LIBERA UNIVERSITÀ INTERNAZIONALE DEGLI STUDI SOCIALI

FAMA-FRENCH RISK FACTORS AND ECONOMIC CYCLE: AN EMPIRICAL STUDY ACROSS MULTIPLE REGIMES

Supervisor: **Prof. NICOLA BORRI**

Co-Supervisor: **Prof. PIERPAOLO BENIGNO**

Author: **GIUSEPPE PISCONTI** - 685781

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Abstract

This empirical work analyses the cyclical behaviour of the 5 Fama-French risk factors plus the Momentum risk factor. The presence of multiple economic regimes is investigated through the implementation of a Markov-Switching model.

Four statistically significant regimes are identified, considering growth and inflation as dependent variables. Some factors, like Conservative-minus-Aggressive (CMA) and Robust-minus-Weak (RMW), seem to perform well even in non-favourable economic conditions and can be used to improve the diversification of a factor-based portfolio.

Other factors, like Market (MKT) and Momentum (MOM) are the best performers across states of rising growth, regardless of a rising or falling inflation. However, their Sharpe ratio is higher when the inflation is decreasing.

Finally, a Black-Litterman portfolio optimization is applied in order to find an optimal factor allocation in each estimated regime. The construction of a dynamic portfolio, that takes in account the regime switching, seems to lead to a better risk-adjusted performance compared with a static multi-factor portfolio. Moreover, whatever multi-factor allocation allows to contain the volatility of the portfolio but only a dynamic allocation outperforms a single-factor allocation in a stable way.

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1. Introduction

The idea that a good portfolio diversification can be obtained by simply distributing wealth among the traditional asset classes is, by now, a myth to dispel. Many evidences, in particular after the Financial Crisis of 2008, show that the economy is characterized by regime changes that can determine a sudden increase in correlation.

A possible solution could be represented by the *Factor Investing* approach: namely, the investment in the risk factors known in literature, such as those of Fama and French. In fact, factors offer considerable risk premiums in the long-run and exhibit a low or negative correlation, even in phases characterized by turbulent markets.

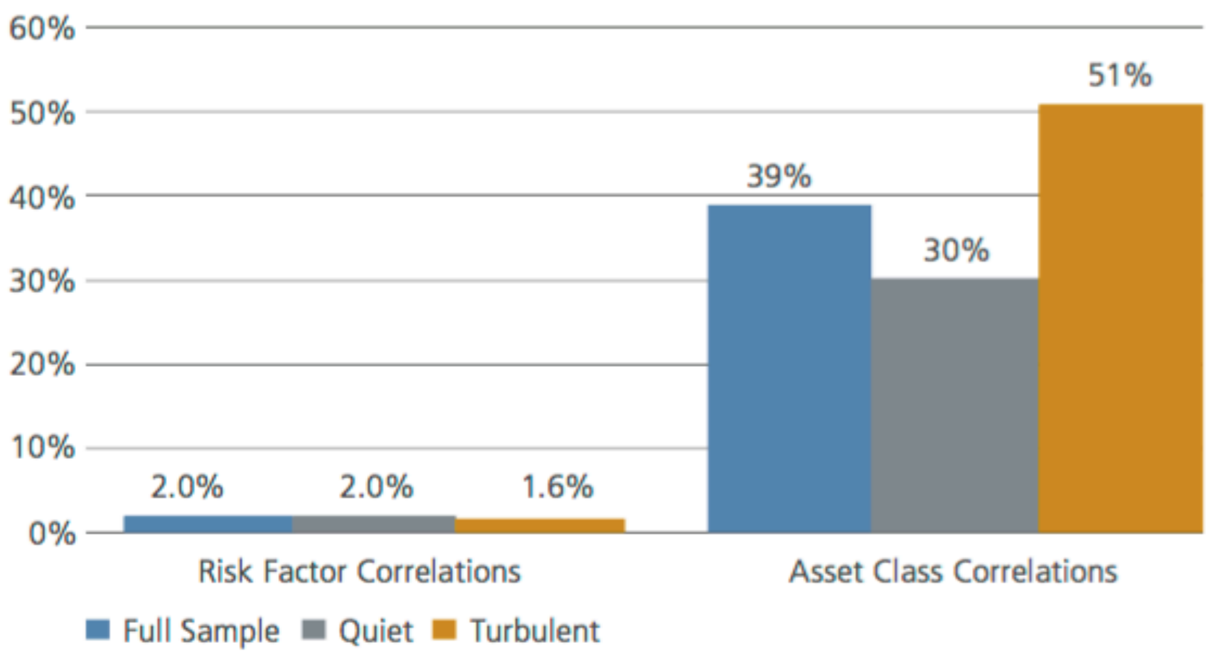

Figure 1 - Average cross-correlation from March 1994 to December 2009
Source: "*The Myth of Diversification: Risk Factors vs. Asset Classes*", Pimco (2010).

However, it seems that the returns on all the factors, taken individually, suffer a certain cyclicality and can experience negative performances even for long periods.

An investment in multiple factors would therefore seem advisable.

In order to understand the criteria of an optimal allocation among factors it is necessary, in my opinion, to study in depth the business cycle and the performance of the most relevant factors in various states of the economy.

Some authors suggest analysing the cycle along two main lines: growth and inflation. This analysis leads to the identification of multiple regimes, going beyond the simple distinction between expansion and recession phases. However, in almost all cases this approach has been limited to counting phases, carried out heuristically and knowing in advance the characteristics that each phase should have.

I will therefore try, in this work, to use a more robust approach from a statistical point of view, adopting a Markov-Switching model to test the presence of multiple regimes, determining their optimal number.

In the first chapter, I briefly review the main evidences against the Efficient Market Hypothesis and the Capital Asset Pricing Model, showing that the idea that we deal with a multi-factor world is, by now, widely acknowledged. Then, I explain why the asset pricing equation should be taken as reference to understand what factors are, which is their nature and why they are cyclical.

Moreover, the knowledge of pricing theory may also help to distinguish between factors and simple anomalies and avoid redundancy. In fact, in recent years, hundreds of factors have been discovered. Therefore, it seems necessary to find more restrictive criteria to assess their statistical significance. The objective is to better guide investment choices and narrow the factors down to a limited number but sufficient to guarantee a good level of diversification, avoiding the danger of investing in apparently different instruments that are, actually, exposed to the same sources of risk.

In the second chapter, I will implement the Markov-Switching model. The dependent variables are the Composite Leading Indicator (CLI) to compute growth and the Consumer Price Index (CPI) to compute inflation. The presence of multiple regimes is investigated from a statistical point of view. Moreover, I will try to determine the optimal number of regimes and find some correspondence between the estimated regimes and some regime classification known in literature. Then, the performance of six risk factors is analysed in every estimated regime.

The selected factors are: Market, Size, Value, Profitability, Investment and Momentum.

In the third chapter, the information collected through the implementation of the MS model and the performance analysis of the factors will be used to formulate investment views to be included in a Black-Litterman model. The goal is to find an optimal allocation in each estimated regime, building a dynamic portfolio with weights that change in anticipation of a regime switch.

Such dynamic portfolio will be compared to a static Black-Litterman portfolio. Finally, I will assess whether multi-factor strategies can outperform single-factor portfolios.

2. Theoretical framework of Factor Investing

2.1 Beyond Efficient Market Hypothesis and Capital Asset Pricing Model

One of the first lessons we learn in finance is that what drives return is risk. Therefore, an investor should investigate which kind of risk will be able to achieve the desired level of return and, consequently, take the most suitable exposure. The Factor Investing approach highlights that the factors behind the assets matter, not the assets themselves.

In fact, a portfolio may contain many assets and each of them can be a bundle of factors. More exactly, there are some assets which can be considered factors themselves and assets that contain many different factors.

Namely, a single asset can make the investor exposed to different risks.

Moreover, there are factors suitable for some investors but not for others, depending on risk preferences. There are factors that behave better over certain phases of the economic cycle and other ones which perform poorly over some moments. In other words, factors are the exposure to different kinds of risk and their origins are in some pillars of the financial economy, such as the Efficient Market Hypothesis (EMH), the Capital Asset Pricing Model (CAPM) and the Arbitrage Pricing Theory (APT).

In a scenario in which all asset classes perform poorly, in which investors look for higher yields, it is crucial to understand what drives excess returns.

The old view of the financial world is based on the Efficient Market Hypothesis and the Capital Asset Pricing Model. The message is that markets are informationally efficient: there is no arbitrage opportunity, no predictable pattern and the only way to achieve higher returns is to take higher risk. Which risk? According to the CAPM, market risk. Namely, the risk due to covariation with market movements. The implication, in terms of asset allocation, is that you need to buy stocks with higher market beta. However, this has not been confirmed by the subsequent empirical studies that have shown, in many cases, a flat or even negative relationship between excess returns and the beta of the portfolio.

According to the EMH, investors are “rational profit-maximisers” and the consequence of their rational behaviour is the random fluctuation of prices.

Prices adjust enough rapidly to the arrival of new information such that no one can achieve above-average returns without taking additional risks.

However, the assumption that all investors are rational and react uniformly and simultaneously to news is not realistic[1].

According to Fama, prices converge to their fundamental value.

In other words, the rational behaviour implies the good evaluation of prices. Therefore, a clear and predictable pattern in prices should not exist and the thesis of Efficient Market Hypothesis should be consistent with the Random Walk Model[2].

The logic of the random walk model is based on the fact that the flow of information is reflected in stock prices and because news is unpredictable, also price changes are unpredictable and random.

At the beginning of the 21st century, many financial economists began to believe in the predictability of stock prices and, consequently, in the chance to earn excess returns. Obviously, when we have "efficiency", markets do not allow investors to earn above-average returns without accepting above-average risks. It seems also obvious that market pricing is not always perfect and that irrational and behavioural factors affect prices.

For example, during a positive trend of a stock, some investors continue to buy it even if the price has already reached a level that is inconsistent with the intrinsic value of the company.

In other cases, the so-called "event studies" such mergers, new exchange listings, initial public offerings, earnings surprises, there are positive excess returns for short periods. However, anomalies disappear when the predictable pattern is published in the financial literature.

Anyway, in the short-run, when price variations are measured over periods of days or weeks, some positive serial correlation exists.

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[1] Pedersen (2017) argues that markets are not completely efficient nor completely inefficient but are "efficiently inefficient." Namely, "just *inefficient* enough that money managers can be compensated for their costs and risks through superior performance and just *efficient* enough that the rewards to money management after all costs do not encourage entry of new managers or additional capital".

[2] "While, Fama argues that today's price is the best estimation of the fair value, Samuelson's conclusion is less ambitious, arguing only that today's price is the best estimation of the tomorrow's price." Thomas Delcey (2018).

Predictability seems to be the consequence of some overreaction, such as optimistic or pessimistic conducts.

In fact, this is the base of the so-called contrarian strategy, that is a technique which invests on stocks out of the favour for long periods of time and disinvest those stocks that have had large ups over the last years.

There are many statistical evidences of return reversals, but those do not imply the inefficiency in the market.

Figure 2.1 - Reversion to the Mean: cumulative performance of "Value" vs. "Growth" Mutual Funds, 1937 - June 2002
Source: *"The Efficient Market Hypothesis and its Critics"*, Malkiel (2003).

There are also other non-random effects or anomalies such as the January effect, the Monday effect and a number of day-of-the-week effects. It seems that these predictable patterns might offer some arbitrage opportunities.

In fact, January seems to be a very good month in terms of return premium, in particular over the first two weeks and for stocks with a small capitalization.

There are also some patterns in returns around the turn of week and month or around holidays. In any case, all these anomalies have to take transaction costs into consideration.

In synthesis, many anomalies have been discovered and empirical studies have shown, at least, a partial predictability of returns[3].

In return forecasting regressions, explanatory variables such as Price-to-Earnings, Price-to-Dividend, Price-to-Book ratios can have a good predicting power, in particular at long horizons. However, this is not necessarily a signal of market inefficiency, but it can represent a failure of the CAPM to capture different sources of risk. Tests on market efficiency are tests of the EMH and the CAPM jointly. In case of rejection it is difficult to disentangle between inefficiency and misspecification of the asset pricing model.

Horizon k	b	$\sigma(b)$	R^2
1 year	-1.04	(0.33)	0.17
2 year	-2.04	(0.66)	0.26
3 year	-2.84	(0.88)	0.38
5 year	-6.22	(1.24)	0.59

Figure 2.2 - OLS regression of excess returns (value weighted NYSE-treasury bill rate) on VW price-dividend ratio
Source: *"New Facts in Finance"*, Cochrane (1999).

[3] As Malkiel pointed out in its paper "The Efficient Market Hypothesis and its critics" (2003), anomalies and irrational bubbles represent the exception, not the rule. Markets are far more efficient than they seem.

Moreover, in such regressions, the R^2 tends to increase with horizon but it is still less than 100%, meaning that the fitting improves but there are no arbitrage opportunities. Hence, investors have to take some risk to exploit these relationships.

The general evidence is that low prices relative to fundamentals seem to predict higher returns in the future.

Many predictable patterns are based on the characteristics of the firm and some valuation parameters. One of the most important of these is the so-called size effect. There are many evidences showing high excess returns achieved by investing in firms with a small capitalization[4].

However, it is also possible that many studies have been affected by the circumstances that the small firms which have gone bankrupt are not included in databases.

Another predictable pattern is related to value stocks. Value stocks appear to earn higher returns than the so-called growth stocks.

P/E ratios and P/BV ratios can help in the identification of them. The two ratios capture companies that have some degree of financial distress whose stocks are traded at low prices relative to the intrinsic value.

We can make other examples of anomalies and statistically predictable patterns. In any case, these patterns simply reflect proxies for measuring different sources of risk.

However, economic agents once believed that the Capital Asset Pricing Model provided a good description of why returns on some stocks were higher than others[5]. Hence, they recognized a unique risk factor.

Therefore, the CAPM assumes that the return on a financial asset only depends on the market excess return, proportionally to the portfolio's beta.

Before the introduction of the CAPM, the risk of an asset was in many cases associated with its own volatility.

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4 Fama and French examined data from 1963 to 1990 and the results showed that the deciles made up of portfolios of smaller stocks earned higher average returns than deciles made up of larger stocks. They demonstrated that also taking into consideration the beta of the stocks, according the capital asset pricing model, the results didn't change. So, they suggested that size might be a better proxy for risk than beta.

5 Cochrane, *New Facts in Financ*e (1999).

The attractiveness of the CAPM is often associated with the fact that is still used to estimate the cost of capital for firms.

However, its greatest intuition is that only systematic risk matters. Idiosyncratic risk does not affect prices; therefore, investors require a compensation only for the portion of risk that cannot be diversified away. In coherence, investors should not be paid for diversifiable risks.

However, empirical evidences show a flatter relationship between beta and average return than the one predicted by the Sharpe-Litner model.

The empirical evidences are not consistent with the model because of many simplifying assumptions.

At this point, it is important to explain the main reasons related to the empirical issues of the CAPM.

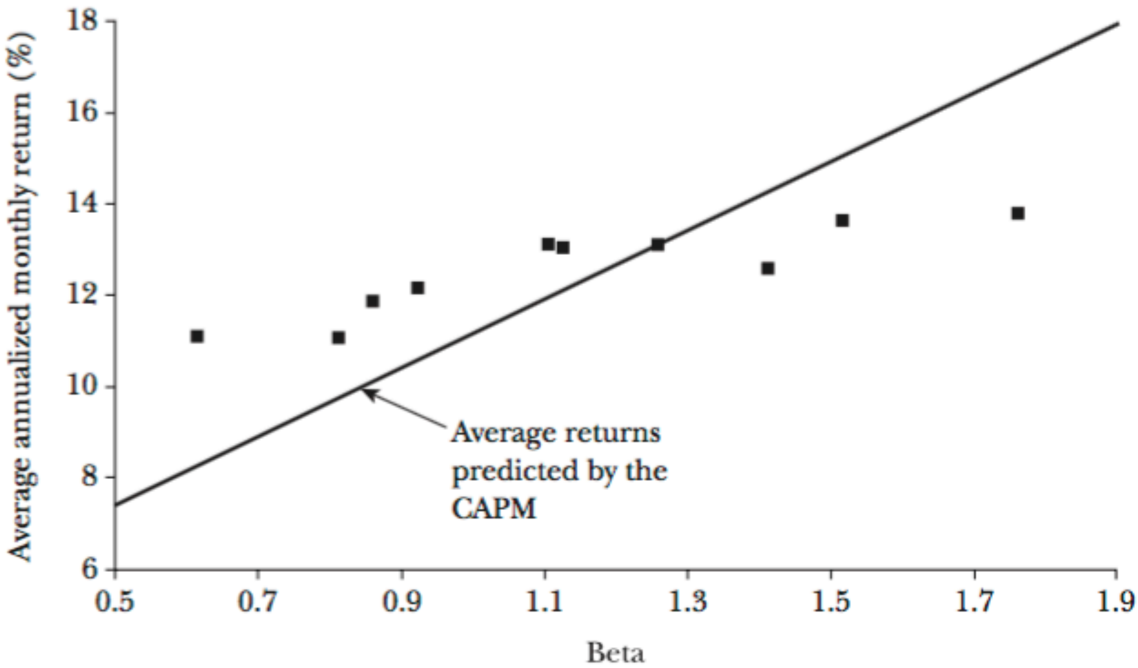

Figure 2.3 - Average Annualized Monthly Return versus Beta for Value Weight Portfolios Formed on Prior Beta, 1928-2003
Source: *"The Capital Asset Pricing Model: Theory and Evidence"*, Eugene F. Fama and Kenneth R. French (2004).

As known, some unrealistic assumptions of the CAPM generate many problems and invalidate most applications. For example, in the CAPM investors only take care about the mean and the variance of one-period portfolio return[6].

They do not take into consideration other important dimensions of risk.

An investor, in fact, given the same Beta, might prefer less correlation with labour income or with the business cycle. In fact, investors don't have only financial wealth. They don't have only one mean-variance utility. And again, they don't have homogeneous expectations. Moreover, the set of assets in the proxy chosen as market portfolio appears incomplete for the lack of foreign assets, bonds, real estate.

Moreover, the CAPM does not take into consideration the liquidity issues for the individual assets and, in general, for the opportunities that investors might have to consume or invest the payoff.

It is evident that both the EMH and the CAPM are not sufficient: investors are active in the search of inefficiencies of the market in order to gain excess returns and they don't consider only one factor.

Ross (1976) extended the CAPM by introducing the Arbitrage Pricing Theory (APT). He postulated the existence of different sources of systematic risk. However, in the APT, the number and the identity of the risk factors is not specified. The concept that investors operate in a multi-factor world became soon well acknowledged and many multi-factor models were proposed. The main reference is the Fama-French 3-factor model (1993), then extended to a 5-factor model (2015).

Moreover, a large literature documented that risk factors are cyclical. Some of them are able to predict macro-economic variables. For example, the HML factor is a good predictor of future GDP growth[7].

Furthermore, Petkova (2005) showed that it is also linked to innovations in term spread, while the SMB factor is mostly related to innovations in default spread.

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[6] "Merton's (1973) intertemporal capital asset pricing model (ICAPM) is a natural extension of the CAPM.(...)In the ICAPM, investors are concerned not only with their end-of-period payoff, but also with the opportunities they will have to consume or invest the payoff." Eugen F. Fama and Kenneth R. French (2004).

[7] Liew and Vassalou (1999).

2.2 Asset Pricing Theory and Multi-Factor Models

"Expected returns vary across time and across assets in ways that are linked to macroeconomic variables, or variables that also forecast macroeconomic events; a wide class of models suggests that a 'recession' or 'financial distress' factor lies behind many asset prices"

John Cochrane, Asset Pricing (2001)

In this paragraph, I want to discuss how the theory related to Asset Pricing can be used to understand the advantages of a multi-factor investing approach. In particular, I want to point out that the interpretation of such theory should lead to a necessary analysis of the cyclicality of risks and returns. The belief that some asset classes perform better than others over some moments because of their characteristics is a wrong concept.

Prices and returns are affected by macro-economic variables. The intuition of Cochrane is that a single equation can be used to price any asset and can be adapted to various circumstances.

Prices equal discounted expected payoffs[8]:

(2.1) $$P_t = E_t(M_{t+1}X_{t+1})$$

where P_t denotes the asset price, M_{t+1} the stochastic discount factor and X_{t+1} the asset payoff.

A first consideration I want to do is the following: the payoff is a random variable. It is a stochastic, not deterministic. Hence, we can only assess the probability of each of its possible outcomes. Intuitively, a random variable can assume different values depending on the realization of a particular state of nature: for example, expansion or recession.

The ability to predict the economic cycle can help to predict payoffs and returns.

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[8] The formula represents the first-order condition of an investor's maximization problem in which the utility function is concave and monotone. Such utility function depends on current and future consumption. The investor has to decide how much to consume and how much to invest in a financial asset. The asset is purchased or sold until the marginal loss equals the marginal gain.

Dividing both sides by P_t we get:

$$\textbf{(2.2)} \qquad 1 = E_t(M_{t+1}R_{t+1})$$

with $R_{t+1} = \frac{X_{t+1}}{P_t}$ denoting the gross return.

Hence, we can think at returns as payoffs of assets with a price equal to 1.

Suppose that an investor borrows 1 dollar at a rate equal to the risk-free R^f and invests it in an asset with return R.

The investor has no commitment of money today and will receive a payoff equal to $R - R^f$ at maturity.

This is a typical long-short strategy. Namely, a strategy that implies the short-selling of an asset and the investment of the proceeds in another asset, betting that, at maturity, the return on the long position will be higher than the interest rate to pay back to close the short position.

The asset pricing equation becomes:

$$\textbf{(2.3)} \qquad 0 = E_t(M_{t+1}R^e)$$

where $R^e = R - R^f$ is the excess return on such zero-cost portfolio.

Hence, we can say that excess returns are payoffs of assets with a price equal to 0.

Borrowing at the risk-free rate and investing in the market portfolio we obtain a payoff equal to the market excess return. This is the market risk factor. All the risk factors known in literature are constructed with long-short strategies and can be treated as excess returns.

For example, the High-minus-Low (HML) risk factor is obtained by buying value stocks and selling short growth stocks.

Therefore, I retain useful to present Asset Pricing Theory to better understand what drives excess return and, consequently, risk factors.

In the asset pricing formula, a crucial role is played by the stochastic discount factor (SDF). In fact, it can be interpreted as a measure of the investor's appetite for money in contingencies.

If investors are risk-neutral, the SDF can be written as the inverse of the gross risk-free rate:

(2.4) $$M_{t+1} = \frac{1}{R^f}$$

Then, recalling that the covariance between 2 random variables can be written as the difference between the expectation of the product and the product of the expectations, the asset pricing equation becomes:

(2.5) $$\mathrm{E_t}(\mathrm{M_{t+1}X_{t+1}}) = \mathrm{E_t}(\mathrm{M_{t+1}})\mathrm{E_t}(\mathrm{X_{t+1}}) + cov_t(\mathrm{M_{t+1}}, \mathrm{X_{t+1}})$$

$$= \frac{1}{R^f}\mathrm{E_t}(\mathrm{X_{t+1}}) + cov_t(\mathrm{M_{t+1}}, \mathrm{X_{t+1}})$$

The first term is the standard discounted present-value formula while the second term is a risk-adjustment.

The SDF can be explicated in terms of marginal utility of consumption as follows:

(2.6) $$M_{t+1} = \beta \frac{u'(C_{t+1})}{u'(C_t)}$$

Therefore, it is a marginal rate of substitution; namely, the rate at which investors are willing to exchange consumption at *t+1* for consumption at time *t.*

The term β captures the investors' impatience: they value current consumption more than future consumption.

Asset prices are lowered if their payoff covaries positively with consumption and, hence, negatively with the SDF:

(2.7) $$P_t = \frac{E_t(X_{t+1})}{R_t^f} + \frac{cov_t[\beta u'(C_{t+1}), X_{t+1}]}{u'(C_t)}$$

Taking the aggregate consumption level as an indicator of the business cycle, an asset that has a positive covariance with consumption is pro-cyclical. In fact, it is reasonable to think that consumption grows during favourable regimes of the economic cycle.

Such asset pays off high when investors' marginal utility is low. In simple words, when they do not need money and feel already wealthy.

Investors' utility always increases with consumption but their marginal utility decreases. In other words, the utility function is concave, and the degree of curvature measures the degree of risk aversion.

In particular, considering a power utility function in the form[9]:

(2.8) $$U(C) = {C^{1-\gamma}}/{(1-\gamma)}$$

The risk aversion is captured by the curvature parameter γ.

Investors desire a level of consumption that is steady over time and across different states of the economy.

Assuming that investors are not risk neutral and that consumption growth is normally distributed, the log of the risk-free rate is written as:

(2.9) $$r^f = \delta + \gamma E_t(\Delta c_{t+1}) - \frac{\gamma^2}{2}\sigma_t^2(\Delta c_{t+1})$$

where δ captures the compensation that investors require for delay of consumption and γ reflects the sensitivity to the expected consumption growth and to its variance.

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[9] This is the so-called Constant Relative Risk Aversion utility function (CRRA).

Therefore, according to such model, interest rates are high when consumption growth is expected to be high. That is intuitive: an investor will require a higher compensation to be willing to save if he believes that a high consumption state will occur.

The third term of the equation captures precautionary savings. When there is uncertainty about future consumption growth and the volatility of consumption is high, investors want to save more.

Hence, they increase the portion of wealth invested in financial assets in the attempt to reach the desired level of consumption in the future. This tendency pushes interest rates down.

The use of the power utility function mentioned above helps to understand the link between risk aversion and consumption. However, such utility function implies that the risk aversion remains constant over time. This is unrealistic: investors are less inclined to take risks when economic conditions get worse.

Smith and Whitelaw[10] (2009) parametrized the coefficient of relative risk aversion as a function of state variables that are related to the business cycle:

1. the dividend yield,
2. the credit spread in the corporate bond market and
3. the term spread in the Treasury market.

They showed that risk aversion exhibits counter-cyclical variation: it increases during contractions and decreases during expansions.

Moreover, another crucial result is the following: it is the variation in risk aversion, rather than the variation in risk itself, that explains most of the variation in equity risk premia.

This evidence should not be surprising if one knows asset pricing theory.

Risk premia are basically excess returns and, according to the theory, should depend on the covariance term between the payoff and the stochastic discount factor, not on the volatility of payoff itself.

[10] *"Does Risk Aversion Change Over Time?"*, Daniel R. Smith and Robert F. Whitelaw (2009).

According to the Consumption Capital Asset Pricing Model (C-CAPM) the expected excess return can be rewritten in terms of the market price of risk $\lambda_{\Delta c}$ and the amount of risk β:

(2.10) $$E_t\left(r_{t+1}^{e,i}\right) = cov_t\left[\Delta_{c_{t+1}}, r_{t+1}^{e,i}\right] / \sigma_{\Delta_{c_{t+1}}}^2 \ \gamma\sigma_{\Delta_{c_{t+1}}}^2$$

The more risk averse people are, or the riskier their environment, the larger is the premium they require to hold risky (high beta) assets.

Pro-cyclical assets make the consumption stream more volatile. An investor that wants to increase its exposure toward one asset included in his portfolio, has to consider the covariance term in determining the effect on the volatility of consumption.

Time-varying risk preferences are needed to explain and match the high variability that we observe in asset prices. Moreover, it is well known that changes of price, price drops in particular, often materialise very rapidly.

The succession of expansion and recession phases is physiological in the economy. However, not all the recessions are equal. Some of them, in particular those that are provoked by financial shocks, may alter investors' risk tolerance more persistently.

According to Guiso (2014), events like the Financial Crisis of 2008 potentially represent a traumatic experience for many investors and, furthermore, such events may lead to emotional contagion effects that can even affect the risk preferences of the following generations[11].

The recovery after the last financial crisis has been much slower than recoveries from other standard recessions.

Moreover, the unconventional monetary policies applied by the main Central Banks have maintained interest rates artificially low, creating a challenging scenario for investors and asset managers that have looked for higher yields trying to avoid, at the same time, excessive risks.

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[11] See also Dohmen et al al. (2011).

In general, it is clear that economic agents are willing to give up a portion of expected return to protect themselves against low consumption states. This is exactly the logic behind an insurance policy: you pay a premium to hedge some risk.

Investors do not like excessive fluctuations in the performance of their portfolio because they do not like fluctuations in their consumption stream.

A portfolio can be composed by different asset classes: stocks, corporate bonds, government bonds, currencies, real estate and so on.

However, when a financial shock materialises, the diversification of such portfolio loses its effectiveness if all the assets positively covary with consumption. In fact, the correlation may increase suddenly and dramatically. This should not be surprising if we have in mind asset pricing theory. One pricing equation holds for every asset, even if many assets have different characteristics. What really matters is the covariance with sources of macroeconomic, systematic risks.

During recessions a portfolio made only of stocks can perform better than a portfolio composed by stocks and bonds if it contains some defensive stocks that react well to the crisis.

The Factor Investing approach has experienced a good success in the last years since there are some risk factors or investment styles, for example Quality and Low Volatility, that have been able to guarantee a positive return even in non-favourable economic conditions.

The essence of factors is strictly linked to consumption and economic cycle. According to Cochrane (2001), and in conformity with the concept of time-varying risk aversion that I stressed before, there are some periods and states of the economy in which investors particularly care about the performance of their portfolio. In such states they are very sensitive to losses and willing to trade-off a sizable portion of return to have an insurance.

Factors are those variables that predict such states. They are able to forecast macro-economic events.

A single asset pricing formula can be used to understand the macro-economic risks underlying each security's value. This is coherent with the idea that many factors can lie behind a single asset.

The equation is unique for every asset pricing model: CAPM, APT, ICAPM and so on. What changes is the specification of the stochastic discount factor.

The SDF reflects the marginal utility growth (equation 1.8). Hence, each pricing model uses a different proxy.

In factor models, the consumption-based expression is replaced as follows:

(2.11) $$m_{t+1} = a + b' f_{t+1} \approx \beta \frac{u'(C_{t+1})}{u'(C_t)}$$

where a and b are free parameters.

The expression is equivalent to a multiple beta model:

(2.12) $$E(R_{t+1}) = \gamma + \beta' \lambda$$

where β is the vector of regression coefficients of returns on factors f.

As in the APT, the number and the identity of factors is not specified. Hence one should look for good proxies of the marginal utility growth.

2.3 Identification of Risk Factors

How many factors exist? Which are they?

These are the questions to which many authors and economic agents tried to answer.

The world of asset management has provided numerous financial instruments to investors that want to take a position in risk factors, especially in recent years. However, in a context in which hundreds of candidates[12] have been identified in literature, it is necessary to use some criteria to better guide investment choices and make investors aware of the risk and return profiles they face by choosing one factor rather than another or a mix of factors. Such scenario could be dispersive for those investors that want to approach a factor-based investment.

The proliferation of risk factors needs more discipline.

Multiple factors, even if called with different names and apparently not similar, may be redundant within a portfolio if their marginal contribution in explaining returns is not statistically different from zero.

Hence, I want to discuss some criteria, reported in recent empirical studies, that can be used to control the identification of factors.

A possible way is related to the estimation and testing of the alpha of a regression of the new factor onto existing factors.

Carhart (1997) improved the predictive power of the standard Fama-French 3-factor model by adding the Momentum factor. Furthermore, Fama and French (2015) extended their original model introducing the Profitability and Investment factors.

For example, the seasonality risk factor of Heston and Sadka (2008) may improve the predicting power of the FF 5-factor model while, since it is correlated with Momentum, would fail considering the Carhart 4-factor model. However, it is clear that such method suffers the arbitrariness related to the choice of the asset pricing model taken as benchmark.

Furthermore, according to Harvey et al. (2015) the usual cut-off level of the t-statistic should be increased from 2 to 3. Since hundreds of papers and factors attempt to explain

[12] In 2001 Cochrane coined the term ‘factor zoo’. Other contributions are provided by Harvey et al. (2015), McLean and Pontiff (2016), and Hou et al. (2017).

the cross-section of expected returns, a new factor needs a much higher hurdle and a multiple-testing framework is needed.

Namely, the simultaneous testing of more than one hypothesis.

They analysed a group of 124 factors discovered no earlier than 2000, showing that almost all the new factors are above a *t*-ratio of 1.96, corresponding to a 5% significance level. However, only 12 out of the 124 considered factors are above a threshold of 3.

The cut-off value of the t-statistic should increase with time and with the introduction of new factors in literature. Using Bonferroni-Holm method[13], the benchmark *t*-statistic starts at 1.96 and increases to 3.78 by 2012. It reaches 4.00 in 2032.

The results show that, in addition to the well-known Value (HML), Momentum (MOM) and Market (MKT) factors, only DCG and SRV[14] are significant across all the *t*-statistic adjustments. Moreover, EP, LIQ, and CVOL[15] are sometimes significant, and the rest are never significant.

Other methods are based on risk premia.

The underlying idea is that factors should provide a sizable and statistically significant premium as compensation for risk, while anomalies should disappear, once discovered and published in literature.

Linnainmaa and Roberts (2016) examined 36 anomalies showing that the returns on the related strategies decreased on average by 58% after the publication. Similar results have been presented by McLean and Pontiff (2016) in 97 strategies.

Berkin and Swedroe (2016) identify some characteristic required to be considered as a risk factor.

A factor must be:

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[13] A method that controls the probability that one or more Type I errors will occur by adjusting the rejection criteria of each of the individual hypotheses. It is used to counteract the problem of multiple comparisons.

[14] Durable Consumption Goods; Yogo (2006) and Short-Run Volatility.

[15] Earnings-Price ratio, Basu (1983); Liquidity, Pastor and Stambaugh (2003); Consumption Volatility, Boguth and Kuehn (2012).

- **persistent**, that is, valid across long periods of time and different economic regimes;

- **pervasive**, that is, valid across countries, regions, sectors and asset classes;

- **investable**, that is, valid after considering the implementation issues, such as trading costs;

- **intuitive**, that is, valid according to a behavioural or risk-based explanation.

In their study the factors that match with such criteria are: Market, Size, Value, Momentum, Profitability and Quality, Term, Carry, Low-Volatility, Default.

However, an analysis based on risk premia, even if statistically significant, would be misleading. In fact, two or more factors that are exposed to the same underlying risk may provide the same premium. What matters is the ability to add pricing information to the existing factors.

Using asset pricing theory as guide, we can say that the exercise aimed at the identification of all the existing risk factors is equivalent to the exercise to identify the true SDF.

Feng, Giglio and Xiu (2019) proposed a new method to assess whether a new factor adds explanatory power for asset pricing, relative to the existing set of factors.

Such method[16] allows to test the marginal importance of each risk factor in explaining the cross-section of returns and, moreover, highlights how a shock to a single factor can affect marginal utility.

The general idea is to construct a multi-factor model to be used as benchmark and see if the addition of other factors adds explanatory power.

Of course, the most difficult aspect is to identify the best low-dimensional benchmark in order to avoid redundancy.

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[16] Double-selection LASSO method of Belloni et al. (2014). It is based on the Fama-Macbeth two-pass methodology applied in high dimensional settings. It selects factors that are either useful in explaining the cross section of expected returns or are useful in mitigating the omitted variable bias.

However, the most innovative aspect is to estimate the role of the additional factor in explaining marginal utility. Namely, the estimation of its factor loading[17].

A positive factor loading implies that high values of the factor correspond to good states of the economy, in which consumption is high and marginal utility is low.

		(1) DS		(2) SS		(3) FF3		(4) No Selection		(5) Avg. Ret.	
id	Factor Description	λ_s (bp)	tstat (DS)	λ_s (bp)	tstat (SS)	λ_s (bp)	tstat (OLS)	λ_s (bp)	tstat (OLS)	avg.ret. (bp)	tstat
136	Cash holdings	-34	-0.42	15	0.17	10	0.54	-18	-0.16	13	0.98
137	HML Devil	54	1.04	-13	-0.25	-100	-2.46**	68	0.84	23	1.46
138	Gross profitability	20	0.48	3	0.06	23	2.00**	13	0.26	15	1.45
139	Organizational Capital	28	0.92	-1	-0.03	20	1.91*	16	0.41	21	2.05**
140	Betting Against Beta	35	1.45	38	1.50	36	2.25**	49	1.49	91	5.98***
141	Quality Minus Junk	73	2.03**	4	0.11	39	3.10***	50	1.04	43	3.87***
142	Employee growth	43	1.36	-4	-0.12	-12	-0.89	18	0.37	8	0.83
143	Growth in advertising	-12	-1.18	0	0.03	12	1.32	-2	-0.13	7	0.84
144	Book Asset Liquidity	40	1.07	5	0.12	20	1.59	20	0.42	9	0.79
145	RMW	160	4.45***	15	0.41	20	1.80*	74	1.48	34	3.21***
146	CMA	38	1.10	0	0.01	3	0.28	7	0.14	26	3.02***
147	HXZ IA	51	2.11**	5	0.21	21	1.94*	40	1.08	34	4.17***
148	HXZ ROE	77	3.37***	23	0.83	33	2.92***	104	2.87***	57	4.99***
149	Intermediary Risk Factor	112	2.21**	60	1.19	4	0.08	22	0.32		
150	Convertible debt	-15	-1.36	-39	-3.22***	26	3.32***	17	1.01	11	1.70*

Table 2.4 - Testing for factors introduced in 2012-2016
Source: *“Taming the Factor Zoo: A Test of New Factors”*, Feng et al. (2019).

The table reports the estimates of the factor loadings and the relative t-statistic. Looking at the last column it is possible to see how a relevant number of new factors provide a statistically significant risk premium but only few of them remain significant if we consider their factor loading estimates.

[17] Its coefficient in the stochastic discount factor. Making inference on the SDF is important in order to have an economic interpretation of results.

The results show that the new factors are spurious or redundant in most cases. The use of the Double-Selection (DS) methodology reduces the number of statistically significant risk factors, as we can see looking at the first column.

It is interesting to notice that, according to such study, the Fama-French Investment factor is not significant even if we take in account the 3-factor model as benchmark. In fact, the third column shows a t-stat equal to 0.28 for the CMA factor loading. However, the Profitability factor (RMW) is highly significant, with a t-stat equal to 4.45.

Considering a threshold value above 3, as discussed before, only RMW and HXZ ROE can be considered able to add pricing information.

3. Investigation of Economic Regimes and Performance Analysis

3.1 Data and methodology

Data are related to United States. The frequency is monthly, and the sample period goes from January 1972 to August 2018.

I considered the time series of the Composite Leading Indicator (CLI) for economic growth and the Consumer Price Index (CPI) for inflation. Moreover, I chose six risk factors: Market (MKT), Value (HML), Size (SMB), Profitability (RMW), Investment (CMA) and Momentum (MOM). The data sources are the OECD site for CLI and CPI and the Kenneth R. French Data Library for the six risk factors.

The objective of this study is to relate the *Factor Investing* approach to the business cycle.

The idea that the economic cycle should be analysed by only looking at growth is, by now, reductive and limiting and would only bring out the simple distinction between expansive and recessive phases.

The multi-regime approach is already widely acknowledged; in particular, the identification of four phases seems to be a choice that achieves huge consensus and diffusion.

Since asset returns are affected by real growth, a natural choice is to separate "good" and "bad" regimes depending on inflation.

In the paper *Index Performance in Changing Economic Environment*[18] (2014) Gupta and other authors identify four regimes in the business cycle depending on how growth is accompanied by inflation.

The relevant indicators are the Consumer Price Index for inflation (3 months minus 36 months rolling variation) and the Composite Leading Indicator for growth (month on month variation). They classify regimes through a bivariate framework, dividing them in four quadrants, depending on whether the CLI and the CPI are rising or falling.

[18] The paper was published for MSCI Market Insight Research. From now on, I will refer to MSCI for the attribution of the classification in 4 regimes taken as reference for this empirical work.

However, their work is limited to the observation and counting of the four following phases: Heating Up, Goldilocks, Stagflation and Slow Down.

Indeed, regimes are identified in a heuristic way, knowing in advance which characteristics each state should have, not relying on a pure statistical evidence principle.

My aim is to implement a Markov-Switching model that allows the testing of the number of regimes in the economy and the statistical significance of the relevant parameters of each regime.

Using the same variables, I want to investigate the presence of four economic regimes from a statistical point of view and assess their characteristics in order to see if there is a correspondence with the classification provided by MSCI.

Following this approach, the choice of the 2 leading indicators seems to be the most appropriate for the analysis of the economic cycle.

In particular, the CLI instead of the GDP is designed to provide early signals of turning points in business cycles showing fluctuation of the economic activity around its long-term potential level. Short-term economic movements are captured in qualitative rather than quantitative terms.

Using the time series of CLI and CPI, realized growth and inflation are estimated. I computed growth by taking a month by month percentage variation in the CLI index.

For each month (date t) the applied formula is:

$$1200 * Log(CLI_t/CLI_{t-1})$$

Therefore, I obtained a monthly series of annualized percentage variations in the CLI index. Moreover, at each date I assessed if there was a price acceleration or deceleration.

First of all, I computed a monthly series of inflation using the same formula applied to the CLI index:

$$1200 * Log(CPI_t/CPI_{t-1})^{19}.$$

[19] Inflation formula suggested by Stock and Watson (2010).

Then, at each date t, I computed the average inflation in the last 3 months and in the last 36 months and then I took the difference.

If the variation is positive there is a price acceleration, otherwise a deceleration.

Both the series were annualized. Hence, they are comparable[20].

The following figure represents the 2 dependent variables, used as input for the Markov-Switching model.

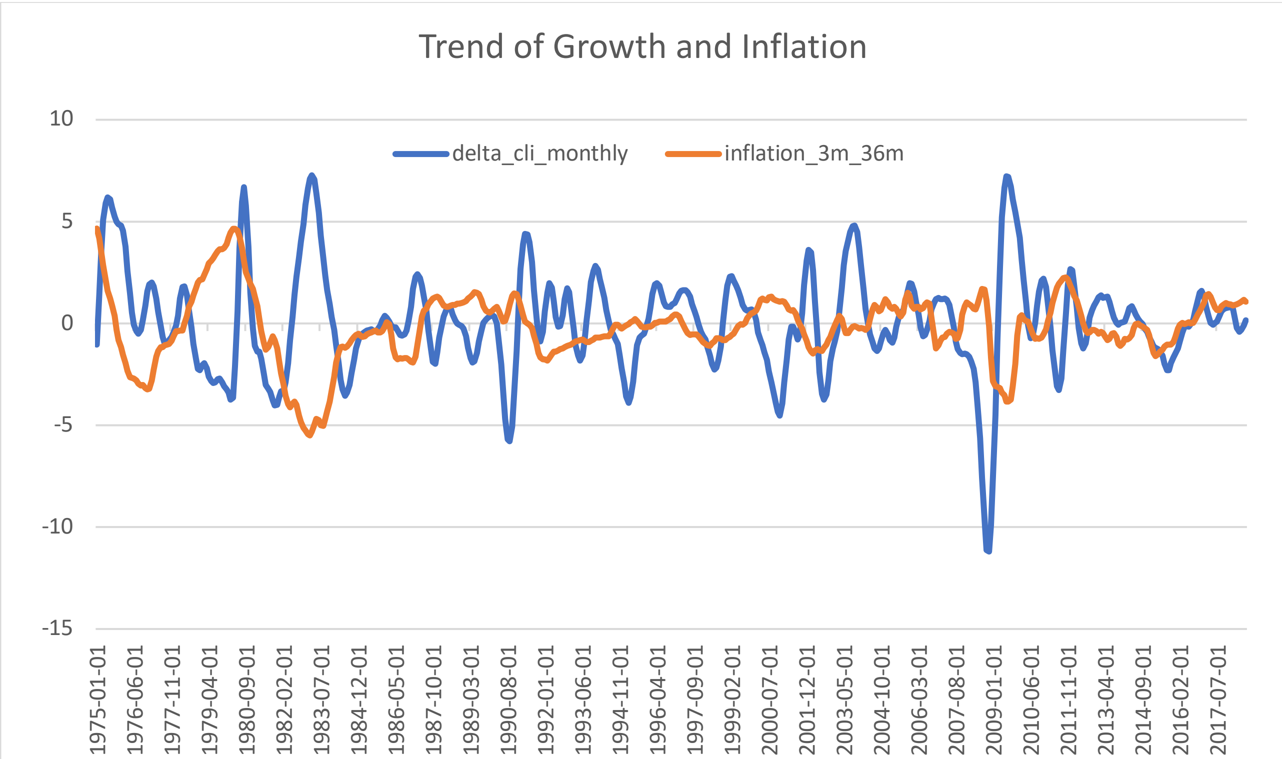

Figure 3.1 - Monthly variation in CLI and 3m-36m rolling moving average of CPI

The plot of the two variables clearly shows that when growth seems to be at its peak, inflation tends to be at the bottom, and vice versa.

[20] In order to have the same length, I had to start from date 1-1975. In fact, the moving average of inflation takes in account observations in the last 3 years and my dataset starts from 1-1972.

This could be due to a physiological delay in the real activity in transmitting its shocks to the price mechanism. Moreover, both the series are negatively skewed (-0.14 CLI and -0.33 CPI) and exhibit a high kurtosis (5.50 and 4.60) in excess with the one of a normal distribution[21].

At this point it would be easy to identify the 4 regimes by looking at the sign of growth and inflation at each month, as suggested by MSCI:

	Growth	Inflation
Heating Up	+	+
Goldilocks	+	–
Stagflation	–	+
Slow Down	–	–

Table 3.2 - Classification of the four regimes according to Gupta et al. (2014) looking at the sign of growth and inflation at each date.

However, as I said before, my aim is to avoid this simplification and follow a more rigorous approach from a statistical point of view.

The application of a Markov-Switching model allows the estimation of a drift parameter and a variance both for CLI and CPI in each state of the economy.

Moreover, I will obtain a transition probability matrix to evaluate which state is the most likely to occur, given the current one, and the persistence of each regime. Namely, its average duration.

Once the classification of the economic regimes is realized, I will analyse the performance of six risk factors through the economic cycle. The objective is to

[21] To have a normal distribution the skewness should be equal to 0 and the kurtosis should be equal to 3.

establish which factors are the most performing in each state, looking at risk-adjusted indicators.

The collection of several information about risk and return characteristics of the risk factors in different phases will be useful to simulate a dynamic asset allocation process, knowing how the regimes tend to switch and to follow one another.

In conclusion, I will assess if such asset allocation process is more efficient than a traditional one that ignores the regime switching.

3.2 Markov-Switching Model

First of all, I wanted to assess whether a model with 4 states is better than a model with a different number of states using information criteria.

I estimated Markov-Switching models with 2, 3, 4, 5 and 6 states[22]. Then, I computed the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC) for each of them using the value of the Log Likelihood function and the number of estimated parameters.

The following equations are used to estimate the AIC and BIC (Stone, 1979; Akaike, 1974) of a model:

$$\begin{aligned} \text{AIC} &= -2 * Ln(L) + 2 * K \\ \text{BIC} &= -2 * Ln(L) + Ln(N) * K \end{aligned}$$

where L is the value of the likelihood, N is the number of observations, and k is the number of estimated parameters.

Obviously, the number of estimated parameters increases with the number of states. AIC and BIC are penalizing criteria, used to compare models with a different number of estimated parameters. In fact, increasing the number of parameters can lead to an over-fitting issue.

The AIC is designed to pick the model that produces a probability distribution with the smallest discrepancy from the true distribution.

The BIC is an estimate of a function of the posterior probability of a model being true, under a certain Bayesian setup.

According to both the information criteria, a model with 4 states should be preferred since a lower AIC or BIC value indicates a better fit. Moreover, a model with 5 states is better than a model with 3 states according to AIC, while the opposite is true according to BIC[23].

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[22] I used the MS_Regress package on MatLab, provided by Marcelo Perlin. The estimation is based on Maximum Likelihood.

[23] AIC and BIC differ by the way they penalize the number of parameters of a model. More precisely, BIC criterion will induce a higher penalization.

A model with 6 states can be preferred only to a model of 2 states according to AIC, while according to BIC it has the worst fit.

	2 States	3 States	4 States	5 States	6 States
AIC	3882.55	3697.06	3545.13	3603.67	3778.46
BIC	3951.92	3811.02	3713.59	3826.63	4075.74

Table 3.3 - AIC and BIC in Markov-Switching models with 2, 3, 4, 5 and 6 states

I implemented a Vector Auto Regressive (VAR) Markov-Switching model in the form:

$$Y_t = \mu(s_t) + Y_{t-1} + \varepsilon_t$$

where:

$s_t =$ regime at time t

$Y_t =$ $[\Delta CLI_t, \Delta CPI_t]$ multivariate dependent variable

$\mu =$ $[\mu_{CLI}(s_t), \mu_{CPI}(S_t)]$ mean for each regime

$\varepsilon_t =$ $[\varepsilon_{CLI,t}, \varepsilon_{CPI,t}]$ stochastic error terms normally distributed with zero mean and constant variance.

Once the output of the process is obtained, I proceed to analyse the estimated parameters in order to identify the 4 regimes.

	ΔCLI (p-value)	ΔCPI (p-value)
State 1	0.19 (0.00)	-0.36 (0.00)
State 2	0.23 (0.00)	0.89 (0.00)
State 3	0.16 (0.00)	0.55 (0.00)
State 4	-0.02 (0.00)	-0.22 (0.00)

Table 3.4 - Estimated drift parameters of growth and inflation in each regime. P-values in parentheses

Expected duration of Regime 1	14.72
Expected duration of Regime 2	10.67
Expected duration of Regime 3	2.26
Expected duration of Regime 4	9.54

Table 3.5 - Average duration (in months) of each regime

The four regimes should be identified by looking at the sign of the estimated intercepts. If the classification adopted by MSCI is correct, one should expect parameters of equal sign in Heating Up and Slow Down (respectively both positive in the former and both negative in the latter) and parameters of opposite signs in Goldilocks and Stagflation (respectively positive CLI and negative CPI in the former and negative CLI and positive CPI in the latter).

In order to classify the 4 states, I also considered the expected duration and the table of estimated transition probabilities at each date.

In synthesis, the MSCI classification, the results of the model and the knowledge of macroeconomic scenarios will be the guide to assign the appropriate economic regime to each state.

Looking at table 3.4, we can immediately notice that all the estimated drift parameters are statistically significant: p-values are equal to 0.

Only State 1 presents opposite signs, while either State 2 and State 3 are characterized by a positive intercept on both the variables of growth and inflation. On the other hand, in State 4 they are both negative.

Hence, either State 2 and State 3 exhibit rising growth and inflation and should be good candidates for the Heating Up regime.

State 1 seems to correspond to the Goldilocks regime (rising CLI, falling CPI) while State 4 should coincide with the Slow Down regime, since the drift parameters are both negative.

Hence, relatively to the intercept parameters, the results of the MS model seem to properly capture 3 out of the 4 regimes identified by MSCI. All except Stagflation, for which the intercept should have been positive for the inflation and negative for the growth.

In State 2 the acceleration is more pronounced than in State 3; the drifts are 0.23 and 0.89 against 0.16 and 0.55. State 2 is also more frequent and more durable: expected duration 10.67 versus 2.26. Therefore, I assigned the Heating Up regime to State 2. For State 3, instead of Stagflation, I decided to use the term Recovery, to indicate a phase in which the economy accelerates but does not reach the peak.

State 1 shows a positive intercept for CLI and a negative intercept for CPI. That is exactly what one would have expected from the Goldilocks regime, in which growth is rising and inflation is falling. Furthermore, according to MSCI, the Goldilocks regime is the one with the longest duration.

Macro-Economic States	State Likelihood
Heating Up	11%
Goldilocks	40%
Slow Growth	32%
Stagflation	17%

Table 3.6 - Relative frequency of economic regimes using bivariate classification
Source: *Index Performance in Changing Economic Environments (2014), MSCI.*

Macro-Economic States	State Likelihood
Heating Up (state 2)	22%
Goldilocks (state 1)	47%
Slow Down (state 4)	25%
Recovery (state 3)	6%

Table 3.7 - Relative frequency of economic regimes according to the Markov-Switching model

This is coherent with the estimation of my model, in which State 1 has the highest expected duration and the highest frequency. Furthermore, if we look at the estimated transition probabilities, State 1 (Goldilocks) is the dominant regime from 2010 to 2016.This result can be interpreted as an effect of the expansive monetary policies of the Federal Reserve. In fact, interest rates have been maintained artificially low and the growth, even if positive, has been not sizable.

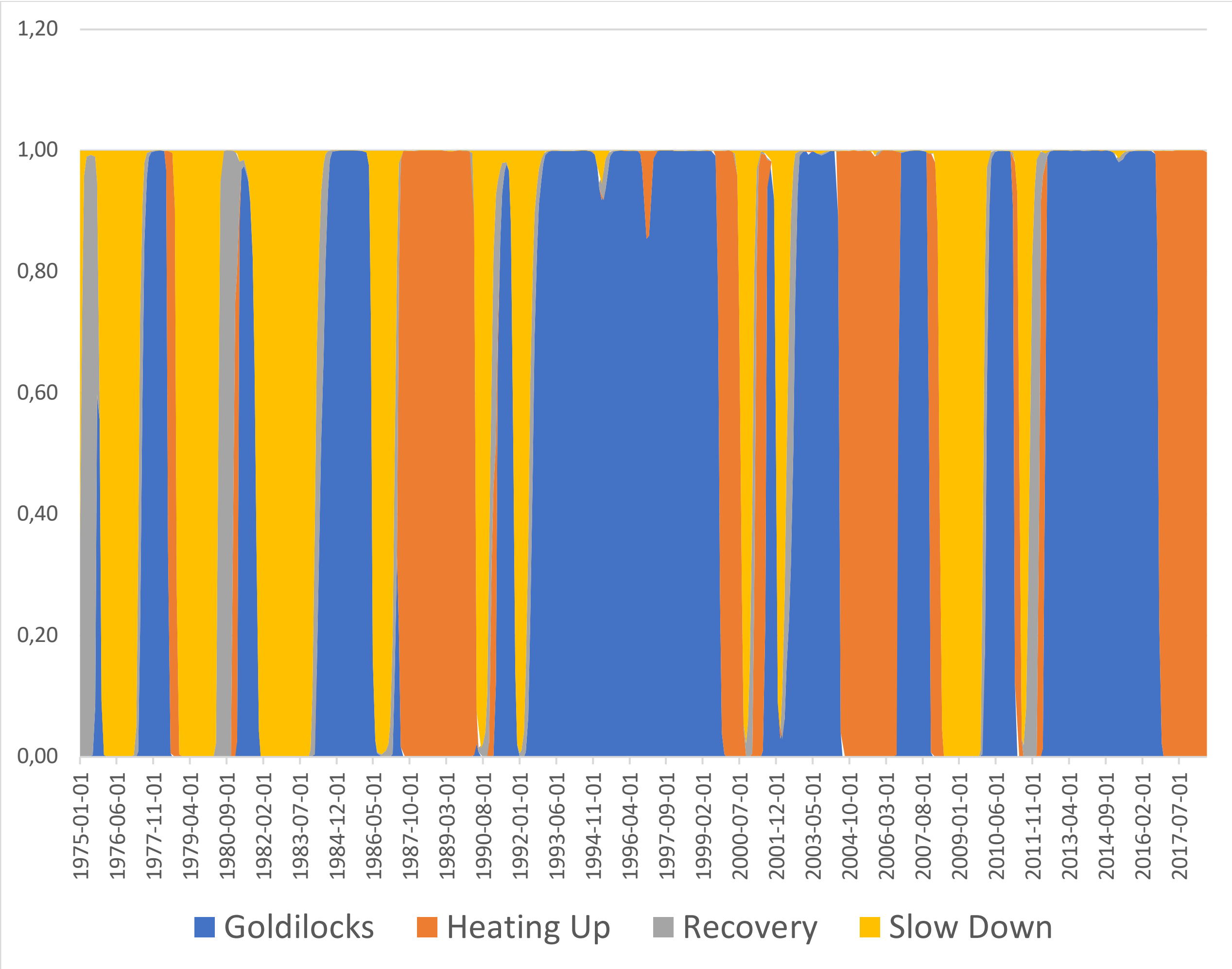

Figure 3.8 - Plot of filtered probabilities of regimes at each date

One can notice that the intercept for CLI in State 1 is lower than in State 2 and State 3. The Goldilocks regime is therefore characterized by a moderate and not very volatile growth and by a falling inflation process. The results of the model seem consistent with the macroeconomic scenario.

From November 2016, price dynamics are once again accelerating, obviously due to the progressive reversal of the central banks' trend, which led to a gradual rise in rates.

One important output of the model is the transition matrix which controls the probability to switch from one state to the other.

In a MS model the transition of states is stochastic and not deterministic. Even if we cannot know with certainty if there will be a switch or not, we can analyse the dynamics of the switching process by looking at the transition matrix:

	Goldilocks	Heating Up	Recovery	Slow Down
Goldilocks	93,2%	4,13%	26,6%	0%
Heating Up	5%	90,6%	17,5%	0,61%
Recovery	0,01%	0,04%	55,8%	9,87%
Slow Down	1,79%	5,2%	0%	89,5%

Table 3.9 - Transition probabilities matrix

The table represents a Markov matrix of transition probabilities. Each column is a probability distribution and, hence, the sum adds up to 100%.

We can immediately notice that all the states are very persistent. In fact, on the main diagonal we find the probabilities to remain in the same regime.

The Goldilocks regime is the most persistent one: 93% probability to remain in the same state and only 5% to pass to Heating Up. There are almost no chances to pass to the Recovery regime and very low probability to pass to Slow Down.

The Heating Up regime shows a certain tendency to switch only to Slow Down (5.20%) or Goldilocks (4.13%), while the probability of persistence is 90.63% and the chance of a switching to Recovery is close to zero.

The Recovery regime is the less persistent one (55.84%) while it shows the tendency to move to Goldilocks (26.61%) or to Heating Up (17.54%). No chance to move to Slow Down.

The Slow Down regime is also very persistent (89.52%) and seems to move almost exclusively towards the Recovery state (9.87%).

In synthesis, the most likely sequence seems to be the following one:

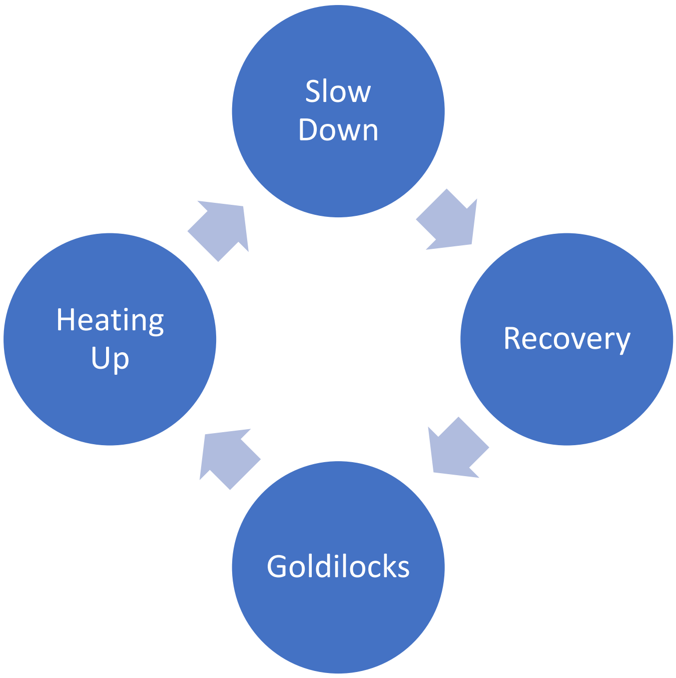

Figure 3.10 - Most likely sequence of states according to the estimated transition probabilities

The model assumes that transition probabilities remain constant over time.

However, the Markov matrix can be useful to make further economic considerations about the estimated regimes.

As I said before, the model does not capture the Stagflation regime.

State 3, besides being a short-lived phase (2.26), seems to be also much less persistent than the others (55.84%).

Once the economy ends up in this state, it therefore tends to remain there for a short time and to flow very quickly into a phase of Goldilocks or Heating Up. Therefore, it can represent the phase that materializes immediately after a period of crisis or general slowdown. In fact, from Recovery it is not possible to switch to Slow Down while the opposite is likely; it is a sort of intermediate state in which both growth and inflation are recovering but not yet “at the peak” as in Heating Up.

Indeed, in the latter, the intercept parameters are greater. Even in Goldilocks, the drift of growth is greater than in Recovery (0.19 versus 0.16).

Actually, Recovery can be reached only from the Slow Down regime, while the chances of passing from Goldilocks and Heating Up are nil.

The Heating Up regime, on the other hand, seems to coincide with periods in which both indices (CLI and CPI) reach their maximum level. For this reason, once the Heating Up phase is reached, if the economy does not remain there then it tends to slow down. The deceleration can affect both growth and inflation to an almost equal extent or it could only concern inflation. In the former case (with probability 5.20%) the Slow Down regimes materializes, in the latter case (with probability 4.13%) Goldilocks: the economy is in deflation but continues to grow with much more moderate rhythms.

3.3 Performance Analysis in Regime Switching Framework

The next step is to relate the performance of the Fama-French risk factors to the 4 estimated regimes. The model gives as output a matrix of filtered probabilities at each date, based on the Hamilton filter. Hence, it assigns to each state its probability of realization.

Using such probabilities, at each month I assumed the most likely state as actually realized[24] and I constructed a matrix of dummy variables that indicates the current state.

In synthesis, for each row of the matrix, only the column correspondent to the most likely state assumes value equal to 1 while the other columns assume value zero. This technique allows to create, for each risk factor, a series of monthly returns in Goldilocks, Heating Up, Recovery and Slow Down. Such output comes out by taking the row by row product between the columns of the dummy matrix and the Fama-French risk factors. Then I run a multivariate regression[25].

As in the previous step of the MS model, the estimated betas represent drift parameters for the Fama-French factors in each regime.

Hence, risk factors are modelled according to a dummy random walk formula:

$$R_t = \mu(s_t) + \sigma(s_t)\varepsilon_t$$

where:

s_t = Current state indicator

R_t = Return on each factor at time t

$\mu(s_t)$ = Average return in each regime

ε_t = Stochastic error terms normally distributed with zero mean and constant variance

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[24] This assumption seems to be reasonable since at each date there is always one state that assumes a probability higher than 65% to materialize and, in most of the cases, the probability is about 90%.

[25] The dependent variable of such regression is represented by the matrix of dummy variables while the regressor is the matrix of risk factors.

For each factor the model estimates the monthly average return and the volatility in Goldilocks, Heating Up, Recovery and Slow Down.

Then, I computed the ratio between mean and volatility in order to obtain a risk-adjusted indicator of performance.

		Mean	Volatility	Sharpe Ratio
MKT	Goldilocks	1,02	0,28	3,66
	Heating Up	0,53	0,40	1,30
	Recovery	0,92	0,81	1,13
	Slow Down	0,25	0,38	0,64
SMB	Goldilocks	0,16	0,19	0,84
	Heating Up	0,28	0,27	1,02
	Recovery	0,69	0,55	1,26
	Slow Down	0,42	0,26	1,62
HML	Goldilocks	0,08	0,18	0,44
	Heating Up	0,23	0,26	0,86
	Recovery	-0,44	0,53	-0,83
	Slow Down	0,91	0,25	3,63
RMW	Goldilocks	0,16	0,14	1,11
	Heating Up	0,23	0,21	1,09
	Recovery	1,08	0,42	2,60
	Slow Down	0,51	0,20	2,58
CMA	Goldilocks	0,09	0,12	0,75
	Heating Up	0,08	0,18	0,47
	Recovery	0,02	0,36	0,07
	Slow Down	0,76	0,17	4,55
MOM	Goldilocks	0,63	0,27	2,29
	Heating Up	0,80	0,40	2,01
	Recovery	1,31	0,80	1,64
	Slow Down	0,22	0,38	0,57

Table 3.11 - Expected return, volatility and Sharpe Ratio of the 6 risk factors in each economic regime

First of all, I notice that all the factors exhibit the lowest volatility during Goldilocks and the highest during Recovery.

As I said before, the Goldilocks regime is the most likely and it is characterized by rising growth and falling inflation.

The market risk factor outperforms the other factors in this regime, followed by Momentum. The worst performers are HML and CMA.

The Value and the Investment factors are the worst performers in 3 out of the 4 regimes but they are the best performers in Slow Down, also in terms of absolute return (HML 0.91 and CMA 0.76).

On the other hand, Momentum and Market are the most pro-cyclical.

They suffer when growth and inflation are falling and, hence, they are the worst performers in Slow Down.

The third best performer in Slow Down is the RMW factor; the risk-adjusted returns in Slow Down and Recovery are very close (2.58 and 2.60) while the performance tapers off moving through the other regimes but remains attractive, in relative terms, even in Goldilocks and Heating Up. This is due to the low volatility and is not surprising since one can expect that firms with high profits outperform firms with low profits, regardless of the economic cycle. Therefore, the performance is stable across the different regimes. Similar considerations can be made about the Investment factor.

About CMA, the logical finding is that, during recessions, firms that invested conservatively outperform firms that invested aggressively. Moreover, this factor is the one with the lowest volatility in each regime.

Concerning the HML factor, its good performance in Slow Down can be explained by the fact that markets seem to be forward-looking and tend to anticipate the business cycle. The Value factor is the premium you receive by investing in undervalued stocks and it is reasonable to think that prices are at the minimum when the economy is slowing down both on growth and inflation. Investors can look at this regime as the most favourable one to take a position in value stocks.

Concerning the Market risk factor, the best regimes are Goldilocks (3.66) and Heating Up (1.30) while, intuitively the worst performance is reported in Slow Down (0.64).

About the Momentum risk factor, the best regime looking at absolute returns is Recovery. However, if we consider the risk-adjusted performance, the

Goldilocks and Heating Up regimes provide a higher remuneration in relative terms (2.29 and 2.01 respectively).

The Momentum risk factor shows a good performance across all the regimes, but it performs quite poorly Slow Down. In a context of recession, for example, there is higher uncertainty and past winners tend to show low persistence in outperforming past losers.

Finally, the Size factor achieves its best risk-adjusted performance in Slow Down. Moreover, the SMB factor is the only one that never appears among the best or the worst performers in each of the regimes.

	Best Performers	Worst Performers
Goldilocks	MKT (3,66) MOM (2,29)	HML (0,44) CMA (0,75)
Heating Up	MOM (2,01) MKT (1,30)	CMA (0,47) HML (0,86)
Recovery	RMW (2,60) MOM (1,64)	HML (-0,83) CMA (0,07)
Slow Down	CMA (4,55) HML (3,63)	MOM (0,57) MKT (0,64)

Table 3.12 - The two most and worst performing factors in each regime according to the risk-adjusted performance

In synthesis, one can make the following classification.

MKT and MOM are pro-cyclical factors: they follow the trend and provide excess returns through trend participation.

SMB and HML are counter-cyclical factors: they move against the trend and provide excess returns through anticipation of the trend reversal.

RMW and CMA are defensive factors: they ensure a risk reduction and provide excess returns through reduction of loss of asset value. Furthermore, a crucial aspect to consider is correlation. In fact, knowing the correlation, potential diversification benefits can be exploited in portfolio construction.

In the following tables it is possible to see how the correlation among the six factors changes depending on the economic regime.

		MKT	SMB	HML	RMW	CMA	MOM
Goldilocks	MKT	1,00					
	SMB	0,15	1,00				
	HML	-0,27	-0,10	1,00			
	RMW	-0,34	-0,31	0,01	1,00		
	CMA	-0,27	-0,08	0,65	-0,08	1,00	
	MOM	-0,19	-0,13	0,00	0,39	-0,08	1,00
Heating Up	MKT	1,00					
	SMB	0,00	1,00				
	HML	-0,42	0,00	1,00			
	RMW	-0,14	0,00	0,55	1,00		
	CMA	-0,46	0,00	0,66	0,11	1,00	
	MOM	0,28	0,00	-0,53	-0,44	-0,30	1,00
Recovery	MKT	1,00					
	SMB	0,24	1,00				
	HML	-0,57	0,12	1,00			
	RMW	-0,27	0,05	0,09	1,00		
	CMA	-0,65	0,16	0,89	0,26	1,00	
	MOM	-0,10	-0,07	-0,07	0,76	0,10	1,00
Slow Down	MKT	1,00					
	SMB	0,39	1,00				
	HML	0,00	0,21	1,00			
	RMW	-0,32	-0,23	0,03	1,00		
	CMA	-0,29	-0,01	0,69	0,29	1,00	
	MOM	-0,20	-0,13	-0,20	0,22	0,22	1,00

Table 3.13 Correlation of returns among the 6 factors across all economic regimes

The most sizable positive correlation coefficients are evidenced in yellow and the most negative in green. The correlation coefficients are all low-to-negative in every regime, with the exception of the correlation between the value and the investment factors. In fact, HML and CMA show a correlation above 0.65 in every regime. As I said before, according to the results obtained by now, the convenience to invest in the value and investment factors is limited to the realization of the Slow Down regime, in which they are the best performers. However, in order to decide how to allocate funds among HML and CMA, one should look at the correlation with the other factors.

With the exception of CMA, the value factor is poorly correlated with the other factors in every regime. However, the correlation with RMW and MOM becomes sizable in Heating Up (0.55 and -0.53 respectively). Hence, value investors can have some benefits by adding exposure to the momentum factor.

Furthermore, while during the Slow Down regime the value and the market factor exhibit a correlation equal to zero and we can say they are independent, during Goldilocks, Heating Up and Recovery investors can exploit a correlation of -0.27, -0.42 and -0.57 respectively.

The momentum and profitability factors show a positive sizable correlation across all the regimes of the cycle, in particular during Recovery (0.76), but in Heating Up the correlation switches to -0.44.

In each regime, even in Slow Down, almost all the factors are negatively correlated with the Market risk factor. This is an evidence in favour of the factor investing approach, as an investment style that is able to offer diversification against market risk.

Moreover, the diversification potential among factors is effective in every regime of the economic cycle, while it is well known that the correlation among the traditional asset classes tend to rise suddenly close to 1 in bad economic regimes.

Eventually, similar considerations can be made by looking at the cumulative performance of the six risk factors. They all achieved remarkable cumulative returns in the sample period.

However, all the factors experienced periods of negative performances. In the following plot it is possible to see that negative peaks of some factors coincide with positive peaks of other factors.

Figure 3.14 - Cumulative returns of the 6 risk factors over all the sample period

In the long-run, it seems that there is no factor able to outperform the Market, with the exception of Momentum.

However, MKT and MOM exhibit the highest volatility. In general, a strategy that invests in a single factor is strongly exposed to cyclical fluctuations. This is true in particular if we consider a short-term investment horizon.

3.4 Regression Analysis

As previously stated, a portfolio consisting only of traditional asset classes would be excessively exposed to changes in the economic cycle.

The thesis I want to support is that, in the field of Alternative Investments, the six factors provide an efficient allocation, able to guarantee a good performance in all the regimes.

Therefore, the natural continuation of this work consists in carrying out an exercise similar to that of the previous paragraph, which however takes the market factor as a reference.

I estimated a one-factor model structured as it follows:

$$R_{i,t} = \alpha(s^{\wedge}{}_{t}) + \beta(s^{\wedge}{}_{t})R_{mkt,t} + \sigma(s^{\wedge}{}_{t})\varepsilon_{i,t}$$

This is a sort of Capital Asset Pricing Model obtained by regressing each factor against the market excess return to assess, in each regime, the sensitivity to the market premium and the ability to generate a positive alpha.

Exactly as in the CAPM, the alpha represents the portion of performance that does not depend on the tendency to move with the market.

If the alpha is statistically significant[26], one can conclude that the extra-performance is due to the skill of the portfolio manager and not to chance.

Hence, the model allows to test whether there are factors able to beat the market persistently or just in some regimes.

The following table reports the results of the performed regressions.

The parameters that are statistically significant at the 95% confidence level are evidenced in yellow.

[26] The test of significance is performed by computing t-statistics and comparing them to the correspondent critical value, considering a 95% confidence level.

		Alpha (t-stat)	Beta (t-stat)	R-Squared
Goldilocks	SMB	0,03 (0.40)	0,09 (3.50)	0,02
	HML	0,11 (1.69)	-0,16 (-6.37)	0,07
	RMW	0,15 (2.89)	-0,16 (-8.25)	0,11
	CMA	0,10 (1.95)	-0,12 (-6.40)	0,07
	MOM	0,38 (3.70)	-0,17 (-4.50)	0,04
Heating Up	SMB	0,04 (0.55)	0,18 (4.63)	0,04
	HML	0,09 (1.53)	-0,33 (-10.62)	0,17
	RMW	0,06 (1.04)	-0,10 (-3.12)	0,02
	CMA	0,04 (1.30)	-0,20 (-11.89)	0,21
	MOM	0,14 (1.70)	0,30 (6.70)	0,08
Recovery	SMB	0,03 (1.19)	0,12 (5.54)	0,05
	HML	0,00 (0.13)	-0,40 (-15.78)	0,32
	RMW	0,07 (2.57)	-0,14 (-6.43)	0,07
	CMA	0,02 (0.87)	-0,31 (-19.50)	0,42
	MOM	0,08 (1.50)	-0,09 (-2.20)	0,01
Slow Down	SMB	0,09 (1.37)	0,24 (9.73)	0,15
	HML	0,23 (2.97)	0,00 (-0.02)	0,20
	RMW	0,13 (2.79)	-0,14 (-7.78)	0,10
	CMA	0,20 (4.23)	-0,12 (-7.00)	0,09
	MOM	0,07 (0.55)	-0,21 (-4.75)	0.04

Table 3.14 - Estimated alpha and beta against the market excess return in Goldilocks, Heating Up, Recovery and Slow Down

In all the regressions the R^2 is very low, meaning that the risk factors movements are poorly explained by the market risk premium. Actually, a one factor model fails in explaining the returns on the selected factors.

This is not a surprising result: in fact, the Fama-French factors are additional sources of systematic risk and, hence, of pricing.

Moreover, all the estimated coefficients are not sizable in absolute value.

In particular, almost all the betas are low-to-negative, indicating the tendency of all factors to be insensitive or to move in the opposite direction with respect to the market risk factor. However, all the betas are statistically significant in all the regimes.

Regardless of the current state, the SMB factor seems to move in the same direction of the market while all the other factors do the opposite. The only exception concerns Momentum in Heating Up, which exhibits the highest beta (0.30). This is consistent with the fact that MOM and MKT are the best performers in Heating Up.

On the other hand, not all the alpha parameters are statistically significant. This result can be also interpreted as a signal of market efficiency.

In particular, according to the model, there is no factor able to generate an alpha that is statistically different from zero during the Heating Up regime.

The results in Slow Down are interesting and consistent with the findings of the dummy random walk model estimated before. In fact, HML, CMA and RMW are able to produce a positive and statistically significant alpha, respectively: 0.23, 0.20 and 0.13. As seen before, these factors are the 3 best performers in the Slow Down regime.

Furthermore, the RMW factor is the only one able to replicate such performance in the other regimes, with the exception of Heating Up.

Again, this result is coherent with the analysis in the previous paragraph, in which I pointed out the propensity of the Profitability factor to produce a stable performance across all the regimes. However, even though the RMW factor is the only one generating a statistically significant alpha in Recovery, the size is not impressive.

Therefore, it seems to be clear that during the regimes characterized by an acceleration on both CLI and CPI, an investor cannot be confident that some risk factor can generate a positive alpha.

In conclusion, the Momentum factor deserves to be mentioned for its alpha parameter in Goldilocks (0.38), that is the highest among all the states.

MOM is the best performer in Goldilocks along the Market risk factor and, in addition, offers good chances of diversification and alpha generation.

This result is crucial since the Goldilocks regime is the most frequent and persistent. If an asset manager had to rely on the results of this work, he should necessarily consider such evidence and allocate a sizable portion of the funds in portfolios based on momentum strategies.

Furthermore, MOM appears among the best performers also in Heating Up and Recovery, but the alpha parameters are not statistically different from zero.

4. Portfolio Optimization and Asset Allocation

4.1 Black-Litterman Model

Until now, many information on how factors behave in different regimes of the economic cycle have been collected.

My intention is to investigate if part of this set of information can be exploited to obtain advantages in the asset allocation process.

The intuitive idea is the following: if an investor knows how the business cycle behaves and how the risk factors perform and interact with each other, then he should achieve a more efficient allocation.

In particular, the portfolio allocation should change as a regime's switch occurs; hence, the crucial point is to predict which state will occur and when the switch will materialise.

The Markov-Switching model provides the information on the probability that a regime will occur given the current one, but there is no certainty if there will be a switch or not. However, it seems realistic to assume that the most likely state is the one that actually materialises. Moreover, the expected duration of the estimated economic regimes can be useful to guide changes in the portfolio composition.

These facts can be taken in account using the Black-Litterman model for portfolio optimization. Indeed, it seems to be the most suitable model in a Markov-Switching framework since it is reasonable to think that views change when the regime changes.

In general, it is well known that the Black-Litterman model allows to overcome the problems of the standard mean-variance portfolio optimization (Markowitz), such as unintuitive, highly concentrated portfolios, input sensitivity and estimation error maximization.

The principal input to estimate in order to perform mean-variance optimization is the vector of expected returns.

According to Best and Grauer (1991) a small increase in the expected return of one of the portfolio's assets can force half of the assets from the portfolio.

The Black-Litterman model allows to improve the stability of optimal weights and lead to final portfolios that are not excessively concentrated.

Furthermore, the problem of estimation error maximization is largely mitigated (Lee, 2000). In fact, the intuition is that expected returns must not be seen as unknown quantities to be estimated; namely, they are implied in the market and can be combined with information and beliefs that investors try to incorporate into views[27].

What is obtained is not only a better optimizer, but a reformulation of the investor's problem. The BL model has a neutral starting point represented by the equilibrium returns implied in the market capitalization weights. The implied equilibrium returns are obtained from known information using reverse optimization.

The investor's problem is to maximize a convex quadratic utility function (4.1) with respect to a vector μ of expected returns. The solution (4.2) can be reversed to find the optimal portfolio weights given the return vector μ.

(4.1) $$U = w^T\mu - \left(\frac{\lambda}{2}\right) * w^T\Sigma\, w$$

(4.2) $$\mu = \lambda\Sigma w$$

(4.3) $$w = (\lambda\Sigma)^{-1}\,\mu$$

Σ and λ represent respectively the covariance matrix of returns and the risk-aversion coefficient.

λ expresses the average risk tolerance as the ratio between the portfolio's excess return and its variance, accordingly to equation (4.4).

(4.4) $$\lambda = (\mu_p - r_f)/\sigma_p^2$$

[27] "In the context of Black-Litterman, the investor is not asked to specify a vector of expected excess returns, one for each asset. Rather, the investor focuses on one or more views, each of which is an expectation of the return to a portfolio of his or her choosing". Bob Litterman, *Modern Investment Management* (2003).

When the market capitalization weights are considered, the vector of weights w is known and is denoted as w_{eq} indicating, according to the CAPM, that the market capitalization weights constitute the equilibium portfolio weights.

Equation (4.2) becomes:

(4.5) $$\Pi = \lambda\Sigma w_{eq}$$

where Π denotes the vector of implied equilibrium returns.

The Black-Litterman model uses a Bayesian approach to combine the implied equilibrium returns (prior distribution) with the distribution of views in order to obtain the so-called Black-Litterman expected returns (posterior distribution).

Views are expressed introducing the matrix P and the vector Q.

The matrix P indicates which assets are involved in the views while Q is the estimated return vector for every different view.

Assuming that there are K views and N assets, each view can be expressed according to the following distribution

(4.6) $$V \sim N(Q, \Omega)$$

where Q is a Kx1 vector and Ω is a diagonal KxK matrix expressing the uncertainty of the views, that are assumed to be uncorrelated. Hence the off-diagonal elements are equal to zero:

$$\Omega = \begin{bmatrix} \tau p_1 \Sigma p_1^T & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \tau p_k \Sigma p_k^T \end{bmatrix}$$

where τ is a scalar intended to stabilize the influence of the uncertainty of each view and p_k is a row vector in the matrix P.

According to the Black-Litterman model it is not necessary to express a view for each asset in the portfolio. Of course, in absence of views, the optimal weights coincide with the equilibrium weights. In general, the higher the number of views and the lower the uncertainty of them, the larger the deviation of the optimal portfolio from the neutral one.

Investors can express absolute or relative views.

Each row of the P matrix corresponds to a view. If a view is absolute the corresponding row adds up to 1 while the row sums equal 0 if the view is relative. The dimension of the P matrix is KxN.

An example, if there are 3 assets and 2 views, could be the following one:

$$P = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & -1 \end{bmatrix}$$

where the first row indicates an absolute view involving asset 2 while the second row indicates the relative view that asset 1 will outperform asset 2.

The quantification of the views is presented in Q:

$$Q = \begin{bmatrix} 0 & 5 & 0 \\ 2 & 0 & -2 \end{bmatrix}$$

The return of asset 2 is predicted to be 5% while asset 1 is assumed to outperform asset 3 by 2%.

I can now introduce the Black-Litterman formula to express the BL expected returns:

(4.7) $$\mu_{BL} = [(\tau\Sigma)^{-1} + P^T\Omega^{-1}P]^{-1}[(\tau\Sigma)^{-1}\Pi + P^T\Omega^{-1}P]$$

The variance of returns is:

(4.8) $$\Sigma_{BL} = (1+\tau)\Sigma - \tau^2\Sigma P^T(\tau V\Sigma V^T + \Omega)^{-1}V\Sigma$$

The BL weights can be derived by inserting (4.7) and (4.8) in equation (4.3):

$$W_{BL} = (\lambda\Sigma_{BL})^{-1}\,\mu_{BL}$$

Hence, the Black-Litterman model provides an optimizer to tilt away from the market portfolio in order to take advantage of perceived opportunities.

Basically, the model leads to a complex weighted average of the Implied Equilibrium return vector (Π) and the View vector (Q) where the weights depend on the scalar factor τ and on the uncertainty of the views, expressed by the matrix Ω.

The k-th view has a degree of confidence equal to $1/\omega_k$ where ω_k is the k-th element on the main diagonal of Ω, representing the variance of the error term that produces a deviation from Q.

(4.9) $$V = Q + \varepsilon$$

where $\varepsilon \sim N(0,\Omega)$.

Intuitively, a high level of confidence in the expressed views will tilt the new return vector μ_{BL} close to Q.

The last equation implies:

(4.10) $$E[V] = Q$$

(4.11) $$Var[V] = \Omega$$

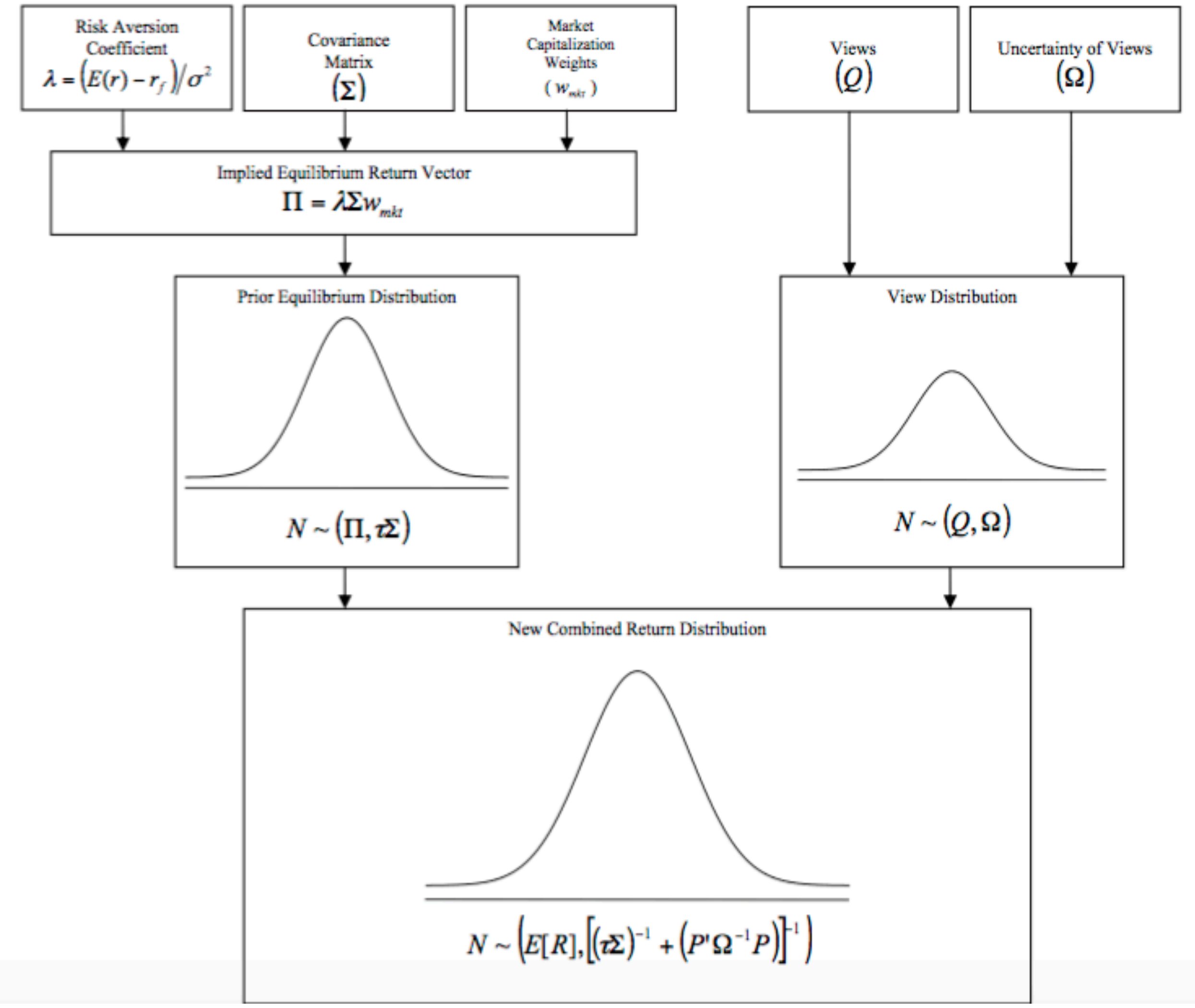

Figure 4.1 - Derivation of the New Combined Distribution
Source: *"A Step-by-Step Guide to the Black-Litterman Model",* Thomas M. Idzorek (2005).

4.2 Implementation in a Markov-Switching Framework

Taking in account the results obtained in the MS model, I performed the Black-Litterman optimization in order to find an optimal allocation in each state, considering a portfolio made of the six risk factors analysed until now.

I started by assuming that, in equilibrium, an investor that has no view will allocate his wealth among the six risk factors in equal parts.

Therefore, the vector of equilibrium weights is:

$$w_{eq} = \begin{bmatrix} 0.1667 \\ 0.1667 \\ 0.1667 \\ 0.1667 \\ 0.1667 \\ 0.1667 \end{bmatrix}$$

Such vector remains equal in each state but, clearly, the vector Π of implied equilibrium returns will change across states. In fact, I estimated a risk aversion coefficient λ and a covariance matrix Σ in each regime.

The former is obtained by taking the average of the Sharpe ratios previously computed for each risk factor in the 4 regimes[28].

Hence, the vector of implied equilibrium returns is:

(4.12) $$\Pi_s = \lambda_s \Sigma_s w_{eq}$$

with s = Goldilocks, Heating Up, Recovery, Slow Down.

[28] See table 3.11.

Σ_S is computed using the correlations and the standard deviations already estimated in each state[29].

	Risk aversion
Goldilocks	1.52
Heating Up	1.13
Recovery	0.98
Slow Down	2.26

Table 4.2 - λ coefficient in each regime

	Goldilocks	**Heating Up**	**Recovery**	**Slow Down**
MKT	0,75%	2,58%	1,54%	4,09%
SMB	0,55%	1,38%	6,32%	3,37%
HML	0,76%	-0,05%	4,38%	3,08%
RMW	0,30%	-0,22%	5,83%	0,90%
CMA	0,34%	0,12%	3,36%	2,12%
MOM	1,59%	2,52%	12,16%	4,20%

Table 4.3 - Implied equilibrium returns in each regime

[29] The MatLab command "corr2cov" allows to pass easy from correlations and standard deviations to covariances.

As I pointed out in the first chapter, it is plausible that risk aversion changes with the economic cycle.

There are periods in which investors are more tolerant than others.

In particular, this happens during favourable regimes while they require a higher premium when the economic conditions get worse.

The estimates in table 4.2 seem to reflect this attitude. The highest λ occurs in the Slow Down regime while in Recovery it is even less than one, meaning that investors require a less than proportional compensation for taking one additional unit of risk.

As I said before, the implied equilibrium returns represent the neutral starting point in the Black-Litterman framework.

After that, I specified the views. Using the results obtained in the Markov-Switching model, I specified 6 absolute views, one for each risk factor.

Recalling that the Q vector represents the expected value of the views' distribution, I constructed Q using the mean returns of the risk factors in each state, previously estimated in the dummy random walk model.

$$Q_{Goldilocks} = \begin{bmatrix} 1.01 \\ 0.16 \\ 0.08 \\ 0.16 \\ 0.09 \\ 0.63 \end{bmatrix} \qquad Q_{Heating\ Up} = \begin{bmatrix} 0.53 \\ 0.30 \\ 0.23 \\ 0.22 \\ 0.08 \\ 0.80 \end{bmatrix}$$

$$Q_{Recovery} = \begin{bmatrix} 0.92 \\ 0.69 \\ -0.44 \\ 1.08 \\ 0.02 \\ 1.31 \end{bmatrix} \qquad Q_{Slow\ Down} = \begin{bmatrix} 0.24 \\ 0.42 \\ 0.91 \\ 0.50 \\ 0.76 \\ 0.22 \end{bmatrix}$$

Figure 4.4 - *Q* vectors in each regime. Each element is the estimated mean return associated to a risk factor in the considered state.

Since there are 6 absolute views, the P matrix is a 6 × 6 identity matrix:

$$P = \begin{bmatrix} 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 \end{bmatrix}$$

Then, I constructed a matrix Ω for each regime, using the rows of P, the Covariance matrix Σ_S and the scaling factor τ.

I set the latter equal to 0.01, in conformity with the suggestions that it should be set close to zero (Black and Litterman, 1990) and between 0.01 and 0.05 (Lee, 2000).

The idea behind is that the uncertainty in the mean is smaller than the uncertainty in the return itself.

Ω Goldilocks

$$\begin{bmatrix} 0.00077003 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.00034743 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.00032839 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.00020318 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.00014752 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.00075388 \end{bmatrix}$$

Ω Heating Up

$$\begin{bmatrix} 0.0016322 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.00073644 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.00069606 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.00043067 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.00031269 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.00159797 \end{bmatrix}$$

Ω Recovery

$$\begin{bmatrix} 0.00658509 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.00297114 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.00280826 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.00173754 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.00126153 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.00644697 \end{bmatrix}$$

Ω Slow Down

$$\begin{bmatrix} 0.00146898 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.00066279 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.00062646 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.0003876 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.00028142 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.00143817 \end{bmatrix}$$

Figure 4.5 - Uncertainty of views in each regime

	μ_{BL} Goldilocks	μ_{BL} HeatingUp	μ_{BL} Recovery	μ_{BL} SlowDown
MKT	0,40	0,25	0,36	0,00
SMB	0,06	0,11	0,37	0,23
HML	0,02	0,05	-0,19	0,63
RMW	0,04	0,05	0,56	0,27
CMA	0,01	0,02	-0,04	0,46
MOM	0,28	0,33	1,03	0,17

Table 4.6 - Black-Litterman returns in each regime

As I pointed out before, BL returns deviate from the implied equilibrium return depending on the number of views and on the level of confidence.

In this case, there is a 6×6 matrix of views' uncertainty for each regime.

The matrix is diagonal since views are not correlated. Hence, each element strongly depends on the volatility of the six risk factors across the states.

After the construction of Ω, I used the Black-Litterman formula to obtain the final combined distribution and the optimal weights of the portfolio.

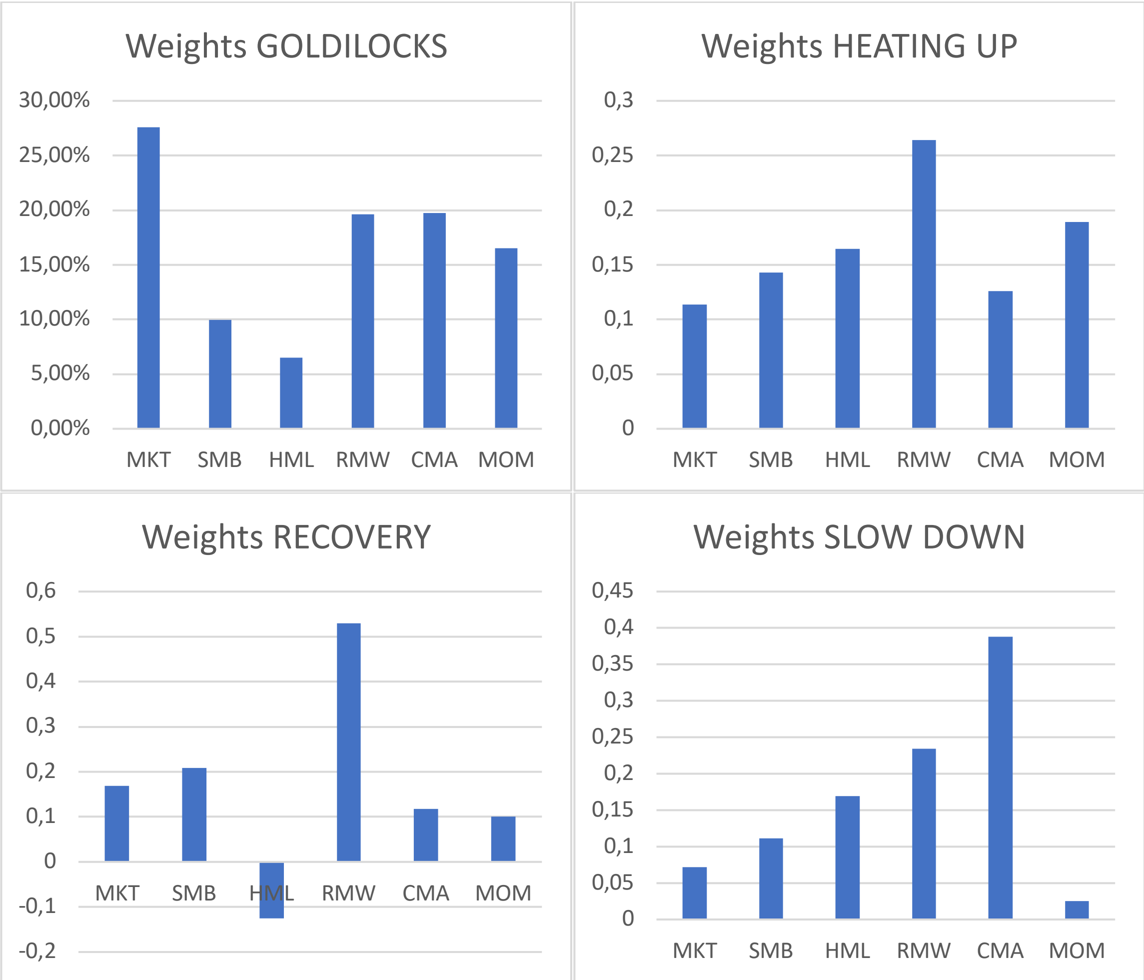

Figure 4.7 - Black-Litterman allocation in each regime

4.3 Analysis of Results and Alternative Strategies

At this point it is possible to analyse the results and assess if a Black-Litterman portfolio of risk factors performs better than a neutrally weighted portfolio.

Moreover, I want to establish whether a dynamic portfolio allocation, that switches when the economic regime changes, should be preferred to static allocation.

In other words, at the end of this work, it is necessary to see if the effort to implement a Markov-Switching model and deeply analyse the economic cycle can be effectively useful to achieve a better performance.

Finally, the performance of a multiple-factor portfolio should be compared with the performance of a single-factor portfolio, in order to assess if the former is able to guarantee relevant diversification benefits and higher risk-adjusted returns.

I considered 3 alternative strategies to construct a multi-factor portfolio:

1) Investment in a portfolio that assigns equal weights to each risk factor,

2) Investment in a Black-Litterman Portfolio with static weights,

3) Investment in a Black-Litterman Portfolio with weights that change across the economic regimes.

Taking strategy **1** as reference allows to determine if there are some regimes or holding periods in which having no view can lead to a better performance or, on the other hand, if Black-Litterman portfolios systematically outperform neutral portfolios.

Taking strategy **3** as reference allows to see if a strategy that ignores economic regimes can outperform a strategy that is based on the identification of them or, on the other hand, if a BL Switching Portfolio systematically outperforms a static BL Portfolio.

I constructed the 3 portfolios considering the Fama-French return series and applying the appropriate weights for each strategy.

At each date I obtained the portfolio return as a weighted average between the Fama-French risk factors' returns and equilibrium weights for strategy **1**, BL static weights for strategy **2** and BL switching weights for strategy **3**.

The BL static weights are estimated by performing again the BL optimization without considering the distinction in regimes. Therefore, the views have been formulated taking in account the performance of the risk factors in all the sample period.

However, in this case the objective is to obtain a unique vector of optimal weights.

The optimal weights are reported in the following figure:

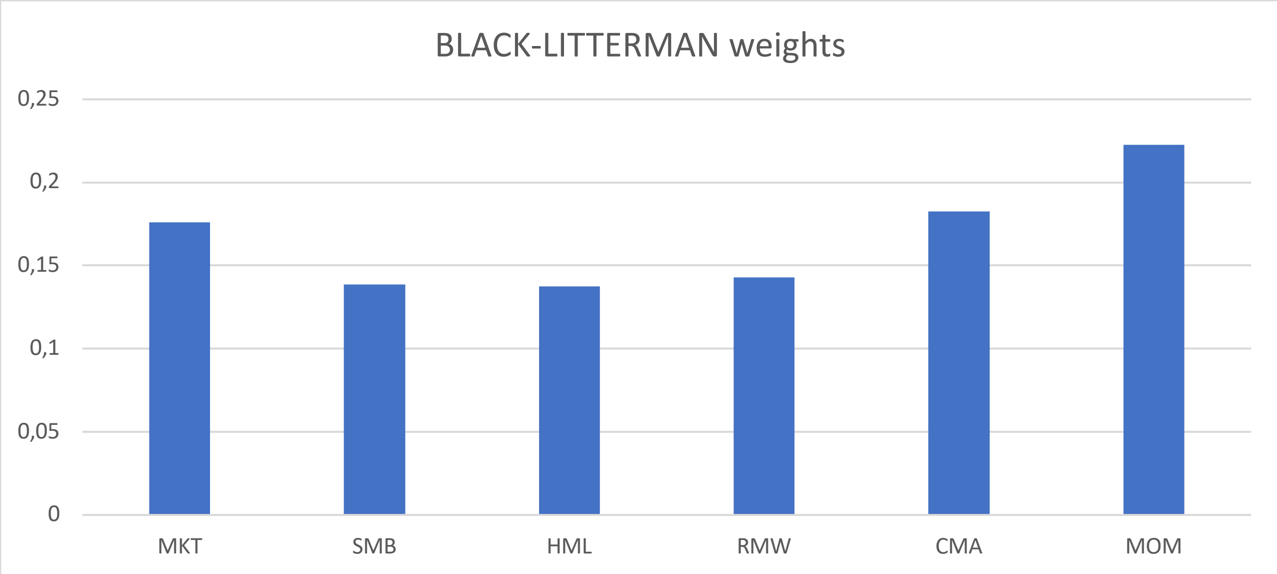

Figure 4.8 - Black-Litterman optimal allocation all over the sample period.

In the selected sample period, there are factors that performed better than others. Indeed, such portfolio composition reflects their cumulative performance. In other words, such strategy over-weights those factors that historically performed better, considering a long horizon and almost ignoring contingent fluctuations due to the economic cycle.

However, the portfolio seems to be well diversified since all the factors receive a weight between 10% and 25%.

Once obtained the 3 portfolios, I plotted their cumulative performance in all the sample period. The BL portfolio with switching weights seems to be the best one, in particular in the last 20 years.

In fact, looking at the following figure, it is evident that the trend of the cumulative return obtained with strategy **3** positively diverge from the other 2 portfolios in a relevant way.

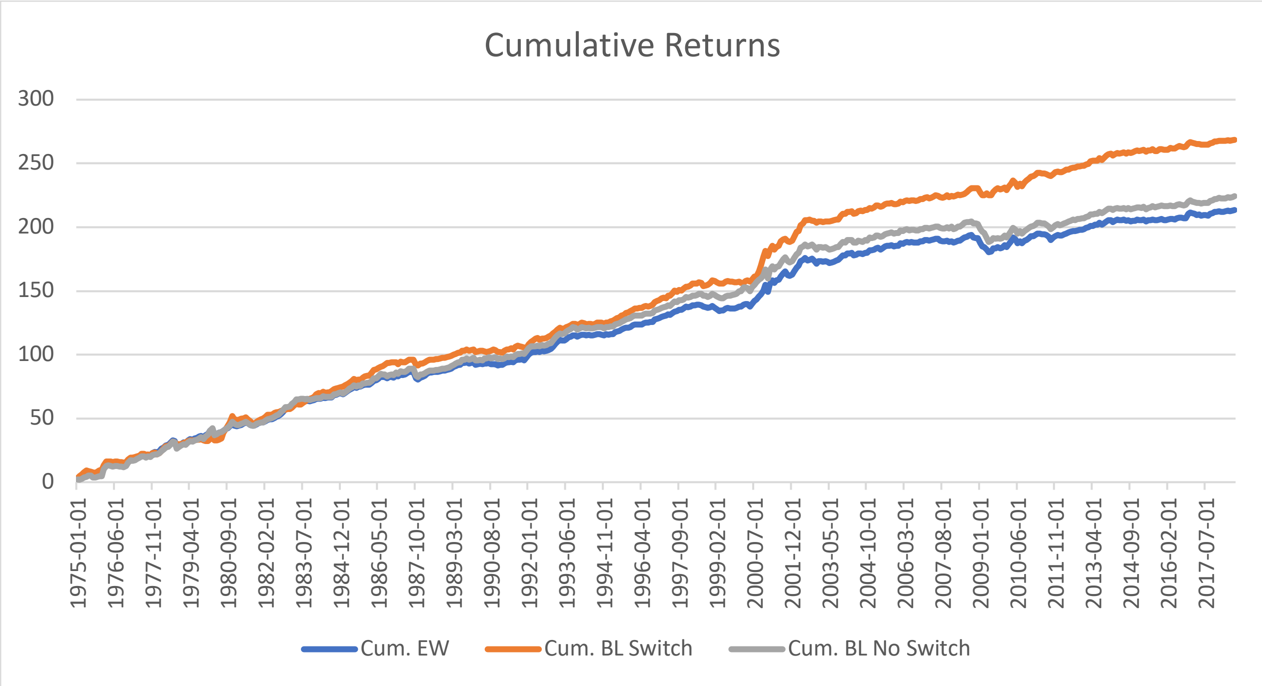

Figure 4.9 - Cumulative returns from January 1972 to August 2018 of the Equally Weighted, Black-Litterman Switching and Black-Litterman Static portfolios

Furthermore, strategy **2** outperforms strategy **1**: the assignment of a slightly higher weight to the best performing factors produce some advantages considering all the sample period.

However, the absolute performance provides no information about the risk taken during the considered period.

Considering again the entire sample period, I compared the risk-adjusted performance of the 3 portfolios in each regime.

I computed a Sharpe ratio in each regime using Black-Litterman weights and equilibrium weights according to the following formulas:

$$SR\_EW_s = \frac{w_{eq}^T\, R_s}{\sqrt{w_{eq}^T \Sigma_s w_{eq}}} \quad \textit{Sharpe Ratio of the equally weighted portfolio}$$

$$SR\_BL_s = \frac{w_{BL}^T R_s}{\sqrt{w_{BL}^T \Sigma_s w_{BL}}} \quad \textit{Sharpe Ratio of the Black-Litterman portfolio}$$

where

s = Goldilocks, Heating Up, Recovery, Slow Down

R_s = vector of risk factors' average returns in each regime

The numerator is the portfolio's average return and the denominator is the portfolio's variance.

	BL Sharpe Ratio		EW Sharpe Ratio
	Switching	Static	
Goldilocks	5,88	5,42	5,16
Heating Up	4,05	3,73	3,67
Recovery	3,47	2,72	2,5
Slow Down	5,53	4,66	4,45

Table 4.10 - Sharpe ratios of the Black-Litterman portfolio and Equally Weighted Portfolio

The table clearly shows that in every regime the Black-Litterman portfolios achieve a higher risk-adjusted performance with respect to the Equilibrium Portfolio.

Furthermore, according to the estimated Sharpe ratios, the BL Switching Portfolio outperforms the BL static portfolio in all the regimes.

It may be interesting to notice that the highest extra-performance of the BL Switching Portfolio materialises in Slow Down. A possible interpretation may be that in Slow Down the portfolio should become more concentrated around those factors that perform well in non-favourable economic conditions. Namely, the most counter-cyclical factors should be overweighed.

Then, I performed a comparison considering holding periods of different length: 3 months, 6 months, 1 year, 3 years, 5 years up to 15 years.

Therefore, at each date, I computed the holding period return that an investor would have achieved by entering at that date and exiting after 3 months, 6 months, 1 year, 3 years, 5 years and so on.

In this way, it is possible to see if time horizon matters in the choice of one of the 3 alternative strategies.

At each date, the best portfolio is clearly the one that achieves the highest holding period return. In the following table it is possible to see how many times during the sample period, each portfolio is considered the best one at different lags.

Moreover, I reported the likelihood as the relative frequency. Namely, the number of favourable cases on the total number cases.

Again, the BL Switching Portfolio seems to be the best choice in most of the cases, regardless of the holding period. However, it is necessary to point out some considerations.

Holding period			EW	BL Switching	BL Static	SUM
3m	Best PTF	Absolute Frequency	95	269	158	522
		Likelihood	18,2%	51,5%	30,3%	100%
6m	Best PTF	Absolute Frequency	70	293	156	519
		Likelihood	13,5%	56,5%	30,1%	100%
1Y	Best PTF	Absolute Frequency	55	311	147	513
		Likelihood	10,7%	60,6%	28,7%	100%
3Y	Best PTF	Absolute Frequency	22	355	112	489
		Likelihood	4,50%	72,6%	22,9%	100%
5Y	Best PTF	Absolute Frequency	0	365	99	464
		Likelihood	0%	78,7%	21,3%	100%
7Y	Best PTF	Absolute Frequency	0	366	74	440
		Likelihood	0%	83,2%	16,8%	100%
10Y	Best PTF	Absolute Frequency	0	363	41	404
		Likelihood	0%	89,9%	10,2%	100%
15Y	Best PTF	Absolute Frequency	0	338	6	344
		Likelihood	0%	98,3%	1,7%	100%

Table 4.11- Equally Weighted Portfolio, Black-Litterman Switching Portfolio and Black-Litterman Static Portfolio compared using different holding periods

An investor cannot be sure that such strategy will outperform an alternative one based on equal weights or static BL weights.

However, it is evident that the likelihood increases with time horizon.

When the holding period is short (less than one year) the dynamic portfolio is outperformed in almost half of the cases but at longer horizons the likelihood to be the best performing one becomes sizable.

On the other hand, when the horizon increases there are less cases in which the Equally Weighted Portfolio and the BL Static Portfolio can be preferred.

In particular, with a holding period of 3 years the Equally Weighted portfolio is outperformed in about 95.5% of the cases while with a holding period of 5 years, or more, it is always outperformed.

Another important consideration is related to the transaction costs to switch the portfolio composition frequently.

Economic cycle and returns are less predictable at short horizons and transaction costs have higher incidence. An investor may be indifferent among the 3 strategies.

However, an investor with longer horizon may take benefits from a dynamic strategy that allows to react in a proper way to changes in the economic cycle and obtain a good performance even in non-favourable environments.

The results obtained until now showed that a multi-factor portfolio that takes in account the economic cycle should be preferred to a multi-factor portfolio with no views or static views that depend on the historical performance of the risk factors.

In the previous chapter, I analysed the performance of each factor in every estimated economic regime.

The evidence is that a long-only investment strategy on a single factor is able to produce remarkable returns in the long-run, but due to the cyclical nature of the returns, the investment on the single factor may not be an optimal allocation for operators with short-term time horizons.

It is well known that the majority of investors is risk-adverse and suffer losses more than it loves gains. Moreover, investors do not want consumption to be excessively volatile, but they desire to stabilize it.

Differently from other asset classes, we have seen that all the risk factors are poorly or negatively correlated, even in non-favourable periods.

In the last years the so-called Smart-Beta indices experienced a large diffusion among investors.

However, the success of these financial instruments may not be due to the idea underlying them (investing in specific risk factors), but rather to the low costs to which they become accessible and to the optimal packaging of these strategies.

Different causes could slow down the adoption of these tools: one of them is the cyclicality of factor returns, which, as we have seen, can under-perform the market even for long periods.

Keeping in mind these concepts, I want to assess if a multi-factor investment approach can lead to better results than a single-factor investment.

After the construction of the earlier mentioned multi-factor portfolios, it is possible to compare the performance of 9 assets: the 6 risk factors plus the 3 new portfolios. The following figures report the cumulative performance in different sample periods: 1975-2018, 2000-2018, 2007-2018, 2007-2009, 2010-2012, 2015-2018. Moreover, in each period I reported mean, volatility, skewness, kurtosis and Sharpe ratio.

The evidence is that, in the very long-run, considering the entire sample period, there are single-factor portfolios that achieve higher cumulative returns than the multi-factor ones: they are MKT and MOM. However, they are characterized by high volatility and, concerning the Momentum risk factor, very high kurtosis (11.49). The relevant indicator to consider is the Sharpe ratio. In fact, all the 3 multi-factor portfolios exhibit lower volatilities and higher Sharpe ratios in the period 1975-2018 but higher kurtosis than all the risk factors with the exception of MOM and RMW.

A high kurtosis indicates that the distribution of returns tends to present heavy tails; namely, the probability of experiencing extreme returns is high.

Moreover, the selected multi-factor portfolios and almost all the single factors tend to be negatively skewed in all the considered periods, with the exception of SMB and CMA. Skewness is a measure of asymmetry of the distribution. Hence, a negative skewness indicates that negative returns are more frequent than positive returns.

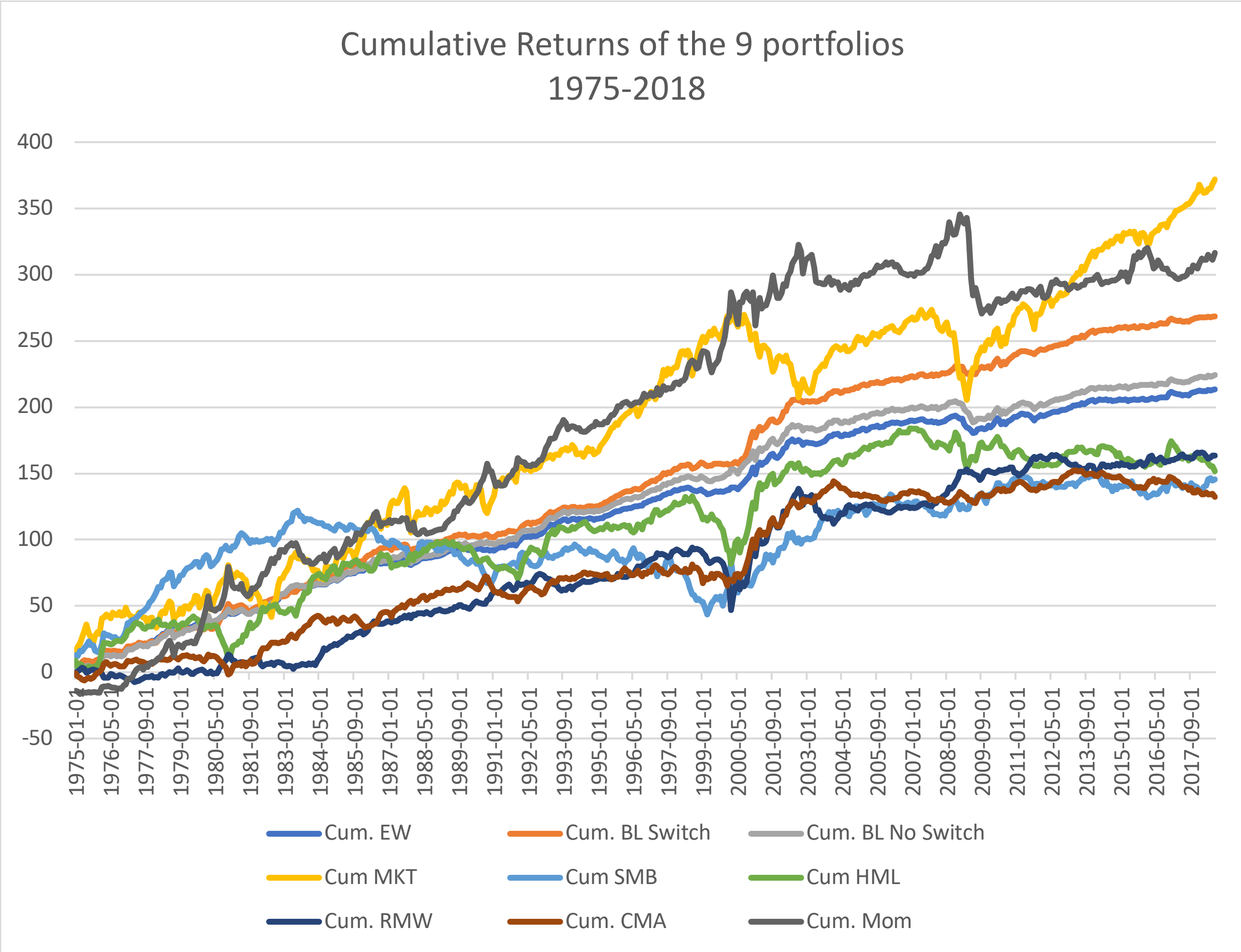

Figure 4.12 - Cumulative returns from January 1975 to August 2018

1975-2018	MKT	SMB	HML	RMW	CMA	MOM	EW Portfolio	BL SWITCHING PTF	BL STATIC PTF
Mean	0,71	0,28	0,29	0,31	0,25	0,60	0,41	0,51	0,43
Volatility	4,39	2,94	2,88	2,26	1,94	4,34	1,16	1,21	1,26
Skewness	-0,67	0,37	0,18	-0,39	0,41	-1,42	-0,48	0,48	-0,77
Kurtosis	2,37	4,40	2,13	12,32	2,13	11,49	5,25	3,55	5,49
Sharpe Ratio	0,16	0,09	0,10	0,14	0,13	0,14	0,35	0,42	0,34

Table 4.13 - Mean, Volatility, Skewness, Kurtosis and Sharpe Ratio of the 9 portfolios from January 1975 to August 2018

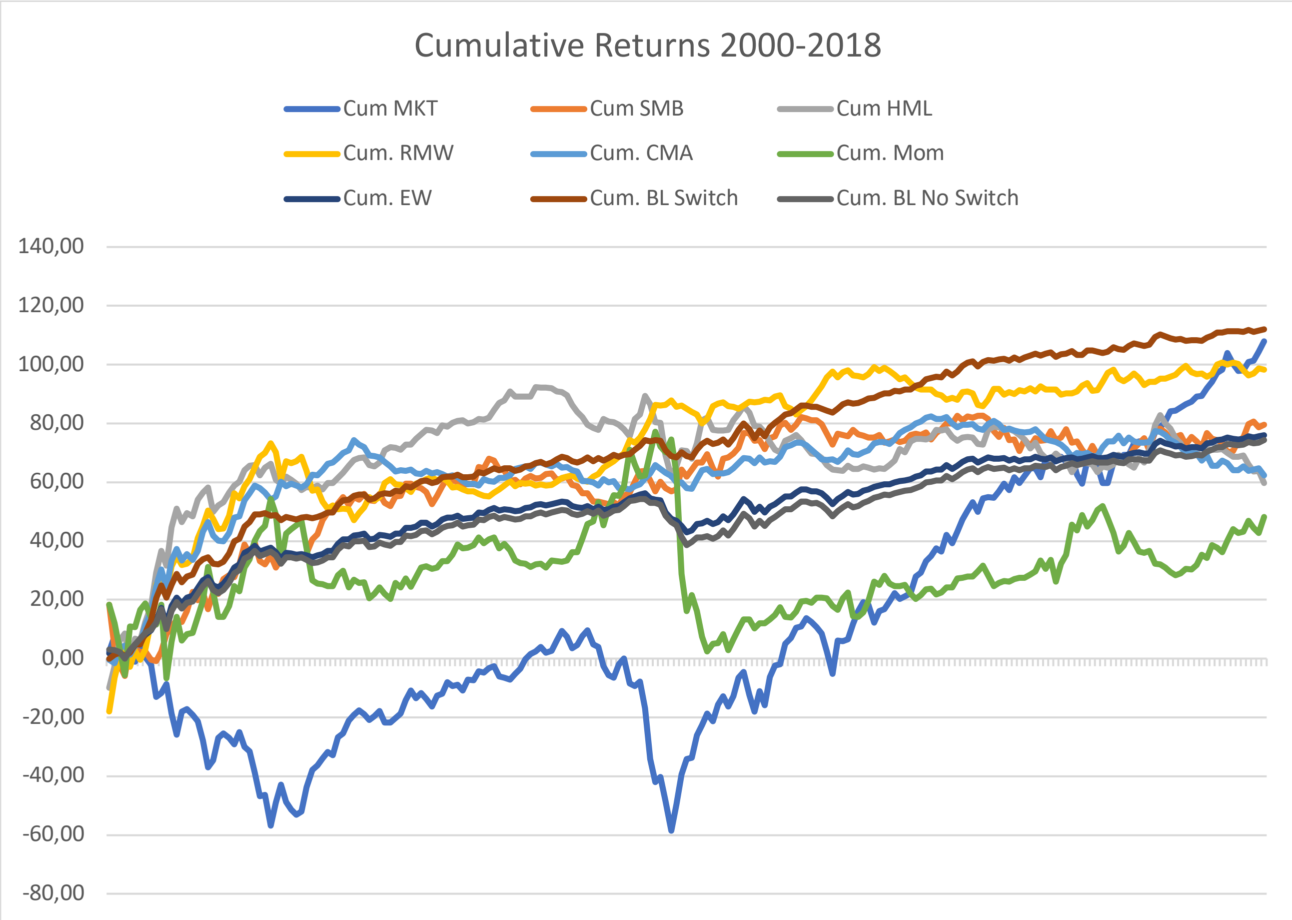

Figure 4.14 - Cumulative returns from January 2000 to August 2018

2000-2018	MKT	SMB	HML	RMW	CMA	MOM	EW Portfolio	BL SWITCHING PTF	BL STATIC PTF
Mean	0,46	0,37	0,27	0,41	0,30	0,22	0,34	0,50	0,33
Volatility	4,30	3,11	3,16	2,97	2,08	5,38	1,32	1,39	1,44
Skewness	-0,63	0,53	0,27	-0,42	0,99	-1,53	-0,04	0,72	-0,40
Kurtosis	1,08	6,68	2,83	8,69	3,01	9,90	3,23	3,59	3,84
Sharpe Ratio	0,11	0,12	0,08	0,14	0,14	0,04	0,26	0,36	0,23

Table 4.15 - Mean, Volatility, Skewness, Kurtosis and Sharpe Ratio of the 9 portfolios from January 2000 to August 2018

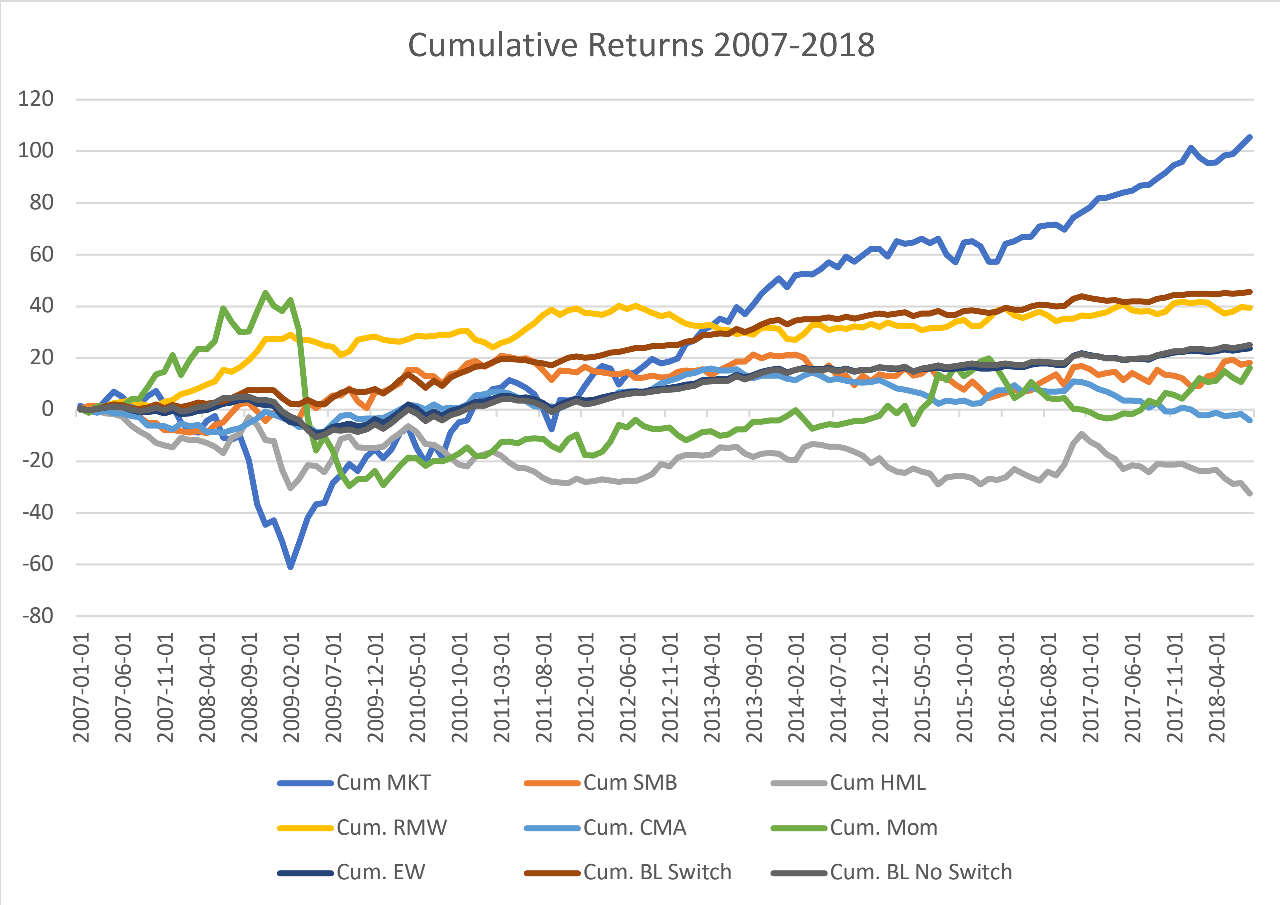

Figure 4.16 - Cumulative returns from January 2007 to August 2018

2007-2018	MKT	SMB	HML	RMW	CMA	MOM	EW Portfolio	BL SWITCHING PTF	BL STATIC PTF
Mean	0,75	0,13	-0,23	0,28	-0,03	0,11	0,17	0,33	0,18
Volatility	4,27	2,40	2,67	1,61	1,44	4,77	1,08	1,09	1,15
Skewness	-0,75	0,28	0,15	0,11	0,21	-2,83	-0,55	-0,28	-0,83
Kurtosis	1,94	-0,03	2,46	0,26	-0,26	19,23	1,71	0,74	1,82
Sharpe Ratio	0,18	0,05	-0,09	0,17	-0,02	0,02	0,16	0,30	0,15

Table 4.17 - Mean, Volatility, Skewness, Kurtosis and Sharpe Ratio of the 9 portfolios from January 2007 to August 2018

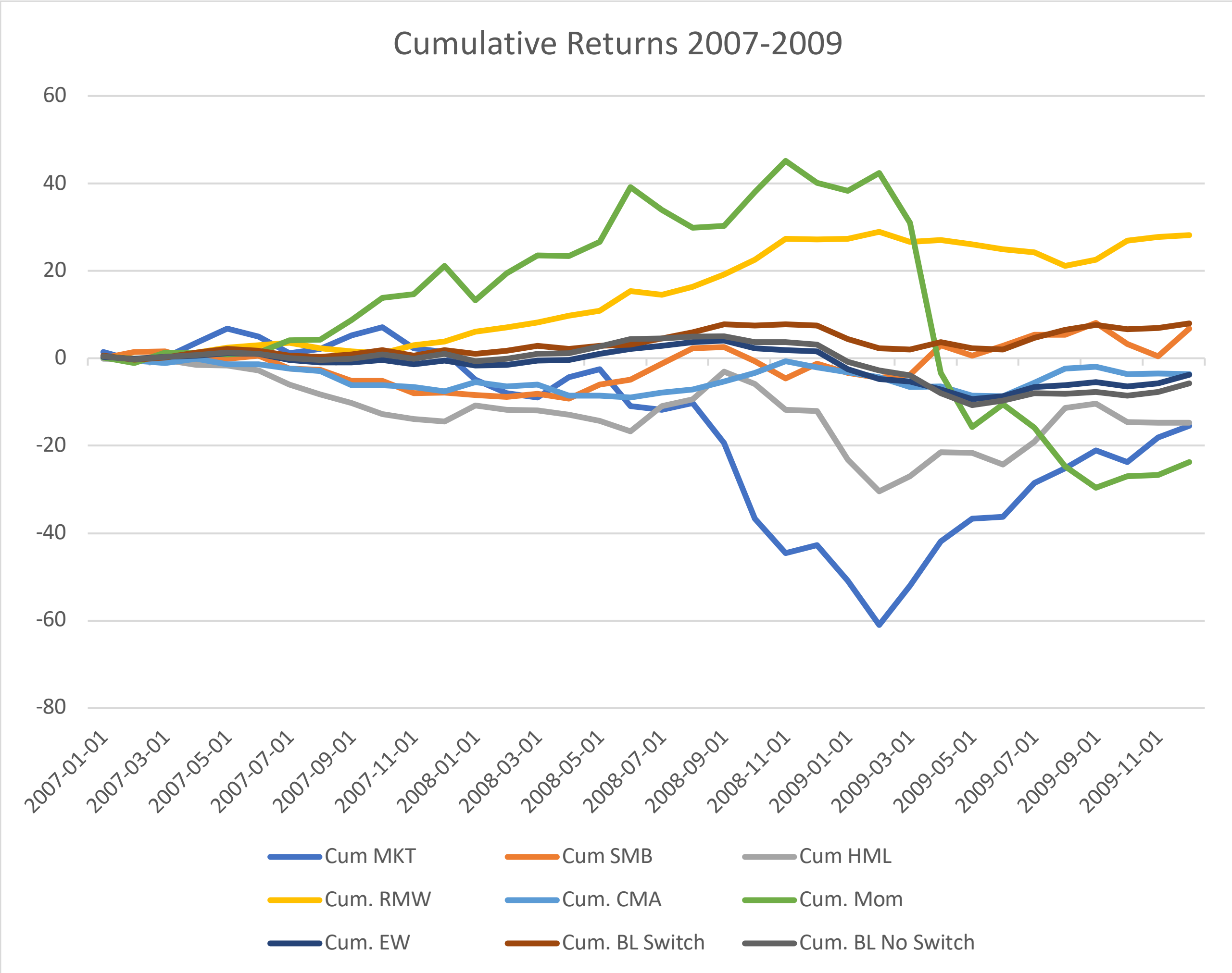

Figure 4.18 - Cumulative returns from January 2007 to December 2009

2007-2009	MKT	SMB	HML	RMW	CMA	MOM	EW Portfolio	BL SWITCHING PTF	BL STATIC PTF
Mean	-0,43	0,19	-0,41	0,78	-0,10	-0,66	-0,10	0,22	-0,16
Volatility	5,82	2,67	3,78	1,75	1,54	7,96	1,27	1,21	1,42
Skewness	-0,71	0,48	-0,13	0,40	0,42	-2,25	-0,94	-0,54	-1,09
Kurtosis	0,74	0,16	1,28	0,53	0,01	8,44	1,45	0,53	1,30
Sharpe Ratio	-0,07	0,07	-0,11	0,45	-0,06	-0,08	-0,08	0,18	-0,11

Table 4.19 - Mean, Volatility, Skewness, Kurtosis and Sharpe Ratio of the 9 portfolios from January 2007 to December 2009

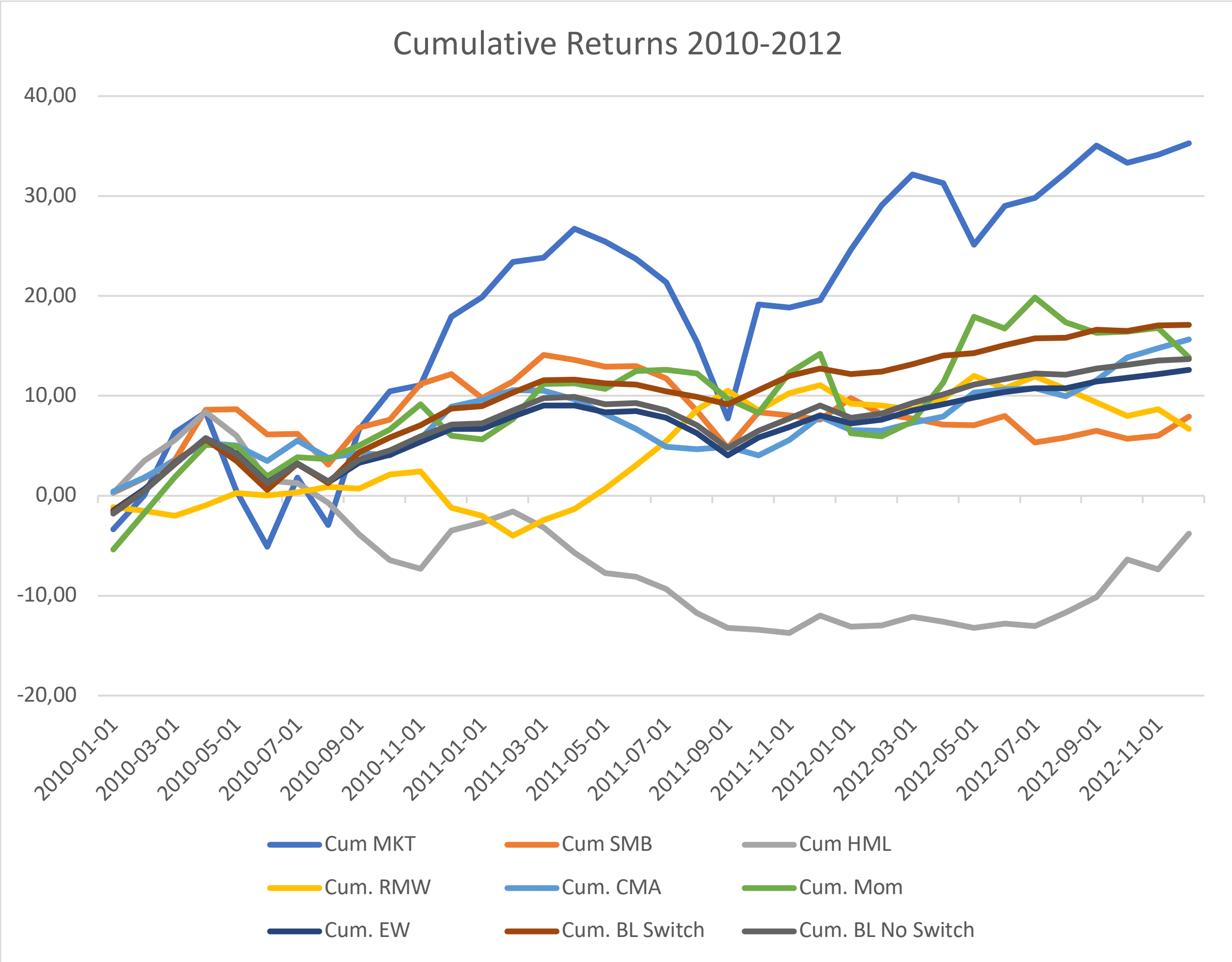

Figure 4.20 Cumulative returns from January 2010 to December 2012

2010-2012	MKT	SMB	HML	RMW	CMA	MOM	EW Portfolio	BL SWITCHING PTF	BL STATIC PTF
Mean	0,98	0,22	-0,11	0,19	0,43	0,38	0,35	0,48	0,38
Volatility	4,58	2,07	2,05	1,56	1,32	2,90	1,32	1,38	1,40
Skewness	-0,05	0,18	0,24	-0,25	0,01	-0,58	-0,56	-0,35	-0,55
Kurtosis	-0,13	-0,08	-0,36	-0,45	-0,75	1,00	-0,02	0,28	-0,23
Sharpe Ratio	0,21	0,11	-0,05	0,12	0,33	0,13	0,26	0,35	0,27

Table 4.21 - Mean, Volatility, Skewness, Kurtosis and Sharpe Ratio of the 9 portfolios from January 2010 to December 2012

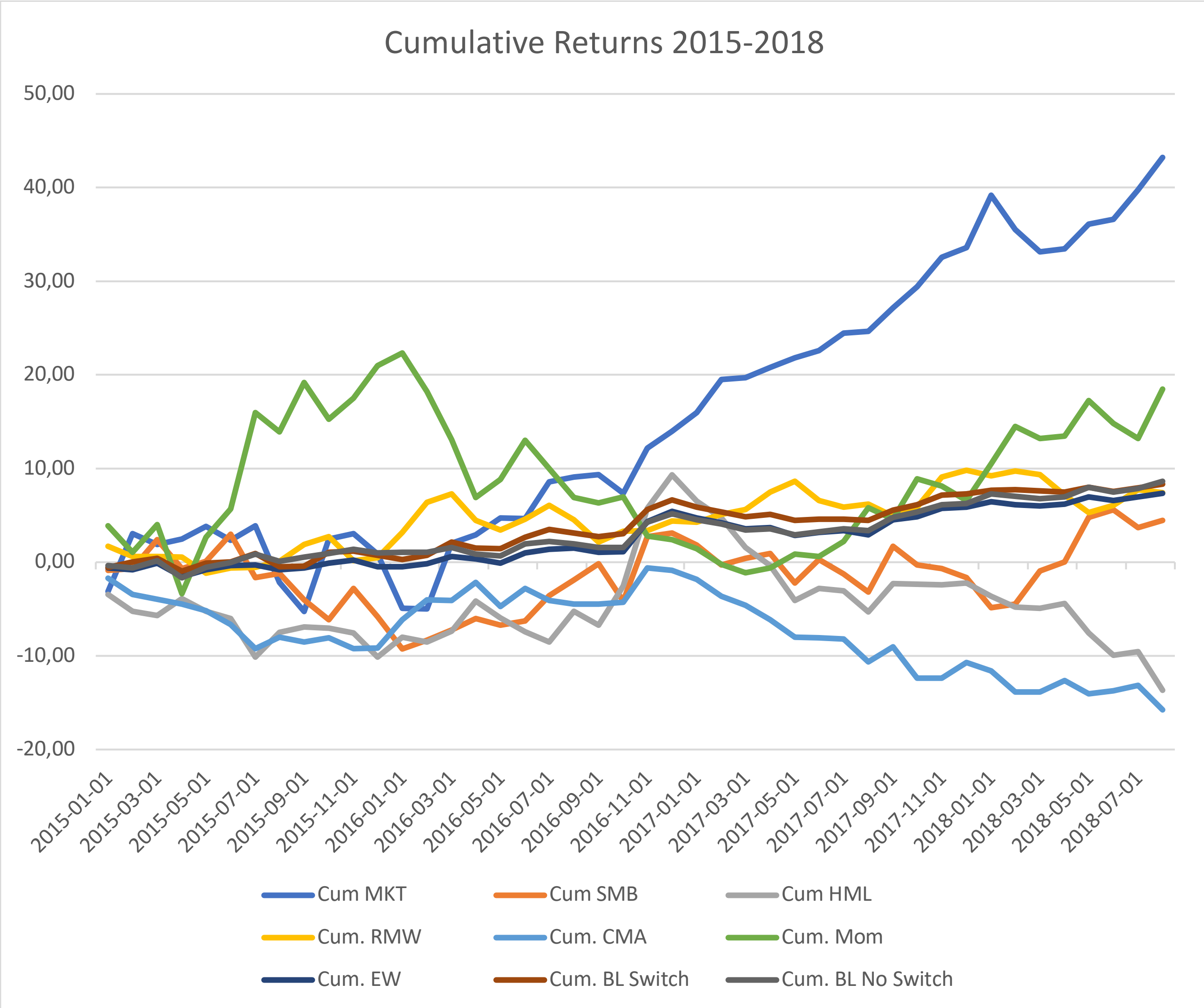

Figure 4.22 - Cumulative returns from January 2015 to August 2018

2015-2018	MKT	SMB	HML	RMW	CMA	MOM	EW Portfolio	BL SWITCHING PTF	BL STATIC PTF
Mean	0,98	0,10	-0,31	0,17	-0,36	0,42	0,17	0,19	0,20
Volatility	2,98	2,56	2,53	1,49	1,57	3,63	0,76	0,77	0,75
Skewness	-0,10	0,38	1,02	-0,14	0,47	0,17	1,47	0,54	0,52
Kurtosis	0,47	-0,04	1,73	-0,31	0,00	0,06	5,47	1,16	2,20
Sharpe Ratio	0,33	0,04	-0,12	0,11	-0,23	0,12	0,22	0,25	0,26

Table 4.23 - Mean, Volatility, Skewness, Kurtosis and Sharpe Ratio of the 9 portfolios from January 2015 to August 2018

Similar considerations can be done restricting the sample period from 2000 to 2018 and from 2007 to 2018. The dynamic portfolio is the best performer in risk-adjusted terms.

Considering the years of the last financial crisis (2007-2009), the BL switching portfolio is outperformed by the RMW factor, but it still achieves a positive Sharpe ratio, while all the other portfolios have negative risk-adjusted returns.

However, after the financial crisis of 2008, the performance of the market risk factor improves significantly. In the period 2010-2012 the multi-factor portfolios still perform better but there is a slightly difference.

Finally, from 2015 to 2018 the MKT factor becomes the best performer.

The last 3 years and the years of the financial crisis provide an evidence that the BL switching portfolio can be outperformed by a single factor over some periods. However, the performance of all the multi-factor portfolios seems to be very stable. Nevertheless, if the economic scenario is particularly negative only the dynamic portfolio remains able to produce non-negative risk-adjusted performance among the multi-factor portfolios.

5. Conclusions

Trusting the results of this work, the presence of multiple regimes in the economy is confirmed from a statistical point of view.

In particular, using the CLI and the CPI as indicators to estimate growth and inflation, the business cycle should be divided in 4 regimes.

An important evidence is that inflation negatively affects returns. Cyclical factors, like MKT and MOM, seem to perform better in regimes characterized by falling inflation even if the growth is moderate (Goldilocks), rather than regimes in which the acceleration is high for both variables (Heating Up). Other factors, like CMA, HML and RMW seem to be counter-cyclical or defensive: they can be used to stabilize the performance of a multi-factor portfolio. In particular, CMA and RMW exhibit very low volatility, regardless of the economic regime.

Moreover, all the factors are poorly or negatively correlated, with very few exceptions over some moments.

Hence, it is possible to assert that *Factor Investing* can represent a solution to the diversification issue concerning the traditional asset classes. However, the investment in a single factor is not recommendable, in particular for short horizons. A portfolio that includes different factors seems to be the most suitable solution for a typical risk-averse investor that wants to achieve a stable performance over time and wants to avoid excessive fluctuations in returns and, consequently, in the portion of his consumption stream that depends on financial wealth.

Nevertheless, the allocation of the available funds among the risk factors cannot be casual. An in-depth analysis of the economic cycle and the application of a suitable portfolio optimization tool, like the Black-Litterman model, lead to a better risk-adjusted performance. In particular, the allocation should be dynamic. The portfolio should anticipate the business cycle and switch its composition accordingly. In doing so, the application of a Markov-Switching model can be very useful. Indeed, the estimated transition probabilities and the expected duration of each regime are the guide to predict regime and portfolio switches.

Obviously, in reality, to evaluate the convenience of such dynamic strategy one has to consider fees and costs related to the active management.

Moreover, as I discussed in the first chapter, the choice among factors should be restricted to a small but sufficient number in order to avoid redundancy. An issue is that in reality it is not easy to exactly replicate the "academic" factors. In fact, the factors defined in academic papers are long-short portfolios with high turnover.

The implementation of long-short strategies is often limited by regulatory constraint or, in general, it is too expensive.

Actually, in most cases, the factor exposure is achieved starting from a market capitalization index and modifying the weights in order to increase the exposure towards those securities that seem to reflect the desired factor characteristic. Such long-only constraint is sometimes necessary to make a factor strategy investable.

In other words, there is a trade-off between investability and exposure to the pure factor. Therefore, the challenge for asset managers is to provide a good compromise.

In synthesis, in reality there are many constraints. However, I retain that understanding theory and statistical relationships between the academic factors can help investors and asset managers to better focus on the desired target. An investor who wants to implement a factor-based strategy needs, first of all, to know what a factor is. Consequently, he has to identify which factors can be considered as such and analyse their characteristics and performances across different states of the economy. Moreover, he has to figure out the optimal portfolio composition with the support of adequate optimization tools. Finally, looking at the available financial products, he has to take the most suitable exposure.

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SUMMARY

Introduction

The idea that a good portfolio diversification can be obtained by simply distributing wealth among the traditional asset classes is, by now, a myth to dispel. Many evidences, in particular after the Financial Crisis of 2008, show that the economy is characterized by regime changes that can determine a sudden increase in correlation.

A possible solution could be represented by the *Factor Investing* approach: namely, the investment in the risk factors known in literature, such as those of Fama and French. In fact, factors offer considerable risk premiums in the long-run and exhibit a low or negative correlation, even in phases characterized by turbulent markets.

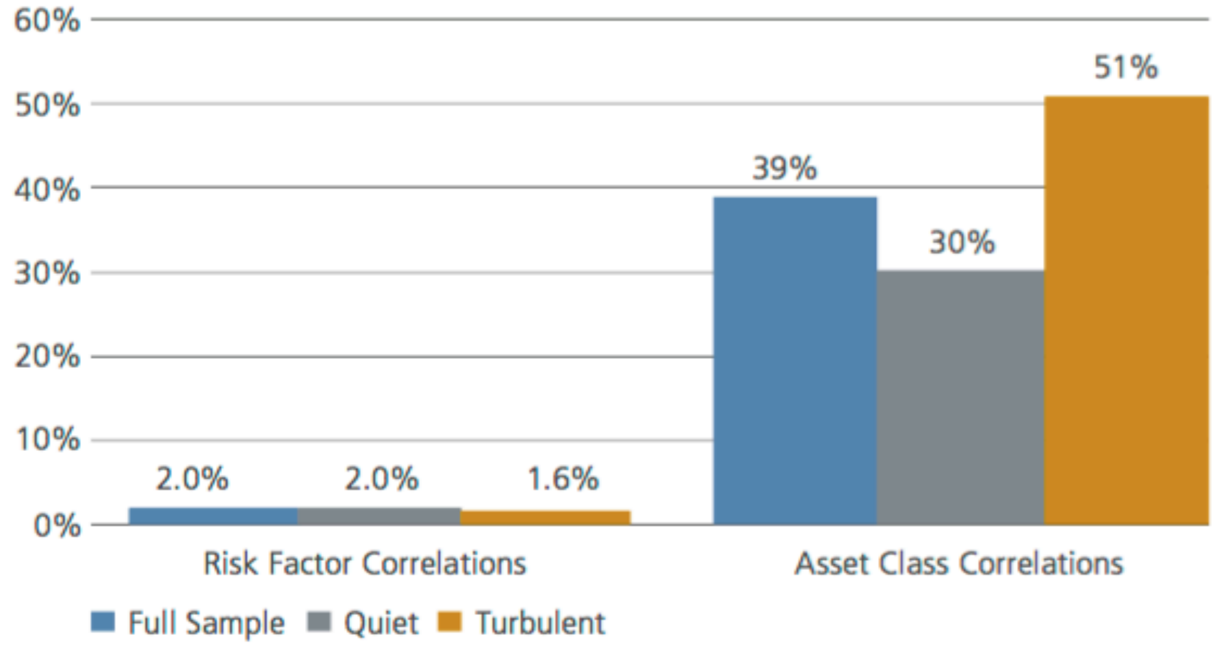

Figure 1 - Average cross-correlation from March 1994 to December 2009
Source: "*The Myth of Diversification: Risk Factors vs. Asset Classes*", Pimco (2010).

However, it seems that the returns on all the factors, taken individually, suffer a certain cyclicality and can experience negative performances even for long periods.

An investment in multiple factors would therefore seem advisable.

In order to understand the criteria of an optimal allocation among factors it is necessary, in my opinion, to study in depth the business cycle and the performance of the most relevant factors in various states of the economy.

Some authors suggest analysing the cycle along two main lines: growth and inflation. This analysis leads to the identification of multiple regimes, going beyond the simple distinction between expansion and recession phases. However, in almost all cases this approach has been limited to counting phases, carried out heuristically and knowing in advance the characteristics that each phase should have.

I will therefore try, in this work, to use a more robust approach from a statistical point of view, adopting a Markov-Switching model to test the presence of multiple regimes, determining their optimal number.

In the first chapter, I briefly review the main evidences against the Efficient Market Hypothesis and the Capital Asset Pricing Model, showing that the idea that we deal with a multi-factor world is, by now, widely acknowledged. Then, I explain why the asset pricing equation should be taken as reference to understand what factors are, which is their nature and why they are cyclical.

Moreover, the knowledge of pricing theory may also help to distinguish between factors and simple anomalies and avoid redundancy. In fact, in recent years, hundreds of factors have been discovered. Therefore, it seems necessary to find more restrictive criteria to assess their statistical significance. The objective is to better guide investment choices and narrow the factors down to a limited number but sufficient to guarantee a good level of diversification, avoiding the danger of investing in apparently different instruments that are, actually, exposed to the same sources of risk.

In the second chapter, I will implement the Markov-Switching model. The dependent variables are the Composite Leading Indicator (CLI) to compute growth and the Consumer Price Index (CPI) to compute inflation. The presence of multiple regimes is investigated from a statistical point of view. Moreover, I will try to determine the optimal number of regimes and find some correspondence between the estimated regimes and some regime classification known in literature. Then, the performance of six risk factors is analysed in every estimated regime.

The selected factors are: Market, Size, Value, Profitability, Investment and Momentum.

In the third chapter, the information collected through the implementation of the MS model and the performance analysis of the factors will be used to formulate investment views to be included in a Black-Litterman model. The goal is to find an optimal allocation in each estimated regime, building a dynamic portfolio with weights that change in anticipation of a regime switch.

Such dynamic portfolio will be compared to a static Black-Litterman portfolio. Finally, I will assess whether multi-factor strategies can outperform single-factor portfolios.

Theoretical framework of Factor Investing

“Expected returns vary across time and across assets in ways that are linked to macroeconomic variables, or variables that also forecast macroeconomic events; a wide class of models suggests that a ‘recession’ or ‘financial distress’ factor lies behind many asset prices”

John Cochrane, Asset Pricing (2001)

The theory related to Asset Pricing can be used to understand the advantages of a multi-factor investing approach. In particular, I want to point out that the interpretation of such theory should lead to a necessary analysis of the cyclicality of risks and returns.

Prices and returns are affected by macro-economic variables. The intuition of Cochrane is that a single equation can be used to price any asset and can be adapted to various circumstances.

Prices equal discounted expected payoffs:

(2.1) $$P_t = E_t(M_{t+1}X_{t+1})$$

where P_t denotes the asset price, M_{t+1} the stochastic discount factor and X_{t+1} the asset payoff.

Then, recalling that the covariance between 2 random variables can be written as the difference between the expectation of the product and the product of the expectations, the asset pricing equation becomes:

(2.5) $$E_t(M_{t+1}X_{t+1}) = E_t(M_{t+1})E_t(X_{t+1}) + cov_t(M_{t+1}, X_{t+1})$$

$$= \frac{1}{Rf} E_t(X_{t+1}) + cov_t(M_{t+1}, X_{t+1})$$

The first term is the standard discounted present-value formula while the second term is a risk-adjustment.

The SDF can be explicated in terms of marginal utility of consumption as follows:

(2.6) $$M_{t+1} = \beta \frac{u'(C_{t+1})}{u'(C_t)}$$

Therefore, it is a marginal rate of substitution; namely, the rate at which investors are willing to exchange consumption at *t+1* for consumption at time *t*.

Asset prices are lowered if their payoff covaries positively with consumption and, hence, negatively with the SDF.

Factors, like the Fama-French ones, are constructed by applying long-short strategies. Hence, their payoff can be considered an excess return. According to the Consumption Capital Asset Pricing Model (C-CAPM) the expected excess return can be rewritten in terms of the market price of risk $\lambda_{\Delta c}$ and the amount of risk β:

(2.10) $$E_t\left(r_{t+1}^{e,i}\right) = cov_t\left[\Delta_{c_{t+1}}, r_{t+1}^{e,i}\right] / \sigma_{\Delta_{c_{t+1}}}^2 \, \gamma \sigma_{\Delta_{c_{t+1}}}^2$$

The more risk averse people are, or the riskier their environment, the larger is the premium they require to hold risky (high beta) assets.

Taking the aggregate consumption level as an indicator of the business cycle, an asset that has a positive covariance with consumption is pro-cyclical. In fact, it is reasonable to think that consumption grows during favourable regimes of the economic cycle.

Investors desire a level of consumption that is steady over time and across different states of the economy. Moreover, their risk aversion exhibits counter-cyclical variation: it increases during contractions and decreases during expansions.

Pro-cyclical assets make their consumption stream more volatile. An investor that wants to increase its exposure toward one asset included in his portfolio, has to consider the covariance term in determining the effect on the volatility of consumption.

In general, it is clear that economic agents are willing to give up a portion of expected return to protect themselves against low consumption states. This is exactly the logic behind an insurance policy: you pay a premium to hedge some risk.

Investors do not like excessive fluctuations in the performance of their portfolio because they do not like fluctuations in their consumption stream.

The Factor Investing approach has experienced a good success in the last years since there are some risk factors or investment styles, for example Quality and Low Volatility, that have been able to guarantee a positive return even in non-favourable economic conditions.

The essence of factors is strictly linked to consumption and economic cycle. According to Cochrane (2001), and in conformity with the concept of time-varying risk aversion, there are some periods and states of the economy in which investors particularly care about the performance of their portfolio. In such states they are very sensitive to losses and willing to trade-off a sizable portion of return to have an insurance.

Factors are those variables that predict such states. They are able to forecast macro-economic events.

A single asset pricing formula can be used to understand the macro-economic risks underlying each security's value. This is coherent with the idea that many factors can lie behind a single asset.

The equation is unique for every asset pricing model: CAPM, APT, ICAPM and so on. What changes is the specification of the stochastic discount factor.

The SDF reflects the marginal utility growth. Hence, each pricing model uses a different proxy.

In factor models, the consumption-based expression is replaced as follows:

(2.11) $$m_{t+1} = a + b' f_{t+1} \approx \beta \frac{u'(C_{t+1})}{u'(C_t)}$$

where a and b are free parameters.

The expression is equivalent to a multiple beta model:

(2.12) $$E(R_{t+1}) = \gamma + \beta'\lambda$$

where β is the vector of regression coefficients of returns on factors f.

As in the APT, the number and the identity of factors is not specified. Hence one should look for good proxies of the marginal utility growth.

How many factors exist? Which are they?

These are the questions to which many authors and economic agents tried to answer.

The world of asset management has provided numerous financial instruments to investors that want to take a position in risk factors, especially in recent years. However, in a context in which hundreds of candidates[30] have been identified in literature, it is necessary to use some criteria to better guide investment choices and make investors aware of the risk and return profiles they face by choosing one factor rather than another or a mix of factors. Such scenario could be dispersive for those investors that want to approach a factor-based investment.

The proliferation of risk factors needs more discipline.

Multiple factors, even if called with different names and apparently not similar, may be redundant within a portfolio if their marginal contribution in explaining returns is not statistically different from zero.

Hence, I want to discuss some criteria, reported in recent empirical studies, that can be used to control the identification of factors.

A possible way is related to the estimation and testing of the alpha of a regression of the new factor onto existing factors.

Carhart (1997) improved the predictive power of the standard Fama-French 3-factor model by adding the Momentum factor. Furthermore, Fama and French (2015) extended their original model introducing the Profitability and Investment factors.

For example, the seasonality risk factor of Heston and Sadka (2008) may improve the predicting power of the FF 5-factor model while, since it is correlated with Momentum, would fail considering the Carhart 4-factor model. However, it is clear that such

[30] In 2001 Cochrane coined the term 'factor zoo'. Other contributions are provided by Harvey et al. (2015), McLean and Pontiff (2016), and Hou et al. (2017).

method suffers the arbitrariness related to the choice of the asset pricing model taken as benchmark.

Furthermore, according to Harvey et al. (2015) the usual cut-off level of the t-statistic should be increased from 2 to 3. Since hundreds of papers and factors attempt to explain the cross-section of expected returns, a new factor needs a much higher hurdle and a multiple-testing framework is needed.

They analysed a group of 124 factors discovered no earlier than 2000, showing that almost all the new factors are above a *t*-ratio of 1.96, corresponding to a 5% significance level. However, only 12 out of the 124 considered factors are above a threshold of 3.

The cut-off value of the t-statistic should increase with time and with the introduction of new factors in literature. Using Bonferroni-Holm method[31], the benchmark *t*-statistic starts at 1.96 and increases to 3.78 by 2012. It reaches 4.00 in 2032.

The results show that, in addition to the well-known Value (HML), Momentum (MOM) and Market (MKT) factors, only DCG and SRV[32] are significant across all the *t*-statistic adjustments. Moreover, EP, LIQ, and CVOL[33] are sometimes significant, and the rest are never significant.

Other methods are based on risk premia.

The underlying idea is that factors should provide a sizable and statistically significant premium as compensation for risk, while anomalies should disappear, once discovered and published in literature.

Linnainmaa and Roberts (2016) examined 36 anomalies showing that the returns on the related strategies decreased on average by 58% after the publication. Similar results have been presented by McLean and Pontiff (2016) in 97 strategies.

Berkin and Swedroe (2016), relying on risk premia, identify some characteristics required to be considered as a risk factor.

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[31] A method that controls the probability that one or more Type I errors will occur by adjusting the rejection criteria of each of the individual hypotheses. It is used to counteract the problem of multiple comparisons.

[32] Durable Consumption Goods; Yogo (2006) and Short-Run Volatility.

[33] Earnings-Price ratio, Basu (1983); Liquidity, Pastor and Stambaugh (2003); Consumption Volatility, Boguth and Kuehn (2012).

In their study the factors that match with such criteria are: Market, Size, Value, Momentum, Profitability and Quality, Term, Carry, Low-Volatility, Default.

However, an analysis based on risk premia, even if statistically significant, would be misleading. In fact, two or more factors that are exposed to the same underlying risk may provide the same premium. What matters is the ability to add pricing information to the existing factors.

Using asset pricing theory as guide, we can say that the exercise aimed at the identification of all the existing risk factors is equivalent to the exercise to identify the true SDF.

Feng, Giglio and Xiu (2019) proposed a new method to assess whether a new factor adds explanatory power for asset pricing, relative to the existing set of factors.

Such method[34] allows to test the marginal importance of each risk factor in explaining the cross-section of returns and, moreover, highlights how a shock to a single factor can affect marginal utility.

The most innovative aspect is to estimate the role of the additional factor in explaining marginal utility. Namely, the estimation of its factor loading[35].

A positive factor loading implies that high values of the factor correspond to good states of the economy, in which consumption is high and marginal utility is low.

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[34] Double-selection LASSO method of Belloni et al. (2014). It is based on the Fama-Macbeth two-pass methodology applied in high dimensional settings. It selects factors that are either useful in explaining the cross section of expected returns or are useful in mitigating the omitted variable bias.

[35] Its coefficient in the stochastic discount factor. Making inference on the SDF is important in order to have an economic interpretation of results.

		(1) DS		(2) SS		(3) FF3		(4) No Selection		(5) Avg. Ret.	
id	Factor Description	λ_s (bp)	tstat (DS)	λ_s (bp)	tstat (SS)	λ_s (bp)	tstat (OLS)	λ_s (bp)	tstat (OLS)	avg.ret. (bp)	tstat
136	Cash holdings	-34	-0.42	15	0.17	10	0.54	-18	-0.16	13	0.98
137	HML Devil	54	1.04	-13	-0.25	-100	-2.46**	68	0.84	23	1.46
138	Gross profitability	20	0.48	3	0.06	23	2.00**	13	0.26	15	1.45
139	Organizational Capital	28	0.92	-1	-0.03	20	1.91*	16	0.41	21	2.05**
140	Betting Against Beta	35	1.45	38	1.50	36	2.25**	49	1.49	91	5.98***
141	Quality Minus Junk	73	2.03**	4	0.11	39	3.10***	50	1.04	43	3.87***
142	Employee growth	43	1.36	-4	-0.12	-12	-0.89	18	0.37	8	0.83
143	Growth in advertising	-12	-1.18	0	0.03	12	1.32	-2	-0.13	7	0.84
144	Book Asset Liquidity	40	1.07	5	0.12	20	1.59	20	0.42	9	0.79
145	RMW	160	4.45***	15	0.41	20	1.80*	74	1.48	34	3.21***
146	CMA	38	1.10	0	0.01	3	0.28	7	0.14	26	3.02***
147	HXZ IA	51	2.11**	5	0.21	21	1.94*	40	1.08	34	4.17***
148	HXZ ROE	77	3.37***	23	0.83	33	2.92***	104	2.87***	57	4.99***
149	Intermediary Risk Factor	112	2.21**	60	1.19	4	0.08	22	0.32		
150	Convertible debt	-15	-1.36	-39	-3.22***	26	3.32***	17	1.01	11	1.70*

Table 2.4 - Testing for factors introduced in 2012-2016
Source: *"Taming the Factor Zoo: A Test of New Factors"*, Feng et al. (2019).

The table reports the estimates of the factor loadings and the relative t-statistic. Looking at the last column it is possible to see how a relevant number of new factors provide a statistically significant risk premium but only few of them remain significant if we consider their factor loading estimates.

The results show that the new factors are spurious or redundant in most cases. The use of the Double-Selection (DS) methodology reduces the number of statistically significant risk factors, as we can see looking at the first column.

It is interesting to notice that, according to such study, the Fama-French Investment factor is not significant even if we take in account the 3-factor model as benchmark. In fact, the third column shows a t-stat equal to 0.28 for the CMA factor loading. However, the Profitability factor (RMW) is highly significant, with a t-stat equal to 4.45.

Considering a threshold value above 3, as discussed before, only RMW and HXZ ROE can be considered able to add pricing information.

Investigation of Economic Regimes and Performance Analysis

Data are related to United States. The frequency is monthly, and the sample period goes from January 1972 to August 2018.
I considered the time series of the Composite Leading Indicator (CLI) for economic growth and the Consumer Price Index (CPI) for inflation. Moreover, I chose six risk factors: Market (MKT), Value (HML), Size (SMB), Profitability (RMW), Investment (CMA) and Momentum (MOM). The data sources are the OECD site for CLI and CPI and the Kenneth R. French Data Library for the six risk factors.

The objective of this study is to relate the *Factor Investing* approach to the business cycle.

The idea that the economic cycle should be analysed by only looking at growth is, by now, reductive and limiting and would only bring out the simple distinction between expansive and recessive phases.

A multi-regime approach is already widely acknowledged; in particular, the identification of four phases seems to be a choice that achieves huge consensus and diffusion. Since asset returns are affected by real growth, a natural choice is to separate "good" and "bad" regimes depending on inflation.

Indeed, regimes are often identified in a heuristic way, knowing in advance which characteristics each state should have, not relying on a pure statistical evidence principle.

My aim is to implement a Markov-Switching model that allows the testing of the number of regimes in the economy and the statistical significance of the relevant parameters of each regime.

The application of a Markov-Switching model allows the estimation of a drift parameter and a variance both for CLI and CPI in each state of the economy.

Moreover, I will obtain a transition probability matrix to evaluate which state is the most likely to occur, given the current one, and the persistence of each regime. Namely, its average duration.

First of all, I wanted to assess whether a model with 4 states is better than a model with a different number of states using information criteria.

I estimated Markov-Switching models with 2, 3, 4, 5 and 6 states[36]. Then, I computed the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC) for each of them using the value of the Log Likelihood function and the number of estimated parameters.

The following equations are used to estimate the AIC and BIC (Stone, 1979; Akaike, 1974) of a model:

$$\begin{aligned} \text{AIC} &= -2 * Ln(L) + 2 * K \\ \text{BIC} &= -2 * Ln(L) + Ln(N) * K \end{aligned}$$

where L is the value of the likelihood, N is the number of observations, and k is the number of estimated parameters.

Obviously, the number of estimated parameters increases with the number of states. AIC and BIC are penalizing criteria, used to compare models with a different number of estimated parameters. In fact, increasing the number of parameters can lead to an over-fitting issue.

The AIC is designed to pick the model that produces a probability distribution with the smallest discrepancy from the true distribution.

The BIC is an estimate of a function of the posterior probability of a model being true, under a certain Bayesian setup.

According to both the information criteria, a model with 4 states should be preferred since a lower AIC or BIC value indicates a better fit. Moreover, a model with 5 states is better than a model with 3 states according to AIC, while the opposite is true according to BIC[37]. A model with 6 states can be preferred only to a model of 2 states according to AIC, while according to BIC it has the worst fit.

Hence, I chose to pick the model with 4 states.

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[36] I used the MS_Regress package on MatLab, provided by Marcelo Perlin. The estimation is based on Maximum Likelihood.

[37] AIC and BIC differ by the way they penalize the number of parameters of a model. More precisely, BIC criterion will induce a higher penalization.

	2 States	3 States	4 States	5 States	6 States
AIC	3882.55	3697.06	3545.13	3603.67	3778.46
BIC	3951.92	3811.02	3713.59	3826.63	4075.74

Table 3.3 - AIC and BIC in Markov-Switching models with 2, 3, 4, 5 and 6 states

I implemented a Vector Auto Regressive (VAR) Markov-Switching model in the form:

$$Y_t = \mu(s_t) + Y_{t-1} + \varepsilon_t$$

where:

$s_t =$ regime at time t

$Y_t =$ $[\Delta CLI_t, \Delta CPI_t]$ multivariate dependent variable

$\mu =$ $[\mu_{CLI}(s_t), \mu_{CPI}(S_t)]$ mean for each regime

$\varepsilon_t =$ $[\varepsilon_{CLI,t}, \varepsilon_{CPI,t}]$ stochastic error terms normally distributed with zero mean and constant variance.

Once obtained the output of the process, I analysed the estimated parameters in order to identify the 4 regimes. In doing so, I taken as reference the regime classification provided by Gupta et al (2014):

-Heating Up: rising growth, falling inflation.

-Slow Down: falling growth, falling inflation.

-Goldilocks: rising growth, falling inflation.

-Stagflation: falling growth, rising inflation.

	ΔCLI (p-value)	ΔCPI (p-value)
State 1	0.19 (0.00)	-0.36 (0.00)
State 2	0.23 (0.00)	0.89 (0.00)
State 3	0.16 (0.00)	0.55 (0.00)
State 4	-0.02 (0.00)	-0.22 (0.00)

Table 3.4 - Estimated drift parameters of growth and inflation in each regime. P-values in parentheses

Expected duration of Regime 1	14.72
Expected duration of Regime 2	10.67
Expected duration of Regime 3	2.26
Expected duration of Regime 4	9.54

Table 3.5 - Average duration (in months) of each regime

The four regimes have been identified by interpreting the estimated parameters. In particular, the main guide is provided by the sign of the intercept parameter (mean value). A positive intercept denotes an acceleration while a negative intercept a deceleration of the variable.

I also considered the expected duration and the table of estimated transition probabilities at each date.

State 1 seems to correspond to the Goldilocks regime (rising CLI, falling CPI) while State 4 should coincide with the Slow Down regime, since the drift parameters are both negative. One can notice that the intercept for CLI in State 1 is lower than in State 2 and State 3. The Goldilocks regime is therefore characterized by a moderate and not very volatile growth and by a falling inflation process. The results of the model seem consistent with the macroeconomic scenario of the last years, characterized by expansive monetary policies that have maintained interest rates artificially low.

Either State 2 and State 3 exhibit rising growth and inflation and should be good candidates for the Heating Up regime.

Hence, relatively to the intercept parameters, the results of the MS model seem to properly capture 3 out of the 4 regimes identified by MSCI. All except Stagflation, for which the intercept should have been positive for the inflation and negative for the growth.

In State 2 the acceleration is more pronounced than in State 3; the drifts are 0.23 and 0.89 against 0.16 and 0.55. State 2 is also more frequent and more durable: expected duration 10.67 versus 2.26. Therefore, I assigned the Heating Up regime to State 2. For State 3, instead of Stagflation, I decided to use the term Recovery, to indicate a phase in which the economy accelerates but does not reach the peak.

Moreover, looking at transition probabilities, we can make additional considerations. Once the economy ends up in this state, it tends to remain there for a short time and to flow very quickly into a phase of Goldilocks or Heating Up. Therefore, it can represent the phase that materializes immediately after a period of crisis or general slowdown. In fact, from Recovery it is not possible to switch to Slow Down while the opposite is likely; it is a sort of intermediate state in which both growth and inflation are recovering but not yet "at the peak" as in Heating Up.

Regime Classification

State 1: Goldilocks

State 2: Heating Up

State 3: Recovery

State 4: Slow Down

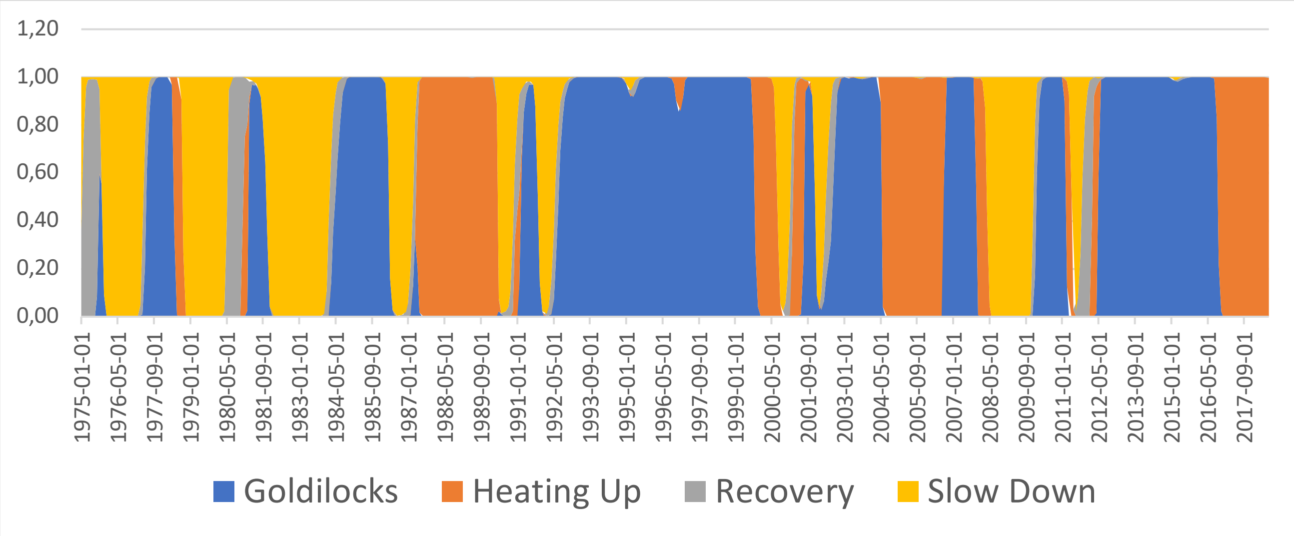

Figure 3.8 - Plot of filtered probabilities of regimes at each date

In a MS model the transition of states is stochastic and not deterministic. Even if we cannot know with certainty if there will be a switch or not, we can analyse the dynamics of the switching process by looking at the transition matrix:

	Goldilocks	Heating Up	Recovery	Slow Down
Goldilocks	93,2%	4,13%	26,6%	0%
Heating Up	5%	90,6%	17,5%	0,61%
Recovery	0,01%	0,04%	55,8%	9,87%
Slow Down	1,79%	5,2%	0%	89,5%

Table 3.9 - Transition probabilities matrix

After the identification of the regimes, I analysed the performance of the 6 factors using Sharpe ratios.

	Best Performers	Worst Performers
Goldilocks	MKT (3,66) MOM (2,29)	HML (0,44) CMA (0,75)
Heating Up	MOM (2,01) MKT (1,30)	CMA (0,47) HML (0,86)
Recovery	RMW (2,60) MOM (1,64)	HML (-0,83) CMA (0,07)
Slow Down	CMA (4,55) HML (3,63)	MOM (0,57) MKT (0,64)

Table 3.12 - The two most and worst performing factors in each regime according to the risk-adjusted performance

In synthesis, one can make the following classification.

MKT and **MOM** are **pro-cyclical factors**: they follow the trend and provide excess returns through trend participation.

SMB and **HML** are **counter-cyclical factors**: they move against the trend and provide excess returns through anticipation of the trend reversal.

RMW and **CMA** are **defensive factors**: they ensure a risk reduction and provide excess returns through reduction of loss of asset value.

Furthermore, a crucial aspect to consider is correlation. In fact, knowing the correlation, potential diversification benefits can be exploited in portfolio construction.

I computed correlations in each regime and the results show that all the risk factors are poorly or negatively correlated, regardless of the economic regime.

The diversification potential among factors is effective in every regime of the economic cycle, while it is well known that the correlation among the traditional asset classes tend to rise suddenly close to 1 in bad economic regimes.

Portfolio Optimization and Asset Allocation

Taking in account the results obtained in the MS model, I performed Black-Litterman optimization in order to find an optimal allocation in each state, considering a portfolio made of the six risk factors analysed until now.

I started by assuming that, in equilibrium, an investor that has no view would allocate his wealth among the six factors in equal parts. Then, I formulated views using the mean returns and variances estimated before. Combining the views' distribution with the distribution of implied equilibrium returns, I obtained the Black-Litterman returns and the final optimal BL weights.

	μ_{BL} Goldilocks	μ_{BL} HeatingUp	μ_{BL} Recovery	μ_{BL} SlowDown
MKT	0,40	0,25	0,36	0,00
SMB	0,06	0,11	0,37	0,23
HML	0,02	0,05	-0,19	0,63
RMW	0,04	0,05	0,56	0,27
CMA	0,01	0,02	-0,04	0,46
MOM	0,28	0,33	1,03	0,17

Table 4.6 - Black-Litterman returns in each regime

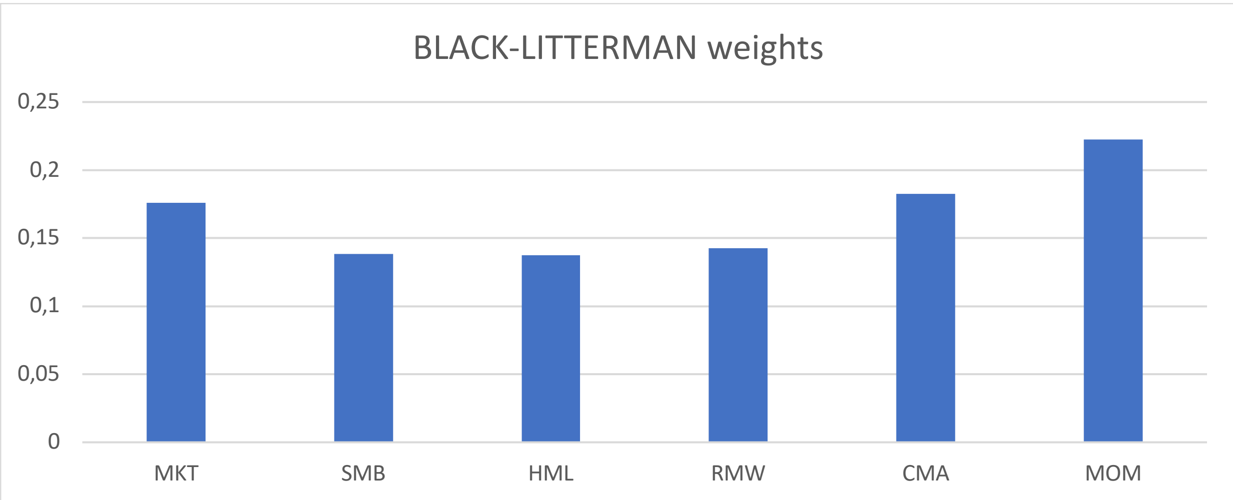

Figure 4.8 - Black-Litterman optimal allocation all over the sample period

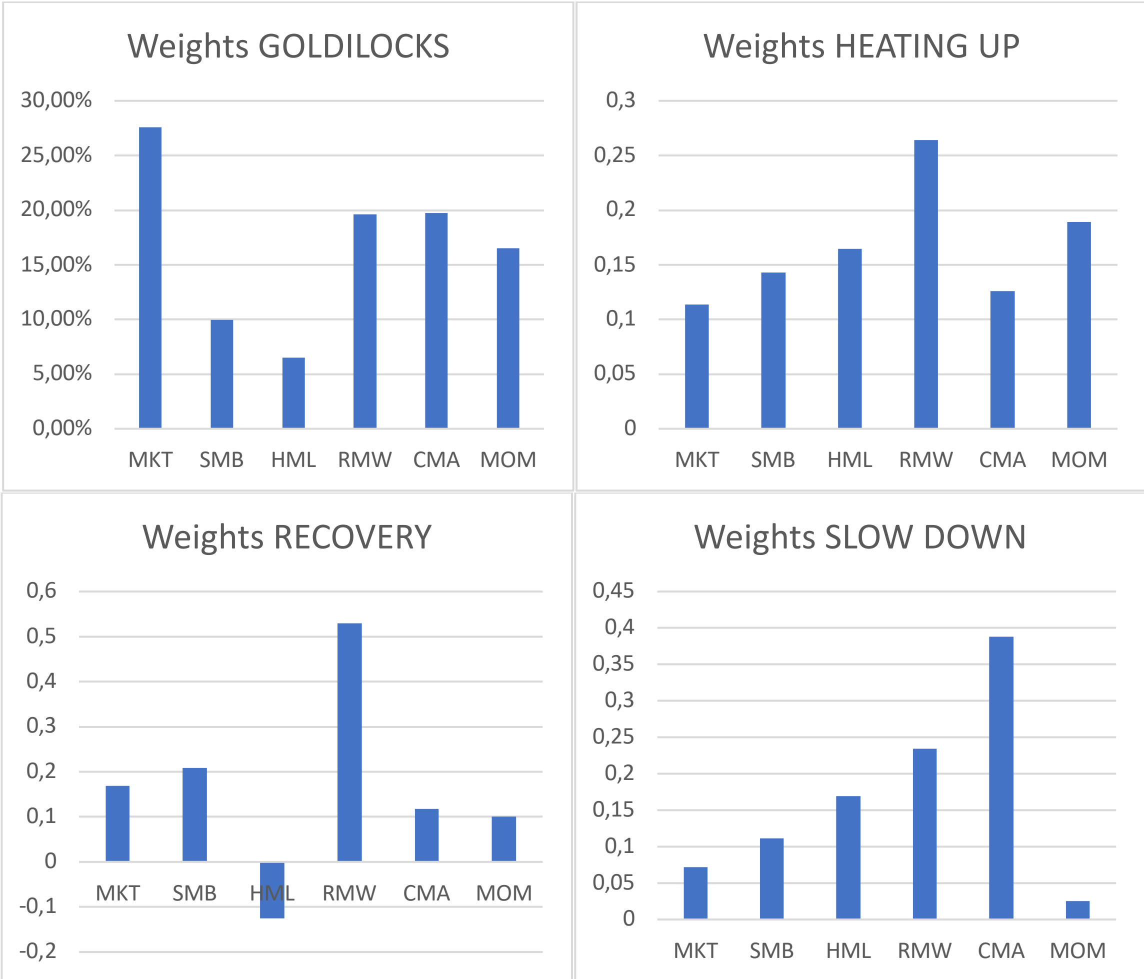

Figure 4.7 - Black-Litterman allocation in each regime

At this point it is possible to analyse the results and assess if a Black-Litterman portfolio of risk factors performs better than an equally weighted portfolio.

Moreover, I want to establish whether a dynamic portfolio allocation, that switches when the economic regime changes, should be preferred to static allocation.

Finally, the performance of a multiple-factor portfolio will be compared with the performance of a single-factor portfolio, in order to assess if the former is able to guarantee relevant diversification benefits and higher risk-adjusted returns.

Alternative Strategies

I considered 3 alternative strategies to construct a multi-factor portfolio:

1) Investment in a portfolio that assigns equal weight to each risk factor,
2) Investment in a Black-Litterman Portfolio with static weights,
3) Investment in a Black-Litterman Portfolio with weights that change across the economic regimes.

	BL Sharpe Ratio		EW Sharpe Ratio
	Switching	Static	
Goldilocks	5,88	5,42	5,16
Heating Up	4,05	3,73	3,67
Recovery	3,47	2,72	2,5
Slow Down	5,53	4,66	4,45

Table 4.10 - Sharpe ratios of the Black-Litterman portfolio and Equally Weighted Portfolio

In every regime the Black-Litterman portfolios achieve a higher risk-adjusted performance with respect to the Equilibrium Portfolio. Furthermore, according to the estimated Sharpe ratios, the BL Switching Portfolio outperforms the BL static portfolio in all the regimes.

Then, I performed a comparison considering holding periods of different length: 3 months, 6 months, 1 year, 3 years, 5 years up to 15 years. Therefore, at each date, I computed the holding period returns.

In this way, it is possible to see if time horizon matters in the choice of one of the 3 alternative strategies. Moreover, I reported the likelihood as the relative frequency. Namely, the number of favourable cases on the total number cases.

Holding period			EW	BL Switching	BL Static	SUM
3m	Best PTF	Absolute Frequency	95	269	158	522
		Likelihood	18,2%	51,5%	30,3%	100%
6m	Best PTF	Absolute Frequency	70	293	156	519
		Likelihood	13,5%	56,5%	30,1%	100%
1Y	Best PTF	Absolute Frequency	55	311	147	513
		Likelihood	10,7%	60,6%	28,7%	100%
3Y	Best PTF	Absolute Frequency	22	355	112	489
		Likelihood	4,50%	72,6%	22,9%	100%
5Y	Best PTF	Absolute Frequency	0	365	99	464
		Likelihood	0%	78,7%	21,3%	100%
7Y	Best PTF	Absolute Frequency	0	366	74	440
		Likelihood	0%	83,2%	16,8%	100%
10Y	Best PTF	Absolute Frequency	0	363	41	404
		Likelihood	0%	89,9%	10,2%	100%
15Y	Best PTF	Absolute Frequency	0	338	6	344
		Likelihood	0%	98,3%	1,7%	100%

Table 4.11- Equally Weighted Portfolio, Black-Litterman Switching Portfolio and Black-Litterman Static Portfolio compared using different holding periods

An investor cannot be sure that such strategy will outperform an alternative one based on equal weights or static BL weights.

However, it is evident that the likelihood increases with time horizon.

When the holding period is short (less than one year) the dynamic portfolio is outperformed in almost half of the cases but at longer horizons the likelihood to be the best performer becomes sizable.

On the other hand, when the horizon increases, there are less cases in which the Equally Weighted Portfolio and the BL Static Portfolio can be preferred.

After the construction of the earlier mentioned multi-factor portfolios, it is possible to compare the performance of 9 assets: the 6 risk factors plus the 3 new portfolios. I computed the cumulative performance in different sample periods: 1975-2018, 2000-2018, 2007-2018, 2007-2009, 2010-2012, 2015-2018. Moreover, in each period I reported mean, volatility, skewness, kurtosis and Sharpe ratio.

The evidence is that, in the very long-run, considering the entire sample period, there are single-factor portfolios that achieve higher cumulative returns than the multi-factor ones: they are MKT and MOM. However, they are characterized by higher volatility and, concerning the Momentum risk factor, very high kurtosis (11.49). The relevant indicator to consider is the Sharpe ratio. In fact, all the 3 multi-factor portfolios exhibit lower volatilities and higher Sharpe ratios in the period 1975-2018 but higher kurtosis than all the risk factors with the exception of MOM and RMW.

Similar considerations can be done restricting the sample period from 2000 to 2018 and from 2007 to 2018. The dynamic portfolio is the best performer in risk-adjusted terms.

Considering the years of the last financial crisis (2007-2009), the BL switching portfolio is outperformed by the RMW factor, but it still achieves a positive Sharpe ratio, while all the other portfolios have negative risk-adjusted returns.

However, after the financial crisis, the performance of the market risk factor improves significantly. In the period 2010-2012 the multi-factor portfolios still perform better but there is a slightly difference.

Finally, from 2015 to 2018 the MKT factor becomes the best performer.

The last 3 years and the years of the financial crisis provide an evidence that the BL switching portfolio can be outperformed by a single factor over some periods. However, the performance of all the multi-factor portfolios seems to be very stable. Nevertheless, if the economic scenario is particularly negative only the dynamic portfolio remains able to produce non-negative risk-adjusted performance among the multi-factor portfolios.

Conclusion

Trusting the results of this work, the presence of multiple regimes in the economy is confirmed from a statistical point of view.

In particular, using the CLI and the CPI as indicators to estimate growth and inflation, the business cycle should be divided in 4 regimes.

An important evidence is that inflation negatively affects returns. Cyclical factors, like MKT and MOM, seem to perform better in regimes characterized by falling inflation even if the growth is moderate (Goldilocks), rather than regimes in which the acceleration is high for both variables (Heating Up). Other factors, like CMA, HML and RMW seem to be counter-cyclical or defensive: they can be used to stabilize the performance of a multi-factor portfolio. In particular, CMA and RMW exhibit very low volatility, regardless of the economic regime.

Moreover, all the factors are poorly or negatively correlated, with very few exceptions over some moments.

Hence, it is possible to assert that *Factor Investing* can represent a solution to the diversification issue concerning the traditional asset classes. However, the investment in a single factor is not recommendable, in particular for short horizons. A portfolio that includes different factors seems to be the most suitable solution for a typical risk-averse investor that wants to achieve a stable performance over time and wants to avoid excessive fluctuations in returns and, consequently, in the portion of his consumption stream that depends on financial wealth.

Nevertheless, the allocation of the available funds among the risk factors cannot be casual. An in-depth analysis of the economic cycle and the application of a suitable portfolio optimization tool, like the Black-Litterman model, lead to a better risk-adjusted performance. In particular, the allocation should be dynamic. The portfolio should anticipate the business cycle and switch its composition accordingly. In doing so, the application of a Markov-Switching model can be very useful. Indeed, the estimated transition probabilities and the expected duration of each regime are the guide to predict regime and portfolio switches.

Obviously, in reality, to evaluate the convenience of such dynamic strategy one has to consider fees and costs related to the active management.

Moreover, as I discussed in the first chapter, the choice among factors should be restricted to a small but sufficient number in order to avoid redundancy. An issue is that in reality it is not easy to exactly replicate the "academic" factors. In fact, the factors defined in academic papers are long-short portfolios with high turnover.

The implementation of long-short strategies is often limited by regulatory constraint or, in general, it is too expensive.

Actually, in most cases, the factor exposure is achieved starting from a market capitalization index and modifying the weights in order to increase the exposure towards those securities that seem to reflect the desired factor characteristic. Such long-only constraint is sometimes necessary to make a factor strategy investable.

In other words, there is a trade-off between investability and exposure to the 'pure' factor. Therefore, the challenge for asset managers is to provide a good compromise.

In synthesis, in reality there are many constraints. However, I retain that understanding theory and statistical relationships between the academic factors can help investors and asset managers to better focus on the desired target. An investor who wants to implement a factor-based strategy needs, first of all, to know what a factor is. Consequently, he has to identify which factors can be considered as such and analyse their characteristics and performances across different states of the economy. Moreover, he has to figure out the optimal portfolio composition with the support of adequate optimization tools. Finally, looking at the available financial products, he has to take the most suitable exposure.