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Valuation of High-Growth Firms: Chasing the Latent Value

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Abstract

High-growth firms (HGFs) generated an ever-increasing concern over the years. The underlying reasons lie in the ability of these companies to positively contribute to the business environment by enhancing productivity and creating jobs. The ability to create a significant gap with the performance of other organisations, and to bring socio-economic benefits, represent elements that many policymakers are trying to emulate and promote. Despite this evidence and the surging interest, there are relevant unsolved issues about their definition and valuation. This research first resumes the existing literature, then emphasises the methodological determinants that influence the identification of HGFs. The given answers contribute significantly to the comparison of different study results until now misaligned due to a lack of definitions' uniformity. The traditional valuation methods are questioned. While they are commonly recognised and applied tools, they deserve sound refinements in HGFs circumstance. Concerning relative valuation, the following empirical analysis develops new standards for comparable companies' choice. Finally, this research provides a solid empirical foundation for the financial analysis and value judgment of high-growth firms. The identification of the essential financial drivers and their influence on value are developed and formulated to intercept more precisely the value of HGFs, through an econometric model.

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Introduction

Growth is an element that expresses itself within real life in different ways. For instance, it may indicate the development of a human being, the advancement of a country's culture, or it may take up the economic concept of industrial plan's development. While for some aspects, it is conceived as a natural fact, for others, it assumes an outstanding conception. The growth of a child is a predictable, determined event. It does not generate a stir or involve the capacity to achieve it and derives in a clear and definite way from identified causes. By contrast, a company that achieves an annualised sales growth rate of 30% and generates an average annual return for shareholders of 50% for three years in a row, gives different perceptions of growth. Here, it is conceived as an extraordinary event, which in the narrowest sense of its definition indicates the ability to exceed a commonly accepted average threshold. Moreover, as the growth cannot be considered deterministic in this case, capability plays a fundamental role in its achievement. Between the two meanings just reported, we can include the idea of making a difference. Taking a quote from Jonathan Ive, "It's very easy to be different but very difficult to be better."[1] Thus, within the corporate framework and competition dynamics that are triggered, the ability to be different does not necessarily means making a difference. In a world where judgements are made based on publicly shared performance, being different only counts when organisations are better. Concerning High-Growth Firms, their diversity lies first in the ability to deliver performance beyond the upper limit of ordinary, which in the economy is often indicated by the return of a market index. Apart from the strictly financial dimension, they also bring several socio-economic benefits, contributing to productivity and job creation, which have developed an increasing interest in these entities. The interest just mentioned improved in institutional circles, through the definition of public policies, and in the academic world, across the enhanced research on High-Growth Firms. Moved by the many challenges and the growing success of these companies, that now are market leaders, I have set up the following research starting with a theoretical approach and then covering the topics that come closest to the academic knowledge acquired over the years about business valuation. This thesis is divided into three different chapters to provide a solid empirical foundation through the development of a valuation model suited to the particularities of these companies. Thus, the first chapter presents the major features of HGFs, first analysing their recent literary progresses and the issues related to their identification and definition. In the middle part, it addresses the relationship between companies and their business environment, starting with a description of the life cycle and factors that influence their survival in the market. Then,

[1] Kahney, Leander. (2018). The Genius Behind Apple's Greatest Products. Retrieved from https://www.interaction-design.org/literature/article/apple-s-product-development-process-inside-the-world-s-greatest-design-organization

the chapter investigates issues related to public policy and the win-win relationship with the business environment in which companies operate. Finally, the analysis moves from external to internal forces, focusing on the strategic value drivers and the role of the entrepreneur during the various life cycle of the company. The second chapter deals with economic and financial issues related to HGFs. First, it examines the main financial characteristics of high growth firms. Subsequently, the most used valuation methods are presented: the discounted cash-flow and the relative valuation. After a careful analysis, their limitations are highlighted, and alternative versions are presented, together with probabilistic valuation methods. These last valuation tools are more able to capture the value of high-growth firms. The final part of the research proposes answers to several questions related to the valuation of HGFs and concerns the empirical contribution that this thesis aims to provide. In particular, it seeks to investigate the financial drivers that most influence the value of companies, in order to develop a model that is appropriate to their characteristics. Thus, a panel of HGFs is analysed through different methodologies and with the support of quantitative tools. Once the answers from this first model have been obtained, the research attempts to question the multiples valuation method. In its traditional meaning, multiples are not considered to capture the value of high-growth firms accurately. Thus, an attempt is made to provide first a concrete demonstration of what has just been said, and then to develop a new selection strategy for comparable companies, which is quite different from the traditional one. Lastly, the results are compared to actual observations and concrete facts to confirm the validity of the research.

1. High-Growth Firms: Analysis and Perspectives

Interest in high-growth firms is increasing sharply over the last few years. The underlying reasons lie in the ability of these companies to contribute positively to the environment by increasing productivity and jobs. Many policymakers tried to create tools to support these entities, albeit with several difficulties. Instead, research has examined the possible determinants of the particular growth trajectory and the common characteristics of these companies. Thus, to better understand the evolution of this interest, it is useful to start with an example. In 2010 a major review of the literature on HGFs was carried out by Henrekson and Johansson, who found only twenty papers published since 1990[2], a result that appeared well below their expectations. If we perform a similar survey today, on a website like Google Scholar, using the terms "high-growth firms" or "gazelles", the outcome of the research shows that the title of more than one hundred documents contains these words. Furthermore, the study of Henrekson and Johansson is cited in nearly a thousand academic studies since 2010, confirming the surging interest in this subject.[3] Despite the growing appetite for a more precise meaning of "high-growth firms", even today we cannot find concrete definition of them. For this reason, the following chapter aims to make the understanding of this phenomenon as clear as possible.

Thus, this chapter starts with a brief introduction, which helps to learn how literature has developed, moving from an interest in small to high-growth firms. It is necessary to understand where several questions started and how they have changed over the years thanks to the findings of the studies.

Subsequently, there is a focus on two critical aspects: the identification and the definition of HGFs. The most widely used research methodologies are presented and analysed to get a result that is as homogeneous as possible, selecting the criteria for the identification and selection of HGFs that, more than others, have led to significant results in previous researches. The central part of the chapter carries-out an external analysis to delve into several factors that can lead to high growth, taking advantage of recent researches that produced noteworthy results. Hence, it takes into account the business environment, organisation and role of public policies to determine their effects on this type of firm.

In the last part of the chapter, the analysis shifts from external to internal factors. There is a study of five strategic drivers that contribute to the high growth of these companies, and then a closer look at the role of the entrepreneur in the various life-stages of HGFs. Finally, the chapter

[2] Henrekson, M., Johansson, D. Gazelles as job creators: a survey and interpretation of the evidence. Small Bus Econ 35, 227–244 (2010). https://doi.org/10.1007/s11187-009-9172-z

[3] Results obtained from https://scholar.google.com

concludes with a summary of the areas of research that deserve further investigation, which this research should not perform.

1.1. From Small to High-Growth Firms

During the literature analysis, more or less recent, it was possible to conclude that most of the authors survey high growth firms, mainly for two reasons: job creation and growth.

Birch was a pioneer in this regard when in 1979, with "The job generation process", he found that smaller US companies, christened “gazelles”, were more important job creators than larger ones which, according to his study, had the biggest job destruction rate.[4] The results at that time, based on a size distinction of the company, seemed convincing.

In subsequent years, Birch's evidence was questioned by Davis, Haltiwanger and Schuh in 1996, when they did not find a clear relationship between the size of the company and job creation for the period 1973–1988 in the United States. The same happened in 2011 when Neumark, Wall and Zhang observed a non-monotonous relationship between the two variables.[5] A monotonic inverse or non-monotonic relationship shows that an increase in the independent variable causes a decrease in the dependent variable. This relationship is developed in 2013 by Haltiwanger et al., who suggested that the focus on job creation should not be applied to the size of enterprises but to their age. Their most important discovery was related to the role of the company's age and its relationship to growth dynamics, specifying that "once we control for firm age, the negative relationship between firm size and net growth disappears and may even reverse sign as a result of relatively high rates of exit among the smallest firms."[6]

This brief introduction helps us to understand how attention, about the job contribution, has evolved over the years, moving interests from small to high-growth firms.

1.1.1. Issues about Identification

During their researches above “A taxonomy for high-growth firms” in 1998, Delmar and Davidsson highlighted that four issues, more than others, need to be considered to identify HGFs. This paragraph analyses and explains these four points to make as clear as possible the question above identification. It is essential for the following description of HGFs definitions.

[4] Birch, D. G. W. (1979). The Job Generation Process. MIT Program on Neighborhood and Regional Change, 302, 1979. Retrieved from http://ssrn.com/abstract=1510007

[5] Coad, A., Daunfeldt, S. O., Hölzl, W., Johansson, D., & Nightingale, P. (2014). High-growth firms: Introduction to the special section. Industrial and Corporate Change, 23(1), 91–112. https://doi.org/10.1093/icc/dtt052

[6] Haltiwanger, J., Jarmin, R. S., & Miranda, J. (2013). Who creates jobs? Small versus large versus young. Review of Economics and Statistics. https://doi.org/10.1162/REST_a_00288

The first issue is the indicator of growth, that is the choice of the variable to observe the growth. Thanks to Henkerson and Johansson's survey in 2010, and the extended version of Daunfeldt, Elert and Johansson in 2014, we know that over 30 analysed studies used *employment* and *sales* as growth indicators. However, the first one is the preferred variable during these studies. The second point is the choice between the measurement of growth in relative or absolute terms. Differently from the first issue, where Daunfeldt et al. found no sensible result differences choosing among the indicators, in this case it is significant to distinguish and to understand the effects of choice.[7] Relative measures of a variable, such as value, volume, height and price, were well explained in 1985 by Tornqvist, P. Vartia and Y.O. Vartia, who defined them as "pure numbers ... independent of the units of measurement". They suggested using log-percentage changing because of its ability to produce better results than the simple change percentage used commonly.[8] While absolute measures refers to raw changes in size between two-time points and are the easiest measures to apply, and also the most popular in the literature. Concerning the effects of choice, Delmar, Davidsson and Gartner (2003) proved that relative growth models lead to a preference towards small firms, while absolute measures towards large firms.[9] There are also indicators combining relative and absolute changes. The most common is the Birch Index, which manages to decrease the bias in identifying small companies such as HGFs, reducing the effect of company size on the growth indicator.

The Birch Index is defined as:

$$(E_t - E_{t-k}) * (\frac{E_t}{E_{t-k}})$$

where E_t is the number of employees in year t and E_{t-k} is the number of employees in a previous period[10]. These distinctions of measurements could lead to the conclusion that many studies are not comparable, and therefore it becomes challenging to outline a universal methodology to identify high-growth firms.

The third issue relates to the selection of an analysis time frame that can reduce the amount of statistical noise over the years. The recent trend is to consider a three- or four- years period

[7] Daunfeldt, S. O., Elert, N., & Johansson, D. (2014). The Economic Contribution of High-Growth Firms: Do Policy Implications Depend on the Choice of Growth Indicator? Journal of Industry, Competition and Trade, 14(3), 337–365. https://doi.org/10.1007/s10842-013-0168-7

[8] Törnqvist, L., Vartia, P., & Vartia, Y. O. (1985). How should relative changes be measured? American Statistician, 39(1), 43–46. https://doi.org/10.1080/00031305.1985.10479385

[9] Delmar, F., Davidsson, P., & Gartner, W. B. (2003). Arriving at the high-growth firm. Journal of Business Venturing, 18(2), 189–216. https://doi.org/10.1016/S0883-9026(02)00080-0

[10] See footnote 4

because these firms tend to change substantially over these periods. Although several authors tried to study the best-fitted period of analysis, this is still an unsolved topic.[11]

Finally, the last identification matter is the difference between internal growth and external growth, and the choice between them for the research. Organic, or internal, growth refers to in-house operations that support the growth of employment inside the company. Acquired, or external, growth is the increase in employment due to external mergers and acquisitions operations. Most researches use a hybrid, or total growth, model due to the lack of mergers and acquisitions data. This model includes both strategies and considers them uniformly.[12]

The problems outlined above are at the heart of political policy debates in Europe and beyond. Several studies are trying to explain what affects the establishment of these companies in one country rather than another, if some of them benefit from a specific policy approach, what effect the entrepreneurial figure has on the performance of high-growth companies.

These issues are addressed in detail later, after a necessary explanation of the problems faced during the identification of HGFs, to set out the most widely used definitions in the literature.

1.1.2. Groups of Definitions

Given the numerous interpretations about the research of high growth firms, this research follows the set of definitions established by the World Bank in 2019. The reason for this choice lies in the clearness and effectiveness that this distinction makes over companies under analysis.

According to this classification, high-growth companies can be grouped into three different definitions:

- absolute;
- relative;
- distributional.[13]

Absolute definitions provide for the choice of a specific growth rate and a predefined period. Most common definitions are the Birch index or that provided by Eurostat-OECD, in 2007, which states that HGFs are "all enterprises with average annualised growth greater than 20% per annum, over a three-year period should be considered as high-growth enterprises. Growth can be measured by the number of employees or by turnover." A minimum threshold of ten workers has been introduced to identify better companies belonging to the HGFs category and to avoid including

[11] See footnote 4
[12] See footnote 4
[13] Grover Goswami, A., Medvedev, D., & Olafsen, E. (2019). High-Growth Firms: Facts, Fiction, and Policy Options for Emerging Economies. High-Growth Firms: Facts, Fiction, and Policy Options for Emerging Economies. The World Bank. https://doi.org/10.1596/978-1-4648-1368-9

too small companies. Furthermore, the OECD has made a supplementary distinction by defining "gazelles" as all companies with the above characteristics, but with five or more years of life.[14]

Relative definitions target a set number of HGFs in a selected percentile of firms in the distribution of revenue or employment growth. Hence, Haltiwanger et al. (2016), during an analysis of the contribution to labour and production growth by HGFs, used as relative measurement systems to classify companies, the weighted deciles of the distribution of different variables. Thus, for example, for the employment growth specifications, they used employment weighted deciles of employment size.[15]

The last set of definitions is *distributional*. They study the right tail of the growth rates distribution, mostly Laplace's, and in particular, a certain threshold by which this tail converts to a power-law distribution. These types of definitions certainly need to be developed in-depth to provide better answers, but they deserve close attention to the potential of their results.

It is necessary to recognize that these definitions have a common objective: the identification of high-performance companies that succeed in achieving above-average growth rates. However, the impact of these different definitions on the identification of high-growth firms is not analysed in this research, although it is worthy of further investigation. This study would be inappropriate concerning the objective of this research, i.e. the valuation of HGFs.

1.2. External Analysis

The following analysis has the purpose of studying three external factors that could shape the growth of the firms. The first area of research is the explanation of the four HGFs life cycle phases and focuses on a company's ability to create growth that is persistent and does not represent a single event. The study then moves to the business environment, which provides answers about the relationship of HGFs with significant matters in the literature, such as job creation and productivity. Besides, the analysis evaluates the influence of different geographical backgrounds among these companies and the effect of spillovers. The third and final part involves public policy. Therefore, the last part focuses not only on the difficulties that today's policymakers are facing in regulating support instruments for HGFs, but also on the recent initiatives that several countries have put in place in favour of them. Hence, it is an analysis that starts from a theoretical point of view and merges into practical examples, to provide the reader with a clear and comprehensive discussion.

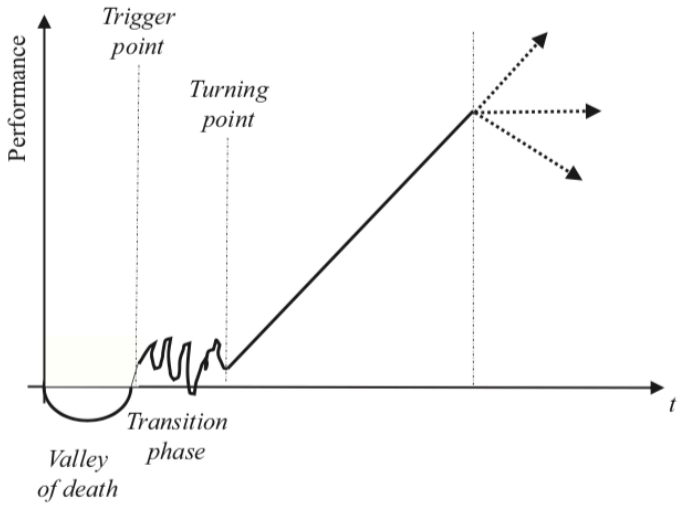
[14] Eurostat-OECD. (2007), Eurostat-OECD Manual on Business Demography Statistics. Office for Official Publications of the European Communities: Luxembourg.

[15] Haltiwanger, J., Jarmin, R. S., & Kulick, R. B. (2016). High Growth Young Firms: Contribution to Job, Output and Productivity Growth. SSRN Electronic Journal. https://doi.org/10.2139/ssrn.2866566

1.2.1. Organization

The HGFs analysis from an external perspective starts with their life cycle. It has four main phases, dependent on each other, with different duration and characteristics. The best way to exhibit these stages is a graphical representation, which shows the breakdown of the life cycle of HGFs: valley of death, trigger point, transition phase and a turning point.

Figure 1: Generic trajectory of an HGF



Source: RAUSP Management Journal[16]

The "valley of death" is the first stage, and it describes the period from fundraising to creation of positive cash flow. During this time, the business may have a prototype of a product or service that needs further development to generate sales. Through funding, companies deploy these improvements that lead to a finished product or service that is ready to be commercialized. Through revenues, they become independent and achieve positive cash flow.[17]

Once reached the latter, the company may face many "trigger points", i.e. crucial events, which have the potential to bring a performance business, or "trundler", into a high-performance, or "flyer".[18]

According to Brown and Mason, to better understand these different phenomena, it is necessary to divide them into three different types: endogenous, exogenous and co-determined. Endogenous

[16] Monteiro, G. F. A. (2019, February 11). High-growth firms and scale-ups: a review and research agenda. RAUSP Management Journal. Emerald Group Publishing Ltd. https://doi.org/10.1108/RAUSP-03-2018-0004

[17] House of Commons Science and Technology Committee. (2013). Bridging the valley of death: improving the commercialisation of research. House of Commons (pp. 1–133). The Stationery Office Limited. https://doi.org/10.1049/et.2013.0910

[18] Storey, D.J. (1992). Should we abandon support for start-up businesses? Working Paper 11. CSME. University of Warwick, Coventry.

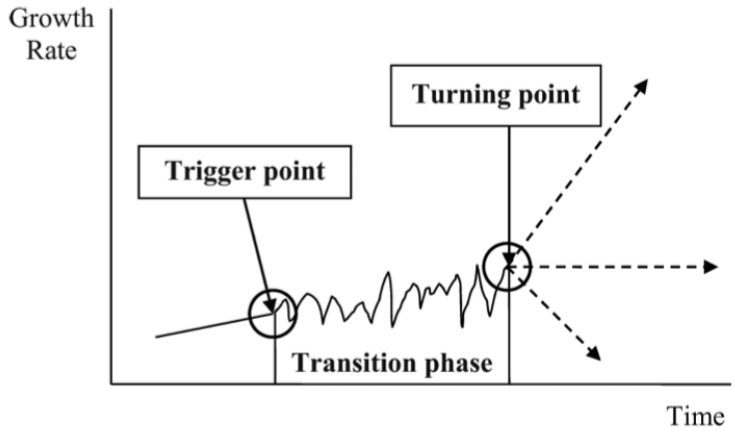
trigger points are events that occur as a consequence of an action taken by the company itself. [19] Thus, they are the effect of the company's use of resources and capabilities to exploit opportunities for growth. One example is the acquisition of another company, as was the case in 2005 when Google acquired Android for $50 million, which later became the world's largest mobile platform. Another example is when Apple revolutionized the world of personal technology in 1984 by launching the first Macintosh.

The exogenous trigger points are events outside the company's control, but which give it the possibility to undertake a path of growth and expansion. They are events stimulated by the business environment such as innovation, macroeconomic factors, access to investments or changes in public policy.[20] A famous example is the market failure of Ford Edsel in 1957, the company invested about 400 million for the development of this model, then withdrawn from the market in 1960.[21] Finally, the codetermined trigger points refer to decisions that are not under the control of the company but over which it can exert influence. They are considered to be a result of both parties. Thus, the clearest examples are entering into a joint venture, acquisition by another company or receipt of a new contract.[22] An example is a joint venture created in 2008 between NBC Universal Television Group (Comcast) and Disney ABC Television Group (The Walt Disney Company) to create a streaming platform called "HULU". It was a success with the offering lining up $1 billion.[23]

As shown in Figure 2, the end of the trigger points marks the beginning of a transition phase that introduces many pitfalls. In fact, within this phase, companies have to try to express the potential of the opportunities offered by the trigger points. These act as a kind of catalyst for ultimate growth.

[19] Brown, R., & Mawson, S. (2013). Trigger points and high-growth firms: A conceptualisation and review of public policy implications. Journal of Small Business and Enterprise Development, 20(2), 279–295. https://doi.org/10.1108/14626001311326734
[20] See footnote 19
[21] Retrieved by https://it.businessinsider.com/?r=US&IR=T
[22] See footnote 19
[23] Retrieved by https://www.educba.com/

Figure 2: The growth "trigger point" process



Source: Journal of Small Business and Enterprise Development[24]

Also, during this period, companies have the opportunity to meet "secondary triggers" that can amplify the growth opportunities offered by the former. Google's previous example represents this phenomenon. The acquisition of Android gave the possibility two years later to the Mountain View company to enter the mobile phone market and become one of the major world leaders.
It is, therefore, a very critical stage for the organization that must try to capitalize on the challenges it faces. For this reason, it is relevant to note that not all companies succeed in doing so, and many have negative secondary triggers. The latter decrease the growth of the society, and many times they can undermine its survival.[25] One example of a potential negative trigger point is a company that has just signed a relevant contract but requires an expansion of production capacity during the transition phase, thus testing the company's organizational resources.[26] The transition phase involves both positive and negative moments that require various factors such as managerial ability and company resources. Once finished, however, the company is faced with a turning point, i.e. the beginning of a new growth trajectory that can be positive, neutral or negative. As with trigger points, these are very significant moments for the company that can be represented by a single event or several factors that culminate after a certain period. Being able to recognize these turning points, after having already passed the transition phase, is essential for the company because they can represent a milestone for profitability and growth.[27] At this point, assuming that the company shows a rapid increase in growth, several questions arise. How long will this growth last? What role will the company play to maintain it? Precisely because growth cannot last forever,

[24] See footnote 19
[25] See footnote 19
[26] Barney, Jay B. 2001. "Is the Resource-Based 'View' a Useful Perspective for Strategic Management Research? Yes." Academy of Management Review. Academy of Management. doi:10.5465/AMR.2001.4011938.
[27] See footnote 19

many authors tried to study this phenomenon, and four hypotheses emerged about the non-persistence of high growth. The first one argues that growth is a purely random effect, and, therefore, cannot persist or even be correlated. This hypothesis means that it is not possible to create models or strategies to support growth because of its random nature.[28]

The second one assumes that after a period of high growth necessarily follows a period of stagnation. This one undertakes that expansion period brings difficulties that a company is not able to solve in the short term. Hence, issues like adjusting its management or its resources, to meet the needs of the organization, could lead to inefficiency.[29] The third hypothesis argues that past growth is a constraint for future growth as it creates rigidity and path dependence within the organization.[30]

The final one argues that it is the power of the established incumbents to disrupt the length of growth of these companies. Think, for example, of the dominance that Amazon exerts and the strength it can use, through a price war, against a high-growth firm. For this reason, the most common strategy in HGFs is not to lower costs but to bring innovations to existing products and better service quality.[31]

Therefore, HGFs have a growth path in which they face many challenges, which sometimes also represent opportunities. It is a destructive way in which these societies must be able to carefully analyse the threats that the market presents to them and possibly turn them into opportunities for growth. However, as we have just said, not all of them can do this, and for this reason, in the next paragraph, we analyse the impact that these companies have on the business environment and vice versa.

1.2.2. Business Environment

The relationship between the business environment and high-growth firms is under the magnifying glass because it has a potential not yet entirely understood. In particular, from this point of view, it is interesting to understand the effects that this factor gives in terms of job creation and productivity.

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[28] See footnote 17

[29] Du, J., & Temouri, Y. (2015). High-growth firms and productivity: evidence from the United Kingdom. Small Business Economics, 44(1), 123–143. https://doi.org/10.1007/s11187-014-9584-2

[30] Baker, D. D., & Cullen, J. B. (1993). Administrative Reorganization and Configurational Context: The Contingent Effects of Age, Size, and Change in Size. Academy of Management Journal, 36(6), 1251–1277. https://doi.org/10.5465/256811

[31] Surowiecki, J. (2016, July 1). Why startups are struggling. Technology Review. Massachusetts Institute of Technology.

The first step is to outline the impact that these companies have on high-income and emerging countries economy. This analysis reviews the most relevant studies made on this issue, which subsequently help us to establish the share of labour created by HGFs.

Regarding the researches carried out on high-income economies, there is a clear difference in the results obtained by authors. A study carried out in 2016 by Bravo Biosca, Criscuolo and Menon showed how the incidence of HGFs varies from 3% in Norway and Austria to 6% in America and England.[32] Choi showed that the incidence in the US varies from 5% to 15%[33], while it goes from 2% in the Netherlands, Austria, Italy and Germany and Norway[34] to 6% in Sweden[35] and UK.[36]

For developing countries, a significant contribution was made by Goswami, Medvedev and Olafsen during a project for the World Bank in 2019. They studied the incidence of HGFs in 11 emerging countries[37] and used the United States as a benchmark. However, the results show that the incidence rates do not change significantly compared to high-income economies. They conclude that there is no clear relationship between the development of the countries, measured by capita income, and the incidence of high growth.[38] Now that the incidence range (5-20%) within the HGFs has been figured out, it is possible to analyse the relationship between them and job creation.

As already mentioned in the previous paragraph (see 1.1), the focus on HGFs evolved following the findings of the capability, of a little percentage of HGFs, to contribute disproportionately to job creation and productivity. According to a study conducted in the United Kingdom, during the period 2005-2008, 6% of all companies created 54% of jobs.[39] During the same period, 6% of Swedish companies contributed more than 40% of jobs.[40] More recent research on the United States showed that a small percentage of companies considered HGFs, around 10-15%, contributed more than 50% to job creation and output.[41] Results anticipating this trend have been already developed by Schreyer in 2000. He found, using different definitions of HGFs between

[32] Bravo-Biosca, A., Criscuolo, C., & Menon, C. (2016). What drives the dynamics of business growth? Economic Policy, 31(88), 703–742. https://doi.org/10.1093/epolic/eiw013

[33] Choi, T., Rupasingha, A., Robertson, J. C., & Leigh, N. G. (2017). The effects of high growth on new business survival. Review of Regional Studies, 47(1), 1–23.

[34] Goedhuys, M., & Sleuwaegen, L. (2010). High-growth entrepreneurial firms in Africa: A quantile regression approach. Small Business Economics, 34(1), 31–51. https://doi.org/10.1007/s11187-009-9193-7

[35] Daunfeldt, S. O., Lang, Å., Macuchova, Z., & Rudholm, N. (2013). Firm growth in the Swedish retail and wholesale industries. Service Industries Journal, 33(12), 1193–1205. https://doi.org/10.1080/02642069.2013.719883

[36] Anyadike-Danes, M., Bonner, K., Hart, M., & Mason, C. (2009). Measuring business growth: high growth firms and their contribution to employment in the UK. London: NESTA, (October), 1–52. https://doi.org/10.1039/c1ob06569f

[37] Brazil, Côte d'Ivoire, Ethiopia, Hungary, Indonesia, India, Mexico, South Africa, Tunisia, Turkey, United States

[38] see footnote 13, pp. 6

[39] See footnote 21

[40] See footnote 19

[41] See footnote 15

countries, that this type of companies contributed 50% of jobs in France, 65% in the Netherlands and 90% in Spain.[42]

These results answer the first question about the relationship between HGFs and the business environment in terms of job creation. Hence, it can be state that only a small percentage of HGFs contribute, more than proportional, to job creation in the countries.

The second point of interest relates to the issue of productivity. According to Krugman, he defined productivity "as a ratio between the output volume and the volume of inputs. In other words, it measures how efficiently production inputs, such as labour and capital, are being used in an economy to produce a given level of output."[43] Although the definition seems straightforward, the measurement could be a harder challenge. Analysts usually compare inputs and output values given by the *price per quantity* formula. However, this computation does not allow us to understand what effectively drives growth (i.e. higher prices, more production, demand shocks). The data on firm-level prices are seldom accessible, the total factor productivity (TFP) is usually defined in terms of revenue (TFPR) rather than quantity (TFPQ). Hence, if a firm increase the prices this reflects rising the TFPR for reasons unrelated to technical efficiency. This because TFPR blends the "true" measure with idiosyncratic demand and factor prices effect.[44] The central literature has not confirmed a strong relationship between productivity and business growth in a consistent way for all companies, mainly for measurement problems. However, here are presented positive results about HGFs and productivity. SBA Office of Advocacy compared three different employee-size segments of HGFs (1-19, 20-499, 500+), in the United States, in four different periods from 1994 to 2008. They showed that HGFs revenue-based labour productivity was higher in HGFs than other companies and, that difference increased over time.[45] Another research, conducted in the United Kingdom in 2012, confirmed that HGFs, especially the foreign ones, are on average more productive than other companies.[46]

Moreover, several studies verified a correlation between productivity and high growth events. Du and Temouri highlighted that TFP growth increases the probability to experience a high-growth period for both new firms (younger than five years) and incumbents (older than five years). They also control several variables that drive productivity and confirmed previous research results like:

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[42] Schreyer, P. (2000). High-growth firms and employment. OECD Science, Technology and Industry Working Papers 2000/03, 139. https://doi.org/10.1787/861275538813

[43] Krugman, P. (1994). Defining and measuring productivity. Organisation for Economic Co-Operation and Development, 1. Retrieved from http://www.oecd.org/std/productivity-stats/40526851.pdf

[44] See footnote 13, pp. 69

[45] Tracy, S. L. (2013). Accelerating job creation in America: The promise of high-impact companies. In Small Business and Job Creation: Analyses and Implications (pp. 27–91). Nova Science Publishers, Inc.

[46] Mason, G., C. Robinson, and C. Rosazza-Bondibene. (2012). Sources of Labour Productivity Growth: Sectoral Decompositions for Britain, 1998–2007. NESTA Research Report. National Endowment for Science, Technology and the Arts, London.

- firm age is more important to explain HGF incidence compared with firm size, previously showed by Bravo-Biosca[47] study;
- the presence of intangible assets increases the probability to reach HGF status, in line with Mason, Bishop and Robinson[48] findings;
- average wages are associated with a higher likelihood to become HGFs;
- cash holdings are negatively correlated with the HGF growing path, taking up a concept studied by Jensen[49]in 1986;
- impact of internationalisation is positive for a certain sector (services) but void for other (manufacturing).[50]

The relationship between productivity and high-growth episodes is also confirmed by the recent study of the World Bank about HGFs in emerging countries.[51] The final consideration of the business environment part is related to the effect of agglomerate spillovers and linkages and the geographical incidence. Several studies confirmed that HGFs produce broader economic and social benefits, including promoting the growth of other firms in the same region and industrial clusters. Especially when the input production market is competitive, they improve overall efficiency by attracting resources from less productive companies.[52] Data from 43 industries in the Netherlands over 12 years reveals that a higher proportion of HGFs in a sector produces a positive impact on following industry growth.[53]

According to the literature, the spillovers effect, and similarly the linkages for HGFs, can be of 3 different natures:

- horizontal (HS), measured as the number of companies that have become HGFs in the same industry or region since year t to t+3;
- forward (FS), showing the average share of HGFs in the supplier industry, weighted by the volume of intermediate goods between industries;
- backward (BS), show the average share of HGFs in the buyers' industry, weighted by the volume of intermediate goods between industries.

[47] Bravo-Biosca, A. (2011). A look at business growth and contraction in Europe. Nesta, 1–31. Retrieved from www.nesta.org.uk/wp11-02

[48] Mason, G., Bishop, K., & Robinson, C. (2009). Business Growth and Innovation - The wider impact of rapidly-growing firms in UK city-regions. Technology, (October), 1–50.

[49] Posner, R. A., & Jensen, M. C. (2009). Agency costs of free cash flow, corporate finance, and takeovers. In Corporate Bankruptcy (pp. 11–16). Cambridge University Press. https://doi.org/10.1017/cbo9780511609435.005

[50] See footnote 30

[51] See footnote 13, pp. 72-74

[52] See footnote 13, pp. 16

[53] Bos, J. W. B., & Stam, E. (2014). Gazelles and industry growth: A study of young high-growth firms in The Netherlands. Industrial and Corporate Change, 23(1), 145–169. https://doi.org/10.1093/icc/dtt050

Hence, an increase of HGFs in the share of suppliers (FS) may lead to an increase in growth or revenue, due to better access to inputs. Conversely, an increase in HGFs in the share of buyers (BS) is associated with an increase in wages and productivity as a result of increased demand that allows higher mark-ups.

1.2.3. Public Policy

At this point, it is necessary to introduce the subject of public policies. Since it has been demonstrated, even within this research, that these companies contribute positively to productivity and job creation, many countries have tried, and still try, to find efficient tools to support HGFs. Indeed, it would be a win-win situation if a country succeeded to find policies that would facilitate the growth path and thus, the expansion of high-growth firms by fostering productivity and job creation.

However, to do this requires knowledge and predictive capacity, as it would be necessary to understand the duration and scale of growth, which does not have to represent a single event. For this reason, the following are several public support initiatives implemented in previous years by many countries that have tried, in different ways, to get positive results. The choice to first set out practical cases and only then the issues related to public policy on HGF reflects the ability of this structure to provide the reader with a complete picture of what has been done and what needs to be improved.

First of all, policies towards HGFs should be considered a subset of the tools for facilitating entrepreneurial growth and healthy ecosystems. The first tools to support these companies, in the 2000s, were difficult to distinguish from the more general ones aimed at promoting entrepreneurship. Although many of these initiatives remained almost similar and very close to those aimed at small and medium-sized companies (SMEs), several countries have put into real practice policies to support HGFs. These interventions can be classified according to the market failure they seek to address. The five areas of interest are: strengthen firm capabilities, improve access to finance, help firms reach new markets, resolve regulatory obstacles, provide critical infrastructure. Among the examples, the most notable programmes are:

- in the Netherlands the Masterclasses, Angel Program, Port4Growth;
- in the United Kingdom the High-Growth Start-Up;
- in Finland the Growth Firm Service programmes;
- in Colombia the iNNpulsa;
- in India the GVFL state-backed venture fund.

However, these public policies present several problems. One of them is often the lack of valuation metrics leading to the reliable measurement of results. It means that public policies are not comparable or transferable to other states because results are not precise. Besides, sometimes the public policies include several supporting tools within it, which are difficult to measure individually.[54]

More generally, the criticism about public policy is that it fails to identify the right target beneficiaries. This issue happens for three main reasons: the belief that the technology sectors are the best source of HGFs and that these types of companies are more likely to grow; the tendency to consider HGFs as recent start-ups; and the assumption that the manufacturing sector is a significant resource of HGFs.[55] Regarding the first belief, results from researches showed that HGFs are in all sectors and that they do not relate only to technological sectors. Moreover, they are not even over-represented in these sectors.[56] The interventions concerning technology companies erroneously focus on traditional research and development (R&D) and not on innovation, intended as the exploration of new solutions that do not necessarily come from scientific laboratories. According to Mason and Brown, "These 'hidden' innovations include innovations in organisational forms and business models and innovations created from the novel combination of existing technologies and processes."[57]

Therefore, policymakers should take into account the role and effect of R&D that may not lead to innovation and end-user-relevant products. The second wrong conception supports familiarity between HGFs and recent start-ups. This association, as we have already seen in the previous sections, appears to be erroneous as start-ups usually need a more extended period to achieve sustained growth than older HGFs. Instruments including pre-incubation policies, such as the use of management buyouts or corporate spinouts, have shown significant effectiveness in creating HGFs, unlike those focused on start-ups. Consequently, it essential to distinguish the different growth paths that these companies follow to establish the tools that best suit their nature and needs. The third point, i.e. the current of thought that manufacturing activity is more valuable than services, presents difficulties both at the conceptual and at the definition level. Nowadays, it is more challenging to distinguish product or service companies due to the ability of many of them to create product-as-service business models. Companies now define themselves as "solution

[54] See footnote 13, pp. 122-124
[55] Mason, C., & Brown, R. (2013). Creating good public policy to support high-growth firms. Small Business Economics, 40(2), 211–225. https://doi.org/10.1007/s11187-011-9369-9
[56] See footnote 2
[57] See footnote 56

providers" and no longer as providers of a product or service. Therefore, a mere policy focused on HGFs in the manufacturing sector would be inappropriate and difficult to apply.[58]

A further criticism of HGFs policies is that the growth of these companies does not persist. This assumption leads to the exclusion of all those instruments that analyse past growth to predict future growth. Since this is not possible, given the not yet encouraging results of such a report, it begs the question of the meaning of these supportive policies.

At this point, an alternative vision to the problem of targeting HGFs comes if one relates business dynamism to institutional elements of the business environment. In 2013, Bravo-Biosca et al. found that some indicators, such as financial development, banking competition and institutions promoting the better implementation of contracts, are associated with a more dynamic distribution of growth and a wider share of growing and shrinking companies. More restrictive measures regarding employment/labour laws and R&D support are associated with smaller shares of HGFs.[59]

This analysis has a different point of view than the support policies that directly address HGFs, but at the same time, it can give further insight into some factors that have a more significant impact on the ability of one country than another to achieve a higher share of HGFs.

It is clear that, on the one hand, there is a need for policymakers to contribute on the growth of these companies to increase jobs and productivity and, on the other hand, there is a challenge in targeting and measuring their future growth. For this reason, the last topic addressed in terms of public policy is a framework proposed by the World Bank to support countries in the development of appropriate policies that do not waste financial resources. At this point, the "ABC" framework proposes a reorientation of policy that does not have to search for potential HGFs but rather implement the ABCs of entrepreneurial growth: improving Allocative efficiency, encouraging Business-to-business spillovers, and strengthening firm Capabilities.

As shown in Figure 3, to achieve the "firm dynamism and growth" objective, not only the ABCs mentioned above but also three more crosscutting elements are needed. One is the importance of data improvement at the firm level. Although the collection of data available on issues such as wealth distribution or poverty has increased and improved over the last few decades, there is still a lack of appropriate data collection on the distribution of companies. Concrete action by international organisations and statistical agencies is necessary to succeed in this objective. [60]

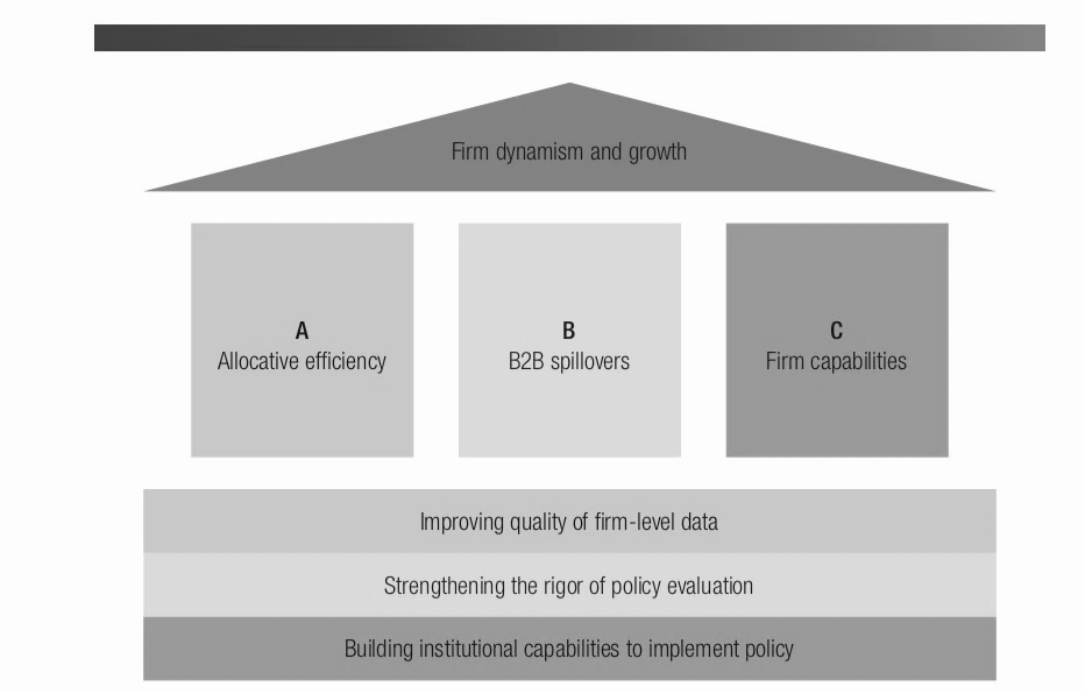
The second element is strengthening the rigour in policy evaluation. As we mentioned earlier, many policies lack a rigorous and logical evaluation practice. Integrating these impact

[58] See footnote 56
[59] See footnote 33
[60] See footnote 13, pp. 133-134

assessments as part of the design and implementation of programmes can ensure that the public resources invested to achieve the objectives and enable policymakers to correct and adapt programmes along the way.

The last theme is institutional capabilities to implement policies. Ensuring that institutions have the right mandate and the necessary financial resources is crucial to achieving this goal. There also need to be cooperation between the various departments and ministries, which should not be a one-off action. Furthermore, they should have the ability to replicate policies adopted by other countries without copying and pasting but analysing them to understand how to integrate with social elements.[61]

Figure 3 - The "ABC" Framework to Support Firm Dynamism and Growth



Source: World Bank Group 2019[62]

After having dealt with the above elements, the analysis of the framework gets to the heart of the discussion. The first key point is *allocative efficiency*, which is the ability to distribute resources optimally. Regarding this concept, the literature has given a fundamental role to competition as a force that pushes the less efficient companies to leave the market and transfer resources to the more efficient companies. Countries that are more productive and dynamic shown to have a high

[61] See footnote 13, pp. 131-135

[62] Grover Goswami, A., Medvedev, D., & Olafsen, E. (2019). High-Growth Firms: Facts, Fiction, and Policy Options for Emerging Economies. High-Growth Firms: Facts, Fiction, and Policy Options for Emerging Economies. The World Bank. https://doi.org/10.1596/978-1-4648-1368-9

rate of industrial turnover.[63] This observation can explain, for example, the differential in 2014 between Indian and Mexican companies against American ones, which grow four times faster than the former. This difference happens because very often emerging economies are characterised by business dynamics that keep resources in inefficient companies without letting them exit or grow.[64]

Because such market distortions are challenging to analyse and recognise, policymakers could analyse productivity growth by breaking it down into three different moments. *Entry* policies should ensure that new, more productive companies can enter the market quickly by reducing regulatory burdens. Instead, *exit* policies should ensure that less productive companies leave the way to more efficient companies. Therefore, they should ensure that incumbent companies, defined in some specific situation as "zombie" firms[65], do not prevent the growth of new ones by increasing allocative inefficiency. Finally, *reallocation* policies should ensure greater flexibility in the labour market and financial services.[66]

The second point of the framework is *B2B spillovers* which, as we have already highlighted in section 1.2.2, could represent a high growth opportunity for companies. Spatial policies can maximise the benefits of agglomeration by reducing problems of coordination or congestion externalities. Urban policies should then either improve land use or increase the use of public transport, which can provide better spatial connectivity and agglomeration formation. About foreign direct investment (FDI), these policies include strengthen the regulatory climate, ensure a stable macro and political environment, and facilitate access to resources. Very often, an incentive to attract foreign investment in one's home country is to ensure lower tax rates than other tax systems. A final issue is direct tools to support B2B spillovers. Examples are the science and technology parks, physical places where companies of different natures collaborate using the facilities provided by institutions.[67]

The last element in the framework analysis represents the *firm capabilities*. Companies cannot find these elements on the market and must, therefore, learn them. Managerial skills and innovation are clear examples that increase the possibility of high growth. Policies that can enhance the capabilities of firms tend to be direct rather than regulatory and include financial incentives, non-market incentives and direct procurement of goods and services. The first type

[63] Haltiwanger, J. (2015). Top Ten Signs of Declining Business Dynamism and Entrepreneurship in the US. Paper prepared for the Kauffman Foundation New Entrepreneurial Growth Conference. Amelia Island, FL. June 17–19.
[64] Hsieh, C. T., & Klenow, P. J. (2009). Misallocation and manufacturing TFP in China and India. Quarterly Journal of Economics, 124(4), 1403–1448. https://doi.org/10.1162/qjec.2009.124.4.1403
[65] McGowan, M. A., Andrews, D., & Millot, V. (2017). Insolvency regimes, zombie firms and capital reallocation. OECD Economics Department Working Papers, 4(2), 10–18. https://doi.org/10.1080/09692290.2016.1262446
[66] See footnote 13, pp. 138
[67] See footnote 13, pp. 139-141

covers vouchers, equity financing and public procurement but also indirect interventions such as tax incentives. These interventions aim to encourage companies to undertake actions that they would not otherwise take. For example, grants, equity financing or tax incentives can address credit market imperfections and lead a company to undertake a series of projects. Non-market incentives are inducement instruments and recognition awards. Although similar to the previous ones, they shift the burden of failure from the government to the participants. This risk is, of course, offset by an award, which encourages companies to participate. The last nature of incentives is technology, extension support, and advisory services. These processes help companies to reduce their information asymmetries regarding their capabilities and address shallow markets. Besides, they serve as advisory firms on workforce training, supply chain development, customer acquisition, exporting, and technology transfer. Famous examples are the Manufacturing Extension Partnership in the United States and SPRING and A*STAR in Singapore.[68]

The analysis of external factors that are influenced and, at the same time, can influence high-growth firms presents many insights and impressive results even though they cannot be considered exhaustive. Besides the need for further researches, in the next paragraph, there is an analysis of these companies from a different point of view. Hence, the internal factors that may determine the success of a company are taken into consideration, also taking up the discussion just ended on managerial skills.

1.3. Firm-Level Analysis

The ability to overcome several challenges along the way and, at the same time, take up the opportunities that a company faces are critical elements to survive in the market. Although the business environment and external factors indeed play an almost crucial role in a company's future, it is also true that a company has the necessary tools to address these pitfalls. These tools are elements within the company, under its control, that can make a difference to the competition. For this reason, this paragraph moves the attention to company level and its elements. Firstly, it focuses on a strategic analysis of HGFs that covers five key management points: human capital, strategy, human resources management (HRM), innovation and capabilities. The examination of these elements concludes with an agenda for future inquiries. Then the study addresses the importance of the entrepreneur in the various phases of HGFs. This section focuses on the skills required to succeed at decisive moments and the dynamism of this role during the HGFs lifecycle.

[68] See footnote 13, pp.142

Finally, the chapter closes with a final review of the results obtained so far from the literature and the areas that deserve further investigation.

1.3.1. The Strategic Management of High-Growth Firms

What differentiates the management of high-growth companies from other companies? What are the most important strategic elements, and how do they contribute to the disruptive growth trajectory of these companies?

Attempting to answer these questions, relevant for a complete understanding of HGFs, it is necessary to review the results produced by the literature until now. An analysis of the last thirty years on the subject of strategic management in HGFs has led to the definition of five growth drivers that more than others play a decisive role in these companies.[69] The following studies start from this significant literary review results by defining these five elements and their characteristics. Subsequently, through the combination of these drivers, the research presents an agenda for the future strategic analysis of high-growth firms.

Human capital is a prevalent theme in the literature of high-growth companies. It was first introduced into the definition of capital by Adam Smith in the 17th century. Today, the human capital of a company means the value created by the set of skills and knowledge of its workers. Within the latter, there is the level of education, management experience, cognitive skills and domain expertise.[70] Concerning the level of education, there is a distinction between founders-managers and employers. A higher level among the first group has a positive correlation with high growth. Also, companies with a team of people with a high level of human capital endowment (i.e. PhD) showed higher growth than the others.[71] While for the group of workers, the results seem to contrast those of the first group. A higher level of education contributed to high growth only in southern and continental European Union member states. While about the new entrant countries, mostly Eastern European, this relationship was negative.[72] Therefore, it is possible to observe that the effect on growth varies between the two groups and that the level of education of key workers, i.e. founders or managers, is a driver of such growth. The level of management experience is a highly underestimated aspect of the research but has led to meaningful results. Indeed, there is a positive relationship between management teams with past,

[69] Demir, R., Wennberg, K., & McKelvie, A. (2017). The Strategic Management of High-Growth Firms: A Review and Theoretical Conceptualization. Long Range Planning, 50(4), 431–456. https://doi.org/10.1016/j.lrp.2016.09.004

[70] See footnote 70

[71] Almus, M. (2002). What characterizes of fast-growing firm? Applied Economics, 34(12), 1497–1508. https://doi.org/10.1080/00036840110105010

[72] Hölzl, W. (2009). Is the R&D behaviour of fast-growing SMEs different? Evidence from CIS III data for 16 countries. Small Business Economics, 33(1), 59–75. https://doi.org/10.1007/s11187-009-9182-x

i.e. not novice, experience and high growth. One of the underlying reasons for this correlation is undoubtedly the capacity of managers, belonging to this category, to have learned in past experiences the "rules of the game" and therefore have a better ability to conduct operations.[73] Besides, a study conducted on US companies confirmed this trend by stating that older and larger HGFs have as chief executive officers (CEOs) with a wealth of experience gained in the past.[74] However, the second part of the paragraph, concerning the figure of the entrepreneur, takes up this particular aspect in greater detail. Concerning cognitive ability, studies show that a higher number of CEOs analytical, practical and creative intelligence support multiple actions and choices that led to high growth. Moreover, another critical successful feature is responsive to training and practice, i.e. the ability to refine their managerial practice over time.[75]

Those elements do not necessarily go hand in hand with higher education. Different findings show that when responding to crises, some highly qualified managers may run the risk of being cognitively stigmatized.[76] Therefore, higher education alone does not have a positive effect on their cognitive ability to respond to high growth. It can be possible to summarize that cognitive skills are a driver of growth when they are developed and refined over time through practice and training. Finally, the last element of human capital is domain expertise. Several studies highlighted some areas of expertise that led to positive results on growth. Previous experiences within the company's sector is a positive feature related to high growth events. It allows founders-managers to have a critical understanding of industry dynamics, such as relationships with suppliers, customers or business partner networks. Moreover, this element is also relevant in the transfer of know-how from previous business initiatives to the new one, affecting its growth and survival.[77]

The *strategy*, defined by Porter as the choice of a set of different activities to provide a unique mix of value, is a recurring element in the analysis of HGF.[78] It presents two macro-areas: strategic planning and differentiation. The company's ability to develop well-defined procedures for the business plan, such as recruitment tests, analysis of competitors, control and coordination departments and allocation plans for resources, is directly related to a high growth rate. Besides, a further contribution comes from the ability of managers to create collaborative structures that

[73] Brüderl, J., & Preisendörfer, P. (2000). Fast-Growing Businesses: Empirical Evidence from a German Study. International Journal of Sociology, 30(3), 45–70. https://doi.org/10.1080/15579336.2000.11770218
[74] Baum, J. R., & Bird, B. J. (2010). The Successful intelligence of high-growth entrepreneurs: Links to new venture growth. Organization Science, 21(2), 397–412. https://doi.org/10.1287/orsc.1090.0445
[75] See footnote 75
[76] Muurlink, O., Wilkinson, A., Peetz, D., & Townsend, K. (2012). Managerial Autism: Threat-Rigidity and Rigidity's Threat. British Journal of Management, 23(SUPPL. 1). https://doi.org/10.1111/j.1467-8551.2011.00790.x
[77] See footnote 70
[78] Porter, M. E. (1996). What Is Strategy? Harvard Business Review, 74 (6): 61–78.

can give a shared vision of the company's objectives. The company must then be able to adapt these procedures to its dynamic path and then adjust its routines and processes during the various phases. Although strategic planning plays an important role to achieve growth, further studies that take into account the speed of growth and the size of the company are needed to exploit the potential of this relationship. In terms of differentiation, to product or market, this is a strong predictor of growth. To understand its positive influence, we need to consider its nature. Indeed, it is a differentiation based on specialization and product development rather than product diversification. Companies that have succeeded in pursuing this strategy have shown higher levels of profitability and have been able to better understand the market because of their need to follow the requirements of customers and suppliers.[79] Hence, quoting Porter it is a strategy "about being different".[80]

Human Resources Management (HRM) is the third strategic driver of growth. In this research, HRM refers to the body of human resources operations within a company that allows achieving its goals. In this vision, it is necessary to enhance the value of resources in order to achieve optimal results. The analysis considers three main aspects of HRM: the selection, training and incentive compensation of employees and their relationship with high growth.

Since the earliest studies in this field, there was a positive relationship between effective management of the recruiting department and company growth. In this direction, successful HGFs have heavily invested in the development of that department to involve high-level professionals. The latter, through elaborate recruitment practices, company orientation and new recruiting, guarantee the employment of right talent and ensure the transmission of the corporate vision.[81] However, another hypothesis that emerged from the literature raises doubts about the willingness of HGFs to hire qualified workers. As shown by a study in Sweden in 2014, HGFs often tend to hire employees with general rather than specialized human capital.[82] At other times, rather than favouring talent, some HGFs have reported that they hire people with a flexible mindset and capacity of adapting to the progress the company was undergoing. In conclusion, it seems that hiring staff is closely linked to the company's willingness to grow and their ability to make a significant contribution. Training and employee development practices also have a positive impact on the company's growth and at the same time, provide an element of distinction from the competition. The latter characteristic stems from the company's ability to train its staff by

[79] See footnote 70
[80] See footnote 79
[81] Hambrick, D. C., & Crozier, L. M. (1985). Stumblers and stars in the management of rapid growth. Journal of Business Venturing, 1(1), 31–45. https://doi.org/10.1016/0883-9026(85)90005-9
[82] Coad, A., Daunfeldt, S. O., Johansson, D., & Wennberg, K. (2014). Whom do high-growth firms hire? Industrial and Corporate Change, 23(1), 293–327. https://doi.org/10.1093/icc/dtt051

providing them with skills that are difficult for competitors to obtain.[83] Finally, the employee incentive system has two parallel streams of thought. One of them argues that an incentive system for companies has a positive correlation with its growth. Main reasons lie in the fact that first of all the employees of these companies are subject to hardworking and consequently, incentives can help to sustain their productivity.[84] Additionally, stock option plans can align their goals to those of the company. Another view is that incentives to managers can sometimes harm the company's growth. Because executives in these companies have sensitive information about them, they may be reluctant to share it if they have vested interests.[85]

The theme of *innovation*, also addressed in the previous paragraphs, is heterogeneous in terms of research results and methods. From a strategic point of view, the literary results suggest a distinction between product, process or market innovation when it concerns the effect on growth. While the former tends to create an initial possibility for the company to exploit a given market or serve a new one, process innovations could help the company to sustain its growth over time. The latter conclusion is based on the fact that growth depends on processes and organisational structures. However, the results of the researches have not analysed the possible contingency effects of different types of innovation until now. Further studies could, therefore, provide more precise outcomes on the ability of a specific type of innovation to lead to high growth within companies.[86]

The last driver of growth is *capabilities*. It is the ability to exploit resources, processes and change them in order to achieve the desired result or benefit for the company. Although very close to the concept of individual capabilities, they refer to processes and coordination relationships. They can be managerial, financial or innovation capabilities. The former relates to the ability to manage different types of strategic challenges, like overcoming some competitive and organisational barriers. Financial capabilities are a determining factor for high growth as defined by Todd and Taylor: "Growth requires funding, and the provision of finance is a particularly important strategic skill".[87] Once obtained, companies need to leverage other types of skills to achieve high growth,

[83] See footnote 70

[84] Barringer, B. R., Jones, F. F., & Neubaum, D. O. (2005). A quantitative content analysis of the characteristics of rapid-growth firms and their founders. Journal of Business Venturing, 20(5), 663–687. https://doi.org/10.1016/j.jbusvent.2004.03.004

[85] Parker, S. C., Storey, D. J., & van Witteloostuijn, A. (2010). What happens to gazelles? The importance of dynamic management strategy. Small Business Economics, 35(2), 203–226. https://doi.org/10.1007/s11187-009-9250-2

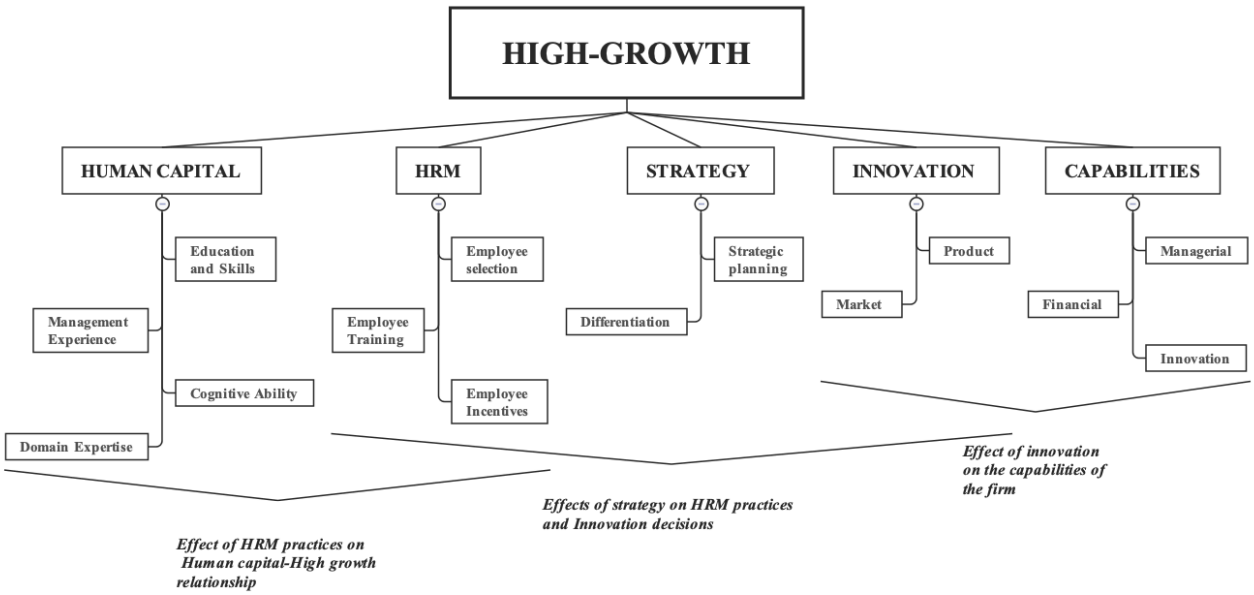
[86] See footnote 70

[87] Todd, A., & Taylor, B. (1993). The baby sharks: Strategies of Britain's supergrowth companies. Long Range Planning, 26(2), 69–77. https://doi.org/10.1016/0024-6301(93)90137-5

and hence financing can be seen as its catalysts. Finally, innovation capabilities reflect the company's ability to reinvent itself to meet environmental and technological challenges.[88]

The previous analysis identified five strategic drivers recurrent in the literature, highlighting their characteristics and implementation possibilities. In this regard, the following section aims to show the possible relationships that exist between the drivers and propose possible future research ideas to understand the nature of these relationships. Since they affect growth, it is worth analysing them not only individually but also in combination with each other. Figure 4 shows these relationships considering strategic drivers as independent variables and high growth as the only dependent variable.

Figure 4 – Strategic drivers' relationships with High-Growth



Source: Personal Elaboration

Assuming that each independent variable affects growth, the effects that some variables may have on others is analysed. The first relationship concerns human capital and human resources management. Efficient management of HRM practices, such as job training, contributes positively to human capital formation and thus to growth. Next, the strategy affects HRM practices, such as the choice to adopt some recruiting or training policies, and on innovation. Since the latter is of various types, the strategy adopted by the company influences the decision of one of them. The last relationship, on the other hand, involves innovation and business skills. When a company is innovation-oriented and reinvents itself to adapt to the needs of the market and the external environment, it has to be able to change its capabilities as well quickly.[89]

[88] See footnote 70
[89] See footnote 70

At the end of this analysis, it is possible to state that, although the research has produced outstanding results regarding the strategy to achieve a high level of growth, it needs more attention in the future. Furthermore, the strategic study needs research that does not identify and analyse the effect of individual drivers but the combined effects of several factors marking growth.

1.3.2. The Role of The Entrepreneur During the Lifecycle of HGFs

Section 1.2.1 presents the various life stages of high-growth companies through the analysis of their unusual trajectory. Every single phase brings with it changes that the company, as an organisation, must be able to embrace and exploit in order to achieve its ambitions. In the meantime, the manager has to answer positively to these changes. Therefore, this section analyses the figure of the entrepreneur of a high-growth company in the various phases considering the changes that he should make to the organisation and its way of managing it.

During the initial phase, the entrepreneur has a fundamental role in structuring the resources supporting the business model. This period is marked by uncertainty and the demand for the entrepreneur to achieve profitability through obtaining financing and hiring employees to implement the various business operations. Besides, he needs to create a flexible structure ready to adapt to several scenarios and competitive contexts.[90] Within this period, entrepreneurial action, recognised as the entrepreneur's judgemental capacity, is shaped. The latter has been defined by Klein as "residual, controlling decision-making about resources deployed to achieve some objectives; it is manifest in the actions of individual entrepreneurs; and it cannot be bought and sold on the market, such that its exercise requires the entrepreneur to own and control a firm."[91]

Moreover, it is set precisely at this stage because the entrepreneur judges in conditions of uncertainty, with scarce resources and looking to satisfy future market preferences. Subsequently, if the company begins to overgrow and reaches one of the turning points (see figure 2), it has to change its organisational capabilities. But how should the entrepreneur respond to these necessary changes? He should begin to reflect on a new role in the organisation, establishing more formalised processes to provide greater hierarchical control. Furthermore, during this phase managers may consider internalising operations that were previously outsourced due to the size of the company. Networking relationships with creditors, investors, suppliers and perhaps trade associations play a significant role in this phase as they can contribute to finding resources and

[90] Sirmon, D. G., Hitt, M. A., Ireland, R. D., & Gilbert, B. A. (2011, September). Resource orchestration to create competitive advantage: Breadth, depth, and life cycle effects. Journal of Management. https://doi.org/10.1177/0149206310385695

[91] Foss, N. J., & Klein, P. G. (2012). Organizing entrepreneurial judgment: A new approach to the firm. Organizing Entrepreneurial Judgment: A New Approach to the Firm (pp. 1–299). Cambridge University Press. https://doi.org/10.1017/CBO9781139021173

innovations. Over time the company stabilises in the market as well as its growth rates. In this phase, the role of the manager is to maintain a competitive advantage over the competition. Therefore, there is a claim to explore new ways to diversify the products or create new developments. Also, he should pay attention to less efficient units in order to allocate their resources on other projects and achieve better results. Management that succeeds in transferring this dynamism into the company's strategy is certainly able to take advantage of new growth opportunities for the company. Those who fail to do so, and therefore show a lack of managerial skills are an obstacle to the company's growth. [92]

1.3.3. Future Perspectives

The research on HGFs contains many relevant questions, outstanding answers and interesting economic issues. The analysis carried out above established some problematic features related to the study of these societies, which surely future researches will be able to clarify. The possibility of defining company growth with different measures brings consequences to the definition of HGFs. Although there is still no homogeneity of definition, this is essential for the comparison of the results that the research brings. Indeed, the current scenario denotes a significant number of impressive results which, due to the different research methodologies, cannot provide a universal conclusion. The use of the definition provided by the OECD has become popular because it allows identification of high-growth companies without the need to obtain micro-data, which very often are not available. However, some authors have questioned this definition because it excludes all companies with less than ten workers, although these may show very high levels of growth. Subsequently, the use of the employment measure could also be a sub-optimal choice. From a public policy point of view, targeting companies according to their employment rate can be flawed if it creates incentives that lower productivity.

Besides the problems of definition, the chapter shown a difference, in terms of HGFs share, between countries. This difference is a phenomenon of keen interest for future research because it has its origins in various institutional environments, public policies and the organisation of economic activities. Finding explanations for the relationship between a given country and the factors that lead its companies to grow faster than others is a significant challenge for policymakers. This difference could explain, for example, the distribution of unicorn companies among the various countries of Europe and then compare them with the much larger number of these companies in the US or China.

[92] See footnote 91

Finally, further attention should go to the internal dynamics of the company and the management that distinguishes successful companies from "normal" ones. Regarding the latter point, an in-depth and combined analysis of several successful companies and their strategies could provide further results for the research. Thus, there is a need to analyse, where possible, the strategic drivers of growth and their inter-relationship to provide comprehensive management answers. Moreover, the contribution of entrepreneurial researches could provide support for the integration between management dynamics and the behaviour of managers during the various phases of the life cycle.

Although these issues are of considerable interest and relevance, the following thesis aims to analyse this type of company from an evaluation point of view and would, therefore, be out of context to pursue such research. Thus, the next chapter analyses the principal valuation methodologies applied and the problems they face with high-growth firms. It then offers alternative valuation tools for this type of societies, providing the basis for the third chapters, which focuses on two empirical studies.

2. Tools and Methods of Business Valuation

The valuation of high-growth firms is the primary topic of this thesis. The need to deepen this subject lies behind the desire to clarify and examine the potential criticalities of these companies in their valuation process. When talking about valuation, we should always remember that this is one of the most critical phases of business decision processes and acquires greater complexity when it comes to particular situations. High growth firms are a clear example of this. As we see later, due to some peculiarities of these companies, traditional methods run into different problems during their valuation process. For this reason, the structure and purpose of this chapter focus primarily on identifying these critical issues and then to give alternative valuation tools, considered more appropriate for high-growth firms.

At the beginning of the paragraph, we first discuss the incorrect categorisations that many practitioners make about these companies. Then, as they are necessary for subsequent evaluation, the principal financial characteristics are presented. A further focus is made on the critical relationship between price and value, which involves several quotations of high-growth companies. During recent years, we have seen several IPOs that have reached prices well above those expressed by the analysis of the fundamentals. Once completed this introduction, traditional valuation methods like discounted cash flow and relative valuation are analysed. While it is assumed that the reader is familiar with the subject, this primary analysis allows for a more transparent discussion of the following sections. Concerning the latter, they first focus on the valuation criticalities of traditional methods. They then provide different solutions to achieve the value of high-growth companies in a more conscious way.

2.1. Measurement and Characteristics

In the last chapter, we have seen how high growth companies can be young, as start-ups with an innovative product that is successful on the market, but also mature companies that try to reinvent their businesses to take advantage of new growth opportunities. De facto, distinguishing between young and mature companies seems wrong to recognize high-growth companies. Despite this evidence, in practice, there are different ways of measuring growth and mature companies by relating them to growth. Some ways of listing high-growth companies are used in the world of finance, although they are quite approximate. Compared to more effective methods, they are very generic and easy to apply. Thus, before analysing the main financial characteristics of high-growth companies, these wrong classifications of HGFs are presented together with a more effective traditional measurement system.

2.1.1. Misleading Categorization of High-Growth Firms

Usually, mature companies are distinguished from high-growth companies. This because there is a false belief that companies with many years of life, can no longer have high growth rates. As seen before, this distinction is entirely wrong because the age of companies does not influence their growth path. Since the role of categorization in the valuation field is to determine the financial characteristics in common among companies, it is necessary to present and analyse them to provide alternative solutions to measure them precisely. Hence, according to Damodaran, the following classifications used in practice are subjective and have numerous flaws. These measurements include:

- *sector-based*: according to which companies are high-growth depending on the sector to which they belong. Thus, for example, companies in the technology sector are high growth, while those in the steel sector are mature. It is immediately noticeable that this definition seems rather sketchy, considering companies like Apple which, despite being in a technology sector, is a mature company;
- *analyst growth estimates/growth history*: they distinguish high growth firms from mature ones according to the future growth in earnings estimated by analysts' forecasts. When earnings growth cannot be forecasted, they use data from the past. However, both situations define the companies with the highest growth rate as high growth companies. Besides, the threshold within which this rate must fall is subjective, so if the market grows by 5%, a company that grows by 15% can be considered high growth. The limitation of this measurement is that it is focused exclusively on earnings and not, for example, on other indicators such as sales.
- *market-based*: classify high-growth versus mature companies according to the level of market multiples at which they trade. Thus, high growth companies are those that trade at higher multiples of their earnings, sales, or book value. Because we aim to analyse the market and its ability to deliver the correct value of a given company and this approach assumes that the market is right, it is against our objective.[93]

Since subjective factors strongly influence the three measurements above, and a perfect classification of HGFs does not exist, it can be compelling to analyse the nature of their balance sheet items. Unlike mature companies, HGFs derive most of their value from growth assets. These assets depend not only on how much growth is anticipated but also on the excess returns that follow this growth. At this point, it is useful to remind that growth investments have no value

[93] A. Damodaran. (2018). The Dark Side of Valuation: Valuing Young, Distressed, and Complex Businesses. Pearson FT Press. ISBN: 9780134854267

when the company has a return on capital equal to its cost of capital. Although these measurements are appropriate for HGFs, they can only be made after valuing companies and their fundamentals.

2.1.2. Main Financial Characteristics

Despite some differences such as sector, growth outlook or size, high-growth companies share some characteristics that make an impact on how we value them. The first feature concerns *dynamic financials*. These companies are valued with the support of their financial statements such as the balance sheet, income statement or cash flows. These are continually evolving for high-growth firms, not only from year to year but also in shorter periods. Thus, sales in the past year can be very different from those in the quarter just ended, only a few months later. [94] An example of these substantial variations is provided by the company Inmode Ltd. which in the period 2017-2020 saw its sales increase from 53 million to 156 million and its net income from 8 to 61 million.[95]

The following distinctions of these companies concern the *public and private capital*. It is common to assume that firms that grow a lot in their first phase become public to raise additional funds on the capital market. In practice, this transition is not standardisable or predictable for all high-growth firms for different reasons. First, it is necessary to consider the difference between economies, the role of institutions and the development of capital markets. Moreover, access to the capital market may vary over the years. In 2008 the number of IPOs registered in America, due to the financial crisis, was only 31 transactions compared to the average of 178 over the last 20 years.[96] Finally, another critical determinant feature is the sector in which the company operates. Some sectors allow access to capital markets earlier than others, like that of healthcare which in 2014 recorded 102 IPOs compared to Business Services, which recorded only 8 out of a total of 275.[97] Therefore, when referring to high-growth firms, both public companies and those with founders or venture capitalists as their primary equity holders are considered.

The fourth characteristic that HGFs have in common is the *size disconnect*, i.e. the difference between the market value and the book value of these companies. The former is very often much higher than the latter because it incorporates the growth prospects of the assets while the book value is often not.[98] Amplifon S.P.A., a high-growth Italian company, specialized in the earing care sector, had a book value of approximately 709 million in 2019 compared to a market value

[94] See footnote 93
[95] Data retrieved by https://www.thomsonreuters.com/en.html
[96] Data retrieved by https://www.statista.com/
[97] Data retrieved by https://www.thomsonreuters.com/en.html
[98] See footnote 93

of 4.5 billion.[99] In addition to this difference, it is also necessary to consider those cases of companies which are valued at millions or billions of dollars and have a low number of sales and negative earnings. It was the case of Tesla, which at the end of 2019 suffered losses of 775 million but had a market cap of approximately 77 billion.[100] The *share of debt* to their value is another common feature among high-growth firms. These very often do not have adequate cash flows to support the introduction of new debt, and so, even in sectors where debt is the preferred source of financing, these companies have a lower debt ratio than the more mature ones. The last point of common ground concerns their *market history*. These companies usually have a short and shifting history. Since the valuation depends strongly on some market values, such as the Beta, assessing data of these companies is challenging due to a short time horizon of analysis and volatility that characterize their numbers.[101] For example, an analyst who wanted to evaluate Univar, an American company specialized in chemical and ingredient distribution, would find it rather complicated considering the short period on the market, from 2015, and the fluctuating variation that its numbers have had.[102]

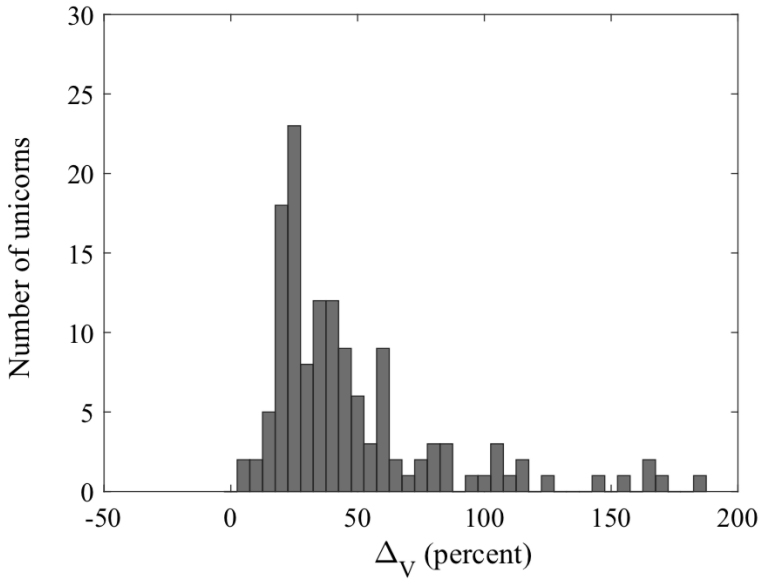
2.1.3. Value vs Price: the Field of Asymmetry

"Price is what you pay, value is what you get". Taking up Benjamin Graham's concept of value investor in the book "The Intelligent Investor", this Warren Buffet's quote can be seen as the primary strategy that made him one of the most famous investors in the world.[103] What is important to note in this quote, in the first instance, is a difference between the market value that a company has, judged arbitrary, and the intrinsic value, which is fundamental. Thus, one can decide to sell a gold bar for ten euros, and since that decision is up to the seller, the price is subjective. It is known to all, however, that the value is very different from the selling price of the bar. Although in the stock market, we are not able to perceive the intrinsic value explicitly, this difference between price and value occurs very often. The market can ignore the value of a company for a while, quoting Benjamin Graham "In the short run, the market is a voting machine but in the long run it is a weighing machine." Trying to figure out when the market converges towards the fair value of a company, although W. Buffett does not find it interesting, would be very difficult to calculate.[104] However, the reasons why the share value may be higher or lower

[99] Data retrieved by https://www.thomsonreuters.com/en.html
[100] Data retrieved by https://www.thomsonreuters.com/en.html
[101] See footnote 93
[102] Data retrieved by https://www.thomsonreuters.com/en.html
[103] Data retrieved by https://www.forbes.com/
[104] Buffett, W. E., & Cunningham, L. a. (1997). The Essays of Warren Buffett : Lessons for Corporate America Essays by Essays by. Business, 219.

than the intrinsic value are numerous and belong to very different semantic fields, from human behaviour to market trends. Unlike the price we see, the intrinsic value of a company is challenging to obtain, and to do so requires accurate analysis and valuation methods that understand as much as possible the drivers that affect this one. While these methods are addressed later, it is also right to consider the market and the fact that many times it seems irrational compared to fundamentals. Even high-growth companies that, enjoying very high expectations and sometimes utopian prospects, very often show huge differences between value and price. A recent study by Will Gornall and Ilyaa Strebulaev examines this asymmetry about price and value of US unicorns.

Figure 5 - Distribution of unicorn overvaluation



Source: Journal of Financial Economics 2020

This figure shows the percentage change between fair value and post money valuation (ΔV). According to a sample of 135 Us unicorns, it was found that their post-money valuation was on average 48% higher than the fair value. In 14 of these companies, it was even 100% higher. Besides, by making value adjustments to these companies, about half, precisely 65, lost their unicorn status, thus falling below one billion of valuation.[105] This research, although not entirely consistent with the type of companies being evaluated in this thesis, is in line with the asymmetric price-value ratio of high-growth companies. Analysing and studying the company before investing is a golden rule for any investor. Thus, the following paragraphs do not refer to the market price of a company and its relationship to fair value. On the opposite, they provide the

[105] Gornall, W., & Strebulaev, I. A. (2020). Squaring venture capital valuations with reality. Journal of Financial Economics, 135(1), 120–143. https://doi.org/10.1016/j.jfineco.2018.04.015

tools to derive this value most reliably, studying the characteristics of HGFs and adapting the most used valuation models.

2.2. Methods of Business Valuation

Before examining the difficulties faced in the assessment of high-growth firms, it is necessary to present and explain the most widely used methods of valuation by practitioners and academics. This explanation merely provides support for the reader since it is assumed that she already has the right background on these topics. The first method is discounted cash flow, which defines that the value of a company on a given date can be represented by the cash flows that it produces during its future life, appropriately discounted to reflect time and risk factors. The second is the relative valuation, which defines the value of a company based on multiples of specific key economic business measures that are expressed by the market and refer to listed companies operating in the same sectors as the company to be valued. Subsequently, once the valuation dynamics are described, it is possible to expose the problems related to the valuation of high-growth companies in terms of both intrinsic and relative valuation.

2.2.1. Discounted Cash Flow Method

The discounted cash flow method is based on the assumption that the value of an asset is the present value of expected cash flows on the asset, discounted at a rate that reflects the riskiness of these flows. Hence, a correct valuation requires the ability to obtain all possible information about such assets and to conduct a valuation model that accurately determines their intrinsic value. This precision is not possible in reality and quoting Damodaran the "intrinsic valuation is, in some sense, an act of faith." In any case, since it is not possible to know the intrinsic value exactly, the only option is to conduct an analysis that manages to understand, as much as possible, the main drivers of this value. Thus, there are four basic inputs needed to conduct the value estimate: the cash flows of existing assets, growth, discount rate and terminal value.[106]

Cash flows to the firm can be measured in two different ways: the first is to add up the cash flows belonging to all claim holders of the company. Therefore, being this one financed by equity and debt, adding the cash flow to equity holders to those of the debtholders. A second way is to start from the operating profits of the company and estimate the cash flows before debt payments but after reinvestment. The formula is as follows:

$$\text{Free Cash Flow to Firm} = \text{After tax operating income} - (\text{Capital Expenditures} +$$

[106] See footnote 93

$$- \text{Depreciation} + \text{Change in non cash Working Capital})^{107}$$

The risk of an asset is another input to consider during the valuation because the riskier cash flows should be valued less than the more stable ones. The way to involve this risk in the valuation is through the discount rate. Thus, a high discount rate reflects a riskier cash flow and vice versa. Before analysing the characteristics of the discount rate, we should be noted that company may have several risks: equity, debt or firm's operations. From a balance sheet perspective, the first two belong to the liabilities' risks of a company, and the last belongs to those of assets. Since they have to balance each other, the equity risk is partly determined by the risk of operations and partly by choice of debt needed to finance the business and vice versa. In terms of discount rates, equity risks are measured by the cost of equity while business risks by the cost of capital. Since the latter is given by the weighted average of the cost of equity and the cost of debt, it is necessary first to consider these two aspects. Analysing equity risk and finding a measurement is difficult for two reasons. The first is that the cost of equity, unlike the cost of debt with interest, has an implicit cost that is not observable. The second reason lies in the different perception of risk among the various equity holders, that have different claims in terms of expected returns. This problem arises in publicly traded companies, where there are many investors, who also differ in terms of risk adversity. Thus, corporate finance theories developed the concept of the marginal investor, i.e. the one that has the largest number of shares and therefore, can influence their price through trading. Assuming that the marginal investor has a diversified portfolio, the risk of the investment is measured as the risk added to his portfolio. Hence, only the portion of the risk attributable to the market or economy should be built into expected returns. From here, the corporate finance theories have developed several alternatives to measure this non-diversifiable risk. The first is the Capital Asset Pricing Model, which incorporates this risk, or exposure to all market components, by measuring the beta assigned to a company.[108] This model defines the expected return as a function of three variables such as risk-free, beta and equity risk premium through the formula:

$$\text{Expected return} = \text{Risk Free Rate} + \text{Beta}_{\text{investment}} \times \text{Equity Risk Premium}$$

where the risk-free rate is the interest expected from a risk-free investment, the beta represents the volatility of the investment security and the equity risk premium is the excess return that investments in stock markets provide over a risk-free rate.

[107] Berk, J., & DeMarzo, P. (2014). Corporate finance; Corporate finance. The Prentice Hall Series in Finance, 1(January), 1104. https://doi.org/10.1016/S1574-0102(03)01011-2

[108] Damodaran, A. (2002). Investment valuation: Tools and techniques for determining the value of any asset. Progress in Brain Research. https://doi.org/978-1-118-01152-2

Another option to estimate the expected return is the arbitrage pricing and the multifactor models, which allows to search for different non-diversifiable risks and estimate the betas against them. The expected return is determined as a function of the various betas and the premium risk through the formula:

$$\text{Expected Return} = \text{Risk Free Rate} + \sum_{i=1}^{i=n} \beta_i \,(\text{Risk Premium i})$$

where $ß_i$ represents the number of betas in the model. Unfortunately, these two models have different limitations. The first concerns risk-free, which to be defined in this way requires two conditions: it must not be subject to either default or reinvestment risk, considering the magnitude of the time horizon. Frequently government bonds are used as risk-free, but some countries contain a default risk. Besides, considering a 1-year treasury bond for a 5-year valuation exposes it to the reinvestment risk mentioned above, in the same way, that a 5-year treasury bond could do so with semi-annual payments subject to reinvestment. Beyond this first challenge, the beta is usually calculated using linear regression of the stock's returns against those of a market index. The slope tells us how much that stock price changes as the market shifts. The problem with this measurement is that it only happens backwards and therefore, cannot work with companies with a limited history, as is often the case with HGFs. Thus, a solution could be that of the bottom-up beta, i.e. an average of the industry in which the company operates, adjusted for differences in leverage.[109]

Next, the cost of debt reflects the risk that borrowers bear if interest and principal payments are not delivered. For this possible risk of default, debtholders require an additional spread on the risk-free rate when lending money to companies. More specifically, to calculate the cost of debt requires three elements. The first is the risk-free rate, which as a general rule should be the same used for the cost of equity. The second element is the default spread, where for its computation, there are three different methodologies:

- If the firm has outstanding tradable bonds, the market rate for bonds is used as the cost of debt, as long as that bonds are liquid, and they represent the majority of the company's debt;
- if the firm has a bond rating established by a rating agency such as S&P or Moody's, the spread can be estimated using that rating;
- if the company does not have a rating but has debt outstanding, it can estimate a synthetic rating using its financial ratios. An efficient method is to use the interest coverage ratio[110], where higher its value higher the rating assigned to the company.

[109] See footnote 93
[110] Interest Coverage Ratio = EBIT/Total Interest Expenses

The last element required to calculate the cost of debt is the tax rate, more precisely the marginal tax rate because interest allows a company to save taxes at the margin. Thus, the cost of debt's formula is:

$$\text{After Tax Cost of Debt} = (\text{Risk} - \text{Free Rate} + \text{Default Spread}) * (1 - \text{Marginal Tax Rate})$$

Once measured the cost of equity and the cost of debt, it is possible to assess a company's cost of capital using the market data of the financial structure to weigh the two rates. Whereas for the former, it is easier to obtain the value[111], for the debt, many practitioners use the book value.
The formula is as follows:

$$\text{WACC} = \frac{\text{Equity}}{\text{Value}} * \text{Cost of Equity} + \frac{\text{Debt}}{\text{Value}} * \text{After tax Cost of Debt}$$

After getting these two values, it is necessary to consider how the capital structure changes as it leads to different debt and equity costs. A common practice is to adjust the financial structure over the years until to reach a final target.[112]
The third input of the DCF method is *growth*. Estimating the growth rate of a company, as we have seen in Chapter 0, is very challenging because it is necessary to consider the future, an unpredictable factor. The two standard methods applied are the historical and forecasted growth rates and the fundamental growth rates. The first approach considers the past pattern as an indicator of future growth. Note that historical and forecasted growth rates for the same company may lead to different estimates for different reasons such as the measurement of earnings, the period of analysis chosen, and the choice of the average. However, since analysing past periods is only meaningful if they are good predictors of future growth, it should be pointed out that many studies found that:

- the relationship between past and future growth is weak;
- there are scaling problems, so as a company grows, its growth rates slow down;
- companies in some sectors, such as commodities, have periods of high growth followed by periods of low growth.

A solution to the lack of confidence in past performance to predict future ones can be provided by analysts' or industry experts' estimates. While this may be a good strategy, it should be considered that these estimates are subjective and very often tend to be too optimistic in good times and pessimistic in bad times. In any case, studies have shown that neither method is a good predictor of growth.

[111] Market value of Equity = Price * N° of shares outstanding
[112] M. Vulpiani. (2014). Special Cases of Business Valuation. McGraw Hill

Instead, taking into account the alternative method of fundamental growth rates, it estimates growth through the analysis of the corporate drivers that generate it. Through the decomposition of growth (see A), it can be argued that growth is determined by new investments that expand the business or by increasing the efficiency of existing assets. These two types of investments are very different from each other. While the former can be measured through comparison with their original cost, the cost of investment versus its return, the second ones do not have concurrent investment costs that impact on cash flows negatively. For this reason, it is useful to consider some conventional constraints for a correct estimate of growth efficiency. The first concerns the ability to achieve higher growth efficiency for a mature company, against a lower willingness to grow according to new investments. One motivation lies in the higher possibility to obtain growth through new investments from companies that have returns below the industry average than a mature firm that outperforms it. Moreover, the effects of investment growth are higher when the cost of capital is low. Moving from a return on capital from 4% to 5% represents a 25% increase in growth efficiency while moving from 26% to 27% represents a growth of only 3.8%. The second limitation concerns the possibility of increasing efficiency growth only for finite periods. Indeed, it does not seem realistic to assume that a company is inefficient for its lifetime. Therefore, during a valuation, both factors should be taken into account in the high-growth period, but only the return on new investments in terminal value computation. [113]

The *terminal value* is the fourth input to consider during the discounted cash flow method. Hence, since the companies can have an infinite life, it would be tough to perform a year-basis valuation across this broad time horizon. Hence, the value of the operations is divided into two parts to overcome this problem: an explicit one, which evaluates the cash flows for a predetermined future period, and the remaining one, which captures the terminal value. Therefore, the latter reflects the value of the company's operations from the end of the explicit period onward. As these are two different periods, it is necessary to adjust the related cash flows downwards. According to Bini Smaghi, this adjustment aims not to overestimate the company in question and not to include an error changing the result of the estimate about the extension of the period. Therefore, forward-looking analysis of market shares, profitability and turnover ratios of invested capital ensures consistency of the results with the historical performance of the company and the industry, with the consensus and, more generally, with external sources of the forecast.[114]

Three approaches are usually used to calculate terminal value. The multiples method, perhaps the most common, requires a multiple to be applied to earnings in the final year to reach the terminal

[113] See footnote 93

[114] T. Aaron, W. Ballwieser, M. Bini, S. Giuliani, E. Teo. (2020). Roundtable: 10 big issues in business valuation. OIV Journal

value. However, this model is not consistent with the intrinsic valuation, as it usually uses current market data from comparable companies. The other two models result more valid to estimate the terminal value.

The first concerns the liquidation value, i.e. the value assuming that the assets are liquidated in the terminal year. However, to be consistent, both market-based numbers and cash-flow-based estimates have to be considered during the assessment. This method can be very effective with companies that have a finite life and marketable assets such as real estate.[115] When it comes to companies, treat as a going concern, it can be much more consistent to consider that cash flows grow at a set rate in the future. So, considering an infinite time horizon, Gordon's perpetual growth model helps to estimate the terminal value, according to the formula:

$$\text{Terminal Value}_t = \frac{\text{Cash Flow}_{t+1}}{\text{Discount Rate} - \text{Perpetual Growth Rate}}$$

Since in this model, slight variations in growth have a significant effect on the valuation, there are three precautions to avoid abusing its effects. Limiting the growth rate is a first coherent choice because it should take into account the lack of a company's ability to grow indefinitely at a higher rate than the economy. Therefore, following a rule of thumb, the stable growth rate, expressed in nominal or real terms, should not exceed the risk-free rate used in the valuation. Subsequently, it is necessary to adapt the characteristics of the firm to those of more mature companies as it grows. In practice, betas should also be shifted for high-risk companies towards stable growth and use a debt ratio that reflects larger and more stable cash flows. Finally, a final matter concerns the reinvestment rate. Since these are usually lower in mature companies, it should be adjusted by considering lower levels that are adequate to sustain a stable growth rate. Considering the relation between reinvestments and growth (see appendix A), when estimating the terminal value, the main assumption is the reinvestment rate that accompanies this growth instead of the growth rate to choose (see Appendix B).[116]

In conclusion, these four inputs and their determination are necessary elements for the efficient use of the discounted cash flow method. Once obtained the present value, after discounting the cash flows at a risk-adjusted rate, further operations must be performed to obtain the equity value at which we are willing to pay for a share. Thus, starting from the enterprise value estimated in the DCF, all cash and marketable securities, crossholdings in other companies, potential liabilities other than debt and finally the employee options should be considered.

[115] See footnote 108
[116] See footnote 93

2.2.2. Relative Valuation Method

When we make purchases or are in the process of doing so, especially if they are relevant, we make sure that what we are paying is the right price. To do this, we compare prices with similar transactions and items, such as buying a house after checking market prices in the neighbourhood. Thus, relative valuation is a method that estimates the value of a company or investment based on how much investors are willing to pay in the market for similar assets or investments. Therefore, it requires two conditions to be applied: that prices are standardized, often converting them into multiples, and that the companies or investments are similar to each other.

However, in today's markets, it is almost impossible to find companies that are similar to each other because they usually differ in risk, growth opportunities and ability to generate returns. While it is a widely used and easy to apply valuation model, the relative method hides pitfalls to consider carefully during its development. Therefore, the objective is to describe and analyse the measures necessary to perform a relative valuation that is consistent with the value of a company. As mentioned earlier, comparing the price of two identical products is easy, but it becomes harder when dealing with companies with different characteristics. So, comparing such entities requires to standardise the values in some way by scaling them to a common variable. Considering at the numerator the market value or a transaction value for equity, and enterprise value, for the entire business, it is possible to distinguish four multiples based on the parameters used in the denominator. They are:

- earnings multiples: which relate the value of assets to their ability to generate earnings - P/E ratio- or other times with financial indicators such as EBITDA - P/EBITDA;
- book value or replacement costs multiples: the former relates the market value of a company to its book value - P/BV ratio. This multiple is affected by the different accounting policies, the type of sector and the growth potential of the company. The second considers the relationship between the replacement cost of the asset to the denominator - P/Replacement cost or Tobin's Q- instead;
- revenue multiples: unlike the first two, these are less affected by the accounting choices of the companies. They relate sales to market prices or the value of the company - P/Sales and EV/Sales. They also allow a comparison of companies in different sectors;
- sector-specific multiples: these apply to companies belonging to specific sectors because of their particularities. An example is the MAU (monthly active user) multiple used for internet companies. The disadvantages of these multiples are that they cannot relate companies in

different sectors or to the market, and very often, they are not linked to company fundamentals.[117]

Once the standardised market pricing is clear, it is possible to proceed with the relative valuation by following the necessary steps to avoid pitfalls. The first stage is to define the multiple used within its possible alternatives. Indeed, multiples can have different versions of themselves, calculated in different ways that therefore lead to different estimates. Considering the PE multiple, it can be assessed by the current price or the average of the last six months. Also, there are those using current data with those using future values, i.e. forward-multiples. Once established that the analysis gives a homogeneous definition of the multiple, it should be considered that this one is consistent. Since the numerator can be expressed both in terms of equity value and enterprise value, also denominator should be defined in the same terms. Hence, if the numerator is the market capitalisation, the denominator should also use a measure that reflects the equity and vice versa. A multiple such as Price-to-Sales that has different measures is not consistent because, even if it is applied equally to all companies, it could lead to overestimating one company with less debt than another with a different financial structure. Next, another measure to be carried out is the application of this multiple to all the companies considered. In this phase, attention should be paid to the different fiscal years of the company and the potential effects on the multiple. After the first step of definition, values of other companies into the industry/market should be analysed. Doing it requires to perform its distribution in the market by analysing the average and median values of other companies. However, some companies cannot be derived through multiple, as can happen with a high-growth firm with negative earnings. Excluding this type of companies causes positive bias in the multiple's values. So, different solutions manage to avoid this error like be aware of this bias and built in the multiple; calculating the aggregate multiple, considering also companies that are losing money; applying a multiple that can be used for all companies in the group.[118]

Against the common perception that discounted cash flow requires more assumptions than relative valuation, the latter also needs an analysis of the corporate fundamentals and the possible effects caused by their changes. Thus, even for this kind of judgment, it is advisable to consider the three value drivers introduced in the DCF: risk, growth and cash-flow generating potential. These three dimensions vary through multiples, so it is necessary when deciding to adopt a multiple rather than another, to analyse its structure and composition to understand which element affect the value. For example, looking at multiples of the company value, thanks to the DCF it can be expressed as:

[117] See footnote 93
[118] See footnote 108

$$V_0 = \frac{FCFF}{k_c - g_t}$$

where k_c is the cost of capital of the company and g_t is its perpetual growth rate.

If we divide by FCFFs both sides, we get the multiple EV/FCFF, which has the cost of capital and growth as determinants. Since the FCFF of a company is net income after taxes net of capital expenditures and capital needs, we can rewrite the previous multiple to obtain it:

$$\frac{V_0}{EBIT * (1 - t)} = \frac{(1 - \text{Reinvstment Rate})}{k_c - g_t} \rightarrow \frac{V_0}{EBIT} = \frac{(1 - \text{Reinvstment Rate}) * (1 - t)}{k_c - g_t}$$

where EV/EBIT is a function of the reinvestment rate. Analyse and understand the determinants underlying the value expressed by multiples is necessary to be able to say and motivate when one company is less expensive than another, i.e. when it trades at a lower multiple.[119]

A final step is the definition of a comparable firm as a company that shares the same risks, growth potential and cash-flows. Since in practice finding identical companies is almost impossible, several techniques help to select panels of comparable companies according to different assumptions. The most common is to consider as similar companies all those that are within the same industry, assuming that they share the previous characteristics. However, this selection rule is not appropriate when considering extensive sectors, for which all companies do not share the above features, and when there are few listed companies within a specific industry. Thus, two alternative methods are: selecting companies according to their fundamentals or considering companies in the market as comparable and checking their differences in fundamentals using statistical analysis. The latter method uses three different techniques to control the differences:

- subjective adjustments: they average the multiples of the comparable companies and then compare it with the value of the company's multiple. If differences between the two values exist, they can be explained by the analysis based on the fundamentals of the company;
- modified multiples: according to which multiples are modified to reflect differences in the variables that determine their value. Thus, for high-growth societies, since they show very different growth rates, a widely used multiple is the PEG ratio, which balances the PE multiple for the different growth outlooks of the companies' EPS. This method is based on two main assumptions, namely that companies are comparable in all their value measures and that there is a linear relationship between multiples and fundamentals. Thus, in the PEG ratio, it implies that as growth doubles, the EP doubles;

[119] See footnote 93

- statistical techniques: these involve the use of linear regressions based on the business sector or market. The advantages of these techniques are that they make it possible to support the strength of the relationship between the multiple and the variable use. Moreover, they can control more variables than modified multiples and if a relationship is non-linear, the regression can be modified to allow for that.[120]

The analysis just concluded has highlighted the adjustments needed during the assessment. More generally, DCF and relative methods can lead to different results on a company's value. Besides, relative approach shows different results depending on the multiple selected during the analysis.
The purpose of this introduction of traditional models and their fundamental principles is to guide the reader through the next steps. These introduce the major critical issues that traditional methods face when assessing high-growth firms. Therefore, the discussion is based on traditional methods' ability to capture the value, despite the peculiarities of these societies. Finally, it is only after understanding their weaknesses that practical solutions for business valuation are presented.

2.3. Valuation Issues of High-Growth Firms

The valuation methods presented in the last sections are widely used in business valuation, due to their ability to deliver truthful enterprise value commonly. However, beyond the necessary measures required to carry them out, some situations place a strain on the soundness of these methods. One of these is high growth firms. Because of their nature and the financial characteristics announced in the first part of this chapter, the most commonly used valuation methods have several critical points, which are explained below. The decision to continue on a dichotomic framework on the evaluation of high growth firms is necessary to identify the limits of each model, and then to try providing ad hoc solutions in the final part of the chapter.

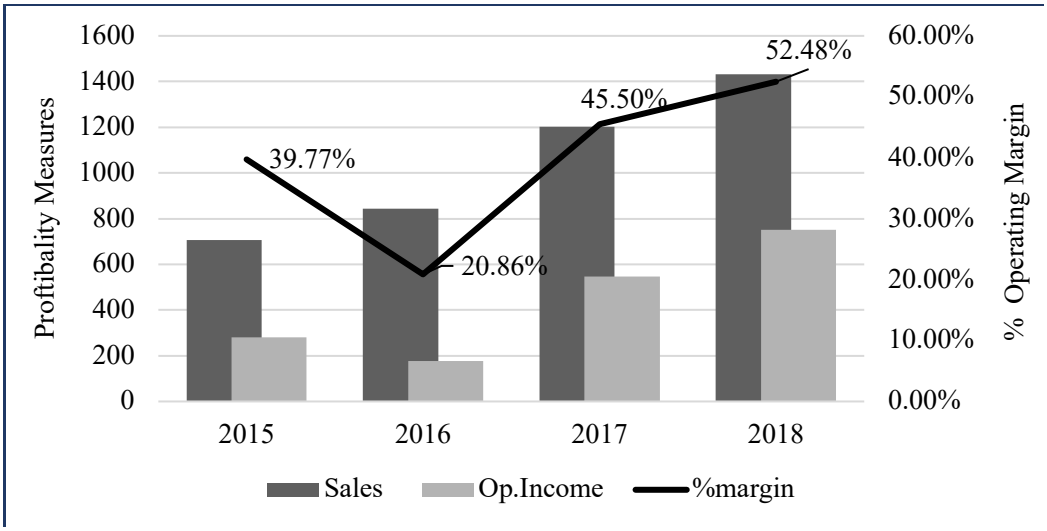
2.3.1. Limits of Discounted Cash Flow

In this section, it is possible to identify the problems affecting the DCF method, breaking it into the main areas of application: existing assets, value of growing assets, risk, terminal value and equity value per share.
Existing assets are measured by their ability to generate future cash flows discounted at a risk-adjusted rate. However, with HGFs, this measurement faces different problems. The first concerns poorly measured earnings, i.e. the value of assets tends to be much smaller than the value of the company. Soundly speaking, operating income, in the discounted cash flow assumption, is the

[120] See footnote 93

income generated by the company's assets, once exceeded costs, used to value them. The perspective of considering the operating costs related to a specific year is the first problem because high growth companies usually are investing today not for current sales, but to nurture and to capture a customer base in the future. Thus, considering all costs as operating expenses leads to underestimating the value of existing assets. Another element of contrast is the shifting profitability that high-growth firms present. Unlike the margins of mature companies, which usually move within a predefined range of values, HGFs have very different values between individual periods, which makes it very difficult to make reliable forecasts.[121]

Figure 6 – Triton International LTD. Profitability over Years



Personal Elaboration, Data in Millions of USD, 22/05/2020

Figure 6 shows the Triton International example. The company showed revenues' compound annual growth rate (CAGR) of 26% from 2015 to 2018, with an operating margin that had several shifts from 39% in 2015 to 20% in 2016 and 52% in 2018, confirming what said before.[122]

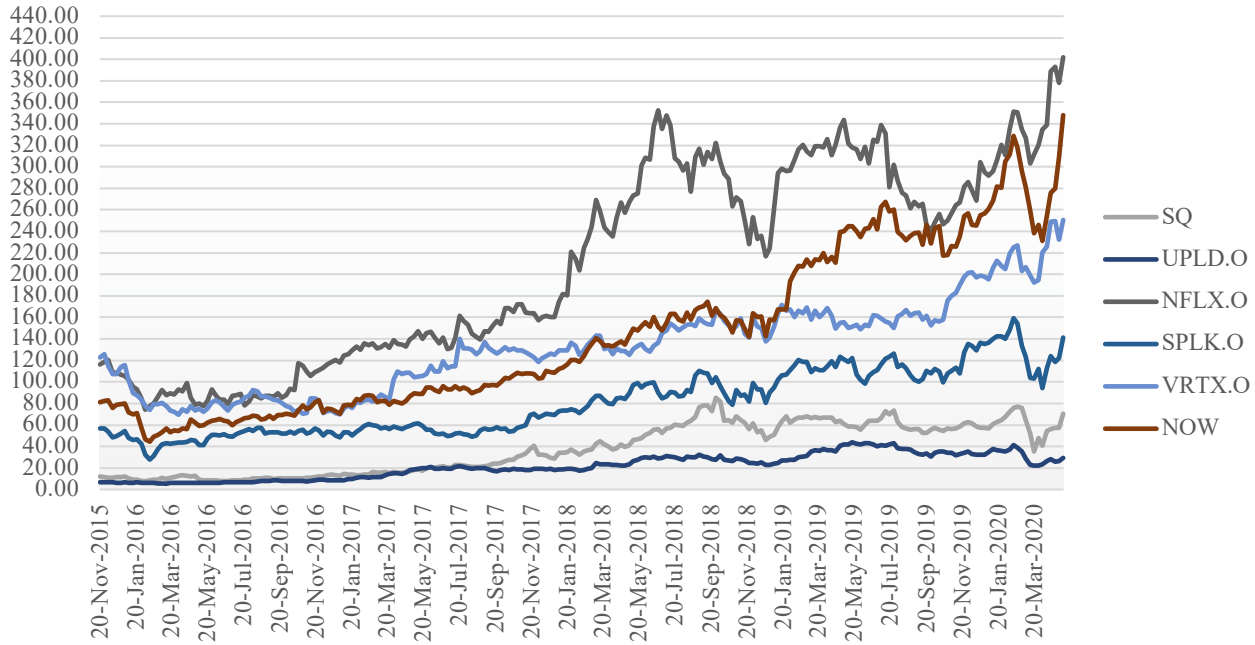
The second area of application concerns *growing assets*. Growth is indeed the central theme of these companies, and it is a challenge to assess it correctly because of specific characteristics. The first is the scaling effect of growth. Considering a company that has achieved high levels of growth in recent years, its size should not be overlooked. As a company grows, it is more challenging to sustain previous levels of growth, and thus it is necessary to consider this aspect very carefully from a forecasting perspective. To support growth, it is also necessary to consider the rate of reinvestment over the years. Thus, growth and reinvestment rates need to be balanced to achieve a return on capital that is viable for the company as it approaches the long term. Furthermore, if a company can act behind the scenes in the early stages, as it achieves success and growth, there is an increase in competition. Hence, since only a few companies, in specific niche markets, can

[121] See footnote 93
[122] Data retrieved by https://www.thomsonreuters.com/en.html

maintain a competitive position while remaining stable, the others must find solutions to avoid perishing in the face of the advance of new competitors. This effect can lead to a decrease in profitability and growth in the long term.[123] Finally, it is also appropriate to consider the effect of macroeconomic decisions on the company and its size. Indeed, smaller growing companies are more affected by economic cycles and their negative phases.

Discount rates are necessary to get the present value of future cash flows generated by assets. Accordingly, their determinants relate to investment risk and the mix of choice between capital or debt for financing the business. In both cases, HGFs present critical issues. Since the value of a high-growth company depends on both existing and growing assets, and the latter are riskier than the former, two separate rates should be used to reflect this difference in risk. This judgement, however, is baffling considering historical performance. Besides, it is common practice to use market values to determine the capital structure. Despite, high-growth firms are most often subject to high volatility, which has implications for the D/E ratio. As can be seen from Figure 7, in a sample of companies that have grown in recent years by at least 25% EPS, it is possible to see price volatility between months, demonstrating the above.[124]

Figure 7 – Price Movements between High-Growth Firms



Source: Personal Elaboration, 29/05/2020

Finally, another critical aspect concerns the change in the risk profile that growth brings. When this profile changes, the underlying discount rate changes. Thus, from a foresight perspective, as a company matures and more secure, it must be considered that existing assets assume a greater

[123] See footnote 113
[124] Data retrieved by https://www.thomsonreuters.com/en.html

portion of the total value and that its cost of capital decreases, reflecting this situation. Thus, for high-growth companies, discount rates should be higher in the early stages and lower over time.[125] At this point, two questions arise spontaneously, how long will the company grow, and what will its characteristics be once the growth is over? To answer these questions, the *terminal value* must be taken into account, understood as the period in which the company has a growth rate that it can sustain forever. Although a problematic phase of any evaluation, as already seen in the previous sections, the construction of the terminal value is even more difficult with HGFs. First of all, because it represents a more substantial portion of the overall value of the company since they often generate low cash flows from existing assets. Besides, the terminal value is subject to the uncertainty that follows the growth of these companies. Hence, it is affected by the characteristics previously stated, such as competition and size reached by the company. A potential solution, which is discussed in more detail later, is to develop several valuation scenarios that leverage different assumptions regarding the characteristics of the firm and so the terminal value. In any case, considering the difficulty of estimating the values of these companies in the short term, it is clear how difficult it can be in an even longer time horizon. [126]

Finally, the last area of application concerns the *value of equity per share*. Usually, to find this value the debt and equity claims is subtracted from the enterprise value. Then, the cash and crossholdings are added, and this value is divided by the number of shares. However, these steps are critical for high-growth firms. Regarding the value of cash, especially HGFs in the early stages of the life cycle, have high reinvestment rates that lead to the dissipation of cash balances. Thus, at the time of the valuation, their value may be very different from the last one stated in the financial statements. For debt, on the other hand, high-growth companies usually require hybrid forms of financing compared to traditional ones. These offer the advantage of keeping interest rates low, but in exchange for an equity option, like in the case of convertible debt. Thus, since only debt is subtracted in the formula, the hybrid structure of these loans has to be broken down by dividing the equity and debt parts. The last aspect concerns voting and non-voting shares. This practice is more frequent in this type of company than the mature ones, especially when they are young, because of the founders' willingness to maintain control when raising equity from the public. Thus, shares are divided into two separate classes, retaining the shares with voting rights. Therefore, to calculate their price this distinction must be taken into account, which may create some problems.[127]

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[125] See footnote 93

[126] T. Koller, M. Goedhart, D. Wessels. (2020). Valuation: Measuring and Managing the Value of Companies. John Wiley & Sons, 7th edition.

[127] See footnote 93

2.3.2. Limits of Relative Valuation

Given the different issues that the traditional discounted cash flow method faces when evaluating high-growth firms, it may be easier to apply the relative method. However, the peculiarity of these companies also has a significant impact on such valuation. Therefore, the purpose of this section is to shed light on the significant issues to outline an alternative method in the next phase. These critical issues can be divided into four macro-areas:

- *Comparable firms*: the common practice of considering listed companies present in the same sector may be totally out of place in the case of HGFs. The reason lies in the fact that these companies are not comparable in fundamentals and other measures with those present in the sector. Also, in terms of growth, they share different risks and opportunities which does not make them comparable in terms of industry. Taking the US company Zoom Video Communications Inc. as an example, it shows EV/Sales and P/E multiples of 41.6 and 332 respectively against an industry average of Software and IT Services of 8.2 and 104.1.[128]
  Besides, many companies cannot be evaluated with multiples using operational indicators. Especially in the early stages, many HGFs have negative EBITDA, EBIT or operating result values, making impossible the use of earnings-based multiples. Therefore, instead of using sales, which cannot assume a negative value, sector multiples have developed over the years. These ones, also called non-financial multiples, use sector-specific operating measures to obtain a value estimation of companies. Examples of sector multiples are those used for internet companies, that estimate the value of the company through the number of visitors to the website, or for telecommunications companies that use the number of subscribers. However, these metrics expose themselves to subjective valuations of their values, i.e. when the value paid for a subscriber can be considered high or low, and also to the difficulty of explaining variations in the multiple values between companies.
- *Choice of multiples and base year values*: as multiples vary depending on the choice of the base year in which they are considered, for high-growth companies, calculating these values at certain stages of their life could be misleading. Thus, an HGF with negative or small earnings, compared to high market value, results in a P/E multiple inconsistent or very high. The same happens for values such as EBIT or EBITDA that do not reflect a company's future potential to date. To overcome this problem, many analysts use forward multiples values. This practice, especially for high growth companies and start-ups, is highly recommended. However, the characteristics of the company should always be taken into account over the years to avoid setting a long-term growth rate equal to the current one.

[128] Data retrieved by https://www.thomsonreuters.com/en.html

- *Different growth potentials*: at this point, it is essential to consider these perspectives when comparing several companies. Consequently, as we have seen in Appendix A, in calculating the value, these differences should take into account not only the growth rate but also its length and the excess returns that accompany it. Indeed, two companies that have the same growth potential may have different multiples values because of other dimensions. Thus, many analysts, instead of dealing with growth and its implications for value, tend to create stories about growth. However, a multiple that considers it, while remaining very simple, is the PEG ratio. This metric requires making assumptions about the relationship between growth and value that are quite unreliable. In fact, given its formula:

$$\text{PEG} = \frac{\text{PE}}{\text{Expected Growth in Earnings per Share}}$$

  it is possible to see at once that it does not consider the risk within it. Thus, two companies that share the same growth expectations, but with different risks, should betray different values of the multiple. Moreover, it assumes that the EP grows in proportion to growth when the value grows less than proportionally to growth.
- *Risks' differences*: given the close relationship between growth and risk, the variation that one implies on the other must also be taken into account, changing the values of the multiple overtime.[129]

2.4. Valuation Corrections and Alternative Methods

To this point, the following research shows the traditional methods of valuation and the peculiarities that high-growth firms share. These two introductions have allowed analysing and deeply study the weaknesses that traditional methods face because of the assumptions made when evaluating HGFs. Since the purpose of this research is to shed light on issues that are still unsolved and full of free interpretation, it was necessary to divide the methods and address the main issues separately. In this last part of the chapter, using the guidance obtained so far, three different evaluation methods are provided, which can evaluate high-growth firms more efficiently. The adjusted method of discounted cash flow and the relative valuation is presented, starting from the traditional method, and adapted to meet the needs of HGFs. To avoid redundancy, some explanations that have already been introduced in the presentation of the models are omitted. Therefore, it is presented a more practical analysis, which lays the foundations for the research in the final chapter. Finally, this section addresses the real options model, an additional valuation method that can be useful to estimate companies with growth opportunities.

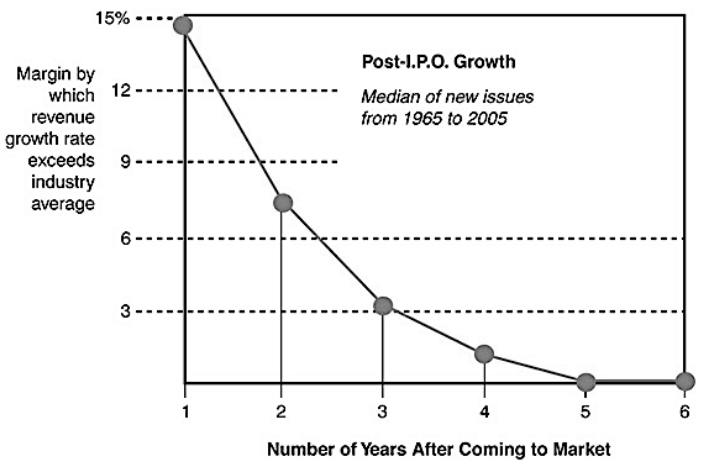
[129] See footnote 93

2.4.1. Adjusted-Discounted Cash Flow

The main objective of discounted cash flow is to obtain reasonable estimates of the company's future cash flows and discount rates. As the traditional method faces several problems, the following are the measures necessary to carry out this method for high-growth companies.

First, there is a choice about the model to adopt in the valuation. The cash flow method can be performed by discounting the cash flows of the entire company at the cost of capital or by valuing equity by discounting cash flows at the cost of equity. While the former requires the estimation of new debt issues and interest payments for each period, the latter option becomes more difficult if there is a change in the company's debt ratio over time. Much attention must be paid to this feature because very often it is assumed that if a company does not use debt today, it does not use it in the future either, leading to an assumption of no debt in perpetuity during the valuation. However, as seen above, in the long term high-growth companies tend to adjust to the characteristics of mature companies in the market, resulting in lower growth rates and more stable cash flows, also caused by the issuance of debt. Therefore, it is necessary to adopt a cash flow model that is flexible and not rigid to avoid blocking the current characteristics of the company. Also, when there are young companies with a little history, this model should try to tie in the best possible way the company's inputs, such as operating margins and risk. Assuming, therefore, that the operating cash-flows model is more efficient with high-growth companies rather than equity one, the first step is to assess the company's operating assets, incorporating both existing and growing ones. Thus, the process starts with an estimate of revenues which involves several factors. One of the most important is the scaling of growth rates as the company expand.[130]

Figure 8 - Revenue Growth in the Years After the Initial Public Offering



Source: Andrew Metrick, The New York Times

[130] See footnote 93

A study conducted by Metrick in 2006, which compared the revenues' growth rate of post-IPO HGFs to that of the industry in which they operated, showed that they initially had much higher rates than the industry. However, after five years, they were in line with the latter.

Although it cannot be considered an absolute rule, it highlights the fact that only a few companies can achieve extraordinary growth rates over very long periods and that most of them have these rates only for a short time.[131]

Then, to find out the length of this first period and the subsequent phases of growth of the company, attention must be paid to company's specific features. Information about the size of the overall market, the presence, strength and quality of the products offered by competitors and their management could give information about the possibility to achieve high growth. So, companies operating in large markets with the protection of the competition, through efficient management can maintain high growth rates over several periods. In order to validate assumptions about future revenue growth rates, the following tools can be used:

- *absolute changes in revenue*: used instead of percentage change as it can help to avoid overestimating revenue growth over time;
- *past growth*: analysing past growth rates to better understand how they changed as the company expanded. This test can be a valuable basis for estimating future growth rates;
- *industry data*: can be used primarily to triangulate results and to understand which target the company is moving towards in the long term.

Subsequently, moving from revenues to operating margin requires an analysis of the company's cost structure. Several assumptions can be made, such as maintaining the company's current operating margins over time or changing them. The latter assumption is the most likely with high growth companies. At this point, consider the scenario in which the company's current margin is negative compared to the long-term sustainable margin. These could happen for three reasons. The first provides high fixed costs incurred in the early stages of life with visible returns in subsequent periods. Usually, these companies are defined as capital-intensive because of their high initial investments that have an impact on operations. The second case concerns the mix of expenses of a different nature: operating expenses and expenses to generate growth. As noted earlier, typically HGFs today suffer expenses that are aimed at consolidating a future consumer base. The third reason is a mismatch between revenues generated and expenses incurred. On the other hand, the opposite scenario, i.e. an initial margin that is too high and decreases over time, is less likely and may occur in specific circumstances. These are niche markets, where companies

[131] Metrick, A., & Yasuda, A. (2011). Venture capital & the finance of innovation. Venture capital and the finance of innovation. (pp. 357–377).

initially benefit from a competitive advantage due to the temporary absence of competitors, or when a company owns a patent or exclusive on a certain product.[132]

Once the company's margin has been analysed, it is necessary to estimate how and when the long-term target is reached. Concerning the "how", it is possible to converge the operating margin towards the average of the company's sector or consider a larger company as a benchmark. About "when", to estimate operating margin's changes until reaching the target, it is necessary to consider not only the sector in which the company operates but also the investments made and their ability to generate a return over time. Since growth is not free and does not result from chance, it is appropriate to consider the effect of reinvestment of the company over the years. As we have seen during the previous parts, considering history can be misleading as it can lead to reinvestment rates that are not suitable for expected growth estimates. Thus, depending on the type of company and its characteristics, three different paths to estimate a company's reinvestments can be determined:

- for growth companies at the beginning of the life cycle, reinvestments can be estimated using the differential of revenues over the years, adjusted for the sales-to-capital ratio, following the formula:

$$\text{Reinvestment}_n = \frac{\Delta\ \text{Revenues}_n}{\frac{\text{Sales}}{\text{Capital}}}$$

using the company or industry data. Besides, future revenues can be used to obtain the current reinvestment, thus creating a time differential between the two items;

- for a growing company with a more consolidated history of earnings and reinvestment, the following formula can be used:

$$\text{Expected growth rate in Operating Income} = \\ = \text{Return on Capital} * \text{Reinvestment Rate} + \text{Efficiency Growth}$$

which relates growth rates to reinvestments. As can be seen from the formula, the second factor regarding efficiency growth can be eliminated once the company has reached sustainable levels[133];

- for companies that have already invested for growth in the coming years, there is the possibility that they can grow using little or no short-term reinvestment. In this case, it can be estimated when the period without reinvestment lasts to determine when the company needs to return to supporting its growth with other investments.

[132] See footnote 93

[133] Efficiency growth in mature phase = 0

Finally, for all three categories of companies, the need for reinvestment that they need should be eliminated once they reach maturity. In this phase, reinvestment should be strictly linked to fundamentals, according to the formula:

$$\text{Reinvestment Rate in Mature Phase} = \frac{\text{Growth Rate}_{\text{stable}}}{\text{Return on Capital}_{\text{stable}}}$$

If the return is estimated independently of the operating income during the growth phase, and without recourse to return on capital, the return on the imputed capital should still be analysed (see appendix C), ensuring that it remains within a specific range.[134]

Then, during the assessment, a risk profile that is consistent with the company's growth and operational numbers must be considered. By briefly analysing the components of the cost of capital, they are the beta, the cost of equity, the cost of debt and the debt ratio, and can be considered the same for any company. However, what changes is the risk profile. Thus, to make a consistent assessment, it becomes necessary to adjust discount rates over time, according to the company's condition. In general, two rules should be respected. The first is that high-growth companies should have high equity and debt costs when sales growth is high, and as sales growth becomes steady and margins improve, interest rate values should decrease.

During the earnings expansion phase, the company may obtain excess cash flows that can be used in a different way, such as repaying debt or paying dividends. However, rather than repaying debt, companies prefer to keep a share or increase it to take advantage of the tax benefits. In any case, this rule suggests that the cost of capital must respect the changes that the company makes over time, and therefore cannot be a fixed number. The second general rule concerns risk parameters or beta. At this point, it is not the right method to use price information due to the very often limited history and substantial standard errors. Instead, it is more precise to use data from other companies that share the same risk and growth profile as the company under evaluation, performing a bottom-up beta approach.[135]

At this point, the assumptions about stable growth need to be considered. Expectations about the terminal value of high-growth companies are more important than those of mature ones because of the weight of this value on the total. So, when will an HGF become a mature company and achieve stable growth?

These questions could be answered in many ways, most of them subjective, that consider many circumstances. To tighten the circle and not get lost in personal interpretations, Damodaran suggests several general propositions for estimating the terminal value. First, one should not wait

[134] See footnote 93
[135] See footnote 93

too long to put the company on a stable growth path. In other words, it means that unlike many analysts do, periods of growths should not be planned to take a too long span. For example, considering a timeframe of 10-15 years with high growth rates is out of the ordinary because only a few companies have been able to achieve this performance. Thus, it makes more sense to adopt stable growth characteristics consistent with the firm's operation. Taking up the point made earlier, this means lower discount rates and more debt. Moreover, the spread between the return on capital and the cost of capital should not exceed 4 or 5% during the stable growth phase.

After obtaining the enterprise value through discounted cash flow, the value of equity needs to derive the share price. Considering the explanations in the DCF paragraphs, this last part of the analysis starts from non-operating assets and liquidity. High-growth companies usually burn cash balances very quickly, meaning that even a recent value can be misleading. Because some specific information may be private, one can estimate a company's cash balance by analysing its capital market activity over a while. For example, it is possible to understand if a company raises funds to pay back a loss-making result or it does so through existing cash. If it repays through existing cash, it is possible to adjust the most recent value of cash balance with the occurred expenses and derive the new value for cash.

Another factor that determines the transition from enterprise value to equity value is debt and non-equity claims. Consider convertible debt as the preferred form of financing for an HGF. It cannot be considered in its entirety as debt but should be split into equity, i.e. the option to convert to equity, and debt. The simplest way is to assess the latter as straight debt. Thus, the difference between its value and the market value of the convertible bond gives us the value of the conversion option. Thus, from the enterprise value, only the part of the "pure" debt should be subtracted to reach the equity value. Finally, again concerning borrowing, a long-standing issue concerns the variation of debt over time. Many analysts argue that today's debt may be of negligible value compared to future issues and that the expected value of the latter should be subtracted. However, a more consistent assessment, according to Damodaran, considers only present debt values.

At the end of the valuation, there are final corrections an adjustment to make. Thus, to arrive at the value of equity per share, it is necessary to divide the equity value by the number of outstanding shares. At this stage, there is one last consideration: differences in voting rights. Concerning this discrimination, it should be specified that shares with voting rights should be priced higher than those without voting rights. Besides, this premium may vary between companies depending on the size of the difference. Therefore, it is necessary to carefully assess

this factor when allocating the value between shares, also taking into account the average premium in the country where the company operates. [136]

2.4.2. Relative Valuation: Alternative Strategies

The section on valuation issues provided the basis for proposing potential alternatives to traditional multiples. Thus, this part analyses the various possibilities and derivation of multiples, based on the characteristics of the high-growth companies involved. Before doing so, it is essential to remember that HGFs are, more often than not, the exception in the sector in which they operate. This fact has implications for the traditional way of thinking about multiples and their use. Thus, one should abandon the idea that a high-growth company in the automotive sector must necessarily be compared with others in the same sector. This practice would not reflect the value of the company. Hence, as a first step, it is necessary to consider comparable high-growth companies from different sectors, using fundamental rather than business analysis.

Then, it is advisable not to consider the use of current book value or earnings numbers in the study because of their volatility, especially in the early stages. While, about the forward values, there are two dangers to avoid. The first regards the multiples of revenues, which do not consider the fact that a company may lose money. At this point, it is prudent to use the expected profit margin. The second danger concerns the multiples of earnings, where the survival factor of the company until the forecast period must be carefully considered.

Despite this advice, the companies included in the analysis inevitably differ in their fundamentals. Indeed, it is impossible to align risk and growth for all companies, and so, a more practical solution is to control these variables through linear regression. Thus, the dependent variable is expressed by the analysed multiple and the risk, growth or another measure that wants to be tested, represent the independent variable. This approach makes possible to relate the growth with the other variables, also using large samples of comparable companies. Concerning this methodology, the initial step is to determine a sample of companies and find some statistical measures such as average and median of the sector, to compare them with the values of the company. Using the PE ratio, for example, it is possible to make a linear regression against expected growth and betas, weighting companies by their market capitalisation. Based on the results, after checking for the significance of the ratios, we can determine the value of the multiple using the company's values in the linear equation. By finally comparing it with the current PE value, this allows analysing their difference to come to different conclusions. When multiples of earnings cannot be used, due to small or negative values, other multiples such as EV/Sales become available. At this point, the

[136] See footnote 93

"melded approach" suggested by Damodaran can be used. The latter involves the use of forwarding measures of revenues/earnings but superimpose two factors. The first is the risk, which is captured in the discount rate used to get the present enterprise value, even considering the eventuality that the company fails. The second factor is the use of a multiple that does not represent the industry average but the characteristics of the company in the coming years. Finding the latter requires to look at the relationship between multiple and fundamentals of the sector in the present. Thus, it is possible to make a linear regression between the EV/Sales of the company against the expected growth and the operating margin of the industry at the current date. Once this relationship is found, entering the company's forward values of sales give the enterprise value at forecasted year t. Taking this value back through appropriate discount rates allows making a more accurate assessment of the company today.[137]

2.4.3. Probabilistic Methods

The characteristics of high-growth companies require the control of many variables to assess the company and its risk. Compared to the methods outlined above, the next ones are based on the ability to find value through a different and potentially more informed way. They do not seek value in a company through the likelihood of achieving certain outputs but instead assign a value to the company for each possible outcome. For clarity, these methods are presented in order of complexity, starting from the simplest to the most difficult-to-perform. Hence, the paragraph starts from the scenario analysis moving then to the decision trees and real options. Thanks to these ones, a valuation method based on Monte Carlo simulation, which is the best way to determine the risk along the spectrum, is provided.

Multiple Scenario Analysis

In line with Mc Kinsey & Company's views, given the high uncertainty associated with high growth companies, it is not possible to rely on a single long-term forecast. Thus, an efficient and simple solution is provided by scenario analysis. Thanks to this method, it is possible to express the evolution of the market in terms of multiple scenarios. The easiest way to conduct this study is to create best and worst-case scenarios. However, because it is considered too naive, we move on to describe a more generalised version directly. Through multiple scenario analysis, the value of an asset can be expressed under several variables, both macroeconomic and asset specific. The fundamental points of this analysis are four:

[137] See footnote 93

- the decision of base factors on which construct scenarios: they include elements that have the most significant impact on the value of the asset. Examples are the state of the economy, the response of competitors or changes in regulation;
- the number of scenarios for each factor: increasing the number of scenarios implements the case study, increasing the probability of obtaining as much information as possible on the value of that asset. However, it is not recommended to develop a large number of scenarios as a basic rule. Indeed, it is better to develop several scenarios for which it is possible to predict cash flows and understand their difference
- estimating the cash flows of assets in each scenario: two or three critical estimation variables are usually identified. Examples could be operating margins or growth rates;
- assigning a probability to each scenario: for some scenarios, it is necessary to take advantage of services' expertise that forecast specific variables, such as interest rates or economic growth. For others' probabilities, one should rely on personal knowledge about company's sector or its competitors. However, the important thing is to consider scenarios that complement each other, in the sense that they cover the full range of probabilities and do not exclude other possibilities.

Final results can be represented in two ways: as a value of each scenario or as an expected value for all probability-weighted scenarios.

Finally, this analysis, although providing a better estimate than the simple/worst case, has some critical points. The fact that the final result is based on the analyst's ability to define scenarios, and carefully estimate cash flows under each of them, puts the focus on developing realistic scenarios. Moreover, this analysis is more suitable for discrete rather than continuous risk variables. In other words, it prefers variables that can change one time rather than others, such as interest rates, which vary continuously.[138]

Real Options and Monte Carlo Simulation

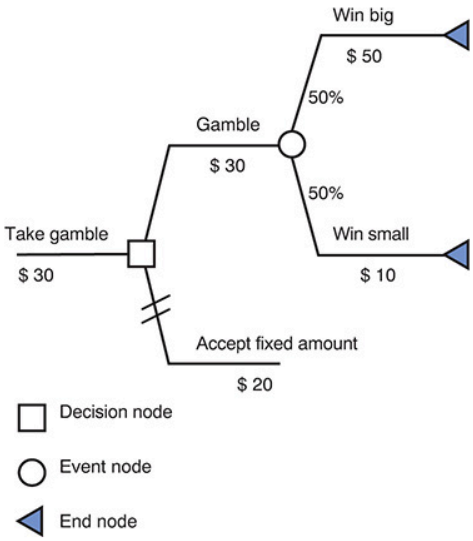
The scenario analysis just concluded is more suitable for discrete rather than continuous risks. However, in some valuations, there are also subsequent risks. It means that a company or a project, to achieve specific outcomes, have to go through several steps that could affect their value. A failure in one of the steps can sometimes also lead to the default of the company. The decision trees aim to consider the risk during the various stages and the right response to it. Understanding the structure of these tools requires a first distinction between:

[138] See footnote 108

- radical nodes: represents the beginning of the decision tree in which the analyst is faced with a decision choice or uncertain outcomes. This phase tries to evaluate the risky investment;
- event nodes: represent the different possibilities that are obtained if the project is undertaken. Thus, for example, one can distinguish between successful or failed outcomes;
- decision nodes: represent the choices that can be made.
- Final nodes: these are the final results of the first risky outcomes and decisions taken.[139]

The figure below shows a simple decision tree related to a gamble and its potential outcomes. The investor could take the gamble o received a fixed amount. If he takes the gamble, he has two possibilities: “Win big” or “Win Small”. Looking at the structure of the decision tree the decision node represents the choice of management.

Figure 9 - Decision Tree



Source: A. Damodaran

In this case, if based on the pure expected value, taking the gamble is more convenient than receiving a fixed sum. This example, although very explanatory, does not represent business complexity in the real world. Thus, it is necessary to introduce another choice available to management before starting a project, i.e. delaying the investment. The option to decide later can be of great value to the company as it can learn new information and make a safer investment. These types of decisions are called real options and belong to the decision nodes.[140] They are located after an information node in the decision tree.[141] This type of argument is closely related to high-growth companies and the uncertain nature of their investments. The idea behind real

[139] A. Damodaran. Facing Up to Uncertainty: Using Probabilistic Approaches in Valuation. (2018). Available at SSRN: https://ssrn.com/abstract=3237778 or http://dx.doi.org/10.2139/ssrn.3237778
[140] See footnote 107
[141] See footnote 139

options is that business decisions involve a certain degree of flexibility. This one can be seen as an option to behave in a certain way. The best-known model for evaluating financial options is that of Black and Scholes (1973). Subsequently, Schwartz and Moon (2001) developed a model to assess the options of high growth companies, taking into account two of the most critical valuation problems of these companies: the cash flow estimation and the growth rate estimation. The model is a Monte Carlo simulation[142] approach based on three stochastic factors. Unlike other models, cash flows are modelled by two separate stochastic processes for costs and revenues. Besides, expectations are assumed to be uncertain and therefore vary between people. This fact is typical of high-growth companies compared to more mature ones, where expectations are homogeneous.[143] The stochastic characteristics foreseen by the model are:

- *first factor*: assumes that the variations in revenues follow a Geometric Brownian motion[144] and are represented by the formula:

$$\frac{dR(t)}{R(t)} = \mu(t) * dt + \sigma(t) * dz_R(t)$$

  where μ(t) indicates the expected income change, $\sigma(t)$ the annual volatility in income growth, dz_R a standard Wiener process for income changes, and dt an infinitesimal time interval;

- *second factor*: represents the movement of variable costs. Together with the first factor, it allows determining the trend of cash flows over time. Since the variable cost process is modelled according to the Ornstein-Uhlenbeck process[145], this implies the following assumption regarding the reversal property of the average, i.e. that HGFs have higher variable costs than mature companies. Also, the variable cost rate is assumed to converge to the rate belonging to the more mature companies with a k_y speed rate. The second factor could be expressed by the formula:

$$dy(t) = k_y * \big(\bar{y} - y(t)\big) * dt + \varphi(t) * \mathrm{d}z_y(t)$$

  Where $\bar{y}$ is the variable cost rate of the industry average, $\varphi(t)$ the volatility of variable costs and $\mathrm{d}z_y$ is a standard Wiener process representing the unexpected component in variable costs.

[142] A Monte Carlo method (MCM) is a computational algorithm that utilizes random sampling in some way during the computation, such as computing an expected value, where the algorithm uses realizations of some random process. (Academic Press Library in Signal Processing. (2014). Volume 3. Pages 297-322)

[143] M. Rudolf. (2004). Valuation of Growth Companies and Growth Options. *Modern Concepts of the Theory of the Firm*. Günter Fandel et al. 449–73.

[144] A stochastic process S_t is said to follow a Geometric Brownian Motion if it satisfies the following stochstic differential equation: $dS_t = uS_t dt + \sigma S_t dW_t$; where W_t is a Wiener process (Brownian Motion) and u, σ are constants. (Z. Yang. (2015). Geometric Brownian Motion Model in Financial Market)

[145] The Ornstein-Uhlenbeck process is a diffusion process that was introduced as a model of the velocity of a particle undergoing Brownian motion. By M. Kozdron. (2014).

Moreover, total costs (C) are represented by a part of fixed costs (F) plus a percentage of revenues as variable costs:

$$C(t) = F + R(t) * y(t)$$

- *third factor*: this shapes the expected income changes according to the Ornstein-Uhlenbeck process, also implying the property of reversing average expectations. In other words, it is assumed that high-growth companies generate higher revenue growth rates than more mature firms. Over time the decline of this rate is uncertain, but since HGFs tend to be similar to value companies in the long term, then this rate is the same as for the latter. The equation that can express the third factor is:

$$d\mu(t) = k * \left(\bar{\mu} - \mu(t)\right) * dt + \eta(t) * \ dz_{\mu}(t)$$

where k is the mean reversal speed rate at which the expected growth $\mu(t)$ converges to the average of the sector $\bar{\mu}$, $\eta(t)$ is the volatility of expected revenue changes and dz_{μ} is a standard Wiener process that represents the unexpected component of expected revenue changes.

Besides, when an HGFs company becomes a valuable company, the expected volatility of its revenues is zero:

$$d\sigma(t) = k_{\sigma} * \left(\bar{\sigma} - \sigma(t)\right) * dt$$

$$d\varphi(t) = k_{\varphi} * \left(\bar{\varphi} - \varphi(t)\right) * dt$$

Where k_{σ} and k_{φ} are the mean reversion speeds the revenues change volatility σ and the variable cost volatility φ.

The model requires the simulation of these three stochastic processes, inherent to the predetermined correlation structure, for 100 future quarters. The three correlations, revenues-expected revenues, revenues-variable costs and expected revenues-variable costs, are:

$$dz_R * dz_{\mu} = \rho_{R\mu} * dt$$

$$dz_R * dz_y = \rho_{Ry} * dt$$

$$dz_y * dz_{\mu} = \rho_{y\mu} * dt$$

Thanks to these calculations, the cash balance is obtained for each quarter and in each simulation cycle. Once the 100 quarters are simulated, it is possible to create a frequency distribution for over 100,000 different cash balances. Thus, the value of the company can be expressed by the equation:

$$\text{Company Value} = \text{Cash Balance in quarter } 100 * \text{RADF} + \text{CV}$$

where CV represents the continuation value, and the first factor is the cash balances discounted for the risk-adjusted discount factor (RADF). Despite the amount of data required and the high

accuracy necessary to specify it, the Schwartz and Moon model allows a more accurate description of the processes that determine the company's value.[146]

[146] See footnote 143

3. Model Development through Empirical Analysis

3.1. Introduction and Objective of the Analysis

The qualitative analysis of the high-growth firms just concluded allows starting the following research aware of the features and peculiarities that these companies present during the business valuation. The main objective of this thesis is to highlight the valuation dilemmas of high-growth firms and try to define a solution. Hence, the following research tries to suggest a solid empirical foundation on the facts that have emerged and been discussed in the previous chapters.

In particular, it focuses on a valuation model: Comparable Companies Analysis (CCA) and its ability to capture the market price of HGFs. Although very interesting and necessary for a complete valuation, the previously presented methods could be the subject of further research.

I believe that, in order to achieve a significant result, it is necessary to focus on a single model and analyse its critical points to make a valuable contribution to research.

Thus, the following quantitative analysis of high-growth companies is developed through two different phases, to precisely answer the following sets of questions:

i. What financial data most reflects the value of these companies? How can analysts' estimates help in the valuation of HGFs?
ii. How to conduct an efficient relative evaluation following previous results? Where to look for the "real" comparable of these companies?

Concerning the first group of inquiries, the aim is to analyse the ability of past performance, i.e. the results reported in the company's financial statements, to reflect the enterprise value of high-growth companies. Since the analysis can also be carried out using the values of estimates over time, they are also taken into account to obtain a complete answer to the first question. This first analysis represents a significant point for research as it provides a practical operational cue for the valuation, identifying the drivers and type of data that most influence the value of HGFs. Moreover, the results obtained represent the starting point for the secondary analysis.

The latter examines the relative valuation method and, more specifically, the dynamics involved in selecting comparable companies. Ideally, to find out how the market prices a specific company, analyses of similar companies within the same sector are used. Since HGFs are usually real exceptions, due to their high growth volumes, this concept is questioned by the results of analysis. Finally, in light of these evaluation matters, there is a need to define a model that allows CCA to capture the market value of HGFs better.

3.1.1. Previous Studies

Before presenting the analysis and how it is carried out, it is required to present past research that outlined the initial route of this work. Starting from past research, understanding its limits and potential is a good point of view for the researcher. So, before starting to analyse an unknown field, with particular dynamics, the consultation of specific researches was essential to identify the type of analysis and the objectives to achieve through this thesis.

Recent studies carried out by Hawkins[147] in 2007, the SL&C Group[148] in 2011 and the consulting firm McKinsey and Co. in 2012, have shown that the analysis of comparable companies can be arbitrary, inaccurate, and that valuations can be significantly improved when using regression analysis.

Thus, to continue these researches and extend them to the field of high-growth companies, a pathway of analysis was developed that could consistently and effectively obtain a result for business valuation. Besides, further analyses, in particular see Damodaran[149] and Fernandez[150], were considered during the development of the initial project to enrich it further. The willingness and determination to seek answers to hitherto unresolved questions was then a further drive that stimulated this work from beginning to end.

3.1.2. Analysis Overview

The starting point of the research, almost personal, is represented by the various interests that emerged during the study of HGFs and the deep desire to somehow explain their diversity compared to the rest of companies.

So, once the questions that this paper aims to answer were formalised, the next step was to find the data required to carry out the analysis.

At this point, the research starts with a definition of panel companies. Because the firms under review had to meet the precise requirements of high-growth firms, the first skimming was done by considering the companies on the Fortune 100 list. From there, a subsequent selection was made by excluding all companies in the "Financials" sector because their business model and characteristics are very different from companies in other sectors.

[147] G. Hawkins. (2007). Regression Analysis in Valuation Engagements. 9th chapter

[148] Securities Litigation & Consulting Group. (2011). Rethinking the Comparable Companies Valuation Method. http://www.slcg.com/pdf/workingpapers/CCV%20paper.pdf

[149] A. Damodaran. (2011). The Little Book of Valuation: How to Value a Company, Pick a Stock and Profit.

[150] Fernández, P. (2019). Valuation using multiples: dispersion. Useful to compare and to negotiate. Ssrn, 1–13.

Once the group of panel companies has been defined, the research moved on to data mining. The research methodology was based on an extensive initial time frame, from 2010 to 2019. This timeframe guarantees to have as much information about the companies as possible.

The same quarters of different fiscal years of companies were considered as reference periods, to ensure continuity and consistency within the research. Thanks to the use of various databases of the Wharton School of the University of Pennsylvania (WRDS), which are presented in the following sections, it was possible to obtain: historical data, relating both to industries and companies and the relative estimates made over the years by analysts. Since the latter were presented on different days and were copious within each quarter, were grouped together for each fiscal quarter and then averaged. The objective of this step was to obtain a dataset comparable over time.

Once composed, the final dataset presents necessary analysis' elements for each quarter from 2010 to 2019, except for companies that were born or became public after 2010.

Then, to answer the first set of questions, related to the financial drivers that influence the value of the company, linear regressions were performed. The need for this first analysis lies in the curiosity to analyse the portion of value that belongs to present, and therefore to values reported quarterly, and that which belongs to future and growth expectations about companies, i.e. the estimates. This research is also significant for other valuation methods, such as discounted cash flow, which base their estimates on growth expectations and future values.

Subsequently, as this first study concerned individual companies, it was decided to analyse one of the most widely used valuation methods: the relative valuation. At this point, a first model was developed connecting five price multiples: Price/Operating Earnings, Price/Earnings, Price/Book Value, Price/Cash Flow, Price/Sales to the companies' market capitalisation. Since data before 2016 was not available for some companies, it was decided to align the time horizon of this second study with that of the Fortune 100 list, analysing data from the first quarter of 2016 to the third quarter of 2019. This adjustment also allows a more precise analysis of the period of high growth shown by the companies.

The study, which aims to answer the second set of questions, was conducted starting from the traditional use of multiples and making improvements at each stage that led to a new method of CCA that would drive to tangible results for the research.

The multiples considered initially refer to industry multiples and have been extrapolated from the "Financial Ratios Suite" database by WRDS, which provides the most used financial ratios for both industries and companies over the years. After conducting a linear regression analysis for

each industry, the first results obtained showed a meagre ability of industry multiples to explain the market capitalisation of high-growth companies.

Then, between the sectors of companies, the ones with several companies have been selected for a second analysis. The data were retrieved from the same database as before, but at the companies' level instead of industries. This second model only takes into account high growth firms within the analysis and therefore, their multiples over the years. Thus, a linear regression analysis was conducted for each company using its capitalisation as a dependent variable and the multiples of companies within the industry as independent variables. The introduction of "growth" variable in this model has led to a statistical improvement of results over the previous model.

Despite this, the level of significance obtained could not be considered exhaustive, and it was decided to improve the model further. Thus, three groups of firms, very similar over the years in terms of market cap, enterprise value and revenue growth were built. Besides, an additional variable, the annualised return of the 2016-2019 period, was added to the model to select more precisely the group of comparable.

The results of this last analysis highlighted the ability to obtain better value estimates, although linking very similar companies but belonging to different sectors, confirming what also stated by Damodaran.[151]

3.1.3. Panel Companies

Before entering into the specific analysis and reporting the methodologies used, it is necessary to present the selection of companies used to create the analysis panel. The first requisite regards the necessity to be a public company, as the analysis of private high growth firms was not included in the objective of the following research, as well as being much more in line with venture capital and private equity issues.

At this point, in order to facilitate the search for high-growth companies, two lists, drawn up annually by Fortune and the Financial Times, were analysed. These lists take into consideration the 100 American companies and the 1000 European companies that have grown more than the others according to their respective selection parameters. Since the second panel contained only 37 listed companies, unlike the first one which contained all 100, being one of its selection criteria, the Fortune 100 list and therefore the American market was chosen as the field of analysis.

Fortune's selection criteria for compiling the high-growth group of companies are as follows. Companies must:

- be listed on a major U.S. stock exchange;

[151] See footnote 93

- submit semi-annual reports to the SEC;
- have a minimum market capitalization of $250 million and a share price of at least $5 on June 28, 2019, with the latter freely traded since June 30, 2016.
- have sales and net income for the four quarters ended April 30, 2019 or before that date, of at least $50 million and $10 million, respectively, and must have reported annual growth in sales and earnings per share of at least 20% per year for the three years ended April 30, 2019 or before that date.

Once these initial selection criteria have been met, companies are ranked according to the growth rate of revenues, earnings per share[152] and total annualised return at 30 June 2019. These three growth factors are then equally weighted, and in the event of a break-even, companies are selected based on the highest growth in sales over the last four quarters to obtain the overall ranking.[153]

Since the selection parameters of the list correspond to the definition of high-growth companies provided by the OECD, seen in Section 1.1.2, the list is an excellent resource for analysis.

A total of 54 companies belonging to 19 different sectors according to the Standard Industrial Classification (SIC) code adopted by the Security Exchange Commission (SEC) in the United States.[154]

Consisting of four digits, the SIC Code identifies the primary sector through the first two digits and the sub-industry through the third and fourth digits for each company. For this research, the primary sector was taken as a reference, excluding the subcategories. This choice made it possible to obtain larger groups of sectors and therefore, also comparable companies for the second part of the analysis.

Thus, the companies involved in the following study are shown in the table below:

Table 1 - *Companies Under Analysis and Industries*

Company Name	Industries	Company Name	Industries
Modine Manufacturing Co.	Autos	On Semiconductor Corp.	Chips
Winnebago Industries	Autos	Green Brick Partners Inc.	Cnstr
Molson Coors Beverage Co.	Beer	Lgi Homes Inc.	Cnstr
Patrick Industries Inc.	Bldmt	Corcept Therapeutics Inc.	Drugs
Pgt Innovations Inc.	Bldmt	Eagle Pharmaceuticals Inc.	Drugs
Amazon.Com Inc.	Bussv	Enanta Pharmaceuticals Inc.	Drugs
Arista Networks Inc.	Bussv	Supernus Pharmaceuticals Inc.	Drugs
Autohome Inc.	Bussv	Planet Fitness Inc.	Fun

[152] To compute the revenue and EPS growth rates, Fortune uses a trailing-four-quarters log linear least square regression fit.

[153] The list excludes real estate mutual funds, limited liability companies, limited partnerships, business development companies, closed-end investment companies and companies that lost money during the quarter ending 30 April 2019 or before that date.

[154] Data retrieved by https://www.sec.gov/info/edgar/siccodes.htm

Etsy Inc.	Bussv	Biotelemetry Inc.	Hlth
Facebook Inc.	Bussv	Irobot Corp.	Hshld
Grubhub Inc.	Bussv	Centene Corp.	Insur
Healthequity Inc.	Bussv	Coherent Inc.	Labeq
Match Group Inc.	Bussv	Nanometrics Inc.	Labeq
Nv5 Global Inc.	Bussv	Wingstop Inc.	Meals
Pra Health Sciences Inc.	Bussv	Wynn Resorts Ltd	Meals
Westlake Chemical Corp.	Chems	Pbf Energy Inc.	Oil
Analog Devices	Chips	Bmc Stock Holdings Inc.	Rtail
Applied Materials Inc.	Chips	Firstcash Inc.	Rtail
Broadcom Inc.	Chips	Five Below Inc.	Rtail
Cabot Microelectronics Corp.	Chips	Malibu Boats Inc.	Ships
Comtech Telecommunications Inc.	Chips	Mastercraft Boat Hldngs Inc.	Ships
FormFactor Inc.	Chips	Adobe Inc.	Softw
Lam Research Corp.	Chips	Paycom Software Inc.	Softw
Micron Technology Inc.	Chips	Salesforce.com Inc.	Softw
Mks Instruments Inc.	Chips	Ss&C Technologies Hldgs Inc.	Softw
Monolithic Power Systems Inc.	Chips	Veeva Systems Inc.	Softw
Nvidia Corp.	Chips	Oneok Inc.	Util

Source: Personal Elaboration

As the number of companies under analysis is substantial, the research skips the company description, the presentation of their main figures and the trend in recent years. In the next section, on the other hand, all the data retrieval methods and databases used to perform these searches are presented.

3.2. Data and Methodologies

3.2.1. Data Research

To make the data processing and search path clearer, the analysis starts with the presentation of all the necessary data for each company involved in the research. They are as follows:

- Quarterly balance sheet values from Q1-2010 to Q3-2019: values reported by companies in their quarterly reports to the SEC. The required items were: "Cash and Short-Term Investments", "Common Shares Outstanding", "Debt in Current Liabilities", "Long-Term Debt - Total", "Depreciation and Amortization - Total", Preferred Stock";
- Quarterly closing price of shares from Q1-2010 to Q3-2019;
- Quarterly Financial Ratios for the period Q1-2010 to Q3-2019 and for the reference industries: the selected multiples are "Price/Earnings", "Price/Sales", "Price/Book Value", "Price/Cash Flow", "Price/Operating Earnings";

- Estimates of values from Q1-2010 to Q3-2019: the items found are "Earnings Per Share (EPS)", "Book Value Per Share (BPS)", "Cash Flow Per Share (CPS)", EBIT (EBI)", "EBITDA (EBT)", "Net Income (NET)" and "Revenue (SAL)";
- Other indicators: "Gross Profit Margin", "Debt/Equity Ratio", "Cash Ratio", "PERMNO".[155], "OFTIC"[156], Standard and Poor's Identifier Number.

The search for these entries was done through the database of the Wharton School of the University of Pennsylvania called Wharton Research Data Services (WRDS). Within it, one can find several databases with company data according to individual research needs. Concerning those of this work, the "Compustat IQ" database was used for the quarterly balance sheet data, more specifically the "Compustat North America" section because the companies were all listed on the major United States stocks exchanges. This database provides balance sheet, income statement and flow of funds data for more than 500 companies and also offers additional data for about 47000 active and 37000 inactive companies.[157]

The “Center for Research in Security Price” U.S. Stock Database (CRSP), which contains end-of-day and end-of-month prices on the NYSE, NYSE MKT, NASDAQ and Arca primary stock exchanges, together with basic market indices, was used for closing price requirements and common shares outstanding.

About the financial ratios of companies and industries, necessary for the second part of the analysis, the database "Financial Ratios Suite by WRDS" was used. It is a web-based search engine that provides over 70 pre-calculated financial ratios for all US companies in eight different categories. As for the industries then, all financial ratios belonging to the "Fama-French 49 Industries" were initially downloaded. Thanks to this industry classification, it was possible to control the specific sector of the companies using the SIC Code. Once the sectors useful for the analysis were obtained, the redundant data were eliminated.[158]

Subsequently, the estimate data were found through the "Institutional Brokers' Estimate System" (I/B/E/S) database, which contains analysts forecast data. Thomson Reuters provide this database, which is a historical profit estimation database containing analysts' estimates for over 20 forecasting measures, including EPS, revenues, price targets, EBITDA and pre-tax profit, available at both consensus and detailed levels.[159] The estimates found refer to values for an annual estimate period (1-Year Forecast).

[155] Permanent ID Number of a Security
[156] Official Ticker of a Security
[157] Data Retrieved by https://wrds-www.wharton.upenn.edu/
[158] Data Retrieved by https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/Data_Library/det_49_ind_port.html
[159] Data Retrieved by https://wrds-www.wharton.upenn.edu/

Once the data collection phase was finished, the study moved on to processing in order to create a final dataset. At this point, it was necessary to obtain a single value for each company, for each item, in each quarter to compare all the retrieved data. While the balance sheet items, share price and financial ratios met this requirement, the estimate values presented numerous data within the quarters, developed by several analysts daily. Thus, they were reorganised, respecting the fiscal year of each company, in the following way. Initially, all estimates belonging to the same fiscal quarter, made on different days, were changed in the date, bringing them to the last available day of the quarter. Once this action was carried out, an average was made within these periods. At the end of this operation, each company had an average yearly forecast in each quarter.

When all the data met the time requirements of the analysis, they were incorporated into a single document, which represented the final dataset, joining them respecting the company name, their permanent number (PERMNO) and the reference date (e.g. Q1-2015 with Q1-2015). From here, it was possible to perform some calculation necessary for the analysis. Firstly, the quarterly market capitalisation for all the companies was calculated. Subsequently, through all the values retrieved explicitly from the "Compustat IQ" database, the enterprise value was obtained.

3.2.2. Linear Regressions: Introduction

Before deepening into the analyses, it is useful to recall some statistical concepts that have been used during the study. In order to provide an empirical contribution to the paper, the analyses are based on the Ordinary Least Square (OLS) linear regression model.

This model, as well as being one of the most widely used in the financial field to determine the relationship between different variables, proposes a clear and effective solution to the questions asked above.

Going into detail, the Ordinary Least Square estimator allows determining the correlation between preselected covariates and a response variable. This correlation can be influenced by numerous factors that make the result inadequate. In order for OLS estimator to generate reliable results, certain assumptions must be respected, which are listed in Appendix D.

Since the two analyses intend to search for the degree of influence of several variables on the response variable, the most suitable analysis was a multivariate linear regression. A multivariate linear regression could be expressed by the following equation:

$$Y = \beta_0 + f_1(x_1)\beta_1 + \cdots + f_n(x_n)\beta_n + e$$

Where Y represents the response variable, which depends on the error (e) and function $f_n(x_n)$where n indicates the number of covariates present in the model. Through the linear regression it is then possible to determine the β model variants, so that the sum of the squares of

the errors (e) is minimal. The slope coefficients are expressed by the coefficient β, while the forecast values through the coefficient $\hat{\beta}$. So, the equation of slope coefficients is:

$$\hat{\beta} = \beta + e$$

About the error term (e), also called residuals, can be defined as the difference between the independent variables multiplied by slope coefficients and the dependent variable. Usually, the goal of models is to obtain low error values, because the closer they are to zero, the better the quality of the model. Also, the errors are assumed normally distributed:

$$e_n \sim N(0, \sigma^2)$$

With an error equal to 0 and a constant variance expressed by the term σ^2.[160] In addition to the description of the model, it is necessary to introduce and define the interpretation of the resulting results in order to understand the quality of the analysis. Besides a description of the model, it is required to explain how to interpret the outcomes. This process allows to understand the quality of the analyses better. Thus, before conducting the statistical test, it is necessary to select the dependent variables, the independent variables and to define hypotheses according to which accept or reject the results generated. At this point, H_0 is defined as the null hypothesis and H_1 as a different hypothesis. The statistical test is used to calculate the probability that the null hypothesis is rejected or not according to a decision rule.

3.2.3. Linear Regressions: Interpretation of Results

Once the necessary steps have been taken, using a calculation program such as MS-Excel or R, the linear regression results are obtained. At this stage, one must pay attention to the following generated outputs:

- R^2 or coefficient of determination is an indicator of the goodness of the regression curve. It can vary from 0 to 1 and represents the ability of independent variables to explain the dependent variable (y). The higher this value, the more the constructed model is able to capture the value of the response variable.
- Adjusted R^2: when the regression model is multivariate, it is advisable to use the Adjusted R^2 value, which is a modified version of R^2 for the number of predictors in the model, which can also take negative values.
- Standard error: it measures the accuracy of the regression model. It represents the average of the errors present in the model and is inversely proportional to its accuracy. Because of this feature, the smaller the standard error, the more accurate the regression model is.

[160] H. Lang. (2015). Elements of Regression Analysis.

- Significance F: it represents the probability that the null hypothesis is valid. In other words, the higher its value, the greater the probability that the model is wrong. Usually, a threshold level of significance F is used. The most common levels are 1%, 5% or 10%.
- P-value: this value belongs to each independent variable and represents the probability that the estimated coefficients are wrong or unreliable. As with significance F, p-value values should be as low as possible. The pre-set cut-off level depends on the type of variables under analysis. The most used levels are 1%, 5% or 10%.[161]

Once the previous parameters have been analysed, it is possible to express an opinion on the quality of the linear model and analyse the possible errors that contradict the assumptions within the linear regression to find solutions. Thus, the main assumptions are endogeneity, homoscedasticity, and absence of multicollinearity.

Since a violation of these assumptions causes an imperfection in the model, it is necessary to check whether they are respected. If they are not, possible errors may be different and may belong individually to one of the three assumptions. About homoscedasticity, when it is not respected in the model, it is called heteroscedasticity. This error implies that the residual variances are not constant to the value of the independent variables. In order to fix this error, some remedies can be performed. One of these is the Breusch-Pagan Test (BP), which checks whether the variance of the residual terms is dependent on the values of the covariates. The null H_0 hypothesis predicts the presence of homoscedasticity, while the alternative H_1 hypothesis does not support it. The statistical test is performed using a chi-squared test, with the result expressed in terms of the p-value. The square of residuals is used as the response variable, while the original covariates of the model as independent variables to perform the test. Once the regression analysis is conducted, Once the regression analysis is conducted, the value of χ^2using the formula:

$$\chi^2 = R^2 * n$$

Where R^2 is the R^2 of the model, *n* the number of observations, and the value $\chi^2$is asymmetrically chi-square distributed under the hypothesis nothing H_0. For the p-value to be accepted, it must be above a certain threshold. Otherwise, the model presents heteroscedasticity.

A possible remedy to this error in the model is the transformation of the variables. To perform this action, one selects the potential variables and execute the natural logarithm of the original value. The outcomes of that operation must necessarily read differently, due to the logarithmic transformation performed.[162]

[161] Sharma, S. C., & Dougherty, C. (1994). An Introduction to Econometrics. The American Statistician, 48(2), 172. https://doi.org/10.2307/2684280

[162] See footnote 14

A second possible error within the model is endogeneity. This term indicates the correlation between error terms and covariates in the OLS model. This error appears when the expected value of the residuals is different from zero, and therefore, when an independent variable is correlated with one of the covariates, the OLS estimator generates inconsistent results. The main reasons leading to the following error are:

- sample selection bias: it occurs when model data is not collected arbitrarily;
- simultaneity: occurs when a dependent variable influences one or more independent variables. Think of the case of an insurance that covers any type of damage. It leads the holder of the contract to take riskier attitudes under the insurance cover;
- lack of significant independent variables: when this event occurs, the model tries to explain this fact by modifying the residue;
- measurement error: it is defined as the variation between the measured and actual result and it is created due to the connections between covariates and the error term (endogeneity).

A remedy for endogeneity is the use of instrumental variables. They need to be correlated with endogenous variables, but not with the error term.[163]

A third potential error in the model is multicollinearity. It generates redundant information within the model, which means that the explanation of a regressor on the response variable is superimposed by what explains one or more independent variables. Therefore, this phenomenon occurs when there is a strong correlation between one or more independent variables, which makes the variance of the OLS estimator very large.[164]

To find out if the model has multicollinearity, we can run a *"Variance Inflation Factor Test"*, which is expressed by the formula:

$$VIF = \frac{1}{(1 - R^2)}$$

This factor is calculated for each covariate present in the model, using it as a response variable and the others as independent variables in a new regression model. Once the values of R^2 are obtained for all covariates, the respective VIFs are calculated. VIF values greater than 10 indicate the presence of multicollinearity in the model.

A remedy to multicollinearity problems is the removal of critical variables, as seen in the subsequent analysis.

[163] See footnote 14

[164] Yoo, Wonsuk, Robert Mayberry, Sejong Bae, Karan Singh, Qinghua Peter He, and James W Lillard. 2014. "A Study of Effects of MultiCollinearity in the Multivariable Analysis." International Journal of Applied Science and Technology 4 (5): 9–19. http://www.ncbi.nlm.nih.gov/pubmed/25664257

Finally, to check if the key assumption of normality is respected, several methods can be used. The one used in this paper is a graphical solution called Quantile-Quantile plot (Q-Q) that allows understanding if errors are distributed normally. Thus, the quantiles of the covariates are graphically plotted against each other. If the covariates come from a normal distribution, a straight line with slope one is generated by the graph. Besides, this graphical representation allows us to understand if the dependent variable is a linear function of the independents when the drawn values follow a straight line.[165]

3.3. Which Drivers Influence the Value of HGFs?

The following analysis model presented is addressed to the first set of questions introduced at the beginning of the chapter. The latter closely involves one of the most complex aspects of evaluation: value estimation. At this point, the following model aims to initially identify which type of factors have the most significant impact on enterprise value among the historical ones and the estimates made by analysts about the future. Subsequently, once the most suitable model for the analysis has been identified, some tests are followed to identify some errors that undermine the OLS estimator assumptions presented in Appendix D.

The final objective is to suggest which elements should take on more considerable attention during the analysis of HGFs and also demonstrate that the value of companies with high growth estimates lies more in future numbers than in historical and current numbers, as repeated several times during the paper.

The data under analysis correspond to 54 companies presented in the Table 1.

3.3.1. Choice of Variables

The dependent variable selected for this model is the enterprise value. The following formula was used for its calculation:

$$\text{EV} = \text{E} + \text{Total Debt} + \text{Preferred stock} - \text{Cash and ST Investments}$$

Where E represents the market value of equity and Total Debt represents the market value of short and medium/long-term debt.

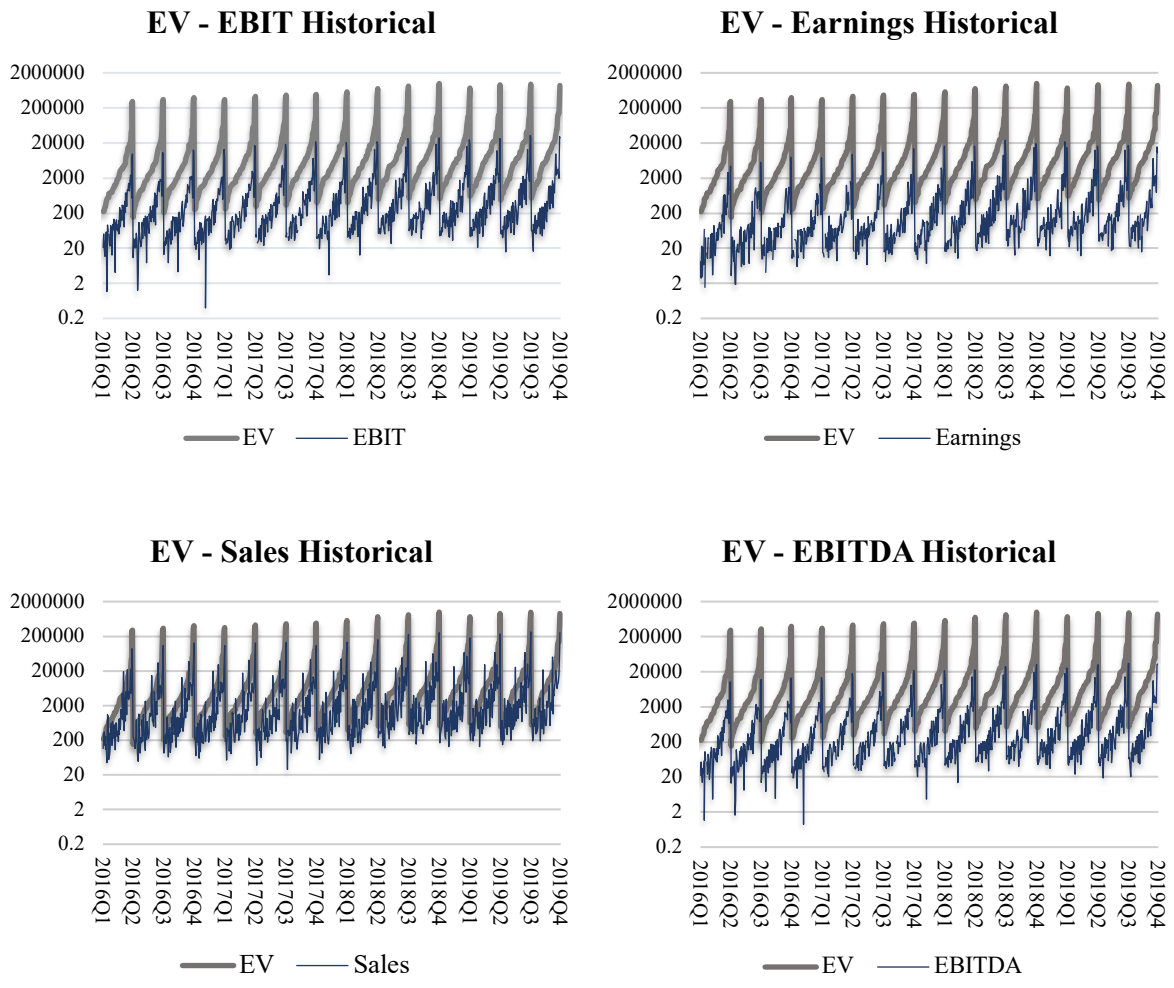
The selected independent variables are the main items of the income statement and represent key elements for the evaluation of the company. They are:

[165] Koenker, R., & Hallock, K. F. (2001). Quantile regression. Journal of Economic Perspectives, 15(4), 143–156. https://doi.org/10.1257/jep.15.4.143

- Revenue: indicate the turnover of a company and are expressed through the SALES ticket within the analysis, in order to maintain consistency with the reference database that makes them explicit in this way;
- EBITDA: represents the gross operating margin that highlights the income deriving from the operations of a company and therefore, without considering interest, depreciation and amortization of assets.
- EBIT: indicates the company's operating income and expresses the company's result before interest and corporate taxes.
- Earnings: represent the company's revenues once tax obligations have been met.

Before analysing the initial model, it is necessary to specify that for the first analysis, the data relating to the items just described have been collected at both historical and estimation level. The following graphs show the trend of the items over the years:

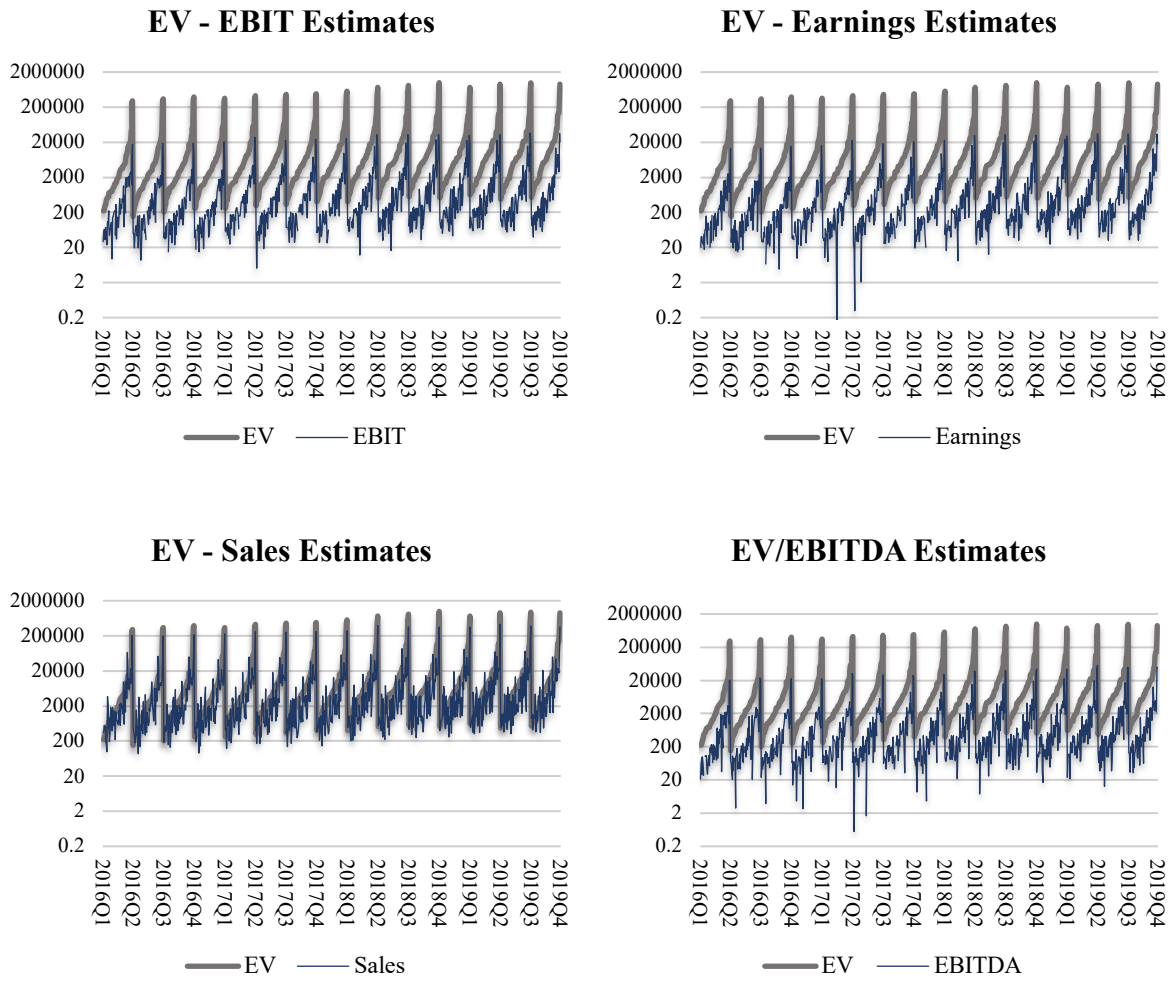
Figures 10 - *EV and Main Value Drivers Historical Performances*



Source: Personal Elaboration

Looking at the first four graphs relating to the trend of historical drivers, it can be seen that sales moved in virtually the same way as dependent variable. The measures that showed some countertrend were EBIT and EBITDA. Turning now to the estimate values, they showed:

Figures 11 - ***EV and Main Value Estimates Historical Performances***



Source: Personal Elaboration

As in the first case, Sales shows more clearly their correlation with the Enterprise value. In this case, earnings and EBITDA represent the most discordant values. This fact may mean that although some companies had negative or very low estimates of these items, enterprise value continued to grow, driven by other factors that influenced it stronger.

3.3.2. Results

The first analysis carried out led to the choice of the most explanatory coefficients to be adopted in the enterprise value analysis model. Thus, this first "screening" was carried out using the four covariates, historical and estimates, in two linear regression models with the same response variable. The results of the first step of the analysis are as follows:

Table 2 - Historical Linear Regression Model Outputs

Regression Statistics	
Multiple R	0.95906
R^2	0.91980
Adjusted R^2	0.91940
Standard Error	29878.56982
Observations	820

Source: Personal Elaboration

Table 3 – Estimates Linear Regression Model Outputs

Regression Statistics	
Multiple R	0.96340
R^2	0.92810
Adjusted R^2	0.92775
Standard Error	28288.0044
Observations	820

As can be seen from the tables, the linear regression model of the estimates has led to more robust results than the historical elements. This output indicates a greater ability of the estimates to explain the current enterprise value, confirming what was said in the second chapter. Since the objective of this first analysis is to identify the type of variables to be selected, the models were analysed by looking at the value of the adjusted R^2, because for multivariate regression models it is more recommended than the classic R^2, and the standard error. In both cases, the analysis of the estimates led to better results, higher R^2 and lower standard error, and therefore the latter was chosen as variables for the analysis model.

Initial Model

The initial model corresponds to that examined for "screening" and uses all four covariates. The results of the regression analysis are as follows:

Table 4 – *Analysis Coefficients' Outcomes*

Independent Variables	Coefficients	Standard Error	t Stat	P-value	Significance
Intercept	-2588.598	1114.223	-2.323	0.0204	
EBIT	14.024	2.062	6.801	2.01E-11	***
EBITDA	11.384	0.858	13.276	1.48E-36	***
Sales	0.813	0.066	12.240	9.65E-32	***
Earnings	-17.479	1.991	-8.777	9.76E-18	***

****P<0.001, **P<0.05, *P<0.1*

The levels of significance of the variables, expressed by the P-value, are all below the acceptance threshold set for this analysis of 0.1. The factors that are more explanatory than the others, also due to their minor standard error, are Sales and EBITDA.

Table 5 – *Initial Model Regression Statistics*

Observations	R^2	Adjusted R^2	Standard Error	Covariates
820	0.92811	0.92776	28288.00445	4

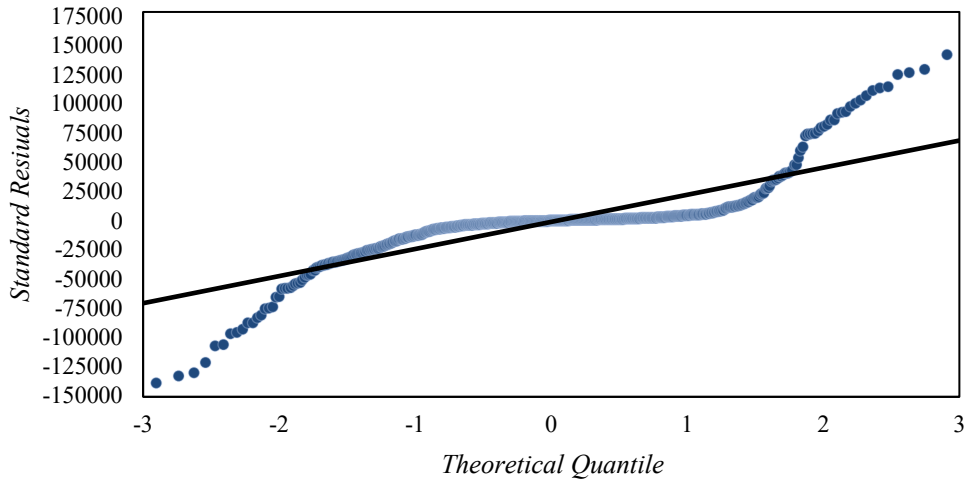
Source: Personal Elaboration

Since it is a multivariate linear regression model, the adjusted R^2 is analysed which, as mentioned above, is an indicator of the goodness of the regression curve and the closer it is to the value of 1, the higher the ability of the independent variables to explain that response. In this case, the value of 0.927 indicates that covariates can explain about 93% of the independent variable.

Analysis of the Initial Model

The analysis of the model allows identifying strengths and weaknesses through the tests before mentioned. Regarding the assumption of normality, the Q-Q graph shows:

Figure 12 - *Normal Q-Q Plot*



Source: Personal Elaboration

In order to check if the residuals derive from a normal distribution, they must follow the straight line. In this case, we can see how, although in the central part they follow it quite precisely, this homogeneity is not present in the tails. Since one of the assumptions of the OLS estimator is normality, the model must be improved in order to respect this assumption.

Then, to examine the possible presence of multicollinearity in the model, the *Variance Inflation Test was* used. The results are as follows:

Table 6 - *Variance Inflation Factor Test*

Independent Variables	VIF	Decision
EBIT	91.51	×
EBITDA	33.73	×
Sales	8.45	∨
Earnings	72.90	×

× – *refused,* ∨ – *accepted*

The reported VIF values differ widely. Following the general rule of thumb involving multicollinearity for VIF values greater than 10, it can be seen that the model has this problem. The explanation can be found in the derivation of the variables. Since they all come from the income statement, they are influenced by each other. Think of the EBITDA that comes from Sales. So, the model also needs further improvement in this respect.

Finally, in order to check for heteroscedasticity problems, the Breusch-Pagan Test was conducted. With a level of significance set at 99% and one degree of freedom, according to the tables the cut-off point of the value of χ^2 is 6.63. So, in order to confirm the null hypothesis H_0, about homoscedasticity, the value of χ^2 must be less than this number.

Table 7 - *Breusch-Pagan Test*

df	***χ^2***	***P-value***
1	258.962	2.889E-58

Source: Personal Elaboration

As can be seen from the table, the value of χ^2 of 258.96 is higher than the value indicated by the tables, which implies the presence of heteroscedasticity at a 99% confidence level. Besides, the value of p-value is lower than the threshold of 0.05, which leads to rejecting the null hypothesis H_0.

Since this first model, despite the first encouraging results, has some imperfections that need to be corrected in order to respect the assumptions of the OLS estimator, a variable selection procedure called "Backward Eliminations" was used. According to its rules, the variable with the highest P-value was eliminated, checking also for the values of VIF test.[166] The choice fell on the EBIT variable, which had the highest values for both parameters.

Second Model

After removing the EBIT variable, a second linear regression model was developed, which produced the following results:

Table 8 - *Second Model Coefficients' Ouputs*

Independent Variables	Coefficients	Standard Error	t Stat	P-value	Significance
Intercept	-349.838	1093.604	-0.320	0.749	
EBITDA	13.715	0.808	16.982	1.29E-55	***

[166] Shi, Lin, Johan A. Westerhuis, Johan Rosén, Rikard Landberg, and Carl Brunius. 2019. "Variable Selection and Validation in Multivariate Modelling." Bioinformatics 35 (6). Oxford University Press: 972–80. doi:10.1093/bioinformatics/bty710.

Earnings	-5.191	0.860	-6.033	2.45E-09	***
Sales	0.718	0.067	10.761	2.42E-25	***

****P<0.001, **P<0.05, *P<0.1*

The p-value values remain valid for all three variables as in the previous case, but there is an improvement in the standard error of all covariates.

Table 9 - *Second Model Regression Statistics*

Observations	R^2	Adjusted R^2	Standard Error	Covariates
820	0.9240	0.9237	29061.7995	3

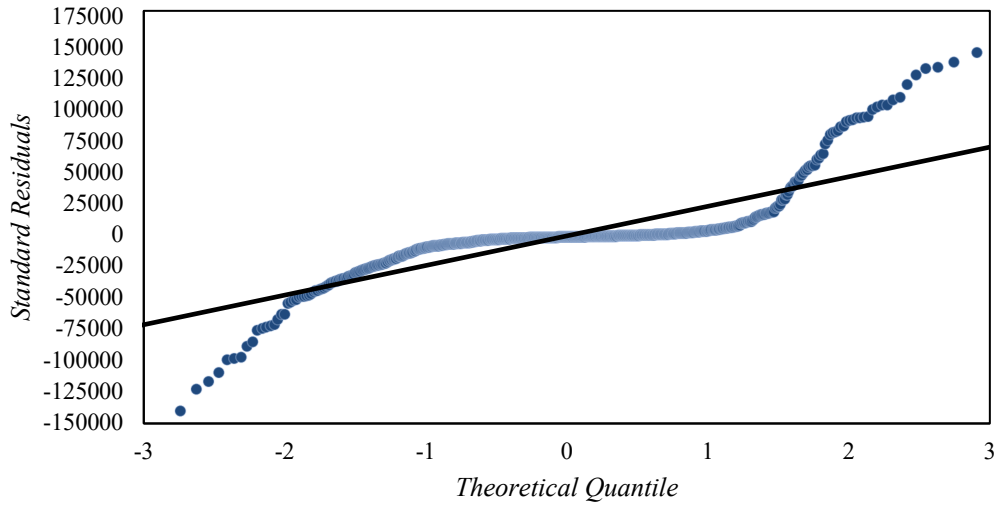
Source: Personal Elaboration

The Adjusted R^2 value of 0.9237 is slightly lower than that of the initial model (0.9277). This indicates that the variable EBIT did not have much influence on the dependent variable EV, as its removal did not significantly alter the value of Adjusted R^2.

Analysis of the Second Model

Proceeding consistently with the first analysis, the Q-Q graph is presented in order to check the presence of normality within the model.

Figure 13 – *Normal Q-Q Plot*



Source: Personal Elaboration

Although the model has some improvements, there is still a problem of normality highlighted by the graph queues that diverge from the straight line.

Concerning multicollinearity, the VIF test produced the following results:

Table 10 - *Variance Inflation Factor Test*

Independent Variables	VIF	Decision
EBITDA	28.35	×
Earnings	12.89	×
Sales	8.08	∨

× – refused, ∨ – accepted

Although VIF values have dropped, there remains a multicollinearity problem for the EBITDA and Earnings variables. It is necessary to adjust the model further, in order to get these values within the threshold value of 10.

Finally, the Breusch-Pagan Test that verifies the presence of heteroscedasticity, at a level of significance of 99% and one degree of freedom, has produced the following results:

Table 11 - *Breusch-Pagan Test*

df	χ^2	*P-value*
1	262.403	5.14E-59

Source: Personal Elaboration

The threshold of χ^2 of 6.63 is not respected by the model, as in the previous case, and therefore it is still in the presence of heteroscedasticity. Also, the P-value does not exceed the threshold of 0.05, so the null hypothesis H_0 is rejected.

Given these results, it is required to improve the model by eliminating another variable. Following the same rule used previously, the "Earnings" variable has the highest P-value in the group, in addition to having a VIF value above the acceptable threshold.

Third Model

The third regression model developed involves only two independent variables, EBITDA and Revenue, and the Enterprise Value as response variable. The linear regression analysis generated the following results:

Table 12 - *Third Model Coefficients' Ouputs*

Independent Variables	Coefficients	Standard Error	t Stat	P-value	Significance
Intercept	-1773.882	1090.707	-1.626	0.104	
EBITDA	9.108	0.269	33.914	4.67E-158	***
Sales	1.037	0.042	24.943	1.52E-102	***

****P<0.001, **P<0.05, *P<0.1*

The linear regression confirms the validity of p-values and it showed a marked improvement of standard error, settling at around 0 for both covariates. Model-specific data are shown below:

Table 13 - *Third Model Regression Statistics*

Observations	R^2	Adjusted R^2	Standard Error	Covariates
820	0.9206	0.9204	29684.5923	2

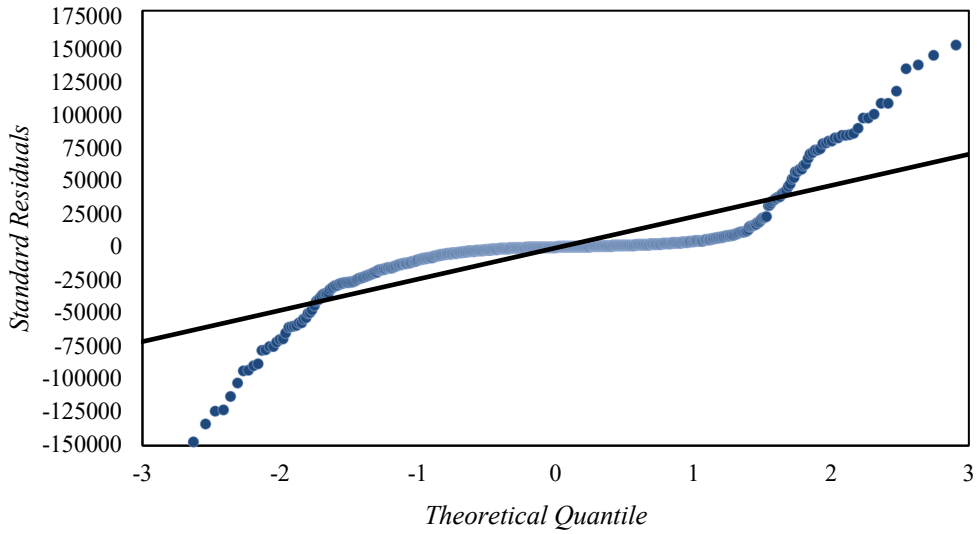
Source: Personal Elaboration

As can be seen from the table, the value of Adjusted R^2 has decreased compared to the second model's one (0.9240), due to the elimination of "Earnings" variable. However, it is still highly satisfactory, considering the two covariates of the model explain about 92% of the dependent variable. Concerning the standard error, the third model is at the previous levels, which cannot be considered satisfactory because of their dimension.

Analysis of the Third Model

Regarding the assumption of normality, the Q-Q Plot generated is as follows:

Figure 14 - *Normal Q-Q Plot*



Source: Personal Elaboration

As can be seen from the graph, although the queues show an improvement over the previous cases, they do not guarantee the normality of the residues. As this assumption is crucial for the accuracy of the model, further improvements need to be made on the variables.

Concerning the problem of multicollinearity presented in previous models, the VIF test on variables produced the following results:

Table 14 - *Variance Inflation Factor Test*

Independent Variables	VIF	Decision
EBITDA	3.00	∨
Sales	3.00	∨

× – refused, ∨ – accepted

The Variance Inflation Factor Test showed essential improvements over previous analysis, excluding multicollinearity problems in the model. Contrary to what one might think, although the two covariates both came from the income statement, they do not correlate with each other. Moving on to the last test to check homoscedasticity intake, the Breusch-Pagan test shows:

Table 15 – *Breusch-Pagan Test*

df	χ^2	***P-value***
1	210.381	1.13E-47

Source: Personal Elaboration

Unfortunately, the model still has heteroscedasticity problems evident through the values of χ^2 and P-value. However, since the latter model showed encouraging analysis results for the achievement of the study objective, it was decided to carry out a refinement on the two covariates. At this point, a logarithmic transformation of the response variable and the two covariates was performed to mitigate the problem of heteroscedasticity present in the model. The logarithmic transformation of the variables is a widely used method to solve different types of situations such as the lack of linear relationship between the variables or the presence of a highly skewed variable to be made approximately normal. In this research, the type of the transformation rule is the "log-log model".[167]

Final Model

After the logarithmic transformation of the variables through the application of natural logarithm on their value, the regression analysis thus constructed presents the following outputs:

Table 16 - *Final Model Coefficients' Outputs*

Independent Variables	Coefficients	Standard Error	t Stat	P-value	Significance
Intercept	2.878	0.164	17.565	7.71E-59	
EBITDA	0.814	0.044	18.354	2.91E-63	***
Sales	0.094	0.047	2.008	4.49E-02	**

****P<0.001, **P<0.05, *P<0.1*

[167] Benoit, Kenneth. 2011. "Linear Regression Models with Logarithmic Transformations." London School of Economics, 1–8. http://www.kenbenoit.net/courses/ME104/logmodels2.pdf.

The result of the linear regression confirms the significance of both independent variables on the enterprise value. Besides, significant improvements can be noted for the standard error, which, although it had previously reached low values, is even closer to 0. As for the regression coefficients, the following table shows:

Table 17 - *Final Model Regression Statistics*

Observations	R^2	Adjusted R^2	Standard Error	Covariates
820	0.7625	0.7619	0.8124	2

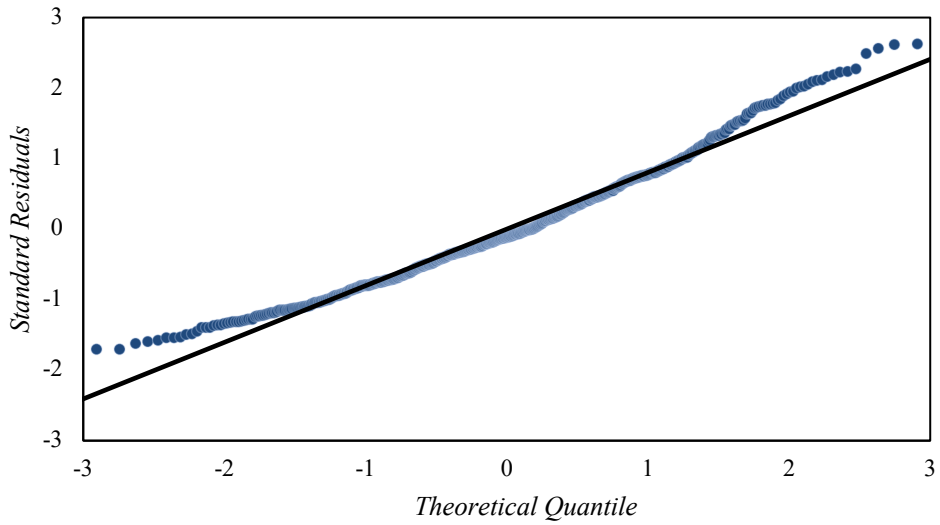
Source: Personal Elaboration

Analysing the proposed results, we can notice the difference in Adjusted R^2 value, which has decreased from about 92% in the previous model to the current 76%. Despite the loss of explanatory power of about 17.4% by the independent variables following the logarithmic transformation, the model remains above the acceptable threshold of 75%. Additionally, it shows a significant improvement of standard error, which reaches a value around zero, confirming the accuracy of the model.

Analysis of the Final Model

Following the same steps carried out during the other analyses, the first test to be carried out was to verify the normal distribution of the residues. At this point, the generated Q-Q Plot results:

Figure 15 - *Normal Q-Q Plot*



Source: Personal Elaboration

As can be seen from the Figure 15, the model has no errors about the OLS estimator's assumption about the normal distribution of residuals. Compared to previous models, the improvement is clear and undeniable from a graphical point of view. The two lines are almost totally superimposed.

About the presence of multicollinearity in the model, the VIF test was conducted, obtaining the following results:

Table 18 - *Variance Inflation Factor Test*

Independent Variables	VIF	Decision
EBITDA	6.418	∨
Sales	6.418	∨

× – refused, ∨ – accepted

The VIF test values for both covariates are below the maximum threshold of 10, which excludes correlation problems between variables in the model.

Finally, regarding the presence of heteroscedasticity, the Breusch-Pagan Test was conducted, with the following results:

Table 19 – *Breusch-Pagan Test*

df	χ^2	*P-value*
1	6.5990	0.0102

Source: Personal Elaboration

At a 99% significance level and one degree of freedom, the model has $\chi 2$ value of 6.59, below maximum threshold expressed on the chi-square tables of 6.63. This evidence indicates that problem of heteroscedasticity has also been solved, confirmed by the P-value of 0.102, higher than the 0.05 threshold value. Therefore, under this last information, it is possible to accept the null hypothesis H_0 and respects the assumption of homoscedasticity required by the OLS estimator.

3.3.3. Analysis of Outcomes

Thanks to the development of the model, the final result was consistent with the assumptions of the OLS estimator. This requirement is fundamental to confirm the validity of the research.

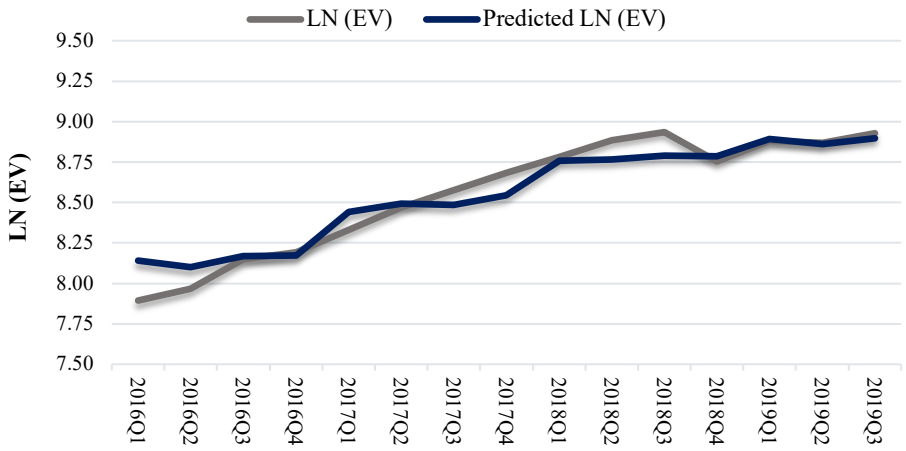
As it was intended to answer the first set of questions concerning the financial data that most influence the value of companies and the role of estimates during the valuation, the following conclusions can be drawn. Concerning the first topic, the models just presented proved the ability of two specific variables to influence and determine enterprise value: EBITDA and Revenue. Concerning the importance of estimates, it has been demonstrated that these elements are more fitted of capturing current levels of enterprise value than historical ones. Therefore, when analysing high-growth firms, one should take more account of estimation values than past or current values. The value of these companies is more represented by future expectations rather

than current or past numbers, confirming what was said during second chapter. Finally, to show the validity of the final model, the EV's equation determined through their coefficients is:

$$\mathrm{LN(EV)} = 2.87784 + 0.81373 * \mathrm{LN(EBITDA)} + 0.09394 * \mathrm{LN(Sales)} + \hat{\varepsilon}$$

Using this equation to obtain the "predicted enterprise values" for each company and comparing them with real values, it is possible to recognise the precision of the model through the following graph:

Figure 16 - ***Real EV vs Model Prediction***



Source: Personal Elaboration

Thanks to this figure, it is possible to notice how the "Predicted EV" of the model can correctly estimate and predict the real EV of the companies during the analysis period. Such a figure further confirms the results of the analysis and the first experimentation of this elaboration.

Finally, thanks to this first study, it is possible to introduce the second part of the analysis, which questions the use of multiples according to the traditional methodology and tries to provide concrete answers to the problems that this method of valuation faces with high growth firms.

3.4. Beyond the Simplicity of Relative Valuation

The second part of the analysis focuses on one of the most used valuation techniques by practitioners: the relative valuation. As introduced and examined in the second chapter, this method has some imperfections when applied to HGFs. Since it is based on the comparison of similar companies within the same sector to determine the value of a company, in the case of high growth firms very often these similar firms, within the same sector, do not exist.

The following analysis aims to answer the second set of questions presented above, concerning the most suitable way to carry out a comparable company analysis and seek for the "real" comparable of high growth firms.

The multiples used during the valuation are price multiples and more precisely: "Price/Earnings", "Price/Sales", "Price/Book Value", "Price/Cash Flow", "Price/Operating Earnings". The

dependent variable of this second part of the study is market capitalisation, calculated through the share price at the end of the quarter and the respective number of shares outstanding. The reference period is the same as the previous analysis, i.e. from the first quarter of 2016 to the third quarter of 2019. The study skips the introduction of regression analysis and starts directly with the presentation of the survey and its results to avoid redundancies.

3.4.1. Data Analysis

The following research is divided into several phases, necessary for the accuracy of the analysis and its results. The first one examines the ability of sector multiples to capture the market capitalisation of the companies included in this paper (see Table 1).

After analysing the results of this first observation, the growth factor, which is crucial for these companies, is introduced into the analysis. Hence, only high-growth firms belonging to the same sector are compared, according to the classification adopted by the SEC, using the same type of multiples as before as independent variables. The results of the second study are compared with those of the first one to examine their differences.

Subsequently, a completely new methodology is developed to perform the relative valuation. It involves high growth companies belonging to different sectors, but similar in risk, financial structure and growth. This attempt, although very innovative, was fundamental for the solution of the questions that this paper sought.

Finally, the best methodology, between the three studies, is compared with the model developed in the first part of the analysis to comment on their strengths and weaknesses, hoping to answer the initial questions as exhaustively as possible.

3.4.2. Results

The first observation is made on a sample of 46 companies from 15 different sectors. Multiples belonging to different sectors were compared with the market capitalisation of the companies to obtain the outcomes. The former data were found through the "Financial Ratios by WRDS" database in the "Industry Ratio" section.

So, once the companies were divided by sector and associated with their respective multiples, linear regressions were conducted to understand how much the independent variables, coming from companies operating in the same business, can explain the level of capitalisation of high growth firms. As the study was carried out in 15 sectors, it is considered appropriate to skip the specific presentation of the results in order to avoid unnecessary delays. Instead, the results are presented in groups, and are as follows:

Table 20 - *Sector Multiples Linear Regression Statistics*

N	Sector	R^2	Adjusted R^2	Standard Error	Significance F	Type	N°of observations
1	Autos	0.0532	-0.1440	310.8472	0.9251	Sector Multiples	**30**
2	Bldmt	0.3567	0.2226	292.7947	0.0474	Sector Multiples	**30**
3	BusSv	0.2439	0.1989	3602.1925	0.0009	Sector Multiples	**120**
4	Chems	0.4642	0.1666	2558.3541	0.2647	Sector Multiples	**15**
5	Chips	0.0579	0.0252	31151.371	0.1226	Sector Multiples	**150**
6	Cnstr	0.1944	0.0266	445.4202	0.3580	Sector Multiples	**30**
7	Drugs	0.5661	0.5260	416.2126	0.0000	Sector Multiples	**60**
8	Fun	0.1357	-0.3445	2149.5984	0.9112	Sector Multiples	**15**
9	Hshld	0.2265	-0.2032	741.2638	0.7512	Sector Multiples	**15**
10	Insur	0.9016	0.8469	2351.2460	0.0003	Sector Multiples	**15**
11	LabEq	0.1144	-0.0781	2020.7880	0.7045	Sector Multiples	**30**
12	Meals	0.0454	-0.1074	6656.1883	0.8771	Sector Multiples	**30**
13	Rtail	0.1961	0.0930	1723.2526	0.1161	Sector Multiples	**45**
14	Ships	0.4763	0.3672	184.5042	0.0057	Sector Multiples	**30**
15	Softw	0.4859	0.3971	15350.024	0.0073	Sector Multiples	**75**
Average		**0.301**	**0.1329**	**4663.604**	**0.3395**		
Total		**30.12%**	**13.29%**	**4663.604**	**0.339**	**Sector Multiples**	**690**

Source: Personal Elaboration

As shown in the table, the individual regressions by sector led to the following values. It can be seen that sector multiples explain market capitalisation by 13.29% out of a total of 690 observations on average. This first result confirms the risk of misusing sector multiples. As we can see from the results, HGFs cannot be compared with companies in the same sector that have lower average growth rates.

Therefore, the traditional method of multiples, which involves the use of sector multiples to determine the value of the company, is approximate according to the above results. Thus, a second attempt was made using only high-growth companies in the same sector within the analysis.

At this point, four industries were selected for analysis: Chips, Drugs, Business Service and Software. These industries were selected because they provided a more reliable comparison than ones with a small number of HGFs and for which the analysis was too limited.

The independent variables of the following analysis are the same multiples as before. However, the data are derived from company statements. The outcomes provided are as follows:

Table 21 - *HGFs Multiples Regression Statistics*

N	Sector	R^2	Adjusted R^2	Standard Error	Significance F	Type	N°of observations
1	Softw	0.7168	0.6524	5218.2063	0.0153	HGFs Multiple	**120**
2	Chips	0.3210	0.2598	4516.5356	0.0612	HGFs Multiple	**360**
3	Drugs	0.4160	0.2579	11162.0426	0.1383	HGFs Multiple	**180**
4	BusSv	0.2440	0.2010	1795.8817	0.0053	HGFs Multiple	**450**
Average		**0.4246**	**0.3428**	**5673.1666**	**0.0550**		
Total		**42.46%**	**34.28%**	**5673.166**	**0.055**	**HGFs Multiple**	**1110**

Source: Personal Elaboration

It is possible to notice a marked improvement in terms of R^2, Adjusted R^2 and the value of Significance F. Although the standard error has changed slightly negative, we can conclude that by making a selection based on growth within sectors, it is possible to obtain better results than the traditional method. As evidence, the Adjusted R^2 value for the second analysis of 0.34 is almost three times that obtained with the traditional analysis of 0.13. Even looking only at the last four sectors, it can be seen that the growth analysis always performs better than sector multiples, except for the "Drugs" sector, which shows different results.

Although the power of the independent variables has improved in this model, it cannot be considered satisfactory. Therefore, other factors strongly influence the market value of companies. The multiples considered so far in the different typologies have failed to reach a significant level of explanation.

Thus, an atypical type of selection has been developed concerning the Comparable Company Analysis. All companies were selected, including those belonging to different sectors, following these parameters: similar revenue growth in the period 2016-2019, a similar annualised rate of return and also comparable market capitalisation and enterprise value.[168]

At this point, according to the selection criteria just presented, three groups of companies comparable to each other have been created. They are summarised in the following table:

[168] For these two last factors an average of the values belonging to the reference quarters (2016-2019) has been used.

Table 22 - *CCA Groups Analysis*

USD Millions	Company Name	Sector	EV	Market capitalization	Revenue *g* a.r.	Return a.r.
Group A	**Nvidia**	Chips	90,038.66	94,846.99	35%	52%
	Salesforce.com	Sotw	86,221.99	86,749.99	26%	24%
	Adobe	Softw	88,702.51	90,839.66	24%	45%
Group B	**Arista Networks**	BusSv	12,532.18	13,971.13	38%	59%
	Veeva Systems	Softw	10,474.02	11,349.63	27%	68%
	Matchgroup	BusSv	10,711.92	9,664.82	20%	67%
Group C	**Oneok**	Util	29,581.42	20,164.66	21%	19%
	Lam Research	Chips	21,387.17	23,634.22	27%	33%
	Micron Technology	Chips	40,123.01	37,682.87	38%	41%

Source: Personal Elaboration

As a result of this classification, a new analysis was conducted to determine whether price multiples, which previously showed little ability to determine market capitalization, could achieve better results in this case. The linear regression analyses, performed separately for the three groups, show the following results:

Table 23 - *Groups Linear Regression Statistics*

	R^2	Adjusted R^2	Standard Error	Significance F	Type	N°of observations
Group A	0.710	0.634	21200.85	9.771E-05	Multiples	90
Group B	0.728	0.657	3558.97	3.905E-04	Multiples	90
Group C	0.669	0.583	7144.99	1.671E-03	Multiples	90
Total	**70.23%**	**62.47%**	**10634.94**	**7.20E-04**	**Multiples**	**270**

Source: Personal Elaboration

The average results of intra-group regressions show a clear improvement over previous analyses. With an Adjusted $R^2$of 62.47%, it is certainly possible to establish a better ability of price multiples to determine the market capitalisation of companies. For example, let us consider a single case like that of ONEOK Inc., which operates in the utility sector as a service provider of natuaral gas liquids. Using multiples of two semiconductor companies, Lam Research Corporation and Micron Technology Inc., they can explain the market capitalisation of ONEOK Inc. for about 77%. The result is surprising because although the companies belong to different sectors, with different growth estimates, they can be considered comparable.

It is fair to remember that this method has been applied to a limited number of companies, due to the lack of other groups available in the list of companies provided by Fortune. Therefore, the results of this analysis cannot be treated as general, but rather as a starting point for future research.

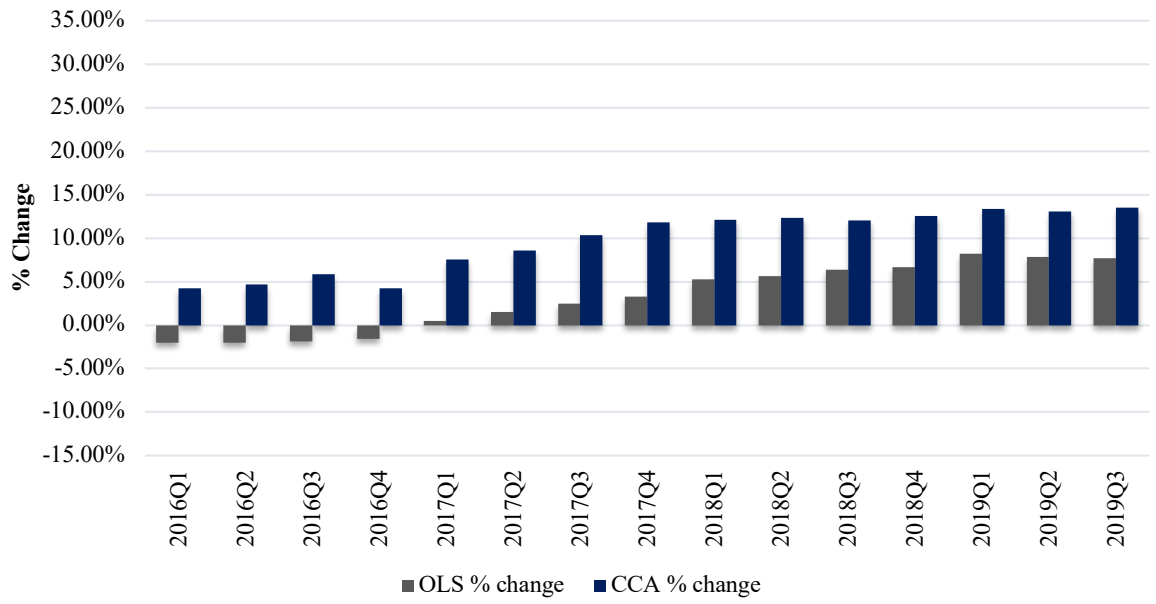
3.4.3. Analysis of Outcomes

Therefore, to answer the second set of questions, we can conclude that this result, still not generalisable, is proving that relative valuation is an instrument that should be used with due care and without abuse. In fact, following a simple comparison between companies, which are considered similar only because they belong to the same sector, one could arrive at misleading results during the analysis of particular companies such as high growth firms. Moreover, perfecting this valuation technique as much as possible by combining similar companies from several features, both financial and non-financial, can lead to meaningful results, as shown by this analysis. Since this last result is fascinating, the study makes a final comparison between the model developed during the first part of the analysis and this last study.

As the last judgment involves estimates values of market capitalisation, in order to compare these ones with the OLS estimator outcomes, it is necessary to find the relative enterprise value using the formula introduced in the previous sections and to execute a logarithmic transformation.[169]

Once the two models became comparable in time and scale, an average was performed for each quarter. The updated data now included enterprise value estimates for the two models on a logarithmic basis. After this step, the % change from real EV was calculated for both studies. The following figure displays the results:

Figure 17 - *OLS Model vs CCA Model*



Source: Personal Elaboration

[169] This is necessary for the comparison between the two methods, as the first method presents the results on a logarithmic basis. Additionally, the reported values were used to get the EVs values, except for market capitalisation, which was estimated by the model.

As we can see, both models are able over time to provide an estimate very close to the real enterprise value of the companies analysed. In particular, the OLS model presents an average distortion of 3.19% compared to the Comparable Companies Analysis model, which has an average value of 9.75%.

This latest demonstration highlights, even more, the accuracy of the first model and proposes a new method for researching comparable companies which moves away from traditional methods and which should be investigated in future research.

3.5. Future Research

The research on HGFs provides many relevant questions, outstanding answers and challenging economic issues. The analysis carried out has established some problematic points related to the study of these companies, which surely future research will be able to clarify. The possibility to define the growth of the company with different measures has consequences on the definition of high growth firms. Even if there is still no homogeneity of definition, this is essential for the comparison of the results that the research produces. The use of the definition provided by the OECD has become popular because it makes it possible to identify high-growth companies without the need to get micro-data, which very often are not available. However, some authors have questioned this definition because it excludes all companies with fewer than ten workers, even though these may show very high levels of growth. Subsequently, the use of the employment measure may also be a sub-optimal choice. From a public policy point of view, targeting companies according to their employment rate can be incorrect if it creates incentives that reduce productivity.

Besides the problems of definition, this research showed a difference in terms of HGFs' shares between countries. The latter is a consideration of great interest for future research because it stems from various institutional environments, public policies and the economic structure. Finding explanations for the relationship between a given country and the factors that lead its companies to grow faster than others is a significant challenge for policymakers.

Further attention must also be paid to the internal dynamics of the company and their management that distinguishes successful companies from "normal" ones. Concerning the last-mentioned point, an in-depth and combined analysis of several successful companies and their strategies could provide further results for the research. Therefore, it is necessary to analyse, where possible, the strategic drivers of growth and their relationships to provide comprehensive management responses. Moreover, the contribution of entrepreneurial research could provide support for the integration between management dynamics and managers' behaviour during the various phases of the life cycle.

Subsequently, as regards valuation issues, the uniqueness of these companies was further highlighted by the lack of traditional valuation methods to trace the value of HGFs.

Future research should implement and spread the use of methods that more closely involve all drivers that influence growth and value. An analysis of several companies, in different countries and with numerous variables, can be an attractive prospect for the results produced by this research. Precisely the latter has indeed demonstrated the feasibility of identifying value drivers that more than others influence the market price. Therefore, an analysis involving both financial and other elements, and examining their inter-relationships, could be of great significance for the literature about the business valuation.

Concerning valuation methods, the need to develop new approaches to value assessment has been pointed out. Finally, it is essential to understand the potential and risks of applying the valuation methods in order to understand that they should be balanced with the features of the companies under analysis, as demonstrating by High-Growth Firms study.

Conclusion

Proceeding chronologically with the questions and targets that the research set out, it is possible to reach the following conclusions thanks to the available results.

Concerning the definition and identification of high-growth firms, a methodological gap has emerged that future research should fill. The analyses carried out so far have not succeeded in defining high-growth companies comprehensively. Therefore, it is argued that research is necessary to take into account more elements such as strategic drivers, the influence of the geographical background or the structure of the company. Regarding their nature, it is considered approximative to select only financial parameters for the desired definition. High growth firms, apart from imposing abnormal performance upon other companies, are also drivers of socio-economic benefits. The absence of the latter could begin to be considered a necessary element. Thus, redefining the paradigms hitherto unsuitable is the first step to achieve uniformity of research. This operation would make it possible to compare the results of different authors using a benchmark.

Subsequently, from the valuator's point of view, the old and abused lenses of traditional methods should be changed. Indeed, these last ones represent the empirical foundations on which research should evolve and find new answers that today are increasingly fast. However, it is essential not to consider them a point of arrival. Understanding the company's dynamism, the factors that determine and drive it represents a turning point for a field such as business valuation, that makes assumptions its starting point. Precisely the search for real value, i.e. observable value, shows the need to implement these models. Therefore, the limits identified are concrete limits that must be solved through the effort of developing new methodologies that require considerable commitment in terms of implementation and dissemination. Although some authors have contributed significantly to the questions that the evaluation of HGFs raises, see Damodaran, Schwartz and Moon, such methods do not yet seem to be generally used by the financial and academic community because of their complexity.

Regarding the empirical results developed by the following research, they aimed to answer the following questions:

I. What financial data most reflects the value of these companies? How can analysts' estimates help in the valuation of HGFs?
II. How to conduct an efficient relative evaluation following previous results? Where to look for the "real" comparable of these companies?

As for the first topic, the econometric model developed by the research has demonstrated the ability of two specific variables to influence and determine the value of the company: EBITDA

and Revenues. Concerning the importance of estimates, it has been demonstrated that these elements are better suited to capture current levels of enterprise value than historical ones. Therefore, when analysing high-growth firms, estimation data should be taken more into account than past or current metrics. Future expectations more represent the value of these companies than by current or past numbers.

Subsequently, to answer the second set of questions, we can conclude that this result, which cannot yet be generalised, is proving that relative valuation is a tool to be used with proper care and without misuse. In fact, following a simple comparison between companies, considered similar only because they belong to the same sector, erroneous results could be obtained in the analysis of particular companies such as those with high growth. Furthermore, perfecting this evaluation technique as much as possible by combining similar companies with different characteristics, both financial and non-financial, can lead to significant results, as demonstrated by this analysis. Thus, the development of a new selection method, although quite divergent from conventional approaches, opens the door to a new interpretation of multiples, which we could define as "non-sector-specific".

The selection of companies, belonging to different sectors, during the comparable' decision led to the conclusion that HGFs can be better analysed in this way. What Damodaran anticipated about the relative valuation and its shortcomings is amply demonstrated in this thesis.

Finally, I hope that the results of this research can be continued to improve the interpretation of these companies wherever possible. Their evolving nature makes them a fascinating subject of study, thanks also to the dominant position these firms have now acquired on the market.

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Appendices

A. *Decomposition of Growth*

According to Damodaran, the growth of earnings is composed by different parts. Define E_t the earnings in period t, I_t the capital invested at beginning of period t and ROI_t the return of those investments:

$$E_t = ROI_t * I_t$$

The relative variation of earnings can be written as:

$$\Delta E_t = ROI_t * I_t - ROI_{t-1} * I_{t-1}$$

Hence, the growth rate, expressed in terms of change in earnings is:

$$g = \frac{\Delta E}{E_{t-1}} = \frac{(ROI_t * I_t - ROI_{t-1} * I_{t-1})}{E_{t-1}}$$

If we assume that the ROI is stable over the period, the expected growth rate in earnings is:

$$g = \frac{\Delta E}{E_{t-1}} = ROI * (I_t - I_{t-1}) = ROI * (\frac{\Delta I}{E_{t-1}})$$

It is possible to notice that the growth rate is a function of two variables: the return on new investments (ROI) and the share of earnings put into these investments ($\frac{\Delta I}{E_{t-1}}$).

A more realistic scenario, where the return on investments changes over period, provide the following growth rate:

$$g = \frac{\Delta E}{E_{t-1}} = ROI * \left(\frac{\Delta I}{E_{t-1}}\right) + \frac{(ROI_t - ROI_{t-1})}{ROI_{t-1}}$$

With the assumption that return on new investments in period t is the same as the return on existing assets in the same period. Generalizing even further, it is possible to distinguish between return on new investments and return on existing assets. In this case, the expected growth rate is:

$$g = \frac{\Delta E}{E_{t-1}} = ROI_{New,t} * \left(\frac{\Delta I}{E_{t-1}}\right) + \frac{(ROI_{Existing,t} - ROI_{Existing,t-1})}{ROI_{Existing,t-1}}$$

Finally, it is possible to define the expected growth rate of earnings like a function of two different variables. The first one is the growth from new investments determined by the marginal return on those investments and the share of earnings used in these investments. The second part, also called *efficiency growth,* provides the effect of a change in the return on investments on the existing assets between the period.[170]

[170] A. Damodaran. 2018. "The Dark Side of Valuation: Valuing Young, Distressed, and Complex Businesses". Pearson FT Press. ISBN: 9780134854267

B. *Reinvestment rate and Terminal Value Calculation*

Reinvestment rate in stable growth about free cash flow to the firm (FCFF), could be written as:

$$\text{Reinvestment rate} = \frac{\text{Stable growth rate}}{\text{Return on Capital in Stable Phase}} \qquad \text{(I)}$$

From here, while changing the stable growth rate and holding other variables constant can change considerably the value, changing the reinvestment rate together with growth rate could create an offsetting effect:

$$\text{Terminal Value} = \frac{\text{EBIT}_{t+1} * (1 - t) * (1 - \text{Reinvestment Rate})}{\text{Cost of Capital}_t - \text{Stable Growth Rate}} \qquad \text{(II)}$$

It is possible to see this offsetting effect because gains from increasing the growth rate are partially or completely balance by the loss in cash flows because of the higher reinvestment rate. So, to understand when value increase or decrease as stable growth rate increases, it should take into account the excess returns. If these ones are higher than the cost of capital during the stable phase, increasing the stable growth rate increases value. While, as Damodaran said "if the return on capital is equal to the stable-growth rate, increasing the stable-growth rate has no effect on value." Let's substitute the (I) into stable growth rate of (II):

$$\text{Terminal Value} = \frac{\text{EBIT}_{t+1} * (1 - t) * (1 - \text{Reinvestment Rate})}{\text{Cost of Capital}_t - (\text{Reinvestment Rate} * \text{Return on Capital})} \qquad \text{(III)}$$

Finally, setting the return on capital equals to cost of capital:

$$\text{Terminal Value}_{RoC=CoC} = \frac{\text{EBIT}_{t+1} * (1 - t)}{\text{Cost of Capital}_t} \qquad \text{(IV)}$$

Confirming that the key assumptions when valuing the terminal value is about the excess returns that accompany the growth.[171]

C. *Imputed Return on Capital*

In order to check for consistency between operating income and reinvestment rate, when they are estimated separately, it is possible to conduct a test finding the imputed return on capital through the formula:

$$\text{Imputed Return on Capital} = \frac{\text{Expected Operating Income After tax}_n}{\text{Capital Invested in Firm}_{n-1}}$$

[171] A. Damodaran. 2018. "The Dark Side of Valuation: Valuing Young, Distressed, and Complex Businesses". Pearson FT Press. ISBN: 9780134854267

Where the numerator is the forecasted operating income and the bottom part is total reinvestment, net capital expenditures and Δ in non-cash working capital, over the previous period added by the initial reinvestment at the time of assessment:

$$\text{Capital Invested in Firm}_{n-1} = \text{Capital Invested in Firm}_0 + \sum_{x=1}^{x=n-1} \text{Reinvestment}_n$$

The Imputed Return on Capital, as approaching the steady state, has to be compared to both the company's steady state return on capital and industry average. When it is higher than those ones, it means that the reinvestment forecast for the firm over the period is insufficient. Conversely, when Imputed Return on Capital is below the other two measures, it means that the reinvestments are too high, given the revenues and earnings forecast.[172]

D. *OLS Estimator Assumptions*

The main assumptions of OLS Estimators are:

- Linearity in alpha and beta parameters: dependent variable is a linear function of a set of independent variables and a random error factor
- The expected value of the error term is zero for all the observations:

  $$E(\varepsilon_i) = 0$$

  If this assumption is not respected, the intercept is biased.
- Homoscedasticity: the variance of the error term is constant in all the variables and over time. The measurement of variable provides the uncertainty of the model. Homoscedasticity implies that this uncertainty is equal across all the observations.

  $$V(\varepsilon_i) = E(\varepsilon_i)^2 = \sigma^2 = \text{constant}$$

  If the assumption is not respected, the model uncertainty varies across the observations. This problem is called heteroscedasticity.
- Error term is independently distributed and not correlated: there is no correlation between the dependent variables' observations:

  $$\text{Cov}(\varepsilon_i, \varepsilon_j) = E(\varepsilon_i, \varepsilon_j) = 0, \qquad i \neq j$$

  If this assumption is not respected, there could be spatial correlation, serial or panel correlation problems.
- X_i is deterministic: there is no correlation between X_i and the error term:

  $$\text{Cov}(X_i, \varepsilon_i) = E(X_i, \varepsilon_i) - E(X_i) * E(\varepsilon_i) = X_i E(\varepsilon_i) - X_i E(\varepsilon_i) = 0$$

[172] A. Damodaran. 2018. "The Dark Side of Valuation: Valuing Young, Distressed, and Complex Businesses". Pearson FT Press. ISBN: 9780134854267

If this assumption is not respected there could be endogeneity and simultaneity problems. If all Gauss-Markov assumptions are respected than the OLS Estimators parameters, alpha and beta, are "BLUE" – Best Linear Unbiased Estimators.[173]

[173] Hallin, Marc. 2014. "Gauss-Markov Theorem in Statistics." In Wiley StatsRef: Statistics Reference Online. John Wiley & Sons, Ltd. doi:10.1002/9781118445112.stat07536.

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High-Growth Firms Valuation - Summary

Introduction

The following research aims to present and empirically verify the problems that arise when estimating the value of a High-Growth Firm. Although this is a very contemporary topic, there is currently no consensus on which methodology is best suited to approach their valuation. Since traditional methods fail to intercept the value of these companies, the purpose of this research is to address this problem through a solid empirical foundation.

1. High-Growth Firms: Analysis and Perspectives

(A. Coad, S. O. Daunfeldt, W. Hölz, D. Johansson, A,Matthias, D. D. Baker, J. B. Cullen, J. B. Barney, B. R. Barringer, J. R. Baum, B. J. Bird, D. L. Birch, A. Bravo-Biosca, R. Brown, S. Mason, F. P. Delmar, Eurostat-OECD, World Bank, J. Haltiwanger, J. W. B. Bos, E. Stam, M. Jensen, P. Krugman, S. C. Parker, D. J. Storey)

Interest in high-growth firms is increasing sharply over the last few years. The underlying reasons lie in the ability of these companies to contribute positively to the environment by increasing productivity and jobs. Despite the growing appetite for a more precise meaning of "high-growth firms", even today we cannot find concrete definition of them. Precisely, there are four issues about the identification of HGFs that need to be analysed before developing a potential definition. The first issue is the indicator of growth, that is the choice of the variable to observe the growth. The most used growth indicators are employment and sales, where the first one is the preferred variable according to recent studies. The second point is the choice between the measurement of growth in relative or absolute terms. There are also indicators combining relative and absolute changes. The most common is the Birch Index, which manages to decrease the bias in identifying small companies such as HGFs, reducing the effect of company size on the growth indicator. The third matter relates to the selection of an analysis time frame that can reduce the amount of statistical noise over the years. The recent trend is to consider a three- or four- years period because these firms tend to change substantially over these periods. Although several authors tried to study the best-fitted period of analysis, this is still an unsolved topic. Finally, the last identification question is the difference between internal and external growth, and the choice between them for the research. Most researches use a hybrid, or total, growth model, due to the lack of mergers and acquisitions data. This model includes both strategies and considers them uniformly. Given the numerous interpretations about the research of high growth firms, this research follows the set of definitions established by the World Bank in 2019. The reason for this choice lies in the clearness and effectiveness that this distinction makes over companies under analysis. According to this classification, high-growth companies can be grouped into three different definitions: absolute, relative and distributional. *Absolute* definitions provide for the choice of a specific growth rate and a predefined period. Most common definitions are the Birch

index or that provided by Eurostat-OECD. *Relative* definitions target a set number of HGFs in a selected percentile of firms in the distribution of revenue or employment growth. The last set of definitions is *distributional*. They study the right tail of the growth rates distribution, mostly Laplace's, and in particular, a certain threshold by which this tail converts to a power-law distribution. These types of definitions certainly need to be developed in-depth to provide better answers, but they deserve close attention to the potential of their results. Besides the problems of definition, the study shows a difference, in terms of HGFs share, between countries. Finding explanations for the relationship between a given country and the factors that lead its companies to grow faster than others is a significant challenge for policymakers. Finally, further attention should go to the internal dynamics of the company and the management that distinguishes successful companies from "normal" ones. Regarding the latter point, an in-depth and combined analysis of several successful companies and their strategies could provide further results for the research.

2. Tools and Methods of Business Valuation

(A. Damodaran, M. Vulpiani, W. Buffett, L. A. Cunningham, W. Gournall, I. A. Strebulaev , M. Bini, A. Metrick, T. Koeller, M. Goedhart, D. Wessels, M. Rudolf, Y. Zhijun, M. Kozdron, F. Black, M. Scholes, E. S. Schwartz and F.Moon)

The valuation of high-growth firms is the primary topic of this thesis. The need to deepen this subject lies behind the desire to clarify and examine the potential criticalities of these companies in their valuation process. A first topic of valuation involves the identification of growth firms among the others. Distinguishing between young and mature companies seems inaccurate to recognize high-growth companies during valuation steps. There is a false belief that companies with many years of life, can no longer have high growth rates. The best-fitting method to categorize HGFs is to analyse the nature of their balance sheet items. Unlike mature companies, HGFs derive most of their value from growth assets, which depend not only on how much growth is anticipated but also on the excess returns that follow this growth. Despite some differences such as sector, growth outlook or size, high-growth companies share some characteristics that make an impact on how we value them. According to Damodaran, the first feature concerns dynamic financials. These companies are valued with the support of their financial statements, such as the balance sheet, income statement or cash flows, which are continually evolving for high-growth firms, not only from year to year but also in shorter periods. A distinction of these companies concerns the public and private capital. It is common to assume that firms that grow a lot in their first phase become public to raise additional funds on the capital market. In practice, this transition is not standardisable or predictable for all high-growth firms for different reasons. First, it is necessary to consider the difference between economies, the role of institutions and the

development of capital markets. Moreover, access to the capital markets may vary over the years and between sectors. An additional characteristic that HGFs have in common is the size disconnect, i.e. the difference between the market value and the book value of these companies. The former is usually much higher than the latter because it incorporates the growth prospects of the assets while the book value is often not. The share of debt to their value is another common feature among high-growth firms. Usually, these companies do not have adequate cash flows to support the introduction of new debt, and so, even in sectors where debt is the preferred source of financing, they have a lower debt ratio than the more mature ones. The last point of common ground concerns their market history. These companies tend to have a short and shifting history. Since the valuation depends strongly on some market values, such as the Beta, assessing data of these companies is challenging, due to a short time horizon of analysis and volatility that characterize their numbers. Moving to valuation matters, the most used methods of business valuation try to reach the fair value through their assumptions and calculations. The *Discounted Cash Flow Method* is based on the assumption that the value of an asset is the present value of expected cash flows on the asset, discounted at a rate that reflects the riskiness of these flows. Hence, a correct valuation requires the ability to obtain all possible information about such assets, and to conduct a valuation model that accurately determines their intrinsic value. This precision is not possible in reality and, quoting Damodaran, the "intrinsic valuation is, in some sense, an act of faith". In any case, since it is not possible to know the exact intrinsic value, the only option is to conduct an analysis that manages to understand, as much as possible, the main drivers of this value. *Relative valuation* is a method that estimates the value of a company or an investment, based on how much investors are willing to pay in the market for similar assets or investments. Therefore, it requires two conditions to be applied: the standardization of prices, often converting them into multiples, and the similarity of the companies to each other. However, in today's markets, it is almost impossible to find exactly comparable companies, because they usually differ in risk, growth opportunities and ability to generate returns. While it is a widely used and easy to apply valuation model, the relative method hides pitfalls to consider carefully in its development.

Valuation Issues of High-Growth Firms

The *discounted cash flow* presents some issues in HGFs case. The perspective of considering the operating costs related to a specific year is the first problem, because high growth companies usually invest today, not for current sales, but to nurture and to capture a customer base in the future. Thus, considering all costs as operating expenses leads to underestimating the value of existing assets. Moreover, the shifting profitability that high-growth firms present, has to be considered. Unlike the margins of mature companies, which usually move within a predefined

range of values, HGFs have very different values between individual periods, which makes very difficult to make reliable forecasts. Growing assets are the second problem faced by DCF method. As a company grows, it is more challenging to sustain previous levels of growth. For this reason, it is also necessary to consider the rate of reinvestment over the years and to balance it with growth rate, in order to achieve a return on capital that is viable for the company, as it approaches the long term. Concerning discount rates, their determinants relate to the investment risk and to the mix of choice between capital or debt for financing the business. In both cases, HGFs present critical issues. Since the value of a high-growth companies depends on both existing and growing assets, and the latter are riskier than the former, two separate rates should be used to reflect this difference in risk. Another critical aspect is the change in the risk profile that growth brings. Thus, from a foresight perspective, as a company matures, it must be considered that existing assets assume a greater portion of the total value and that its cost of capital decreases, reflecting this situation. Thus, for high-growth companies, discount rates should be higher in the early stages and lower over time. Despite this is a problematic phase of any evaluation, the construction of the terminal value is even more difficult with HGFs. First of all, because it represents a more substantial portion of the overall value of the company since they often generate low cash flows from existing assets. Moreover, it is clear that the terminal value is subject to the uncertainty that follows the growth of these companies. Finally, the value of equity per share represents a last trigger point. In order to find this value, debt and equity claims are subtracted from the enterprise value, while cash and crossholdings are added. Then, this value is divided by the number of shares. These steps are critical for high-growth firms. The value of cash, especially HGFs in the early stages of the life cycle, has high reinvestment rates that lead to the dissipation of cash balances. Thus, at the time of the valuation, their value may be very different from the last one stated in the financial statements. For debt, on the other hand, high-growth companies usually require hybrid forms of financing, which offer the advantage of keeping interest rates low, but in exchange for an equity option, like in the case of convertible debt. Thus, since only debt is subtracted in the formula, the hybrid structure of these loans has to be broken down by dividing the equity and debt parts. Given the different issues that the traditional discounted cash flow method faces when evaluating high-growth firms, it may be easier to apply the *relative method*. However, the peculiarity of these companies has a significant impact even on such valuation. These critical issues involve four macro-areas of relative valuation. Concerning *comparable firms*, the common practice of considering listed companies in the same sector may be totally out of place in the case of HGFs, because their fundamentals and other measures are not similar to the sector averages. Also, in terms of growth, they share different risks and opportunities which makes them not

comparable in terms of industry. About the *choice of multiples and base year values*, as multiples vary depending on the choice of the base year in which they are considered, for high-growth companies, calculating these values at certain stages of their life could be misleading. To overcome this problem, many analysts use forward multiples values. However, the characteristics of the company should always be taken into account over the years to avoid long-term growth rate setting equal to the current one. Regarding *different growth potentials*, it is essential to consider many perspectives when comparing several companies. When calculating the value, in fact, these differences should take into account not only the growth rate but also its length and the excess returns that accompany it. A multiple that considers it, while remaining very simple, is the PEG ratio. This metric requires making assumptions about the relationship between growth and value that are quite unreliable. Lastly, considering *risks' differences,* given the close relationship between growth and risk, the variation that one implies on the other must also be taken into account, changing the values of the multiple overtime.

Valuation Corrections and Alternative Methods

In this last part of the chapter, using the guidance obtained so far, three different evaluation methods are provided, which can evaluate high-growth firms more efficiently. Focusing on *Adjusted Cash Flow Method*, the main objective is to obtain reasonable estimates of the company's future cash flows and discount rates. The cash flow method can be performed by discounting the cash flows of the entire company at the cost of capital or by valuing equity and discounting cash flows at the cost of equity. While the former requires the estimation of new debt issues and interest payments for each period, the latter option becomes more difficult if there is a change in the company's debt ratio over time. In the long term, HGFs tend to adjust to the characteristics of mature companies in the market, resulting in lower growth rates and more stable cash flows, also caused by the issuance of debt. Therefore, it is necessary to adopt a cash flow model that is flexible and not rigid, to avoid blocking the current characteristics of the company. Moreover, one of the most important estimation is the scaling of growth rates as the company expand. To find out the length of the first period and the subsequent phases of growth of the company, attention must be paid to information about the following factors: size of the overall market; presence, strength and quality of the products offered by competitors; management structure of the firm. In order to validate assumptions about future revenue growth rates, some tools can be used. One is to consider the absolute change in revenues, instead of percentage change, as it can help to avoid overestimating revenue growth over time. Then, past growth analysis can help to better understand how they changed as the company expanded. This test can be a valuable basis for estimating future growth rates. Finally, industry data can be used primarily to triangulate results and to understand

which target the company is moving towards in the long term. Moving to the company's cost structure, different assumptions can be made, such as maintaining the current operating margins over time or changing them. The latter assumption is the most likely with high growth companies. Once the company's margin has been analysed, it is necessary to estimate how and when the long-term target is reached. Since growth is not free and does not result from chance, it is appropriate to consider the effect of reinvestment of the company over the years. Then, for the assessment, a risk profile that is consistent with the company's growth and its operational numbers must be considered. Thus, to make a consistent estimation, it becomes necessary to adjust discount rates over time, according to the company's condition. Concerning terminal value, Damodaran suggests several general propositions for a correct estimation. First, one should not wait too long to put the company on a stable growth path. Moreover, the spread between the return on capital and the cost of capital should not exceed 4 or 5% during the stable growth phase. At the end of the valuation, there are final corrections an adjustment to make. Thus, to arrive at the value of equity per share, it is necessary to divide the equity value by the number of outstanding shares but considering a higher price for the ones with voting rights, and vice versa. Moving the discussion to *Alternative Strategies for Relative Valuation*, it is essential to remember that HGFs are, more often than not, the exception in the sector in which they operate. Thus, one should abandon the idea that a high-growth company in the automotive sector must necessarily be compared with others in the same sector. Hence, as a first step, it is necessary to consider comparable high-growth companies from different sectors, using fundamental rather than business analysis. Despite this advice, the companies included in the analysis inevitably differ in their fundamentals. Indeed, it is unreachable to align risk and growth for all companies, and so, a more practical solution is to control these variables through linear regression. Thus, the dependent variable is expressed by the analysed multiple, while risk, growth or other measure that wants to be tested, represent the independent variable. Referring to *Probabilistic Methods,* it can be stated that, compared to the methods outlined above, the next ones are based on the ability to find value through a different and potentially more informed way. The first approach is the multiple scenario analysis, through which the value of an asset can be expressed under several variables, both macroeconomic and asset specific. The final results can be represented as a value of each scenario or as an expected value for all probability-weighted scenarios. However, the scenario analysis is more suitable for discrete rather than continuous risks. Since in some valuations there are also subsequent risks, the decision tree is a useful model to implement. It requires that a company or a project, to achieve specific outcomes, have to go through several steps that could affect their value. A failure in one of the steps can sometimes also lead to the default of the company. The decision trees aim to

consider the risk during the various stages and the right response to it. Understanding the structure of these tools requires a first distinction between the different “nodes”: radical nodes represents the beginning of the decision tree, in which the analyst is faced with a decision choice or uncertain outcomes; event nodes represent the different possibilities that are obtained if the project is undertaken; decision nodes indicate the choices that can be made; final nodes are the final results of the first risky outcomes and decisions taken. Thus, the decision tree strategy introduces another choice available to management before starting a project: delaying the investment. The option to decide later can be of great value to the company as it can learn new information and make a safer investment. These types of decisions are called real options and belong to the decision nodes. They are located after an information node in the decision tree. This type of argument is closely related to high-growth companies and to the uncertain nature of their investments. The idea behind real options is that business decisions involve a specific degree of flexibility, which can be seen as an option to behave in a certain way. The best-known model for evaluating financial options is that of Black and Scholes (1973). Later, Schwartz and Moon (2001) developed a model to assess the options of high growth companies, taking into account two of the most critical valuation problems of these companies: the cash flow estimation and the growth rate estimation. The model is a Monte Carlo simulation approach based on three stochastic factors. Unlike other models, cash flows are modelled by two separate stochastic processes for costs and revenues. However, expectations are assumed to be uncertain and therefore vary between people. This fact is typical of high-growth companies compared to more mature ones, where expectations are more homogeneous.

3. Model Development through Empirical Analysis

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The main objective of this thesis is to highlight the valuation dilemmas of high-growth firms and try to define a solution. Hence, the following research tries to suggest a solid empirical foundation on the facts that have emerged and been discussed in the previous chapters. Thus, the next quantitative analysis of high-growth companies is developed through two different phases, to precisely answer the following sets of questions: What financial data most reflects the value of these companies? How can analysts' estimates help in the valuation of HGFs? How to conduct an efficient relative evaluation following previous results? Where to look for the "real" comparable of these companies? Concerning the first group of inquiries, the aim is to analyse the ability of past performance, i.e. the results reported in the company's financial statements, to reflect the enterprise value of high-growth companies. Since the analysis can also be carried out using the values of estimates over time, they are also taken into account to obtain a complete answer to the

first question. The second questions examine the relative valuation method and, more specifically, the dynamics involved in selecting comparable companies. Ideally, to find out how the market prices a specific company, analyses of similar companies within the same sector are used. Since HGFs are usually real exceptions, due to their high growth volumes, this concept is questioned by the results of analysis.

Panel Companies and Data Research

At this point, to facilitate the search for high-growth companies, the Fortune 100 Index was analysed as a source of selection. The list groups the 100 US companies that have grown more than 20% annually in terms of Revenue, EPS and return rate over the last three years. To make the data processing and search path clearer, the analysis starts with the presentation of research databases that provided all the necessary data for each company involved in the research.

The data collection was made through the databases of the Wharton School of the University of Pennsylvania called Wharton Research Data Services (WRDS). Concerning those used in this study, the "Compustat IQ" database was used for the quarterly balance sheet data. The “Center for Research in Security Price” U.S. Stock Database (CRSP), which contains end-of-day and end-of-month prices on the NYSE, NYSE MKT, NASDAQ and Arca primary stock exchanges, together with basic market indices, was used for closing price requirements and common shares outstanding. About the financial ratios of companies and industries, the database "Financial Ratios Suite by WRDS" was used. Subsequently, the estimate data were found through the "Institutional Brokers' Estimate System" (I/B/E/S) database, which contains analysts forecast data. The estimates found refer to values for an annual estimate period (1-Year Forecast). When all the data met the time requirements of the analysis, they were incorporated into a single document, which represented the final dataset, joining them respecting the company name, their permanent number (PERMNO) and the quarterly reference date (e.g. Q1-2015 with Q1-2015).

Which Drivers Influence the value of HGFs?

The following model aims to initially identify which type of drivers have the most significant impact on enterprise value between the historical and the estimates made by analysts about the future. Subsequently, once the most suitable parameters for the model analysis has been identified, some tests are followed to identify errors that undermine the OLS estimator assumptions presented in Appendix D. The final objective is to suggest which elements should take on higher consideration during the analysis of HGFs, and also demonstrate that the value of companies within HGFs lies more in future numbers than in historical and current ones. The data under analysis correspond to 54 companies, belonging to 19 industries, for the period 2016-2019 (quarterly). The dependent variable selected for this model is the enterprise value. The selected

independent variables are the main items of the income statement and represent key elements for the evaluation of the company. They are: Revenue, EBITDA, EBIT, Earnings. The first analysis carried out led to the choice of the most explanatory coefficients to be adopted in the enterprise value analysis model. Thus, this first "screening" was carried out using the four covariates, historical and estimates, in two linear regression models with the same response variable. The results of the first step of the analysis are:

Table 1 - Historical Linear Regression Model Outputs

Regression Statistics	
Multiple R	0.9590
R^2	0.9198
Adjusted R^2	0.9194
Standard Error	29878.56982
Observations	820

Table 2 – Estimates Linear Regression Model Outputs

Regression Statistics	
Multiple R	0.9634
R^2	0.9281
Adjusted R^2	0.9277
Standard Error	28288.0044
Observations	820

Source: Personal Elaboration

The linear regression model of the estimates has led to more robust results than the historical elements. Since the objective of this first analysis is to identify the type of variables to be selected, the models were analysed by looking at the value of the adjusted R^2, because for multivariate regression models it is more recommended than the classic R^2, and the standard error. In both cases, the analysis of the estimates led to better results, higher R^2 and lower standard error, and therefore the latter was chosen as variables for the analysis model. The following model has been developed in several phases that allowed to eliminate the variables that undermined the assumptions of the OLS estimator. Every phase provides: a linear regression; a Normal *Quantile-Quantile Plot* to check for normality assumption; a *Variance Inflation Factor Test* to check for multicollinearity error; a *Breusch-Pagan Test*, to check for homoscedasticity assumption. Whenever a model did not meet the fundamental assumptions, a variable selection procedure called "Backward Eliminations" was performed. According to its rules, the variable with the highest P-value was eliminated, checking also for the values of VIF test. Since the procedures are the same from the initial model to the final one, the full results will be presented only for the final model, instead providing a summary description for the others.

Initial Model

The initial model corresponds to that examined for "screening" and uses all four covariates. The levels of significance of the variables, expressed by the P-value, are all below the acceptance threshold set for this analysis of 0.1. The factors that are more explanatory than the others, also due to their minor standard error, are Sales and EBITDA. The Q-Q Plot shows the absence of normality between residuals. Since one of the assumptions of the OLS estimator is normality, the

model must be improved in order to respect this assumption. The results of VIF Test differ between variables. Following the general rule of thumb, involving multicollinearity for VIF values greater than 10, EBIT, EBITDA and Earnings variables present this error. About the Breusch-Pagan Test, with a level of significance set at 99% and one degree of freedom, according to the tables the cut-off point of the value of χ^2 is 6.63. The value of χ^2 of 258.96 is higher than the value indicated by the tables, which implies the presence of heteroscedasticity. Besides, the value of p-value is lower than the threshold of 0.05, which leads to rejecting the null hypothesis H_0. Following "Backward Eliminations" elimination EBIT variable was eliminated from the model.

Second Model

The p-value values remain valid for all three variables and there is an improvement in the standard error of all covariates. The Adjusted R^2 value of 0.9237 is slightly lower than that of the initial model (0.9277). This indicates that the variable EBIT did not have much influence on the dependent variable EV. Although the model has some improvements, there is still a problem of normality. Concerning multicollinearity, there remains this problem for the EBITDA and Earnings variables. Finally, the Breusch-Pagan Test confirms the presence of heteroscedasticity. Following "Backward Eliminations" process, Earnings variable was eliminated from the model.

Third Model

The third regression model developed involves two independent variables, EBITDA and Revenue, and the Enterprise Value as response variable. The linear regression confirms the validity of p-values and it showed a marked improvement of standard error, settling at around 0 for both covariates. The value of Adjusted R^2 has decreased slightly, due to the elimination of "Earnings" variable, but remains still satisfactory. The Q-Q plot shows strong improvement but still not guarantees normality within the model. The Variance Inflation Factor Test also showed essential progresses over previous analysis, excluding multicollinearity problems in the model. However, the model still has heteroscedasticity problems evident through the values of χ^2 and P-value. At this point, a logarithmic transformation of the response variable and the two covariates was performed to mitigate the problem of heteroscedasticity present in the model. In this research, the type of the transformation rule is the "log-log model".

Final Model

After the logarithmic transformation of the variables through the application of natural logarithm on their value, the regression analysis thus constructed presents the following outputs:

Table 3 - *Final Model Coefficients' Outputs*

Independent Variables	Coefficients	Standard Error	t Stat	P-value	Significance
Intercept	2.878	0.164	17.565	7.71E-59	
EBITDA	0.814	0.044	18.354	2.91E-63	***
Sales	0.094	0.047	2.008	4.49E-02	**

****P<0.001, **P<0.05, *P<0.1*

The result of the linear regression confirms the significance of both independent variables on the enterprise value. Besides, significant improvements can be noted for the standard error, which, although it had previously reached low values, is even closer to 0. As for the regression coefficients, the following table shows:

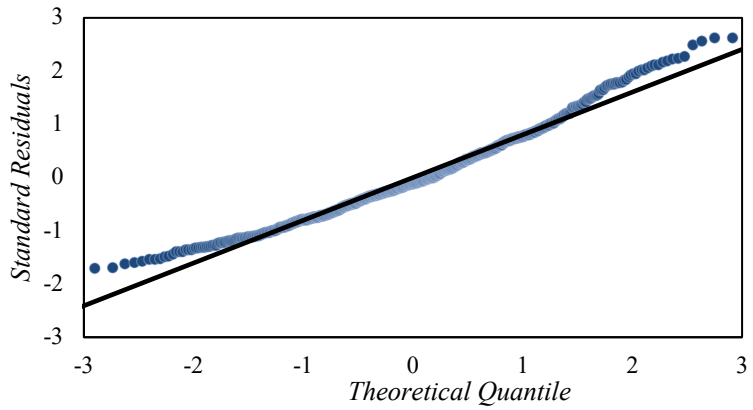
Table 4 - *Final Model Regression Statistics*

Observations	R^2	Adjusted R^2	Standard Error	Covariates
820	0.7625	0.7619	0.8124	2

Source: Personal Elaboration

Adjusted R^2 value decreases from about 92% in the previous model to the current 76%. Despite the loss of explanatory power of about 17.4% by the independent variables following the logarithmic transformation, the model remains above the acceptable threshold of 75%. Q-Q Plot results are:

Figure 1 - *Normal Q-Q Plot*



Source: Personal Elaboration

About the presence of multicollinearity in the model, the VIF test excludes correlation problems between variables in the model:

***Table 5* - Variance Inflation Factor Test**

Independent Variables	VIF	Decision
EBITDA	6.418	∨
Sales	6.418	∨

× – refused, ∨ – accepted

Finally, regarding the presence of heteroscedasticity, the Breusch-Pagan Test was conducted, with the following results:

Table 6 – *Breusch-Pagan Test*

df	χ^2	*P-value*
1	6.5990	0.0102

Source: Personal Elaboration

At a 99% significance level and one degree of freedom, the model has $\chi 2$ value of 6.59, below maximum threshold of 6.63. This evidence indicates that problem of heteroscedasticity has also been solved, confirmed by the P-value of 0.102, higher than the 0.05 threshold value. Therefore, under this last information, it is possible to accept the null hypothesis H_0 and respects the assumption of homoscedasticity required by the OLS estimator.

Analysis of Outcomes

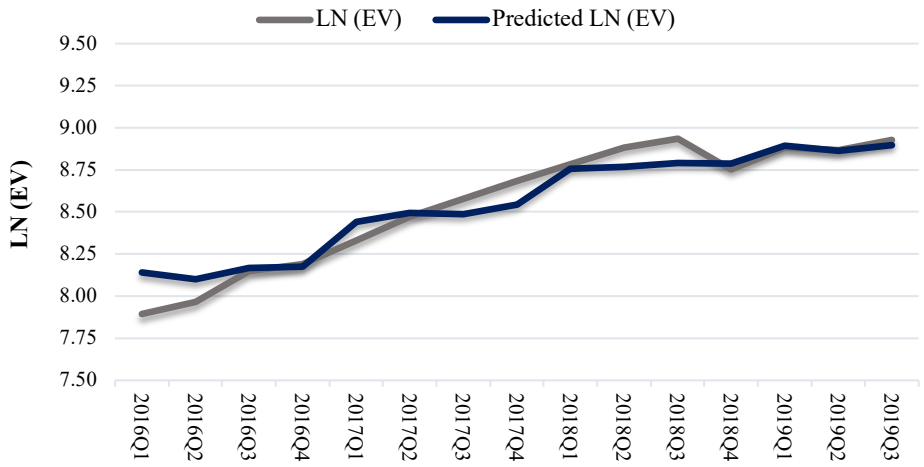
Thanks to the development of the model, the final result was consistent with the assumptions of the OLS estimator. This requirement is fundamental to confirm the validity of the research.

As it was intended to answer the first set of questions concerning the financial data that most influence the value of companies and the role of estimates during the valuation, the following conclusions can be drawn. Concerning the first topic, the models just presented proved the ability of two specific variables to influence and determine enterprise value: EBITDA and Revenue. Concerning the importance of estimates, it has been demonstrated that these elements are more fitted of capturing current levels of enterprise value than historical ones. Finally, to show the validity of the final model, the EV's equation determined through their coefficients is:

$$\mathrm{LN(EV)} = 2.87784 + 0.81373 * \mathrm{LN(EBITDA)} + 0.09394 * \mathrm{LN(Sales)} + \hat{\varepsilon}$$

Using this equation to obtain the "predicted enterprise values" for each company and comparing them with real values, it is possible to recognise the precision of the model through the following figure, where it is possible to notice how the "Predicted EV" of the model can correctly estimate and predict the real EV of the companies during the analysis period:

Figure 2 - *Real EV vs Model Prediction*



Source: Personal Elaboration

Such a figure further confirms the results of the analysis and the first study of this thesis.

Beyond the Simplicity of Relative Valuation

The second part of the analysis focuses on one of the most used valuation techniques by practitioners: the relative valuation. Since it is based on the comparison of similar companies within the same sector to determine the value of a company, in the case of high growth firms very often these similar firms, within the same sector, do not exist. The multiples used during the valuation are price multiples and more precisely: "Price/Earnings", "Price/Sales", "Price/Book Value", "Price/Cash Flow", "Price/Operating Earnings". The dependent variable of this second part of the study is market capitalisation. The reference period is the same as the previous analysis, i.e. from the first quarter of 2016 to the third quarter of 2019. The following research is divided into several phases, necessary for the accuracy of the analysis and its results. The first one examines the ability of sector multiples to capture the market capitalisation of the companies. After analysing the results of this first observation, the growth factor, which is crucial for these companies, is introduced into the analysis. Hence, only high-growth firms belonging to the same sector are compared, according to the classification adopted by the SEC, using the same type of multiples as before as independent variables. The results of the second study are compared with those of the first one to examine their differences. Subsequently, an alternative companies' selection strategy is developed to perform the relative valuation. It involves high growth companies belonging to different sectors, but similar in risk, financial structure and growth. The first observation is made on a sample of 46 companies from 15 different sectors. Multiples belonging to different sectors were compared with the market capitalisation of the companies to obtain the outcomes. Linear regressions were conducted to understand how much the independent variables, coming from companies operating in the same business, can explain the market capitalisation of HGFs. The first summary result is the following:

Table 7 - *Sector Multiples Linear Regression Statistics*

	R^2	Adjusted R^2	Standard Error	Significance F	Type	N°of observations
Average	0.3012	0.1329	4663.604	0.339	Sector Multiples	690

Source: Personal Elaboration

Sector multiples explain market capitalisation by 13.29% out of a total of 690 observations on average. This first result confirms the risk of misusing sector multiples. As we can see from the results, HGFs cannot be compared with companies in the same sector t. At this point, four industries were selected for second analysis: Chips, Drugs, Business Service and Software. The independent variables of the following analysis are the same multiples as before. However, the data are derived from company statements. The outcomes provided are as follows:

Table 8 - *HGFs Multiples Regression Statistics*

N	Sector	R^2	Adjusted R^2	Standard Error	Significance F	Type	N°of observations
1	Softw	0.7168	0.6524	5218.2063	0.0153	HGFs Multiple	**120**
2	Chips	0.3210	0.2598	4516.5356	0.0612	HGFs Multiple	**360**
3	Drugs	0.4160	0.2579	11162.042	0.1383	HGFs Multiple	**180**
4	BusSv	0.2440	0.2010	1795.8817	0.0053	HGFs Multiple	**450**
Average		**42.46%**	**34.28%**	**5673.166**	**0.055**	HGFs Multiple	**1110**

Source: Personal Elaboration

It is possible to notice a marked improvement in terms of R^2, Adjusted R^2 and the value of Significance F than fist analysis. Hence, we can conclude that by making a selection based on growth within sectors, it is possible to obtain better results than the traditional method. As evidence, the Adjusted R^2 value for the second analysis of 0.34 is almost three times that obtained with the traditional analysis of 0.13. Since the result were still not satisfying, an innovative type of selection has been developed concerning the Comparable Company Analysis. All companies were selected, including those belonging to different sectors, following these parameters: similar revenue growth in the period 2016-2019, a similar annualised rate of return and also comparable market capitalisation and enterprise value. At this point, according to the selection criteria just presented, three groups of companies comparable to each other have been created.

Table 9 - *CCA Groups Analysis*

USD Millions	Company Name	Sector	EV	Market capitalization	Revenue *g* a.r.	Return a.r.
Group A	Nvidia	Chips	90,038.66	94,846.99	35%	52%
	Salesforce.com	Sotw	86,221.99	86,749.99	26%	24%
	Adobe	Softw	88,702.51	90,839.66	24%	45%
Group B	Arista Networks	BusSv	12,532.18	13,971.13	38%	59%
	Veeva Systems	Softw	10,474.02	11,349.63	27%	68%
	Matchgroup	BusSv	10,711.92	9,664.82	20%	67%
Group C	Oneok	Util	29,581.42	20,164.66	21%	19%
	Lam Research	Chips	21,387.17	23,634.22	27%	33%
	Micron Technology	Chips	40,123.01	37,682.87	38%	41%

Source: Personal Elaboration

The linear regression analysis, performed separately each group, shows the following results:

Table 10 - *Groups Linear Regression Statistics*

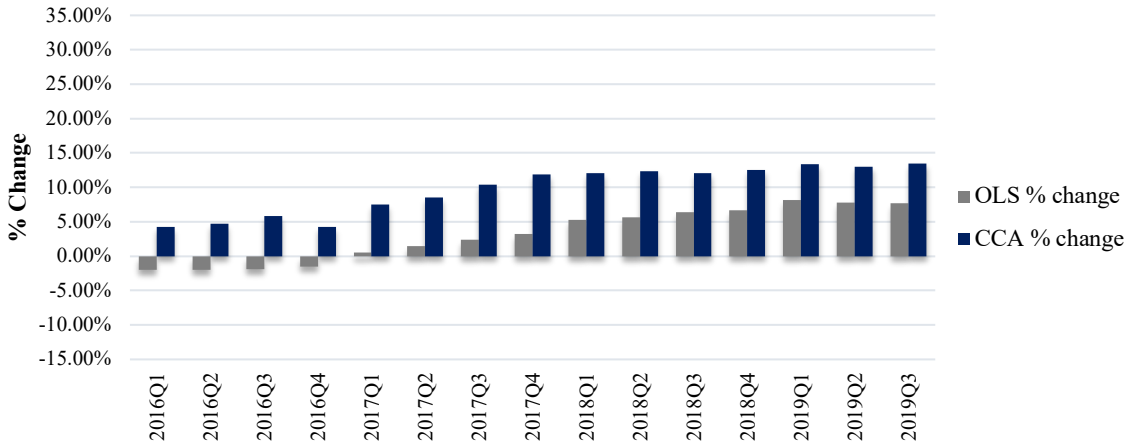
	R^2	Adjusted R^2	Standard Error	Significance F	Type	N°of observations
Group A	0.710	0.634	21200.85	9.771E-05	Multiples	90
Group B	0.728	0.657	3558.97	3.905E-04	Multiples	90
Group C	0.669	0.583	7144.99	1.671E-03	Multiples	90
Total	**70.23%**	**62.47%**	**10634.94**	**7.20E-04**	**Multiples**	**270**

Source: Personal Elaboration

The average results of intra-group regressions show a clear improvement over previous analyses. With an Adjusted R^2 of 62.47%, it is certainly possible to establish a better ability of price multiples to determine the market capitalisation of companies. It is fair to remember that this method has been applied to a limited number of companies, due to the lack of other groups

available in the list of companies provided by Fortune. Therefore, the results of this analysis cannot be treated as general, but rather as a starting point for future research. Since this last result is fascinating, the study makes a final comparison between the model developed during the first part of the analysis and this last study. As the last judgment involves estimates values of market capitalisation, in order to compare the two methods, it is necessary to find the relative enterprise value to execute a logarithmic transformation on the CCA outcomes. After this step, the % change from real EV was calculated for both studies. The following figure displays the results:

Figure 3 - *OLS Model vs CCA Model*



Source: Personal Elaboration

As we can see, both models are able over time to provide an estimate very close to the real enterprise value of the companies analysed. In particular, the OLS model presents an average distortion of 3.19% compared to the Comparable Companies Analysis model, which has an average value of 9.75%. This latest demonstration highlights, even more, the accuracy of the first model and proposes a new method for researching comparable companies which moves away from traditional methods and which should be investigated in future research.

Conclusion

Through the identification of the financial drivers that most influence the value of HGFs, the developed model represents a stronger valuation method to evaluate High-Growth Firms. Moreover, the use of a new strategy for the selection of comparable companies has shown that traditional methods, and in particular traditional multiples, are not consistent with the characteristics of a high-growth company. Both results provide empirical proof, confirmed by the comparative analysis with the results observed over the years.