LUISS

Financials in Football: Evaluations, Bonds and Derivatives

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I would like to begin this thesis by thanking Deloitte&Tousche for the opportunities that provides, with summer internships, to many young and inexperienced undergraduates, which have the opportunity to discover the working environment and to develop skills that will be fundamental in future experiences. Thanks to that experience I was able to analyze in depth topics of my area of interest and also understand the logic of the consultant sector, in which would be fascinating to work in the near future. Especially due to the team-building activities and stimulating atmosphere, favored by the Head of Financial Evaluation Department, Mr. Marco Vulpiani, I was able to establish sincere relationships with some of Deloitte's employees, that supported me in my experience. I would like also to thank Deloitte&Tousche for allowing to include the work brought on for "Money League" project in Chapter 1 of this Thesis.

I would like to thank also AS Roma S.P.A, that gave me the opportunity to present my work in a dedicated meeting with the Chief Executive Office Mr Guido Fienga and the Chef Financial Officer Mr Giorgio Francia. I hope that my work has contributed to stimulate additional investigation in a sector in continuous financial development.

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Introduction / Executive Summary

This study starts with a **comparative analysis of the principal European football systems**. It includes a **cost analysis** and **economic/financial indexes** useful to understand **comparative evaluations** and **financial instruments based on new Deloitte algorithm**.

The role of **financial statements in the financial fair play imposed by UEFA** and recent movements in the transfer markets in the final part of the analysis try to identify a trend for the future of the business.

Deloitte is at currently at the 22nd edition of the Deloitte Football Money League in which the team compiled by Dan Jones, Timothy Bridge, Samuel Boor, Chris Hanson and Calum Ross profiles the highest earning clubs in the world's most popular sport. Published just eight months after the end of the 2017/18 season, the **Money League is the most up to date and reliable analysis of the clubs' relative financial performance**.

There are a **number of metrics**, both Financial and non-financial that can be **used to compare clubs, including attendance, worldwide fan base, broadcast audiences and on-pitch success**. In the Money League Deloitte focuses on clubs' **ability to generate revenue from match days** (including ticket and corporate hospitality sales), **broadcast rights** (including distribution from participation in domestic leagues, cups and European club competitions) and **commercial sources** (including sponsorship, merchandising, stadium tours and other commercial operations) which are then ranked on that basis.

The ever-changing financial landscape of football over the past 20 years has been both extraordinary and fascinating in equal measure. When the Deloitte Football Money League was first published covering the 1996/97 season, Manchester United topped the table with a revenue of £88m. Fast forward 20 years to the 2018 edition, United have regained top spot from Real Madrid following 11years of Spanish dominance, but in 2019 edition the Galatticos recorded revenues for €750m over six times greater than in 1997.

To gain a place in the top 20, a club must now generate €200m, which represents an increase of 21% on the amount needed in the 2014/15 edition when 20th position was secured with revenue of €165m, an amount very different from that of season 1996/97, where the 20th club generated just €36m in total revenue. The 1997 revenue ratio between a top earning and bottom-earning club in the top 20 was 3,2 vs 3,4 in the last edition.

In the 22 editions of Deloitte Money League to date, there have been 42 different teams from 11 different leagues across the world taking a spot in the top 20, with only ten teams managing to remain present in the top 20. Whilst clubs from outside the big five European leagues have made occasional appearances in the Money League top 20, the **dominance of clubs from England, Germany, Italy, Spain and France has become more apparent, particularly in the most recent editions**. This dominance reflects the growing trend of polarization, common not only in Money League but across much of the football world. Even the biggest clubs in Europe outside the big five leagues struggle to break into our top 20, some of the trends seen outside Europe assess the possibility of a non-European club gaining a spot in the Money League.

CHAPTER 1 concludes by showing how the comparison in the next few years will develop precisely on the financial equilibrium introduced through the new regulation of the Financial Fair Play of UEFA which will develop clubs' business and financial equilibrium and the acknowledgement in the financial statements.

The introduction of financial fair play beyond the evident sporting value aimed at guaranteeing a more level playing field by aiming to avoids leagues dominated by magnates with inexhaustible capital has a financial interest in guaranteeing interest around the football system that remains dependent of the competitiveness of clubs in different leagues. As has been highlighted in the analysis of the major leagues, in fact, it's precisely where there is competition that the customer is interested in spending money to follow a sporting event at the stadium or on television. For example a better redistribution of the television rights among the different clubs increases competition and interest of the fans that will then increase the "broadcast cake" to be served in the following years.

The **periodic analysis carried out by Deloitte focuses on the performance of the sector** and the competitive advantage, **based on the comparison of the different components of the revenues** and on an analysis on the single macro areas of revenues (stadia, broadcast divisions, commercial activities, capital gains/capital losses from the sale of the multi-year rights to the players' performances).

S**occer clubs are valued** based upon numerous **factors not limited to the main multiple of Enterprise Value (equity plus debt) on revenues (CHAPTER 2)**. Income is generated from broadcast agreements, premium seating and match day ticket sales, media, brand and product licensing, and merchandise and concessions sales. Crucial is the identification of the **value of the team players that depends on performance**, **role, age as well as behaviour**. As seen in the accounting analysis there are different way to represent that in the balance sheet, but what is **difficult is to predict the value of the players for the future** because of the numerous variables involved in the analysis.

Starting from the analysis obtained from the documentation produced by Deloitte **summarized on a three-year basis** and also **trying to identify a prospective trend**, (**in CHAPTER 3**) a comparison is made between economic and financial evidence on aggregated economic data of the leagues and **analyzing the items that characterize the budget of a football firm** also **including a series of industry efficiency indicators**.

Also in relation to the entry into force of the financial fair play, in order to understand what are the appropriate methods of evaluation for football clubs (and in particular for the player park), an **investigation has been carried out on** possible financial instruments that according International Accounting Standards IAS 39 (**CHAPETR 4**) could be able to book in the balance sheet the mark to market of the **value of the team players.**

It helps also in debt raising capacity because when the club performance is constantly at high levels the financial capacity will accordingly increase in the long run. But if the performance is not satisfying may be difficult to earn stable revenues to devote to the debt service and for this reason the capital markets look with diffidence at financing football club on unsecured basis.

A possible solution has been discovered by Kick-bonds that link in a positive relation the remuneration of the bond with the performance of the team **(CHAPTER 5)**. In this way the clubs can attract financial interest from the market and can benefit of a sort of natural hedge because it commits themselves to pay more only if the performance is good.

The platform Kickoffers, created in partnership with Deloitte, has structured an algorithm in order to quantify the performance of the team and of the football player linked to remuneration of these kick-bonds. If the player's disposal would generate a gain in respect of initial evaluation coming from the Deloitte algorithm, an extra bonus will be paid to the investors. The first experience of Deloitte algorithm application has been the one of Italian soccer club Chievo which listed its financial bonds and not its shares as Juventus, Lazio and Roma.

In my 2018 experience in Deloitte when I analyzed the financial assessments of the football teams on multiples, I understood as predictable may be the component of linear growth of a player's value, but also that a random component is part of soccer players evaluation (for example used as a reward on the coupon of a bond issue parameterized to the surplus value on a specific player).

On the other hand in 2019, both in my “Analysis and Management of Financial Risk” course at the London School of Economics and in my “Financial Mathematics” studies at the Mathematics Department of Edinburgh, I studied how, under certain conditions that seem to be able to apply also on the performance of the players' value, you can price the value of a derivative contract which has as its underlying the price of an asset that moves over time according to an identified trend but also in relation to its average value and volatility.

The Black & Scholes model, that I analyzed with the help of Professor Biagini and her notes (**CHAPTER 6**), is applied continuously and involves the use of statistical tools and probability distribution that I have had the opportunity to deepen in my studies. I therefore thought I could extend the contents of my thesis also to an analysis of the now widespread tools of options on players (both in terms of the right to buy and to sell) with high statistical-mathematical value.

From an economic-financial modelling point of view, the Merton model identifies the **market value of a company (Equity Value, or Market Capitalization if listed) as a call option on the value of the assets (entire corporate value or Enterprise Value) with strike price equal to the value of the debt and with expiry the debt maturity**. The present value of the Equity is expressed as the present value of the average probable difference between the value of the entire asset at the maturity of the debt and the value of the debt itself. Equity can have a positive present value even if the current value of the assets is lower than the current value of the debt because, at the maturity of the debt, the market value of the assets (Enterprise Value) can exceed, with a certain probability, the debt value that grows only with the interest rate.

The basic assumption of the methodology is that, if the asset company value should be lower than the debt value, the property would have no economic interest in continuing in the company activity and therefore the value would be at the minimum zero given it is not obliged to proceed with capital increases, unless there is a prospective value to be protected. If this difference were instead positive, the expected value of the payoff of this difference discounted according to a risk-neutral approach would be the value of the Equity itself. Basically, from this approach, it emerges just how **the current value of the company can be higher than the current difference between assets and liabilities since, with a certain probability, the assets will grow more than the debt capitalized for its contractual interest rate**.

It is an approach based on market evidence that leads to an evaluation of an equivalent company to that of discounting the company's future cash flows by means of a discount factor that takes into account of the uncertainty of these flows according to an adequate relationship between risk (variance of cash flows) and return (discount rate) which must be on the **Security Market Line**. If this value were contractually divisible into sub-sets of assets, the **diversification effect that allows to optimize the risk / return profile** by determining the optimal portfolios (which represent the efficient Markowitz frontier) must also be considered.

If the valuation of the Equity can be conceptualized as a call option on the underlying corporate value compared to a strike represented by the value of the debt at maturity, by extension the **value of the Equity can be divided into subsets represented specific categories of assets** with which to associate specific values: i) the most certain part using, instead, for the enhancement the approach of cash flows discounted at an interest rate that reflects the modest variance of these flows; ii) the most random (but also most market and therefore verifiable) part following the approach of the call options to evaluate their value.

Certainly the part with the most certain cash flows lends itself better than the random one to be able to sustain the debt in terms of repayable prospect but, if debt or mezzanine funds are identified in the market with appetite for individually transferable assets and with a market value that can be parameterized, the advantages in using this valuation method would be different. From the point of view of the company's debt capacity, access to the specialized, alternative or additional debt market to the banking and bond market could be obtained. Investors interested in the finance ability of even a single asset with a formula similar to that of the pledge (if not of the financial leasing) could be found in which the repayment is collateralized by the value of the asset at maturity of the debt and whose remuneration could be indexed the payoff at maturity equal to the difference between the value of the asset and the capitalized debt (kick bond).

From an Equity value point of view, **the future value of the more random assets could emerge immediately, despite the International Accounting Standards do not allow to be valued before their realization according to the logic of prudence (IAS 39), unless this derives from a market exchange contract of this value**.

For example, you could enter into forward contracts that fix the price of the asset at maturity (without predicting its sale) or even assign call options on certain assets that would collect premiums and reveal the unexpressed values of these assets as: i) the cash flow method cannot capture the specificity of these assets, necessarily having to reason for average values; ii) the international accounting standards do not allow their valuation in the financial statements until realization / marketing.

If soccer companies do not want to set forward prices or transfer the rights on the value of the assets with respect to a certain strike (which may even be higher than the value of the specific debt that you can allocate on these assets), they could also consider issuing shares privileged with special rules that factor the rights regarding the future sale of certain assets with respect to a specific strike assigned to them. In this last way, differently from the previous ones, the value linked to the optionality of these assets could not be reported in the financial statements (which would instead be very functional for the issue of new debt), but the value not expressed in specific terms could still be valorized of greater equity, the enhancement of which occurs through free trade between the parties.

The **market equity value is already, per se, an expression of the presence of an asset value not reflected in the financial statements** due to prudential constraints imposed by the IAS, however a more specific identification of this subset would have a positive effect on the capitalization suffering from penalizing weightings and discount for uncertainty.

The assets to be valued according to the optional approach are those with greater uncertainty and for which a linear motion can be distinguished, identifiable in parameterization algorithms that simply identify the trend of variation of the value over time, and in a stochastic motion that determines a future value independent of time and closely linked to parameters the current value and the variance of the probability distribution which is assumed to be of the normal type. **Through the Black and Scholes equation, the values of any derivative on these assets can be determined**, whether they are linear (forward or short selling) or optional (call / put long / short) or digital (all-or-nothing). In relation to the type of derivative to be valued, a specific formula will be determined which can be used to quantify the contract value that is assumed to be able to finalize.

Different derivatives have different economic functions. While a forward sale of the asset defines the future value of the asset itself, a **forward contract lends itself to transferring the difference between the future value of a given asset with respect to an identified strike allowing to monetize the difference,** and therefore implicitly to remain neutral with respect to the strike identified. A call option, on the other hand, represents the positive present value of the sale of the difference between the future value of the asset and a given strike and, if sold, call option allows you to collect the reward and set the forward value and, **if acquired, call option allows you to focus on the appreciation of a certain asset and to possibly guarantee the purchase of the asset in the event of appreciable appreciation**. The purchase of a **put option ensures that the value of a given asset is not lower than its strike at maturity** because with this instrument the right to be reimbursed is acquired for the negative difference between the value of the asset and the strike, at the same time it can allow profit to be made from valuations considered excessive for an asset that is estimated to depreciate.

The digital option allows instead to place bets on the future value of the asset which will involve the payment of an initial premium against the possible collection of another value in the event of a favorable event, or of nothing in the event of an opposite event. They can also be seen as a fixed contribution to a future purchase, which however does not completely neutralize the position.

Conditional to a labour law check on the transferability of the economic rights referred to above in terms of capital gain / loss of the underlying asset compared to an identified strike (which may or may not coincide with the value of the debt allocated to this asset), the possibility could be assessed to treat the players' park as a set of assets whose value is characterized in part by a linear trend and in part in stochastic random motion. This set could be divided by individual asset in order to facilitate its enhancement according to the specific linear growth algorithm already developed and through the random component to be identified.

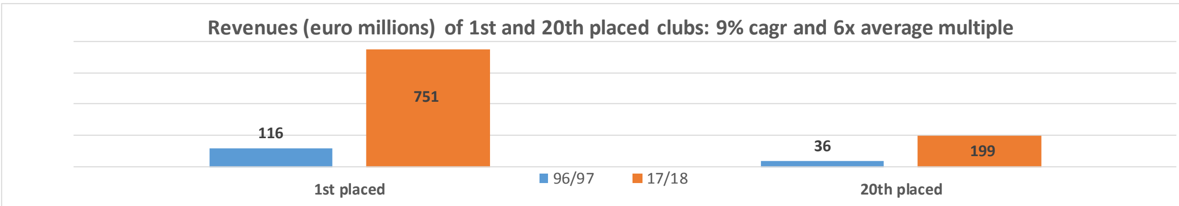
The correlation between the different assets, distinguished by role and age, not being equal to one, would also allow o build a portfolio that optimizes the risk / return profile (less risk for the same return or greater return for the same risk), and therefore, it also optimizes the valorization function of these assets both from a point of view of greater debt capacity and to merge the unexpressed value of equity for the reasons mentioned above.

Derivatives instruments on soccer payers values may become an interesting financial instrument tradable also in respect of absence of underlying, it means that may evolve in a **market dimension not only for people in the know** of soccer but also for funs or speculations.

CHAPTER 1 - Deloitte Money Football League today

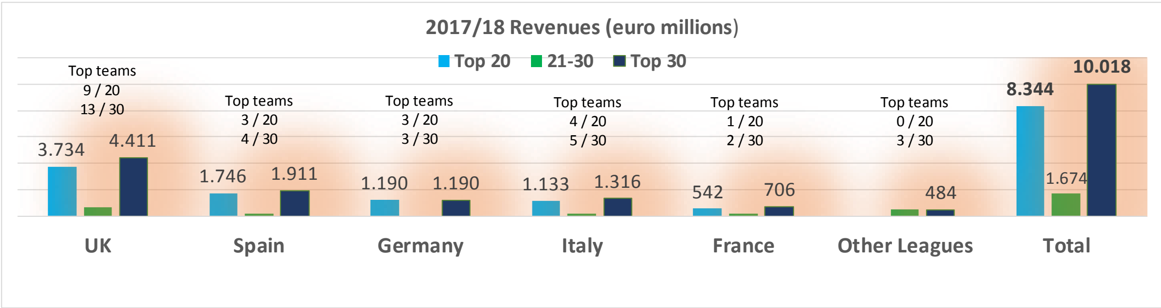
GENERAL OVERVIW

The ever-changing financial landscape of football over the past 20 years has been both extraordinary and fascinating in equal measure. When the Deloitte Football Money League was first published covering the 1996/97 season, Manchester United topped the table with a revenue of £88m. Fast forward 20 years to the 2018 edition, United have regained top spot from Real Madrid following 11years of Spanish dominance, but in 2019 edition the Galatticos recorded revenues for €750m over six times greater than in 1997.



To gain a place in the top 20, a club must now generate €200m, which represents an increase of 21% on the amount needed in the 2014/15 edition when 20th position was secured with revenue of €165m, an amount very different from that of season 1996/97, where the 20th club generated just €36m in total revenue. The 1997 revenue ratio between a top earning and bottom-earning club in the top 20 was 3,2 vs 3,4 in the last edition.

In the 22 editions of Deloitte Money League to date, there have been 42 different teams from 11 different leagues across the world taking a spot in the top 20, with only ten teams managing to remain present in the top 20. Whilst clubs from outside the big five European leagues have made occasional appearances in the Money League top 20, the **dominance of clubs from England, Germany, Italy, Spain and France has become more apparent, particularly in the most recent editions**. This dominance reflects the growing trend of polarization, common not only in Money League but across much of the football world. Even the biggest clubs in Europe outside the big five leagues struggle to break into our top 20, some of the trends seen outside Europe assess the possibility of a non-European club gaining a spot in the Money League.



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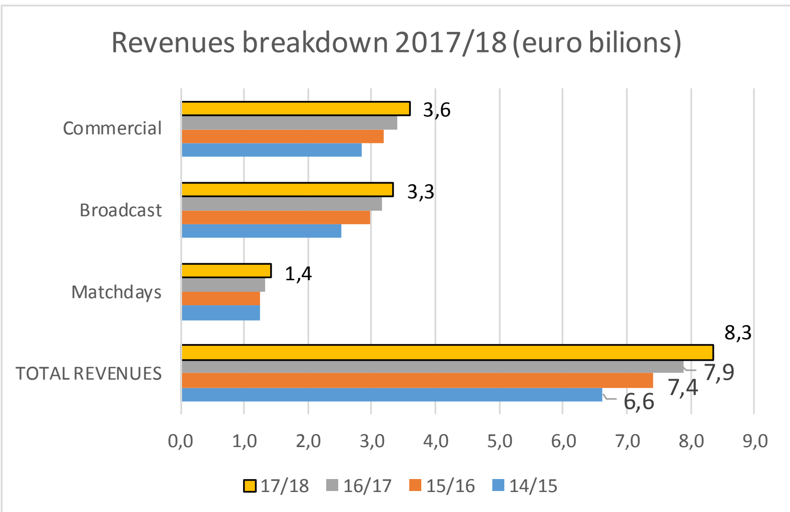
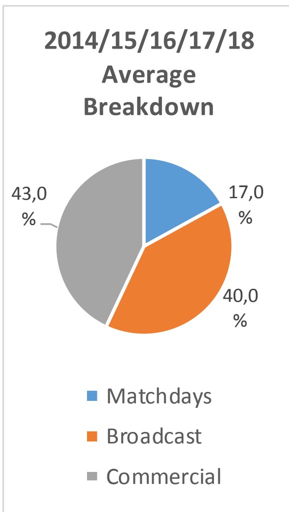
Last year **the total revenue for the top 20 was** another record totaling €7,9 billion. This was a 7% increase on the previous year's top 20 with the uplift of €0,5 billion coming from 49% in broadcast revenue, 42% from commercial revenue and with only 9% of the increase due to match day revenue.

This break down in revenues evolution has been confirmed also **in 2017/18 edition** with **€8,3 billion of revenues increasing by 5%** in respect of 2016/17 season which leaves an **expectation for next season to break the €8,5 billion roof**.





Year after year Money League sees records been broken. The 20th edition saw the €600m revenue barrier broken for the first time, and by three clubs, with Manchester United, Barcelona, and Real Madrid all achieving that feat. For the third consecutive year, the **top ten contains the same clubs, but five of the top ten have changed position**.

As well as Real Madrid and Manchester United swapping places. Paris Saint Germain slipped two places to sixth, with Bayern Munich jumping a place to fourth and Manchester City also climbing one place to fifth, this being their highest ever position.

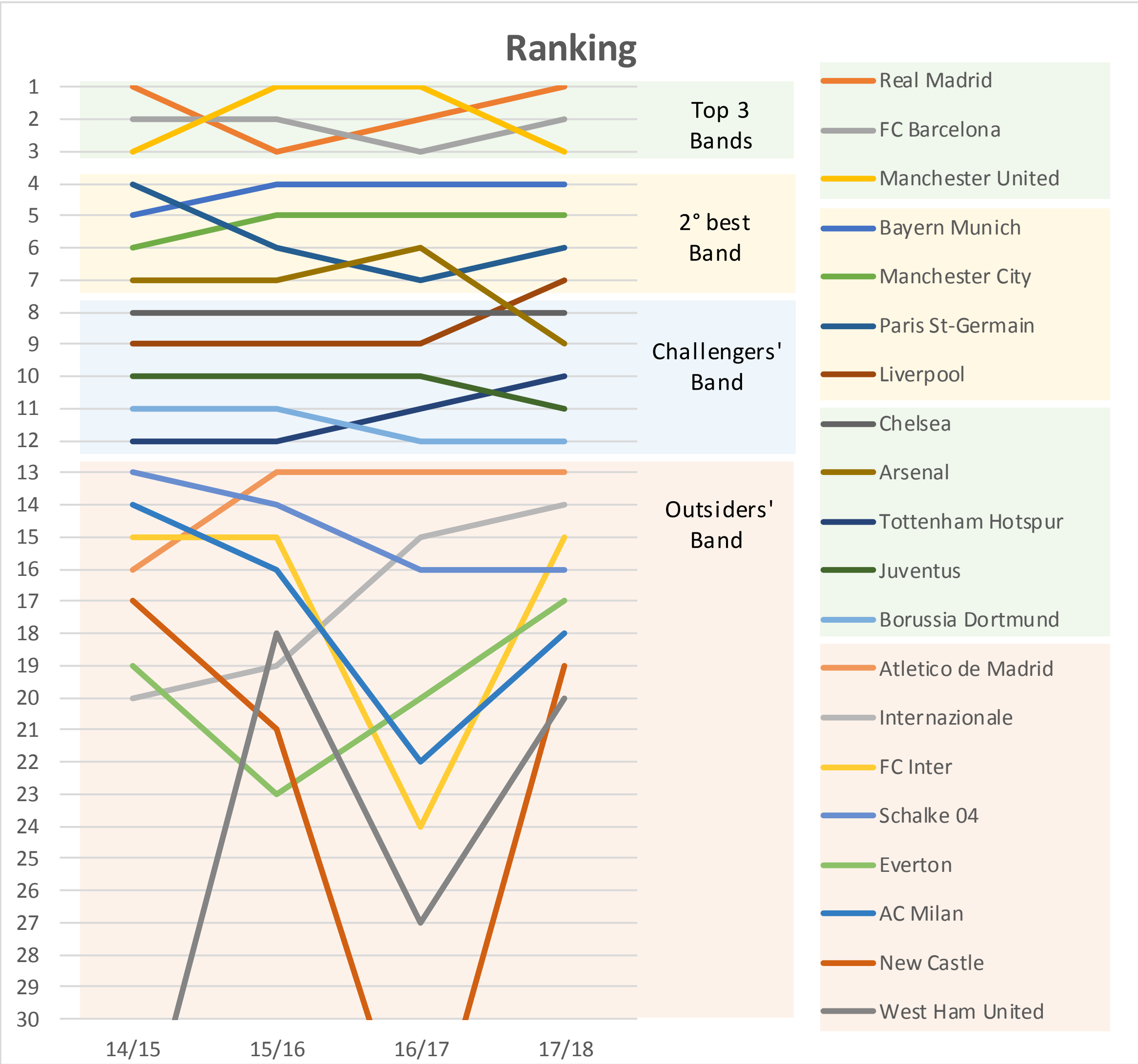
There was only one debutant in top 20, with Premier League champions Leicester City making their first appearance in the Money League to end an incredible year.

	Revenues 2017/2018 (M€)	ΔR 17/16	ΔR %
Real Madrid	751	76	10,2%
FC Barcelona	690	42	6,1%
Manchester United	666	-10	-1,5%
Bayern Munich	629	41	6,6%
Manchester City	568	41	7,2%
Paris St-Germain	542	56	10,2%
Liverpool	514	90	17,4%
Chelsea	506	78	15,4%
Arsenal	439	-48	-11,0%
Tottenham Hotspur	428	73	17,0%
Juventus	394	-11	-2,9%
Borussia Dortmund	317	-15	-4,9%
Atletico de Madrid	304	32	10,5%
Internazionale	281	19	6,7%
FC Inter	250	78	31,3%
Schalke 04	244	14	5,6%
Everton	213	14	6,4%
AC Milan	208	16	7,7%
New Castle	202	102	50,5%
West Ham United	198	-15	-7,8%

In 2016/17 season and in spite of the weakened Pound due to external factors such as Brexit, **Manchester United retained its first place** in Money League and they make their tenth appearance at the top with **as revenue of £581m (€676m)**. The Red Devils' revenue pound increased +13%, with broadcast revenues up £54m after enhanced Premier League distributions.

Their crown was critical only because of pound depreciation (in 2016 equal to -15%) with result that the **gap between them and Real Madrid (with revenues increase of + 8%) was just €1,7m**, and the importance of the €3m additional amount received from winning the Europa League Final is clearly evident. Commercial revenue growth was limited in 2016/17, but Manchester United with this revenue stream remained the club's largest and is over 30% more than closest domestic rivals, Manchester City.

Manchester United's strong commercial growth coupled with a return to UEFA Champions League saw them take the Money League crown keeping 5% far the Barcelona who lost in the Money League 'El Clasico' by piping Real Madrid to second position, by the smallest of margins.



As predictable and said, the combined revenue for 2017/18 season of the top 20 clubs sets a new record, exceeding €8 billion for the first time.

The rises and falls in this year's Money League highlight the continuing evolution of the financial landscape in the world of football. Whilst the top 20 retains many familiar names from Europe’s ‘big five’ leagues, 14 clubs see their position change from last year, including three new entrants.

Broadcast revenue remains the most prominent source of revenue for Money League clubs with a 43% share, compared to 40% and 17% for commercial and matchday revenue respectively. Nonetheless, a noticeable trend in this year's edition has been the commercial growth achieved by Europe's largest clubs, particularly for those in the ‘big five’ leagues with no new broadcast cycles commencing in 2017/18 (i.e. England, France, Italy and Spain).

Real Madrid become the first club to break the €700m barrier en route to returning to the top of the Money League, generating revenue of more than €750m. Barcelona complete a first Spanish one-two since 2014/15, with revenue of €690m. The gap between the top two has grown to the second widest in Money League history (€60m) after being the narrowest ever in last year’s edition. Including Manchester United, who drop to third this year, the top three clubs in the Money League collectively generated revenue in excess of €2,1 billion, just 5% short of the combined revenue of the top 20 clubs in 1999/00.

Tottenham Hotspur break into the top ten for only the second time in Money League history replacing Italian giants Juventus and marking a change in the composition of clubs in the top ten for the first time since 2012/13. The revenue of the tenth placed club increased 6% to €428m in 2017/18 (Tottenham Hotspur) from €406m in 2016/17 (Juventus). Only three of the top ten clubs (Bayern Munich, Manchester City and Chelsea) remained in the same position as in the previous edition.

This year’s Money League continues to underline the importance of qualification for, and performance in, UEFA club competitions. The UEFA Champions League and UEFA Europa League have played an important role in improving, or retaining, Money League positions for Liverpool, Chelsea, Tottenham, AS Roma, Everton and AC Milan, whilst Arsenal, Juventus, Leicester City and Southampton have lost positions or even their top 20 status altogether after reduced European competition revenue in 2017/18. The impact of UEFA club competitions looks set to be even greater in next year’s edition with distributions to participating clubs increasing to a reported €2,6 billion in 2018/19 from €1,8 billion in 2017/18 following the start of a new broadcast cycle.

As well as this increase in the overall amount to be distributed by UEFA, changes to the Champions League qualification process and distribution mechanism have also been introduced. This will notably benefit many of the clubs in the Money League and bring the issue of financial polarisation to the fore once again. The four top-ranked national associations in UEFA's country coefficients (currently Spain, England, Italy and Germany) are now guaranteed four automatic Group Stage qualification places and alterations to the distribution model mean that a portion of the total amount available for distribution will be allocated based on a club's historical performance in UEFA competition over a ten-year period.

For the second consecutive year, no clubs outside of the 'big five' leagues in Europe appear in the Money League top 20 and only three are in the top 30.

FUTURE PREDICTIONS

Last year it was expected that the 16/17 record year, driven by increased broadcast and commercial revenues would be eclipsed in 17/18 season. New domestic broadcast deals starting in 2016/17 for Premier League and La Liga Clubs (as well as international broadcast deals for the Premier League), could have meant the 22nd edition could have seen the €8 billion barrier broken. The **weakened pound** could have helped ensure a close **three-way fight between Real Madrid, FC Barcelona and Manchester United for the top spot**. After what was predicted last year and what happened during the season 2017/18 season reported in March 2019, one would certainly **expect the €8,5 billion barrier to be broken next season 2019/20**, but revenue growth shoul be around5% and not as significant as seen in 2016/17 (+13%). **Germany's new domestic broadcast** deal commences and **will increase** revenue, but **Premier League and La Liga distributions will remain** relatively **stable**, as both enter the second year of existing TV deals.

With most domestic leagues in the same broadcast rights cycle in 2018/19 as 2017/18, excluding Serie A who have secured a modest increase on their domestic and international rights commencing in 2018/19, it is unlikely that broadcast revenue will increase at a similar rate as we have seen previously. Therefore, **revenue growth in next year's edition will be driven by improved**

UEFA distributions and increases in commercial revenue of the Money League's top clubs.

The longer term composition of the Money League is a fascinating point of discussion. With the Premier League’s lack of substantial increases for the next broadcast cycle, we may see other **clubs from the other ‘big five’ leagues begin to narrow the gap to their English counterparts**. However, the Premier League’s decision to award a small package of rights to Amazon and the continued emergence of over the top (OTT) platforms elsewhere in the market could provide a level of competition that could potentially deliver substantial increases in broadcast rights values in future cycle negotiations.

As we approach the UEFA club competition broadcast rights sales process for the next cycle from 2021/22, the European football environment could also be subject to change. It was recently announced that a new third-tier competition will be introduced, running alongside the existing Champions League and Europa League, which could assist in addressing the issue of financial polarisation in European football and helping to strengthen the development of football across UEFA's member associations.

Looking further ahead, the long term composition of the Money League is an intriguing topic. The dominance of English clubs will very much depend on the outcome of the Premier League's ongoing tender for the next three-year TV deal starting from 2019/20. Further increases would maintain, if not improve, the positions of English clubs. However, if growth is marginal, other countries may have the opportunity to close the gap, particularly in Spain who will also be negotiating new broadcast deals.

European football dominates financially, and has done for the first 20 years of the Money League though the **landscape may change** considerably in the longer term, with attractive and emerging football markets looking to become the next football powerhouse. In Rising stars there is a snapshot of the key opportunities and challenges for high-profile clubs operating in **China** and the **USA**, these being two leagues that **could aspire** to see a member club enter the Money League in the future due to the attractiveness of the attention given by younger generations.

What remains **crucial is a diversification of revenues** and real estate, **good customer relationship management**, **discouraging wage growth** through system agreements (salary cap, benefits plan, highly variable forms), **reducing**

the role of agents with concerted actions Added to this is the need develop **revenues from the exploitation intangibles** (image rights, various rights) and **new media.**

In terms of single team bet, the **rights gained thanks to the victory of the 2018 Champions League** will be brought in the next few years and could permitt **Real Madrid try to win again the Deloitte Money League in season 2018/19** reported in March 2020 but, at the same time, the **sale of CR7 after 8 years of its participation in the growth of Real Madrid,** despite **being an extraordinary income** activity that does **not directly affect the revenues considered by the Deloitte Money League**, It will probably have a **catastrophic effect on merchandising and television rights** that will make the results of the ranking perhaps more unpredictable in the coming years, **giving a boost to Juventus** and Paris Saint Germain (another king of the market) who will both need to consolidate it with a careful management which is able to strengthen the link between sporting and economic results of well run football clubs.

CHAPTER 2 - Evaluation methods of soccer clubs

Methodological approaches

After having seen the historical evolution and having defined some economic-financial aspects typical of professional football clubs, we now focus on the determination of their value in relation to which are, or could be, the most **suitable evaluation methods for an accurate and truthful estimate** of economic capital taking which takes into account **synthetic value** and **corporate governance**. Exploring the case of professional football teams (Business Systems Review, 2013) "**professional football teams** as this **special business combination** provides an evident example of companies **whose performance cannot be evaluated considering only financial returns** on shareholders' value. The **investments of a professional football team are mainly in intangible resources**, first and foremost in the skills and the competences of players, coaches, the general manager, and the medical staff. At the same time, the final outcome will include both financial income, and intangible assets, like experience, popularity, reputation". The same Dan Jones, partner of Deloitte, defined the process of evaluation of a football association as "as much of an art as a science".

Among the methods of evaluating the economic capital of professional football clubs, reference is made to the main valuation methods that can be subdivided into equity (simple and complex), income, financial, mixed, **stock market multiples and** some **hybrid methods specific to the business sector**.

As for football clubs and more generally for all sports clubs, they have two particularities that make it difficult to evaluate and identify the most suitable methodology. The first lies in the fact that companies mainly pursue sporting success, sometimes even at the expense of economic and financial equilibrium, although this phenomenon is mitigated thanks above all to UEFA policies (UEFA license, but above all Financial Fair Play).

KPMG Approach

In this regard, KPMG summarizes the **virtuous circle that is created when a club achieves sports success** that i**ncreases** the involvement of **fans**, **media** and **sponsors**, which causes an **increase in revenues** that can in turn be **used to**

improve the team in order to have more chances continued of success the sports field.

Source: KPMG

A second important problem in assessment consists in the fact that **for football clubs there is no direct correlation between investments (input) and sports results (output)**.

Given the characteristics of sports clubs in general, the **adoption of the simple equity method does not seem ideal in search of a correct estimate of the economic value of the company** as this method lacks an allowance of the intangible elements, typical of football budgets mainly consisting of rights to the players' performances and from the corporate brand. In this context, the adoption of **a complex capital method seems more appropriate as it considers the identification and subsequent valuation of the intangible elements**.

However, as part of this methodology, we recall that it is particularly difficult and discretionary to attribute values to elements such as the player 'names', thus risking to removal of the income element from the assets generated which is fundamental for investors.

In any case, the **income methods do not seem adequate to the particularity of the clubs, since the difficulties linked to the punctual determination of future incomes**, the choice of the discount rate as well as the historical problem of losses in the football sector cannot be neglected. For companies in crisis in fact, the same considerations given in the assessment for "in health" companies do not apply first of all as regards the state of operating uncertainty in which these companies are located.

In terms of financial methods, T. Markham in his 2013 paper, inserts the **DCF methodology among the ideal models**. In his work the DCF approach is suggested **even if it presents similar difficulties to the income one due to the lack of reliable future forecast plans** (regarding the volatility of the typical performance of the football businesses), while the valuation of the stock market values is considered unsuitable. The latter, although they represent market values, represent a small number of exchanges due to the lack of liquidity that makes them subject to speculative behavior.

According to Markham himself, the **multiples method of turnover is not a reliable model of assessment for clubs**, since this methodology, **even if it allows a quick sanity check of the enterprise value**, is likely to lead to unsteady values due to an **underestimation of effects that the capital structure** (leverage / access to credit and related costs) could have on the ability to generate income for the most "unbalanced" clubs from an economic-financial point of view.

Currently, however, **a whole series of "hybrid" methods, based on particular algorithms**, such as those **provided by** various **specialized** companies such as **Forbes and KPMG**, as well as the same soccer information provider **Deloitte**, which, precisely on the evaluation of the players and their non-recourse financing, it **is also a market consolidation**.

Over the years, among the various methodologies formulated and proposed, the one used by the American specialized company Forbes has met with great success among the insiders of the main American sports, or teams participating in the NFL (National Football League), NBA (National Basketball Association), NHL (National Hockey League) and MLB (Major League Baseball).

Only since 2004, following the huge follow-up achieved by the football phenomenon in the world and the increasingly important figures that characterized the business, **Forbes thought to draw the first ranking of value** in the sector, or **the 20 most valuable football teams in the world** using **multiples based** on sector **data published by Deloitte**.

Over time, Forbes has developed an algorithm that has replaced, or better integrated, the previous traditional model focused mainly on market multiples as summarized by Mike Ozanian (Forbes staff member): "Operating income is earnings before interest, depreciation and amortization, player trading and disposal of player registrations. Most Revenue figures are from Deloitte's Money League report. "In this regard, Markham argues that "Historically, the technique favored by Forbes in valuing clubs was based on multiples of revenue."

FORBES Approach

In essence, the **Forbes method breaks down the economic value of the football** companies (equity plus net debt) **based on the current agreement of the stadium** (unless the new stadium is in progress) **in the following four parts**: i) **match day** portion of the value of a club deriving from the revenues of the various activities on the day of the match; ii) **broadcasting** for the portion of the value of a club deriving from the distribution of domestic and international television rights regarding the championship, the national cups and possibly the international ones; iii) **commercial** for the portion of the value of a club deriving from sponsorships, merchandising and revenues from other commercial activities; iv) **brand** for the portion of the value of a club in excess of the sum of the three previous parameters.

The report published by Deloitte this year and referred to the 2017/2018 financial statements shows that the revenues of the 20 football companies with the highest turnover in the world generated revenues of € 8,3 billion, an increase of 5% compared to the survey of last year.

All the clubs in the ranking have seen their revenues increase, which Deloitte divides into match days, television rights and commercial revenues, compared to the previous year. In this regard Marco Vulpiani, partner at Deloitte and Sector Leader, states that "the British clubs record high positions in the rankings mainly thanks to the excellent infrastructure and additional services they are able to offer to their public. In this sense, the results of Juventus also demonstrate how strategic it is to invest in a stadium owned by the club and in its image and the recent rebranding of the club shows the will to continue on this path ".

The Forbes ranking shows that for the **first time in five years, with a value of 3,4 billion euros, Manchester United** managed to **return to the top of the list of the most valuable clubs in the world** drawn up by Forbes. Barcelona and Real Madrid have to be satisfied in completing the podium next to the "Red Devils" with the values respectively of 3,35 billion euros for the Catalans and 3,3 billion euros for the Merengues. The reasons for the success of United are due, as highlighted by the author of the report, to the **strength of the brand and marketing**. No one, as Deloitte also points out, in the 2016/2017 season, was able to generate revenues like the team of the red half of Manchester.

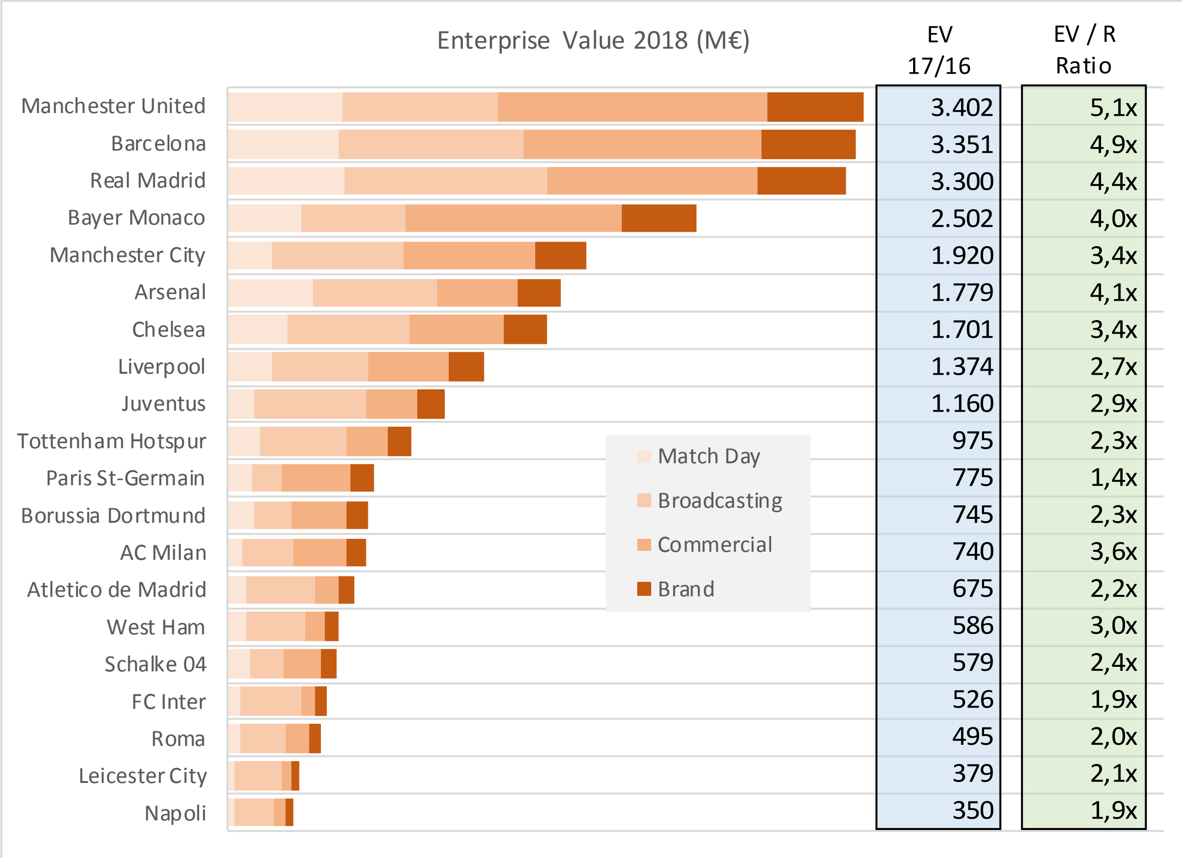
Forbes: The World's Most Valaublе Soccer Teams 2018

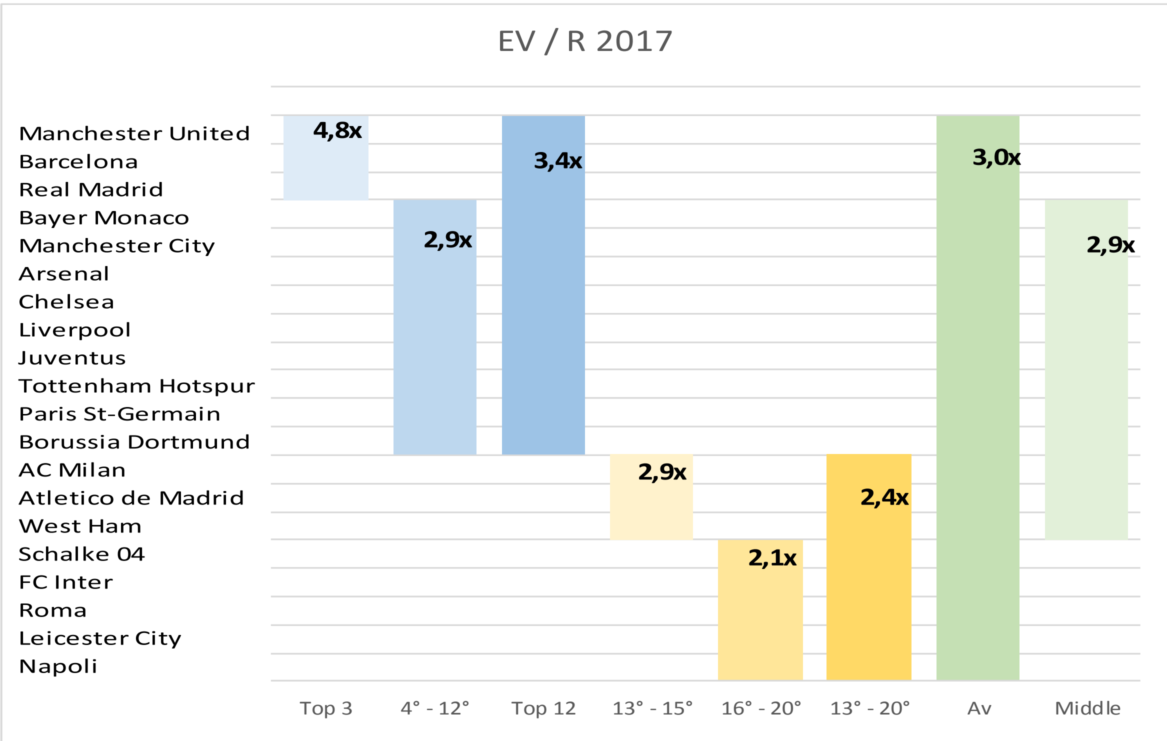
	Teams	Match Day	Broadcasting	Commercial	Brand	Value
1	Manchester United	611	834	1.437	520	**3.402**
2	Barcelona	590	986	1.280	495	**3.351**
3	Real Madrid	618	1.091	1.121	470	**3.300**
4	Bayer Monaco	387	562	1.158	395	**2.502**
5	Manchester City	231	711	699	279	**1.920**
6	Arsenal	456	657	434	232	**1.779**
7	Chelsea	319	654	496	232	**1.701**
8	Liverpool	232	515	435	192	**1.374**
9	Juventus	134	599	277	150	**1.160**
10	Tottenham Hotspur	172	463	216	124	**975**
11	Paris St-Germain	124	165	363	123	**775**
12	Borussia Dortmund	144	195	294	112	**745**
13	AC Milan	80	273	278	109	**740**
14	Atletico de Madrid	96	371	125	83	**675**
15	West Ham	99	317	99	71	**586**
16	Schalke 04	119	174	203	83	**579**
17	FC Inter	62	334	69	61	**526**
18	Roma	64	245	121	65	**495**
19	Leicester City	30	252	53	44	**379**
20	Napoli	34	205	68	43	**350**
	Totale	**4.602**	**9.603**	**9.226**	**3.883**	**27.314**
	incidence	17%	35%	34%	14%	

Multiples EV / Revenues

Using a **sanity check** method based on **revenues multiples**, because it is **suitable to provide a homogeneous order of size** even if it does not take into account the net financial position of the companies and the corporate assets, comes out that **Top 20 average Enterprise value / Revenues in the range of 2,9x – 3x** (depending if without or with extreme values), **Top 3 one of 4,8x** and **Top 12 of 3,4x**.

It means that should be applied a different multiple for each revenues level.





In addition to the methodology proposed by Forbes, the **KPMG network also offers** through its annual report "Football clubs' valuation - The European Elite 2017" **the measurement of the economic value of the most important football clubs** in Europe. For the same KPMG analysts, the simple application of **the revenue multiple method is however considered too simplistic** to be adopted for all clubs **due to differences in the markets in which they operate**, revenues from television rights and their distribution or at the level of competitiveness. KPMG has also developed its own algorithm based on **five P parameters** typical of the football industry, with different weight and meaning:

1. **profitability** related to the ratio of personnel costs over the last two years compared to revenues.
2. **popularity**, given the strong correlation between sports successes and the involvement of media and fans considered as followers on social media (Instagram, Facebook and Twitter) as a good indicator of popularity.
3. sports **potential** (including players' property) and performance expectations as a driver of revenue generation in commercial terms, match days and television rights.
4. League TV **property rights** and their redistribution method as a key factor in the potential revenues of a football club.
5. **property of the stadium** which, together with the players, represents one of the most important activities for generating revenues.

The results of this report show that the **value of the 32 major clubs in Europe has grown, marking a significant + 14% compared to the 2016 survey**. In this regard, Andrea Sartori, KPMG's global head of sports and author of the report, states: "The aggregate value of the 32 major European football clubs has increased in the last year by more than 3 billion euros, a clear indicator of the fact that the overall value of football, as a sector, has increased.

This **growth is explained by the boom in football matches on pay-tv**, following the **internationalization of the commercial operations** of the clubs, but above all thanks to **investments in private and modern stadia** and following the adoption of more sustainable management practices ".

As for the ranking, you can see how Manchester United is considered, in front of the Spanish Real Madrid and Barcelona that are held respectively at 2,97 and

2,75 billion euros, the club with the highest economic value, exceeding for the first time ever the threshold of 3 billion euros.

Even here, as in previous rankings, **the British are the boss by placing six of the top ten places**. Italy is still very far from the top and only places Juventus in the top 10. AC Milan (fifteenth), AS Roma (eighteenth), FC Internazionale (nineteenth), Napoli (twentieth) and SS Lazio (twenty-ninth) are the other Italians present in the ranking. With regards to Italian football, Andrea Sartori underlines how in Italy: "Juventus are the club for which sports performance is strictly related to the trend in revenues and business value. It is also interesting to note that the value of Juventus (€ 1.2 billion) is higher than the sum of Milan (€ 547 million), Inter (€ 429 million) and Lazio (€ 227 million). "

Also from the differences in the various assessments which, **even if they lead to substantially similar results**, are based on different methodologies, it is clear that for the football companies the process of determining the value is far from simple multiples. They are **based on Deloitte data** necessary **for multiples but also implying adjustments into so-called hybrid models** formulated by Forbes, and only two years ago, by KPMG.

To help understand the similarities and the possible differences between the two different evaluation algorithms proposed by the two companies it may be useful to compare the 12 football clubs included in all the classifications dealt with in terms of overall economic value, economic value of the respective brands and finally of the revenues obtained.

The champion refers to the following clubs belonging to the respective national leagues: Manchester United, Manchester City, Arsenal, Chelsea, Liverpool and Tottenham Hotspur (Premier League), Barcelona and Real Madrid in the Spanish Liga, Bayern Munich and Borussia Dortmund in the Bundesliga , Juventus in Serie A and Paris St-Germain Ligue 1.

Comparison KPMG vs FORBES

From a very first observation we can deduce how, for both models, **the positions in the ranking are substantially identical**; the only difference lies in the inversion of the two Spanish teams since the American study positions the Barcelona behind Manchester United, while KPMG inserts Real Madrid. All the remaining positions, even if with different evaluations, more or less important, are the same for both reports.

If, when regarding positions, **the rankings are almost mirrored**, this is **only partially valid with reference to the economic values of all the clubs** taken into consideration. On average, the ratings of the first twelve clubs diverge 10,5%. However, if we exclude from this calculation the average of the differences between the last two associations (Paris Saint-Germain and Borussia Dortmund), **the difference in value of the two methods** decreases to a more **acceptable 6,7%.** But where does this discrepancy come from? In light of all these considerations made, it can be said that **the discrepancy is animated by the different algorithms underlying the two evaluation models**, as well as from the **weight of the various parameters** within the algorithms themselves and finally from a **possible diversity of availability of the same information**, recalling that Forbes takes into account the value deriving from match days, television rights, commercial revenues and the brand, while KPMG considers profitability, popularity, sporting potential, television rights and finally ownership of the stadium.

As for the valuations of Deloitte, which have taken care to estimate respectively the value of the brand and the revenues of the various clubs, it can be stated that they also substantially follow the positions outlined by Forbes and KPMG with the exception of Paris Saint-Germain, who are out of the top 10 in both classifications of overall value, while they are relatively in the seventh and sixth place.

CHAPETR 3 - Financial Economics of Football Teams

Indexes of Football Teams

The **main objective of accounting analysis** is to point out the **financial situation** and its **equilibrium** through the most **appropriate synthetic economic financial indexes** for the kind of business under **evaluation**. These indexes have to be directly valuable for the management and a key element for their effectiveness is their ability to **compare historical results of the same company and/or results coming from other companies** active within the same business sector.

There are a number of metrics that can be used to compare clubs, including **attendance**, **worldwide fan base**, **broadcast audience**, **revenues and profitability**. In the Money League Deloitte focus on a club's **ability to generate revenue from match day, broadcast rights, commercial sources** (including sponsorship, merchandising, stadium management) **and extraordinary result from buying and selling football players** (indicated as components of extraordinary management). Over and above plain vanilla analysis and comparison among soccer companies of revenues breakdown, a first useful business **sector index is the ratio between total collection per match day income and the number of attendances**. It gives an immediate idea of attractiveness of the team and does not require a particular accounting reclassification and may be easily provided by the companies for an unique comparison.

The **indexes point the incidence of costs on revenues** and the **breakdown of costs themselves**, and are able to identify the **level of efficiency of the organization**. The **main principal costs typical of the management of soccer companies**, that not include real estate business itself, are related to the football players and staff both in terms of **salary** as is the **amortization of intangible rights of use** of professional athletic performance (players' registration rights or right to use).

It's necessary make clear that there exist different ways to account for the two kind of costs related to sports performances of the players and, independently from what has been adopted from the companies, a **careful reclassification is indispensable for a fair comparison among clubs**.

Whilst for the salaries there are not doubt a question of timing for cost recognition, there is not agreement on multi-year costs linked to right of use.

For the intangible item pertaining the right of use it is possible to record the transaction in Profit and Losses with a full, clear, prudent and objective adherence to the cash flow approach as well as being able to capitalize these costs and differentiate through amortization in Profit and Losses the weight of extraordinary transactions. Applying the second methodology it would be possible record the value of the team in the balance sheet and in the accompanying notes. It gives a fair representation of the team value through periodical adjustments (increase for appreciation /decrease for depreciation and amortization) that, according IAS principles (16,19,32), will have an impact in both the Profit and Loss and in the Balance Sheet. Differently from the first, according this second methodology of fair value representation, the Return on Equity ratio would be impacted by higher return and higher asset values with an effective market return also avoiding volatility of the index otherwise linked at the period of the transaction.

Before we begin the analysis of the main economic/financial indexes it is important to point out that **other reclassification matters may arise and have to be rendered uniformed for comparison purpose** (from the treating of training field/stadia/buildings amortization criteria and provisions for risks).

Soccer clubs' Income Statement and economic indexes

A	**Revenues – Sales**		
A1	Matchday	***All-in cash ratio :***	***(A1+A2+A3+F1) / (B1+B2+ F2)***
A2	Broadcast rights		
A3	Commercial sources		
B	**Costs**		
B1	Staff salaries		
B2	Other cost including real estate		
C	**GROSS RESULT – Ebitda**	***Profit ratio (1/C) :***	***(A1+A2+A3) / (B1+B2)***
C/A	*Return on Sales*		
D	Depreciation & Amortization	***Investments' elasticity :***	***D / (B1 + D)***
E	**Ebit (Earning before interests and taxes)**		
E/A	*EBIT MARGIN*		
F	**Extraordinary net results**	***Ext/Ord ratios :***	***F / A or F / C***
F1	Gain on players' disposals		
F2	Loss on players' disposals		
G	**Total Operating Result**	***All-in profit ratio :***	***(A1+A2+A3+F1) / (B1+B2+D+F2)***
H	**Interest**		
I	**EBT (Earning before taxes)**		
L	**Taxes**		
M	**NET RESULTS – Profit/Loss**	**NET RESULTS ratios :**	M / A or M / C or M / E or M / I

The **first economic index** to study and understand during a financial analysis, **independently from the business sector**, is the **Gross Result of ordinary activities** (excluding any atypical result from extraordinary activities as the transaction on intangible right of use) **on Revenues form typical activities**. The return on sales (also called Ebitda margin) gives an **immediate perception of acompany's ability to make a profit form the investments** spent, **while including in the costs also the amortization** it is possible to obtain the **net result (Ebit) on sales (ebit margin)** as an **indicator of company profitability net of the cost of its investments**.

Among those economic indexes useful for a full understanding of financial equilibrium there are those that show **relations among specific positive and negative economic items**. Starting from typical operation analysis it' common to **compare** at first all **revenues including collection from trading** (match days collection + commercial sources + broadcast rights + capital gain on trading) **with all costs** (salaries + amortization + capital losses on trading). A second view of the same index may be obtained **removing from the index those items pertaining to extraordinary economic** components, or directly measuring the incidence of the extraordinary components in respect of an ordinary one, **comparing capital gain/losses on trading with ordinary revenues** (match days collection + commercial sources + broadcast rights), **or comparing capital gain/losses on trading with gross margin (Ebitda)**.

Another useful index of operating costs able to measure the **elasticity of investments is the comparison of amortization of right of use with total costs**. In in this comparison **if we decide to add also the costs for salaries** then it becomes possible understand the **incidence of** the overall cost afferent the **human resources on total costs**.

Financial Statements as information and measure instrument

The introduction of the regulation of the Financial Fair Play by the UEFA and the scarcity of the available resources will force a rethinking of management dynamics and will push management to planning for interventions that will not exceed the real availability of financial resources. Inserted in this new frame of reference, the financial statement document completes, for professional football clubs, its transmutation from an element of pure and correct information to a fundamental component in the business strategy.

In short, from the balance sheet, they will not only have to draw numbers and indices, but mainly strategic directions. Investments, in particular, will have to be oriented mainly towards the consolidation of a club's assets. By carefully evaluating the balance between revenues and costs, it will be possible to operate according to sustainable sports programs in the short and medium term. These and other key factors can be developed only if the management will have an agile tool available in the time and immediate readability for all its users. The latter will not only be the third parties interested in controlling an economic and financial solidity aimed at maintaining the commitments (suppliers, other clubs, customers or fans), but also the governing institutions in charge of verifying regular participation in competitions.

The balance, no longer a simple best practice but a real regulatory obligation, will be a sort navigation tool to allow management to develop projects and programs (sports and financial) consistent with the economic balance.

The same introduction of the Financial Fair Play will require an interpretative and adaptive effort, both by the UEFA and the National Federations, in order to allow a uniform application within the 52 countries involved. In fact, in the accounting and, above all, tax matters, the UEFA area is characterized by a very high heterogeneity of the regulations in force and if this evident application discrepancy is not exceeded, any attempt at reform risks giving way to an extreme and prolonged conflict between the different legal jurisdictions involved.

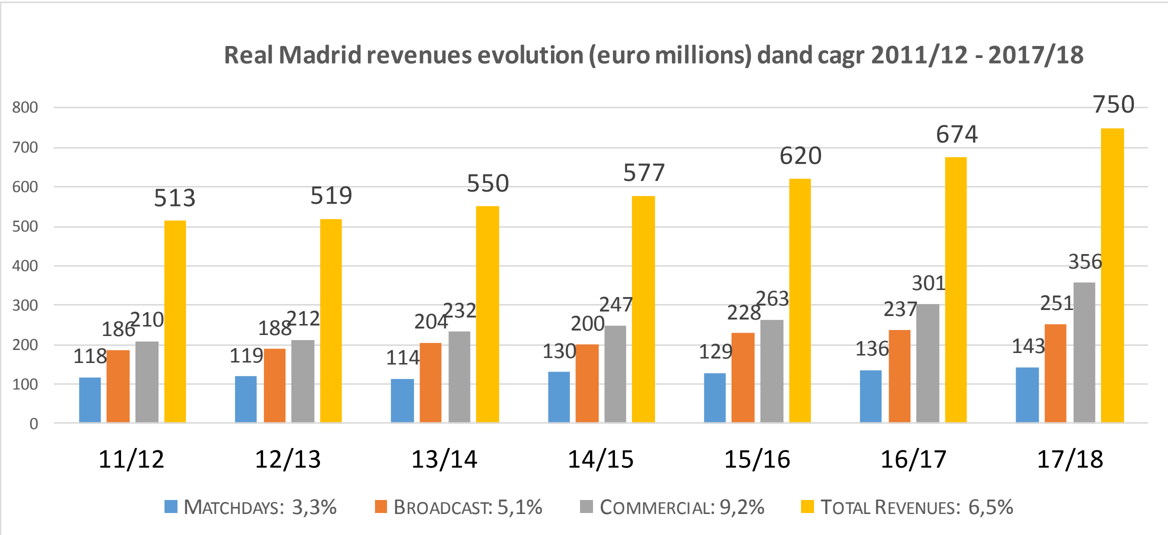
Case study: Real Madrid, still the team to beat?

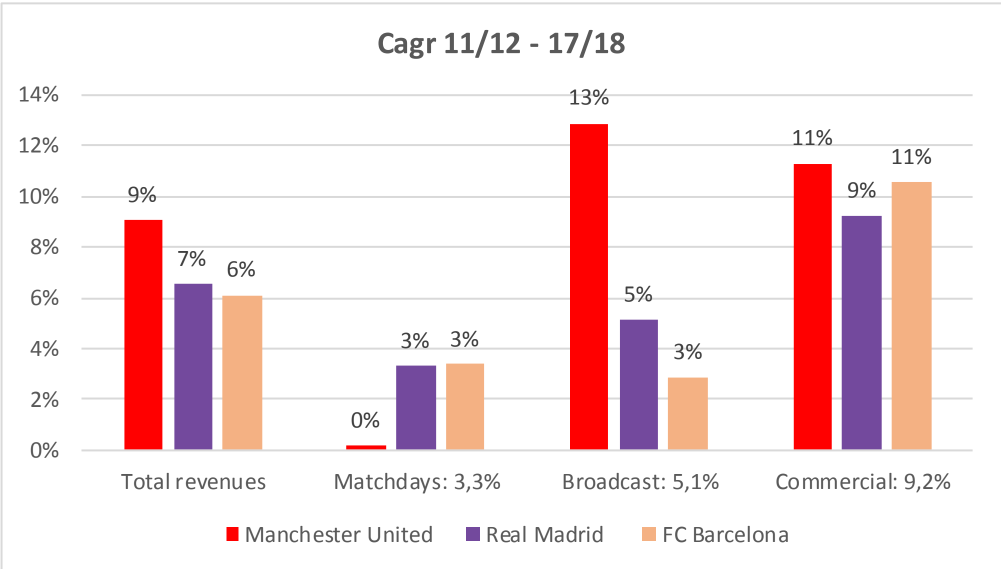
To conclude the work carried out attempts to identify a prospective trend and it was decided to carry out the analysis of the benchmark club as excellence (Real Madrid), which more than any other sports team results with excellent economic-financial performance that allows it to occupy from the very first years the top ranking position drawn up every year by Deloitte regarding clubs by turnover.

For Real Madrid, considered by Forbes as a sports club with a top and stable rating, the Merengue brand is equal to over 3 billion dollars, a technical analysis was carried out aimed at deepening its structure of the income statement with attention to the net financial indebtedness ratios and the break-even result. In addition, some efficiency indicators have been explained and analysed for the

Galatticos, including the total solvency index, the indebtedness index, the net financial position and the cost of employees.

Operating revenues, excluding the gains on the sale of players, amounted in 17/18 to € 750 million, also for preceding year, placing itself first in the world since 2011/12 and constantly above the threshold of € 500 million, representing the amount of sector revenues as being the highest in the world.





The sources of revenue (match days, television rights and marketing) have historically been fairly distributed (30% of revenues from ticketing and membership fees, 30% from television rights and 40% from trade revenues which have increased significantly compared with the past). The average annual growth rate is 10% and the diversification of sources confers economic stability, mitigating the impact of any fluctuations caused by sporting results or other cyclical causes.

The cost of personnel amounts to approximately 300 million euros and, if compared to operating income excluding capital gains, determines a value of 50% of the indicator used internationally to measure the operating efficiency of football clubs. The maximum level recommended by the European Club Association is 70%, thus we can appreciate how the Madrid club is considered to be of excellence within the sector. It therefore follows that Real Madrid can afford a squad of football players with a high level of income because its economic management generates an equally high turnover. EBITDA, is the gross result before the calculation of depreciation and interest and taxes, again with the exclusion of the minus / capital gains, amounts to approximately 250 millions euros, while the pre-tax result amounts to approximately 150 millions euros and a lows it to comply with the set breakeven result parameters according to which the cumulative sum of the last three years cannot be lower than a certain defined year-on-year threshold (also negative as cumulative loss). Basically according to the financial fair play the important thing is that you do not spend more than your receipts, but in the case of the Real Madrid Polis port which also includes other sports with a negative result, the breakeven of the football sector must also offset the results of the other disciplines sports.

The Debt / Ebitda ratio is one of the most used indicators of the credit market as it provides an approximate idea of how many years is necessary for the company to be able to repay its net financial debt. Obviously, it is an indicator whose sustainability is closely linked to the predictability and certainty over time of the operating financial flows that the company is able to produce and markets with high uncertainty will allow debt with a low multiples while others are more stable and will allow higher multiples. The football sector has multiples ranging from 2 to 3 and Real Madrid underlines its financial strength with a multiple of only 0,5. That proves that, if managed professionally, the football sector is highly remunerative and does is one which doesn’t require the use of huge amounts of capital (and indebtedness) if over the years one is able to generate cash flows capable of repaying and remunerating the capital invested.

The debt must be sustainable and the same financial fair play requires that there are no problems related to expired debts towards other clubs, tax authorities and their employees (with the exception of those for bonuses not yet achieved).

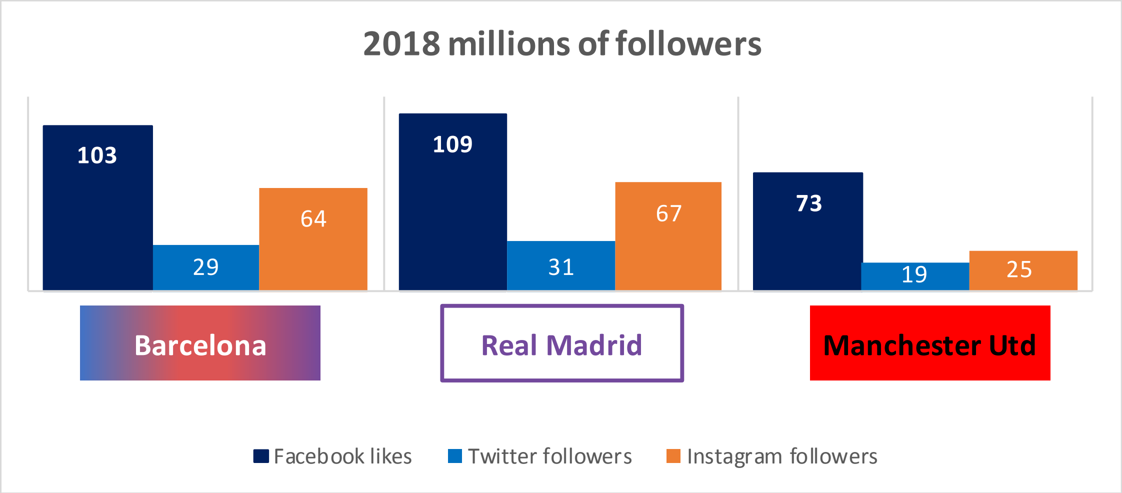
While the debt ratio equal to the ratio between shareholders' equity and total liabilities is equal to 0.67 which implies 40/60 covered loans between equity and total indebtedness, the D / E ratio between net financial indebtedness and net equity for Real Madrid it is equal to about 0,25 (net financial indebtedness equal to a quarter of the means) which means that the resources (PN / CIN equal to about 80%) fund a large part of the net invested capital.

The equity, which according to the financial fair play must be non-negative and growing, amounts to over 400 million euro and therefore the total assets, are estimated to be more than one billion euro.

The solvency ratio is equal to the ratio between total assets and total debts and is 1,7 million for Real Madrid, which means that the Merengues are solvent since their asset value can fully cover (if = 1) and over (> 1) of their total debt.

Specifically, we note that the component of the most risky assets relating to players' registration rights, if compared to shareholders' equity, results less than 90%, proving that even an unlikely complete devaluation of the entire player park would not jeopardize the ability of Real Madrid to repay its debts and partially absorb the equity. Finally, it is worth noting in terms of the evolution of investments that the Madrid club has invested over 300 million euros in the last decade on its stadium and the modernization of the facilities to improve its functionality and consequently creating a very significant economic return. Financially everything seems perfect, but is it enough?

The rights accrued with the victory of the 2018 Champions League that will be brought in the next few years will certainly return to compete for the Deloitte Money League Real for next season but at the same time the sale of CR7 after 8 years of his role in the growth of Real, despite being an extraordinary income activity that does not directly affect the revenues considered by the Deloitte Money League, would probably have a catastrophic effect not only on followers, but also on merchandising and television rights that will make the results of the ranking in the next few years more unpredictable, giving a boost to Juventus.



However, this change will need to consolidate things with careful management capable of strengthening the link between sporting and economic results of well-run football clubs. In short, financial **fair play will benefit not only the big five but probably also the Deloitte Money League competition in the coming years**.

CHAPTER 4 - IAS 39 and IFRS 9

SCOPE OF APPLICATION of IAS 39

The purpose of this principle is to establish the criteria for the detection, evaluation and disclosure relating to financial instruments in the financial statements of companies.

Football teams have their assets value "limited" by international accounting standards, but using specific financial contracts there can be an assets value increase due to IAS 39 principle.

In fact, the **objective** of IAS 39 is to define the principles for the recognition and measurement of financial instruments.

A **financial instrument** is a contract resulting in a financial asset for one company and an equal financial liability (or equity instrument) for another company. While a financial asset may consist of: cash, contractual rights to receive cash or another financial asset from another enterprise, contractual rights to exchange financial instruments with another enterprise under potentially favourable conditions or equity instrument of another company, a **financial liability** is a contractual obligation to deliver cash or another financial asset to another enterprise or to exchange financial instruments with another enterprise under potentially unfavourable conditions.

Particular financial instruments are **derivatives**, that are characterized by:

(a) Value changes in relation to changes that occur on a predetermined "underlying", **as can be football players value**, that may either be interest rate, price of a security, price of a commodity, exchange rate in foreign currency, index of prices or rates, credit rating,

(b) Which requires no initial net investment or a minimum initial net investment relating to other types of contracts which have a similar reaction to changes in market conditions;

(c) Which will be settled in the future.

IAS 39 requires that **all financial assets and financial liabilities are recognized in the financial statements**. This also applies to all derivative contracts.

Initially a financial asset or financial liability is recognized for the first time in the financial statements as a cost, which is also equal to the fair value of the consideration given in exchange (in the case of an asset) or received (in the case of a liability).

Financial assets can be classified into four categories:

1) loans and receivables originated by the company and not held for negotiation;

2) investments held to maturity;

3) financial assets available for sale;

4) financial assets held for trading

Subsequently the financial instruments are valued at fair value or at amortized cost, according to the category in which they are classified

As with the other IAS, the scope is defined with an exclusion procedure. The following financial instruments are excluded from the application of this principle:

1) Equity investments in subsidiaries, associates and companies subject to joint control, to which IAS 27, 28 and 31 apply;

2) Rights and obligations deriving from leasing contracts, to which IAS 17 applies;

3) Assets and liabilities held by an employer and tied to a pension plan, at which IAS 19 applies;

4) Rights and obligations contained in insurance contracts.

However **IAS 39 does apply to in insurance derivatives incorporated contracts**; equity instruments, including warrants, options and other instruments, classified by the issuer among the components of equity. This means that IAS 39 must apply to these instruments:

1) Financial guarantee agreements, such as sureties, letters of credit, which provide for payments to be made in the event of default by the principal;

2) Contracts concerning price adjustments on the sale of companies; contracts that prescribe payments related to climatic, geological or other variables

IAS 39 applies, therefore, to the following financial instruments:

- Debt titles;
- Equity securities (except for investments in subsidiaries, associates and joint ventures);

- Other financial assets, such as investments and credits (whether they "originated" from the company or "acquired" from other companies);
- Financial liabilities (whether they "originated" from the company or "acquired" from other companies);
- Financial derivative instruments (be they derivative assets or liabilities);
- Derivative financial instruments ("embedded") in other financial instruments;
- Assets and liabilities used as hedging instruments.

Of particular importance is the fact that **IAS 39 provides for the recognition in the financial statements of all derivative instruments while today these instruments are often not recognized.**

The definitions contained in IAS 39 are indispensable for understanding the entire Standard. With regard to financial instruments, IAS 39 provides the following fundamental definitions:

Financial instrument is any contract that gives rise to a financial asset for a company and to a financial liability or a representative instrument for another company.

Financial activity is any activity that is; cash and cash equivalents, a contractual right to receive cash or another financial asset from another enterprise, a contractual right to exchange financial instruments with another enterprise on potentially favourable terms; or an equity instrument of another company.

Financial liability is any liability that is a contractual obligation to deliver cash or another financial asset to another enterprise, or to exchange financial instruments with another enterprise under potentially unfavourable conditions.

Equity instrument is any contract that represents a residual investment in the assets of a company net of all its liabilities, that is, an equity investment.

Financial derivative instrument is a financial instrument whose value changes in relation to changes that occur in a predetermined interest rate, price of a security, price of a commodity, exchange rate in foreign currency, index of prices or rates, credit rating or other variable (sometimes called "underlying"), which requires that there is no initial net investment or a minimum initial net investment relating to other types of contracts which have a similar reaction to changes in market conditions and which will be settled in the future.

For the application of the above definitions, another accounting principle (IAS 32) states that the term **"enterprise"**, that is is inclusive of natural persons, partnerships, legal entities and public bodies.

In IAS 39 the financial instruments defined above are classified into **four categories**, to which different valuation criteria apply. The definitions of the four categories below are therefore fundamental for fully understanding IAS 39.

The four categories identified are:

1. **Financial asset or liability held for trading**
   Financial instruments fall into this category when a financial asset was purchased or a financial liability was incurred primarily for the purpose of generating a profit from short-term price fluctuations or margin of profit of the operator.
   A financial asset must be classified as held to be negotiated if, regardless of the reason for which it was purchased, it is part of a portfolio for which there is evidence of recent and effective manifestation of a profit realization in the short term.
   According to IAS 39, derivative financial instruments (be they derivative assets or derivative liabilities) are always held to be traded unless they are designated and are effective as hedging instruments.
2. **Investments held to maturity**
   They are financial assets with fixed or determinable payments and fixed maturity that a company has the actual intention and ability to possess to maturity with the exception of the loans and credits originated by the company, which constitute a category apart from.
3. **Loans and receivables originating from the company:**
   They are financial assets that are created by the company by supplying money, goods or services directly to a debtor, with the exception of those that originated with the intention of being sold immediately or in the short term, which they must be classified as owned to be negotiated. Loans and receivables originating from the company are not included among the investments held to maturity but, rather, are classified separately according to the provisions of this Standard.
4. **Financial assets available for sale**
   Is a residual category that includes financial assets that are not:

   (a) Financial assets held for trading.

(b) Investments held to maturity, or

(c) Loans and receivables originating from the company.

Recognition and measurement of financial instruments

The definitions provided by IAS 39 are as follows:

Fair value: is the consideration at which an asset can be exchanged, or an extinguished liability, between aware and available parties, in a fair transaction.

Amortized cost of a financial asset or liability: is the value at which the financial asset or liability was valued upon initial recognition net of principal repayments, increased or decreased by the overall amortization of the differences between the initial value and that at maturity, and net of any write-down (made directly or through the use of a fund) following a lasting reduction in value or insolvency.

Effective interest criterion: it is a criterion for calculating amortization based on the use of the effective interest rate of a financial asset or liability. The effective interest rate is a rate **that accurately discounts the expected flow** of future cash payments up to the expiration date or the next date of recalculation of the price based on market value, so as to obtain the current net book value of the asset or financial liability. This calculation must include all fees and points paid or received between the parties involved in the contract. The effective interest rate is sometimes qualified as the level of return on maturity or on the next price recalculation date, and is the internal rate of return on the asset or liability in that year.

IAS 39 provides rules for the accounting of financial instruments used to hedge other assets and liabilities. Hedging for accounting purposes, it means designating one or more hedging instruments, so that their changes in fair value lead to offsetting, in whole or in part, of the change in fair value or the cash flows of a hedged item.

An hedged item is an asset, liability, irrevocable commitment, or expected future transaction that exposes the company to the risk of changes in fair value or future cash flows, and which for hedge accounting purposes, is designated as a hedge.

On the other hand **hedging instrument** is a designated derivative or (in limited circumstances) another financial asset or liability whose fair value or cash flows are expected to offset changes in fair value or cash flows of a designated covered element. According to the provisions of IAS 39, a non-derivative financial asset or liability can be designated as a hedging instrument for hedge accounting purposes only if it hedges the risk of changes in foreign exchange rates.

To value the efficiency of the hedging strategy we use the **Hedging effectiveness level** that is the level at which compensation for changes in fair value or cash flows attributable to a hedged risk is obtained from the hedging instrument.

DETECTION OF FINANCIALS

An enterprise must recognize a financial asset or liability in its balance sheet **when, and only when, the enterprise itself becomes a party to the contractual clauses of the instrument.** As a consequence of the foregoing, an enterprise recognizes all contractual rights and obligations arising from derivative contracts as assets or liabilities in its balance sheet.

DATE OF NEGOTIATION / DATE OF REGULATION

A "regular" purchase or sale 2 of financial assets must be recognized using:

accounting at the settlement date method. The method used must be **applied consistently for all purchases and sales** belonging to the same category (assets / liabilities held to be negotiated; investments held to maturity; loans and receivables originated by the company; financial assets available for sale).

The **negotiation dat**e is the date on which an enterprise undertakes to buy or sell an asset. Accounting on the trade date refer to the recognition of the asset that must be received and the liability that must be paid on the date of negotiation; or the elimination of an asset sold and the recognition of a credit towards the buyer for payment on the negotiation date.

The **settlement date** is the date on which an activity is delivered to the company or by a company, which implies the detection of an asset on the day it is transferred to an enterprise; the elimination of an activity on the day it is transferred from the company.

In the case of accounting at the settlement date, the company will record - in the period between the negotiation date and the settlement date - any change in the fair value of the asset.

ELIMINATION or TRANSFER

A company must reverse a financial asset when it loses control of the contractual rights represented by that asset. **Control is lost when the company exercises or collects the rights or benefits under the contract, the rights expire, or the company transfers these rights**.

The determination of the loss of control over a financial asset by an enterprise depends on the position of the enterprise or that of the assignee. **If the position of both companies indicates that the transferor has maintained control, they must not remove the asset from its balance sheet.**

When the assignee has the right to repurchase the asset at a predetermined price that could be different from the current value, or there are call options for the assignor and put options for the assignee with substantially similar exercise price, there is no loss of control. **The transferor generally loses control of the business when the transferee is able to obtain the benefits associated with the property by selling or pledging the business.**

SOME SPECIAL CASES

a) Partial transfer of a financial asset, that happens if one company transfers to another a part of a financial asset, while retaining ownership of the rest, then the book value of the financial asset must be divided between the asset sold and that maintained based on their current value relating to the date of the sale.

b) Sale of assets associated with new financial assets or liabilities occurs if a company transfers a financial asset and simultaneously creates a new financial asset or assumes a liability, it will have to account for the current value of the new asset or liability and will also have to account for the profit o the loss deriving from the transaction as the difference between the consideration and the current value of the asset sold plus the current value of the new liability assumed less the current value of the new asset acquired more or less any value adjustments that had previously been accounted for in equity.

c) A company will **eliminate a financial liability** (or part of it) from its balance sheet only when it is extinguished, that is, after the fulfilment, cancellation or expiry of the contractual obligation.

The replacement between an existing debtor and his lender of debt instruments under substantially different conditions is an extinction of the old debt, which should result in the elimination of that debt and the accounting of the new one. Even a substantial change in the conditions of an existing debt instrument should be accounted for as an extinction of the old debt.

If the replacement or modification of the terms of a debt instrument entails the payment of **commissions or other costs, these must be accounted for as part of the profits or losses on the payments.** If the contractual changes are not accounted for as debt settlement, the costs and commissions are adjustments to the book value of the liabilities and must be amortized over the remaining term of the loan.

The contractual conditions are substantially different if the present value of the cash flows envisaged by the new contract, including the costs paid and net of any commissions collected, differs by at least 10% from the present value of the residual cash flows envisaged by the contract native to.

The difference between the amount paid for the elimination or transfer of a liability and its book value must be included in the result of the year. Partial elimination of a liability or elimination associated with new financial assets or liabilities The elimination of partial liabilities or associated with the creation of new assets or liabilities is accounted for as the corresponding transactions connected with the total or partial elimination of assets.

EVALUATION

INITIAL ASSESSMENT OF ACTIVITIES AND FINANCIAL LIABILITIES

When a financial asset or financial liability **is recognized for the first time**, an enterprise must evaluate it at cost, or at the fair value of the consideration given in exchange (in the case of an asset) or received (in the case of a liability). Transaction costs are included in the initial recognition of all financial assets and liabilities.

The fair value of the consideration given or received can normally be determined by referring to the cost of the transaction itself or to other market values. If these market values cannot be reliably determined, the fair value of the consideration is **estimated as the sum of all future cash payments or collections, discounted,** if the effect of discounting is significant, using the prevailing interest rates in the market for a similar instrument (similar in currency, in contractual terms, in the type of interest rate and other factors) of an issuer with a similar credit rating.

SUBSEQUENT ASSESSMENT OF FINANCIAL ACTIVITIES

For the purpose of evaluating a financial asset subsequent to initial recognition, this Standard classifies financial assets into different categories:

1) Loans and credits originated by the company and not held for negotiation; investments held to maturity;

2) Financial assets available for sale;

3) Financial assets held for trading.

After initial recognition, an enterprise must evaluate financial assets, including derivatives that constitute financial assets, at their fair value, without any deduction for the transaction costs that may be incurred in the sale or other disposal, except for loans and credits originated by the company and not held for negotiation, investments held to maturity, any financial asset that does not have a market price listed on an active market and whose fair value cannot be reliably determined.

These financial assets must be valued on the basis of the amortized cost criterion. However, fair value is a more appropriate method of measuring the amortized cost for most financial assets. To fully understand the **fair value** principle we also have to dig in the **amortized cost** and the **problems connected with holding the investments until maturity**.

THE FAIR VALUE

The fair value can be reliably determined for most of the financial assets **classified as available for sale or held for trading**.

The fair value of a financial instrument can be measured reliably if and only if, the variability in the range of reasonable fair value estimates is not wide for this instrument, if the probabilities of the various estimates within the range can be reasonably and is valued and used in the estimate of fair value.

The situations in which the fair value can be measured reliably include a financial instrument for which a public price quotation exists in an active publicly accessible securities market for that instrument, or a representative debt instrument that has been rated by an independent rating agency and whose cash flows can be reasonably estimated, or even a financial instrument for which there is an appropriate valuation model and for which the input information in this model can be reliably valued since the data come from active markets.

The fair value of a financial asset or financial liability can be determined using one of the many generally accepted methods. The valuation techniques must incorporate the assumptions that would be formulated in the fair value estimates by those who participate in the market, including the assumptions made regarding advance payment rates, presumed credit loss rates and interest and discount rates.

Underlying the definition of fair value there is the presumption that the company is in operation and that it has no intention or need to liquidate, significantly reduce the scope of its activities or undertake an operation under unfavourable conditions.

The existence of public price quotations in an active market is usually the best indication of fair value. The most appropriate price quoted on the market for an **asset held or a liability that must be issued is the current bid price (bid)** while for an **asset that must be purchased or for a liability held is the current offer or the asking price (offer).** When bid and offer prices are not available, the price of the most recent transaction can provide an indication of the current fair value, assuming that there have been no significant changes in the economic circumstances between the date of the transaction and the date of allocation in balance.

If the market for a financial instrument is not an active market, public price quotes may need to be adjusted to arrive at a reliable fair value measurement. If market movements are not frequent, the market is not very stable or only small volumes are exchanged compared to the units in circulation of the financial instrument that must be valued, the listed market prices may not be indicative of the fair value of the instrument.

If there is no market price for a financial instrument as a whole but there are markets for the parts that make it up, the fair value is obtained on the basis of the relevant market prices. If there is no market for a particular financial instrument but there is a market for such a financial instrument, the fair value is obtained on the basis of the market price of the similar financial instrument.

There are many situations in addition to those listed in the previous points where it is likely that the variability in the range of reasonable fair value estimates is not significant. Normally, the fair value of a financial asset that a company has purchased can be estimated by third parties. It is not it is likely that a company purchases a financial instrument for which it does not believe it can obtain a reliable fair value measurement after the purchase.

Gains and losses deriving from fair value measurement

A recognized gain or loss resulting from a change in the fair value of a financial asset or financial liability that is not part of a hedging relationship must be recognized in the financial statements either a profit or loss on a financial asset or financial liability held for trading must be included in the income statement of the year in which it occurred, or an income or expense on a financial asset available for sale must be included in the income statement of the year in which it occurs and recognized directly in equity, through the statement of changes in equity.

If an enterprise recognizes the purchases of financial assets using the accounting on the settlement date, any change in the fair value of the asset that must be received in the period between the negotiation date and the settlement date is not recognized for the assets entered on the cost or at amortized cost. For assets revalued at fair value, however, the change in fair value must be recognized in the income statement or in equity, depending on how it is most appropriate.

THE AMORTIZED COST

Those financial assets that are excluded from fair value measurement and have a predetermined maturity must be measured at amortized cost using the effective interest rate criterion.

Those that do not have a fixed deadline must be valued at cost. All financial assets are subject to review to check whether there has been a lasting loss in value.

Some examples might be, short-term loans without a pre-established interest rate are normally valued at their original value unless the effect of imputing the interest is significant; or loans and credit originated by a firm cannot be negotiated are valued at their amortized cost regardless of the firm's intention to own them until their maturity.

For variable rate financial instruments, the periodic estimate of the financial flows that can be determined to reflect the changes in interest rates on the market changes the effective return of a monetary financial asset. These changes in cash flows are recognized over the residual duration of the asset, or until the next price revision date if the asset is subject to price change (reprising). In the case of a variable rate financial asset initially recognized at a value equivalent to the repayable principal at maturity, re-estimating future interest on payments normally has no significant effect on the book value of the asset.

INVESTMENTS HELD TO MATURITY

An enterprise does not have the actual intention of owning an investment in an asset to maturity financial with a fixed maturity if one of the following conditions is present:

1) The enterprise intends to own the financial asset for an undefined period only;

2) The company is ready to sell the financial asset (except in the event of a non-recurring situation that could not reasonably have been foreseen by the company) following changes in interest rates or market risks, liquidity needs, changes in the availability and return of alternative investments, changes in sources and terms of financing or changes in risk on foreign currency;

3) The issuer has the right to extinguish the financial asset at an amount significantly lower than its amortized cost.

An enterprise must not classify any financial asset as held to maturity if the enterprise has sold, transferred or exercised a sale option during the current financial year or the previous two for an amount which is not exactly insignificant. Held to maturity prior to maturity (not insignificant in relation to the overall portfolio held to maturity), except if **sales fairly close to the expiration or exercise date** of any call option sold, so that changes in market interest rates did not have a significant effect on the fair value of the financial asset; or sales after the company has already collected substantially all the original capital of the financial asset through ordinary scheduled or advance payments; or sales due to an event isolated beyond the control of the company and which is non-recurring and may not have been reasonably foreseen in advance by the company.

On the contrary, most equity securities cannot be considered an investment held to maturity or because it does not have a limited life span (such as, for example, ordinary shares) or because the amounts that the owner can receive may vary in a non-predetermined method (such as, for example, options, warrants and rights on shares).

An enterprise is not in a position to possess an investment in a financial asset with a fixed maturity to maturity if has no financial resources available to continue to support the investment to maturity, and is subject to legal, supervisory or other constraints that prevent it from holding the financial asset to maturity.

A company must assess its intention and ability to own its investments until maturity at each balance sheet date and classified as held to maturity until maturity not only when such financial assets are purchased but also at each balance sheet date.

If, following a change in intention or capacity, it is no longer appropriate to record an investment held to maturity at amortized cost, this must be re-evaluated at fair value, or if it becomes possible to reliably measure a financial asset, the asset must be measured at fair value.

SUBSEQUENT ASSESSMENT OF FINANCIAL LIABILITIES

After the initial evaluation, a company must evaluate all financial liabilities depending on the asset type.

Derivative liabilities which are connected to an unlisted equity instrument and which must be settled with the delivery of the same instrument whose fair value cannot be reliably measured at cost and liabilities held for trading and derivatives which are liabilities valued at fair value.

On the other hand the financial liabilities which are designated as hedged items valued on the basis of the provisions on hedge accounting contained in the paragraphs relating to the hedges of this standard, while all remaining liabilities at amortized cost.

DURABLE LOSSES OF VALUE AND IRRECUPERABILITY OF ACTIVITIES

A financial asset has suffered a lasting loss of value if its book value is greater than its presumed realizable value. An enterprise must determine at each balance sheet date whether there is any objective indication that a financial asset or group of assets may have suffered a lasting reduction in value.

The objective evidence that a financial asset or group of assets has suffered a permanent loss of value or is unrecoverable, includes information that reaches the attention of the owner of the asset about the following points:

a) Significant financial difficulties of the issuer;

b) An effective breach of the contract, such as a default or failure to pay interest or principal;

c) The extension from the lender to the debtor for economic or legal reasons relating to the financial difficulty of the beneficiary, of a concession that the lender would not otherwise have considered;

d) A high probability of bankruptcy or other financial reorganization of the issuer;

e) The recognition of a lasting loss of value on that activity that occurred in a previous year;

f) The disappearance of an active market for that financial asset due to financial difficulties;

g) A historical trend of collections of receivables from customers indicative of the fact that the full nominal amount of a portfolio of such receivables will not be recovered.

The disappearance of an active market, due to the fact that a company's securities are no longer publicly traded, is not evidence of a lasting loss of value. A downgrade in the creditworthiness of a company does not in itself constitute evidence of a lasting loss of value, although this may be indicative of a lasting loss of value if considered together with other available knowledge.

If it is **probable that an enterprise will not be able to recover all the amounts due** (principal and interest) according to the contractual agreements for financing, credit or investments held to maturity recorded at amortized cost, **it means that there has been a lasting loss of value or a bad debt loss.** The amount of the loss is represented by the difference between the book value of the asset and the present value of the expected future cash flows discounted at the original effective interest rate of the financial instrument (realizable value). Cash flows relating to short-term loans are generally not discounted. The book value of the asset must be reduced to its presumed realization value directly or through the use of a fund. The amount of the loss must be recognized in the income statement for the year.

The **permanent loss** of value and the **loss due to non-collection** can be individually assessed and recognized for financial assets that are individually significant. The permanent loss of value and the loss due to non-collection can be assessed and recognized on the basis of a portfolio of similar financial assets.

A **lasting loss** in value of a financial asset recognized at amortized cost is measured using the original effective interest rate of the financial instrument since discounting at the current market interest rate would, in effect, require the fair value measurement for those assets that this Standard would otherwise evaluate at amortized cost. If a **loan, credit or investment held to maturity has a variable interest rate, the discounting rate for assessing the realizable value is the effective current interest rate determined according to the contract.** As a substitute calculation of fair value, a creditor can evaluate the permanent loss of value based on the fair value of an instrument using a detectable market price. If an asset is pledged as collateral and is likely to be foreclosed, then the holder assesses the permanent loss of value based on the fair value of the collateral.

If, **in a subsequent financial year**, the amount of the permanent loss of value or a loss on credits decreases and the decrease can be objectively connected to an event that occurred after the devaluation (such as an improvement in the financial solvency of the debtor), it must the original value of the financial asset can be restored directly or through adjustment of the fund. The **restoration of value** must not determine, on the date on which the original value of the financial asset is restored, a book value of the financial asset that exceeds the amortized cost that would have occurred if the loss of value had not been recognized. **The transfer amount must be included in the income statement for the year.**

The carrying amount of each financial asset that is not recognized at fair value since the fair value cannot be measured reliably must be re-examined in order to have an indication of lasting loss of value at each balance sheet date based on an analysis of the flows expected net financial income. If there is any indication that a **lasting loss of value has occurred**, the amount of the loss of value of this asset is represented by the difference between the book value and the present value of the expected future cash flows and discounted at the current interest rate market value of a similar financial asset (realization value).

Once the value of a financial asset has been reduced to its presumed realizable value, interest income is subsequently recognized based on the interest rate used to discount future cash flows in determining its realizable value. In addition, after the initial recognition of a lasting loss of value, the company will review whether this activity has suffered further lasting losses of value at each subsequent date of preparation of the financial statements.

If a loss relating to a financial asset recognized at fair value **(the realizable value is lower than its original purchase cost)** has been recognized directly in equity and there is objective evidence that the asset has suffered a lasting loss in value, the overall net loss that had been recognized directly in equity must be made to pass from equity to the income statement for the year even if the financial asset has not been reversed.

The amount of the loss that must be made to pass from the shareholders' equity to the income statement is represented by the difference between its purchase cost and the current fair value or the realizable value, less any lasting loss of value on the asset previously recognized at income statement. The realizable value of a representative debt instrument revalued at fair value is represented by **the present value of expected future cash flows discounted at the current market interest rate for a similar financial asset**.

If, in a subsequent financial year, the fair value or realizable value of the financial asset recognized at fair value increases and the increase can be objectively related to an event that occurs after the loss had been recognized in the income statement, the loss must be reversed, with the reversal amount included in the income statement for the year.

COVERAGE OPERATIONS

Coverage operations can be of three different types:

Fair value hedge, a hedge of the exposure to changes in fair value of a recognized asset or liability or an identified portion of that asset or liability which is attributable to a particular risk and which will affect the net profit of balance;

Cash flow hedge, a hedge of the exposure to the variability of cash flows that is attributable to a particular risk associated with a recognized asset or liability (such as all or only some variable future interest payments on a debt) o to a planned transaction (such as a planned purchase or sale in advance) and which will be charged to the income statement. A hedge of an unrecorded corporate commitment to buy or sell an asset at a fixed price in the company's reporting currency is accounted for as a cash flow hedge even if it is exposed to fair value risk.

Hedging of a net investment in a foreign entity, in which the definition of IAS 21 applies, which includes the associated entities, subsidiaries, joint ventures or permanent foreign organizations whose activities are not an integral part of those of the company which draws up the budget.

ACCOUNTING OF HEDGING REPORTS

Accounting for hedges depends primarily on whether or not the hedge meets the requirements to qualify as a special hedge.In this Standard, a hedging relationship qualifies as special hedge accounting if, and only if, all of the following conditions are met:

(1) At the beginning of the hedge there is formal documentation of the hedging relationship and corporate risk management objectives and hedging strategy. This documentation must include the identification of the hedging instrument, the element or related transaction hedged, the nature of the risk hedged and how the company will assess the effectiveness of the hedging instrument in compensating the exposure to changes in fair value the hedged item or cash flows from the hedged transaction attributable to the hedged risk;

(2) It is assumed that the hedge is highly effective in obtaining compensation for changes in the fair value or in the cash flows attributable to the hedged risk, consistent with the corporate risk management strategy originally adopted for that particular hedging relationship;

(3) For cash flow hedges, an expected transaction that is hedged must be highly probable and must present an exposure to changes in cash flows which could ultimately affect the economic result of the year;

(4) The effectiveness of the hedge can be measured reliably, that is, the fair value or cash flows of the hedged item and the fair value of the hedging instrument can be measured reliably; is

(5) The hedge was assessed on the basis of a recurring criterion and is considered to be highly effective throughout the year.

Hedging relationships that satisfy the above conditions to be classified as special hedges are accounted for with different rules depending on the type of relationship used.

When using **fair value hedging**, the profit or loss deriving from the valuation of the hedging instrument at fair value must be recognized immediately in the income statement; is the gain or loss on the hedged item attributable to the hedged risk must adjust the carrying amount of the hedged item and must be recognized immediately in the income statement. This provision also applies if the hedged item is measured at fair value with the changes in fair value recognized directly in equity. It also applies if the hedged item is otherwise valued at cost.

On the other hand when **using cash flow coverage** the portion of the gain or loss on the hedging instrument that is qualified as one effective hedging must be recognized directly in equity using the prospectus movements in equity; is the ineffective portion must be entered eithe immediately in the income statement if the hedging instrument is a derivative; or in accordance with the general fair value valuation rules in the limited circumstances in which the hedging instrument is not a derivative.

If a **hedge does not meet the requirements for special hedge accounting** because it cannot satisfy the above conditions, the profits and losses deriving from changes in the fair value of a hedged item that is valued at fair value that occurred after the initial valuation they are recorded according to the general rules (in the income statement or directly in equity).

The **fair value adjustments** of a hedging instrument which is a derivative should be **recognized in the income statement.**

IFRS 9

It is important to mention also IFRS 9 the second most important accounting standard for corporate treasurers because addresses how to account for financial instruments, or how they are measured on an on-going basis, evolving and adjusting IAS 39. In fact **IFRS 9** is the more recent Standard released on 24 July 2014 that will replace most of the guidance in IAS 39.

Treasurers should examine these standards closely to understand their implications on risk management, tax, internal controls and processes.

Under IAS 39, many preparers, auditors and users of financial statements had given feedback that the requirements for reporting financial instruments, analysed above, were **too complex and difficult to apply.**

There were too many exceptions in the application of IAS 39, and companies struggled to apply the Standard correctly and consistently. Therefore, after the Global Financial Crisis, with strong interest from the G20, the Financial Advisory Groups decided to accelerate the project for IFRS 9 to supersede IAS 39.

The new Standard, IFRS 9 is more **principles-based**, and therefore requires more judgment in its application. In contrast with IAS 39, it applies a **two-step approach** to classify all types of financial assets, which are either measured at **fair value or amortized cost**.

Under IFRS 9, the classification of financial assets are **dependent on the 'business model'** test and **'contractual cash flow'** test to determine whether they are measured at fair value or amortized cost.

CHAPETR 5 – Innovative liabilities & Mark to Market values

Kick-Bonds indexed to performance

As was shown by Forbes and KPMG, **soccer clubs are valued** based upon numerous **factors not limited to the enterprise values (equity plus debt) based on multiple sources of revenue**. Income is generated from broadcast agreements, premium seating and match day ticket sales, media, brand and product licensing, and merchandise and concessions sales. **The valuation of a team is also based on the value of its roster, including transfers fees** (both incoming and outgoing) and, on an individual basis, **length of contact** (for purposes of depreciation). Also taken into consideration is the **value of the club's stadium and facilities**.

Crucial is the identification of the **value of the team players that depends on performance**, **role, age as well as behaviour**. As seen in the accounting analysis there are different way to represent that in the balance sheet, but what is **difficult is to predict the value of the players for the future** because of the numerous variables involved in the analysis.

Of course **when the club performance is constantly at high levels the economic results will accordingly increase** in the long run. **If the performance is not satisfying may be difficult to earn revenue to devote to the debt service** and for this reason the **capital markets look with diffidence at financing football club** on unsecured basis. A **possible solution has been discovered by Kick-bonds that link in a positive relation the remuneration of the bond with the performance of the team**. In this way the **clubs can attract financial interest** from the market and can benefit of a sort of **natural hedge** because it commits themselves **to pay more only if the performance is good** and in the other scenario id may offer a low remuneration or nothing to the bondholders that lend on the performance of the club. In other terms, there is a **sharing of economic results between the club and the bondholders**, and for this reason this **instrument is closer to the concept of equity share that than a debt instrument** issued to pay more for riskier business and not for successful ones.

In this sense, they are **speculative instruments similar to listed shares**, exchanged on listed platforms (Kick offers) that allow financial supporters and traders to invest in buying and selling bonds **with a remuneration depending on the performance of the team and of its representative players**. It's a new financial instrument to support a club's target that **give the possibility to earn from clubs successes**, players' performance and their increase in valuation.

It's a bond with recourse on the club and generally linked to a young football player. For the future the goal is to make it non recourse and thus suited to help the observance of financial fair play imposed by Uefa. If at the moment it is more of a bet than a traditional investment, **for the future the evolution could evolve in a off balance sheet instrument** able to address financial fair play constrains.

The finance becomes close to fantasy football, so one yellow card on a Sunday match may decrease the bond value on a Monday morning as well as one goal or the player disposal with capital gain would increase the price. The platform Kickoffers, created in partnership with **Deloitte, has structured an algorithm in order to quantify the performance of the team and of the football player linked to remuneration of these kick-bonds**. In particular, the **remuneration will depend on the performance of the team in the league, results in national cups, goals/assists/penalties and value of disposals of representative football player associated with the bond issuance**. If the player's disposal would generate a gain in respect of initial evaluation coming from the Deloitte algorithm, an extra bonus will be paid to the investors. After subscription, the monitoring of the market will be essential for trading decision of buying/selling and take profit of volatility, and during vesting period the investors could earn interests matured during the period.

The first experience of Deloitte algorithm application has been the one of Italian soccer club Chievo which listed its financial bonds and not its shares as Juventus, Lazio and Roma. The minimum size has been 100 euro and maximum 5.000 euro, the tenor is of for five years and at maturity the bond will be reimbursed at its nominal value.

The yield is linked to the performance of the team in the national league and cup competition, while the football player performance would be the one of Valter Birsa at the club since 2014. The instrument will be listed at Malta stock exchange, at par with an expected – though not guaranteed - yield of 7-8%.

Merton model & Security Market Line applications

From an economic-financial modelling point of view, the Merton model identifies the **market value of a company (Equity Value, or Market Capitalization if listed) as a call option on the value of the assets (entire**

corporate value or Enterprise Value) with strike price equal to the value of the debt and with expiry the debt maturity. The present value of the Equity is expressed as the present value of the average probable difference between the value of the entire asset at the maturity of the debt and the value of the debt itself. Equity can have a positive present value even if the current value of the assets is lower than the current value of the debt because, at the maturity of the debt, the market value of the assets (Enterprise Value) can exceed, with a certain probability, the debt value that grows only with the interest rate.

The basic assumption of the methodology is that, if the asset company value should be lower than the debt value, the property would have no economic interest in continuing in the company activity and therefore the value would be at the minimum zero given it is not obliged to proceed with capital increases, unless there is a prospective value to be protected. If this difference were instead positive, the expected value of the payoff of this difference discounted according to a risk-neutral approach would be the value of the Equity itself. Basically, from this approach, it emerges just how **the current value of the company can be higher than the current difference between assets and liabilities since, with a certain probability, the assets will grow more than the debt capitalized for its contractual interest rate**.

It is an approach based on market evidence that leads to an evaluation of an equivalent company to that of discounting the company's future cash flows by means of a discount factor that takes into account of the uncertainty of these flows according to an adequate relationship between risk (variance of cash flows) and return (discount rate) which must be on the **Security Market Line**. If this value were contractually divisible into sub-sets of assets, the **diversification effect that allows to optimize the risk / return profile** by determining the optimal portfolios (which represent the efficient Markowitz frontier) must also be considered.

If the valuation of the Equity can be conceptualized as a call option on the underlying corporate value compared to a strike represented by the value of the debt at maturity, by extension the **value of the Equity can be divided into subsets represented specific categories of assets** with which to associate specific values: i) the most certain part using, instead, for the enhancement the approach of cash flows discounted at an interest rate that reflects the modest variance of these flows; ii) the most random (but also most market and therefore verifiable) part following the approach of the call options to evaluate their value.

Certainly the part with the most certain cash flows lends itself better than the random one to be able to sustain the debt in terms of repayable prospect but, if debt or mezzanine funds are identified in the market with appetite for individually transferable assets and with a market value that can be parameterized, the advantages in using this valuation method would be different. From the point of view of the company's debt capacity, access to the specialized, alternative or additional debt market to the banking and bond market could be obtained. Investors interested in the finance ability of even a single asset with a formula similar to that of the pledge (if not of the financial leasing) could be found in which the repayment is collateralized by the value of the asset at maturity of the debt and whose remuneration could be indexed the payoff at maturity equal to the difference between the value of the asset and the capitalized debt (kick bond).

From an Equity value point of view, **the future value of the more random assets could emerge immediately, despite the International Accounting Standards do not allow to be valued before their realization according to the logic of prudence (IAS 39), unless this derives from a market exchange contract of this value**.

For example, you could enter into forward contracts that fix the price of the asset at maturity (without predicting its sale) or even assign call options on certain assets that would collect premiums and reveal the unexpressed values of these assets as: i) the cash flow method cannot capture the specificity of these assets, necessarily having to reason for average values; ii) the international accounting standards do not allow their valuation in the financial statements until realization / marketing.

If soccer companies do not want to set forward prices or transfer the rights on the value of the assets with respect to a certain strike (which may even be higher than the value of the specific debt that you can allocate on these assets), they could also consider issuing shares privileged with special rules that factor the rights regarding the future sale of certain assets with respect to a specific strike assigned to them. In this last way, differently from the previous ones, the value linked to the optionality of these assets could not be reported in the financial statements (which would instead be very functional for the issue of new debt), but the value not expressed in specific terms could still be valorized of greater equity, the enhancement of which occurs through free trade between the parties.

Market Value trough Black & Scholes derivatives evaluation

The **market equity value is already, per se, an expression of the presence of an asset value not reflected in the financial statements** due to prudential constraints imposed by the IAS, however a more specific identification of this subset would have a positive effect on the capitalization suffering from penalizing weightings and discount for uncertainty.

The assets to be valued according to the optional approach are those with greater uncertainty and for which a linear motion can be distinguished, identifiable in parameterization algorithms that simply identify the trend of variation of the value over time, and in a stochastic motion that determines a future value independent of time and closely linked to parameters the current value and the variance of the probability distribution which is assumed to be of the normal type. **Through the Black and Scholes equation, the values of any derivative on these assets can be determined**, whether they are linear (forward or short selling) or optional (call / put long / short) or digital (all-or-nothing). In relation to the type of derivative to be valued, a specific formula will be determined which can be used to quantify the contract value that is assumed to be able to finalize.

Different derivatives have different economic functions. While a forward sale of the asset defines the future value of the asset itself, a **forward contract lends itself to transferring the difference between the future value of a given asset with respect to an identified strike allowing to monetize the difference,** and therefore implicitly to remain neutral with respect to the strike identified. A call option, on the other hand, represents the positive present value of the sale of the difference between the future value of the asset and a given strike and, if sold, call option allows you to collect the reward and set the forward value and, **if acquired, call option allows you to focus on the appreciation of a certain asset and to possibly guarantee the purchase of the asset in the event of appreciable appreciation**. The purchase of a **put option ensures that the value of a given asset is not lower than its strike at maturity** because with this instrument the right to be reimbursed is acquired for the negative difference between the value of the asset and the strike, at the same time it can allow profit to be made from valuations considered excessive for an asset that is estimated to depreciate.

The digital option allows instead to place bets on the future value of the asset which will involve the payment of an initial premium against the possible collection of another value in the event of a favorable event, or of nothing in the event of an opposite event. They can also be seen as a fixed contribution to a future purchase, which however does not completely neutralize the position.

Conditional to a labour law check on the transferability of the economic rights referred to above in terms of capital gain / loss of the underlying asset compared to an identified strike (which may or may not coincide with the value of the debt allocated to this asset), the possibility could be assessed to treat the players' park as a set of assets whose value is characterized in part by a linear trend and in part in stochastic random motion. This set could be divided by individual asset in order to facilitate its enhancement according to the specific linear growth algorithm already developed and through the random component to be identified.

The correlation between the different assets, distinguished by role and age, not being equal to one, would also allow o build a portfolio that optimizes the risk / return profile (less risk for the same return or greater return for the same risk), and therefore, it also optimizes the valorization function of these assets both from a point of view of greater debt capacity and to merge the unexpressed value of equity for the reasons mentioned above.

An analysis of the financial statements of the sector companies in fact shows that the debt is equal to the sum of the multi-year rights to the sports performances of the players, whose valuation is however conservatively conserved at the amortized purchase values, or if lower than the presumed ones I realize, without therefore being able to highlight the value of the pool of players who are not only functional to the company business, but that their own transfer and acquisition activity is increasingly part of the management characteristic of a football club. This has no longer been the case for extraordinary income but rather for ordinary activities whose correct valuation in the financial statements is prevented by the application of the IAS.

CHAPTER 6 - Black & Scholes Equation

Probability spaces, filtrations, processes and conditional expectation

Probability space and filtration

A probability space sums up the intuition we have about the uncertain (random) future behavior of some key phenomena of interest. It is in fact our mathematical model for the phenomena. Once the model has been chosen, we can compute all the relevant quantities, such as average values, dispersion around the average, and dependence structure among the various variables involved.

Formally, a probability space is a triplet (Ω,F,P) where each of Ω, F and P is selected by the modeler:

Ω is the set of possible outcomes. If we are modeling the price evolution of some football players, Ω will be a path space.

F is a collection of subsets (the events) of Ω, which are relevant to us and which we would like to measure. F is also referred to as 'the information'. This collection contains Ω and is required to be closed for countable unions and complement (the mathematicians say F is a sigma-algebra).

P is the probability, a normalized, non negative measure on the events. If A is an event, P(A) is our measure of the chances of A to happen. The properties of P are:

P : F → [0,1]

P(Ω) = 1 (normalization property)

P is countable additive: if (An)n is a sequence of disjoint events, then

$$\mathbb{P}(\sqcup_n A_n) = \sum_n \mathbb{P}(A_n)$$

From the properties above, we get

P(Ac) = 1 − P(A),

Where Ac denotes the complement Ω \ A of the event A, and that P(∅) = 0. This can be easily seen by writing Ω = A t Ac and applying the normalization and the additives properties. Also, if two events A,B are not disjoint, the probability of A ∪ B can be obtained as:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

In addition, the countable additivity implies the following **monotone convergence** properties of P:

$A_n \downarrow A \Rightarrow P(A_n) \downarrow P(A)$, namely if a sequence $(A_n)_n$ of events shrinks to A, then their probabilities decrease to the probability of the limit event A.

$A_n \uparrow A \Rightarrow P(A_n) \uparrow P(A)$, namely if a sequence $(A_n)_n$ of events increases to A, then their probabilities increase to the probability of the limit event A.

Example. Area on the unit square Q.

Definition . A real random variable on (Ω,F) is a function on Ω which takes values in R:

$$X : \Omega \to R$$

and is F-measurable, i.e. the counter-image of any half line $(-\infty,x]$ is an event:

$$\{X \leq x\} \in F \quad \text{for all } x \in R$$

It may happen that F is 'quite big' and for certain purposes we only need a subcollection of events. A collection G of events is a sub-sigma-algebra of F if it contains Ω and is closed for countable unions of events and complement. That is, a sub-sigma algebra is a collection of events which is a sigma-algebra in its turn. When a random variable X verifies the more stringent condition

$$\{X \leq x\} \in G \quad \text{for all } x \in R,$$

we say X is G-measurable.

Example The sigma-algebra generated by a rv X is

$$\sigma(X) := \sigma(\{X \leq x\} \mid x \in R)$$

and it is a sub-sigma algebra of F. It is clear that X is σ(X)-measurable, and σ(X) is in fact the smallest sigma-algebra on Ω for which X is measurable. Any event which can be written in terms of X.

$$\{a \leq X \leq b\}$$

belongs to σ(X).

Probability Theory would be just a branch of Analysis were it not for the concept of filtration. Suppose we are modelling the future evolution of a random quantity which changes with time.

When time passes, more information will be progressively revealed and the filtration in fact tracks the increase of information with time. Technically, a filtration is an increasing collection of sub-σ-algebras F_t of F, i.e. $F_{t1} \subseteq F_{t2}$ if $t1 < t2$. Here of course t is a time parameter, which may be discrete or continuous.

If t1 is fixed (say, t1 = 3 months from now) then F_{t1} contains all the information about what may happen in [0,t1], e.g. the knowledge of the paths up to time t1. If t2 = 6 months from now, then F_{t2} describes what may happen in [0,t2]. As any event happened by time t1 may be seen as an event happened before t2, F_{t2} is a bigger collection of events than F_{t1}.

Often, in Finance, a finite time horizon T is fixed. We may select as T the longest maturity of the derivatives in our portfolio, so we are not interested in what will go on after T. Also, we may assume $F_T = F$. We then have a filtered probability space $(\Omega,(F_t)_{t\leq T},P)$, which is also called a **stochastic basis**.

Stochastic Processes

Given a filtered space $(\Omega,(F_t)_{t\leq T},P)$, a real-valued stochastic process $S = (S(t))_t$ is simply a collection of real valued random variables, measurable functions from (Ω,F_T) to R (or to R^d for d-dimensional processes). An important question we can ask is: how does our process S behave as time flows? If we are still alive at time t from now, will we know S(t) or not?

If the answer is: yes, for every t the variable S(t) is known at time t (i.e. F_t-measurable), the process S is said to be adapted to the filtration, or simply adapted. In other words, S is adapted if the following two conditions hold for any fixed time t,

$$S(t) : \Omega \rightarrow R$$

for all fixed reals x, the set $\{S(t) \leq x\}$ belongs to F_t (= is known at time t).

A process S can be thought as a function of two variables, time t and outcome ω:

$$S = S(t)(\omega) = S(t,\omega)$$

The sections along time t and outcome ω are of different nature: when $t = t^*$ is fixed, the result is the random variable,

$$S(t^*)$$

the future stock price at time t^*. This is also called the $t*$-marginal of the process S.

When $\omega = \omega^*$ is fixed, the result is a function of time

$$S(\cdot,\omega^*) : [0,T] \to R$$

which is called a path of the process.

There may be some legitimate confusion at this point. We have said that Ω itself is often a path space. The paths ω are those of some underlying fundamental quantities, which contain all the uncertainty. The process S is going to be a deterministic function of this quantity.

In the Black-Scholes market model, we will see that the logreturns of the stock price process S are modeled as a Brownian motion, which is one of the most famous stochastic processes. The Brownian motion paths are continuous functions ω on [0,T], and therefore Ω in this case is the space of continuous paths on [0,T].

Distribution of a process

For any t we know the distribution of S(t) (the so-called t-marginal distribution). In fact, in the definition of adapted process, we can compute the Cumulative Distribution Function of S(t)

$$P(St \leq x)$$

Marginal distributions are not enough to reconstruct the distribution of the whole S. Suppose we are monitoring the process at two specific dates t1,t2. This may be the case for a path dependent option (discrete barrier, asian, etc). Suppose we also need the joint distribution of S(t1),S(t2). A joint distribution can never be recovered from the marginals without extra hypotheses (such as independence, specific correlation structure). But in the construction of the model: there are a filtration and an adapted process. We can thus compute the joint distribution

$$P(S(t1) \leq x1, S(t2) \leq x2)$$

as the sets {S(t1) ≤ x1} ∗ {S(t2) ≤ x2}, for any xi belong to Ft2 ∗ FT = F. Thus they are events, and events can be measured by P. In the same way, we know the distribution of any subfamily (finite or countable) of variables of the process:

$$[S(t1), S(t2), ...S(tn)]$$

Even though actual computation may be very difficult.

Expectation and expectation of transforms

E[X], also known as the mean of X, is a well known concept for a random variable X. It is the average of the outcomes X(ω), weighted according to the probability of happening. If X is discrete, E[X] = Pi xiP(X = xi); if X is a continuous random variable, with density pX, then P(x < X ≤ x + dx) = pX(x)dx and the expectation is Z:

$$E[X] = xpX(x)dx$$

Note that the distribution, and thus the expectation of a random variable, strongly depends upon P. A change in our probabilistic views results in a change of distribution and expectation, for all the random X on Ω.

Expectation is a linear operation, in that the expectation of a linear combination of random variables is the linear combination of the expectations:

$$E[aX + bY] = aE[X] + bE[Y]$$

A consequence of the linearity is that the joint distribution of X,Y is not needed to compute the expectation of a linear combination of X and Y . Recalling however that the knowledge of the joint distribution (e.g. knowledge of the joint density pX,Y (x,y) if (X,Y) are jointly continuous) is essential to compute quantities which involve both random variables. Examples of these quantities are the product E[XY], or probabilities of intersections of events generated by X and events generated by Y like P(a < X ≤ b,c < Y ≤ d).

In the rest of this paragraph, X is a continuous random variable and Y is a deterministic function of X, say

$$Y = g(X)$$

It is extremely important to be able to compute the expectation of Y given the distribution of X. Suppose in fact X is the price of a stock at time T and g is the payoff function of an option on this stock. Then, the price of the call Y = g(X) is a expectation.

Even if X has a density, in general it is not true that Y has a density.

In the financial interpretation Y is a digital option written on X (as could be a football player value).

If however g is invertible and differentiable, then Y has a density given by the formula:

$$p_Y(y) = p_X(g^{-1}(y))\frac{1}{|g'(g^{-1}(y))|}$$

We derive pY as above (since g is differentiable and invertible, it is strictly monotone either increasing or decreasing. So, either g0 > 0 or g0 < 0. Assume you are in one of the two cases, and note P(Y ≤ y) = P(g(X) ≤ y) = P(X ≤ g−1(y)) or P(Y ≤ y) = P(g(X) ≥ y) = P(X ≥ g−1(y)); then differentiate wrt y...)

The lognormal density

$$p_Y(y) = \frac{1}{y\sigma\sqrt{2\pi}}\exp\left(-\frac{1}{2\sigma^2}(\ln y - \mu)^2\right)$$

So, when the transform Y of X has a probability density, its expectation is

E[Y] = R y pY (y)dy which, when g is regular and invertible can be written as

$$\int y\, p_X(g^{-1}(y))\frac{1}{|g'(g^{-1}(y))|}dy$$

Equivalently, using the change of variable x = g−1(y) in the above integral:

E[Y] = E[g(X)] = g(x)pX(x)dx

in which we now weight the g- transformed outcomes of X with the density of X. The latter formula is sometimes more convenient to use, and it is always valid, even if Y does not have a density (like in the digital option case). Note that when g(x) = x2 − E[X]2,

E[Y] = E[g(X)] = E[(X − E[X])2]

is the variance of X. The covariance of two random variables is denoted by Cov(X,Y) and is defined as E[(X − E[X])(Y − E[Y])]. The correlation ρX, Y is defined as

$$\rho_{X,Y} = \frac{Cov(X,Y)}{\sigma_X \sigma_Y}$$

X and Y are said uncorrelated if ρX,Y (or, equivalently, their covariance) is zero.

Independence

Two real random variables X,Y are independent if for any couple of intervals (or halflines) I1,I2 the probability of the intersection X ∈ I1,Y ∈ I2 factorizes into the product of the probabilities

P(X ∈ I1,Y ∈ I2) = P(X ∈ I1)P(Y ∈ I2)

If the couple (X,Y) has a joint probability density, the above condition is equivalent to the following two:

1. the marginals X and Y have probability densities pX, pY
2. the joint density is the product of the marginal densities:

p(x,y) = pX(x)pY (y)

So, if X,Y are independent, then

E[XY] = E[X]E[Y]

and therefore they are uncorrelated:

E[(X − E[X])(Y − E[Y])] = 0

For a Gaussian bivariate (X,Y) independence is equivalent to uncorrelation. In fact, the general bivariate Gaussian density is:

$$p_{X,Y}(x,y) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}} \exp\left(-\frac{1}{2(1-\rho^2)}\left[\frac{(x-\mu)^2}{\sigma_1^2} - 2\rho\frac{(x-\mu)}{\sigma_1}\frac{(y-\nu)}{\sigma_2} + \frac{(y-\nu)^2}{\sigma_2^2}\right]\right)$$

where the parameter ρ is the correlation between $X \sim N(\mu, \sigma_1^2$ and $Y \sim N(\nu, \sigma_2^2$. It is immediate then to check that the joint density factorizes into the product of the marginal densities if and only if ρ = 0. These results generalize to the case of Gaussian n-variate (X1,...Xn): independence is equivalent to uncorrelation. If µ is the vector of the means, and Σ denotes the variance covariance matrix, the n-variate Gaussian N(µ,Σ) distribution has the following probability density::

$$p(x_1, \dots x_n) = \frac{1}{(2\pi)^{\frac{n}{2}}\sqrt{det(\Sigma)}} \exp\left((\underline{x} - \underline{\mu})'\Sigma^{-1}(\underline{x} - \underline{\mu})\right)$$

in which 0 denotes the transpose operation. The joint density can be factorized into the product of the n- Gaussian marginals if and only if the VarCov matrix Σ is diagonal, if and only if the correlations are null. This is the reason why often independence is confused with uncorrelation. Out of a Gaussian context however, the two notions are not equivalent: as shown above, independence always implies uncorrelation, but the viceversa is not true.

Conditional expectation (C.E.) of a random variable X is an expectation = averaging of outcomes procedure

conditional = given some extra knowledge

The extra information to which we condition the average is the key point. The result is not a number in general, but a random variable which is measurable with respect to the information we consider, namely known when this information is revealed.

C.E. is the best prediction of X which can be made given the extra information.

Suppose we are modeling a process, for two dates only: tomorrow, t = 1 and the day after, T = 2. Our process is just a couple of random variables S =(S1,S2). Assume F1 = σ(S1), F2 = σ(S1,S2). Then S is obviously adapted. We put a probability on F = F2 such that S1 and S2 have a known bivariate Gaussian distribution, with density

$$p(x,y) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}} \exp\left(-\frac{1}{2(1-\rho^2)}\left[\frac{(x-\mu)^2}{\sigma_1^2} - 2\rho\frac{(x-\mu)}{\sigma_1}\frac{(y-\nu)}{\sigma_2} + \frac{(y-\nu)^2}{\sigma_2^2}\right]\right)$$

Our best guess on the value of S2, is its expectation, the parameter ν.

Let us pretend for a while it is already tomorrow, when S1 is revealed. Looking at S1, and depending on the correlation ρ, our views on S2 are subject to a change. First, we need the so called ‘conditional density of S2 given S1’. That is, we pretend S1 = x is known, and compute the density

$$p_{S_2}(y|S_1 = x) = \frac{p(x,y)}{\int p(x,y)dy}$$

which is the conditional density of S2 given S1 = x. This is the new density for S2 with respect to we compute expectation. Explicitly, in our bivariate Gaussian case, the conditional density is:

$$p_{S_2}(y|S_1 = x) = \frac{1}{\sqrt{2\pi}\sigma_2\sqrt{1-\rho^2}} \exp\left(-\frac{1}{2(1-\rho^2)\sigma_2^2}\left(y - (\nu + \rho\frac{\sigma_2}{\sigma_1}(x-\mu)\right)^2\right.$$

and is thus a Gaussian $N(\nu + \rho\frac{\sigma_2}{\sigma_1}(x-\mu), \sigma_2^2(1-\rho^2$

So,

$$E[S_2 \mid S_1 = x] = \int y\, p_{S_2}(y|S_1 = x)dy = \nu + \rho\frac{\sigma_2}{\sigma_1}(x-\mu)$$

Now, we of course do not know exactly the outcome for S1. So x must be replaced with S1 and finally we get the C.E.

.

$$E[S_2 \mid S_1] = \nu + \rho\frac{\sigma_2}{\sigma_1}(S_1 - \mu)$$

This is a deterministic function of S1 only, and thus known when S1 is revealed. Its interpretation is the best prediction we can make for S2 given the knowledge of S1.

Since in our model the information is generated by the evolution of the process S (the natural filtration of S), the C.E. above coincides with the conditional expectation of S2 given the information at time 1:

E[S2 | S1] = E[S2 | F1]

Note that E[E[S2 | S1]] = E[S2] = ν. This is a general property of C.E.: its expectation is equal to the original mean.

Also, if ρ = 0, the C.E. is identical to the expectation:

E[S2 | S1] = ν

For a joint Gaussian distribution, uncorrelation is equivalent to independence. If two variables are independent, information on the outcome of one (S1 here) does not help us in predicting the second, and in particular the conditional expectation boils down to expectation.

These are general results, which are stated among other properties in the following proposition.

Properties of C.E.

$$E[E[Y \mid X]] = E[Y]$$

additivity: $E[Y1 + Y2 \mid X] = E[Y1 \mid X] + E[Y2 \mid X]$

Let f be a deterministic function; then,

$$E[f(X)Y \mid X] = f(X)E[Y \mid X]$$

that is, random variables which are known when X is known can be treated like constants and taken out of the conditional expectation sign. In particular, by taking Y = 1,

$$E[f(X) \mid X] = f(X)$$

If two variables (X,Y) are independent then the conditional expectation of one with respect to the other is constant, and coincides with the mean

$$E[Y \mid X] = E[Y];E[X \mid Y] = E[X]$$

Conditional expectation E[Y |X], similarly to its relative the expectation, is a notion which heavily depends upon P. If we changed our probabilistic views, the resulting average of outcomes would change: we would get a different random variable.

Conditional expectation with respect to information and martingales Let us go back to our filtered space (Ω,(Ft)t≤T ,P).

It is the best prediction of the random variable Y given the information we (will) possess in t1. As for the conditional expectation with respect to random variables, this conditional expectation is an average of the outcomes of Y given the knowledge we are supposed to have in t1. So, E[Y | Ft1] is a random variable, known in t1.

So given a filtered space, fix 0 ≤ t0 < t1 < t2 ≤ T, and let Y,W be random variables (know at least in T).

$$E[E[Y \mid Ft1]] = E[Y];$$

if Y is known by time t1, then \E[Y | Ft1] = Y ;

additivity: $E[Y + W \mid Ft1] = E[Y \mid Ft1] + E[W \mid Ft1]$

for any Z known by time t1,

$$E[ZY \mid Ft1] = ZE[Y \mid Ft1]$$

If Y is independent of Ft1, then

$$E[Y \mid Ft1] = const = E[Y]$$

tower law:

$$E[Y \mid Ft0] = E[E[Y \mid Ft1] \mid Ft0]$$

which means the best prediction of Y in t0 can either be made directly or in two steps: 1) first compute the best prediction of Y in t1, Ye := E[Y | Ft1]; 2) and then compute the best prediction in t0 of Ye.

The practice in Finance is to set the initial information F0 equal to the so-called trivial σ-algebra

$$F0 = \{\emptyset,\Omega\}$$

This means in 0 we know nothing about the future specific realizations, we can only describe them as a whole, by Ω seen as a whole piece. The only variables known at this date are the constants. So,

$$E[Y \mid F0] = E[Y \mid c] = E[Y]$$

as any rv Y is independent of a constant.

An adapted process M is a martingale if

$$E[M(t) \mid Fs] = M(s)$$

for all $0 \leq s < t \leq T$.

A martingale is the formalization of the price process for a fair game. In fact, suppose M(t) models the price we have to pay at time t to play a random game. Then the process M is the value process of the game, and it must obviously be adapted. If M is a martingale, the conditional average of the future value of the game M(t) at time s is exactly the current price M(s). So, on average, we are paying a fair price.

Brownian motion and Ito's Lemma

Let (Ω,(Ft)t∈[0,T],P) be a filtered space. Note now t is a continuous time parameter. The process W = (W(t))t≤T is a (standard) Brownian Motion on this space if

$$W(0) = 0$$

W is adapted to the filtration

for any s < t, the increment W(t) − W(s) is independent of Fs, and has distribution N(0,t − s) the paths W(·,ω) are continuous

Marginal distributions are Gaussian. For any t, W(t) can be written as W(t) − W(0), so from 3) above it has distribution N(0,t).

Independent, identically distributed increments (BM is a process with IID increments). Since W(u), for any u ≤ s is known at time s, as the Brownian Motion is adapted, from property 3) above it follows that for any u ≤ s < t the variables

W(u),W(t) − W(s)

are independent, and therefore jointly Gaussian $N(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} u & 0 \\ 0 & t-s \end{pmatrix}$. This argument can be extended to any number of increments, i.e. if 0 ≤ t1 < ...tn ≤ T then

W(t1),W(t2) − W(t1),...W(tn) − W(tn−1)

are independent and thus jointly Gaussian with distribution

$$N\left(\begin{pmatrix} 0 \\ \vdots \\ 0 \end{pmatrix}, \begin{pmatrix} t_1 & 0 & \dots & \dots & 0 \\ 0 & t_2 - t_1 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & t_n - t_{n-1} \end{pmatrix}\right)$$

This shows the distribution of any family of increments depends only on the time distance at which the increments are sampled (identically distributed increments).

Brownian Motion W is a martingale.

Proof. Fix two arbitrary dates s < t and write W(t) = W(t) − W(s) + W(s). Then,

E[W(t) | Fs] = E[W(t) − W(s) + W(s) | Fs] = W(s) + E[W(t) − W(s)] = W(s)

given the properties of the conditional expectation.

A Brownian motion with drift b and volatility σ > 0 is the process

B(t) = bt + σW(t)

which is a linear transform of the (standard) Brownian motion W.

The processes L which, like the BM, have IID increments and satisfy

P({L(t) = L(t−)}) = 1 for all t ∈ (0,T]

are called L´evy processes. These generalizations of BM may admit jumps, but only at stochastic, 'unpredictable' times.

Some transforms of W

Squared BM

A relevant process is the squared Brownian Motion process Z: Z(t) = W2(t). Write W(t) = (W(t) − W(s)) + W(s) and compute the conditional expectation:

E[W2(t) | Fs] = E[(W(t) − W(s))2 + 2W(s)(W(t) − W(s)) + W2(s) | Fs],

from which

E[W2(t) | Fs] = (t − s) + 2W(s)E[W(t) − W(s)] + W2(s) = W2(s) + (t − s)

and therefore W2 is not a martingale. However, the above computation shows that modulo a deterministic correction, W2 leads to a martingale Y :

Y (t) = W2(t) − t

Geometric BM

Consider BM with drift b and volatility σ. The exponential transform process Y

Y (t) = exp(B(t)) = exp(bt + σW(t))

is called **Geometric Brownian Motion**. For any fixed t, it is the exponential of a Gaussian variable, so the marginal distributions are lognormals.

Is Geometric Brownian motion a martingale? Using the same trick of writing W(t) as an increment plus W(s), let us compute the expectation

E[exp(bt + σW(t)) | Fs] = exp(bt + σW(s))E[exp(σ(W(t) − W(s))]

The last expectation is the σ-exponential moment of a Gaussian variable with mean 0 and variance t − s, and thus equals

$$e^{\frac{\sigma}{2}(t-s)}$$

so that

$$E[\exp(bt + \sigma W(t)) \mid \mathcal{F}_s] = \exp(bt + \frac{\sigma^2}{2}(t - s) + \sigma W(s))$$

As the right hand side in general is not equal to

Y (s) = exp(bs + σW(s))

the GBM is not a martingale for any choice of the parameters. The GBM is a martingale if and only if the parameters satisfy

$$b = -\frac{\sigma^2}{2}$$

the only GBMs which are martingales are those of the type

$$Y(t) = e^{-\frac{\sigma^2}{2}t} e^{\sigma W(t)}$$

Jensen's inequality for conditional expectation Let us fix a convex function f of W and consider the transformed process Y = f(W), Y (t) := f(W(t)). There is a general result for conditional expectations in these cases, known as Jensen's inequality, which reads as follows

f(E[W(t) | Fs]) ≤ E[f(W(t)) | Fs]

As W is a martingale, for any convex f

f(W(s)) ≤ E[f(W(t)) | Fs]

so we have an inequality instead of an equality. Since the inequality is strict when f is strictly convex, the f-transform will not be a martingale. Using this result, we could a priori have obtained that W2 is not a martingale; and also that the GBM with b = 0, Y = exp(σW), is not a martingale as well - but our calculations prove we obtain a martingale if the growth in expectation is reduced by the multiplicative factor $e^{-\frac{\sigma^2}{2}t}$.

A Markov process S is an adapted process such that, for every deterministic function g = g(x), and for any arbitrary dates s < t, the conditional expectation of g(S(t)) satisfies

E[g(S(t)) | Fs] = E[g(S(t)) | S(s)] = ge(S(s))

That is, only the information contained in the present value of the process, at s, is needed to make the best prediction on the future value g(S(t)). Nothing else is necessary, of the whole past history from 0 to s.

This does not mean that g(S) must be a martingale! the C.E. on the right hand side may be a different function ˜g of S(s).

An adapted process W with IID increments (not necessarily a Browian Motion) is a Markov process. Moreover, any transform

S(t) = F(t,W(t))

via a deterministic function F(t,x) which is invertible wrt the x variable for every fixed t is also a Markov process.

Fix s < t and consider any deterministic g = g(x). Then,

$$g(W(t)) = g((W(t) - W(s)) + W(s))$$

The increment Z = W(t) − W(s) is independent of Fs and the only other variable showing up is W(s). Then

$$E[g(W(t)) \mid Fs] = E[E[g(W(t)) \mid W(s)]] = g(z + W(s))pt{-}s(z)dz$$

in which pt−s indicates the density of the increment. Note also that the function ge can be read in the above formula: ge(x) = g(z + x)pt−s(z)dz

For demonstration, fix a deterministic function g and consider g(S). Fix t > s and consider

$$g(S(t)) = g(F(t,W(t)) = h(W(t))$$

where we set h(x) = g(F(t,x)) and therefore the C.E. of g(S(t)) is

$$E[g(S(t)) \mid Fs] = E[h(W(t)) \mid Fs] = eh(W(s))$$

W(s) can be written in terms of S(s) since F(s,·) is invertible in x! If this inverse is denoted by F−1(s,x), finally

$$E[g(S(t)) \mid Fs] = ge(S(s))$$

where ge(x) := eh(F−1(s,x)).

This is the reason why deterministic functions F(t,x) are called 'Markovian': then (F(t,W(t))t is a Markov process.

The BM with drift and the GMB are Markov processes.

Ito's formula

Ito's formula gives the dynamics of any smooth Markovian function of the BM W.

Knowing the dynamics (=the infinitesimal variations of the process) is quite useful, especially for simulation purposes.

Consider a deterministic, smooth function F of two variables (t,x). Then, F varies only in response to changes in (t,x), according to the following equation

$$dF(t,x) = Ft(t,x)dt + Fx(t,x)dx$$

which represents the infinitesimal change in the value of F,

dF(t,x) := F(t+dt,x+dx)−F(t,x)

when we move from (t,x) to (t+dt,x+dx) as a sum of the infinitesimal changes in the variables amplified by the proper partial derivatives.

A second order approximation would be

$$dF(t,x) = F_t(t,x)dt + F_x(t,x)dx + \frac{1}{2}\left(F_{xx}(t,x)(dx)^2 + 2F_{tx}(t,x)dtdx + F_{tt}(t,x)(dt)^2\right)$$

but as the terms between parentheses are negligible wrt the others, we never see them explicitly when writing infinitesimal variations.

Interpret t as a time parameter, and consider the stochastic process

Y (t) = F(t,W(t))

in which W is a standard BM. The variation of Y can be described in terms of the variations of time and W using :

$$\begin{aligned} dF(t,W(t)) &= F_t(t,W(t))dt + F_x(t,W(t))dW(t) + \\ &+ \frac{1}{2}\left(F_{xx}(t,W)(dW(t))^2 + 2F_{tx}(t,W(T))dtdW(t) + F_{tt}(t,W(t))(dt)^2\right) \end{aligned}$$

The reason why we go to the second order is that the paths of W are very wild. They are continuous, yes, but non-differentiable, so we are not sure about the order of the last group of terms between parentheses. In fact, it turns out that:

(dW(t))2 ∼ dt

while of course (dt)2 and the product dW(t)dt are of higher order than dt.

The intuition is that

dW(t) = W(t + dt) − W(t) ∼ N(0,dt)

and so we approximate the squared increment with its mean dt.

Ito's Lemma

Let F(t,x) be a smooth function (the minimal regularity required is C1,2(t,x)). The Markov process defined by

F(t,W(t))

has dynamics given by the (stochastic) differential equation

$$dF(t,W(t)) = \left(F_t(t,W(t)) + \frac{1}{2}F_{xx}(t,W(t))\right)dt + F_x(t,W(t))dW(t$$

Note the difference with ordinary calculus: the blue, extra term above is known as It¯o's term.

Diffusion processes

An It¯o process is any adapted process Y whose dynamics may be written as

$$dY(t) = \alpha(t)dt + \beta(t)dW(t)$$

where α and β are adapted processes which are called the coefficients of the stochastic differential equation. The first coefficient, α, is called the drift - though in Finance, when Y is a positive price process, only the fraction $\frac{\alpha(t)}{Y(t)}$ is called 'the drift'. The second coefficient, β, is the diffusion coefficient.

Thanks to the It¯o's formula, any Markovian function Y (t) = F(t,W(t)) of W is a diffusion.

In the BM and GBM cases, we have shown that these processes are martingales if and only if the drift is 0. For a general diffusion, we state the following result.

If a diffusion Y is a martingale, necessarily the drift vanishes. Conversely, if the drift vanishes and the diffusion coefficient satisfies an integrability condition:

$$E[\int_0^T \beta^2(t)dt] < +\infty$$

then the diffusion is a martingale.

Therefore, the only diffusions candidates to the martingale property are those of the form

$$dY(t) = \beta(t)dW(t)$$

and these are true martingales if β is good. Without extra conditions on β a diffusion with zero drift is only a local martingale.

An intuitive idea of why the drift must vanish when the diffusion is a martingale is the following. By definition, any martingale M has conditional increments ΔM = M(t + Δt) − M(t) with zero C.E. :

$$E[M(t + \Delta t) - M(t) \mid Ft] = 0$$

If the time difference becomes infinitesimal, then E[dM(t) | Ft] = 0 for any t. For a general diffusion Y we have

$$E[Y(t + dt) - Y(t) \mid Ft] = E[dY(t) \mid Ft] = \alpha(t)dt + \beta(t)E[dW(t) \mid Ft] = \alpha(t)dt$$

since α and β are adapted, thus known at time t, and dW(t) is an infinitesimal centered Gaussian increment, independent of Ft. If Y is a martingale, then

$$E[dY(t) \mid Ft] = 0 = \alpha(t)dt$$

for all t, which means α = 0.

The converse is more complicated. In fact, if α = 0 then we must control how the infinitesimals add up, in order to be sure that the resulting process is a martingale. This is the reason why an integrability condition on β is needed to ensure the martingale property in Proposition

It¯o's formula for diffusions Suppose a diffusion S is assigned:

$$dS(t) = \alpha(t)dt + \beta(t)dW(t)$$

and consider a deterministic, Markovian transform of the diffusion S

$$P(t,S(t))$$

If S models a stochastic price process of a stock, then the price process of any European call option is exactly a Markovian transform of S.

Therefore, we are interested in the dynamics of P and we wonder if there is a formula: the answer is positive. Exactly as we have seen in the Markovian transforms of the standard BM W, there is an It¯o's formula for the Markovian transforms of any diffusion S:

$$dP(t, S(t)) = P_t(t, S(t))dt + P_s(t, S(t))dS(t) + \frac{1}{2}P_{ss}(t, S(t)(dS(t))^2$$

Since (dS(t))2 ∼ β(t)2dt, we can equivalently write

$$dP = (P_t + \alpha P_s + \frac{1}{2}\beta^2 P_{ss})dt + P_s \beta dW$$

The Black-Scholes' world

The famous Black-Scholes(-Merton) 1973 formula for option prices is a milestone in Finance. It can be found everywhere, from Wikipedia and practitioners' booklets to the Financial press and all graduate texts on Mathematical Finance.

The pricing of derivatives is a relative pricing: derivatives are priced in terms of the assigned price dynamics of the basic financial instruments.

The basic financial instruments in the Black-Scholes model are only two. This is the minimal number of basic assets to have in order to be able to construct portfolios (mixed investments). The first instrument is the money market account (B, the 'bond') and the second is the 'stock' S. The bond is assumed to be riskless, and continuously paying a constant interest rate r ≥ 0:

$$dB(t) = rB(t)dt \quad B(0) = 1$$

or B(t) = ert. B should be regarded as a capitalization factor.

The other basic instrument is the risky stock, which is required to satisfy the SDE + initial condition (Cauchy problem)

$$dS(t) = \mu S(t)dt + \sigma S(t)dW(t)$$

$$S(0) = S0$$

where S0 is the observed initial market price, and μ, σ are constants (σ > 0). These latter parameters are called drift and volatility respectively. From the general theory on SDEs, the above Cauchy problem has a unique solution, and since we already know a solution, S must be:

$$S(t) = S_0 e^{(\mu - \frac{\sigma^2}{2})t + \sigma W(t)}$$

In particular, the marginals S(t) satisfy

$$\ln \frac{S(t)}{S_0} \sim N((\mu - \frac{\sigma^2}{2})t, \sigma^2 t)$$

That is, mean and variance of the stock logreturn grow linearly with time.

$$E[S(t)] = S0e\mu t,$$

We realize μ is the exponential growth of the average stock price. We may then be concerned with arbitrage possibilities. What if μ ≥ r, or μ < r? In the first case, one may be tempted to borrow money from the bank, and invest everything in stock; in the other case, one may think of selling short the stock, and invest in the bond. However, neither of these strategies leads to an arbitrage. In fact, while B deterministically increases at rate r, the stock increases at rate μ only in average (in expectation). So, depending on the particular path, and as a consequence of the noise effect, for any t the marginal S(t) (which is a lognormal) can be below S0ert even if μ > r; and it can be above S0ert even if μ < r.

We end by noting that μ and σ have also an 'instantaneous' meaning. The ratio $\frac{dS}{S}$ is the return of the stock on the infinitesimal interval (t,t + dt) and thus: μ is the conditional rate of return of the stock

$$E[\frac{dS(t)}{S(t)} \mid \mathcal{F}_t] = \mu dt$$

and σ2 is the conditional rate of variance:

$$E[(\frac{dS(t)}{S(t)} - \mu dt)^2 \mid \mathcal{F}_t] = E[(\sigma dW(t))^2 \mid \mathcal{F}_t] = \sigma^2 dt$$

Portfolio strategies

A portfolio in stock and bond is the most intuitive concept of all. In order to develop rigorous Maths, we are going to formalize it a bit.

A portfolio strategy, or simply a strategy, is a couple of adapted processes (K,H) where H(t) is the number of stock shares held at t and K(t) is the number of bond shares. Therefore, at any instant t the portfolio value V is given by

V (t) = K(t)B(t) + H(t)S(t)

The simplest example is a buy-and-hold strategy, consisting of a couple of constant processes (K0,H0). In practice, we choose at time 0 the positions in stock and bond and keep them till T.

The initial cost of this strategy is V (0) = K0 +H0S0 and the value process of the corresponding portfolio is:

V (t) = K0ert + H0S(t)

Of course, buy-and-hold strategies form a very small set in a continuous time market: active trading occurs only at 0 and at the final date T, when the positions are closed. Suppose now that trading occurs along a pre-specified finite time grid

t0 = 0 < t1 < ... < tn = T

Now, a strategy is piecewise-constant and can be identified with K(ti),H(ti) for i = 0...n − 1 , where for each i K(ti),H(ti) are the positions in bond and stock to be held from time ti up to time ti+1:

K(t) = K(ti),H(t) = H(ti) if t ∈ [ti,ti+1)

For times t before t1, 0 ≤ t < t1, the value V clearly is

$$V(t) = K(t0)B(t) + H(t0)S(t) = K0B(t) + H0S(t)$$

At the initial time, to implement the strategy we must start with initial wealth w0 equal to V (0) = K0 + H0S0. We arrive in t1 with wealth

$$w_1 = \lim_{t \to t_1 -} V(t) = K_0 B(t_1) + H_0 S(t_1)$$

since the processes B,S are continuous. To implement the strategy in the second step we should possess the wealth

$$w2 = V(t1) = K(t1)B(t1) + H(t1)S(t1)$$

This other wealth can be obtained by external infusion of money in case w1 < w2, or by consumption if the opposite inequality holds. If w1 = w2, at the date t1 and at all the subsequent trading dates, the portfolio strategy is called self-financing. With continuous processes, the self-financing condition on a finite grid amounts to the continuity of V globally on [0,T], trading dates included.

A self-financing strategy is thus obtained by starting with an initial endowment w0, and then (re-)allocating money between stock and bond at the trading dates with no infusion and no withdrawal of money.

The self-financing equations, which must hold at any trading date ti,i > 0 are

$$K(ti-1)B(ti) + H(ti-1)S(ti) = K(ti)B(ti) + H(ti)S(ti)$$

$$dV(t) = K(ti)dB(t) + H(ti)dS(t)$$

$$dV(t) = K(t)dB(t) + H(t)dS(t)$$

This means that the variations in value of a self-financing portfolio are proportional to the changes in value of bond and stock, the coefficients being the number of shares held.

This is the limit obtained when portfolio re-adjustments are infinitesimally close to each other.

A general portfolio strategy (H,K) is called self-financing if the above equation holds on [0,T]. In particular, any self-financing portfolio is a diffusion.

For a self-financing portfolio it is equivalent to know:

the couple (H,K), V (0) and one of H, K.

The choice is usually to give V (0) (the initial wealth) and H, the position in the risky stock.

The reason is the following. If we work with discounted quantities, i.e. with the processes

$$\widetilde{S}(t) = \frac{S(t)}{B(t)}, \widetilde{V}(t) = \frac{V(t)}{B(t)}$$

then this latter verifies dVe = HdSe.

Therefore, if one knows V (0) = Ve(0) and H, thanks to the above SDE also Ve is known and K can then be computed using the relation

K(t)B(t) + H(t)S(t) = V (t) = ertVe(t)

Arbitrage and martingale measures: the First Fundamental Theorem of Asset Pricing

Finite trading dates market

An arbitrage strategy in a finite time market model is a self-financing strategy V (0),H such that

V (0) = 0

V (T) ≥ 0 and P(V (T) > 0) > 0

Thus an arbitrage is a strategy which makes money out of nothing: it starts with zero money, and in a self-financing way leads to a non-negative wealth, which is positive with positive probability. So, if such strategies existed, one would get a positive gain with positive probability at no cost and without risk.

If an arbitrage opportunity were present in a real market, all the (smart) investors would try to take advantage of it simultaneously. Therefore, prices of the underlying financial instruments would not be at equilibrium, as there would be an almost instantaneous move of the prices as a response to a supply-demand imbalance. This price movement would continue until any opportunity for riskless profit is no longer available.

This is the rationale behind the construction of models were No Arbitrage (mathematically defined above, NA for short in the sequel) strategy is present.

In the Cox-Ross-Rubinstein (binomial tree) model, with capitalization factor m = er and stock multipliers eu,ed, the NA condition holds if an only if

d < r < u

and this is exactly the condition one puts on the parameters in order to develop option pricing.

Dalang-Morton-Willinger

Given a finite time filtered probability space (Ω,(Fti)i=0...n,P) and a market model (B,S), where S is the (possibly multidimensional) risky asset and B is the bond, NA is equivalent to the existence of a probability Q on FT with the properties:

Q(A) = 0 if and only if P(A) = 0 [Q is equivalent to P, notation Q $\sim$ P]

the discounted stock price process $\widetilde{S} = \frac{S}{B}$ is a Q-martingale, i.e.

$$E^{\mathbb{Q}}[\frac{S(t)}{B(t)} \mid \mathcal{F}_s] = \frac{S(s)}{B(s)}$$

Such Q, which is not necessarily unique, is called an equivalent martingale measure (EMM) for Se.

The second property above better reads as current prices for S are expected values of discounted future values under Q:

$$S(s) = E^{\mathbb{Q}}[B(s)\frac{S(t)}{B(t)} \mid \mathcal{F}_s] = E^{\mathbb{Q}}[e^{-r(t-s)}S(t) \mid \mathcal{F}_s]$$

which when s = 0 reads

S(0) = EQ[e−rtS(t)]

This fundamental relation has the financial meaning that Q calibrates S.

The FTAP is the basis of the use of martingale methods in Finance. The philosophy behind it goes as follows.

Suppose the model bond-stock (B,S) verifies the NA condition. Then, by the FTAP implication: NA ⇒ existence of an EMM, there exists (at least) one EMM Q.

Suppose we would like to launch a derivative on S, with payoff Ψ and maturity T (with no early exercise feature). We would like to place the derivative on the market with a fair price P. By fair price we mean that the extended market (B,(S,P)) must also verify NA, in the sense that there exists no arbitrage strategy (K,H,J) based on stock-bond-derivative, and this latter with price P. Obviously, at the maturity T P(T) = Ψ. What about indermediate dates? The solution is to set

$$P(t) = E^{\mathbb{Q}}[B(t)\frac{\Psi}{B(T)} \mid \mathcal{F}_t] = E^{\mathbb{Q}}[e^{-r(T-t)}\Psi \mid \mathcal{F}_t]$$

or, equivalently, dividing by B(t)

$$\widetilde{P}(t) := \frac{P(t)}{B(t)} = E^{\mathbb{Q}}[\frac{\Psi}{B(T)} \mid \mathcal{F}_t]$$

If the price P verifies the above condition, the probability Q, which is an EMM for Se, is an EMM also for the discounted Pe thanks to its definition above and to the terminal condition P(T) = Ψ.

So, Q is an EMM for the bidimensional risky process (S,P). Therefore, the FTAP, converse implication, existence of an EMM ⇒ NA, implies the extended market verifies NA.

In particular, a fair initial price is

P(0) = EQ[e−rT Ψ]

that is prices must be computed as Q-expectations of discounted payoffs.

As a final remark, remember that Q may be very different from P and it should be regarded only as a pricing tool.

Portfolio selection (optimization) and, more generally, any risk management issue must all be addressed under P, that is

compute P(S(T) > K)

compute E[U(V (T))] if U is the agent's utility function, and E is P-expectation

consider the supremum over a class V of portfolios of some performance criterion, e.g. expected utility maximization supV ∈V E[U(V (T))] or Markowitz's mean-variance optimization.

General markets

The FTAP in the version we presented holds for markets open only on a finite number of dates (like the Cox-Ross-Rubinstein model).

In general, for market models open on infinite trading dates (discrete but infinite dates, like in asymptotic problems, or continuous time as Black-Scholes) we must add a condition to the arbitrage portfolios.

An arbitrage strategy is a self-financing strategy V (0),H such that

V (0) = 0

there exists a constant $c > 0$ such that $V(t) \geq -c$ for all $t \in (0,T)$ c here is a finite credit line which must be respected during the trading, and it may depend on H

$$V(T) \geq 0 \text{ and } P(V(T) > 0) > 0$$

The market verifies No Arbitrage (NA) if no such strategy exists.

For generale markets, to get a consistent pricing rule one needs to exclude not only arbitrage strategies, but also certain limits of self-financing strategies, which are 'asymptotic arbitrages' and are called Free Lunches with Vanishing Risk (FLVR).

So, if we can exhibit an EMM Q for Se, then the same reasoning we made above for the pricing of a new derivative applies.

Pricing principle Suppose Q is an EMM for S and consider a European option payoff Ψ, with maturity T. Then, if

$$P(t) = E_Q[e^{-r(T-t)}\Psi \mid F_t]$$

the extended model (B,S,P) verifies NFLVR and also the weaker condition NA.

This approach seems quite abstract, but it is powerful as it is very general. In fact, it applies to any market model (not only to Black-Scholes) and to all types of payoffs with no early exercise features, including path-dependent ones like Asian options:

$$\Psi = \left(\frac{\int_0^T S(t)dt}{T} - K\right)^+ \text{ or } \left(S(T) - \frac{\int_0^T S(t)dt}{T}\right)^+$$

like lookbacks:

$$\Psi_1 = S(T) - \min_{t \leq T} S(t) \text{ or } \Psi_2 = \max_{t \leq T} S(t) - S(T)$$

and barrier options. We will see this is not the case for the PDE based methods, which work only for Markovian derivatives of the type g(S(T)), functions of S at the maturity date only.

Pricing and hedging are closely related: the Second FTAP

There is evidently a link between pricing and hedging. Suppose a derivative Ψ with maturity T can be replicated/perfectly hedged, there exists a self-financing portfolio V in stock and bond such that

$$V(T) = \Psi$$

then there is only one way of pricing the derivative without introducing arbitrage in the extended market (stock, bond, derivative):

$$P(t) = V(t)$$

In some sense, the replicable derivatives are redundant: they can be synthetically reproduced using stock and bond. The number of stock shares used in the replication, H, in jargon is often called the Delta (Δ) of the derivative.

Of course, the martingale method and the hedging-based method must give the same result. This is due to the fact that any self-financing portfolio verifies

$$dVe = HdSe$$

and if H is good enough then Ve is a martingale when Se is. So, under any EMM Q we have:

$$\frac{V(t)}{B(t)} = E^{\mathbb{Q}}[\frac{V(T)}{B(T)} \mid \mathcal{F}_t] = E^{\mathbb{Q}}[\frac{\Psi}{B(T)} \mid \mathcal{F}_t]$$

and therefore

$$P(t) = B(t)E^{\mathbb{Q}}[\frac{\Psi}{B(T)} \mid \mathcal{F}_t]$$

coincides with V (t).

An easy example of replicable contract is a forward contract. The strategy is of buy-and-hold-type, with H(t) = H0 ≡ 1.

The market is complete if all the derivatives are replicable.

So, if a market is complete, any fixed derivative has only one fair price, corresponding to the value of the replicating portfolio.

Any real market is incomplete: one cannot perfectly hedge out all conceivable risk. Nevertheless, in this course we consider only two models, and both are complete: the Cox-Ross-Rubinstein model, and the Black-Scholes one. For arbitrage free market models, the Second FTAP characterizes completeness.

Consider a market model verifying NFLVR. Then, the following are equivalent

the market is complete there exists only one Q EMM for Se.

Valuation of options in Black-Scholes

Let us go back to the Black-Scholes model, and let us exhibit a Q which is the unique EMM for Se, the discounted stock price. Since

$$dS = \mu S dt + \sigma S dW,$$

then $\widetilde{S} = \frac{S}{B}$ has dynamics given by

$$d\widetilde{S} = (\mu - r)\widetilde{S}dt + \sigma\widetilde{S}dW$$

So, Se is not a martingale under P in general.

$$\lambda = \frac{\mu - r}{\sigma}$$

The market price of risk financially represents the instantaneous excess return over the riskless rate μ − r per unit of volatility, and should remind the reader of the Sharpe Ratio. Set

$$W*(t) = W(t) + \lambda t$$

and note

$$dW* = dW + \lambda dt$$

Written in terms of W∗, the SDE for Se reads as

$$dSe = \sigma S dWe*$$

If there existed a probability Q ∼ P such that W∗ were a Q-Brownian motion, we would immediately conclude Q is a martingale probability for Se. And this is in fact the case.

An important probabilistic result, called Girsanov Theorem, ensures that there exists a unique Q with this property, and gives the explicit transform which links Q to P. So, by the second FTAP, we also know that the market is complete: any derivative with no early-exercise feature is replicable. The transform provided by the Girsanov theorem is the Radon-Nikodym density of Q wrt P, which is a random variable equal to

$$\frac{d\mathbb{Q}}{d\mathbb{P}} = \exp(-\lambda W(T) - \frac{\lambda^2 T}{2}) > 0$$

The strict positivity of such function ensures that Q is equivalent to P. The knowledge of the density function in fact allows to compute the Q probabilities in terms of deformed P probabilities

$$\mathbb{Q}(A) = E^{\mathbb{Q}}[I_A] = E[\frac{d\mathbb{Q}}{d\mathbb{P}} I_A]$$

as the indicator IA is weighted by the 'deformation' density function $\frac{d\mathbb{Q}}{d\mathbb{P}}$ prior to taking Pexpectation. Anyway, the best thing to do when computing prices here is to think only about the effect of the change of measure on Se, which becomes martingale GBM under Q, i.e.

$$Se(t) = \exp(-\sigma 2t/2 + \sigma W*(t))$$

which is equivalent to say that S is

$$S(t) = Se(t)ert = S0 \exp((r - \sigma 2/2)t + \sigma W*(t))$$

or also

$$dS = rSdt + \sigma SdW*$$

The drift of S under Q is r. So, on average, under Q the stock is a GBM with the same σ, but grows at the rate of the riskless bond. This is the reason why Q is called a risk neutral measure. Even if the investor is risk averse, when pricing she behaves as if she were risk neutral.

If the derivative has maturity T and is of the simple Markovian form g(S(T)), then its price today is

$$P(0) = e^{-rT} E^{\mathbb{Q}}[g(S(T))] = e^{-rT} E^{\mathbb{Q}}[g\left(S_0 \exp((r - \sigma^2/2)T + \sigma W^*(T))\right)]$$

whence we are left with the computation of the one dimensional integral

$$P(0) = e^{-rT} \int g(S_0 \exp((r - \sigma^2/2)T + \sigma\sqrt{T}x)) \frac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi}} dx$$

which may be impossible to compute analytically. In any case, it can be approximated via simulation. If we wish to know the price at time 0 < t < T, then

$$P(t) = EQ[e-r(T-t)g(S(T)) \mid Ft]$$

This also leads to the computation of an integral. The idea is to write

$$S(T) = S(t)\exp((r - \sigma 2/2)(T - t) + \sigma(W*(T) - W*(t))),$$

so that the only dependence of g(S(T)) from the past up to t is contained in the term S(t), while the factor exp((r − σ2/2)(T − t) + σ(W∗(T) − W∗(t))) is independent of Ft. So,

$$E^{\mathbb{Q}}[g(S(T))e^{-r(T-t)} \mid \mathcal{F}_t] = e^{-r\tau} \int g(S(t) \exp((r-\sigma^2/2)\tau + \sigma\sqrt{\tau}\, x)) \frac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi}} dx$$

where τ = T − t is the time to maturity . The formula looks the same, only S0 must be replaced by S(t) (random, but known at t) and T by τ. If we call G the deterministic function

$$G(t,s) = e^{-r(T-t)} \int g(s \exp((r-\sigma^2/2)(T-t) + \sigma\sqrt{T-t}\, x)) \frac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi}} dx$$

then we realize the price process is a Markovian function of S:

G(t,S(t))

If the payoff is more complicated, like in a path dependent case, often the only way is resorting to simulation.

The payoff is of the form for Asian options.

$$\left(\frac{\sum_{i=1}^{n} S(t_i)}{n} - K\right)^{+}$$

Note that the conditional expectation of the discounted payoff say at any t4 > t > t3 depends not only on S(t), but also on S(t1),S(t2),S(t3).

Suppose for simplicity the average of the stock is made only on two dates, t1 and t2 = T.

Since $\frac{S(t_1)+S(T)}{2} = \frac{1}{2}S(t_1)(1 + \exp((r - \sigma^2/2)(T - t_1) + \sigma(W^*(T) - W^*(t_1)$, simulate two independent standard Gaussian variables Z,Y and call z1,y1 the outcomes. Set

$$v_1 = \frac{S_0 e^{(r-\sigma^2/2)t_1 + \sigma\sqrt{t_1}z_1}(1 + e^{(r-\sigma^2/2)(T-t_1)+\sigma\sqrt{T-t_1}y_1})}{2}$$

and compute the max m1 between v1 −K and 0. Repeat the algorithm sufficiently many times' and average the values. This average, discounted, is the approximate value of the option:

$$e^{-rT}\frac{\sum_{i=1}^{N} m_i}{N}$$

Stochastic discount factor In many books for economists, pricing is made by referring to the 'stochastic discount factor' (SDF) and then taking P-expectations. The price of any derivative with payoff Ψ is given by

P(t) = EQ[e−r(T−t)Ψ | Ft]

and $\frac{d\mathbb{Q}}{d\mathbb{P}} = \exp(-\lambda W(T) - \frac{\lambda^2 T}{2})$.

Q-conditional expectation via P. Note first that if L denotes the process

$$L(t) = \exp(-\lambda W(t) - \frac{\lambda^2 t}{2})$$

then L is a P-martingale with terminal value $L(T) = \frac{d\mathbb{Q}}{d\mathbb{P}}$ and with L(0) = 1. Thanks to the abstract version of Bayes' Theorem for conditional expectation, we can write:

$$E^{\mathbb{Q}}[e^{-r(T-t)}\Psi \mid \mathcal{F}_t] = \frac{E[e^{-r(T-t)}L(T)\Psi \mid \mathcal{F}_T]}{E[L(T) \mid \mathcal{F}_t]} = \frac{E[e^{-r(T-t)}L(T)\Psi \mid \mathcal{F}_T]}{L(t)} = E[e^{-r(T-t)}\frac{L(T)}{L(t)}\Psi \mid \mathcal{F}_t]$$

in which the second equality follows from the martingale property of L. So, if D(t;T) denotes the variable

D(t) = e−rtL(t)

then the process M with M(t) := P(t)D(t) is a P martingale with terminal value ΨD(T) (the martingale property follows from the tower law of conditional expectation) and the price at any t can be computed as

$$E[\frac{D(T)}{D(t)}\Psi \mid \mathcal{F}_t]$$

and at time 0, P(0) = E[D(T)Ψ]. This is why the process D is called the SDF: it incorporates both the discounting and the change of probabilistic views. The moral is: we can compute prices as P expectations, if we first let the SDF act as a multiplicative deformation on the payoff Ψ.

The formula for the price of a call option

The final formula for the price C(0) of a call with maturity T and strike K is

C(0) = S0Φ(d1) − Ke−rT Φ(d2)

where Φ is the standard Gaussian CDF,

$$d_1 = \frac{\ln \frac{S_0}{K} + (r + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}}, d_2 = d_1 - \sigma\sqrt{T}$$

and, at any intermediate t,

C(t,S(t)) = S(t)Φ(d1(t)) − Ke−rτΦ(d2(t))

where τ = T − t and di(t),i = 1,2 are

$$d_1(t) = \frac{\ln \frac{S(t)}{K} + (r + \frac{\sigma^2}{2})\tau}{\sigma\sqrt{\tau}}, d_2(t) = d_1(t) - \sigma\sqrt{\tau}$$

Note the 'historical' drift µ has vanished from the valuation formula, as expected.

The price of the corresponding put option can be found either by direct calculations, similar to the ones seen for the call, or more easily via the put-call parity.

The valuation PDE in Black-Scholes and the Delta

The derivation method of the Black-Scholes valuations PDE is very important, as it gives as a byproduct the structure of the replicating portfolio for any given Markovian payoff g(S(T)). The scheme is the following:

Fix a payoff g(S(T)); its unique price at time t will be a Markovian function of time and S(t) only: let us call it

G(t,S(t))

Apply It¯o's formula to G

Identify the self-financing strategy (K,H) which replicates g(S(T))

Derive a valuation PDE with terminal condition solve the PDE, and get the Delta of an option Let us go through the steps.

This point is heuristically assumed in the original article by Black and Scholes, but we already know it is correct on the martingale method.

It¯o's formula gives:

$$dG = G_t dt + G_s dS + \frac{1}{2} G_{ss} (dS)^2$$

that is

$$dG = (G_t + \frac{1}{2} G_{ss} \sigma^2 S^2) dt + G_s dS$$

If the derivative admits a replicating portfolio, necessarily its value V (t) = H(t)S(t) + K(t)B(t) must coincide with G, so that dG(t,S(t)) = dV (t) = K(t)dB(t) + H(t)dS(t) = K(t)rertdt + H(t)dS(t) therefore, we get two equations, one for H and one for K: 1) H(t) = Gs(t,S(t)) and 2)

$$K(t)re^{rt} = G_t(t, S(t)) + \tfrac{1}{2}G_{ss}(t, S(t))\sigma^2 S^2$$

.

As Kert = K(t)B(t) = V (t) − H(t)S(t) = G(t,S(t)) − H(t)S(t) then

$$r[G(t, S(t)) - G_s(t, S(t))S(t)] = G_t + \frac{1}{2}\sigma^2 S^2(t) G_{ss}$$

This equation must be satisfied for every 0 ≤ t < T. Since when t is fixed S(t) is a lognormal, which can assume any positive real value, we can replace the random S(t) with the positive real parameter s and write the famous, parabolic valuation PDE:

$$G_t(t, s) + rsG_s(t, s) + \frac{1}{2}\sigma^2 s^2 G_{ss}(t, s) = rG(t, s)$$

which is the same for all the derivatives. It is a linear, parabolic PDE where only the interest rate and volatility enter, while µ has vanished.

The specific derivative enters only in the terminal condition, so we face a Cauchy problem:

$$\begin{cases} G_t(t, s) + rsG_s(t, s) + \frac{1}{2}\sigma^2 s^2 G_{ss}(t, s) = rG(t, s) \\ G(T, s) = g(s) \end{cases}$$

with the terminal condition G(T,s) = g(s). Such Cauchy problem has a unique solution.

Since the Cauchy problem has a unique solution, we also prove that any Markovian derivative can be replicated in a self financing way using stock and bond. In fact, if we can solve the above Cauchy problem, and explicitly find G, then we can immediately find H (and K by difference), thanks to the self-financing condition:

$$K(t) = \frac{G(t,S(t)) - H(t)S(t)}{B(t)}).$$

The number of shares of the risky stock which are necessary for the replication (a.k.a. the Delta of the option) is simply the derivative of the function G with respect to the stock s, composed with the current level of the stock S(t):

H(t) = Gs(t,S(t))

It is the sensitivity of the option price wrt stock movements.

How to solve the Cauchy problem? there are three possible solutions:

1) **analytical methods**, by reducing the valuation PDE to the heat equation;

2) **numerical methods** (discretization of the PDE via numerical schemes like finite differences, etc)

3) **the Feynman-Kaˇc probabilistic representation**, which gives the same representation obtained by the martingale method.

The Delta for a call option at time t is Φ(d1(t)). The easiest way to show it directly from the formula for the option price (4), substituting S(t) = s and taking the s derivative of C. Note this is a positive number, smaller than 1 .

For a put, the situation is quite different: its Delta is negative, and equals −Φ(−d1). This can be derived either by differentiation of the put price, or using the put call parity:

$$P(t,s) = C(t,s) - s + Ke^{r(T-t)}$$

and differentiating wrt s the right hand side.

An example of Delta hedging can be a trader sells a call option at a price c∗, which may differ from the Black Scholes price C0. The trader has now an (extra) exposition −Φ(d1) to the stock movements (this is the Delta of a short call). To neutralize the effects of stock movements, she invests in stock and bond so to keep the global portfolio: call, positions in stock and bond Delta neutral. So, she starts following the replicating portfolio V in stock and bond for the call. This portfolio exactly prescribes to buy Φ(d1) shares of S. So, globally the Delta will be zero.

The initial cost of the replicating strategy is the Black-Scholes price of the call C0. By adjusting the Delta continuously, i.e. continuously buying and selling shares in order to possess Φ(d1(t)) at any time t, she will finally end up at T with:

$$V(T) - (S(T) - K)^+ + (c^* - C_0)e^{rT} = (c^* - C_0)e^{rT}$$

So, whatever is c∗, the final payoff is non-random: the risk has been eliminated. As long as a portfolio is self-financed and Delta neutral, there are no other risks.

And if c∗ > C0 the trader has even made a positive gain - so, we have described an arbitrage! This is a faithful picture of what happens in practice. ‘Fair’ prices are computed on a hedging basis, and then an ‘add-on’ is charged by the trader/financial institution (here, the difference c∗ − C0). Of course, in the real world the hedging is never perfect: there are transaction costs, thus the portfolio cannot be rebalanced frequently so to stay Delta-neutral; there is a discretization error due to the fact that we cannot possibly hedge continuously even with zero transaction costs; and there are for sure other sources of risk because the stock S is not the only one in the world.

The 'add-on' is a compensation for the exposure to residual risk.

Put & Call Parity

Knowing the value of a call option at time 0, strike price and value of underlying at time 0, we are able to determine the value of the put option at time 0. In fact if C(S,t) is the value of a call option at security value S and time t<T, then C(S,t) satisfies the Black-Scholes equation, and has terminal value max(S−K,0). If P(S,t) is the value of a put option at security value S and time t<T, then P(S,t) also satisfies the Black-Scholes equation, and has terminal value max(K−S,0). Therefore by linearity, C(S,t)−P(S,t) is a solution and has terminal value

$$C-P=S-Ke-r(T-t).$$

This relationship is known as the put-call parity principle between the price C of a European call option and the price P of a European put option, each with strike price K and underlying security value S.

Merton Model

In 1974, economist Robert C. Merton proposed a model for assessing the structural credit risk of a company by modeling the company's equity as a call option on its assets. This allows for easier valuation of the company and also helps analysts determine if the company will be able to retain solvency by analyzing maturity dates and total debts.
The Merton Model under the following assumptions assumptions:

- All options are European and are exercised only at the time of expiration.
- No dividends are paid out.
- Market movements are unpredictable (efficient markets).
- No commissions are included.
- Underlying stocks' volatility and risk-free rates are constant.
- Returns on underlying stocks are regularly distributed.

Lead to the following formula to value the equity of of a company:

$$E=V_t N(d_1)-Ke_{-r\Delta T}N(d_2)$$

where:

$$d_1 = \frac{\ln \frac{V_t}{K} + \left(r + \frac{\sigma_v^2}{2}\right) \Delta T}{\sigma_v \sqrt{\Delta T}} \qquad d_2 = d_1 - \sigma_v \sqrt{\Delta t}$$

E = Theoretical value of a company's equity
Vt= Value of the company's assets in period t
K = Value of the company's debt
t = Current time period
T = Future time period
r = Risk free interest rate
N = Cumulative standard normal distribution
e = Exponential term
σ= Standard deviation of stock returns

CHAPTER 7 – New financial frontier: A.S. Roma case study

An analysis of the financial statements of the sector companies shows that the net equity is close to zero while the net debt is approximately equal to the book value sum of the multi-year rights to the sports performances of the players, whose valuation is however conservatively conserved at the amortized purchase values, or if lower than the presumed ones of realization. Under this prudential approach, therefore, is not possible to highlight the value of the pool of players because it is not only functional to the company business, despite their own transfer and acquisition activity is increasingly part of the management characteristic of a football club. This has no longer been the case for extraordinary income but rather for ordinary activities whose correct valuation in the financial statements is prevented by the application of the International Accounting Standards (IAS).

According a preliminary analysis of soccer players audited balance sheet values compliant with the IAS, emerges that the extra market value of football players park is approximately equal to unexpressed value of balance sheet related to the football players park itself.

In the analysis of A.S. Roma Calcio S.p.A., the only difference between the low level of net equity (cured by progressive capital increases) and the positive stock market capitalization of over 400 million euros at the current brilliant prices, gives a first idea of what could be positively valued by making use of derivative instruments on multi-year rights to the players' sports performances. In other words, the market value already values, albeit in a non-analytical way, in part what cannot be reported in the financial statements in compliance with international accounting standards and it implies that the lenders (banks; bondholders and debt funds) find difficult to consider totally this market value because not certifiable. At current market conditions, the ratio between corporate debt and multi-year rights to the players' sports performances market value would be 91% against the substantial equivalence of 60% that emerges from the book values.

The same financial clauses negotiated on the occasion of the refinancing of 08/08/19 by means of a non-convertible senior bond issue, secured but not on the multi-year rights to the sports performances of the players that can be transferred towards third parties without breaching any covenant, seems to allow the use of derivative instruments on these rights which could open up new financial opportunities in terms of resources (debt and equity) and market transparency, also increasing the rating of the same unsecured and subsequent issue.

In compliance with financial fair play, on the other hand, it is noted that the use of collateralized debt to assets can allow to invest more in companies without having to resort to capital increases (of 150 mil) that must comply with both minimum rules and maximum capitalization ceilings that, often, other companies football tend to circumvent with sponsorship or image rights management contracts.

Trough Black & Scholes model applied to two different players (underlyings) on two different derivative instruments (forward and call option), appears evident as predictable and more manageable my become the market value of football players and soccer companies.

Below is attached the company presentation and the case study mentioned above, to the Chief Executive Office Mr Fienga and to the Chef Financial Officer Mr Francia of A.S. Roma S.p.A. occurred at beginning of 2020. I hope that my work would point out new financial frontier on which may be the case to investigate further.

Ev/Revenues multiples comparison

AS Roma		Market Cap	390	
price@10/02/20	0,628	Debt	240	

2017-18	**Total**	Match days	Broadcasting	Commercial
Enterprise Value	**495**	64	245	186
Revenues	**251**	77	140	33
EV/Revenues	**2,0**	0,8	1,7	5,6

Sources: Forbes 2018 and AS Roma Financial Statements 18-19 (mil euro)

2018-19	**Total**	Match days	Broadcasting	Commercial
Enterprise Value	**630**	75	326	229
Revenues	**233**	66	136	30
EV/Revenues	**2,7**	1,1	2,4	7,7
delta year % EV/R	37%			

Sources: MIB and AS Roma Financial Statements 18-19 (mil euro)

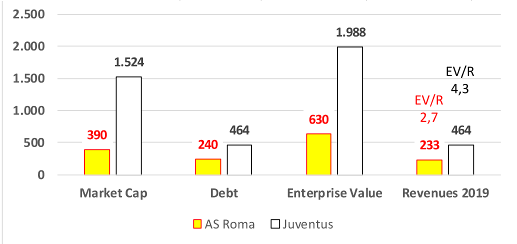
Juventus		Market Cap	1524	
price@10/02/20	1,149	Debt	464	

2017-18	**Total**	Match days	Broadcasting	Commercial
Enterprise Value	**1.160**	134	599	427
Revenues	**402**	56	287	59
EV/Revenues	**2,9**	2,4	2,1	7,2

Sources: Forbes 2018 and Juventus Financial Statements 18-19 (mil euro)

2018-19	**Total**	Match days	Broadcasting	Commercial
Enterprise Value	**1.988**	248	972	768
Revenues	**464**	70,6	315	78
EV/Revenues	**4,3**	3,5	3,1	9,8
delta year % EV/R	48%			

Sources: MIB and Juventus Financial Statements 18-19 (mil euro)



According the **EV/Revenues multiples method breakdown comparison**, is possible rounds that Match Days & Broadcasting values are approxmaley equal to the value of "Diritti" (multi-years rights of sport performances of players), so that:

- Football players value for ASRoma would be of 400 mil euro (as confirmed by the Market Transfert Market Evaluation)
- Commercial value (merchandising and revenues from all other commercial activities) strictley linked to the brand and managed by controlled subsidaries (mainly Soccer SAS di Brand Management and ASR Media and Sponsorship) would be of approximatley 230 mil euros

2

AS Roma 2018-19 Financial Statements analysis

AS Roma 30/06/2019

	ASSETS	**LIABILITIES**	
EV net of WC	**630**	**630**	
Market Premium	150	380	**(Market Cap. - Net Worth)**
(MV-BV) brand/subs	94		
(MV - BV) Diritti	136		
		10	Net Worth
BV Diritti	264	240	Net Debt
Brand/subs	135	149	Net Working Capital
Adjuted BV	399	399	
Adjusted EV	779	779	

'Diritti'' = Multi-years rights to the sports performances of players

Net Debt	**/**	**BV Diritti**	**=**	**91%**
Net Debt	**/**	**MV Diritti**	**=**	**60%**

AS Roma 30/06/2019

Profit & Losses

	2017-18	**2018-19**
Matchdays	**77**	**66**
Broadcasting rights	**140**	**136**
Commercial Sources	**33**	**30**
Total Revenues	**251**	**233**
Costs & amortizations	-289	-352
Ordinary margin	**-39**	**-119**
Results from disposals	**46**	**132**
Operating results	**7**	**13**

In accordance with financial fair play and prudence management of the companies, the disposal activity is often drive by financial needs

3

AS Roma players Transfert Market Evaluations

Player	Net Book Value@ **30/06/19**	Player Market Value@ **30/06/19**	Delta	**30 June 2018**	**30 June 2017**	**30 June 2016**	**30 June 2015**	**30 June 2014**
ANTONUCCI	0,1	0,5	0,4	0,5	0,2	0,1	0,0	0,0
BIANDA	5,0	2,0	-3,0	2,0	0,0	0,0	0,0	0,0
BUSO	0,0	0,5	0,5	0,0	0,0	0,0	0,0	0,0
CAPRADOSSI	0,1	2,0	1,9	0,8	0,5	0,2	0,3	0,1
CELAR	0,4	0,8	0,4	0,3	0,1	0,0	0,0	0,0
CENGIZ UNDER	10,7	35,0	24,3	27,0	5,0	2,0	1,2	0,1
CORIC	7,2	6,0	-1,2	7,5	10,0	7,0	2,5	0,0
CRISTANTE	22,3	25,0	2,7	25,0	3,0	2,5	4,0	3,0
DEFREL	12,0	12,0	0,0	10,0	9,0	6,0	6,0	1,0
DZEKO	4,6	14,0	9,4	25,0	22,0	12,0	20,0	27,0
EL SHARAAWY	3,3	30,0	26,8	20,0	17,0	17,0	14,0	20,0
FAZIO	1,1	6,0	4,9	13,0	9,0	5,0	8,0	8,0
FLORENZI	1,9	25,0	23,1	30,0	20,0	20,0	18,0	14,0
GERSON	8,1	13,0	4,9	10,0	8,0	10,0	0,0	0,0
GONALONS	4,1	4,0	-0,1	7,5	14,0	15,0	16,0	14,0
JUAN JESUS	4,0	9,0	5,0	13,0	7,5	9,0	11,0	12,0
KARSDORP	10,0	7,0	-3,0	9,0	9,0	4,5	0,8	0,0
KLUIVERT	17,0	20,0	3,0	15,0	3,0	0,0	0,0	0,0
KOLAROV	1,9	8,0	6,1	12,0	10,0	11,0	11,0	12,0
MARCANO	1,3	4,0	2,7	8,0	8,0	6,0	4,7	4,0
MIRANTE	3,0	0,8	-2,2	1,0	2,0	2,8	3,0	6,0
NZONZI	22,9	23,0	0,1	30,0	30,0	20,0	7,5	6,0
OLSEN	9,2	7,5	-1,7	1,3	1,5	1,5	1,3	0,3
PASTORE	20,5	15,0	-5,5	15,0	20,0	20,0	25,0	18,0
L. PELLEGRINI	7,9	35,0	27,1	28,0	12,0	2,5	0,3	0,2
PERES	6,1	4,0	-2,1	8,0	13,0	5,5	5,5	2,0
PEROTTI	4,2	12,0	7,8	20,0	17,0	15,0	10,0	3,3
SADIQ	1,0	1,4	0,4	1,8	2,0	2,5	0,4	0,1
SANTON	7,5	5,0	-2,5	3,0	5,0	6,0	7,0	8,0
SCHICK	29,6	15,0	-14,6	20,0	15,0	3,0	0,2	0,2
SPINAZZOLA	29,5	15,0	-14,5	12,0	8,0	0,8	0,6	0,7
VERDE	0,0	2,5	2,5	3,5	1,5	0,9	1,0	0,0
ZANIOLO	4,6	40,0	35,4	1,0	0,0	0,0	0,0	0,0
Others *	3,4	0,9	-2,5	0,5	0,5	0,4	0,2	0,1
Sum	**264,4**	**400,9**	**136,5**					

* others include: Agostinelli, Anocic, Bamba Mory, Besuijen, Boer, Bouah, Calafiori, Fuzato, Coly Sidy Keba, Masangu, Nani, Persson, Riccardi and Seck

4

Derivative approach for equity value determination

From an economic-financial modelling point of view, the Merton model identifies the market value of a company (Equity Value, or Market Capitalization if listed) as a call option on the value of the assets (entire corporate value or Enterprise Value) with strike price equal to the value of the debt and with expiry the debt maturity. <u>The present value of the Equity is expressed as the present value of the average probable difference between the value of the entire asset at the maturity of the debt and the value of the debt itself. Equity can have a positive present value even if the current value of the assets is lower than the current value of the debt because, at the maturity of the debt, the market value of the assets (Enterprise Value) can exceed, with a certain probability, the debt value that grows only with the interest rate.</u>

The basic assumption of the methodology is that, if the asset company value should be lower than the debt value, the property would have no economic interest in continuing in the company activity and therefore the value would be at the minimum zero given it is not obliged to proceed with capital increases, unless there is a prospective value to be protected. Basically, merges just how the current value of the company can be higher than the current difference between assets and liabilities since, with a certain probability, the assets will grow more than the debt capitalized for its contractual interest rate.

It is an approach based on market evidence that leads to an evaluation of an equivalent company to that of discounting the company's future cash flows by means of a discount factor that takes into account of the uncertainty of these flows according to an adequate relationship between risk (variance of cash flows) and return (discount rate).

<u>If the valuation of the Equity can be conceptualized as a call option on the underlying corporate value compared to a strike represented by the value of the debt at maturity, by extension the value of the Equity can be divided into subsets represented specific categories of assets with which to associate specific values</u>: i) the most certain part using, instead, for the enhancement the approach of cash flows discounted at an interest rate that reflects the modest variance of these flows; ii) the most random part following the approach of the call options to evaluate their value.

5

Derivative approach under IAS 39 and IFRS 9

From an Equity value point of view, the future value of the more random assets could emerge immediately, despite the International Accounting Standards do not allow to be valued before their realization according to the logic of prudence (IAS 39 and IFRS 9), unless this derives from a market exchange contract of this value.

For example, you could enter into forward contracts that fix the price of the asset at maturity (without predicting its sale) or even assign call options on certain assets that would collect premiums and reveal the unexpressed values of these assets considering that:

- the cash flow method cannot capture the specificity of these assets, necessarily having to reason for average values;
- the international accounting standards do not allow in the financial statements until realization/marketing.

The market equity value is already, per se, an expression of the presence of an asset value not reflected in the financial statements due to prudential constraints imposed by the IAS, however a more specific identification of this subset would have a positive effect on the capitalization suffering from penalizing weightings and discount for uncertainty.

Assets valuable according to the optional approach are those with greater uncertainty for which can be distinguished:

a. a linear motion, identifiable in algorithms that simply identify the trend of variation of the value over time, and in
b. a stochastic motion that determines a future value independent of time and closely linked to parameters the current value and the variance of the probability distribution which is assumed to be of the normal type.

6

IAS 39 and IFRS 9 derivatives possible investigation

Conditional to a labour law check on the transferability of the economic rights referred to above in terms of capital gain / loss of the underlying asset compared to an identified strike (which may or may not coincide with the value of the debt allocated to this asset), the possibility could be assessed to treat the players' park as a set of assets whose value is characterized in part by a linear trend and in part in stochastic random motion.

This set could be divided by individual asset in order to facilitate its enhancement according to the specific linear growth algorithm already developed and through the random component to be identified. The correlation between the different assets, distinguished by role and age, not being equal to one, would also allow o build a portfolio that optimizes the risk / return profile (less risk for the same return or greater return for the same risk).

An analysis of the financial statements of the sector companies in fact shows that the debt is equal to the sum of the multi-year rights to the sports performances of the players, whose valuation is however conservatively conserved at the amortized purchase values, or if lower than the presumed realization.
It is not possible, therefore being able to highlight the value of the pool of players who are not only functional to the company business, but that their own transfer and acquisition activity is increasingly part of the management characteristic of a football club. This has no longer been the case for extraordinary income but rather for ordinary activities whose correct valuation in the financial statements is prevented by the application of the IAS.

Crucial could be to investigate together with Auditors if the use of a derivative contract on a specific underlying could be able to write at its fair value the item in the balance sheet because traded at a specific market value.

7

Pricing through Black and Scholes formula

Through the Black and Scholes equation, the values of any derivative on these assets can be determined, whether they are linear (forward or short selling) or optional (call / put long / short) or digital (all-or-nothing).

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0$$

*Where V is the price of the option as a function of stock price S and time t, r is the risk-free interest rate, and σ is the volatility of the stock.

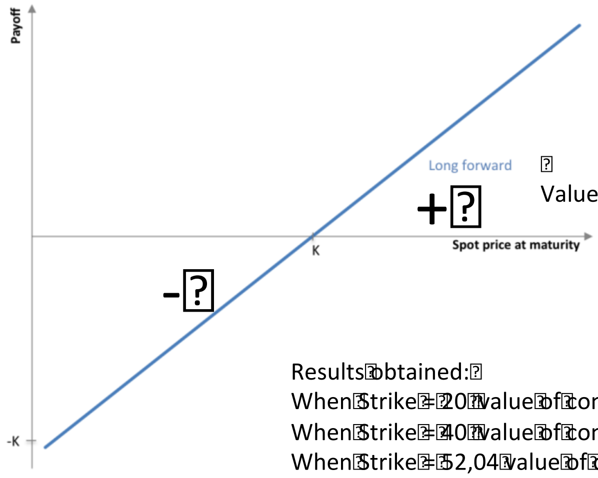
In relation to the type of derivative to be valued, a specific formula will be determined which can be used to quantify the contract value that is assumed to be able to finalize. **Different derivatives have different economic functions:**
A **FORWARD** sale of the asset defines the future value of the asset itself, a **forward contract lends itself to transferring the difference between the future value of a given asset with respect to an identified strike allowing to monetize the difference,** and therefore to remain neutral with respect to the strike identified. (See following example on ZANIOLO)
A **CALL OPTION**, on the other hand, represents the positive present value of the sale of the difference between the future value of the asset and a given strike and, if sold, call option allows you to collect the reward and set the forward value and, **if acquired, allows to focus on the appreciation of an asset and to guarantee the purchase of the asset in the event of appreciable appreciation**. (See following example on PELLEGRINI)
The purchase of a **PUT OPTION ensures that the value of a given asset is not lower than its strike at maturity** because with this instrument the right to be reimbursed is acquired for the negative difference between the value of the asset and the strike, at the same time it can allow profit to be made from valuations considered excessive for an asset that is estimated to depreciate.
The DIGITAL OPTION allows instead to place bets on the future value of the asset which will involve the payment of an initial premium against the possible collection of another value in the event of a favorable event, or of nothing in the event of an opposite event.

8

Forward Contract written on Nicolò Zaniolo's Value

A forward is a non-standardized contract between two parties to buy or sell an asset at a specified future time at an agreed price at the time of conclusion of the contract.

Payoff Forward Contract = S_t – K



Value of Forward contract = $S0 - \frac{K}{e^{2*0,02}}$

Assumptions:
Volatility of 39,23%
Stock price of 50 mil
Risk Free Rate of 2%
Time to Expiration of 2 years

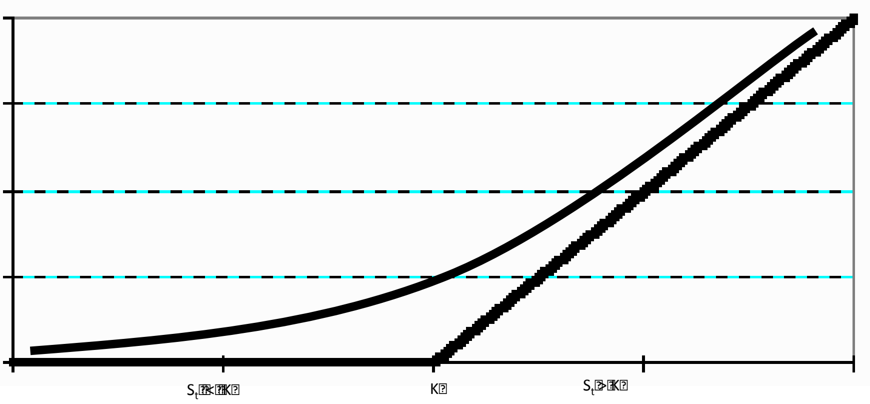
Results obtained:
When Strike = 20 value of contract = 30,78 mil
When Strike = 40 value of contract = 11,57 mil
When Strike = 52,04 value of contract = 0 mil
When Strike = 60 value of contract = -7,65 mil
When Strike = 80 value of contract = -26,86 mil

9

Call Option written on Lorenzo Pellegrini’s value

Call options are financial contracts that give the option buyer the right to buy an asset or instrument at a specified price at a specific maturity.

Payoff Call Option = max (0, S_t – K)



Assumptions:
Volatility of 36,4%
Stock price of 40 mil
Risk Free Rate of 2%
Time to Expiration of 2 years

Call Option Formula under Black & Scholes Method:

$$C(S_t, t) = N(d_1)S_t - N(d_2)PV(K)$$

$$d_1 = \frac{1}{\sigma\sqrt{T-t}}\left[\ln\left(\frac{S_t}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)(T-t)\right], \quad d_2 = d_1 - \sigma\sqrt{T-t}, \quad PV(K) = Ke^{-r(T-t)}$$

Results obtained:
When Strike = 60 value of call = 3,37 mil
When Strike = 40 value of call = 8,73 mil
When Strike = 20 value of call = 21,26 mil

10

Appendix - Calculation of Derivatives Values

	31 January 2020	30 June 2019	31 January 2019	30 June 2018	31 January 2018	30 June 2017	Standard dev.	Volatility
N. Zaniolo	50,00	40,00	10,00	1,00	0,05	0,05	22,35	44,70%
M. Ødegaard	50,00	15,00	4,00	4,00	3,50	1,50	18,75	37,50%
M. Mount	45,00	12,00	10,00	4,00	1,00	0,40	16,81	37,35%
D. van de Beek	55,00	40,00	20,00	14,00	9,00	4,00	19,79	35,97%
D. Olmo	35,00	30,00	15,00	6,50	3,50	1,50	14,22	40,64%
Mean Value @ c.date	47,00	27,40	11,80	5,90	3,41	1,49		39,23%
% Difference closest c.date	72%	132%	100%	73%	129%			
Average Difference	101%							
Standard Dev Differences	29,2%							

Volatility obtained comparing Nicolò Zaniolo with 4 other football players suggested by Transfer Market is 39,23%

	31 January 2020	30 June 2019	31 January 2019	30 June 2018	31 January 2018	30 June 2017	31 January 2017	30 June 2016	31 January 2016	30 June 2015	31 January 2015	30 June 2014	31 January 2014	Standard dev.	Volatility
L. Pellegrini	40,00	35,00	35,00	28,00	23,00	12,00	6,00	2,50	1,00	0,25	0,20	0,18	0,00	15,73	39,34%
N. Barella	45,00	36,00	30,00	25,00	15,00	5,00	2,50	1,30	0,80	0,80	0,50	0,20	0,20	16,02	35,60%
S. Sensi	30,00	18,00	7,00	3,50	4,00	4,50	4,00	4,00	3,50	0,60	0,45	0,45	0,40	8,51	28,37%
B. Cristante	25,00	25,00	25,00	25,00	15,00	3,00	2,00	2,50	3,50	4,00	4,50	3,00	3,00	10,38	41,51%
Mean Value @ c.date	35,00	28,50	24,25	20,38	14,25	6,13	3,63	2,58	2,20	1,41	1,41	0,96	0,90		36,20%
% Difference closest c.date	23%	18%	19%	43%	133%	69%	41%	17%	56%	0%	48%	6%			
Average Difference	39%														
Standard Dev Differences	36%														

Volatility obtained comparing Lorenzo Pellegrini with 3 other football players suggested by Transfer Market is 36,20%

11

CONCLUSION

As said in the work, trough Black & Scholes model applied to two different players (underlyings) on two different derivative instruments (forward and call option), appears evident as predictable and more manageable my become the market value of football players and soccer companies.

The company presentation and the case study mentioned above, to the Chief Executive Office Mr Fienga and to the Chef Financial Officer Mr Francia of A.S. Roma S.p.A. occurred at beginning of 2020, would point out new financial frontier on which may be the case to investigate further.

In particular could be invested time in better understanding with auditors if and how would be possible trough financial derivatives to express the mark to market value of football payers remaining compliant with International Accounting Standards, but benefiting of the transparency able to eliminate market discount in evaluation and attract considerable more financial lenders to soccer world.

What it is important to clarify is how to predict the expected market future value of the multi-year rights to the sports performances of the players and its relation with the salary of players, their age and potentiality.

The Transfer Market game gives a good proxy of market value of players if compared with the most recent transaction because it allow to identified the comparative value of the players. Given that the salary is a stable percentage of multi-year rights to the sports performances, the latters could be representative of the yearly all in cost the soccer companies may be prepared in to engage a player. The limits that immediately come out pertain the revenues coming from merchandising or value of ticket sold that are, indeed, already included in the fair play rule of being net income positive in he medium run.

Would be useful to better investigate and understand if there is a trend in value increase/decrease of multi-year rights to the sports performances in relation with the age and the football player’s role.

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