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Department of Business Management

Chair of Asset Pricing

An Empirical Evaluation of Value at Risk and Expected Shortfall Models during the 1997-98 Asian Financial Crisis

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Contents

Abstract

During the *Third Encuentro Financiero Internacional* hosted by Caja Madrid in Madrid July 2003, Gertrude Tumpel-Gugerell, member of the Executive Board of the European Central Bank, addressing the recent development of stock markets, made the following statement:

Before 1997, the volatility on the leading stock indexes hovered around 15% in France and Germany and in the United States, both in terms of historical volatility and of implied volatility. From 1997 onwards, the typical value of those volatilities doubled. This doubling was the result of a slow but steady rising trend, lasting more than six years. (…)
Euro area stock market volatility increases at well-known times of financial turmoil. This is particularly visible at the times of oil shocks, on Black Monday in October 1987, in correspondence with the Latin American-Russian-Asian crises in 1997-98 and after the terrorist attacks on September 2001[1].

In times where financial markets are characterized by high uncertainty, it is crucial for financial institutions to be able to forecast losses. This thesis aims at evaluating the two main methodologies for market risk estimation, Value at Risk (VaR) and Expected Shortall (ES) under some of the most widely used approaches (Simple Moving Average, Exponentially Weighted Moving Average, GARCH (1,1), GJR GARCH and Historical Simulation). More specifically, we will compute the 1 – day VaR and ES estimates at 95% confidence level for an equally weighted portfolio composed by the Hang Seng Index and the FTSE Bursa Malaysia KLCI Index under each approach. The analysis will be carried out before and after the 1997 – 98 Asian Crisis and, at the end, the performances of each model during the two subperiods will be assessed using the Conditional Coverage mixed test (VaR), the Traffic Light test (VaR) and the Acerbi & Szekely test (ES).
For what concerns the structure, the research project is divided in two Chapters. In Chapter 1 we will introduce the theoretical framework of Value at Risk and Expected Shortfall under different approaches as well as the main backtesting methods. In Chapter 2 we will apply the theory illustrated in the first chapter to the empirical study we mentioned in the previous paragraph.

[1] https://www.ecb.europa.eu/press/key/date/2003/html/sp030702.en.html

Chapter 1: Value at Risk, a theoretical framework

1.1 Introduction

In this chapter we will be discussing some of the main methodologies used in Financial Risk Management to estimate market risk, namely Value at Risk and Expected Shortfall. In particular, such models can be classified into two categories: parametric approaches and non-parametric approaches.

In the first approach, we assume that stock returns follow a given statistical distribution (e.g. Gaussian distribution). The main models belonging to this class that we will be discussing in the following paragraphs are

- **Simple Moving Average** method, where volatility at time *t* is computed as the simple standard deviations of stock returns *n* days ahead;
- **Exponentially Moving Average** method, where volatility is the squared root of the weighted average of squared returns such that exponentially declining weights are assigned to each return going back further in time;
- **Stochastic volatility models** with a focus on GARCH (1,1) and GJR GARCH models where we use historical data to estimate the parameters of the model and then use them to forecast future volatility

As we will discuss later, the parametric approach for Value at Risk calculation is relatively simple to implement, however it suffers some major drawbacks (above all non-normality of returns and *fat-tails*).

In the non-parametric approaches we do not make any assumptions on returns distributions because we "let the data talk". The model belonging to this class that will be explained in more details is **Historical Simulation**, where the Value at Risk at a given level of confidence is computed by ranking the first *n* days ahead returns, sorting them from smallest to largest and then picking up the quantile that corresponds to the desired confidence level.

The benefits of such approach rely on the fact that historical data are used in order to estimate Value at Risk and Expected Shortfall, thus overcoming the issue of distributional assumptions on financial data. However, nothing comes to a cost: in choosing the length of the window size, we must carefully evaluate the trade – off between accuracy and adaptability of the model. On the other hand, historical data are not always suitable to describe asset prices movements, especially in periods of crisis.

After describing the main models, we will move onto the potential applications of Value at Risk, its strengths and weaknesses and then some of the main backtesting methodologies.

Finally, at the end of the chapter we will also describe the Basel Regulatory Framework and its guidelines regarding market risk evaluation.

1.2 Financial Market Risk

With the term *financial risk* we usually refer to three types of risks (Jorion, Financial Risk Manager Handbook, 2009):

1. **Market risk**, which refers to the risk of fluctuation of asset prices with respect to movements in market variables (e.g. interest rates, exchange rates and other prices);
2. **Credit risk**, the risk of losses associated to the fact that one or more contracting parties might not be able to fulfill their contractual obligations;
3. **Operational risk**, which denotes the risk of losses resulting from "*the risk of loss resulting from inadequate or failed internal processes, people and systems or from external events*" (Basel Committee on Banking Supervision, 2002)[2].

In this dissertation, we will be focusing on the concept of market risk and the risk metrics associated to it.

1.2.1 Coherent risk metrics

A risk metric is an algorithm that essentially estimates the underlying risk of a financial portfolio. The first question that we should ask ourselves when trying to quantify the exposure is: what are the features and properties that a good risk metric should posses?

According to Artzner, Delbaen, Eber, & Heath (1998) a coherent risk measure should have the following four properties:

1. *Monotinicity.* If asset A has *weak dominance* over asset B then A is riskier than B[3].
2. *Sub-additivity.* A coherent risk measure ϑ should be sub additive meaning that the risk of a diversified portfolio should not be more than the weighted average of the risks of the single components.

   $$\vartheta(A+B) \leq \vartheta(A) + \vartheta(B)$$

   In other words, the sub-additivity property takes into account the diversification effect when we add to our portfolio securities with different risk profiles.
3. *Homogeneity.* For any *k>0*, the homogeneity assumptions requires that

   $$\vartheta(kA) = k\vartheta(A)$$

[2] The definition of operational risk given by the Basel Committee is very broad and encompasses a wide range of areas: from internal/external frauds and damages of physical assets to business disruptions and system failures.

[3] The term *weaklly dominance* was mutuated from game theory: a strategy A is weakly dominant if, regardless of what the other players do, it will result in a payoff that is equal or greater than all the other strategies.

In other words, the homogeneity assumption states that if we want to double the investment in a specific asset, the risk will be doubled as well.

4. *Risk free condition.* Suppose that we invest a portion of our savings in a risky asset A and the remaining amount δ in a risk-free asset. Then, the risk-free condition implies that

$$\vartheta(A+\delta) = \vartheta(A) - \delta$$

Let's suppose that I have a \$500.000 of capital of which \$100.000 is invested in a risk-free asset and \$400.000 in a risky asset so that $\vartheta(\$400.000)$. According to the risk-free assumption, the capital at risk is \$300.000 since the risk-free capital balances out the position in the risky asset, thus decreasing the overall exposure.

Artzner, Delbaen, Eber, & Heath (1998) also found out that risk measures such as Expected Shortfall (ES) are coherent risk metrics, while some of the most common metrics (e.g. simple Value at Risk) are coherent only under certain assumptions. These findings will be discussed more in detail in paragraph 1.6.

1.2.2 An introduction to VaR

Even though the predecessors of VaR can be traced back to the late 19th century[4], the credit for the use of current VaR is attributed to US investment bank JP Morgan with the release during October 1994 of a technical document detailing the *RiskMetrics*™ methodology[5].

As defined by *RiskMetrics™* (J.P. Morgan; Reuters, 1996), Value at Risk is defined as:

(…) a measure of the maximum potential change in value of a portfolio of financial instruments with a given probability over a pre-set horizon.

Oftentimes, in VaR calculation it is assumed that the distribution of a portfolio of securities follows a normal distribution. Let's define the return of a portfolio of securities at time $t+1$ as

$$(1)\ r_{t+1} = \frac{V_{t+1} - V_t}{V_t}$$

[4] The first attempts to measure risks is attributed to Francis Edgeworth who stressed on the importance of using past data to estimate future probabilities.

[5] *RiskMetrics™* contains essentially market risk measurement techniques and data sets of volatility and correlations used to estimate the aforesaid risk. In other terms, it assumes that the return of a portfolio of securities is normal and used to compute the VaR of a portfolio of investments using the variance-covariance method.

where $V_{t+1}$and V_t are the values of the portfolio at times $t + 1$ and t respectively. By assuming normality of returns we can write

$$(2)\ r_{t+1} \sim N(\mu, \sigma^2)$$

Where:

- μ is the mean or expectation of the distribution (also median and mode);
- σ^2 is the variance of the distribution.

The Value at Risk for a probability α can be defined as follows:

$$(3)\ Prob(r_{t+1} \leq VaR_{t+1}^{1-\alpha}) = \alpha$$

In other terms the $VaR_{t+1}^{1-\alpha}$represents the value of r_{t+1} such that there is only α probability that the random variable assumes a value that is smaller or equal than the aforesaid value.

Visually speaking, the idea can be described by Figure 1.1[6]

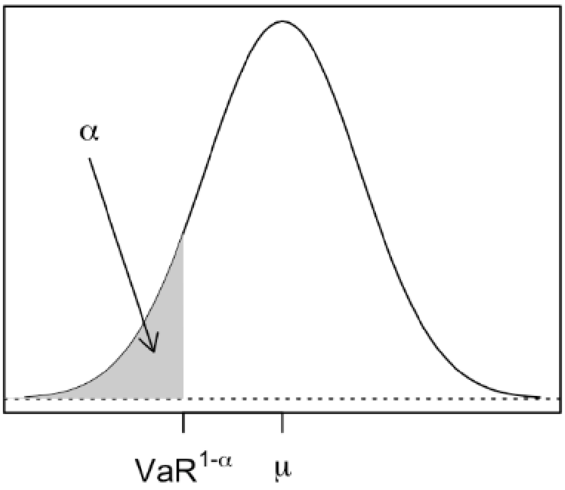

Figure 1.1: VaR at $1 - \alpha$ level

Since $r_{t+1} \sim N(\mu, \sigma^2)$, it is possible to standardize $R_{t+1}$by subtracting μ and dividing $R_{t+1} - \mu$ with σ such that

$$(4)\ \frac{R_{t+1} - \mu}{\sigma} \sim N(0,1).$$

As a consequence, eq. (3) can be rewritten as $Prob(Z \leq z_\alpha) = \alpha$ where Z is the standard normal distribution and

[6] Source http://faculty.baruch.cuny.edu/smanzan/FINMETRICS/_book/index.html

$$(5)\ z_\alpha = \frac{VaR_{t+1}^{1-\alpha} - \mu}{\sigma}.$$

Consequently from eq. (5) we can write

$$(6)\ VaR_{t+1}^{1-\alpha} = z_\alpha * \sigma + \mu$$

For example, for $1 - \alpha = 95\%$, $z_\alpha = -1.64$, so that $VaR_{t+1}^{95\%} = -1.64 * \sigma + \mu$[7].

[7] From now on we will assume that for stock returns μ=0

1.3 VaR methods: Parametric Approaches

One of the most popular approaches to VaR calculation are the parametric approaches where the volatilities of the portfolio's underlying assets are estimated using historical data and an assumption about the portfolio's return's distribution is made (e.g. normal distribution). The models for volatility forecast that will be introduced in the following paragraphs are:

- **Simple Moving Average** (SMA) method, where volatility at time *t* is computed as the simple standard deviations of stock returns *n* days ahead;
- **Exponentially Weighted Moving Average** (EWMA) method, where volatility is the squared root of the weighted average of squared returns so that exponentially declining weights are assigned to each return going back further in time;
- **Stochastic volatility models** with a focus on GARCH (1,1) and GJR GARCH models where we use historical data to estimate the parameters of the model and then use them to forecast future volatility

1.3.1 Parametric Approach and Simple Moving Average Method

If the portfolio is made up by only one asset, then the procedure for VaR calculation has been already explained in paragraph 1.2.2. In the case of a multi-asset portfolio, VaR can be computed using matrix notation.

The return R of a portfolio of n assets can be written using matrix notation in the following form:

$$(6)\ R_p = \overline{w}' \times R$$

Where

- $\overline{w}$ is a $n \times 1$ matrix containing the portfolio's securities' weights;
- $\overline{w}'$ is the transpose of $\overline{w}$;
- R is a $n \times 1$ matrix containing the portfolio's securities' individual returns.

The variance of the portfolio is

$$(7)\ Var\left(R_p\right) = Var(\overline{w}' \times R) = E(\overline{w}' \times \tilde{R})^2$$

where $\tilde{R} = R - ER$ and E denotes the expectation operator.

Using linear algebra, eq. (7) becomes:

$$(8)\ Var\left(R_p\right) = \overline{w}' \times (E\tilde{R}\tilde{R}') \times \overline{w}$$

Where $E\tilde{R}\tilde{R}'$ is the Variance Covariance matrix.

$$(9)\ E\tilde{R}\tilde{R}' = \begin{pmatrix} Var(R_1) & \cdots & Cov(R_1, R_n) \\ \vdots & \ddots & \vdots \\ Cov(R_n, R_1) & \cdots & Var(R_n) \end{pmatrix} = \Sigma$$

Having estimated the variance of the portfolio's returns, it is possible to easily compute the VaR for a given level of confidence by using the following formula:

$$(10)\ VaR^{1-\alpha} = -\overline{w}'\vec{\mu} + z_\alpha\sqrt{\overline{w}\Sigma\overline{w}'}$$

Where

- $\overline{w}'\vec{\mu}$ is the mean return of the portfolio;
- $\sqrt{\overline{w}\Sigma\overline{w}'}$ is the standard deviation of the portfolio.

As we can see, the parametric approach is relatively easy and simple to implement: all we have to do is the estimation of the variance-covariance matrix and then assume that returns are multivariate normally distributed. However, this simplicity comes to a cost: empirical studies have shown indeed that actual returns are characterized by fatter tails than the ones of the normal distribution so that VaR might underestimate the downside risk. In fact, as Mandelbrot (Mandelbrot, 1963) pointed out " (…) *the empirical distributions of price changes are usually too "peaked" to be relative to samples from Gaussian populations*" An explanation to this is provided by the fact that the assumption of constant volatility is violated in the real world. In other words, volatility is time-varying (Jorion, Financial Risk Manager Handbook, 2009).

So, in order to take into account volatility changes through time, we can use the Simple Moving Average Method. The procedure is the following:

1. Choose the length rolling window size *n*;
2. Calculate the volatility of the portfolio's returns at time *t* as the standard deviation of returns between *t-1* and t-*n-1*, the volatility at time *t+1* as the standard deviation of returns between *t* and *t-n*, the volatility at time *t+2* as the standard deviation of returns between *t+1* and *t-n+1* and so on[8]. In other words, we update the sample period after period by removing the oldest observation and adding the newest one;
3. Compute Value at Risk, assuming that returns are normally distributed

$$VaR_{t+1}^{1-\alpha} = z_\alpha * \sigma_t$$

[8] As we can see, the time difference between the newest observation and the oldest one is constant and equal to the rolling window *n*.

1.3.2 EWMA and RiskMetrics

The exponentially weighted moving average method (EWMA) is the technique implemented by RiskMetrics for VaR calculation. It assumes that data collected more recently convey more information. The model can be formalized as follows:

$$(11)\ \sigma_t^2 = \frac{\sum_{i=0}^{n} \lambda^i r_{t-i}^2}{\sum_{i=0}^{n} \lambda^i}$$

As we can see, σ_t^2 is simply the weighted average of past squared returns.

The λ term is also known as *decay factor* and expresses the relative weight put on past observation. In practice, the EWMA places geometrically decreasing weight so recent observations have greater impact compared to data collected far away in the past: the lower is the value of λ and the quicker past observation are forgotten. The parameter is estimated via the Maximum Likelihood method and RiskMetrics often set the value of lambda at 0.94.

Moreover, it can be shown that by factorizing the numerator by $(1-\lambda)$ eq. (11) converges to

$$(12)\ \sigma_t^2 = \lambda\sigma_{t-1}^2 + (1-\lambda)r_{t-1}^2 \text{ if } n \to \infty$$

Having estimated the volatility, the procedure for VaR calculation is the same as the one for the Simple Moving Average, assuming that returns are normally distributed

$$VaR_{t+1}^{1-\alpha} = z_\alpha * \sigma_t$$

As we will see in the following paragraph, EWMA is just a case of the GARCH (1,1) model where we set $\alpha_0 = 0$, $\alpha_1 = 1-\lambda$ and $\beta_1 = \lambda$. In EWMA *Persistence* is equal to 1 meaning that shocks to volatility are permanent.

1.3.3 The GARCH model for volatility estimation

As seen in paragraph 1.3.1, one of the major problems of the simple parametric approach is that it assumes constant volatility so that risk can be potentially underestimated / overestimated when the actual volatility is high/low. This issue has been tackled by Engle (1982) and Bollerslev (1986) with the introduction of the GARCH model for volatility estimation.

By taking into account time variations in risk, what we assume is that the return of a given security at a certain time $t+1$ follows a normal distribution with μ_t=0 and σ_t^2 as parameters. So, this idea can be formalized as follows[9]

[9] See (Jorion, Financial Risk Manager Handbook, 2009)

$$r_{t+1} \sim N(0, \sigma_t^2)$$

As we can see, both mean and variance are a function of time which means that they can change form period to period. In particular, σ_t^2 is referred to as "*conditional variance*" as opposed to the "*unconditional variance*" which is constant through time.

The GARCH (m, s) can be represented as follows:

$$(13)\ \sigma_t^2 = \alpha_0 + \sum_{z=1}^{m} \alpha_z r_{t-z}^2 + \sum_{i=1}^{s} \beta_i \sigma_{t-i}^2$$

Where

- r_t denotes the return at time t;
- σ_t the portfolio's variance at time t;
- the alphas reflect the impact of past random deviations on σ_t^2; the betas measure the portion of past realized variances that are conveyed in the current period. Both parameters are estimated via the Maximum Likelihood method.

In other terms, current variance is a function of past returns and past variances.

For the purpose of this dissertation, it is of interest to analyze the GARCH (1,1) model

$$(14)\ \sigma_t^2 = \alpha_0 + \alpha_1 r_{t-1}^2 + \beta_1 \sigma_{t-1}^2$$

$$r_t = \sigma_t z_t \ and\ z_t \sim N(0,1)$$

To derive the unconditional (average) variance we take the expectation of eq. (14) and set $E(r_{t-1}^2) = \sigma_t^2 = \sigma_{t-1}^2 = \sigma^2$.

$$(15)\ \sigma^2 = \frac{\alpha_0}{1 - (\alpha_1 + \beta_1)}$$

With $\alpha_0, \alpha_1, \beta_1 \geq 0$ and $(\alpha_1 + \beta_1) \leq 1$. The second condition is essential for stationarity.

$\alpha_1 + \beta_1 = \gamma$ is also called *persistence* and denotes the speed at which the model reverts back to the long run average after a shock. In fact, we can use the GARCH (1,1) to forecast the volatility n steps ahead. Rearranging eq. (14) and iterating for n times we can write

$$(16)\ E_{t-1}(r_{t+n}^2) = \alpha_0(1 + \gamma + \gamma^2 + \dots + \gamma^{n-1}) + \gamma^n \sigma_t^2 =$$

$$\alpha_0 \frac{(1 - \gamma^n)}{1 - \gamma} + \gamma^n \sigma_t^2$$

So, for $n \rightarrow \infty$, eq. (16) becomes

$$\lim_{n\to\infty} E_{t-1}(r_{t+n}^2) = \lim_{n\to\infty}\left(\alpha_0 \frac{(1-\gamma^n)}{1-\gamma} + \gamma^n \sigma_t^2\right) \quad = \frac{\alpha_0}{1-(\alpha_1+\beta_1)}$$

The GARCH model assumes that returns are not i.i.d. since they exhibit volatility clustering.

In conclusion, the 1-α level VaR is computed as follows

$$VaR^{1-\alpha} = z_{\alpha} * \widehat{\sigma_t}$$

Where

- z_{α} the percentile at α level;
- $\widehat{\sigma_t}$ the standard deviation estimated by the GARCH (1,1) model

1.3.4 The GJR GARCH model for volatility estimation

The GJR (Glosten-Jagannathan-Runkle) GARCH model is a variation of the standard GARCH models introduced by Glosten-Jagannathan-Runkle (1993).

$$(17)\ \sigma_t^2 = \alpha_0 + (\alpha_1 + \gamma I_{t-1}) r_{t-1}^2 + \beta \sigma_{t-1}^2$$

Where

- $r_t = \sigma_t z_t \ and \ z_t \sim N(0,1)$
- $I_{t-1} = \begin{cases} 0, & r_{t-1} \geq 0 \\ 1, & r_{t-1} < 0 \end{cases}$

I_{t-1} is a dummy variable which is 0 if $r_{t-1} \geq 0$ and is 1 if $r_{t-1} < 0$. This implies that the impact on volatility of a negative shock on stock returns is greater than the impact of a positive shock.

Note that if $\gamma = 0$, we have the standard GARCH (1,1) model.

1.4 Non – Parametric Approaches

As opposed to the parametric approaches to Value at Risk calculation, we have another class of models, the non – parametric approaches. Such category have been given this name since no assumptions about the returns' distribution is made. By *letting the data speak* the issues linked to the parametric approaches are partially overcome. However, as we delve deeper in the discussion, we will discover that new problems arises if we choose this methodology.

The models for volatility forecast that will be introduced in the following paragraphs are:

- **Historical Simulation**, where the Value at Risk at a given level of confidence is computed by ranking the first n days ahead returns, sorting them from smallest to largest and then picking up the quantile that corresponds to the desired confidence level;
- **Monte Carlo Simulation**, where, instead of using historical data, we artificially generate stock prices and returns.

1.4.1 Historical Simulation

Historical simulation is a non - parametric approach since no assumptions about returns distribution are made. Essentially, with this simulation model we are assuming that the empirical distribution of returns estimated from historical data is a good approximation of the distribution of future returns.

Generally speaking, what we need to do is to take the historical n returns of the latest n periods and sort them from smallest to largest into a histogram. The $VaR^{1-\alpha}$ is the quantile that corresponds to the desired level of confidence, $(1-\alpha)$.

$$(18)\ x^{th} = \alpha n$$

For example, if $n = 100$, $VaR^{95\%}$ is the fifth smallest observation out of 100.

The main advantage of historical simulation is that it avoids the parametrization problem: fat-tails, skewness and other implications are directly accounted for by the empirical distribution of observations.

However, this method has several shortcomings as well. First of all, extreme values are very difficult to estimate in the distribution and large estimation errors are possible if the period of analysis is too short. As a result, large observation periods are required. However, as Hendricks pointed out (Hendricks, 1996), large sample periods results in flatter VaR implying less reactivity to new market conditions. Secondly, with historical simulation we violate the assumption that more recent data are more meaningful than observations made far away in the past since we implicitly assume that each observation has the same weight. In order to give more weights to more recent returns, we could shorten the period of analysis. However, in doing so we would meet the same problems as described earlier in the paragraph. As we can see, the selection of the length of the sample period is a major concern (Boudoukh, Richardson, & Whitelaw, 1998).

1.4.2 Monte Carlo simulation

Monte Carlo simulation is one of the most widely used tool to simulate assets' return in finance. For the case a portfolio with only one asset, the process can be summarized in the following steps:

1. Establish the statistical distribution that can better approximate the distribution of returns (e.g a normal distribution);
2. Draw a random number ε_t between 0 and 1 using a uniform distribution with a random number generator for k times, where k represents the time-steps from t to the horizon $t+k$;
3. Compute the value of the cumulative distribution function of the related function defined in 1 in correspondence of ε_t;
4. Compute stock prices from $t+1$ to $t+k$ by assuming that they follow a given random process. Normally, a Geometric Brownian Motion is assumed:

   (19) $S_{t+i} = S_{t+i-1} + \mu S_{t+i-1}\Delta t + \sigma S_{t+i-1} z_{t+1-1}\sqrt{\Delta t}$ with i=1, 2, …, $t+k$.

   Where:

   - S_t is the stock price at time t;
   - $S_{t+1}$is the simulated value of the stock price resulted from the process;
   - μ is the sample mean of the stock price;
   - σ is the standard deviation of the stock price;
   - z_t is the value of the cumulative distribution function computed with the random number generator;
   - Δt is the time horizon we consider.
5. Compute the simulated returns from $t+1$ to the end of the analysis period, $t+k$;
6. Compute the portfolio value at horizon;
7. Repeat steps 2, 3, 4, 5 for as many times a necessary in order to generate *n* paths for stock prices (Figure 1.2);
8. Rank the *n* terminal portfolio simulated values from smallest to largest and find the VaR associated to the desired confidence level.

Figure 1.2 : Monte Carlo simulation for Apple Inc. stocks with 1000 scenarios for $\mu = 23.09\%$ and $\sigma^2 = 42.59\%$ (annual volatility)[10].

Monte Carlo is one of the most widely used method in the financial industry because of its flexibility. In particular, it can model financial instruments with non-linear payoffs, namely derivatives.

The major drawbacks of the method are:

- it is subject to model risk, since analysts are required to make assumptions about the stochastic process underlying the assets;
- it is time consuming since we might need to simulate a large number of price paths in order to converge to a stable VaR measure;
- it is resource consuming since we need reasonable computing power to complete the task. Jorion (2009) discussed the problem regarding the *curse of dimensionality*: the covariance matrix has N(N+1)/2 dimension and a portfolio made up of 100 assets would require 5050 entries. Since the risk manager deals with large portfolios, one of the methods that he can rely on in order to make the process less time-consuming is the *principal-component analysis* which finds the most relevant risk factors;
- as the time steps decrease, the variance decreases as well, which implies that large discontinuities cannot occur over short periods of time. The model should be adjusted in

[10] Source: https://www.pythonforfinance.net/2016/11/28/monte-carlo-simulation-in-python/

order to take these complications into account especially when we deal commodities, since discrete jumps can be observed.

1.5 VaR: applications

According to Sironi (2008) there are three main areas where VaR is implemented:

1. **Benchmarking different types of risks.** VaR provides a common risk evaluation framework regardless of the nature of the financial asset we are dealing with. It can be considered as a *lingua franca* between different trading desks taking positions in different assets (e.g. bonds, derivatives, stocks etc…). The importance of the VaR is relevant in the following example: think about a world where VaR does not exists. Assuming that we have taken a position in government bonds and stocks, how can we compare different risk metrics such as the duration and the beta of a stock portfolio? Thanks to VaR we are able to encompass these obstacles.
2. **Limiting risk exposure.** VaR can be used to set the operating limits of the trading units within a bank. Suppose for example that Bank X has two trading desks: desk 1 (stocks) and desk 2 (bonds). The VaR limits for each unit given a confidence level and a trading horizon is \$200,000 and \$100,000 respectively. Since the maximum amount that can be invested in a certain position depends on its VaR limit, by changing the latter we can change the capital allocation strategy among different business units.
3. **Designing risk-adjusted performance (RAP) metrics.** Finally, VaR can be used to design risk adjusted performance metrics. One of the most commonly used metrics is the RAROC (*Risk-adjusted Return on Capital).*

- $RAROC_{(ex-ante)} = E(P)/CaR_{(ex-ante)}$
- $RAROC_{(ex-post)} = P/CaR_{(ex-post)}$

where

- $E(P)$ is the expected profit;
- P the realized profit;
- $CaR_{(ex-ante)}$ the capital allocated to a single unit;
- $CaR_{(ex-post)}$ the undertaken risk;

RAP metrics have three main different purposes:

1. Support traders in making investment decisions by analyzing the *ex-ante* profitability and the risk profile of the alternatives;
2. Establish an incentive scheme that is not profit-based only:

3. Compare the *ex-post* performance of the different units within a financial institution to determine which units are allocationg resources more efficiently and hence deserving more capital to invest.

1.6 VaR: weaknesses

Having described the main VaR models that are relevant for the scope of this dissertation, we can now ask ourselves: what is the best VaR model in terms of accuracy that we can use for financial risk assessment? The answer is simple: by far, there is no one best model for every circumstance and it depends on the purpose of the risk manager. For example, if the purpose of the research is to analyze the daily risk exposure and the risk-adjusted profitability of a trading unit within a bank, then the variance-covariance approach is the most appropriate approach. In fact, the limitations led by the assumptions that returns are normally distributed (namely *fat tails*) is not relevant if the purpose is to estimate and limit the risk exposure and the risk-adjusted performance on a daily basis. However, it is relevant if the aim is to measure the capital adequacy of the bank since risk is underestimated in such assumption.

According to Sironi (2008), there are three four main critiques to VaR which derive from misunderstanding the scope of the model.

1. **Extreme events are not accounted for in VaR models.** It is true that extreme events are not accounted for in VaR models. However, as mentioned earlier, the main purpose of VaR models is not to measure the capital adequacy of banks, but to estimate the risk exposure and the operational limits of each trading desk on a daily basis. In other words, VaR is a ordinary risk management tool. On the other hand, such rare events must be taken in account in capital adequacy assessment.
2. **The magnitude of losses is not accounted for.** VaR does not take into account the magnitude of losses if a violation occurs. To fix this issue alternative risk metrics were introduced, such as Expected Shortfall, which will be discussed in more details in paragraph 1.8.
3. **VaR models yield divergent outputs.** If we change the underlying assumptions of a VaR model (e.g. we assume that returns are distributed as a *t-student* distribution instead of a Gaussian distribution and/or we change extend or shorten the sample period etc…) we will certainly get different results. However, if, as mentioned before, the aim of the model is to evaluate the risk-adjusted performance of the trading units within a bank for capital allocation in the units themselves, this issue is not very relevant. In fact, what we need here is not an assumptions-independent model but a risk assessment framework that is uniformly implemented in all the business units. So even if the model underestimates or overestimates the potential losses, there would not be issues at all since the over/under estimation is reflected through out all the trading units, not affecting the capital allocation strategy.

4. **VaR models can potentially decrease the stability of financial markets.** If everyone in the financial sector has adopted VaR as a risk management tool, then this practice can potentially amplify the volatility of the market. This is true because every trader would get the same result and try to decrease their exposure thus worsening market conditions. However, this should not be considered as a direct implication of VaR models rather than as a consequence of human nature.

Other than these critiques, which as we have just seen are mainly driven by misunderstandings about the scope of the model, there is another major issue that has been mentioned at the beginning of the chapter: VaR is not a coherent risk measure, namely the *sub-additive* property is violated if the joint distribution of risk factors is not normally distributed (Artzner, Delbaen, Eber, & Heath, 1998). So, in this case the following must be true:

$$VaR(A + B) > VaR(A) + VaR(B)$$

Let's consider the following example: We have two identical bonds A and B. Each defaults with a probability of 3% and, in that case, the loss is 150. If no default occurs, the loss is 0. The 95%VaR of each bond is therefore 0. Hence VaR(A)=VaR(B)=VaR(A)+VaR(B)=0. The example can be summarized in the table below:

Probability	Bond A	Bond B
3%	150	150
97%	0	0
95%VaR	**0**	**0**

Let's now consider a portfolio made up of one bond A and one bond B. We also suppose that defaults are independent. In this case, we get a loss of 300 with probability 0.03*0.03=0.0009, a loss of 0 with probability 0.97*0.97=0.9409 and a loss of 150 with probability 1-0.9409-0.0009=0.0582. Hence VaR(A+B)=150>0=VaR(A)+VaR(B). As a result, VaR is not sub – additive.

1.7 Backtesting VaR

The reliability of a specific VaR model can be tested using a backtesting method. The methodology provided by the Basel Committee is thought to be too simplistic and more accurate and rigorous approaches are needed.

For the purpose of this dissertation, in the following paragraphs we will be discussing two different statistical tests to evaluate VaR models: the *Proportion of Failure* (POF) test by Kupiec (1995) and the *Conditional Coverage Independence* (CC) test by Christoffersen (1998).

Both these approaches rely on the hypotheses testing methodology: if the null hypotheses is rejected, then the VaR model is regarded as inaccurate and *vice-versa*, if not as accurate.

1.7.1 The Proportion of Failure test

With the POF test we analyze if the frequency of the violations π is significantly different from the theoretical one, α.

If the model is accurate, then $\pi = \frac{x}{n} = \alpha$, where *x* is the number of exceptions in the sample over a pre-determined time horizon and *n* the number of observations. So, the null hypothesis that we want to test is H_0: π= α. Under H_0, the likelihood function is

$$(20)\ L_0(\alpha) = \alpha^x(1-\alpha)^{n-x}$$

Under the alternative hypotheses, the likelihood function is

$$(21)\ L_0(\pi) = \pi^x(1-\pi)^{n-x}$$

In order to carry out the test, we must take the ratio of the aforementioned likelihood functions

$$(22)\ LR_{POF}(\alpha) = -2\ln\frac{\alpha^x(1-\alpha)^{n-x}}{\pi^x(1-\pi)^{n-x}}$$

The likelihood ratio is asymptotically distributed as a chi-squared distribution with 2 degrees of freedom.

Generally speaking, the null hypothesis that the empirical frequency of VaR breaches is equal to the theoretical one has to be rejected if the value of $LR_{POF}(\alpha)$ lies beyond the critical value of χ_1^2.

So, if the value of the likelihood ratio is 0.80 for $\alpha = 1\%$, the null hypothesis is not rejected and the VaR model is regarded to be accurate since the corresponding value of a chi-squared distribution with the same significance level is 6.635.

1.7.2 The Conditional Coverage Independence test

The CCI test aims at testing whether VaR exceptions are serially independent or not. In other words, a desirable property of VaR models is that the probability for an exception to occur in day *t* should be independent from an exception that had occurred in the previous day.

Let's define the following variables:

$$(23)\ I_t = \begin{cases} 1 \text{ if VaR limit is hit in } t \\ 0 \text{ if VaR limit is not hit in } t \end{cases}$$

and

$$(24)\ \pi_{i,j} = \Pr\left(I_{t-1} = i | I_t = j\right) \text{ where } i \text{ and } j = 0{,}1$$

Eq. (23) is a variable that is 0 if there is an exception at time *t* and is 1 if not.

Eq. (24) defines the probability that Eq. (23) assumes value *j* in day *t* given that in the previous day its value was *i*. So,

- $\pi_{1,1}$ is the probability that an exception in *t-1* is followed by another one in *t*;
- $\pi_{1,0}$ is the probability that an exception in *t-1* is not followed by another one in *t;*
- $\pi_{0,1}$ is the probability that no exceptions in *t-1* is followed by an exception in t;
- $\pi_{0,0}$ is the probability that there are not exception neither in *t-1* nor in *t*.

If VaR events are serially independent then the following must be true

$$\pi_{1,1} = \pi_{0,1} = \pi_1$$

$$\pi_{0,0} = \pi_{1,0} = \pi_0$$

The first identity means that the probability that an exception occurred a t time *t* given that it had occurred in *t-1* is the same as the probability that an exception occurred at time *t* given that it had not in *t-1*. In other words, the probability that an exception occurred or did not occur in *t* is independent from the fact that it had occurred or had not occurred in *t-1*.

Let's consider a sample of *T* observations for time *t* and *t-1*. Let's define the following variables:

- $T_{1,1}$ the number of exceptions that are preceded by another exception;
- $T_{0,1}$ the number of exceptions not preceded by another exception;
- $T_{1,0}$ the number of missed exceptions preceded by a period where no exceptions happened;
- $T_{0,0}$ the number of missed exceptions preceded by a period of no exceptions.

With these data we can estimate the empirical probabilities:

$$P_{0,1} = \frac{T_{0,1}}{T_{0,1}+T_{0,0}} \rightarrow P_{0,0} = 1 - P_{0,1}$$

$$P_{1,0} = \frac{T_{1,0}}{T_{1,0}+T_{1,1}} \rightarrow P_{1,1} = 1 - P_{1,0}$$

The likelihood ratio used to test the null hypothesis is

$$(25)\ LR_{CC} = -2 \ln \frac{L_{H_0}}{L_{H_1}}$$

Where

1. $L_{H_0} = (1 - \pi_1)^{T_{0,1}+T_{0,0}} {\pi_1}^{T_{1,0}+T_{1,1}}$. It is the likelihood function under the null hypothesis and $\pi_1 = \dfrac{T_{0,1} + T_{1,1}}{T_{1,1} + T_{0,1} + T_{1,0} + T_{0,0}}$
2. $L_{H_1} = (1 - \pi_{0,1})^{T_{0,0}} {\pi_{0,1}}^{T_{0,1}} (1 - \pi_{1,1})^{T_{1,0}} {\pi_{1,1}}^{T_{1,1}}$. It is the likelihood function under the alternative hypothesis[11].

LR_{CC} is asymptotically distributed as a χ_1^2. Generally speaking, the null hypothesis of no serial correlation has to be rejected if the value of LR_{CCI} lies beyond the critical value of χ_1^2.

Notice that LR_{CC} does not depend on the confidence level. This means that we must combine the two likelihood ratios in order to test that the expected number of VaR violations is correct and that these violations are independent from one another.

The statistic to be used for the conditional coverage mixed test is the following one

$$(26)\ LR_{ccmixed} = LR_{POF} + LR_{CCI}$$

Which is asymptotically distributed as a χ_2^2.

In conclusion, we must combine Christoffersen's and Kupiec's test in order to have a more comprehensive assessment. Plus, by splitting the statistic in its two components we are able to analyze the reason that caused a given VaR model to be rejected.

[11] Notice that we can derive L_{H_0} from L_{H_1} by putting $\pi_{1,1} = \pi_{0,1}$

1.8 Expected Shortfall

The VaR of a portfolio is a risk measure that only tells us the potential losses over a specific period of time given a pre-determined confidence level. But what happens if the VaR limit is hit? Are losses equal or greater than the limit itself? For example, if we say that the 5 days VaR is $200 with a confidence level of 99%, it means that in 100 simulations, we expect that in 99 cases we won't see a loss that is greater than $200. However, what happens if a *tail event* occurs? Is the loss $201 or $300? The simple VaR method cannot answer this question and we should introduce some changes to the original model if we want to know the answer.

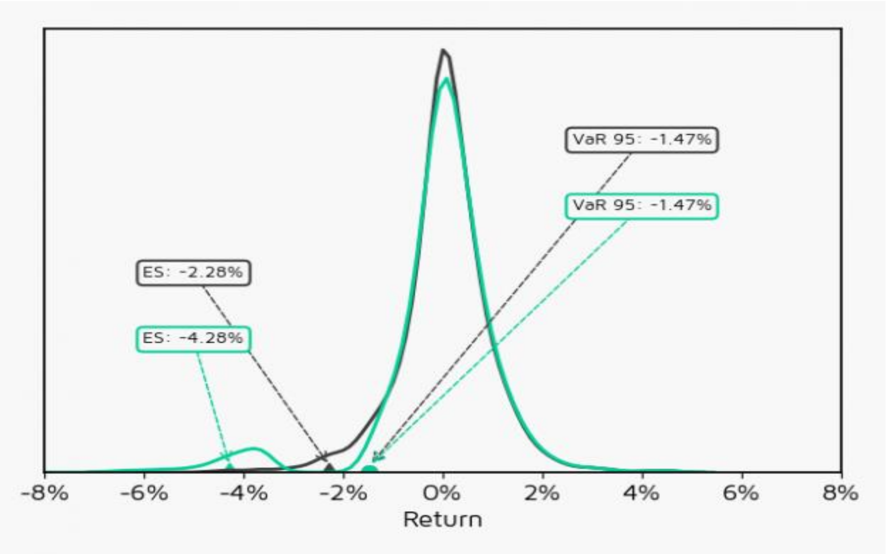

Figure 1.3: Expected Shortfall for two different distribution with same 95%VaR[12]

As we can se from Figure 1.3, the 95%VaR is the same. However, the expected losses are greater in the green colored function when the limit is hit.

The *Expected Shortfall* (ES) at α level over a specific period of time is the expected portfolio's loss in the worst α cases. In formulas we have

$$(27)\ ES^{1-\alpha} = E(X|X > VaR^{1-\alpha})$$

Where X is the portfolio's loss.

Alternatively, the expected shortfall can be computed as follows:

- If we assume that losses are continuously distributed, then

$$(28)\ ES^{1-\alpha} = \frac{1}{\alpha}\int_{-\infty}^{VaR} xf(x)$$

[12] Source: https://quantdare.com/value-at-risk-or-expected-shortfall/

- If we assume that losses are discretely distributed

$$(29)\ ES^{1-\alpha} = \frac{1}{\alpha}\sum_{x_i<VaR} x_i P(X = x_i)$$

As mentioned at the beginning of the chapter, ES is also a coherent risk measure. The *sub-additive* property is indeed satisfied[13], so that

$$ES^{1-\alpha}(A) + ES^{1-\alpha}(B) \geq ES^{1-\alpha}(A + B)$$

Now let's see how the expected shortfall is computed in the approaches described above.

In the models seen in the previous paragraphs, ES is estimated as follows:

1. ***Simple Moving average*** approach assuming normality of returns[14]

$$(30)\ ES^{1-\alpha} = -\overline{w}'\overrightarrow{\mu_t} + \sqrt{\overline{w}\Sigma_t\overline{w}'}\,\frac{\phi(z_\alpha)}{\alpha}$$

Where

- $\overline{w}$ is a $n \times 1$ matrix containing the portfolio's securities' weights;
- $\overline{w}$' is the transpose of $\overline{w}$;
- $\vec{\mu}$ is the $n \times 1$ vector containing the portfolio's average returns of the n assets
- z_α is the quantile at α level;
- $\phi(z_\alpha)$ is the value of the standard normal distribution at α level;
- $\overline{w}'\overrightarrow{\mu_t}$[15] and $\sqrt{\overline{w}\Sigma_t\overline{w}'}$ are the mean and standard deviation of the portfolio at time t respectively.

If the portfolio is made by one asset only, then eq. (11) collapses into

$$(31)\ ES^{1-\alpha} = -\mu_t + \sigma_t\frac{\phi(z_\alpha)}{\alpha}$$

Where

- μ_t and σ_t are the mean and standard deviation of the asset at time *t*;

2. ***GARCH and EWMA*** assuming normality of returns

$$ES^{1-\alpha} = -\mu_t + \sigma_t\frac{\phi(z_\alpha)}{\alpha}$$

Where

- μ_t and σ_t are the mean and standard deviation of the asset at time *t*

[13] For more details please see Acerbi & Tasche (2002)
[14]Khokhlov, V. (2016). *Conditional Value-at-Risk for Elliptical Distributions*
[15] For simplicity we will assume that $\overrightarrow{\mu_t} = 0$

3. ***Historical simulation***

   As described earlier, in historical simulation we take the n returns of the sample period and sort them from smallest to largest. The $1 - \alpha$ level VaR is the observation so that its position is computed as follows:

$$x^{th} = \alpha n$$

   Given that VaR limit is hit, since we want to know the average loss exceeding VaR, ES is calculated as below:

$$(32)\ ES^{1-\alpha} = \frac{\sum_{i=x^{th}}^{n} X_i}{n - x^{th}}$$

   Where $X_1, X_2, \ldots, X_n$ are the returns exceeding VaR.

4. ***Monte Carlo simulation***

   ES calculation for Monte Carlo simulation is similar to historical simulations'. What we need to do is to filter out the losses exceeding the VaR and compute the average using the same formula.

1.8.1 Backtesting Expected Shortfall

Acerbi & Szekely (2014) proposed a simple backtesting method for Expected Shortfall. The test statistic used for this test is

$$(33)\ Z = \frac{1}{Np_{VaR}} \sum_{t=1}^{n} \frac{X_t I_t}{ES_t} + 1$$

Where

- *N* is the number of periods the test is carried out onto;
- X_t is the portfolio return for period *t*;
- p_{VaR} is the probability of VaR failure defined as 1-VaR level;
- ES_t is the estimated expected shortfall for period *t*;
- I_t is the VaR failure indicator on period *t* with a value of 1 if $X_t < -VaR_t$, and 0 otherwise.

The test statistic is expected to be 0 and it is negative when risk is underestimated. In fact, as we can see from eq. (33) that Z is sensitive to the number of VaR violations and the magnitude of the losses exceeding VaR relative to the ES estimate. As a consequence

1. One large VaR violation relative to the estimated ES can cause the model to be rejected;
2. One large VaR violation on a day where the ES is large as well might not cause the model to be rejected;
3. A period with many relatively small VaR breaches can cause the model to be rejected.

1.9 The Basel Accords

The Basel Accords are a set of agreements and principles on banking supervision established by the Basel Committee which is formed by 45 members (central banks and authorities with formal authority for the supervision of the banking sector) from 28 countries all over the world. Its main purpose is to guarantee the stability of the financial sector by providing guidelines to financial institutions regarding risk management procedures and processes.

The need of such supranational regulation can be traced back to 1974, in the aftermath of Bankhaus Herstatt bankruptcy in West Germany which failed due to bad investments in the forex market. Herstatt bank accumulated cumulative losses 10 times higher than its capital (DM 470 millions vs DM 44 millions) and was liquidated on June 26^{th} 1974. Because of time zones, the offices located in New York continued to operate as usual until the closing of the US market and the counterparties with which the Bank had commercial agreements had never received any kind of payment.

The failure of Herstatt Bank proved the inadequacy of the existing national regulatory frameworks to deal with transnational affairs and the Basel Committee on Banking supervision was established in order to tackle the challenges of an already globalized financial market.

Since its foundation, the Basel Accords have been constantly updated. Up to now, there are three different versions of the agreements, namely Basel I, Basel II and Basel III.

Basel I was set in 1988 and it was focused on dealing with credit risk. More specifically, a minimum on the Capital (C) to Risk Weighted Assets (RWA) was introduced:

$$\frac{C}{\sum_i^n A_i RW_i} \geq 8\%$$

Where

- C is the capital (Tier 1+Tier 2);
- A_i is asset i;
- RW_i is the risk associated to asset i

In practice, the ratio means that the total risk-weighted assets cannot exceed the 8% of the capital. If a given financial institution wanted to increase its asset base, it had to increase its capital as well in order to comply with the target of 8%.

As for the numerator, Tier 1 is composed by, paid-up share capital/common stock and disclosed reserves. Tier 2 is made up of undisclosed reserves, asset revaluation reserves, general provisions/general loan-loss reserves, hybrid (debt/equity) capital instruments, subordinated debt.

As for the denominator, the risky asset weights are determined according to the asset class. For example, the weight for cash is 0% since it is considered as a risk-free asset[16].

However, the evaluation method suffered some major limitations: the same weights were applied for different categories of borrowers (banks, firms, government…) each of which has clearly different risk exposures and other types of risks were not accounted for.

In order to solve these problems; Basel II was established in 2004. The New Basel Accord is made up of three main pillars:

- In the **First Pillar** market risk and operational risk are accounted for the calculation of the capital to risk weighted asset ratio; moreover, the risk evaluation system is more flexible since two alternative approaches are defined:
  i) The *Standard approach* where the weighting coefficient are based on the ratings provided by rating agencies regarding specific borrowers;
  ii) The *Internal-Rating Based approach* which distinguishes two different types of losses:
    - *Expected Loss* (*EL*)
    - *Unexpected loss (UL)*

  The *Expected Loss* is computed as follows

  $EL = EAD \times LGD \times PD$ where

  EAD=loss incurred by the bank if the borrower fully defaults on his debt;

  LGD= loss incurred by the bank when the borrower declares default;

  PD= borrowers' default probability

  The *Unexpected Loss* can be interpreted as the standard deviation of the loss distribution.

  Clearly, the IRB approach relies on the VaR logic: the minimum level of capital must be such that the probability of unexpected losses is smaller than a threshold over a pre-determined time horizon. Moreover, the choice of the preferred VaR model to implement for market risk assessment is given to the bank.

- The **Second Pillar** provides guidelines regarding the supervisory activities of regulatory authorities;

[16] For more details see Basel Committee on Banking Supervision (1998). *International Convergence of Capital Measurement and Capital Standards.* Basel.

- The **Third Pillar** regulates the obligation of financial institutions to report information to the public regarding their financial soundness.

The 2007-2008 financial crisis has highlighted the faults of Basel II: insufficient capital in terms of quantity and quality, lack of regulation regarding specific financial instruments (e.g. subprime mortgages) which brought to insufficient reserves to cover the risks deriving from trading activities, liquidity deficit and excessive financial leverage.

Basel 3, which was announced in the post-crisis years, aims at solving these criticalities: Tier 1 capital must be at least 6% of the total risk-weighted assets and a *liquidity coverage ratio* and a *net stable funding ratio* have been introduced to withstand liquidity risk. In order to decrease financial leverage, Tier 1 capital must be at least 3% of the total assets.

VaR and capital requirements

In 1996, the Basel Committee set out a simple backtesting framework to test the accuracy of VaR models for financial institutions adopting the internal models approach[17]. According to the Committee,

- VaR is to be calculated on a daily basis
- A 99% one tailed confidence level is to be used for the purpose
- VaR is to be estimated by assuming a holding period of no less than 10 days

Similarly to the Conditional and Unconditional coverage tests, the backtesting methodology proposed by the Basel Committee lies on comparing the number of actual VaR violations with the theoretical one. For example, let's suppose that the theoretical daily VaR is 200 with a confidence level of 99%. This means that I am expecting losses greater or equal than 200 in 1% of the cases, namely 2.5 days over 250 trading days. If the number of days with a VaR violation is equal or just over 2.5 days than, we can conclude that the model is reasonably accurate. Vice versa, if the number of days with a VaR violation is much greater than 2.5, than we must investigate the validity of the model. Based on this logic, the Committee introduced a multiplier which increases as the number of exceptions increases (Figure 1.2)

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[17] Basel Committee on Banking Supervision. (1996). *Supervisory Framework for the use of "Backtesting" in conjunction with the Internal Models Approach To Market Risk Capital Requirement.* Basel.

zone	number of exceedances	multiplier k	cumulative probability assuming q^* = 0.99
green	0	3.00	.0811
	1	3.00	.2858
	2	3.00	.5432
	3	3.00	.7581
	4	3.00	.8922
yellow	5	3.40	.9588
	6	3.50	.9863
	7	3.65	.9960
	8	3.75	.9989
	9	3.85	.9997
red	10 or more	4.00	.9999

Figure 1.2: VaR scaling factors (Basel Committee on Banking Supervision, 1996)

The capital requirement relative to market risk (*C)* is thus computed

$$(34)\ C = \max\left(VaR_{t-1}; M \times \frac{1}{60}\sum_{i=1}^{60} VaR_{t-i}\right) + SR$$

Where:

- VaR_{t-1} is the 99% 10-day VaR of the previous day;
- M is the multiplier whose value varies between 3 and 4 according to the accuracy of the model (Figure 1.2);
- SR (*specific risk*) is a risk component that is not accounted for in VaR models.

Eq. (12) basically says that the capital requirement for market risk is the highest between the previous day VaR and the average of the last 60 days VaR, which average is scaled by the multiplier.

In 2013 the Basel Committee stressed the importance of moving from VaR to Expected Shortfall because *a number of weaknesses have been identified with using VaR for determining regulatory capital requirements, including its inability to capture "tail risk"* (Basel Committee on Banking Supervision, 2013). The confidence interval set by the Committee is 97.5% for the IRB methodology

Chapter 2: An empirical analysis of Value at Risk during the 1997-98 Asian Financial Crisis

2.1 Introduction

In this chapter of the research project we will analyze the an equally weighted portfolio made up by two East Asian stock market indexes, the Hang Seng Index (HSI) and the FTSE Bursa Malaysia KLCI Index (FBMKLCI) during the 1997 – 1998 Asian Financial Crisis.

More specifically, the daily Value at Risk and Expected Shortfall will be computed at a confidence level of 95% using the approaches seen Chapter 1 and a backtest will be carried out at the end in order to evaluate the performance of the models during the pre – crisis and crisis periods.

2.2 Background

During the 90's the economies of several South East Asian countries - Malaysia, Indonesia, Thailand, South Korea and the Philippines - were characterized by rapid economic growth. The expansion was made possible by a number of reasons, some of which turned out to be the causes of the following crisis (Radelet, Sachs, Cooper, & Bosworth, 1998):

- Capital inflows across the South East Asian countries averaged over 6% of GDP between 1990 and 1996 which increased the dependence of the economies on such inflows causing them to be more vulnerable in case of capital flow reversal;
- Exchange rates pegged to the US dollar. If, on one hand, exchange risk was absorbed by central banks encouraging capital inflows, on the other it became a serious issue when the Federal Reserve started to increase interest rates and foreign reserve began to scarce;
- Financial deregulation which led to loan provisions without sufficient scrutiny and build up of foreign debt;
- Slowing export growth due to the devaluation of the Chinese Yuan and Mexican Peso in 1994.

In the early months of 1997 the Thai baht came under speculative attack after a major property developer Somprasong Land failed to meet a foreign debt repayment signaling a worsening economy. Thailand government attempted to defend the peg but without success: on July 2 1997 after depleting the Central Bank's foreign reserves, the currency was left to free – float in the market and was drastically devaluated due to capital flight. The devaluation made foreign debt repayment more expensive and firms began to default. Soon after the negative sentiment of the market quickly turned into panic which spread into other countries.

The IMF intervened to stabilize the crisis through a program of emergency lendings in combination with economic reforms which turned out to be ineffective. It is only when the IMF carried out a debt rollover plan at the end of January 1998 that the situation began to normalize.

2.3 The Data

The data represented in the dissertation consist of the daily arithmetic returns of an equally weighted portfolio made up by two East Asian stock market indexes, the Hang Seng Index (HSI) and the FTSE Bursa Malaysia KLCI Index (FBMKLCI) from January 1st 1996 to December 31st 2001. The daily returns have been calculated using the daily closing prices of the indexes for the sample period. Such prices have been downloaded from Bloomberg.

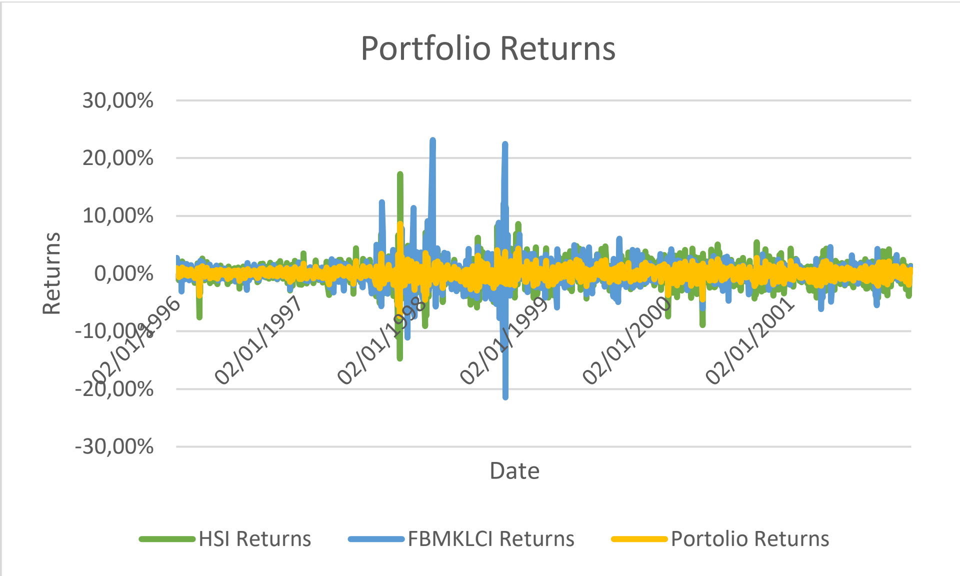

Figure 2.1: Portfolio returns 1/1/1996 – 31/12/2001

We decided to use the aforementioned indexes for a number of reasons:

1. HSI and FBMKLCI are composite indexes. As a consequence, they comprise the 50 and 30 largest companies listed on the Hong Kong and Malaysian stock markets in terms of market capitalization. Therefore, they can be analyzed to track and monitor the performance of the constituent companies as well as a benchmark for the national economy overall;
2. Financial data time series needed for the purpose of such dissertation were fully available for HSI and FBMKLCI for the period 01/01/1996-31/12/2001;
3. HSI and FBMKLCI are one of the most popular indexes analyzed in the scientific literature when dealing with the economy of South-East Asia.

As we can see from Figure 2.1, we can clearly see volatility clustering: volatility changes through time.

As we have mentioned before, the purpose of this dissertation is to evaluate the accuracy of VaR models described in Chapter 1 during the pre-crisis and crisis periods. In order to do so, we need to analyze the Hong Kong 3 month Interbank Offered Rate (HIBOR) and the Kuala Lumpur 3 month

Interbank Offered Rate (KLIBOR) which will give us a hint on the start and end of the financial crisis.

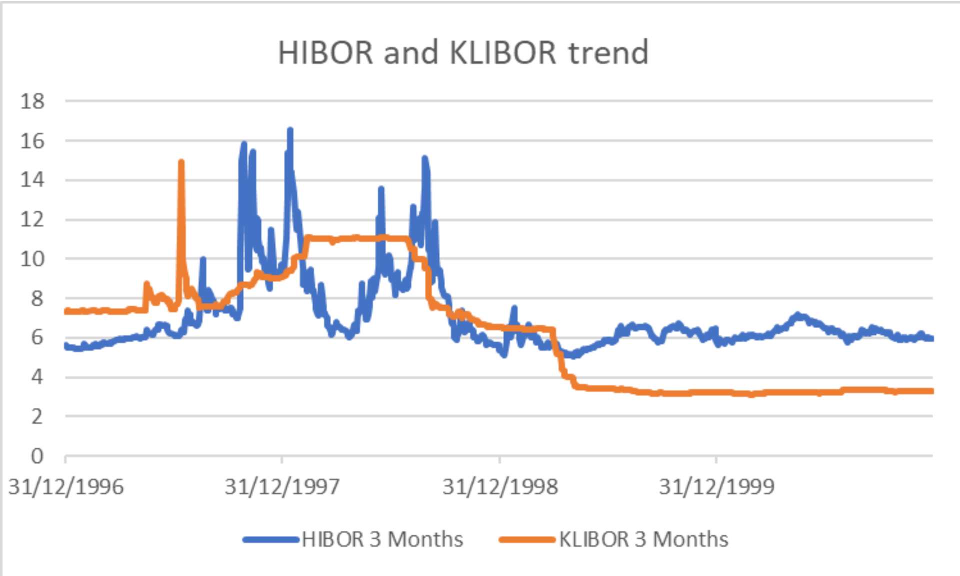

Figure 2.2: 3 month HIBOR and LIBRO trend 1/1/1996 – 31/12/2001

More specifically, the 3 month HIBOR and KLIBOR are short term (3 months) rates for interbank lendings in the Hong Kong and Malaysian interbank markets. According to Furfine (2001) the interbank market has two critical roles in the financial system:

- It is the market where central banks intervene to set their policy rates;
- It allows the transfer of funds from banks in surplus to banks in need.

Morevover, such rate is also used as a reference rate for debt instruments.

Therefore, a hike in the interbank rate can signal the fact that lending banks see their borrowing counterparts as riskier, hence increasing the cost of borrowing and inevitably decreasing the stability of the financial system. Instability can also lead to decreases in business loans thus affecting the real economy overall.

As we can see from Figure 2.2, the 3-month HIBOR and KLIBOR hiked between May 1997 and April 1999. Consequently, we can divide the sample period in two sub-periods: Pre-crisis (1/7/1996-30/04/1997) and Crisis (1/5/1997-30/4/1999).

2.3.1 Forecast window size

In two of the approaches we implemented for Value at Risk calculation, namely Simple Moving Average and Historical Simulation, we used rolling windows of 20, 50, 125 days for volatility estimation. The purpose of this differentiation is to investigate the models' reactivity to new market conditions. While it is intuitive that a shorter window quickly adapts to changes in financial markets, we must also be aware of the *echo – effect* (Figlewsky, 1994): if there is a noticeable change in stock prices, volatility estimation will be susceptible of fluctuations as well. The shorter the window, the greater the fluctuation. When the outlier moves out of the sample being replaced by newer data, volatility will be subject to the same change seen some periods ago, but in the opposite direction. While the first fluctuation is motivated by stock prices variation, the second one occurs simply because the outlier is replaced by other data and has no financial meaning.

2.4 Pre - Crisis

2.4.1 Backtesting Value at Risk

VaR Level=95%, Approach	CC test result	LR ratio CC	P value CC	POF test	LR ratio POF	P value POF	CCI test	LR ratio CCI	P value CCI	Observations	Failures
EWMAlambda094	'accept'	1.40	0.50	'accept'	0.19	0.66	'accept'	1.21	0.27	193	11
EWMAlambda097	'accept'	1.40	0.50	'accept'	0.19	0.66	'accept'	1.21	0.27	193	11
GARCH	'accept'	0.83	0.66	'accept'	0.05	0.83	'accept'	0.79	0.38	193	9
GJR_GARCH	'accept'	1.99	0.37	'accept'	1.67	0.20	'accept'	0.32	0.57	193	6
HistSim125days	'accept'	0.83	0.66	'accept'	0.05	0.83	'accept'	0.79	0.38	193	9
HisrSim20days	'accept'	2.46	0.29	'accept'	0.19	0.66	'accept'	2.27	0.13	193	11
HistSim50days	'accept'	1.16	0.56	'accept'	1.11	0.29	'accept'	0.05	0.83	193	13
SMA125days	'accept'	1.37	0.50	'accept'	0.84	0.36	'accept'	0.53	0.47	193	7
SMA20days	'accept'	1.16	0.56	'accept'	1.11	0.29	'accept'	0.05	0.83	193	13
SMA50days	'accept'	2.85	0.24	'accept'	1.11	0.29	'accept'	1.74	0.19	193	13

Figure 2.3: Pre – crisis VaR backtest results

Figure 2.3, shows the outputs for Kupiec's Proportion of Failures test (POF), Christoffersen's Conditional Coverage Indipendence test (CCI) and the Conditional Coverage mixed (CC) test.

Recalling the backtest methodology described in Chapter 1, Kupiec's POF test aims at evaluating whether the proportion of failures relative to the number of observations is consistent with the model's confidence level. Christoffersen CCI test aims at detecting whether the VaR breaches are independent from one another or not. The CC mixed test is a combination of the first two tests used to assess if the average number of breaches is correct and if such breaches are independent from one another.

In Figure 2.3 we can see that the null hypothesis according to which the number of VaR violations is consistent with the confidence level and that these violations are serially independent is not rejected in any of the models. In essence, the best performing models with the lowest Likelihood ratio value are the GARCH (1,1) model and Historical Simulation with rolling window of 125 days.

On the other hand, the worst performing model with the highest Likelihood ratio value is represented by the Simple Moving Average method with rolling window of 50 days followed by the Historical Simulation with rolling window of 20 days. By decomposing the CC test Likelihood ratio in its two main components, we see

- A high LR value for the POF test and an even higher value for the CCI test regarding the Simple Moving Average with rolling window of 50 days
- A relatively low LR value for the POF test which is counterbalanced by a high value for the CCI test.

Let's also take a look to the Traffic Light (TL) test as proposed by the Basel Committee

VaR Level=95%,				
Approach	Observations	Failures	TL test results	Probability
EWMAlambda094	193	11	'green'	73.97%
EWMAlambda097	193	11	'green'	73.97%
GARCH	193	9	'green'	50.03%
GJR_GARCH	193	6	'green'	14.71%
HistSim125days	193	9	'green'	50.03%
HistSim20days	193	11	'green'	73.97%
HistSim50days	193	13	'green'	89.42%
SMA125days	193	7	'green'	24.66%
SMA20days	193	13	'green'	89.42%
SMA50days	193	13	'green'	89.42%

Figure 2.4: Pre – crisis Traffic Light backtest results

According to Figure 2.4, all the approaches fall into in the *green zone*. The results of the TL test confirm the output of the CC mixed test.

2.4.2 Backtesting Expected Shortfall

VaR Level=95%,				
Approach	Test result	Pvalue	Test Statistic	Critical Value
EWMAlambda094	'reject'	0.01	-0.78	-0.54
EWMAlambda097	'reject'	0.00	-1.05	-0.54
GARCH	'accept'	0.50	0.06	-0.54
GJR_GARCH	'accept'	0.50	0.16	-0.54
HistSim125days	'accept'	0.18	-0.31	-0.54
HistSim20days	'reject'	0.01	-0.79	-0.54
HistSim50days	'reject'	0.00	-1.24	-0.54
SMA125days	'accept'	0.50	0.04	-0.54
SMA20days	'reject'	0.02	-0.68	-0.54
SMA50days	'reject'	0.03	-0.65	-0.54

Figure 2.5: Pre – crisis ES backtest results

As we can see from Figure 2.5, the results of the Acerbi & Szekely test are quite different from the outputs of the Conditional Coverage mixed test: while the conjoined hypothesis according to which the outlined Value at Risk models are coherent with the confidence level and the VaR breaches are independent from one another is not rejected for almost all the approaches, the Expected Shortfall backtest shows that only four out of ten models are significant, namely GARCH (1,1), GJR GARCH, Historical Simulation and Simple moving Average with rolling windows of 125 days. To understand what happened in more details, let’s take a look at the following table.

VaR Level=95%,						
Approach	Confidence Level	Observed Confidence Level	Expected Severity	Observed Severity	Expected # of failures	# of failures to expected # of failures
EWMAlambda094	0.95	0.95	1.25	2.03	10	1.10
EWMAlambda097	0.95	0.95	1.25	2.33	10	1.10
GARCH	0.95	0.92	0.16	0.99	10	1.60
GJR_GARCH	0.95	0.97	1.25	1.76	10	0.60
HistSim125days	0.95	0.92	-0.16	1.07	10	1.60
HisrSim20days	0.95	0.91	0.22	1.13	10	1.80
HistSim50days	0.95	0.90	0.16	1.29	10	2.00
SMA125days	0.95	0.97	1.25	1.71	10	0.70
SMA20days	0.95	0.94	1.25	1.62	10	1.30
SMA50days	0.95	0.94	1.25	1.59	10	1.30

Figure 2.6: Pre – crisis ES backtest results details

Where

- *Observed Confidence Level* is the ratio between the number of periods without failures and the number of observations;
- *Expected Severity* is defined as the average ratio of Expected Shortfall to Value at Risk over the periods with VaR violations;
- *Observed Severity* as the average ratio between the portfolio losses and Value at Risk over the periods with VaR violations

Let's also recall eq. (33). Regarding the Acerbi and Szekely test statistic, the following points where mentioned:

1. A period with many relatively small VaR breaches can cause the model to be rejected;
2. One large VaR violation relative to the estimated ES can cause the model to be rejected;
3. One large VaR violation on a day where the ES is large as well might not cause the model to be rejected.

With respect to point 1, the models that have been rejected are characterized by:

❖ A lower *Observed Confidence Level* relative to theoretical Confidence Level meaning that the actual periods with VaR violations is greater than the predicted ones;

❖ An *Observed Number of Failures* to *Expected Number of Failures ratio* greater than 1 which implies that the number of actual failures is greater than the predicted one;

With respect to points 2 and 3, the models that have been rejected are characterized by:

- A relatively larger difference between the *Observed Severity* and *Expected Severity*, meaning that, all else being equal, actual average losses exceeding VaR are greater than the predicted ones.

Of course, these factors cannot be analyzed individually since the rejection / non rejection of the models depends on the combination of the three.

In essence, Expected Shortfall seems to be a more conservative risk measure relative to Value at Risk since out of ten approaches that have been accepted by the Conditional Coverage Mixed test, six have been rejected by the Acerbi and Szekely test, suggesting that VaR models suffer from risk underestimation relative to ES. In other words, this means that all VaR approaches were able to correctly predict the actual number of VaR violations, but if we also consider the average magnitude of losses through Expected Shortfall calculation, only four out of the ten approaches are deemed to be accurate.

2.5 Crisis

2.5.2 Backtesting Value at Risk

Approach	CC test	LR ratio CC	P value CC	POF test	LR ratio POF	P value POF	CCI test	LR ratio CCI	P value CCI	Observations	Failures
EWMAlambda094	'accept'	3.31	0.19	'accept'	3.00	0.08	'accept'	0.31	0.58	468	32
EWMAlambda097	'accept'	2.40	0.30	'accept'	1.81	0.18	'accept'	0.60	0.44	468	30
GARCH	'reject'	35.31	0.00	'reject'	33.11	0.00	'accept'	2.20	0.14	468	55
GJR_GARCH	'reject'	28.71	0.00	'reject'	27.73	0.00	'accept'	0.98	0.32	468	52
HistSim125days	'accept'	3.90	0.14	'accept'	3.70	0.05	'accept'	0.21	0.65	468	33
HistSim20days	'accept'	0.22	0.90	'accept'	0.11	0.74	'accept'	0.10	0.75	468	25
HistSim50days	'reject'	7.55	0.02	'reject'	7.13	0.01	'accept'	0.42	0.52	468	37
SMA125days	'reject'	8.89	0.01	'reject'	4.46	0.03	'reject'	4.43	0.04	468	34
SMA20days	'reject'	7.36	0.03	'reject'	5.29	0.02	'accept'	2.07	0.15	468	35
SMA50days	'reject'	6.03	0.05	'reject'	5.29	0.02	'accept'	0.75	0.39	468	35

Figure 2.7: Crisis VaR backtest results

Figure 2.7 shows the outputs of the CC mixed test for the different VaR approaches. What is interesting to point out is that only four of the ten models are significant – namely the Exponential Moving Average Method with lambda equal to 0.94 and 0.97 and the Historical Simulations with rolling windows of 125 and 20 days respectively.

By decomposing CC test statistic in its two components, it seems that most of the non-significant approaches were rejected because of failures of the POF test.

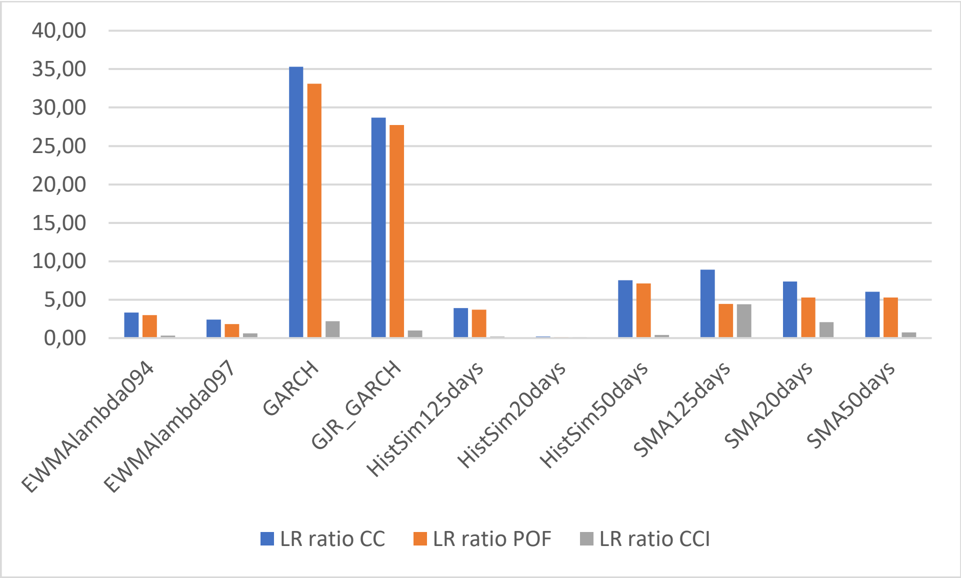

Figure 2.8: Likelihood ratios comparison

As we can see from Figure 2.8, all the non-significant models feature extremely high LR ratios for the POF test relative to the LR ratios for the CCI test with the GARCH (1,1) as the worst performing one.

In other words, the results suggest that VaR breaches are independent from one another, but on the other hand, some of the models were rejected because the actual frequency of the violations are not consistent with the confidence level.

It is also interesting to note that the worst performing approaches belong to the GARCH family, with the GJR GARCH model showing a slightly better performance compared to simple GARCH model.

Let’s also take a look at the traffic lights test

VaR Level=95%,				
Approach	**Observations**	**Failures**	**TL test**	**Probability**
EWMAlambda094	468	32	'yellow'	96.84%
EWMAlambda097	468	30	'green'	92.95%
GARCH	468	55	'red'	100.00%
GJR_GARCH	468	52	'red'	100.00%
HistSim125days	468	33	'yellow'	97.97%
HistSim20days	468	25	'green'	68.11%
HistSim50days	468	37	'yellow'	99.74%
SMA125days	468	34	'yellow'	98.73%
SMA20days	468	35	'yellow'	99.23%
SMA50days	468	35	'yellow'	99.23%

Figure 2.9: Crisis Traffic Light backtest results

The analysis in Figure 2.9 confirms the results of the CC test with the GARCH family models as the worst performings ones and the EWMA and historical simulations with rolling windows of 125 and 20 days as the least inaccurate.

2.5.2 Backtesting Expected Shortfall

VaR Level=95%,					
Approach	**Test**	**Pvalue**	**Test Statistic**	**Critical Value**	**Observations**
EWMAlambda094	'reject'	0.0166	-0.4690	-0.3496	468
EWMAlambda097	'reject'	0.0345	-0.3941	-0.3496	468
GARCH	'reject'	0.0001	-1.6864	-0.3496	468
GJR_GARCH	'reject'	0.0001	-1.4796	-0.3496	468
HistSim125days	'reject'	0.0038	-0.6024	-0.3496	468
HistSim20days	'reject'	0.0108	-0.5026	-0.3496	468
HistSim50days	'reject'	0.0002	-0.8264	-0.3496	468
SMA125days	'reject'	0.0004	-0.7583	-0.3496	468
SMA20days	'reject'	0.0011	-0.6906	-0.3496	468
SMA50days	'reject'	0.0013	-0.6829	-0.3496	468

Figure 2.10: Crisis ES backtest results

Figure 2.10 shows the result of the Acerbi & Szekely test for expected shortfall backtest during the crisis period. It is clear that all the approaches are not significant at a confidence level of 95%, pointing out the inability of the models to capture increased risk. Again, the least accurate models are GARCH (1,1), GJR GARCH and the most accurate EWMA with lambda equal to 0.97 and 0.94 respectively.

Approach	Confidence Level	Observed Confidence Level	Expected Severity	Observed Severity	Expected # of failures	# of failures to expected # of failures
EWMAlambda094	0.95	0.93	1.25	1.35	23.4	1.37
EWMAlambda097	0.95	0.94	1.25	1.36	23.4	1.28
GARCH	0.95	0.88	1.25	1.43	23.4	2.35
GJR_GARCH	0.95	0.89	1.25	1.40	23.4	2.22
HS125dd	0.95	0.93	1.32	1.49	23.4	1.41
HS20dd	0.95	0.95	1.00	1.41	23.4	1.07
HS50dd	0.95	0.92	1.26	1.43	23.4	1.58
SMA125dd	0.95	0.93	1.25	1.52	23.4	1.45
SMA20dd	0.95	0.93	1.25	1.42	23.4	1.50
SMA50dd	0.95	0.93	1.25	1.41	23.4	1.50

Figure 2.11: Crisis ES backtest results details

To understand the reasons underneath the rejections of the models, let's analyze Figure 2.11. All the models are characterized by

- ❖ A lower *Observed Confidence Level* relative to theoretical Confidence Level meaning that the actual periods with VaR violations is greater than the predicted ones;
- ❖ An *Observed Number of Failures* to *Expected Number of Failures ratio* greater than 1 which implies that the number of actual failures is greater than the predicted one;
- ❖ A relatively larger difference between the *Observed Severity* and *Expected Severity*, meaning that, all else being equal, actual average losses exceeding VaR are greater than the predicted ones.

Again, Expected Shortfall seems to be a more conservative risk measure relative to Value at Risk since out of four approaches that have been accepted by the Conditional Coverage mixed test, all have been rejected by the Acerbi and Szekely test, suggesting that VaR models suffer from risk underestimation relative to ES. In other words, this means that all VaR approaches were able to correctly predict the actual number of VaR violations, but if we also consider the average magnitude of losses through Expected Shortfall calculation, none of the ten approaches are deemed to be accurate.

2.6 Analysis

The findings of the empirical study can be summarized in the following points:

1. From **Figure 2.3** and **Figure 2.5**, we can see that during the ***pre – crisis*** period all the ten approaches under the Value at Risk methodology are significant, while only four out of ten approaches are significant under Expected Shortfall (namely GARCH (1,1), GJR GARCH, Historical Simulation and Simple moving Average with rolling windows of 125 days). This means that all VaR approaches were able to correctly predict the actual number of VaR violations, but if we also consider the average magnitude of losses through Expected Shortfall calculation, only four out of the ten approaches are deemed to be accurate;
2. From **Figure 2.7**, we can see that during the ***crisis*** period only four out of ten approaches under the Value at Risk methodology are significant (namely the Exponential Moving Average Method with lambda equal to 0.94 and 0.97 and the Historical Simulations with rolling windows of 125 and 20 days respectively). Indeed, by decomposing the CC test statistic in its two components, it seems that most of the non-significant approaches were rejected because of failures of the POF test. As we can see from **Figure 2.8**, all the non-significant models feature extremely high LR ratios for the POF test relative to the LR ratios for the CCI test with the GARCH (1,1) as the worst performing one. In other words, the results suggest that VaR breaches are independent from one another, but on the other hand, some of the models were rejected because the actual frequency of the violations are not consistent with the confidence level. Finally, as expected, even though rejected, the GJR GARCH model performed slightly better than the standard GARCH (1,1).
   On the other hand, from **Figure 2.10**, we can see that none of the ten approaches are significant under Expected Shortfall. Again, this means that only four of all VaR approaches were able to correctly predict the actual number of VaR violations, but if we also consider the average magnitude of losses through Expected Shortfall calculation, none of the ten approaches are deemed to be accurate.

The failure of VaR and ES approaches can be ultimately be motivated by two factors:

1. The performance of parametric approaches depends on the underlying assumptions regarding returns distribution which distributions are not a good approximation of financial returns behavior;
2. The performance of non – parametric approaches depends on the length of the window size which is the result of a trade – off between accuracy and adaptability of the model. However, all the models did not react fast enough to accommodate changes in the market.

Hence, as stressed in **paragraph 1.5**, Value at Risk should not be used to define a financial institutions' capital requirements but as an ordinary risk management tool for benchmarking different types of risks, limiting risk exposure and designing risk-adjusted performance metrics.

The danger of using VaR as a determinant for capital adequacy clearly emerges from the failure of hedge fund Long-Term Capital Management (LTCM) in 1998 for which VaR was heavily blamed. According to Jorion (2000) the default of the hedge fund relies on a series of incorrect assumptions made by the managers.

First of all, they assumed that the fund's volatility was constant and not greater than an unleveraged investment in the S&P500. Therefore, the resources were allocated so as to maximize expected returns subject to the constraint that risk is no greater than that of US equities. This assumption is inaccurate since volatility is time – varying and can easily double in periods of turmoil. Secondly, the focus on a portfolio's standard deviation is more appropriate when the distribution of a random variable is symmetric which assumption is not accurate for describing the actual behavior of financial returns since they exhibit *fat tails*.

After reporting significant losses in May and June 1998, LTCM tried to reduce its risk profile. However, instead of selling the less liquid positions, they sold the most liquid since they were less profitable. If the model was correct, the daily volatility should have decreased from $45 million to $35 million, however its actual value was closer to $100 million. According to the author, this was due to the fact that the model was biased and that the market was becoming more volatile.

The case of LTCM proves that the VaR model heavily underestimated risk, leading to an inappropriate capital base.

Conclusions

The aim of this research project is to assess the accuracy of Value at Risk and Expected Shortfall models in evaluating market risk. More specifically

- In Chapter 1 we have covered some of the most widely used market risk estimation methods, namely
  - **Value at Risk** (**VaR**) with the following approaches: Simple Moving Average, Exponentially Weighted Moving Average, GARCH (1,1), GJR GARCH and Historical Simulation;
  - **Expected Shortfall (ES)** under the same approaches;
  - **Backtesting methodologies** to evaluate model accuracy, namely the Conditional Coverage mixed test and Traffic Light test for Value at Risk models and the Acerbi & Szekely test for Expected Shortfall.
- In Chapter 2 we have implemented the models described in the previous chapter before and after the 1997 – 98 Asian Financial Crisis. The 1 – day VaR and ES estimates were computed on an equally weighted portfolio composed by the Hang Seng Index and the FTSE Bursa Malaysia KLCI Index and the related backtests were carried out.

In particular, from the findings of Chapter 2 we have discovered that

1. During the ***pre – crisis*** period all the ten approaches under the Value at Risk methodology are significant, while only four out of ten approaches are significant under Expected Shortfall (namely GARCH (1,1), GJR GARCH, Historical Simulation and Simple moving Average with rolling windows of 125 days). This means that all VaR approaches were able to correctly predict the actual number of VaR violations, but if we also consider the average magnitude of losses through Expected Shortfall calculation, only four out of the ten approaches are deemed to be accurate;
2. During the ***crisis*** period only four out of ten approaches under the Value at Risk methodology are significant (namely the Exponential Moving Average Method with lambda equal to 0.94 and 0.97 and the Historical Simulations with rolling windows of 125 and 20 days respectively) while none of the ten approaches are significant under Expected Shortfall. Again, this means that only four of all VaR approaches were able to correctly predict the actual number of VaR violations, but if we also consider the average magnitude of losses through Expected Shortfall calculation, none of the ten approaches are deemed to be accurate. As expected, even though rejected, the GJR GARCH model performed slightly better than the standard GARCH (1,1).

From these findings we can infer that

1. Expected Shortfall is a more conservative risk measure than Value at Risk;
2. Value at Risk approaches were not suitable to forecast losses during the financial crisis period after taking into account the average magnitude of losses with Expected Shortfall

A financial institution would have faced serious problems if it only relied on VaR estimates to determine capital requirements since such methodology

- Depends on the underlying assumptions about returns' distributions in case of parametric approaches[18];
- Depends on the trade – off between accuracy and adaptability when choosing the length analysis window in case of non – parametric approaches. However, in many cases the models did not react fast enough to new market conditions.

As a consequence, as Sironi (2008) pointed out, VaR methodology should not be used to define capital requirements, but it should be considered as an ordinary risk management tool to determine the operational limits of trading desks on a daily basis.

Further Studies

The thesis aimed at evaluating the performance of Value at Risk and Expected Shortfall before and after the 1997 – 98 Asian Crisis by analyzing a portfolio composed by the Hong Kong and Malaysian stock indexes.

The analysis can be replicated for different periods of financial turmoil and with using different approaches. In particular, the analysis of the following topics could be interesting:

- The application of Machine Learning to market risk estimation and assessment;
- Implement the *GARCH model with Jumps* (Sidorov, Revutskiy, Faizliev, Korobov, & Balash, 2014) for volatility modeling during periods of financial distress and compare it with the standard GARCH model.

[18] Which assumptions are not coherent with the actual behavior of financial data.

Summary

3.1 Introduction

The research project aims at evaluating the two main methodologies for market risk estimation, namely *Value at Risk* (VaR) and *Expected Shortall* (ES) under some of the most widely used approaches (*Simple Moving Average, Exponentially Weighted Moving Average, GARCH (1,1), GJR GARCH and Historical Simulation*). More specifically, we will compute the 1 – day VaR and ES estimates at 95% confidence level for an equally weighted portfolio composed by the Hang Seng Index and the FTSE Bursa Malaysia KLCI Index under each approach. The analysis will be carried out before and after the 1997 – 98 Asian Crisis and, at the end, the performances of each model during the two subperiods will be assessed using the Conditional Coverage mixed test (VaR), the Traffic Light test (VaR) and the Acerbi & Szekely test (ES).

For what concerns the structure, the research project is divided in two Chapters. In Chapter 1 we have introduced the theoretical framework of Value at Risk and Expected Shortfall under different approaches as well as the main backtesting methods. In Chapter 2 we have applied the theory illustrated in the first chapter to the empirical study.

3.2 Chapter 1

In Chapter 1 we have discussed some of the main methodologies used in Financial Risk Management to estimate market risk - Value at Risk and Expected Shortfall.

As defined by *RiskMetrics*^{TM} (J.P. Morgan; Reuters, 1996), Value at Risk is defined as:

(…) a measure of the maximum potential change in value of a portfolio of financial instruments with a given probability over a pre-set horizon.

Oftentimes, in VaR calculation it is assumed that the distribution of a portfolio of securities follows a normal distribution. In formulas

$$(35)\ VaR_{t+1}^{1-\alpha} = z_\alpha * \sigma + \mu$$

Where

- z_α is the quantile of the normal distribution at α level
- σ is the standard deviation of returns
- μ is the mean of returns

Even though VaR is a relatively simple and intuitive risk metric, it has some disadvantages as well the most peculiar one being the inability to capture the magnitude of losses. Indeed, the VaR of a portfolio is a risk measure that only tells us the potential losses over a specific period of time given a pre-determined confidence level. But what happens if the VaR limit is hit? Are losses equal or greater than the limit itself? In other words it does not answer the question *if things go bad, how bad can they get?* Therefore, we have also introduced Expected Shortfall as an alternative risk metric. In essence, *Expected Shortfall* (ES) at α level over a specific period of time is the expected portfolio's loss in the worst α cases. In formulas we have

$$(36)\ ES^{1-\alpha} = E(X|X > VaR^{1-\alpha})$$

Where X is the portfolio's loss.

There exists different approaches to VaR and Expected Shortfall calculation which can be classified into two main categories: parametric approaches and non-parametric approaches.

In the first approach, we assume that stock returns follow a given statistical distribution (e.g. Gaussian distribution). The main models belonging to this class that we have discussed are

- **Simple Moving Average** method, where volatility at time *t* is computed as the simple standard deviations of stock returns *n* days ahead;
- **Exponentially Moving Average** method, where volatility is the squared root of the weighted average of squared returns such that exponentially declining weights are assigned to each return going back further in time;
- **Stochastic volatility models** with a focus on GARCH (1,1) and GJR GARCH models where we use historical data to estimate the parameters of the model and then use them to forecast future volatility

After forecasting volatilities with the approaches described above, we can simply implement eq. (35) and (36) for VaR and ES calculation.

The parametric approach is relatively simple to implement, however it suffers some major drawbacks (above all non-normality of returns and *fat-tails*).

In the non-parametric approaches we do not make any assumptions on returns distributions because we "let the data talk". The model belonging to this class that have discussed in more details is **Historical Simulation**, where the Value at Risk at a given level of confidence is computed by ranking the first *n* days past returns, sorting them from smallest to largest and then picking up the quantile that corresponds to the desired confidence level. Then, for ES calculation we can simply take the average of losses exceeding the VaR.

The benefits of such approach rely on the fact that historical data are used in order to estimate Value at Risk and Expected Shortfall, thus overcoming the issue of distributional assumptions on financial data. However, nothing comes to a cost: in choosing the length of the window size, we must carefully evaluate the trade – off between accuracy and adaptability of the model. On the other hand, historical data are not always suitable to describe asset prices movements, especially in periods of crisis.

In Chapter 1 we have also discussed the potential applications of VaR that we can summarize in the following points:

- **Benchmarking different types of risks.** VaR provides a common risk evaluation framework regardless of the nature of the financial asset we are dealing with. It can be considered as a *lingua franca* between different trading desks taking positions in different assets (e.g. bonds, derivatives, stocks etc…). The importance of the VaR is relevant in the following example: think about a world where VaR does not exists. Assuming that we have taken a position in government bonds and stocks, how can we compare different risk metrics such as the duration and the beta of a stock portfolio? Thanks to VaR we are able to encompass these obstacles.
- **Limiting risk exposure.** VaR can be used to set the operating limits of the trading units within a bank. Suppose for example that Bank X has two trading desks: desk 1 (stocks) and desk 2 (bonds). The VaR limits for each unit given a confidence level and a trading horizon is $200,000 and $100,000 respectively. Since the maximum amount that can be invested in a certain position depends on its VaR limit, by changing the latter we can change the capital allocation strategy among different business units.
- **Designing risk-adjusted performance (RAP) metrics.** Finally, VaR can be used to design risk adjusted performance metrics. One of the most commonly used metrics is the RAROC (*Risk-adjusted Return on Capital).*

    - $RAROC_{(ex-ante)} = E(P)/CaR_{(ex-ante)}$
    - $RAROC_{(ex-post)} = P/CaR_{(ex-post)}$

  where

    - $E(P)$ is the expected profit;
    - P the realized profit;
    - $CaR_{(ex-ante)}$ the capital allocated to a single unit;
    - $CaR_{(ex-post)}$ the undertaken risk;

RAP metrics have three main different purposes:

1. Support traders in making investment decisions by analyzing the *ex-ante* profitability and the risk profile of the alternatives;
2. Establish an incentive scheme that is not profit-based only:
3. Compare the *ex-post* performance of the different units within a financial institution to determine which units are allocationg resources more efficiently and hence deserving more capital to invest.

We have also argued about the weaknesses of VaR. More specifically

1. **Extreme events are not accounted for in VaR models.** It is true that extreme events are not accounted for in VaR models. However, as mentioned in the chapter, the main purpose of VaR models is not to measure the capital adequacy of banks, but to estimate the risk exposure and the operational limits of each trading desk on a daily basis. In other words, VaR is a ordinary risk management tool. On the other hand, such rare events must be taken in account in capital adequacy assessment.
2. **The magnitude of losses is not accounted for.** VaR does not take into account the magnitude of losses if a violation occurs. To fix this issue alternative risk metrics were introduced, such as Expected Shortfall.
3. **VaR models yield divergent outputs.** If we change the underlying assumptions of a VaR model (e.g. we assume that returns are distributed as a *t-student* distribution instead of a Gaussian distribution and/or we change extend or shorten the sample period etc…) we will certainly get different results. However, if, as mentioned before, the aim of the model is to evaluate the risk-adjusted performance of the trading units within a bank for capital allocation in the units themselves, this issue is not very relevant. In fact, what we need here is not an assumptions-independent model but a risk assessment framework that is uniformly implemented in all the business units. So even if the model underestimates or overestimates the potential losses, there would not be issues at all since the over/under estimation is reflected through out all the trading units, not affecting the capital allocation strategy.
4. **VaR models can potentially decrease the stability of financial markets.** If everyone in the financial sector has adopted VaR as a risk management tool, then this practice can potentially amplify the volatility of the market. This is true because every trader would get the same result and try to decrease their exposure thus worsening market conditions. However, this should not be considered as a direct implication of VaR models rather than as a consequence of human nature.

5. **VaR is not a coherent risk measure**, namely the *sub-additive* property is violated if the joint distribution of risk factors is not normally distributed (Artzner, Delbaen, Eber, & Heath, 1998). So, in this case the following must be true:

$$VaR(A + B) > VaR(A) + VaR(B)$$

Finally, the main backtesting methodologies were introduced:

❖ For Value at Risk

- **Kupiec's Proportion of Failure** (POF) test which aims at testing whether the number of actual VaR violations is consistent with the confidence level;
- **Christoffersen's Conditional Coverage Indipendence** (CCI) test that assesses whether VaR breaches are serially independent or not;
- **The Conditional Coverage mixed** (CC) test which is a combination of the Kupie's and Christofferesen tests. It aims at evalulationg that VaR models accurately predict the actual number of violations and that they are independent from one another.

The test statistics in all three cases are distributed as chi – squared distributions with one (POF and CCI tests) and two degrees of freedom (CC test). As a consequence, the null hypothesis will be rejected if the value of the test statistic is higher than the critical value of the corresponding confidence level.

❖ For Expected Shortfall

- Acerbi & Szekely (2014) test which aims at testing whether the average loss estimated by the model is accurate or not.

Finally, in paragraph 1.9 we briefly discussed the Basel Regulatory Framework and its guidelines regarding Value at Risk and capital requirements. In particular, an alternative backtesting method for VaR is provided, the Traffic Light test which, analogously with the POF and CCI tests, aims at testing if the models can reasonably predict the number of actual violations in relation to the chosen confidence level by classifying the approaches in zones according to their ex – post performance. Ultimately, in 2013 the Basel Committee stressed the importance of moving from VaR to Expected Shortfall because *a number of weaknesses have been identified with using VaR for determining regulatory capital requirements, including its inability to capture "tail risk"* (Basel Committee on Banking Supervision, 2013)

3.3 Chapter 2

In Chapter 2 we analyzed an equally weighted portfolio made up by two East Asian stock market indexes, the Hang Seng Index (HSI) and the FTSE Bursa Malaysia KLCI Index (FBMKLCI) during the 1997 – 1998 Asian Financial Crisis. More specifically, the daily Value at Risk and Expected

Shortfall has been computed at a confidence level of 95% using the approaches seen Chapter 1 and a backtest has been carried out at the end in order to evaluate the performance of the models during the pre – crisis and crisis periods.

More specifically, we started this chapter by briefly outlining the context and the root causes under the Asian Crisis which can be summarized in the following points:

- Capital inflows across the South East Asian countries averaged over 6% of GDP between 1990 and 1996 which increased the dependence of the economies on such inflows causing them to be more vulnerable in case of capital flow reversal;
- Exchange rates pegged to the US dollar. If, on one hand, exchange risk was absorbed by central banks encouraging capital inflows, on the other it became a serious issue when the Federal Reserve started to increase interest rates and foreign reserve began to scarce;
- Financial deregulation which led to loan provisions without sufficient scrutiny and build up of foreign debt;
- Slowing export growth due to the devaluation of the Chinese Yuan and Mexican Peso in 1994.

Hence, when a major property developer Somprasong Land failed to meet a foreign debt repayment signaling a worsening economy, in the early months of 1997 the Thai baht came under speculative attack. Thailand government attempted to defend the peg but without success: on July 2 1997 after depleting the Central Bank's foreign reserves, the currency was left to free – float in the market and was drastically devaluated due to capital flight. The devaluation made foreign debt repayment more expensive and firms began to default. Soon after the negative sentiment of the market quickly turned into panic which spread into other countries.

The IMF intervened to stabilize the crisis through a program of emergency lendings in combination with economic reforms which turned out to be ineffective. It is only when the IMF carried out a debt rollover plan at the end of January 1998 that the situation began to normalize.

We then moved on describing the data used in the thesis which consist of the daily arithmetic returns of an equally weighted portfolio made up by two East Asian stock market indexes, the Hang Seng Index (HSI) and the FTSE Bursa Malaysia KLCI Index (FBMKLCI) from January 1st 1996 to December 31st 2001. The daily returns have been calculated using the daily closing prices of the indexes for the sample period. Such prices have been downloaded from Bloomberg. Since the purpose of this dissertation is to evaluate the accuracy of VaR models described in Chapter 1 during the pre-crisis and crisis periods, we analyzed the Hong Kong 3 month Interbank Offered Rate (HIBOR) and the Kuala Lumpur 3 month Interbank Offered Rate (KLIBOR) which has given us a hint on the start and end of the financial crisis. Hence, in **Figure 2.2**, we can see that the 3-month HIBOR and KLIBOR hiked between May 1997 and April 1999. Consequently, we have divided the sample period in two sub-periods: Pre-crisis (1/7/1996-30/04/1997) and Crisis (1/5/1997-30/4/1999).

3.3.1 Outputs of the empirical study

As mentioned earlier, in Chapter 2 we have implemented the models described in the previous Chapter 1 before and after the 1997 – 98 Asian Financial Crisis. The 1 – day VaR and ES estimates have been computed on an equally weighted portfolio composed by the Hang Seng Index and the FTSE Bursa Malaysia KLCI Index and the related backtests were carried out.

The findings of the empirical study can be summarized in the following points:

1. From **Figure 2.3** and **Figure 2.5**, we can see that during the ***pre – crisis*** period all the ten approaches under the Value at Risk methodology are significant, while only four out of ten approaches are significant under Expected Shortfall (namely GARCH (1,1), GJR GARCH, Historical Simulation and Simple moving Average with rolling windows of 125 days). This means that all VaR approaches were able to correctly predict the actual number of VaR violations, but if we also consider the average magnitude of losses through Expected Shortfall calculation, only four out of the ten approaches are deemed to be accurate;
2. From **Figure 2.7**, we can see that during the ***crisis*** period only four out of ten approaches under the Value at Risk methodology are significant (namely the Exponential Moving Average Method with lambda equal to 0.94 and 0.97 and the Historical Simulations with rolling windows of 125 and 20 days respectively). Indeed, by decomposing the CC test statistic in its two components, it seems that most of the non-significant approaches were rejected because of failures of the POF test. As we can see from **Figure 2.8**, all the non-significant models feature extremely high LR ratios for the POF test relative to the LR ratios for the CCI test with the GARCH (1,1) as the worst performing one. In other words, the results suggest that VaR breaches are independent from one another, but on the other hand, some of the models were rejected because the actual frequency of the violations are not consistent with the confidence level. Finally, as expected, even though rejected, the GJR GARCH model performed slightly better than the standard GARCH (1,1).

   On the other hand, from **Figure 2.10**, we can see that none of the ten approaches are significant under Expected Shortfall. Again, this means that only four of all VaR approaches were able to correctly predict the actual number of VaR violations, but if we also consider the average magnitude of losses through Expected Shortfall calculation, none of the ten approaches are deemed to be accurate.

From these findings we can infer that

- Expected Shortfall is a more conservative risk measure than Value at Risk;
- Value at Risk approaches were not suitable to forecast losses during the financial crisis period after taking into account the average magnitude of losses with Expected Shortfall

A financial institution would have faced serious problems if it only relied on VaR estimates to determine capital requirements since such methodology

- Depends on the underlying assumptions about returns' distributions in case of parametric approaches[19];
- Depends on the trade – off between accuracy and adaptability when choosing the length analysis window in case of non – parametric approaches. However, in many cases the models did not react fast enough to new market conditions.

As a consequence, as Sironi (2008) pointed out, VaR methodology should not be used to define capital requirements, but it should be considered as an ordinary risk management tool to determine the operational limits of trading desks on a daily basis. Indeed, the risk and dangers of using VaR as a determinant for the capital base has been discussed in Jorion's paper (Jorion, 2000) which was mentioned at the end of paragraph 2.5.

[19] Which assumptions are not coherent with the actual behavior of financial data.

Appendix A – GARCH (1,1) parameters estimates

Pre crisis HSI parameters				
Parameter	Value	Standard Error	t-statistic	p-value
Constant	5.30E-06	3.07E-06	1.7285	0.0839
GARCH{1}	0.9064	0.0284	31.8904	3.62E-223
ARCH{1}	0.0612	0.0182	3.3735	7.42E-04

Pre crisis FBMKLCI GARCH parameters				
Parameter	Value	Standard Error	t-statistic	p-value
Constant	8.91E-07	1.61E-06	0.5517	0.5812
GARCH{1}	0.9336	0.0113	82.5053	0
ARCH{1}	0.0594	0.0128	4.641	3.47E-06

Figure 3.1: Pre – crisis GARCH (1,1) parameters estimates

Crisis HSI GARCH parameters				
Parameter	Value	Standard Error	t-statistic	p-value
Constant	5.36E-06	4.44E-06	1.2086	0.2268
GARCH{1}	0.9	0.0404	22.2874	4.90E-110
ARCH{1}	0.05	0.0178	2.8072	0.005

Crisis FBMKLCI GARCH parameters				
Parameter	Value	Standard Error	t-statistic	p-value
Constant	3.29E-06	2.07E-06	1.5882	0.1122
GARCH{1}	0.8934	0.0303	29.5092	2.19E-191
ARCH{1}	0.0683	0.0196	3.4787	5.04E-04

Figure 3.2: Crisis GARCH (1,1) parameters estimates

Appendix B – GJR GARCH parameters estimates

Pre crisis HSI GARCH parameters				
Parameter	Value	Standard Er	t-statistic	p-value
Constant	7.00E-06	1.26E-06	5.5621	2.67E-08
GARCH{1}	0.9005	0.0283	31.765	1.97E-221
ARCH{1}	0.0107	0.0231	0.4611	0.6447
Leverage{1}	0.0946	0.0327	2.89	0.0039

Pre crisis FBMKLCI GARCH parameters				
Parameter	Value	Standard Er	t-statistic	p-value
Constant	2.67E-07	1.46E-06	0.1829	0.8549
GARCH{1}	0.9451	0.0102	92.7526	0
ARCH{1}	0.0295	0.01	2.9606	0.0031
Leverage{1}	0.0508	0.0182	2.7991	0.0051

Figure 4.1: Pre – crisis GJR GARCH parameters estimates

Crisis HSI GARCH parameters				
Parameter	Value	Standard I	t-statistic	p-value
Constant	5.33E-06	4.48E-06	1.1896	0.2342
GARCH{1}	0.9	0.0403	22.3113	########
ARCH{1}	0.05	0.0314	1.5937	0.111
Leverage{1}	-	-	-	-

Crisis FBMKLCI GARCH parameters				
Parameter	Value	Standard I	t-statistic	p-value
Constant	2.65E-06	1.92E-06	1.3817	0.1671
GARCH{1}	0.9035	0.0289	31.2741	########
ARCH{1}	0.0431	0.0179	2.4131	0.0158
Leverage{1}	0.0513	0.0199	2.58	0.0099

Figure 4.2: Crisis GJR GARCH parameters estimates

Appendix C – MATLAB scripts

Value at Risk backtest script (using MATLAB's Risk Management Toolbox™)

```
vbt=varbacktest(Returns,[EWMAlambda094,EWMAlambda097,GARCH,GJR_GARCH,HS125dd,HS20dd,HS50dd,SMA125dd,SMA20dd,SMA50dd],'VaRID',{'EWMAlambda094','EWMAlambda097','GARCH','GJR_GARCH','HS125dd','HS20dd','HS50dd','SMA125dd','SMA20dd','SMA50dd'})

cc_test_results=cc(vbt)

tl_test_results=tl(vbt)
```

Expected Shortfall backtest script (using MATLAB's Risk Management Toolbox™)

```
ebt=esbacktest(Returns,[EWMAlambda094,EWMAlambda097,GARCH,GJR_GARCH,HS125dd,HS20dd,HS50dd,SMA125dd,SMA20dd,SMA50dd],[ES_EWMAlambda094,ES_EWMAlambda097,ES_GARCH,ES_GJR_GARCH,ES_HS125dd,ES_HS20dd,ES_HS50dd,ES_SMA125dd,ES_SMA20dd,ES_SMA50dd],'VaRID',{'EWMAlambda094','EWMAlambda097','GARCH','GJR_GARCH','HS125dd','HS20dd','HS50dd','SMA125dd','SMA20dd','SMA50dd'})

summary=summary(ebt)

test_results=unconditionalNormal(ebt)
```

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