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Sovereign Default: Theory and Evidence

Master's Degree Thesis in Asset Pricing

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INTRODUCTION

The topic of the present Master's Degree Thesis is sovereign default. We deal with this issue on two levels: by developing a review of the main theoretical and empirical findings literature has accomplished, focusing also on definitions and legal and institutional facets, and by developing a model that we simulate using MATLAB.
The focus of the first chapter is on defining the different aspects the phenomenon of sovereign default can take: we review both how market players define the issue and definitions given by economic literature. We focus also on international policy aspects, reviewing the main institutional mechanisms developed in the international community to efficiently handling this recurring problem, and focusing on the mechanics of the Greek and Argentine restructurings.
The second chapter is focused on literature review: we introduce the main theoretical models that illustrate how sovereign lending can be sustained and the main mechanisms that induce repayments in the context of the anarchy that characterize international relations, lacking of a judicial entity that can enforce repayments of sovereigns to private parties. We explore patterns of sovereign borrowing and timing of default, and review the costs associated to default empirical economic literature has estimated, that at a high level of generality can be divided in output costs, costs associated to the exclusion from international capital markets, higher future borrowing costs and losses associated with trade reductions after the event of not honoring international financial obligations.
The third chapter of the work develops the theoretical model we present in this thesis: it is a slight modification of the first model presented by Aguiar and Gopinath (2006) [2]. The model is a reputational model of sovereign borrowing, with output losses in case of default and nonpermant exclusion from capital markets after default. We change the output process' specification designed in the original paper by removing the deterministic time trend from the output equation: in our version output is generated only by a lognormal $AR1$ shock. We derive some properties under the alternative assumption of permanent exclusion from capital markets after the credit event.
Chapter 4 presents the results of the simulation, briefly describing the structure of the codes, that are provided in the appendix.

1

SOVEREIGN DEFAULT: CATEGORIES AND DEFINITIONS

In order to deal properly with the problem of sovereign default we need to give a precise definition of the phenomenon. As several authors point out, it is surprisingly hard to get a precise definition of sovereign default [4].

1.1 RATING AGENCIES DEFINITIONS

As a starting point we can mention the definitions of sovereign default used by the main Credit Rating Agencies (CRAs), that are followed by many economists and are the foundations of the wide majority of datasets on sovereign default.
Standard and Poor's Global Ratings "generally defines "default" as the failure to meet a principal or interest payment on the due date contained in the original terms of a debt issue" [50]. S&P counts two more precise conditions in order to characterize the state of default of a sovereign. "We consider a sovereign to be in default under any of the following circumstances:

- For local -and foreign- currency bonds, notes, and bills issued by the central government and held outside the public sector of the country, a sovereign default occurs when the central government either fails to pay scheduled debt service on the due date or tenders an exchange offer of new debt with less-favorable terms than the original issue;
- For private-sector bank loans incurred by the central government, a sovereign default occurs when the central government either fails to pay scheduled debt service on the due date or negotiates with the bank creditors a rescheduling of principal or interest at less-favorable terms than in the original loan" .

As we see in the definition of Standard and Poor's we have both the case of payment failure and restructuring, both for bonds and bank loans
The definition that is provided by Moody's Investors Service counts four possible conditions that trigger sovereign default:

- "a missed or delayed disbursement of a contractually-obligated interest or principal payment (excluding missed payments cured within a contractually allowed grace period), as defined in credit agreements and indentures;
- a bankruptcy filing or legal receivership by the debt issuer or obligor that will likely cause a miss or delay in future contractually obligated debt service payments;

- a distressed exchange whereby 1) an issuer offers creditors a new or restructured debt, or a new package of securities, cash or assets, that amount to a diminished value relative to the debt obligation's original promise and 2) the exchange has the effect of allowing the issuer to avoid a likely eventual default;

- a change in the payment terms of a credit agreement or indenture imposed by the sovereign that results in a diminished financial obligation, such as a forced currency re-denomination (imposed by the debtor, or the debtor's sovereign) or a forced change in some other aspect of the original promise, such as indexation or maturity" [16].

1.2 SOVEREIGN DEFAULT TAXONOMY

Even if most markets participants stick up to the definitions provided by Credit Rating Agencies that we have mentioned above, reinforcing their role as informational intermediaries in financial markets, the notion of sovereign default can be generally wider. Actually, if we interpret default as a breach of a contract, we can consider a spectrum of events that constitute default, ranging from a simple administrative delay or mistake to a huge restructuring.
Ams, Baqir, Gelpern and Trebesch (2018) propose a useful analytical approach to default episodes [4]. They distinguish among Technical Default, Contractual Default and Substantive Default.

1.2.1 *Technical Default*

Under this label the authors include any contractual event of default on public financial obligations that does not constitute default under relevant third party definitions, by which they mean primarily those used by Credit Rating Agencies. Administrative errors and some covenant default could be labeled as Technical Default episodes by markets participants.

1.2.2 *Contractual Default*

Under this label the authors include any event of default that also constitutes default under third party definitions, so basically default episodes as defined by CRAs. Typically, Contractual Default episodes involve missed payments that persist for a duration that exceeds a certain grace period. According to the authors predefault debt exchanges and restructurings that follow modifications to the conditions of the debt contract do not fit the definition.

1.2.3 *Substantive Default*

Under this label are counted default episodes that are mentioned in the relevant third parties definitions of default but do not constitute an event of default according to the letter of the relevant debt contracts. Exchange of distressed debt and restructurings properly fit this definition.

Common to the three categories is the circumstance of Cross Default, that links two otherwise unrelated debt contracts: if a credit event occurs with respect to a group of loans or bonds, where Cross Default clauses are inserted in other debt instruments' contractual terms, those outstanding instruments of the same issuer are deemed to be in state of default, so that creditors can deploy remedies to preserve their position.

Following the authors, we can give a classification of the most relevant types of events of default

1.3 REPUDIATION OF DEBT

As we have seen sovereign default can occur when there is a failure to pay on the due date, or even a minor event as a covenant breach or an administrative delay. Repudiation takes place when a government declares the illegitimacy of a certain bond series or bank loan and rejects the obligation to pay.
The case of repudiation of debt is different from a moratorium, in which a government, with a a public act, as an announcement or a piece of legislation stops unilaterally to service a certain amount of its debt without questioning the validity of the obligation to pay [4]. Repudiation typically occurs in the aftermath of regime changes in a certain country: typically the new government will refuse to honor the obligations taken on by the previous regime.
A famous example of repudiation of debt after a regime change is the Soviet repudiation of Tsarist debts in 1918 [27]. Even if after the Soviet Revolution the provisional Soviet government declared that it would repay the outstanding debt of the former regime, the subsequent year the Soviets defaulted on that debt declaring their decision to repudiate it. In a decree on January 28, the government declared that the debts would not be honored since they were concluded "by the government of the Russian landlords or of the Russian bourgeoisie". A minor case of sovereign default entailed a doctrinal elaboration of the category of the odious debt. This category emerged in the doctrine of international financial relations following an arbitration occurred in 1923 involving Great Britain and Costa Rica [21]. In 1917 the government of Costa Rica was overthrown by a military coup lead by Federico Tinoco that during its two years of dictatorship was able to borrow money from The Royal Bank of Canada as sovereign debt of Costa Rica. The raised money was then exported by Tinoco for his personal use when he was leaving the country in 1919. In a subsequent arbitration Great Britain claimed that the successor government of Costa Rica was tied to the payment of the external debt provided by the Royal Bank of Canada, as states and governments inherit the debt incurred by their predecessors. Costa Rica replied that the Tinoco government could not be viewed as the legitimate government of Costa Rica and so could not bind successor Costa Rica's government to payments of financial obligations taken on during its activity.

William H. Taft, the arbitrator, disagreed with the view of the delegate from Costa Rica, claiming that a change of government does not waiver a country from its external financial obligations. However, Costa Rica was able not to repay Tinoco loans, that were judged to be "transactions full of irregularities" [26], as the bank knew that the money "was to be used by the retiring president F. Tinoco for its personal support after he had taken refuge in a foreign country. It could not hold his own government for the money paid to him for this purpose." The Taft arbitration did not question the legal status of the Tinoco government or the doctrine of state succession, but ruled that Costa Rica could avoid responsibility to repay the debt since the Royal Bank of Canada knew that that the proceeds of the loans were beneficial only for Tinoco himself, not the people of Costa Rica.[21]
Tinoco arbitration was the cornerstone case for the definition of the class of the "odious debts", as defined by the Russian jurist Alexander Sack [41]. A sovereign debt is deemed to be odious under Sack's formulation if it is contracted by a despotic power, the purpose of the borrowing is not in the general interest of the state but for personal interest and the lender knows that the amounts will not benefit the whole country.
The Sack's interpretation of the above mentioned Soviets repudiation of debt was generally critical, as the debt built up during the Tsarist regime could not be viewed as odious, since it did not fit the three requirements of his definition. This occurrence is an exception to the general rule of state succession, but it is limited as we see to personal debt taken on by a dictator in the name of his country, and therefore can lead to repudiation of debt.
The doctrine of odious debt was applied implicitly also in the Versailles Treaty [1]: the Reparation Commission decided to refuse to charge on liberated Poland the amount of debt taken by German government to colonize the country. After being neglected in the doctrine for a long time, the concept of odious debt came back to the fore after the American invasion of Iraq. During its 25 years of controlling his country, the government of Saddam Hussein managed to build up an amount of unpaid debt of $125 billions. [21]
A number of commentators claimed that a relevant amount of this accrued debt was to be deem as odious *à la Sack*, and therefore it was to be written off [1]. The case of the Iraqi debt led to a certain degree of resurgence of the odious debt doctrine to justify repudiation.

1 "There is a widespread acknowledgment that the debts created by Saddam Hussein's regime bought weapons, palaces, and instruments of repression. Iraqi legislators should, as a first order of business, establish an arbitral process to determine the legitimacy of the estimated $120 billion in claims against their people. Only after Iraqis have an accurate accounting of these claims against their nation, and determine which are legitimate, should they appeal to creditors for debt relief, if any is required. To do otherwise would allow creditors to evade responsibility for financing Saddam's regime against its people. An odious debts arbitration would demonstrate to Iraqis that justice can be served by the rule of law. An arbitration would also expose the role of foreign creditors and thus help establish accountability in other countries." [1]

1.4 OFFICIAL LENDING AND DEFAULT

A sovereign borrower can raise funds by issuing bonds or by taking bank loans entering in financial contracts with the private sector. In some circumstances the lender is not a private financial actor, but actually a public institution, a sovereign state or a multilateral institution. By official lending we mean the credit granted by states and multilateral financial institutions as the World Bank and the International Monetary Fund to sovereign borrowers.
The IMF and the World Bank were founded in Bretton Woods in 1944, with different mandates.
The fundamental role of the IMF as envisioned in Bretton Woods was "to guard an adjustable peg exchange rate system and provide short term finance to deal with temporary current account deficits in advanced countries" [6]. As the Bretton Woods system collapsed in 1973, the IMF did not cease to exist, but actually changed its operational role, adding to its fundamental task of lender to countries that experience Balance-of-Payment critical imbalances a role of "crisis manager and development financier for developing countries".
The International Bank for Reconstruction and Development (IBRD) and the International Development Association (IDA) are the two agencies that make up the World Bank, whose goal is to promote long term growth in its member countries and help reducing poverty [12]. The IBRD is a major partner to the middle income countries, that represent more than the 60% of its portfolio [2], providing financial resources, knowledge, and technical services, helping governments reform to improve services and encourage more private investment in those countries.
The focus of the IDA are the poorest countries, to which it provides grants and credits (interest free loans) [3]. Participation to IMF programs lies on sticking up to some agreements on a program of economic reform the borrower has to realize: typically, a country's government and the IMF must agree on a list of economic policies before the IMF provides lending to the country. A country's commitments to undertake certain policy actions, known as policy conditionality, are in most cases an integral part of IMF lending. This policy program underlying an arrangement is in most cases presented to the Fund's Executive Board in a "Letter of Intent" and further detailed in a "Memorandum of Understanding" [4]. The number of such conditions to be met has sharply risen from the foundation of the institution: if the consideration of intrusive conditionalities was not envisioned at the Bretton Woods conference, in programs publicly available between 1999 and 2001 the number of performance criteria to be evaluated averaged to 9 and total conditions averaged to 21 [12]. Also the World Bank lending is tied to conditionalities that have to be met by the borrowing countries, that have risen over time.
Circumstances that can lead to an event of default on official debt can be found in the General Conditions of the IBRD and IDA and in the Articles of Agreements of the IMF. Those conditions are incorporated

2 https://www.worldbank.org/en/who-we-are/ibrd
3 https://ida.worldbank.org/
4 https://www.imf.org/en/About/Factsheets/IMF-Lending

by reference in transaction-specific legal agreements between the institution and the borrower.
In order to shed light on how an event of default on official debt could be triggered it can be useful to examine the article VII of the General Conditions of the World Bank, "Cancellation, Suspension, Refund, Acceleration" . According to the wording of the article, the Bank can suspend and cancel in whole or in part the right of the borrower to make withdrawals from the loan account if one of the following events occur, that can be seen as a default on official lending:

- *Payment Failure*, that is essentially the same condition of the event of default in the CRAs definition;
- *Performance Failure*, that occurs if "a Loan Party has failed to perform any other obligation under the Legal Agreement to which it is a party or under any Derivatives Agreement": this clause is directly related to the conditionalities imposed by the Bank to borrowing countries;
- *Fraud and Corruption*: The Bank can suspend and cancel the loan if the borrowing party "has engaged in corrupt, fraudulent, coercive or collusive practices in connection with the use of the proceeds of the Loan"
- *Cross Suspension*, that is the analogous of Cross Default for the market based lending, in which the Bank suspend and cancel a Loan to a country after an episode of default on another Agreement with the Bank;
- *Misrepresentation*: any representation made made by the borrowing party on which the Bank had to rely on making the Loan that reveals to be incorrect can determine suspension and cancellation of financing;
- *Withdrawal from membership in the World Bank or the IMF* [24].

Refund of past disbursement can be requested by the Bank mainly in case of fraud and corruption, while acceleration of interest and principal is associated with payment failure, subject to a 30 days period of grace, and to performance failure, subject to a 60 days period of grace [4].
In the IMF's Articles of Agreements can be found the relevant Official Default taxonomy for the Fund's lending.

1.5 DEBT RESTRUCTURING

Following the definition of Das, Papaioannou and Trebesch (2012) we define a sovereign debt restructuring as an exchange of outstanding sovereign debt instruments, issued or guaranteed by the government, such as loans or bonds, for new debt instruments or cash through a legal process [47].
We have to distinguish between distressed debt restructuring, on which we focus for our analysis, that is a debt restructuring at terms less favourable than the original bond or loan terms, typically during a crisis or in its aftermath, from restructurings that are part of

routine liability management operations (LMOs), such as debt swaps, that occur in normal times. As the authors point out, there are two main elements in a debt restructuring:

- *Debt Rescheduling*, that is the lengthening of maturities of the old debt, possibly at lower interest rates. This of course implies a certain amount of debt relief, by shifting payments in the future;

- *Debt Reduction*, that is a reduction in face value of the old debt instruments;

- *Debt Buyback*, defined as the exchange of outstanding debt instruments for cash, offered at discount.

This type of operations implies a haircut, that is a reduction in present value of creditor claims.
A restructuring can occur after a payment failure (or default *stricto sensu*). If this is the case, we define the process as *Post Default Restructuring*. The restructuring of the sovereign debt of Argentina between 2005 and 2010 is a prominent example of a restructuring after a government has ceased to honor interest and principal of a certain series of bonds.
Conversely if a restructuring prevents a default, and the debt exchange or reduction takes place before a sovereign stops servicing its debt, we speak of *Pre-emptive Restructuring*. A recent and notorious example is the Greek debt restructuring of 2012.
Worth mentioning is the definition of Credit Event by the International Swaps and Derivatives Association (ISDA), that is incorporated by reference in standard Credit Default Swap Contracts (CDS): if a debt repudiation, a failure to pay or a distressed debt restructuring happens, this constitutes a Credit Event that triggers a payment under a CDS.
Under the definition of the ISDA a restructuring is deemed to be a credit event if it occurs after a deterioration of the creditworthiness of the sovereign borrower, and it is binding for all holders.
As Das, Papaioannou and Trebesch (2012) point out, a restructuring could be designed to avoid the triggering of a credit event, if it is designed as voluntary and if Collective Action Clauses (CACs) are not used in the process.
CACs are conditions in the contractual terms of a sovereign debt contract that "mandate that if a debt restructuring offer is supported by a supermajority of creditors, it becomes binding on all investors—irrespective of their individual preferences. The goal of these provisions is to prevent individual creditors from freeriding on the debt relief granted by others and to remove the risk of protracted holdout litigation in court." [42] Avoiding the use of CACs may avoid triggering the credit event, as the process of restructuring would not be binding anymore for all investors, but would imply a more costly overall process for restructuring, protecting the position of holdouts: since 2013 there is a general obligation of including CACs in all sovereign bonds' contractual terms of the Euro area, to make eventual restructuring processes more straightforward and quick.

1.5.1 *Haircut definitions*

A haircut is defined as a measure of creditor losses in a debt exchange process [47]. Two methods of calculation prevail in the literature to get an estimation of a haircut.
Cruces and Trebesch (2013) propose the following formula to get such an estimate [11]:

$$H_{CT} = 1 - \frac{PV(New\ debt, r_e)}{FV(Old\ debt)}$$

where $FV(Old\ debt)$ stands for the face value amount of the old outstanding debt (including past due interest on the old debt but no penalties), and $PV(New\ debt, r_e)$ is the present value of new debt instruments (plus possible cash repayments), discounted at the interest rate r_e, that is the interest rate prevailing in secondary markets at the exit from default.
The rationale behind this formula is that after a default episode there is acceleration of debt, so principal and interests come due immediately and thus it makes sense to evaluate them at face value, without discounting the cash flows.

Another measure for haircuts has been proposed by Sturzenegger and Zettelmeyer (2008) [44], according to the following formula:

$$HC_{SZ} = 1 - \frac{PV(New\ debt, r_e)}{PV(Old\ debt, r_e)}$$

where $PV(Old\ debt, r_e)$ stands for the present value of the remaining contractual payments of the old debt instruments, inclusive of eventual interest or principal arrears, and $PV(Old\ debt, r_e)$ is the present value of the new debt instruments after the operation of restructuring.
Both the old and the new debt are discounted at the same interest rate, that is as above the yield immediately prevailing after the debt exchange becomes public information and the debt instruments start trading in the secondary market.
The rationale the authors state for this haircut measure is as follows. They do not consider debt acceleration, but compare the value of old and new debt as default and restructuring did not happen, so they prefer to evaluate the loss under the original contractual terms. Moreover they claim that in a world of perfect foresight the value of the debt before default/restructuring and the value of the outstanding debt right after the exchange should be equal, and so basing the valuation on present value terms "the measured gain or loss will reflect the extent to which the result of the exchange was incorrectly anticipated".

1.5.2 *Key steps in a restructuring process and International Forums*

In this section we introduce a stylized timeline for the process of debt restructuring, then we focus on the main international forums that address restructurings. The typical steps are the following:

- The process is started by a payment failure episode, or by an announcement of debt restructuring by the government;
- The government of the restructuring country starts negotiations with the creditors, that can be bilateral, multilateral with syndicates of creditors, eventually with the monitor of international financial institutions, for instance the IMF, or informal negotiation clubs on debt restructuring, like the Paris Club or the London Club (see below). The goal of the negotiation is to get an agreement on the magnitude of the haircut in order to restore a form of debt sustainability of the restructuring country, the currency and law of the new debt instruments, and a set of conditions on a macroeconomic adjustment policy agenda;
- After the necessary rounds of negotiations, an offer is presented to creditors, that can decide to accept or reject the offer: as noted above, CACs can play a major role in the outcome;
- If the offer is accepted by all creditors or by a qualified majority of them the actual exchange takes place.

There are two multilateral forums for debt restructuring: the Paris Club and the London Club.
The Paris Club is an informal forum created by creditor governments to deal with rescheduling and restructuring of sovereign bilateral external debt [20]. Established in 1956 from an ad-hoc meeting to reschedule Argentina's debt with various Western countries [49], the Club witnessed an evolution from a role of simple debt collector, with agreements that until the 1980s could entail only a debt rescheduling, without weakening debtors' moral and legal obligation to repay their debts in full, to a role of relief provider with the adoption of Naples (1994) and Evian (2003) terms of restructuring.
In its current configuration, the Paris Club is composed of 22 permanent members, that share the common interest of being largely exposed creditors of other countries. Since 1961 representatives of the IMF and the IBRD are invited to Club meetings as observers, to provide information and technical advice, and also OECD, UNCTAD, the European Commission and four multilateral development banks (the African Bank of Development, the Asian Bank of Development, the European Bank for Reconstruction and Development, and the Inter-American Development Bank) attend negotiation meetings with their representatives. However, despite having reached a stable role in the international financial architecture, the Paris Club does not have an official charter to date, and has only a small secretariat in the French Capital.
The Paris Club operates according to the principles of *solidarity*, meaning that members avoid taking actions that could harm the positions of other member creditors; *consensus*, implying that Paris Club rescheduling deals must be accepted by all of its members; *conditionality*, meaning that in order to approach to Paris Club for a rescheduling program, debtor countries must agree to implement a program of macroeconomic adjustment with the IMF; *case-by-case approach* in dealing with rescheduling processes, taking into account that the effective members that take part to the negotiations vary with respect to

the debtor exposition and *comparability of treatment*, that means that a debtor who gets an agreement with the Paris Club has to seek comparable conditions with other eventual creditors, and can accept stricter condition only from the official sector.
As stated before the Paris Club initially worked only in the role of a debt collector. The agreements were modeled under the so called "Classic terms", that were only "flow treatments", meaning that debt stock reductions were not considered in the Paris club operational horizon, but only reschedulings that could not alter the NPV profile of sovereign debt. Under the Classic terms, the repayment profile is negotiated with debtors on a case-by-case basis, although it has tended to include a 3-year grace period and a 10-year repayment period [20]. With the adoption of the Toronto terms in 1988 the Club gave itself a more flexible toolkit to deal with debt rescheduling, extending maturity horizons, but with the adoption of Naples terms for the first time it could deal with stock treatments, providing the possibility of debt relief in a comprehensive debt restructuring process.
Another step further in the development of the Paris Club toolkit was taken in 1996, in the context of the HIPC (Heavily Poor Countries) initiative by the IMF and the IDA: the initiative called for a coordinated action of multilateral and bilateral creditors to reduce sovereign external debt of HIPC countries to a level of sustainability, and to channel the resources of debt relief to reducing poverty [49].
The Paris Club was involved in a multi-stage approach in the mechanism of the initiative, also with the introduction of the Cologne Terms. To be able to participate to HIPC initiative program, a country's debt had to be considered to be unsustainable after a traditional debt relief flow treatment procedure (Naples Terms), and after implementing a conditional macro adjustment program with the IMF and the IDA, also realizing a Poverty Reduction Strategy.

The schedule was as follows: after a classic flow treatment the IMF and IDA executives came to the *Decision Point*, to provide debt relief to the country that still suffered an unsustainable debtor position. *Completion point* was reached when the relevant country had achieved macroeconomic stability under a Poverty Reduction and Growth Facility supported program and it had carried out structural and social reforms, successfully implementing a PRS for at least one year. At this point the country could receive further assistance to get to debt sustainability, and the Paris Club could provide debt stock treatment.

In 2005 the HIPC initiative was coupled with the Multilateral Debt Relief Initiative (MDRI), that in a similar multi-stage framework, with the involvement also of the the African Development Fund and the Inter-American Development Bank, granted countries reaching the *Completion Point* a full official debt cancellation on multilateral obligations.

With the adoption of the Evian terms in 2003 was introduced the possibility of providing debt relief to nonHIPC countries in order to restore the sustainability of their debt stock.
The cost of debt relief to creditors under the HIPC Initiative is currently estimated at US $76.2 billion, while the cost to the four multilaterals providing debt relief under the MDRI is estimated at US $43.3 billion (both in end-2017 present value terms) [25]. The Paris Club's

involvement in the HIPC initiative was substantial: the Club has provided 37% of the total debt debt relief under the program, with some of its members agreeing for even more favourable conditions.
Up to date we can mention a total of 433 agreements reached within the Paris Club for debt relief programs of 90 countries [49].

The debt restructuring process between a sovereign borrower and commercial banks is labeled as "London Club" restructuring. Differently from Paris Club, that has an office and codified policies, even at a low level of instituzionalization, the London Club does not have a Secretariat or fixed venues, and the costs of its meetings are usually borne by the debtor: the name denotes the case-by-case restructuring practices developed between major Western banks and developing countries' governments in the late 1970s and early 1980s [47]; moreover the label can be somehow further misleading, because actually the majority of the meetings took place in New York and not in London.
The London process takes place with a negotiation between the government that declares not to be able to honor its obligations and a Bank Advisory Committee (BAC), or Creditor Committee, or Steering Committee, that is a group of banks with the largest exposure to the sovereign, negotiating on behalf of all the banks affected by the restructuring, with the goal of avoiding coordination problems between the potentially wide number of creditors, and to concentrate the responsibility of negotiation with the largest institutions that can have better expertise.
The main elements of a London Club restructuring can be reported as follows. A government that is experiencing financial distress contacts a number of banks among its creditors and asks them to build and chair a BAC. Once the Steering Committee is established the negotiations start, with the possibility to deploy the full array of instruments for a debt restructuring we have mentioned above: there can be solvency and liquidity consideration with the possibility for the committee to provide flow treatments by extending maturities and providing short term liquidity support to the governments, and stock treatments as well by giving outright reduction in the face value of the loans or bonds.
If the negotiations are successful an "agreement in principle" is signed between the government's officials and the representatives of the banks in the Steering Committee. Once the terms are approved they are sent to other creditor banks involved in the restructuring that are not included in the BAC for approval. Typically to get the final approval of such terms of restructuring unanimity is required among all creditors, leading to intra creditor disputes that can significantly extend the spell of time to make the terms of the restructuring operational.
Das, Papaioannou and Trebesch (2012) report as examples of quick and successful settlement within the London Club the cases of Pakistan (1999) and the Dominican Republic (2005), that took only a few meetings to get to a final approval.
Examples of troublesome deals are given by the above mentioned case of post-Saddam Iraq: the government had to settle more than 13,000 individual claims on Saddam's era debt, a process that took

more than two years. The authors claim that we can give a positive overall judgement of the restructuring processes under the London Club label: in the 1980s and 1990s over 100 restructuring agreements in principle were reached and made operational.

As prominent examples of recent sovereign restructurings, that have many different implications and some common features, we report the main elements of the the cases of Greece and Argentina.

1.5.3 *The restructuring of the sovereign debt of Argentina (2001-2010)*

The history of Argentina's sovereign default counts many episodes and it is not a concluded affair, however it is interesting to analyze the mechanics of the first Argentina's restructuring of the XXI century. Martin Guzman (2020) gives a punctual analysis of the default and restructuring process the government of Argentina faced between 2001 and 2010 [22]. After a decade of reforms led by the Washington Consensus view of economic liberalization and a currency peg of the peso to the dollar, the country witnessed an unsustainable increase in external debt in both public and private sector. International investors were willing to lend at low interest rates believing to the program of reforms that was being deployed by the government of Argentina, and the currency peg in their analysis substantially lowered devaluation risk.
A deep recession started in 1998, leading to an aggregate drop of GDP of the country between the kick in of the crisis and 2001 of 15.7%, and the impossibility for the government to defend the fixed exchange rate, and to honor its financial obligations. After a deep political crisis, the details of which we omit, on the last day of 2001 Argentina declared default on $81.3 billion of sovereign debt with private creditors, and abandoned the convertibility system. After declaring default, the country started a process of restructuring, aiming to a strong debt relief in order to regain a status of debt sustainability, so it is a classical example of what we defined above as *Post Default Restructuring*. The default affected 150 different bonds, denominated in six different currencies, and issued under eight different legal jurisdictions, and was at the time the largest episode in the history of sovereign defaults. The process was characterized by an absence of CACs, that as we will see gave more force to the position of holdouts.

The first step in negotiations was the so called Dubai offer, presented by the government of Argentina in the 2003 IMF-World Bank Annual Meeting in Dubai.
Argentina's government promised to run budget surpluses of at least 4% starting in 2004 and full repay of preferred creditors.
The proposal entailed a writedown of 73% on the $81.3 billion on which the country had defaulted 2 years before with no recognition of due accrued interests.
The restructuring proposal was based on the possibility to exchange existing old bonds with a menu of new securities:

- a new discount bonds with 75% discount on principal and an increasing interest rate in the range of 1% to 5%, and a maturity of 8 to 32 years;
- a new par bond with no discount on principal, and fixed interest rate in the range of 0.5% to 1.5%, and a maturity of 20 to 42 years;
- a new quasi-par bond with a 30% discount on principal, a fixed interest rate in the range of 1% to 2%, and a maturity of 8 to 32 years.

The first offer was not accepted by creditors, leading to further rounds of negotiations, culminated in the 2005 Buenos Aires offering, that was instead accepted by a majority of creditors.
Argentina promised to run budget surpluses of at least 2.7% from the moment of the agreement with creditors. The exchange bond program of the new offer involved the substitution of old debt securities with the issue of:

- new par bonds with no writedown of principal, a maturity of 35 years and a reduced annual interest rate of 1.33% over the first five years, which would then increase over time up to 5.25%, for a total face value of $15.0 billion;
- new discount bonds with a writedown of 66.3% on principal, a maturity of 30 years and an annual interest rate of 8.25%, for a total a face value of $11.9 billion;
- new quasi-par bonds for local bondholders, issued in Argentine pesos adjusted by a proxy of consumer price index (CPI) inflation, with a maturity of 42 years and a fixed annual interest rate of 3.31%, for a total face value of $8.3 billion [23].

Moreover, Argentina proposed to issue GDP-linked securities, that promised payments if some conditions were met: if in the considered year actual real GDP exceeded base case GDP; if in the considered year annual growth in actual real GDP exceeded the growth rate in base case GDP, where the base case GDP growth was set to 4.26% for 2005, 3.55% for 2006, 3.42% for 2007, 3.3% for 2008, 3.29% for 2009, 3.26% from 2010 to 2012, 3.22% for 2013, 3.03% for 2014 and 3% from 2015 to 2034; if GDP-linked securities' payments did not exceed a certain cap. The payment on each unit was set to 5% of the difference between the actual real GDP and the base case GDP for each reference year. The participation to this exchange was of 76.15%, entailing an exchange of $62.32 billion out of the $81.84 billion of old bonds, included due interests until December 2001, without considering interests due in 2002 and 2003.
Collective Action Clauses were not generally deployed in the terms of the exchange bonds, but limited at bond series level.
Negotiations for a second swap took place in 2010, in order to get a restructuring for the left $18.3 defaulted debt, given the limited participation on the 2005 offer. The participation to the new exchange was of the 70.74% over the remaining eligible debt, increasing the total participation to 92.4%: $13.1 billion of old debt was exchanged

for $2.1 billion in par bonds, $4.8 billion in discount bonds and $957 million in a new the global bond due in 2017, for a total writedown of 40% in face value [22].
Among the others, two issues are worth analyzing. As noted above, the absence of CACs in the defaulted bonds' contractual provisions has given a powerful position to holdouts that decided not to agree to the conditions of the exchange offers. Among them, a group of hedge funds specialized in purchasing sovereign distressed debt, called "vulture funds" by Argentine press, purchased at consistent discount Argentina's defaulted debt, litigated in courts claiming for full repayment, inclusive of accrued interest and a compensation for delayed payments, and in many cases won. A famous ruling by judge Thomas P. Griesa from the New York Southern District Court (Republic of Argentina v. NML Capital, Ltd., 09-28-2011), extended the extremely favourable treatment distressed debt specialized hedge funds could receive by sustaining litigations in front of the courts to a group of bondholders labeled as "me too" that did not agree with the restructuring terms, nor litigated to get full reimbursement, applying a form of *pari passu* clause. Therefore emerged issues regarding equality of treatment among groups of bondholders: hedge funds that were able to get astonishing returns by litigation, after buying distressed debt at extreme discount, exchange bondholders who agreed on less favourable terms and "me too" bondholders who received a better treatment than investors who participated in the restructuring process. This heterogeneity violates the general *pari passu* clause that is reported in the sovereign debt instruments contract terms, and highlights the relevance of the CACs for an efficient and quick restructuring process, by reducing uncertainty, minimizing legal costs and assuring equal treatment among investors.

The second element worth mentioning is the actual haircut and debt relief that was granted to the country. There are different metrics to judge the debt reduction Argentina could achieve. Using the measure proposed by Sturzenegger and Zettelmeyer (2008) [44], Cruces and Trebesch (2013) estimated a haircut of 73% of the Argentina's debt following the restructuring [11]. Guzman wants to give a measure for the effective debt relief taking into account payments to holdouts and GDP-linked warrants (between 2005 and 2011 about $10 billion) according to he following formula:

$$Debt\ Relief = 1 - \frac{FV(New\ debt) + GDPW\ + holdouts}{FV(Old\ debt)}$$

where $FV(New\ debt)$ is the face value of new debt instruments, $FV(Old\ debt)$ is the face value of old defaulted debt instruments, $GDPW$ represents payments under the GDP-linked warrants, and $holdouts$ stands for payments done to holdouts, getting a much lower estimate of 20.5%.

Even if the question of Argentina's debt is still today an open affair, examining the key elements of the first stage of the restructuring is a good example of the key elements in a Post Default restructuring process.

1.5.4 *The restructuring of the sovereign debt of Greece (2012)*

If the Argentine case is a prominent case of Post Default restructuring, conducted without the possibility to rely on CACs and with legal litigations resulting in disparities of treatment between groups of bondholders, the case of Greece is different under many respects. Also in this case we will focus on the mechanics of the offers and the actual exchange, without facing other important implications the crisis has had in the political and economic debate in EU and not.
Following the wave of the global financial crisis, Greece started to face a deep financial crisis, determined by several distortions its economy witnessed since the beginning of its Eurozone membership: a debt-driven growth with a negligible amount of FDIs due to the reduction of interest rates it could benefit of, an inflation of the nontraded sectors at the expense of exports and tradables, lack of necessary structural reforms, excessive current account and budget deficits [30]. The Greek economy was extremely weak with a current account deficit of 15% in 2008, 15% fiscal deficit and 127% public debt-to-GDP in 2009. The crisis had multifaceted aspects. The sovereign debt aspect entailed investors starting perceiving debt issued by Greece as much riskier and unsustainable as a whole: Greek sovereign bond yields continued to rise until spreads over German bunds shot up from 300 to almost 900 basis points during April 2010. The second shock was a banking crisis that resulted in difficulties for Greek banks to finance themselves in the interbank market, and doubts on their liquidity and solvency were seriously posed: in 2008 the Greek government had to shore up banks making available €28 billion [30]. The revelation of the systematic underestimation of deficit and debt figures that the Greek government committed led to a deeper deterioration: ultimately Greece experienced a sudden stop, with international lenders no more willing to finance the country as a whole, not just its public sector, and had to turn to the official sector for financial assistance [33].
Zettelmeyer, Trebesch and Gulati (2013) provide a precise illustration of the main elements of the crisis and of its resolution [55].
A first emergency line of credit was provided in 2010 by EU loans amounting to €80 billion, and by the IMF, with a disbursement of €30 billion, to be paid in three tranches subject to conditionalities with respect to a fiscal adjustment of the magnitude of 11% of GDP, and structural reforms to restore the environment for growth. The Greek crisis led to institutional innovation, with the creation within the European Union of the European Financial Stability Facility (EFSF), with a lending capacity of €440 billion at the time for troubled sovereign debt. There was an intervention by the European Central Bank, that under its Secondary Market Purchase Program started to buy Greek bonds to stabilize their price, operations that made it the single largest creditor of the country, with €42.7 billion in February 2012.
Despite a certain correction in Greek public finance imbalances, there was skepticism about the sustainability of the Greek public debt, that was certified by the downgrade of Greece by Moody's just after the first package of financial support was deployed.
A certain amount of Private Sector Involvement, started to seem un-

avoidable, following the Dauville Statement by Chancellor Merkel and President Sarkozy in October 2010, who claimed for the necessity of a permanent sovereign resolution mechanism within the EU that would have to substitute the EFSF, and the letter of Minister Schauble to the ECB and the IMF, proposing an involvement of holders of Greek bonds in a debt restructuring process.
After an initiative by EU and IMF of additional official lending for €64 billion a proposal of debt restructuring was built in July 2011 by 39 financial institutions through the Institute of International Finance (IIF). Old privately held sovereign bonds bonds could be voluntary exchanged with;

- a 30-year par bond with no face value reduction paying 4% in the first 5 years, 4.5% in the next 5 years, and 5% thereafter;
- a 30-year discount bond with a 20% face value reduction but slightly higher coupon rates (6%, 6.5%, and 6.8% in the same time schedule as above);
- a 15-year discount bond with a 20% face value reduction and 5.9% coupon;
- a par bond in lieu of cash repayment at the time of maturity of the bond held by the creditor [55].

Par bonds' principal according to this proposal were to be fully collateralized by the purchase by Greece of zero coupon bonds issued by the EFSF.
Zettelmeyer, Trebesch and Gulati (2013) give an estimate of the haircut that would have been realized, assuming a 90% participation of private holdings of Greek debt using the Sturzenegger and Zettlemeyer metric and a discount rate of 9%, the same proposed by IIF, amounting to 11.5%.
Debt relief was instead calculated to be zero or even negative, using as discount rate a rate in between the international risk free rate, and the estimated rate at which Greece would have returned to get access to international capital markets (the authors calculate a negative debt relief using a discount rate of 5% used by IMF in its assessment of debt sustainability).
However, this proposal was not implemented, since the deteriorating conditions of the Greek economy led to another plan with a deeper Private Sector Involvement. The Euro Summit statement of October 26, 2011, requested for a PSI quantified in a nominal discount of 50% on notional Greek debt held by private investors, and disposed an additional amount of official lending to guarantee the recapitalization of the Greek banking system.
The negotiation that resulted in the March-April 2012 debt exchange are under many respects similar to the ones under a London Club restructuring: in fact a steering committee was formed by a group of 12 banks representing a large group of creditors that held roughly 40% of privately held Greek debt. At the end of negotiations a take-it or-leave-it proposal was offered to investors: newly issued bonds offered much lower face value than proposed in the IIF offer and lower coupons, but were disposed large upfront payment to creditors. The offer was built with the following instruments:

- "One and two year notes issued by the EFSF, amounting to 15% of the old debt's face value;
- 20 new government bonds maturing between 2023 and 2042, amounting to 31.5% of the old debt's face value, with annual coupons between 2% and 4.3%, issued under English law;
- A GDP-linked security which could provide an extra payment stream of up to 1% of the face value of the outstanding new bonds if GDP exceeded a specified target path;
- Compensation for any accrued interest still owed by the old bonds, in the form of six-month EFSF notes" [55].

The offer involved all privately held sovereign and sovereign-guaranteed bonds issued before 2012: the total face value of the securities considered was of €195.7 billion.
In order to realize the exchange one major problem existed at the time of the proposal: the 86% of the eligible debt was issued under Greek law, and did not had CACs in the contractual structure, that were instead clearly stated at single issue level for the bonds issued under English law the country had outstanding.
In this situation the Greek legislature instead of imposing an authoritative restructuring decision, that could have been viewed as controversial under several legal and political aspects, or waiting for a bond by bond series negotiation, passed a law that introduced the CACs in the bonds' contractual provisions on an ex-post basis. With the Greek Bondholders Act, 4050/12, 23 of February 2012, it was established that a restructuring of Greek sovereign debt issued under national law could have been performed if two conditions were met: the consent should have been given by a qualified majority of bondholders, with a participation level of at least 50% of the total face value; if this quorum was achieved, the qualified majority was of the two-thirds of the face-value taking part to the vote. Moreover this thresholds were not set at a single bond level, but were to be computed with respect to the whole Greek sovereign debt issued under local law.
Greece, IMF and EU institutions decided to set another condition: in order to unequivocally go forward with the exchange an overall participation threshold was posed, that had to be met considering both the foreign law and domestic law bonds.
The outcome of the vote were published on March 9: 82% of total face value of debt issued domestically had accepted the restructuring conditions, that adding the foreign law participation rate was sufficient to realize the exchange. By the end of the process, at the end of April, Greece announced that the total final participation was of €199.2 billion, or 96.9% of eligible principal; the short term payouts through EFSF short term notes amounted to €29.7 billion, and the new outstanding long term debt had a total face value of €62.4 billion, so the face value of Greece's debt declined by €107 billion, or 52%. Only holders of €6.4 billion in face value held out.

Using the measure of haircut proposed by Sturzenegger and Zettelmeyer, Zettelmeyer, Trebesch and Gulati (2013) give and estimate of the haircut Greek debt has received after the exchange: the estimate is in a range between 59% and 65%, using different discount rates derived by the yields on the new debt securities after they started trading in the secondary markets.

In order to get an estimate of the actual debt relief Greece could benefit of, the authors considers the following computations, using several discount rates, taking into account the disbursements Greece had to make in EFSF notes, and the €25 billion the country borrowed from the EFSF to compensate Greek banks for PSI related losses:

$$Debt\ Relief = 100 \times \frac{PV(old) - PV(newb) - PV(efsf) - PV(bnk) - PV(gdp)}{PV(old)}$$

where $PV(old)$ is the present value of the exchanged debt, $PV(newb)$, is the present value of the new outstanding debt, $PV(efsf)$ is the present value of the short term notes issued bt EFSF Greece had to purchase, $PV(bnk)$ is the present value of the loan Greece had taken with EFSF to recapitalize domestic banks to compensate them for the involvement in the exchange and $PV(gdp)$ is the present value of the GDP-linked securities.
Using different discount rates the authors get to very different estimates: at 3.5% discount rate the debt relief in present value amounted to 45.3%, while discounting at 15.3% the relief was estimated in 75.9%: this wide margin reflected uncertainties with respect to the rate at which Greece would have been able to borrow once out of the status of default.

This prominent restructuring process, that has seen other episodes and political developments and another bailout in 2015, marks many differences with the case of Argentina.
First of all in the magnitude, the amount of debt restructured being more than two times larger.
As mentioned above, it was not a post default restructuring, but a preemptive restructuring: creditors suffered a huge loss, but there was no unilateral stop in servicing interests or principal. Moreover, it is a case of sovereign insolvency within the European Union and within developed countries, that before the European sovereign debt crisis was considered an event with really small probability attached. The institutional innovation within the EU determined the involvement of the EFSF as lender, with the cooperation of the IMF and the ECB: this process has led to the establishment of the ESM, which is nowadays the main creditor of the country.
Then we can mention how relevant the CACs have been: by adding a retroactive condition a more efficient, quick and participated restructuring process has been achieved, with little room for holdout position, and no room at all for hedge funds starting lawsuits in front of courts.

1.6 SOVEREIGN DEFAULT: THE BIG PICTURE

Before turning our attention to the literature review in this section we show some aggregate statistics on sovereign default. Using data provided by Bank of England (BoE) and Bank of Canada (BoC) database

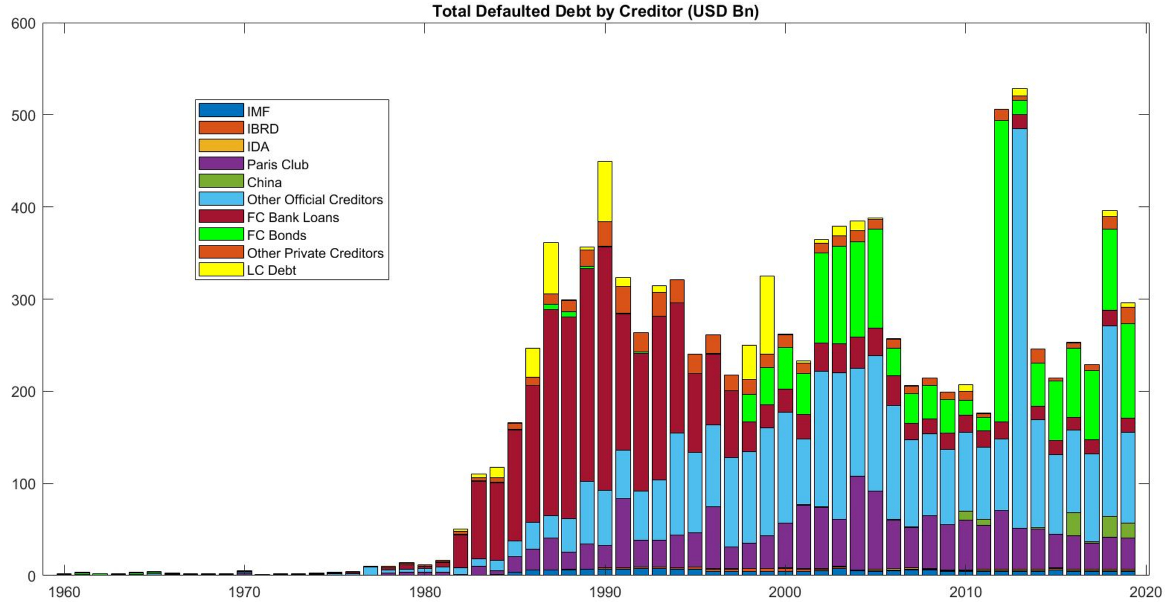

Figure 1: Total defaulted debt in state of default disagreggated by affected creditor. Source: BoE-Boc

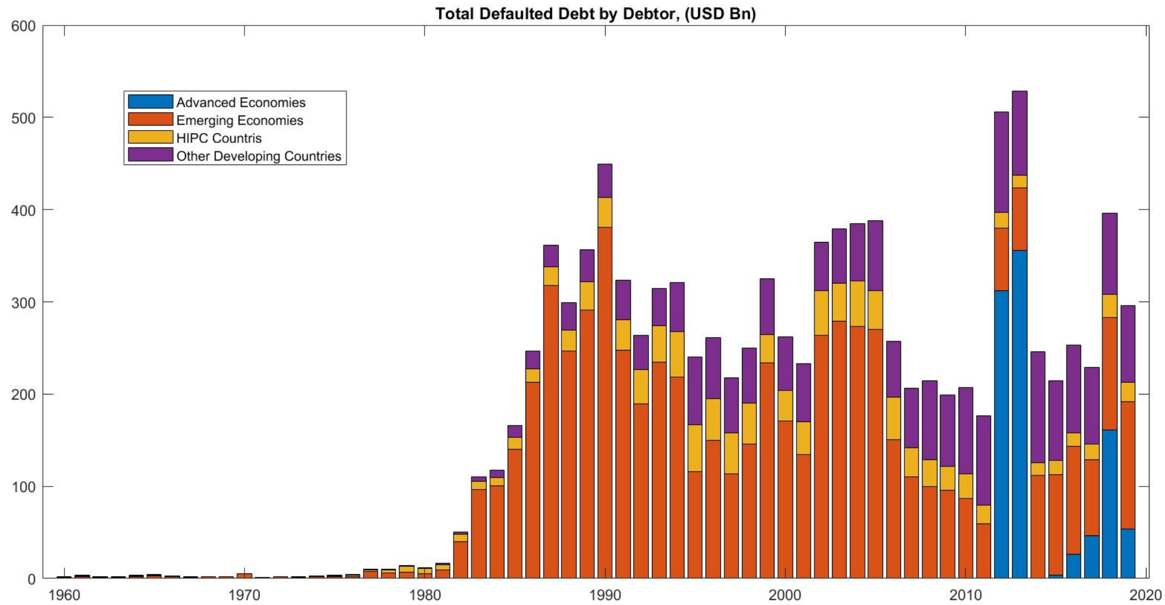

Figure 2: Total defaulted debt in state of default disaggregated by category of debtor. Source: BoE-BoC

on sovereign default we can show general trends dating back to 1960. As the data clearly show and Beers and de Leon-Manlagnit point out [7] there is a clear tendency for countries to default on loans and securities denominated in foreign currency: default on local currency securities is a residual item. The strict capital controls and the general fragmentation of international financial markets after WWII made external cross-border financing from private actors really limited: this is clearly shown in the low default rate in 1960s (see Figure 3). As capital controls started to be removed external financing from the private sector became available in greater magnitude, default rates started to rise, reaching peaks of more than 50% of sovereign issuers in state of default between the end of the 1980s and the first half of the 1990s.

From Figure 1 we see that cross-border financing was obtained in the 1980s recurring to bank loans in foreign currency. Reiterated defaults on such loans determined the banks reducing their involvement, and the relative weight of financing through the issue of bonds is reflected in the larger proportion of default on those instruments from the end of 1990s onward. We see from the graph the constantly reducing role of the Paris Club as bilateral forum to manage debt restructurings, with a rise in bilateral loans provided by China, India and Gulf Countries: those creditors have not yet joined the Paris Club generating a fragmentation of the bilateral restructuring institutional framework [7]. The non-Paris Club official defaulted debt has sharply risen after the Great Recession and the European sovereign debt crisis, with Greece, Ireland and Portugal fallen in state of default and the role of the ESFS-ESM come to the fore. The European debt crisis is clearly visible also from Figure 2 that shows how in 2012-2013 advanced countries had the largest share of debt in state of default, reversing the secular prevalence of emerging economies defaulting on debt. Figure 4 shows how the total defaulted debt has changed its relative weight with respect to the world total public debt and the world GPD: if around the last part of the 1980s and the beginning of the 1990s accounted for more than 6% of the total world public debt, it has reduced its share to less than 1% of the total outstanding.

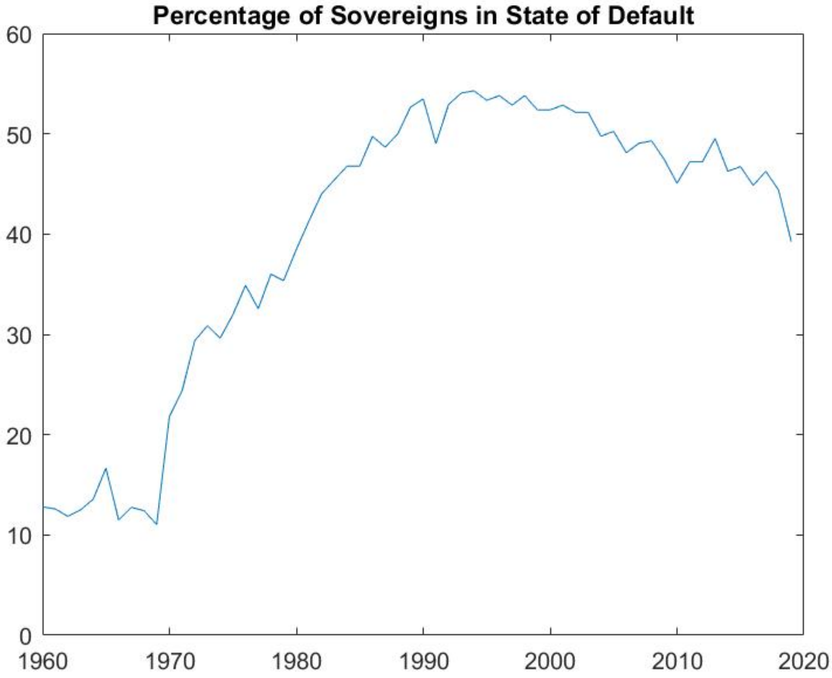

Figure 3: Percentage of sovereigns in state of default. Source: BoE-BoC

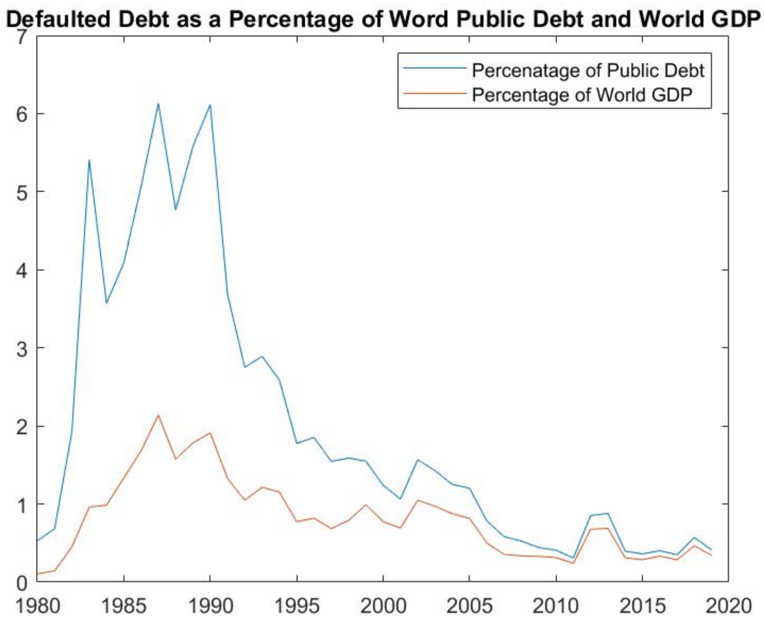

Figure 4: Total defaulted debt to total world public debt ratio and total defaulted debt to world GDP ratio. Source: BoE-BoC

2

LITERATURE REVIEW ON SOVEREIGN DEFAULT

So far we have discussed definitions and institutional aspects of the phenomenon of sovereign default. In this chapter we will examine how the economic literature has dealt with theoretical and empirical problems related to sovereign default.

2.1 LENDING TO SOVEREIGNS AND SOVEREIGN IMMUNITY

Lending to sovereigns is different from lending to a firm. In the corporate world contracts are enforced by national courts: repudiation of debt as we have described above, i.e. a unilateral decision of a firm not to honor its financial obligations, cannot take place, since it would certainly trigger a lawsuit, and a sentence that would force the firm to hand over its assets to creditors, through a liquidation procedure.
An enforcement mechanism is what really marks a difference between corporate debt and sovereign debt: given the anarchy of international relations there is no international court that can force to repayment a sovereign debtor that defaults, even if as we have seen there are forums and institutions that give a well established procedural frameworks to ease negotiations among parties to get a deal on restructurings. Moreover sovereign lending is not collateralized: in case of default a seizure of the sovereign debtor's asset cannot take place since few sovereign assets are located in foreign jurisdictions, typically amounting to small share of due debt [31].

In addition, over the centuries legal doctrines have been developed to protect foreign assets from seizure. Panizza, Sturzenegger and Zettelmeyer (2009) give a review of the evolution of such principles [31].
The ancient principle of Sovereign Immunity has protected sovereign debtors' assets: under this doctrine sovereigns cannot be sued in foreign courts without their consent. There is the possibility to decide a waiver to this principle, if the sovereign while entering in a contractual relationship with a foreign private party voluntarily includes in the covenants of the loan or the bond the submission to the ruling of foreign courts in case of a dispute between the parties.
Absolute sovereign immunity was the prevailing doctrine in the nineteenth and in the first half of twentieth century: under this view of international relations, absent explicit covenants the only solution a creditor could pursue was to address its own government to persuade it to make pressure on the sovereign debtor.
After WWII the sovereign immunity doctrine was restated in less strict terms: during the Cold War US government did not want to grant such a protection to Soviet Union states and its allies. So

it claimed that foreign sovereigns were denied immunity for commercial activities with direct effect on the US. This view was officially stated in the Foreign Sovereign Immunities Act (FSIA) of 1976, which allowed private parties to sue foreign governments if a dispute emerged and was related to commercial activities, a subset of which are sovereign debt contracts.
Under this doctrine formalized in the FSIA central banks and its asset and reserves are immune from seizure and attachment, the central bank generally being viewed as a separate legal entity that cannot be held responsible for the acts of the sovereign.
Beside the Sovereign Immunity principle the authors mention the Act of State doctrine, that states that courts "should not judge the validity of a foreign sovereign's act committed in its territory" [31].
Finally the International Comity principle as defined in a Supreme Court sentence of 1895 is defined as the recognition which one nation allows within its territory to the legislative, execution or judicial acts of another nation.

Given the presence of those legal principles in the doctrine, even in their lighter version, that protects the debtor, and the absence of international courts that may enforce contracts, economic theory has researched the motivation that let sovereign international lending exist at all. We review some of the main theories in the next section.

2.2 WHY DO COUNTRIES REPAY AND HOW CAN SOVEREIGN LENDING EXIST? A REVIEW

Since there is not a direct legal power to enforce repayment, there must be a mechanism of incentives that determines the borrower's choice to honor its obligation. Many economists have analyzed the issue.
Eaton and Gersovitz (1981) in a famous paper have proved that under certain conditions a sovereign debt market can exist even if the only way creditor has to react to an episode of default is by denying future credit to the borrower, i.e. by forcing the borrower in financial autarky [13].
The main driver that pushes the borrower to repayment is in this setting the smoothing function that capital markets can have on consumption: if the only way the borrowing country has to insure against bad states of the world (i.e. negative output shocks) is to access international capital markets, the threat of ending up in financial autarky is sufficient to assure repayment, up to a certain level. This level is higher the higher the variance of the output shock, so the more valuable is to the country retaining access to capital markets.
Eaton and Gersovitz propositions are based on the threat of a permanent embargo, that would be triggered if reputation for repayment of the borrower was undermined by a sovereign default episode: in this setting there are only reputation-for-repayment contracts.
In a reputation contract the creditor cannot have legal recourse in case of default, it cannot seize any debtor's assets, it cannot interfere with the country's trade: the worst scenario a country that is in a reputation contract could face is that it will never again be allowed to write

those kind of contracts.
The feasibility of those kind of contracts in a setting in which the only retaliation a creditor can have with respect to insolvent debtors is to exclude them from international markets has been criticized by many economists.
Bulow and Rogoff (1989) observe that if international lending is not the only means to smooth consumption over time in fact pure reputational contracts cannot exist [39]. Their critique is based on the assumption that the borrowing country can exchange intertemporal consumption by either taking loans and issuing bonds, or by purchasing cash-in-advance insurance contracts. If this is the case a country could default on the due debt and use the proceeds to collateralize an insurance contract that can provide a schedule of payments not smaller than the loan or bond in present value: this possibility would jeopardize the sovereign international debt market and make impossible the very existence of pure reputational contracts.
They claim in the same paper that the only way in which reputational contracts could be sustainable is to allow for the possibility of direct punishments to the debtor: "if there are some direct costs that the lender can impose on a country in an event of default those contracts can be sustained, but on the base of these costs" [39]. So under a certain point of view Bulow and Rogoff save the possibility for reputational contracts by a redefinition of what a reputational contract is: a contract that is now sustained by a threat of direct punishment.
In another paper published the same year Bulow and Rogoff go further in the critique to the pure reputational model à la Eaton and Gersovitz [38]. They specify again that the main reason for repayment in an international sovereign loan is the threat of direct sanctions the lenders can impose by suing the defaulted debtor and by putting pressure for action on the their domestic legislature. They argue that the sanctions could entail the impairment of the conditions of free trade in the good markets for the insolvent borrower: it would be forced to conduct somehow roundabout in order to avoid possible seizure. Moreover according to the authors the country could be blocked from the access to short term trade credits, important to reduce transaction costs in international trade.
The defaulted debtor could be punished further by making consumption smoothing even more difficult: the country could not openly hold foreign asset in foreign countries allied to the creditor fearing a seizure (in the Bulow and Rogoff's specification the borrower is a Less Developed Country and the lender an agent of an industrialized country: the nonsolvent country could not hold assets in industrialized countries).
The authors' model is a bargaining model: if the threat in Eaton and Gersovitz model was the permanent exclusion from international capital markets, Bulow and Rogoff allow for renegotiations, that is clearly closer to real outcomes.
In this setting contracts can be renegotiated at any time: and the present value of future repayments, i.e. the amount a country can borrow depends on the likelihood and the probable outcome of future negotiations. As Panizza, Sturzenegger and Zettelmeyer (2009) point out [31] the fact that creditors can extract anything from the

negotiations depends critically on the fact that the punishments illustrated above not only harm debtors, but benefit creditors, that could for example receive a share of the creditor trade share: credibility of the punishment is the key to enforce repayments.

Kletzer and Wright (1998) build a model of sovereign lending as an intertemporal barter of a nonstorable good using an infinitely-repeated game with at least two participants [51].
In this paper the authors adopt an endogenous means of enforcing repayments. In the Bulow and Rogoff model the reason to repay is to avoid the threat of impairing international trade, and in the Eaton and Gersovitz model the concern of financial autarky forces to repay up to a certain level: those threats are exogenous means or enforcing repayments, that "are characterized as dependent on positive and negative awards administrated by a third party whose credibility is assumed" [51]. In the Kletzer and Wright model each payment is purely voluntary, and the only incentive to repay is given by the increase in the surplus of the payee in continuing the relationship with the other party, and the same holds for the lender. Intertemporal exchange is sustained by punishment threats that only reallocate the surplus in the long-term relationship, being the default in the view of the authors a possible contingent outcome in the long-term relationship between borrower and lender: they claim that abrupt interruptions in international financial relations typically do not occur, instead there is renegotiation on the time schedule and magnitude of payments. In their setting agents can always renegotiate the terms of any relationship, including the punishments. The presence of incoming competitive lenders is envisioned, but equilibria are renegotiation proof by coalition. A short lived payments moratorium is imposed to any party that does not honor any scheduled payment, and the long term financial relationship is restored as soon as the the punished party makes a sufficient payment to restore the surplus of the other party.
As anticipated the defaulted party in this setting cannot simply start another contract with a competing lender: the original creditor in this case adopt a "cheat the cheater" strategy of inducing the borrower to default on the new contract with the party that cheated the moratorium in exchange of restoring the original financial relationship. Wright (2002) builds another model in which what really matters is not only the borrower's reputation for future borrowing, but the bank's reputation [52]. After showing that in presence of multiple lenders and the possibility to write contracts à la Bulow pure reputational lending is impossible, thus accepting the Bulow and Rogoff critique, the author sets up a model in which syndicated multiple lending is provided to the borrower. In such a scenario, if the borrower defaults there is a threat on a credit institution that decides to enter in new financial relationships with the debtor: if it provides new contracts to the borrower it will be excluded from future syndicated lending. This threat in necessary for the lenders to collude on the punishment of the country that defaults: absent this credible threat to creditors no equilibrium would be sustainable with multiple lenders. So in this setting lenders are willing to preserve reputation

of firmly punishing defaulters, since cheating on the silent agreement would impair future possibilities of profits.

Cole and Kehoe (1998) revive the reputation based argument, by noting that the Bulow and Rogoff critique is based on the reject of models in which the "action of agents in one arena of behaviour affect reputation in that arena only" [10]. In the Eaton and Gersovitz setting the defaulter loses a good reputation for repayment, and is excluded from international capital markets, but if the country could enter in cash in advance contracts that would not be deemed a major issue. But Cole and Kehoe notice that there are many examples of situations in which governments are engaged in trust relationships: they mention the case in which a foreign investor performs an FDI in a country with the implicit understanding that the government will not tax returns on the investment beyond a certain level, or domestic relationship, in which, for instance, residents make up upfront investments to open a shop while trusting the government would not decide ex-post to tax their return more heavily than promised.
Cole and Kehoe argue that reputation still matters, in the sense that losing a good reputation standing in one arena tarnishes good reputation in other arenas. The will to preserve good reputation in those other arenas is a sufficient incentive for repayment and to let a sovereign debt market to exist.
They build a model of incomplete information in which the government of the borrowing country can be normal, that sometimes can default, or honest, that attaches much more disutility to defaulting on financial obligations. The government is involved in two reputation relationships, one with the lenders, and the other with domestic employees (domestic labour relationship), to whom the government promise a certain salary to work for it. Cole and Kehoe show that workers will be willing to work for the government only if they do not have clues that it could default on its debt: in that case there would be consistent risk of financial exploitation at their damage; losing good reputation in one arena has negative spillovers in other arenas.

Broner, Martin and Ventura (2010) explain the repayment of sovereign international debt with market structure consideration, leaving aside punishment arguments [9]. They argue that if foreign creditors could resell the debt they purchased from the borrowing country's government in the secondary market to domestic residents, and if the government cannot distinguish if a given debt contract is currently held by a resident or a foreigner, repayment takes necessarily place, even if there is no threat of punishment.
A government that maximizes the residents' utility in fact would not default on amounts it dues ultimately to its private sector, and of course attaches more utility to repay its residents than foreigner creditors. But with a sufficiently developed secondary market the government would decide to repay, since government could not perform targeted default, letting the sovereign international lending possible and not making sovereign risk to impair consumption smoothing function of capital markets and lowering overall welfare.

2.3 IS SOVEREIGN BORROWING COUNTERCYCLICAL?

As we have seen above, in the conventional view à la Eaton and Gersovitz, the main reason that induces countries to repayment on international financial obligation is the fear to be excluded from international capital markets, so losing the consumption smoothing function that sovereign borrowing can provide.

Several papers highlight that the pattern of sovereign borrowing of emerging countries is not aimed at business cycle balance, accessing international markets to borrow during economic downturns and running budget surpluses during boosts, as both the Keynesian and Neoclassical theories predict, but it is quite the opposite: procyclical budget policies are observed. Levy Yeyati (2009) performs panel regressions of the net financial flows to developing countries disaggregated by lender on the real GDP gap and current and past default dummies, with annual data [53]. The default dummy equals 1 if the country is in state of default according to S&P in the given year, and 0 elsewhere; the past default dummy equals 1 if the country has defaulted at least once in the sample 0 elsewhere. The author finds a strongly procyclical private net lending to sovereigns, while official lending (multilateral and bilateral) and IMF lending is strongly countercyclical. Levy Yeyati finds also a negative correlation between past default and debt flows: this suggests that having defaulted once on international financial obligations could affect future access to external funds.

Gavin and Perotti (1997) derive the same result by comparing Latin American countries to industrial economies, using data from 1968 to 1995 [18]. They regress the change in the fiscal surplus of the general government, measured as share of GDP, on the rate of growth of real GDP, the percentage change in the terms of trade and the lagged fiscal surplus. They interpret the coefficient on the output growth as the impact on fiscal outcomes of changes in real output. The authors distinguish between good times and bad times: bad times are years during which a country's rate of output growth is less than the average minus one standard deviation, good times are the other years. Major asymmetry in the fiscal response is found for industrial economies,: during good times on average the budget surplus increases by 0.25% for each percentage point of GDP growth, during bad times a 1% GDP negative growth is associated with 1% growth in deficit. For Latin America countries this is not the case: during good times the fiscal response is negligible, and during bad times the coefficient on GDP growth is negative, suggesting fiscal consolidation during recessions, but not statistically significant. The difference is sharper and statistically significant when the authors focus on deep recessions, in which output decreases by more than 4% for Latin American countries: the average change in the fiscal surplus during those episodes (18 in the sample) is 2%. Gavin and Perotti explain this procyclicality by intensified borrowing constraints that fiscal policymakers in Latin America have to face during economic downturns: they have to cope with loss of confidence and precarious creditworthiness, losing market access to run countercyclical fiscal policies.

Alesina, Campante and Tabellini (2008) criticize this view, by observing that if the main problem were the precarious access to capital markets, countries would self-insure by accumulating reserves in good times, in order to avoid the binding constraints during recessions, and this is not the case [3]. Moreover lenders should provide developing countries funding during recession, knowing that this would smooth out properly the cycle. They explain this procyclicality by the presence of corrupt politicians that can appropriate part of tax revenues for unproductive public consumption. In their model voters observe the state of the economy but not the whole government borrowing, there can be hidden off-balance liabilities, so during booms they demand more resources for themselves, in a "starve the Leviathan" effort to reduce political rents.

2.4 WHEN DO COUNTRIES DEFAULT?

2.4.1 *Default and macroeconomic environment*

In the economic models we have mentioned above the main reason to repay international sovereign debt was not to lose the possibility to access capital markets to smooth consumption during major economic downturns; but as we have seen above for emerging economies the pattern of sovereign borrowing seems to be procyclical, not assuring this smoothing function. So does default occur when macroeconomic outcomes of a given country are particularly poor? Empirical literature has examined this issue.

Levy Yeyati and Panizza (2011) examine 23 default episodes between 1982 and 2003, using quarterly data [54]. Considering the full sample they show that GDP starts contracting on average two years before the default episode; restricting their attention to credit events related to emerging countries they find that the default episode occurs after an average recession of 3 years: their study seems to validate the idea that defaults occur in bad times.

Tomz and Wright (2007) study the relationship between sovereign default and economic activity using data from 1820 to 2004 [46]. To measure good and bad times in overall economic performance they construct a measure of business cycle by comparing actual GDP with Hodrick-Prescott filtered trend GDP. They find a correlation coefficient between an indicator variable of default and this measure of business cycle of -0.08 when considering the full sample, that rises to -0.11 when restricting the analysis to the subset of countries that defaulted at least once during the examined period: the relationship is negative, but substantially weaker than one could expect. They find also that on average default began when output was 1.6% below trend, and that economic performance was below trend during the spell of time of default state; conversely they estimate that on average output reached the trend level on the first year after exit from default state, and was about 0.2% above trend during nondefault years. Nonetheless only 62% of the 169 default episodes of the their sample actually started during economic downturns, and in more than 39% of the observations countries managed to avoid default in periods of below-trend GDP, and in about 44% of the observed years in which countries were in state of default they remained in default even if

output was above trend [46]. Moreover Tomz and Wright estimate that more than one third of defaults in their sample began in good times, and more than one half of defaults ended during bad times. So the relationship between default and domestic economic activity is negative, but weaker than expected: there must be other factors to take into account.
Panizza, Sturzenegger and Zettelmeyer (2009) highlight the role played by exogenous factors that could trigger default [31], in particular the tightening of global credit conditions, as shown by Reinhart and Rogoff (2011) that illustrate a clear secular regularity of defaults occurring in clusters following the reversals of global expansionary capital flows, dating back to the start of the nineteenth century [36].
The inability to borrow abroad in domestic currency is one of the major drivers of sovereign insolvencies. This inability is what Eichengreen, Hausmann and Panizza (2005) call "The Original Sin"[15]. Emerging countries that have to deal with this issue will accumulate foreign debt issued in foreign currency, "having a currency mismatch in their balance sheet" [15] and exposing them to real exchange rate risk with respect to their capacity to repay: as the authors point out the real exchange rate tend to be quite volatile and to depreciate in bad times, thus undermining the creditworthiness of the country.
Moreover the presence of dollar-denominated debt makes difficult for central banks to resort to expansionary monetary policies that could trigger a depreciation of domestic currency reducing net worth and letting the weight of debt of foreign currency debt become even heavier.
The authors highlight how the volatility of the real multilateral exchange rate to which developing countries with dollar-denominated debt are tied can affect sovereign solvency by computing the percentage gap between the maximum and the minimum value of a 5 year moving average of the real exchange rate for a sample of developed and developing countries for the period between 1980 and 2000. They show that the real exchange rate moved by more than 60% over the examined horizon for developing countries: thus the 5-years average GDP would have moved by more than 50% in dollar terms through the real exchange rate channel. To further show evidences on the role of the exchange rate on sovereign solvency the authors select a sample of cases in which the dollar value of GDP of emerging countries over a two-year period fell by more than 30%, between 1980 and 1999. In the vast majority of those cases this reduction in the dollar value of GDP was linked to a sovereign default. But if the average change in dollar value of GDP amounted to 46%, the reduction using constant local currency units is less that one twentieth than that: according to the authors the inability to pay was due more to real exchange rate movements than to output decline. So the inability to pay can be caused by runs on emerging countries currencies in presence of dollar denominated debt, making countries affected by the "Original Sin" with relatively large amount of debt denominated in dollars riskier than counterparties with similar fiscal indicators. This result is confirmed by the authors that regress foreign currency credit ratings on debt-to-GDP ratio, debt-to-tax-revenues ratio, on the level of foreign debt and on an index for Original Sin, computed as the

proportion of debt issued on foreign currency, finding a large, negative, statistically significant value on the coefficient for the foreign currency issued debt.

2.4.2 *Predicting default*

Literature has investigated the role of macroeconomic fundamentals in affecting the risk of sovereign default, in the attempt to identify what set of economic fundamentals' misalignments is likely to determine an event of default.
Manasse, Roubini and Schimmelpfenning (2003) build an early-warning model of sovereign debt crises by selecting potential explanatory variables from a set with a deep theoretical foundation [28]. They base their model on data from 1970 to 2002, including information on 47 economies that have experienced in the covered period episodes of sovereign insolvency or restructuring. A country is in default according to their analysis if it is declared to be so by S&P, or it has received assistance by the IMF. In their paper they consider 50 variables, that can be broadly split in:

- Measures of solvency, such as public or external debt relative to capacity to pay, captured by GDP, revenues, reserves;
- Liquidity measures, such as short term and external debt services, expressed in relation to GDP; reserves or exports;
- Macroeconomic control variables, such as GDP growth and inflation;
- Variables associated with currency crises, such overvaluation, current account balance as a percentage of GDP, trade balance as a percentage of GDP, import growth, REER growth;
- Social commitment of the government variables, such as health expenditures as a percentage of GDP and social expenditures as a percentage of GDP.

The means of the various measures of external debt in the examined time period start from a relative low levels in noncrisis periods, then tend to increase in the years of the kick in of the crises, dropping at the exit. This path holds for public and private external debt measures. Overall macroeconomic variables show a worsening in the run up to a default episode, and an improvement in the aftermath of the exit.
Beside the analysis of descriptive statistics the authors perform an event study analysis, by regressing each variable of interest on seven dummies, for the three years that precede a crisis, the year of the outset of the crisis and the three years following the start of the crisis. They do the same for the exit from state of default. The estimated coefficients give the difference from nondefault mean of the respective event, whereas the constant of the model is the mean of all nondefault episodes. The analysis shows that the total external debt and the public external debt-to-GDP ratio increase in the run up to a crisis and are higher than during noncrisis episodes, with a spike in the exit year. In particular short-term external debt rises significantly in the

years that precede the default episode and at the moment of exit goes back to the level of nondefault years. The same dynamics holds for current account deficits and external financing variables. The authors find also large devaluation movements with respect to the US dollar in the years leading to a default episode, and large appreciation in the years following exit from the status of insolvency.
The Early Warning System is developed by the authors with a logit model, allowing for different coefficients on the variables between entering and exiting a status of default. This is done by multiplying each regressor by the lagged state of default indicator of Standard and Poors's. The model the authors implement is:

$$P_t = f((1 - SPD_{t-1}) \times X_{t-1};\ SPD_{t-1} \times X_{t-1})$$

where SPD is the default indicator of Standard and Poor's, and X is the set of regressors. The coefficients on the first set of variables describe the relationship between the explanatory variables and the probability of entering in a sovereign debt crisis at time t given that the country was not in status of default in the previous period. The coefficients on the second argument describe the relationship between the explanatory variables and the probability of being in status of default at t, given that the country was already experiencing a default episode in the previous period. The model provided by the authors correctly predicts 74% of all debt crises occurred between 1976 and 2001, while sending only 6% of false signals.

Reinhart (2002) finds a deep linkage between sovereign debt and currency crises for emerging markets, by analyzing data from 1979 to 1999 [34]. By simply looking at the joint occurrence of default and currency crises in the sample, the author finds that the empirical probability of having a currency crisis within 24 months of defaulting (with the crisis either before or after the episode of default) is about 84%. Since defaults are rarer than currency crises, that in the analyzed sample occur also to industrial countries that have an empirical probability of 0% of defaulting on debt, the probability of having a default within 24 months of a currency crisis is about 58% for the whole set of countries, going up to 66% for the subset of the developing countries. The economic conclusion provided by the author is straightforward: since much of the emerging countries' sovereign debt is denominated in dollars a deep depreciation can lead to the impossibility to honor due external financial obligations (balance sheet effect), that is in line with the conclusions of Eichengreen, Hausmann and Panizza (2005) we have reported above[15].
Having found a strong connection between currency crises and default in emerging markets, Reinhart in the same paper tries to asses the predictive ability of sovereign ratings. The author deploys a probit estimation technique, using as dependent variable a currency crisis dummy, and as independent variable the 12-months change in sovereign credit rating, lagged one year, using ratings data from Moody's and Standard and Poor's. The same exercise is repeated substituting the dependent variable with a default dummy. The coefficient of both the rating agencies' credit ratings are statistically insignificant for predicting currency crises, while they perform better

in predicting default episodes: circumstance that is explained by the author by the excessive weight given to debt-to-export ratios to build the ratings, and too light attached to indicators of liquidity, currency misalignment and asset price behavior.

2.4.3 *Serial defaulters*

Some countries are more prone to default than others. Reinhart, Rogoff and Savastano (2003) report that between 1824 to 2001 debts of Argentina and Brazil were either in default or subject to a process of restructuring for a quarter of time, those of Venezuela and Colombia about 40% of the time, and Mexico's public debt was in status of default for almost half of the years since its independence [35]. They introduce the concept of *Debt Intolerance*, that stands for the tendency of some countries to default with external debt-to-GNP ratio otherwise considered safe: as they report, Mexico defaulted on its debt in 1982 with a debt-to-GNP ratio of 47%, Argentina decided to stop honoring its obligations in 2001 with a debt-to-GNP ratio of about 50%, threshold that for industrial countries would be deemed safe. Between 1970 and 2001 53% of middle income countries' defaults occurred with external debt-to-GNP ratio smaller or equal to 60%. The authors find that emerging countries with a history of default tend to have on average larger external debt-to-GDP ratios and external debt-to-exports ratio with respect to emerging countries without a history of default. Using the external debt-to-GNP ratio (both private and public), and (1-IIR), where IIR is the average institutional investor rating as a proxy for default risk, they subdivide 53 countries in 3 regions, using data from 1970 to 2000. Club A is composed by countries with continuous access to capital markets, that have IIR rating greater or equal to 67.7. Club C is composed by the most debt intolerant countries, with IIR smaller or equal to 24.2, that receive external funding mainly from the official sector. Club B is composed by countries that have intermittent access to capital markets, and have IIR in between 24.2 and 67.7 (club B is then composed by 4 subsets based on external debt and IIR). Then they propose the following cross sectional regression to determine safe thresholds for different countries:

$$IIR_i = \alpha + \beta_1 INF_i + \beta_2 DEFRES1_i + \beta_3 DEFRES2_i + \beta_4 LDEF_i + \\ + \beta_5 EXTDEBTGNP_i \times NOTA_i + \beta_6 DEBTGNP_i \times A_i + \epsilon_i$$

where IIR_i is the average Institutional Investor Rating for country i between 1970 and 2000, INF_i is the percentage of 12-month periods of average inflation at or above 40% since 1948, $DEFRES1_i$ is the percentage of years in state of default or restructuring since 1824 for country i, $DEFRES2_i$ is the percentage of years in state of default or restructuring since 1946 for country i, $LDEF_i$ is the number of years since the last default or restructuring for country i, $EXTDEBTGNP_i$ is the average external debt-to-GNP ratio of country i between 1970 and 2000, $NOTA_i$ is a dummy variable that takes value 1 if the country is not part of club A in the partition presented above, $DEBTGNP_i$ is the average debt-to-GNP ratio from 1970 to 2000, A_i is a dummy variable taking value 1 f the country i is part of club A. Negative coefficients are estimated on default and inflation variables, and seem to be the same for club A and non club A countries. Significantly, the debt ratios enter the estimated equation with opposite sign: coefficient on average external to GNP ratio is negative and statistically significant for countries belonging to club B and C, whereas debt-to-GNP ratio has a positive coefficient for countries of club A. With the estimated coefficients the authors suggest a way to compute safe external debt-to-GNP ratios, that would be comparable to the safest region of club B. They perform this exercise for Argentina, showing that the country would be safe with an external debt-to-GNP ratio lower or equal to 15%, while defining a general minimal safety ratio of 35% for Club B countries.

2.5 THE COSTS OF SOVEREIGN DEFAULT

In this section we briefly review some empirical literature that assesses the costs of sovereign default across several dimensions.

2.5.1 *Exclusion cost*

As highlighted above describing the main theoretical models for sovereign borrowing and default, one of the main driver that enforces repayment on external obligations is the threat of losing access to international markets. The original model of Eaton and Gersovitz is based on the threat of permanent exclusion from financial markets. Empirical evidence seems to be quite different from what theorized by the authors: there seems to be a period of exclusion from financial markets, but it appears to be relatively short.
Gaston Gelos, Sahay and Sandleris (2011) work with data for 144 developing countries from 1982 to 2000. They want to assess which factors determine the availability of markets access, defined by the authors as "public or publicly guaranteed bond issuance or public or publicly guaranteed borrowing through a private syndicated bank loan, that results in an increase in the country's indebtedness" [19]. By implementing a logit model regressing a market access variable on GDP and GDP per capita, IMF program dummies, inflation, liquidity variables, exchange rate and terms of trade changes among the others, they find as expected that default affects negatively access probability, but this effect is weak: the probability of market access after a default is only about 3% lower than otherwise. Moreover they show that the median number of years that took countries in state of default to regain access to international markets was 4 in the 1980s and fell to 0 in the 1990s, with the resumption of gross private flows level of two years before the default episode in only two years after default on average.
Richmond and Dias (2009) derive different estimates of average and median duration of exclusion periods [37]. They apply for default the definition developed by S&P, and define market access as the first of the following events after exit from status of default (again, accordingly to S&P): "net positive transfers in the form of bonds and commercial bank loans to the public and publicly guaranteed sector;

or positive net transfers from bonds and commercial bank loans to the private sector" [37]. So they consider a broader definition of markets access than the one used by Gelos, Sahay and Sandleris. Examining data from 1980 to 2005, with 128 default episodes according to S&P, they find a median time span to partial access after exit from status of default of 3 years, and a median time span to get again full access of 7 years. On average the time to get resumption to partial access in their estimates is 5.7 years, the average time to full access post default is 8.4 years. These estimates offer a picture much more severe than what described in the previously examined paper, and a period of exclusion much larger, even if the authors find substantial geographical differences: on average a European emerging country experiences a period of complete exclusion of 3.4 years after exiting from default, while a country from Africa or Middle East gets partial market access on average after 8.2 years. Cruces and Trebesch (2013) focus on exclusion from international capital markets after episodes of restructuring [11]. They examine 67 restructuring cases from 1980 to 2010, finding an average duration from the completion of the restructuring process of 5.1 years, with a median of 3 years. Interestingly, they find that the average time until partial reaccess (net flows to the country > 0) is increasing in the haircut size: on average partial reaccess occurs after 2.3 years if the haircut (defined as proposed by Sturzenegger and Zettelmeyer above) is less than 30%, while the average duration is 6.1 years if the haircut size is greater than that threshold.

2.5.2 *Higher borrowing costs*

Defaulting on debt is typically associated to future higher yields to maturity when access to market is regained by the defaulting country. Several papers have tackled this issue, and have tried to quantify the effect of having defaulted on sovereign financial obligations on future borrowing.

Borensztein and Panizza (2009) use an unbalanced panel of 31 countries to regress the yearly average of EMBI global spreads over a set of controls, among which the log of GDP per capita, the log of inflation, the fiscal balance scaled by GDP, the current account balance scaled by GDP and the ratio of external debt over exports, and a set of variables that track default history [8]. Those are dummy variables that take value 1 if the country was in state of default at certain time periods. They consider data from 1997 to 2004. The authors' analysis leads to the conclusion that having defaulted at $t-1$ has a large and statistically significant effect on spreads amounting to 400 basis points on average, and this effect reduces to 250 basis points on average two years after default; the effect loses statistical significance after this point in time and then it fades away. According to their analysis the market seems to punish defaulters charging on them larger spreads, but it has short memory.

Flandreau and Zummer (2004) perform a similar exercise for the gold standard period (1880-1913) finding that a spread over UK government bonds was charged of about 500 basis points due to default during rescheduling negotiations, going down to 90 points in the year in which an agreement was reached and thereafter lowering to a level of

45, losing statistical significance [17].
Eichengreen and Mody (1998) examine the spreads charged on bonds issued by 37 emerging countries both by sovereigns and private parties [14]. They select data based on issuances that took place between 1991 and 1996, years in which there was a boom in the total face value of those instruments; they examine only instruments denominated in a developed country's currency, mainly in US dollars. They perform an analysis of the determinants of such spreads by estimating the coefficient on a certain number of dependent variables, mainly macro variables and rating variables associated to ratings provided by Institutional Investor, and find a positive and statistically significant coefficient on a dummy for debt rescheduling: spreads are remarkably higher if the issuing country has defaulted the previous year.

2.5.3 *Output losses*

A standard assumption in sovereign default models is that not honoring financial obligations may entail a loss on output for the period that follows the decision to default. Empirical literature has tried to estimate the impact of default of the growth rate of the economy of the defaulting country: here we review some of such studies. The main problem in addressing this issue is endogeneity: as we have seen above, default on average occurs when output is below trend, but it is default that determines the output loss or a major economic downturn that determines the governments' decision to default? As we will see there is mixed evidence.
Sturzenegger (2004) estimates the impact of default on growth by running a cross-country growth regression for 100 different countries, using data from 1974 to 1999 [43]. The baseline regression the author performs is as follows:

$$GROWTH_i = \alpha + \beta X_i + \gamma DEF_i + \epsilon_i$$

where $GROWTH_i$ is the average growth rate of real GDP per capita in country i between years 1974 and 1999, X_i is a vector of controls that includes among the others GDP per capita, population, investment to GDP ratio. To disentangle the independent role of macroeconomic instability and that of default decision the author includes in the vector of controls inflation, volatility of inflation and a dummy for banking crises. DEF_i is a dummy variable that takes value 1 if the country has ever defaulted, 0 otherwise. In another specification of the model the author deploys another dummy to track default, $DEFPLU_iS$, that takes value 0 if the country has never defaulted, 1 if the country has defaulted either in the 1980s or in the 1990s, 2 if the country has defaulted both in the 1980s and in the 1990s. The author finds that there is a large and negative impact of default on growth, accounting to about 0.6% on average. This implies that considering the whole period analyzed by the author nondefaulters have a cumulated growth that exceeds the levels of defaulters by 14% on average, that is a remarkable effect.
De Paoli, Hoggarth and Saporta (2006) using data from 1970 to 2000 assess the cost of a debt crisis by comparing the actual realization of GDP for defaulting countries to a counterfactual GDP that is an esti-

mated value of GDP as if the crisis did not happen [32]. For the output counterfactual the authors assume that GDP would have followed its precrisis trend, that they estimate using the Hodrick-Prescott filter on the available past output data. Then they present average output losses on a per annum basis, distinguishing four scenarios, by type of sovereign default: default only, default and currency crisis, default and banking crisis, triple crisis. For the "default only" scenario they have only four observations, that behave at odds with the theory: they find a negative median loss of 5.2% per year and an average negative cost per year of 1%: in this cases the output after default appears to be consistently above average, but the number observation as the authors claim is too small to draw conclusions.

Considering the average of the four scenarios the authors find a median loss of 7% per year and an average of 15.1% per year. They find moreover that the output loss from twin crises is more severe if default comes with a banking crisis rather than with a currency crisis, being the triple crisis the worst possible scenario. Finally they find that average per annum output loss is increasing in the length of the crisis: the longer it takes to reduce arrears or complete a restructuring, the more output falls relative to its estimated potential. Again endogeneity issues are easily visible.

Panizza and Levy Yeyati (2006) examine 20 default cases between 1982 and 2003 reaching opposite conclusions: they find that, contrary to what is typically assumed, output downturns precede default, and that the trough of the contraction occurs in the quarter of default, and GDP starts to grow after default [54]. They show this results by using quarterly GDP data. First they plot the quarterly GDP data for defaulting countries in a window of 6 years centered on the default event: they find that GDP starts falling in the three years before the event, and keeps falling in the quarter after the event, immediately reverting the trend thereafter. They validate this result by regressing GDP growth on a market pressure index and some dummies that track the quarter of default and three quarters before and after default: growth is always significantly lower in the quarters leading to default but not in the quarters following default, pattern that they claim being hidden by the customary use of annual data. To further validate that default marks the beginning of a recovery they compare cumulative growth before and after a default event for different windows centered on the event: cumulative growth goes from negative to positive. Clearly they do not want to state causality: in their findings as default does not cause further GDP contractions, it is not even the cause of economic recovery. As claimed above, default tends to occur during major economic downturns, that are followed by steeper recoveries.

Borensztein and Panizza (2009), using cross-country annual panel data for 83 countries from 1972 to 2000 explore the dynamic structure of the impact of default on growth [8].

Specifically they run the following regression:

$$GROWTH_{it} = \alpha + \beta X_{it} + \gamma DEF_{it} + \sum_{j=0}^{3} \delta_j DEFB_{it-j} + \epsilon_{it}$$

that has the same specification of the Sturzenegger regression, but it is time indexed. $GROWTH_{it}$ is the growth rate of real per capita GDP, X_{it} is the typical vector of controls used in growth cross-countries regressions, DEF_{it} is a dummy variable taking value 1 if the country i is in state of default at time t, and $DEFB_{it}$ is a dummy that takes value 1 if the country entered in state of default in the selected year. They estimate $\gamma = -1.184$, and $\delta_i = -1.388, 0.481, 0.337, 0.994$. So there is large effect in the first year of the default episode, with a drop in growth of 2.6%, and they find no significant effect of lagged default variables. As Uribe and Schmitt-Grohé (2017) [48] highlight, this implies that if the growth rate goes back to its predefault trend, there is a permanent output loss, that will be always be lower to its nondefault counterfactual, according to the findings of Borensztein and Panizza. Again the endogeneity problem does not let us to say if this permanent output loss is due to the default decision per se, or the default decision is determined by economic disruptions.

2.5.4 *Trade costs*

We have seen that the theoretical model developed by Bulow and Rogoff bases the reason for repayment on punishments inflicted by the affected lender to the borrower, that harm the defaulter while at the same time giving some payoff to the creditor, so being credible. One of the typical arenas in which this can happen is international trade, through trade sanctions and embargoes. Beside direct punishment by the lender, sovereign default can affect trade by simply resulting in a drying up of short term trade credit, that is used to finance international trade.
Rose (2005) gives an estimate of the effect of default on international trade without disentangling between retaliation and the trade credit effect [40]. The author uses data from IMF and World Bank, focusing on 283 Paris Club deals from 1948 and 1997 among 217 countries, at annual frequency. The estimation is performed through a gravity equation, as follows:

$$logT_{ijt} = \alpha + \beta X_{ijt} + \sum_{k=0}^{K} \gamma_k IMF_{ijt-k} + \sum_{m=0}^{M} \delta_m RENEG_{ijt-m} + \epsilon_{ijt}$$

where T_{ijt} denotes the average value of real bilateral trade between country i and country j at time t, X_{ijt} is a vector of typical regressors used in gravity equations' estimation, among which there is real GDP of country i and country j, population of both countries, distance between country i and country j, dummies for common language and common currencies. IMF is a dummy variable that takes value 1 if the two countries began a program with IMF at time t, 2 if both countries began the program at time t, 0 otherwise. This dummy variable is included in the model since Paris Club restructurings are always coupled with IMF programs. $RENEG$ is the dummy that captures the effect of default: it takes value 1 if country i and country j renegotiated debt at time t, and 0 otherwise. The coefficients of interest are the δ_m that capture current and lagged debt renegotiation's effects on trade. The model is estimated with $K = 5$ and $M = 15$.
The author estimates an effect associated to the inception of IMF pro-

grams with a drop on bilateral trade of about 10%, that dies away after 3 years. The average effect on the renegotiations dummy variables is about 8% per year, and it seems to be persistent up to 15 years.
The Rose's regression finds a negative impact on bilateral trade between the defaulting country and the creditor countries affected by default, but it does not aim to disentangle the drivers of those reductions in trade. Martinez and Sandleris (2011) address the issue, by giving an estimate of bilateral, multilateral and general effects of sovereign default on trade flows [29]. They define bilateral effect as a decline in trade that is stronger with respect to the affected creditors counterparties than with all other countries; a multilateral effect as a stronger decline in trade flows with respect to all creditor countries than with non creditor countries; general effect as a general decline in trade with the whole set of trading partners. To establish the role of sanctions (overt or covert) in international trade flows decline they do not work with changing policy signals, which may be different to interpret or ambiguous, but directly on trade flows data, basing their analysis on the same dataset used by Rose.
Their line of reasoning is that if in the aftermath of a default the countries that suffered the insolvency impose sanctions, the bilateral effect should prevail; if all creditors impose sanctions, then the multilateral effect should prevail and "the decline effect should be at least as severe with affected creditors as with non affected countries". So bilateral effect and multilateral effect paired with the "relative severity condition" are seen as proxies for sanctions. In order to disentangle the general effect from the bilateral effect they run the following regression:

$$logT_{ijt} = \alpha + \beta X_{ijt} + \sum_{k=0}^{K} \gamma_k IMF_{ijt-k} + \sum_{m=0}^{M} \delta_m RENEG_{ijt-m} + \sum_{l=0}^{L} \eta_l RG_{ijt-l} + \epsilon_{ijt}$$

that has the same variables of the Rose's regression, with the add of RG, a general default dummy that takes value 1 if either country i or country j were involved in debt renegotiations as debtors. In order to disentangle the general effect from the multilateral effect the authors run a second regression replacing $RENEG$, that is a dummy for bilateral effect, with another dummy, $CRED$, that takes value 1 if one of the two countries has defaulted on international debt and is negotiating within the Paris Club, and the other is member of a specific creditor coalition, namely the Paris club itself or the OECD, as follows:

$$logT_{ijt} = \alpha + \beta X_{ijt} + \sum_{k=0}^{K} \gamma_k IMF_{ijt-k} + \sum_{m=0}^{M} \phi_m CRED_{ijt-m} + \sum_{l=0}^{L} \eta_l RG_{ijt-l} + \epsilon_{ijt}.$$

The relative severity condition to assess the prevailing of multilateral effect is tested by running

$$\begin{aligned} logT_{ijt} = \alpha + \beta X_{ijt} + \sum_{k=0}^{K} \gamma_k IMF_{ijt-k} + \sum_{m=0}^{M} \delta_m RENEG_{ijt-m} + \\ + \sum_{l=0}^{L} \zeta_l OC_{ijt-l} + \sum_{n=0}^{N} \theta_n NC_{ijt-n} + \epsilon_{ijt} \end{aligned}$$

where OC is a dummy variable taking value 1 if one of the countries is a debtor involved in renegotiations at time t and the other one is a creditor country not involved in the renegotiations, and 0 otherwise; NC is another dummy that takes value 1 if one of the two countries is a debtor involved in international public debt restructuring and the other is not a creditor. The three models tested by the authors tell us the same result: there is an remarkable decline in overall trade when a country defaults on international public debt, but there is no evidence of sanctions imposed by countries that suffered default or by coalitions of creditors. Using 10 lags in the first regression the authors find a negative impact on overall trade of 5.5% per year during the first 5 years following default, while the effect on affected creditors is positive. The coefficient on bilateral default variable turns negative after 7 years, but it is not statistically significant. Same results are found in the second regression, where again the general default variable has a negative and statistically significant coefficient using 5, 10 and 15 lags, whereas the coefficient on the creditors coalition's dummy is again positive and turns negative only after some years without being statistically significant. In the third model the evidence against sanctions is provided by the fact that the sharper reduction on trade is accounted by flows with noncreditor countries, that leads the authors to a strong reject of the sanctions' hypothesis. Borensztein and Panizza (2009) tackle the issue of the role of the reduction of availability of trade credit for the country's exporting firms after default, that may arise from the risk of imposition of capital or exchange controls [8].

They use OECD data on net credit extended by OECD countries to developing countries and economies in transition: the OECD definition encompasses loans issued or guaranteed by the official sector for the purpose of trade not represented by negotiable instruments. This definition according to the authors may underestimate the whole volume of trade credit. They run the following regression:

$$NTC_{it} = \alpha + \beta DEF_{it} + \gamma X_{it} + \epsilon_{it}$$

where NTC_{it} is the net trade credit scaled by international trade in country i at time t, DEF is a dummy that takes value 1 if the country is in state of default at time t, X is a vector of controls. Estimation of this model finds a negative and statistically significant value on the coefficient for the default dummy, suggesting that net trade credit falls when the country does not honor its international financial obligations.

They then run an equation relating bilateral trade and default, controlling for trade credit as follows:

$$logT_{ijt} = \alpha + \beta DEFDI_{ijt} + \gamma TCDI_{ijt} + \delta X_{ijt} + \epsilon_{ijt}$$

where $logT_{ijt}$ is the log of bilateral trade between country i and country j at time t, $DEFDI_{ijt}$ is a dummy variable that takes value 1 if either country i or country j is in state of default, and 0 otherwise, and the pair is composed by a developing and an industrial country, $TCDI_{ijt}$ is the total trade credit received by the developing country in the pair, and X_{ijt} is a vector of controls. By estimating this second model the authors find a negative and statistically significant value for the coefficient on the dummy variable, but the coefficient on the trade credit variable is positive and statistically significant. So the evidence is mixed: default episodes are associated with a decline in trade credit, but the relationship between trade and default is not affected when controlling for trade credit.

As Panizza, Sturzenegger and Zettelmeyer (2009) observe, the channel through which default affects trade flows is still a puzzle.

3

THE MODEL

In this chapter we develop the theoretical model we will simulate in the next chapter using MATLAB, considering first the case of positive probability of redemption after default, then the case of perpetual financial autarky.

3.1 THE MODEL ECONOMY

We consider a model of a small open economy (SOE) as developed in Arellano (2008) [5], Aguiar and Gopinath (2006) [2] and Uribe and Schmitt-Grohé (2017) [48], modifying the model proposed by Aguiar and Gopinath (2006) by removing the time trend from the output process.

The model economy is populated by identical individuals whose preferences are described by the utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$$

where c_t stands for the consumption at time t, $\beta \in (0,1)$ is the subjective discount factor, that captures impatience and u is the period utility function.

The representative agent has CRRA preferences: the period utility is of the form

$$u = \frac{c^{1-\gamma}}{1-\gamma}$$

and it is strictly increasing, reflecting a desire for more consumption, and strictly concave, reflecting diminishing marginal value of extra consumption. The curvature parameter γ generates aversion to risk and to intertemporal substitution.

3.1.1 *Output process*

Our economy is designed to trade a single good and a single one-period bond with the rest of the world. Each period $t \geq 0$ the representative country is endowed with y_t units of goods. The endowment process is given by

$$y_t = e^{z_t}.$$

In our setting the output process is entirely determined by a shock z_t, that follows an AR(1) process:

$$z_t = \mu_z(1-\rho_z) + \rho_z z_{t-1} + \epsilon_t^z$$

where $|\rho_z| < 1$ is the autoregressive parameter, $\epsilon_t^z \sim N(0, \sigma_z^2)$ is the disturbance term.
Assuming stationarity of the process we easily find that $E[z_t] = \mu_z$ and $var(z_t) = \frac{\sigma_z^2}{1-\rho_z^2}$. The process is assumed to be lognormal in order to prevent the realization of negative level of the endowment, that would not have economic meaning.

3.1.2 *Budget constraint*

We assume that the government acts in a benevolent way, i.e. it maximizes households' utility. Each period t it can buy or sell one period zero coupon bonds in a quantity a_{t+1} for a price of $q(a_{t+1}, z_t)$. We denote with a_{t+1} net foreign asset position entered in at time t that matures at $t+1$, with a_t net foreign assets purchased at time $t-1$ that mature in the current period. A negative value for asset means positive debt. Therefore if the government wants to take positive debt at time t it will receive $q(a_{t+1}, z_t)a_{t+1}$ units of good at time t with the promise to deliver a_{t+1} units of good at $t+1$. If the government runs a budget surplus it invests $q(a_{t+1}, z_t)a_{t+1}$ units of good at t to receive a_{t+1} units the next period.
We model the price to be endogenous to the level of debt assumed in the current period and to the realized shock to the economy, but the currently due quantity of debt is not relevant for the pricing function. This is due to the fact that in our environment there are only zero coupon bonds, and so at each period the whole stock of the country's debt is negotiated again: there is only a one period dependence in the debt position. The shock to the economy is instead relevant, since it is informative of the business cycle and, given the autoregressive structure of the endowment process, of the future financial strength and solvency of the country.
At the beginning of each period the government decides to honor its financial obligations or to default.
The budget constraint for a country that stays in the contract is given by:

$$c_t - a_t = y_t - q(a_{t+1}, z_t)a_{t+1}.$$

To avoid Ponzi Games we require a lower bound for asset holdings:

$$a_t \geq a_{lb}.$$

A default decision by the government entails some consequences according to our model: current debts are erased form the budget constraint, international saving and borrowing is no more allowed, i.e. the country is excluded from financial markets for a certain (stochastic) number of periods. The budget constraint for a country that defaults on its financial obligations is:

$$c_t = y_t^{def}$$

where $y_t^{def} \leq y_t$, that means that we assume an output loss in autarky.

3.1.3 *Recursive structure*

At the beginning of each period the economy can be either in a good financial status of in a bad financial status. The bad financial status

is associated with having defaulted on debt, the good financial status is associated with having repaid debt in the previous period.
We define the model with a recursive structure following the Bellman principle. Default will occur if the value associated to honor debt obligations is lower than that attached to default on debt payments and entering a bad financial status.
Thus, the value function associated with being in good financial standing at the start of period t is defined as follows:

$$V^G(a_t, z_t) = max(V_t^C, V_t^B)$$

where the superscript B denotes bad financial status, the superscript G good financial status and the superscript C honoring financial obligations and continuing to participate to capital markets
Under the specification of our model default will occur if at beginning of period t for an economy in good standing $V^B(z_t) > V^C(a_t, z_t)$ (significantly V^B does not depend on a, signaling that debt has been repudiated). The decision to default entails an output loss, but not perpetual exclusion from the financial markets: a country with bad financial standing will be redeemed with probability λ and start next period with good financial status and no debt. If $\lambda = 0$ there is perpetual exclusion from international capital markets, and some characteristics of the model with such specifications will be developed as a special case. The recursion associated to the bad financial status is defined by

$$V^B = u((1-\delta)y_t) + \lambda\beta E_t V^G(0, z_{t+1}) + (1-\lambda)\beta E_t V^B(z_{t+1})$$

where δ is the parameter that determines the output loss in autarky.

Significantly in defining the recursive structure of V^B we do not maximize with respect to the control: in fact as specified above all the (penalized) output is consumed in a bad financial state, so there is no optimal control problem to solve, since the value of the control is already given in the budget constraint. Moreover we assume that the probability λ of regaining access to financial markets is constant each period, and assumed to be independent from the realization of y.
The spell of time T until regaining access to financial markets after a default episode is modeled as a random variable of time of first success, i.e. with a modified geometric distribution: $T \sim ModGeom(\lambda)$. Therefore we easily find the average time of financial autarky as the expected value of the modified geometric distribution:

$$E(T) = \sum_{k=1}^{\infty} k\lambda(1-\lambda)^{(k-1)} = \frac{1}{\lambda}$$

which is decreasing in the probability of redemption.

The Bellman equation associated with honoring the obligations and continuing to participate in the capital markets is

$$V^C = max_{c_t}[u(c_t) + \beta E_t V^G(a_{t+1}, z_{t+1})]$$

subject to te budget constraint defined above.

3.1.4 *Lenders and bond pricing*

Following the framework of Cochrane (2005) the price of the asset i in a world with no uncertainty is given by

$$q_t^i = \frac{1}{R^f} x_{t+1}^i$$

where $R^f = 1 + r^\star$ is the gross risk-free rate of return, the prevailing risk free rate is denoted by $r^\star$, and x_{t+1}^i is the payoff of the asset i, that will be gained with certainty in such a state of the world.
Introducing uncertainty we have a payoff that is a random variable, and so we must consider its expected value, and a proper discount factor, as follows:

$$q_t^i = \frac{1}{R^i} E_t[x_{t+1}^i].$$

A one period risk free bond is defined as an asset with a payoff of one, that will come with certainty, therefore we can write:

$$q_t^{rf} = \frac{1}{R^f}$$

that is also the price of a risk free one period bond in our model economy.
The sovereign bond of the economy that we are considering is not however a risk free asset, as we have have seen it has a positive probability of default. We can define a default indicator random variable:

$$I_{D(a_t,z_t)} = \begin{cases} 1, & \text{if } V^B(z_t) > V^C(a_t, z_t) \\ 0, & \text{otherwise} \end{cases}$$

that is a Bernoulli random variable with success parameter equal to $P(V^B(z_t) > V^C(a_t, z_t))$.
In such a setting the default probability is equal to the expected value of the default indicator variable:

$$P(V^B(z_t) > V^C(a_t, z_t)) = P(default) = E[I_{D(a_t,z_t)}] = \zeta$$

So the expected payoff of such a bond is not equal to 1 with certainty.
The price of the zero coupon bond is given by discounting the expected payoff:

$$q(a_{t+1}, z_t) = \frac{E_t(1 - I_{D(a_t,z_t)})}{R^f} = \frac{1-\zeta}{R^f}$$

using the same risk free discount factor implied risk by neutral pricing
As we can see the sovereign risky bond price is decreasing in the expected value of the default indicator variable.

The international capital market is modeled as an environment of perfectly competitive risk neutral investors, that are assumed to have perfect information about the economy's endowment process and can correctly observe the income level every period. Every period lenders choose a_{t+1} to maximize expected profits ϕ, as price takers:

$$\phi = q(a_{t+1}, z_t)a_{t+1} - \frac{1-\zeta}{1+r^\star}a_{t+1}$$

that is a zero expected profit condition.

The default probability $\zeta \in [0,1]$, and so the bond price $q \in [0,(1+r^\star)^{-1}]$: when default occurs with certainty the price is zero; for a positive level of asset holdings instead the probability of default is zero and so the price of the bond will be equal to the inverse of the gross risk free rate. The country gross interest rate is given by the inverse of the sovereign bond price:

$$1 + r^c = \frac{1}{q(a_{t+1}, z_t)}$$

and the country spread s is the difference between the country interest rate and the international risk free rate:

$$s = r^c - r^\star.$$

3.1.5 *Decision timing*

The schedule of the government decision is structured as follows: at the beginning of period t it inherits the asset position a_t, it observes the endowment realization y_t, that in our setting depends in a certain part on the output at $t-1$, and decides to honor its debt or to default. If the government decides to stay in the contract, it decides the asset level that matures next period a_{t+1} subject to the budget constraint. The difference between the new asset position and the inherited asset position is transferred to the households for consumption.

3.2 THE DEFAULT SET

The default set is the set of all the endowment levels at which a country chooses to default given a certain level of debt:

$$D(a_t) = \{y_t \in Y | V^B(z_t) > V^C(a_t, z_t)\}$$

Given this definition of the default set we can list some useful properties, following the lines of Arellano (2008).

Default sets are shrinking in assets: for all $a_t \le a_t^\bullet$, if default is optimal for $a_t^\bullet$ for some realizations of the endowment process, then default will be optimal for higher debt levels for the same realizations of the endowment, i.e. $D(a_t) \subseteq D(a_t^\bullet)$.

Proof. For all $y_t \in D(a_t^\bullet)$, default is the optimal policy so

$$u((1-\delta)y_t) + \lambda\beta E_t V^G(0, z_{t+1}) + (1-\lambda)\beta E_t V^B(z_{t+1})$$

$$> u(y_t + a_t^\bullet - q(a_{t+1}, z_t)) + \beta E_t V^G(a_{t+1}, z_{t+1})$$

By assumption $a_t \le a_t^\bullet$, so we have:

$$y_t + a_t^\bullet - q(a_{t+1}, z_t)a_{t+1} \geq y_t + a_t - q(a_{t+1}, z_t)a_{t+1}, \forall a_{t+1}.$$

If the optimal policy is to stay in the contract, and to honor financial obligations:

$$u(y_t + a_t^\bullet - q(a_{t+1}, z_t)a_{t+1}) + \beta V^G(a_{t+1}, z_{t+1})$$

$$\geq u(y_t + a_t - q(a_{t+1}, z_t)a_{t+1}) + \beta V^G(a_{t+1}, z_{t+1}).$$

This establishes that under no default condition the value of the contract is increasing in asset holdings.
Since we have assumed that for the asset level $a_t^\bullet$ default is the optimal choice

$$u((1-\delta)y_t) + \lambda\beta E_t V^G(0, z_{t+1}) + (1-\lambda)\beta E_t V^B(z_{t+1})$$

$$> u(y_t + a_t - q(a_{t+1}, z_t)a_{t+1}) + \beta E_t V^G(a_{t+1}, z_{t+1})$$

□

3.3 CASE OF PERPETUAL FINANCIAL AUTARKY

Now we turn our attention to the case in which $\lambda = 0$, i.e. the case in which the status of bad financial standing is absorbing and there is no positive probability to regain access to international capital markets. In such a scenario we additionally remove the output loss in autarky, setting $\delta = 0$. Under these assumption we can prove two useful results.

3.3.1 *Trade balance implications*

We define the trade balance as

$$tb_t = y_t - c_t.$$

From the budget constraint we can see that equivalently

$$tb_t = q(a_{t+1}, z_t)a_{t+1} - a_t.$$

We can show that at debt levels for which the default set is not empty, an economy that chooses not to default will run a trade surplus. In other words, for positive default probabilities there are no contracts available such that the economy can experience positive capital inflows. Formally stated, if $D(a_t) \neq \emptyset$, then $tb_t = q(a_{t+1}, z_t)a_{t+1} - a_t > 0$, $\forall a_{t+1}$ satisfying the No-Ponzi-Game condition.

Proof. Assume that $D(a_t) \neq \emptyset$, and that for some a_{t+1}^* the trade balance is nonpositive, i.e. $q(a_{t+1}^*, z_t)a_{t+1}^* - a_t \leq 0$.
So we have

$$V^C = max_{c_t}\{u(c_t) + \beta EV^G(a_{t+1}, z_{t+1})\}$$

$$= max_{c_t}\{u(y_t + a_t - q(a_{t+1}, z_t)a_{t+1}) + \beta EV^G(a_{t+1}, z_{t+1})\}$$

$$\geq u(y_t + a_t - q(a^*_{t+1}, z_t)a^*_{t+1}) + \beta EV^G(a^*_{t+1}, z_{t+1})$$

$$\geq u(y_t) + \beta EV^B(z_{t+1})$$

$$= V^B(y_t),$$

$$\forall y_t \in Y.$$

The first inequality follows from the definition of a maximum, the second from the fact that by assumption $a_t - q(a^*_{t+1}, z_t)a^*_{t+1} \geq 0$, and $V^G(a_{t+1}, z_{t+1}) \geq V^B(z_{t+1})$. But if for all realizations of the stochastic endowment it is optimal to stay in the contract and to honor the country's financial obligations the default set is empty, that contradicts our assumption, and proves the statement. □

This proposition states that a country with a level of debt that puts it at risk of default and that chooses to continue participating in international capital markets will devote part of its endowment to the debt service, by running trade balance surpluses.
The proof can be given precisely only assuming $\lambda = 0$. In the general model the value function associated with the bad financial financial standing has also the term associated with redemption, $\beta\lambda EV^G(0, z_{t+1})$, that for a sufficient large λ can result in a value of default that exceeds the value of staying in the contract , even with positive capital inflows: in principle we cannot state the last inequality of the above proof if there is positive probability of redemption.

3.3.2 *Default incentives*

Now we turn to characterize when the default incentives are stronger. We can prove that the lower is the realization of the income process, the higher the incentives not to honor debt. This rather intuitive conclusion is based on the fact that the utility function is concave and increasing in consumption. Under no default, i.e. if a country can access international capital markets to roll over its debt and experience net capital inflows, it will decide to stay in the contract, because this will determine an increase in current consumption and in overall utility. Conversely, if the default set is not empty the contracts available are not useful insurance instruments, because they cannot increase consumption with respect to the endowment, so default will be preferable in recessions.

Formally: $\forall y^*_t \leq \bar{y}_t$, if $\bar{y}_t \in D(a_t)$, then $y^*_t \in D(a_t)$.

Proof. If $\bar{y}_t \in D(a_t)$, by definition the value of default is greater than the value of staying in the contract, i.e.

$$u(\bar{y}_t) + \beta EV^B(z_{t+1}) > u(\bar{y}_t + a_t - q(a_{t+1}, z_{t+1})a_{t+1}) + \beta EV^G(a_{t+1}, z_{t+1}).$$

Letting $a^{\diamond}_{t+1}$ be the asset holdings under the endowment realization $\bar{y}_t$ and a°_{t+1} the asset holdings under y^*_t, with $a^{\diamond}_{t+1} \leq a^{\circ}_{t+1}$ if

$$u(\bar{y}_t + a_t - q(a^{\diamond}_{t+1}, z_{t+1})a^{\diamond}_{t+1}) + \beta EV^G(a^{\diamond}_{t+1}, z_{t+1}) - \{u(y^*_t + a_t - q(a^{\circ}_{t+1}, z_{t+1})a^{\circ}_{t+1}) + \beta EV^G(a^{\circ}_{t+1}, z_{t+1})\}$$

$$> u(\bar{y}_t) + \beta EV^B(z_{t+1}) - \{u(y^*_t) + \beta EV^B(z_{t+1})\}$$

then $\bar{y}_t \in D(a_t)$ implies $y^*_t \in D(a_t)$.

To end the proof we have to show that the above inequality holds. The right hand side simpifies to $u(\bar{y}_t) - u(y^*_t)$.
Due to utility maximization, given that $y^*_t \leq \bar{y}_t$,

$$u(\bar{y}_t + a_t - q(a^{\diamond}_{t+1}, z_{t+1})a^{\diamond}_{t+1}) + \beta EV^G(a^{\diamond}_{t+1}, z_{t+1})$$

$$\geq u(\bar{y}_t + a_t - q(a^{\circ}_{t+1}, z_{t+1})a^{\circ}_{t+1}) + \beta EV^G(a^{\circ}_{t+1}, z_{t+1}).$$

We first prove the following

$$u(\bar{y}_t + a_t - q(a^{\circ}_{t+1}, z_{t+1})a^{\circ}_{t+1}) + \beta EV^G(a^{\circ}_{t+1}, z_{t+1}) - \{u(y^*_t + a_t - q(a^{\circ}_{t+1}, z_{t+1})a^{\circ}_{t+1}) + \beta EV^G(a^{\circ}_{t+1}, z_{t+1})\}$$

$$> \{u(\bar{y}_t) - u(y^*_t)\}$$

The above inequality can be simplified as

$$u(\bar{y}_t + a_t - q(a^{\circ}_{t+1}, z_{t+1})a^{\circ}_{t+1}) - u(y^*_t + a_t - q(a^{\circ}_{t+1}, z_{t+1})a^{\circ}_{t+1})$$
$$> \{u(\bar{y}_t) - u(y^*_t)\},$$

that implies

$$u(\bar{y}_t) - u(\bar{y}_t + a_t - q(a^{\circ}_{t+1}, z_{t+1})a^{\circ}_{t+1}) < u(y^*_t) - u(y^*_t + a_t - q(a^{\circ}_{t+1}, z_{t+1})a^{\circ}_{t+1}).$$

Since $\bar{y}_t \in D(a_t)$, then $a_t - q(a_{t+1}; z_{t+1})a_{t+1} < 0$ for all contracts available. Therefore the above inequality holds, since the utility function is strictly increasing and concave: the reduction in utility that occurs subtracting the same magnitude of utility is greater the lower the starting point. Therefore our initial inequality is proved a fortiori, being $a^{\diamond}_{t+1} \leq a^{\circ}_{t+1}$, and we have $y^*_t \in D(a_t)$. □

3.4 RISKY LENDING AND RISKY BORROWING

3.4.1 *The risky lending set*

We can give a characterization of the risky lending set by noticing that for a given realization of the output process, as assets decrease the value of staying in the contract decreases, with the value of default that stays constant: the value function for the bad financial status does not depend on a_t. Therefore there is a level of asset holdings for

which the default set is the entire set, i.e. default is always preferred. As we noted before, default is chosen only with negative assets: there is a level of assets $a \leq 0$ such that default set is empty, a level of asset holdings such that investing in the country bond is exactly as investing in the risk free asset.
Denoting as $\underline{a}$ the upper bound of assets for which the default set constitutes the entire set, i.e. the smallest level of debt for which default occurs with certainty and is always preferable, and as $\bar{a}$ the lower bound of assets for which the default set is empty, i.e the highest level of debt such that default never occurs and is always preferable to stay in the contract we can write:

$$R = (\underline{a} = sup\{a|D(a) = Y\}, \bar{a} = inf\{a|D(a) = \emptyset\})$$

where Y is the set of the possible realizations of the endowment process.
Risky lending is then an open interval whose endpoints are the smallest level of debt for which the government will decide always not to repay, and of course for this level of debt there will not be contracts available, and the lowest asset level for which lending is not risky anymore.

3.4.2 *Risky borrowing*

Since we know that default incentives are higher the lower the realization of the output process, and that the value of staying in the contract is increasing in assets we can characterize again the default set as an interval $[\underline{y}, y^*(a))$, where only the upper bound is a function of assets, and $\underline{y}$ is the worst possible realization of the output process. The default boundary is the value for which the value of repayment and default are equal, i.e. $V^B(y^*(a)) = V^C(y^*(a), a)$.
Now we turn to characterize the effectively relevant region for risky borrowing. We have that for $a \geq \bar{a}$ the country bond prices are equal to the inverse of the risk free rate; for $a \leq \underline{a}$, the bond prices falls to zero, since for such levels of debt default occurs with certainty. For asset levels that belong to the risky lending set we have that bond prices are increasing in assets, but $q(a_{t+1}, z_{t+1})a_{t+1}$ first decreases until a certain point a^* and then increases. The borrower will never choose a bond contract such that $a_{t+1} < a^*$: he can always find a contract that entails lower financial obligations for the future while increasing consumption by the same amount. The asset level a^* is the one that increases the most current consumption. So we find that the risky borrowing region is $B = [a^*, \bar{a}]$, and it is a subset of the risky lending region: the bond contracts that the borrower rationally decides to enter in are only a fraction of the contracts available on international capital markets.

4

MATLAB SIMULATION

In this chapter we present a simulation of the model developed in the previous chapter, performed using MATLAB. The codes, reported in the appendix, are slight modifications of the ones attached to the above mentioned paper of Aguiar and Gopinath (2006) [2]. [1]

4.1 VALUE FUNCTION ITERATION

The first part of the code is based on a value function iteration process. The values selected fo the simulation parameters are the same used by Aguiar and Gopinath:

- The coefficient of relative risk aversion γ is as customary in the literature set equal to 2;
- The quarterly world risk free rate is set equal to 1%;
- The discount factor β is set equal to 0.8;
- The percentage output loss in autarky δ is set equal to 2%;
- The probability of redemption λ is set equal to 0.1 per quarter;
- The autocorrelation coefficient of the AR1 process ρ_z is set equal to 0.9;
- The standard deviation of the innovation term σ_z is set equal to 0.034.
- The long run mean of the autoregressive process for the shock is set equal to $-0.5\sigma_z^2$.

The possible asset holding values are discretized in a grid of 400 points, with the maximum level representing no debt, and the lowest level of holdings representing a debt level equal to 30% of GDP. The continuous AR1 process is discretized in a 25 states Markov chain using the method proposed by Tauchen (1986) [45]. The author proposes to choose a certain number N of possible states of the variable of interest z, and to choose a maximum value z_N that is a multiple of the unconditional standard deviation of the process. The lowest value the discretized process can take, z_1 is set equal to $-z_N$ and the remaining values $\{z_2, \dots, z_{N-1}\}$ are located in an equispaced manner, with pace p. The transition probabilities are given by

$$\pi_{jk} = F(\frac{z_k + p/2 - \rho_z z_j}{\sigma_z}) - F(\frac{z_k - p/2 - \rho_z z_j}{\sigma_z})$$

1 Original codes are available at https://scholar.harvard.edu/gopinath/pages/data-and-codes

for $j, k \in [2, \ldots, N-1]$, where $F()$ is the standard normal CDF. The transition probabilities at the boundaries, for $k = 1$ and $k = N$ are given by

$$\pi_{j1} = F\left(\frac{z_1 + p/2 - \rho_z z_j}{\sigma_z}\right)$$

and

$$\pi_{jN} = 1 - F\left(\frac{z_N - p/2 - \rho_z z_j}{\sigma_z}\right).$$

The process of the code starts by initializing the variables, setting the price of the zero coupon bond to a starting level equal to the inverse of the riskfree rate, and guessing an initial value for the value functions of being in a bad financial standing, participating to international capital markets and being redeemed to international lending after default. The optimization process is performed using a nested while loop: the inner loop is the proper value function iteration loop, the outer loop is on the interest rate. The inner loop starts by computing the expected value of the value functions, by multiplying the properly reshaped value functions' vectors by the transition matrix obtained through the Tauchen algorithm. Once we have the expected value function for the next period for each value of the shock and each asset level we select the maximum through current and future consumption, and we store the index for the asset level that provides the max for each realization of the shock in two distinct policy function vectors, one associated with good financial standing, the other associated with bad financial standing. Moreover a logical vector for default is filled during the process, taking value 1 for the indices in which the elements of the value function of bad financial standing are greater than the ones of the value function associated to participating to capital markets. Then the originally guessed value functions are updated, and the process continues until convergence.
The outer loop is performed by computing the expected value of default, multiplying the transition matrix derived from the Tauchen algorithm by the default vector, that is reshaped as a $Z \times A$ matrix, where Z is the number of possible values for the shock, and A is the number of elements in the grid for asset holdings. The original default vector is again reshaped in a $Z \times A$ matrix and the elements are summed up through the columns: if for a given asset level default occurs with certainty, no matter what the realization of the shock, the sum of the elements of the given column equals to the number of possible states of the shock. The column indexes of the reshaped default vector associated with certain default are then used to select the corresponding columns of the expected default matrix, that are filled by row with ones. Then the price of the sovereign bond is updated by multiplying the starting value by $1 - Edef$, with $Edef$ being the matrix of expected default, in a risk neutral fashion. The so obtained new price schedule is then compared to the previously stored one, and the process is iterated until convergence. Below we present the plot of the price schedule for the sovereign bond with a high level of the shock and with a low level of the shock for all possible level of asset holdings: we clearly see from the graph obtained by the simulation that a good performance of the economy make sustainable a slightly higher level of debt, that in any case according to our model cannot never be as higher as 27% of GDP.

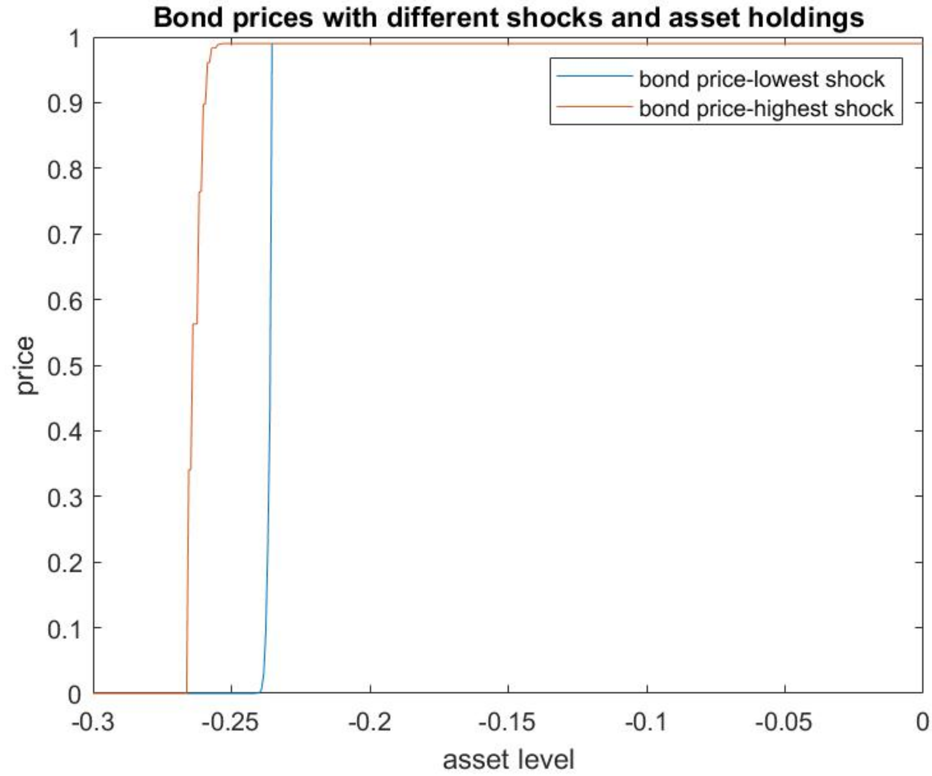

Figure 5: Price schedule of the ZCB with different shocks

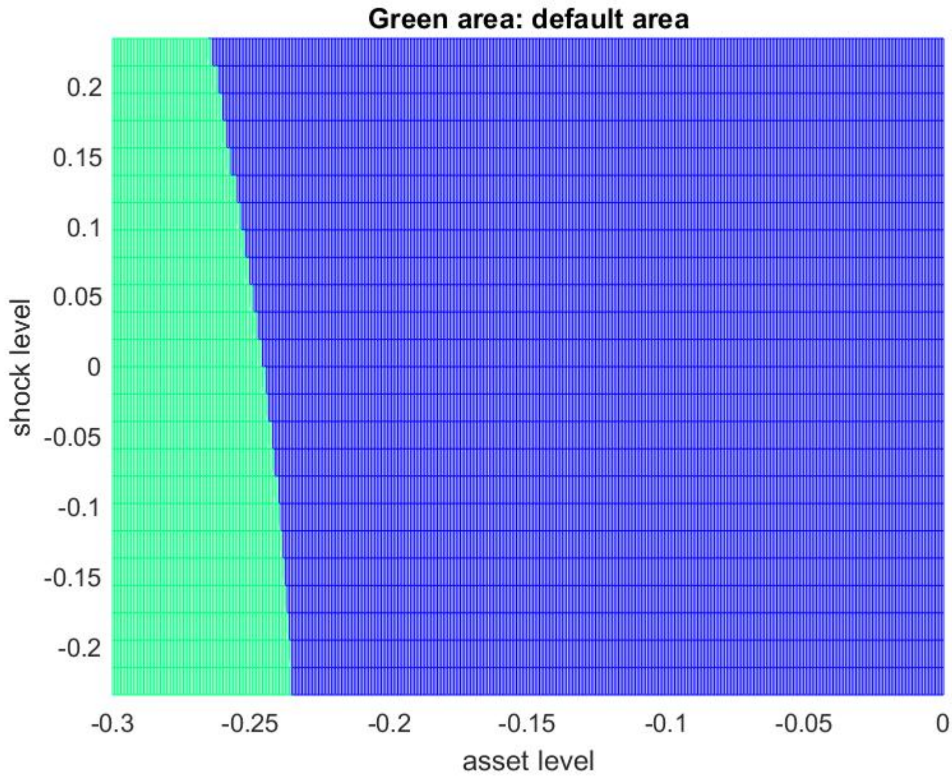

Figure 6: Default area, nondefault area, default boundary

The second graph represents the default and nondefault areas, and the default boundary we clearly see that the worse the shock the lower the possible debt level that can be held honoring financial obligations.

4.2 BUSINESS CYCLE SIMULATION

Once we have the policy vectors that store the indexes associated with the asset holdings level at $t+1$ that maximize utility in both the bad financial standing scenario and in the case of participation to capital markets, we run the code for the business cycle simulation. We simulate 100 times 1000 quarters of our model economy, then we select the last 500 observation for each iteration and take averages to then derive summary statistics.
The business cycle is simulated each time by generating a standard uniform random variable, that gives actually a probability, and comparing the realization of this random variable to the conditional CDF of the shock to the economy z_t given z_{t-1}, obtained by summing over a column a given row of the transition matrix obtained by Tauchen algorithm: if the realization is smaller or equal to the first value of the CDF vector, the cyclical position is the one associated to the lowest realization of the shock to the economy (the most negative one), if it is equal to 1 the business cycle at t is at its peak, otherwise it assumes one of the 23 intermediate values. Then it is simulated a logical vector for redemption of the same length as the business cycle's vector: it takes value 1 if there is redemption after default, 0 otherwise. Next step is initializing vectors for paths of our variables of interest: asset holdings, default state, that will be equal 1 if the economy is in state of default a t time t and 0 otherwise, expected default, and a vector for default history, that at time t takes value 1 if the economy was in state of default at $t-1$ and 0 otherwise, and finally a path vector for the price of the zero coupon bond internationally traded. Those vector are filled with values according to the three possible cases that can occur over the years: participation to capital markets, default with redemption, default without redemption. A positive level of debt will be held only in case of participation to capital markets and will be derived by the policy vector associated with good financial standing, both the cases of default will obviously have zero debt at $t+1$. The price of the zero coupon bond in each case will be determined using again the policy vector derived from value function iteration. Having filled our simulated time series for the level of debt and price of the sovereign bond we derive the simulated time series for log output, log consumption, trade balance, trade-balance-to-GDP ratio, and quarterly spread directly from the definitions of the theoretical model. Once we have averaged through iterations as mentioned above, we display volatilities and correlations for the statistics of interest. The average occurrence of default is shown for the complete simulation of 10000 quarters. Table 1 reports the results of the simulation.

Simulation Results		
Variable	Simulated Value	Argentina 1983.1-2000.2
$\sigma(y)$	7.72	4.08
$\sigma(R_s)$	0.03	3.17
$\sigma(tb/y)$	0.31	1.36
$\sigma(c)$	7.77	4.85
$\rho(y, R_s)$	0.1431	-0.59
$\rho(y, tb/y)$	-0.1455	-0.89
$\rho(y, c)$	0.9985	0.96
$\rho(R_s, tb/y)$	-0.1082	0.68
$\gamma_1(y)$	0.8922	
$Rate\ of\ default$	3.85	75

Table 1:
Averages of the 100 simulations, taking for each simulation the last 500 observations. R_s is the quarterly sovereign spread. Standard deviations are expressed in percentage terms. Rate of default is the number of quarters the country is in state of default over 10000 quarters. Business Cycle statistics for Argentina are taken from Aguiar and Gopinath (2006) [2].

The simulation of the model predicts a countercyclical trade balance, consistently with theory and data, even if real data for Argentina show a much highly negative correlation. The model gives counterfactual predictions relating to correlation between sovereign spread and output: if in real data spread tends to rise when output goes down and to move down during boosting economy, our model incorrectly predicts a spread that is positively correlated with output. This is turn leads to an incorrect prediction for the direction of the comovements between spread and trade balance: the data clearly show prociclicality, while in our simulated model trade balance and sovereign spread are negatively correlated. Output and consumption are much more volatile in the simulated model than in real data for Argentina, while interest rates are remarkably more volatile in the data than in the model. The output process of the model is highly persistent reflecting the value of the autoregressive parameter of 0.9 set in the optimization iteration. The model predicts one default in 2500 years, that is absolutely at odds with the rate of default for Argentina estimated by Aguiar and Gopinath, set to 75, while the default of 2001 was the fifth Argentine default or restructuring episode in the previous 180 years [2]. Finally the simulated model predicts a maximum debt-to-GDP ratio between 24% and 27%, depending on the state of the economy, that seems to be low when compared to real data for Argentina, that could sustain higher levels of debt, but it approaches the even stricter safe level estimated by Reinhart and Rogoff (2003) [35].

CONCLUSIONS

Government debt insolvencies are a recurring feature of public finance [7], that involve not only financial and economic issues, but also, as we have seen, juridical and political aspects, being one of the great cleavages and issues that characterize international economic relations. There are secular regularities that characterize this phenomenon, as highlighted by Reinhart and Rogoff (2011) [36]: yesterday's developing countries defaulted on their international financial obligation as today's emerging countries do. The epicenter of sovereign default has just moved from Europe (Spain defaulted on its external debt 13 times between 1500 and 1900 [35]) to Latin America. But we have seen that actually "this time could be different" as after the Great Recession and the European sovereign debt crises developed countries were involved in sovereign insolvencies, reverting the secular regularity of emerging countries representing the largest shares of debt in state of default. Political and juridical innovations can also lead to more efficient handling of debt crises, as we have seen with the role of CACs and the emerging of new institutional players as the EFSF-ESM, while the oldest forums as the Paris Club have started to lose their pivotal role.

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A

MATLAB CODES

Code for the value function iteration:

```
%Code for the Model I of the Paper by Aguiar and Gopinath without trend

%the endowment is given by y=exp(z)
%the preferences are given by u(c)=c^(1−s)/(1−s)


%z is the log productivity; Z possible states, spaced by the interval intz
%a is the financial asset vector; A possible levels of asset holdings
%(negative asset means debt) spaced by the interval inta

%h is the credit history: it can be good or bad;


%lambda  is the probability of transiting from h=bad to h=good
%s is the coefficient of rel risk aversion
%cost stands for the additional output cost if the country is in au
%beta is the discount factor
% rbase is world riskfree rate


%z follows the following AR(1) process:
%z(t)=mi_z∗(1−rho_z)+rho_z∗z(t−1)+u(t);
%mi_z is the expected value of the process (long run mean);
%rho_z is the AR(1) coefficient;
%u(t)~normal(0,sdz^2) is the innovation term; sdz is the standard deviation
%of the innovation

% q is the vector of prices for the domestic debt: it has size (A∗Z,A);
% the rows are A∗Z because we account for all the possible combinations of
```

```
% asset holdings and shock state at time t; the columns are A because we
% consider each possible asset level choice for time t+1;
%q is the inverse interest rate once we choose a column for asset holdings
%at t+1;

%Vgood is value of having good credit rating given a,z. Size:(AxZ,1)

% Vbad is value of having bad credit rating given a,z. Size:(AxZ,1)
%a always equals zero under bad crediit conditions

%  Vbadgood is value of having good credit history but zero
% assets (redemption)
%  policygood and policybad is index of choice of (a') given
%  state (a,z). Size (A*Z,1)



%parameters
lambda=0.1;
s=2;
sdz=0.034; %standard deviation of the innovation term
mi_z=-0.5*sdz^2;
rho_z=0.9;
sdpr=sdz/(1-rho_z^2)^(0.5); %standard deviation of the AR1 productivity process
A=400; %number of states for asset holdings
Z=25; %number of states for log prod
rbase=0.01;
cost=0.02; %percetage of output loss in financial autarky
beta = 0.8;%discount rate

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%state space

%limits of asset holdings
amax=0.0;
amin=-0.3;
%grid for asset holdings
inta=(amax-amin)/(A-1); %interval of the grid values;
a=amin:inta:amax; %grid written as a row;

a(find(a==min(a(a>=0))))=0; %ensures zero is a state: it assigns the value
% of zero to the smallest nonnegative element of a
a=a(:); %rewrites the grid as a column vector
azero=find(a==0); %identifies where a==0
```

```
%Tauchen discretization for the shock process: z, values of the shock,
%pdfz, transition matrix
[z,pdfz]=mytauchen1(mi_z*(1-rho_z),rho_z,sdz,Z);
pdfz=pdfz.*((1./(sum(pdfz,2)))*ones(1,Z)); %ensures each row adds up to one across columns



%Initial guess for the price of the domestic bond: the riskfree rate:
q0 = 1/(1+rbase)*ones(A*Z,A);

%Initial values for the value functions: we start from an initial guess of
%all ones

Vgood = ones(A*Z,1); %good credit rating
Vbad = ones(A*Z,1); %bad credit rating (autarky)

%Vbadgood picks out value of being in good credit standing with zero assets (redemption)
Vbadgood = reshape(Vgood,A,Z);
Vbadgood = Vbadgood(azero,:); %selects the row that has as index the one in which a==0
%from the grid of a
Vbadgood = ones(A,1)*Vbadgood;
Vbadgood = reshape(Vbadgood,A*Z,1); %ripristinates the initial proper size;

%income
y=exp(z);
y = kron(y,ones(A,1));
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Interest rate iteration

diffq=1;
tolq=1e-6; %arbitrary very small number

while diffq>tolq

%Savings given the choice of the asset holdings at t+1; future assets
%discounted by q0
S = (ones(A*Z,1)*a').*q0-(kron(ones(Z,1),a)*ones(1,A));


% current consumption is the endowment net of savings (case of repayment)
c=y*ones(1,A)-S;

%consumption under the defaul scenario
```

```
cdefault = (1-cost)*y*ones(1,A);

%utility functions
u=((c).^(1-s))./(1-s); %case of good financial standing
u(find(c<=0))=NaN; %ruling out cases of negative or zero consumption
udef=((cdefault).^(1-s))./(1-s); %in case of default
udef(find(cdefault<=eps))=NaN;



%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%value function iteration
diff=10;
tolV=min(max(diffq,tolq),1e-6); %tighten tolV as diffq declines


while diff>tolV



%compute the (initialiazed) expected values of the value function for the successive period, using the
%transition matrix pdfz

EVgood = pdfz*(reshape(Vgood,A,Z)'); %size(Z,A)
%Vgood will be next period's V assuming no default and the choice of
%next period's a  (across the columns).
%with reshape and transpose we have the z across rows and a
%(next period across cols;

EVgood = kron(EVgood,ones(A,1)); %This adds the current a across the rows. %size(A*Z,A);

%same for EVbad which is V assuming default
EVbad = pdfz*(reshape(Vbad,A,Z)'); %size(Z,A)
EVbad = kron(EVbad,ones(A,1)); %size(A*Z,A);


%EVbadgood is the value associated with redemption to
%good credit standing
EVbadgood = pdfz*(reshape(Vbadgood,A,Z)');
EVbadgood = kron(EVbadgood,ones(A,1));

%maximum found over each row, associated column index stored in
%policygood and policybad
[Vbad1,policybad] = max(udef + beta.*lambda.*EVbadgood + beta.*(1-lambda).*EVbad,[],2);
[Vgood1,policygood] = max(u + beta.*EVgood,[],2);
```

```
%(A*Z,1) vector of logical 1 if the value of default is greater of
%the value of repayment and staying in good financial status, zero
%elsewhere
default=Vbad1>Vgood1 | isnan(Vgood1)==1;

%replacing the values of the value function in good financial
%standing  that are smaller than the ones of the value function in bad
%financial standing with the values of the latter
Vgood1(find(Vbad1>Vgood1 | isnan(Vgood1)==1))=Vbad1(find(Vbad1>Vgood1| isnan(Vgood1)==1));


Vbadgood = reshape(Vgood1,A,Z);
Vbadgood = Vbadgood(azero,:);
%extract the value of row of Vgood1 associated with zero asset (as
%before): good financial status and zero debt (redemption)
Vbadgood = ones(A,1)*Vbadgood;
Vbadgood = reshape(Vbadgood,A*Z,1);

diff=max([max(max(abs(Vgood-Vgood1))),max(max(abs(Vbad-Vbad1)))])
%Subtract to our initial guess the result of maximization for the good
%and the bad state, then we label as diff the greatest absolute value of
%divergence


%Then we restart the maximization using as starting values the max
%find in the previous iteration that will be multiplied after reshaped
%by the pdfz and again optimized and checked for convergence
Vgood=Vgood1;
Vbad=Vbad1;

end %end value function iteration


%we now compute q1

%expected value of default, given the choice of a for the next period,
%the current level of assets and the value for the shock
Edef = pdfz*(reshape(default,A,Z)');

```

```
ind = reshape(default,A,Z)'; %restructuring the default logical vector in a
%(Z,A) matrix (shock levels trough the columns)

%if the sum through the cols equals Z=25, i.e. we find for a given
%level of assets only ones, the default occurs with certainty
ind = sum(ind)==Z;
%ind will be  logical vector with ones if the default is certain,
%and zero elsewhere;

for j=1:A

if ind(j)==1
Edef(:,j)=1;
end
end
%replaces the rows associated with the index of certain defaul of the
%matrix of expected value of default


Edef = kron(Edef,ones(A,1)); %we adjust the matrix of expected value of default to account
%for the current level of assets;

q1=1/(1+rbase).*(1-Edef); %price of the domestic bond accounting for the
%expected value of default


q1=max(q1,0);


diffq = max(max(abs(q1-q0))) % check for convergence of q

q0=q1; %price update for the successive iteration

end



%policy functions

policy=(1-default).*policygood+default.*azero;
%(1-default).*policygood selects the column index of the maxs when default
%does not occurr,if default occurs we select the index of azero

```

```


%calculate x_t(state)
for i=1:A*Z
ct(i,1)=c(i,policy(i));
qt(i,1)=q0(i,policy(i));
end


figure(1);
plot(a,q0(1,:),a,q0(end,:));
title('Bond prices with different shocks and asset holdings');
legend('bond price-lowest shock','bond price-highest shock');
xlabel('asset level');
ylabel('price');


figure(2);
mesh(a,z,abs(reshape(default,A,Z))');
axis([min(a),max(a),min(z),max(z),0,1]);
view(2)
xlabel('asset level');
ylabel('shock level');
title('Green area: default area')
colormap winter
```

Code for the Tauchen algorithm:

```
function [s, Pi] = mytauchen1(mu,rho,sig,N)

m       = 3;
s       = zeros(N,1);
Pi      = zeros(N,N);
s(1)    = mu/(1-rho) - m*sqrt(sig^2/(1-rho^2));
s(N)    = mu/(1-rho) + m*sqrt(sig^2/(1-rho^2));
step    = (s(N)-s(1))/(N-1);

for i=2:(N-1)
s(i) = s(i-1) + step;
end

for j = 1:N
for k = 1:N
if k == 1
Pi(j,k) = normcdf((s(1) - mu - rho*s(j) + step/2) / sig);
elseif k == N
Pi(j,k) = 1 - normcdf((s(N) - mu - rho*s(j) - step/2) / sig);
else
Pi(j,k) = normcdf((s(k) - mu - rho*s(j) + step/2) / sig) - ...
```

```
normcdf((s(k) - mu - rho*s(j) - step/2) / sig);
end
end
end
```

Code for the Business Cycle simulation:

```

%Business cycle simulation
sim=100;
nos=500;
T=10000;

for s=1:sim




%generate path of z:
%We generate realizations from an uniform probability
    distrubution to
% simulate business cycle: given the result of each
    realization we will
% check along the conditional CDF derived from the Markov
    Matrix in which
% one of 25 states of the economy we are at time t
%temp will be the cdf of z_t given z_t-1
%prob(temp(z(i-1))<shockz<=temp(z(i))) = pdf(z(i))


shockz=rand(T,1);
zpath=zeros(T,1);
zpath(1,1)=find(z==max(z(find(abs(mi_z-z)==min(abs(mi_z-z)))
    ))); %start zpath at z closest to muz

for t=2:T;
temp=cumsum(pdfz(zpath(t-1),:));
if shockz(t)<temp(1);
zpath(t,1)=1;
elseif shockz(t)>temp(Z);
zpath(t,1)=Z;
else
zpath(t,1)=find(shockz(t)<=temp(2:Z) & shockz(t)>temp(1:Z-1)
    )+1;
end;
end;








%generate redemption realization (1=redemption):
```

```
redemp = ones(floor(lambda*T),1);

redemp = [redemp; zeros(T-length(redemp),1)];
temp = randperm(T)';
redemp = redemp(temp);

%starting values
apath=zeros(T,1);
defpath=zeros(T,1);
history=zeros(T,1);
state=zeros(T,1);
qpath=zeros(T,1);
Edefpath=ones(T,1);

apath(1)=azero;

for t=1:T-1;
state(t) = (zpath(t)-1)*A + apath(t);

if history(t)==0 | (history(t)==1 & redemp(t)==1);
if default(state(t))==0;
apath(t+1)=policygood(state(t));
qpath(t) = q0(state(t),policygood(state(t)));
Edefpath(t)=Edef(state(t),policygood(state(t)));
Vpath(t)=Vgood(state(t));
elseif default(state(t))==1;
apath(t+1)=azero;
history(t+1)=1;
defpath(t)=1;
qpath(t) = 1/(1+rbase);
Edefpath(t)=0;
Vpath(t)=Vbad(state(t));


end;
elseif history(t)==1 & redemp(t)==0;
apath(t+1)=azero;
history(t+1)=1;
qpath(t) = 1/(1+rbase);
Edefpath(t)=0;
Vpath(t)=Vbad(state(t));

end;
end;

qpath(T)=NaN;


ypath = exp(z(zpath));
ay=a(apath(2:T))./(ypath(1:T-1));
```

```
logy = log(ypath);
assets = a(apath);
dassets = assets(2:T)-assets(1:T-1);
dassetsy = dassets./exp(logy(1:T-1));
nx = assets(2:T).*qpath(1:T-1)-assets(1:T-1);
nx(find(defpath(1:T-1)==1))=0;
c = exp(logy(1:T-1)) - nx;
c(find(defpath(1:T-1)==1))=exp(logy(find(defpath(1:T-1)==1)));
tbty=nx./ypath;
sovr = 1./qpath - 1;
spread = sovr-rbase;


sample = defpath==0 & (history==0 | redemp==1);


sovrt=sovr(T-nos:T-1);
logyt=logy(T-nos:T-1);
nxt = nx(T-nos:T-1);
nxy = nxt./exp(logyt);
lconst = log(c(T-nos:T-1));
spreadt=spread(T-nos:T-1);
spreadtannual=(1+spreadt).^4-1;




% note that the spread in the model is quarterly while it is annual in the
% data. need to make appropriate adjustment
STD=std([logyt, spreadt, nxt, lconst, tbtyg]);
CC=corrcoef([logyt, spreadt, nxt, lconst]);
AC=corrcoef(logyt(1:end-1),logyt(2:end));
stdyrnxc(s,:)=STD;
ccr(s,:)=[CC(1,2:4),CC(2,3),AC(1,2)];
defaultpc(s)=mean(defpath(find(apath<azero)))*100;

end

disp('mean std y r nx c')
disp(mean(stdyrnxc,1))
disp('mean correlation yr ynx yc rnx yy')
disp(mean(ccr,1))
disp('mean default')
disp(mean(defaultpc))
```

B

SUMMARY

The topic of the present Master's Degree Thesis is sovereign default. As a starting point we can mention the definition of sovereign default used by the main Credit Rating Agencies (CRAs), that are followed by many economists and are the foundations of the majority of datasets on sovereign default. Standard and Poor's Global Ratings "generally defines "default" as the failure to meet a principal or interest payment on the due date contained in the original terms of a debt issue" [50]. S&P counts two more precise conditions in order to characterize the state of default of a sovereign. "We consider a sovereign to be in default under any of the following circumstances:

- For local -and foreign- currency bonds, notes, and bills issued by the central government and held outside the public sector of the country, a sovereign default occurs when the central government either fails to pay scheduled debt service on the due date or tenders an exchange offer of new debt with less-favorable terms than the original issue;
- For private-sector bank loans incurred by the central government, a sovereign default occurs when the central government either fails to pay scheduled debt service on the due date or negotiates with the bank creditors a rescheduling of principal or interest at less-favorable terms than in the original loan" .

Ams, Baqir, Gelpern and Trebesch (2018) propose a useful analytical approach to default episodes [4]. They distinguish among Technical Default, Contractual Default, and Substantive Default. The authors define as Technical Default any contractual event of default on public financial obligations that does not constitute default under relevant third party definitions, by which they mean primarily those used by Credit Rating Agencies. Administrative errors and some covenant default could be labeled as Technical Default episodes by markets participants. Contractual Default is defined as any event of default that also constitutes default under third party definitions, so basically default episodes as defined by CRAs. Typically, Contractual Default episodes involve missed payments that persist for a duration that exceeds a certain grace period. According to the authors predefault debt exchanges and restructurings that follow modifications to the conditions of the debt contract do not fit the definitions. As Substantive Default the authors include default episodes that are mentioned in the relevant third parties definitions of default but do not constitute an event of default according to the letter of the relevant debt contracts. Exchange of distressed debt and restructurings properly fit

this definition. As we have seen sovereign default can occur when there is a failure to pay on the due date, or even a minor event as a covenant breach or an administrative delay. Repudiation takes place when a government declares the illegitimacy of a certain bond series or bank loan and rejects the obligation to pay.
The case of debt repudiation is different from a moratorium, in which a government, with a a public act, as an announcement or a piece of legislation stops unilaterally to service a certain amount of its debt without questioning the validity of the obligation to pay [4]. Repudiation typically occurs in the aftermath of regime changes in a certain country: typically the new government will refuse to honor the obligations taken on by the previous regime.
A famous example of repudiation of debt after a regime change is the Soviet repudiation of Tsarist debts in 1918 [27]. Following the definition of Das, Papaioannou and Trebesch (2012) we define a sovereign debt restructuring as an exchange of outstanding sovereign debt instruments, issued or guaranteed by the government, such as loans or bonds, for new debt instruments or cash through a legal process [47]. We have to distinguish between distressed debt restructuring, on which we focus for our analysis, that is a debt restructuring at terms less favourable than the original bond or loan terms, typically during a crisis or in its aftermath, from restructurings that are part of routine liability management operations (LMOs), such as debt swaps, that occur in normal times. As the authors point out, there are two main elements in a debt restructuring:

- *Debt Rescheduling*, that is the lengthening of maturities of the old debt, possibly at lower interest rates. This of course implies a certain amount of debt relief, by shifting payments in the future;

- *Debt Reduction*, that is a reduction in face value of the old debt instruments;

- *Debt Buyback*, defined as the exchange of outstanding debt instruments for cash, offered at discount.

This type of operations implies a haircut, that is a reduction in present value of creditor claims.
Cruces and Trebesch (2013) propose the following formula to get such an estimate of a haircut on a restructuring process [11]:

$$H_{CT} = 1 - \frac{PV(New\ debt, r_e)}{FV(Old\ debt)}$$

where $FV(Old\ debt)$ stands for the face value amount of the old outstanding debt (including past due interest on the old debt but no penalties), and $PV(New\ debt, r_e)$ is the present value of new debt instruments (plus possible cash repayments), discounted at the interest rate r_e, that is the interest rate prevailing in secondary markets at the exit from default. Another measure for haircuts has been proposed by Sturzenegger and Zettelmeyer (2008) [44], according to the following formula:

$$HC_{SZ} = 1 - \frac{PV(New\ debt, r_e)}{PV(Old\ debt, r_e)}$$

where $PV(Old\ debt, r_e)$ stands for the present value of the remaining contractual payments of the old debt instruments, inclusive of eventual interest or principal arrears, and $PV(Old\ debt, r_e)$ is the present value of the new debt instruments after the operation of restructuring. A restructuring can occur after a payment failure (or default *stricto sensu*). If this is the case, we define the process as *Post Default Restructuring*. The Restructuring of the sovereign debt of Argentina between 2005 and 2010 is a prominent example of a restructuring after a government has ceased to honor interest and principal of a certain series of bonds.

Conversely if a restructuring prevents a default, and the debt exchange or reduction takes place before a sovereign stops servicing its debt, we speak of *Pre-emptive Restructuring*. A recent and notorious example is the Greek debt restructuring of 2012.

There are two multilateral forums for debt restructuring: the Paris Club and the London Club.

The Paris Club is an informal forum created by creditor governments to deal with rescheduling and restructuring of sovereign bilateral external debt. The Paris Club is an informal forum created by creditor governments to deal with rescheduling and restructuring of sovereign bilateral external debt [20]. Established in 1956 from an ad-hoc meeting to reschedule Argentina's debt with various Western countries [49], the Club witnessed an evolution from a role of simple debt collector, with agreements that until the 1980s could entail only a debt rescheduling, without weakening debtors' moral and legal obligation to repay their debts in full, to a role of relief provider with the adoption of Naples (1994) and Evian (2003) terms of restructuring.

The debt restructuring process between a sovereign borrower and commercial banks is labeled as "London Club" restructuring. Differently to Paris Club, that has an office and codified policies, even at a low level of instituzionalization, the London Club does not have a Secretariat or fixed venues, and the costs of its meetings are usually borne by the debtor: the name denotes the case-by-case restructuring practices developed between major Western banks and developing countries' governments in the late 1970s and early 1980s [47]; moreover the label can be somehow further misleading, because actually the majority of the meetings took place in New York and not in London.

The London process takes places with a negotiation between the government that declares not to be able to honor its obligation and a Bank Advisory Committee (BAC), or Creditor Committee, or Steering Committee, that is a group of banks with the largest exposure to the sovereign, negotiating on behalf of all the banks affected by the restructuring, with the goal of avoiding coordination problems between the potentially wide number of creditors, and to concentrate the responsibility of negotiations with the largest institutions that can have better expertise.

The main elements of a London Club restructuring can be reported as follows. A government that is experiencing financial distress contacts a number of banks among its creditors and asks them to build and chair a BAC. Once the Steering committee is established the negotiations start, with the possibility to deploy the full array of instru-

ments for a debt restructuring we have mentioned above: there can be solvency and liquidity consideration with the possibility for the committee to provide flow treatments by extending maturities and providing short term liquidity support to the governments, and stock treatments as well by giving outright reduction in the face value of the loans or bonds.
If the negotiations are successful an "agreement in principle" is signed between the government's officials and the representatives of the banks in the steering committee. Once the terms are approved they are sent to other creditor banks involved in the restructuring that are not included in the BAC for approval.

Lending to sovereigns is different from lending to a firm. In the corporate world contracts are enforced by national courts: repudiation of debt as we have described above, i.e. a unilateral decision of a firm not to honor its financial obligations, cannot take place, since it would certainly trigger a lawsuit, and a sentence that would force the firm to hand over its assets to creditors, through a liquidation procedure. Eaton and Gersovitz (1981) in a famous paper have proved that under certain conditions a sovereign debt market can exist even if the only way creditor has to react to an episode of default is by denying future credit to the borrower, i.e. by forcing the borrower in financial autarky [13]. Eaton and Gersovitz propositions are based on the threat of a permanent embargo, that would be triggered if reputation for repayment of the borrower was undermined by a sovereign default episode: in this setting there are only reputation-for-repayment contracts. Bulow and Rogoff (1989) observe that if international lending is not the only means to smooth consumption over time in fact pure reputational contracts cannot exist [39]: there could be cash in advance insurance contracts providing the same payoffs in bad states of the world. In another paper published the same year Bulow and Rogoff go further in the critique to the pure reputational model à la Eaton and Gersovitz [38]. They argue that sanctions of the affected lender could entail the impairment of the conditions of free trade in the good markets for the insolvent borrower, and only the existence of this credible threat can sustain sovereign borrowing and induce repayment. Kletzer and Wright (1998) build a model of sovereign lending as an intertemporal barter of a nonstorable good using an infinitely-repeated game with at least two participants [51]. A short lived payments moratorium is imposed to any party that does not honor any scheduled payment, and the long term financial relationship is restored as soon as the the punished party makes a sufficient payment to restore the surplus of the other party. The defaulted party in this setting cannot simply start another contract with a competing lender: the original creditor in this case adopts a "cheat the cheater" strategy of inducing the borrower to default on the new contract with the party that cheated the moratorium in exchange of restoring the original financial relationship. Broner, Martin and Ventura (2010) explain the repayment of sovereign international debt with market structure considerations, leaving aside punishment arguments [9]. They argue that if foreign creditors could resell the debt they purchased from the borrowing country' s government in the secondary market to domestic residents, and if the government cannot distinguish if a given debt contract is currently held by a resident or a foreigner, repayment takes necessarily place, even if there is no threat of punishment. Cole and Kehoe (1998) build a model of sovereign lending based on reputation, in which repayment is sustained by the fear of the government to lose good reputation in other relationships in which trust matters: potential negative trust-related spillovers that would derive from default induce honoring financial obligations [10]. Several authors have analyzed if defaults come with major economic downturns. Levy Yeyati and Panizza (2011), using data from 1982 and 2003, find strong evidence that defaults occur after remarkable output contractions [54]. Tomz and Wright (2007) consider a much wider time horizon, examining data from 1820 to 2004, and finding the same relationship, but much weaker: only 62% of the 169 episodes they examine actually starts after major economic downturns [46]. Eichengreen, Hausmann and Panizza (2005) highlight the role of debt denominated in foreign currency held outstanding by emerging countries in harming their solvency [15].

Economists have tried to build models to predict sovereign defaults from macro fundamentals. Manasse, Roubini and Schimmelpfenning (2003) build an early-warning model of sovereign debt crises by selecting potential explanatory variables from a set with a deep theoretical foundation [28]. The model provided by the authors, based on a logistic regression, correctly predicts 74% of all debt crises occurred between 1976 and 2001, while sending only 6% of false signals. Reinhart (2002) finds a deep linkage between sovereign debt and currency crises for emerging markets, by analyzing data from 1979 to 1999 [34]. Costs of sovereign default are examined through 4 dimensions: costs arising from exclusion from financial markets, higher borrowing costs, output losses, and trade costs. Gaston Gelos, Sahay and Sandleris (2011) work with data for 144 developing countries from 1982 to 2000. They show that the median number of years that took countries in state of default to regain access to international markets was 4 in the 1980s and fell to 0 in the 1990s. Richmond and Dias (2009) derive different estimates of average and median duration of exclusion periods [37]. Examining data from 1980 to 2005, with 128 default episodes according to S&P, they find a median time span to partial access after exit from status of default of 3 years, and a median time span to get again full access of 7 years. On average the time to get resumption to partial access in their estimates is 5.7 years, the average time to full access post default is 8.4 years. Cruces and Trebesch (2013) focus on exclusion from international capital markets after episodes of restructuring [11]. They examine 67 restructuring cases from 1980 to 2010, finding an average duration from the completion of the restructuring process of 5.1 years, with a median of 3 years. Interestingly, they find that the average time until partial reaccess (net flows to the country > 0) is increasing in the haircut size.

Borensztein and Panizza (2009) give an estimate of the higher borrowing costs after default: using data from 1997 to 2004 the authors' analysis leads to the conclusion that having defaulted at $t-1$ has a large and statistically significant effect on spreads amounting to 400 basis points on average, and this effect reduces to 250 basis points on average two years after default. Eichengreen and Mody (1998) examine the spreads charged on bonds issued by 37 emerging countries both by sovereigns and private parties [14]. They find that spreads are remarkably higher if the issuing country has defaulted the previous year.

Sturzenegger (2004) estimates the impact of default on growth by running a cross-country growth regression for 100 different countries, using data from 1974 to 1999 [43]. The author finds that there is a large and negative impact of default on growth, accounting to about 0.6% per year on average. De Paoli, Hoggarth and Saporta (2006) using data from 1970 to 2000 assess the cost of a debt crisis by comparing the actual realization of GDP for defaulting countries to a counterfactual GDP that is an estimated value of GDP as if the crisis did not happen [32]. Then they present average output losses on a per annum basis, distinguishing four scenarios, by type of sovereign default: default only, default and currency crisis, default and banking crisis, triple crisis. For the "default only" scenario they have only four observations, that behave at odds with the theory: they find a negative median loss of 5.2% per year and an average negative cost per year of 1%: in this cases the output after default seems to be consistently above average, but the number observation as the authors claim is too small to draw conclusions.

Considering the average of the four scenarios the authors find a median loss of 7% per year and an average of 15.1% per year. They find moreover that the output loss from twin crises is more severe if default comes with a banking crisis rather than with a currency crisis, being the triple crisis the worst possible scenario. Finally they find that average per annum output loss is increasing in the length of the crisis: the longer it takes to reduce arrears or complete a restructuring, the more output falls relative to its estimated potential. Panizza and Levy Yeyati (2006) examine 20 default cases between 1982 and 2003 reaching opposite conclusions: they find that, contrary to what is typically assumed, output downturns precede default, and that the trough of the contraction occurs in the quarter of default, and GDP starts to grow after default [54].

Rose (2005) gives an estimate of the effect of default on international trade without disentangling between retaliation and the trade credit effect [40]. The author uses data from IMF and World Bank, focusing on 283 Paris Club deals from 1948 and 1997 among 217 countries, at annual frequency. The estimation is performed through a gravity equation. The Rose's regression finds a negative impact on bilateral trade between the defaulting country and the creditor countries affected by default, but it does aim to disentangle the drivers of those reductions in trade. Martinez and Sandleris (2011) address the issue, by giving an estimate of bilateral, multilateral and general effects of sovereign default on trade flows [29], aiming at capturing the effects of bilateral or multilateral sanctions after default. The three models tested by the authors tell us the same result: there is an remarkable decline in overall trade when a country defaults on international public debt, but there is no evidence of sanctions imposed by countries that suffered default or by coalitions of creditors. Boresztein and Panizza (2009) tackle the issue of the role of the reduction of availability of trade credit for the country's exporting firms after default, that may arise from the risk of imposition of capital or exchange controls [8]. The evidence authors show is mixed: default episodes are associated with a decline in trade credit, but the relationship between trade and default is not affected when controlling for trade credit.

We develop the following model, on the lines of Aguiar and Gopinath (2006) removing the time trend in the output process they use [2]. The model is then simulated using MATLAB. The model economy is populated by identical individuals whose preferences are described by the utility function,

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$$

where c_t stands for the consumption at time t, $\beta \in (0,1)$ is the subjective discount factor, that captures impatience and u is the period utility function.

The representative agent has CRRA preferences: the period utility is of the form

$$u = \frac{c^{1-\gamma}}{1-\gamma}$$

and is strictly increasing, reflecting a desire for more consumption, and strictly concave, reflecting diminishing marginal value of extra consumption. The curvature parameter γ generates aversion to risk and to intertemporal substitution.

Our economy is designed to trade a single good and a single one-period bond with the rest of the world. Each period $t \geq 0$ the representative country is endowed with y_t units of goods. The endowment process is given by

$$y_t = e^{z_t}.$$

In our setting the output process is entirely determined by a shock z_t, that follows an AR(1) process:

$$z_t = \mu_z(1-\rho_z) + \rho_z z_{t-1} + \epsilon_t^z$$

where $|\rho_z| < 1$ is the autoregressive parameter, $\epsilon_t^z \sim N(0, \sigma_z^2)$ is the disturbance term.

Assuming stationarity of the process we easily find that $E[z_t] = \mu_z$ and $var(z_t) = \frac{\sigma_z^2}{1-\rho_z^2}$. The process is assumed to be lognormal in order to prevent the realization of negative level of the endowment, that would not have economic meaning.

We assume that the government acts in a benevolent way, i.e. it maximizes households' utility. Each period t it can buy or sell one period zero coupon bonds in a quantity a_{t+1} for a price of $q(a_{t+1}, z_t)$. We denote with a_{t+1} net foreign asset position entered in at time t that matures at $t+1$, with a_t net foreign assets purchased at time $t-1$ that mature in the current period. A negative value for asset means positive debt. Therefore if the government wants to take positive debt at time t it will receive $q(a_{t+1}, z_t)a_{t+1}$ units of good at time t with the promise to deliver a_{t+1} units of good at $t+1$. If the government runs a budget surplus it invests $q(a_{t+1}, z_t)a_{t+1}$ units of good at t to receive

a_{t+1} units the next period.
We model the price to be endogenous to the level of debt assumed in the current period and to the realized shock to the economy, but the currently due quantity of debt is not relevant for the pricing function. This is due to the fact that in our environment there are only zero coupon bonds, and so at each period the whole stock of the country's debt is negotiated again: there is only a one period dependence in the debt position. The shock to the economy is instead relevant, since it is informative of the business cycle and, given the autoregressive structure of the endowment process, of the future financial strength and solvency of the country.
At the beginning of each period the government decides to honor its financial obligations or to default.
The budget constraint for a country that stays in the contract is given by:

$$c_t - a_t = y_t - q(a_{t+1}, z_t)a_{t+1}.$$

To avoid Ponzi Games we require a lower bound for asset holdings:

$$a_t \geq a_{lb}.$$

A default decision by the government entails some consequences according to our model: current debts are erased form the budget constraint, international saving and borrowing is no more allowed, i.e. the country is excluded from financial markets for a certain (stochastic) number of periods. The budget constraint for a country that defaults in its financial obligations is:

$$c_t = y_t^{def}$$

where $y_t^{def} \leq y_t$, that means that we assume an output loss in autarky.
At the beginning of each period the economy can be either in a good financial status of in a bad financial status. The bad financial status is associated with having defaulted on debt, the good financial status is associated with having repaid debt in the previous period.
We define the model with a recursive structure following the Bellman principle. Default will occur if the value associated to honor debt obligations is lower than that attached to default on debt payments and entering a bad financial status.
Thus, the value function associated with being in good financial standing at the start of period t is defined as follows:

$$V^G(a_t, z_t) = max(V_t^C, V_t^B)$$

where the superscript B denotes bad financial status, the superscript G good financial status and the superscript C honoring financial obligations and continuing to participate to capital markets
Under the specification of our model default will occur if at beginning of period t for an economy in good standing $V^B(z_t) > V^C(a_t, z_t)$ (significantly V^B does not depend on a, signaling that debt has been repudiated). The decision to default entails an output loss, but not perpetual exclusion from the financial markets: a country with bad financial standing will be redeemed with probability λ and start next period with good financial status and no debt. The recursion associated to the bad financial status is defined by

$$V^B = u((1-\delta)y_t) + \lambda\beta E_t V^G(0, z_{t+1}) + (1-\lambda)\beta E_t V^B(z_{t+1})$$

where δ is the parameter that determines the output loss in autarky. The Bellman equation associated with honoring obligations and continuing to participate to capital markets is

$$V^C = max_{c_t}[u(c_t) + \beta E_t V^G(a_{t+1}, z_{t+1})]$$

subject to te budget constraint defined above. We define a default indicator random variable:

$$I_{D(a_t,z_t)} = \begin{cases} 1, & \text{if } V^B(z_t) > V^C(a_t, z_t) \\ 0, & \text{otherwise} \end{cases}$$

that is a Bernoulli random variable with success parameter equal to $P(V^B(z_t) > V^C(a_t, z_t))$.
In such a setting the default probability is equal to the expected value of the default indicator variable:

$$P(V^B(z_t) > V^C(a_t, z_t)) = P(default) = E[I_{D(a_t,z_t)}] = \zeta$$

The price of the zero coupon bond is given by discounting the expected payoff:

$$q(a_{t+1}, z_t) = \frac{E_t(1 - I_{D(a_t,z_t)})}{R^f} = \frac{1-\zeta}{R^f}$$

using the same risk free discount factor implied risk by neutral pricing. The international capital market is modeled as an environment of perfectly competitive risk neutral investors, that are assumed to have perfect information about the economy's endowment process and can correctly observe the income level every period. Every period lenders choose a_{t+1} to maximize expected profits ϕ, as price takers:

$$\phi = q(a_{t+1}, z_t)a_{t+1} - \frac{1-\zeta}{1+r^\star}a_{t+1}$$

that is a zero expected profit condition.

The default probability $\zeta \in [0,1]$, and so the bond price $q \in [0,(1+r^\star)^{-1}]$: when default occurs with certainty the price is zero; for a positive level of asset holdings instead the probability of default is zero and so the price of the bond will be equal to the inverse of the gross risk free rate. The country gross interest rate is given by the inverse of the sovereign bond price:

$$1 + r^c = \frac{1}{q(a_{t+1}, z_t)}$$

and the country spread s is the difference between the country interest rate and the international risk free rate:

$$s = r^c - r^\star.$$

The schedule of the government decision is structured as follows: at the beginning of period t it inherits the asset position a_t, it observes the endowment realization y_t, that in our setting depends in a certain part on the output at $t-1$, and decides to honor its debt or to default. If the government decides to stay in the contract, it decides the asset level that matures next period a_{t+1} subject to the budget constraint.

The difference between the new asset position and the inherited asset position is transferred to the households for consumption.
Simulation using MATLAB is based on two main scripts. The first part of the code is based on a value function iteration process. The values selected fo the simulation parameters are the same used by Aguiar and Gopinath (2006):

- The coefficient of relative risk aversion γ is as customary in the literature set equal to 2;
- The quarterly world risk free rate is set equal to 1%;
- The discount factor β is set equal to 0.8;
- The percentage output loss in autarky δ is set equal to 2%;
- The probability of redemption λ is set equal to 0.1 per quarter;
- The autocorrelation coefficient of the AR1 process ρ_z is set equal to 0.9;
- The standard deviation of the innovation term σ_z is set equal to 0.034.
- The long run mean of the autoregressive process for the shock is set equal to $-0.5\sigma_z^2$.

The possible asset holding values are discretized in a grid of 400 points, with the maximum level representing no debt, and the lowest level of holdings representing a debt level equal to 30% of GDP. The continuous AR1 process is discretized in a 25 states Markov chain using the method proposed by Tauchen (1986) [45]. Below we present the plot of the price schedule for the sovereign bond with a high level of the shock and with a low level of the shock for all possible level of asset holdings: we clearly see from the graph obtained by the simulation that a good performance of the economy makes sustainable a slightly higher level of debt, that in any case according to our model cannot be higher than 27% of GDP.

The second graph represents default area and nondefault area, and default boundary: we clearly see that the worse the shock the lower the possible debt level that can be held honoring financial obligations. Business cycle simulation is performed by simulating 100 times 1000 quarters of our model economy, then by selecting the last 500 observation for each iteration and taking averages to then derive summary statistics. Table 2 reports the results of the simulation. The simulation of the model predicts a countercyclical trade balance, consistently with theory and data, even if real data for Argentina show a much highly negative correlation. The model gives counterfactual predictions relating to correlation between sovereign spread and output: if in real data spread tends to rise when output goes down and to move down during boosting economy, our model incorrectly predicts a spread that is positively correlated with output. This is turn leads to an incorrect prediction for the direction of the comovements between spread and trade balance: the data clearly show prociclicality, while in our simulated model trade balance and sovereign spread are negatively correlated. Output and consumption are much more volatile

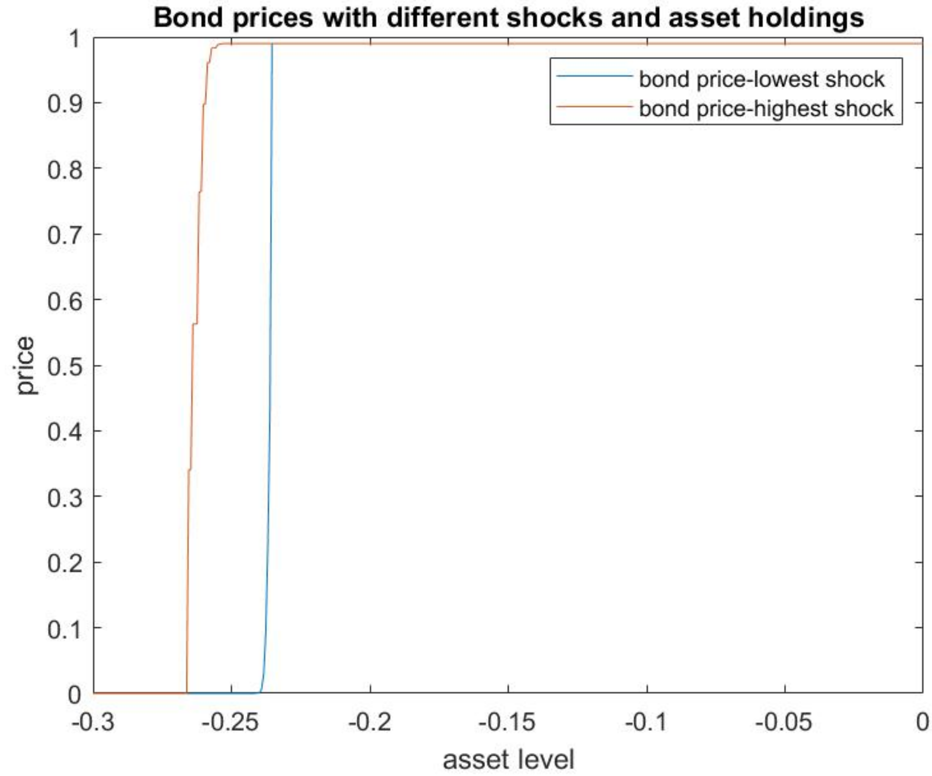

Figure 7: Price schedule of the ZCB with different shocks

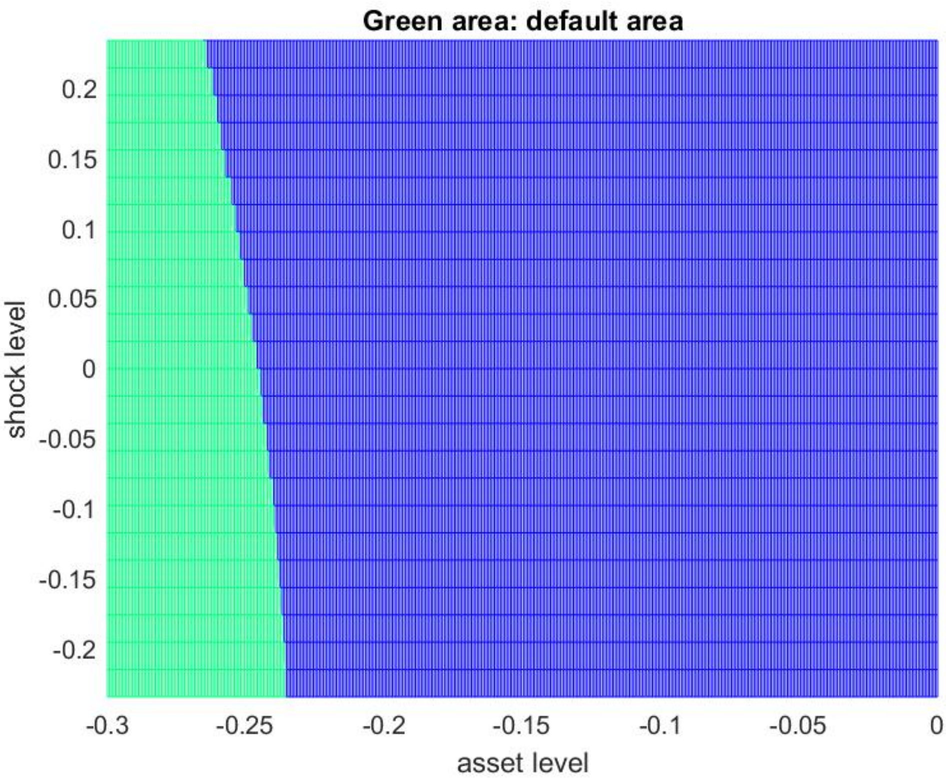

Figure 8: Default area, nondefault area, default boundary

in the simulated model than in real data for Argentina, while interest rates are remarkably more volatile in the data than in the model. The output process of the model is highly persistent reflecting the value of the autoregressive parameter of 0.9 set in the optimization iteration. The model predicts one default in 2500 years, that is absolutely at odds with the rate of default for Argentina estimated by Aguiar and Gopinath, set to 75, while the default of 2001 was the fifth Argentine default or restructuring episode in the previous 180 years [2]. Finally the simulated model predicts a maximum debt-to-GDP ratio between 24% and 27%, depending on the state of the economy, that seems to be low when compared to real data for Argentina, that could sustain higher levels of debt, but it approaches the even stricter safe level estimated by Reinhart and Rogoff (2003) [35].

Simulation Results		
Variable	Simulated Value	Argentina 1983.1-2000.2
$\sigma(y)$	7.72	4.08
$\sigma(R_s)$	0.03	3.17
$\sigma(tb/y)$	0.31	1.36
$\sigma(c)$	7.77	4.85
$\rho(y, R_s)$	0.1431	-0.59
$\rho(y, tb/y)$	-0.1455	-0.89
$\rho(y, c)$	0.9985	0.96
$\rho(R_s, tb/y)$	-0.1082	0.68
$\gamma_1(y)$	0.8922	
$Rate\ of\ default$	3.85	75

Table 2:

Averages of the 100 simulations, taking for each simulation the last 500 observations. R_s is the quarterly sovereign spread. Standard deviations are expressed in percentage terms. Rate of default is the number of quarters the country is in state of default over 10000 quarters. Business Cycle statistics for Argentina are taken from Aguiar and Gopinath (2006) [2].