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A General Equilibrium Analysis on the Consequences of Demographic Transition in Italy

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1 Introduction

It is no secret that most countries in the *OECD* face serious changes regarding the demographic composition of their economy. Declining growth rates of populations reach values below zero and developments in the field of medicine increase life expectancies. As a matter of fact, as time goes by, the number of senior citizens in those countries grows quickly, without being matched by a similar increase in the number of working individuals. This demographic shift towards an older population is causing some serious concerns regarding the future of the systems of Social Security being employed. As the tax base shrinks, and as the number of retirees grows, governments all around the world start questioning the structure of their pension systems, as they struggle to gather funds to sustain them.

The main question being argued is one that relates to the way in which Social Security should be funded and organized. Many pension systems, including Italy, belong to the structure defined as *Pay-As-You-Go*. In *The Economics of Pensions*, Barr and Diamond (2006) define an *Unfunded*, or *Pay-As-You-Go* (*PAYG*), pension system in the following way: "As an individual contributor, a worker's claim to a pension is based on a promise from the State that, if he pays contributions now, he will be given a pension in the future. The terms of the promise are fairly precise, being set out in each country's Social Security legislation (although subject to legislative change). From an aggregate viewpoint, the State is simply taxing one group of individuals and transferring the revenues to another, whether viewed on an annual or a lifetime basis." Under certain conditions, *PAYG* systems can guarantee a mostly satisfactory outcome for everyone. Samuelson (1958), in a simple overlapping generations model, showed that, as long as the economy has a growing population and wages are growing, each generation is expected to receive a greater benefit than the prior, forever.

This, of course, is not what happens in reality.

In this thesis, an analysis of the current and future situation of the Italian Social Security system will be provided, by running a simulation employing a finite horizon life-cycle model, evaluating the steady state in the year 2015 and in the year 2075, in order to be able to make an inference regarding what the situation will be like in the future.

The main building block of this thesis, as well as the main inspiration behind it, is the model described by Kitao (2015). In her paper, Kitao follows her assessment of the current pension system in Japan with a policy proposal, suggesting a transition towards a *Funded* scheme, and, consequently, a simulation of the transition towards an alternative steady state. Japan resembles Italy in a few key ways. First of all, both countries are plagued with negative population growth rates and ever-increasing life expectancies. As a consequence, both countries have a large portion of senior citizens. Life expectancy in Japan is 85 years, while in Italy it is 84. Median age is 47.3 in Japan, which is second only to Monaco, and 45.5 in Italy. Furthermore, both are characterised by *Unfunded* Social Security systems, which differ only in the way benefits are calculated and distributed. While Japan's pension system takes the form of a *Defined Benefit* (*DB*) scheme, in which benefits are calculated as a function of earnings, Italy's pension system, after the 1995 *Riforma Dini*, which marked the passage from a *DB* to a *Defined Contribution* (*DC*) pension system, can be defined as *Notional Defined Contribution* (*NDC*). In an *NDC* pension system, benefits are instead a direct function of the contributions of a single individual. The amount contributed by the individual is part of a fictitious, or *Notional*, account, which awards the owner with an equally fictitious interest rate. In Italy, notional accounts produce each year an interest rate that equals a five-year moving average of nominal *GDP* growth.

Any discussion concerning policy reform is often complicated and needs to address many different issues at the same time in order to be worthwhile. Kitao, as many before her, argue that a privatization of the pension system would bring about not only some solace to the finances of the government, but would also increase the overall welfare of individuals. Feldstein and Samwick (1997) are in favor of the same argument, but even they are not safe from criticism. As Aiyagari points out, one needs to be careful in choosing the correct returns to be used in simulations, and needs to keep in mind the welfare of all generations involved in the transition. Comparing steady states with different premises is not particularly useful and the techniques necessary to simulate a transition between steady states are burdensome and time consuming (e.g. Conesa and Krueger (1999) and Nishiyama and Smetters (2005)). As a consequence, such an analysis will be omitted from this thesis, and the focus will be put solely on the consequences of the policy that is currently adopted.

This thesis will be structured in the following way: in Section 2, the simple overlapping generations model developed by Samuelson will be employed in order to give a rough sense of the way in which *PAYG* Social Security Scheme are organized, in Section 3, a thorough description of the model used by Kitao and in this thesis will be provided, in Section 4 there will be an explanation concerning the method used to solve numerically the model, in Section 5 a justification of the calibration of the model to the Italian economy will be detailed, consequently, in Section 6 there will follow a comment of the numerical results. The simulations run in this thesis show that, if the formula used to calculate benefits is kept constant, Social Security expenditure as a percentage of *GDP* will increase from 23% to 40% and the taxes levied on consumption will increase from 22% to more than 50%. In Section 7 a short mention concerning the robustness of the results will be given, and, lastly, in Section 8 the thesis will be concluded with a brief recap of all its Sections.

2 Samuelson's Model

In this section, Samuelson's OLG model, as described by Feldstein and Liebman (2002) will be used to give a brief overview of how *PAYG* social security schemes are structured.

In this model at each period t two generations overlap: the generation which is young at t and which is able to work and to generate an income which is necessary to consume, and the generation which is old at t which, instead, has worked in the previous period and has no way to generate an income for itself. It will be assumed that the economy has full employment and that the population grows according to

$$L_{t+1} = L_t(1+n) \tag{1}$$

Where n represents the growth rate. There is no technology to transfer consumption across periods. For this reason, a simple system of *PAYG* Social Security is in place. At each period the young generation pays a certain percentage of its wage as taxes in order to finance the consumption of the old generation. The aggregate tax expenditure at period t, T_t, will then equal

$$T_t = \theta w L_t \tag{2}$$

where θ represents the tax rate and w represents the wage. For the purpose of this example at first, the level of wages will be considered to be constant. Since this is a *PAYG* scheme, the pension benefit that the old generation will be receiving at time t, B_t, will equal T_t.

At period *t+1* the previously young generation will receive its pension benefit,

$$B_{t+1} = T_{t+1} = \theta w L_{t+1} = \theta w L_t(1+n) \tag{3}$$

Thus, in this very simple model, the ratio of benefits received to the taxes paid by any given generation, or the Implicit Rate of Return, is constant and it equals:

$$\frac{B_{t+1}}{T_{t+1}} = \frac{T_{t+1}}{T_t} = \frac{L_{t+1}}{L_t} = (1+n) \tag{4}$$

If, instead, technological progress is added to the model, it is assumed that the increase in productivity will increase wages at a rate of g

$$w_{t+1} = w_t(1+g) \tag{5}$$

With this simple modification, it is clear that the ratio between benefits and taxes equals

$$\frac{B_{t+1}}{T_t} = (1+g)(1+n) \approx 1+n+g \tag{6}$$

For simplicity of notation, γ will be defined as:

$$\gamma \equiv (1+g)(1+n) - 1 \tag{7}$$

In this model the implicit rate of return of social security between each period equals $1+\gamma$.

The table in the next page shows in a schematic way the way in which the Social Security system in this very simple model taxes one generation and operates the distribution of benefits to the other.

Generation	Period t	Period $t+1$	...	Period $t+n-1$	Period $t+n$
t	$-T_t$	$+T_t(1+\gamma)$			
$t+1$		$-T_t(1+\gamma)$	$+T_t(1+\gamma)^2$		
...					
$t+n-1$				$-T_t(1+\gamma)^{n-1}$	$+T_t(1+\gamma)^n$
$t+n$					$-T_t(1+\gamma)^n$

Now that the way in which *PAYG* system work has been described, in the following Section, a more complex complex model is used in order to perform a more accurate analysis. It is already clear that population growth represents a key factor in the functioning of *PAYG* Social Security systems. How big its role is will be further analyzed in the following sections.

3 The Model

3.1 Economic environment

Demography

The economy represented in this model is composed of overlapping generations of individuals, who enter the working force at age $j = 1$ and live up to the age of J. The coefficient s_j represents the survival probability of an individual, that is, the probability that an individual aged j will survive one more year. Once an individual dies, his asset portfolio is assumed to be completely lost, differently from the way in which this is modeled in Kitao's paper, where assets are distributed as a lump-sum bequest to all survivors. This economy, for the sake of simplicity, is assumed to be closed.

Endowments

Individuals are endowed at each period with one divisble unit of time, which they can choose to allocate to work or to leisure. Earnings are computed as $y_t = \eta_j h w_t$. In this formula, h represents endogenously chosen hours of work, w_t represents wages, determined in a competitive market and set to equal the marginal product of labor in period t, and η_j is the deterministic age-specific level of productivity of an individual.

Individual's Preferences

In each period, individuals obtain utility from consumption and leisure, $u(c, h)$. Individuals maximize the sum of the discounted utilities they obtain each period of their lifetime, $\sum_{j=1}^{J} \beta^{j-1} u(c_j, h_j)$, where β represents their subjective discount factor and c_j and h_j represent the level of consumption and the hours of work at age j, respectively.

Government

In the economy presented in this thesis, the government runs a *Pay-As-You-Go* public pension system, in which it awards a pension $ss_t(y)$ to individuals at, or above, age j_R at which they retire and leave the work force. Benefits are defined as a function of average income during the lifetime of the individual.

To finance this public pension system, the government taxes individuals in four ways:

1. Labor income is taxed at a fixed rate τ^l;
2. Income from capital rented to firms is taxed at a fixed rate τ^k;
3. Income from the interest rate on government debt at a rate τ^d;
4. Consumption is taxed at the rate τ_t^c.

Another way in which the government is able to finance itself is through the issuance of public debt D_{t+1}, which pays a riskless interest rate r_t^d. These various financing tools are used to fund the government's expenditure. The government's expenses are defined as the purchase of goods and services, G_t, repayments on the interest and principal of previously issued public debt, D_t, and, as already mentioned, payment of public pension benefits.

The interest rate that is paid on government debt and the one that is paid on capital rented to firms are distinguished, as in Kitao (2015) and Braun and Joines (2014). It is assumed that individuals will allocate an exogenous fraction ϕ_t of the assets in their portfolio to government debt and a fraction $(1-\phi_t)$ to firms' capital. The gross after-tax rate of return that will be generated by each unit of investment is defined as:

$$R_t = 1 + (1 - \tau^k) r_t^k (1 - \phi_t) + (1 - \tau^d) r^d \phi_t$$

The government budget constraint is satisfied at every period and it is defined as:

$$G_t + (1 + r^d) D_t + S_t = \sum_x \{ \tau^l y_t(x) + [\tau^d r^d \phi_t + \tau^k r^k (1 - \phi_t)] a_t(x) + \tau_t^c c_t(x) \} \mu_t(x) + D_{t+1}$$

where S_t represents total public pension expenditure, $S_t = \sum_x ss_t(x) \mu_t(x)$ and $\mu_t(x)$ is a measure of individuals in state x at time t. The original model included a variable M_t, representing medical expenditure, which, for the sake of simplicity was omitted for the purposes of this thesis.

When computing an equilibrium it is necessary that at least one variable adjusts to balance the government's budget constraint. In the computation of the first steady state, τ^l will be such variable. All other variables will be kept fixed and exogenous. In the computation of the second steady state, τ_l will be fixed at the level determined in the previous step, while τ_t^c will adjust to balance the government's budget constraint. The increase in government expenditure due to a relative increase in Social Security spending, S_t, will be funded with the use of indirect taxation since it is shown by Kitao (2015) that using the labor income tax would be distortionary, shrinking labor supply, which would already be scarce due to the negative growth rate of the population. Conversely, using budget deficits to finance it would cause the government debt to explode and become quickly uncontrollable, as shown by İmrohoroğlu, Kitao and Yamada (2016).

3.2 Individual's Problem

In each period, individuals buy and accumulate a one-period riskless asset, a_t, which contains both government-issued public debt as well as capital rented to firms. This asset earns a return equal to R_t, which was defined in the previous subsection. Individuals face a tight debt limit, $a_t \geq 0$, therefore they cannot borrow against future income and their asset balance must be greater or equal than zero in each period.

The state vector of an individual $x = \{j, a\}$ is composed of two elements: j, age, and a, assets carried over from previous period. Individuals in each state optimally choose consumption, the allocation of their unit of time to work and leisure, and saving, in order to maximize their lifetime utility. The individual's problem is described by the Value Function, $V(x)$, which is defined as follows:

$$V(j, a) = max_{c,h,a'}\{u(c, h) + \beta s_j V(j + 1, a')\}$$

subject to:

$$(1 + \tau^c)c + a' = Ra + (1 - \tau^l)y + ss$$

$$y = \eta_j hw$$

$$a' \geq 0$$

Pension benefits are zero for individuals which are younger than j_R.

3.3 Equilibrium

In this section a competitive equilibrium will be defined. A competitive equilibrium consists of individuals' policy functions $\{c_t(x), h_t(x), a_{t+1}(x)\}$ for each state vector

x, factor prices $\{r_t^k, w_t\}$, consumption tax τ_t^c, labor income tax τ^l, the measure of individuals over the state space $\mu_t(x)$, for a given sequence of demographic parameters $\{s_j\}_{j=1}^J$, and government policy variables $\{G_t, D_t, \tau^k, \tau^d, S_t\}$, such that:

1. Individual's decisions represent the solution to the aforementioned optimization problem;

2. Factor prices are determined in a competitive market and set to equal the marginal productivity of capital and labor:

$$r_t^k = \alpha \left(\frac{K_t}{L_t}\right)^{\alpha-1} - \delta$$

$$w_t = (1-\alpha)\left(\frac{K_t}{L_t}\right)^{\alpha}$$

3. The capital and labor markets clear:

$$K_t = \sum_x a_t(x)\mu_t(x) - D_t$$

4. The consumption tax or labor income tax satisfy the government's budget constraint;

5. The goods market clears:

$$\sum_x c_t(x)\mu_t(x) + K_{t+1} + G_t = Y_t + (1-\delta)K_t$$

4 Methodology

The task of solving this model presented itself to be challenging in two different ways. The first concerns finding a solution to the individual's problem. The second, instead, concerns the search for the two steady states. In this section an explanation will be provided as to how such problems were solved.

4.1 Individual's Problem

Being in the context of finite horizon dynamic programming, it was not possible to find an analytical solution by guessing a functional form, or by employing backward recursion and finding a solution with pen and paper. Instead, the problem was solved through backward and forward recursion following İmrohoroğlu et al (1995), Sargent (1987) and Stokey and Lucas (1989). The policy function of each individual was computed by operating a single recursion beginning from the last period of life $j = J$ until the first period was reached $j = 1$.
First, it is assumed that the functional form of the value function in the first period after death, $j = J + 1$, is known a priori to be identically equal to 0. That is:

$$V_{J+1}(\cdot) \equiv 0$$

Beginning here, the solutions are found by computing a recursion starting from age $j = J$ until age $j = 1$, following:

$$V_j(a) = max_{\{c,h,a'\}}\{u(c, h) + \beta s_j V_{j+1}(a')\}$$

subject to:

$$(1+\tau^c)c + a' = Ra + (1-\tau^l)y + ss$$

$$y = \eta_j hw$$

$$a' \geq 0$$

Once we substitute for c using the budget constraint, the problem reduces to the choice of the two control variables a and h. Then, the possible values of the policy functions were discretized. That is, it is assumed that $a \in S \equiv \{s_1, s_2, ..., s_n\}$, with S being a discrete set of dimension $n \times 1$, with $n = 200$, and $s_1 = 0.001$ and $s_n = 10$. Therefore, it is composed of equidistant points with each step set to equal 0.0503. Conversely, $h \in W \equiv \{w_1, w_2, ..., w_m\}$, with W being a discrete set of dimension $m \times 1$, with $m = 10$, and $w_1 = 0.05$ and $w_m = 0.8$. For what concerns the size of these vector spaces, n was chosen to equal 200, since higher values would slow down the process considerably, and lower values led to numerical mistakes. İmrohoroğlu et al (1995) chose a value of 602. On the other hand, a much lower size was required for m, as the problem did not appear to be as sensitive to the finesse of the grid being employed in the grid-search. The minimum and maximum values, instead, were chosen as to avoid any boundary solution in both cases.

The policy function takes the form of an $n \times 1$ vector for individuals aged j_R or above, and it contains the utility-maximizing asset portfolio for each level of beginning-of-period assets. For individuals aged j_R or below, instead, the policy functions are two $n \times 1$ vectors. The first contains optimal asset holdings and the second, instead, contains optimal labor supply.

After assuming that every individual dies with certainty after age J, the value function in period J equals:

$$V_J = max_{\{c_J, a_{J+1}\}}\{u(c_J, 0)\}$$

subject to:

$$(1+\tau^c)c_J = Ra_J + ss$$

$$a_{J+1} \geq 0$$

The solution to this problem is an $n \times 1$ vector policy function for age-J individuals, A_{J+1}. This is a vector of zeros, since there is no bequest motive and death is assumed to be certain after age J is reached. The value function at J is an $n \times 1$ vector whose entries equal the value of the utility function as evaluated at $Ra_J + ss$, with a_J taking on values $s_1, s_2, .., s_n$. The value function is then passed on to the previous period $J-1$. The policy function is found by solving:

$$V_{J-1}(a_{J-1}) = max_{\{c_{J-1}, a_J\}}\{u(c_{J-1}) + \beta s_{J-1} V_J(a_J)\}$$

subject to:

$$a_J + (1+\tau^c)c_{J-1} = Ra_{J-1} + ss$$

$$a_J \geq 0$$

The policy function is found as follows: for $a_{J-1} = s_1$, the value of $a_J \in S$ that solves the above problem is obtained by evaluating the objective function at each point in the grid S and finding the maximum. This value is then reported as the first element of the $n \times 1$ policy function A_J. By repeating this procedure for all possible initial levels of asset holdings $a_{J-1} = s_1, s_2, .., s_n \in S$, the entire vector A_J is filled. At the same time, the age-$J-1$ value function V_{J-1} is found as an $n \times 1$ vector, with entries corresponding to the right-hand-side of the above objective function evaluated at the policy function A_J. In practice, this process amounts to evaluating the objective function for each possible combination of initial asset holding and investment decision at period $J-1$, creating an $n \times n$ matrix, and then, for every entry of the initial holding level, finding the investment decision which maximizes utility.

Working backwards, the process arrives at age $J_R - 1$, the age which comes immediately before the beginning of mandatory retirement, J_R. The problem to solve is:

$$V_{J_R-1}(a_{J_R-1}) = max_{\{c_{J_R-1},h_{J_R-1},a_{J_R}\}}\{u(c_{J_R-1}, h_{J_R-1}) + \beta s_{J_R-1}V_{J_R}(a_{J_R})\}$$

subject to:

$$a_{J_R} + (1+\tau^c)c_{J_R-1} = Ra_{J_R-1} + (1-\tau^l)y_{J_R-1}$$

$$y_{J_R-1} = \eta_{J_R-1}h_{J_R-1}w$$

$$a_{J_R} \geq 0$$

At this point, the problem needs to be solved to find a second policy function vector, the $n \times 1$ vector H_{J_R-1}. The policy functions are found in a similar way as was previously described: for $a_{J_R-1} = s_1$, the value of $a_{J_R} \in S$ and of $h_{J_R-1} \in W$ are obtained by evaluating the objective function at each point in the grid $S \times W$. This process is repeated for each possible level of initial asset holdings $s_1, s_2, ..., s_n$, until the vectors A_{J_R} and H_{J_R-1} are filled. The value function V_{J_R-1} is found as an $n \times 1$ vector with entries corresponding to the right-hand-side of the above objective function evaluated at the policy functions A_{J_R} and H_{J_R-1}. This was put in practice by evaluating the objective function for each possible combination of initial asset holding, investment decision and labor supply, creating an $n \times n \times m$ matrix for every $j < J_R$. For every entry of the initial asset holding, the combination of investment and hours of work were extracted.

Once the matrices of the objective functions for each period were constructed, forward recursion was used, starting at an initial asset holding at age $j = 1$, $a_1 = 0$, and the optimal paths of investment and labor supply were obtained.

4.2 Steady State

The way in which the Steady State was computed in both demographic scenarios is based on an algorithm described by Auerbach and Kotlikoff (1993), known in the literature as *Gauss-Seidel* method. The algorithm starts by making a guess on a subset of endogenous variables, and treats those variables as exogenous in some of the equations. Through this simplification it becomes easier to solve for the system, as well as for the initially guessed exogenous variables. Once the solutions to these guessed exogenous variables equals the guesses made in the first place, a stationary equilibrium is considered to be found. The variables that were guessed in this process are: the initial level of aggregate capital stock, K, and the initial level of aggregate labor supply, L. This exogenous guesses are then used to compute the marginal product of capital, r^k, and the marginal product of labor, w. These variables play a focal role in the individual's problem, as they determine both the amount of assets the individual will hold, as well as the labor she will supply. Once the optimal choice of the individual is computed, those choices are aggregated, and new values of aggregate labor supply and aggregate capital stock are found. If those values match those of the guesses previously made the initial guess can be considered a "true" solution. If, instead, the solution found and the initial guess do not match, the initial guess is corrected by some amount, depending on the solution obtained, and the process starts over from the newly made guess.

The process used in this thesis follows approximately İmrohoroğlu et al (1993):

- Step 1: a tolerance level, or convergence criterion, $\epsilon > 0$ is chosen. The tolerance chosen for this study was $\epsilon = 0.01$. The value was established after a process of experimentation. Using greater values would have led to significant numerical errors, whereas smaller values would have led the number of iterations necessary to find a solution to increase significantly.

- Step 2: two learning coefficients are chosen, θ_L and θ_K. These coefficients are used to adjust the initial guess of Capital, K, and Labor, L. The values chosen for the evaluation of the first steady state are $\theta_L = 0.4$, and $\theta_K = 0.1$. On the other hand, since the second steady state proved to be more sensible to the size of the learning factor, the values had to be adjusted along the way. The adjustment value of Labor θ_L, was kept fixed at 0.4, whereas the one for Capital θ_K was set to equal 0.1 until the twentieth iteration, to 0.01 until the forty-fifth iteration, and to 0.0005, until it reached the steady state. Once again, the values of the coefficients were chosen through experimentation. Especially θ_K needed careful calibration, as a lower number would have increased greatly the number of iterations necessary for convergence, while a greater number would have led the level of capital to alternate between values close to the steady state level, but never quite meeting the convergence criterion, due to the step size being too large.

- Step 3: two guesses are made for the initial candidates of steady state values of capital and labor, K_0 and average hours of work $\bar{h}$, which are then used to compute L_0. The initial values chosen were $K_0 = 1$ and $\bar{h} = 0.2$.

- Step 4: the guesses are used to compute factor prices w and r^k as the marginal product of labor and capital respectively. The values of factor prices that are found are consequently plugged in the individual's budget constraint.

- Step 5: the individual's problem is solved using the factor prices evaluated in Step 4.

- Step 6: individuals' choices are aggregated to compute the new levels of Capital and Labor, K_1 and L_1.

- Step 7: a check is made to see whether convergence was achieved. If $|K_0 - K_1| < \epsilon$ and $|L_0 - L_1| < \epsilon$ the algorithm stops and a solution is found. Otherwise the algorithm continues to Step 8.

- Step 8: initial guesses are adjusted with the learning factor as follows:

$$K_2 = (1 - \theta_K)K_0 + \theta_K K_1$$

$$L_2 = (1 - \theta_L)L_0 + \theta_L L_1$$

  These new values substitute the initial guesses K_0 and L_0 and the algorithm returns to Step 4. The two figures in the next page describe the path of capital towards convergence as the number of iterations increases in the first and second steady state respectively.

Figure 1: Iterations to Steady State (2015).

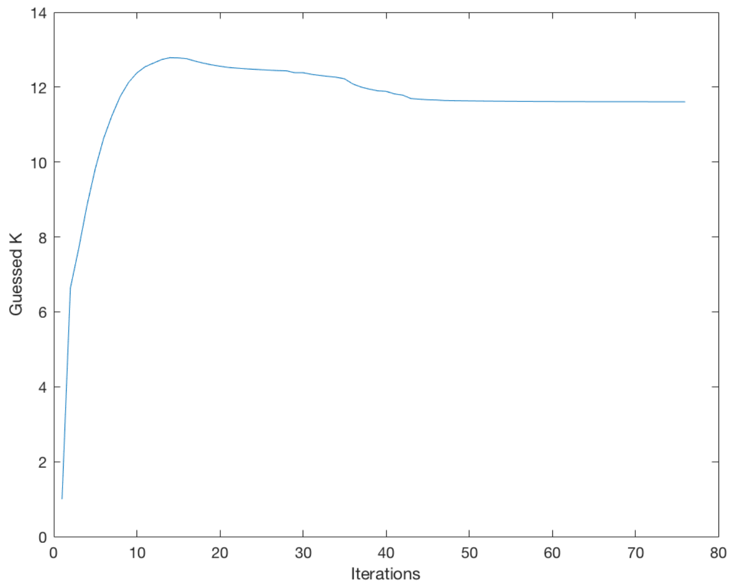
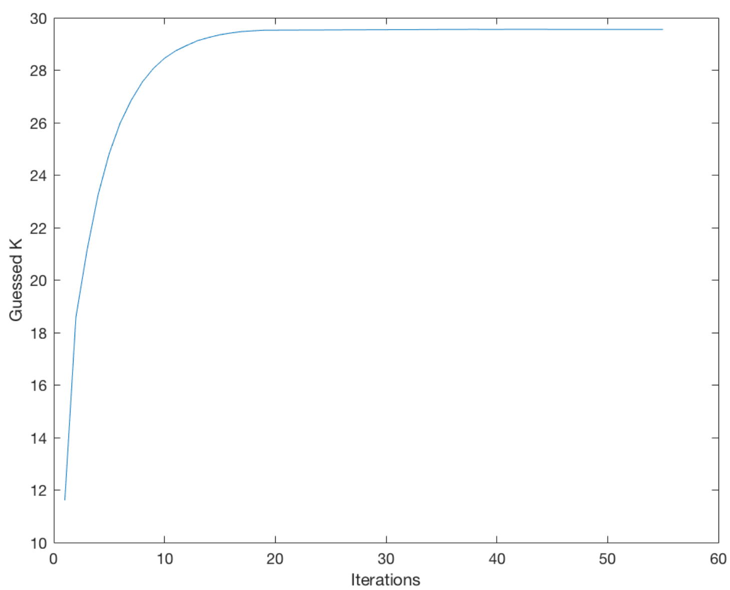


Figure 2: Iterations to Steady State (2075).



5 Calibration

In this section an explanation is provided as to how different variables of the model were parametrized. Some variables were chosen to reflect the specific empirics of the Italian economy, whereas some others follow the original paper of Kitao (2015).

Preliminaries

The unit of this model is an individual, representing a household as the head of the family. The two steady states represent a snapshot of two years: the first represents an approximation of the stationary equilibrium of the Italian economy in the year 2015, whereas the second represents some time after 2075, the last year for which demographic projections were available, once the economy reaches its new stationary equilibrium.

Demographics

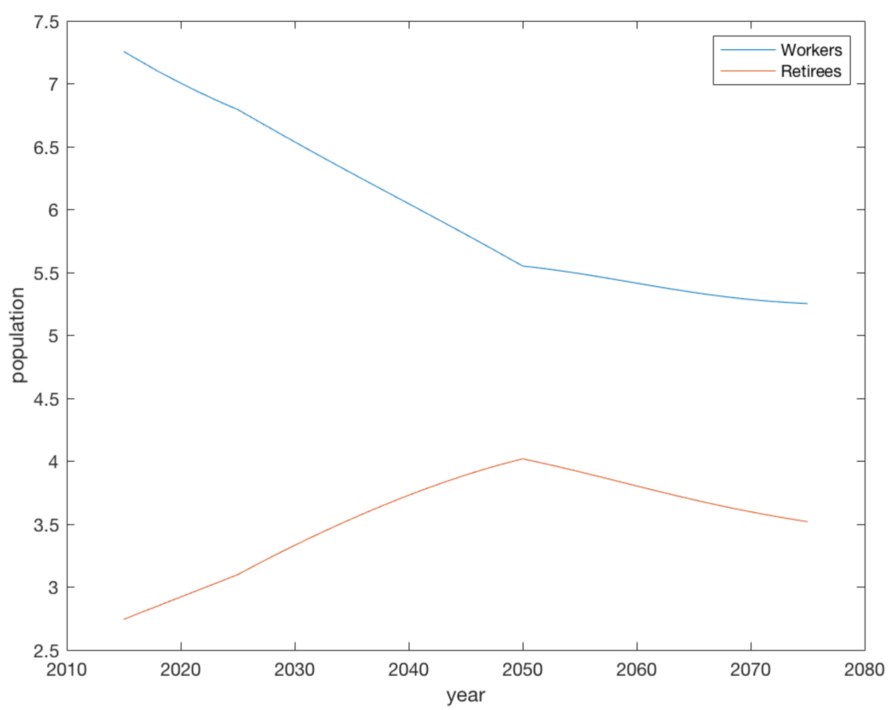
Individuals enter the working force at age 20 and live up to the maximum age of 90, after which they automatically die. Within the model, the age of 20 years old equals an age of $j = 1$. Once individuals reach the age of 66, or $j_R = 46$, they retire. The maximum age of 90 is represented by $J = 70$. Conditional survival rates, s_j, were obtained from the life tables published by *ISTAT* on their database. Only the data for 2018 was available, so that is what was used in both instances of steady state. Life tables would differ in the year 2075, but this was a necessary simplification. Population growth projections were instead obtained from Eurostat's database.

The dependency ratio, defined as the ratio of the number of individuals aged 65 and older to the number of individuals aged 20-64, was obtained from *OECD*'s database. Such projections were only available for the years 2015, 2025, 2050, and 2075. The values for the missing years were constructed with linear interpolation. Dependency ratios start in 2015 at 37.8%, quickly grow to 72.4% in 2050, only to fall back to 67% in 2075.

Population size was normalized at 10 in the year 2015 and dependency ratios were adapted in order to divide the population in two age groups: 65 years old and above, and 64 years old and below. This implies that the working age group includes individuals aged 0 to 19 years old as well. The simplification that was made has the effect of softening the picture that the original paper shows.

The graph below shows the evolution of the two age groups, according to the projections.

Figure 3: Interpolated demographic evolution.



Preferences, endowments, and technology

All variables of this section were assigned following the model of Kitao (2015), as a consequence all parameters are calibrated for the Japanese economy.
A non-separable utility function was assumed. The function takes the form:

$$u(c,h) = \frac{[c^{\gamma}(1-h)^{1-\gamma}]^{1-\sigma}}{1-\sigma}$$

The parameter γ determines the preference weight the individual puts on consumption relative to the one she puts on leisure and σ is related to the level of risk aversion. γ was set at 0.352, following Kitao, so that individuals aged 20 to 64 years old would spend approximately 40% of their disposable time for market work. In spite of this, in the simulations run for this thesis, individuals only spend approximately 30% of their time working. On the other hand, σ was set to 3.0, implying a relative risk aversion of 1.70 and an intertemporal elasticity of substitution of 0.59, in the range of estimates in the literature (e.g. Mehra and Prescott (1985)).
There is a lot of discussion concerning the value that β should take. In an overlapping generations setting, economic theory does not impose any restrictions on the size of this factor (e.g. Benninga and Protopapadakis (1990), Kocherlakota (1990) and Deaton (1991) discuss the restrictions on the subjective discount factor in economies with infinitely lived individuals), although traditionally it was assumed to be lesser than one. To cite some examples, Auerbach and Kotlikoff (1987), in their representative agent life-cycle model with certain lifetimes, set the discount factor β equal to 0.9852. Hubbard and Judd (1987), too, used the same discount factor with uncertain lifetimes. Nevertheless, more recent empirical evidence on the value of the discount factor suggest that a subjective discount factor greater than one is also a possibil-

ity (e.g Hansen and Singleton (1983), Hotz et al (1988), Hurd (1989)). Hence, the discount factor β was set to equal 1.021, so that the simulation would produce a capital-output ratio of approximately 2.5 in the first steady state, as was estimated for Japan by Hansen and İmrohoroğlu (2016). The capital-output ratio that was obtained in this simulation was equal to 2.67.

The age-specific labor productivity η_j is calibrated based on income data from *ISTAT*'s database. The data extracted reflect income, aggregated across the types of employment of the main income provider of the household by *ISTAT*, in 2017. Income levels reported were only available for four age groups: 34 years old and below, 35 to 44 years old, 45 to 54, and 55 to 64. Values were normalized in such a way to have an efficiency of 1.0 for the first age group.

Capital share in the production function α is set to 0.362 and the depreciation rate δ is set to 0.089, based on Hayashi and Prescott (2002).

The *Total Factor Productivity*, TFP, in the first steady state is assumed to equal 1.0. In the second steady state, instead, the TFP was calibrated as follows. Firstly, the projections of Italian output from the year 2015 to the year 2060 were obtained from the *OECD* databases. Secondly, growth rates were calculated. Next, the growth rates in the years 2061-2075 was assumed to be constant and equal to the mean of the growth rates in the years 2046-2060. Lastly, the product of all the growth rates obtained as described was used in place of TFP. In order to perform the robustness checks described in Section 7, this value of TFP was modified by multiplying each growth rate by a given coefficient, to show that the outcome obtained in this work is independent of growth.

Government - Pension system

In the economy described in this thesis, the government operates a *Pay-As-You-Go* pension system. Benefits, ss, are provided for each individual once he reaches the retirement age $j_R = 46$ (66 years old).

Benefits are determined as a function of average labor income throughout the individual's career. It is assumed that a fixed fraction of his income, ρ, is taken by the government and put in a notional account, which will in turn generate a pre-defined interest rate, κ, which in real life would equal a five-year moving average of GNP growth. The formula which was employed to perform the calculation of benefits is:

$$ss = \frac{\rho \bar{y} \sum_{j=1}^{j=j_R-1} (1+\kappa)^{j-1}}{j_R - 1}$$

The parameters used to compute pension benefits were calibrated as follows.

Contribution rates in Italy equal approximately 33%, with one third being contributed by the worker, and the remaining two thirds being contributed by the employer. It is assumed that the full burden of contributions is placed on the worker. Hence, the contribution parameter ρ is set to 0.33.

The interest rate being applied to each worker's contributions, κ, was set to equal 0.5% per year.

Once all parameters are set, the gross replacement rate, which is defined as the ratio of average pension benefits to average pre-tax earnings, equals 73.65%, not too far off from the value of approximately 76%, as reported by *OECD*.

Government - Expenditures, Debt, and Taxes

According to *ISTAT*, government expenditures accounted for approximately 20% of the Italian *GDP* in 2015. Therefore the ratio G_t/Y_t was set to match the data.

Government debt, D_t, was set to 130% of *GDP*, not too far off from the value reported by Eurostat, of approximately 135%. This was accomplished by adjusting the portion of the individual's asset portfolio that holds government issued debt, ϕ, until demand was equal to supply. The risk-free interest rate earned by public government debt, r^d, was set at 1%, despite the fact that in 2015 the 10-year Italian government bond's yield ranged between 1.1% and 2.5%, according to the Ministero dell'Economia e delle Finanze.

Following Kitao (2015), the capital income tax rate is set at 40%, in the range of estimates of effective tax rates in the literature. Similarly, the tax rate on interest income from government debt is set at 20%. The labor income tax rate was set to adjust to balance the government's budget constraint in the evaluation of the first steady state, settling at a value of 43.53%, a value which comes close to the average tax wedge for a single worker of 48%, as reported by *OECD*. In the second steady state, the tax on labor income was kept fixed at the previously obtained value. The consumption tax rate was set at 22% in the first steady state, to reflect the *Imposta sul Valore Aggiunto* (*IVA*), which equals 22% for most goods in the market. In the second steady state the tax rate was adjusted to balance the budget constraint of the government. The equilibrium value that was obtained under the new demographic scenario is 53.89%.

A combination of all previously mentioned taxes are used in order to finance the public pension system described in the previous subsection.

6 Numerical Results

In this section a brief comment on the numerical results of the two simulations is provided. The simulations were computed in their entirety through the use of ad hoc original Matlab scripts and functions.

The model is simulated in the demographic situation of 2015 in the initial steady state, whereas the second represents the year 2075. The transition between the two steady states was not computed.

The first steady state of 2015 was reached, without incurring in too many problems, in 76 iterations of the simulation, meeting successfully the convergence criterion of $\epsilon = 0.01$. The level of capital accumulation, K, under this first parametrization was found to be equal to 11.61. The level of labor that was found was equal to 2.48.

The second steady state of 2075, on the other hand, proved to be a little more complicated to reach. The simulation arrived in proximity of the steady state in approximately 20 iterations. However, once the level of capital stock fell in the interval $[29.50, 29.60]$, the simulation started performing steps that were likely too large, probably due to the learning factor being slightly too big. Several adjustments had to be made in order to allow the simulation to meet the convergence criterion that was set, reaching it at iteration 55. The value of capital accumulation to which the simulation converged to was 29.55. The labor supply, on the other hand, equaled 1.81.

The aggregated variables of output, Y, and capital, K, more than double in size, whereas labor supply, L, drops by approximately 30%. This is mostly due to the fact that while output and capital enjoy the benefits of a higher *Total Factor Productivity* in the last steady state, increasing both output in general, as well as the marginal product of capital, while the labor supply struggles to match the increase in wages due to a severe fall in the size of the population. These results differ significantly with respect to those of Kitao (2015), as she reports huge drops in all three macro-variables, due to a population that shrinks in 2200 to 22% of its initial value in 2010. The government's Social Security spending, S/Y, was to found to be equal to 22.98% in the first steady state, missing the mark of 16.2% of *GDP*, as reported by *OECD* in their database. However, this amount almost doubles in the second steady state, reaching 40.44%, due to the worsening of the dependency ratio in the time elapsed between the two. The government, while receiving a higher income from taxes, evaluates benefits in the first and second steady states according to the same formula, without modifying the contribution rates imposed on individuals. Following this, individuals, who are now richer due to the increase in TFP and, as a consequence, their wages, are able to invest more and consume more but, having paid higher contributions to Social Security, are also expecting higher benefits once they retire. The necessity of paying those benefits forces the government to increase Social Security spending and to finance such increase with an increase in indirect taxes, which reach 51.82%.

The original paper by Kitao (2015), upon which this simulation is based, presents results with mostly similar movements in the main variables. Kitao's simulation, calibrated on the Japanese economy, started with the first steady state in the year 2010 with the final one being in 2200. As a result, the magnitude of the changes in these variables is more severe than in the simulation performed in this instance. The estimated dependency ratio in the final steady state is 53%, as opposed to the value of 67% that is used in this thesis. Kitao shows a drop of approximately 75% in all aggregated variables. Similarly as was shown, wages increase, while the interest rate falls. Average work hours increase. Lastly, social security spending increases from 10% of *GDP* to 12.7%.

In the next page follow two tables representing the main variables in Italy and Japan in the two steady states, calculated in this thesis and in Kitao (2015), respectively.

Variables	Initial SS^{IT}_{2015}	Final SS^{IT}_{2075}
Consumption tax τ^c	22%	51.82%
Labor Income tax τ^l	45.3%	45.3%
Aggregate Output Y	1.000	2.536
Aggregate Capital K	1.000	2.546
Aggregate Labor L	1.000	0.729
Aggregate Consumption	1.000	1.666
Interest rate r^k	4.63%	4.57%
Wage w	1.000	3.879
Avg work hours	1.000	1.012
SS spending S/Y	22.98%	40.44%
Dependency Ratio	37.8%	67%

Variables	Initial SS^{JP}_{2010}	Final SS^{JP}_{2200}
Consumption tax τ^c	5%	17.6%
Labor Income tax τ^l	33.5%	33.5%
Aggregate Output Y	1.000	0.224
Aggregate Capital K	1.000	0.243
Aggregate Labor L	1.000	0.214
Aggregate Consumption	1.000	0.206
Interest rate r^k	5.58%	4.49%
Wage w	1.000	1.046
Avg work hours	1.000	1.050
SS spending S/Y	10.0%	12.7%
Dependency Ratio	40.0%	53.0%

7 Robustness

The results presented in the previous section were obtained by performing a simulation in which the growth of *GDP* follows exactly the projections made by *OECD*.
In order to check how sensitive the results are to the accuracy and size of the growth predictions, the same simulation was performed four more times, in such a way to accomodate for different realizations of growth predictions. The simulation was run under a zero growth scenario, in which the TFP in 2075 is equal to the TFP in 2015, then it was run again at a TFP growth that equals 50% the TFP growth in the baseline model, at 150% of growth and at 200% growth. These modifications were achieved by multiplying the growth in each year by a coefficient (0.0, 0.5, 1.5, 2.0) before evaluating the compounding.

It is possible to observe that, while all macro-variables vary significantly in the different growth scenarios, the main variables of interest in this analysis, namely the tax on consumption and the change in the ratio of Social Security expenditure to output, remain approximately equal across every simulation. Since the formula used to compute benefits is the same across all scenarios, and the contribution rate used is also the same, as growth differs, and with it wages, individuals now pay higher contributions in absolute terms and, as a consequence of the *Defined Contribution* structure of the Italian pension system, expect higher benefits once they retire. Hence, the only variable that affects significantly the government's expenditure and the related adjustment of financing, the tax on consumption, is the composition of the population: the dependency ratio.

The calculation used in the robustness checks are the same as the ones used in previous sections, with the same trial and error approach with the choice of the learning factor θ. It was set equal to 0.1 in the first iterations, but was often dropped to lower values to decrease the size of the step when the algorithm struggled to reach convergence. The finesse of the grid used in the individual's problem, that is, the number of points on the grid, and the maximum amount of asset holdings were increased progressively as the multiplier of growth increased. The highest number of points in the grid reached was 900, which significantly slowed down the speed of the calculations.

The table shown below shows the main variables found by the simulations in the various scenarios of growth.

Variables	$SS_{0\%}$	$SS_{50\%}$	$SS_{150\%}$	$SS_{200\%}$
Consumption tax τ^c	53.89%	52.66%	52.66%	52.70%
Labor Income tax τ^l	45.3%	45.3%	45.3%	45.3%
Aggregate Output Y	0.725	1.362	4.688	8.641
Aggregate Capital K	0.735	1.366	4.706	8.683
Aggregate Labor L	0.719	0.728	0.728	0.728
Aggregate Consumption	0.763	0.872	3.002	5.527
Interest rate r^k	4.44%	4.59%	4.57%	4.56%
Wage w	1.008	2.086	6.436	11.864
Avg work hours	0.987	1.006	1.006	1.006
SS spending S/Y	40.99%	40.48%	40.48%	40.48%
Dependency Ratio	67.0%	67.0%	67%	67%

8 Conclusion

All the work described in the previous sections detailed the consequences that the serious demographic shift that is taking place in Italy could have if no changes are made to the way in which the Social Security system is structured.

In the first Section, Samuelson's model was used to explain the way in which *PAYG* pension system function from a financing perspective. In the next Section, the model employed by Kitao, and in this thesis, was described in all its parts. In Section 4, the numerical methods and framework necessary to solve the individual's problem, namely, a backward recursion and the discretization of the vector space containing the policy functions of the individual, and the research for a steady states, following the *Gauss-Seidel Method*, were detailed. In Section 5, the calibration necessary to solve numerically the model was provided, with references to either the literature or, for certain variables, data related to the Italian economy. In the next Section there was an overview of the results of the baseline model, with simulations of the economy in the years 2015 and 2075. In order to keep benefits in line with the formula mandated by the Riforma Dini, and funding the increased pension expenditure, equal to 23% of *GDP* in 2015 and to 40% of *GDP* in 2075, through the use of indirect taxes, an increase of the Imposta sul Valore Aggiunto (*IVA*) of almost 30% is going to be necessary. In Section 7, the same experiment was run under different scenarios of growth, with respect to the baseline simulation, which was calibrated around the projections provided by *OECD*.

Despite the fact that almost all macro-variables change significantly, as was expected, with the changing growth, the main variable of interest, the tax on consumption, τ^c, stays more or less constant across all simulations and growth scenarios. This is likely due to the fact that the main driver of Social Security expenditure, and of government income is the composition of the population, much more skewed towards retirees in the 2075.

The outcome pictured by the experiment run in this thesis is clearly an unrealistic one, as it is unlikely that such a high taxation would be politically feasible. However, it gives a small insight on the seriousness of the situation the government finds itself in. Outcomes that are be more probable are an increase in the retirement age, or a decrease in benefits. All of those solutions, however, treat the symptoms, rather than the illness. While this thesis does not enter in the wider discussion concerning Social Security policy, it is a fact that such reforms will be in the spotlight in the upcoming years.

9 References

1. Auerbach, A.J. and L.J. Kotlikoff (1993). Dynamic Fiscal Policy. *New York, N.Y. : Cambridge University Press.*

2. Barr, N. and P. Diamond (2006). The Economics of Pensions *Oxford Review of Economic Policy Vol. 22, No. 1, Pensions,15-39.*

3. Benninga, S. and A. Protopapadakis (1990). Leverage, time preference, and the equity premium puzzle. *Journal of Monetary Economics 25, 49-59.*

4. Braun, A. R. and H. Joines (2014). The implications of a graying Japan for government policy. *Journal of Economic Dynamics and Control 57, 1-23.*

5. Conesa, J. C. and D. Krueger (1999). Social Security Reform with Heterogeneous Agents. *Review of Economic Dynamics 2, 757-795.*

6. Deaton, A. (1991). Saving and liquidity constraints. *Econometrica 59, 1221-48.*

7. Feldstein, M. and A. Samwick (1997). The Economics of Prefunding Social Security and Medicare Benefits. *NBER Macroeconomics Annual 1997, Volume 12, 115-164.*

8. Feldstein, M. and J. Liebman (2002). Social Security. *Handbook of Public Economics, Vol. 4, Capter 32.*

9. Hansen G. D. and S. İmrohoroğlu (2016). Fiscal reform and government debt in Japan: A neoclassical perspective. *Review of Economic Dynamics 21, 201-224.*

10. Hansen, L. P. and K. J. Singleton (1983). Stochastic consumption, risk aversion, and the temporal behavior of asset returns. *Journal of Political Economy 91, 249-265.*

11. Hayashi, F. and E. C. Prescott (2002). The 1990's in Japan: A lost decade. *Review of Economic Dynamics 5(1), 206-235.*

12. Hotz, J. V., F. E. Kydland and G. L. Sedlacek (1988). Intertemporal preferences and labor supply. *Econometrica 56, 335-360.*

13. Hubbard, R. G. and K. L. Judd (1987). Social Security and individual welfare: Precautionary saving, liquidity constraints and the payroll tax. *American Economic Review 77, 646-660.*

14. Hurd, M. D. (1989). Mortality risk and bequests. *Econometrica 57, 779-813.*

15. İmrohoroğlu, A., S. İmrohoroğlu and D. H. Joines (1993). A numerical algorithm for solving models with incomplete markets. *The International Journal of Supercomputing Applications 7, 212-230*

16. İmrohoroğlu, A., S. İmrohoroğlu, and D. H. Joines (1995). A Life Cycle Analysis of Social Security. *Springer : Economic Theory Vol. 6, No. 1, Symposium: The Discipline of Applied General Equilibrium (Jun., 1995), pp. 83-114.*

17. İmrohoroğlu, S., S. Kitao and T. Yamada (2016). Achieving fiscal balance in Japan. *International Economic Review 57, 117-154.*

18. ISTAT's database. *http://dati.istat.it/.*

19. Kitao, S. (2015). Fiscal cost of demographic transition in Japan. *Journal of Economic Dynamics and Control 54, 37-58.*

20. Kitao, S. (2015). Pension Reform and Individual Retirement Accounts in Japan. *Journal of The Japanese and International Economies Vol. 38, 111-126.*

21. Kocherlakota, N. R. (1990). On the 'discount' factor in growth economies. *Journal of Monetary Economics 25, 43-47.*

22. Legge 8 agosto 1995, n. 335. *Gazzetta Ufficiale.*

23. Mehra, R. and E. C. Prescott (1985). The Equity Premium. *Journal of Monetary Economics 15, 145-161.*

24. Ministero Dell'Economia e Delle Finanze (2016). Public Debt Report.

25. Nishiyama, S. and K. Smetters (2005). Consumption Taxes and Economic Efficiency with Idiosyncratic Wage Shocks. *Journal of Political Economy, Vol. 113, No. 5,1088-1115.*

26. OECD (2017). Pensions at a glance: Country Profiles - Italy. *https://www.oecd.org/els/public-pensions/PAG2017-country-profile-Italy.pdf.*

27. OECD (2020). Taxing wages - Italy. *http://www.oecd.org/tax/tax-policy/taxing-wages-italy.pdf.*

28. Samuelson, P. A. (1958). An exact consumption-loan model of interest with or without the social contrivance of money. *Journal of Political Economy Vol. 66, No. 6, 467-482.*

29. Sargent, T. J. (1989). Dynamic Macroeconomic Theory. *Cambridge: Harvard University Press.*

30. Stokey, N., J.R. Lucas Jr. and E. C. Prescott (1989). Recursive methods in economic dynamics. *Cambridge: Harvard University Press.*