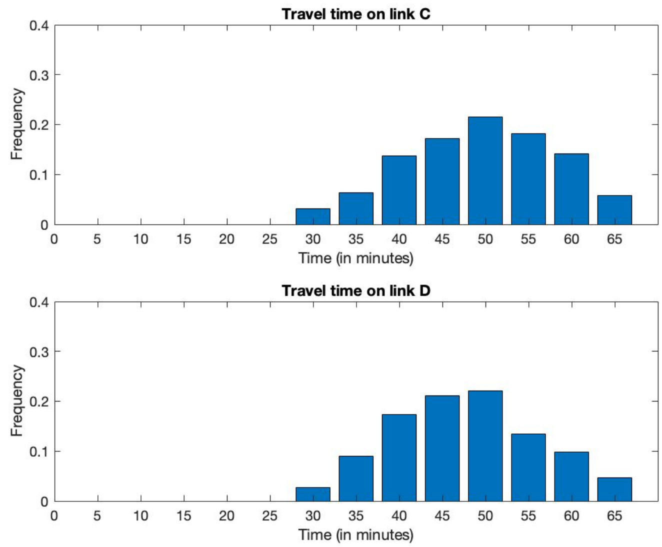

Figure 8.14: Combinations of variables (set 8)

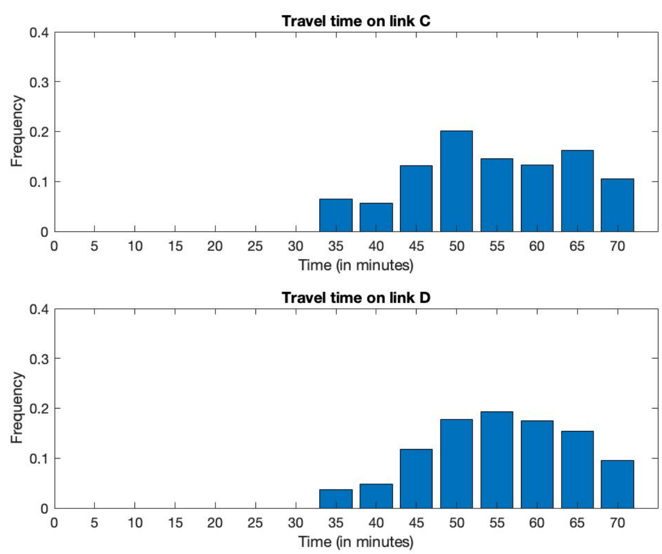

Figure 8.15: Combinations of variables (set 11)

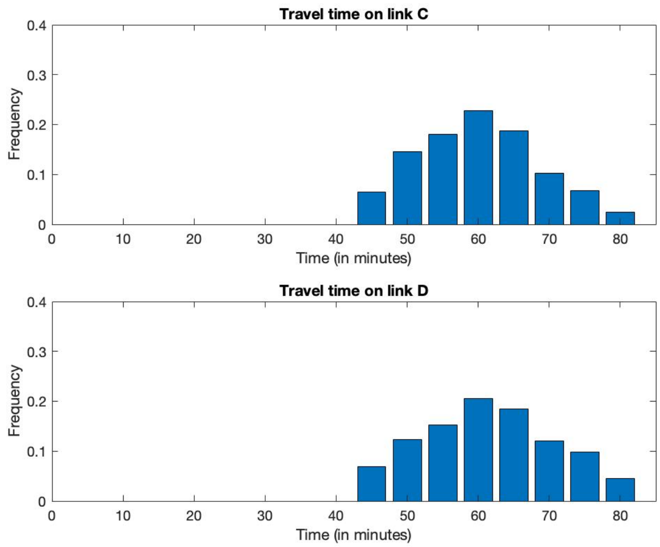

Figure 8.16: Combinations of variables (set 1)

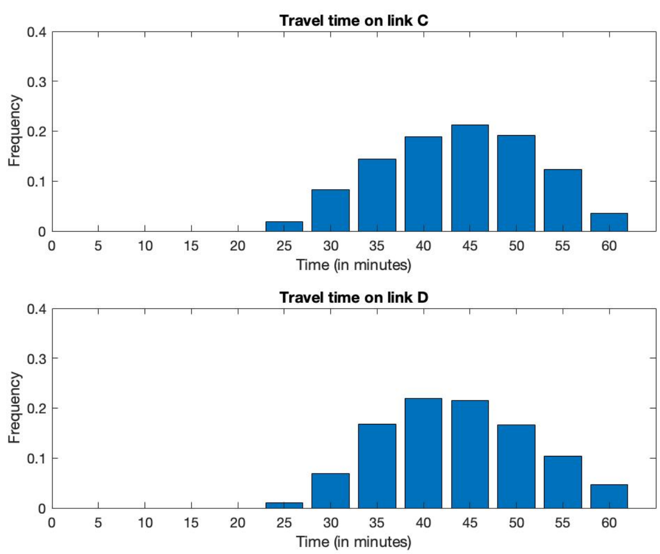

Figure 8.17: Combinations of variables (set 4)

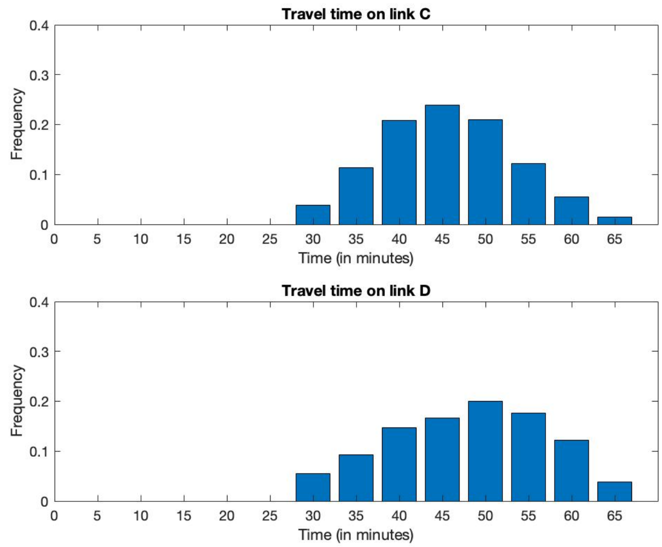

Figure 8.18: Combinations of variables (set 2)

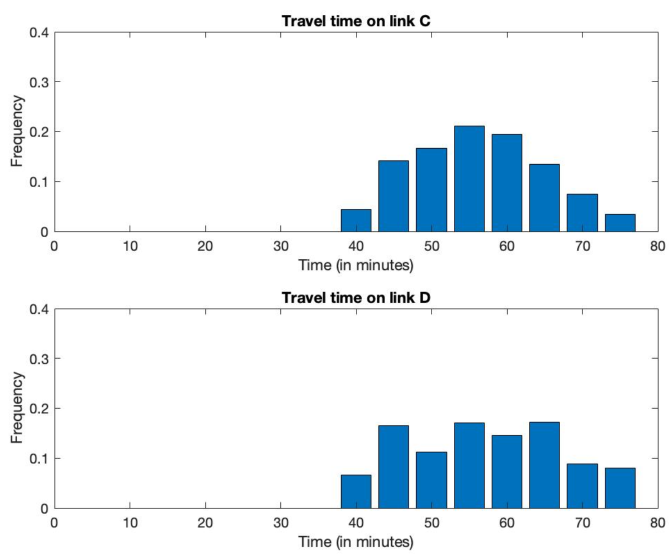

Figure 8.19: Combinations of variables (set 5)

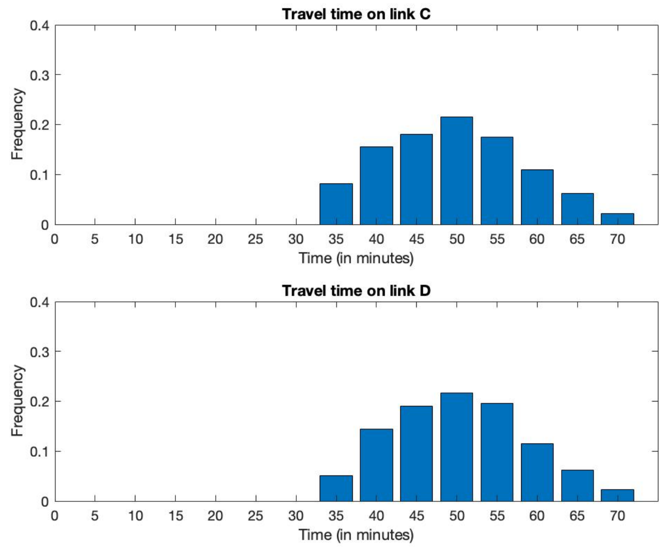

Figure 8.20: Combinations of variables (set 9)

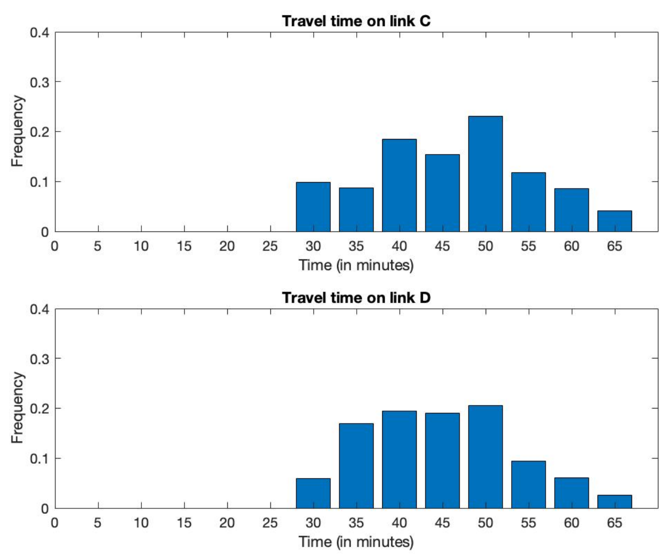

Figure 8.21: Combinations of variables (set 6)

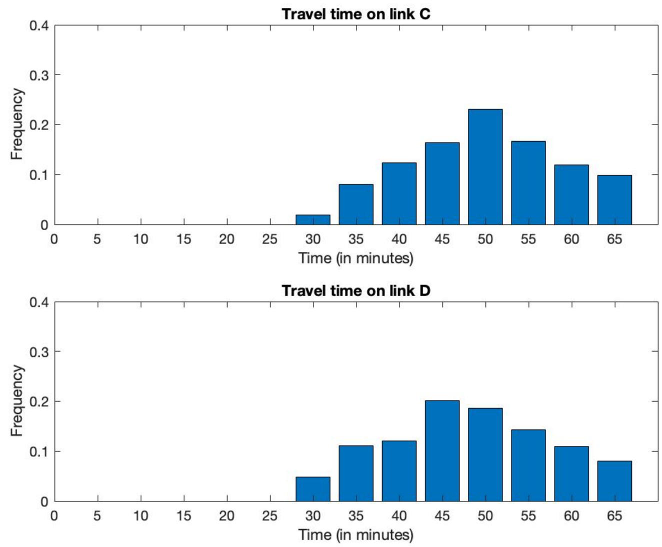

Figure 8.22: Combinations of variables (set 10)