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An empirical analysis of aggregate risk: isidiosyncratic volatility still increasing?

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Abstract

Following the methodology employed by Campbell et al. (2001), I construct three volatility measures, relative to market, industry and firms - specific level returns. I find that the positive trend persisting from 1962 until the early 90s for the last volatility component, namely the one relative to idiosyncratic risk, is reversed and subsequently disappears; this is mainly due to a structural break in the series, accountable to the 2002 recession. By replicating the analysis, dividing firms on the basis of size, I reveal that the big firms group is the main driver of all volatility measures for the complete sample. Moreover, the change in pattern and the structural change described above come from big firms aggregate volatility measures.

Contents

1 Introduction

Stocks volatility, mostly defined as the variation of returns over time, is a commonly used measure to assess the risk of a security; namely, the higher the volatility, the riskier the security is considered. It appears in portfolio theories and optimization models, measured through the standard deviation of a security's returns, together with returns, to estimate the profitability of an investment, given its riskiness, and to determine the degree of diversification of a portfolio. It has been extensively studied to grasp its intrinsic characteristics, such as its mean reverting property, the fact it tends to form clusters or its negative relationship with returns. In addition, several model were constructed to help economists and experts to forecast volatility, such as the ARCH and GARCH model.

In particular,the aggregate volatility, the main focus of this analysis, is defined as the volatility faced by investors of aggregate index funds and, as highlighted in Campbell et al. (2001), concentrating in just one of the components, the one capturing market - wide returns is not enough to have an extensive understanding of this aggregate measure.

Indeed, the authors are able to prove that the increasing historical volatility registered in that time period, from 1962 to 1997 in their analysis, is mainly driven by idiosyncratic firm-level volatility. The latter presents a distinct positive trend throughout the entire time - span they considered. Their result is remarkably meaningful, as they underlined the importance of firm - specific shocks, emphasising the substantial influence they have on aggregate volatility as a whole.

In the last twenty years, global economy experienced a considerable amount of downturns, first among all the 2008 financial crisis, in a limited period of time; in addition, the shape of economy and financial market profoundly changed to adjust to new and more strict regulations.

The objective of this thesis is to extend some key elements of Campbell et al. (2001), to investigate the effect of such dramatic changes on the three aggregate volatility measures, with a particular focus on the idiosyncratic - level components. First, I find that the trend discovered by the authors of the paper disappeared, after a brief reversal in the early 2000s; conversely, market and industry - level components did not experience any major changes during the last two decades.

Second, I further investigate on the causes of the abrupt changes firm - level volatility underwent in the time - span analysed; the output of my search clearly detect a structural break in the series, which took place during the early 2000s recession.

Third, I run again the analysis using daily frequency data, as a robustness check.

Fourth, I repeat my analysis for small and big firms, discovering the latter group drives the pattern of all three components of aggregate volatility; so, the structural break detected for the idiosyncratic - level components is directly accountable to firms with a high market capitalization. This is mainly due to the high influence big firms have on total aggregate volatility; the group registers a weight of over 0.9 throughout the sample considered.

This thesis is organized as follows: in section 2, I will describe the dataset I used to conduct my research and how I divided my sample in several industries; in section 3, I constructed the three volatility measures and studied their characteristics by plotting them, by obtaining their descriptive Statistics and serial correlation coefficients. I also investigated their impact on total volatility, the correlation and any lead/lag effect among them. Additionally, I explored the possibility of a structural break for the idiosyncratic volatility component. Finally, I replicated my previous analysis, dividing my sample in small and big firms. Here, I studied any difference among the two groups and compared each of them to the overall output obtained in the previous section. I also studied the impact the two sub - groups play on total aggregate volatility ; in Section 4, I present a summary of my results and some concluding comments.

2 Data Description and Empirical Approach

2.1 Data

In order to extend Campbell, Lettau, Malkiel and Xu paper, I used CRSP data for firm returns, including firms listed in AMEX and Nasdaq, over the period from January 1962 to December 2019. [1] Despite the differences with the above - mentioned paper, the main characteristics the authors found in their sample, such as trends of the volatility components, remained unchanged during the time span they considered.

In order to divide the sample into different industries, I referred to the industry classification employed in Fama and French (1997) based on SIC codes and HSIC codes when the former were not available, which leaves me with 48 distinct industries; The industry with more individual firms is Financial Services, with a total of 7826 over the entire sample, while the industry with the lowest number of firms is Tobacco Products, with 46 companies overall. The biggest industry in terms of market capitalization is Petroleum and Natural Gas, accounting for almost 9 percent of the total sample market cap; it is followed by Financial Services, that represent more than 7 percent of the total market value. Lastly, in terms of market cap, the industry the smallest impact is the Agricultural sector, with an average of 0.06 percent over the total market capitalization.

2.2 Empirical Approach

In this subsection, I will explain the decomposition procedure employed by Campbell et al. (2001) and use it to empirically estimate the three components of aggregate volatility.

[1] The sample I used is slightly different from the one employed by Campbell et al., as the total number of firms in the period covered by the paper are not the same. Indeed, the total number of firm in my sample is 1147 in January 1962, 9113 in December 1997 and 7534 in December 2019. Campbell et al.(2001) sample goes from 2047 unique firms in July 1962 to 8927 in December 1997

Specifically, I decompose total volatility of a stock into three components, starting from market returns, industry-specific and firm-level returns in excess of a risk-free measure. The next step is to proceed to break down industry and idiosyncratic returns. The most straightforward way to do so is by implementing the CAPM equation, expressing industry returns as:

$$R_{jt} = \beta_j R_{mt} + \epsilon_{jt} \tag{1}$$

Where R_{jt} is the excess return for industry j at time t. Similarly, firm-specific returns are decomposed as follows:

$$\begin{aligned} R_{ijt} &= \beta_{ij} R_{jt} + \eta_{ijt} \\ &= \beta_{ij}[\beta_j R_{mt} + \epsilon_{jt}] + \eta_{ijt} \end{aligned} \tag{2}$$

As higlighted in Campbell et al. (2001), η_{ijt} is, by construction, orthogonal to R_{jt}; as a consequence, it is reasonable to assume R_{mt} and ϵ_{jt} are orthogonal to as well. These orthogonality allows me to discard any covariance components from the volatility computation. Our volatility components can then be defined in the following way for industry volatility:

$$\mathbb{V}[R_{jt}] = \beta_j^2 \mathbb{V}[R_{mt}] + \mathbb{V}[\epsilon_{jt}] \tag{3}$$

For firm - level volatility, we have:

$$\mathbb{V}[R_{ijt}] = (\beta_{ij}\beta_j)^2 \mathbb{V}[R_{mt}] + \beta_{ij}^2 \mathbb{V}[\epsilon_{jt}] + \mathbb{V}[\eta_{ijt}] \tag{4}$$

In order to employ this method, we need to estimate firm-specific betas, which is not straightforward, especially since this estimates are likely to be estimated with errors and are not likely to remain stable over time. For these reasons, the decomposition used to estimate the three volatility components in this thesis, is a simplified version of the former, coming with the advantage of getting rid of the firm betas estimation, but at the cost of having the covariances terms different from zero. The simplified model fro industry returns is:

$$R_{jt} = R_{mt} + \tilde{\epsilon}_{jt} \tag{5}$$

where the fundamental difference is the dropping of β_j from (1); the former equation is defined as "market-adjusted return model" by Campbell et al. (1997). From (1) and (5), we can express $\tilde{\epsilon}_{jt}$ as:

$$\tilde{\epsilon}_{jt} = \epsilon_{jt} + (\beta_j - 1)R_{mt} \tag{6}$$

Computing the variance of (5) and substituting (6), we obtain:

$$\mathbb{V}[R_{jt}] = \mathbb{V}[R_{mt}] + \mathbb{V}[\tilde{\epsilon}_{jt}] + 2(\beta_j - 1)\mathbb{V}[R_{mt}] \tag{7}$$

in the context of this simplified decomposition, $R_{mt} \not\perp \tilde{\epsilon}_{jt}$, therefore we cannot ignore the covariance term between R_{mt} and $\tilde{\epsilon}_{jt}$.

In a similar manner, firm-specific returns are defined as:

$$R_{ijt} = R_{jt} + \tilde{\eta}_{ijt} \tag{8}$$

where, through (2) and (8), we can express $\tilde{\eta}_{ijt}$ as follows.

$$\tilde{\eta}_{ijt} = \eta_{ijt} + (\beta_i j - 1)R_{jt} \tag{9}$$

The argument uncovered for (5) applies to (9) as well: $R_{jt} \not\perp \tilde{\eta}_{ijt}$. As a consequence, we will not discard the covariance term between R_{jt} an $\tilde{\eta}_{ijt}$.

Following the same procedure employed for industry returns, we get:

$$\mathbb{V}[R_{ijt}] = \mathbb{V}[R_{jt}] + \mathbb{V}[\tilde{\eta}_{ijt}] + 2(\beta_{ij} - 1)\mathbb{V}[R_{jt}] \tag{10}$$

One way to cancel the covariance terms in (7) and (10) is to formulate them in terms of weighted average across industries and firms within the same industries. That way, we will obtain:

$$\sum_j w_{jt}\mathbb{V}[Rjt] = \mathbb{V}[R_{mt}] + \sum_j w_{jt}\mathbb{V}[\tilde{\epsilon}_{jt}] \tag{11}$$

for industry volatility and:

$$\begin{aligned} \sum_j w_{jt} \sum_{i \in j} w_{ijt}\mathbb{V}[Rijt] &= \sum_j w_{jt}\mathbb{V}[R_{jt}] + \sum_j w_{jt} \sum_{i \in j} w_{ijt}\mathbb{V}[\tilde{\eta}_{ijt}] \\ &= \mathbb{V}[R_{mt}] + \sum_j w_{jt}\mathbb{V}[\tilde{\epsilon}_{jt}] + \sum_j w_{jt} \sum_{i \in j} w_{ijt}\mathbb{V}[\tilde{\eta}_{ijt}] \end{aligned} \tag{12}$$

for firm-specific volatility.

I will employs this methodology throughout the entire thesis.

In order to empirically estimate the three measures of aggregate volatility, I mainly follow Campbell, Lettau, Malkiel and Xu (2001), starting from the monthly individual firms' returns (in excess of the U.S. Treasury monthly-adjusted average returns), I constructed industry returns as follows:

$$R_{jt} = \sum_{i \in j} w_{ijt} R_{ijt} \tag{13}$$

w_{ijt} being the weight of the firm i belonging to industry j at time t and is expressed in terms of market capitalization. Similarly I obtained the market excess return in the same manner:

$$R_{mt} = \sum_{i} w_{jt} R_{jt} \tag{14}$$

With w_{jt} being the weight assigned to the industry j at time t. At this point, we are ready to construct our three volatility measures; in order to avoid any inconvenience stemming from the estimation of the firm-specific betas, we employ the simplified methodology described above. The market sample volatility in period t is obtained as follows:

$$MKT_t = (R_{mt} - \mu_m)^2 \tag{15}$$

For Industry sample volatility, we start from:

$$R_{jt} = R_{mt} + \epsilon_{jt} \tag{16}$$

we obtain an estimate of the industry-specific residual, we square it and we average across industries to make all the individual industries' covariance cancel out. Pursuing this methodology, we obtain an estimate for the industry volatility:

$$IND_t = \sum_{j} w_{jt} \hat{\epsilon}_{jt}^2 \tag{17}$$

Finally, to construct the firm volatility measure, we derived the firm specific residuals from

$$R_{ijt} = R_{jt} + \eta_{ijt} \tag{18}$$

then we average their squares within each industry:

$$\hat{\sigma}^2_{\eta jt} = \sum\nolimits_{i \in j} w_{jt} \eta^2_{ijt} \tag{19}$$

finally, we average over industries to construct out sample firm volatility:

$$FIRM_t = \sum\nolimits_{j} w_{jt} \hat{\sigma}^2_{\eta jt} \tag{20}$$

Similarly to industry volatility, this procedure has, as the main objective, to cancel firm-specific covariances from the equation.[2]

3 Results

3.1 Replication & Extension

This section of the thesis mainly focuses on replicating some key aspects of Campbell et al. (2001), expanding the time horizon analyzed to December 2019. I will begin by plotting market, industry and firm - specific volatility and analyzing the pattern followed by each of them, any difference and similarity bringing them together.
Next, I will look into the serial correlation that characterizes the three components and study the possibility that they contain a unit root.
Subsequently, I will derive a summary of the series' descriptive statistics and examine their moments, including some measure for the detrended series; I will also study the

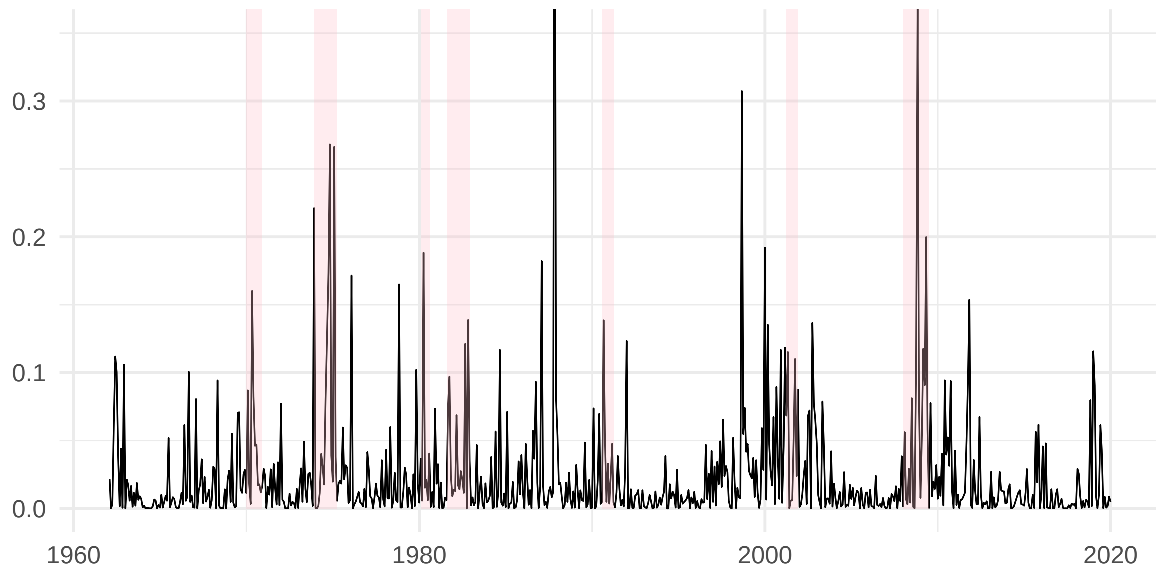
[2]The estimation of the three sample variances I described in this section is slightly different from the one shown in Campbell et al. paper; This is due to the fact that, while, throughout their analysis, the authors mainly used daily data, I decided to use monthly data. Since I took into consideration a longer time horizon, I found it computationally more convenient to carry out my analysis this way.

impact that each component has on the total volatility, assessing the relevance of each in terms of both mean and standard deviation.
Lastly, I will further investigate the relationship between the three, by looking at their correlation and establishing any cause-effect relationship between them.

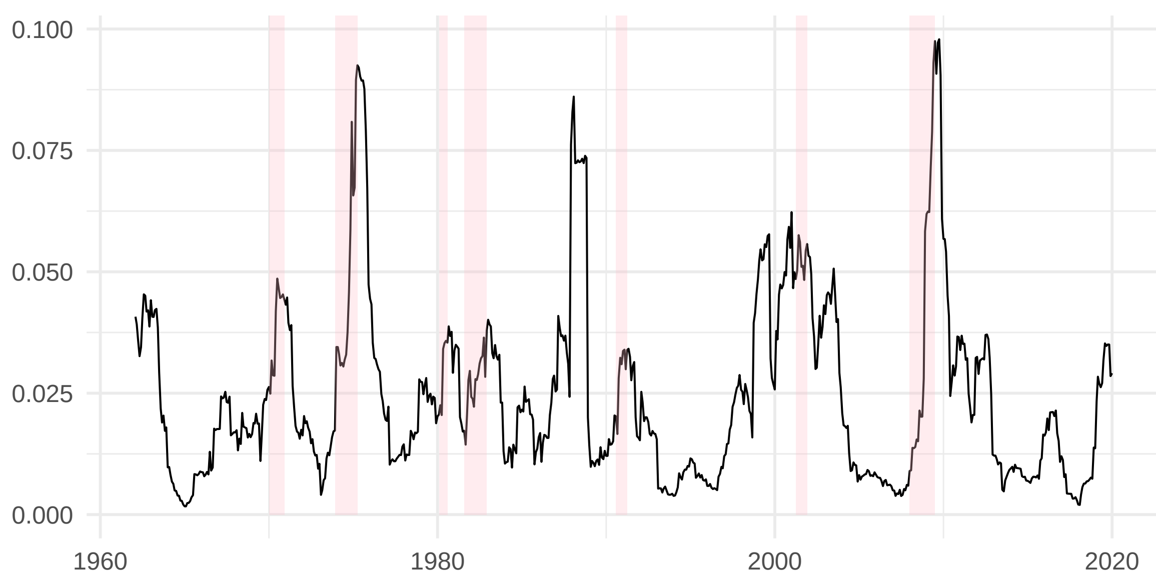
3.1.1 Preliminary Analysis

To have a clear and straightforward idea of volatility components' pattern evolution, I plot the movements over time of the annualized monthly market, industry and firm - level volatility, respectively in figure 1, 2 and 3.
In all three graphs, I let some of the biggest peaks go outside of the Figure. I also add a plot of an MA(12), to illustrate in a clearer manner the progression of the three series. As we can see from Figure 1, Market volatility pattern did not change much in the last two decades: the series does not show any trend, with two major spikes corresponding, in order of magnitude, to the the 1987 crash and the 2008 financial crisis.
In particular, MKT reached 0.6418 in October 1987, a massive increase compared to the second highest observation, 0.3694, recorder in October 2008. Market volatility also showed significant increases at the end of the 90s, in connection to the Dot-com bubble, and in the mid-70s, probably related to the 1975 recession. Similarly to Campbell et al.(2001), the volatility estimate shows a significant high-frequency noise.
For what concerns industry volatility, we immediately perceive a marked difference with respect to the former graph; Figure 2 looks far less noisy, with a reduced amount of high-frequency spikes through the time span covered. Moreover, is the volatility with the lowest values among the three.
In this graph, IND experiences a moderate spike in the mid-70s and in the early 80s; after that, there is a major increase in February 2000, reaching a value of 0.2680 and finally, a modest rise in 2008, in connection with the financial crisis. Leaving aside the big spike corresponding to the Dot-com bubble, IND does not exhibit any trend, and its characteristics as a series did not significantly change in the last two decades.

Figure 1: Market Volatility



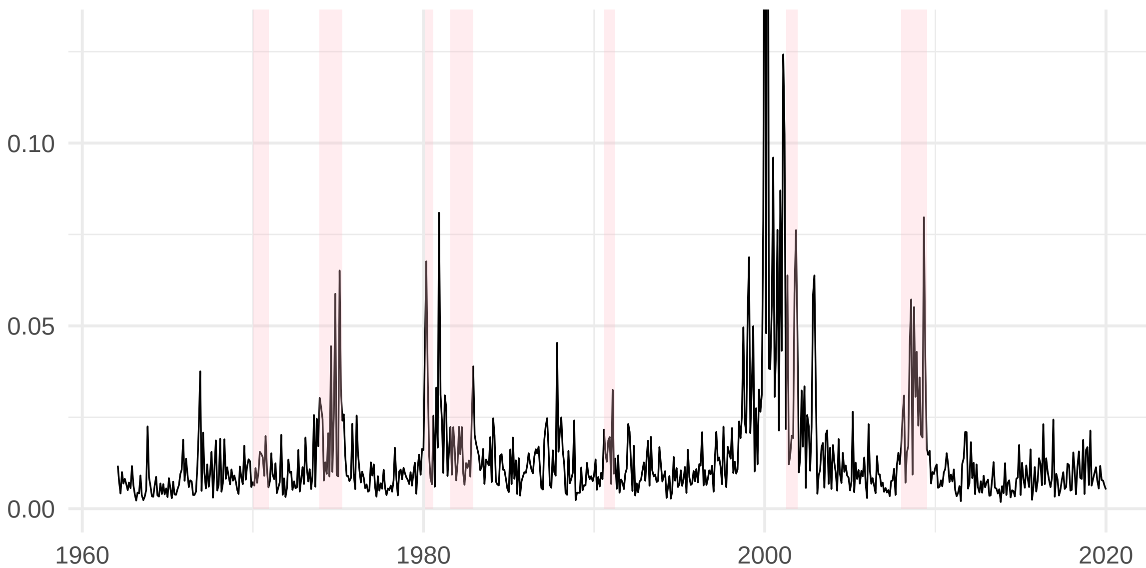
(a) market volatility



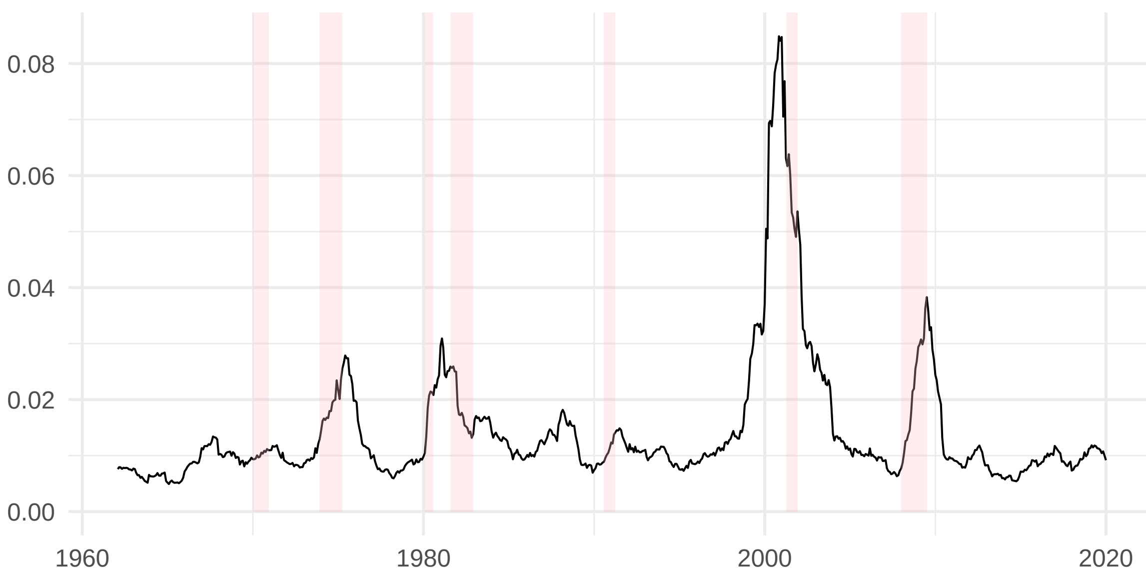
(b) Moving Average of order 12

Notes: This Figure shows the pattern of annualised market volatility, calculated using equation (15), in panel a, and an MA(12) in panel b. In the first plot, highest value of the series are cut out of the graph, to highlight the micro - dynamic of the former.

Figure 2: Industry Volatility



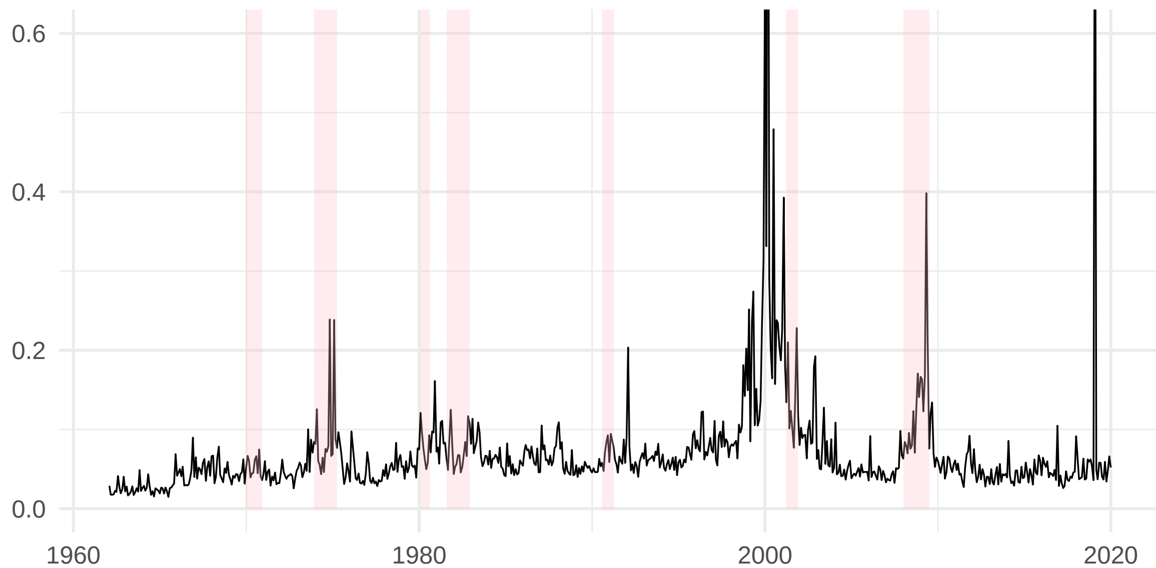
(a) Industry volatility



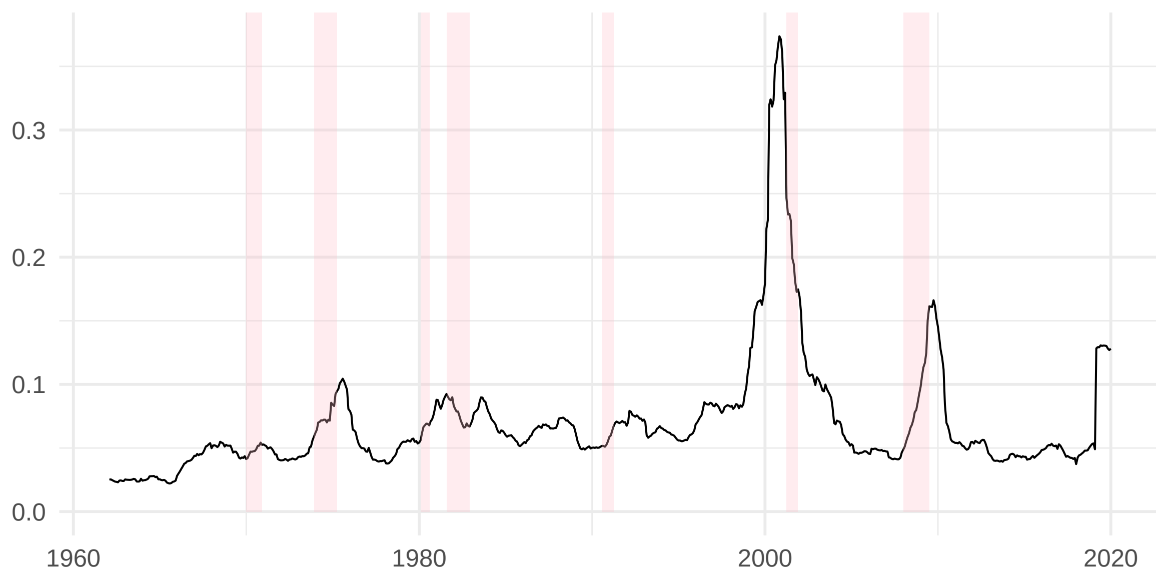
(b) Moving Average of order 12

Notes: This Figure shows the pattern of annualised industry volatility, calculated using equation (17), in panel a, and an MA(12) in panel b. In the first plot, highest value of the series are cut out of the graph, to highlight the micro - dynamic of the former.

Figure 3: Firm Volatility



(a) market volatility



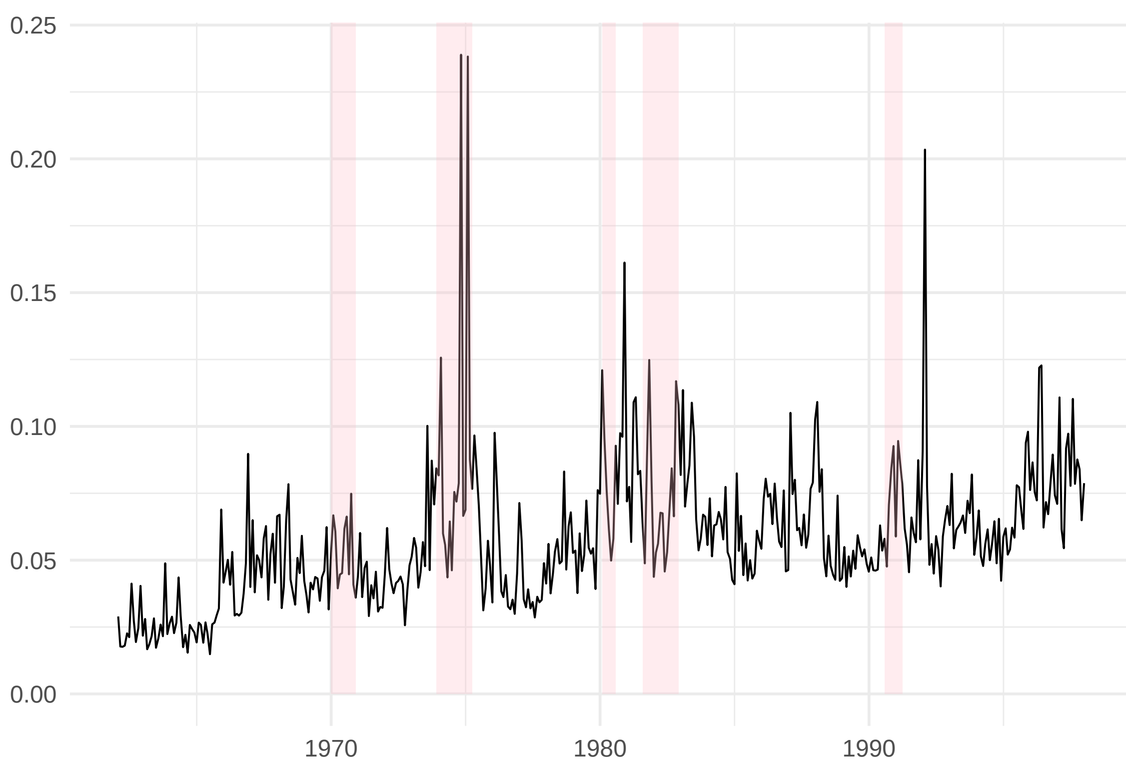
(b) Moving Average of order 12

Notes: This Figure shows the pattern of annualised firm volatility, calculated using equation (19) and (20), in panel (a), and an MA(12) in panel (b). In the first plot, highest value of the series are cut out of the graph, to highlight the micro - dynamic of the former.

In contrast with Campbell et al.(2001), IND does not show an equally important spike in 1987; it shows some deviation from the average value or high-frequency series movement, but it not as significant as the one in 1980.

The very same remark can be done for FIRM, plotted in Figure 3; the two series show a similar pattern, especially in the last two decades. In particular, firm volatility plot captures a strong increase between the 1998 and the 2000, with a maximum value of 1.1740 in February 2000; the biggest spike after this cluster is recorded in January 2019, probably due to the high uncertainty and the weakening of the of the global expansion, as reported by the IMF, followed by the 2008 crisis.

Figure 4: Firm Volatility - Original Sample



Notes: This Figure exhibit the pattern of annualised firm volatility, calculated using equation (19) and (20), excluding the last twenty years, to highlight the upward trend in the series.

As pointed out by Campbell et al., FIRM shows a visible positive trend between the

1962 and 1997,persisting until the year 2000. Indeed, after the above-mentioned series of spikes and excluding the increase due to the financial crisis and the 2019 spike, FIRM stabilizes, moving between a narrow interval, approximately between 0.03 and 0.4.

In order to provide a more straightforward graphic idea of the positive trend in FIRM, I plotted the volatility series excluding the last 21 years in Figure 4.

This is particularly interesting; for more than 30 years, FIRM maintained a positive trending pattern, regardless of the several downturns and shocks the economy and financial markets experienced. Then, after the early 2000s, it changed dramatically and at a completely unexpected speed, considering the stable development the idiosyncratic components displayed until that time. So, it come natural to ask why such a deep change took place and what influence had on aggregate volatility as a whole.

As we will discover shortly, FIRM has a great impact on aggregate volatility; as a consequence, this deviation particularly interesting to the scope of this paper and we will later investigate its causes.

To sum up, throughout this preliminary analysis, I reveal the following main findings: First, MKT and IND did not experienced any relevant change in the extended time period. On the contrary, they kept the same characteristics throughout the entire horizon analysed.

More in details, market volatility components show a several number of spikes, an indication of a rather high volatility; on the other hand, IND, with the exception of the peaks cluster in the early 2000s, did not exhibit any remarkable variation from its mean.

Second, I find that, contrary to the above - mentioned components, FIRM was more heavily altered during this extended time sample, as witnessed by the reversal in trend shown by Figure 3.

Despite the differences I just mentioned, the three series tend to move together most of the time and all of them seem to increase during economic downturns. However, it emerges from the plots that IND and FIRM have a stronger correlation with respect to the market-level volatility measure; this also appears from the Figures in Campbell et al.(2001), probably due to the higher high-frequency movements, though modest, registered from Figure 1. The volatility with the highest values appears to be FIRM, while IND is the one with the most modest value among the three.

3.1.2 Series Analysis

In this part of the analysis, I will inspect the serial correlation embedded by the three volatility estimates, in order to better understand they nature as time series and, afterwards, I investigated the possibility of a unit root.

Table 1: Serial correlation Structure

	MKT	IND	FIRM
ρ_1	0.1507	0.4187	0.4269
ρ_2	0.1009	0.5154	0.4505
ρ_3	0.1098	0.3862	0.3772
ρ_4	0.0817	0.3292	0.3870
ρ_5	0.0781	0.2820	0.2758

Notes: This table displays the serial correlation coefficients for market, industry and firm - level volatility components, from the first to the firth lag. I obtained the coefficient by running the ACF of the three series and storing its output.

Finally, I will study their statistics in order to spot any significant change in the last two decades. The results are exhibited in the following tables.
As shown on Table 1, all volatilities exhibit relatively higher serial correlation. Nevertheless, the overall picture did not change much with respect to the original restricted

sample, with MKT being the series with lowest coefficients; FIRM is still fairly persistent, despite the overall decrease in its serial correlation coefficients. The industry - specific volatility, on the other hand, preserved almost the same persistence reported in Campbell et al. (2001).

Given this results, it comes natural to run an Augmented Dickey-Fuller tests on all three measures, to discard the unit root scenario.

The main objective of the ADF test is to check whether a time series, in particular in an autoregressive model, contains a unit root or, to put it differently, if the series is integrated of order one, I(1).

The test is implemented on AR(p)s; the general version of the model looks like the following:

$$\Delta y_t = \alpha + \beta t + \theta y_{t-1} + \sum_{j=1}^{\rho-1} \delta_j \Delta y_{t-j} + \epsilon_t \tag{21}$$

Where $\Delta y_t = y_t - y_{t-1}$. The test is run on θ, with the null and alternative hypothesis being

$$H_0 : \theta = 0$$

$$H_1 : \theta < 0$$

Hence, we are dealing with a one-sided test, whose null hypothesis is that the series contains a unit root, while the alternative is that the series is stationary or trend-stationary, depending on the model we are dealing with. Namely, the former definition applies to model with $\beta = 0$, the latter is instead applied to model with $\beta \neq 0$. The test produced by this test follow a Dickey - Fuller distribution, whose critical values are reported under Table 2.

I run three different types of ADF tests: the first one is ran on a model with no drift and trend of any kind, meaning both α ad β are set to zero; the second one is a done on a model with a constant but without a trend ($\alpha \neq 0$ and $\beta = 0$); lastly, the third model includes both a trend and a drift, namely both α and β are different from zero.

The result are presented in Table 2. [3]

Table 2: Unit Root Testing

	MKT	IND	FIRM
Simple Model			
t-statistics	-12.2558	-6.5052	-6.50030
p-value	$< 2e-16$	$1.49e-10$	$5.13e-10$
Constant			
t-statistics	-15.8117	-9.1379	-10.0155
p-value	$< 2e-16$	$< 2e-16$	$< 2e-16$
Constant and Trend			
t-statistics	-15.8071	-9.1778	-10.1936
p-value	$< 2e-16$	$< 2e-16$	$< 2e-16$

Notes: This table describes the results of an Augmented Dickey-Fuller test for the Market, industry and firm volatilities series in three variations: A simple model, one with a drift and lastly,a model with both drift and trend. The 1% critical values are: -2.58 for the simple model; -3.43 the model with only the drift and -3.96 for the model with both a drift and a trend.

My volatility series proved to be stationary. The unit root hypothesis is rejected at the 1 percent confidence interval for all models, with the second model being the one who most strongly reject the I(1) hypothesis for MKT, and the last one for FIRM and IND.

[3] I chose the number of lags to include in the models described in Table 2 by the BIC, as this thesis does not have a predictive goal; for each model, the BIC included 1 lag.

Having proved the stationarity of our volatility estimates, we can carry on with an examination of the descriptive statistics of MKT, IND and FIRM, provided in Table 3. Our estimates' statistics experienced some variations with respect to the original sample: the mean of the series slightly increased, probably as a result of the dot-com bubble that, as we saw in Figure 2 and 3, heavily influenced both IND and FIRM and the 2008 crisis for MKT, that represented the second highest observation observed in Figure 1. Similarly, the standard deviation of all volatilities, with exception of MKT,experiencing slightly decreased, increased during the last 21 years, with FIRM's value more than doubling. The reason behind this phenomenon is quite straightforward: Starting from the latest 90s until present-day, many economic events, such as the dot-come bubble and the 2007 financial crises, contributed to undermine the stability of the financial markets, leading to more volatile stock prices.

Table 3: Summary Statistics

	MKT	IND	FIRM
Mean*10^2	2.4322	1.4364	7.0300
Std.Dev.*10^2	4.5913	1.8294	7.6147
Std.Dev. Detrended*10^2	4.5912	1.8227	7.5160
Min	1.1e-09	0.0018	0.0149
Max	6.4e-01	0.2680	1.1740

Notes: This table shows a summary of the main descriptive statistics for market, industry and firm - specific volatility. I report their mean and standard deviation, multiplied by a hundred and their minimum and maximum values. Also, I add the standard deviation for the three detrended series, to highlight any significant difference with the one obtained from raw data.

As concluded by Granger (1969), economic recessions have a radical effect on stock returns' volatility, accounting for more than 60 percent of the latter; as a consequence, the several downturns global economy experienced led to an inevitable increase in all volatility components, boosting the mean of the three series. Moreover, being these recessions alternated by recovery periods, such as the 2004-2007, caused returns to pass from unstable periods to more steady phases; this is probably the main driver of the evident increase in IND and FIRM standard deviations. The fact that MKT standard deviation is not affected by this increase, it brings more evidence that the rise in the latter comes from the time-horizon spanning from the early 2000s until 2007 approximately. Indeed, MKT is the only volatility components which is not massively influenced by the economic downturn experienced in that time-frame.

Going forward, as anticipated in the graphical analysis section, MKT is the series with the lowest and the second highest values among the three volatility. As a matter of fact, the market-level volatility records the second highest standard deviation, as an additional evidence of its high frequency noise. On the other hand, FIRM is the estimate with the highest value for min and max entries and its standard deviation is the highest. For what concerns the industry-level volatility, it has the lowest standard deviation among the three estimates. Finally, we see that the standard deviation of the detrended series has no relevant differences in none of the volatility estimates in Table 3; on the other hand, the divergence between the standard and detrended standard deviation of FIRM in the Campbell's paper is the greatest, though still modest. This is a direct cause of what we observed in Figure 3: after the more recent developments of FIRM, which lost its positive trend in the last 20 years, the role of the latter became relevant.

Table 4: Mean - Variance Decomposition

	MKT	IND	FIRM
Mean			
1962 - 1997	0.2509	0.1257	0.6235
1998 - 2019	0.1919	0.1387	0.6694
1962 - 2019	0.2231	0.1318	0.6451
Variance: 1962 - 1997			
MKT	0.5034	0.0809	0.1643
IND		0.0181	0.0741
FIRM			0.1592
Variance: 1998 - 2019			
MKT	0.0725	0.0420	0.1631
IND		0.0271	0.1890
FIRM			0.5064
Variance: 1962 - 2019			
MKT	0.1591	0.0494	0.1611
IND		0.0253	0.1675
FIRM			0.4377

Notes: This table displays the fraction of the total volatility's mean or variance is due to each of the three estimates. To construct the table, I followed the steps highlighted in Campbell et al.(2001): first of all, I derived the total volatility as $\sigma_t^2 = MKT_t + IND_t + FIRM_t$. Following this, the fraction volatility accountable to, for instance, the market components is simply $\mathbb{E}[MKT_t]/\mathbb{E}[\sigma_t^2]$, same for the other two components. Next, for the variance of volatility, we have that $1 = \mathbb{V}[MKT_t]/\mathbb{V}[\sigma_t^2] + \mathbb{V}[IND_t]/[\sigma_t^2] + \mathbb{V}[FIRM_t]/\mathbb{V}[\sigma_t^2] + 2Cov[MKT_t, FIRM_t]/\mathbb{V}[\sigma_t^2] + 2Cov[MKT_t, IND_t]/\mathbb{V}[\sigma_t^2] + 2Cov[IND_t, FIRM_t]/\mathbb{V}[\sigma_t^2]$.

Next, in order to find out the fraction of each variance attributable to each of the three components, I decompose the mean and variance of the total volatility; the procedures implemented is reported under Table 4, where its output is exhibited.[4] Looking at the top panel, where the fraction of each components contribute to the mean of the total volatility, we immediately observe that FIRM is responsible for the bigger portion of the mean in all the time horizon considered in the table. In particular, in the period going from 1998 to 2019, firm-level volatility accounts for more than 66 percent of the total volatility mean, an increase of 7.4%, leading to an overall increase of the importance of FIRM in the entire time horizon, namely 0.6451. On the other hand, MKT impact decreased in the last two decades, causing its extended sample impact to be around 0.22. Speaking of IND, its influence on total volatility is modest in all time horizons considered, compared to the other two measures, although it experienced a slight increase in the last twenty years. In the complete sample, it accounts for almost 14 percent of the total volatility mean.

Switching to the bottom part of the table, we see that, from 1962 to 1997, MKT variance alone accounts for more that 50 percent of the total variation, for a overall impact of 0.7486 when adding the covariance terms. Moving forward, FIRM has a modest relevance on the total variation, namely slightly less than 16 percent, while IND accounts for less than 2 percent.

The results are basically reversed in the second variance panel, where the firm - level volatility accounts for more than 50 percent, with a total of 0.8585. On the other hand, MKT variance has a limited impact, especially if compared to the previous time-span; indeed, it represents less than 8 percent of the total variation. As before, IND influence is rather low, around 0.2581, when considering the covariance terms.

Given the previous discussion, the results for the entire time horizon are not surprising:

[4] In contrast to Campbell et al.(2001), I did not downweight nor detrend the three series with the aim of capturing the real impact MKT, IND and FIRM have on the total volatility of a stocks' return.

MKT and FIRM are the components influencing the most the total variation, respectively 0.37 and 0.77, adding the covariance terms. As a consequence, IND is not a particularly relevant component in terms of variance of total volatility.
This results are an additional evidence supporting the fact that, even without downweighting any economic downturns or detrending the series, and even though the positive trend spotted by Campbell, Lettau, Malkiel and Xu (2001) in FIRM is not present in the last twenty years, the firm component still plays a fundamental role in total returns' volatility, both in terms of its mean and variance, and should therefore be carefully taken into account. In addition, this analysis gave an important motive to investigate the reason behind the change underwent by FIRM, as its variation seem to have a deep impact on the pattern of the total volatility of a stock, both in terms of mean and standard deviation.
I will examine more in-depth and in more relevant manner what occurred to the firm-specific volatility in section 3.5.

3.1.3 Correlation and Lead - Lag relationships

As highlighted in the previous section of the thesis, despite the differences in high-frequency movements and divergences and the impact economic recessions had on the three series, MKT, IND, and FIRM tend to co-move for the majority of the time span considered. As a consequence, it seems natural to further investigate the strength of the link connecting the three components. Therefore, in this section of the thesis, I will study the relationship existing between the three series. The first indicator of the strength of the relationship among MKT, IND and FIRM is their correlation. Looking at table 5, where the correlations are exhibited, we can see that IND and FIRM are strongly correlated; the correlation among them increased after the last two decades, probably due to the similar pattern the two series show in the beginning of 2000. On the other hand, the correlation between the market and industry - level volatility is rather low with respect to the former, though they still show a fair correlation of almost 0.4.

Lastly, the correlation between MKT and FIRM is the weakest among the three, with a value of approximately 0.3. This is not striking; as a matter of fact, it was clear from section 3.2 that the series share slightly less with respect to the former and the industry-specific volatility.
I also included the correlation matrix for the detrended series, in order to study any significant deviation from the original data.

Table 5: Correlation Matrix

	Raw Data			**Detrended Data**		
	MKT	IND	FIRM	MKT	IND	FIRM
MKT	1.0000	0.3894	0.3053	1.0000	0.3924	0.3121
IND		1.0000	0.7963		1.0000	0.7958
FIRM			1.0000			1.0000

Notes: This table displays the correlation among market, industry and idiosyncratic - level volatility, for both raw and detrended data.

As highlighted from Table 5, the correlation between the firm and industry volatility is not remarkably affected from the de-trending of data; it experienced a small decrease, but the overall relationship between the two is not relevantly influenced by that.
On the other hand, both correlations between MKT and IND, together with the one between the market and firm - level series experience an increase after de-trending the data, as proof that, apart from FIRM's positive trend, the three series tend to move together most of the time.

Subsequently, I proceeded by performing a Granger - causality test; its main objective is to assess whether a series "causes" or forecasts another series. It considers two different models:

$$y_t = \alpha + \beta Y_{t-p} \tag{22}$$

and

$$y_t = \alpha + \beta Y_{t-p} + \gamma X_{t-p} \tag{23}$$

where Y_{t-p} and X_{t-p} are matrices of p lagged vectors of, respectively, dependent and independent variables.

The null hypothesis of this test is:

$$H_0 : \gamma = 0 \tag{24}$$

while its alternative is the following:

$$H_1 : \gamma \neq 0 \tag{25}$$

Granger test controls for lagged causality of the regressors with respect to the dependent variable.

It is particularly useful in this context, as we are analyzing three series that, according to both a graphical and correlation analyses, appear to be tightly connected. Following, I conducted the Granger - causality test on the three linearly detrended series[5], putting them in a bivariate VAR(p) model; the output of the test is reported in Table 7.[6]

In the table, on the rows we have the cause variable, on the columns the dependent variables. As outputs, we have the F-statistics produced by the test and the relative

[5]I also conducted the test on the raw series; I did not report the results, nor I will discuss them in the thesis, as they are pretty similar to the ones reported in table 7, leaving the overall causality between the series unchanged.

[6]In Campbell et al. (2001), other than de-trending the series, the authors downweight the 1987 crisis from all the volatility series, being the highest value for both MKT, IND and FIRM. Due to the difference in my sample, both in terms of time horizon and values, The highest values in the three volatility do not coincide in terms of time; Given that, and to keep my analysis as close as possible to reality, I did not downweighted any of the series.

p-value. In the last entry of each panel I put the number of lags chosen to construct the VAR model; specifically, I chose them using the Akaike Information Criteria.

Table 6: Granger - Causality Test

	MKT	IND	FIRM
MKT			
F-statistics		2.0120	4.1594
p-value		0.0348	2.4e-03
# Lags		9	4
IND			
F-statistics	2.2694		6.5473
p-value	0.0160		3.5e-09
# Lags	9		9
FIRM			
F-statistics	1.2851	1.5345	
p-value	0.2738	0.1305	
# Lags	4	9	

Notes: This table shows the output of a Granger causality test for market, industry and firm - specific volatility measures. Before running the tests, I constructed 6 different bivariate VAR(p) models, whose lags where chosen through the AIC. On the row of the table, we find the variable used as independent in the VAR(p) model, while its regressand is in the column of the table. I reported the F- statistics produced by the test and its relative p - value, plus the number of lags of x and y included in each model.

as Table 7 highlights, without downweighting the three series, MKT shows forecasting powers over both IND and FIRM, fairly stronger for the latter; the test is indeed significant at 5 percent for IND and at 1 percent for the firm-level volatility. For what concerns the industry-level volatility, we observe that it Granger-causes both MKT and FIRM; in particular, it has a really strong forecasting power on FIRM. On the other hand, FIRM does not appear to have any kind of forecasting abilities for none of the series, whose granger causality test is not significant neither for MKT or IND.
Comparing this table with the one produced in Campbell et al. (2001), the results of this analysis are clear: The last two decades deeply changed the causality relationship between the three volatility components. Now, this is not surprising; just by looking at Figures 1, 2 and 3, it is easily seen that the various economic downturns global economy experienced in the last twenty years profoundly influenced stock returns' volatility, with a different impact for each component of the latter. This changed the relationships registered so far between MKT, IND and FIRM, including their lead/lag connection.

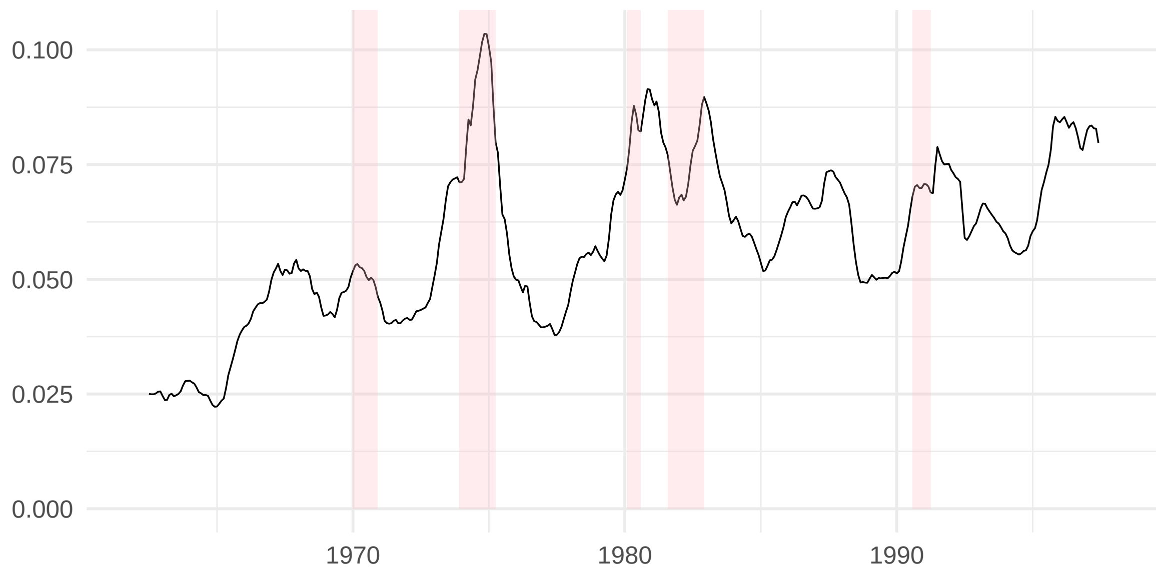
Most of the output of this thesis points to the same direction: From 1998 until the end of 2019, the relationship between IND and FIRM has got stronger, while MKT seems to be more detached from the other two volatility components. Nevertheless, the overall situation in terms of mean and standard deviation remain almost unchanged: FIRM and MKT are the more volatile components, moreover, FIRM is the one with highest mean. On the other hand, the industry-level volatility presents more modest statistics as it is the smallest series, in terms of magnitude and deviation.
Another important results uncovered in this analysis is the change in FIRM trend. As shown in section 2.1 of the thesis, the positive trend characterizing the former seems to disappear after the peak coinciding with the Dot-com bubble. This is in line with both U.S. and E.U. stock price index volatility that, in fact, do not show a trend of any kind in the last two decades. The same applies to other volatility measures, such as the CBOE volatility index, the VIX, which measures a 30-day expectation of U.S. stock

market volatility, based on S&P 500 mid-quote prices of both call and put option with the former as underlying. Being one of the most relevant volatility measures worldwide, it is probably among the index and measure worth analyzing. Indeed, excluding major economic downturns cited above, the index did not show any trend or relevant increase from the early 90s until December 2019, consistently with the overall output of this thesis.

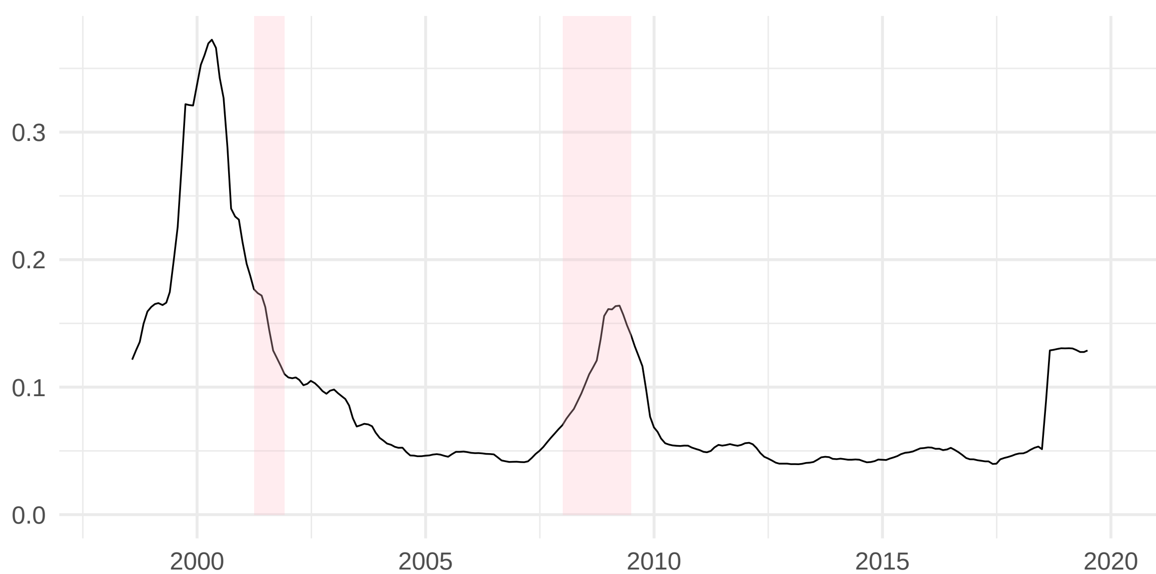
3.2 Trend Reversal and Structural Break Investigation

Having assessed the plausibility of my results compared to historical data and relative volatility patterns, I will further investigate the nature of the prominent change registered for FIRM in a more rigorous and analytical manner.
Considering the output of figure 3, showing the deep difference in the pattern of FIRM before and after the 2000s spikes cluster, I proceed with an in-depth analysis of the trends in the two time-span.
In Figure 5, I plot the trend components of the firm - specific volatility, dividing it into two parts: Figure 5 (a) depicts FIRM trend from January 1962, the starting point of my sample, to December 1997, a couple of years before the Dot-com bubble took place; meanwhile, Figure 5 (b) shows the same for the last twenty two years. For both graphs, I shade the NBER recession periods for the relative horizons, to highlight the economic downturns experienced so far.
It is easily seen that, apart from the spikes exhibited in the plot, figure 5 (a) displays a firm upward trend throughout the whole time-frame, leaving little room for doubts. On the other hand, the bottom plot shows a dramatically different scenario: Despite the steep rise connected with the influence of the early 2000s recession, FIRM's trend component seems to experience a brief but strong reversal, even though partially explained by the preceding peak, persisting until 2005. After that, As foreseen in section 3.2, the trend seems to disappear, with Figure 5 (b) looking almost fully flat, excluding the peak related to the 2008 financial crisis from my assessment and, as expected from

Figure 5: FIRM - Trends



(a) Trend component in FIRM from 1962 to 1997



(b) Trend component in FIRM from 1998 to 2019

Notes: This figure displays the trend of idiosyncratic - specific volatility measures. In particular, Panel (a) shows the trend the series experienced from 1962 to December 1997; conversely, panel (b) exhibit the same for the last two decades, until December 2019.

the preliminary analysis, in January 2019, which is the second highest value for the series considered.
These results comes as a firm confirmation of the guess proposed during the graphical analysis, namely that FIRM has deeply changed during the last two decades. Given the impact it has on total variance, as proved in Table 4, studying this pattern deviation may be of major importance in order to understand more deeply total variance dynamics.

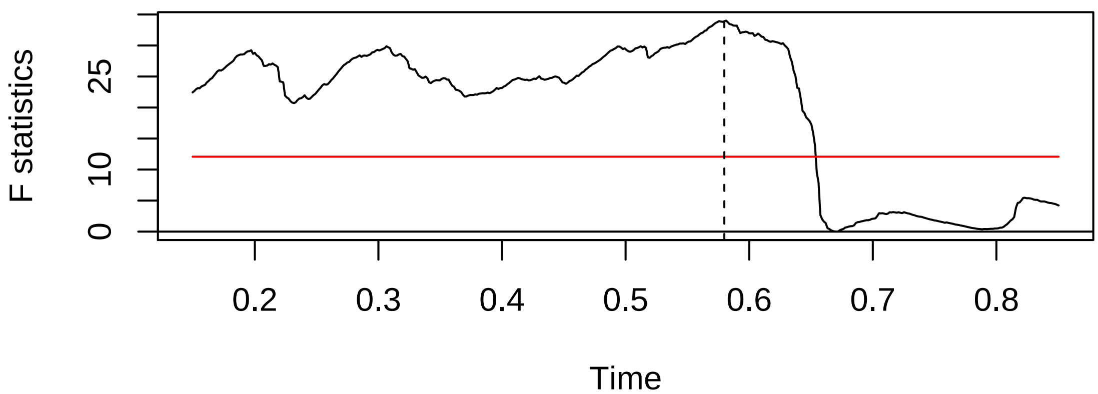
Considering Figure 3 and observing the change discussed above, it comes natural to test for a structural break in the series, which can be defined as an unforeseen variation of one or more parameters of a model, in this case a time series, over time.
The First test I run is the SupF test. It consists in running an F-statistics for every point considered, whose null Hypothesis is that the model before each point is the same as the one constructed after that; putting it differently, it tests the stability of the model and its parameters over time. I run the test in two different versions: for the first one, I consider a model with FIRM as independent variable and a constant, to test whether the series stays constant or if it incurred in a structural change. On the other hand, for the second version, I decided to model FIRM as an AR(3); I opted for this type of model after a rough splitting of data with the year 2000 as a threshold. Indeed, I hypothesised the break happened during that period, as confirmed after from the test I run. After a careful examination of ACF and PACF of the two splits, both seem to follow an AR(3) process.[7] In Figure 6, I plot a graphical representation of the two tests, with 1 percent significance level.
As highlighted from the figure, both versions of the test pick a breakpoint and, as a

[7] I also run a SupF test on an AR(4) model, which the whole series seems to follow and the breakpoint detected by the latter is roughly the same as the one I described. Indeed, the breakpoint picked with the AR(3) is at observation 454, the one chosen with the AR(4) is 453. Given this, I consider the model chosen as consistent.

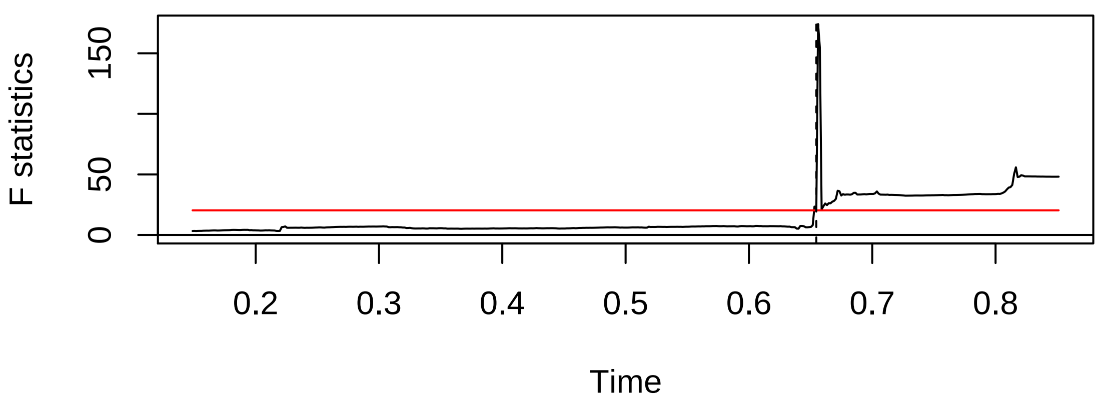
consequence, detect a structural change in the model. The main difference between the two versions is the point chosen as a breakpoint: as a matter of fact, Figure 6 (a), plotting the F- statistics for the model with the constant only, rejects the null hypothesis of the stability of the series, identifying the point in which the model changed in September 1995. Conversely, Figure 6 (b), plotting the F-statistics for the AR(3) model, locates the breakpoint in March 2000, deeming the Dot-com bubble accountable for the structural break in FIRM.
Subsequently, I run the SupF test on the AR(3) model: as expected, the stability hypothesis is strongly rejected, with an F - statistics of 174 and a p - value of zero.
My conclusions are robust to different methodology; one other test I employ is in the appendix.

Summing up, the results of these tests leaves no room for doubts: idiosyncratic - level volatility did actually experience a structural break in the time horizon I consider. In particular, focusing on the the AR(3) model, which I believe is the more appropriate to describe FIRM series based on the ACF and PACF plot output, the break in the above - mentioned volatility measure took place in March 2000, accordingly to what we have seen through the preliminary analysis.
This is a fairly remarkable result: the Dot-com bubble is the cause of the break in firm-specific volatility and, as a consequence, the reason lying behind the trend reversal in the latter. It is very important to acknowledge a structural break in a model, as ignoring it can be quite dangerous, in the contest of modelling; indeed, as highlighted in Hansen (2001), not caring about it could lead to misleading model inferences, poor forecasts and wrong or misleading policy recommendation. Once again, keeping in mind the impact that FIRM has on total aggregate volatility, ignoring the break discovered in the series may have significant negative repercussion on studies focused on the whole volatility measure.

Figure 6: SupF test plot



(a) SupF plot for the simple model



(b) SupF plot for the AR(3) model

Notes: This figure shows a graphical representation of the SupF test employed on a simple model with a constant in panel (a) and for an AR(3) model in panel (b).The red lines denote the 1 percent confidence interval of the test, while the horizontal dotted lines highlight the point where the structural break took place.

3.3 Robustness Check - Daily Data

In this part of the thesis, I re - run the main elements of the previous section's analysis. The main difference between the previous and current part of the thesis is the frequency of firms' returns; as a matter of fact, in this section I will employ data collected in a daily frequency in order to construct the three monthly volatility measures. Due to the considerable increase in the amount of data, I decided to cut my sample; indeed, in this section, I will consider data from January 1980 to December 2005, in order to speed up the computation process, but keeping the crucial points needed to compare the two different analyses.

First of all, I will describe the modified empirical approach applied to obtain MKT, IND and FIRM from daily companies stock returns; After that, I will exhibit the result of such modified analysis and compare them to the ones obtained in the previous section.

3.3.1 Empirical Estimation

Similarly to section 2.2, I follow Campbell, Lettau, Malkiel and Xu (2001) approach: my starting point is the daily individual firms' returns (in excess of the U.S. Treasury monthly-adjusted average returns, divided by the number of trading days in a month). From that, I obtain industry returns the same way I did in the empirical approach section:

$$R_{js} = \sum\nolimits_{i \in j} w_{ijs} R_{ijs} \tag{26}$$

w_{ijs} being the weight of the firm i belonging to industry j at time s and is expressed in terms of market capitalization. In a similar way, I calculate daily market returns:

$$R_{ms} = \sum\nolimits_{i} w_{js} R_{js} \tag{27}$$

With w_{js} being the weight assigned to the industry j at time s.

From this point, the construction of the three volatility measures slightly differs from the one described in the previous part of the analysis, although we will still employ the simplified methodology described in 2.2.

For what concerns the market volatility component, I obtain an empirical measure of the former in period t, where t represents the months, as follows:

$$MKT_t = \hat{\sigma}^2_{mt} = \sum\nolimits_{s \in t} (R_{ms} - \mu_m)^2 \tag{28}$$

Focusing on industry sample volatility, starting from:

$$R_{js} = R_{ms} + \epsilon_{js} \tag{29}$$

I obtain an estimate of the industry-specific residual, I square it sum it within a month:

$$\hat{\sigma}^2_{\epsilon it} = \sum\nolimits_{s \in t} \epsilon^2_{js} \tag{30}$$

After that, I average $\hat{\sigma}^2_{\epsilon it}$ across industries to eliminate all the individual industries' covariance. Doing this, I obtain a monthly estimate for the industry volatility:

$$IND_t = \sum\nolimits_j w_{jt} \hat{\epsilon}^2_{jt} \tag{31}$$

Lastly, to obtain the idiosyncratic volatility component, I obtain firm specific residuals from the following equation:

$$R_{ijs} = R_{js} + \eta_{ijs} \tag{32}$$

Thereafter, I sum η_{ijs} within each month for each firm in my sample:

$$\hat{\sigma}^2_{\eta ijt} = \sum\nolimits_{s \in t} \eta^2_{ijs} \tag{33}$$

Having obtained a monthly measure of idiosyncratic - specific residuals, I average them within each industry, as illustrated below:

$$\hat{\sigma}^2_{\eta jt} = \sum\nolimits_{i \in j} w_{jt} \eta^2_{ijt} \tag{34}$$

Finally, we average $\hat{\sigma}^2$ over industries to construct my monthly sample firm volatility measure, without having to take into account covariances terms in the equation:

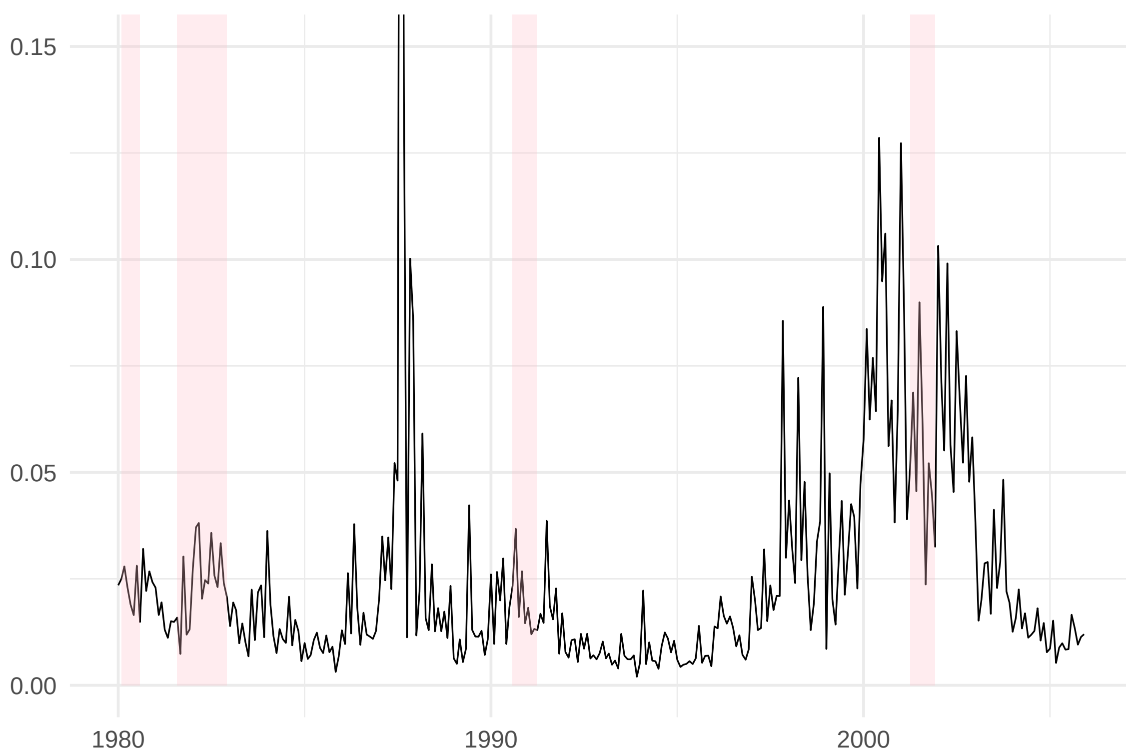
$$FIRM_t = \sum\nolimits_j w_{jt} \hat{\sigma}^2_{\eta jt} \tag{35}$$

3.3.2 Analysis Results

Having thoroughly illustrated the empirical steps to follow in order to obtain the three aggregate volatility components when dealing with daily frequency stock returns, I will display the results of the analysis in this context and compare it with the previous section's output.

Firstly, I plot the graphical pattern of the three series over time, in order to replicate the graphical analysis computed in 3.1.1, taking into account the differences due to the dissimilarity in the time horizon considered for the two analyses. The above - mentioned graphs for market, industry and idiosyncratic components are displayed, respectively, in Figure 7, 8 and 9.

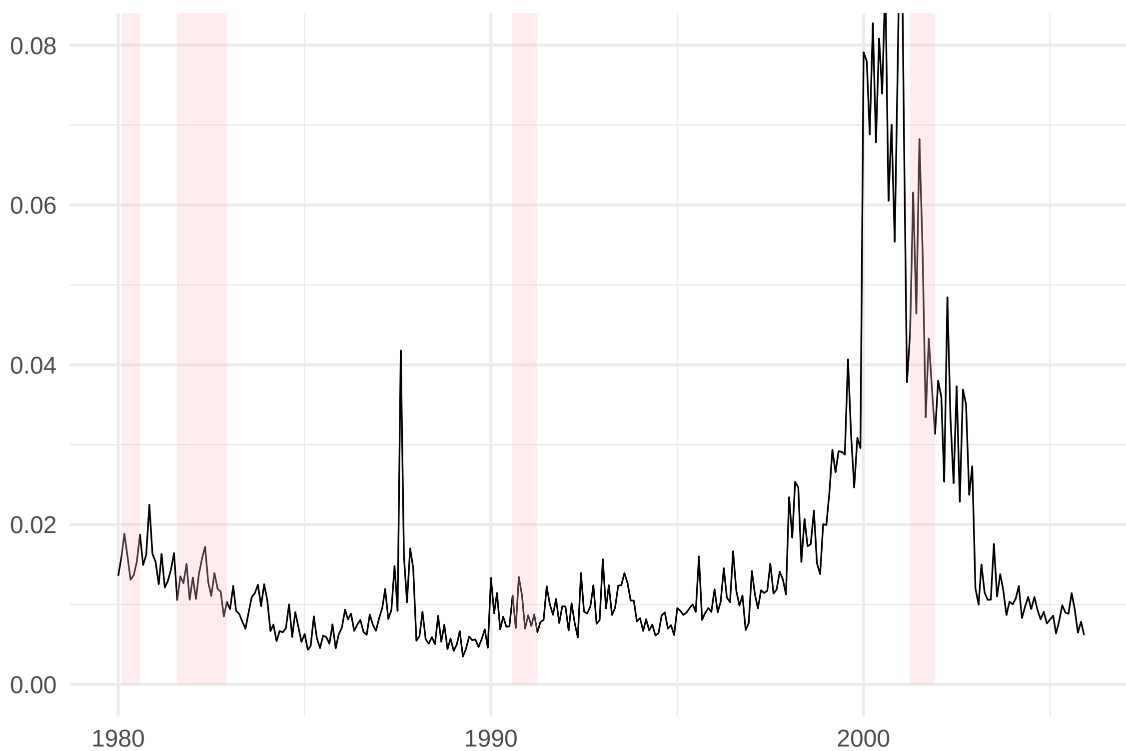
Figure 7: Market Volatility



Notes: This Figure exhibits the pattern of annualized market volatility, calculated through equation (28), using stock returns in a daily frequency in a reduced sample, from 1980 to 2005.

Similarly to the Figures reported in the preliminary analysis, I add NBER recession dates, shaded in pink, to highlight downturn periods; in addition, the highest value of the three series are cut out of the graphs, to underline the micro - dynamics of the total aggregate volatility components.
Focusing on market volatility, shown in Figure 7, we see that the pattern followed by the series is very similar to the one displayed in Figure 1, with the highest value recorded in the end of 1987; the smaller cluster of peaks captured around the early 2000s is a detailed shared with the previous section's picture. Moreover, the volatility seems to rise with recessions and it looks the noisiest series among the three. Nevertheless, we can spot some differences with respect to the volatility obtained through monthly

Figure 8: Industry Volatility

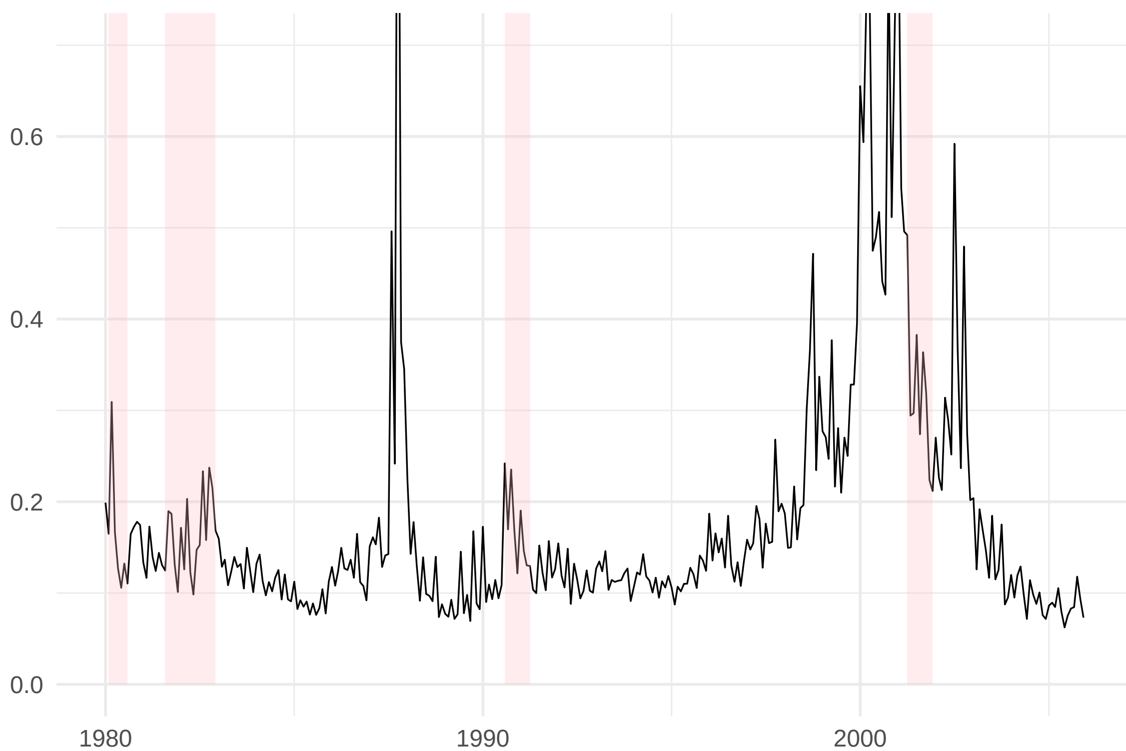


Notes: This Figure exhibits the pattern of annualized industry volatility, calculated through equations (30) and (31), using stock returns in a daily frequency in a reduced sample, from 1980 to 2005.

frequency data: as a matter of fact, Figure 7 seems to exhibit slightly more modest spikes with respect to Figure 1 and, in general, presents somewhat smaller values. Indeed, in terms of spikes and values, this series recalls more closely Campbell's market volatility which, as I highlighted in the previous section, is calculated using daily returns.

Moving to industry - specific volatility component, exhibited in Figure 8, it is easily noticed that this series also increases with recession and, as a consequence, tends co - move with MKT. Similarly to Figure 2, it registers a few number of spikes and more modest values with respect to Figure 7. Moreover, it captures the same big cluster of peaks in the early 2000s, in parallel with the Dot - com bubble, where the series

Figure 9: Firm Volatility



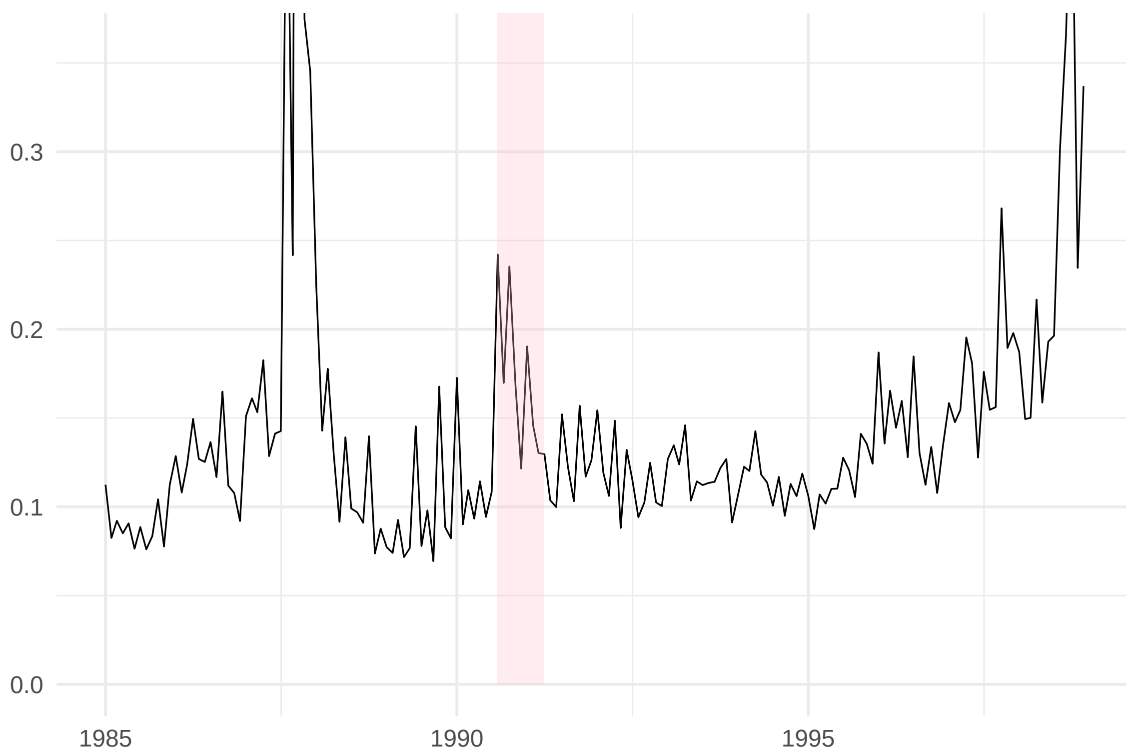
Notes: This Figure exhibits the pattern of annualized idiosyncratic volatility, calculated through equations (33), (34) and (35), using stock returns in a daily frequency in a reduced sample, from 1980 to 2005.

registers its highest value as well. Also in this case, the value displayed in Figure 8 seems slightly lower with respect to industry volatility obtained through the first empirical methodology illustrated on this thesis.

For what concerns Figure 9, relative to idiosyncratic volatility, the co - movement with the other two series and the tendency to rise with recessions also applies to this volatility component. The general pattern of the series is maintained, despite the frequency change, with a positive trend, more clearly exhibited in Figure 10, persisting until the cluster of spikes captured in the early 2000s. Lastly, as underlined in 3.1.1, FIRM is the only component registering any trend, similarly to what we have seen so far.

Notwithstanding this, there are some significant divergence with respect to previous

Figure 10: Firm Volatility - Positive trend

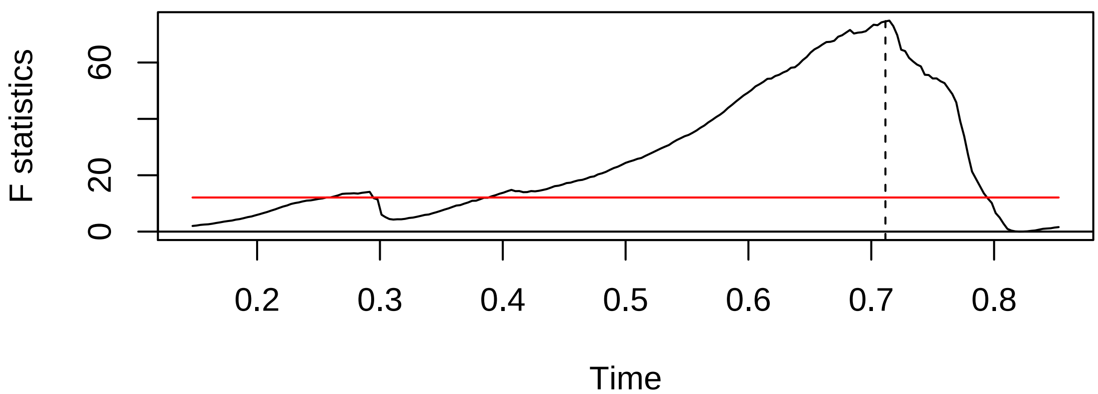


Notes: This picture displays the pattern of annualised idiosyncratic volatility from January 1985 to December 1998, in order to highlight the positive trend registered by the series before the Dot - com bubble.

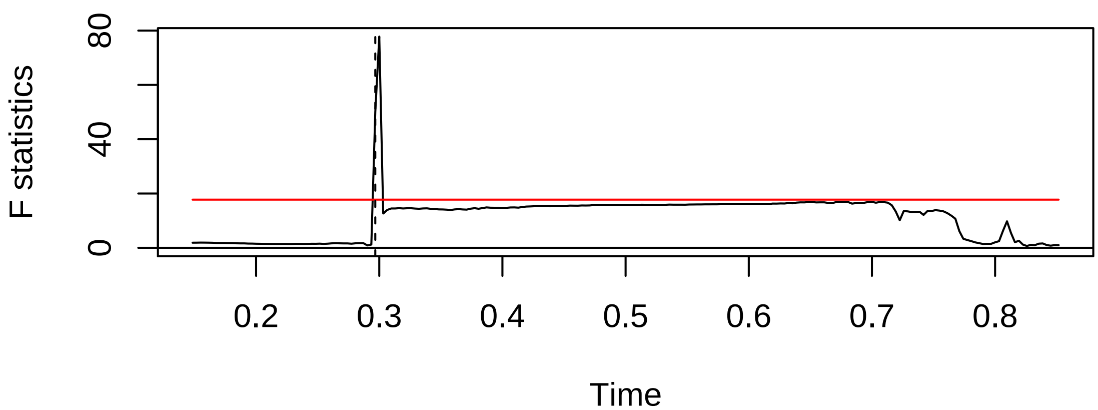
section's FIRM: In particular, the most evident change is the remarkable spike registered on October 1987, which is not present in Figure 3. Indeed, similarly to market volatility, this particular brings together this series with the one exhibited in Campbell (2001).
Summing up what I discovered so far, all three volatility measure analysed in this section do not significantly differ from the ones obtained in 3.1 in terms of pattern and general characteristics, but there are some minor differences that are worth noticing, specifically regarding the overall magnitude of the spikes and the mean value of the series for market and industry - specific volatility, and appearance of a new peak for FIRM.
The output of this graphical analysis suggests that, although some variation arose throughout this section, especially concerning the mean value of the series and, in the case of FIRM, the emergence of a new significant peak in the series pattern, the overall development follows the one displayed in the previous section. As a consequence, I can affirm that the results displayed in 3.1.1. are robust to different data - frequency, in this case daily.

Having completed the preliminary analysis for this section, I move forward to the second critical point of this study: check whether a structural break took place in the series capturing idiosyncratic - specific volatility. The idea is to investigate if the results exhibited in Figure 6 are in line with what we will find using daily stock returns, taking into account the differences in the time - period considered.
Following the same steps illustrated in 3.2, I verify the possibility of a structural break in FIRM by running a SupF test on the volatility series; the output of the latter is displayed in Figure 11. The upper graph represents the result of the test ran on a simple model with a constant, to check for the stability of the series. What is immediately perceived from it is that the model presents a structural break at 1 percent confidence interval close to the end of the sample, namely in July 1998, right before the Dot - com bubble started; the test gives a supf of more than 74 and a p - value of 3.331e-16.

Figure 11: SupF test plot - Daily data



(a) SupF plot for the simple model



(b) SupF plot for the AR(2) model

This figure shows a graphical representation of the SupF test employed on a simple model with a constant in panel (a) and for an AR(2) model in panel (b).The red lines denote the 1 percent confidence interval of the test, while the horizontal dotted lines highlight the point where the structural break took place

Similarly, the same test run on an AR(2), chosen after taking into consideration the acf and pacf of FIRM, also detects a structural break in firm - specific volatility component. As a matter of fact the breakpoint, at 1 percent confidence interval as well, is captured on November 1987, a month after the stock market crash and coincides perfectly with the peak registered in the series which was not present in the first FIRM obtained throughout this thesis. The SupF registered is 77 and the p-value is almost zero.
Comparing Figure 11 with Figure 6, it is immediately noticed that both upper and lower graphs look very different among them. As a matter of fact, in the previous section the breakpoints for (a) and (b) were, respectively, on September 1995 and March 2000. Based on the output of Figure 11, the simple model with a constant instability recorded in the graph seems very connected with the Dot - Com bubble, as it arises at the beginning of the cluster of spikes I described before, and seems to validate the thesis built in the previous section, where I attributed to the bubble the cause of the sudden change in FIRM's positive trend. On the other hand, when modeling the series as an AR, it looks like the appearance of the new peak strongly influenced the output of the SupF test, and as a consequence partially undermine the above - mentioned argument. Despite this, this changes in output could be due to the change in the time horizon considered in the two analyses, as this test highly depends on the sample tested.

Summing up, what we discovered so far is that, when obtaining the three volatility components starting from daily frequency data, the overall pattern of market, industry and firm volatility closely follows the one obtained with monthly data, with some minor differences in the magnitude of the spikes and the mean of the series in a slight manner. In addition, for what concerns FIRM, a new spike is captured by the series, relative to the 1987 stock market crash. According to the output of the SupF test on the series, modeled as an AR(2), this event seems to be the trigger of a structural break in the component, partly attenuating the robustness of the thesis built so far. The question is whether this is influenced by the difference in the two samples used, which I leave for subsequent studies to answer.

3.4 Small vs. Big Firms

After shedding light on the abrupt change in firm-specific volatility and having identified the early 2000s as the main driver of the latter, I considered interesting to proceed by further study the impact of the Dot - Com bubble over my sample in a more specific context. As a matter of fact, I divided firms in two sub - sample, using the median market capitalization in each period as a threshold; following this methodology, I obtained a division between small and big firms.

The empirical motivation behind this choice is given by the fact that, in the last twenty years, firm size skyrocketed, re-shaping the size distribution of the latter.

Figure 12: Mean of firms' market capitalization

Notes: This picture displays the development in average market cap of firms in my sample, from 1962 to 2019.

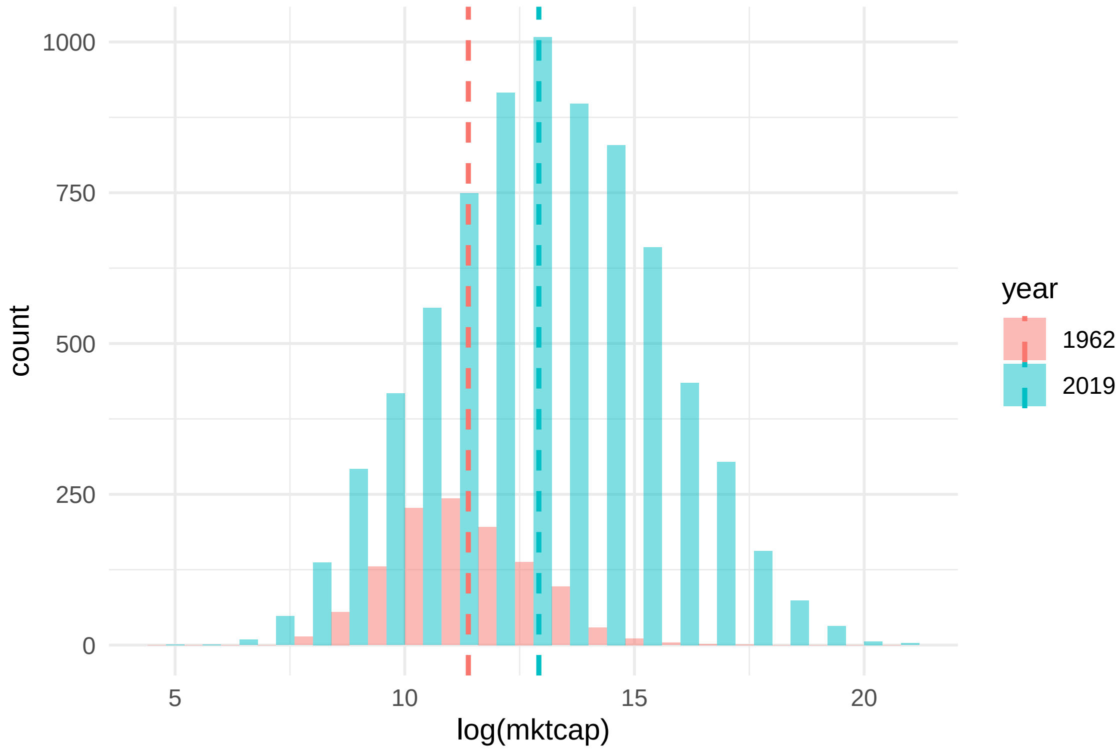
In Figure 12, I plotted the average market capitalization in each period in my sample, and it clearly exhibits the scenario I just described; starting from January 1962 until the early 90s, the mean size of the firms in the sample kept a steady pace, with a slight upward trend in the last decade included in the sample. After that, the picture shown abruptly changes: firms' size starts to increase, more and more rapidly, reaching a value more than 5 times bigger with respect to the starting point. This dramatic mutation raised the question of whether the results obtained in the previous section were somehow influenced by it, or whether the latter applies to both sub - samples, and with which magnitude.

The first thing I did was to study firms' size distribution at the start and the end of the sample, to have a clearer idea of the changes incurred throughout the time - span considered. Both distribution resemble a log - normal, and were both highly right-skewed, suggesting a significant amount of firms are considered of small size, while there are a limited number of firms with a remarkably big market capitalization. Considering the characteristics of the two distribution, which made the overall picture difficult to read, I decided to plot them in terms of logarithm in Figure 13. As we can see from the two plot, the pink distribution, representing the distribution of firms' size in the starting point of the sample, is slightly right - tailed and with a lower mean with respect to the blue distribution, capturing the distribution in December 2019. Indeed, as underlined by the dotted lines, the former distribution has a mean of approximately 11, conversely, the blue distribution shows a mean of almost 13.

In addition, the distribution for December 2019 looks more like a normal distribution, suggesting a less polarized size distribution.

Lastly, it is interesting to note that the distribution for the ending point of the sample is fairly bigger, when compared to the other one, underlining the remarkable increase in number of firms through the time sample considered.

Figure 13: Firms size - Histogram



Notes: This picture displays the distribution of firms' size in January 1962 and December 2019. The dashed lines highlight the mean of the two series.

Afterwards, to quantify the shift observed in Figure 13, I reported a summary statistics of market cap in both years, with the addition of December 1999, exhibited in Table 7. The table reports each entry in million USD. By observing the number shown in the latter, we immediately notice the shift experienced by the market capitalization in these 57 years: indeed, all measure, with the exception of the minimum value, are significantly higher in the right - hand side of the table, confirming what we observed from the histograms. The table suggests that the shape of firms changed radically, with an increased size and a more spanned size distribution, despite in all periods is highlighted a dramatic polarization towards the left side of the distribution, as implied by the gap between the 90% percentile and the maximum value in both sides of the table. Despite the noticeable increase in size between the right and central part of the table, the gap between December 1999 and December 2019 is outstanding, with a 2.68 bigger mean for the end point with respect the the 1999.

Table 7: Summary Statistics - Market Capitalization

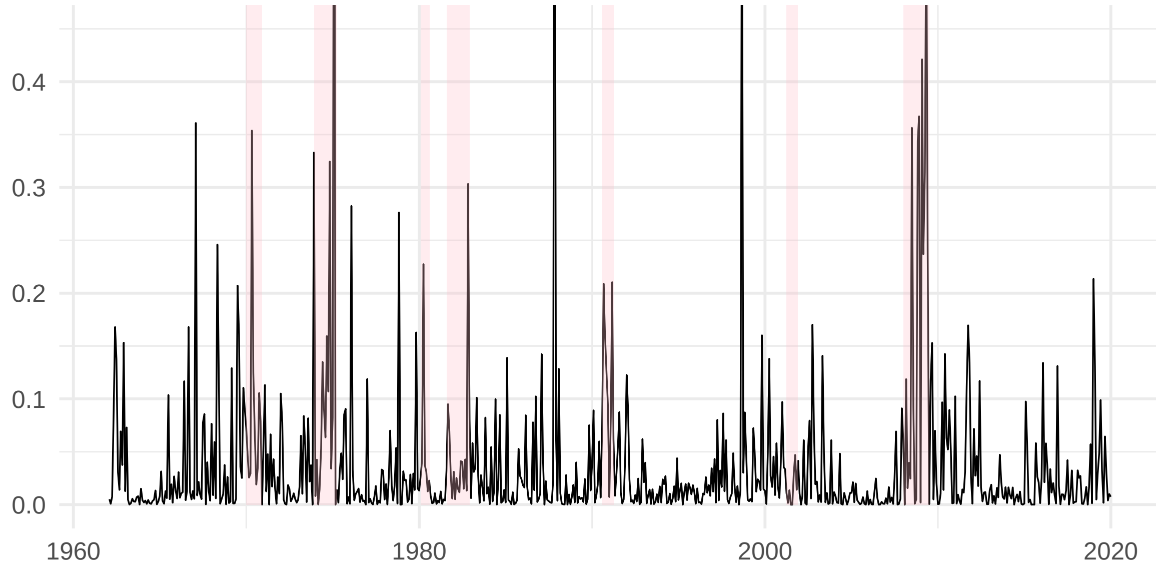
	January 1962	December 1999	December 2019
Mean	0.3231	2.107	5.677
Min	0.0020	4.3e-05	1.8e-04
50% Quintile	0.0802	1.4e-01	3.9e-01
90% Quintile	0.6197	2.490	9.210
Max	30.1989	6e+02	1.3e+03

Notes: This table displays a summary of firms' size descriptive statistics in January 1962, December 1999 and December 2019.

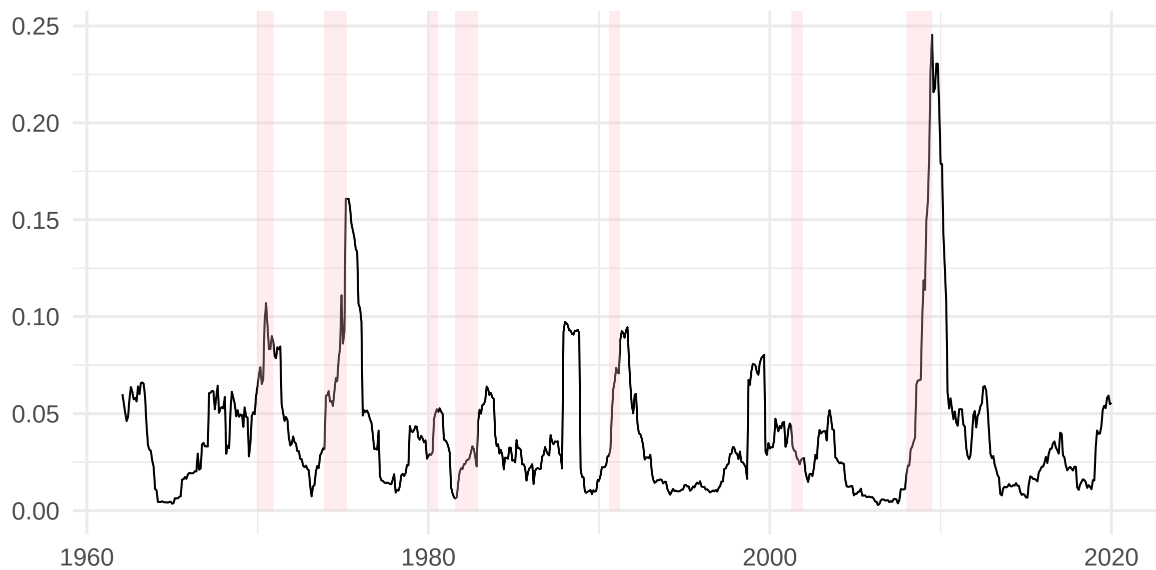
3.4.1 Plot Comparison

Having carefully motivated the empirical motivation behind this section, I proceed with a graphical analysis of the three volatility components, which I construct implying the same methodology described in section 2.2, to compare MKT, IND and FIRM for small and big firms, besides spotting any deviation with respect to the overall pattern analysed in section 3.1.1. In all plots, volatilities are annualised or, in other words, multiplied by 12; in addition, I let the biggest spike go outside of the graph, to highlight the micro - dynamics of each series. Finally, I added to the plots the NBER recession, which are shaded in pink. In addition, I added an MA(12) plot, to more clearly highlight the dynamic of the three aggregate volatility components. The plots for the market volatility are exhibited in Figure 14 and 15; in particular, the former shows the pattern of MKT for small firms, while the latter depicts the latter for the

Figure 14: Market Volatility - Small Firms



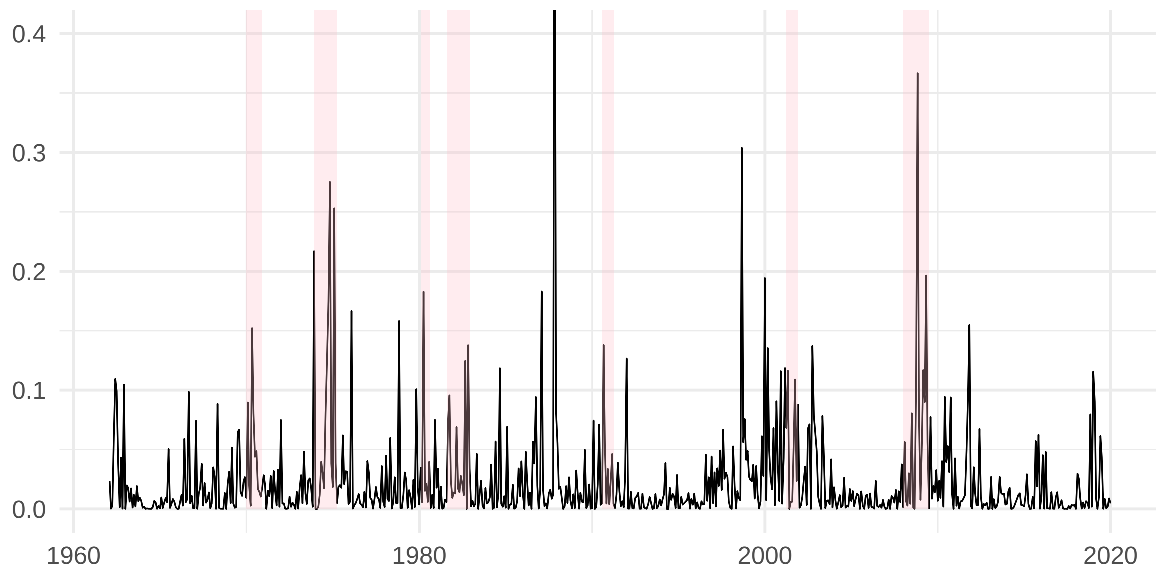
(a) Market Volatility



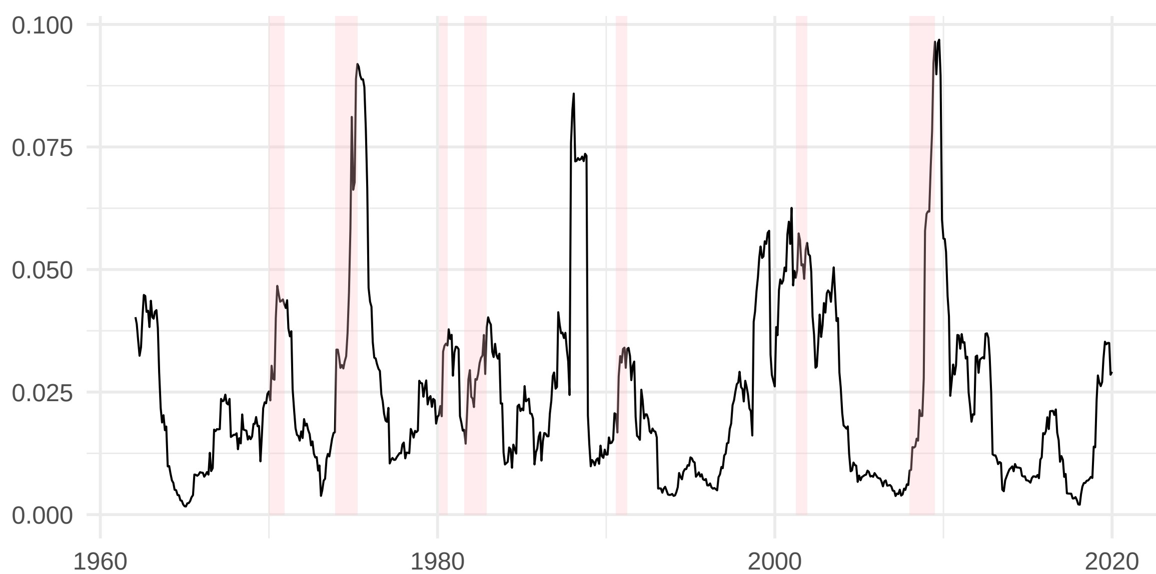
(b) Moving Average of order 12

This Figure shows the pattern of annualised market volatility for small firms, calculated through equation (15). In panel (a), the pattern of the series is displayed, while panel (b) exhibit an MA(12). The highest value of the series in panel (a) are cut out of the graph, to highlight the micro - dynamic of the former.

Figure 15: Market Volatility - Big Firms



(a) Market Volatility



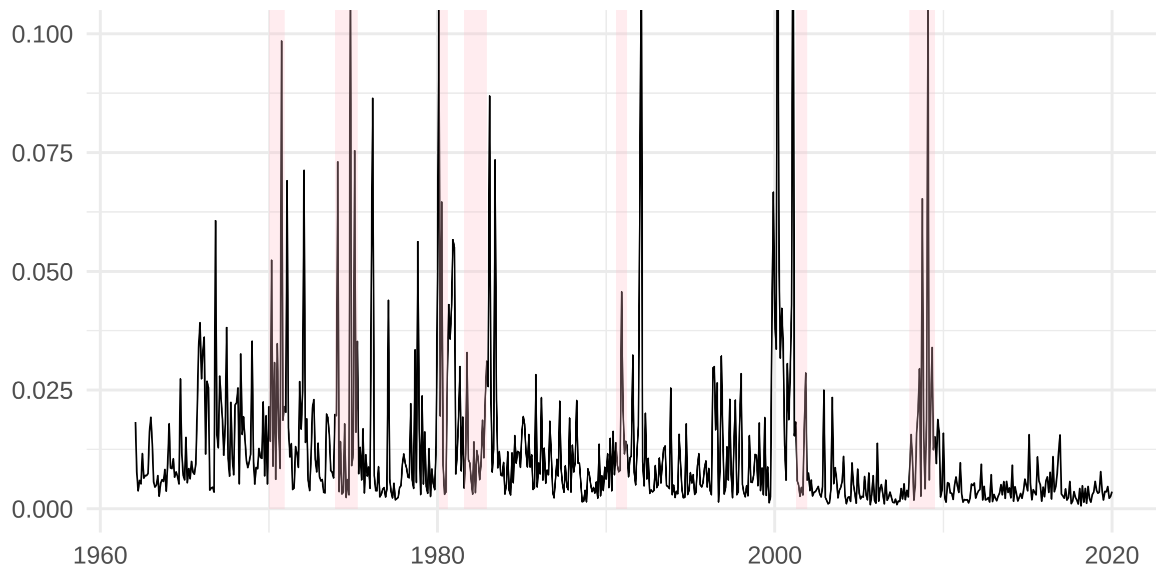
(b) Moving Average of order 12

This Figure shows the pattern of annualised market volatility for big firms, calculated through equation (15). In panel (a), the pattern of the series is displayed, while panel (b) exhibit an MA(12). The highest value of the series in panel (a) are cut out of the graph, to highlight the micro - dynamic of the former.

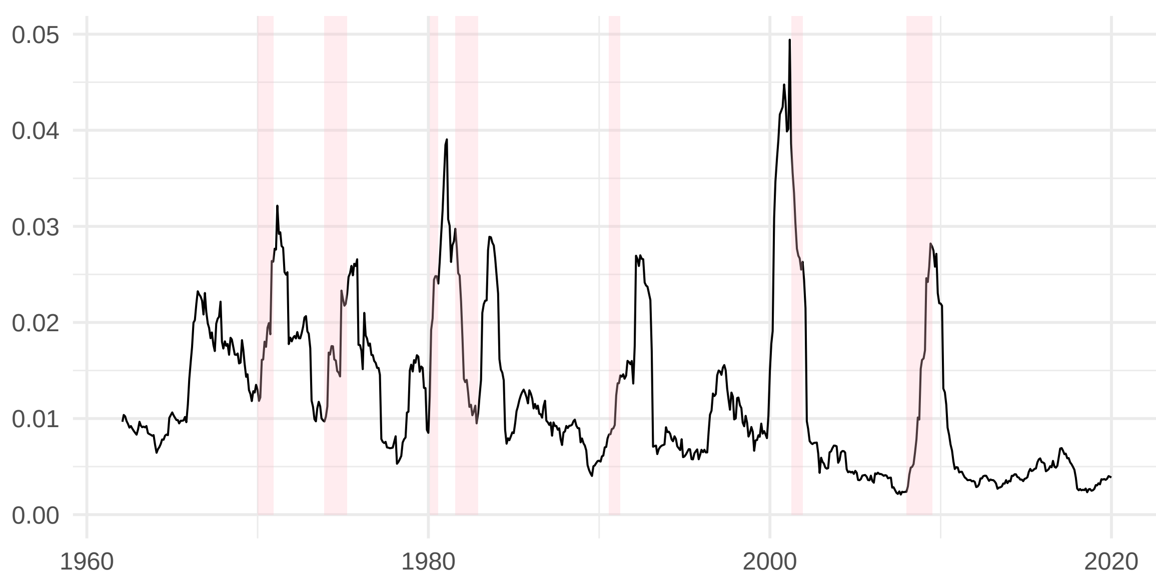
second sub - sample. It is immediately clear the two plots reveal several differences, even though, as we will shortly discover, MKT is going to be the series with less diversities among the three.
Indeed, the two graph seem to co - move almost perfectly, but the magnitude of the spikes for small firms is significantly higher. In addition, even though not perceived by the plots, the impact the various economic downturns had on the two series differs: the peak reached by MKT in Figure 14 is identified on January 1975 with a value of 0.8654, followed by October 1987, where the market volatility component reached 0.8495. On the other hand, big firms' MKT registered its maximum value on October 1987, 0.6361, followed by the 2008, with a value of 0.3666. These graphs suggest that, broadly speaking, the sub - sample of small firms reacts more deeply to economic recession and has in general a higher volatility with respect to big firms. Moreover, it looks like the market volatility for big firms follows very closely, both in terms of pattern and magnitude, the overall MKT graph exhibited in Figure 1, following practically the same impact order for the spikes registered, and with almost identical value for the two biggest peaks.

The scenario is deeply distant when switching to industry - specific volatility, shown in Figure 16 and 17: the upper and lower plot looks significantly different, especially when considering the left part of the two. Indeed, Figure 16, exhibiting IND for small firms is dramatically more volatile than the lower plot, recording massive spikes especially between the 1970 and the early 80s. Moreover, small firms seem to be more sensitive to both the early 90s recession and to the 2008 financial crisis with respect to Figure 17; nevertheless, the peak is reached in both during the early 2000s recession, where the two series reached value of, respectively, 0.1450 and 0.2697. Even in this case, Figure 17 is almost identical to Figure 2.
In addition, while big firms' IND looks pretty flat throughout the horizon considered, the series shows a slight decreasing trend for small firms, steeper in the first part of the

Figure 16: Industry Volatility - Small Firms



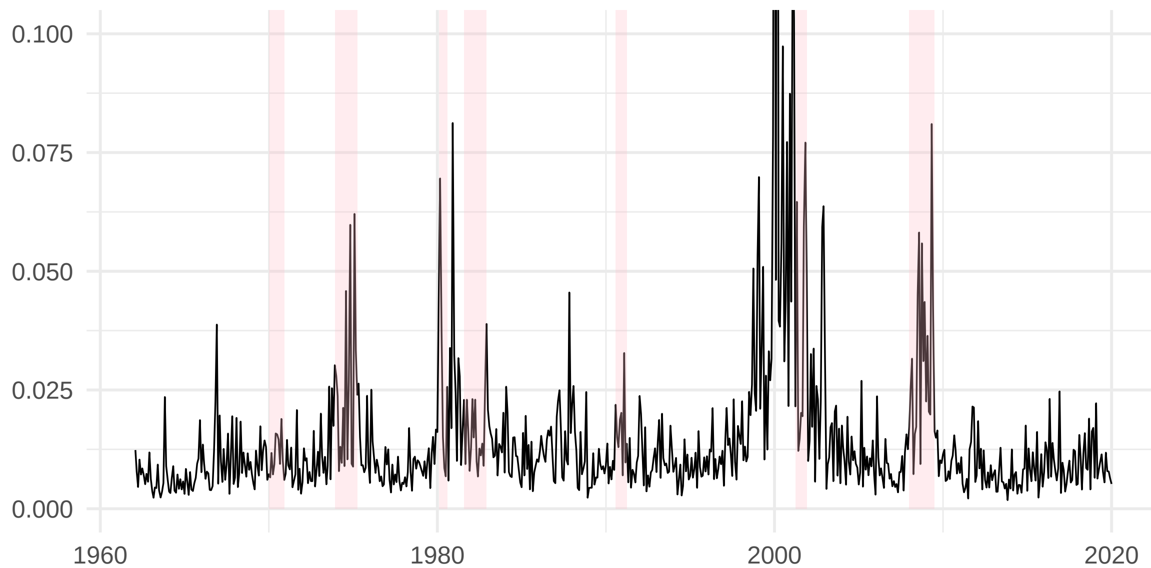
(a) Industry Volatility



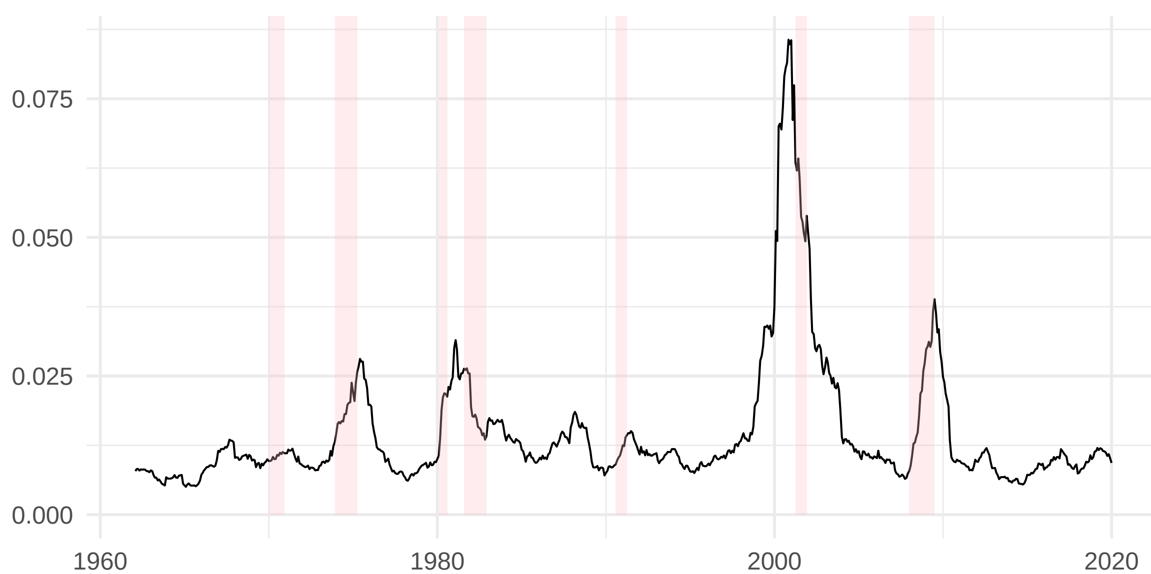
(b) Moving average of order 12

Notes: This Figure displays the pattern of annualised industry volatility for small firms, calculated through equation(17). In panel (a), the pattern of the series is shown, while panel (b) exhibit an MA(12). The highest value of the series in panel (a) are cut out of the graph, to highlight the micro - dynamic of the former.

Figure 17: Industry Volatility - Big Firms



(a) Industry Volatility



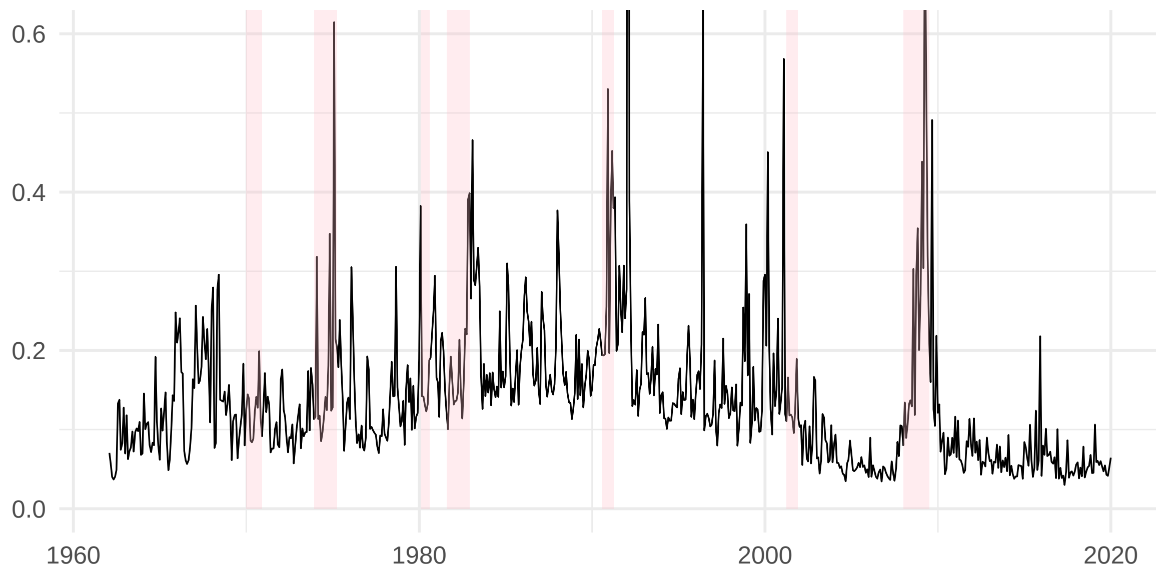
(b) Moving average of order 12

Notes: This Figure displays the pattern of annualised industry volatility for big firms, calculated through equation(17). In panel (a), the pattern of the series is shown, while panel (b) exhibit an MA(12). The highest value of the series in panel (a) are cut out of the graph, to highlight the micro - dynamic of the former.

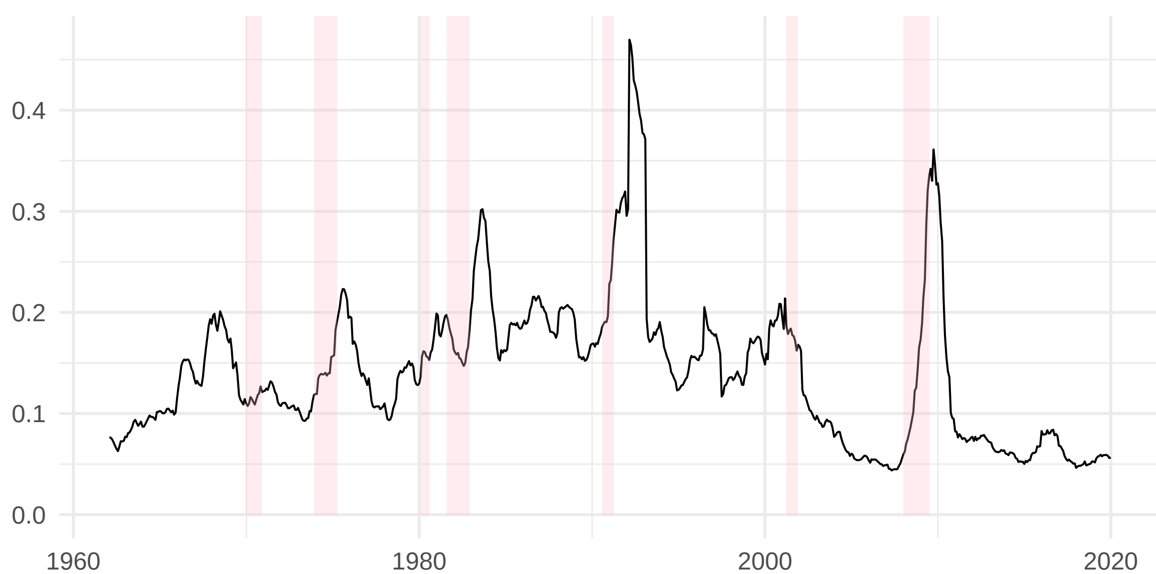
plot.
On the other hand, MKT and IND for small firms looks more similar to each other than in both the complete sample and the big firms sub - sample, suggesting a higher correlation between the former two.

Even looking at figure 18 and 19, we observe several differences in the firm - specific volatilities for the two sub - sample. Similarly to what observed for IND, Figure 18 has registered far more volatility in the first part of the graph, right before the early 2000s; in addition, it seems that Figure 18 has a higher mean in the first 40 years, which rapidly decreases after the 2002 recession. On the other hand, the plot exhibiting the firm - specific volatility for big firms looks significantly more stable than the other, registering a modest spike in 1975. After that, it displays its peak in February 2000, with a value of 0.9723. Instead, switching the Figure 18, we see there is no remarkable increase in that same period; conversely, small firms highest value for firm volatility is found in January 1992, where it reached an amount of over 2. Moreover, the financial crisis visibly had a more deep impact on the latter, while the early 2000 recession did not cause any spike. Even in this case, Figure 19 looks definitely more similar to Figure 3 than the plot for small firms; it also reports the same big spike in January 2019 that, as underlined in the preliminary analysis, is probably accountable to the weakening in the global expansion and high uncertainty.
For what concerns any deviation from the initial pattern for the two FIRM series, we observe that in both plots, the two components underwent some visible changes, and both seems to take place in parallel to the early 2000s downturn. In particular, as exhibited in Figure 19, firm - specific volatility for big firms shows a firm positive trend until the spikes cluster registered in the 2000s; on the other hand, despite revealing a modest upward slope, Figure 18, displays FIRM for the small market capitalization sub - sample looks more flat than the lower part graph. Moreover, it seems like the series experienced a reversal in trend in the early 90s, approximately ten years before the Dot

Figure 18: Firm Volatility - Small Firms



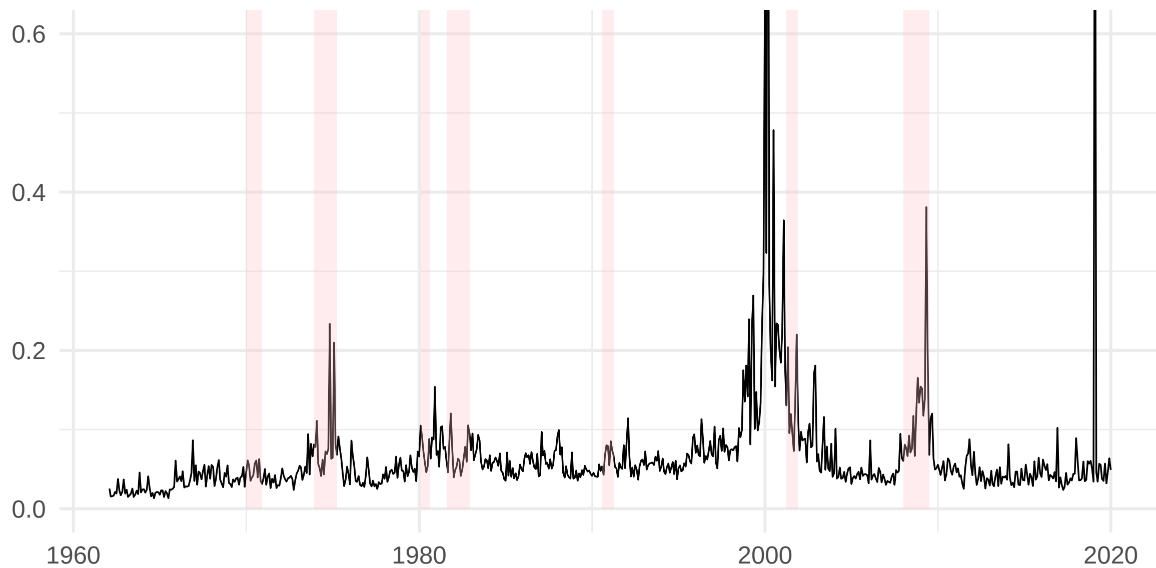
(a) Firm Volatility



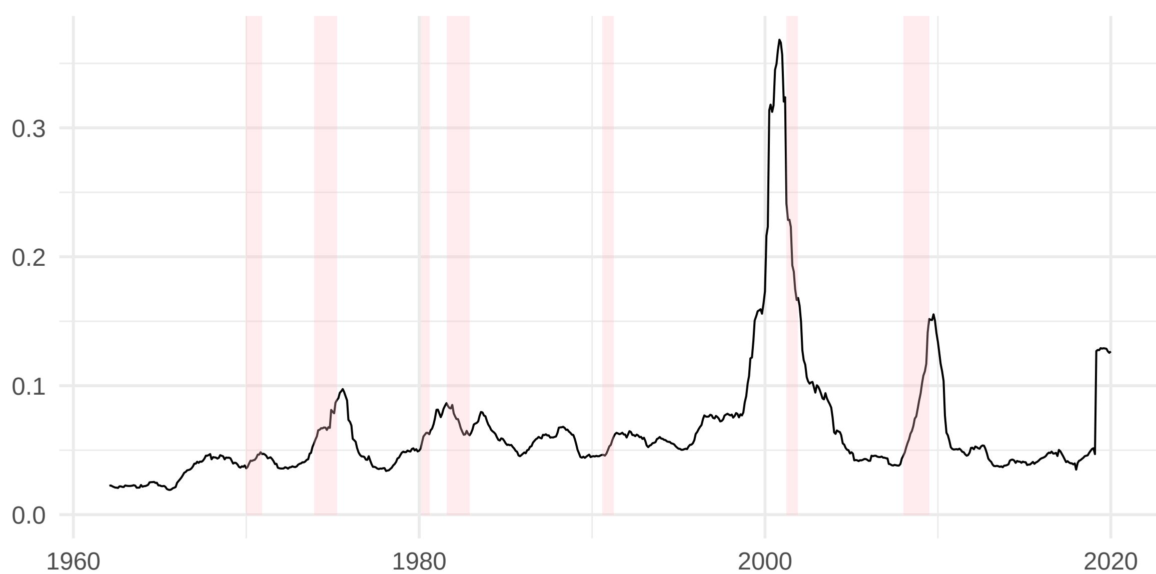
(b) Firm volatility for big firms

Notes: This Figure shows the pattern of annualised industry volatility for small firms, calculated through equation(19) and (20). In panel (a), the pattern of the series is displayed, while panel (b) exhibit an MA(12). The highest value of the series in panel (a) are cut out of the graph, to highlight the micro - dynamic of the former.

Figure 19: Firm Volatility - Big Firms



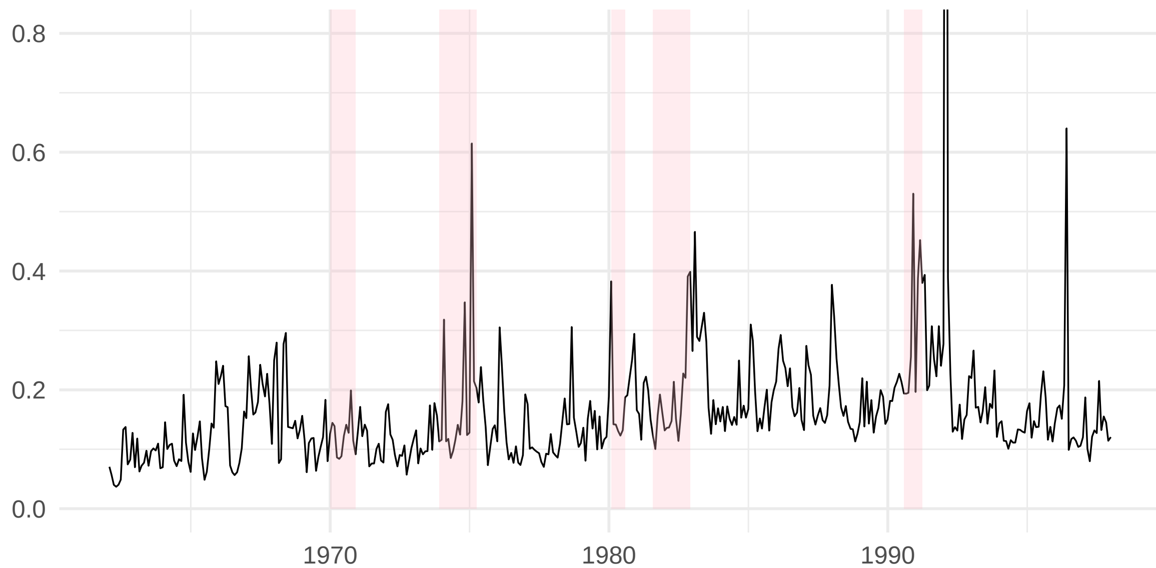
(a) Firm Volatility



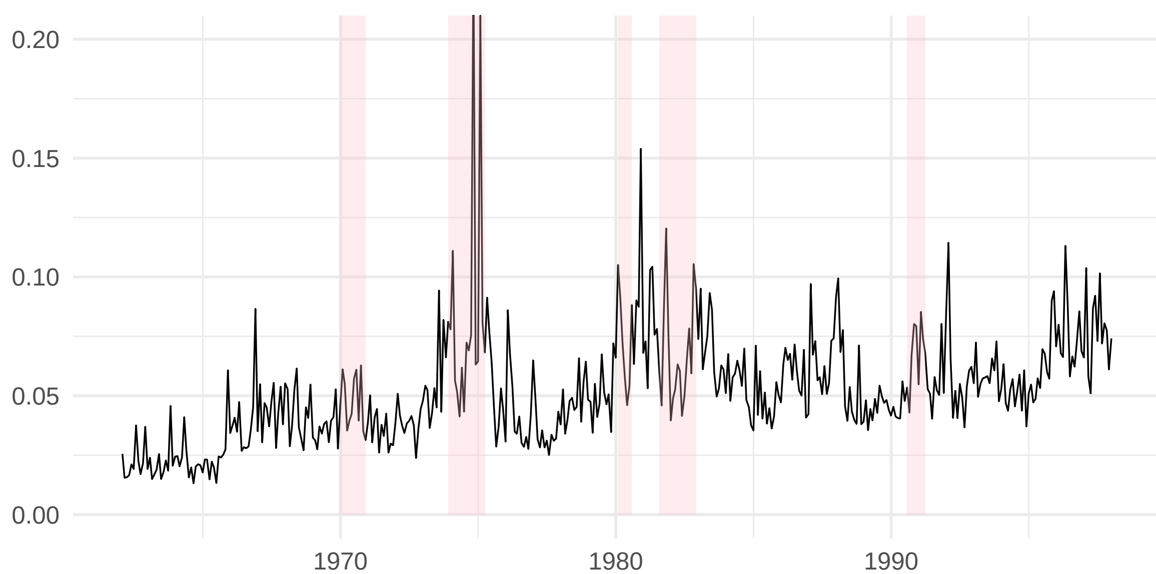
(b) Moving Average of order 12

Notes: This Figure shows the pattern of annualised industry volatility for big firms, calculated through equation(19) and (20). In panel (a), the pattern of the series is displayed, while panel (b) exhibit an MA(12). The highest value of the series in panel (a) are cut out of the graph, to highlight the micro - dynamic of the former.

Figure 20: Firm Volatility - Restricted Sample



(a) Firm volatility for small firms



(b) Firm volatility for big firms

Notes: This Figure shows the pattern of annualised firm volatility (in the restricted sample) for small firms in panel a, and for big firms in panel b. The highest value of the series are cut out of the graph, to highlight the micro - dynamic of the former.

- com bubble, that had the effect of steepening it.

To sum up, the sub - sample containing the small firms group is more sensitive to economic downturns for all the three series; in addition, their values are, in general, higher than the ones registered for big firms. On the other hand, MKT, IND and FIRM for big firms are way more stable than the ones constructed for the other sub - sample, and are remarkably similar to the ones plotted for the whole sample. It is noticeable that, for both sub - sample, the three volatility components seem to co - move, registering increases for approximately the same periods in time. Lastly, as highlighted in the previous section, all three series increase with recessions.

3.4.2 Serial Correlation and Unit Root Testing

Table 8: Autocorrelation Structure

	Small Firms			**Big Firms**		
	MKT	IND	FIRM	MKT	IND	FIRM
ρ_1	0.1721	0.3057	0.3945	0.1525	0.4184	0.4216
ρ_2	0.0962	0.2199	0.3146	0.1037	0.5158	0.4521
ρ_3	0.1856	0.2337	0.3029	0.1117	0.3863	0.3756
ρ_4	0.1266	0.2065	0.2550	0.0817	0.3302	0.3908
ρ_5	0.1626	0.0960	0.2419	0.0762	0.2833	0.2776

Notes: This table displays the serial correlation coefficients for market, industry and firm - level volatility components, from the first to the firth lag. I stored the results for both big and small firms. I obtained the coefficient by running the ACF of the three series and storing its output.

In this subsection, I will inspect the serial correlation of the three series for both sub - sample, to highlight any differences between them or similarities with the whole sample; Subsequently, I will conduct an ADF test, to discard the possibility of a unit root in the series. The serial correlation structure is displayed in Table 8; there is no significant difference between the left and right - hand side of the table, merely a slightly higher

Table 9: Unit Root Testing

	Small Firms			**Big Firms**		
	MKT	IND	FIRM	MKT	IND	FIRM
Simple Model						
T-Statistics	-12.647	-9.466	-6.647	-12.211	-6.479	-6.719
P-value	$< 2e\text{-}16$	$< 2e\text{-}16$	6.1e-11	$< 2e\text{-}16$	1.8e-10	3.8e-11
Constant						
T-Statistics	-15.786	-13.464	-11.946	-15.759	-9.132	-9.999
P-value	$< 2e\text{-}16$	$< 2e\text{-}16$	$< 2e\text{-}16$	$< 2e\text{-}16$	$< 2e\text{-}16$	$< 2e\text{-}16$
Trend & Constant						
T-Statistics	-15.795	-14.074	-12.189	-15.751	-9.172	-10.197
P-value	$< 2e\text{-}16$	$< 2e\text{-}16$	$< 2e\text{-}16$	$< 2e\text{-}16$	$< 2e\text{-}16$	$< 2e\text{-}16$

Notes: This table describes the results of an Augmented Dickey-Fuller test for the Market, industry and firm volatilities series of the two sub - samples in three variations: A simple model, one with a drift and lastly,a model with both drift and trend. The 1% critical values are: -2.58 for the simple model; -3.43 the model with only the drift and -3.96 for the model with both a drift and a trend.

Like in the previous section, I chose the number of lags to include in the model by the Bayesian Information Criterion; for each model, the BIC included 1 lag.

coefficients in IND and FIRM for small companies, while market volatility autocorrelation is lightly lower. As expected, the right - hand side if the table shows very similar results with respect to Table 1. Nonetheless, considering the rather high serial correlation revealed by IND and FIRM, running and Augmented Dickey - Fuller test seems necessary. Following the same approach employed in section 3.3, I ran the test for three models, to account for every feasible scenario and exclude the possibility that the series contain a unit root.

As highlighted in Table 9, where the results of the tests are stored, in all models, for all the series, the null hypothesis of a unit root is strongly rejected, with all p-values very close to zero. In particular, the complete model, with the inclusion of both a trend and a constant, seems to be the one rejecting more strongly the I(1) hypothesis for small firms; the same applies to the left - hand panel of Table 9, with the exception of MKT, that displays an higher t - statistics for the model with just the constant.

As in the whole sample, having proved that our volatility measures are stationary for all firms' size, we can proceed with the analysis, by treating the variables in level rather than in differences.

3.4.3 Descriptive Statistics and Variance Decomposition

In this section of the thesis, I will describe each of the three series for the two groups, in terms of their descriptive statistics and the impact they have on the total variance, to assess the importance each of the component has for small and big companies in my sample and compare the results.

In Table 10, I stored a summary of the descriptive statistics for MKT, IND and FIRM; it is immediately clear that the two group, as anticipated in section 4.1, apart from sharing the ranking in terms of magnitude for each entry, show significant differences. As a matter of fact, each element of MKT and FIRM reported in the left - hand part of the table is remarkably higher, especially for the latter, whose statistics are almost 2 times bigger with respect to the big firms group. The statistics for industry volatility are, instead, smaller with respect to big firms; once again, the latter presents numbers closer, almost identical, to the whole sample statistics.

Table 10: Summary Statistics

	Small Firms			**Big Firms**		
	MKT	IND	FIRM	MKT	IND	FIRM
Mean*100	3.8622	1.2133	13.9837	2.4166	1.4574	6.5214
Std.Dev.*100	7.8746	1.6685	12.3250	4.5519	1.8435	7.5248
Std.Dev. Detrended*100	7.8712	1.6306	12.1944	4.5519	1.8367	7.4184
Min	9.2e-07	0.0006	0.0301	3.3e-11	0.0018	0.0132
Max	8.7e-01	0.1450	2.3981	6.4e-01	0.2698	1.1668

Notes: This table shows a summary of the main descriptive statistics for market, industry and firm - specific volatility, for both small and big firms. Namely, I report their mean and standard deviation, multiplied by a hundred and their minimum and maximum values. In addition, I add the standard deviation for the three detrended series, to highlight any significant difference with the one obtained from raw data.

Focusing on the comparison within the three series, firm - specific volatility component confirms itself as the component with the highest mean, while IND has the more modest one. Similarly, FIRM and MKT are, respectively, the series with the highest standard deviation, both for raw and detrended data; for what concerns the standard deviation for the detrended series, it shows a negligible deviation from the one for raw data, slightly more evident for FIRM. Finally, market volatility component keeps being the one with the lowest minimum value, followed by IND, and the second highest maximum value.

These results are an additional proof to the theory that small firms are more sensitive, with the outcome of higher standard deviation and mean in the volatility components, especially for FIRM. This discrepancy in the latter, suggests that this augmented sensitivity is particularly enhanced for the above - mentioned component.

Next, I studied the influence each of three volatility components has on the total volatility of a stock; to do so, I decomposed both mean and variance of the latter and reported the output of this procedure in Table 11.[8] For what concerns the mean decomposition, firm - specific volatility has, once again, the biggest influence on total volatility; it accounts for almost 63 percent of it for big firms and more than 0.73 for small firms.

Table 11: Volatility Decomposition

	Small Firms			**Big Firms**		
	MKT	IND	FIRM	MKT	IND	FIRM
Mean	0.20264	0.063661	0.73370	0.23247	0.14020	0.6273
Variance						
MKT	0.20819	0.023927	0.17129	0.15896	0.05025	0.1613
IND		0.009346	0.07724		0.02607	0.1691
FIRM			0.51000			0.4343

Notes: This table displays the output of the total volatility mean and standard deviation decomposition for small firms on the right and big firms on the left. The procedure employed to construct and decompose total volatility is the same described in the notes of table 4.

[8]Contrary to Table 4, I did not report the decomposition dividing for different time horizons; the main reason I skipped it is that, in this section, we are mainly interested in the differences between the two groups constructed on the basis of size of companies, rather than deviation stemmed over time. As a consequence, I found replicating the exact same step of Table 4 redundant.

Subsequently, MKT is the second series in terms of its impact on total volatility mean, with an impact of approximately 20 and 23 percent for small and big firms, respectively. Finally, industry - specific components has the more modest influence on total volatility, slightly bigger for big firms, where it account for 14 percent of total volatility mean; conversely, its impact in the left - hand side of the table is slightly higher than 0.06.
Switching to the lower part of the table, we have the decomposition in terms of variance, divided in variance and covariance terms; even in this context, FIRM dominates, with a total influence of approximately 76 percent for both the small and big firms sub - sample, adding the covariance with IND and MKT. Afterwards, market volatility is the second series in terms of variance's impact; indeed, its variance accounts for more than 0.20 of total variation for small firms and almost 16 percent for big ones. Lastly, IND has, once again, a particularly small influence, namely of 0.11 for the left side of the table and almost 25 percent for big companies, the majority of which is due to the covariance terms with the other two series.

The results obtained through this last analysis highlight a finding already hypothesised in the previous subsection: firms with a relatively small market capitalization show a distinct increased sensitivity in firm - specific volatility that, as a consequence, is the most important component of the total variation of a stock return, more important than for both big firms or in the whole sample.

3.4.4 Correlation and Granger - Causality Test

Having analysed the series' moments and descriptive statistics, and having assessed the impact each of them has on total variance, this section will focus on uncovering the relationship between the series, with the aim of shedding light on any difference coming from the size of the firms in my sample.
First of all, I obtain the correlation between them for both small and big firms, and stored the results in Table 12; it is immediately clear that the relationship among the

Table 12: Correlation Matrix

	Small Firms			Big Firms		
	MKT	IND	FIRM	MKT	IND	FIRM
MKT	1.0000	0.2712	0.2628	1.0000	0.3904	0.3070
IND		1.0000	0.5594		1.0000	0.7945
FIRM			1.0000			1.0000

Notes: This table displays the correlation among market, industry and idiosyncratic - level volatility, for both small and big firms.

three components is quite weaker for small firms in all entries, with highest value registered for the correlation between IND and FIRM, being approximately 55 percent. Conversely, the same correlation in the right - side part of the table is almost 0.80, definitely higher than the former. Nevertheless, this is the more remarkable difference; as a matter of fact, the other entries, despite showing a stronger relationship, are not so distant from the small firms group results. In any case, the hierarchy is intact: in both sub - sample, FIRM and IND have the stronger correlation, while MKT and FIRM are the series with the lowest correlation among the three. Again, the big firms group output is not distant from the one displayed whole sample.

Subsequently, I analyse the Lead - Lag relationships among the three series for both sub - samples through a Granger - causality test; to run this test, I construct several VAR(p) model using the detrended series. The output of the test is exhibited in Table 13.[9] It is easily seen that the right and left - hand results of test do not significantly

[9]As for Table 7, I decided to not downweigh the series, as the peak each of them experienced is different.

Table 13: Granger - Causality Test

	Small Firm			Big Firms		
	MKT	IND	FIRM	MKT	IND	FIRM
MKT						
F - Statistics		3.3023	3.3560		2.0372	3.831
P - value		0.00572	0.0051		0.0323	4.2e-03
# Lags		5	5		9	4
IND						
F - Statistics	3.0989		2.8556	2.30221		6.596
P - value	0.0087		0.0024	0.0145		2.9e-09
# Lags	5		9	9		9
FIRM						
F - Statistics	0.4853	0.6078		1.3503	1.5407	
P - value	0.7875	0.7912		0.2491	0.1285	
# Lags	5	9		4	9	

Notes: This table reports the output of a Granger - Causality on a VAR(p) composed by the three detrended series. The dependent variable of the VAR is the one in the column, while the independent one is on the row of the table. The number of Lags included in the VAR are chosen by the AIC.

differ, not even from the whole sample output displayed in the previous section, underlining the strength of these results. As a matter of fact, market volatility shows a strong forecasting power on both IND and FIRM, with a remarkably lower p-value for FIRM in the right-hand side of the table. The very same applies to industry volatility, that appears to Granger - cause MKT and FIRM, with a higher statistics for the latter in the big firms group. On the other hand, firm volatility component does not have any forecasting power for any of the other two series; in particular, the p - value is particularly high for the small firms sub - sample.

The output highlighted throughout this section serves as additional evidence and enhance the credibility of the theory suggested in 4.1: small firms sample is more reactive to changes in market conditions and, as a consequence, show a highest and more volatile volatility for both MKT and, in an amplified manner, for firm - specific variation. Nevertheless, it looks like each series accounts for a little part of the respective overall variance component; as a matter of fact, it looks like MKT, IND and FIRM obtained through the big firms group are the main driver of the whole sample components obtained in section 3.
This is a major finding, as it could mean the pattern followed by the three series, including FIRM trend reversal and path evolution, may be mainly caused by volatility in companies with a high market capitalization.

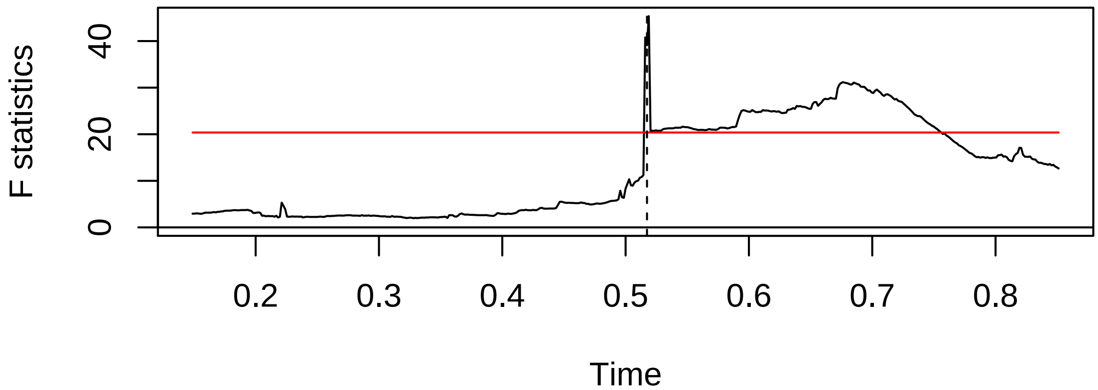
3.5 Structural Break test

Considering the argument mentioned above, and in the light of the findings reported in section 3.5, I decided to run, similarly to what I did for the overall sample, a number of tests to verify the presence of structural break in the firm - specific volatility for both size groups. The main aim of this section is to check whether one of the two groups' volatility component presents the same characteristics discovered for FIRM in the previous part of the thesis, to support or dismiss in a more analytical manner the thesis established so far.

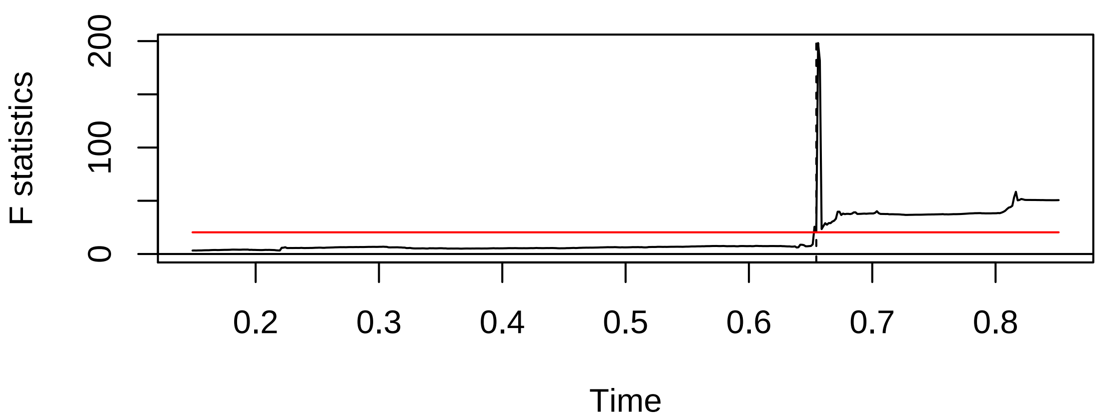
Following the same order defined in 3.2, I start by running a SupF test on both series; even in this case, after a study of the ACF and PACF of the two FIRM volatility, I determine both series are appropriately described by an AR(3) process, so I run the test on the above-mentioned model in both cases. A graphical output of the test is shown in Figure 21; as we can see, both series exhibit a structural break at the one percent confidence interval, but at different points in time. Specifically, Figure 21 (a), depicting the SupF test for small firms indicates the presence of a structural break in mid - 1992; this is perfectly in line with the analysis carried out in section 4.1, where I highlighted a reversal in the trend shown from that year by the series, apparently exacerbated during the early 2000s. In addition, running the test gave a F-statistics of over 45 and a p - value tending to zero.
Conversely, the plot for big firms paints a different picture: unsurprisingly, Figure 21 (b) is easily comparable to the SupF run in the previous section, both in terms of pattern and of breakpoint. As a matter of fact, the point detected by the test is exactly the same one found in Figure 6 (b), namely on March 2000. The test rejects even more strongly than the previous one the null hypothesis of model stability over time, with an f - statistics of almost 200 and a very small p - value.
Similarly to the structural break analysis carried out in the previous section, the results are robust to different methodologies, one of which reported in the appendix.

Interestingly, all the results obtained throughout this section, both graphical, descriptive and analytical, strongly point at one argumentation: volatility of firms with a higher market capitalization seem to drive the overall volatility of the stock market, which follows almost identically its pattern for all the three components analysed along the entire time horizon taken into consideration. As a consequence, the vast majority of the events and changes underwent by MKT, IND and FIRM in the complete sam-

Figure 21: SupF Test Plot - Small and Big Firms



(a) Small Firms Graph



(b) Big Firms Graph

Notes: This figure shows a graphical representation of the SupF test employed on an AR(3) for small firms in panel (a) and big firms in panel (b). The red lines denote the 1 percent confidence interval of the test, while the horizontal dotted lines highlight the point where the structural break took place.

ple are largely caused by the big firms sub - group relative variance components; this means that the trend registered by the idiosyncratic variation element, its reversal and, as supported by both the SupF and Chow test, the structural change triggered by the Dot - Com bubble are entirely accountable to big firms.
Conversely, despite being fairly more volatile and, in general, with higher values with respect to the former, the three volatility components for small firms appear to have a much more modest impact on the overall sample, almost unperceived when comparing the different plots, except for some spike attributable to them. As an additional proof, the firm - specific volatility for the above - mentioned sub - sample does not appear to experience the same structural changes discovered for the overall FIRM; indeed, it only registers a breakpoint, coinciding with its reversal in trend, in 1992 which is not observable in the complete sample FIRM volatility.

3.6 Aggregate Volatility Decomposition

Having confirmed the heavy influence the big firms sub - sample has on the overall aggregate volatility measures, in this section I carry on my analysis by trying to assess the reason leading to such polarized results.

I start by computing monthly weights for each sample; specifically, in line with the weighting type chosen to obtain MKT, IND and FIRM in section 2.2, I calculate the weight for small companies in each time period as:

$$w_{st} = \sum_{k \in s} Mktc_{kt} / \sum_{n} Mktc_{nt} \tag{36}$$

where $Mktc_{kt}$ is the market capitalization of firm k, belonging to the small firms group s, while $Mktc_{nt}$ is the market capitalization of firm n, in the complete sample. Similarly, the weight for big companies in each period is computed as follows:

$$\begin{aligned} w_{bt} &= \sum_{z \in b} Mktc_{zt} / \sum_{n} Mktc_{nt} \\ &= 1 - w_{st} \end{aligned} \tag{37}$$

with $Mktc_{zt}$ is the market capitalization of firm z, belonging to the big firms group b. The output of this analysis depicts an interesting yet not surprising framework: big firms' weight is very high throughout all the time period, with values ranging from 0.9474, in January 1962, to slightly more than 0.99 in December 2000. Conversely, the weight for small firms is particularly small and decreased over time.
The plots depicting the dynamics of the weights over time can be found in the Appendix.

Subsequently, I obtain the the total aggregate volatility for the complete sample, decomposing it in terms of the two sub - sample, in the following way:

$$\sigma_t^2 = w_{st}^2\sigma_{st}^2 + w_{bt}^2\sigma_{bt}^2 + 2w_{st}w_{bt}cov(s,b) \tag{38}$$

where $\sigma_{st}^2 = MKT_{st} + IND_{st} + FIRM_{st}$ and $\sigma_{bt}^2 = MKT_{bt} + IND_{bt} + FIRM_{bt}$
This way, by dividing each term of equation (28) by σ_t^2, I obtain the fraction each of them account for, the impact each term has on the complete sample total aggregate volatility. The results are in line with what we found when computing the two sub - samples' weight; indeed, the term representing the variance of big firms accounts for the biggest fraction of total aggregate volatility through the entire period. Its maximum impact is registered in January 2019, of almost one, while the minimum impact is found on February 1962, when big firms' variance accounted for 95 percent of σ_t^2. The covariance term is the second in terms of its influence of aggregate volatility, whose peak is registered in March 1962 with a value of approximately 0.42 percent. Lastly, the first term, referring to the small firms group is the one with the lowest impact, whose maximum is reached in May 1968 with a value slightly higher than 1 percent.

The bottom line of this analysis is that, as suggested by the previous analysis, big firms sub - sample has an enormous influence on overall volatility. More precisely, it accounts for around 95 percent (or more) of it throughout the entire horizon I consider; On the other hand, small companies have an almost negligible impact on the complete sample volatility.

Moreover, I suggest that what we discovered by running this decomposition heavily comes from the weight assigned to two groups, computed in (27) and (28). We employed similar weights when constructing MKT, IND and FIRM; as a consequence, the particularly high weights assigned to big companies are likely to be the cause of the significant influence the latter have on the three volatility measures for the overall sample.

4 Conclusions

In this thesis, I started by replicating the study of Campbell, Lettau, Malkiel and Xu (2001): I obtained the three volatility measures through a simple and efficient methodology employed in the above - mentioned paper, that allowed me to ignore the covariance components and, at the same time, avoiding to estimate any firm or industry - related beta coefficient. Then, I extended their study in several dimensions: First, I updated their analysis for the last two decades; Second, I repeated the analysis, obtaining the three volatility measures from daily data; Third, I run tests for a structural break; Finally, I replicated the previous analysis for two sub - sample, dividing firms in terms of market capitalization.

My main findings are the following. First of all, the results of this thesis revealed that the trend in the firm - specific volatility, discovered by Campbell et al., persists until the early 2000s and, after a cluster of peaks registered around that period, it briefly reverses. Finally, the series stabilizes, showing no trend until December 2019. Market and industry volatility components keep a stable pattern throughout the entire time horizon considered, displaying no significant deviation in the updated sample period.

Next, I discovered that the change in firm - level volatility is mainly caused by a structural change in mean with breakpoint on March 2000, right before the Dot - Com bubble and in the middle of the spike this volatility measure experienced.

Afterwards, when using daily data, I discovered that the overall pattern of the three series is maintained, while the structural break test partly discards the argument made above, even though it could be due the the divergence in samples arising from the complication of computation arising from the increasing frequency of data.

Lastly, when repeating the analysis for small and big firms, I found that the former has a low influence on the complete sample volatility measures, has a higher magnitude and it is more volatile. on the other hand, high market capitalization firm stocks' volatility is the main driver of the three series in the complete sample. In addition, the structural break uncovered above is entirely coming from the big firms group.

I additionally found that the main reason why big firms have such a significant influence on the overall aggregate volatility stems from the high weights assigned to this sample, whose lowest value is higher than 94 percent and show an increasing pattern in almost the entire time horizon considered.

I believe that an interesting topic for future studies will be to investigate further developments in idiosyncratic - level component pattern, in particular during the outbreak of the COVID - 19, an event without precedents, that shocked financial markets and the global economy as a whole.

In addition, I expect that broadening the study carried out through this thesis by analysing different sub - sample or investigating whether the finding highlighted applies to different volatility estimates (and to what extent), is particularly compelling. Due to time constraints, I leave these topics to future researches.

5 Appendix

Chow Test

Once confirmed the role the early 2000s recession played on the deviation FIRM had with respect to the pattern it followed from 1962 until the XXI century, I decided to run another kind of test, as a double check of the thesis uncovered with the SupF test. In particular, I run a Chow test; it takes into account a model split in two or more different samples, marked by structural breakpoints. Following this, and allowing for one structural break point as in our case, the model described by the test would look like:

$$y_1 = x_1\beta_1 + \epsilon_1 \tag{39}$$

and

$$y_2 = x_2\beta_2 + \epsilon_2 \tag{40}$$

With the breakpoint as a threshold. Starting from the difference in this two model, under the assumption of the independence between ϵ_1 and ϵ_2, Chow constructed a test with null hypothesis:

$$H_0 : \beta_1 = \beta_2 = \beta \tag{41}$$

Similarly to the SupF, the test uses F-statistics; the. main difference between the two lies in the more severe assumptions that have to be met in order to run the Chow test, namely the exogeneity of the breakpoint considered and the prior knowledge of the latter.

First, I run the test for the complete sample, on the breakpoint found through the SupF test (March 2000) and found that, even in this case, the null hypothesis is strongly rejected by the Chow test, with an F-statistics of more than 43 and a p-value very close to zero.

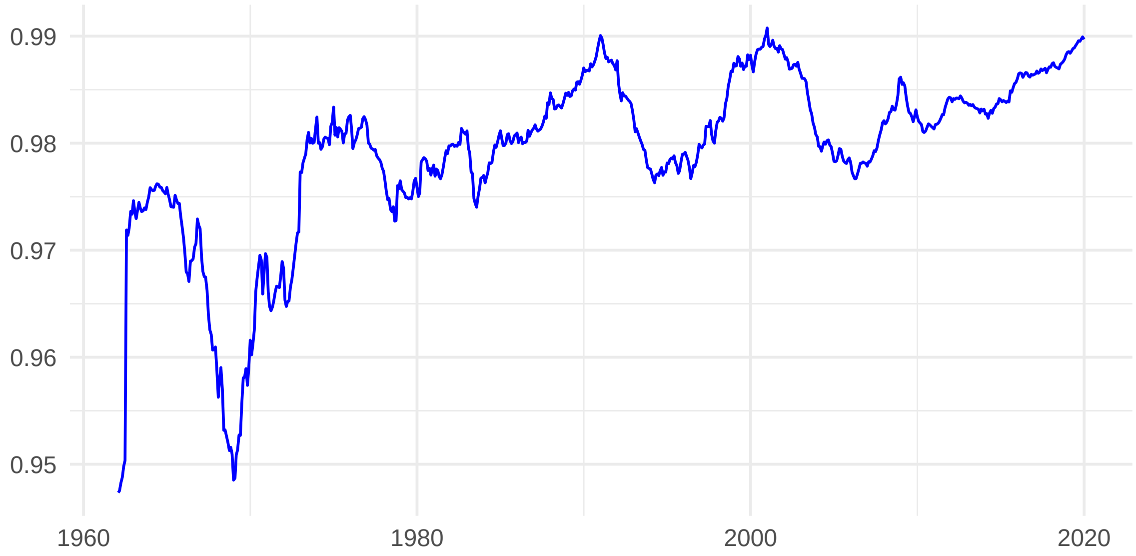
Subsequently, I run a Chow test for the big and small firms series, using the point found by the SupF test and, as expected, it confirmed what we discovered before: both test strongly rejected the model stability hypothesis and confirmed the breakpoints determined by the previous test for both groups. In particular, small firms group test considered the structural break occurred in April 1992 to be significant at percent confidence level, with an F - Statistics of approximately 11 and and p - value of 6e-09; similarly, the big firm group breakpoint proposed was found significant at the one percent confidence level as well, with an F - Statistics of 49.5 and a p - value extremely close to zero.

Small and Big companies Weight - Graphs

Figure 22: Weight - Small Firms

Notes: This Figure exhibit the pattern of the weight for small firms obtained through equation (26).

Figure 23: Weight - Big Firms



Notes: This Figure exhibit the pattern of the weight for big firms obtained through equation (27).

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