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"Predictability of Return"

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Abstract

This thesis tries to find an answer to a fundamental problem of financial theory: is excess return predictable? To reply to this question we will study both bonds and stocks, using reliable and stable instruments, applied to the US market. We will also utilize a "scientific method" in action, the kind of reasoning that could lead to the creation and the constant improvement of a portfolio of assets, over time, using the past to predict the future and, in the same time, positioning ourselves in the right spot on the market, between "riskless" and aggressive strategies. In doing so, we will not forget the true aim of a work like this: the application of studied economic theory to the reality of the word, a learning process that could lead a simple student to a better understanding of financial markets.

Introduction: Predictability of Return

This dissertation has the objective to analyze a possible predictable evolution in the markets, both for long-maturity government bonds and stocks, or even to try and apply a strategy that could trade a combination of the two. If a trading signal, or a combination of some proxies, could predict excess return consistently, using economic theory, with positive and remarkable results, this could, hopefully, be exploited in a trading strategy, made less risky and more remunerative as possible as time goes on. The first idea comes from a paper made by Antti Ilmanen, published in June 1995 in the The Journal of Finance called "Time-Varying Expected Returns in International Bond Markets"[1]. Ilmanen says that when we try to understand foreseeable changes in U.S. and international asset returns there are two ways to consider the origin of this "soothsaying"[2]. A right assumption could be an evidence of low efficiency in the market or otherwise it could be a rational response to changes in the economic situation. We will see that a combination of these two not mutually excluding factors is maybe the best explanation.

We should focus on two questions:

- Is it possible to predict excess return using a proxy specific to the single country, but still relevant on a broad economic sense?
- Is this proxy consistent with concepts like rationality or market efficiency or in opposition to them? Can we use this approach across different financial markets?

I will start this paper, mimicking the reasoning used by the economist, studying government bond markets, concentrating my personal study on the United States. In comparison to other markets, bonds, especially government ones, are much safer and also easier to analyze. An interesting taking from the paper written by Ilmanen is that the excess returns of a long-term government bond is dependent only on interest rate risk without uncertainty related to liquidity, no default risk while risk coming from the

[1] Antti Ilmanen, *Time-Varying Expected Returns in International Bond Markets*, The Journal of Finance, Vol 50, No. 2 (Jun., 1995), pp. 481-506.
[2] Alan Zimmerman and Jim Blythe, *Business to Business Marketing Management: A Global Perspective*, Routledge (2005).

forex market could be hedged, (for example using forwards and futures)[3] and this factor will be important in my reasoning because I, an Italian student, used to reason with Euros, could easily approach a trading strategy of this kind. In reality we will see that our strategy will also partially depend on the price of this bonds over time, but as we will demonstrate later, interest rate will remain the dominant factor.

The return awarded for investing in a set of the financial market like bonds, considered as a safe one, especially during bad times, will probably give us only small positive earnings overall, but if we are able to find a good proxy to predict how the market will develop, we will gain positive results nonetheless and this is the most important thing on the long run, especially if the simplicity of our analysis will lead to a clear picture, trading period after trading period, of the whole market. Predictability and reliability will be concepts to count on, in particular during recessions or economic difficulties of the market. If for any kind of reason we fail in our intent, or at least we are not satisfied by the total results, we should try different methods of trading, hopefully combining different markets together, like for example the stock market, a market that we will indirectly study from the first moment of our strategy.

In his paper Ilmanen analyzed the foreseeable change in the excess returns of government bonds of long maturities between January 1978 and June 1993 in the United States, Canada, Japan, Germany, France, and the United Kingdom, using the month as the basic trading period. The situation regarding these six markets was, at that moment in time and is even more today, that of the world's most liquid and developed markets.[4] Hence, in my work I put myself in the shoes of someone who wants to analyze these markets after the release of this paper using it as a kind of first step for a simple model that I will try to study, improve and develop over time. I am not as inclusive as Ilmanen and I will favour concentrating my studies on a single country, but applying a strategy that could be, in my personal opinion, easily expandable for any country, or at least the major ones.

[3] Frank K. Reilly and Keith C. Brown, *Investment Analysis & Portfolio Management*, South-Western CENGAGE Learning (1970), Chapter 21.

[4] Antti Ilmanen, *Time-Varying Expected Returns in International Bond Markets*, The Journal of Finance, Vol 50, No. 2 (Jun., 1995), pp. 481-506.

This dissertation is organized as follows. In the first section we analyze the Model presented by Ilmanen, which gave me the first idea for this thesis. Following that, I present the first structure of my personal analysis, with the specific choices of assets and proxies I made to try and tail the reasoning made by the economist. In the third part I will explain the results I have found applying this interesting concepts to the real world and the problems I faced with possible explanations of why they happen. In the Section 4 I will improve the strategy, searching for new alternatives, that will be put together and expanded, step after step in Section 5, to reach a “Final” portfolio. Section 6 will offer some more food for thought. At last I will conclude this work with my personal take on the subject.

Section 1: Ilmanen's Model

1.1 Some basis

Ilmanen reports that other analysis were published before his paper, that tried to understand how to predict excess bond returns with an approach to fluctuations based on different time periods, citing Lauterbach (1989), Zhang (1992), and Boudoukh (1993) in particular.

They had only mixed results, thus other economists regarded this excess returns as an expression of something more than merely risk compensation of the specific market, as said by Mankiw (1986), DeBondt and Bange (1992).

In contrast to other studies, time-varying risk aversion is the way chosen by the author to explain this unidentified component.[5]

In his search, understanding his previous reasoning, Ilmanen wanted to project the profit obtained by a bond in a zero-investment scenario, using a proxy that could be a key factor of the risk premium between the long government bond and the "risk-free" asset, so he used the Inverse Relative Wealth signal, that stated simply is the the ratio of exponentially weighted past wealth to current wealth, which is potentially, for the economist, able to include details about changes in anticipated bond risk premia, thus leading to excess return if this pattern is fully understood and exploited. For Ilmanen a factor that measures the reluctance of the average investor to take a certain risk, contingent to a particular state of the economy, causes a countercyclical movement in bond returns, and this fact is confirmed also by more recent studies that show "a fear-based explanation of the increase of risk aversion during the financial crisis"[6].

In fact, relative risk aversion (called RRA by the author) and relative wealth clearly have an inversely proportional relationship: in simple terms an agent on the market is less likely to risk when his wealth is decreasing, with utility of a single dollar increasing as the total amount is lowering. The fact that investors are more risk averse and demand a

[5] Antti Ilmanen, *Time-Varying Expected Returns in International Bond Markets*, The Journal of Finance, Vol 50, No. 2 (Jun., 1995), pp. 481-506.

[6] Luigi Guiso, Paola Sapienza and Luigi Zingales, *Time varying risk aversion*, Journal of Financial Economics, 128 (2018) 403-421.

higher compensation for bearing more risk in their assets when their wealth is relatively low, a pretty logical concept, was stated by Sharpe[7] and Chen[8] in 1989 and 1991 respectively.

Here comes the inverse relative stock market wealth (INVRELW), the magic pill, intended as a proxy for the relative risk aversion level. Further developments of his theory convinced Ilmanen that a trading strategy, based on the inverse relative stock market wealth, made to exploit this return predictability of world bonds he observe, could earn annual excess returns between 3% to 8%, at least in his calculations for the passed 30 years.[9]

This approach really shocked me: I was always inclined to an opposite one, with the bond market as a possible "tutor" for an investment intuition in the stock market. I was intrigued by this reading, which is why I will explain his process, step after step.

1.2 Starting from Marcus

In 1989 Marcus presented a model based on two periods and two assets where analogous actors have the same utility fuction:

$$U(W) = \frac{1}{1-\gamma}\,(W - W_{min})^{1-\gamma}$$

with wealth (W), "the floor on wealth that might correspond to a subsistence value"[10] (W_{min}) and a positive constant (γ).

Relative Risk Averse formula, called by Marcus simply "A", was closely related to total wealth with the formula:

[7] William Sharpe, *Investor Wealth Measures and Expected Return*, from Quantifying the Market Risk Premium Phenomenon for Investment Decision Making, ICFA (September 1989).

[8] Nai-fu Chen, *Financial Investment Opportunities and the Macroeconomy*, The Journal of Finance, Vol. 46, No. 2 (Jun., 1991), pp. 529-554.

[9] Antti Ilmanen, *Time-Varying Expected Returns in International Bond Markets*, The Journal of Finance, Vol 50, No. 2 (Jun., 1995), pp. 481-506.

[10] Alan .J. Marcus, *An Equilibrium Theory of Excess Volatility and Mean Reversion in Stock Market Prices*, NBER Working Paper No. 3106.

$$A = \frac{-WU(W)}{U(W)} = \gamma \times \frac{W}{W - W_{min}}$$

Agents' risk aversion goes to inifinity when $W = W_{min}$, hence, assuming rationality, they will never allow their wealth to fall below a certain point, the target for subsistence. As soon as we come closer to this amount, the risk aversion increases. In the opposite direction, as the actor becomes wealthier, relative risk aversion approches the constant γ in an asymptitocal way from above. If we erase the subsistence level from the equation we obtain a regular utility function, with a constant relative risk aversion. As a consequence, a positive W_{min} decreases the risk aversion.

In particular, in the model presented by Marcus, the floor under which an agent would never let his wealth go beyond, was fixed while absolute wealth had an inverse relationship with Risk Aversion and the premium needed to risk.

Ilmanen challenges this model presenting an approach based on a target of subsistence that changes with time and a relative risk aversion inversely related to relative wealth.[11] As we can easily imagine, in fact, the survival target changes for various reasons like the necessities of everyday's life of an average consumer. For example the study of Sundaresan (1989) is based on consumption smoothing[12], Constandinides (1990) focused on habit persistence[13], Campbell and Cochrane (1994) linked excess stock and bond returns to fluctuations of the economy[14]: all these studies are based on subsistence wealth considered as a consequence of past consumption, leading to the countercyclical pattern in RRA, as expected by Ilmanen. For now all seems to be working pretty fine.

[11] Antti Ilmanen, *Time-Varying Expected Returns in International Bond Markets*, The Journal of Finance, Vol 50, No. 2 (Jun., 1995), pp. 481-506.

[12] Suresh M. Sundaresan, *Intertemporally Dependent Preferences and the Volatility of Consumption and Wealth*, Review of Financial Studies 2, (1989) 73-89.

[13] George M. Constantinides, *Habit Formation: A Resolution of the Equity Premium Puzzle*, Journal of Political Economy Vol. 98, No. 3 (Jun., 1990), pp. 519-543.

[14] John Y. Campbell and John H. Cochrane, *By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior,* Journal of Political Economy Vol. 107, No. 2 (April 1999), pp. 205-251.

1.3 The proxy: Inverse Relative Wealth

Ilmanen uses the Inverse Relative Wealth, intended as the ratio between past and current relative wealth, as the indicator for the situation of relative risk aversion. In particular he uses an Exponential Moving Average of past wealth levels, hence giving less importance to the more remote history of the economic cycle.

His Inverse Relative Wealth equation is:

$$INVREL\ W_t = \frac{ewaW_{t-1}}{W_t} = \frac{(W_{t-1} + 0{,}9\ W_{t-2} + 0{,}9^2\ W_{t-3} + \cdots) \times 0{,}1}{W_t}$$

Where 0,9 is a factor to weight the relative importance of past observations, W_t is the value of the stock market at the current time t, $ewaW_{t-1}$ is the Exponential Moving Average of the stock market until the previous moment in time considered, usually with a lag of 1 month.[15]

The Relative Risk Aversion here has a stronger correlation coefficient with variations in wealth when the denominator is smaller, because he structured it as a relative measure. Previous studies on the expected premia to compensate for higher risk showed a direct proportion with the Relative Risk Aversion formulation, giving as a result a significant direct proportion between the Inverse Relative Wealth function and the returns of the assets we will receive in the future too.

As an indicator for global wealth Ilmanen's choice is the stock market index. Years before two economists tried to describe a "World Portfolio" using many assets, ranging between stocks, bonds, "cash", metals and real estate[16], hence Ilmanen already knows, and he states it clearly, that this is an approximation, because stocks are a lonely percentage of the total "World Portfolio", but he is more interested in the changes of aggregate wealth than wealth itself and stock market is really volatile with a strong direct relationship with the other sections that constitute the economy. Another reason is that stock prices are measured, precise and easier to understand than other important

[15] Antti Ilmanen, *Time-Varying Expected Returns in International Bond Markets*, The Journal of Finance, Vol 50, No. 2 (Jun., 1995), pp. 481-506

[16] Roger G. Ibbotson and Laurence B. Siegel, *The World Market Wealth Portfolio*, The Journal of Portfolio Management, (Sep., 1983) 5-17

variables, which are more an approximation than a clear picture, which, in fact, in his studies have already failed in the roles of possible proxies. Moreover, to prove his reasoning, he examined the Inverse Relative Wealth showing that it really reflectes Relative Risk Aversion with a distinct countercyclic pattern in regards to the bond market movement in the United States. Moreover he finds the presence of a temporal lag that allows to predict the excess bond returns, hence allowing us to trade with it.[17]

Here comes my first observation: the stock market is a notoriously wild horse. It could, maybe, be intended as a good proxy but will it be one whole month ahead of the bond market? Or may it be possible an opposite view? Are not bonds the safe assets during crisis[18]? If a crisis has already happened, is it not already too late?

We will find our answers much later.

In the graph below, taken from the Ilmanen's paper, we have the Inverse Relative Wealth, the excess bond return expressed as the monthly return of long-maturity government bond in excess of a one-month Treasury Bill Rate in percentage and a total market index deflated by the consumer price index to represent nine similar cycles happened between 1953 and 1992. Although the period of expansion (exp) is usually longer than the period of contraction (con) he divided them in three more subsamples of equal graphical size for a better understanding of the process.[19]

[17] Antti Ilmanen, *Time-Varying Expected Returns in International Bond Markets*, The Journal of Finance, Vol 50, No. 2 (Jun., 1995), pp. 481-506.
[18] Gary B. Gorton and Guillermo Ordoñez, *The Supply and Demand for Safe Assets*, NBER Working Paper No. 18732 (Jan., 2013).
[19] Antti Ilmanen, *Time-Varying Expected Returns in International Bond Markets*, The Journal of Finance, Vol 50, No. 2 (Jun., 1995), pp. 481-506.

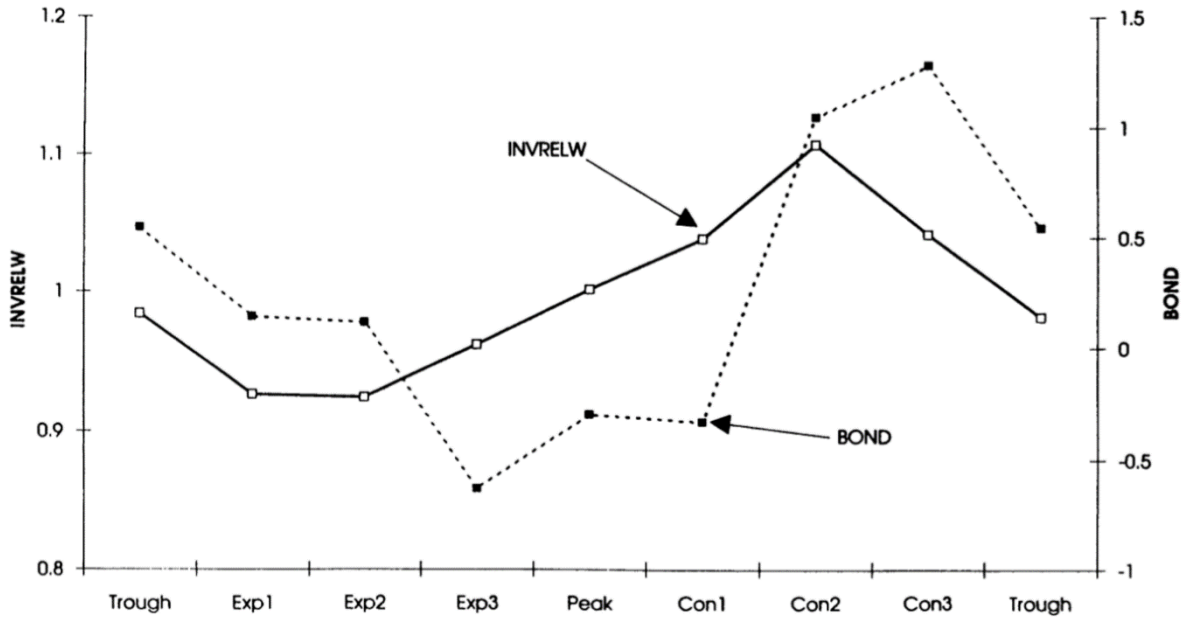

Exhibit 1. Average cyclical behavior of the U.S. inverse relative wealth and excess bond return, 1/1953 to 12/1992.

The following figure shows the average value of U.S. INVRELW and the U.S. bond return at different stages of the business cycle. As Ilmanen expected, INVRELW shows a distinct countercyclic pattern. Furthermore, the excess bond return exhibits a similar pattern with a temporal lag, so INVRELW appears to track well the business cycle pattern of the expected bond risk premium.[20]

1.4 His Dataset

Going to the geographical choice of his studies, Ilmanen analyzed only bonds with a reliable pricing and with no default risk studying the predictability of excess government bond returns in six different countries, the most advanced ones: United States, United Kingdom, Canada, France, Germany and Japan. These countries represented 80% of the global bond market of the period and the access for data was optimal and easily available.

He structured the time period between January 1978 and June 1993, concentrating his studies to ten-year government bonds, and shorting one-month treasuries, to create a

[20] Antti Ilmanen, *Time-Varying Expected Returns in International Bond Markets*, The Journal of Finance, Vol 50, No. 2 (Jun., 1995), pp. 481-506.

zero-investment portfolio. The reliability of predictions in North America (United States and Canada) was superior to the other countries involved and some markets reached a comparable fluidity only years later like Japan (1970s) or France (1980s).[21]

It is useful to point out here that in later studies Ilmanen expanded the time-period to whole 30 years, from 1965 to 1995, even if limiting them from a geographical perspective.[22] This factor will be much useful later, especially with clear tables about his results that could be easily understanded and compared to our results.

Ilmanen finds that the markets of Canada, United States and the United Kingdom are much more unstable. The correlation between returns in different countries is positive, for the government bond market, ranging from 0,33 to 0,55, but between United States and Canada, so closely related, it is quite higher reaching a value of 0,79. Inverse Relative Wealth used in the role of forecasting istrument, was really persistent while the excess return series have autocorrelations positive even if in a weakly way. He also tried to create a "World" bond market weighting the results of these six countries to their comparative total global importance. All these results are presented in Exhibit 2, taken directly from his paper.[23]

[21] Antti Ilmanen, *Time-Varying Expected Returns in International Bond Markets*, The Journal of Finance, Vol 50, No. 2 (Jun., 1995), pp. 481-506.
[22] Antti Ilmanen, *Forecasting U.S. Bond Returns (Understanding the Yield Curve, Part 4),* Salomon Brothers, August 1995.
[23] Antti Ilmanen, *Time-Varying Expected Returns in International Bond Markets*, The Journal of Finance, Vol 50, No. 2 (Jun., 1995), pp. 481-506.

Panel A: Sample Means, Standard Deviations, and First-Order Autocorrelation Coefficients							
	US	*CA*	*JA*	*GE*	*FR*	*UK*	*WO*
Bond							
Mean	0.082	0.012	0.087	0.020	−0.017	−0.021	0.061
Std.	2.917	3.054	1.743	1.554	1.666	2.813	1.983
AC1	0.134	0.211	0.175	0.176	0.140	0.095	0.191
INVRELW							
Mean	0.948	0.971	0.961	0.968	0.942	0.939	0.955
Std.	0.072	0.107	0.136	0.115	0.120	0.077	0.076
AC1	0.834	0.903	0.922	0.902	0.878	0.795	0.885

Panel B: Sample Correlations Between Excess Bond Returns							
	US	*CA*	*JA*	*GE*	*FR*	*UK*	*WO*
US BOND	1.000						
CA BOND	0.794	1.000					
JA BOND	0.417	0.398	1.000				
GE BOND	0.509	0.541	0.549	1.000			
FR BOND	0.340	0.410	0.365	0.518	1.000		
UK BOND	0.389	0.375	0.393	0.388	0.332	1.000	
WO BOND	0.951	0.825	0.623	0.663	0.491	0.540	1.000

Exhibit 2. Summary Statistics for International Bond Returns and Inverse Relative Wealth, 1/1978 to 6/1993[24]

1.5 Predictability of the Regression

Now Ilmanen wants to practically see whether the Inverse Relative Wealth function he created is a useful predicting strategy both for singular countries and a simulated global market. Ilmanen reported the slope coefficients, t-statistics, the R^2, adjusting for their degrees of freedom, both in-sample and out-of-sample. The out-of-sample is used to avoid a possible "data-snooping-bias": it is created a model based on all the past results and it is tried on the following period, to see if it is able to understand how the real value will become. This process is also called Cross-Validation.[25]

The results renovate the idea that Inverse Relative Wealth as a proxy seems to foresee the pattern that the bond market will follow in a statistically significant way. He also

[24] Antti Ilmanen, *Time-Varying Expected Returns in International Bond Markets*, The Journal of Finance, Vol 50, No. 2 (Jun., 1995), pp. 481-506.

[25] Ronald Christensen, *Thoughts on prediction and cross-validation*, Department of Mathematics and Statistics, University of New Mexico (May 21, 2015).

performed an out-of-sample analysis finding that even in this case Inverse Relative Wealth has a consistent ability to predict future returns, with a R^2 value of the out-of-sample actually stronger than the R^2 of the in-sample result in United States, Canada and Germany.[26]

The Inverse Relative Wealth could predict, in-sample, a range between 4% and 11% of the excess return in the bond market, while in the out-of-sample it goes between 0% and 7%. Moreover the R^2 of the World Bond Market is even higher, finding that Inverse Relative Wealth could be an even better predictor taken as a global proxy than in isolated bond markets.

In 1991 Harvey studied 17 different countries and found that a single source of risk could describe the return variation in 16 of this countries, excluding only Japan that was not integrated in the "global" system (and as said before probably was affected by liquidity problems. Hence a good worldwide signal, related to the overall situation of the "world" market, could understand the pattern of all (or at least the most important) states best than a strict focus on a singular region: integration between markets was becoming a reality.[27]

It makes sense, it is clear that our suspect seems wrong. Inverse Relative Wealth could be a good proxy for the bond market: all Ilmanen's data points in that direction.

It appears to be possible to predict the way bond markets will evolve, even if this an ability also depending by the situation in which the market finds itself, in fact Ilmanen noted that excess returns were better predicted during financial crisis than in a situation of bull market as stated by Tierens in 1994 about the U.S. stock market.[28]

From regular economic theory, we can also add that during a financial crisis there will always be a run-to-safety, represented by less risky investments, such as the bond market. For the period of growth of the economy, especially the middle section in which he divided these years, it standed out as the least predictable section of his study. In my personal opinion it may well be possible that the investor will avoid a safer bond

[26] Antti Ilmanen, *Time-Varying Expected Returns in International Bond Markets*, The Journal of Finance, Vol 50, No. 2 (Jun., 1995), pp. 481-506.
[27] Campbell R. Harvey, *The World Price of Covariance Risk*, The Journal of Finance, Vol. 46, No. 1. (Mar., 1991), pp. 111-157.
[28] Antti Ilmanen, *Time-Varying Expected Returns in International Bond Markets*, The Journal of Finance, Vol 50, No. 2 (Jun., 1995), pp. 481-506.

investment, waiting for a peak in the stock market, but we will clearly demonstrate what happens later.

Independent Variable	Dependent Variables						
	US BOND	*CA BOND*	*JA BOND*	*GE BOND*	*FR BOND*	*UK BOND*	*WO BOND*
WO INVRELW							
b	6.76	6.49	3.85	3.31	2.10	5.60	5.30
$[t]$	[2.41]	[1.97]	[2.01]	[2.31]	[1.21]	[1.78]	[2.64]
R^2	2.6	2.1	2.3	2.1	0.4	1.8	3.6
($oos\ R^2$)	(5.6)	(3.1)	(1.1)	(6.8)	(−0.4)	(1.0)	(6.4)

Exhibit 3. Regressions of Excess Government Bond Returns

Summary statistics for the United States (US), Canada (CA), Japan (JA), Germany (GE), France (FR), the United Kingdom (UK), and the "world" (WO), a GNP-weighted average of six country-specific variables, while INVRELW stands for Inverse Relative Wealth. b is the regression slope coefficient, and t is its t-statistic. R^2 is the in-sample coefficient of determination (adjusted for degrees of freedom and expressed in percent), while "oos R^2" is the out-of-sample coefficient of determination.[29]

1.6 Economic significance

Of course Ilmanen needed to study a little more in depth how predictable was excess return overall, comparing the effectiveness of the dynamic trading strategy he utilized, based on his world instrument, to the results of a strategy based on a simple “World” long government bond portfolio in a static way, as showed in the following table.

[29] Antti Ilmanen, *Time-Varying Expected Returns in International Bond Markets*, The Journal of Finance, Vol 50, No. 2 (Jun., 1995), pp. 481-506.

	Static: Always Bond	Dynamic In-Sample		Dynamic Out-of-Sample	
		Scaled	1/0	Scaled	1/0
Mean	0.73	8.09	3.94	6.84	3.14
Std.	6.87	6.48	4.45	9.49	4.66
Sharpe ratio	0.03	0.36	0.26	0.21	0.19

Exhibit 4. Static and Dynamic Trading Strategies on World Bonds: performance between 1/1978 and 6/1993.

The in-sample and out-of-sample are the performances of a zero-investment strategy based on the world long-maturity bond portfolio. Mean represents the annualized sample mean of excess returns computed monthly, in percentage. Std is the standard deviation of monthly excess returns, annualized. Sharpe Ratio is obtained dividing the sample mean by the standard deviation of the portfolio's excess return (not annualized). The results are based on the period between 1/1978 and 6/1993.[30]

The strategy he wants to utilize is funded with the world one-month Eurodeposit, thus resulting in a zero-investment strategy. Of course this approach is possible only in a theoretical situation, given the restrictions posed by the Federal Reserve, but it is an interesting scenario nonetheless.[31] Moreover, based on the expected results of the bond market, given by the Inverse Relative Wealth proxy, he recalibrated the dynamic strategy each month.

In reality there are two different dynamic strategies: a "1/0" approach and a regular "scaled" tactic. For 1/0 it is intended a situation where the investor uses all his cash holding the portolio, made by the combination of government bonds in this peculiar case, if he expects a good progression in the market for his strategy, while abstaining from the market completely if he fears losses. In the scaled strategy he takes a position that is sized to the portfolio's expected excess return as a percentage of 100. He applied

[30] Antti Ilmanen, *Time-Varying Expected Returns in International Bond Markets*, The Journal of Finance, Vol 50, No. 2 (Jun., 1995), pp. 481-506.

[31] Gordon J. Alexander, *On Back-Testing "Zero-Investment" Strategies*, The Journal of Business, Vol. 73, No. 2 (April 2000), pp. 255-278.

this methods both using out-of-sample and in-sample data of the world excess bond return as predicted by his model.[32]

While the in-sample are estimates based on the predicted results of the regression using Inverse Relative Wealth, the out-of-sample are forecasted values obtained by rolling all real historic data from 1978, beginning from the start of 1980, one-month-ahead.

Comparing the results, the scaled approach obtains a positive return of around 8,09% in the in-sample while out-of-sample is 6,84%, the 1/0 strategy gains 3,94% and 3,14% respectively while the static strategy stabilizes itself on a poor 0,73%. For the Sharpe Ratio, a financial indicator that helps understand how much risk is rewarded in the investment,[33] the static approach does not compare in a good way, attesting itself on a barely positive 0,03, while for the 1/0 it ranges between 0,19 in-sample and 0,26 in the other case and for the scaled it goes to 0,21 and 0,36 respectively.

The theory seems to work, the world Inverse Relative Wealth shows here a good predictive ability and the strategy is significant from an economical point of view. However Ilmanen did not adjust the results for risk variations during the time period or for trading fees (and the scaled portfolio is really dependent on transactions). Moreover some earnings obtained on paper during the 1980s could not have been obtained cause lack of shorting bond market ability, hence the returns are overrated. The 1/0 approach relies much less on trading transactions, without the possibility to go short, basing itself on the sign of the prediction about return of the bonds, without any necessity to change the proportion between assets in the portfolio, lowering fluctuations, in a simpler and hence more feasible way.

To conclude this paragraph it is still significant to see that for all the six countries the results of the dynamic strategies are always better than the static ones, holding long-maturity bonds or conversely short-term deposits, both confronting Sharpe ratios and average returns.[34]

[32] Antti Ilmanen, *Time-Varying Expected Returns in International Bond Markets*, The Journal of Finance, Vol 50, No. 2 (Jun., 1995), pp. 481-506.
[33] William F. Sharpe, *The Sharpe Ratio*, The Journal of Portfolio Management, 21 (1), (Fall 1994) 49-58.
[34] Antti Ilmanen, *Time-Varying Expected Returns in International Bond Markets*, The Journal of Finance, Vol 50, No. 2 (Jun., 1995), pp. 481-506.

1.7 Recurrence between nations

An important consideration: Ilmanen noted that the predictability of bond markets across the different nations examined develops in similar ways and a proxy related to the "World" is superior to his singular regional components taken as lonely signals.[35]
Although my personal analysis will soon be much different from the one proposed by the economist, it is still an interesting finding, which could be useful for later expansions of my search in a worldwide perspective. For example other studies suggested than even a proxy related only to the US market could be a better predictor than a local one (applied to his own country of origin).[36]
Ilmanen noted that international bond markets became more and more integrated during his studies, with a correlation between national bond markets increasing over time. Dividing the period he studied in two different parts, until September 1985 and after this date, the predicted bond returns in these six nations averaged 82% in the first half and 91% then. This results were even more important for France and Japan, where, in these years, the markets become much more trustworthy and structured.
In the following image we can appreciate the similar path between predicted bond returns across nations. The six "portfolios" reached a maximum for each recession of the 1980s, in particular 1982, 1984, 1987 (Black Monday) and during the Gulf War of 1990.[37]
This factor could become important for our later studies on the subject.

[35] Antti Ilmanen, *Time-Varying Expected Returns in International Bond Markets*, The Journal of Finance, Vol 50, No. 2 (Jun., 1995), pp. 481-506.
[36] Benton E. Gup, *Capital Markets, Globalization, and Economic Development*, Innovations in Financial Markets and Institutions (1989).
[37] Antti Ilmanen, *Time-Varying Expected Returns in International Bond Markets*, The Journal of Finance, Vol 50, No. 2 (Jun., 1995), pp. 481-506.

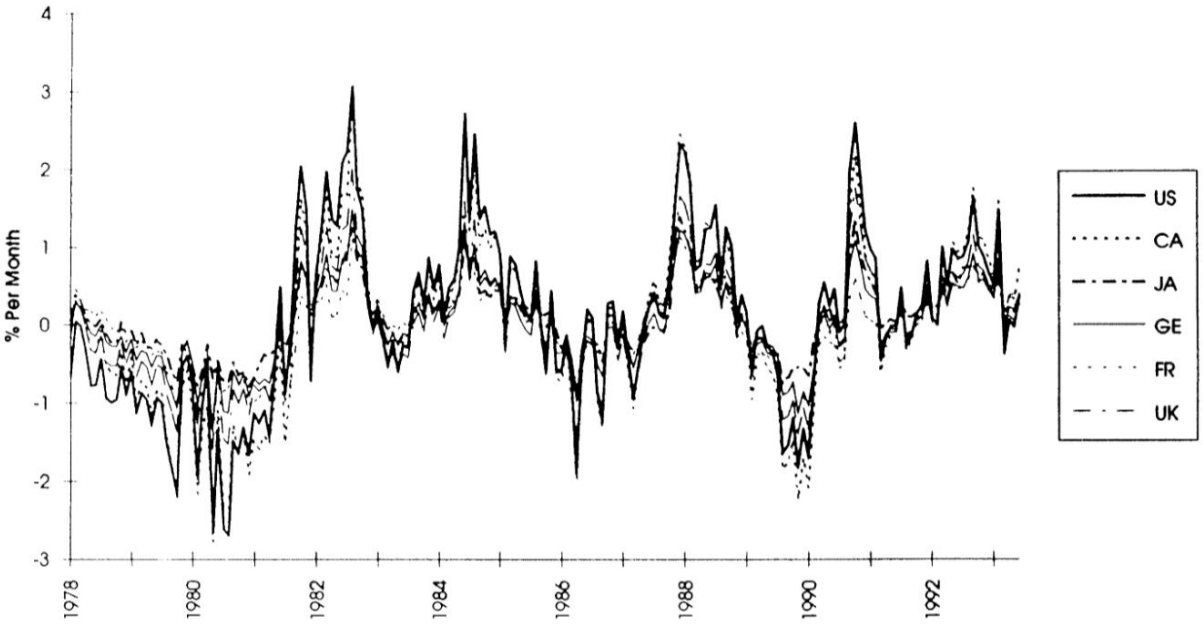

Exhibit 5. Relation between predicted bond returns from the regression in the six nations.

United Stated (US), Canada (CA), Japan (JA), Germany (GE), France (FR) and United Kingdom (UK) between January 1978 and June 1993.[38]

1.8 Ilmanen's Conclusion

Ilmanen tried to search an answer for the obvious question, the "elephant in the room", of this paper. How is it possible to explain the origin of this predictability in the pattern of asset prices and returns? Can the economic theory explain it through the theory and the models developed until now?

Ilmanen noted that the Capital Asset Pricing Model is not able to motivate the positive results obtained on the bond market.[39]

Moreover, in his researches, the stock market, that obviously shows an inverse relationship to Inverse Relative Wealth, as checked also by other studies[40], was not strongly related to the bond market, at least it was not a good proxy taken alone. In my

[38] Antti Ilmanen, *Time-Varying Expected Returns in International Bond Markets*, The Journal of Finance, Vol 50, No. 2 (Jun., 1995), pp. 481-506.
[39] Antti Ilmanen, *Time-Varying Expected Returns in International Bond Markets*, The Journal of Finance, Vol 50, No. 2 (Jun., 1995), pp. 481-506.
[40] Benton E. Gup, *Capital Markets, Globalization, and Economic Development*, Innovations in Financial Markets and Institutions (1989).

opinion this is a little bit weird: Inverse Relative Wealth is created using the stock market. If the stock market is not related to the bond market, how Inverse Relative Wealth could possibly predict a strategy based on the bond market? It is true that an instrument based on something will have a correlation different from his original source, otherwise there will be no reason to create signals. Maybe this difference could be strong, but it should still be related to the true starting point of the prediction.

In any case Ilmanen concluded that one of the varables that could try and forecast the bond market is the adversion to risk, that changes with the level of wealth of the investor. In synthesis, the efficiency of Inverse Relative Wealth is low when aggregate wealth is high and viceversa. But this is probably not enough: even here, from these results, we could see that Inverse Relative Wealth could be a useful instrument, but taken alone, it could not work (in later studies he will weight and combine this signal to others to create more complete proxies).[41]

A more complicated multifactor model, more structured, comprising other assets, not only bonds, but also stocks and forex, could be a better proxy, maybe across the world, especially if market integration continues to reduce the differences across countries, with more possibilities to analyze both stock and government bonds.[42]

[41] Antti Ilmanen, *Forecasting U.S. Bond Returns (Understanding the Yield Curve, Part 4)*, Salomon Brothers, August 1995.

[42] Antti Ilmanen, *Time-Varying Expected Returns in International Bond Markets*, The Journal of Finance, Vol 50, No. 2 (Jun., 1995), pp. 481-506.

Section 2: Structure of my Analysis

2.1 The Time Period

Now we actually want to try to build a trading strategy, analyzing the past stock market performance to predict a possible movement in the bond market, using some of the intuitions deriving from the work of Ilmanen, analyzing them and seeing if they are valid till the present time.

The first step is to decide a period to study for our research: I personally think that the most interesting way to approach this scenario is to put ourselves in the shoes of an investor who wants to build his portfolio, using the informations gained by Ilmanen's studies, hence my starting date will obviously be related to the final one used by Ilmanen.

"Time-Varying Expected Returns in International Bond Markets" was published by Ilmanen in June 1995, hence I could start "trading" in July 1995, a month or so after the release.

Although the publication happened in June 1995, Reading some of his tables it seems that at least some of his studies ended in reality in June 1993.

This could leave a gap of 2 whole years without any true analysis about the pattern of the market. For this reason I decided to start before this date: from other results reported by the economist he gives us clear data on the trading strategy in the period between 1990 and 1995. This should be the final aim of our interest, that is why I intend to study the whole period between 1990 and February 2020, the end date before this thesis will be uploaded. In reality I will let my portfolio start a little bit later: inflation, shocks regarding oil, fear for the Gulf War, led to a recession in the US economy between July 1990 and March 1991. I decided against a starting point during the heat of the financial instability, as the first stage is a possible factor deciding the results of our strategies over time, as a small change here will affect the portfolio for more than 30 years, amplifying his magnitude running onwards. From my analysis it appears that, although the recession ended only 3 months later, as soon as the year changes to 1991 both bond and stock US markets, as usual, had already fully recovered. This aspect in contrast, could lead to

better results over time, but we still need a period easily relatable to that used by the economist, hence I will go for a 1990-1991 dichotomy.

Thus, if the period analyzed by the economist was 30 years, as we could understand not by the Journal of Finance, but by Ilmanen's "Forecasting U.S. Bond Returns", published slightly after, in August 1995, my paper will focus on 30 whole years and 2 months: a good target to analyze, in my personal opinion. For instance, I will use excel as a basis of my thesis, as the perfect platform to build, step after step, a new model, enabling me to see with numbers and graphs the results of what I am doing. I will mainly use Bloomberg data, escorted, situation by situation, by other databases, at the necessity of the moment.

2.2 Structure of the Analysis: Stock Market and Moving Average

The second step consists in forecasting this bond risk premium, applying the concept of the Inverse Relative Wealth, as our proxy for time-varying risk aversion, as a determinant of the expected bond risk premium.

Remembering the reasoning used by Ilmanen, Relative Risk Aversion and asset returns are correlated, hence, Inverse Relative Wealth should be approximated using the stock market, the most volatile segment of the real economy.

Using the stock market alone is, in theory, not a good strategy: the market is really volatile, day-by-day, so it is not the perfect proxy to use for building a pretty adaptive strategy over-time, at least regarding a totally different market, even if strongly related as the governative bond one.

A first refinement strategy could be to use a Simple Moving Average of the stock market over the period we want to study but we will demonstrate that there is an even better way to do just that: the Exponentially Weighted Average of past wealth levels. At last we will describe the Inverse Relative Wealth.

Starting from the basis, a Moving Average is a useful indicator, utilized especially for technical analysis, that filters the noise from short-term fluctuations in prices which, as said before, could be pretty random, smoothing prices over time. It is based on past prices so it establishes a trend.

It is a signal but a lagging one: for instance, in our case, it shifts from his pattern only after a significant and consistent over time change in our stock market target.
As a starting point I will use the US S&P 500, a stock market index really useful to study the US market, made by the 500 larger companies listed on the stock exchange of the United States. It is one of most used one and in general it is considered as a good indication for the whole equity market. That is also the main reasoning behind my choice.

2.3 The Simple Moving Average

The first main alternative is the Simple Moving Average (SMA), an average calculated by adding consecutive prices, in our case stock prices, and then dividing by the number of prices we have taken into consideration.
The general formula, then, is:

$$SMA_t = \frac{P_1 + P_2 + \cdots + P_{N-1} + P_N}{N}$$

Where P_t is the price of the asset at time t and N is the total number of Prices we have taken into account.
It is pretty clear to understand that a short timeframe will lead to a pattern much more similar to the one obtained by the regular stock market, but it allows us to react easily to changes in the stock market. A longer time period will smooth out much more volatility, creating safer indications, but also it will be slower and lag our graph even more from the real-time market, leaving behind many important informations.
From an analytical point of view it clearly gives you a pattern upward or downward looking, easy to understand and easy to use for a proxy.
In our case, I opted for a one-year period average, to create a movement smoother enough to be understood but also not so lagging behind in a continuously developing market as are local and global stock markets.

$$SMA_{12} = \frac{P_1 + P_2 + \cdots + P_{11} + P_{12}}{12}$$

A possible problem arising for the simple average is that more recent days are still more important in our analysis than times passed long ago, thus many economists believe that we should prefer certain periods of time, nearer to us, and not rely so much on a good percentage of prices that could be already outdated. Moreover, if Efficient Market Hypothesis are realistic, all available informations are already counted in the equity valuations at the present time, thus invalidating the whole concept of Simple Moving Average, totally based on historical data, which can not add, in their opinion, anything on future trends of the market. Although I am not totally convinced by concepts like strong market efficiency, I am neither inclined to this solution too.
In fact, in contrast to Simple Moving Average we have the Exponential Moving Average (EMA).

2.4 The Exponential Moving Average

This average, also called Exponential Weighted Moving Average (EWA as said by Ilmanen), reacts in a more meaningful way to fresh statistic evaluations, placing different importance on the data considered, with greater stress placed on more nearer values. Still a lagging indicator, it is a good one, used in financial studies to build buy and sell signals, or to show how market moves. In fact, it could happen, with a regular Simple Moving Average, to react to changes in the market when it is already too late while, with the Exponential Moving Average, which places more attention to the last signals, it is more plausible to choose the optimal point of action, especially in trading.
Like the Simple Moving Average it shows when the pattern in prices is upward or downgoing, but unlike his counterpart it flattens much earlier so it is useful to put an eye on the rate of change from a period to the following, thus being able to observe in an easier way if, for example, prices in the stock market are moving with opposite sign.
Going to the mathematical formula it is:

$$EMA_t = \left[P_t \times \left(\frac{s}{1+N}\right)\right] + EMA_{t-1}\left[1 - \left(\frac{s}{1+N}\right)\right]$$

Where t stands for the present time, P_t represents the price of the stock at time t and N is the total number of datas we have considered. In particular calculating the Exponential Moving Average is not a straightforward task.

The first thing to do is calculate the Simple Moving Average, then we need to find $\left(\frac{s}{1+N}\right)$, the factor that allows us to weight and smooth our formula. Only after this we can focus on our true result. Clearly it depends on the chosen s parameter that is usually considered as the number 2, for convention.

With this method the weight attributed to more recent prices is greater for shorter-period Exponential Moving Averages, so if we take an average based on 20 results, the most recent data weights around 9,52% while for a 10 period this value rises to 18,18%.[43]

Also in this case I went for a 12 months timeframe, for the same reasoning used in the "regular" average.

The main difference between the two moving averages is represented by the sensitivity to changes in prices. The Simple Moving Average smooths fluctuations in an equal way for all the results, while the Exponential Weighted Moving Average is a more reactive way to proceed, a good compromise between the pattern given by the simple one and the excessive variability of an every-day change in prices.

But it is still not all sunshine and puppy dogs! The Exponential Moving Average could lead to possible problems too. Others economists are against putting too much bias on recent data, for the opposite reasoning that led others prefer this solution to the simple one. Moreover the Exponential Moving Average relies again on history of past prices, thus it is, at least in part, in opposition to a concept like market efficiency.

As a starting point I will go with the idea that moving averages are useful instruments to predict at least a part of the variation in the market, in particular choosing the latter average as a middle approach to the argument.

Thus I will go for the following formula, using the Simple Moving Average we already applied and developing it on a 12 months period:

$$EMA_{12} = \left[P_{12} \times \left(\frac{2}{1+12}\right)\right] + EMA_{11}\ \left[1 - \left(\frac{2}{1+12}\right)\right]$$

[43] By Adam Hayes, *Exponential Moving Average - EMA Definition*, Investopedia (Jul., 8, 2019).

Even if Ilmanen started directly using the Exponential Moving Average of the past performances of the stock market, probably considering it a more reliable proxy than the direct stock market performance, I will try in the table below and with the following graphs to demonstrate this point, using yearly moving averages.

In particular I will compare the S&P 500 index and the two moving averages.

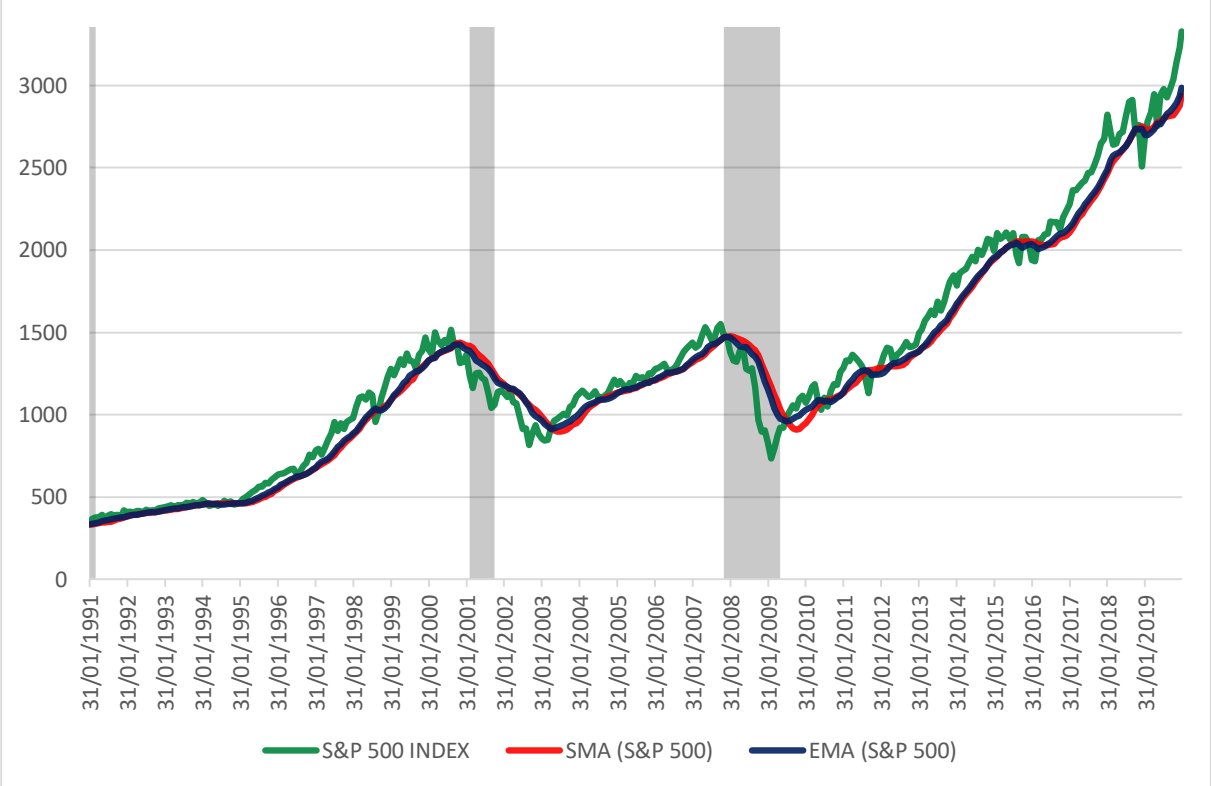

Exhibit 6a. Graphical comparison between S&P 500, SMA and EMA based on the S&P 500. January 1991 to February 2020.

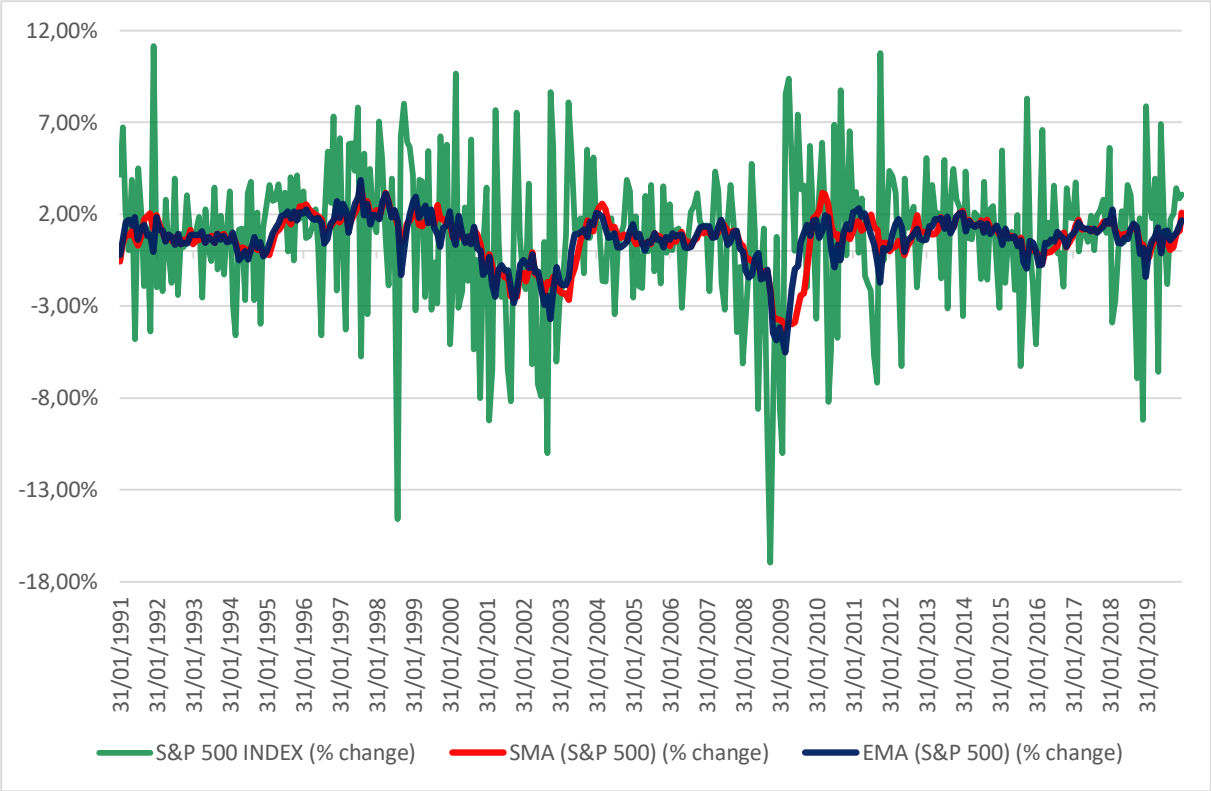

Exhibit 6b. Percentage change comparison between S&P 500, SMA and EMA based on the S&P 500. January 1991 to February 2020.

Exhibit 6c. Comparisons between S&P 500, SMA and EMA based on the S&P 500. January 1991 to February 2020.

From the Graphs it is clear to see that the market prices are much more volatile over the period considered while the moving averages are similar: this is pretty clear in the % change graph. However the Exponential Moving Average reacts a little more to prices than his counterpart.

Correlation	*S&P 500 INDEX*	*SMA (S&P 500)*	*EMA (S&P 500)*
S&P 500 INDEX	1		
SMA (S&P 500)	0,986	1	
EMA (S&P 500)	0,990	0,999	1

	S&P 500 INDEX	*SMA (S&P 500)*	*EMA (S&P 500)*
Mean	1306,00	1255,11	1259,59
Variance	452797,29	403414,28	404710,90
Standard Deviation	672,90	635,15	636,17

	S&P 500 (% change)	*SMA (% change)*	*EMA (% change)*
Mean	0,747%	0,633%	0,638%
Variance	0,001642	0,000169	0,000173
Standard Deviation	0,04052	0,01302	0,01314

In particular, from the correlation table we can see how closely the two moving averages are related between them, while the S&P 500 index is a little more related to the exponential one, as we expected: both the correlation and the mean of the Exponential Moving Average is in the middle between the index and the other average. The standard deviation, σ, a measure of the dispersion of the numbers assumed by a variable, is obviously higher in the S&P 500 showing us that a day-by-day stock market is much more "riskier" also as an instrument than a more conservative and ponderate analysis of the initial data.

Considering the % change over time, the standard deviation of the moving averages is more than 3 times lower than the index, especially in the exponential case, once again. Indeed, the mean, given the positive trend of the three graphs, is a little bit higher in the index in comparison to the two averages.

To conclude the simple average is more reliable than raw datas and the smoothest of the three. But the exponential peer, weighting the recent past more, could be more useful, especially when the pattern of the market changes between bullish and bearish.

But now it is time to go to the real Ilmanen's proxy.

2.5 Inverse Relative Wealth

I think I have already spent much time talking in a good way of this instrument so I will do a quick recap. It wants to capture the relationship between the past stock prices and the current ones, as a way to understand the economic situation, both in the equity market and also to predict the real economy. For the reasons said above, it will not have at the numerator the S&P 500 values or the Simple Moving Average computation, but the exponential one.

Going to the formulas, the mathematical equation is the following:

$$INVRELW_{t+1} = \frac{EMA_t}{W_{t+1}} = \frac{\left[P_t \times \left(\frac{s}{1+N}\right)\right] + EMA_{t-1}\left[1 - \left(\frac{s}{1+N}\right)\right]}{W_{t+1}}$$

The mathematical explanation is the same one used for the Exponential Moving Average. The only difference is that, as a denominator we take W_t, intended as the real value of the stock market at the current time.

From a computational stand point this formula slightly changes to:

$$INVRELW_{13} = \frac{EMA_{12}}{S\&P500_{13}} = \frac{\left[P_{12} \times \left(\frac{2}{1+12}\right)\right] + EMA_{11}\left[1 - \left(\frac{2}{1+12}\right)\right]}{S\&P500_{13}}$$

It is obvious for the formula that it all translates to a whole 13 months continuous evaluating process.

2.6 The Bond Market

After having decided a proxy to use as a trading signal, we need to choice the main focus of our action on the market. As could be predicted by the previous chapter, I will go for the bond market: if Ilmanen used Inverse Relative Wealth to predict the bond movements, we can try to do the same.

A quick reminder, a bond is simply a loan between the agent and the debtor, usually a corporation or a government. This tool of basic fixed income is used to finance activities and projects both by companies and states.

Between corporate and government bonds I will go for the latter, because a corporate one is usually riskier than a government bond. In fact, although there are many types of government bonds, emitted by states, municipalities and regions, the safest bet is always the "Nation" option. Moreover, companies changes, in performance, in situation, in market position and evaluation. It is a way more difficult "system" to study over time in comparison to the US bond emissions.

In fact, government bonds are usually denominated in the local currency and in this case the government will be never forced to default (unless it chooses to do that) because it could always "print" more money, thus it is an unlikely event to occur. Many economists consider only some countries as risk-free assets, but others think that, for this same reasoning, all government bonds, at the higher level of the national hierarchy, are riskless assets.

In any case I will focus only on the most important and consolidated economy of the globe, the United States of America, so this debate is a purely theoretical question. Let's dive a little bit more in the concept of bond risk components

Risk, in our case, could be decomposed in four major components, credit risk, currency risk, inflation risk and interest rate risk.

- Credit risk: as said, if needed, the country could print more money at maturity to repay his debt. There are some notable exceptions, such as the ruble crisis in 1998 when Russia "opted" for bankruptcy. But if we stick to the ratings of statistical agencies, in particular the "Big Three" (Standard & Poor's, Moody's and Fitch) we should always be pretty safe.
- Currency risk: is the risk that the value of the currency a bond pays out will fall regarding the exchange rate with the currency of the nation we live in. If we are an Italian that invests in the US market we should consider the foreign exchange market (Forex) between Euro and Dollar, thus taking into account a component of currency risk. For the moment we will ignore this factor.

- Inflation risk: this component depends on a possible increase in the general price level, called inflation. Today the standard expectation for a good economy is a positive and pretty constant low inflation: a good inflation rate is considered to be around 2% or 3%, favouring investments and the stability of the market. But if the economic situation worsens or the credibility of the national central bank falls, events like hyperinflations are always a possibility. In this case the value of the bond we own will decline rapidly. As said, we will focus only on big and strong enough economy that will have a generally positive but low inflation rate target.
- Interest rate risk: last but not least, is the risk of losing money from movements in changes of the interest rate. For instance, when interest rates rise, many financial instruments such as bonds and stocks will decline in their market value. To be considered, shorter maturities of the bonds will decrease the interest rate risk. But we will use as a proxy a stock, with the same relation to interest rate risk as the bond market, hence our position will hopefully already consider this component of risk before making our plan for the future investment.

My first idea will hence be to use the stock market as a signal to invest in one bond and short another one, hopefully reaching a return with a zero-investment on our part. As said by Ilmanen, even if the Exponential Moving Average, contained in the Inverse Relative Wealth, will cancel out part of this lag, there should be still a temporal "distance" between our signal and the market. I will go for a one month period, as unitary measure for our study of market interactions, especially because, in my humble opinion, if one month was a good lag in 1995, probably today it will be the same (and in reality probably much less).

As a starting stage I will consider both the 3-month and 30-year bonds, because they are quite common bonds used pretty much worldwide, at least by some of the the major economies. My idea will be that the stock market will overreact to economical variations, a 3-month bond, a more stable investment than stocks, will react but less and a longer bond will react even less to changes in the economy, because it will be considered a safer investment with long-maturity. When the economy is close to reaching his peak it will put more interest in the short-run, not expecting something to

happen pretty soon, with higher yield especially for the long bond. The opposite will happen during crisis, when the importance of a safe asset will be much higher, dragging down yields, as prices become higher for the bond market and stock market crash. Only then the Federal Reserve will try to decrase the short-term interest to stimulate the economy, making the yield fall even lower and the price increase.

To demonstrate this I will compare the pattern between the stock market, represented by the moving average, and some bonds (3-month, 30-year, 5-year taken as a middle ground) within the US market.

From the pictures below (Exhibit 7) it is pretty clear that bond yields always move in the same direction, whatever the maturity might be. But a shorter maturity will lead to more reaction in yields, hence in prices, both when prices are going up and when they are going down, in particular during bubbles, when a safer investment is much more valuable and the percentage changes in a small fraction of time. In fact in the Internet Bubble burst of 2001 and the financial crisis of 2007, and both the periods following these two recessions, the yield change is pretty significant. From the pattern of the 3 graphs in the USGG Indexes Comparison we can see that, in particular when yields are low, the difference of perceived value of the bonds changes pretty widely between them, while in periods of higher yields they are pretty similar, even sometimes higher for the short or intermediate maturity bonds in comparison to a 30-year bond. In synthesis, as we expected, when prices are increasing and yield is lowering, the short bond yield lowers more than the 30-year, instead when the yield is going up, the 3-month increase is much higher than the 30-year counterpart while the 5-year pose itself at an half-way pattern.

However all of this reasoning seems to matter much less than we could hope, in fact it seems that the long-maturity bond will pretty much always have a higher yearly yield (thus return) than the short counterpart. This is a problem, because it could seem that a fixed long-government bond strategy will always be preferable to a short-maturity one. Of course we need to study how the price changes in those particular years, maybe compensating the effect of the yields. We will see later.

To be noted, spoilering some concepts useful later, there are periods where yield curve will become “humped” or even “inverse”. At first glance, it seems to be fully inverse

right before a major crisis like in 2001 or 2007, thus opening space for different strategies, and still humped in many other occasions, when shorter maturities will be more remunerative than longer ones, the more recent is clearly in the middle of 2019.

From the % change graph of the yield, to show the "stability" of the bonds over-time, it is apparent to see that they all move in the same direction, but longer maturities have slower patterns than shorter ones. For a better view I have excluded the short-maturity bond, showing only the 5-year and 30-year ones but the observer could be certain that the comovement is similar, only much more "intense", thus not really adapted to be shown on a paper like this, in my humble opinion.

Let us continue nonetheless our study.

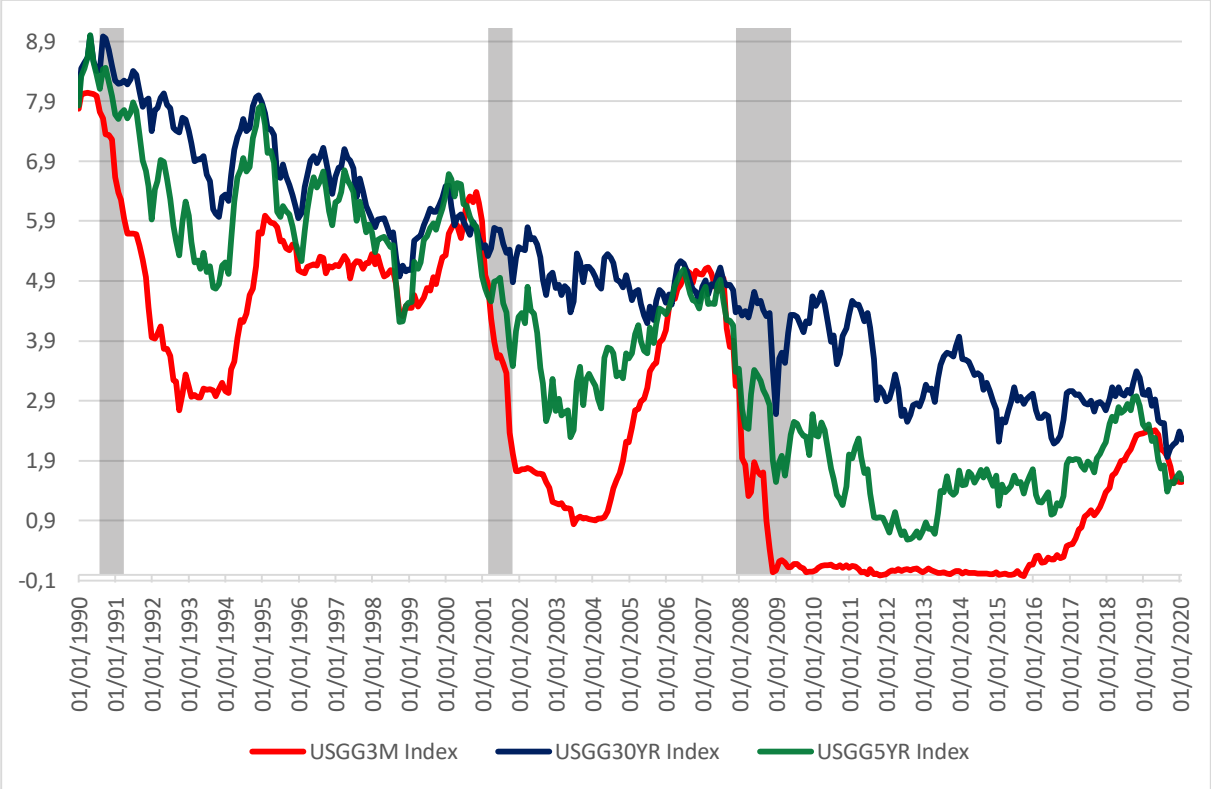

Exhibit 7a. USGG Indexes Comparison between bonds of different maturities: US 3-month, US 5-year and US 30-year.

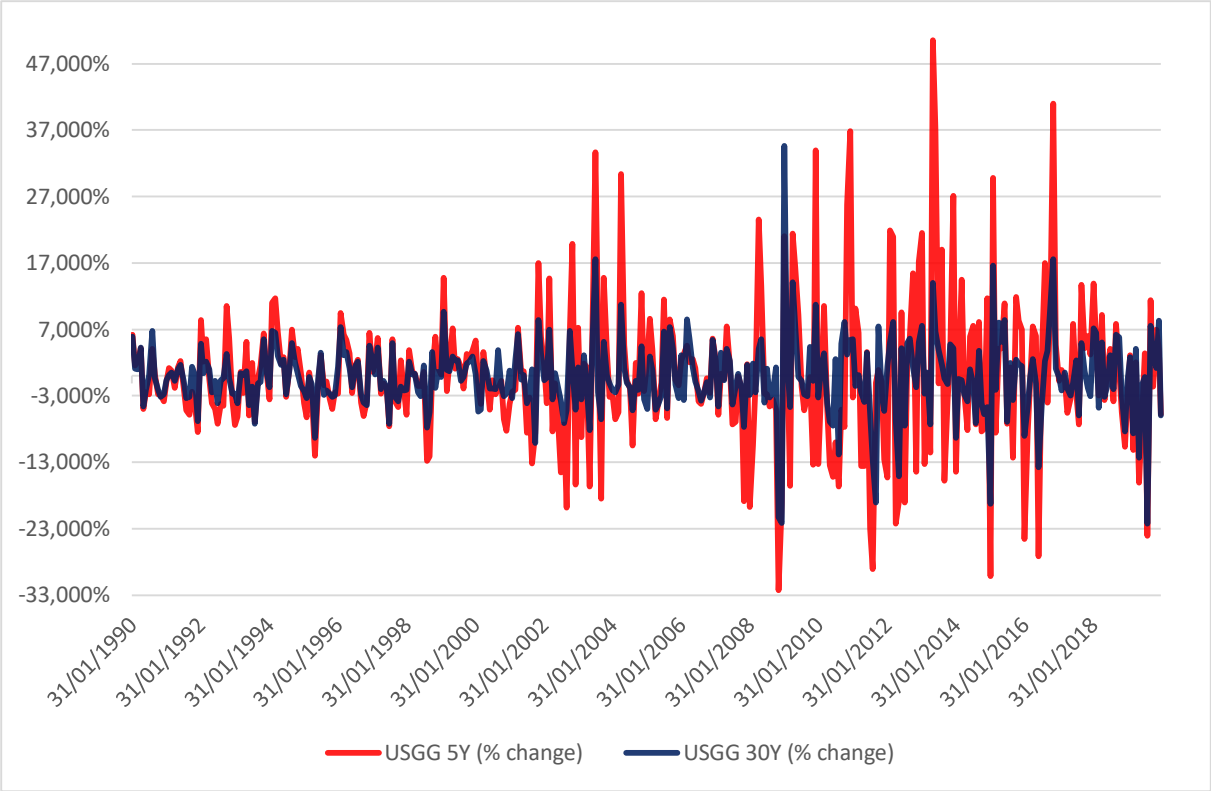

Exhibit 7b. USGG Indexes Comparison (% change) between US 5-year and US 30-year Generic Government Bonds.

Correlation	*USGG3M Index*	*USGG2YR Index*	*USGG5YR Index*	*USGG30YR Index*	*SPX Index*
USGG3M Index	1				
USGG2YR Index	0,98	1			
USGG5YR Index	0,93	0,98	1		
USGG30YR Index	0,80	0,87	0,95	1	
SPX Index	-0,49	-0,56	-0,67	-0,83	1

	USGG3M Index	*USGG2YR Index*	*USGG5YR Index*	*USGG30YR Index*	*SPX Index*
Mean	2,76	3,32	3,94	5,01	1273,80
Variance	5,24	5,45	4,64	3,06	468083,71
Standard Deviation	2,29	2,34	2,15	1,75	684,17

(% change)	*USGG3M Index*	*USGG2YR Index*	*USGG5YR Index*	*USGG30YR Index*	*SPX Index*
Mean	7,32%	0,40%	0,10%	-0,19%	0,71%
Variance	0,82	0,02	0,01	0,00	0,00
Standard Deviation	0,91	0,13	0,11	0,06	0,04

Exhibit 7c. Comparison between S&P Index, USGG3M, USGG2YR, USGG5YR and USGG30YR.

From Exhibit 7 we can see again how the longer the maturity, the lower the volatility of the asset, in this case the Government Bonds of the United States. The only exception, we did not expected is the 2-year government bond that has a slightly higher variance than the 3-month one. For "fun" I have hence done the same reasoning for the % change over time, and here things come back as normal, with a % monthly change of the yield of 3-month so high that the average shift is 7,32%, that is positive, but ranges from +358% to -100%, thus invalidating any real economic meaning. Still, variance is pretty useful, showing a high standard deviation of around 0,9 in the percentage change, much higher than 0,13 of the 2-year bond, as we could expect. Of course in the real value the standard deviation will still be (incredibly) higher for the 2-year than the 3-month counterpart (2,335 for the 2-year versus 2,29 of the 3-month).

In any case, the stock market demonstrates itself as a high risk market, with a variance of 468 084 in comparison to the values of the bond between 3 and 5,4 over a 30-years timespan. Of course the smallest standard deviation is 1,75 and it comes from the long-maturity bond, totally not comparable is the 684,17 of the stock market, on a totally different class.

But the % change shows us that the price of the stocks are not that bad as an investment on the long run, with a positive percentage change of 0,71 for the market index, while the much less risky 2-year bond obtains still a good 0,4% yierly return appreciation. The 30-year bond, in the period considered is lowering in yield even if in a small percentage per month, but still remaining the much more profitable bond option with a yierly average of 5% against the lowering 3,9 of the 5-year, 3,3 of the 2-year and 2,7 of the 3-month one. Usually, only from this table, we can easily say that we will probably apply a fixed long 30-year bond, short 3-month Treasury bill strategy, with maybe only some small differencies.

The gap between bonds has still decreased over-time.

Moreover, as expected, if we find ourselves at the extremes of the yield curve the correlation to other bonds will become much less important when we move in the other direction, thus the intermediate maturities are useful middle ground and maybe safer approaches to an investment strategy using bonds, but also much less remunerative. Remembering that a zero-investment strategy money, even if is at risk-free rate, still

cost us money, going to the extreme maturities seems the best option for overall profitability of a trading portfolio.

The 3-month treasury shows a strong 0,98 correlation with the 2-year, 0,93 with the 5-year bond and "only" 0,8 with the 30-year. On the contrary the 5-year is surely the middle ground of the 4, with a high 0,98 with the 2-year and 0,95 with the 30-year. Remembering that it has still a positive 0,93 relation with the 3-months, we will surely find a way to utilize it, especially with yield curves different from the normal pattern. To conclude the study on bond correlations, the results between the 2-year and 30-year is a reasonable 0,87. Also this fact could be exploited later.

Last but surely not least, going to the stock market, the relationship between it and the bond yield is that we could have expected before. The yield change is nearly opposite for the most different asset, the 30-year bond with an "opposite" relationship at around -0,83. For the shorter maturities it increases to -0,67 for the 5-year, the 2-year shows a -0,56 and the 3-month "only" a -0,49.

It is true that the markets seem to move in opposite directions, but we are talking about yields, and bond yield move in opposite direction to bond prices. Hence long-maturity government bond could probably be strictly correlated to stock prices. This is really interesting. In reality what matters to us it excess rate of return coming from bond. If we will discover that it comes mainly from bond prices, our whole understanding of the subject will be shaken. But we will see that in reality yield is the major component of excess bond returns, hence we will demonstrate how in effect our reasoning is safe, at least for the moment. Even the fact that the two prices are sometimes considered as moving in opposite directions is not always true, thus maybe explaining at least in part some less clear result.

But in general we can interpret this data as a demonstration that when the stock market is going up, bonds are a perceived as less attractive investment, thus leading to a lowering in their real value, while when the market falls, the importance of the stability of the bond market is much higher, with the 30-year as the most opposite to the stock market among the three. In the figure below we can observe the three patterns of the stock market, the 3-month bond and the 30-year one combined. As said before we can see how the Inverted Yield Curve preceeds a fall of the stock market index S&P 500

and this happens right before a major recession (dot-com bubble and mortgage crisis). One small inversion of the yield curve occurred in the middle of 2019 and we will try to understand what happened later and what we could expect for the future (from the last year, a near-future recession could theoretically be expected). To conclude this part, we notice how bonds are becoming less and less remunerative over the 30-year time period, as the stock market exploded to unprecedented levels, subtracting capitals and investors' focus from the bond market.

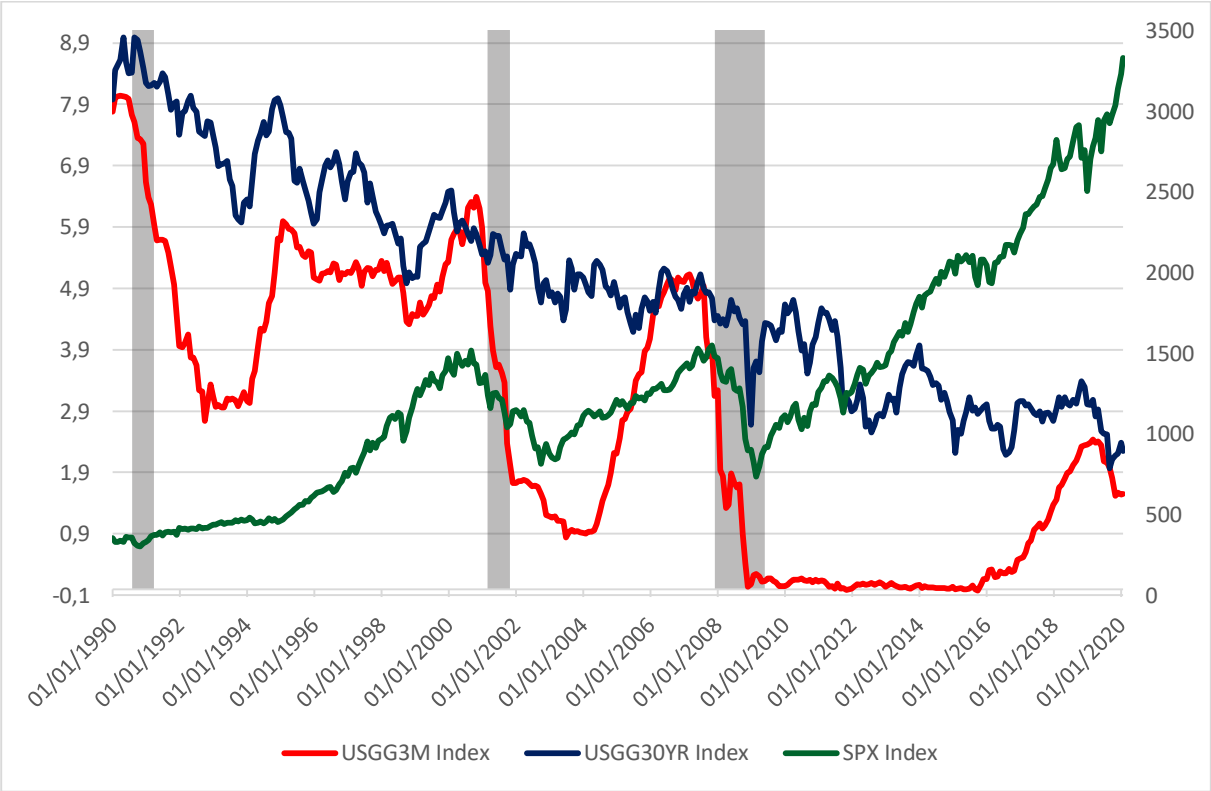

Exhibit 8. Comparison between yields of US 3-month bond and US 30-year bond, with the S&P 500 stock market.

2.7 Zero-investment strategy

This is another concept I want to try to apply to our model. It usually entails two set of securities, one in which you want to go long and the other you want to short, using a certain trading rule: the net value of the two investments should be zero.

The "winners" is represented by the instruments you want to go long, usually expecting an appreciation in their market values and in contrast, the "losers" are the ones you want to short, usually thinking they will depreciate or at least appreciate less.[44]

For example we could short-sell a 3-month US Treasury for a value of 100$ and invest that money for 100$ in a US 30-year bond (USYC3M30).

The economic profitability associated with this strategy should be none if the markets are perfectly integrated, but as demonstrated by many studies, such as contrarian and momentum trading rules used by De Bondt and Thaler in 1985 (over long periods of time)[45] and 1987 (adding seasonal patterns)[46], Jegadeesh and Titman in 1993 (concentrating their studies on a yearly timespan)[47], this is not the case.

It could be considered an arbitrage if the result of this operation is higher than the riskless rate of return.

There are many problems in implementing zero-investment portfolios, such us the limit posed by the Federal Reserve on short-selling, but, for example, for this event Markowitz, in 1983 and 1987, tried to resolve these issues. In any case, in a real world scenario this strategy is probably not achievable, both for regulations and costs of transaction, but it could be considered an interesting academical study.

To summarize, if we call L and S, the long and short portfolios of instruments respectively, and $r_{L,t}$ and $r_{S,t}$ their returns, with unrestricted short-selling, the return of the strategy will be:

$$LMS_t \equiv r_{L,t} - r_{S,t}$$

With LMS I mean Long minus Short. The weight of both L and S is 100% so they cancel out to 0, thus zero-investment on our part. We need to underline that, if at the following

[44] Gordon J. Alexander, *On Back-Testing "Zero-Investment" Strategies*, The Journal of Business, Vol. 73, No. 2 (April 2000), pp. 255-278.

[45] Werner F. M. De Bondt, Richard Thaler, *Does the Stock Market Overreact?*, The Journal of Finance, Vol 40, Issue 3 (July 1985), pp. 793-805.

[46] Werner F. M. De Bondt and Richard H. Thaler, *Further Evidence on Investor Overreaction and Stock Market Seasonality*, The Journal of Finance, Vol. 42, No. 3, (Jul., 1987), pp. 557-581.

[47] Narasimhan Jegadeesh and Sheridan Titman, *Returns to Buying Winners and Selling Losers: Implications for Stock Market Efficiency*, The Journal of Finance Vol. 48, No. 1 (Mar., 1993), pp. 65-91.

time the signal changes, and we want to go long on S and short on M, our equation becomes[48]:

$$SML_t \equiv r_{S,t} - r_{L,t}$$

But we need to remember that this zero-investment strategy will also present a much more expensive bargain over the time period. The cost of money, gained by borrowing it at risk-free rate with bank deposits or short-term government bonds, may seem low, over a short period of time. But we will rebalance our budget monthly considering any financial gain as still a component of a zero-portfolio investment, thus reducing even more the total excess return of the investment over a long timeframe as could be a 30 years period. An interesting theoretical argument, but owning some cash could be a better way to approach this strategy for a common man of the street, especially when our strategy will inevitably face low performances, reducing our capital even more than in a simpler scenario. Thus I will analyze both the results with a zero-investment strategy, considering it my "base" approach to the portfolio, and a regular owned cash investment. Remembering that the Federal Reserve does not allow for a pure zero-investment, limiting short-selling, this could be also an interesting real-world adaptation of the economic and financial reasoning presented here.
In short, I want to find a signal from the market of a particular country to build up a zero-investment strategy long-short using the bonds of that country with a lag period of one month.

2.8 Some considerations

From January 1991 onwards I will use the Exponential Moving Average one period before, in relationship with the stock market, intended as a comparable way to proceed with the Inverse Relative Wealth proxy, which I will calculate later, to predict a pattern in the bond market. If the market is bullish, the stock prices are going up, thus the Exponential Moving Average is moving upward, and then also the Inverse Relative

[48] Gordon J. Alexander, *On Back-Testing "Zero-Investment" Strategies*, The Journal of Business, Vol. 73, No. 2 (April 2000), pp. 255-278.

Wealth will do the same, sooner or later. After the increase of the signal I will expect for the following month an increase in percentage of the one-month bond yield higher than the 30-year one, but a long excess return higher overall for the 30-year, in which we expect a higher capital gain. Hence, considering the temporal lag of one month, in this case I will go long for the month on the 30-year bond and short sell the one-month, expecting the first return one to grow more than the other. If the market signal is the opposite and the market is bearish, I will go long in the short maturity bond and short the long one. I will start from the United States economy, as said by our sources this should be sufficient to create a strategy efficient on a broader range in the future, possibly to the some of the strong markets in the world, both for reliability of the data and for the stability of the portfolio.[49] After analyzing the results of my portfolio I will try to improve any possible weakness, to increase the profitability and hopefully reduce the risk we face from this monthly transaction. The portfolio will be rebalanced monthly. This will be a "1/-1" investment strategy, resulting in a 0 investment from our equity.
I will stress again in the future, with real numbers and changes in the value of two portfolios, that a zero-investment strategy is also a cost on the long-run, especially if we consider the monthly gain as only a basis for an increase in the zero-investment position itself and not relate it to a merely simple increase in personal equity to be reinvested in the same pattern (but with higher returns overall).

2.9 Using De Bondt's Studies

Our previous considerations, led us to understand that markets do not always move in a smooth and rational way. They can go up, down without any real reason to change, moved by emotions, speculations or news. That is why we are avoiding the stock market itself as a proxy. That is why, later, we will use other proxies to base our portfolio investment, even when we will switch to a mainly stock market trade.
Our objective is not to oppose the market, but understand his dynamic, his movement, his *"zeitgeist"*[50], starting to utilize it at our own advantage, without, hopefully, allowing

[49] Benton E. Gup, *Capital Markets, Globalization, and Economic Development*, Innovations in Financial Markets and Institutions (1989).
[50] Robert J. Shiller, *Speculative Asset Prices, Prize Lecture,* Yale University, New Haven, (December 8, 2013).

components of randomness to affect our strategies and balancing a good positive volatility with a more conservative approach during periods of high variability. That is why we need to study a little more this concept.

From the studies of De Bondt in "The Psicology of Underreaction and Overreaction in World Equity Markets", published in 2000, we can say he supports a behavioral pattern to news of both under and overreaction, depending on the situation, stating that investors are not totally rational and their psicology could be easily influenced. To be noted, this paper was published slightly before the 2001 recession, when the market was convinced that the new technology would lead to a potentially infinite growth.

Markets such as the German and Swiss were bullish even if the economic situation was plagued by low employment rates and the real growth of the GDP was low. Already in May 1998 Solow expressed concerns about the rapid increase in the US stock market.[51] While from an economical standpoint this attitude is dangerous, for the man of the street this is no-sense. Markets should be efficients and the prices should reflect the real value of the assets, this is the argument used by Fama in 1970[52]: random error in prices could occur, but not consistent ones hence "you can't beat the market".[53] That is why Fama concluded, in a similar way in 1991, that "if it is real, it is rational".[54]

For Keynes "day-to-day fluctuations in the profits of existing investments, which are obviously of an ephemeral and nonsignificant character, tend to have an altogether excessive, and even an absurd, influence on the market".[55] In the Keynesian idea it is a totally random market, like an animal instinct and this is sustained by various arguments such as the studies of Pring in 1991 and 1993 or Soros in 1998.

But behavioral finance says that "what goes up must come down". This means that prices should be on average around the real value of the stock. In the short period, however, there is no guarantee about a real equality between the two, hence you should

[51] Robert Solow, comment in the *"International Herald Tribune"*, (May 7, 1993).
[52] Eugene F. Fama, *Efficient Capital Markets: A Review of Theory and Empirical Work*, The Journal of Finance, Vol. 25, No. 2, (May, 1970), pp. 383-417.
[53] Werner F. M. De Bondt, *The psychology of underreaction and overreaction in world equity markets*, Security market imperfections in worldwide equity markets. Cambridge Univ. Press, 2000, pp. 65-89.
[54] Eugene F. Fama, *Efficient Capital Markets II*, Journal of Finance Vol 46, No. 2 (Dec., 1991), pp. 1575-1617.
[55] John M. Keynes, *The General Theory of Employment, Interest and Money*, (Feb. 1936) Chapter 12.

still go for the real value obtained by the fundamental analysis as said in 1934 by Dodd and Graham.[56]

In the first case, even if a singular investor could be irrationatal, the majority of them will be a "*homo economicus*" and the minority will never lead to changes in price at market level. This is the most prevalent theory used in finance. But there are still anomalies this approach cannot explain such as a divergence between the price of the stock and the dividends obtained investing in it, seasonal and monthly patterns in prices, initial public offerings prices or excessive reactions to decisions of the management of a company.[57]

Mehra and Prescott noted in 1985 that over the period between 1889 and 1978, for 90 years, and I would add this is even more true today, as also said by my tests about mean of S&P 500 and US General Government bonds, excess return from the equity market has prevailed on default-free debt[58], (I noted it especially for longer bond maturities, while for the one-month bond this difference was much less important).

The comment made by De Bondt is pretty interesting, saying that if this difference in performance depends on the risk aversion, in the flip of a coin scenario between a 50 000 dollars and 100 000 dollars, with 0,5 probability of head and obviously the same for the tail, the average investor should be indifferent with only a 51 209 dollars offer.[59] This is called "Equity Premium Puzzle". This leads to hypothesis such as irrational loss aversion or investing through consolidated habits.

Moreover there are systematic evidences that the investor is not rational in decision-making process as affirmed by Kahneman and Tversky[60] and there is also a lack of diversification in the assets an investor usually choose.

De Bondt is against the concepts of perfect markets and perfectly rational investors.[61]

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[56] Benjamin Graham and David Dodd, *Security Analysis*, McGraw-Hill, New York (1934).

[57] Werner F. M. De Bondt, *The psychology of underreaction and overreaction in world equity markets*, Security market imperfections in worldwide equity markets. Cambridge Univ. Press, 2000, pp. 65-89.

[58] RajnishMehraEdward C.Prescott, *The equity premium: A puzzle, Journal of Monetary Economics*, Vol. 15, Issue 2, (Mar., 1985) pp 145-161.

[59] Werner F. M. De Bondt, *The psychology of underreaction and overreaction in world equity markets*, Security market imperfections in worldwide equity markets. Cambridge Univ. Press, 2000, pp. 65-89.

[60] Daniel Kahneman and Amos Tversky, *Prospect Theory: An Analysis of Decision under Risk*, Econometrica, Vol. 47, No. 2 (Mar., 1979), pp. 263-291.

[61] Werner F. M. De Bondt, *The psychology of underreaction and overreaction in world equity markets*, Security market imperfections in worldwide equity markets. Cambridge Univ. Press, 2000, pp. 65-89.

In 1983 Simon expressed the concept of "bounded rationality"[62]. While with a totally logical reasoning each asset has a certain value for the investor and the single person will maximize his utility, minimizing the risk and reaching his desidered output, under bounded rationality, the individual is not able to understand the stunning possibilities of investments he has and goes for the easier option. This could depend by habit, emotion, opportunity cost of his time or other factors. This fact create a significant lag between what is done by the investor on the market and what is his real best option using economic theory.

In 1979 Kahneman and Tversky refused the axioms of expected utility, stating that "it is not an adequate descriptive model"[63] posing higher importance on risk aversion, fear of losses and irrational weight to improbable events. Past experience are usually more important for the way of investing than rationality creating false possible scenarios in the mind of the "*homo non-economicus"*: in a simple way, risk affects massively problems about choices.

In this way the value of the asset is influenced by the system of beliefs that affects a real estimation of the economic reality.[64]

Thus there is a common path between investors, a pattern influenced by conversations, friendship or social Medias, an illusion of the personal economic situation as too good in comparison to the real average situation of the US market, a true detachment from the real world. People are easier to influence than we usually think.

An example of falsity in this mindset is the skyrocketing perceived value of accomodations in California during the 1980s: Shiller in 1990 explained that there was a superstition of prices increasing forever for a lack of availability of houses.[65]

New data will be always adapted to be put in the usual system of knowledge, avoiding to change our expectations for as long as we can. In the 1970s no one expected a really prolonged high inflation leading in the 1980s to high real returns without a real economic reason. Experience thus not lead to good learning, it only reinforce our

[62] Herbert A. Simon, *Rational decision making in business organizations*, Nobel Memorial Lecture 1978. p. 502.
[63] Daniel Kahneman and Amos Tversky, *Prospect Theory: An Analysis of Decision under Risk*, Econometrica, Vol. 47, No. 2 (Mar., 1979), pp. 263-291.
[64] Werner F. M. De Bondt, *The psychology of underreaction and overreaction in world equity markets*, Security market imperfections in worldwide equity markets. Cambridge Univ. Press, 2000, pp. 65-89.
[65] Robert J. Shiller, *Speculative Prices and Popular Models,* Journal of Economic Perspectives, Vol. 4, No. 2 (Spring 1990) Pages 55–65.

personal “ideology”, giving excessive overconfidence, expecially when the real level of knowing is pretty low, as demonstrated today by the real world, where everyone can talk about everything, from the crisis in Ukraine to the perfect investment in bitcoins. The interpretation given to data is more important than the information itself, giving stress to non-existent bad luck and overcompensating limited success with imaginatory skills. All this shared system of ignorance between common people can really affect the real market in a bad way.

For example it was only a series of errors that led to a constant stream of merger and acquisition in the US market in the 1960s and this, of course, was followed by divisions in the following decades: simply there was overconfidence on the importance of diversification, like the multitasking idea reaching prominence in the last years, that is becoming a clear and demonstrated detriment to our society. Even if the mergers were errors in the 1960s prices went steadily up as said by Matsusaka in 1990, while they inexplicably went down during the economical right decisions to break this unholy acquisitions in the 1980s. Only in April 1991 The Economist recognized it was the right decision to do to cut the losses and increase productivity.[66]

All these factors lead us to affirm that substantial weak form market inefficiencies are present in the market, in contrast with the usually accepted “truth”, or, as said by Shleifer and Summers: “If the efficient markets hypothesis was a publicly traded security, its price would be enormously volatile”.[67]

2.10 The concept of Overreaction

A question could be about how an investor predict the future. It is pretty clear to imagine they are not used to “invent” something really divergent from the recent experience and this is evident in both optimism and excessice confidence, or on the contrary to overreaction to news about negative performances. A non-rational actor on the market tends to accept only sensationalist predictions about an “exponential” market growth or

[66] Werner F. M. De Bondt, *The psychology of underreaction and overreaction in world equity markets*, Security market imperfections in worldwide equity markets. Cambridge Univ. Press, 2000, pp. 65-89.
[67] Andrei Shleifer and Lawrence H. Summers, *The Noise Trader Approach to Finance*, Journal of Economic Perspectives, Vol. 4, No. 2 (Spring 1990) Pages 19–33.

a “looming” disaster. These patterns are only distractions, “noise” that can only drag us away from the reality of the situation.

In 1993 Kahneman and Lavallo studied this forecasting process explaining that an event that occurs in the present, for a certain amount of time, will cause both “bold forecasts and timid attitudes to risk”[68] to different investors.

Thus past experience will overcome the reality of the situation causing error of prediction. In 1985 De Bondt and Thaler showed consistent forecasting errors on the part of the average investor, characterized by excessive high expectations and overreactions to events.[69] The American Association of Individual Investors asked to members selected by chance to foresee changes in the equity market every week and the result was even bullish when the market was rising and a too bearish expectations when it was lowering in prices. In the same way, past success of certain IPOs like Microsoft or Intel, led to unrealistic anticipations about companies in already consolidating markets.

As said by Shiller “it is hardly plausible that speculative prices make effective use of all information”[70], thus the two related concepts of overreaction and underreaction.

The term “overreaction”, intended as related to the economic reality, was probably coined in 1982 by David Dreman.[71] He believed that there was too much positive expectation in the market for high price-earnings ratios and on the contrary too much negative anticipation on low ones, only temporarily undervalued, after reports about difficulties or negative returns, creating excessive alarmism and pessimism. Of course, in the following periods of time these firms obtained way better results than expected by the market, causing a rise in prices. On the contrary, for high Price-Earnings, this overpriced companies will probably decline in the future.

[68] Daniel Kahneman and Dan Lovallo, *Timid Choices and Bold Forecasts: A Cognitive Perspective on Risk Taking*, Management Science, Vol. 39, No. 1 (Jan., 1993), pp. 17-31.
[69] Werner F. M. De Bondt, Richard Thaler, *Does the Stock Market Overreact?*, The Journal of Finance, Vol 40, Issue 3 (July 1985), pp. 793-805.
[70] Robert J. Shiller, *Speculative Asset Prices, Prize Lecture*, Yale University, New Haven, (December 8, 2013).
[71] David N. Dreman, *The New Contrarian Investment Strategy*, Random House, New York (1982).

In general we should hear news in the media, but be careful to change our whole strategy for some headlines, how the market moves is really difficult to predict, and if a certain pattern exists today, it will be probably replaced by the opposite-one tomorrow.[72]

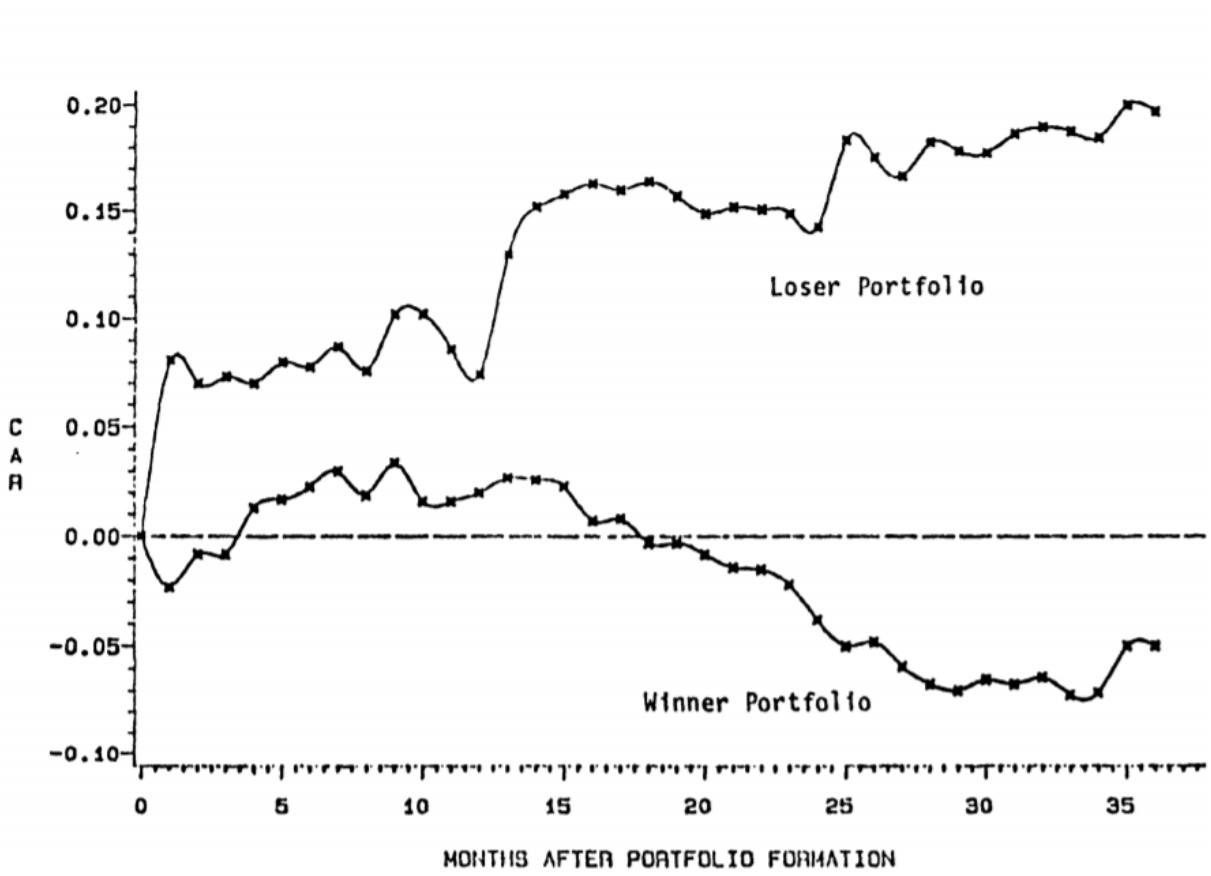

Exhibit 9. Cumulative Average Residuals for Winner and Loser Portfolios of 35 Stocks (1-36 months into the test period). January 1933 – December 1980.[73]

De Bondt, with Thaler, affirmed that there is a true overreaction bias. In the Winner-Loser graph they summarized the situation of all the companies presented in New York Stock Exchange since December 1925. The portfolio of 50 NYSE stocks that appreciated the most in the first 5 years taken into account were outperformed in the

[72] Werner F. M. De Bondt, *The psychology of underreaction and overreaction in world equity markets*, Security market imperfections in worldwide equity markets. Cambridge Univ. Press, 2000, pp. 65-89.
[73] Werner F. M. De Bondt, Richard Thaler, *Does the Stock Market Overreact?*, The Journal of Finance, Vol 40, Issue 3 (July 1985), pp. 793-805.

following years by the 50 stocks that resulted as losers in the first years.[74] Price reversal shows that assets maintained a "memory" of the past and this is an anomaly for the standard financial theory. Even if this occurrence has been explained (at least as a test) by taxation, seasonality, risk varying with time or other factors, many studies demonstrated the phenomenon of overreaction.

After the Great Depression, in 1934 Graham and Dodd used value and growth as concepts useful to build a remunerative set of assets. In this model Price-Earnings, cash flow and equity ratios are really important.[75] It is true that value stocks obtain more positive results than growth stocks but value stocks must not always bear higher risk, as demonstrated by studies conducted by De Bondt, Thaler, Fama, French and Lakonishok. It is demonstrated that past winnners should be short-selled in favour of past losers, and this fact is also applied at a national level. Creating a "World" index, countries that scored more in the first years, will do worse later on.

The argument used by De Bondt to explain is, of course, overreaction. He refers to "mental frames[76]" that transform popularity and attractiveness into perceived optimal investment methods. This condition is reinforced, probably involuntarily, by public opinion and financial news, causing imitation behavior between investors, amplifying this effect. Hence many traders will continue to act as the recent past performance will become a long-lasting trend. It is like an Exponential Moving Average at the extreme of the spectrum, with a lack of interest in the real data and a strong stress on recent informations, giving excessive weight to news.

Moreover what is known is usually considered in a better light: familiarity, lack of international focus, interest in local companies are important for an everyday transaction. Thus for De Bondt it exists an arbitrage possibility, investing in 20% of corporations for which analysis are pretty low and financing it with the 20% of firms with higher expectations and this return increases with higher forecast horizons.

Quoting from "The Intelligent Investor" of Graham published in 1959 "no one really knows anything about what will happen in the distant future, but analysts and investors

[74] Werner F. M. De Bondt, Richard Thaler, *Does the Stock Market Overreact?*, The Journal of Finance, Vol 40, Issue 3 (July 1985), pp. 793-805.
[75] Benjamin Graham, David Dodd, *Security Analysis*, McGraw-Hill, New York (1934).
[76] Werner F. M. De Bondt, *The psychology of underreaction and overreaction in world equity markets, Security market imperfections in worldwide equity markets*, Cambridge Univ. Press, 2000, pp. 65-89.

have strong views on the subject just the same".[77] This comment, pretty much taken alone, can summarize all this argument: that is why contrarian investing is a good trading strategy.

In 1995 Berry and Dreman stressed the importance of two kinds of occurrences: events that confirm previous expectations and surprises that are in contrast with the past. Studying the period between 1973 and 1993 they discovered that good news for losers and the contrary for winners cause excessive reactions in the market.[78] Hence a positive result for a firm that is already going well does not cause the same response than a bad one that obtains the same results, but a far less reaction of the investors on average. Bauman and Miller in 1997 focused on value and growth stocks showing that the first category both outperform the second and reacts far less to negative surprises than the counterpart.[79]

However, on the long run, both categories reach comparable results.

For many investors overreaction is a common miscalculation, thus they stress the importance of the present, like a rapid growth in the recent past, underreacting the possibility of future positive returns in other companies. This anomaly causes a constant higher price for corporations with significant reputations that will obviously lead to a poor performance on the market in the future, while a good investor should put more money in companies that are not on the crest of the wave.

Fama in the 1970 said that faced with the prospect that prices could follow a random walk pattern, the economist try constantly to find an explanation and a model.[80]

De Bondt is convinced that "people are human"[81] and psychology is important also for finance, altering the suggestions of rationality. Finance is a science but a social one because both the individual process of making a decision and the study to operate in the market are influenced by recurring errors and misjudgement. As said by Kahneman and

[77] Benjamin Graham, *The Intelligent Investor*, Harper Business Essentials (1949).
[78] David N. Dreman and Michael A. Berry, *Overreaction, Underreaction, and the Low-P/E Effect*, Financial Analysts Journal, Vol. 51, No. 4 (Jul. - Aug., 1995), pp. 21-30.
[79] W. Scott Bauman, Robert E. Miller, *Investor Expectations and the Performance of Value Stocks versus Growth Stocks*, The Journal of Portfolio Management 23 (3), (Mar., 1997) pp 57-68.
[80] Eugene F. Fama, *Efficient Capital Markets: A Review of Theory and Empirical Work*, The Journal of Finance, Vol. 25, No. 2, (May, 1970), pp. 383-417.
[81] Werner F. M. De Bondt, *The psychology of underreaction and overreaction in world equity markets*, Security market imperfections in worldwide equity markets. Cambridge Univ. Press, 2000, pp. 65-89.

Tversky "The predicted value is selected so that the standing of the case in the distribution of outcomes matches its standing in the distribution of impressions".[82]

Graham and Dodd said in 1934 that the financial market is not a perfect scale, but it is like a "voting machine".[83]

We can conclude that: going against the crowd is sometimes a good strategy, when the crowd is acting foolishly.

[82] Daniel Kahneman and Amos Tversky, *Prospect Theory: An Analysis of Decision under Risk*, Econometrica, Vol. 47, No. 2 (Mar., 1979), pp. 263-291.

[83] Benjamin Graham, David Dodd, *Security Analysis*, McGraw-Hill, New York (1934).

Section 3: Application of the Strategy

3.1 Let's start trading!

Trying to recreate the strategy used by Ilmanen, for limited availability of data I will substitute the shorting 1-month Treasury investing in a long-maturity US government bond with a strategy based on going long for a 30-year US bond and going short on the 3-month Treasury-Bill between 1991 and September 2001 and shortening a 1-month Treasury bond after this date. In the first period we will be able to compare our results to those obtained by Ilmanen until 1995 and we will procede alone after this date.
As a starter, let's analyze the graph of Inverse Relative Wealth and the positive or negative gains of our fixed implemented strategy over-time.

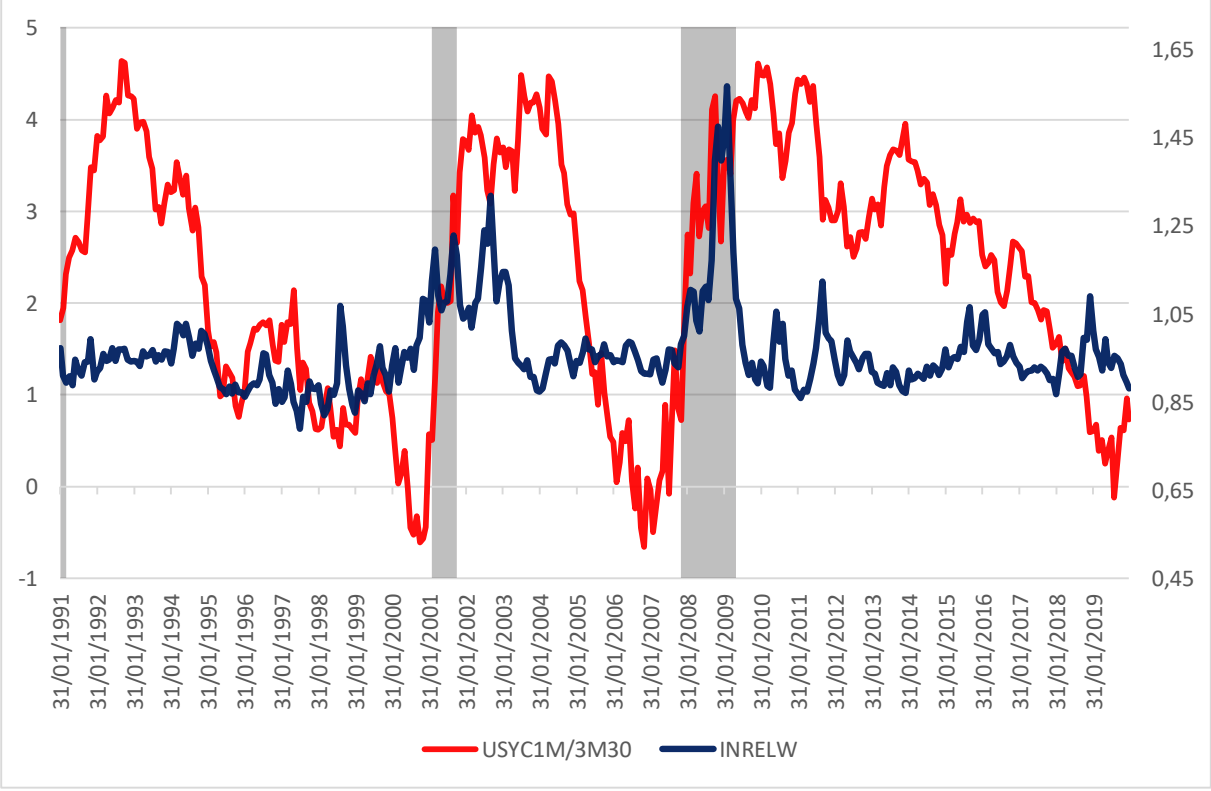

Exhibit 10. Comparison between US Inverse Relative Wealth and the fixed implemented strategy. Data about the strategy are in excess bond return calculated monthly as a percentage of the money invested during that month.

Shockingly we find out that the graph of the Inverse Relative Wealth is much different from the fixed strategy results. For once, the strategy appears to be pretty good taken alone, earning a series of basis points clustered between -100 and +500. The Inverse Relative Wealth Curve has a pattern pretty much opposite to the one of the fixed strategy, not recognizing many changes in the market situation and following, not preceeding, the bond market in 2001 and 2008. A scarce result in comparison to what we hoped. We could clearly notice as the Inverse Relative Wealth peaks during the recession, in the period right before the recover of the stock market, as we could understand from later studies, thus being unable to predict neither the fall, nor the rise of our bond strategy, always pursuing, not forerunning it. Clearly nor a good proxy, nor a bad one.

Now let's compare the fixed strategy to the US stock market itself, especially we want to visualize the S&P 500 and the Exponentially Weighted Moving Average.

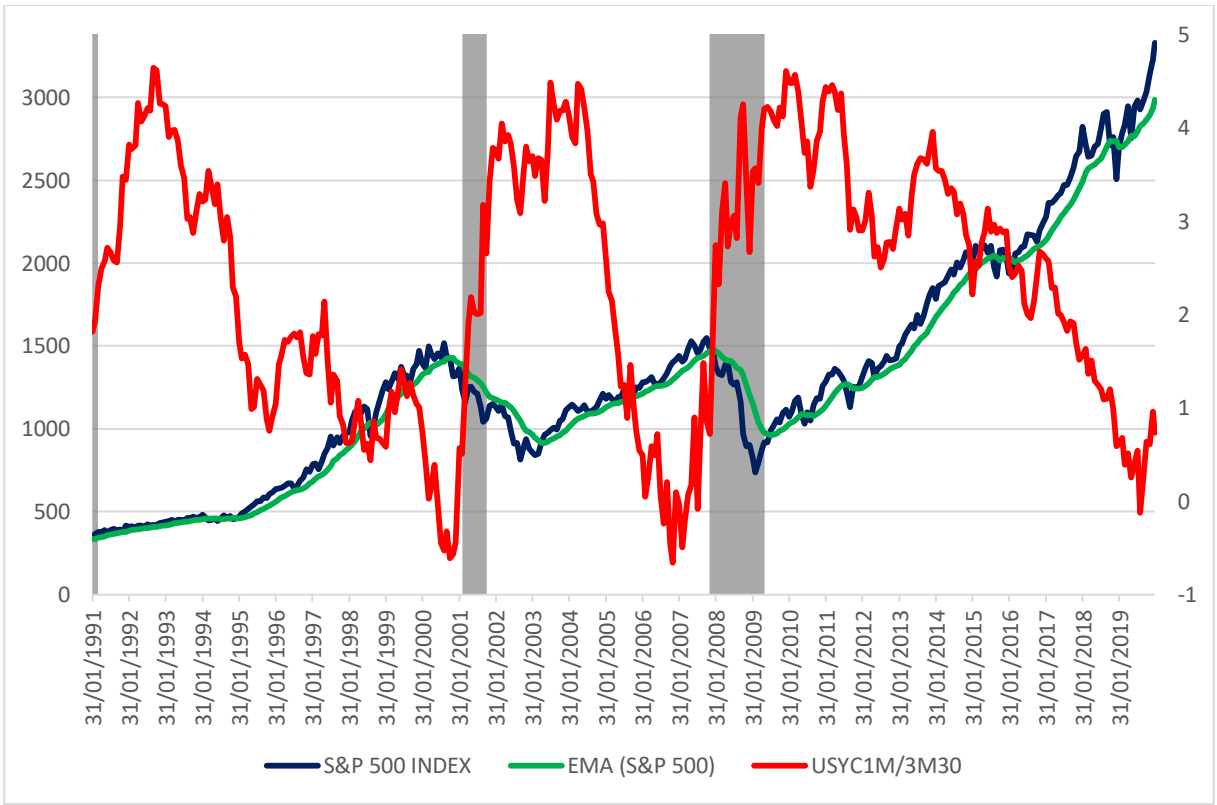

Exhibit 11. Comparison between US stock market and the fixed implemented strategy. Data about the strategy are in excess bond return calculated monthly as a percentage of the money invested during that month. In grey we can see the three recessions of the period.

It appears that the Stock Market and the fixed strategy are inversely related, especially after 1993. When the S&P 500 crashed in 2000, this movement was accompanied by a sharp rise in the excess bond return. Then the profitability of the fixed strategy slowed-down, and actually became negative during the market expansion of 2006 and 2007. The rise of the profitability was soon followed by the crash on the market that culminated in 2009. After that, the revenue from the strategy lowered reaching 0 monthly gain in 2019, when the S&P 500, in contrast, exploded as never seen before. Now it is rising once again.

Clearly the stock market is not a good signal for the strategy. But the contrary may be true: it appears that the bond market is able to predict a stock crash, and even a recession by lowering the yield spread between long and short governative bonds. But this reasoning is based only on graphs, let's go to some basic statistics

Correlation Matrix	*USYC1M/3M30*	*S&P 500 INDEX*	*EMA (S&P 500)*	*INRELW*
USYC1M/3M30	1			
S&P 500 INDEX	-0,30	1		
EMA (S&P 500)	-0,28	0,99	1	
INRELW	0,21	-0,11	0,024	1

Exhibit 12. Correlation between the fixed strategy and the Stock Market with the three possible proxies, S&P 500, the Exponentially Weighted Moving Average and the Inverse Relative Wealth.

Also from the correlation analysis we can admire as the stock market and the fixed strategy are inversely related. This is pretty clear from our theory: when the market is expected to rise, a stock is a much more rewarding instrument. It is only when the prediction are negative that the common investor will shift intensively on bonds, anticipating a fall in the S&P 500, and a rise in bond profitability.

The Inverse Relative Wealth has a positive relationship with the profitability of the investment in long-maturity bonds funded with short-ones, but this is only a positivity around 0,2. It is now apparent that the reasoning behind the Inverse Relative Wealth

makes sense, however it is too little and too late to be a good instrument for the bond yield spread.

On the contrary, taking the opposite of the stock market signal, with a positive correlation of 0,28 for a proxy based on the opposite of the weighted average and 0,3 for the opposite of the S&P 500, is around 50% better than the Inverse Relative Wealth. A possible explanation could be that the recent importance of the stock market was not fully anticipated by the author. As seen in the graph about the relationship between Inverse Relative Wealth and excess bond return, he expected an Inverse Relative Wealth able to predict what will happen with a one-month lag. My results are not supporting this prediction.

	USYC1M/3M30	*S&P 500 INDEX (%)*	*EMA (S&P 500) (%)*	*INRELW(%)*
Mean	2,34479	0,747%	0,638%	0,048%
Variance	1,88752	0,0016	0,0002	0,0018
Standard Deviation	1,37387	0,0405	0,0131	0,0425

Exhibit 13. Mean and Variance return comparison between the fixed strategy and the stock market with the three possible proxies (expressed in monthly percentage change).

Obviously seeing mean and standard deviation of the percentage variation of our objects of interest (let us remember that USC1M/3M30 is constructed as the percentage gain of the fixed strategy) we can observe that in theory Inverse Relative Wealth could be a good predictor, with a positive mean and a low standard deviation, not so different with the ones of the bond strategy. The S&P 500 and the moving average could be analyzed only in this percentage form, otherwise the results will be totally nonsense.

Now it is time to compare the results of our fixed strategy to the ones obtained by the three dynamic strategy, using as a proxy the positive variation of the stock market, his moving average and, of course, the Inverse Relative Wealth function.

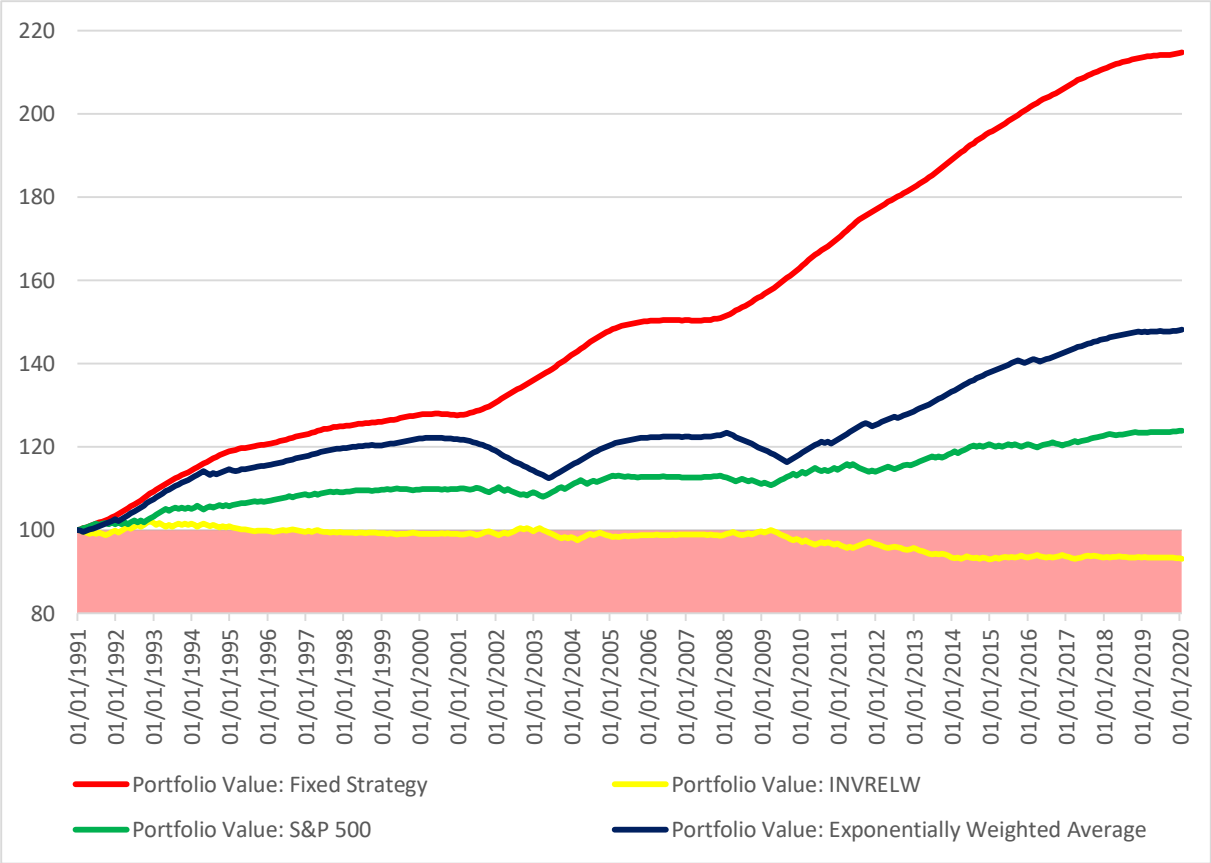

Exhibit 14. Portfolio Value comparison of Fixed Strategy, S&P 500 and ewaS&P proxies dynamics and the Inverse Relative one.

Note: the red part signifies that we are currently in a debt position. I have started the graph from 100 but it is clear to understand that in a zero-investment scenario, the true starting point is a "0".

From the comparison it is notable that Inverse Relative Wealth is the worst proxy in the long-run. Obviously in a portfolio scenario it is important to gain momentum and use it over time, but the value of the Inverse Relative Wealth portolio goes from 100$ to 93,15$ in February 2020 with a net loss of nearly 7%. Third best is the S&P 500 index taken as a proxy, reaching in the same time period a value of 123,88$, a gain of nearly a quarter on the starting value. With 148,11$ we have the Exponentially Weighted Average, that seems a good smoothing value for a rough stock market, obtaining nearly 50% net gain from what we shorted in 1991 to fund our strategy. Last but not least, the fixed strategy. As unbelievable as it may seem, it appears that our long-short government bond strategy does not need a stock market proxy to operate. It is pretty functioning

without any human intervention, earning a net amount of 114,72 dollars over 29 years, for a percent gain of 114,7% in the time period (and a total position of 214,72$).
The best results were graphically obtained between 1991 and 1994, 2000 and 2005, and finally between 2008 and 2017. It is a long stretch of time with only minor setbacks over the interval. Obviously the compounding effect of our cash gain helps to increase the difference as years pass by between the "good" strategy and the others.
Taking into account the recessions, it is obvious to realize that during 2002 and 2008 the competition for capital was totally in favour of the bond market, helping excess bond returns as soon the stock market prices dropped, leading to lower yields but higher bond prices.

	Fixed Strategy	*INVRELW*	*S&P 500*	*EWA*
Average monthly (%)	0,219%	-0,020%	0,062%	0,113%
Best Performance	0,503%	0,503%	0,500%	0,503%
Worst Performance	-0,058%	-0,461%	-0,503%	-0,414%

Exhibit 15. Comparison between the four strategies in monthly performance reported in percentage change.

From the table about monthly performances it is clear to see that the 0,22% monthly gain is not a bad result, with an average yearly gain of about 395 basis points. The Exponentially Weighted Average scored about half the monthly gain of the fixed strategy and the stock market index a bit more than half of the average. It is intriguing to find out that the best monthly performance of our strategy was predicted by all the strategies with the exception of the equity market, resulting in his worst performance.
The fixed strategy obtained only a small negative result of less than 6 monthly basis point, in the worst performance scenario.

3.2 Some differences

How can we explain this fundamental difference with the results obtained by Ilmanen? Let us compare the two set of results, especially in the overlapping period, between 1990 and 1995 when we have both pretty reliable data on our part as well as the economic results of the author.

	1965-74	1975-84	1985-94	1990-95
Dynamic Strategies				
Scaled Strategy				
Average Excess Return	**4.09%**	**15.29%**	**6.40%**	**5.30%**
Volatility	5.65	20.30	7.32	4.80
Sharpe Ratio	0.72	0.75	0.87	1.10
1/0 Strategy				
Average Excess Return	**0.89%**	**3.20%**	**7.27%**	**6.73%**
Volatility	5.56	8.92	8.66	7.79
Sharpe Ratio	0.16	0.36	0.84	0.86
Static Strategies				
Always-Bond				
Average Excess Return	**-3.13%**	**-1.68%**	**5.62%**	**5.25%**
Volatility	8.35	12.36	10.05	8.28
Sharpe Ratio	-0.38	-0.14	0.56	0.63

Exhibit 16. Results of Ilmanen's strategy over time.

Results of the Static always long government bond strategy in comparison to the two dynamics 1/0 strategy (if his proxy indicate to invest he would do so, otherwise do not) and scaled strategy (investing in each period following the proxy).[84]

It is fascinating to watch that the best performance obtained by Ilmanen was during the years between 1965 and 1984 for his dynamic strategy, especially after 1975, with high volatility in the market and in his portfolio, but a good reward in excess bond returns terms.

Over time the scaled strategy was not so powerful in comparison to the other two and it was surpassed by the 1/0 strategy for 10 years consistently. Cleary 20 years of predominance for the scaled strategy are a good result, but we have to keep in mind that all the last 10 were "against" his strategy. Comparing these results to the static

[84] Antti Ilmanen, *Forecasting U.S. Bond Returns (Understanding the Yield Curve, Part 4)*, Salomon Brothers, August 1995.

strategy, in fact, we can admire as the worst performance for the static portfolio was in the first recorded period, between 1965 and 1974, improving but still negative in the second period and following pretty much the pattern of the scaled strategy in the last two ones, with an average yearly return difference of only 0,05% yearly between the two, with the scaled strategy still slightly above but with a performance gap much reduced from the starting years taken into account.

Excluding the years between 1975 and 1984 it does not appear to me that his results were as staggering as expected.

But considering the static strategy it is pretty clear that the results, especially in the last 5 years were pretty good, with an excessive bond return of around 5,25% yearly.

In the end probably the same Ilmanen's strategy unfolded over-time as a much more preminent fixed long-bond minus short-maturity one portfolio, mirroring our totally fixed long-short strategy. In my personal opinion this could signify that in general the fixed strategy was a good and safe bet, overall. Maybe it did not work in the 1970s or 1980s, but those years had peculiar problems and characteristics that a "rational" scenario could not anticipate ex-ante, thus reducing the efficiency of the predictions made by the economist.

Ilmanen gives us also the volatility and the Sharpe ratio. Luckily we have our own data on the last section of the table and we can calculate and compare all these results to the ones obtained by the economist starting from 1990 (since we have also considered this first year in our analysis but excluded it from a portfolio starting point due to the economic recession).

Let us go back to my results and calculate the average yearly return, the volatility (represented by the standard deviation) and the Sharpe Ratio, with the method explained above and used by the economist. Results are presented in the Table below.

Static Strategy	***1990-1994***	***1995-1999***	***2000-2004***	***2005-2009***	***2010-2014***	***2015-2020***
Av. Excess Return	**3,53%**	**1,49%**	**3,16%**	**2,03%**	**3,99%**	**1,94%**
Volatility	5,91	2,23	5,29	2,70	5,76	3,10
Sharpe Ratio	0,60	0,67	0,60	0,75	0,69	0,63

Exhibit 17. Static Strategy Results, 1990-2020.

As expected by the graphical study there are three periods were the yearly performance is quite good, after 1990, 2000 and 2010. The yearly excess return does never fall behind 1.49% and the volatility is obviously higher during the good periods than the more stable ones. Sharpe Ratio, is always between 0,6 and 0,75, a little higher but comparable to the results obtained by Ilmanen. The best period was surely 0,75 Sharpe Ratio in 2005-2009, but this depends mainly by a relative low variability of the market.

However, there is an important discrepancy to be considered. My results and those obtained by Ilmanen does not match for the 1990-1995 period at all. His 5,25% average excess return his 1,71% yearly higher than mine. Thus consists in an higher Sharpe Ratio for his Table than mine, with a slight difference between 0,6 of mine and 0,63 of his results. Clearly the Average Excess Return is pretty much compensated in the Sharpe Ratio by an increase in his calculated standard deviation for the portfolio.

The Average Excess Return difference is really important because it is nearly 50% more profitable in his calculation than mine.

For instance, let us compare my results using a coefficient of proportionality comparable to his study for my Excess Return statistics.

From empirical analysis I have found out that a monthly coefficient of 1,32 can clearly predict this difference. Let us find out how it impacts on our static long-government bond zero-investment strategy.

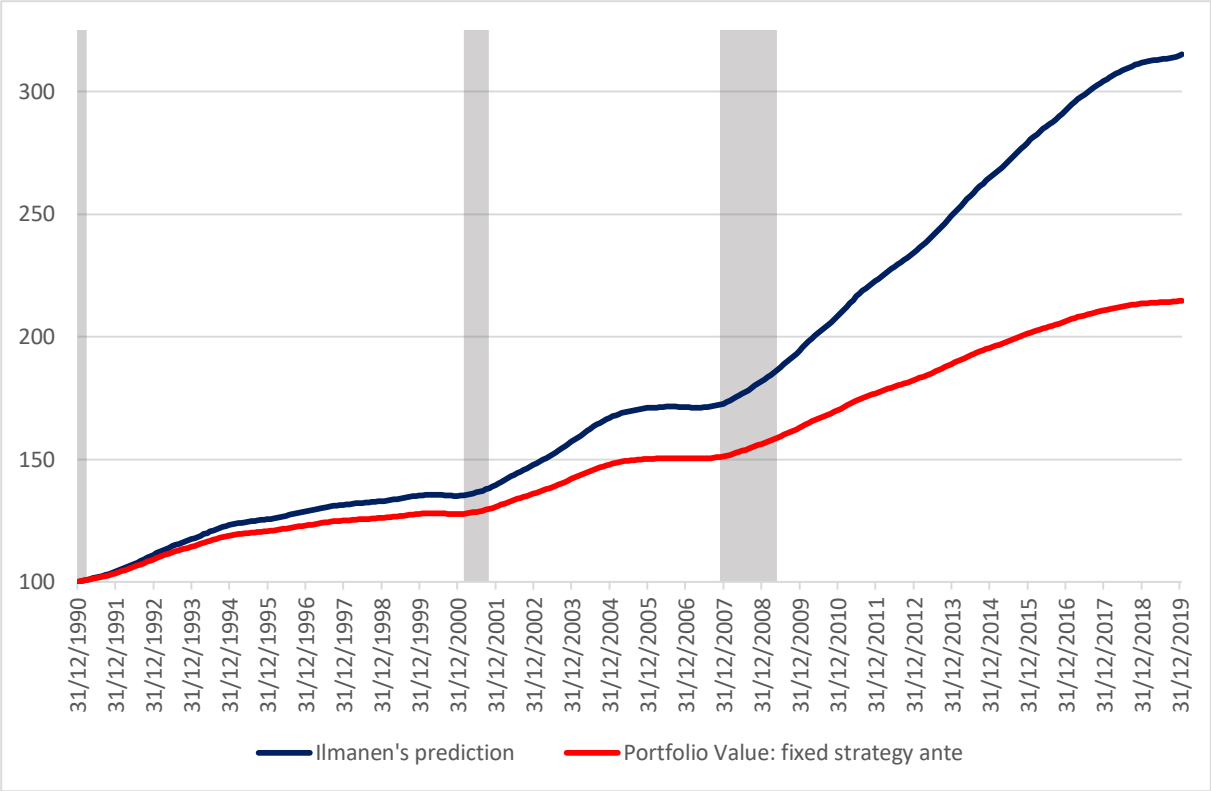

Exhibit 18. Static Strategies comparison.

Value of the portfolio using both data from Ilmanen and mine 1990-2020.

The results are really interesting. Remembering that our net investment is 0, because all what we use for the fixed strategy comes from the shortening on the market, the end results are 214,72 for our results and 315,14 for the ones using Ilmanen. Clearly over time what could appear a limited bias has become a net gain difference of 111 dollars and we obtained after 29 years and 1 month only 53% of what we could expect using Ilmanen's data. This coefficient magnifies excess bond returns in particular during the two recessions of the 2000s, creating a gap that will only grow over time.

For instance, after 30 years the average yearly excess return will be "only" 3,945% for our database and 7,398% for results coming for a database comparable to the one used by the economist.

How is it possible? Our data comes from Bloomberg and they should pretty accurate, given the importance of the database we used in our search. But Ilmanen had access to CRSP database and many other good informations about the market. Counting only the

fix strategy this is really strange, but considering his predictions about Inverse Relative Wealth it becomes really unexpected.
There is a possibility, a positive bias in the excess return library he used to formulate his strategy.

3.3 Caveat Compounder

The Center for Research in Security Prices is a really important database when we come to US markets. It was used by Ilmanen but also by many other searches and studies.
However there are possible errors in his computations, at least there were for the 30 years ago informations that the economist used to fuel his analysis.
In February 1998, again in "The Journal of Finance", Linda Canina, Roni Michaely, Richard Thaler and Kent Womack published an article called "Caveat Compounder: A Warning about Using the Daily CRSP Equal-Weighted Index to Compute Long-Run Excess Returns". In the text they argued that using the Daily CRSP index could lead to a large misleading information about excess returns, calculated by them at a positive increase of the real excess return of about 6% each year or 0,43% each month. In their calculation this error was around 33% of the total calculated return, hence it was of true impact in calculating the profitability of a set of investments.[85] Their study was obviously more focused on a portfolio based on stocks, but their conclusion is that the CRSP index was a pretty good approximation only for a small period of time. My arguing is that if the CRSP index produced an average gain for a strategy inflated by 33%, something similar could have happened three years before with at least some of the computations used by Ilmanen.
Weirdly enough a one-third is pretty much the exact prediction I made to compare my results to the Ilmanen's ones. Let us deflate the economist excess return by 33% and see graphically what we obtain.

[85] Linda Canina, Roni Michaely, Richard Thaler and Kent Womack, *A Warning about Using the Daily CRSP Equal-Weighted Index to Compute Long-Run Excess Returns,* The Journal of Finance, Vol. 53, No. 1 (Feb., 1998), pp. 403-416.

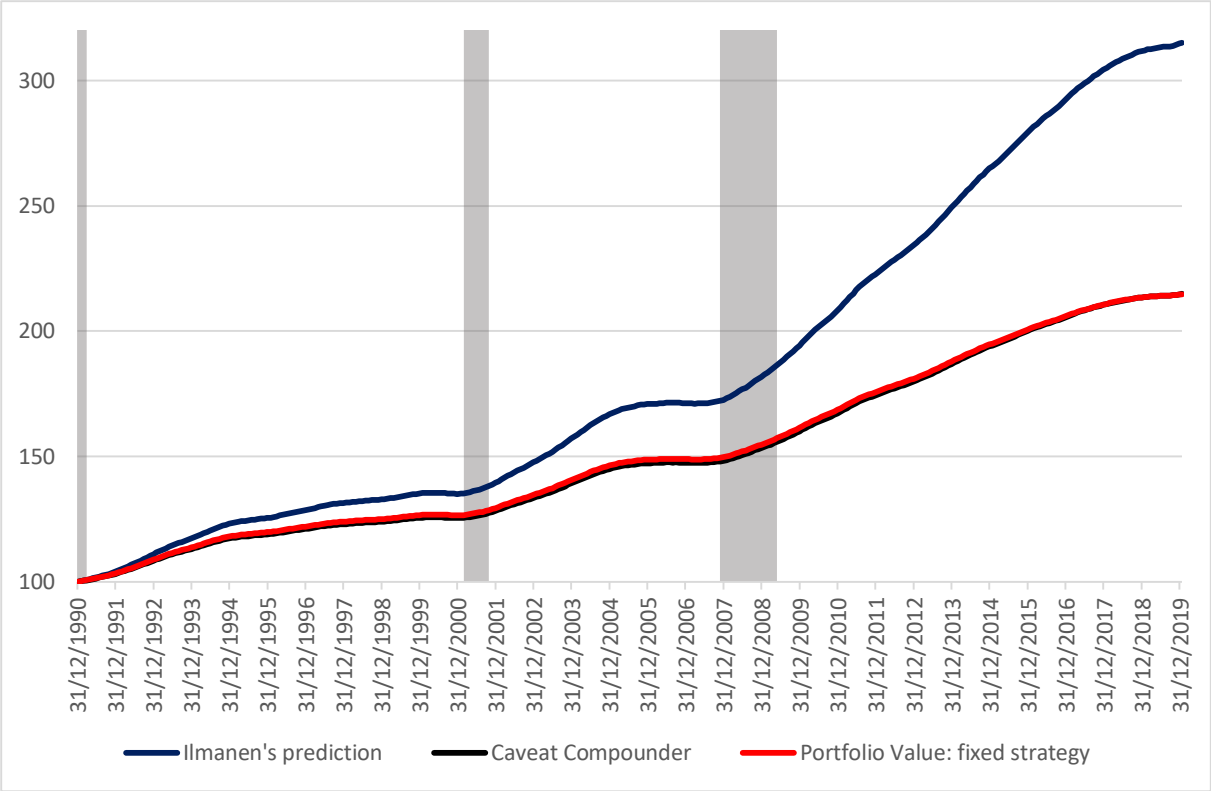

Exhibit 19. New Static Strategies comparison.

Analysis after the considerations made by the Caveat Emptor paper of 1998. As said before, in grey we see the recessions happened over time.

It is notable to see how much the two graphs are related. The Caveat Compounder inspired one is even a little less profitable than my expectation, at least in the middle of the period taken into account, while at the end it recovers the small difference and ends at pretty much the same point as our calculations have suggested: 214,87$ against 214,72$ of our previous computation.

Clearly the difference between the graphs is not given by a difference of investment, but probably in the calculation of the results. I think that 30 years of data have improved the computational system of excess bond returns, lowering it significantly, and even if Ilmanen had access to the best informations available at that time and he is much more experienced and smart than any reasoning I could use, probably the data bias is sufficient to explain this unexpected difference, even in a "simple" long static strategy.

Section 4: “Improving” the Strategy

4.1 The search for a new market signal

Now it is clear. The Inverse Relative Wealth is not a good predictor for bond market returns, at least not consistently. Maybe it was so during the years between 1965 and 1985, probably caused by a bias enforced from the many recessions happened during those years, going from oil shortages to wars abroad, from high inflation to stagnation in the market. But today I can not appreciate how it could be transformed into a good market indicator.

Also the stock market itself, with his moving averages, has changed over time, especially from the years taken into account by the economist. I do not think Ilmanen, or anyone else, for what matters to our study, anticipated the strong importance of stocks as an overwhelming attractor of cash and investments. The unprecedented heights reached by this market maybe have changed the form of the relationship between stocks and bonds, influencing the results of any possible strategy.

But if nothing has worked from the stock market, how can we possibly improve an already pretty reliable static strategy? We need another proxy or at least a signal that times are changing and we need to shift on a new position. This can probably come only from the bond market itself, creating a recursive loop, which is not a true rule but can still anticipate what we will find one month from now, at least in the period we analyze. Of course this rule must be studied over time in the future, because we cannot rely forever on a proxy so dependent on the dataset it must analyze year after year.

But we still have at least a good starting point: a fixed strategy, with a return higher than the risk free investment and a positive and significant Sharpe Ratio during a quite long span of 30 years.

Analyzing the performance of the strategy, we find that it acted poorly only in limited circumstances: in the last part of 2000, in some limited months of 2006 and 2007 and at last one month in August 2019. The first two of these spans were shortly after followed by a recession in the market, and about the last one, actually there is a possible expectation of a stock market decrease, at least made by some actors. The total number

of months of losses in the portfolio strategy is 15 on the whole period between 1990 and February 2020, for a total of 371 months (4,04% of the total number)

Moreover it should be noted that the fall in the excess return always preceded, not followed, the market crash.

We could ask why this happens.

4.2 Inverse Yield Curve

Let us analyze the last period where the portfolio obtained a loss, August 2019. As said before there were expectations about a possible recession, predicted by Forbes, using averages of the past, as late as April 2021, and this data is consistent with the other negative results we have. But there is also another interesting result.[86] In May 2019 the Yield Curve started inversing and this culminated in August with the following two figures (the first one is taken from seekingalpha.com while the latter is my reconstruction of Forbes data).

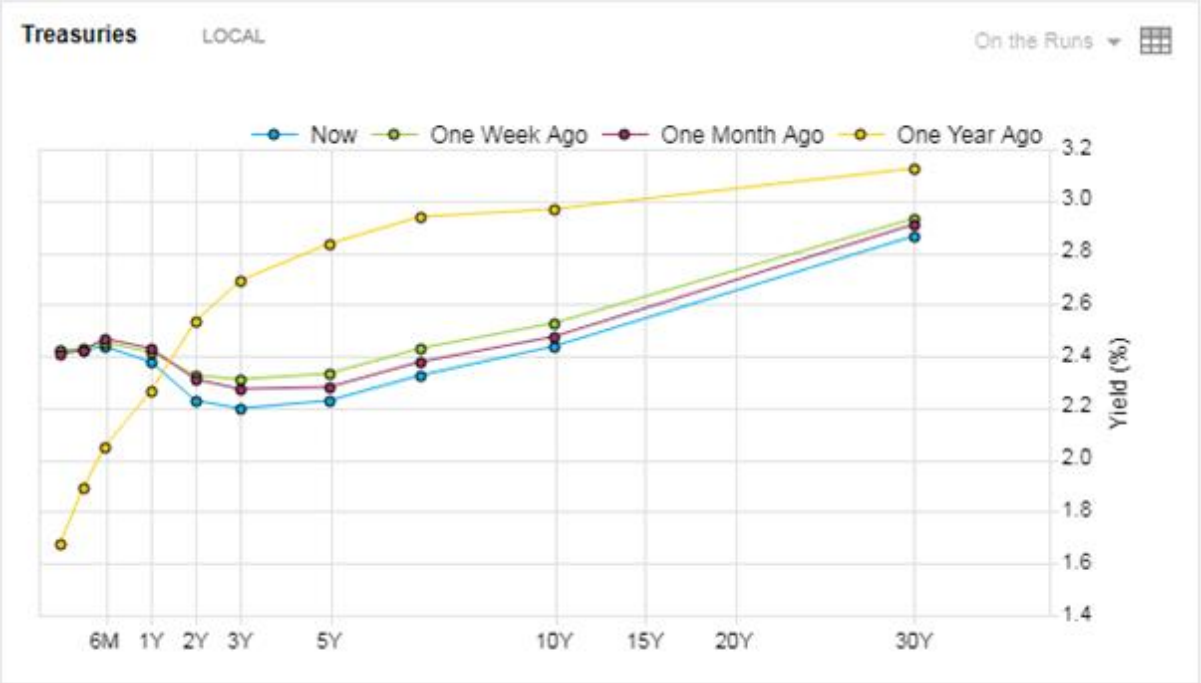

Exhibit 20. Yield Curve evolution.

Factset 12/5/2019, comparison between a regular Yield Curve (One Year Ago) and an inverted one.[87]

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[86] Data taken from Trefis Team, *Inverted Yield Curve Suggesting Recession Around The Corner?*, Forbes (Oct. 2, 2019).

[87] taken from Hiu Fat Tse, *Long Bond Butterfly Spread and Convexity to Gain on Yield Volatility and Negative Butterfly Twist* (May 23, 2019).

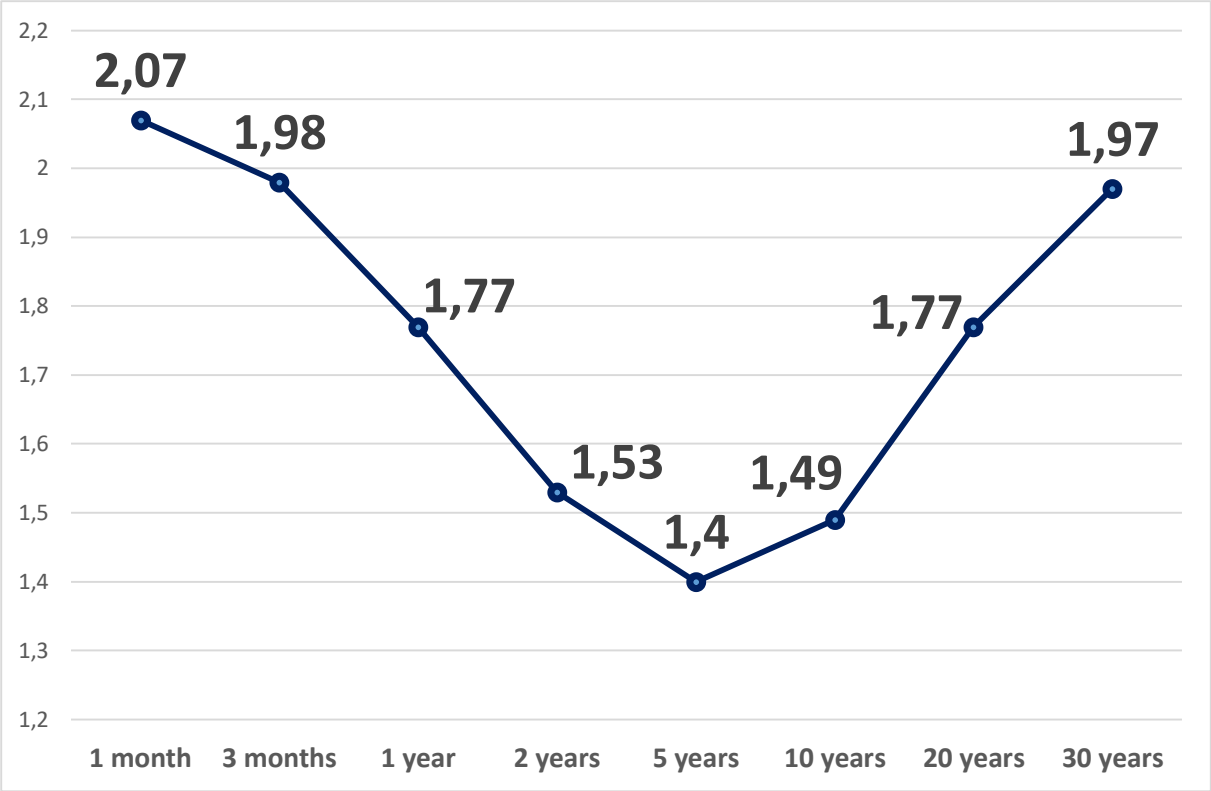

Exhibit 21. Yield of US treasury bonds, 27 August 2019.[88]

The yield curve is the plot of different yields obtained at different maturities, from the shortest, usually 1 month, to the longest, like 30 years. The graph allows for a quick analysis of bonds of different maturities and the return on investment we could obtain. The first part of the arc is determined by the monetary policy expectations about the Federal Reserve: when the Central Bank of the United States raises interest rate, it goes up, on the opposite case, of course, it goes down. When we shift towards longer maturities the effect of other events like the amount of pressure from the investors, inflation or variations in expected growth gain more and more influence.

In general a yield curve is expected to be upward looking, with rates of 1 month lower than 3 months and this reasoning goes on until the 20 and 30 years maturities. Hence a normal yield curve is the regular situation in the market with a positive slope of the graph moving from nearer maturities towards longer horizons. The reasoning is that a long term focus of the investor should be compensated more for the increase in liquidity

[88] Data taken from Trefis Team, *Inverted Yield Curve: Is Recession Coming In Next 20 Months?*, Forbes (Oct. 2, 2019).

premium that the investment bears. Moreover it makes sense, from a rationality standpoint that the longer you are willing to wait, the more money you should receive in return. Risk of losses is extremely low, especially for a country like the United States, which we are talking about right now.

When the yield curve is inverted, short yields are higher than long term ones, because there are expectations of a recession, as the *"homo economicus"* becomes more and more concerned about medium to long term investments, and he is more willing to smooth his consumption, as the benefit of one dollar in a good environment is much less than the utility of a dollar during a recession, trading consumption today for more useful consumption tomorrow. This kind of reasoning happened for the last 5 recessions, where the graph inverted in August 1978, September 1980, December 1988, June 1998, January 2006 (and now in August 2019).

The concept of risk aversion, together with overreaction to market expectations, will put pressure on the market. For supply-demand basic theory, as soon as the investors' fear rise, as more people expect an economic downturn, they will start investing in long term bonds, rising the price but lowering the yield. The lack of interest for short maturities, in contrast, will lead to more and more rise in short term yields, to try and attract new capital. In general long government bonds will be more costly but the run to safety will be a clear trend before the (possible) apocalypse, with a higher rate of substitution between consumption over time. A flatter or inverse curve is the clear expression of this process.

However we should note that it is not possible to predict the intensity of the recession, the date it will begin or end only by the slope of the yield curve. When the recession starts, probably the yield curve has already become regular with long-term higher than short-ones. In this time gap, between the inversion and the recession, usually the stock market continues to grow and it even peaks right before the following fall.

For the stock market and the economy more in general this is still a clear indication of distress in the financial situation: in August, in fact, there were many fears, from US-Chinese poor relationship on trade issues, protests in Hong Kong, fear of a new economic crisis and similar huge concerns. Like cold precedes a bigger illness, an inverted yield curve becomes a symptom of a problem in the real world (or less likely

local) economy: that is why yield curve inversions suscitate attention or even "panic" of the financial world. As soon as the stock market begins to shake, bonds become even more attractive to investors, rising the price of bonds and reducing yield even more. After sometime, however, longer maturities will be again compensated with higher return, hence this event is not a long-lasting one, but a minor and (still) a significant occurrence that easily can foresee a turning point in the business cycle.

4.3 Humped Yield Curve

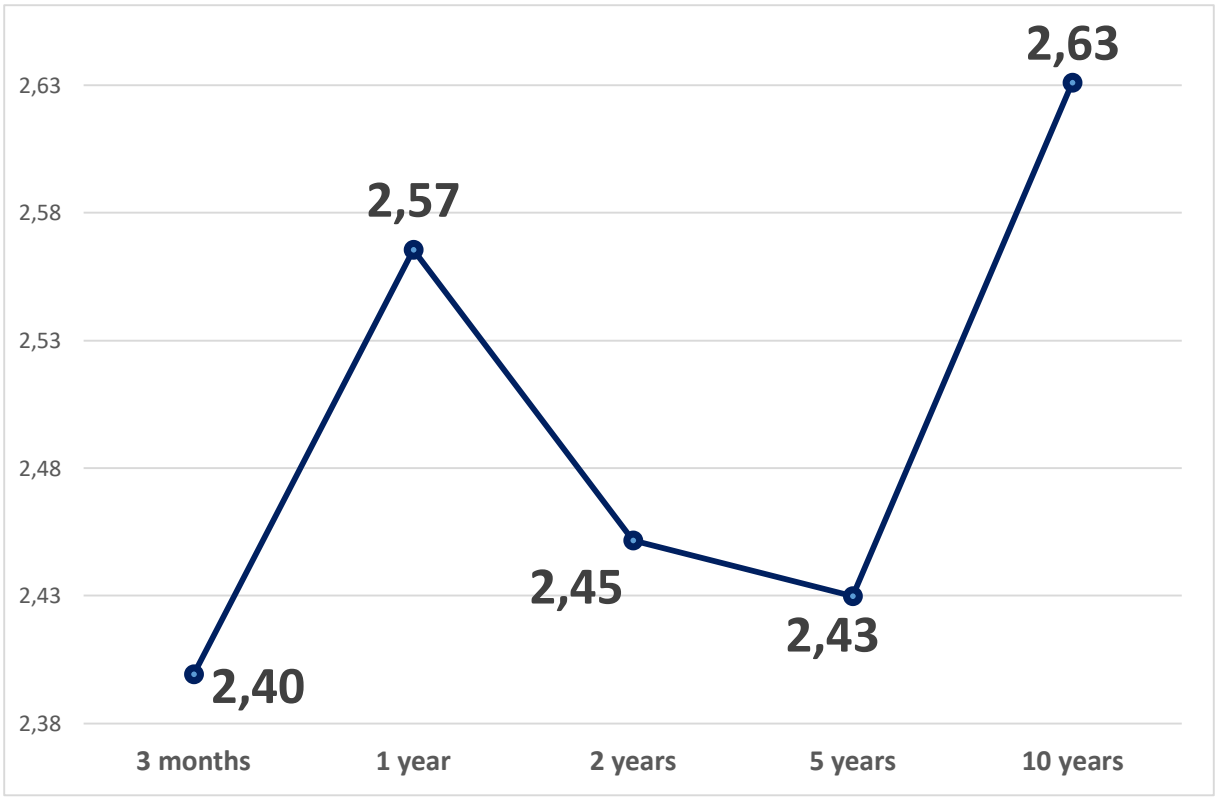

Exhibit 22. My analysis of "What does a Humped Yield Curve Mean for Future Stock Market Returns" data, referring to the 11/2/2019.[89]

In the figure above we can see a Humped Yield Curve (also called Bell-Shaped). Although not a beautiful sight, it happens when short and long yields are lower than in the middle part of the trajectory. In this event, at a short maturity the arc is upward,

[89] Bryce Coward, *What Does a Humped Yield Curve Mean for Future Stock Market Returns*, Advisor Pespectives (Feb. 11, 2019).

similar to the regular one (but maybe too much increasing), and in the last part the slope is downward looking.

It signifies that the investor will not receive higher return to compensate for the longer maturity, hence they will move to more competitives mid-term portfolios. This will cause an increase in the yield of the long-maturity bond, rebalancing the normal shape over time, as done for the Inverse Yield Curve.

Some observers think that it is an uncommon occurrence, in reality it happens, at least in part, more than we could expect. It is considered an indication of uncertainty or variability in the economy. An inverse yield curve could be predicted by bell-shaped one as the hump may be the first consequence of a slowing-down economy.

This creates an opportunity to apply the negative butterfly strategy as we will see below.

As seen in the Exhibit above, in February 2019 the yield curve became humped. And as we have seen in the previous paragraph, it has evolved in an Inverse Curve. It appears that the hump could be the first suggestion to "take cover" against a possible recession.

From 1986 onwards, each time we saw a hump, the stock market has always lost momentum, but as for the Inverse Curve, it was not possible to predict the timespan.

For example it humped in May 1986, 15 months before the peak in the market. In July 1988 it humped again but the market reached his climax 2 whole years later.

The yield arc showed the belly in October 1997, but the Internet Bubble exploded only 2 and a half years later. In 2008 the pattern was reversed with a burst of the stock market of only around 5 months. Some economists predicted, correctly, that the belly in 2019 could lead to a full inversion of the yield curve forcing an "obvious" conclusion: "the clock marking the end of the cycle starts ticking.

"Of course, unless this time is different".[90]

4.4 The Butterfly

In general, in Finance, a butterfly spread is a trading strategy regarding stocks, using options to limit risk and to let you hopefully earn a profit from a change of volatility in the future. Although the profit is constrained and limited, it allows to earn a return both

[90] Bryce Coward, *What Does a Humped Yield Curve Mean for Future Stock Market Returns* (Feb. 11, 2019).

for expectations of high volatility and for a decrease in the same indicator. Depending on how we construct it, when volatility is in a downturn trend, we earn a profit with a long butterfly, while the short one is the solution for the opposite case. But this strategy is based on call or put options of the stock market, and we are not interested in this argument for now.

For the bond market we have similar reasoning. The secondary market allows to trade and earn a return in many (possibly infinite) ways. We already analyzed the case of a zero-investment portfolio, financed with governative bonds, used to buy other bonds, limiting the focus on two bonds per time. Now we will focus on three different maturities: a short, a long but also a middle ground maturity. Here, instead of focusing on the yield, we focus much more our attention on the shape of the trajectory of yields across maturities.

We still want to relate our findings of an Inverse Yield Curve to our initial fixed long-government bond strategy. It is important to understand that with such an arc in the market, we could gain only limited amounts of money, and probably earn also some losses. A way to avoid this problem is substitute in those moments another possible strategy, the long butterfly.

A long (or positive) bond butterfly is a trade consisting in a long position in both long and short bonds (government ones in our case) called long barbell, going selling some medium term bonds (short bullet). This is the case when after a shift in the bond market, the two yields at the extreme of the curve are much higher than the middle ground, creating a shape similar to a “U”. This trade dynamic is perfect to react to an Inverse Yield, helping investors to profit from lateral positive switches of the yield curve, decreasing the regular bend of yields. It is profitable when the trajectory changes upward in one direction in particular, or towards 0 in the center (always in comparative terms to other maturities). This is a good way to profit during a deterioration of the economy.

The term butterfly is a consequence of the connotation of the middle bond as a part of the body, while the short and long maturities are the two wings.

A negative butterfly happens, after a variation in the yields, in this peculiar case when the intermediate rate is greater than what we could expect in a regular scenario, creating the hump between the extreme maturities. In contrast to the positive counterpart, the

negative butterfly could for example profit when a recession finally comes and the pessimism shifts from the medium governative bond to other yields.

In both cases we could profit from trading with still another zero-investment on our part. I will focus my attention to the Butterfly based on the 2-year, 5-year and 30-year US government bonds. This strategy is based on selling the current 5-year bond two times and buying the other two one time (or the inverse strategy is also viable). Clearly the first position expect an Inverse Yield curve, while the latter a Humped one.

My reasoning is that this is a pretty large difference between maturities that will amplify both successes and failures of the strategy, but given the fact the fixed strategy is really reliable, if we can find the good signal to implement, instead of the regular long-short scenario, we can obtain much greater results. A pretty common butterfly is for example the 2, 5 and 10 but it is less volatile, and this is not what we are aiming for here. Hopefully if the situation lasts for some months we can obtain a net excess return.

The size between the two extreme maturities will be usually sized to maintain a cash and duration neutral position, both in the positive and negative cases. But as a basic approach that could be also used as a signal in itself I will take a 50/50 stand, as seems plausible studying the Bloomberg data. It will make also easier to study the two different butterflies in the same instant of time.

For instance my idea is that if for one month the positive butterfly earns more than the fixed strategy, it is possible that the inverse curve will last for at least one other month. In the opposite case, if the negative butterfly is becoming interesting, I can shift to that position in the market.

To avoid that a sudden shift between a positive butterfly to a negative one, I will pose the limit that the signal could not pass between positive to negative over a 1 month period (and the viceversa will be hopefully true). Sometimes it can happen that high variability will provoke a significant shift between opposite signals over a short period of time. This is quite common in the stock market, especially when taken as a raw data, without any smoothing average, but could happen to all segments of our financial market, considering the concept of overreaction and the following adjustments over time.

4.5 A Useful Proxy

Let us apply the positive butterfly, the negative one, and both of them together and compare our results with the fixed strategy.

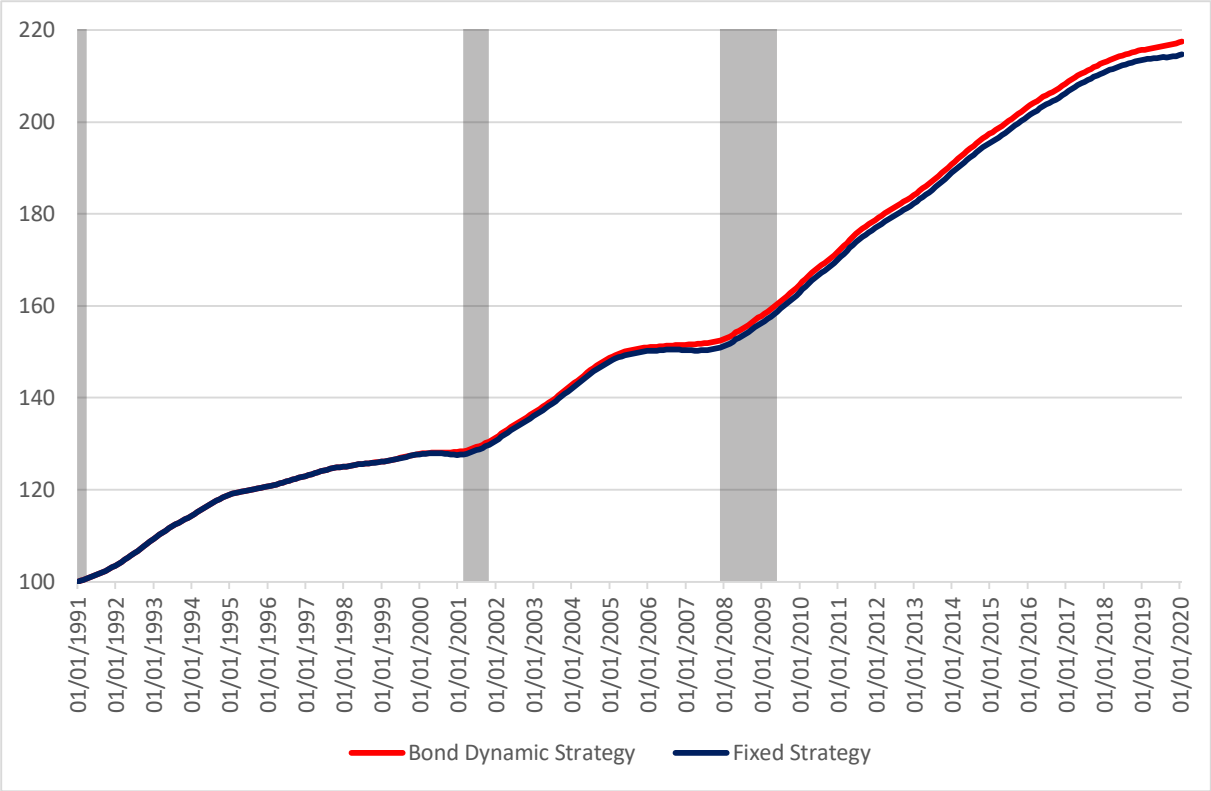

Exhibit 23. Comparison between Fixed Strategy and Dynamic strategy based on both positive and negative butterflies combined.

While the Fixed Strategy reaches a portfolio value of 214,72, the applying of the positive butterfly alone helped the portfolio reach 215,34, the negative without the counterpart reached 216,96 and the two combined reached 217,48. It is clear, the difference is really small in the total significance for our wallet.

However we can appreciate from the graph that the butterfly enters in action right before a recession, as we could expect, helping us understand the change of sign and covering our losses, even if small, that a changing pattern can cause to our portfolio.

To be noted that in the fixed strategy the "bad" trades were 15 (15 months on the total).

With the butterflies we can still seriously decrease this number. The negative, although slightly better from an excess return prospective, decreased this number from 15 to 14 months: nothing special. The positive, although not so profitable overall, is a serious opportunity to cover Inverse Yield risk, obtaining a good proportion on only 3 months of losses. The combined strategy was the best one, with only two months of negative performance, June 2000 and July 2007, where it fails to anticipate a hump/inverse yield change and the end of the Inverse yield respectively.

So while the fixed strategy was a bad investment in the 4,16% of the time period, the combined strategy reduce it to 0,55% obtaining a positive earning in the 99,45% of the interval taken into consideration. A maybe slow but astonishing performance overall.

Now let us compare the Fixed strategy (restricted on the time period of 1991 to 2/2020) with the Bond Dynamic strategy in the same time period in the two tables below: to clear the situation, volatility is calculated using the standard deviation.

Static Strategy	***1991-1994***	***1995-1999***	***2000-2004***	***2005-2009***	***2010-2014***	***2015-2020***
Av. Excess Return	**4,72%**	**1,49%**	**3,16%**	**2,03%**	**3,99%**	**1,94%**
Volatility	6,06	2,23	5,29	2,70	5,76	3,10
Sharpe Ratio	0,78	0,67	0,60	0,75	0,69	0,63

Bond Dynamic	***1991-1994***	***1995-1999***	***2000-2004***	***2005-2009***	***2010-2014***	***2015-2020***
Av. Excess Return	**4,72%**	**1,50%**	**3,27%**	**2,14%**	**3,99%**	**2,00%**
Volatility	6,06	2,24	5,40	2,85	5,76	3,14
Sharpe Ratio	0,78	0,67	0,61	0,75	0,69	0,64

Exhibit 24. Comparison between Static Bond Strategy and Dynamic Strategy using Bonds, 1991-2020

It is pretty clear that the strategies are overall similar. They start in the first years as the same because the two fixed butterflies do not “activate” as the yield curve is a regular one during the first period. Then in the second period the Average Excess Return is slightly higher for the Dynamic Strategy, but also the volatility, so the Sharpe Ratio is the same. This pattern repeats in the third period and in the fourth one while the Sharpe Ratio is lower in the third period, because the higher variability is even more weighted

than excess return improvement. 2010-2014 the two strategies follow the same course, as expressed by the perfectly matching 3 values for each period. In the last section the the Dynamic Strategy is a little bit more interesting in comparison to the static strategy, especially for the higher excess return,

Overall the Average Excess Return is always higher for the Dynamic strategy, even if the Sharpe Ratio is always the same or only a little lower for the Static. Useful, in my opinion, but not the final investment strategy I will choose in my personal life. Moreover, Sharpe ratio shows a pretty stable pattern over time: this could be a good signal about the evolution of the real bond market and the applicability of this strategy alone in the next 30 years.

A good conservative strategy that earns some gains without losses and is consistent over-time, will always be a safe-asset to keep in mind, especially during financial crisis or for very risk-averse investors.

Section 5: Expanding the Strategy

5.1 Going to the Stock Market

But now let us imagine that we are not interested in this good but limited excess return: we want more. It is time to go back to the stock market.

I will again use a zero-investment strategy, putting myself in the shoes of someone who does not invest his own money at any time, but only scales the money invested by the excess return he gained over time. Although interesting and probably less risky, this will always drag a little bit down our results.

So we want to fund 100$ dollars at the risk free rate, using short government bonds, investing them in the stock exchange, in a fixed position, without thoughts or monthly rebalances of any kind, apart from the money we need to borrow.

We will start in 1991 and track our monthly performance as done before. Below we can see the graph:

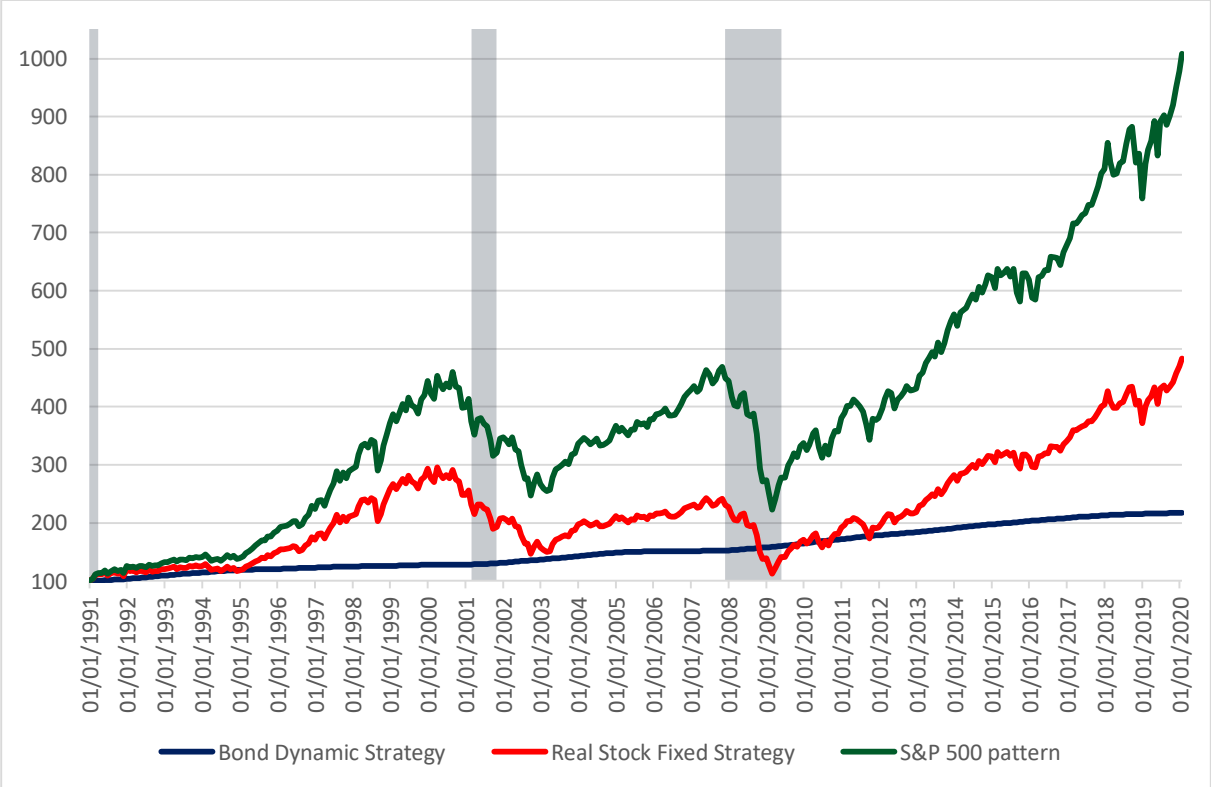

Exhibit 25. Comparison between Bond dynamic strategy, Stock exchange pattern and our Fixed S&P strategy.

We can admire that of course the stock exchange is reaching the stars, with a max final value of over 1000 at the end of the period. From 100 dollars invested we could gain in the time span over 900, more than 900% gain from what we invested. But here it was possible only without a zero-investment strategy, needing no money to be borrowed to function. Borrowing the total amount of money of our portfolio over time will surely decrease significantly this value.

Let us dive a little bit more in the relationship between our fixed stock strategy based on shortening 1-month or 3-month US Treasury Bills and our "old" dynamic bond strategy.

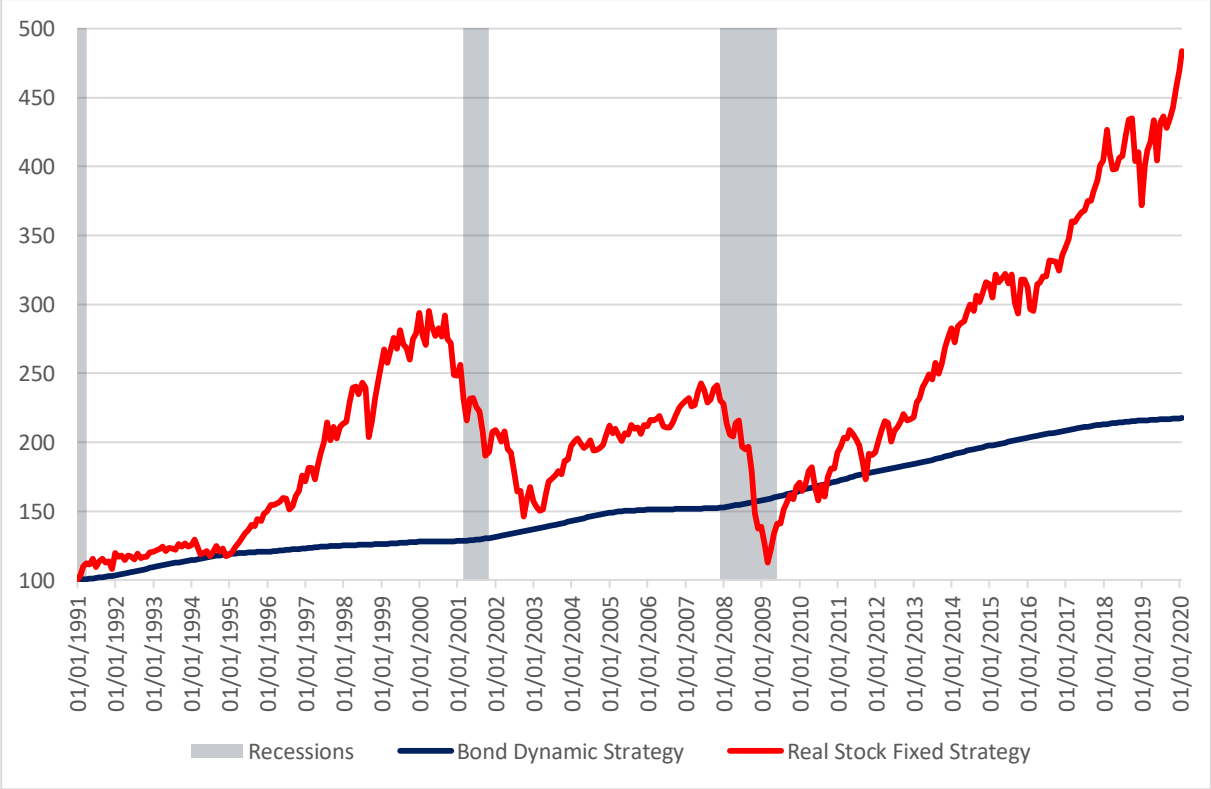

Exhibit 26. Easier comparison between the Bond Dynamic Strategy and the Real Stock Fixed one.

From the Exhibit above is not so clear to call for an easy winner. In the long-run, of course, as we could expect from all we said above, the Dynamic Bond appears to be surclassed by the stock market, funded with our equity or with borrowed money.

In fact, while our loved Bond Dynamic Strategy reaches 217,5$, with a net gain of 117,5 dollars from our zero-investment position, for the counterpart this value reaches in the end 486,66$, a net gain 3,29 times bigger than the precedent, without doing any real active trading in the market. However it is possible to see that the Bond strategy reacted way better during financial crisis, as said before.

In particular the dynamic strategy obtained a net advantage on the stock market during the last two months of 1994, then after the dot-com bubble burst, in 2003 it nearly reached his counterpart in money value. But the moment where the Dynamic Strategy shined was surely during the period covering the Financial Crisis of 2007, the Eurozone Crisis and the following months. In particular, from October 2008 to October 2009, one whole year and one month, the portfolio value of the dynamic strategy was higher. It also surpassed stocks in January 2010, between May and August of the same year, and at last in September 2009. After this date the gap between the two increased, all in favour of the S&P 500 fixed strategy, but as said, it is possible that we will see a new big financial crisis in the next months, so the Bond Dynamic could still recover, at least somewhat, with a risk exposure always much less than the counterpart.

In fact, in raw data, if the Bond Dynamic lost momentum only in two months, the total number where the stock acted poorly is 122 whole months, the 34,86% of the total (while for the bond, I remember again, is 0,56%).

To be noted, from our studies we understand how the stock market is predicted to crash before the end of 2021 (maybe sooner). This could be the best overall performance of the stock market, hence our results could already show a positive bias towards this strategy, in comparison to the same reasoning made a couple of years from now.

However, let’s get back to our 1991-2020 table for a simple analysis.

Bond Dynamic	*1991-1994*	*1995-1999*	*2000-2004*	*2005-2009*	*2010-2014*	*2015-2020*
Av. Excess Return	**4,72%**	**1,50%**	**3,27%**	**2,14%**	**3,99%**	**2,00%**
Volatility	6,06	2,24	5,40	2,85	5,76	3,14
Sharpe Ratio	0,78	0,67	0,61	0,75	0,69	0,64

Fixed Stock	*1991-1994*	*1995-1999*	*2000-2004*	*2005-2009*	*2010-2014*	*2015-2020*
Av. Excess Return	**4,54%**	**29,72%**	**-5,57%**	**-3,88%**	**16,83%**	**10,56%**
Volatility	5,89	43,34	14,10	16,13	26,88	16,32
Sharpe Ratio	0,77	0,69	-0,39	-0,24	0,63	0,65

Exhibit 27. Comparison between Static Stock Strategy and Dynamic Bond Strategy, 1991-2020

Comparing the two strategies, the "end" of the Dynamic Bond reasoning with the fixed stock, we could see that the most striking period is maybe the first one, when both markets and excess returns move in a comparable way, and strangely the volatility of Bond Portfolio is even a little higher than for the Stock S&P 500 investment.

However the higher Average Excess Return compensates for the higher variability with the very interesting 0,78 Sharpe Ratio. In the following period, before the dot-com bubble, of course the performance of the stock market is really astonishing, and in the third one the burst strikes the stock market.

For the fouth period, the first good half is compensated by a large margin by the second financial crisis. So the way it is structured the period, does not "allow" us to see the intermediate recovery, at least on a simple table, without other data or graphics. Of course, after this one, the 2010-2014 has a much higher Average Excess Return for the Stock Fixed Strategy even if the Sharpe Ratio is dragged down by a really high standard deviation. About the last years, the Average Excess Return somewhat decreases and the Sharpe Ratio (0,65) is only slightly above the Bond Dynamic Strategy. Two different options, for two totally different types of investors.

5.2 Combining the strategies

Now we want to try and combine the two strategies, to hopefully obtain a much more interesting result.

A possible signal for our decision to invest could be the recession. If we analyze the graph we could easily understand how a big part of our losses (using the stock market) occurs during the two big recessions of the 2000s. But from a perspective focus based on both these two, in particular the latter, and the first months of 1991, when we started trading, it seems certain that a recover of the market before the end of the recession is a strong possibility, in the same way like the healing process of the bond market is completed even before the manifestation of the downturn of the real economy.

So let us pose a new trading rule: if in the trading period before, we were in a recession and the value of the S&P 500 the previous month decreased from the one before it, we will rely on a "1/0" strategy, without any position on the market till the situation on the financial stage becomes much more clear.

Our results show that the position on the market will be of 812,21 dollars, a net gain of 712,21$. But let us now analyze the same situation, with only a small difference: when our signal is negative, instead of abstaining from any trade, we will transfer our focus on the bond markets, in the same way we have seen before. The results are pretty interesting. With a still limited number of transactions (our 0 position was maintained for a total of 18 months) we reach 848,65$, an increase in the net gain of 5,05%.

It is time to plot the new serie.

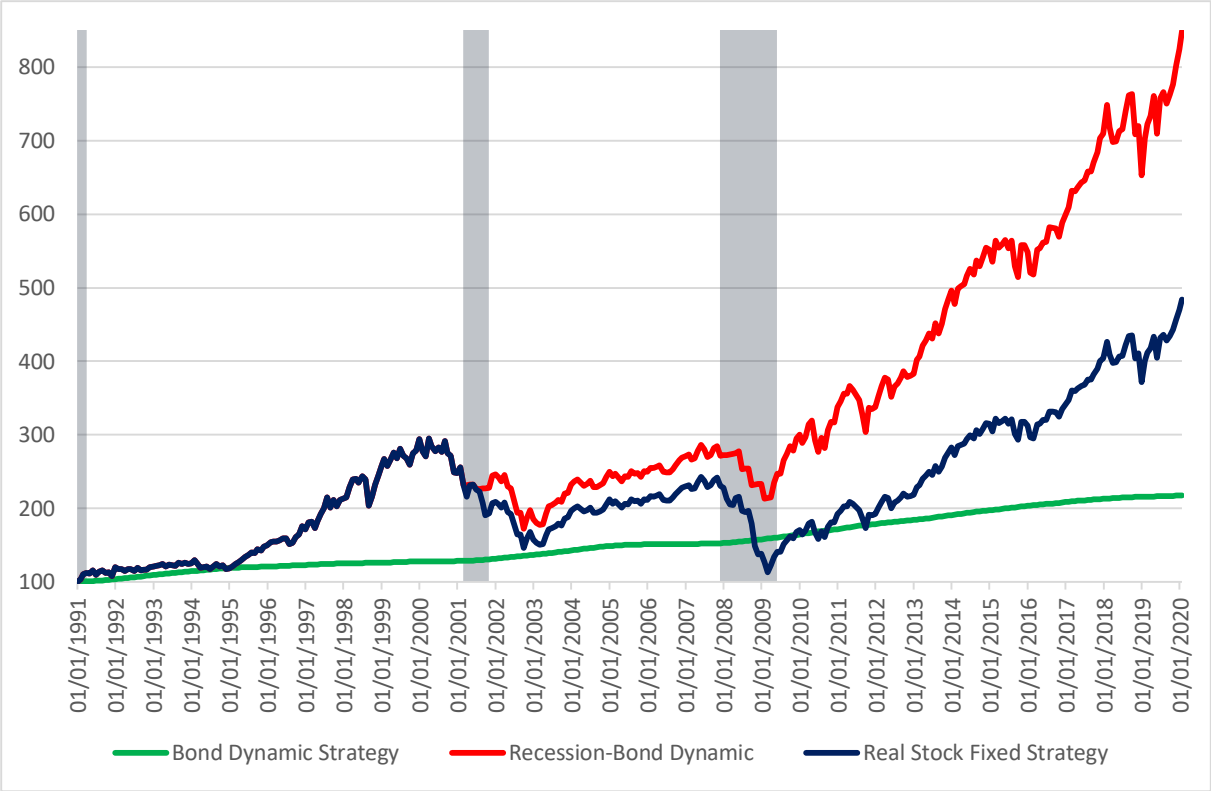

Exhibit 28. Comparison between the Bond Dynamic Strategy, the Fixed Stock one and the Dynamic combination of the two, 1991-2020.

For simplicity in a relatively small space like this paper I will plot only the final Dynamic Strategy, but the "1/0" stock is slightly below the Dynamic Strategy. It is simple to see that especially in 2002 and 2009 the advantage of the combination between the two strategies is significant.

The majority of losses that were not recognized by this signal are concentrated after the 2000 peak of the market, that anticipated by many months the real recession. Other losses happened during the high instability periods right before and especially after the 2008 crisis. However, usually the pattern on a two months period will recover many of these losses, so it is against our own interest to take a different position in those occurrences.

Overall, the new bond-stock combination allows us to decrease losses, still capturing the good trends in the market, with a rough comparison between the two more "risky" possibilities with a nearly double excess overall return in the more structured one, and 6,4 times the bond dynamic strategy.

Even in this case, we could study the tables for "1/0" strategy and Recession-Bond Dynamic one:

Fixed Stock	*1991-1994*	*1995-1999*	*2000-2004*	*2005-2009*	*2010-2014*	*2015-2020*
Av. Excess Return	**4,54%**	**29,72%**	**-5,57%**	**-3,88%**	**16,83%**	**10,56%**
Volatility	5,89	43,34	14,10	16,13	26,88	16,32
Sharpe Ratio	0,77	0,69	-0,39	-0,24	0,63	0,65

1/0 Strategy	*1991-1994*	*1995-1999*	*2000-2004*	*2005-2009*	*2010-2014*	*2015-2020*
Av. Excess Return	**4,54%**	**29,72%**	**-3,17%**	**3,22%**	**16,83%**	**10,56%**
Volatility	5,89	43,34	10,63	7,81	26,88	16,32
Sharpe Ratio	0,77	0,69	-0,30	0,41	0,63	0,65

Recession-Bond	*1991-1994*	*1995-1999*	*2000-2004*	*2005-2009*	*2010-2014*	*2015-2020*
Av. Excess Return	**4,54%**	**29,72%**	**-2,97%**	**3,97%**	**16,83%**	**10,56%**
Volatility	5,89	43,34	10,39	7,67	26,88	16,32
Sharpe Ratio	0,77	0,69	-0,29	0,52	0,63	0,65

Exhibit 29. Comparison between a Fixed stock strategy and our dynamic strategies, 1/0 and Recession-Bond (using the Bond market during our signal based on a recession in the market).

Comparing the tables with the "old" simple Fixed Stock stance, as expected the two starting periods are exactly the same. In the 2000-2004 section, both strategies improve on the fixed, with the Recession-Bond trumping the 1/0 both for lower variability and lower losses, even if the result is still negative overall. The same is true in the following period: while the Fixed is again negative and pretty volatile, the Recession-Bond is the best of the three in all aspects, with a Sharpe Ratio of 0,52 in comparison to 0,41 of the 1/0. In the following two periods, the results for Average Excess Return is the same across the three, with the same exact volatility. Now the real applied strategy is the regular Fixed Stock, with the second and third best returns respectively, and the same is true for volatility. The only negative Sharpe Ratio is obtained between 2000 and 2004, and all other results about reward-to-variability are on the same range as the results for bonds. Only the results between 2005 and 2009 are positive but a little disappointing.

For a better understanding of the comparison between our strategy and the raw value of the stock market, and also to comprehend even more the opportunities, but also the costs on the long-run of a zero-investment portfolio, let us compare again the results, between the S&P 500, the strategy we decide to apply, and the results offered by the same strategy made by own equity.

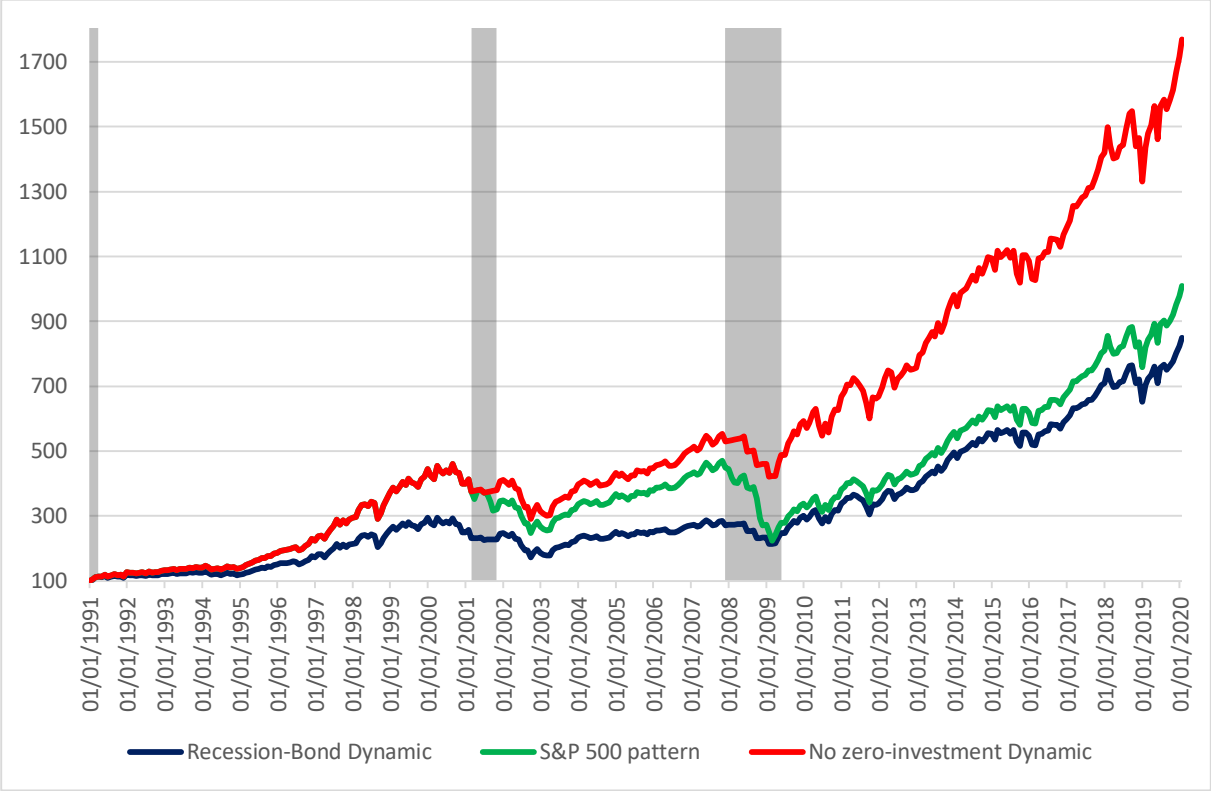

Exhibit 30. Comparison between our zero-investment Dynamic Strategy, the same strategy with own capital, and S&P 500.

From the figure is clear: investing 100$ taken from our pockets on the 1st January 1991 we would have obtained 1768 dollars and 9 cents. While the stock market result is clearly interesting, it reaches only 1008 dollars and 30 cents at the end of the period. Our own dynamic strategy recovers quite a lot during the period of crisis, especially from the difficult situation it was in 2000 in comparison to the stock market. The difference, in fact, drops from 184,2 dollars in October 2007 to only 8,6 dollars in February 2009. However, on the long-run, the cost of money, even if it is at risk-free rate, creates again a small gap, that builds up month after month, with our strategy

closing at 848,65 dollars (159 less than the market). It is clear: every cent counts on a marathon and our reasoning is starting to work in a very competitive way for all the possible preferences of investors, with a strategy that could be based on bonds, stocks or a combination of the two, adjusted at each period, following the predicted pattern of the markets.

5.3 The Final Strategy

But this is not enough. Let us impose two more rules, the first one about months of consecutive negative performance and the second about introducing even more bond market in our strategy.

About the first one we can notice how, in general, negative performances are possible in every situation without much warning, but if the negative pattern becomes clear over a period of at least two whole months, it could be considered safer to switch to the bond-dynamic strategy for a while, until the market stops his instability, preferring a constant (or pretty much so) positive stream of income, even if it will be slow, to avoid possible losses over longer periods of time. In this way we can clearly reduce risk in the timeframe, and hopefully, at least in some circumstances, even increase total return from the portfolio.

If the first one was for periods without warning, we know that both humped and inverse yield curves are recurrent factors before any crisis. Thus, if we see in succession a possible gain both from a negative and positive butterfly, we know that it is possible a financial crisis in a few months time. To be noted that it is not possible to predict the precise moment when it happens, but from empirical studies, we can clearly see that it will be after a two months period (at least). In this occurrence we will again switch to our classical Bond-Dynamic strategy, increasing the effect of butterfly proxies on our model.

Hence we will have a triple protection: in case of a recession we could use the stock market performance as an indicator; in case of no-warning we will still see if perseverance on the market increases only the losses we receive going on with our starting plan ; at last, plan C, the more structured one, the proxy of both butterflies, joint

together to stop trading on the stock market, to gain momentum from the governative bonds, anticipating the same pattern other investors will surely follow sooner or later.

From data obtained from Forbes we can see that the starting point of the yield curve inversion will anticipate the stock market crisis between a minimum of 10 months (September 1980) to a full 33 in June 1998. But we must remember that the starting point of the recession will be proxied by the possible profitability of the butterfly, hence some months will probably pass between the first sight of a future inversion of the yields and a real application of the bond strategy in substitution to our stock positioning, thus letting us to come much near to the peak of the market, without risking too much from a possible anticipation of the stock market crash, when we will probably have already abandoned the more risky approach to the investment portfolio.

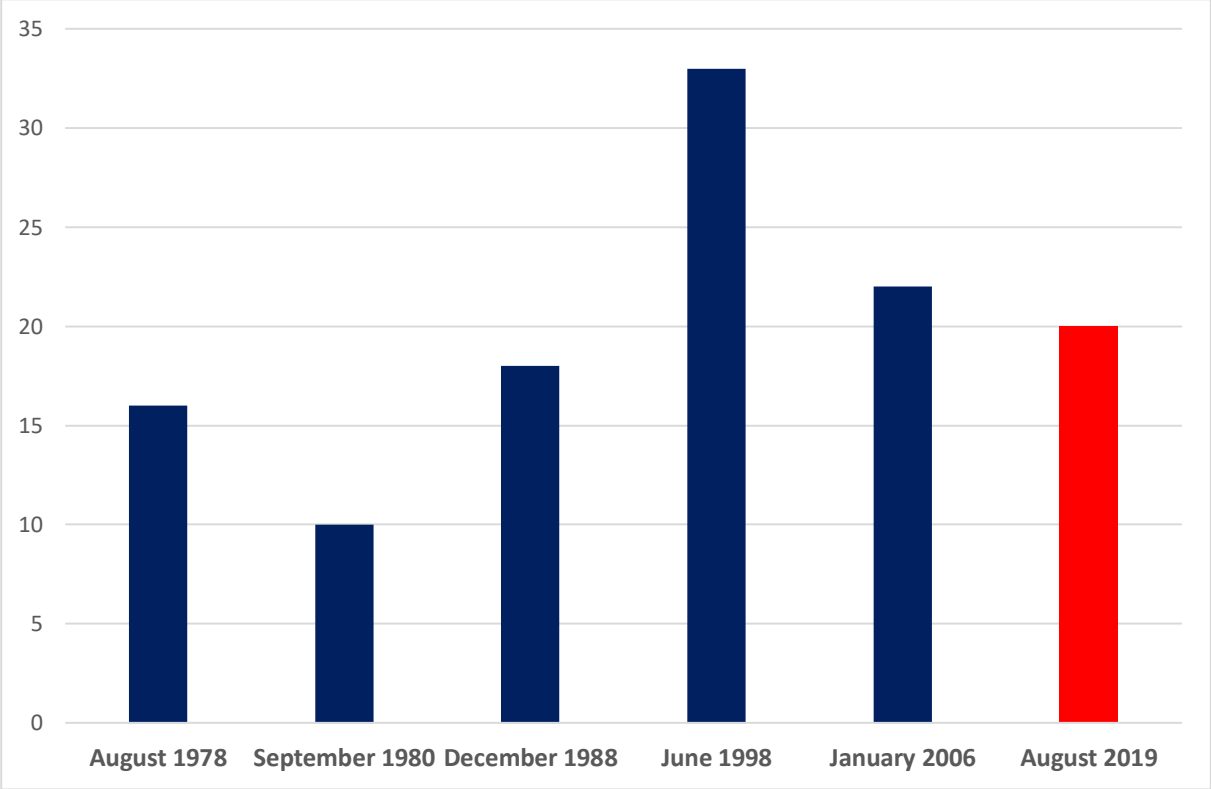

Exhibit 31. Months between the start of the Inversion in Yield Curve and the moment where the Recession finally kicks in.

The value for August 2019 is a Forbes prediction based on the average of the past experiences (April 2021 will therefore be the date for the next recession).[91]

[91] Data taken from Trefis Team, *Inverted Yield Curve: Is Recession Coming In Next 20 Months?*, Forbes (Oct. 2, 2019).

In this way we will reach a total number of 110 (9,17 years) trading months using the bond-dynamic strategy we considered as too "conservative" alone. On 349 total active trading months it is a 31,52% of the total. Not a low amount, especially if we consider that the only stock strategy had around 33% of negative performance months. In this way the negative returns obtainend from our positioning in the market are reduced to 72 total months (only 20% of the total). Of course trading in a market, in particular a stock one, will be always risky. But from this combination we will surely both reduce negative volatility and increase the total return, our two primary objectives.

In the end our zero-investment strategy will reach 1395,04 dollars, a net gain of 1295% on a money we never really owned but only borrowed in the studied time-period.

For an even more satisfying result, let us quickly go back to an owned-equity portfolio scenario. The "Final" result will be an astonishing 2901,072 dollars, a net gain of 2801$. Not bad on 100$ invested for a little more than 29 years.

Now let us see graphically the results.

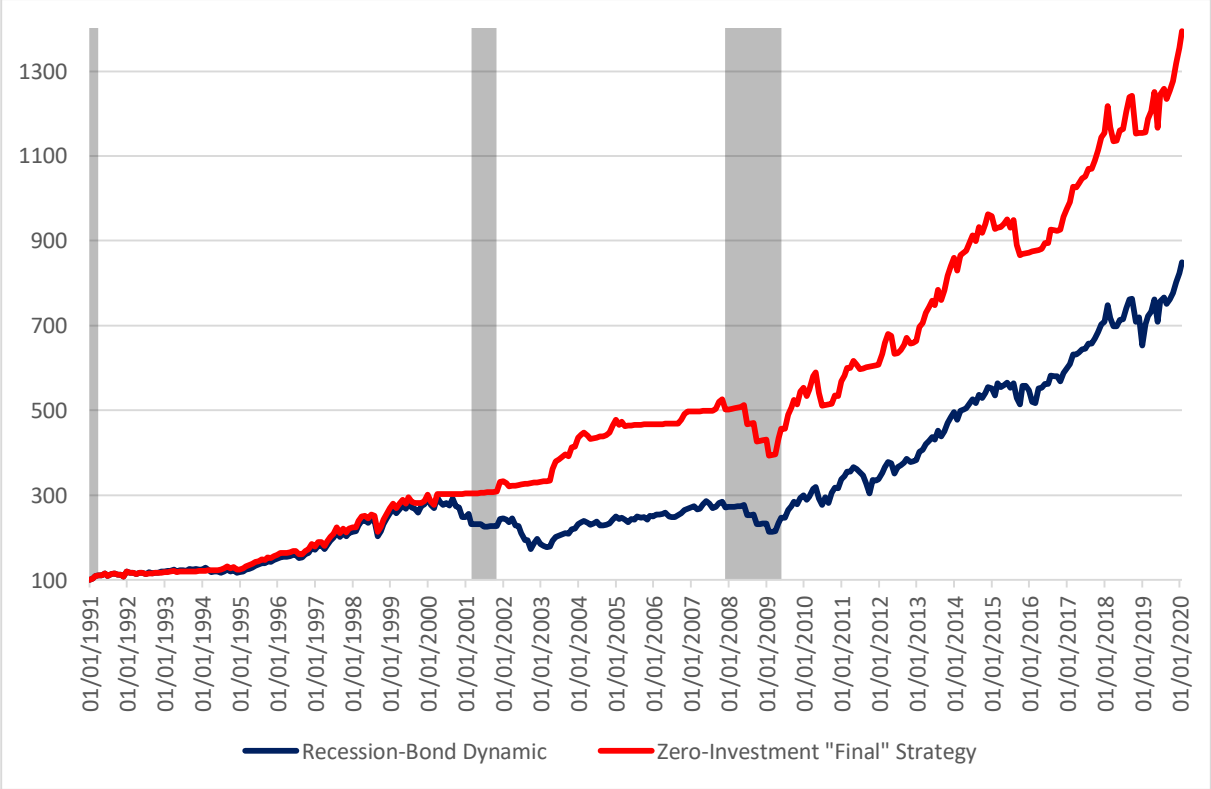

Exhibit 32. Comparison from the Dynamic combination of Stocks-Bonds using a Recession-based signal and the Final more conservative (and more successful strategy).

The two strategies mirror themselves until the year 2000. In 1994 the Recession-Bond Dynamic rises a little above the "Final", but, in general, the "Final" option stays consistently above the peer. In 2000, until 2003, the Final creates a first decisive gap with the other, a discrepancy that remains more or less as a constant. During the recession of 2008 and 2009 the gap starts decreasing, because a stronger position, overall, losing a similar percentage of money, decreases the total difference. After that, the gap between the two strategies increases again over-time, without any real setback of the Final portfolio,

From the following table analysis we can try and say something more:

Recession-Bond	***1991-1994***	***1995-1999***	***2000-2004***	***2005-2009***	***2010-2014***	***2015-2020***
Av. Excess Return	**4,54%**	**29,72%**	**-2,97%**	**3,97%**	**16,83%**	**10,56%**
Volatility	5,89	43,34	10,39	7,67	26,88	16,32
Sharpe Ratio	0,77	0,69	-0,29	0,52	0,63	0,65

Final Strategy	***1991-1994***	***1995-1999***	***2000-2004***	***2005-2009***	***2010-2014***	***2015-2020***
Av. Excess Return	**6,35%**	**28,07%**	**11,75%**	**3,15%**	**14,61%**	**8,96%**
Volatility	6,25	42,03	18,95	6,84	24,23	15,38
Sharpe Ratio	1,02	0,67	0,62	0,46	0,60	0,58

Exhibit 33. Comparison between the previous Recession-Bond dynamic strategy and the Final one.

The reward perceived by each unit of volatility is clearly higher for the first period, with a Sharpe ratio above 1. In the second period the return is slightly higher for the Recession-Bond Dynamic Strategy: the Final, more conservative, stops right before the peak in the market, focusing on a more safe approach, but the difference is limited in Average Excess Return, and is in part compensated for the Sharpe Ratio by the reduced volatility.

In the third period, for the first time we obtain a positive return, since we started trading with stocks, with a yierly gain of 11,75% and a good Sharpe of 0,62. Than the portfolio is a little more risk-averse in results than his counterpart, betting less on the stock market, but the value has increased so much that the final profit will be much more high

in comparison to the other aggressive strategy, that is a little more volatile, but also more remunerative on the last periods. Overall the 3rd period is the deciding factor that let the Final Strategy prevail over the previous dynamic.

Now it is clear: while the excess return is a valid instrument over the whole period of time, volatility seems to do not fully compensate a lower overall risk in a more conservative strategy, thus a ratio as the Sharpe could tend to overrate a full stock strategy in a good economic situation, leading an investor to think that a strategy like this will be always preferable. This is clearly false, because, when the market "finally" falls, the gap will become so high that nothing could recover the lost ground. This is true at least for a middle ground approach between risk and reward, for risk aversions at the extremes of the spectrum, in my opinion, we still need to consider both the first Bond Dynamic, for high risk-aversion, and Recession-Bond, for a low one. But, in a common economic sense, I think the Final should be considered superior.

5.4 The Final Comparison

Now I want to compare all the six strategies we used and see in a more compact way the results:

Static Strategy	*1991-1994*	*1995-1999*	*2000-2004*	*2005-2009*	*2010-2014*	*2015-2020*
Av. Excess Return	**4,72%**	**1,49%**	**3,16%**	**2,03%**	**3,99%**	**1,94%**
Volatility	6,06	2,23	5,29	2,70	5,76	3,10
Sharpe Ratio	0,78	0,67	0,60	0,75	0,69	0,63

Bond Dynamic	*1991-1994*	*1995-1999*	*2000-2004*	*2005-2009*	*2010-2014*	*2015-2020*
Av. Excess Return	**4,72%**	**1,50%**	**3,27%**	**2,14%**	**3,99%**	**2,00%**
Volatility	6,06	2,24	5,40	2,85	5,76	3,14
Sharpe Ratio	0,78	0,67	0,61	0,75	0,69	0,64

Fixed Stock	*1991-1994*	*1995-1999*	*2000-2004*	*2005-2009*	*2010-2014*	*2015-2020*
Av. Excess Return	**4,54%**	**29,72%**	**-5,57%**	**-3,88%**	**16,83%**	**10,56%**
Volatility	5,89	43,34	14,10	16,13	26,88	16,32
Sharpe Ratio	0,77	0,69	-0,39	-0,24	0,63	0,65

1/0 Strategy	*1991-1994*	*1995-1999*	*2000-2004*	*2005-2009*	*2010-2014*	*2015-2020*
Av. Excess Return	**4,54%**	**29,72%**	**-3,17%**	**3,22%**	**16,83%**	**10,56%**
Volatility	5,89	43,34	10,63	7,81	26,88	16,32
Sharpe Ratio	0,77	0,69	-0,30	0,41	0,63	0,65

Recession-Bond	*1991-1994*	*1995-1999*	*2000-2004*	*2005-2009*	*2010-2014*	*2015-2020*
Av. Excess Return	**4,54%**	**29,72%**	**-2,97%**	**3,97%**	**16,83%**	**10,56%**
Volatility	5,89	43,34	10,39	7,67	26,88	16,32
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Final Strategy	*1991-1994*	*1995-1999*	*2000-2004*	*2005-2009*	*2010-2014*	*2015-2020*
Av. Excess Return	**6,35%**	**28,07%**	**11,75%**	**3,15%**	**14,61%**	**8,96%**
Volatility	6,25	42,03	18,95	6,84	24,23	15,38
Sharpe Ratio	1,02	0,67	0,62	0,46	0,60	0,58

Exhibit 34. Comparison between all the strategies presented in this paper.

Now the picture is more understandable. For the first period the best performance is obtained by the Final strategy, with a true ability to combine the good of both stock and bond markets. This is true simultaneously for Sharpe Ratio and capital gain.
The second best in this period is the bond market, slightly above in Sharpe, but also higher in excess return: a better strategy overall compared to stocks in that moment. In the second one the three "fully" stocks obtain a 29,72% increase in value (yearly). Astonishing result, which is followed closely by the Final, like a "back marker" in a cycling peloton that is afraid of a possible future cliff. Bonds are too conservative in this exploding situation, but their Sharpe is not so lagging behind (and is the same with the "Final" one).
In the third period, Sharpe are closely related between Final, Bond Dynamic and Bond Static, in a decreasing order, but, if we talk about yearly gains, Final is totally above the competition. Nothing to say about stocks, better avoiding this period completely.
2005-2009 is a section much less clear overall, with a reducing difference between patterns, with the "complete" stock approach slightly prevailing on the "Final", that leads the "complex" bond and is followed closely by "simple" bond. "Simple" stock is again in the red side of the "candlestick".
After this uncertainty, the path forms again a wedge between stocks (a more remunerative portfolio) and bonds (a less-risky one), with the Final again in the middle, but nearer to stocks than bonds. Sharpe Ratio is again higher at the extremes of the spectrum, in particular for bonds (0,69). Given the fact that bonds gains less than a fourth of a stock strategy and 27% of the Final, Sharpe Ratio does not depict the real situation for a rational (and not too risk-averse investor). This is again the reality of the last period, with the less interesting result for Sharpe of the Final strategy, but between stocks and bonds, the ratio moves us in the direction of stocks.
In particular Sharpe Ratio seems to reduce consistently during the timeframe for the Final strategy. We start from the high Sharpe Ratio of 1,02, to the second best 0,67 of the second period, that shows us both the higher return and the higher standard deviation. Next is the third best Sharpe and third best excess return with 11,75% yearly. The worst period, both for Sharpe (0,46) and for Average Excess Return is 2005-2009, with the financial crisis started by the mortgage bubble burst. Then the second best Average

Excess return with 14,61% and the second higher variability too. At last, we see a more calm 8,96% of the last section, but with a Sharpe Ratio comparable to the other good periods of the strategy. Overall, it seems that the variability is strictly related to good Average Excess Return, not in opposition to it. Hence a risk-averse investor could be scared, but for an active trader, at least using this strategy, risk always means opportunities and positive results.

In my opinion a ratio, at least not this one, can not explain the real picture of the market. Data must always be interpreted and it is never a universal reality.

To summarize, it is clear that only bond related strategies (and the Final one) always obtained positive results. Final strategy is the best one in the fist and third periods, with the highest return and Sharpe Ratio, but gains a little less than Recession-Bond in the other periods, as a little more conservative approach. However, avoiding huge losses, increases overall return in the long-run. So, while the first two strategies are only bond driven, the following two only stock driven, the fifth mantains a stock pattern, merging it with a portion of the bond market, the last one starts to be a good balance between the two markets, not as risky as the stocks, not as conservative as bonds.

Static Strategy	*1991-1994*	*1995-1999*	*2000-2004*	*2005-2009*	*2010-2014*	*2015-2020*
Max (Month)	*9/1992*	*5/1997*	*7/2003*	*12/2009*	*3/2010*	*6/2015*
Max (%)	**0,50%**	**0,21%**	**0,41%**	**0,40%**	**0,40%**	**0,27%**
Min (Month)	*1/1991*	*8/1998*	*10/2000*	*11/2006*	*7/2012*	*8/2019*
Min (%)	**0,20%**	**0,04%**	**-0,06%**	**-0,06%**	**0,22%**	**-0,01%**

Bond Dynamic	*1991-1994*	*1995-1999*	*2000-2004*	*2005-2009*	*2010-2014*	*2015-2020*
Max (Month)	*9/1992*	*5/1997*	*7/2003*	*12/2009*	*3/2010*	*6/2015*
Max (%)	**0,50%**	**0,21%**	**0,41%**	**0,40%**	**0,40%**	**0,27%**
Min (Month)	*1/1991*	*8/1998*	*6/2000*	*7/2007*	*7/2012*	*12/2018*
Min (%)	**0,20%**	**0,04%**	**0,00%**	**-0,01%**	**0,22%**	**0,05%**

Fixed Stock	*1991-1994*	*1995-1999*	*2000-2004*	*2005-2009*	*2010-2014*	*2015-2020*
Max (Month)	*12/1991*	*10/1998*	*3/2000*	*4/2009*	*10/2011*	*10/2015*
Max (%)	**10,83%**	**7,67%**	**9,18%**	**9,39%**	**10,77%**	**8,30%**
Min (Month)	*6/1991*	*8/1998*	*9/2002*	*10/2008*	*5/2010*	*12/2018*
Min (%)	**-5,26%**	**-14,98%**	**-11,13%**	**-16,95%**	**-8,21%**	**-9,38%**

1/0 Strategy	*1991-1994*	*1995-1999*	*2000-2004*	*2005-2009*	*2010-2014*	*2015-2020*
Max (Month)	*12/1991*	*10/1998*	*3/2000*	*4/2009*	*10/2011*	*10/2015*
Max (%)	**10,83%**	**7,67%**	**9,18%**	**9,39%**	**10,77%**	**8,30%**
Min (Month)	*6/1991*	*8/1998*	*9/2002*	*9/2008*	*5/2010*	*12/2018*
Min (%)	**-5,26%**	**-14,98%**	**-11,13%**	**-9,10%**	**-8,21%**	**-9,38%**

Recession-Bond	*1991-1994*	*1995-1999*	*2000-2004*	*2005-2009*	*2010-2014*	*2015-2020*
Max (Month)	*12/1991*	*10/1998*	*3/2000*	*4/2009*	*10/2011*	*10/2015*
Max (%)	**10,83%**	**7,67%**	**9,18%**	**9,39%**	**10,77%**	**8,30%**
Min (Month)	*6/1991*	*8/1998*	*9/2002*	*9/2008*	*5/2010*	*12/2018*
Min (%)	**-5,26%**	**-14,98%**	**-11,13%**	**-9,10%**	**-8,21%**	**-9,38%**

Final Strategy	*1991-1994*	*1995-1999*	*2000-2004*	*2005-2009*	*2010-2014*	*2015-2020*
Max (Month)	*12/1991*	*10/1998*	*3/2000*	*4/2009*	*12/2010*	*6/2019*
Max (%)	**10,83%**	**7,67%**	**9,18%**	**9,39%**	**6,53%**	**6,72%**
Min (Month)	*6/1991*	*8/1998*	*1/2000*	*9/2008*	*5/2010*	*10/2018*
Min (%)	**-5,26%**	**-14,98%**	**-5,56%**	**-9,10%**	**-8,21%**	**-7,12%**

Exhibit 35. Best and worst trading month for each period considered.

Seeing the best and worst performance for each period, we can see how neither good nor bad results are in percentage consistently higher than the counterpart. At least for the Final strategy they are divided equally, 3 and 3. However, we can see how each best performance always follows the "disaster" of the position by some months, ranging from only 2 (2nd and 3rd periods) to a maximum of 8 (5th and 6th).

As a note, in June 2000 the result of Bond Dynamic is -0,00%, as the first of the two months with negative performance. Unfortunately I was not able to show it in the table above.

Going to non-zero investment strategy, in the graph we can admire the same change seen before: a small increase each month provocates a revolution in the situation at the end.

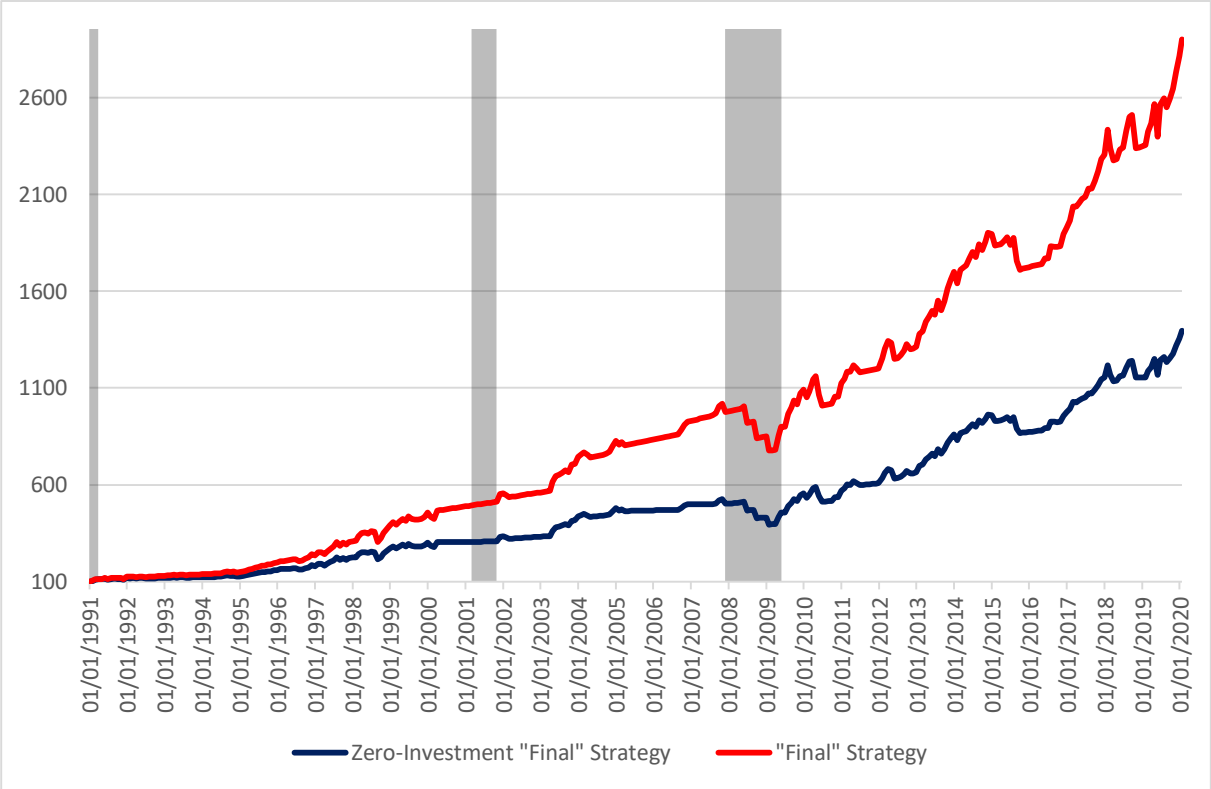

Exhibit 36. Difference between the Final strategy in the zero-investment scenario and the "regular" own money portfolio.

In short we could say that this strategy performs in a very good way with a long-positive trend, with a rising arc pattern over time, doing worse in periods of unpredicted uncertainty, but still encouraging overall.

Studying more thoroughly the Final strategy we can see how it starts slowly, with a period of partial instability, that culminates at the end of June 1992, with a total gain of around 14,88% from the starting month of January 1991 but with 8 months of poor performance overall. This could be one of the worst periods, but it still gains positive, even if not exceptionally high results.

Going on the results are pretty positive and constant and, as we can see, it starts going upward in total gain at a decent rate, with only a couple of bad months per year until 1999 where the anticipation of the dot-com bubble causes some financial instability, with a rising number of negative results (5 between the last half of 1999 and the start of 2000).

To be noted that the worst month of the total period we studied is, maybe surprisingly, August 1998, with a loss in value of 14,98% of the total (from 250 to 213 dollars).

Nonetheless the strategy starts the New Millennium with a total portfolio value of around 301,45 dollars, increasing two times his original value from the start of 1991 with the "more conservative" zero-investment strategy.

After this period the upward looking trajectory is constant and positive, until at least 2002, where a pattern of 1 or 2 months negative per year finally kicks in again. But at the start of 2005 the net excess total return is already around 378,59 increase of value (from the 100 dollars we started from). To be noted that the strategy does not appear to be affected by the dot-com bubble burst, resisting to it and increasing his value as usual.

In the following years, it all proceeds in a good way, until 2008/2009 when 3 months of losses ranging from 8,5% to 9% are a serious blow to the performance, during the subprime mortgage crisis.

However, ruling this period out is not that easy for the trading strategy. But, as soon as they happens, the strategy suddently shifts to the bond alternative investment strategy, gaining a little bit of breath once again. In a real world scenario, with our presence at the "trading table" much more often than once a month, we could surely avoid those bad stages much easier.

After this section, the situation improves once again reaching, without real setbacks, the new decade with a total value of 553,87$ as the 1st January 2010. In particular in 2009 it performs really well in contrast to the recession of the real economy, capturing the recover of the stock market after the crash.

During 2010, the two months in the middle, saw a period of uncertainty, with a total setback of 77$ in total portfolio value. But after this small shock the five years total returns grows at a good positive rate, reaching the end of 2014 with 958,38$.

In 2015, between June and September, a non-anticipated period of uncertainty causes some losses. In 2017 and 2018 there is a pretty sharp rise in excess return and in the end, at last, the portfolio grows to reach the final value of 1395$, as the "Final" new global maximum value obtained by the portfolio.

Now I will visualize all the six "main" zero-investment strategies we applied in this paper. The only strategy that is clearly more difficult to understand is the Bond Static one, but it is slightly below the Bond Dynamic one, as explained in the first section only about bond trading and butterfly proxies.

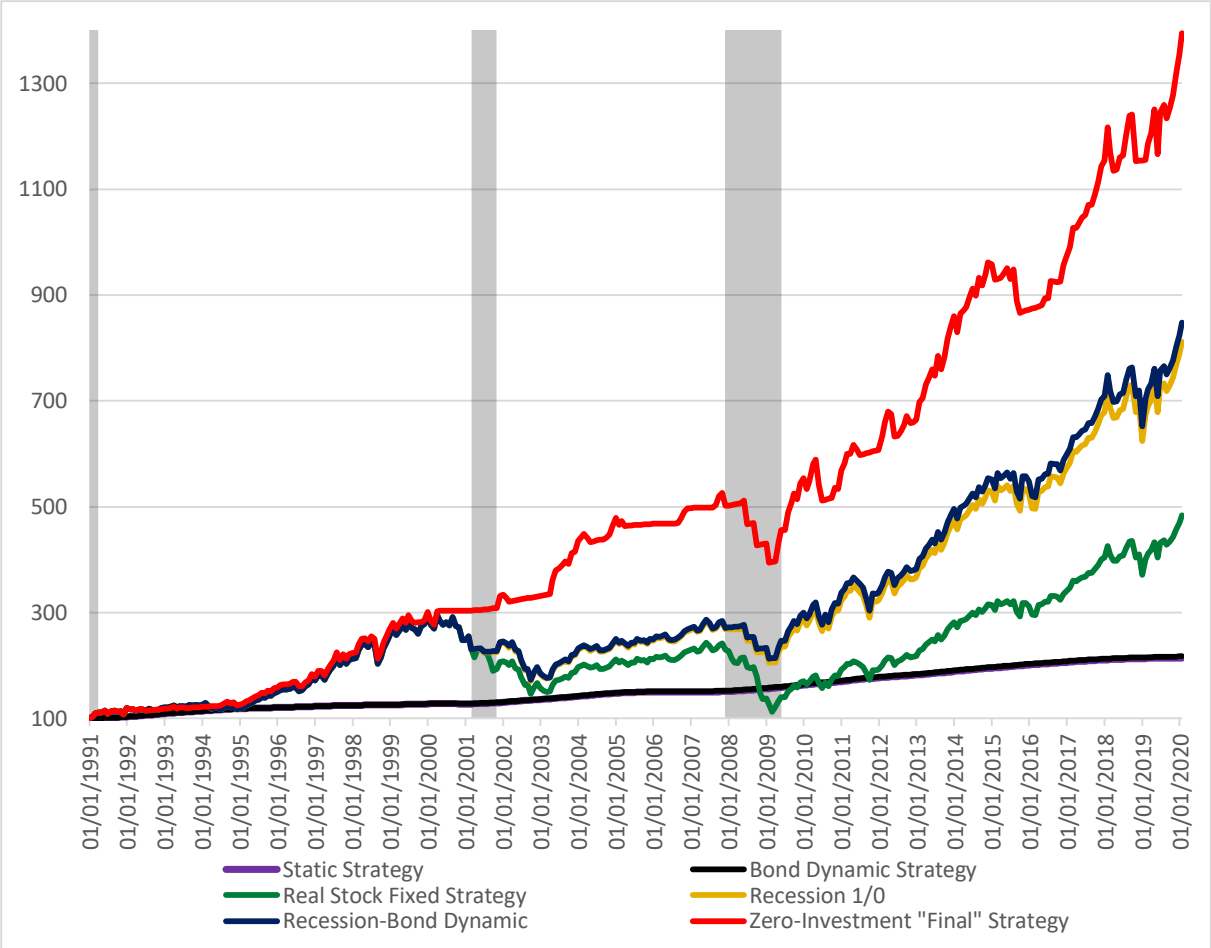

Exhibit 37. Comparison between the six zero-investment strategies, both static and dynamic, with the Final one pretty above the others.

Section 6: Some Final Notes and Recaps

6.1 Stocks and Bonds

Going back to some theory, I want to explain a little more some concepts and informations I used during this studies, ranging from structure of the database to formulas, from markets to personal observations.

Let us dive a little bit more in the relationship between the two markets of bonds and stocks. Now, ex-post, after a direct approach to this subject, our understanding of their day-by-day interactions could be much more clear and complete than before. They usually tend to compete for the available capital and with different peculiarities (and different investment opportunities) the usually move in opposite directions. For example, during periods of financial instability, it is expected an increase in the value of the bonds, with higher prices, thus lower yields, with money moving to this market from the high selling pattern in the stock one, during a bear market situation. When the economy is improving, the safer but less remunerative bond market investment, appears less interesting for the average investor, thus bringing capital from this one to the stocks. In this case, while stock prices rise, yields of bonds increase and their prices start to diminish.

This is the simplest explanation of the relation between the two markets.

In reality, soon after a crisis, inflation, jointly with low interest rates, cause both prices to move together in a similar path. A central bank will usually try to stimulate the economy with measures of expansive monetary policy, in an attempt to encourage the markets, causing a rise in prices of bonds and stocks. This “exception” in the relationship between the two asset classes ends when monetary policy shifts to a regular stance.

But these relationship is not so static and general for any circumastance, in particular it changes with the bond maturity you select. For example a significative difference exists between the stock market and a 30-year maturity bond yield. The stock market is positively related to the industrial production while, for example, long-maturity bonds have an inverse relationship with it or with inflation and are driven for the most part by long-term expectations.

For short maturities the importance of the monetary policy or the economic cycle we are in is much higher, with a pattern more similar to the one of the stock market: this is why they are actually considered as a kind of middle ground between stocks and long-governative bonds.

From our first simple bond fixed strategy, we can also understand that excess returns are higher for long-maturities than short ones, at least on average, and this is caused by the long-run risk premium. It is not static over-time, as an inverse yield curve could also predict a reduction in the return of a bond strategy and, on the contrary, an important growth of upward-looking yields could signal a boost of long-term bond returns.

In 1986 Chen, Roll and Ross investigated the relation between returns from the stock market and systematic risk.[92] In their studies the yield spread between different maturities of government bonds, with other factors such as Gross National Product growth and inflation, could explain stock market movements. In fact, stocks are seen as a good indicator for the real state of the economy: this is the kind of reasoning led Ilmanen to the use of Inverse Relative Wealth in the first place, but this is also why I think an approach coming from the bond market and affecting an equity strategy could be more successful.

Of course, if a recession, as for today, is expected, stock prices are expected to stagnate or decrease in the medium future. But we must also remember that concepts like overreaction and imperfect rationality of the average investor will drive prices differently from what we should expect in a "laboratory" perfect scenario. If stock is in general a good indicator of the reality, it cannot be taken alone.

A joke by Paul Samuelson says: "the stock market correctly forecasted nine of the last four recessions".[93]

[92] Nai-Fu Chen, Richard Roll and Stephen A. Ross, *Economic Forces and the Stock Market*, The Journal of Business, Vol. 59, No. 3 (Jul., 1986), pp. 383-403.

[93] Taken from Brian Domitrovic, *The Stock Market Has Predicted Nine Of The Past Five Recessions*, Forbes (Nov. 22, 2018).

6.2 The Importance of the Dataset

Now that we are approaching to the end of this paper, I want to focus our attention a little more on the bond strategy, explaining some methodologies and observations made during this learning process.

We should remember that the nominal yield, also called coupon rate, is the amount of money, or interest, that the bond gives back as coupons plus the amount paid at maturity. The current yield, which is "quoted" on the market, is not based on the face value of the bond but on the real price that is paid on the market to buy that bond. It is clear that the apparent quotation of the yield is in reality the inverse relationship to the real price of the bond over time. If the price changes in comparison to his theoretical discounted value of today, the investor will incur in a capital change, gain or loss, depending on the market movement. Hence, when we will trade this bond on the market we will obtain the realized yield, the excess return from our market investment.

Using Bloomberg data we can compare the difference between the yields of the strategies we use, both as real excess bond return and as a reaction to current yield changes. In fact, the database gives us both possibilities, helping us understand a market, that is not so easy to comprehend at first glance, given the fact that the "Generic Government Bond Index", independently by his peculiar maturity, will rarely refer to a specific bond emissed in that particular month, but it will be a succession of different bonds, that will lower their maturities, being substituted by new emissions of bonds with a new real "matching" maturity, while the old ones could become the target of a shorter maturity index. For example a 30 year emission will be the target of the 30 year index for some months, and after 20 years, for more or less the same amount of months, it will become the target of a 10 year government bond index.

Bloomberg rules out much of this noise, helping us with clear and understandable data. If for fun, in fact, we compare the difference in current yield between the 30-year index and the 3-month one with the reality of the market (taking into consideration the real price at each moment in time), we obtain.

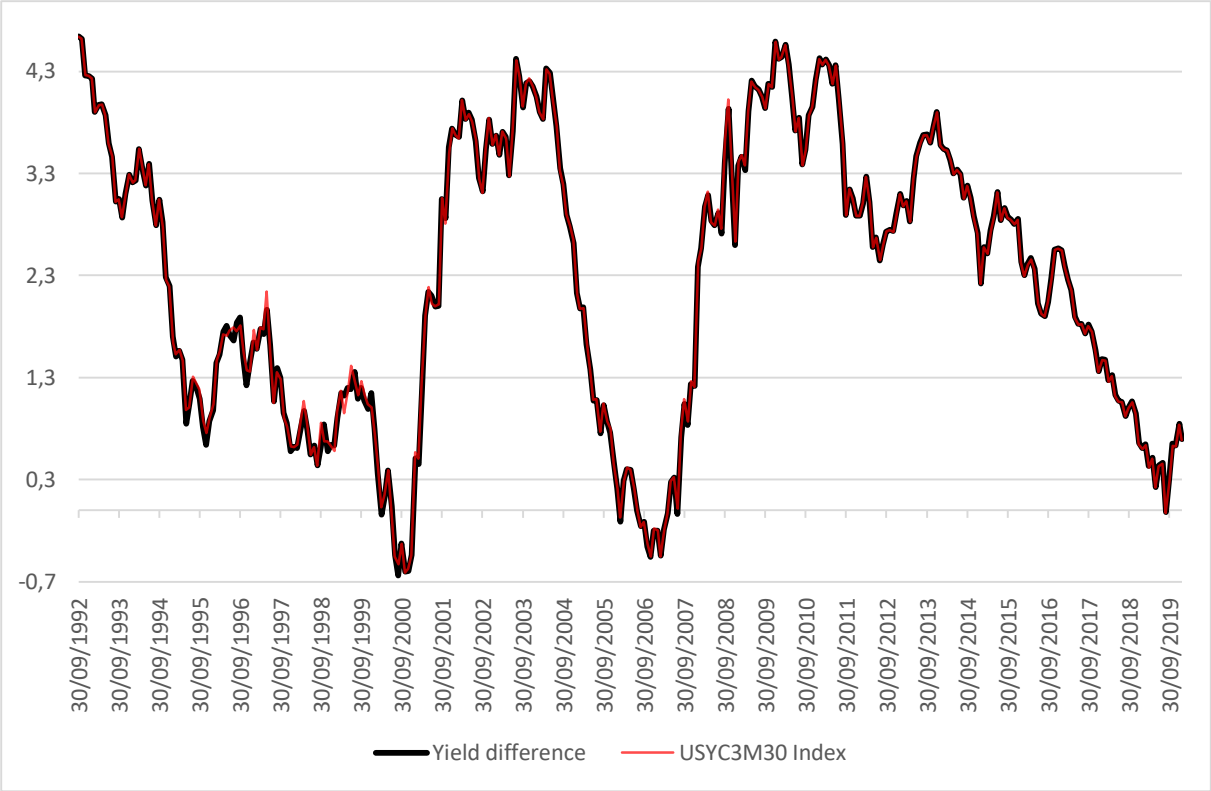

Exhibit 38. Comparison between the real 30-year long minus 3-month short strategy in the period considered.

From the graph it is clear that the majority of the difference is given by a change of yield, thus it could be totally possible to approximate it with the difference between yields. The prices, probably, pretty much rule out each other, in fact the majority of differences between the approximation and the real value is in the years before the 2000. After this date it is still clear to see some differences, but in a "small" figure like this, it will be difficult to do so, and mainly limited to only some trading months.

Going to later years, where we will use the monthly bond to short the 30-year one, we clearly obtain a similar pattern.

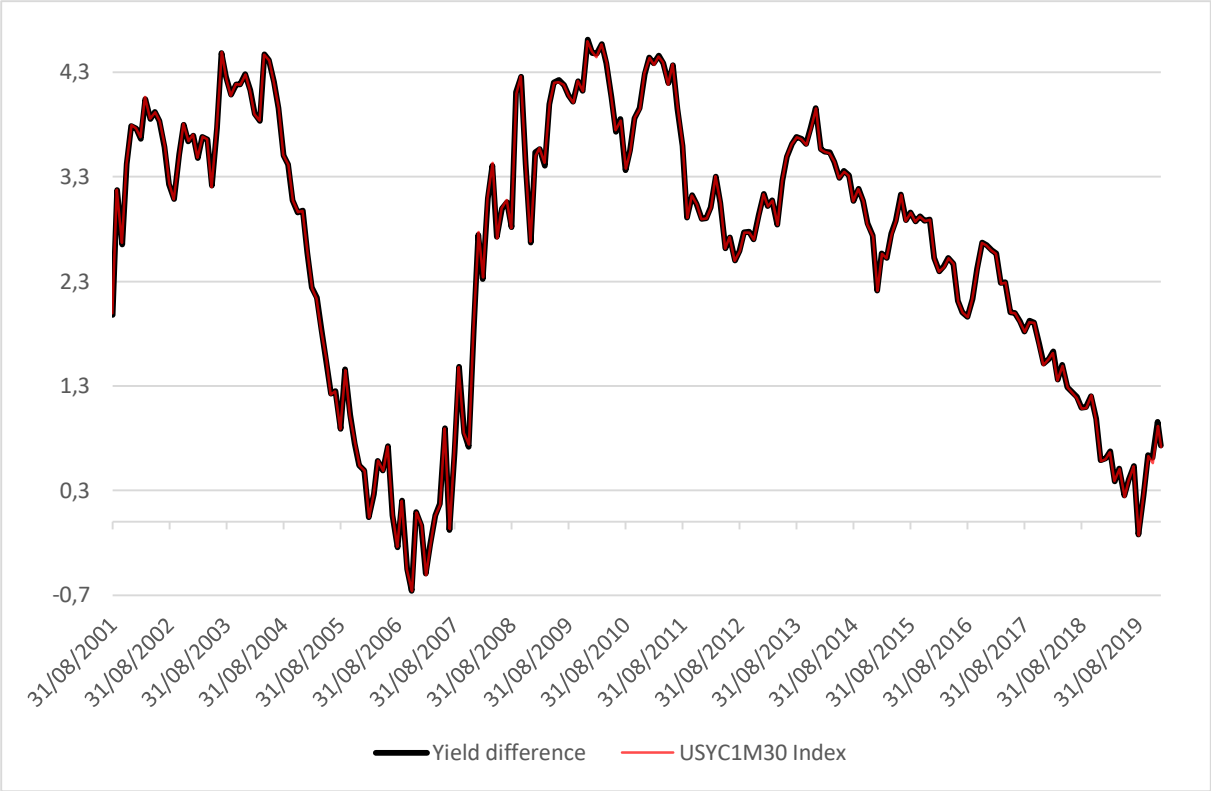

Exhibit 39. Comparison between the real 30-year long minus 1-month short strategy in the period considered.

Even in this case all our ability with transparent graphical analysis should be utilizied to show the clearest possible picture of the results. It is unequivocal to see how the patterns are pretty much the same, showing only some small "perspective" changes over time. Going to the butterflies we used before, the negative one in this particular case, it is also easy to visualize how things were different in the first half of the path, where prices did not cancel each other out really well, and the second par,t when the yield is the true and only dominant factor.

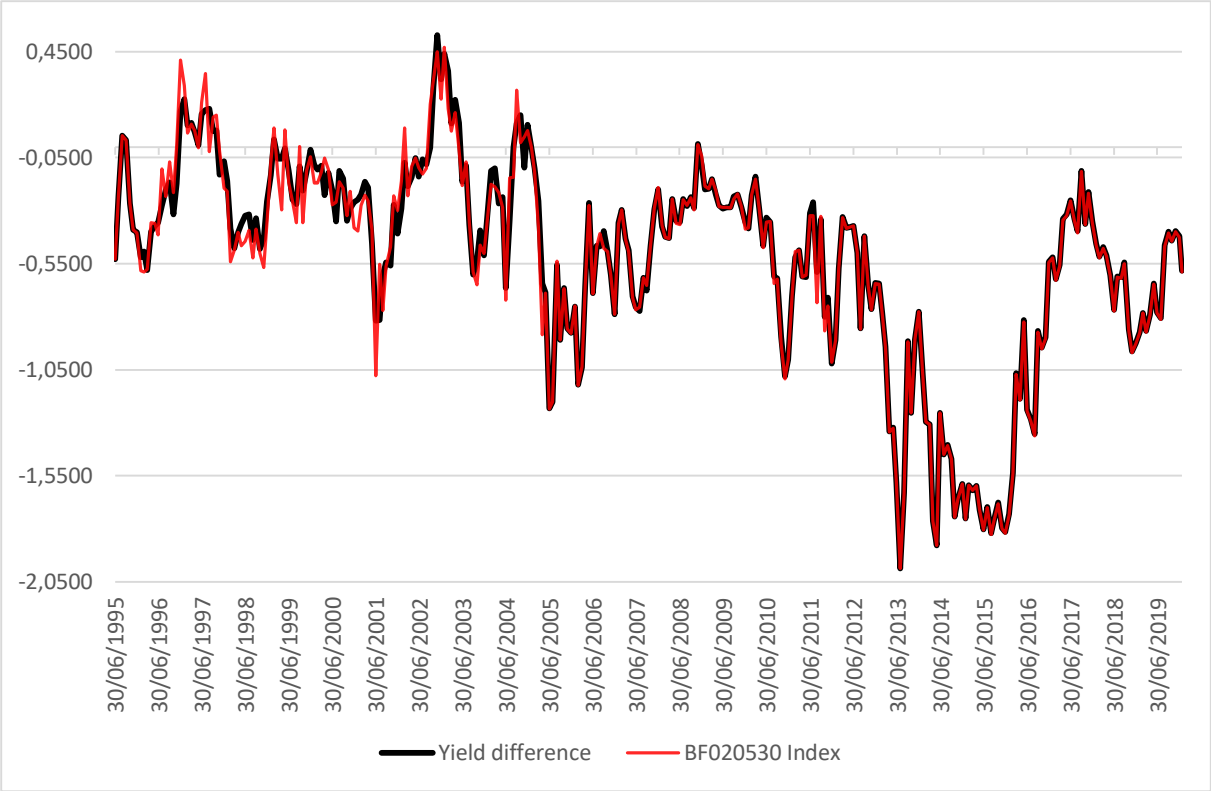

Exhibit 40. Comparison between the excess return of a Negative Butterfly, going long two times the 5-year Government bond and shortening once both the 2-year and the 10-year ones, and the simple yield difference.

In any case, the fact that Bloomberg calculates for us the real excess return for each period is really important, in fact, price of the bonds is not so easily available or understandable for the many reasons said before about the way the bond market works. Moreover there are also many slighty conflicting ways to calculate returns from bonds. I will recap some of these methods now.

6.3 Excess Return

As an example to compute excess return between $t-1$ and t we can use:

$$R_t = \frac{P_t + I_t + C_{t-1,t}}{P_{t-1} + I_{t-1}} - 1$$

Where R_t is the return, P_t is the price at time t, the ending period, P_{t-1} is the starting price at $t-1$, I_t and I_{t-1} are the accrued interests at the time t and $t-1$ respectively and finally $C_{t-1,t}$ are the payments obtained from coupons between the two periods considered.

And this is only the process to calculate the return of one of the two bonds considered.

If we know Duration and Convexity we could use another formula to approximate them:

$$R_t = yield\ income - duration\ \times\ \Delta y + \frac{1}{2}\ convexity\ \times (\Delta y)^2$$

Here the yield income is the yearly yield divided by the time period (for example a 10% yield will become 10/12 percent). Δy is the variation of yield in the time period. But this formula uses duration and convexity, depending on clear statistics, or once again by prices.

6.4 Duration

As a short reminder to measure the sensitivity of price to a change in interest rate we can use Duration, measuring how much time is needed to repay the investor of his starting cash investment. A longer maturity will increase the Duration and expand the importance of the interest rate. A lower coupon, hence a lower yield, will also boost the Duration, and escalate even more the risk.

The formula is:

$$Dur_t = \frac{\sum_{t=1}^{n} \frac{C_t}{(1+y)^t} \times t}{\sum_{t=1}^{n} \frac{C_t}{(1+y)^t}} = \frac{\sum_{t=1}^{n} \frac{C_t}{(1+r)^t} \times t}{P_t}$$

Where C_t is the cash you receive at time t, $\sum_{t=1}^{n} \frac{C_t}{(1+r)^t}$ is the sum of all discounted cash flows of the bond, thus it is the present value of the bond, hence P_t and y are interest rates in a yearly compounding formula. We can say that Duration is strongly dependent on cash flows and prices.

6.5 Convexity

Convexity is a degree of curvature that approximates the gap across the linear relationship between interest rate and price, and the real variation, related to the second derivative of the function.

The convexity formula is:

$$Con_t = \frac{1}{P_t \times (1+y)^2} \sum_{t=1}^{T} \left[\frac{CF_t}{(1+y)^t} \times (t^2 + t) \right]$$

Where P is the price of the bond, T is the final time period (expressed in years as for the regular t), CF_t is the cash received at time t, y is the yield to maturity (YTM).

These are the most important factors to calculate excess returns. I will state again that this is the way to proceed only for one single bond per month.

The database is a blessing to help us study the period considered. Without it, much of this strategy could be only approximated by less precise computations. This is also why the same economist we took our inspiration from is using CRSP data and not computations done all by himself.

6.6 Standard Deviation

$$St.Dev._5 = \sigma_5 = \sqrt{\frac{\sum_{t=1}^{n}(x_t - \bar{x})^2}{t-1}} = \sqrt{\frac{\sum_{t=1}^{5}(x_t - \bar{x})^2}{4}}$$

This is the Standard Deviation over the 5-year timeframe used by me and Ilmanen in our analysis. x_t is the value in the particular year while $\bar{x}$ is the average value. t is each of the 5 years that compose one section. σ of course is the Standard Deviation while σ^2 is Variance. Standard Deviation is used to calculate how is distributed the data in the time set: a higher number, means a higher dispersion. As a simple measure of risk, higher variance means less concentrated and predictable results over time.

6.7 Covariance

Describing it simply, in finance it is a way to understand how returns of two assets or markets move together. If positive, they move in the same direction, hence we will obtain a positive correlation, if negative, the opposite is true. In formula:

$$Cov(x,y) = \frac{\sum_{t=1}^{n}(x_t - \bar{x}) \times (y_t - \bar{y})}{t-1} = \frac{\sum_{t=1}^{5}(x_t - \bar{x}) \times (y_t - \bar{y})}{4}$$

The symbols are in coherence to those used in other formulas before. And now, we have the Correlation.

6.8 Correlation

$$\rho_{xy} = \frac{Cov(x,y)}{\sigma_x \sigma_y}$$

The correlation between two assets represents how two variables change in relationship to each other. It ranges between -1 (perfect negative correlation) to 1 (perfect positive correlation). For instance if x increases by 1 and y does the same, they are perfectly related. If the correlation is 0 they are independent, because a move from one variable does not affect the other. Of course this is really important for our studies, especially searching for a proxy in the market who could predict how well could our portfolio react.

6.9 Sharpe Ratio

The Sharpe Ratio formula is:

$$Sharpe\ Ratio = \frac{R_p - R_f}{\sigma_p}$$

Where R_p is the return of the portfolio, R_f the return of a risk-free investment, σ_p is the volatility of the return of the portfolio using the Standard Deviation. A regular Sharpe

Ratio calculates the return of a portfolio in comparison to the risk-free asset, hence the real profitability of the investment in comparison to a risk-free cash deposit, on the Standard Deviation. In our case, with a zero-investment strategy we already have a value of the return discounted by the risk free.

The Sharpe Index allows to capture the remuneration obtained by the investor per "unit" of risk. Similar to the concept of utility curve, a lower Sharpe Ratio, keeping one of the two components as the same at each moment, could depend from a decrease in excess return or by an increase in volatility.

However, as noted in my studies, Sharpe Ratio confuses positive volatility, related to an increase in excess returns, to a factor of risk. A much higher earning should not be penalized, in my opinion, in the same way as a loss of the same amount. For example in the starting part of the six strategies, bonds resulted more risky than stocks, and this does not make much sense, given the fact that our portfolio based on bonds had a negative performance only in 2 months over 30 years of trading. Problems like this could limit the efficiency of this reward-to-variability ratio.

But "properly used, it can improve investment management."[94]

[94] William F. Sharpe, *The Sharpe Ratio*, The Journal of Portfolio Management, 21 (1), (Fall 1994) 49-58.

Concluding the Thesis

Personal considerations

To conclude about the differencies between my results and those obtained by Ilmanen, I personally think that he could have been in front of a particular data dredging, as he feared since the beginning of his study, or maybe to a particular contingency in the market that forced out results and conclusions that were specific of one single and lonely moment in time. Moreover, I think he had already some results about the fact that the fixed strategy long-government bond, shorting at risk-free rate, was the way to go forward, or at least an important improving possibility for the future, but the whole "*schwerpunkt"* (focal point) was on the first years, thus reducing the objective study of the last years. As said, this is a regular "confirmation bias", a dangerous happening that could strike even at my results. This cognitive partiality is the same as explained for the market overreaction, where people use their past experiences and studies to expand and search for the same results also in the present or even in the future, ignoring informations that could move the mind in an opposite direction. While a regular emotional bias could be a natural even if exaggerated response to signals in the market, the confirmation one transfigure a set of preexisting ideas and beliefs into a valid portfolio strategy.

I think that my reasoning develops as an empirical study, based on the real situation of the economy and the markets, with the true complexity of a human being, always searching for ways to improve the results, thus I hope to avoid at least a portion of this problem from my thesis.

About differencies in methodology, Ilmanen used slighty different yields, the one-month Treasury bond and the long-term government bond of 20 years. I went for the 30 years long-government bond and the 3-month for the first years and only then switched to the 1-month (as said this choice was motivated by a lack of reliable data for the first years). But in general my choices are at positions in the yield curve comparable for the first years, and even more extreme later, thus producing an effect even higher than that studied by Ilmanen. Even if in a particular trading period, his options could theoretically be more profitable than mine, the rarity of this happening (humped or inverse yield curve

specifically), could let me think to this possibility as only marginal, and even if it happened on some occasions, the more than 29 years of time-span, will obviously rule out any miscalculation and data error, with much more importance to a positioning on the extremes of the yield curve, as explained by simple theory, as in my case. Hence, using only Ilmanen's data, returns could be, at least theoretically speaking, much lower, overall, never higher.

If the problem is a different time lag, Ilmanen used a one-month lag 25 and plus years ago. I tried to increase this lag to a number of months ranging from 2 to 4, only for a better understanding of the process, without any confidence that this could really work, and in fact it did not. Of course, after the market integration of recent years it was a hopeless task. We should maybe try and reduce this lag to a matter of days, especially trying to use the Inverse Relative Wealth proxy, but a proxy structured on a daily basis, with a predictive ability limited to some periods, is much more difficult to study, focusing on every single move of the market, without any possibility to tune out the day-by-day inevitable noise and extreme volatility, emotions and various kinds of happenings or news, even some not strictly related to the market, and practically it will be useless for a trading strategy. A proxy valid for the following day, is not a real proxy, in my personal opinion, both as a theoretical exercise and as an investment position.

I think that 30 years are not a small amount of time. Many things happened during this period. Many new technologies are available. Many new possibilities, many new crisis, many new unprecedented peaks of the financial market and the real economy have arisen. It is even possible for a Master's Degree student to study, analyze and maybe change some of the results of a great economist, trying to confirm or refuse his searches. The market of the United States, even if reliable in the past, is much more developed today, and this will be even more important for less technologically advanced countries at the time of Ilmanen's study. Bond markets have probably changed over-time, the patterns of yield evolutions suggest clearly that. Stocks are much more important and profitable than anyone could expect. Market integration is a reality. We could confidently use our findings on a world basis, continuing to study each different reality, until a future time, where an information will be so available and relevant across the globe, to create a real "World" market, with the need of real worldwide proxies, as

predicted by Ilmanen with his paper. This is the final taking from the combination of our studies. Search for the future, using the past, developing strategies that will surely become obsolete on the long-run, but will help us understand each year a little bit more the true core of financial markets.

Conclusion

The question from which we started was: is excess return predictable?
The answer is simply a positive one, it is possible to obtain an excess return, using economic theory and empirical analysis to our own advantage on the market.
In the end, we can say that at least in the US case, this pretty developed strategy, ranging from stocks to bonds, from proxies to recessions, from butterflies to a simple stock market strategy, performs in a pretty positive and usually reliable pattern. It clearly shows a weakness in unexpected changes in the short-term, but it recovers soon after, limiting losses and shifting investments to avoid risk and increase returns. Of course, on a day-by-day real world scenario, we could dive much deeper in the both positive and negative trends. In particular we could hear news, see charts hour-by-hour, understand any possible problem in our system in real-time, thus increasing by much, hopefully, the stability of the investment, and reducing losses by a very high percentage. Short-term market shocks are pretty "random" and the possibility of obtaining a negative result in a particular day is not laughable, but the magnitude and the probability has decreased over-time. Sometimes sticking to the plan could be the best option, revising it only periodically and after clear signals from the market.
Moreover we are using an instrument like the S&P 500 that is pretty reliable and stable, more than any single stock or firm that is included in the index as a lone element. This could be an easy way to study the efficiency of the model on the long-run, but in the short period, a more in-depth analysis of particular corporations and changing them as years pass by, could lead to much more important gains over-time. In a scenario covering up a period of about 30 years, especially a monthly one, is pretty much impossible to understand every single movement and influence, but a clear overall

picture will always be the perfect basis to start investing. Knowing the past will not always lead to an anticipation of the future, but surely it will at least offer a better understanding of consolidated patterns, opening much more interesting ways to trade in the real world.

It should also be said that this is a general model, studied in the US case, but that could work for every other country with some changes. The US-scenario is probably both the most interesting and the safer way to find a portfolio strategy that could be expanded worldwide, while another option will probably be much more country-specific and less applicable overall.

Many studies about financial markets demonstrated that trading rules are a possibility to provide superior excess returns and this was our starting objective. Going on through our studies we found many different ways of trading. We tried to combine them to obtain a much better scenario each time, but it is clear that in general we have found lots of courses that different people could choose to invest in the market.

For a person that retired from active labour, it probably will fit more a low-risk Dynamic-Bond portfolio, if he could be aimed to the right investment for his everyday economic interest (as said before, this is not likely to happen). A day-by-day trader, on the contrary, could be much more inclined towards an active strategy, using financial tools to his own advantage, further expanding this structured but still simple strategy.

As a middle ground, an investor could choose to apply a 1/0 concept, acting only when uncertainty is expected to be pretty low, with low volatility and a strong, clear trend in the financial market.

But there are other problems that could be related to this strategy. First, regarding the zero-investment portfolio, as said before, the US Federal Reserve has put a limit to his possibility. The Securities Exchange Act of 1934, with the Federal Reserve's Board Regulation T, has put constraints on short-selling assets.[95] As said before this is a limit from a theoretical standpoint and for an investor that owns no-capital of his own, but in general this is also an opportunity and it avoids many small expenses over-time: the cost of money, even if it is at a simple and low risk-free rate.

[95] Gordon J. Alexander, *On Back-Testing "Zero-Investment" Strategies*, The Journal of Business, Vol. 73, No. 2 (April 2000), pp. 255-278.

Another big problem is a possible confirmation-bias. It is clear that this strategy was structured ex-post, using past data, related to the same past time period we used to trade in this thesis. This is true. We cannot be sure that in 30 years time this strategy will work once again, perfectly, without problems or necessities to be adjusted month-by-month. Of course, step after step, it should surely be checked and improved.

As said before, my results even show that the work of Ilmanen could be an exaggerated response to biased informations on the market, related to a small period, and a continuously developing market.

But, as shown for Sharpe Ratio before, a cognitive bias is always a possibility. Data is not always as objective as we might think, but is indirectly interpreted every time, in every situation, creating "subjective" outcomes, or at least interpretations of the same inputs. Like overreaction to real world events causes distortions in the market, irrationality or judgement errors, based on wrong assumptions, could always strike.

It is like the classical survivorship bias. We tend to focus on what we see, not understanding what we can not visualize at a first glance. This is why I think my reasoning could seem naïve to a professional that works on the market, but could still help, in my opinion, a student, that has never realized a paper like this at a previous time in his life, to better understand and apply economic and financial theory to the real world, a process that could seem easy and objective, but it must always be analyzed and studied in a critical way, to avaid simple errors or miscalculations.

I think that this, is the true aim of our study, the true focus that impregnates this pages: it is, hopefully, the representation of a learning process, a critical method to apply real financial concepts in our future careers.

I honestly cannot hope to rule out all the problems and instabilities, it is out of the scope of this paper. But I can say that if I become the investor depicted here, I will try to continue to study this strategy, possibly applying at least some concepts over-time. I think I could use it as a solid basis, a reference point, to start working in the real-world markets, maybe even expanding to other sectors of the economy, like forex or oil, hopefully learning from these experiences and adapting any possible change, increasing the overall performance and closing the time-gap between a study on the past and a prediction on the future.

For example, as said before, we could expect from many real world data, like humped and inverse yield curves, opinions of experts and economic realities, that a financial, if not economic crisis, will probably hit the US. In that case a shift to a bond scenario will be a safe bet, as seen for the past and as said by all economic theory on the argument. Then we could expect a real-world fall of US stock market, followed by a positive but volatile period of uncertainty. It will not be a single negative day, or a couple of hours, to lose us all our portfolio value, and it will not be, on the contrary, a too conservative approach to lead as to bankrupt over a longer time period (maybe the contrary could be true).

To summarize: another accurate concept will always be the first sentence heard in my first lesson, in the first year of my Bachelor's Degree: "do not put all your eggs in one basket."[96]

[96] Miguel de Cervantes, *Don Quixote* (Jan. 16, 1605).

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