Department of Economics and Finance

Chair in Asset Pricing

STATISTICAL ARBITRAGE IN FOREIGN EXCHANGE MARKETS

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Introduction

This work is focused on studying the currency markets understanding its determinants the currency excess returns and the reason behind them ending with the presentation of a statistical arbitrage strategy based on the evidence found and the analysis of what this strategy implies in terms of subjects involved into the trades.

At first we will investigate the different forces that reflects the movements in exchange rates. Indeed, the analysis showed that exchange rates absorb shocks to future interest rates. Central point of this work is going to be the concepts of Covered Interest Rate Parity and Uncovered Interest Rate Parity understanding them and divergences from them. We will define deviations from Uncovered Interest Rate Parity also called foreign currency risk premia and how it contributes to determine the level of exchange rates. The last force we have underlined is deviations from Covered Interest Rate Parity. We will arrive to the conclusion that these three forces all together can determine the level of the exchange rate.

Later, we will focus a lot on understanding the reason behind the presence of currencies excess returns, decomposing them, and trying to motivate those currencies excess returns investigating the existence of some compensation for risk in currency markets. During the work we have tried to understand how risk is priced in currency markets. In the analysis we have shown the evidence from the data about the questions we have just said. In the last chapter we will start by defining what is the meaning of statistical arbitrage and the history behind it. Then, we used all the evidence and theory found to create an arbitrage strategy in currency markets. At the end of this research we will try to find a reason for our findings trying to understand our trading strategy and the flows of funds behind that type of trades.

1. Exchange rate determinants

Currency markets and bond markets are tightly connected. Indeed, exchange rates tend continuously to adjust to ensure that returns are in line with risk for both foreign and domestic bond market investors. If we agree on that relation, we can easily see that exchange rates in currency markets are priced in a very similar way as stocks or other instruments, and they reflect a set of different forces. Exchange rates absorb shocks to future interest rates, and we can think about that as the cash flow component that accrues to an investor who takes a long position in the stock market compared to a long position in the foreign currency. Moreover, exchange rates reflect news about future discount rates and this can be called deviations from uncovered interest rate parity that can be called foreign currency risk premia. Finally, we can add a third force which is deviations from covered interest rate parity coming from convenience yields and balance sheet constraints that financial intermediaries can experience. These three forces that we summarized all together can determine the level of the exchange rate. After, we are going to see that shocks to variation of the discount rate explain a lot of FX variance over the short-term period. Therefore, the concept of uncovered interest rate parity (UIP) and later covered interest rate parity (CIP) will be very useful in understanding better the variations of the discount rate. In this chapter we are going to focus on different models that have the aim to capture and understand changes in exchange rate analyzing different forces that can contribute to that.

1.1 Exchange rates and Short Yields

To understand the relationship between exchange rates and short term interest rates, we can start by looking at the Euler equation based on a foreign investor who wants to take a long position in the US bond market:

$$E_t\left(M_{t+1}^{\#}\,\frac{S_{t+1}}{S_t}\,e^{y_t^{\$}}\right) = 1 \tag{1}$$

$$E_t\left(M_{t+1}^{\#}\,e^{y_t^{\#}}\right) = 1 \tag{2}$$

The long position of the foreign investor in a US treasury will allow him to earn a yield in dollars or otherwise he can invest in his domestic bond market earning a yield equal to y_t^d. With M_{t+1} we are referring to a stochastic discount factor. The exchange rate which appears in the Euler equation is denoted in units of local currency per one unit of foreign currency, then an increase means an appreciation of the domestic currency. We can study these two Euler equations trying to understand the implications for the exchange rates. The assumptions that we are going to make are joint log normality of exchange rates and the discount factor. Lower case symbols on the equations below denote logarithms

$$0 = E_t\left[m_{t+1}^{\#}\right] + \frac{1}{2}var_t\left[m_{t+1}^{\#}\right] + y_t^{\#} \tag{3}$$

$$0 = E_t\left[m_{t+1}^{\#}\right] + \frac{1}{2}var_t\left[m_{t+1}^{\#}\right] + y_t^{\$} + E_t[\Delta S_{t+1}] + \frac{1}{2}var_t[\Delta s_{t+1}] - RP_t \tag{4}$$

$$RP_t = -cov_t(m_{t+1}^{\#}\,, \Delta S_{t+1})$$

We get from equation (4) the mean of the discount factor the conditional variance and a covariance term which can be seen as the risk premium. Therefore, the currency risk premium is equal to minus the covariance of the foreign logarithm of the discount factor and the rate of appreciation or depreciation of the domestic currency.

Setting the logarithm of the currency risk premium equals to $rp_t = RP_t - \frac{1}{2}var_t[\Delta s_{t+1}]$ and combining equation (3) with equation (4) we get:

$$E_t[\Delta s_{t+1}] + (y_t^{\$} - y_t^{\#}) = rp_t = E_t(r^{\$/\#}_{t+1}) \tag{5}$$

Where $E_t\left(r^{\$/\#}_{t+1}\right) = \Delta s_{t+1} + (y_t^{\$} - y_t^{d})$

Combining equation (3) with equation (4) some terms cancel out and the result is in equation (5) which says that the amount of expected appreciation or depreciation of the dollar plus the interest rate differential that a foreign investor can get going long on the dollar[1] and borrowing in its domestic currency, that expression is the logarithm of the currency risk premium. From this point, we can write this expression as a difference equation for s_t and we get:

$$s_t = E_t[s_{t+1}] + (y_t^{\$} - y_t^{d}) - E_t(r^{\$/\#}_{t+1}) \tag{6}$$

Therefore, the dollar exchange rate will reflect the expectation of future exchange rate plus the interest differential between the dollar and the other currency analyzed, minus the currency risk premium. Keeping iterating equation (6) we end up with the fundamental exchange rate valuation equation:

$$s^{\$/d}_t = E_t \sum_{i=0}^{\infty}(y^{\$}_{t+i} - y^{d}_{t+i}) - E_t \sum_{i=1}^{\infty} r^{\$/d}_{t+i} + E_t[\lim_{i\to\infty} s^{\$/d}_{t+i}] \tag{7}$$

The expression tells us that the dollar exchange rate[2] will reflect future interest rate differential minus future currency risk premium plus a sort of noise term[3] which reflects the investor's expectations at time *t* of future exchange rate.

Equation (7) was originally derived by Campbell and Clarida[4] but it is still used to analyze the determinants of the exchange rate. That equation tends to hold very generally. However, we actually didn't make too many assumptions about for example market structure but we have just assumed that the bond investor's Euler equation hold, both for the foreign bond market and for the U.S. bond market and not on Euler equations in other markets. What we are not assuming is that

[1] We are assuming the foreign investor's domestic currency is not the dollar

[2] Exchange rate in terms of foreign currency per dollar, then in this example amount of domestic currency per dollar

[3] The expectation at time t of the long-run log real exchange rate. We assume that this limit exists (which requires, for example, that ex-post interest rates differentials follow a stationary stochastic process with mean zero)

[4] John V. Campbell, Richard H. Clarida, "The Dollar and Real Interest Rates,“ *National Bureau of Economic Research,* (1987)

market are not complete[5], which means that the foreign stochastic discount factor equals the domestic discount factor times the ratio of the exchange rates, this is correct if we assume that markets are complete[6]. In equation (7) there are two main forces. The first one is that the dollar exchange rate at time *t* is going to reflect a cash flow term:

$$E_t \sum_{i=0}^{\infty} \left(y_{t+i}^{\$} - y_{t+i}^{d} \right)$$

Which is the difference in short term interest rates in future periods and the second one which is a discount rate term:

$$E_t \sum_{i=1}^{\infty} r_{t+1}^{\frac{\$}{\#}}$$

$$E_t \left(r_{t+i}^{\frac{\$}{\#}} \right) = E_{t+i} \Delta s_{t+i+1} + (y_{t+i}^{\$} - y_{t+i}^{\#})$$

that reflects the currency risk premium and can be seen as short-term deviations from uncovered interest rate parity[7]. Therefore, we should expect that if uncovered interest rate parity holds, the second term should disappear. After we have defined the fundamental exchange rate valuation equation, we can focus on how interest rates movements can affect the spot exchange rate. If the US short-term interest rate goes up the exchange rate will increase, while an increase in the currency risk premium that a foreign investor will demand for holding domestic currency risk, that will cause a decrease of the spot exchange rate. However, now we switch the position in the domestic currency and U.S. dollar, setting U.S. dollar as our domestic currency and a foreign currency with on average greater interest rate than U.S. such as Canadian dollar, Indian rupia, New Zealand dollar. The new equation is:

$$s_t^{\frac{\#}{\$}} = E_t \sum_{i=0}^{\infty} \left(y_{t+i}^{\#} - y_{t+i}^{\$} \right) - E_t \sum_{i=1}^{\infty} r_{t+i}^{\frac{\#}{\$}} + E_t [\lim_{i \to \infty} s_{t+i}^{\frac{\#}{\$}}] \tag{8}$$

[5] $M_{t+1}^{d} = M_{t+1} \frac{S_{t+1}}{S_t}$

[6] In Economics a complete market or also known as Arrow-Debreu market is a market that satisfies these two conditions

- Negligible transaction costs, therefore the assumption of perfect information
- Exist a price for every asset in whatever state of the world

[7] Uncovered interest rate parity states that the change in foreign exchange rates between two countries is equal to the difference in the interest rates of the two countries

In this perspective, we have a U.S. investor who borrows in U.S. dollar and can invest in a foreign currency. We can think of this equation as if the U.S. interest rate is consistently lower than the foreign interest rate now the exchange rate in order to be stationary the currency risk premium should offset the difference in interest rates. From the data is very easy to see that there are persistent differences in interest rate differentials across developed and emerging market countries[8] and we will see later in this work that it can be explained by the existence of a currency risk premium. In other worlds, what we are saying is that countries with low interest rates seemed to offer their currencies as a safer asset than the currencies of the countries with higher interest rates. Moreover, from a different perspective what we have just said and what we will see later is what actually is behind a currency carry trade[9].

[8] Lustig, H., Roussanov, N., & Verdelhan, A. (2011). Common Risk Factors in Currency Markets. Review of Financial Studies;
Tarek A Hassan & Rui C Mano, 2019. "Forward and Spot Exchange Rates in a Multi-Currency World," The Quarterly Journal of Economics

[9] Currency carry trade is a strategy where the investor goes long on the high yield currency funding the trade with a low yield currency. The trader finds to profit from the difference between interest rates

1.2 Exchange rates and Long Yields

In the previous paragraph, we have seen the relationship between short-term interest rates and exchange rates, now we will focus on understanding what type of relationship exists between long term interest rates and exchange rates. Looking back at what we have said in the previous paragraph and what we are going to see in this one, it is clear that there is a strong connection between currency markets and bond markets, and as we pointed out before exchange rates adjust continually to ensure that returns are in line with risk for both foreign and domestic investors. In order to derive a new fundamental exchange rate valuation equation, we now need to consider long-term interest rates as the cash flow component which before was captured by short term interest rate. In this framework we have three forces that contribute to determine and influence exchange rates that are:

- Long-term interest rates
- Short-term deviations from Uncovered Interest Rate Parity in currency markets
- deviations from Expectations Hypothesis in bond markets

it easy to see that now we have introduced a new component that affects the exchange rates, and it is deviations from Expectations Hypothesis[10] in bond markets. The Expectations Hypothesis can be explained as if investors are risk-neutral in bond markets then the expected excess return on a bond of any maturity should be zero. We can think about it as the equivalent of Uncovered Interest Rate Parity in the currency market. We will analyze this component and its importance for determining a new fundamental exchange rate valuation equation. However, it is important to notice that when exchange rates are stationary meaning that there is no long-term risk in the exchange rate which tends to revert to its mean the second and third component that we presented above cancel out and we arrive at the notion of long-run Uncovered Interest Rate Parity[11].

Before focusing on the derivation of the new fundamental exchange rate valuation equation based on long-term interest rates, we need to analyze bond risk premia. We can start from the definition of holding period return for an investor who wants to invest in the bond market. The holding period

[10] The Expectations Hypothesis states that current and future expected short term interest rates determine long term interest rates, therefore the expected return from an investment in a longer maturity bond should be equal to invest in a sequence of short term bonds. Formally, it can be defined as the return on a long-term bond is equal to the geometric mean of the returns on a series of short-term bonds

$$(1 + i_{(0,n)})^n = (1 + i_{(0,t)})^t \left(1 + i_{(t,t+1)}\right) \ldots . (1 + i_{(n-1,n)})$$

[11] Hanno Lustig & Andreas Stathopoulos & Adrien Verdelhan, 2019. "The Term Structure of Currency Carry Trade Risk Premia," American Economic Review

return can be defined as the return received from holding an asset or portfolio of assets over a period of time, known as the holding period. The logarithm of the holding period return on a bond[12] investment with maturity *T* can be written as:

$$hpr_{t+1}^{T} = p_{t+1}^{T-1} - p_{t}^{T} \tag{9}$$

The expression (9) states that the holding period return is equal to the bond price that we will face in the next period which maturity will be *T-1* minus the price that we see today. However, expression (9) is a difference equation, from which we can derive two useful expressions:

$$p_{t}^{T} = -\sum_{i=0}^{T-1} hpr_{t+i+1}^{T-i} \tag{10}$$

$$y_{t}^{T} = \frac{1}{T}\sum_{i=0}^{T-1} hpr_{t+i+1}^{T-i} \tag{11}$$

The equation (9) can be solved by iterating forwards to express the log price of the bond as minus the sum of all the holding period returns that an investor should earn over the maturity of the bond[13], in this way we get equation (10). Similarly, we can derive an expression for the yield of the bond if we think about the average holding period return over the maturity of the bond, in this way we get equation (11). Having defined these equations we can look at excess returns, defining it as:

$$xr_{t}^{\$,T} = hpr_{t}^{T} - y_{t}^{T} \tag{12}$$

Then it is the difference between the holding period return and the short-term interest rates. Now we can rewrite this expression as the sum of the short-term interest rates equals the long-term interest rates minus the bond risk premium as shown below.

$$Ty_{t}^{\$,T} = \sum_{s=0}^{T} y_{t+s}^{\$} + \sum_{s=1}^{T} xr_{t+s}^{\$,T-s+1} \tag{13}$$

[12] We are considering a Zero Coupon Bond, it is a bond that repays the face value at the time of maturity, without any payments during its life

[13] For example, if we have a 20 years bond we are looking at the holding periods returns during the nineteen years

$$\sum_{s=0}^{T} y_{t+s}^{\$} = T y_t^{\$,T} - \sum_{s=1}^{T} xr_{t+s}^{\$,T-s+1} \tag{14}$$

If we take back the fundamental exchange rate valuation equation (7):

$$s_t^{\frac{\$}{d}} = E_t \sum_{i=0}^{\infty} \left(y_{t+i}^{\$} - y_{t+i}^{d}\right) - E_t \sum_{i=1}^{\infty} r_{t+i}^{\frac{\$}{d}} + E_t [\lim_{i\to\infty} s_{t+i}^{\frac{\$}{d}}] \tag{7}$$

We can see immediately that we can modify and substitute the first term in the equation, and we get a new expression of the log of the exchange rate:

$$s_t^{\frac{\$}{d}} = \lim_{T\to\infty} \left(y_t^{\$,T} - y_t^{d,T}\right) + \lim_{T\to\infty} E_t \sum_{i=1}^{T} (xr_{t+i}^{d,T-i+1} - xr_{t+i}^{\$,T-i+1}) - \lim_{T\to\infty} E_t \sum_{i=1}^{T} xr_{t+i}^{\frac{\$}{d},FX} + E_t [\lim_{i\to\infty} s_{t+i}^{\frac{\$}{d}}] \tag{15}$$

Now the dollar exchange rate will reflect not short-term interest rate differences anymore but the long-term interest rate differences. The first term represents the difference in long-term interest rates, for example, the 10 years US Treasury and the 10 years German bond. The second term represents the difference in bond risk premia and this term can be seen as a factor that captures deviations from the Expectations Hypothesis. The third term represents deviations from short term Uncovered Interest Rate Parity, the same term that we encountered before. Finally, the last term which captures investor's expectations at time *t* of the future exchange rate should converge to the unconditional mean if the exchange rate remains stationary. In equation (15) we derived the fundamental exchange rate valuation equation in terms of long interest rates.

From the previous equation, it is possible to analyze how the different forces can impact the exchange rate. We can say today at time *t* that the dollar will appreciate if long-term interest rates, on the long end of the yield curve, go up today relative to foreign yields. What we have just said is intuitive, because it reflects the cash flow component, it becomes more appealing for foreign investors to invest in a foreign currency if it offers a greater yield then other countries, in this example U.S. dollar, pushing dollar exchange rate up, in our case, we remember the investor borrows in a domestic currency and invest in U.S. Treasury. Moreover, the dollar will also tend to appreciate when the U.S. bond risk premium decreases, and finally, the dollar will appreciate when future dollar foreign currency risk premium[14] decreases which is the force that we have said before. It is very important to summarize the forces to which we have just arrived:

- $\left(y_t^{\$,T} - y_t^{d,T}\right)$ Cash flow component reflecting opportunities for investors to enter into a carry trade
- $E_t \sum_{i=1}^{T} (xr_{t+i}^{d,T-i+1} - xr_{t+i}^{\$,T-i+1})$ Deviations from the Expectations Hypothesis. If we remember the equation (13) we can derive from it the definition of bond risk premia as the difference between long-term interest rates and short term interest rate, now if we expect shrinkage of the bond risk premia than we can expect an increase in the dollar exchange rate since a lack of yield in the long end of the yield curve would push investors to prefer cash also considering the fact that a currency with low interest rates embed smaller credit risk[15] compared to other countries with higher interest rates and considering for example a period in which bond risk premia are low so we can imagine a nearly flat yield curve, the country could easily see its currency to appreciate, because it can be seen as the safest asset in form of cash in periods with high uncertainty.
- $E_t \sum_{i=1}^{T} xr_{t+i}^{\frac{\$}{d},FX}$ the currency risk premia factor reflects the existence of risk in holding a specific currency with respect to another one.

Finally, we can easily say that currency markets and bond markets are tightly connected and as we have seen above bond risk premia have a direct effect on the exchange rate.

Let assume that exchange rates are stationary[16] or in other words shocks to long-run exchange rates are not priced in the market. Then what we would expect is that the exchange rate will reflect differences in long term interest rates today.

$$\frac{1}{T}(s_t - s_0) = \lim_{T\to\infty} \left(y_t^{\$,T} - y_t^{d,T}\right) + \frac{1}{T}\lim_{T\to\infty} E_t \sum_{i=1}^{T} (xr_{t+i}^{d,T-i+1} - xr_{t+i}^{\$,T-i+1}) - \frac{1}{T}\lim_{T\to\infty} E_t \sum_{i=1}^{T} xr_{t+i}^{\frac{\$}{d},FX} \quad (16)$$

$$\frac{1}{T}(s_t - s_0) = \lim_{T\to\infty} \left(y_t^{\$,T} - y_t^{d,T}\right) \quad (17)$$

[14] The dollar is the foreign currency for our investor

[15] The level of interest rates in a country is a good proxy of credit risk, indeed an investor is exposed to credit risk and interest rate risk when he is long in a government bond

[16] This is more reasonable when we think about real exchange rate

From equation (17), we can see that with the above assumptions the second and third term in our original equation (16) cancel out and so the only term with which we remain is the difference in long-term interest rates which will reflect exchange rate movements away from its mean. We can try to explain this effect with the fact that in the long run Uncovered Interest Rate Parity restores and this happen because if we assume the exchange rate is stationary then is fair to say that in the long term there is no additional risk for a foreign investor from investing in a foreign bond market[17] relative to its domestic bond market[18]. Therefore, if in the long term as we just said investors are risk-neutral than Uncovered Interest Rate Parity will hold in the long run.

In literature have been found evidence to suggest that if we look at currency markets, high currency risk premium are offset by local currency bond risk premium and this way the result is long run Uncovered Interest Rate Parity, which we derived in equation (17),

$$\frac{1}{T}(s_t - s_0) = \lim_{T\to\infty} \left(y_t^{\$,T} - y_t^{d,T}\right) \tag{17}$$

is a good benchmark model to analyze exchange rates. Recently in literature, there have been some works in this field by for example Backus, Boyarchenko and Chernov (2018), moreover Gourinchas, Ray, and Vayanos (2019), lastly Greenwood, Hanson, Stein and Sunderam (2020), all of them have tried to build models based on this type of version of long-run Uncovered Interest Rate Parity.

These results that we have just got will be useful to get some interesting evidence about the existence or not of currency carry trade because we will analyze currency excess returns from currency portfolios created based on the level of interest rates assuming we are a U.S. investor who borrows in the U.S. bond market and invests in a foreign currency. Remaining on the relation between spot exchange rate and long-term interest rates, it is very interesting to underline the fact that in a recent work[19] there have been found evidence of the fact that high currency risk premia are offset by negative bond risk premia. In other words, there is no currency carry trade to be earned because if we go long in longer maturity bond of countries that have high interest rates and short longer maturity bond of countries that have low interest rates this type of trade will not give us a large excess return. That because the foreign currency risk premia which are low for low interest rates countries is going to be offset by higher term premium so in the end choosing high interest

[17] In our example, US bond market

[18] Hanno Lustig & Andreas Stathopoulos & Adrien Verdelhan, 2019. "The Term Structure of Currency Carry Trade Risk Premia," American Economic Review

[19] Hanno Lustig & Andreas Stathopoulos & Adrien Verdelhan, 2019. "The Term Structure of Currency Carry Trade Risk Premia," American Economic Review

rates on the long end of the yield curve does not seem to pay and there is evidence in literature[20] as we said before that confirm this point and so long run Uncovered Interest Rate Parity holds.
If we go back to the equation (17)

$$(s_t - s_0) = T \lim_{T\to\infty} \left(y_t^{\$,T} - y_t^{d,T}\right) \tag{18}$$

It is important to remark the fact that if we believe in the long run Uncovered Interest Rate Parity and also Purchasing Power Parity[21], long term interest rates, on the long end of the yield curve, should span changes in the exchange rate. In other words, if we regress the changes in exchange rates with long-term interest rate differences, we should get a beta equals to one. However, we will see in chapter two the relevance of Uncovered Interest Rate Parity in explaining the variation of exchange rates. Nonetheless, there are some works and exist a sort of debate in literature[22] about the relevance of long run Uncovered Interest Rate Parity and the relevance of considering only interest rates on the short end of the yield curve to explain variation in exchange rates.
What we will see and also has been found in literature is the fact that this type of models cannot capture all the changes in exchange rates because there are permanent deviations of the exchange rate that are priced by investors in currency and bond markets that leads our past model to a no "offset effect" between bond risk premia and currency risk premia, explaining deviations from long run Uncovered Interest Rate Parity, as we can see below.

$$\lim_{T\to\infty} E_t \sum_{i=1}^{T} \left(xr_{t+i}^{d,T-i+1} - xr_{t+i}^{\$,T-i+1}\right) \neq \lim_{T\to\infty} E_t \sum_{i=1}^{T} xr_{t+i}^{\frac{\$}{d},FX} \tag{19}$$

What causes these differences are permanent country specific innovations to the pricing equation that lead to deviations from long run Uncovered Interest Rate Parity and also long run Purchasing Power Parity. In other words, it will not hold because long run risk in exchange rates exists.

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[20] Menzie Chinn, Guy Meredith 2005. Testing Uncovered Interest Parity at Short and Long Horizons during the Post-Bretton Woods Era. National Bureau of Economic Research

[21] Purchasing Power Parity is an economic theory that compares different countries' currencies through an approach based on a basket of goods. This theory states that two currencies are in equilibrium when the basket of goods considered is priced the same in both countries, considering the exchange rates.

[22] Mikhail Chernov & Drew D. Creal, 2018. "International Yield Curves and Currency Puzzles," NBER Working Papers 25206, National Bureau of Economic Research, Inc.

A recent work[23] has shifted the attention towards the bond supply for understanding movements in the exchange rate. They developed a model in which bond investors must absorb shocks to the supply and demand side for long-term bonds in two different currencies. We have seen that foreign exchange is exposed to unexpected changes in short-term interest rates but; it is important to notice that also long-term bonds are exposed to unexpected changes in short-term bonds. A shift in the supply of long-term bonds in one currency influences the exchange rate between the two currencies, and bond term premia in both bonds' markets. The segmented bond market model is based on the fact that an increase in, for example, U.S. supply of long-term bonds, then arbitrageurs demand higher bond risk premia, keeping all other variables constant. Therefore, an increase in the long-term bond risk premia in the U.S. lead to an increase in the U.S. long yields that cause an appreciation of the dollar immediately and following depreciation overtime to offset the effect of the higher yield. We can see this effect through the equation (18)

$$(s_t - s_0) \uparrow = T \lim_{T\to\infty} \left(y_t^{\$,T} - y_t^{d,T}\right) \uparrow \tag{18}$$

What we have just said happen because the increase in bond risk premia tends to be offset, from the perspective of a foreign investor who borrows in his domestic currency and lends in U.S., by a decrease in foreign currency risk premia. In other words, in the long-term foreign investor does not need a higher return for holding a U.S. dollar bond, because there is no long-run risk, because the investor understands that the dollar will mean revert back to its long-run mean in the long run. The equation below summarizes the model we have just presented.

$$\lim_{T\to\infty} E_t \sum_{i=1}^{T} xr_{t+i}^{d,N-i+1} = \lim_{T\to\infty} E_t \sum_{i=1}^{T} \left(xr_{t+i}^{\$,N-i+1} \uparrow + xr_{t+i}^{\frac{d}{\$},FX} \downarrow \right) \tag{18}$$

These types of models are very useful in understanding the flows of money in the markets in determining exchange rates.

[23] Robin Greenwood, Samuel G. Hanson, Jeremy C. Stein, Adi Sunderam,2020. " A Quantity-Driven Theory of Term Premia and Exchange Rates " Harvard University and NBER, National Bureau of Economic Research

1.3 Exchange rates and convenience yields

A third major force that can help us understand better changes in exchange rates are convenience yields. As we have seen before, an investor has two options: invest in his domestic bond market or otherwise invest in a foreign bond market. We can define a new set of Euler equations to start this analysis of a new determinant of the exchange rate.

$$E_t\left(M_{t+1}^{\#}\,\frac{S_{t+1}}{S_t}\,e^{y_t^{\$}}\right)=e^{-\lambda_t^{\$,\#}} \tag{20}$$

$$E_t\left(M_{t+1}^{\#}\,e^{y_t^{\#}}\right)=e^{-\lambda_t^{\#,\#}} \tag{21}$$

The equation (20) is the Euler equation of a foreign investor who is going long in for example US Treasurys, but now we are making a new assumption which is that the investor gets a convenience yield $\lambda_t^{\$,\#}$ from a long position in US Treasurys, while $\lambda_t^{\$,\#}$ is the convenience yield that an investor gets from a long position in a foreign bond market (#), for example, the German Bund . To make thinks easier, we can assume that $\lambda_t^{\$,\#}$ is big enough, meaning that a foreign investor has a particularly high demand for dollar denominated safe assets. We use log normality and we rewrite the Euler equations as:

$$-\lambda_t^{\$,\#}=\ E_t\left[m_{t+1}^{\#}\right]+\frac{1}{2}var_t\left[m_{t+1}^{\#}\right]+\ y_t^{\#} \tag{22}$$

$$-\lambda_t^{\#,\#}=E_t\left[m_{t+1}^{\#}\right]+\frac{1}{2}var_t\left[m_{t+1}^{\#}\right]+\ y_t^{\$}+E_t[\Delta S_{t+1}]+\frac{1}{2}var_t[\Delta s_{t+1}]-RP_t \tag{23}$$

$$RP_t=\ -cov_t(m_{t+1}^{\#}\,,\Delta S_{t+1})$$

We set the logarithm of the currency risk premium equals to: $rp_t=RP_t-\frac{1}{2}var_t[\Delta s_{t+1}]$ then we can derive a new expression:

$$E_t[\Delta s_{t+1}]+\left(y_t^{\$}-y_t^{\#}\right)=rp_t-(\lambda_t^{\$,\#}-\lambda_t^{\#,\#}) \tag{24}$$

Since the equation (24) is actually a difference equation we can iterate forward for s_t and we get:

$$s_t = E_t[s_{t+1}] + (\lambda_t^{\$,\#} - \lambda_t^{\#,\#}) + (y_t^{\$} - y_t^{\#}) - E_t(xr_{t+1}^{\frac{\#}{\$}}) \tag{25}$$

In this way we arrive to a new expression of the fundamental exchange rate valuation[24] equation which embedded a new term reflecting this concept of convenience yield.

$$s_t^{\frac{\$}{\#}} = E_t \sum_{i=0}^{\infty}(\lambda_{t+i}^{\$,\#} - \lambda_{t+i}^{\#,\#}) + E_t \sum_{i=0}^{\infty}(y_{t+i}^{\$} - y_{t+i}^{\#}) - E_t \sum_{i=1}^{\infty} xr_{t+1}^{\frac{\$}{\#}} + E_t[\lim_{i\to\infty} s_{t+i}^{\frac{\$}{\#}}] \tag{26}$$

From the equation (26) we can see that if the future convenience yield that the investor can get from a long position in US Treasurys goes up we expect the dollar to appreciate today to generate an expected future depreciation and thus lower the perceived return for the foreign investor from holding a long position in US Treasurys to restore equilibrium. This equation we have just presented has an issue coming from the fact that we need a proper way to measure these convenience yields. However, it useful to think about the fact that if the investor can obtain these convenience yields then Covered Interest Rate Parity does not hold and deviations from Covered Interest Rate parity can be measured with the difference between those convenience yields. What we have just said has been proved in some recent works, starting from how to compute those convenience yields. In some literature papers has been used the currency basis to compute these convenience yields interpreting their difference as deviations from Covered Interest Rate Parity. The currency basis, in our example a treasury basis, can be expressed as:

$$b_t^{Treasury} = y_t^{\$} - (y_t^{\#} - (f_t - s_t)) \tag{27}$$

We will go into much more depth about the meaning of currency basis and its relationship with the Covered Interest Rate Parity but now we can think about that as the difference between the yield obtained from a long position in Treasury, and a position in a foreign yield hedged into US dollars. In other words, the difference between a cash position in Treasury and a synthetic position in Treasury. We are using the treasury basis as a proxy for the convenience yields. Then if we set β [25]as the fractional amount of convenience gained with respect to the long position in US Treasurys.

[24] Jiang, Zhengyang, Arvind Krishnamurthy, and Hanno Lustig. 2018. "Foreign Safe Asset Demand for US Treasurys and the Dollar." AEA Papers and Proceedings

[25] Jiang, Zhengyang, Arvind Krishnamurthy, and Hanno Lustig. 2018. "Foreign Safe Asset Demand for US Treasurys and the Dollar." AEA Papers and Proceedings
The beta will have a value between 0 and 1. If β = 0, then the benefit of the synthetic position in hedging into dollar created by adding the forward position to the foreign bond gives no incremental benefits to the foreign investor. In this

Then we can rewrite equation (27) in order to finally get an expression for the difference in convenience yields and plug it into the equation (26)

$$b_t^{Treasury} = -(\lambda_{t+i}^{\$,\#} - \lambda_{t+i}^{\#,\#})(1-\beta) \tag{28}$$

$$(\lambda_{t+i}^{\$,\#} - \lambda_{t+i}^{\#,\#}) = \frac{b_t^{Treasury}}{1-\beta} \tag{29}$$

Now we can put equation (29) into equation (26) and we have an expression for those convenience yields which express deviations from Covered Interest Rate Parity. For example, if we assume that the difference between US interest rate convenience yield and a foreign interest rate convenience yield is positive then we can expect a negative basis (28) which will lower the cash yield on a U.S. treasuries relative to the synthetic yield going towards a balance, a basis equals to zero, eliminating deviations from Covered Interest Rate Parity.

Then we can conclude that the safe asset demand will lead to changes in the exchange rates. It is very interesting to note that recently the run to safe assets due to the Covid 19 crisis has led to some operation on the currency market from the Federal Reserve with for example swap lines which can be seen in this contest as a measure to prevent an explosion of that term $\lambda_{t+i}^{\$,\#}$ in a moment in which is very reasonable to think of very high convenience yield in holding assets denominated in us dollars, thinking about them as a "safe heaven", preventing so a shortage of the dollar in the market.

case, both U.S. Treasury bonds and foreign bonds are valued for their liquidity and safety properties in their respective currencies. If β = 1, then the benefit of the synthetic position in hedging into dollar converts the foreign bond to the equivalent of a U.S. Treasury, in this case the investor care more about the safe and liquid bonds whose payoffs are in dollars

2. Currency returns and risk

In this chapter, we will continue to study the exchange rate. From the previous decomposition of the determinants of the exchange rates, we understood what are the forces that drive changes in the exchange rate, and we have seen that there is some risk factor behind the behavior of the exchange rate. In this part of the work, we will focus on how currency returns can be decomposed and seen them also as a source of aggregate risk. In other words, we will see how risk is priced in currency markets. We have already seen some risk terms that enter into the fundamental exchange rate valuation equation however now we will see through the data if there is some evidence about that. Later we will see how to construct currency portfolios and how they can help us understand better currency returns. Afterward, once we have analyzed the determinants of currency returns and currencies portfolios performances we will see later how we can start to build a statistical arbitrage strategy based on the evidence we are going to find in this chapter.

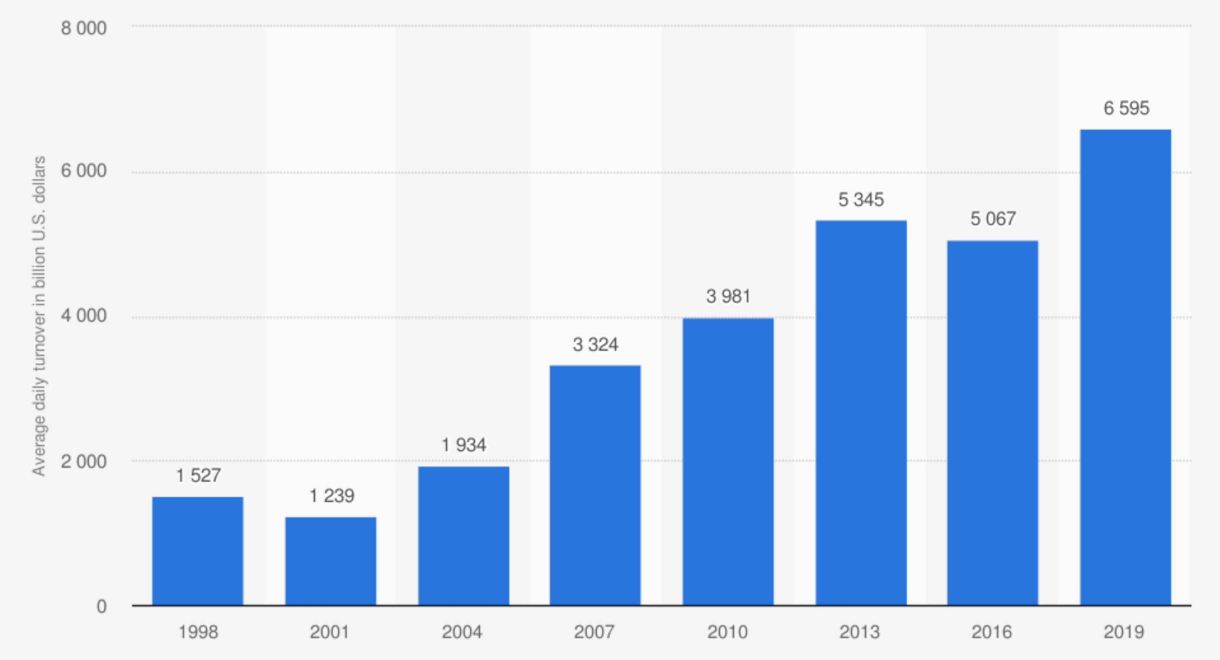
2.1 Importance of exchange rates

Exchange rates reflect many economic factors and variables. As we have studied before, a force that drives exchange rates is interest rates. If interest rates can cause movements in exchange rates, it is important to notice that also inflation impacts currency value and exchange rates. A higher level of expected inflation tends to depreciate the currency which we can think about it as a balancing effect opposed to the interest rate force. Government debt is another component that can affect exchange rates because a higher government debt can be more difficult to finance with foreign capital that can lead to higher inflation because of loose monetary policies. Political stability is another component that can affect exchange rates since a country with uncertainty in its political stability can lead to a runaway of investors from the domestic currency which can cause a huge depreciation of the domestic currency. A country's current account balance affects currency value and exchange rates. It is important to say that it can have a competitive mean because a country with a competitive advantage in exports can have advantages in having a weaker currency making its products more appealing, leaving all other economic and political assumptions constant. On the other hand, the same country will have a greater demand for its currency, leading to an appreciation of the exchange rates relative to other foreign currencies. Other relevant aspects are recession that influence the willingness of foreign investors in holding assets denominated in the domestic currency. Exchange rates are also very relevant for every international firm that have to manage their asset and liabilities that are probably denominated in different currencies. Last but not least, speculations are another topic that is relevant when thinking about exchange rates.

All the factors we have just described are taken under very careful consideration from central banks of the specific country. Every central bank most of the time have the aim of price stability through a mandate of keeping inflation near a specific target to take under control the economy. One of their principal instruments is monetary policies which try to keep the economy away from hyperinflation and deflation, which are the major fears of every central bank. Influencing and operating on these forces central banks have a tremendous impact on the currency markets. Moreover, it is very important to notice that the global economy is based on a dollar system therefore also how the central banks manage their domestic and foreign reserves is a very important topic, especially during recession periods. Then investors should be very careful about central banks' actions because they are one of the most important players in the market.

It is relevant to notice the size of the currency market; it is one of the biggest financial markets in the world. From the picture below, we can see a measure of the size of the currency market.

Figure 1.1: Average daily turnover in the global foreign exchange market from 1998 to 2019



Source: BIS, Statista 2020

The graph shows the average daily turnover in the global foreign exchange market from 1998 to 2019, and it is easy to see that in 2019 the foreign exchange market turnover is greater than $6 trillion a day, the trading volume all over the world in equity markets has been about 18 times lower than the currency market. What we have just said is another example of the relevance of studying currency markets.

2.2 From UIP condition to currency risk premia

Before starting our analysis of currency portfolios we need to define the currency excess return. Then it is possible to determine the currency excess return as:

$$E_t(r_{t+1}) = i_t^{\#} - i_t - E_t(\Delta s_{t+1}) \tag{30}$$

In equation (30) s_t is the spot exchange rate in foreign currency (#) per unit of the domestic currency. Therefore, when s_t increases the domestic currency will appreciate while the foreign currency will depreciate. With $i_t^{\#}$ we represent the interest rate in foreign currency while with i_t the interest rate in the domestic one.

The above equation (30) can be seen as an investor who borrows, for example, in U.S. dollars for three months and then lends at the foreign interest rate $i_t^{\#}$ for three months. In this example our investor has liabilities denominated in U.S. dollars and asset denominated in a foreign currency, therefore he is exposed to the exchange rate, he knows what is the exchange rate at time *t* but he does not know what will be the exchange rate three months from now. Therefore, if the foreign currency appreciates in the time period in which he has asset denominated in the foreign currency and liabilities denominated in dollars, then his excess return will be higher, otherwise, if the foreign currency depreciates during the time in which he has asset denominated in the foreign currency and liabilities denominated in dollars then his excess return will be lower, he will actually lose money.

Looking back to equation (8) we have already seen that the main forces of the exchange rate are the difference in interest rates between the two countries taken under consideration and a risk premium. This introduction of a currency risk premia is relatively new because, for a long time, there has been thought that there was any risk premium in currency markets and so the main relation has always been between exchange rate and interest rates difference. Moreover, if we try to use a regression model to understand a relation between interest rate differences and exchange rates, we get nothing relevant, then the power of interest rates to explain changes in the exchange rate is very little. Therefore, in this chapter, we will go through the analysis of these risk premia in currency markets, trying to find if exists and in case understand it.

The reason why for a long time the literature focused a lot on just the differences in the interest rates to explain changes in the exchange rate is for the famous relation called Uncovered Interest Rate Parity. The relation between the interest rates and the expected changes in the exchange rate was noted by William Douglass (1740), which equation is:

$$E_t(\Delta s_{t+1}) = i_t^{\#} - i_t \tag{31}$$

Where Δs_{t+1} is the exchange rate in foreign currency per domestic currency, then the foreign currency depreciates when s increases, while $i_t^{\#} - i_t$ denotes the interest rate difference between the countries taken under exam. If we think about this relationship between interest rate differences and exchange rate, it seems very reasonable because for example if the U.S. 3 month interest rate is 0.5% and the U.K. 3 month interest rate is 1%, then I can borrow in U.S. dollar for three months and lend in the U.K. for three months and gain a positive return but the shortcoming come from the fact that the positive difference in the interest rates is equal to the expected depreciation of the Pound in this example about 0.5% depreciation. Therefore, if the interest rate difference tends to reflect the amount that the exchange rate is going to depreciate or appreciate, an investor should not consider this type of investment and so the interest rate difference should persist. If our investors, in the example start to borrow in U.S. dollars and immediately invest in the British pound than we expect the U.S. interest rate to go up and the U.K. interest rate to go down, then it is reasonable to think at the end of the day the British pound is going to depreciate.

Therefore, as we have just seen the expression (31) seems really reasonable and correct in explaining the changes in exchange rates. However, later after the work of Douglass, first Tyron and then Fama (1984) tested the Uncovered Interest Rate Parity. Assuming rational expectations where:

$$\Delta s_{t+1} = E_t(\Delta s_{t+1}) + \varepsilon_{t+1}$$

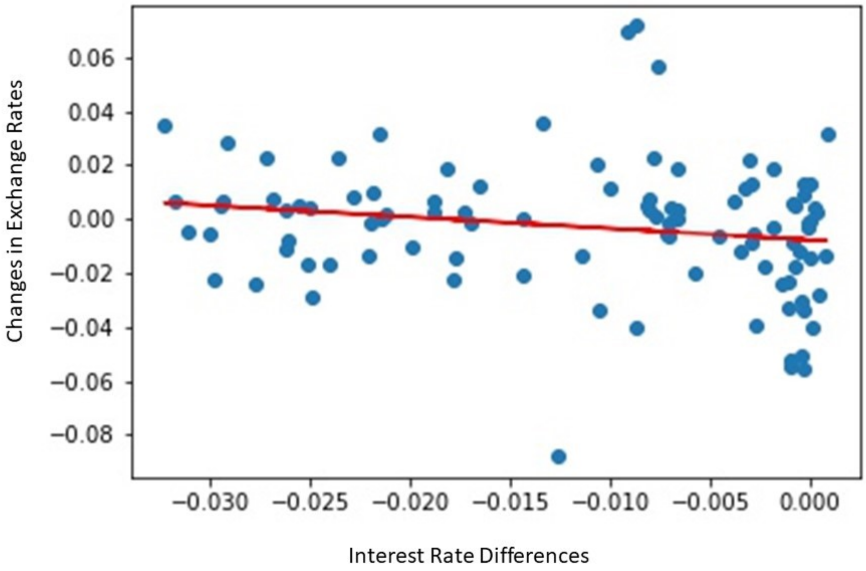
Then it is possible to define a model for the Uncovered Interest Rate Parity puzzle:

$$\Delta s_{t+1} = \alpha + \beta\left(i_t^{\#} - i_t\right) + \varepsilon_{t+1} \tag{32}$$

Then if Uncovered Interest Rate Parity holds this regression should result in a $\beta = 1$. However running this regression the only result is a $\beta < 1$, moreover most of the time it is usually negative, with a $R^2 \cong 0$.

Using real data, monthly exchange rates and 1 month interest rate differences in 17 different countries, the model gives actually no real significant results, with a $\beta < 0$ and most importantly an $R^2 \cong 0$. As we can see from the graph below:

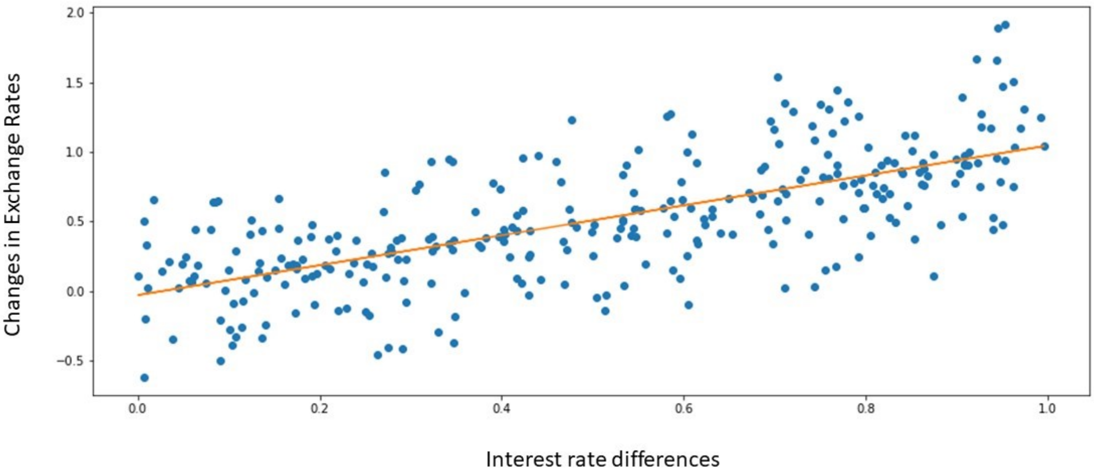
Figure 2.1: test of Uncovered Interest Rate Parity



Source: Python, Eikon Thomson Reuters

Instead of a line with an almost negative slope, we should have expected that changes in the exchange rate should be equal to interest rate differences in a way similar to the one presented below:

Figure 2.2: Expected test of Uncovered Interest Rate Parity



Source: BIS, Statista 2020

The real interesting result is the fact that the result in figure 2.2 is very similar to the result that we can get if we would have added a noise term in the variable of changes in exchange rate capturing volatility of exchange rate. However, as we have seen the result in figure 2.2 is not the reality in the data because figure 2.1 shows us a different result.

Now if we look at the time series of the changes in EUR/USD exchange rate is reasonable to think that the changes in the exchange rate are random, it seems that it is really difficult to determine some factors that can explain movements in the exchange rate. However, it has been shown in the first chapter with the definition of different fundamental exchange rate valuation equation and as we are going to see in this chapter with evidence from the data, is not true that exchange rates movements are completely random.

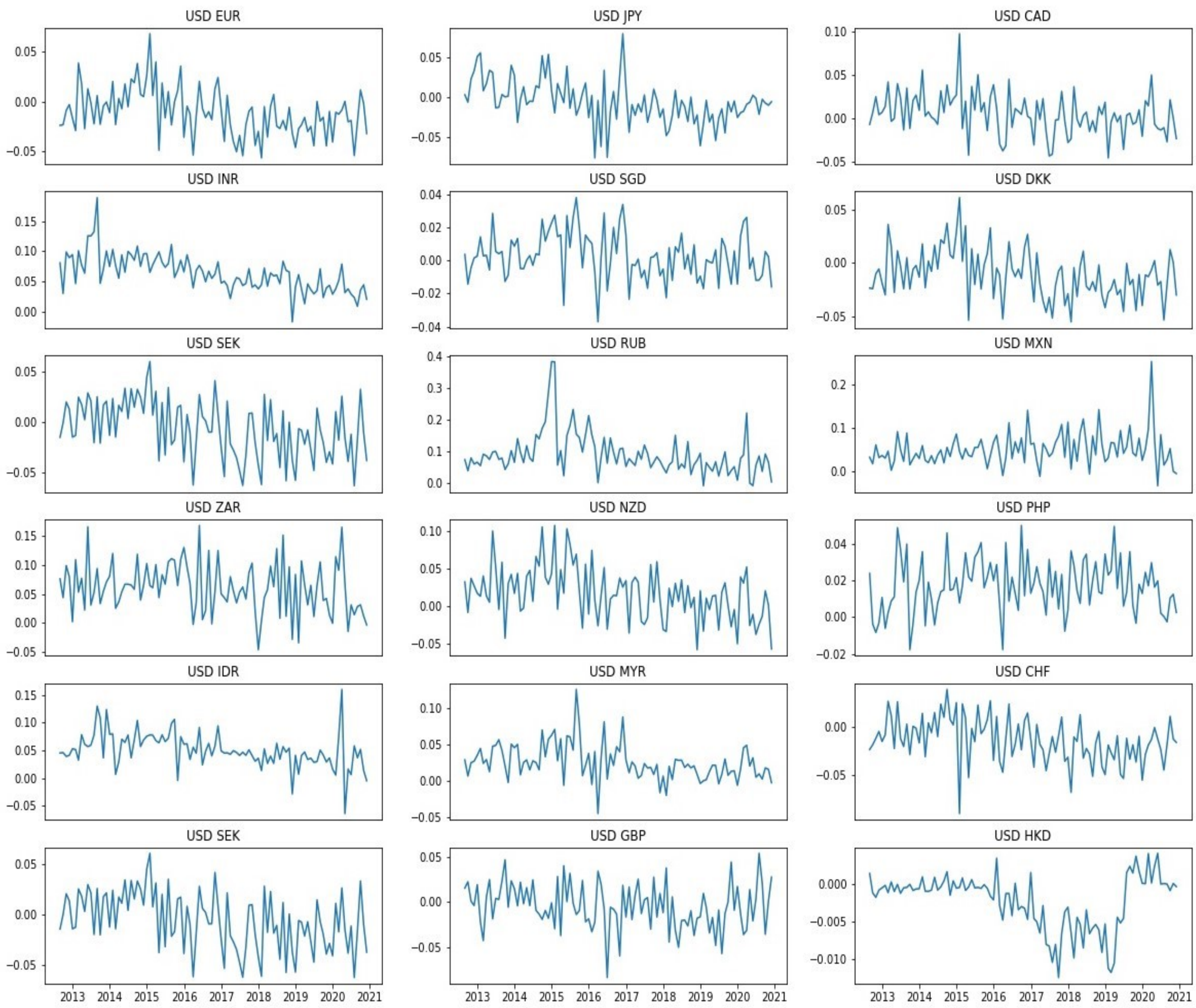
Before starting, it is important to define some expressions that we will need for our analysis of exchange rates. In the first chapter we already presented the assumption on risk neutrality but now it is important to say that Uncovered Interest Rate Parity assumes risk neutrality therefore:

$$E_t(\Delta s_{t+1}) = i_t^{\#} - i_t \quad (33)$$

$$E_t(r_{t+1}) = i_t^{\#} - i_t - E_t(\Delta s_{t+1}) = 0 \quad (34)$$

When the Uncovered Interest Rate Parity model is chosen, equation (33) it is made an assumption of risk neutrality, equation (34), in the sense that the expected excess return is assumed to be 0, because the positive or negative difference in interest rates differences should be offset by changes in the exchange rates which then solves as a balance function. We have shown before that if we regress the changes in the exchange rate on interest rates difference the $\beta \neq 1$ it is actually negative, we can think about this evidence as the expected excess return is time-varying and interest rates difference should be one of the forces, as we have seen in the first chapter in the case of the exchange rate, of the expected excess return, these facts has been shown in literature, remarking the importance of the difference in interest rates but making it not the only driver of expected excess returns. The time-varying feature of the expected currencies excess returns can be seen on the graph below:

Figure 2.3: Expected currencies excess returns from 2012 to 2020



Source: Python, Eikon Thomson Reuters

The figure 2.3 shows the monthly expected currencies excess returns using the expression (39) on 18 countries taking as domestic currency the U.S. dollar. It is very clear that the expected currencies excess returns are time-varying. Therefore, our investor in each subplot borrows for 1 month U.S. Treasuries and lends in a foreign currency for 1 month, exposing him to changes in the spot exchange rate during each specific month.

2.3 Currency portfolios

In the previous paragraph, it has been shown that interest rate differences have some power in explaining movements in the exchange rate. In this paragraph, we will use the analysis we have developed in the previous steps trying to extract information from foreign exchange markets using currency portfolios. The approach used is something we can find in literature in different works. The main idea is that now we are going to switch from a time series approach to a cross-sectional method building currency portfolios. This approach of using currency portfolios is relatively unusual because the main idea of portfolio construction is based on asset classes such as equities and bonds, and think about different currencies and most importantly of exchange rates in terms of assets' exposure to different countries through the main asset classes we said before. In literature, the concept of portfolio construction has been used a lot, for example the famous work of Fama and French[26] where they started a sort of revolution in the asset pricing theory trying to understand the different factors that can explain excess returns, for example, the main factors are book to market value, high minus low, small minus big. Later in this chapter, we will use a similar approach to study currency excess returns, trying to determine some risk factors that can explain the existence of currency risk. However, when we study currency portfolios is not very easy to use a similar approach, because is very difficult to collect data about exchange rates and interest rates in every country for the same time window around the world while for example is very easy to come up with a very strong dataset composed of a hundred of different stocks, is for this reason that we are limited to study some countries instead of others and we will change the sample of countries during this work when needed. These problems will lead us to make some assumptions during this analysis that will be underlined in every step. One assumption that we will maintain through all our analysis is the hypothesis that a foreign investor can buy risk-free bond in the different countries in our sample. This assumption can be seen as easy to make, but in this way we are assuming free capital movements between countries then no capital controls. Moreover, we will not consider default risk as a variable to measure we will consider it but as an internal component of our discussion not as a measure in our models. An important step in our analysis is also the choice of the number of portfolios and most importantly the portfolio allocation. This choice is relevant because we need a number of portfolios which is representative of the different countries with their specific features, moreover the portfolio allocation is important because the weight of each country within a single portfolio can have a big influence on the performance and representativeness of the results we are going to get. The logic of using portfolios is that all the idiosyncratic risk averages out, but if we

[26]Eugene F. Fama, and Kenneth R. French. 1992. "Common risk factors in the returns on stocks and bonds" University of Chicago USA.

invest in different countries are we exploiting the traditional concept of diversification? Later in this work, we will discuss this issues and shows some evidence from the data. These problems are common in portfolio construction in general no matter what asset classes we are considering. However, at the beginning we will use the portfolios as a mean for the analysis of currency excess returns trying to explain their determinants, if exist some opportunities in this market, and if so the reason why.

Our first dataset is composed of 18 countries with the respective spot exchange rates and 3 months interest rates. It is possible to see them in the table below

Figure 2.4: Countries in the first dataset

Mexico	Singapore	Japan	Switzerland
Indonesia	New Zeland	Great Britain	Germany
Sud Africa	Philippines	Hong Kong	Denmark
India	Malaysia	Canada	Sweden
U.S.A.		Russia	

Source: Python

Since the number and the use of the U.S. dollar as the domestic currency, we have 17 available countries to build currency portfolios. The first rule we set is to sort the countries by the level of their average interest rate in the time period considered, from 07-31-2012 to 11-30-2020. At first, the portfolios are built on the concept of extracting information about exchange rates therefore we have constructed 6 portfolios with 3 countries each except for the first one; the reason is that we gave much more importance to the countries with high-interest rates. After all, we are much more interested in understanding the behavior and the determinants of the exchange rate in this type of countries and hoping a better averaging out effect in that portfolios. The portfolios are represented in the table below:

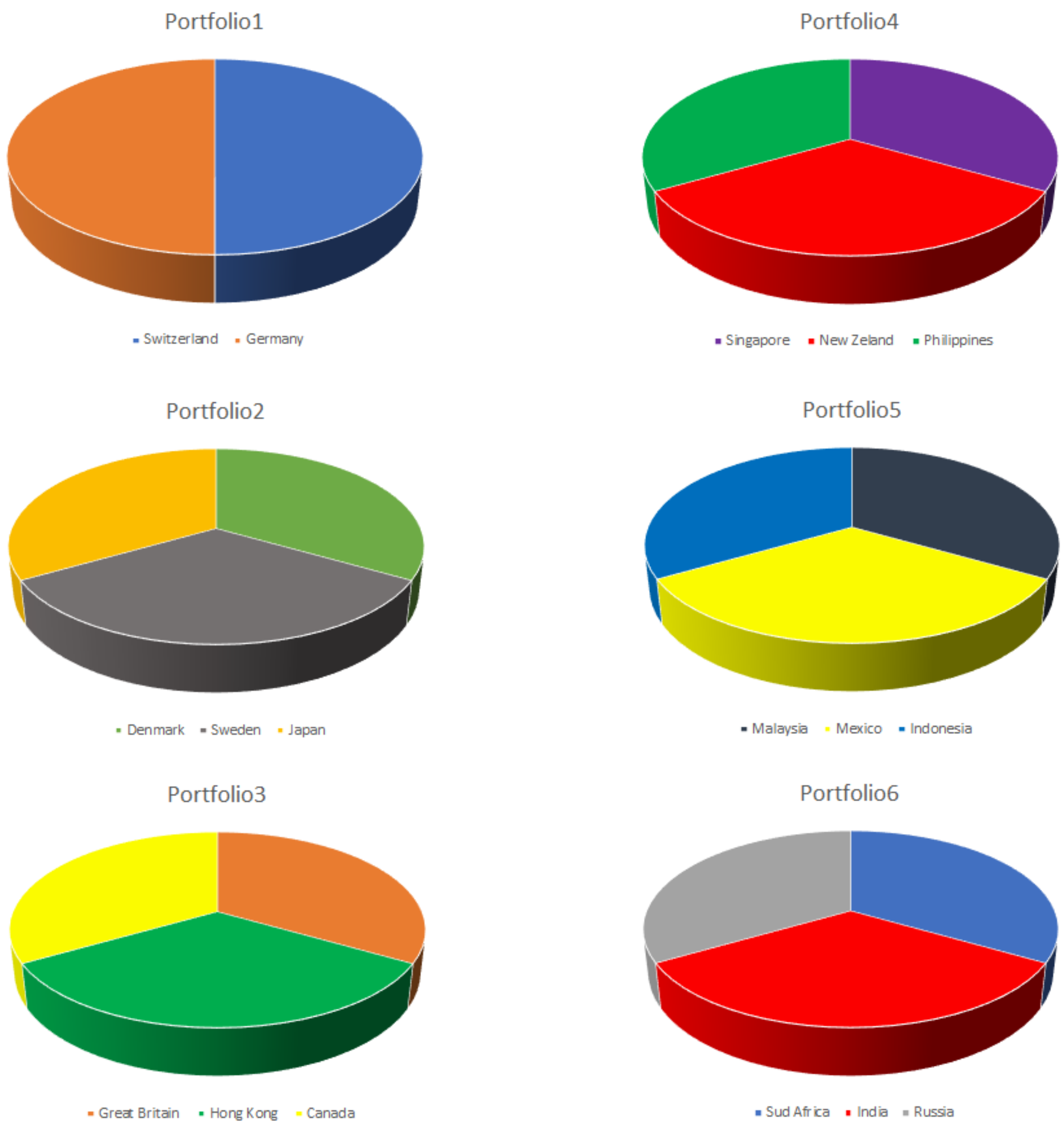
Figure 2.5: Portfolios

Portfolio1	Portfolio2	Portfolio3	Portfolio4	Portfolio5	Portfolio6
Switzerland	Denmark	Great Britain	Singapore	Malaysia	Sud Africa
Germany	Sweden	Hong Kong	New Zeland	Mexico	India
	Japan	Canada	Philippines	Indonesia	Russia

Source: Python

An important decision is how to weight each country within each portfolio. We could use data about international trade to weight each country or otherwise weights could be result from international flows. However, for our analysis is better to build equally weighted portfolios. Therefore, each country will contribute to the same percentage as the other countries within the same portfolio. The table below shows the asset allocation and our definitive portfolios.

Figure 2.6: Portfolios countries allocation



Source: Python, Microsoft Excel

It has been used the method of sorting countries by interest rates as in the paper[27] of Lusting and Verdelhan 2007 to build these portfolios of currency excess returns.

Before looking at the result of this analysis we have to remember the definition of currency excess return from the equation (34)

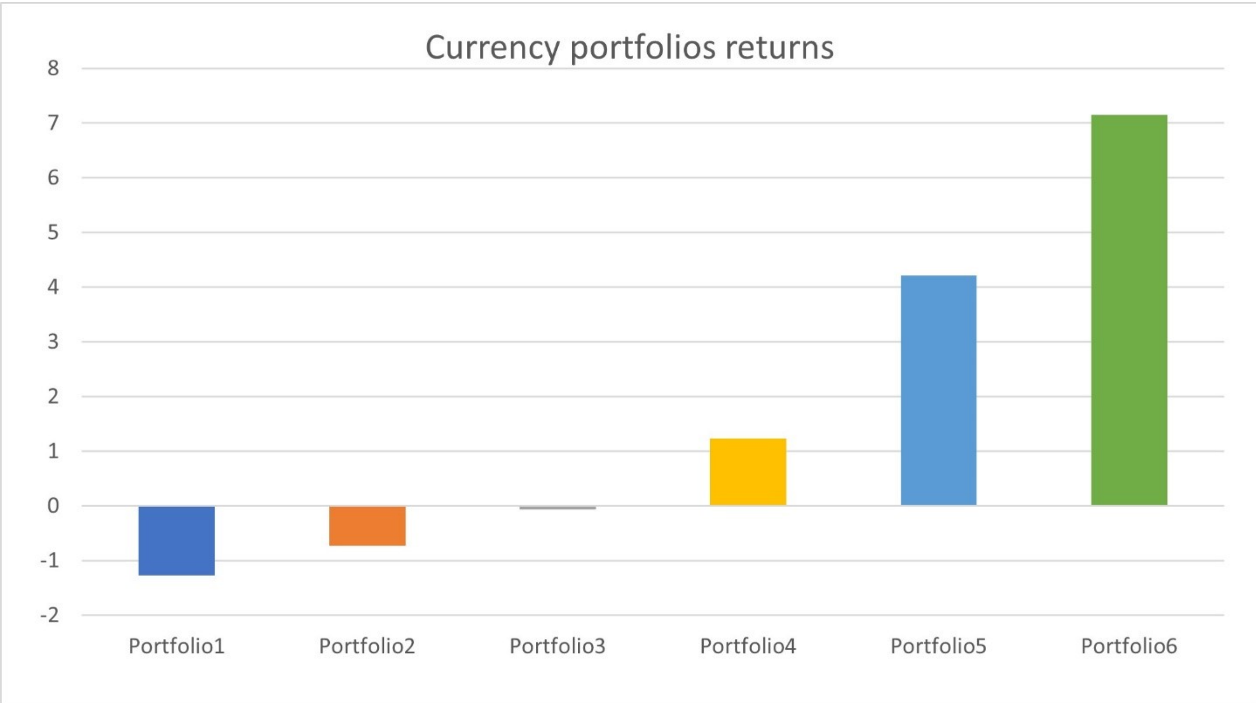
$$E_t(r_{t+1}) = i_t^{\#} - i_t - E_t(\Delta s_{t+1}) = 0 \tag{34}$$

[27] Lustig, Hanno, and Adrien Verdelhan. 2007. "The Cross-Section of Foreign Currency Risk Premia and Consumption Growth Risk." American Economic Review.

We are considering a U.S. investor who borrows in his domestic currency, 3 months U.S. Treasury bills, denoted in the equation by i_t , and invest for 3 months in a foreign currency denoted in the equation by $i_t^{\#}$. The only source of risk for the investor, as we have already seen previously, is the change in the exchange rates in the time period in which the investor is exposed to the foreign currency risk of depreciation. It is clear that the components in equation (34) are all easy to know except for the last term; It is important to notice that with the last we have already introduced a risk component in the study of the exchange rates, this is a first step that will allow us to develop a better understanding of exchange rate risk.

In the table below we can see the statistics of the six portfolios we have built

Figure 2.7: Portfolios statistics



	Portfolio1	Portfolio2	Portfolio3	Portfolio4	Portfolio5	Portfolio6
Portfolios Returns	-1.268503	-0.730688	-0.068523	1.234277	4.208459	7.147591
Portfolios Sharpe Ratio	-0.598627	-0.335019	-0.069907	0.678871	1.732230	1.897811

Source: Python, Microsoft Excel, Eikon Thomson Reuters

From the table, we can see the returns of the six different portfolios and the Sharpe Ratio, which shows the return of each portfolio per unit of risk. We remember that the first portfolio has the countries with the lowest interest rates, while the last portfolio has the countries with the largest interest rates. It is very easy to detect a pattern in the portfolio returns because the average excess returns increase from low interest rate countries to high interest rate countries. However, these results do not give us much information about exchange rates because following these statistics here the investor is just collecting the interest rate difference, but the exchange rates can still be considered completely random. In the next result, we are going to see that the investor is not getting just the interest rate difference but something different. In other words, we will see that these average excess returns could be a sign of some compensation for risk. For example, in the last portfolio, our investor can make money on average by borrowing with U.S. Treasury bills and invest in the three countries with the largest interest rates.

2.4 Currency excess returns and compensation for risk

As the literature has always shown there are no free lunches in the market, so we should expect that the returns we have shown in figure 2.7 embed some compensation for risk. Therefore, it is reasonable to believe that high interest rate currencies will depreciate in bad times. An easy way to define bad times, useful for our analysis, and coherent with our assumptions is consumption growth. Therefore, when consumption growth is low, we have a proxy of a bad time for our investor the opposite when high. In the table below we have the results of our first study of currencies portfolios returns.

Figure 2.8: Consumption Betas

Portfolios	1	2	3	4	5	6
			2012-2019			
Consumption Durables Goods Beta	0.52	0.54	0.33	0.7	0.36	1.28

Source: Python, Microsoft Excel

As it is possible to see from the table, each column represents the OLS estimates of the betas in this time-series regression of portfolios of currencies' excess returns on consumption growth, consumption based on durable goods

$$R_{t+1}^{p} = \alpha_0 + \beta_1 C + \varepsilon_{t+1} \tag{35}$$

The estimates are based on monthly data from 2012 to 2019. We used six currencies portfolios built as described before, sorted on interest rates. The main idea of this test we have just presented is that when consumption growth in the U.S. decreases signaling bad times for the economy the U.S. investor will lose money by investing in high interest rate currencies and it is possible to see this relation in the opposite way, our U.S. investor will make money on average by investing in low interest rate currencies in bad times because in that case, low interest rate currencies will appreciate, thinking about them as safe heaven during periods of high uncertainty or recession for example. The portfolios four five and six consumption betas are significant at the 10% level. We have used the Newey-West or HAC heteroskedasticity-consistent standard errors with an optimal number of lags following the paper Newey & West (1987, Econometrica). A very interesting think is that it is possible to see a small tendency of larger consumption betas in portfolios of countries with higher interest rates. This can be seen as a sort of greater sensitivity of these portfolios towards changes in consumption data. This idea is also confirmed from the fact that the last portfolios have more countries than the first one therefore it should embed a diversification effect that should balance this sensitivity. However this is not the case because the results show us a greater loading on the risk factor.

After these first results, we can continue our analysis in trying to extract information about the currency market building currencies portfolios, underlining the fact that at the monthly frequency there are some factors that can allow us to understand better if there exist some compensation for risk within currency market explaining those portfolios returns. Now we are going to rely also on forward markets, but the most important thing is that the logic of our analysis will remain the same. Avoid thinking about the only exchange rate alone because it is very difficult to get something from it due to the fact that there is a lot of noise that makes it not easy to explain. As an example, the change in the exchange rate as we mentioned before can be explained by dividing it into two parts, systematic shocks, and idiosyncratic shocks. On the other way, building currency portfolios allow us to average out idiosyncratic shocks and study exchange rates from a better perspective understanding that they are not random.

Now we use the same portfolios we created before with the same logic, sorting the countries by the level of their interest rates, and the same investment logic thinking about a U.S. investor who borrows in the U.S. market and invests in a foreign currency for one month. What we are going to discover is if there exist some new risk factors that can explain the currencies' portfolios' excess returns.

At first, we have to present a new definition of currency excess return using forward rates

$$E_t(r_{t+1}) = f_t - s_{t+1} \tag{36}$$

With a forward contract, we decide to exchange some foreign currency for U.S. dollars on a specific date in the future, in our example one month. Therefore, in our example, the future date is one month. In this framework, our investor can sign this contract at the end of the month receives some foreign currency converts it into dollars at the end of the month. It is clear that the only thing we know at time *t* is the forward contract; we do not know what it will be the exchange rate at the end of the month. The investor's excess return is going to be the difference between the forward rate f_t

and the spot exchange rate that he will know at the end of the month s_{t+1} . What we have just explained is the equation (36). At this point, we can rewrite the equation (36) in a different way

$$E_t(r_{t+1}) = f_t - s_{t+1} = f_t - s_t - \Delta s_{t+1} \tag{37}$$

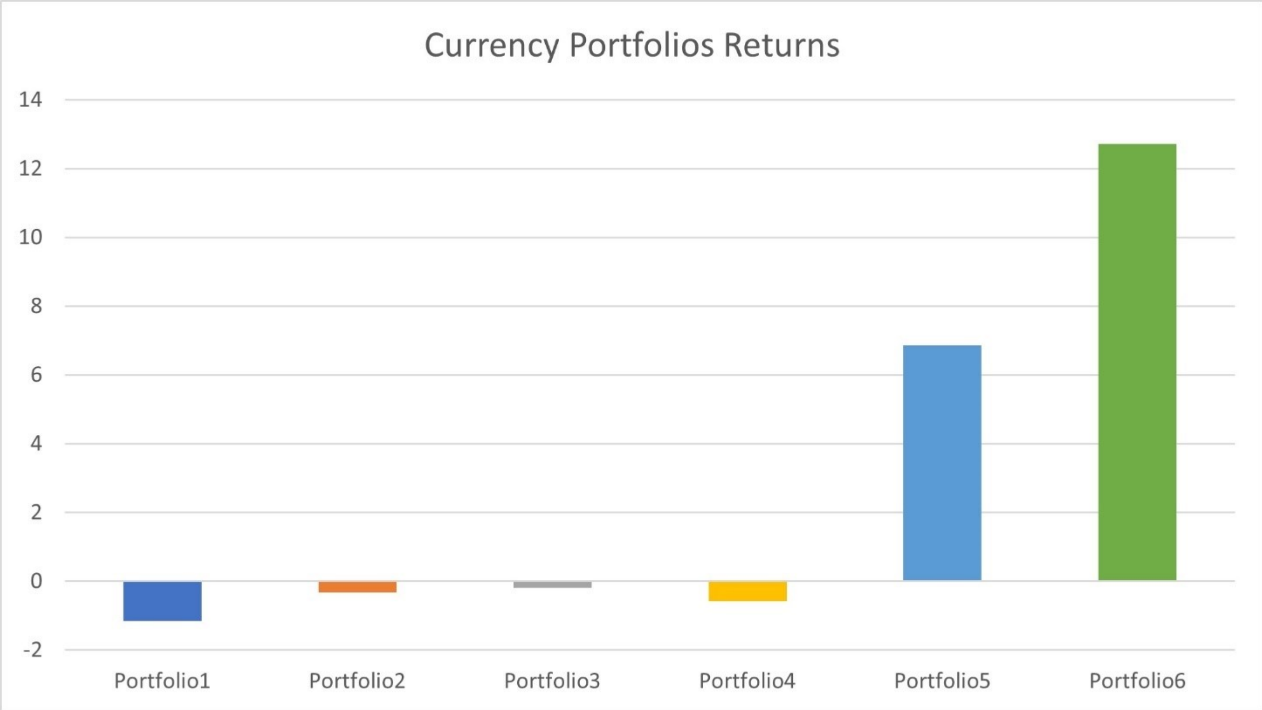
In equation (37) we have rewritten the equation (36) by subtracting the spot exchange rate at time *t* and then subtracting the change in the spot exchange rate between time *t* and the end of the month. Therefore, in equation (37) we have the forward spot difference minus the change in the exchange rate. We can notice that if Covered Interest Rate Parity holds we have a different way to compute a currency excess return

$$E_t(r_{t+1}) = f_t - s_{t+1} = f_t - s_t - \Delta s_{t+1} = E_t(i_t^{\#} - i_t - E_t \Delta s_{t+1}) \tag{38}$$

The only source of risk is the value of the spot exchange rate at the end of the month. It is important to consider the same questions we asked before about the possibility to invest in the countries we have considered and so problems with capital controls, but with forward contracts, we are freer to release these assumptions because with it we just need a counterparty willing to exchange some foreign currency for dollars later in the future. In our analysis, we did not take into account transaction costs and considered just the closing price at the end of each month, not considering the bid/ask spreads. On the opposite there are problems with this approach because we still have a limited sample, it will be difficult to think about the possibility to sign forward contracts in whatever country around the world. The second problem is that Covered Interest Rate Parity does not hold, as we have seen before. There is some evidence also in the literature about the fact that from the fact that after the financial crisis Covered Interest Rate Parity does not hold. The result of this is that the carry is more profitable on forward markets compared to the cash markets, we will discuss in more depth about this topic later in this work.

In the next analysis, we are going to exploit the equation (38) using forward rates and spot exchange rates of all the countries we have considered before, sorting them on the level of their interest rate, using the same currencies portfolios, and in the end running some asset pricing experiments. The time period considered is of 14 years from 2006 to 2020 using monthly data. In the table below we can see the portfolios statistics

Figure 2.9: Portfolios statistics



	Portfolio1	Portfolio2	Portfolio3	Portfolio4	Portfolio5	Portfolio6
Portfolios Returns	-1.148854	-0.316371	-0.189981	-0.575446	6.850121	12.711913
Portfolios Sharpe Ratio	-0.092112	-0.011499	-0.018484	-0.052633	0.224747	0.322707

Source: Python, Microsoft Excel, Eikon Thomson Reuters

From the table, we can see the returns of the six different portfolios and the Sharpe Ratio, which shows the return of each portfolio per unit of risk. We remember that the first portfolio has the countries with the lowest interest rates, while the last portfolio has the countries with the largest interest rates. It is very easy to detect a trend in the portfolios' returns because the average excess returns increase from low interest rate countries to high interest rate countries. It is very interesting to see that in this analysis even if we take into account the 2008 Great Financial Crisis which we did not consider in the previous analysis with interest rate differences the carry in the forward market is greater than on the cash market in the post-crisis period, as we can see with higher portfolios returns on average across the different portfolios.

After having shown the portfolios statistics in the previous figure now we are going to run some asset pricing experiments starting from those portfolios. Our aim is to understand if those portfolios' excess returns are compensation for risk and so risk premia exist in currency markets. The table below shows the results of our analysis based on the six currency portfolios

Figure 2.10: Portfolios excess returns and betas

Portfolios	1	2	3	4	5	6
2006-2020						
Excess returns	-1.15	-0.32	-0.19	-0.57	6.8	12.7
β_{HML}	-0.010	-0.13	0.060	-0.05	0.45	0.9
s.e.	0.03	0.07	0.02	0.03	0.1	0.07
β_{VOL}	0.01	0.01	-0.01	0.1	-0.025	-0.026
s.e.	0.011	0.17	0.01	0.006	0.02	0.02

Source: Python, Microsoft Excel, Eikon Thomson Reuters

From the table, we can see the two empirical tests we ran. In the first test, we have used the intuition from the famous paper of Fama and French where they sort the firms based on some factors such as "Book To Market" or based on their size "Small Minus Big" and they got a cross-section of equity excess returns and they showed that those excess returns correspond to the covariances between the returns and different risk factors, some of them are the ones we mentioned. In other words, they used the same returns to build a risk factor, because for example the risk factor HML is built taking the difference between the returns of the companies with high Book To Market ratios and the returns of the companies with low Book To Market ratios or for the Small Minus Big factor they built the factor making the difference between the returns of the small firm and the returns of the big firms. These risk factors become a proxy for risk in the equity market and they contributed to a big change in asset pricing theory, from single factor models to multifactor models. At this point, we can now use the same approach and build our risk factor from the returns of the currency portfolios. The HML factor in our analysis is constructed by taking the difference between the returns on the portfolio with high interest rate countries and the returns on the portfolio with low interest rate countries. We run the OLS regression of each currencies' portfolios' excess returns on the HML factor built as said before. In figure 2.9 we can see the result. It is clear from the table above the differences in the β_{HML} from negative betas to positive betas across the different portfolios. The result is a sign of the existence of risk premia in currency markets because the β_{HML} can explain on average the currency portfolios excess returns as the covariance between the returns and the risk factor we have built. However, in this way we are trying to explain portfolios return with portfolios returns, it is based on how we built our risk factor. After this test, now we are going to build a new risk factor that is not based on exchange rates and so on the currencies returns. For this new risk factor, we focused on the volatility of the equity markets. Since our dataset is composed of 17 different countries we had a problem with finding a coherent proxy for the global equity markets' volatility due to the limited sample and absence of equity indices in some regions of the world. We used as a proxy of the volatility of the equity markets the CBOE VIX INDEX[28] even if it represents the volatility of the S&P500 and so the volatility of the first 500 public company sorted on their market capitalization in the U.S.A. it can be seen as a good proxy for the volatility of the equity markets around the world because this allows us to build a good dataset with a large sample. It is possible to see the results in figure 2.9 where we regressed the currencies portfolios excess returns on the change in the monthly volatility in the equity market. It is common to think that in periods where volatility is high there is a lot of uncertainty in the market so we can think about it as bad times and vice versa[29]. The betas β_{VOL} show a very interesting result. The betas decrease as we move towards the currencies portfolios with high interest rate countries whose excess returns are positive on average in the time period considered. This result shows that when volatility is high or there is a spike in the equity volatility the returns on the portfolios with low interest rates currencies remain positive, therefore exposing a positive correlation between the firsts portfolios and the equity volatility. On the other hand, the last portfolios made by countries with high interest rates show negative betas presenting a negative correlation between their returns and volatility, meaning that when there is a spike in equity markets' volatility these portfolio will tend to lose money compared to the firsts ones. Summarizing, there is a different amount of risk between the firsts currencies portfolios and the last ones because when volatility is high the investor who invested in high interest rate countries will lose money because those currencies will depreciate. Instead, the investor who invests in countries with low interest rates will make money in bad times because those currencies will tend to appreciate.

---

[28] The VIX Index is an index which is the result of a calculation designed to produce a measure of constant, 30-day expected volatility of the U.S. stock market, derived from real-time, mid-quote prices of S&P 500 Index call and put options. It is one of the most accepted measures of volatility.

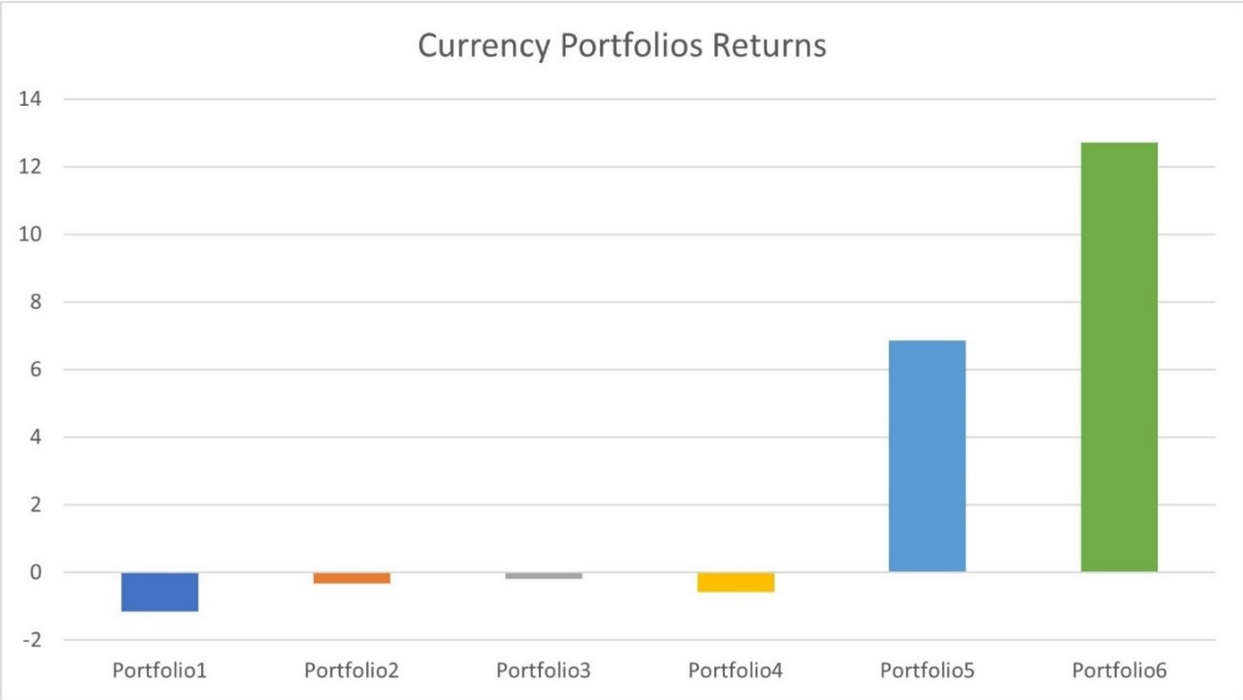
[29] A quantitative reason for the inverse relationship between market prices and volatility can be explained from the traditional present value formula, which is the basic formula where it is possible to start for pricing a risky asset:

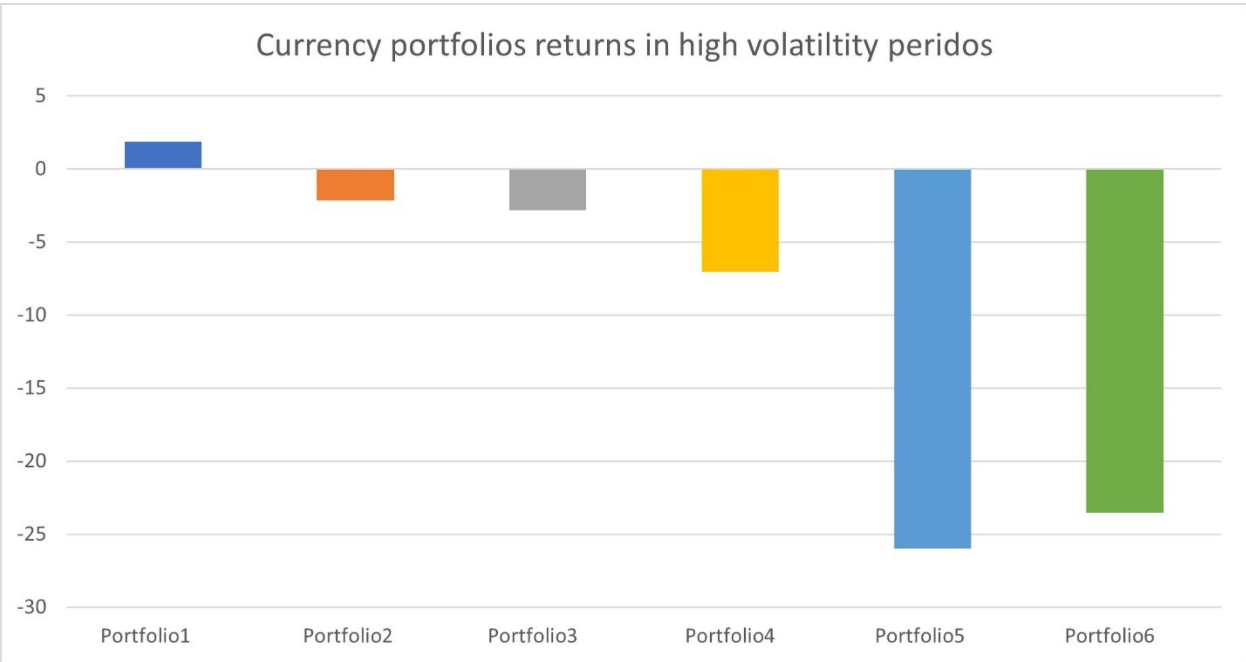
$$PV = \sum_{t=1}^{n} \frac{CF_t}{(1+i)^t}$$

The key point is the role of the term *i* in the equation. That term represents the expected return demanded on the risky investment considered. In periods of high uncertainty, when it is expected that the volatility of the asset price will increase, it is reasonable to think that investors will require a higher expected return on that investment. The result is an increase of the term *i* and so the decrease of the present value of the risky asset, explaining the inverse relation between market prices and volatility.

After we have investigated the existence of some compensation for risk in the currency markets, we can see from a different perspective the same concept we have tried to explain but not running asset pricing experiment anymore. Instead of running OLS regressions, we can study the behavior of our portfolios during periods of high volatility. At first we can compute the mean and the standard deviation of our equity volatility index in the time period considered. If we assume a normal distribution of the data, we can compute what is the value above more than one standard deviation from its mean of our volatility index with a 90% confidence level. Having set this parameter we can extract from our dataset all the currency portfolios returns in the period in which the volatility index was between one and two standard deviations from its mean. The result is summarized in the graph below where we can see the portfolios' returns in the first bar plot and the portfolios returns' resulted from what we have just explained.

Figure 2.11: Portfolios statistics



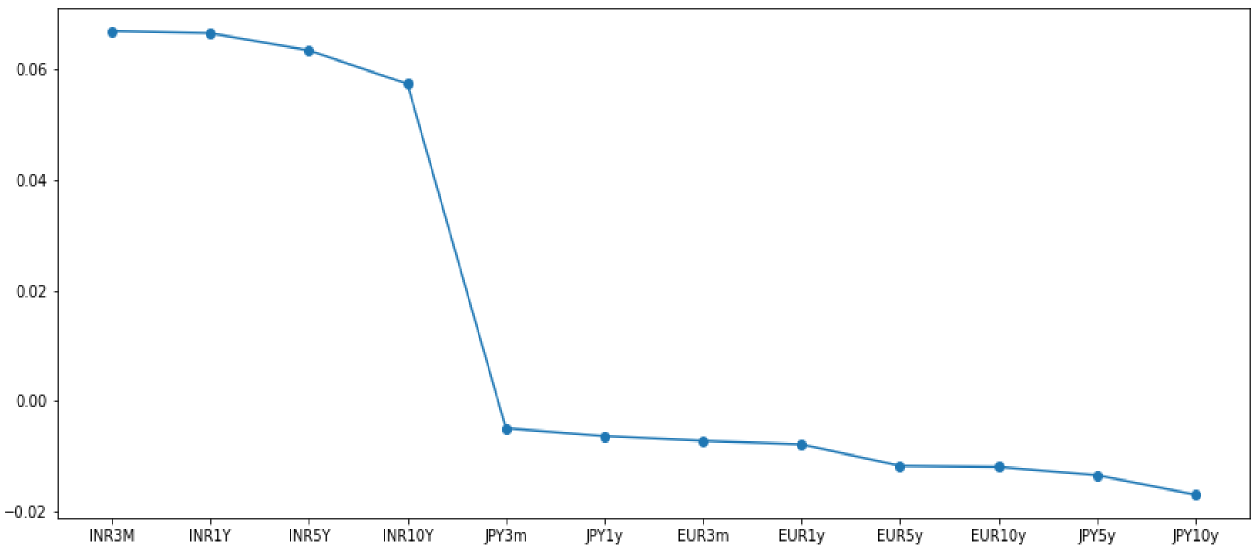


Source: Python, Microsoft Excel, Eikon Thomson Reuters

In the figure above we can see a very interesting comparison of the currencies' portfolios' returns against the same returns in periods of high equity markets' volatility. As we have noticed before, the currencies' portfolios' average excess returns increase as we move towards the portfolios with high interest rate countries. However, if what we discussed before is true, we should expect that the trend in the excess returns showed in the first bar plot should be the opposite in periods of high volatility as a proxy of bad times for investors. In other words, we expect that during periods of high uncertainty the investor who holds the lasts currency portfolios should lose money on average. Therefore, the second bar plot in figure 2.11 is a perfect picture of what we have said before because when there are big shocks the firsts portfolios can be seen as a means of insurance because the investor who invests in those portfolios lose money on average but make money on average when volatility spikes up. On the opposite, in the lasts portfolios, the investor who invests there makes money on average but loses a lot of money on average when volatility increases. This last evidence is in perfect coherence with what we have found before because it strengthens our hypothesis of the existence of some compensation for risk in currency markets explaining those excess returns once we build currency portfolios.

In the first chapter, we have discussed the relationship between exchange rates and short-term interest rates, and also the relationship between exchange rates and long-term interest rates. In the figure below there is an interesting result which links what we have studied about the relationship between exchange rates and long-term interest rates and the evidence we have found in this last chapter.

Figure 2.12: Currencies excess returns for different maturities



Source: Python, Eikon Thomson Reuters

In figure 2.12 we have considered the G4 countries, Brazil excluded, and we have computed the carry trade returns using the same logic we have explained before. Therefore, there is an investor who borrows for one month in the U.S. bond market and invests for the same time period in a foreign currency bond market, as we noticed before the only source of risk in this trade is the change in the exchange rate between the time period in which our investor is exposed to the foreign currency movements in the market. We remember the equation is:

$$E_t(r_{t+1}) = i_t^{\#} - i_t - E_t(\Delta s_{t+1}) = 0 \tag{34}$$

Now instead of be focus only on the possibility to borrow and invest in the money market, U.S. money market and foreign currency money market respectively, we assume the possibility to use bonds with higher maturities, 3 months, 1 year, 5 years, 10 years, but with the same holding period for our investor. Therefore, our dataset is composed of monthly data of the last 10 years India, Europe[30], U.S.A. and Japan bond prices and the respectively exchange rates. The result is shown in the figure 2.12 where we can see that the curve has a negative slope. One reason for that is the choice of the countries because we have a country with high interest rates such as India and countries with lower interest rates such as Europe and Japan, but this is not the point we want to emphasize. The interesting point is that there is a clear negative trend as we move towards the average carry trade returns made with longer maturities bonds for every country we have considered. For example, the trade made with borrowing the 3 months U.S. Treasury Bill and investing in the 3 months Japan bond is higher than the one made with borrowing the 1 year U.S. Treasury bond and investing in the 1 year Japan bond which is higher than the one made with the 5 years which is higher than the one made with the 10 years. Therefore, is clear that this type of strategy is more profitable with short-term bonds than with long-term bonds. The reason for that is the concept we have studied in the first chapter. If we consider bonds with longer maturities we are adding a new source of risk, interest rate risk. From the figure above it turns out that this compensation for interest rate risk also called term premium moves in the opposite direction with the currency risk premium. When our investor goes long in foreign currency and short in its domestic currency he is taking some currency risk but also an additional source of risk, the interest rate risk, and this last two source of risk move in opposite directions and as we have seen before they tend to cancel out each other. We can say that risk premia exist in currency markets and with this last study we have seen how different risk premia interact between each other.

[30] As a proxy for European bond market has been considered the Germany bond market

3. From empirical evidence to statistical arbitrage

We have seen in chapter 1 that currency markets and bond markets are tightly connected. We have investigated the different forces that reflect the movements in exchange rates. Indeed, the analysis showed that exchange rates absorb shocks to future interest rates. Moreover, exchange rates reflect news about future discount rates and we have noticed that this effect can be called deviations from Uncovered Interest Rate Parity also called foreign currency risk premia. The last force we have underlined is deviations from Covered Interest Rate Parity. We arrived at the conclusion that these three forces all together can determine the level of the exchange rate.

In chapter 2 we have focused a lot on understanding the reason behind the presence of currencies' excess returns, decomposing them, and trying to motivate those currencies' excess returns investigating the existence of some compensation for risk in currency markets. We have tried to understand how risk is priced in currency markets. During our analysis, we have shown the evidence from the data about the questions we have just said. Now in chapter 3, we are going to start by defining what is the meaning of statistical arbitrage and when it comes from. Later, we are going to use all the evidence and theory found to create an arbitrage strategy in currency markets. At the end of this research, we will try to find a reason for our finding with the help of some new findings in the literature and then create additional questions and denote further improvements of our strategy and analysis.

3.1. Statistical arbitrage

With the term “statistical arbitrage” investors denote a set of trading strategies that use mean reversion analyses to invest in different portfolios of securities with a very short holding period. These types of strategies rely on mathematical modeling and analysis making it a rigorous approach to investing. In the word of finance, the strategies within the term statistical arbitrage are also known as pairs trading.

The first who introduces the statistical pairs trading or statistical arbitrage is recognized to the quant Nunzio Tartaglia. He was at Morgan Stanley during the 1980s. Already at that time, he built a team of mathematicians, physicists, and computer scientists. They had the goal to develop quantitative arbitrage strategies using mathematical and statistical techniques. As we have said before they introduce pairs trading, they used this technique for trading, which involves trading securities in pairs. The process consisted of identifying pairs of financial securities whose prices tended to move together. When a strange or new deviation different from the relationship between the two assets was noticed, the two securities would be traded with the assumption that the divergence would correct itself. From that time this type of strategy was called pairs trading. Nowadays, pairs trading has become a popular strategy in the industry used by hedge funds and different investors.

In general, the most common concept for investing in the market is to sell overvalued securities and buy undervalued securities. However, it is not easy to determine when security is over or undervalued because the real question that can help in understanding that is the real or fair value of the security we are considering. Pairs trading or statistical arbitrage tries to solve this problem using the concept of relative pricing. Relative pricing means that if the two securities considered have similar features, then the price of both of them must be almost the same. The important aspect is that the price itself is not really important. The only thing that matters is that the prices of the two securities are similar or follow a specific relationship. In the case in which the prices diverge from what we have said, then it could be the case in which one of the two securities is overvalued while the other one is undervalued. Statistical arbitrage tries to exploit these prices deviations with the aim to sell overpriced securities and buy underpriced securities with the assumption that the deviation that causes the mispricing will correct itself. With term spread, we define these deviation in the price relationship between the securities we consider. The greater the spread and the bigger is the deviations between the securities’ prices and of course the greater is the potential profit once the mispricing will correct. Sometimes these types of strategies are also called long-short because when it is the right time to enter in the market the investor will have a long position in the undervalued security and a short position in the overvalued security. This type of trade is usually built in a way in which it has a negligible beta and then a small exposure to the market. When this type of trade construction has been achieved, the performances from the trade are uncorrelated to the market returns. This type of trade feature allows this type of strategy to enter into the group of the market neutral strategies. One important thing that is relevant or the key in this type of strategies and make them profitable is the identification of the security pairs. In our strategy, we are going to build a statistical arbitrage strategy in the currency market using the statistical technique known as cointegration based on the evidence found in the previous chapters.

3.2. Statistical arbitrage in currency market

Before we have mentioned that statistical arbitrage or pairs trading tries to trade the spread between two securities, which reflects the mispricing between the two of them. We are going to use the cointegration technique to build our trading strategy. Before going deep into our strategy, we have to introduce three mathematical concepts that are stationarity, integration, and cointegration. At first, we are talking of time series and there are two definitions of stationarity.

- Weak stationarity: the time series is said to be weakly stationary if the second moments of the process are time-invariant
- Strict stationarity: the time series is said to be strictly stationary if the joint distribution of any subset of the time series observation is independent of time with respect to all moments.

Therefore, we can say that stationarity implies that the time series do not have trends or seasonal effects. Sometimes, time series are modeled as stochastic processes, we can see that from the autoregressive equation below.

$$z_t = b_1 z_{t-1} + b_2 z_{t-2} - b_3 z_{t-3} + \cdots + b_n z_{t-n} \quad (39)$$

In the expression above, we can see that the value of the process at time *t* is a weighted sum of values at times before *t* plus a random disturbance. The process we have defined in expression (39) has the following characteristic equation:

$$L^n - L^{n-1} b_1 - L^{n-2} b_2 - L^{n-3} b_3 - \cdots - b_n = 0 \quad (40)$$

If one of the *n* roots of the polynomial in the equation (40) equals 1, then the process is said to have a unit root. The result is that the time series is not stationary. If all the roots of the polynomial are less than 1 in absolute value except for one root, then the first difference of the process we have presented is stationary, and the process is said that it is integrated of order 1 or I(1).
We can test for stationarity with a statistical hypothesis test called Augmented Dickey-Fuller test (ADF test). It estimates the null hypothesis that the time series considered has a unit root against the alternative hypothesis of stationarity. The test is based on regressing the differenced time series on a

time trend, the first lag, and all lagged differences, then calculate a test statistic of the coefficient on the time series built. We can write the regression as:

$$\Delta z_t = \alpha + \beta t + \gamma z_{t-1} + \delta_1 \Delta z_{t-1} + \delta_2 \Delta z_{t-2} + \cdots + \delta_{n-1} \Delta z_{t-n+1} + \varepsilon_t \quad (41)$$

In the equation above, we can see the time series z_t and the regression for the ADF test where α is a constant, β is a coefficient on a time trend, and *n* refers to the number of lags used in the model. This test is based on testing the sample coefficient γ where we have under the null hypothesis the unit root so non stationarity while equals zero or is negative on the alternative hypothesis.

The last theoretic concept we have to define before starting with the building of our trading strategy is the concept of cointegration. It is possible to define two time series y_t and x_t as cointegrated if both of them are I(1) and we could find a coefficient β that allows us to write:

$$y_t - \beta x_t = \mathrm{I}(0) \quad (42)$$

In other words, the distance between the two time series, once we have considered the scaling factor, is constant. Therefore, when equation (42) holds and so it is a stationary process, we can say that the two time series are cointegrated.

There are a lot of tests that check if two series are cointegrated. We are going to use the Engle-Granger test which involves:

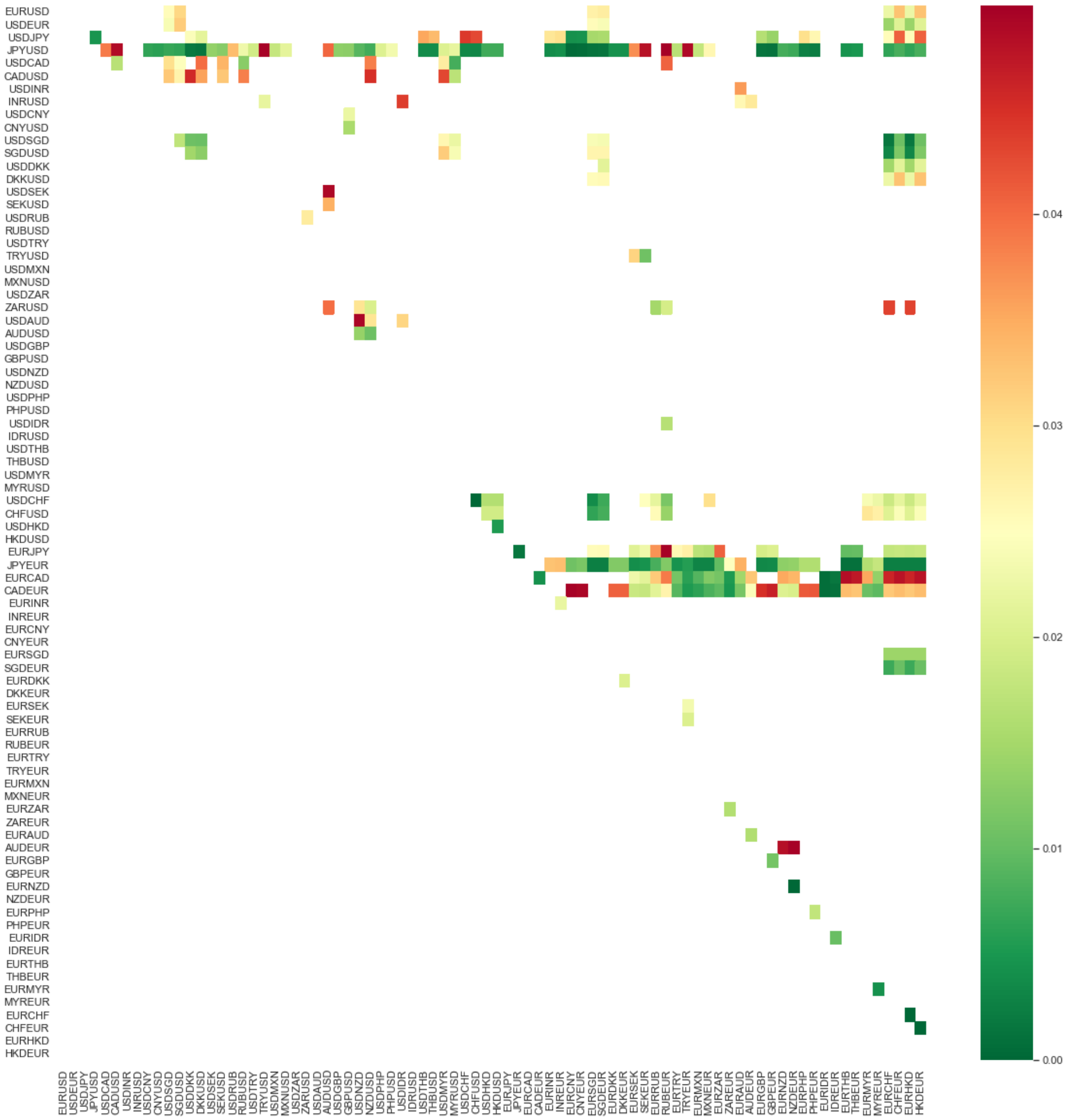
- Regressing by using the OLS the two time series considered to evaluate the stationarity long term relationship
- Using the Augmented Dickey-Fuller test to the regression residual

We have a hypothesis test where under the null we have that the residuals have a unit root. In the case we can reject the null hypothesis, then we can say that the residuals are stationary and so the series are cointegrated. It is important to notice that this approach when used shows the multiplier that makes the combination of the two series stationary and it is the regression coefficient. The weakness of this model is that the result we obtain is different depending on what series we consider as the independent variable.

At this point, we can now implement a forex pairs trading strategy, using the concept of cointegration based on statistical arbitrage using a mean reversion logic. At first, we built a dataset of daily spot exchange rates in the period between 2012 and 2020, both included. We looked through 82 spot exchange rates, considering 42 different countries, to see if any of them were cointegrated if there was a possibility for a statistical arbitrage strategy. The first step is to test if some pairs are cointegrated. We used the ADF test presented before where the null hypothesis says that there is no cointegration between the series considered while the alternative hypothesis says that there is cointegration between them. The P-values and critical values were obtained through regression approximation from MacKinnon 1994 and 2010[31]. We present our result, after having run the test on our dataset, in the map below where we can see the ADF results:

[31] Use of the Statsmodels.tsa.statstols python library
MacKinnon, J.G. 1994 "Approximate Asymptotic Distribution Functions for Unit-Root and Cointegration Tests." Journal of Business & Economics Statistics, 12.2, 167-76.
MacKinnon, J.G. 2010. "Critical Values for Cointegration Tests." Queen"s University, Dept of Economics Working Papers 1227

Figure 3.1: Heatmap ADF test on cointegration



Source: Python, Eikon Thomson Reuters

The ADF test listed some pairs that are cointegrated, resulting in a p-value less than 0.05 rejecting the null hypothesis of no cointegration between the two series with a significance level of more than 95%. After further investigation, the pair we decided to study is USD/JPY with INR/EUR. We remember from the last chapter that arbitrageurs can exploit deviations from Uncovered Interest Rate Parity by investing in high interest rate currencies and borrow in low interest rate currencies by taking some extra risk in their portfolios. Therefore, the choice of this pair is motivated also by the fact that in this way we can study if it is possible to get currency excess returns using statistical arbitrage strategies exploiting deviation from Uncovered Interest Rate Parity. The ADF test resulted in a p-value of 0.029 proving that the two exchange rates time series are indeed cointegrated. After having noted that the pair is cointegrated, we can see the spread between the two time series. We computed the spread using what we presented before with the Engle-Granger method, linear regression to get the coefficient to construct the linear combination between the two exchange rates. In the figure below we can see the spread between the two time series:

Figure 3.2: USD/JPY and INR/EUR spread

Source: Python, Eikon Thomson Reuters

We can already see that there is a sort of mean-reverting trend on the series, the red line in the graph shows the mean of the process. However, we can do a little more than just compute the spread between the two exchange rates. Now, we examine the ratio between the two time series, that is what we will use in the construction of our trading strategy. The ratio between the two time series is presented in the figure below:

Figure 3.4: USD/JPY and INR/EUR ratio

Source: Python, Eikon Thomson Reuters

As we have noticed before with the spread between the two time series also in this case with the ratio we can see a clear mean-reverting trend from 2014 to 2021, the red line shows the mean of the series. Therefore, it is possible to see that no matter what we use for our analysis both in the spread and in the ratio, the pair USD/JPY and INR/EUR tends to move around the mean. At this point, we standardize the ratio to have a better series to study than just the absolute ratio. For this purpose, we used the z-score. It is calculated:

$$z_j = \frac{x_j - \bar{x}}{std} \quad (43)$$

It represents the number of standard deviations a given data point is from the mean. The figure below shows the result of the z-score computation on the ratio we got before.

Figure 3.5: USD/JPY and INR/EUR ratio z-score

Source: Python, Eikon Thomson Reuters

The black line in figure 3.5 shows the mean of the series the red and green line are set at the z-score level of 1 and -1. This representation shows us in a better way the fact that deviations from the mean tend to move back and this is what we wanted for our trading strategy. In building any type of trading strategy is very important to decide the time when enter in the market, therefore the first question is what is a good signal to use that can indicate when it is the right time to trade. In our trading strategy, we are going to use the ratio time series created before, it is in figure 3.4, that is going to help us I building the indicator we need. The signal we have created is based on the expression below:

$$I_t = signal(ratio_{t+1} - ratio_t) \quad (44)$$

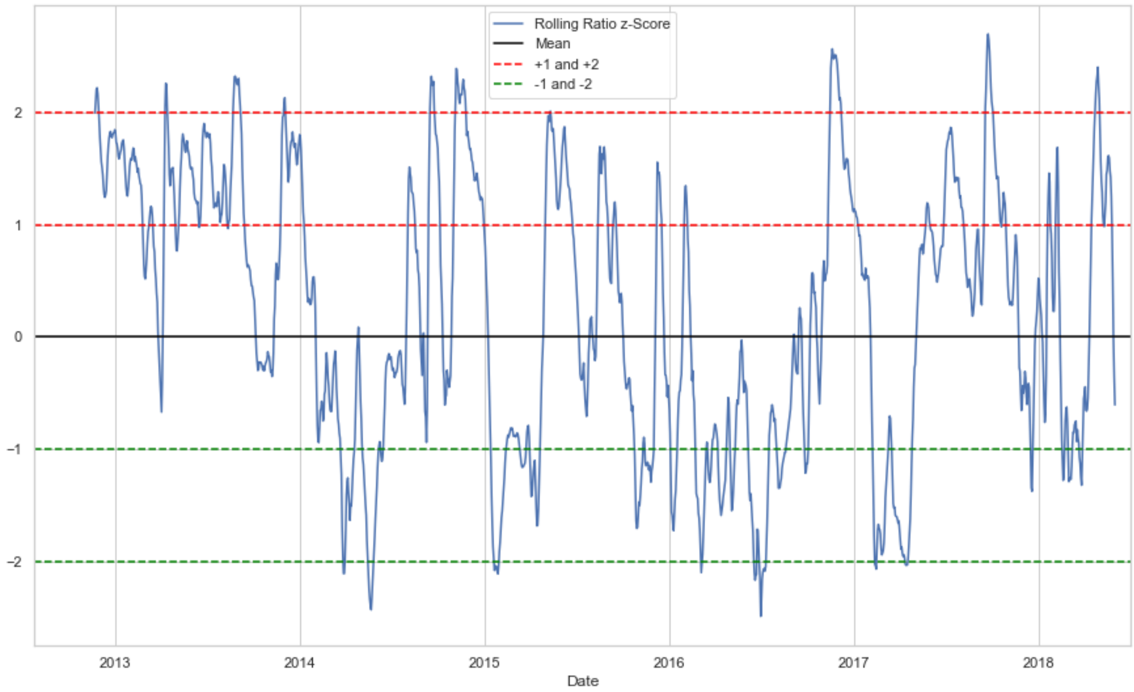
When the variable I_t is positive it indicates a time we have to buy, on the opposite when it is negative it indicates a signal to sell. It is important to notice that in this type of strategy we do not care about where is the price level of the two exchange rates we care only about their relationship and where it is going. before going in more depth about the strategy we first divided our sample in two sets one is the training set and the other one is the test set. We split our dataset in 70% training set and 30% test set. Having defined our dataset now we can move towards the problem of how to determine where the ratio is going because even if we know there is a mean-reverting trend we need to understand the ratio in $t+1$ as we have written in the equation (44). Starting from the fact that the ratio tends to move around its mean we are going to use indicators based on the mean of the time series and one of them is the moving average. The figure below shows the ratio and the 60 and 5 days moving average of the ratio and the 60 days standard deviation

Figure 3.6: USD/JPY and INR/EUR ratio, 5 and 60 days moving average

Source: Python, Eikon Thomson Reuters

Now we can plot the graph of the rolling ratio z-score which is the ratio between the difference of the 5 days and 60 days ratio moving average and the 60 days standard deviation, we can see that in the figure below

Figure 3.7: USD/JPY and INR/EUR rolling ratio z-score



Source: Python, Eikon Thomson Reuters

It is very clear from the graph above that the time series when has shocks which move it more or less than 1 or 2 standard deviations far from its mean tends to revert back to its mean. Therefore, we can create a model that allow us to exploit this feature and then buy when the z-score is below -1 standard deviations because we expect it will come back up towards its mean and sell when z-score is above 1 standard deviation because we expect it will come back down towards its mean. In the figure below we can see what we have just said till now with actual data

Figure 3.8: USD/JPY and INR/EUR trading signals

Source: Python, Eikon Thomson Reuters

It is important to notice that when we buy the ratio we are going long USD/JPY and short INR/EUR. Having said that, now since we have created our trading strategy we can see the performance during the time. If we invested 1 $ dollar at the start of the time period considered that is in 2018-06-01 we could have gotten 460 $ dollars today. We can see what we have just said in the graph below:

Figure 3.9: USD/JPY and INR/EUR pnl in the test data

Source: Python, Eikon Thomson Reuters

However, this result is from the dataset we decided to be the test. In the train set the strategy resulted in a negative profit and loss, but in the time window considered, the last 9 years, the strategy was profitable.

3.3. Evidence on currency statistical arbitrage strategies

In the previous paragraph, we built a trading strategy that is not based on the price dynamic of the spot exchange rates but the long or short position implied in the strategy take advantage of deviations from Uncovered Interest Rate Parity by investing in low or high interest rate currencies, because we have four different countries represented in the pair chosen: U.S.A., Japan, India, and Europe. The countries in the first pair had lower interest rates on average in the time period considered then the other two countries, making them representative of the idea of exploiting deviations from Uncovered Interest Rate Parity and obtain currency excess returns. This strategy is a perfect representation of our study that begun with the analysis of the determinants of the exchange rates then shift towards the understanding of the existence of currencies' excess returns explaining them as compensation for risk. At this point, the last but not least concept that is relevant to examine is who is on the opposite side of our trade. In other words, when with our strategy we are in the situation in which we go long INR/EUR, countries with higher interest rates and short USD/JAPAN countries with lower interest rates, in a simple way we can say that the return on this trade could be explained as compensation for risk if we agree with what we discussed before, but who is on the opposite side of our trade short INR/EUR ad long USD/JAPAN?

The answer to this question is based on the difference between Covered Interest Rate Parity and Uncovered Interest Rate Parity. Before we need to summarize the concept of CIP and go much deeper than before on the concept of cross-currency basis, then we will answer the question.

It is possible to express the CIP relation with the expression below:

$$(1 + i^{\$}_{t,t+n})^n = (1 + i^{foreign}_{t,t+n})^n \frac{S_t}{F_{t,t+n}} \qquad (45)$$

We can imagine having 1 dollar at time *t* and investing in a risk free rate instrument $i^{\$}_{t,t+n}$ for *n* periods or we can decide to exchange that dollar in S_t unit of foreign currency then invest in a foreign currency risk-free rate instrument $i^{foreign}_{t,t+n}$ for *n* periods but we do not want the next period exchange rate risk so we can sell that amount of foreign currency in the forward market at a predetermined exchange rate $F_{t,t+n}$. There is no credit risk because we have assumed risk-free rate and there is no currency risk because we invest on the left-hand side directly in the U.S. risk free rate and that must be equal to the right-hand side where we have a sort of synthetic dollar rate. In the industry the deviations from CIP are defined with an expression that is cross-currency basis so we can add this term on the previous equation and we get:

$$(1 + i^{\$}_{t,t+n})^n = (1 + i^{foreign}_{t,t+n} + z_{t,t+n})^n \frac{S_t}{F_{t,t+n}} \tag{46}$$

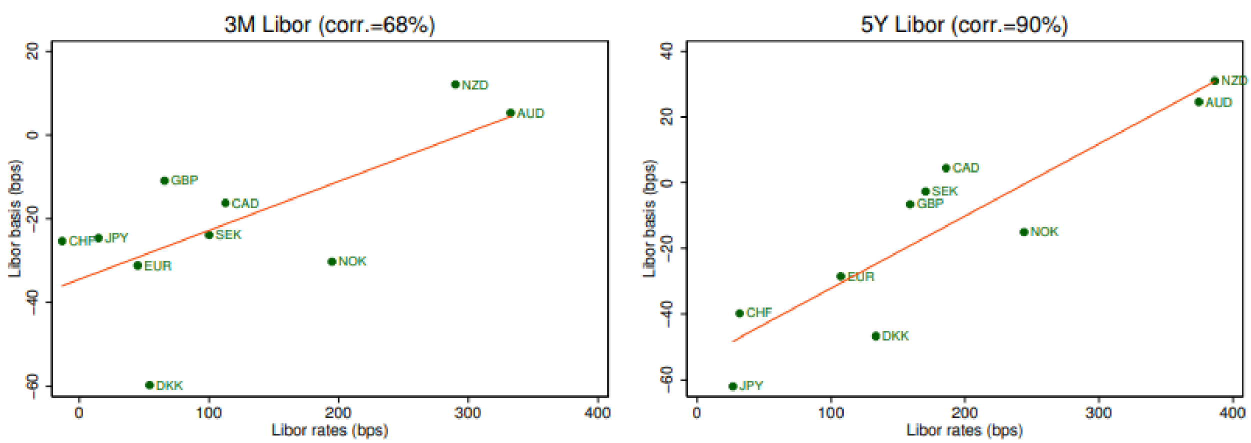
That term $z_{t,t+n}$ represents deviations from CIP and can be defined as:

$$z_{t,t+n} = i^{\$}_{t,t+n} - (i^{foreign}_{t,t+n} - \rho_{t,t+n}) \tag{47}$$

In the equation above there is a difference between two rates, both of them are two dollar rates but different. The first one $i^{\$}_{t,t+n}$ is the rate on the cash market the second term between the parenthesis is the synthetic dollar interest rate which is the rate on a foreign currency minus the rate we need to pay in the forward swap market to hedge or swap our foreign currency position and get back dollars. Therefore, it is clear that the two terms should cancel each other out. We can say that whenever the term $z_{t,t+n}$ is not zero then there are opportunities in the market for arbitrageurs investing at the higher rate and borrowing at the lower rate with currency risk hedged.

After the presentation of these concepts now we can introduce a really interesting result from a paper by Wenxin Du and Alexander Teper, and Adrien Verdelhan[32]. They did a cross section of the variations in cross currency-basis from 2010 to 2016. The result is shown in the figure below:

Figure 3.10: cross currency-basis



Source: Wenxin Du, Alexander Tepper, and Adrien Verdelhan "Deviations from Covered Interest Rate Parity." Journal of

[32] Wenxin Du, Alexander Tepper, and Adrien Verdelhan "Deviations from Covered Interest Rate Parity." Journal of Finance

The figure above shows on the y axis the cross-currency relationship between different cross-currency bases and on the x-axis the nominal interest rate. They found and is also possible to see that on the graph that low interest rate currencies have the most negative bases and high interest rate currencies have the less negative bases or positive. Therefore arbitrageurs will tend to take long position in low interest rate countries and short in high interest rate countries, hedging the currency risk of their positions with exchange rates swaps for example. At this point, we have to remember what we said before about the direction of our trading strategy which relies on deviations from Uncovered Interest Rate Parity, which was the opposite of what we have just said because in that case we went long high interest rate currencies and short low interest rate currencies with an also called unhedged carry trade. This can be explained by who are the subjects on the two sides of the trade. On the one hand, we have hedge funds and big investors who want to collect the UIP through the carry trade they are the client demand side of the global banks and these intermediaries have to take the opposite side of the trade collecting the CIP charging fees for this service. In the end clients take the currency risks while the global banks or dealers take the CIP hedging their risk. The last point we can underline is the reason why the excess returns in the forward markets are larger than on the cash markets, as we have seen before.

$$\begin{aligned} f_t - s_{t+1} &= f_t - s_t + s_t - s_{t+1} \\ &= y_t^{foreign} - y_t^{\$} - \Delta s_{t+1} + z_t \end{aligned} \tag{48}$$

The equation (48) shows the unhedged carry trade. If an investor borrows in U.S. dollars or some low interest rate currency and invests in a high interest rate currency the cross-currency basis is positive $z_t > 0$. The result is that the excess return that our investor can obtain is greater on the forward market $f_t - s_{t+1}$ than on the cash market $y_t^{foreign} - y_t^{\$} - \Delta s_{t+1}$. What we have just explained clarifies the result we have found when we were analyzing the portfolios of currency excess returns where the portfolios on forward markets performed better than the ones on the cash market.

Conclusion

In this work, we started studying the forces that determine the level of exchange rate explaining why the exchange rates are not random. In the second part, we tried to decompose the currency excess return and explain the existence of some risk factors that can explain those returns. However, the factors that we have found are just some of more variables that could explain currency risk premia and so understand new sources of risk in currency markets and nonetheless raising more questions about the predictability of the exchange rates. Further developments of this work could be focused on finding a time-varying currency risk premia which could have the power to be a driver of the exchange rate. It is clear that these topics are really relevant for investors, financial intermediaries and firms who are exposed to currency risk or would exploit new opportunities in the FX market. In the last part of this work we presented a trading strategy which could be improved using more currency pairs and more varied time periods, other improvements could be dealing in a better way with overfitting with like validation, moreover the strategy could be modified creating new and maybe better trading signals and not just rely on the ratio of the pair considered. In the end, we investigated who are the subjects behind the two sides of a long-short strategy in the FX market understanding better the concept of UIP and CIP. Most importantly understanding the big players in the FX market allows us to know the flow of funds within the market that is a very important concept and crucial if you are an investor.

References

John V. Campbell, Richard H. Clarida, “The Dollar and Real Interest Rates,“ National Bureau of Economic Research, (1987)

Lustig, H., Roussanov, N., & Verdelhan, A. (2011). Common Risk Factors in Currency Markets. Review of Financial Studies;

Tarek A Hassan & Rui C Mano, 2019. "Forward and Spot Exchange Rates in a Multi-Currency World," The Quarterly Journal of Economics

Hanno Lustig & Andreas Stathopoulos & Adrien Verdelhan, 2019. "The Term Structure of Currency Carry Trade Risk Premia," American Economic Review

Menzie Chinn, Guy Meredith 2005. “Testing Uncovered Interest Parity at Short and Long Horizons during the Post-Bretton Woods Era”. National Bureau of Economic Research

Robin Greenwood, Samuel G. Hanson, Jeremy C. Stein, Adi Sunderam,2020. " A Quantity-Driven Theory of Term Premia and Exchange Rates " Harvard University and NBER, National Bureau of Economic Research

Jiang, Zhengyang, Arvind Krishnamurthy, and Hanno Lustig. 2018. "Foreign Safe Asset Demand for US Treasurys and the Dollar." AEA Papers and Proceedings.

Eugene F. Fama, and Kenneth R. French. 1992. "Common risk factors in the returns on stocks and bonds" University of Chicago USA.

Mikhail Chernov & Drew D. Creal, 2018. "International Yield Curves and Currency Puzzles," NBER Working Papers 25206, National Bureau of Economic Research, Inc.

Hanno Lustig & Nikolai Roussanov & Adrien Verdelhan, 2011. "Common Risk Factors in Currency Markets," Review of Financial Studies, vol 24(11),

Lustig, Hanno, and Adrien Verdelhan. 2007. "The Cross Section of Foreign Currency Risk Premia and Consumption Growth Risk." American Economic Review, 97 (1)

Tarek A Hassan & Rui C Mano, 2019. "Forward and Spot Exchange Rates in a Multi-Currency World*," The Quarterly Journal of Economics, vol 134(1)

Wenxin Du, Alexander Tepper, and Adrien Verdelhan “Deviations from Covered Interest Rate Parity.” Journal of Finance

Summary

This work can be divided into three parts. In the first one, we have seen that currency markets and bond markets are tightly connected. Indeed, exchange rates tend continuously to adjust to ensure that returns are in line with risk for both foreign and domestic bond market investors. If we agree on that relation, we can easily see that exchange rates in currency markets are priced in a very similar way as stocks or other instruments, and they reflect a set of different forces. Exchange rates absorb shocks to future interest rates, and we can think about that as the cash flow component that accrues to an investor who takes a long position in the stock market compared to a long position in the foreign currency. This concept is expressed in the first model we presented where we arrived at the first fundamental exchange rate valuation equation, an expression which tells us that the exchange rate will reflect future short term interest rate differential minus future currency risk premium plus a sort of noise term which reflects the investor's expectations at time *t* of the future exchange rate. That equation was originally derived by Campbell and Clarida but it is still used to analyze the determinants of the exchange rate. That equation tends to hold very generally. In this model, there are two main forces. The first one is that the dollar exchange rate at time *t* which is going to reflect a cash flow term which is the difference in short term interest rates in future periods and the second one which is a discount rate term that reflects the currency risk premium and can be seen as short-term deviations from uncovered interest rate parity. We can think of this first model as if the interest rate on a low interest rate country is consistently lower than the foreign interest rate, the exchange rate to be stationary the currency risk premium should offset the difference in interest rates. Therefore, it is reasonable to say that exchange rates adjust continually to ensure that returns are in line with risk for both foreign and domestic investors.

Then we presented a second model that tries to understand the type of relationship that exists between long term interest rates and exchange rates. Now we considered long-term interest rates as the cash flow component which before was captured by short term interest rate. In this framework, we have three forces that contribute to determine and influence exchange rates that are: long-term interest rates, short-term deviations from Uncovered Interest Rate Parity in currency markets, and deviations from Expectations Hypothesis in bond markets. We arrived at a new expression of the fundamental exchange rate valuation equation where the first term represents the difference in long-term interest rates, the second term represents the difference in bond risk premia and this term can be seen as a factor that captures deviations from the Expectations Hypothesis, the third term represents deviations from short term Uncovered Interest Rate Parity, the same term that we encountered before, the last term captures investor's expectations at time *t* of the future exchange rate. The first conclusion which can be derived from this model is intuitive because the domestic exchange rate will appreciate if long-term interest rates, on the long end of the yield curve, go up today relative to foreign yields. Moreover, the domestic exchange rate will tend to appreciate when the domestic bond risk premium decreases, and finally, the foreign exchange rate will appreciate when the future foreign currency risk premium decreases. From that point, we assumed that exchange rates are stationary or in other words shocks to long-run exchange rates are not priced in the market. Then what we saw is that the exchange rate will reflect differences in long term interest rates today. At the end, the only term with which we remained was the difference in long-term interest rates which reflect exchange rate movements away from its mean. We explained this effect with the fact that in the long run Uncovered Interest Rate Parity restores and this happen because if we assume the exchange rate is stationary then is fair to say that in the long term there is no additional risk for a foreign investor from investing in a foreign bond market relative to its domestic bond market. Therefore, if in the long term as we just said investors are risk-neutral then Uncovered Interest Rate Parity will hold in the long run. These results were useful to get some interesting evidence about the existence or not of currency carry trade because in the second part of this work we analyzed the currency excess returns from currency portfolios based on the level of interest rates. Remaining on the relation between spot exchange rate and long-term interest rates, it is very interesting to underline that in literature there have been found evidence of the fact that high currency risk premia are offset by negative bond risk premia. In other words, there is no currency carry trade to be earned because if we go long in longer maturity bond of countries that have high interest rates and short longer maturity bond of countries that have low interest rates this type of trade will not give us a large excess return. That because the foreign currency risk premia which are low for low interest rates countries is going to be offset by higher term premium so in the end choosing high interest rates on the long end of the yield curve does not seem to pay and there is evidence in the literature as we said before that confirm this point and so long run Uncovered Interest Rate Parity holds. However, this type of models cannot capture all the changes in exchange rates because there are permanent deviations of the exchange rate that are priced by investors in currency and bond markets that leads our models to a no "offset effect" between bond risk premia and currency risk premia, explaining deviations from long run Uncovered Interest Rate Parity. These differences are permanent country specific innovations to the pricing equation that lead to deviations from long run Uncovered Interest Rate Parity and also long run Purchasing Power Parity. In other words, it will not hold because long run risk in exchange rates exists. Another type of model of the exchange rate is the segmented bond market model which is based on the fact that an increase in, for example, U.S. supply of long-term bonds, lead arbitrageurs to demand higher bond risk premia, keeping all other variables constant. Therefore, an increase in the long-term bond risk premia in the U.S. leads to an increase in the U.S. long yields that cause an appreciation of the dollar immediately and following depreciation over time to offset the effect of the higher yield. In other words, in the long-term foreign investor does not need a higher return for holding a U.S. dollar bond, because there is no long-run risk, because the investor understands that the dollar will mean revert back to its long-run mean in the long run. The last model we presented in the first part of this work is based on the concept of convenience yields. The model tries to capture the fact that if the future convenience yield that the investor can get from a long position in US Treasurys goes up we expect the dollar to appreciate today to generate an expected future depreciation and thus lower the perceived return for the foreign investor from holding a long position in US Treasurys to restore equilibrium. This model we have just presented has an issue coming from the fact that we need a proper way to measure these convenience yields. We used some works from the literature to come up with a formula to compute convenience yields based on the concept of covered interest rate parity and currency basis. This last point allowed us to show also why if we think about safe asset demand as a convenience yield, it will lead to changes in the exchange rates. The first part of this work lets us understand that there are forces that drive changes in the exchange rate and that there is some risk factor behind the behavior of the exchange rates.

In the second part of this work, we started by explaining the importance of studying exchange rates and by defining the currency excess return. When the Uncovered Interest Rate Parity model is chosen it is made an assumption of risk neutrality, in the sense that the expected excess return is assumed to be 0 because the positive or negative difference in interest rates differences should be offset by changes in the exchange rates which then solves as a balance function. We have shown that if we regress the changes in the exchange rate on interest rates difference the $\beta \neq 1$ it is actually negative, we can think about this evidence as the expected excess return is time-varying and interest rates difference should be one of the forces, as we have seen in the first chapter in the case of the exchange rate, of the expected excess return. Later, we sorted 17 countries and built 6 currency portfolios with the countries with lower interest rates in the firsts portfolios and the ones with higher interest rates in the last ones. We considered a U.S. investor who borrows in his domestic currency, 3 months U.S. Treasury bills, and invests for 3 months in a foreign currency. The only source of risk for the investor is the change in the exchange rates in the time period in which the investor is exposed to the foreign currency risk of depreciation. We showed the performances of the portfolios and then started some asset pricing experiments. It is reasonable to believe that high interest rate currencies will depreciate in bad times. An easy way to define bad times, useful for our analysis, and coherent with our assumptions is consumption growth. Therefore, when consumption growth is low, we have a proxy of a bad time for our investor, the opposite when high. The main idea of this test we presented is that when consumption growth in the U.S. decreases signaling bad times for the economy the U.S. investor will lose money by investing in high interest rate currencies and it is possible to see this relation in the opposite way, our U.S. investor will make money on average by investing in low interest rate currencies in bad times because in that case, low interest rate currencies will appreciate, thinking about them as safe haven during periods of high uncertainty or recession for example. A very interesting thing is that it is possible to see a small tendency of larger consumption betas in portfolios of countries with higher interest rates. This can be seen as a sort of greater sensitivity of these portfolios towards changes in consumption data. This idea is also confirmed by the fact that the last portfolios have more countries than the first one therefore it should embed a diversification effect that should balance this sensitivity. However, this is not the case because the results show us a greater loading on the risk factor. The second step was to introduce a different expression of currency excess returns based on forward markets. It was very interesting to see that in this analysis even if we take into account the 2008 Great Financial Crisis which we did not consider in the previous analysis with interest rate differences the carry in the forward market is greater than on the cash market in the post-crisis period, as we saw with higher portfolios returns on average across the different portfolios. Then we run some asset pricing experiments, building two risk factors with the same goal that is to investigate if those portfolios' excess returns are compensation for risk and so if risk premia exist in currency markets. The HML factor in our analysis is constructed by taking the difference between the returns on the portfolio with high interest rate countries and the returns on the portfolio with low interest rate countries. We run the OLS regression of each currencies portfolios' excess returns on the HML factor built. The differences in the β_{HML} resulted is clear, the betas went from negative betas to positive betas across the different portfolios. The result is a sign of the existence of risk premia in currency markets because the β_{HML} can explain on average the currency portfolios excess returns as the covariance between the returns and the risk factor we have built. About the second risk factor, we focused on the volatility of the equity markets. We used as a proxy of the volatility of the equity markets the CBOE VIX INDEX. We regressed the currencies portfolios excess returns on the change in the monthly volatility in the equity market. It is common to think that in periods where volatility is high there is a lot of uncertainty in the market so we can think about it as bad times and vice versa. The betas β_{VOL} shows a very interesting result. The betas decrease as we move towards the currencies portfolios with high interest rate countries whose excess returns are positive on average in the time period considered. This result shows that when volatility is high or there is a spike in the equity volatility the returns on the portfolios with low interest rates currencies remain positive, therefore exposing a positive correlation between the firsts portfolios and the equity volatility. On the other hand, the last portfolios made by countries with high interest rates show negative betas presenting a negative correlation between their returns and volatility, meaning that when there is a spike in equity markets' volatility these portfolios will tend to lose money compared to the firsts ones. To summarize, there is a different amount of risk between the firsts currencies portfolios and the last ones because when volatility is high the investor who invested in high interest rate countries will lose money because those currencies will depreciate. Instead, the investor who invests in countries with low interest rates will make money in bad times because those currencies will tend to appreciate. We then studied the behavior of our portfolios during periods of high volatility. At first, we can compute the mean and the standard deviation of our equity volatility index in the time period considered. If we assume a normal distribution of the data, we can compute what is the value above more than one standard deviation from its mean of our volatility index with a 90% confidence level. Then, we extracted all the currency portfolios returns in the period in which the volatility index was between one and two standard deviations from its mean. The results showed that when there are big shocks the firsts portfolios can be seen as a means of insurance because the investor who invests in those portfolios loses money on average but makes money on average when volatility spikes up. On the opposite, in the lasts portfolios, the investor who invests there makes money on average but loses a lot of money on average when volatility increases. This last evidence is in perfect coherence with what we found before because it strengthens our hypothesis of the existence of some compensation for risk in currency markets explaining those excess returns.

In the last part of the second chapter, we used the relationship between exchange rates and long-term interest rates to show something really interesting. We considered the G4 countries, Brazil excluded, and we computed the carry trade returns using the same logic we have explained before. Therefore, there is an investor who borrows for one month in the U.S. bond market and invests for the same time period in a foreign currency bond market, as we noticed before the only source of risk in this trade is the change in the exchange rate between the time period in which our investor is exposed to the foreign currency movements in the market. Now, we assumed the possibility to use bonds with higher maturities, 3 months, 1 year, 5 years, 10 years, but with the same holding period for our investor. We showed that the curve of excess returns has a negative slope. One reason for that is the choice of the countries because we have a country with high interest rates such as India and countries with lower interest rates such as Europe and Japan, but this is not the point we want to emphasize. The interesting point was that there is a clear negative trend as we move towards the average carry trade returns made with longer maturities bonds for every country we have considered. For example, the trade made with borrowing the 3 months U.S. Treasury Bill and investing in the 3 months Japan bond is higher than the one made with borrowing the 1 year U.S. Treasury bond and investing in the 1 year Japan bond which is higher than the one made with the 5 years which is higher than the one made with the 10 years. Therefore, is clear that this type of strategy is more profitable with short-term bonds than with long-term bonds. The reason for that is the concept we have studied in the first chapter. If we consider bonds with longer maturities we are adding a new source of risk, interest rate risk. It turned out that this compensation for interest rate risk also called term premium moves in the opposite direction with the currency risk premium. When our investor goes long in foreign currency and short in its domestic currency, he is taking some currency risk but also an additional source of risk, the interest rate risk. We can say that risk premia exist in currency markets and we have seen how different risk premia interact with each other.

In the last part of this work, we started by defining what is the meaning of statistical arbitrage and where it comes from. Later, we used all the evidence and theory found to create an arbitrage strategy in currency markets. At first, we defined the meaning and history behind the statistical arbitrage and pairs trading, and then we introduced the mathematical concepts of stationarity, integration, and cointegration useful for the building of our trading strategy. We implemented a forex pairs trading strategy, using the concept of cointegration based on statistical arbitrage using a mean reversion logic. The first step was to build a dataset of daily spot exchange rates in the period between 2012 and 2020, both included. We looked through 82 spot exchange rates, considering 42 different countries, to see if any of them were cointegrated if there was a possibility for a statistical arbitrage strategy. We used the ADF test to see if there were some exchange rate pairs cointegrated, where the null hypothesis says that there is no cointegration between the series considered while the alternative hypothesis says that there is cointegration between them. In all the cointegrated series, we decided to study the pair USD/JPY with INR/EUR. The choice of this pair is motivated also by the fact that in this way we can study if it is possible to get currency excess returns using statistical arbitrage strategies exploiting deviation from Uncovered Interest Rate Parity because we have a pair that represents countries with low and high interest rates. We computed the spread using the Engle-Granger method, the linear regression to get the coefficient to construct the linear combination between the two exchange rates, and then we examined the ratio between the two time series, which is what we used in the construction of our trading strategy. We saw something we wanted that there is a sort of mean-reverting trend in the series. We standardized the ratio to have a better series to study than just the absolute ratio. For this purpose, we used the z-score. This representation showed us in a better way the fact that deviations from the mean tend to move back, and this is what we wanted for our trading strategy. Then we created a signal that allow us to understand when enter into the market and in what direction. We used the rolling ratio z-score which is the ratio between the difference of the 5 days and 60 days ratio moving average and the 60 days standard deviation. From the graph, we showed it was very clear that the time series when has shocks which move it more or less than 1 or 2 standard deviations far from its mean tends to revert back to its mean. Therefore, we can create a model that allows us to exploit this feature and then buy when the z-score is below -1 standard deviation because we expect it will come back up towards its mean and sell when z-score is above 1 standard deviation because we expect it will come back down towards its mean. After having run our model both on the training dataset and then on the test dataset we realized that in the time window considered, the last 9 years, the strategy was profitable. Then we presented a question when with our strategy we are in the situation in which we go long INR/EUR, countries with higher interest rates and short USD/JAPAN countries with lower interest rates, in a simple way we can say that the return on this trade could be explained as compensation for risk if we agree with what we discussed before, but who is on the opposite side of our trade short INR/EUR ad long USD/JAPAN?

The answer to this question is based on the difference between Covered Interest Rate Parity and Uncovered Interest Rate Parity. Before we summarized the concept of CIP and then we move to the concept of cross-currency basis. The work of Wenxin Du, Alexander Tepper, and Adrien Verdelhan "Deviations from Covered Interest Rate Parity" studied the cross section of the variations in cross currency-basis from 2010 to 2016 and it showed that low interest rate currencies have the most negative bases and high interest rate currencies have the less negative bases or positive. Therefore arbitrageurs will tend to take long position in low interest rate countries and short in high interest rate countries, hedging the currency risk of their positions with exchange rates swaps for example. At this point, we have to remember what we said before about the direction of our trading strategy which relies on deviations from Uncovered Interest Rate Parity, which was the opposite of what we have just said because in that case we went long high interest rate currencies and short low interest rate currencies with an also called unhedged carry trade. This can be explained by who are the subjects on the two sides of our trade. On the one hand, we have hedge funds and big investors who want to collect the UIP through the carry trade they are the client demand side of the global banks and these intermediaries have to take the opposite side of the trade collecting the CIP charging fees for this service. Last but not least we explained the result we found when we were analyzing the portfolios of currency excess returns where the portfolios on forward markets performed better than the ones on the cash market. If an investor borrows in U.S. dollars or some low interest rate currency and invests in a high interest rate currency, the cross-currency basis is positive. The result is that the excess return that our investor can obtain is greater on the forward market than on the cash market, and we showed that in the last pages of this work.

References

John V. Campbell, Richard H. Clarida, “The Dollar and Real Interest Rates,“ National Bureau of Economic Research, (1987)

Lustig, H., Roussanov, N., & Verdelhan, A. (2011). Common Risk Factors in Currency Markets. Review of Financial Studies;

Tarek A Hassan & Rui C Mano, 2019. "Forward and Spot Exchange Rates in a Multi-Currency World," The Quarterly Journal of Economics

Hanno Lustig & Andreas Stathopoulos & Adrien Verdelhan, 2019. "The Term Structure of Currency Carry Trade Risk Premia," American Economic Review

Menzie Chinn, Guy Meredith 2005. “Testing Uncovered Interest Parity at Short and Long Horizons during the Post-Bretton Woods Era”. National Bureau of Economic Research

Robin Greenwood, Samuel G. Hanson, Jeremy C. Stein, Adi Sunderam,2020. " A Quantity-Driven Theory of Term Premia and Exchange Rates " Harvard University and NBER, National Bureau of Economic Research

Jiang, Zhengyang, Arvind Krishnamurthy, and Hanno Lustig. 2018. "Foreign Safe Asset Demand for US Treasurys and the Dollar." AEA Papers and Proceedings.

Eugene F. Fama, and Kenneth R. French. 1992. "Common risk factors in the returns on stocks and bonds" University of Chicago USA.

Mikhail Chernov & Drew D. Creal, 2018. "International Yield Curves and Currency Puzzles," NBER Working Papers 25206, National Bureau of Economic Research, Inc.

Hanno Lustig & Nikolai Roussanov & Adrien Verdelhan, 2011. "Common Risk Factors in Currency Markets," Review of Financial Studies, vol 24(11),

Lustig, Hanno, and Adrien Verdelhan. 2007. "The Cross Section of Foreign Currency Risk Premia and Consumption Growth Risk." American Economic Review, 97 (1)

Tarek A Hassan & Rui C Mano, 2019. "Forward and Spot Exchange Rates in a Multi-Currency World*," The Quarterly Journal of Economics, vol 134(1)

Wenxin Du, Alexander Tepper, and Adrien Verdelhan “Deviations from Covered Interest Rate Parity.” Journal of Finance