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Regime-Switching Models and Covid-19: A MATLAB Implementation on Ratti S.p.A.

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To my parents, who have been supporting and trusting me in each and every step of my life. Thanks for conveying me values such as commitment and dedication.

To my brother Federico, for his infinite patience in revealing me MATLAB secrets.

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Introduction

An option is a possibility, power of choice, or the freedom of alternatives. An option is a right, however not an obligation, as an example, to follow through on a business decision. In financial markets, it is the freedom of choice after revelation of additional information that increases or decreases the value of the asset. A “real” option is an option “relating to things”, fixed or permanent, as opposed to abstract things. A financial call option gives the owner the right, but not the obligation, to purchase the underlying stock in the future for a price fixed today. A financial put option gives the owner the right, but not the obligation, to sell the stock in the future for a price fixed today. The managerial operating flexibility is likened to financial options. As new information arrives and uncertainty about market conditions is gradually resolved, management may have valuable flexibility to alter the initial operating strategy. They may be able to defer, expand, abandon, or otherwise change the project during its operating life in order to capitalize on favourable future opportunities or to react in order to mitigate losses. Managerial decisions create call and put options on real assets that give management the right, but not the obligation, to employ those assets to achieve strategic goals and in the long run maximize the value of the firm. The key advantage and value of real option analysis is to integrate managerial flexibility into the valuation process and thereby assist in making the best decisions.[1] The actual marketplace, characterized by change, uncertainty and competitive interactions, the realization of cash flows will probably differ from what management expected. Nowadays, in the face of omnipresent real options, most theorists and practitioners believe that real options should be considered when analysing corporate decisions. In this thesis I will employ the real option approach to study firms’ optimal strategic decisions in the event of pandemics. In details, I will use a MATLAB implementation to determine the optimal suspension-reactivation triggers, where reactivation decision is viewed as a call option and the suspension as a put option, using a dynamic programming method to obtain optimal switching thresholds applied to a real firm.

Operational risk is defined by the Basel II regulations as "the risk of a change in value caused by the fact that actual losses, incurred for inadequate or failed internal processes, people and systems, or from external events differ from the expected losses"[2]. An influenza pandemic is one of those examples. In the event of pandemics, businesses play a special role in protecting

[1] Brach, M. 2003. *Real options in practice*
[2] Basel Committee on Banking Supervision. 2004. *International convergence of capital measurement and capital standards*

employees' health and, at the same time, minimizing economic losses. The Department of Health and Human Services (HHS) and the Centers for Disease Control and Prevention (CDC) have developed a checklist in order to assist businesses to plan in case of outbreak of a pandemic. This checklist identifies the necessary activities for large businesses to establish policies for implementations during a pandemic. In particular, it requires firms to establish procedures for activating and temporarily shutting business operations.

This work is motivated by the concern of a new pandemic now that the problems it can give rise to are real and tangible.

In the course of Chapter I, I will analyse the importance of developing epidemiological models in the context of corporate finance and I will describe the different models that have been developed so far. Finally, the importance of adding stochasticity to the models and their connection with the world of real options will be examined through a theoretical example.

In Chapter II, I will provide a rough calibration to Covid-19 using a modified SIR model allowing for deaths, which will be useful to obtain parameters which come close to reality. Those parameters will be used in the second part of Chapter II.
Then I will propose a two-stage model to address the following research questions:

- In the event of an epidemic, should the firm keep operating considering the loss of productivity of its employees or temporarily suspend operations to circumvent contagion?
- Is the company's objective to maximize value in conflict with its desire to control the disease?
- What are the optimal triggers for businesses to implement the suspension-reactivation strategy?

Within the first stage, I will adapt an epidemic model to explain the stochastic dynamics of an ailment that spreads in a given organization, taking into consideration external contagion and deaths from the virus.
This work assumes that the productiveness of a worker decreases when he gets the disease. With the spread of the ailment, the proportion of infective workers tends to increase day by day, thus harming the firm's productivity and, by consequence, its revenues.
The "mothballing threshold" is the percentage of infected employees over which it is better to temporarily shut down the business and dismiss employees whether they are infected or not.

On the other side, when the percentage of infected employees drops below a certain low threshold ("reactivation threshold" thereafter), the business can be resumed, and employees can be called back to work[3].

In the second stage, I will use a regime-switching model with MATLAB to find these optimal thresholds, based on the theory of real option valuation. This model will be applied to a real-world company operating in the silk fabric sector, the Ratti S.p.A. I will describe how this particular sector reacted to Covid-19 and the reasons that led me to choose this company.

In Chapter III, analysis of results and graphs will be provided.

Final results showed that it is optimal for the active regime to suspend operations when the fraction of infected employees reaches the threshold of 51%, while when the company is already in the inactive regime, it is optimal to reactivate operations when the fraction of infectives drops below 4%, thus leading to a Mothballing Threshold equal to 51% and a Reactivation Threshold equal to 4%.

Since the parameters were somewhat approximate and not entirely certain, it was necessary to "play" with data and analyse the results as those parameters changed. Results showed interesting features.

Firstly, I analysed the results changing the model from stochastic to deterministic, thus giving the volatility coefficient a value equal to 0. Mothballing Threshold remained unchanged to 51% but the Reactivation Threshold increased to 5%. This shows that when there is no uncertainty regarding the dynamics of the virus, firms are less conservative in making the suspension-reactivation strategy, because businesses will be put back in operation with a slightly higher fraction of infectives.

Then, I analysed the behaviour of thresholds by changing switching costs and some basic parameters: the external infection rate, the internal infection rate, and the recovery rate in the active regime.

Concerning the changing switching costs, results showed that by increasing one of the two costs at a time, the Reactivation Threshold decreases, and the Mothballing Threshold increases,

[3] Chen, H. and S.H. Cox. 2010. *An Option-Based Operational Risk Management Model for Pandemics*

meaning that managers will be more reluctant in the decision of temporarily shut down the business when the Mothballing Cost increases, and will be less willing to reopen when the Reactivation Cost increases. This conclusion is quite straight and in line with my expectations.

For what concerns the effect of diseases control strategies, results showed that with increasing both internal and external infection rates at a time, both thresholds were decreasing, meaning that firms will are more prudent and wait for less workers to get infected before both temporarily suspending and reopening operations.

The opposite effect was given by changing the recovery rate in the active regime, because both thresholds increased with the recovery rate. This is quite intuitive, because as the internal recovery rate increases, managers will be less prudent and wait for more workers to get infected before suspending the business and will reopen with more infected workers.

Results show that managers have actually a great responsibility over their businesses because they can control the main contagion parameters within their firm. It is on them that the decision-making power regarding the optimal suspension-reactivation triggers resides. And it is upon them that these thresholds change depending on how they keep internal contagion under control. Their aim should be that of increasing the value of their firm, which is guaranteed by keeping operations open but at the same time take care of workers' health, especially because their productivity is damaged when they are infected. This is not possible when security-measures are weak, and the safety of employees takes second place.

Chapter I

An overview of epidemiological models and real options

1.1 The importance of developing epidemiological models applied to businesses

In the realm of infectious diseases, a pandemic is the worst-case scenario. An epidemic becomes pandemic when it spreads beyond a country's borders. The spread global epidemics has gone hand in hand with the globalisation of mankind. in this modern era, outbreaks are nearly constant, though not every outbreak reaches pandemic level as Covid-19[4] has, which we are currently fighting against. To date, 32 pandemics have occurred in the past 500 years and three in the past century. Historic data reveals that influenza pandemics occur with frightening regularity every 30 to 50 years[3].

Given this pattern, it is reasonable to develop economic models in order to be better prepared in the event of other outbreaks. Impacts on almost all kind of business organizations are staggering: businesses would have to shut down for quarantine, firm's earnings would plunge and leading to default on corporate debts. Furthermore, consumers' confidence might crash, thus deteriorating financial distress even more. Without any improvement in our techniques for fighting this invisible war, sacrifices by households and businesses will be startling[5].

According to searches carried out by Area Studi Legacoop[6], Italian economy due to the pandemic has lost €150 billion in 2020, with a collapse of 8,9%, a percentage twice that of the average world GDP (-4,4%). The loss can be divided into 108 billion in consumption, 16 billion in investments and 78 billion in exports. Concerning employment, the report indicates that at the end of 2020 those employed in Italy were 435,000 fewer than the previous year. The greatest losses were concentrated among fixed-term employees (-412.000), self-employed workers (-141.000). The crisis has also widened the differences in economic dynamics between households and businesses. For Italian households, it has been estimated that disposable income is down by a total of 30 billion against savings that have grown (131 billion, they had been 71 in 2019) and an average propensity to save that has almost doubled (from 8,2% in 2019 to 15,6% in 2020). A similar difference was also recorded among businesses. In fact, the report

[4] Visual Capitalist. 2020. *Visualizing the History of Pandemics*
[5] Mulligan, C. 2020. *Economic Activity and the Value of Medical Innovation during a Pandemic*
[6] Legacoop. 2020. *Covid-19: Prometeia-Area Studi Legacoop; nel 2020 persi 150 miliardi di PIL, 108 di consumi, 78 di esportazioni*

pointed to the phenomenon that - for precautionary reasons and linked to the uncertainty of the outlook - companies have increased their recourse to loans, while keeping the funds acquired on current accounts.

The lost surplus from market activity, while massive, understates the true costs of sacrifices that households and businesses are making. This is why better techniques for fighting the war are highly beneficial.

1.2 Basic Epidemiological Concepts

Mathematical models can provide precious tools to public health authorities for the management of epidemics, potentially contributing to limiting the portion of infected people and victims. These models can be used to reap long and short-term forecasts, which allow decision-makers to optimize accessible control policies, such as lockdowns and vaccination campaigns. Models are also very beneficial in other duties which include the estimation of transmission parameters, analysing the contagion mechanisms and simulation of different epidemic scenarios.
History of epidemiological models can be traced back to pioneers such as Kermack and McKendrick (1927). Since the publication of Bailey in 1957, mathematical epidemiology has become a meticulous discipline. The wide diversity of models developed to date can be categorized into two major streams: deterministic models and stochastic models. Within the real world, the unfold of viruses through a population is a stochastic process but deterministic models are used to obtain satisfactory approximations for relatively vast populations. New researchers have used epidemic modelling to design most effective control policies which include immunization, quarantines, and worker furloughs. However, these models only evaluate effectiveness of epidemic control strategies based on country-wide desires and lead cost-benefit analysis from the macroeconomic standpoint. They do not deliver directives for large corporations to prepare for pandemics nor provide instructions about the implementation of optimal control policies.

All epidemic models have a common feature, which dividing the modelled population into two different health states: susceptibles (S) and infectives (I) and studying the disease transmission among these classes. Beside the simple SI model, other elaborated models incorporate other groups, of which the most used is the recovered (R).

The group S represents the group of people who are healthy but vulnerable to the disease. The class I denotes the individuals who have been infected and can contaminate others. The class R represents people who have recovered from the infection and have acquired immunity[3].

Scientists use a basic measure to track the infectiousness of a disease called the basic reproduction number (o reproduction ratio) — also known as R_0, that indicates how contagious an infectious disease is. It tells the average number of people who will contract a contagious disease from one person with that disease. It applies to a group of people who was previously free of infection and have not been immunized through vaccination[7]. For example, if a disease has an R_0 equal to 3, a person who has the disease will transmit it to an average of 3 other people. That repetition will go on if no one is vaccinated against the disease or is already immune to it.

Three possibilities exist for the potential transmission or decline of a disease, depending on its R_0 value:

- If $R_0 < 1$, every existing infected individual causes less than one new contamination. In this case, the ailment will decline and ultimately die out. The purpose of policymakers is actually to maintain this term below 1.
- If $R_0 = 1$, each existing infection causes one new infection. The disease will stay alive and stable, but there will not be an epidemic outbreak.
- If $R_0 > 1$, each existing infected individual causes more than one new infection. The disease will be transmitted between people quickly, and there it is probable that there will be an outbreak.

1.3 Three simple deterministic epidemiological models: SI, SIR and $SEIR$ models

1.3.1 The SI Model

The Susceptible-Infective model considers a moderate epidemic in a closed population, which means that there is no entrance nor leaving from the population. In this sense, the demographic

[7] Healthline. 2020. *What is R*

turnover, so births and deaths, is not contemplated. Furthermore, the SI model assumes that the contamination does not guarantee immunity, meaning that recovered individuals may get infected again. In Figure 1 the transition dynamics of the ailment in the SI model is provided.

The dashed line shows how infection does not confer immunity. Individuals have reoccurring infections, and infected individuals return to the susceptible state. The transition rate β between the states S and I represents the infection rate within the population, which is the average effective contacts with other persons per unit time[3].

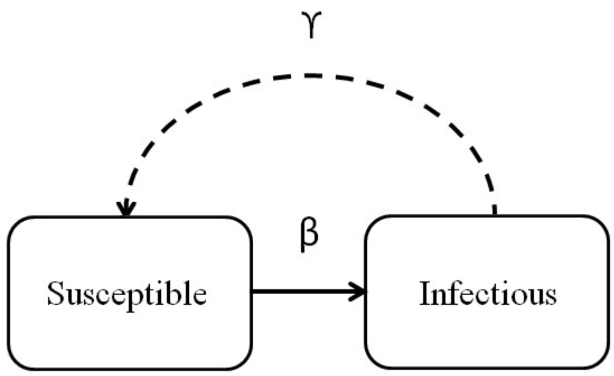

Figure 1: Transition dynamics of the SI model

Individuals in this model cannot get immunity, so once recovered they become susceptible again. This model matches the behaviour of diseases like herpes or cytomegalovirus (CMV).

Let St and It be the proportions of population that reside in the states S and I respectively, as a function of time t, measured in days. The following ordinary differential equation (ODE) represents the SI model:

$$\begin{cases} dIt = (\beta ItSt - \gamma It)dt \\ St = 1 - It \end{cases}$$

$$dIt = [\beta It(1 - It) - \gamma It]dt$$

As described above, β represents the conversion rate between classes S and I per unit time. Therefore, the prompt increase in the fraction of infectives It should be equal to $\beta ItSt$, due to the interaction between infectives and susceptibles. The transition rate γ represents the recovery

rate, which is the proportion of infectives leaving the state I per unit time. Hence, the instantaneous decrease in the proportion of infectives I is equal to γIt[3].
This model is the simplest form of all epidemiological models.

1.3.2 The SIR model

This model adds the class R ("recovered") to the simple SI model analysed before. This model again ignores deaths but assumes that everyone removed from the pool of susceptible pool recovers.
Again, the variables St, It and Rt represent the portion of population in each compartment as a function of time. The model is dynamic because the rates in each class can fluctuate over time, as the variable function t implies.

Each member of the population typically progresses from susceptible to infectious to recovered. This can be shown as a flow diagram, as in Figure 2, in which the boxes represent the different compartments and the arrows the transition among them.

Figure 2: Transition dynamics of the SIR model

The dynamics of a mild epidemic, such as the flu, is often much faster than the dynamics of birth and death, that is why the so-called vital dynamics (the demographic turnover) is sometimes emitted. In Chapter II, I will use this simple SIR model augmented to include deaths (D) to calibrate its effect to Covid-19.

The SIR model can be expressed by the following set of ODEs:

$$\frac{dS}{dt} = -\frac{\beta IS}{N}$$

$$\frac{dI}{dt} = \frac{\beta IS}{N} - \gamma I$$

$$\frac{dR}{dt} = \gamma It$$

Where S is the stock of susceptibles, I is the stock of infectives, R is the stock of recovered and N is the sum of these three. This model was first proposed by William Ogilvy Kermack and Anderson Gray McKendrick as a special case of what we now call Kermack–McKendrick theory.

We note that from:

$$\frac{dS}{dt} + \frac{dI}{dt} + \frac{dR}{dt} = 0$$

it follows that:

$$St + It + Rt = N\ (constant)$$

Furthermore, the following ratio depicts the dynamics of the infected population:

$$R_0 = \frac{\beta}{\gamma}$$

Which depends on the reproduction ratio, as already discussed.

1.3.3 The $SEIR$ model

The Susceptible-Exposed-Infective-Recovered is widely used to analyse infection data during the different stages of an epidemic outbreak. This model adds the category E (exposed), which includes all those individuals who have been infected but are not yet infectious themselves. It is one of the most updated mathematical models to represent the epidemic dynamics and to forecast feasible contagion scenarios. It can be useful to assess the effectiveness of government measures, like lockdowns or quarantines since the outbreak of the infectious disease[8].

It is based on a series of dynamic ODEs that consider the trend of individuals who recover or unfortunately die over time.

The basic SEIR model (Allen 2017) is described in Figure 3. As in the previous models, time is denoted by t and is measured in days. An initial total population of N_0 individuals is divided into the first infectious individuals ($I_0 = 1$) and $S_0 = N_0 - 1$ vulnerable individuals. In each successive day, some susceptibles become exposed. The daily amount of new exposed that become new infectious after an incubation period is determined by the net reproduction number times the number of existing infectious. The net reproduction number varies with time and depends on three elements: the basic reproduction number R_0 , the average number of days in which a subject is infectious $Tinf$ and the fraction of susceptibles to the total population $\frac{St-1}{Nt-1}$, so in each period we have:

$$NewEt = \frac{R_t}{Tinf} It - 1 \quad ;$$

$$R_t = R_0 \frac{St - 1}{Nt - 1}$$

The exposed, after an incubation period of $Tinc$ days, become infectious. The outflow of susceptibles becomes the inflow of exposed in each period while the outflow from the exposed is the inflow into the infectious, who will fall into the category of those who recover or those who die[9].
The allotment to these classes is headed respectively by two probabilities: $1 - p^{fat}$ and p^{fat}, which are fixed exogenously.

[8] Godio, A., F. Pace, and A. Vergnano. 2020. *SEIR Modelling of the Italian Epidemic of SARS-CoV-2 Using Computational Swarm Intelligence*
[9] Favero, C., A. Ichino and A. Rustichini. 2020. *Restarting the economy while saving lives under Covid-19*

Survivors to the disease are then removed and become recovered (REM_RECt) after a period of $Tsrec$ days from the first illness manifestations to recovery. Instead, those who unfortunately pass away are removed as fatalities, REM_FATt after a period of Tsd days from first symptoms to death. In this model, the lethality of the virus is measured by:

$$\lambda^{seir} = \frac{REM\_FATt}{Et + REM\_RECt + REM\_FATt}$$

This parameter will converge to two values: if $R_0 > 1,\ \lambda^{seir}$ converges to p^{fat} .

Instead, if $R_0 \leq 1$, the virus diffusion is inhibited, and the lethality parameter goes to 0.

We notice that, in the first case, the total number of deaths will be the same independently of the size of the reproduction number, because it only dictates the speed at which the number of victims is reached. Secondly, the net reproduction number of the disease changes only as a function of the ratio of the susceptibles to the total population. It is instead affordable to assume that the variable will be dependent on policies and by the behavioural response of people to the unfold of contamination[8].

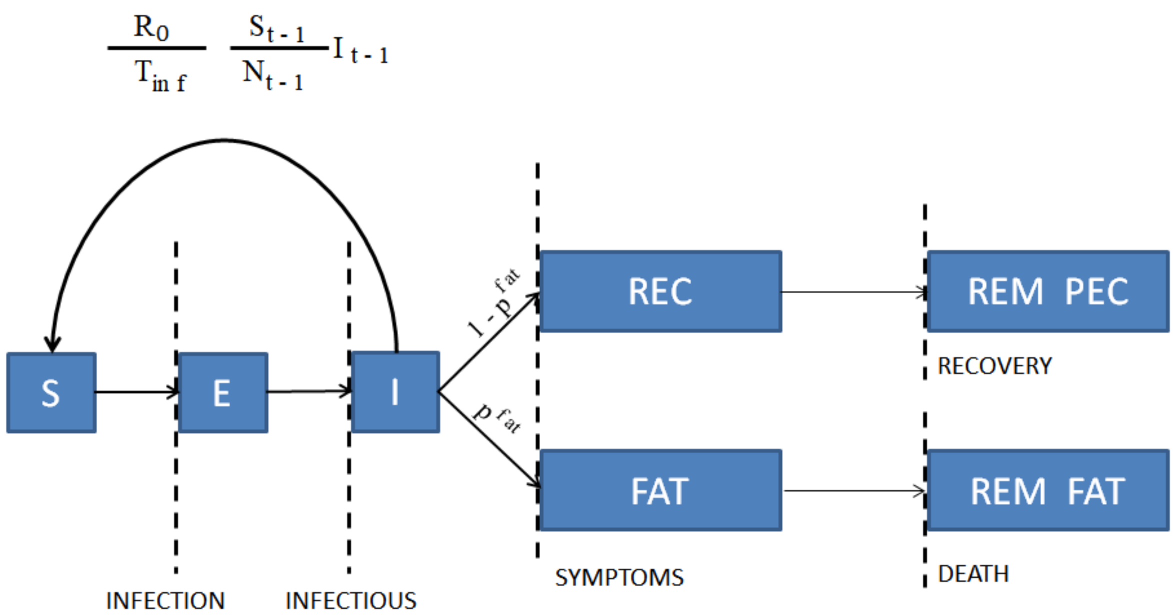

Figure 3: The SEIR Model

1.4 Real options

In the financial markets, an option is the freedom of choice after revelation of additional information that increases or decreases the value of the asset. A "real" option is an option "relating to things", fixed or permanent, as opposed to abstract things. A financial call option gives the owner the right, but not the obligation, to purchase the underlying stock in the future for a price fixed today. A financial put option gives the owner the right, but not the obligation, to sell the stock in the future for a price fixed today.[10] The managerial operating flexibility is analogized to financial options. As new information arrives and uncertainty about market conditions is gradually resolved, management may have valuable flexibility to alter the initial operating strategy.[11] They may be able to defer, expand, abandon, or otherwise change the project during its operating life in order to capitalize on favourable future opportunities or to react in order to mitigate losses. Managerial selections create call and put options on real assets that provide management the right, but not the obligation, to utilize those assets to achieve strategic goals and eventually maximize the value of the company. The key advantage and value of real option analysis is to integrate managerial flexibility into the valuation process and thereby assist in making the best decisions.[10] The real marketplace, pervaded by change, uncertainty and aggressive interactions, the realization of cash flows will likely be different from what management was expecting. That is why an increasing number of academics have been dissatisfied with the existing methods of resource allocation. It is now widely recognized that traditional Discounted-Cash-Flow (DCF) approaches such as standard Net-Present-Value rule (NPV), cannot capture management's flexibility to adapt decisions in response to changing and developing market conditions. Traditional DCF approaches make implicit assumptions concerning an expected scenario and presume management's passive commitment to a certain static operating strategy. Instead, real option methods examine at each step in the decision-making technique the freedom of preference to embark on the next step in the climb, or to pick towards doing so primarily based on the examination of additional data. This freedom of choice is embedded in most, if not all, funding decisions, because they are hardly ever now-or-never choices and infrequently, once a decision has been taken, cannot be abandoned, or modified for the duration of the entire project. In most cases, the choice can be postponed or extended, and frequently it results in consecutive steps with various decision points. All of these picks are actual managerial options and impact on the value of the investment opportunity.

[10] Brach, M. 2003. *Real options in practice*
[11] Trigeorgis, L. 1996. *Real Options: Managerial Flexibility and Strategy in Resource Allocation*

Imagine that you are going to build a new house and you have many options about how to heat it. One option involves the use of a heating oil or natural gas furnace. Another involves the use of an electric or natural gas range in your kitchen for cooking. You do not know how prices for either energy will develop in the future. You may look up historic prices of gas, oil and electricity or the past decade, that may give you some indication on volatility of prices, or which one tended to be cheaper. However, this study cannot ensure you that prices will follow the same price movements. For this reason, it may be of value for you to install a furnace that allows you to switch between energy sources without any problem. It is likely that this additional flexibility will come at a price because a furnace that is able to switch energy source will cost more than a simple one that is able to use one only energy form. However, depending on your annual energy demand and your expectations about future volatilities of each energy source and how they may correlate with each other, this option may well be in the money for you.
While this example may be intuitive and may invite to consider real option to value managerial options, there are different opinions about the topic, some of them are summarized here:

"To be sure, this much-vaunted alternative to the conventional method of evaluating capital-spending decisions using net present value (NPV) is catching on with more and more senior finance executives."
R. Fink. CFO.com, September 2001.
"In ten years, real options will replace NPV as the central paradigm for investment decisions."
Tom Copeland & Vladimir Antikarov. Real Options, A Practitioner's Guide, 2001.
"Information for evaluating real options is costly or unavailable, and asking for more money later is difficult and may be interpreted as a lack of foresight. Projects are selected by financial managers, who do not trust operational managers to exercise options properly."
Fred Phillips, Professor, Oregon Graduate Institute of Science & Technology, Portland. Business Week Online, 1999.
"The myth of Option Pricing—Fine for the stock market and oil exploration, option pricing models don't work in valuing life sciences research."
Vimal Bahuguna, Bogart Delafield Ferrier. In vivo—The Business and Medicine Report, 2000.

Table 1: Opinions on Real Options
Source: Real options in practice, Brach M.

However, real option evaluation does not by definition preclude or substitute traditional DCF and NPV evaluation. Real options theory actually develops these tools and the underlying hypotheses, integrates them into a new valuation pattern, and finally takes them to the next level of strategic analysis[1]. The use of real option evaluation does not defend against funding selections leading to the acquisition of options which might be worthless and, as a result, have a certain probability of expiring valueless. Like any other financial and strategic evaluation device, real option analysis is by no means better than the assumptions that move into the analysis. However, it provides an alternatively secure option area any investment decision to be made. As time goes on and extra information arrives, the thresholds of uncertainty become better defined and the option space safer and less limited. "The key issue is not avoiding failure but managing the cost of failure by limiting exposure to the downside"[12]. Real option analysis is a strategic tool. It involves a cross-organizational workout mapped out to put out the options, find out the risks, and dictate the variety of managerial flexibilities. It provides the framework and shape for real option pricing, and it is the benchmark against which to degree the real alternative execution.

In finance the net present value (NPV) method is used as the basis for most analysis, such as investment decisions, valuation of firms and capital structure decisions. However, as recognized by most practitioners, this method fails to consider real options embedded in business strategic decisions, thus underestimating the firm value. Implicitly, the NPV method assumes that either the "investment is reversible" or "if the investment is irreversible, it is a now or never proposition"[13]. This last statement means that if the firm does not undertake the investment now, it will not be able to implement it in the future. However, in the real world, investment opportunities do not always meet these conditions since most investments are partially or completely irreversible. This is because most investment become sunk costs once the firms make the move and firms cannot recover the costs in full later. Most importantly, in the real-world firms can analyse the desirability of the investment project using newly acquired information. Therefore, firms have the managerial flexibility to defer the investment until new information arrive. The possibility to delay investment can affect the decision to invest and thus, undermines the simplistic NPV approach[13].

A firm's investment opportunity can be seen as a call option. It can either exercise the option and, so make the investment, or hold the option and wait for new information to arrive. The decision of investing now makes the firm immediately lose the option value of waiting until the

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[12] McGrath, R. 1999. *Falling Forward: Real Options Reasoning and Entrepreneurial Failure*
[13] Dixit, A. K., and R. S. Pindyck. 1994. *Investment under uncertainty*

arrival of new information. This lost value is analogous to an opportunity cost that should be included in the calculation of the NPV of the project.

Real options are inherently present in any strategic decision where firms have the managerial flexibility to alter its course, namely though expansion, contraction, delay or abandonment.[14]

In the next paragraphs, I will explain the connection between the world of real options and epidemiological models.

1.4.1 Real options and Lockdown Strategies for Covid-19

Real options models have been used in the realm of business economics to investigate the impact of various policies on businesses and individuals. Giving the huge amount of uncertainty involved in the event of a pandemic, striking the balance between economic costs and lives saved is a challenging task[15]. For example, decision-makers can decide to impose the lockdown to specific regions or business sectors, or reverse their decision depending on the transmission of the virus (here, the decision can be plainly seen as a call option).

Many strategies are available, but which how do we select the best one? Clearly, by acting gradually authorities can learn more about the spread of the disease and adjust future actions, because learning has value[15]. Real options analysis provides a useful framework to structure an optimal policy, because it allows to dismantle the course of action into sub-steps and to assess value and costs of such decisions along a decision path.

For many different motives, the utilization of real options did not gain popularity within the business community, although it dispenses useful intuitions to firms and governments who constantly face trade-offs and make decisions in highly volatile circumstances.

Currently, our governments are designing strategies to deal with the Covid-19 outbreak. Given the amount of uncertainty and the irreversibility of many decisions, creating optimal alternatives is key. Creating options means investing today to be ready to act when conditions become unsustainable, for example hospital supplies and having personnel ready to be employed.

There are three features that define the circumstances for applying real options: uncertainty, irreversibility, and flexibility[15].

[14] Brealey, R., Myers, S. and Marcus, A. 2001. *Fundamentals of Corporate Finance*
[15] Boyer M. and E. Gravel. 2020. *Looking at the Management of the COVID-19 Lockdown strategies Through the Lens of Real Options Analysis*

- *Uncertainty* stands for the unpredictability about the future situation. For example, the outcome of a clinical trial or the evolving of the basic reproduction number of an infectious disease.
- *Irreversibility* of a project means that committed assets cannot be recovered when things do not go as predicted.
- *Flexibility* is the most crucial characteristic of real options. It represents the option to delay, to restart or stop an action, to invest in stages, to abandon or to switch.

Flexibility is valuable if we have time and do not need to act straight away. Managing Covid-19 is all about real options. Decision-makers face the following sources of *uncertainty*:

- Degree of contagion of the disease, likelihood of future waves, fatality rate and healthcare costs.
- Future economic effects linked to lockdowns and social distancing.

Irreversibility in this context can be determined as follows:

- Direct and indirect losses, both economic and emotional, resulting from prolonging the lockdown.

Finally, decisions of policymakers are characterised by *flexibility* because they can choose:

- When and where to impose a total or partial lockdown.
- When and where to lift s total or partial lockdown.

Modelling uncertainty is a crucial mission, which can be fulfilled through suitable stochastic techniques whose development relies upon parameters to be calibrated in conformity with scientific expertise, which is evolving.

Managing Covid is similar to managing a series of options. In the course of last and present year, many options have been exercised by governments. The first was employed when the first lockdown was imposed. The second option exercised was the lockdown lift for daycare centers, schools and part of the economy that was considered of “primary” importance. In the last months, the Italian government has been exercising the option of targeted lockdowns across the regions of Italy.

For lockdown alternatives, thresholds would be conditioned on the number of positive individuals or victims. As mentioned above, real option analysis would help decision makers to achieve an optimal equilibrium between the costs and benefits of lockdowns considering the evolution of the disease as well as the behavioural response of individuals.

1.4.2 Regime-switching models

The history of these models can be traced back to Mossin in 1968, who studied optimal portfolio policies extended to multiperiod by means of dynamic programming.

Regime-switching models arise from real options valuation. The simplest form of these models are optimal stopping problems, and American options are a perfect example of it because holders can exercise their right at any time before the expiration, allowing them to capture profit as soon as the stock price moves favourably.

In the realm of corporate finance, Brennan and Schwarz in 1985 applied this approach to evaluate the value of active and inactive firms through the use of the Black-Scholes formula. They argued that the inactive firm holds the opportunity to invest, whose value is equivalent to that of a call option with a strike price equal to the entry cost. Similarly, the active firm holds the option to abandon because it can exit the market when conditions reveal it is convenient. In this context, the value of the firm should include the value of option to entry and abandon, respectively.

Later in 1994, Dixit and Pindyck extended the model considering that a firm has the option to suspend or scrap a project, and considered four price thresholds for investment, suspending, reactivation and scrapping.

In the next Chapter, I will analyse the world of regime-switching models more deeply and a practical example will be provided.

1.5 Real options and epidemiological models: what a difference a stochastic process makes

Real options have been used to analyse the effect of uncertainty on the timing of policies implementation for disease outbreaks. Viewing the ailment management as an option that may be exercised to reduce harm cause by the disease, the real options approach can be a beneficial advice to determine the best timing of control, that is answering to the questions: when is it optimal to use the option to control the virus and how long should we wait to implement control measures in order to learn more?

To incorporate uncertainty in the decision-making process, the spread of the infection can be described more realistically by a stochastic process[16]. Traditionally, real options framework assumes the change in the level of infected population follows a Geometric Brownian Motion (GBM), (Saphores 2000; Sims & Finnoff 2012) which is well understood and allows for closed-form solutions. However, it works properly for the initial part of the epidemic because it assumes that the mean level of infection grows exponentially, but it is not the case in the long term. In particular, GBM ignores the effect of the size of susceptible population in the epidemic outbreak, which is unrealistic because the transmission rate also depends on the current proportion of susceptible individuals. In fact, the rate of infection fastens when the proportion of susceptible population is high and slows down when the proportion is low.

As we have seen, the evolution in the level of infection over time is described by a logistic-type term in the deterministic formulations of epidemiological models. These models can be extended to include uncertainty by adding a drift term representing the random fluctuations caused by variability in the level of infection spread (Keeling & Rohani 2008).

1.5.1 Set up of epidemiological models of uncertainty in the disease spread

To include uncertainty in the decision-making process, let us assume that the level of infection I follows a stochastic process[16]. Traditionally, the increase in the level of infection can be assumed to follow a GBM, so the dynamics in the level of infection can be described by the following stochastic differential equation (Saphores 2000; Sims & Finnoff 2012):

$$dI = \beta I dt + \sigma I dW$$

Where β is the transmission rate of infection, σ is the volatility coefficient, I is the current level of infected individuals and dW is a Wiener increment.

As the limit of $\sigma \to 0$, the above equation is equivalent to assuming a deterministic exponential growth in the infected area, but this is plausibly true only in the early stages of an epidemic.

In the simple SI model, the increase in the number of infected individuals is given by the rate at which a susceptible individual contracts the infection multiplied by number of vulnerable

[16] Dangerfield, C.E., A.E. Whalley and C.A. Gilligan. 2018. *What a Difference a Stochastic Process Makes: Epidemiological-Based Real Options Models of Optimal Treatment of Disease*

individuals. The rate at which a susceptible individual is contaminated is given by the transmission rate β times I/I_{max} which is the probability of contact with an infectious individual, where I_{max} is the maximum number of potential infected individuals.

Assuming that the population is constant, I_{max} is equivalent to the total population size, so $S = I_{max} - I$. Ignoring the uncertainty in the spread of infection, the evolution of its level is given by the following ODE[16]:

$$\frac{dI}{dt} = \beta I \left(1 - \frac{I}{I_{max}}\right)$$

Uncertainty in the epidemic spread is integrated by assuming there is variability within the transmission rate driven by external forces, such as temperature and climate (Sturrock et al. 2011). These fluctuations can be deemed to be stationary.

We can assume that the 'corrected apparent infection rate' is disturbed, leading to $\beta\left(1 - \frac{I}{I_{max}}\right) \rightarrow \beta\left(1 - \frac{I}{I_{max}}\right) + \sigma\xi$ (Marcus 1991), where ξ is white noise and σ is a constant that controls the extent of that noise. The uncertain evolution of the disease can be described by the following 'mean-reverting SDE':

$$dI = \beta\left(1 - \frac{I}{I_{max}}\right)dt + \sigma I dW$$

This equation has been used in the real options framework for example by Ndeffo Mbah et al. in 2010 to describe the increase in the infected area and by Marten & Moore in 2011 to study the growth in pest populations[16].

When the level of infection outreaches I_{max}, the extent of the diffusion term in non-zero and so the infection level might exceed I_{max} , which is unrealistic in a fixed population.

On the other hand, we can assume that the transmission rate is itself perturbed, leading to $\beta \rightarrow \beta + \sigma\xi$ so the evolution in the level of infection is given by the following 'logistic SDE':

$$dI = \beta I \left(1 - \frac{I}{I_{max}}\right) dt + \sigma I \left(1 - \frac{I}{I_{max}}\right) dW$$

As the level of infected individuals reaches I_{max}, both the diffusion term and the drift approach 0, so the SDE remains in within the interval [0, I_{max}], thus preserving the natural upper boundary of the total population size[16].

The logistic SDE offers an approach of relating in future levels of contamination to the randomness of transmission evolution due to environmental elements, therefore delivering an epidemiological-based approach to incorporating randomness into the decision problem.

1.5.2 The Decision Problem

Since this problem cannot be easily applied to Coronavirus for the reasons I will explain later, I assume that the infection is referred to a forest disease outbreak in a particular tree species. I will assume that the number of trees (the population) is constant and that the control policies can be implemented at any time for an on-the-spot fixed cost C and that this treatment is irreversible. A decision-making authority is faced with the following choice: should the cure be dispensed straight away, or should the decision-maker wait to learn more about the evolution of the disease? Waiting allows the authority to check whether the level of infection degenerates or improves over time.

Classical NPV approach would suggest undertaking the remedy if it provides a value greater than the cost C. However, the uncertainty in the ailment dynamics combined with the irreversibility of treatment, it is worthy to delay the treatment.

I assume that the most effective impact of treatment is to eliminate infection and I assume that it is implemented straight away. If we just consider the economic benefit thanks to timber or crop saved, the value of applying the treatment is:

$$Vt = pIt$$

Where p is the gain yield per unit of infected area cured, which is assumed to be fixed and that the extent of infection It varies stochastically according to GBM, mean-reverting SDE or logistic SDE described above.

Viewing the treatment application as a funding with value Vt, the problem can be seen as a real option as it gives the right but not the obligation to make an investment for a predetermined price in the future[13]. The payoff from applying the treatment at time t is $Vt - C$, so we need to maximize the expected present value:

$$F = \max E[(Vt - C)e^{-rt}]$$

Where t is the time in the future at which the decision is made, r is the discount rate and E is the expectation, which must be considered due to the stochasticity of It and Vt. This is an optimal stopping problem, where the threshold at which the value from applying the treatment immediately is at its maximum must be found.

Using standard methods from dynamic programming, the value of option to apply the treatment F(V) must satisfy the following Bellman equation[13]:

$$\frac{1}{2}b(V)^2\frac{d^2F}{dV^2} + a(V)\frac{dF}{dV} - rF = 0$$

In Table 2, the functions $a(V)$ and $b(V)$ are described according to each stochastic process seen so far.

Stochastic process	$\boldsymbol{a(V)}$	$\boldsymbol{b(V)}$
Geometric Brownian Motion	βV	σV
Mean-reverting SDE	$\beta V\left(1 - \frac{V}{pI_{max}}\right)$	σV
Logistic SDE	$\beta V\left(1 - \frac{V}{pI_{max}}\right)$	$\sigma V\left(1 - \frac{V}{pI_{max}}\right)$

Table 2: For of the functions in the Bellman equation for each stochastic evolution of infection

$F(V)$ must also satisfy the following boundary conditions[16]:

$$F(0) = 0$$

$$F(V^*) = V^* - C$$

$$\frac{d}{dV}F(V^*) = 1$$

Where V^* is the value at which the treatment should be applied immediately.

The fact that the infection cannot be reintroduced from an outside source gives rise to the first condition.

The second condition is called the 'value matching condition'[13] stating that when the investment is undertaken straight away the net gain is $V^* - C$.

The last called 'smooth pasting condition' ensures optimality of the choice of V^*, since if F were not continuous at V^*, then one could do better by investing at a different point[16].

This is a 'free-boundary problem' because the boundary region needs to be determined as part of the problem.

1.5.3 Results

The solution to the 'free boundary problem' associated with each SDE provides the value of the option to treat as a function of the treatment value (V).

If the value of the option to treat $F(V)$ is greater than the NPV of immediate treatment, $F(V) > V - C$, there is value in the option to wait.

When $F(V) = V - C$ there is no additional gain in waiting, so the treatment should be applied as soon as possible. The threshold V^* represents the frontier between the delaying and the immediate investment areas.

The threshold value of treatment for each SDE corresponds to the threshold level of infection I^*, ($V^* = pI^*$) at which the treatment should be applied immediately[16].

Using an epidemiologically based SDE to describe uncertainty in the ailment unfold, such as mean-reverting or logistic SDE, decreases the edge value compared with the traditional Geometric Brownian Motion, so using the first two strategies might result in the treatment deployment when a lower fraction of region is contaminated.

The difference in the most beneficial time to treat among the three procedures increases when volatility is large.

Comparing the three models, we can find the following differences[16]:

- In case uncertainty is very high or the disease is unfolding quickly, the boundaries for GBM and mean-reverting model are unachievable implying treatment should never be implemented.
- The crucial distinction among the logistic SDE, the GBM and the mean reverting SDE is that the former allows for a natural upper boundary to be included endogenously into the problem. This appears in the forms of drift and diffusion coefficients $a(V)$ and $b(V)$ to make sure that the level of infection will not exceed the maximum host population. GBM and mean-reverting SDE allow for the level of contamination to rise above the natural upper threshold, hence overestimating the value that can be obtained from the treatment when the level of infection is high.
- Implementing cure at an inaccurate threshold (derived from GBM or mean-reverting SDE), leads to a loss in value, which arises because treatment boundaries are higher under mean-reversion or GBM than under the logistic method, because ignoring asymptotic boundaries for infection may result in treatment being deployed too late.
- The differences in the thresholds at which to treat between the three models increases with increasing volatility and transmission rate.
- There is value to be obtained from waiting to implement treatment. Nonetheless, the critical difference is that the logistic SDE implies the treatment to be applied in advance and that it is always valuable to implement the cure before the whole region turns into infected, even if the uncertainty is very substantial.

In conclusion, if the model used does not adequately incorporate uncertainty in the disease spread, the imprudent delay before treatment also implies that the full value of option to apply the treatment is not realised[16].

As previously mentioned, this problem cannot fit Covid-19 well, because the context in which it is developed is really different. First of all, the model presented shows a situation in which the cure already exists and the only question to answer is the optimal treatment timing.

In the context of Coronavirus (and human epidemics in general), when a new vaccine is developed, it means that there is already urgency of using it.

At least in the early stages of the epidemics, when the consequences of the infection on society were ignored, the initial choice of developing or not the vaccine could be seen as a real option. Later, after having waited to see how the infection was spreading fast, pharmaceutical companies have begun the race to find a new vaccine, since there was no uncertainty that a cure was needed as early as possible.

Having considered that, once a cure is developed and tested, there is no reason in waiting further to its administration to the population. Actually, we can say that real options of waiting in this context has completely lost value.

Other real options still exist in this context, such as the optimal level of population to be vaccinated, but this has already been analysed by many academics.

In the next Chapter, I will develop a real option model in the context of Coronavirus, considering the optimal regime switching thresholds within a real-world firm.

Chapter II

Regime-Switching Models and Calibration to Covid-19

2.1 Stage I: The Regime-Dependent Stochastic Epidemic Model

In this section, I will adapt a SI model to simulate the spread of the virus, which depends on the regime the firm is currently in. Later, I will modify the model to allow for external contagions and deaths, providing both the deterministic and the stochastic equations. Finally, I will provide a regime-dependent stochastic SI model allowing for external contagions and deaths.

2.1.1 The Modified SI model allowing for external contagions and deaths

Note that I am not considering a whole population, I am focusing only on a large firm with many employees. The spread of the epidemic within the company is influenced by the interaction between vulnerable and infective workers and by outside sources. In fact, susceptible employees are in touch with the external world, thus they are inevitably exposed to infection. The infection rate from external sources will be denoted by β.

Considering an epidemic such as Covid-19, the classical SI model has to be modified to include for deaths caused by the infection, because the original model assumes the disease is not fatal. Assume the group of susceptibles S is composed of the existing susceptible workers and the new employees hired in order to maintain a constant workforce. The constant rate of fatality among the class of infectious I is denoted by δ. The modified SI model that includes external contagions and deaths will be expressed as follows:

$$dI_t = [\alpha I_t(1 - I_t) - \gamma I_t + \beta(1 - I_t) - \delta I_t]dt$$

In Figure 4 the SI model allowing for external contagions and deaths is shown.

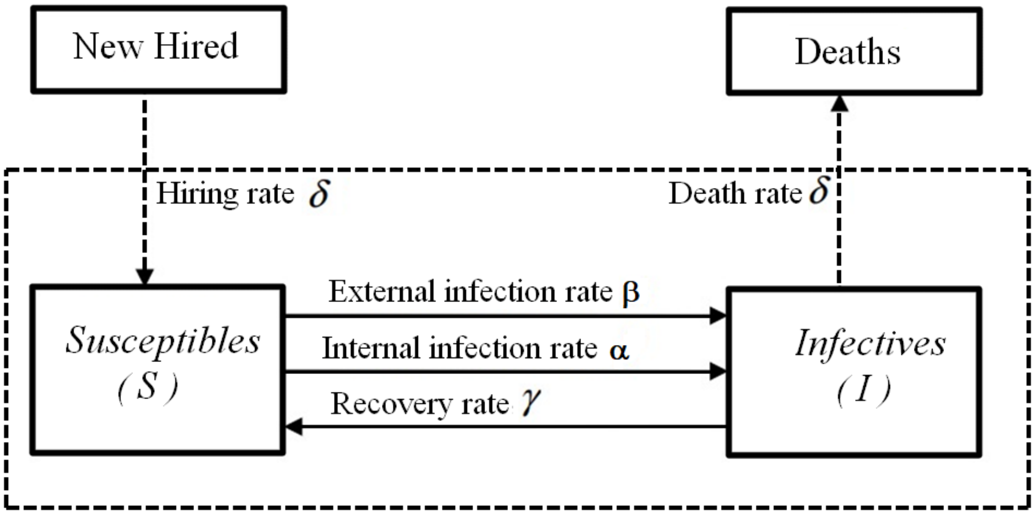


Figure 4: SI model allowing for external contagions and deaths

2.1.2 The Modified Stochastic *SI* model allowing for external contagions and deaths

The model now should be converted into a stochastic model to include uncertainty. To do so, a diffusion term needs to be introduced and some assumptions should be made. According to Cobb (1998), the most reasonable assumption to make is that the random variable is greater in the centre region than in extreme cases, suggesting that this variation is proportional to $I_t(1 - I_t)$. Therefore, the equation will include the variation in I_t in the following way[16]:

$$dI_t = [\alpha I_t(1 - I_t) - \gamma I_t + \beta(1 - I_t) - \delta I_t]dt + \sqrt{cI_t\,(1 - I_t)}\,dW_t$$

Where c is a small positive constant and W_t is a standard Brownian motion.

2.1.3 The Regime-Dependent Stochastic SI model allowing for external contagions and deaths

The above equations are valid if the firm is active and does not implement any disease control policy. Nevertheless, strategic decisions that managers make have an impact on the evolution of the disease. Such decisions might be used to reduce the spread of the contagion, thus altering the parameters α and γ.

The company may adopt disease control programs to lower the infection rate between infectives and susceptibles, for example screening the suspected infectives and ordering a full paid leave or testing for the disease all employees every week or month.

In particular, if the number of infectives rises above a certain threshold, the firm can temporarily shut-down operations and send all employees home[16].

In this thesis, I will analyse the optimal suspension-reactivation strategy, therefore adapting the model to further accommodate the active and inactive regimes.

I will denote r as the regime of the firm, thus $r = 1$ if the firm is active and $r = 2$ if the firm is inactive. γ_1 represents the recovery rate when the firm is active. The corresponding epidemic model will be:

$$dI_t = [\alpha I_t(1 - I_t) - \gamma_1 I_t + \beta(1 - I_t) - \delta I_t]dt + \sqrt{cI_t\,(1 - I_t)}\,dW_t \qquad \text{if r = 1}$$

In the inactive regime, the internal transmission of the disease is cut off, thus $\alpha = 0$. Infectives will recover at a greater recovery rate $\gamma_2 > \gamma_1$ and the contagion will be kept under control. Assuming that the external infection rate β and the death rate δ remain unaltered, the dynamics of the disease when the company is inactive will be:

$$dI_t = [-\gamma_2 I_t + \beta(1 - I_t) - \delta I_t]dt + \sqrt{cI_t\,(1 - I_t)}\,dW_t \qquad \text{if r = 2}$$

Finally, the epidemic model can be expressed in the following manner:

$$dI_t = \mu(I_t, r)\, dt + \sigma(I_t, r)\, dW_t$$

In which,

$$\mu(I_t, r) = \begin{cases} \alpha I_t(1 - I_t) - \gamma_1 I_t + \beta(1 - I_t) - \delta I_t & if\ r = 1 \\ -\gamma_2 I_t + \beta(1 - I_t) - \delta I_t & if\ r = 2 \end{cases}$$

And

$$\sigma(I_t, r) = \sqrt{c I_t\,(1 - I_t)}$$

2.2 Stage II: Real Options and Regime-Switching Models

2.2.1 Practical examples of Real Options

The theory of real options has been deeply analysed in the former Chapter, now some practical examples will be provided.

There are several types of real options and each of them is applicable in a different situation and has a different function. Real options can be classified as with and without strategic value. Strategic value is value locked inside a company, that cannot yet be converted into cash, but can possibly be converted into cash at some time in the future. They are related to possible future projects that do not derive their value primarily by cash inflows. However, real options literature is mostly concerned with real options without strategic value. These are closer related to financial option theory and concerned with the current business. They are also described as cash-generating options (Trigeorgis, 1988) or flexibility options (Triantis, 1999). Options without strategic value mostly have a clear payoff and are typically related to operating decisions. The classification of real options by Trigeorgis (1988) is depicted in Figure 5. This classification uses similarities between real and financial options. Trigeorgis concluded that real options without strategic value (cash-generating) are usually structured as simple options, while options with strategic value (not cash-generating) are structured as compound options.

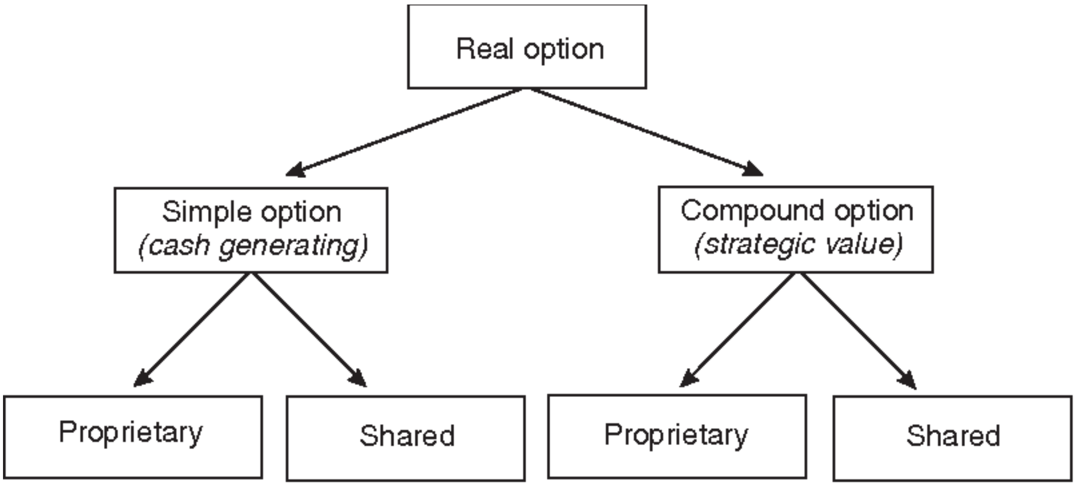


Figure 5: Classification for real options
Source: A conceptual framework for capital budgeting, Trigeorgis L.

In the second layer, both simple and compound options are again divided into proprietary or shared real options. Shared real options are opportunities jointly held by a number of competing firms or industries and can be exercised by any of the collective owners, as opposed to proprietary ones.
Examples of real options in each category are:

- **Simple proprietary real option:** potential expansion of capacity for a product protected by patents.
- **Simple shared real option:** expansion decisions in competitive industries.
- **Compound proprietary real options:** exploration investments protected by government licenses.
- **Compound shared real options:** pilot project proving the market and creating customer acceptance.

The insights from Triantis and Trigeorgis are similar and here defined:

1. *Flexibility real options,* i.e., without strategic value. They are related to a company's current assets in place. They are more generally simple options with a clear payoff. Over time, as uncertainty resolves, it becomes clearer whether they should be executed or not.

2. *Strategic real options*, i.e., with strategic value. They can either be related to projects already undertaken or may be strategic opportunities not related to any project in place. These are growth options, where firms position themselves favourably to potentially exercise profitable growth options in the future, for example by investing in R&D or IT expertise. The potential payoff of these real options is often unclear at the time of execution and they are more technically referred as compound options.

In Table 3 there are examples of growth and flexibility options for different types of industries.

Industry	**Growth option**	**Flexibility option**
Airline	Aircraft delivery option	Contingency rights
Computer hardware	New model under brand name	Assembly configuration
Financial services	IT infrastructure	Abandon service or divest
Internet	Marketing investments	Outsource services
Oil and gas	Lease blocks	Delay production
Pharmaceuticals	Research and development	Outsource production or sales
Power	Global expansion	Peak generating plants
Real estate	Undeveloped lands	Redevelop with adjusted mix
Telecommunications	Merger and acquisitions	Re-deploy assets

Table 3: Growth and Flexibility Options in different industries
Source: A real options approach to company valuation, Aarle R.

Dixit and Pindyck (1994) classify real options primarily by their type of flexibility. They indicate five types or real options:

1. *Option to defer:* the right to delay the start of a project.
2. *Option to stop:* the option to sell or close down a project before completion.
3. *Option to abandon:* the option to sell or close down a project after completion.
4. *Option to temporarily stop producing:* the option to close down for a certain period of time.

In the next paragraphs, some practical simple examples of real options will be provided.

a. Example 1: NPV with simple Option

Suppose that a firm has the opportunity to invest in a project whose commercial success is not clear yet. There are two main phases in the project:

- Phase 1 (Pilot production and test marketing): costs €125.000 and takes one year.
- Phase 2 (Implementation): this phase is carried through only if Phase 1 turns out to be a success. Build €1 million plant which generates after-tax cash flows of €250.000 per year forever.

In the standard approach, NPV would be used with a discount rate of ca. 25% if the project is considered to be risky. Thus, the value of the project calculated with the NPV method will be:

$$NPV = -125 - \frac{500}{1.25} + \sum_{t=2}^{\infty} \frac{125}{(1{,}25)^t} = -125$$

The project seems unprofitable but consider that we are faced with options – contingent decisions – which increase the value of the project.

The different phases of the project entail different risks which should not be combined. In particular, consider that Phase 1 will settle most of the risk, because in case it fails, risk will be eliminated, and the project is certain to be worthless. Assume there is 50% of probability that Phase 1 will turn out a success, thus we will have:

$\frac{1}{2}$

Success: $NPV = -1000 + \sum_{t=1}^{\infty} \frac{250}{(1{,}1)^t} = 1500$

Failure: $NPV = 0$

The project has a forecasted payoff of 0,5(1500) + 0,5(0) = €750 after one year and an investment of €125. Using a 30% discount rate:

$$NPV = -125 + \frac{750}{1,3} = 452$$

Now the project seems advantageous.

b. Example 2: The Value of Waiting

Assume now building a widget factory that will produce one widget per year forever. The widget now costs €100, but from next year the price will increase or decrease by 50% and then remain fixed forever:

$$
\begin{array}{lcccc}
 & t=0 & t=1 & t=2 & \dots \\
0,5 & & \nearrow P_1 = \text{€}150 \longrightarrow & P_2 = \text{€}150 \longrightarrow & \\
 & P_0 = \text{€}100 & & & \\
0,5 & & \searrow P_1 = \text{€}50 \longrightarrow & P_2 = \text{€}50 \longrightarrow &
\end{array}
$$

The factory could be built in just one week for €800. Is it better to invest now or see if in one year the price will increase or decrease?

Suppose the investment is made now:

$$NPV = -800 + \sum_{t=0}^{\infty} \frac{100}{(1,1)^t} = -800 + 1.100 = \text{€}300$$

The NPV tells we should invest now.

Now suppose we wait one year and then invest only in case the price inflates. The value of the project turns out to be:

$$NPV = 0{,}5\,[-\frac{800}{1{,}1} + \sum_{t=1}^{\infty} \frac{150}{(1{,}1)^t}] = \frac{425}{1{,}1} = €386$$

Showing clearly that it is better waiting than investing immediately.

The value of the option to wait is €386 $-$ €300 = €86.

2.2.2 Regime-Switching Models: A Guide to Literature

As already discussed in the previous Chapter, regime-switching models often emerge in real option valuation. The pioneering article on the joint decisions to invest and abandon was written by Brennan and Schwarz in 1985. They construct a general model of the decision to open, close and mothball a mine producing a natural resource whose price fluctuates over time. They obtain a system of equations characterizing the price thresholds for investment and abandonment and provide the entry and exit threshold prices.

In 1968 Mossin worked on optimal mothballing decisions. He developed a model in which operating revenue follows a trendless random walk with upper and lower reflecting barriers and in which there is no possibility of scrapping. He calculated the optimal revenue levels at which it is optimal to mothball and reactivate the project.

The regime-switching models analysed by Dixit and Pindyck solve general problems of optimal switching among a series of alternatives in response to changing economic conditions. Each switch is an exercise of an option, and each switch yields an asset that combines a payoff flow with the option of switching again. Thus, they obtain a set of compound options and price them simultaneously. There is a large body of literature analysing such compound options. Geske (1979) is an early example of this kind in financial economics, followed by Geske and Johnson (1984) and Carr (1988).

Turning to real investment decision, Kutilaka and Marcus (1988) develop a model of switches between two modes with three time periods and they indicate how the model can be extended to many modes and switches.

Fine and Freund (1990) analyse a two-period model in which the firm must choose its capacity before uncertainty is resolved and it can choose either specific capital or flexible capital. Triantis and Hodder (1990) have an alike model in continuous time. In these models, option value is fundamental, because by waiting the firm preserves the opportunity of making a better investment later, and not just that of not investing at all.

2.2.3 Regime Switching models: Practical examples

a. Combined Entry and Exit Strategies

In the following examples, I will confine the discussion to the case of demand uncertainty, assuming a Geometric Brownian Motion price process.

Assume that investment and abandonment decisions are made by a firm that takes the prices as given, as I assume that the price follows a GBM:

$$dP = \alpha P dt + \sigma P dz$$

If the firm enters the market (i.e., invests), it obtains a project that produces one unit of output per period and lasts forever until abandoned. The variable costs of operations C are known and fixed. The risk-free rate of interest is exogenously fixed at r_f. The risk-adjusted discount rate for the corporate revenues is:

$$r = r_f + \theta \rho_M \sigma$$

where θ is the market price of risk and ρ_M is the coefficient correlation between the price P and the entire market portfolio. Let $\delta = r - \alpha$ denote the rate of return shortfall on price, and δ be greater than 0.

The firm incurs a lump-sum cost L to invest in the project and a lump-sum cost to abandon it. For example, this latter could include the termination payments to workers. It might be the case that the investment cost L is not sunk, so that E is negative, reflecting the portion of investment that can be recouped on exit. Of course, $L + E$ must be > 0 to rule out a "money machine" of rapid cycles of investment and abandonment[17].

If the firm is active and exercises the option to abandon, it goes back to the inactive state, thus acquiring another asset: the option to invest. When this option is exercised in turn, it leads back to a live project. Thus, the values of a live firm and an idle firm are interlinked and must be determined simultaneously.

Intuition suggests that an idle firm will invest when demand conditions become sufficiently favourable, and an active firm will abandon when they become sufficiently adverse[17]. The optimal strategy for investment and abandonment (or holding or exercising the two options) will take the form of two threshold prices, say P_H and P_L with $P_H > P_L$. An idle firm will remain idle as long as P remains below P_H and it will invest as soon as P reaches the threshold P_H. An active firm will remain active as long as P remains above P_L and will abandon when P will fall to P_L.

a.1 Valuing Two Options

The value of the firm is now a function of the exogeneous state variable P, and of the discrete state variable that indicated whether the firm is currently inactive (2) or active (1). I will let $V_2(P)$ denote the value of the option to invest (that is, the value of an idle firm) and letting $V_1(P)$ denote the value of an active firm. Note that $V_1(P)$ is the sum of two components, the entitlement to the profit from operation, and the option to abandon should the price fall too far.

Over the range of prices $(0, P_H)$, an inactive firm holds its option to invest. $V_2(P)$ needs to satisfy a differential equation over this interval. The boundary conditions link values and derivatives of $V_2(P)$ to those of $V_1(P)$ at P_H. Likewise, over the span of prices (P_L, ∞), an active firm remains active, holding its option to abandon. $V_1(P)$ satisfies a corresponding differential equation, and the boundary conditions link the values and the derivatives of $V_1(P)$ to those of $V_2(P)$ at P_L.

I begin from the inactive firm. To obtain a differential equation for $V_2(P)$, we need to construct a portfolio with one unit of the option to invest, and a short position of $V_2{}'(P)$ units of output.

The resulting equation is:

[17] Dixit, A. K. 1989. *Entry and exit decisions under uncertainty*

$$\frac{1}{2}\sigma^2 P^2 V_2''(P) + (r-\delta)PV_2'(P) - rV_2(P) = 0$$

That has the general solution

$$V_2(P) = A_1 P^{\beta 1} + A_2 P^{\beta 2}$$

Where A_1 and A_2 are constants to be determined, and β_1 and β_2 are the roots of the quadratic equation[17]:

$$\beta_1 = \frac{1}{2} - \frac{\rho-\delta}{\sigma^2} + \sqrt{\left[\frac{\rho-\delta}{\sigma^2} - \frac{1}{2}\right]^2 + \frac{2\rho}{\sigma^2}} > 1$$

$$\beta_2 = \frac{1}{2} - \frac{\rho-\delta}{\sigma^2} - \sqrt{\left[\frac{\rho-\delta}{\sigma^2} - \frac{1}{2}\right]^2 + \frac{2\rho}{\sigma^2}} < 0$$

The option to invest gets greatly far out of the money and therefore becomes nearly worthless as P goes to 0, thus the coefficient A_2 corresponding to the negative root β_2 must be zero. That leaves:

$$V_2(P) = A_1 P^{\beta 1}$$

Which is valid over the interval $(0, P_H)$ of prices.

Next consider the value of an active firm. The calculation is similar, except that the live project part of the portfolio pays a net cash flow $(P-C)dt$. Then we get

$$\frac{1}{2}\sigma^2 P^2 V_1''(P) + (r-\delta)PV_1'(P) - rV_1(P) + P - C = 0$$

The general solution to this equation is

$$V_1(P) = B_1 P^{\beta 1} + B_2 P^{\beta 2} + \frac{P}{\delta} - \frac{C}{r}$$

The last two terms can be viewed as the value of the live project when the firm is required to keep it operating forever despite any losses, and the first two terms as the value of the option to abandon. The likelihood of abandonment in the not-too-distant future becomes extremely small as P goes to ∞, so the value of abandonment option should go to zero as P becomes very large. Hence the coefficient B_1 corresponding to the positive root β_1 should be zero, thus leaving[17]:

$$V_1(P) = B_2 P^{\beta 2} + \frac{P}{\delta} - \frac{C}{r}$$

Which is valid for P in the range (P_L, ∞).

At the investment threshold P_H, the firm pays the lump-sum cost L to exercise its funding option, giving up the asset of value $V_2(P_H)$ to get the live project which has value $V_1(P_H)$. For this we have the conditions of value-matching and smooth-pasting:

$$V_2(P_H) = V_1(P_H) - L$$

$$V_2{}'(P_H) = V_1{}'(P_H)$$

Similarly, at the abandonment threshold P_L, the value-matching and smooth-pasting conditions are:

$$V_1(P_L) = V_2(P_L) - E$$

$$V_1{}'(P_L) = V_2{}'(P_L)$$

Using the above equations, these conditions can be written as

$$-A_1 P_H^{\beta 1} + B_2 P_H^{\beta 2} + \frac{P_H}{\delta} - \frac{C}{r} = L$$

$$-\beta_1 A_1 P_H^{\beta 1-1} + \beta_2 B_2 P_H^{\beta 2-1} + \frac{1}{\delta} = 0$$

$$-A_1 P_L^{\beta 1} + B_2 P_L^{\beta 2} + \frac{P_L}{\delta} - \frac{C}{r} = -E$$

$$-\beta_1 A_1 P_L^{\beta 1-1} + \beta_2 B_2 P_L^{\beta 2-1} + \frac{1}{\delta} = 0$$

These four equations determine the four unknowns – the thresholds P_L, P_H and the coefficients A_1 and B_2 in the option values.

These equations are very nonlinear in the thresholds, so that the analytic solution in closed form is impossible. However, it can be proved that a solution exists, is unique, and has economically intuitive basic properties. The thresholds satisfy $0 < P_L < P_H < \infty$ and the coefficients of the option value terms, A_1 and B_2 are positive. Some other important general economic insights can be inferred by analytic methods, but further results require numerical solution.

2.3 Regime-Switching in My Example

As we have seen, the essence of regime-switching models is to determine the optimal switching thresholds across different regimes to maximize a certain value function[3].

Suppose the manager can use some methods to estimate the fraction of infectives I_t, because it is too costly to tell if an individual employee is actually infected or not. For instance, he could use public daily released data of disease cases or the hospitalization levels in his region. His goal is to determine two optimal thresholds: the mothballing I_H and the reactivation I_L. The mothballing threshold tells the manager which is the percentage of I_t above which it is better to suspend operations temporarily and offer full paid leave to all workers, independently if they are infected or not, as suggested by the HHS and the CDC in the Business Pandemic Influenza Planning Checklist. If, on the other hand, the fraction of infectives I_t is lower than the reactivation threshold I_L, the manager can call back all workers and resume operations. The full paid leave avoids adverse selection and moral hazard issues, because infectives could pretend to behave normally if they could not get paid during the suspended period[3].

I assume that the productivity of a worker drops to a certain level ξ (0 < ξ < 1) once he gets the disease, normalizing the productivity of a healthy worker to unity. The total number of employees in the firm is denoted by N. The variable and fixed costs are denoted by VC and FC, respectively. Among the fixed costs, employees' wages are included because I assume a full paid leave. Additionally, there is a penalty cost E for every infected worker when the firm is active. This cost may include the firm's reputational damage or employees' unwillingness to work.

Hence, the cash flow function can be defined as:

$$\pi(I_t, r) = [\xi \ * I_t * N \ + \ (1 - \ I_t) * N] * (P - VC) - FC - E * I_t * N \qquad \text{if r = 1}$$

$$\text{And} \quad \pi(I_t, r) = \ -FC \qquad \text{if r = 2}$$

The above equations tell us that the cash flow to the firm $\pi(I_t, r)$ depends both on the fraction of infectives and the regime variable. Clearly, if the company is inactive, the cash flows are only represented by an outflow due to fixed costs (wages, insurance etc.). When switching regime, the firm inevitably faces some costs, otherwise there would be the possibility of an "infinite money machine". The mothballing cost when switching from active to idle is denoted by M and the reactivation cost when switching from idle to active is denoted by A and there is no cost in remaining in the current regime. Given the above assumption, $M + A > 0$. Let C_{ij} be the lump-sum cost of switching from state i to j. The cost matrix is defined as $C = \begin{pmatrix} 0 & M \\ A & 0 \end{pmatrix}$. The discount rate is ρ and it is given. Based on these assumptions, the manager wants to optimize its cash-ins less outflows, including any switching costs, by choosing the optimal regime at each period, which is:

$$max_r V(I,r) = E\,[\int_0^{\infty} e^{-\rho t}\pi(I_t\,,r)dt - \sum_k \sum_{i=1}^{2} \sum_{j=1}^{2} e^{-\rho t_k^{ij}} C_{ij}]$$

Where t_k^{ij} is the time of the *k*th change from regime i to j.

2.3.1 The Methodology: Dynamic Programming

This approach allows us to derive optimal decision by beginning from the last period and going backwards, if the planning horizon is finite, since it can be broken up into a series of choices over a single-period horizon[3]. On the other hand, if the planning period is infinite, a recursive formula for every period can be developed, but using an exogenous discount rate ρ, which is assumed to be constant. Suppose that there are m regimes (i.e., $r = \in \{1,2,\dots,m\}$). A manager obtains cash flow streams $\pi(x,r)$ per unit time, which depends both on the regime r, which is a discrete variable, and on a continuous state variable x, which can also be a vector of state variables.

The dynamics of the continuous state variable x is described by[3]:

$$dx_t = \mu(x,r)dt \; + \; \sigma(x,r)dW_t$$

The agent can switch from regime i to j at a lump-sum cost C_{ij}, but there is no cost to remain in the current regime ($C_{ii} = 0$). As explained in the former paragraph, to avoid the probability of infinite profits, I assume $C_{ij} + C_{ji} > 0$. The discount rate is ρ.

As Bellman says, "An optimal policy has the property that, whatever the initial state and decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision"[18]. Dynamic programming is indeed based on the principle of optimality and can conventionally be expressed on the form of Bellman equation.

Let us consider a short time period Δt. $V(x,r)$ is the maximum achievable sum of current and forecasted future cash flows of time t, given that the firm is in regime r. Within the non-switch regions, the Bellman equation in discrete time can be written as:

$$V(x,r) = \pi(x,r)\Delta t + \frac{1}{1+\rho\Delta t} E_t[V(x_{t+\Delta t}, r)]$$

Multiplying both sides of the equation by (1+ρΔt)/ Δt and through rearrangement, we get:

$$\rho V(x,r) = \pi(x,r)(1+\rho\Delta t) + \frac{E_t[V(x_{t+\Delta t}, r) - V(x,r)}{\Delta t}$$

Taking the limits of this equation at $\Delta t \to 0$ provides the continuous time version of the Bellman equation:

$$\rho V(x,r) = \pi(x,r) + \frac{E_t dV(x,r)}{dt}$$

V can be seen as the value of an asset on a dynamic project. The Bellman equation predicts that ρV, which is the rate of return on the asset on the dynamic project in regime r is equal to the

[18] R. Bellman. 1957. *Dynamic programming*

sum of current cash flow on the project $\pi(x,r)$ and the expected capital rate gain, $E_t dV(x,r)/dt$.[3]

Let V_x and V_{xx} represent the first and the second derivative of V, respectively. By Itô's Lemma,

$$dV(x,r) = V_x(x,r)dx_t + 0{,}5\, V_{xx}(x,r)dx_t dx_t$$

$$= V_x(x,r)[\mu(x,r)dt + \sigma(x,r)dW_t] + 0{,}5\, V_{xx}(x,r)\sigma^2(x,r)dt$$

$$= [\mu(x,r)V_x(x,r) + 0{,}5\, \sigma^2(x,r)V_{xx}(x,r)]dt + \sigma(x,r)V_x(x,r)dW_t$$

Taking expectations on both sides of the equation and dividing them by dt, leaves us with:

$$\frac{E_t dV(x,r)}{dt} = \mu(x,r)V_x(x,r) + 0{,}5\sigma^2(x,r)V_{xx}(x,r)$$

Substituting this last equation into the continuous time version of the Bellman equation, results in the following form of the Feynman-Kac equation[19]:

$$\rho V(x,r) - \mu(x,r)V_x(x,r) - 0{,}5\sigma^2(x,r)V_{xx}(x,r) = \pi(x,r)$$

Suppose that at the boundary point x^* it is optimal to switch from regime i to regime j. At that point, the value function must satisfy two conditions: the value-matching condition and the smooth-pasting condition. The former is a condition which holds independently on the optimality of switching points. This means that the value before switching must be equal to the value after switching less the switching cost:

$$V(x^*, i) = (V_x(x^*, j) - C_{ij}(x^*)$$

[19] The Feynman-Kac equation, named after R. Feynman and M. Kac, establishes the relationship between the solution to certain partial differential equations (PDEs) and stochastic processes.

The latter condition is satisfied at the optimal switching points, meaning that the marginal value before switching equals the marginal value after switching less the marginal cost of switching. Let C' denote the marginal cost function. The smooth pasting condition will be expressed as follows[3]:

$$V_x(x^*, j) - C'_{ij}(x^*)$$

2.4 Theoretical Parameter Calibration

The purpose of this thesis is to construct a theoretical structure and develop a quantitative approach for business managers to prepare in the event of pandemics. The precise calibration of the model is important, thus I will try to adapt the model to the current Covid-19, although it is difficult to assess consistent pandemic data from a real firm. I will use current epidemiological parameters about the spread of the virus in Italy. Although, those parameters have been fluctuating much and additionally they are not the same for every pandemic. The beauty of the model though, is that parameters can be changed very easily, so I will start from a set of parameters but then I will analyse the effect of implementing control strategies, thus decreasing the internal infection rate α, increasing the recovery rate γ and decreasing the external infection rate β.

2.4.1 Disease Dynamics in the $\boldsymbol{SIRD}$ model

In the recently published study by researchers at the La Jolla Institute for Immunology, the scientists analysed the individual components of the immune system of 188 people with Covid-19, including 44 who had been infected for more than six months[20]. Specifically, the study quantified over time the concentration of antibodies to the Spike protein and the concentration of both B and T lymphocyte memory cells. The analysis showed that the concentration of IgG antibodies remained constant at 6 months after contact with Sars-Cov-2. On the other hand, B

[20] Fondazione Umberto Veronesi. 2020. *Covid-19: quanto dura l'immunità?*

lymphocyte memory cells increased after the first month and then settled at 6 months. Regarding the cellular response, memory T lymphocytes had an average half-life of 3-5 months. Based on the findings, the scientists stated that in 95% of the cases analysed, the natural infection generates a robust immunological memory that persists 8 months after infection.

Based on these findings and in order to simplify the calibration, I will assume that recovering from Covid guarantees immunity.

I take the simple SIR model and add one equation to include deaths. Setting the starting population to $N_0 = 1$, so that the class variables are measured as fractions of the population, the model can be written as[21]:

$$\frac{dS}{dt} = -\beta S_t I_t$$

$$\frac{dI}{dt} = \beta S_t I_t - (\gamma_r + \gamma_d) I_t$$

$$\frac{dR}{dt} = \gamma_r I_t$$

$$\frac{dD}{dt} = \gamma_d I_t$$

Where S_t is the fraction of population that is susceptible, I_t is the fraction of infected, R_t the fraction of those who have recovered and D_t the proportion of the deceased. Note that at $t = 0$, $R_t = D_t = 0$, so $S_0 + I_0 = N_0 = 1$. However, for the epidemic to begin, I_0 needs to be greater than 0, so I will assume I_0 is very small at the beginning. As already mentioned, an important assumption here is that a person who recovers becomes immune, so he is no longer susceptible. The other parameters can be interpreted as follows. The *contact rate* or the degree of contagion is denoted by β,[22] which measures how the interaction between susceptibles and infectives causes a reduction in S_t and an increase in I_t, namely how many susceptibles become infected. Next, the *removal rate* is denoted by $\gamma \equiv \gamma_r + \gamma_d$, which represents the rate at which people leave the pool of infectives either by recovering $(\gamma_r I_t)$ or dying $(\gamma_d I_t)$. The ratio $\rho = \gamma/\beta$ is

[21] Pindyck, R.S. 2020. *Covid-19 and the Welfare effects of reducing contagion*
[22] Note that here I am not distinguishing between internal and external transmission rates

the *relative removal rate* and $1/\rho$ is the *reproduction rate* and is denoted by R_0. This last parameter is treated as the key policy variable, so the parameter that social distancing and related policies seek to control together with β, because with γ constant, changing R_0 changes β.[21] If $R_0 \leq 1$, removals from the class of infectives (as infectives recover or die) exceeds entry into the pool, so that the pandemic cannot take off. This was roughly the case of Ebola pandemic, since infectives were contagious only when very sick or dead, and the fatality was very high, so β was low and γ high, making $R_0 < 1$. $\frac{dI_0}{dt} > 0$ needs the initial fraction of vulnerable indivuduals S_0 to exceed $1/R_0$. So, if not everyone is susceptible ($S_0 < 1$) a greater degree of contagion is needed ($R_0 > 1/S_0$) for the epidemic to take off[21].

The $SIRD$ model is simplistic and ignores many aspects of Covid-19. Firstly, it ignores the design of policies to control it and most importantly, it treats the epidemic as occurring within one large mass of homogeneous individuals, while actually outbreaks are regional with each region consisting of heterogeneous individuals. Nevertheless, this model provides rough answers to several interesting questions and helps clarifying the dynamics of Covid-19.

2.4.2 Some Basic Analytics

Assume starting with a fraction of infectives I_0 very close to zero and thus a fraction of susceptible S_0 close to 1. The speed, duration and intensity of the epidemic depend on the parameters β and γ. I will address the following question: What is the maximum number fraction of population that will be infected I_{max} and how does it depend on the degree of contagion β, taking γ_r and γ_d as fixed?

a. The Pool of Infectives

The following equation denotes the behaviour of I_t and I_{max}:

$$\frac{dI}{dS} = -1 + \rho/S_t$$

which is obtained by dividing $\frac{dI}{dt} = \beta S_t I_t - (\gamma_r + \gamma_d) I_t$ by $\frac{dS}{dt} = -\beta S_t I_t$.

So,

$$I_t = \int_0^t \left[-1 + \frac{\rho}{S}\right] dS = S_0 + I_0 - S_t + \rho \log\left(\frac{S_t}{S_0}\right) = 1 - S_t + \rho \log\left(\frac{S_t}{S_0}\right)$$

I_t will reach a maximum when $\frac{dI}{dS} = 0$, so at the point where $S^* = \rho$. Then $\frac{dI}{dt} > (<) 0$ when $S_t > (<) \rho$. The maximum number of infectives is:

$$I_{max} = 1 - \rho + \rho \log\left(\frac{\rho}{S_0}\right) \approx 1 - \rho + \rho log\rho$$

Recall that $\rho = \gamma/\beta$ and note that $\frac{\partial I_{max}}{\partial \rho} = log\rho$. So as long as $\rho < 1$ (so $R_0 = \frac{1}{\rho} > 1$) a decrease in the contact rate β will reduce the maximum number of infectives. Note that if $R_0 = 1, I_{max} = 0$, the epidemic cannot take off[21].

b. The Dead and the Susceptibles

At the end of the epidemic, the total number of victims, denoted by D_∞, depends on the number of infectives at each period of time and on the rate at which infected people recover or die, i.e., the variables γ_r and γ_d. The total number of deaths is a function of the remaining number of susceptibles S_∞. Dividing equation $\frac{dS}{dt} = -\beta S_t I_t$ by $\frac{dR}{dt} = \gamma_r I_t$, we obtain $\frac{dlogS_t}{dR_t} = -\beta/\gamma_r$, so $\log\left(\frac{S_\infty}{S_0}\right) = \left(-\frac{\beta}{\gamma_r}\right) R_\infty$.

But $R_\infty = N_0 - D_\infty - S_\infty = 1 - D_\infty - S_\infty$, so:

$$\log\left(\frac{S_\infty}{S_0}\right) = -\left(\frac{\beta}{\gamma_r}\right) S_\infty - \frac{\beta}{\gamma_r} - \left(\frac{\beta}{\gamma_r}\right) D_\infty$$

Note that $\frac{dlogS_t}{dD_t} = -\beta/\gamma_d$ so that $D_\infty = -\left(\frac{\gamma_d}{\beta}\right)\log\left(\frac{S_\infty}{S_0}\right)$. Substituting above for D_∞ gives the fundamental equation for the final number of susceptibles S_∞:

$$\left(\frac{\gamma}{\beta}\right)\log\left(\frac{S_\infty}{S_0}\right) - S_\infty + 1 = 0$$

This equation lets us determine the fraction of population still vulnerable when the epidemic ends. Note that reducing $R_0 = \beta/\gamma$ raises S_∞, and $S_\infty \to S_0$ as $R_0 \to 1$. Since S_0 is close to 1 and using the last equation, the total number of victims can be written as:

$$D_\infty = \left(\frac{\gamma_d}{\gamma}\right)(1 - S_\infty)$$

How does the final number of susceptibles and deaths depend on the transmission rate β? From the last equation, $\frac{dD_\infty}{d\beta} = \left(-\frac{\gamma_d}{\gamma}\right) S_\infty/d\beta$. Taking the total differential of equation $\left(\frac{\gamma}{\beta}\right)\log\left(\frac{S_\infty}{S_0}\right) - S_\infty + 1 = 0$, with respect to S_∞ and β,

$$\frac{dS_\infty}{d\beta} = \frac{S_\infty logS_\infty}{\beta(1 - S_\infty\ )} \leq 0$$

A higher β means that a higher number of people get infected during the epidemic, thus lowering the final number of susceptibles S_∞.

Government policies aim at reducing the contact rate and are expressed, as already mentioned, in terms of reproduction rate $R_0 = \beta/\gamma$. Considering that $\frac{dD_\infty}{dR_0} = \frac{\gamma dD_\infty}{d\beta}$, we obtain:

$$\frac{dD_\infty}{dR_0} = -\frac{\gamma_d S_\infty logS_\infty}{\gamma R_0(1 - S_\infty)} \geq 0$$

Which can be used to determine how many deaths are avoided if R_0 is reduced, once solved for S_∞.

2.5 Rough Calibration to Covid-19

The calibration of the $SIRD$ model involves only three parameters: β, γ_d and γ_r. The drawback of this method applied to Covid-19 is that the exact number of infectives is not known, because many infected people show mild or no symptoms. At the same time, we do not know the exact number of deaths from the disease, due to limited testing and no autopsies. This might cause an increase in the recorded number of actual deaths since many of the deceased people already had some other diseases or were very old, and Covid-19 unfortunately caused their early death. On the other hand, the cause of death for many Covid-19 might have been recorded as something else, at least in the early phases of the outbreak, thus causing an underestimation of the actual deaths due to the virus. With this premise, the calibration is inevitably rough but provides some interesting insights.

To start, I will select values for β, γ_d and γ_r based on the limited information we have for Italy. I will select data as of November 2020, because much more information was available compared to the very beginning of the virus outbreak.

Firstly, I will take the population to be $N_0 = 1$ and assume that the initial number of infectives is $I_0 = 2 * 10^{-6}$. Given an Italian population of about 60 million people[23], this corresponds to 121 infected individuals at the outset.[24] The initial number of susceptibles is $S_0 = 1 - I_0$.

The time interval Δt is one day. I will set the removal rate γ at 0,07, based on the assumption that the average duration of the virus is 14 days. Assuming that the average illness duration is the same whether the patient recovers or dies, γ_d depends only on the fraction of patients that die. Based on data from Italian Ministry of Health[25], on November 9, 2020, 41.394 people had died since the beginning of the virus outbreak. The recovered were 335.074 and the infected were 558.636. Thus, the total number of cases at that date was 935.104 people, given by the

[23] According to Istat data, the Italian population on 1st January 2020 was 60.317.000 people
[24] This shows that the simple SIRD model is unrealistic since the spread took off at specific points in Italy and not from a pool of people spread out evenly across the country.
[25] opendatadpc.maps.arcgis.com

sum of total removed, infected and deaths. Based on these numbers, the estimated fraction of deaths was 4,43%.

Now, there are some factors to be considered. First, this fraction probably overestimates the true death rate, because the denominator is an underestimation of the actual number of cases, reasonably assuming a high level of asymptomatic infected. The other consideration regards hospital congestion. It is a well-known fact that Italian hospital were highly congested and overwhelmed due to the sudden surge of cases. Congestion should be taken into account as part of the death rate, so for example if the death rate is estimated to be 1% with no congestion, it is significantly higher with congestion. Considering concurrently these facts, I will assume a death rate δ lower than the estimated fraction, but only slightly, and I will set it to be 4%.

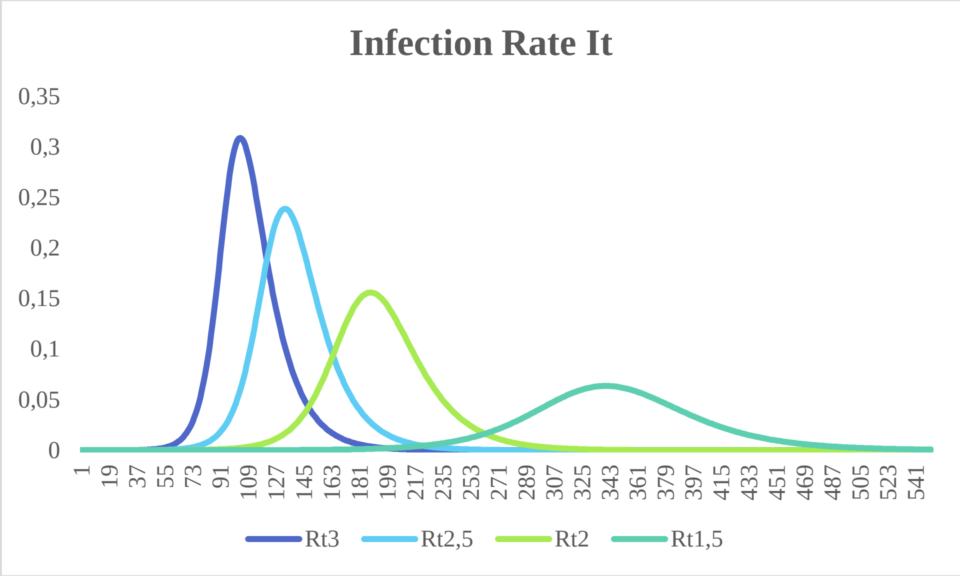
Since $\gamma_d = \delta\gamma$, we get $\gamma_d = 0{,}04 * 0{,}07 = 0{,}29\%$.

Thus, γ_r will be equal to 6,86%[26].

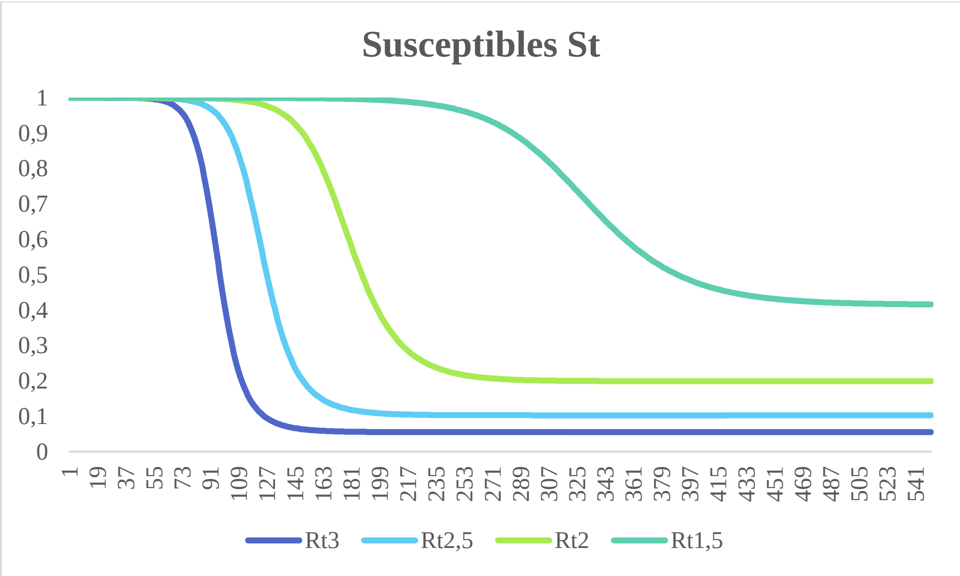
Given γ and its components, we are left with the contact rate β or equivalently, with the reproduction number $R_0 = \beta/\gamma$ which is a function of the social distancing policy implemented. Clearly, at the outset the reproduction number is high because there is still no distancing policy applied. Atkenson estimated from 8 studies based on data from China, Italy and US that R_0 at the outset oscillated between 2,2 and 3,3. Since my study is based on November data, I assume that in that period the reproduction number was around 1,5, considering that the social distancing policies were strongly applied all over Italy. But to start, I will take the base value of R_0 with no social distancing policy to be 3.0 and then explore what happens when R_0 is reduced, assuming that the death rate and the removal rate do not change with social distancing policies.

Figure 6 shows solutions to the SIRD model with $\gamma_d = 0{,}29\%$, $\gamma_r = 6{,}86\%$, $R_0 = 3, 2{,}5, 2$ and 1,5, corresponding to $\beta = \gamma R_0 = 0{,}214, 0{,}179, 0{,}143, 0{,}107$ and with starting number of infectives $I_0 = 2 * 10^{-6}$.

[26] Remind that $\gamma = \gamma_r + \gamma_d$



Infection Rate It
0,35
0,3
0,25
0,2
0,15
0,1
0,05
0
1 19 37 55 73 91 109 127 145 163 181 199 217 235 253 271 289 307 325 343 361 379 397 415 433 451 469 487 505 523 541
Rt3
Rt2,5
Rt2
Rt1,5



Susceptibles St
1
0,9
0,8
0,7
0,6
0,5
0,4
0,3
0,2
0,1
0
1 19 37 55 73 91 109 127 145 163 181 199 217 235 253 271 289 307 325 343 361 379 397 415 433 451 469 487 505 523 541
Rt3
Rt2,5
Rt2
Rt1,5

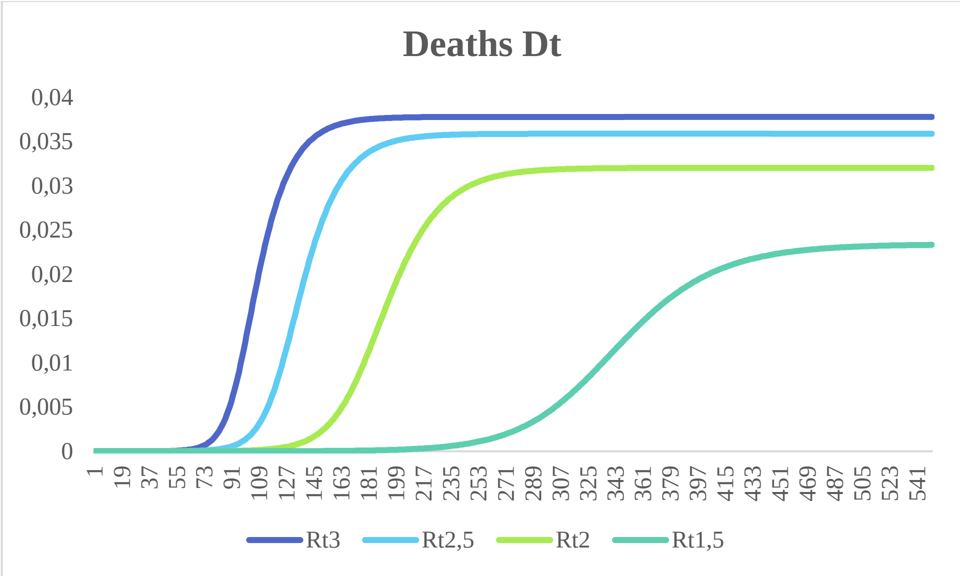


Figure 6: Solution of SIRD Model

Source: own calculations

The middle and bottom panels show the fraction of people that is susceptible and the fraction that has died. The figure suggests that new infections and deaths begin and end at specific points in time, but in fact new infections begin on day 1 and drop to zero only asymptotically[27].

To measure the time span of the disease spread, I will take its beginning (end) to be date at which I_t first reaches (falls back to) 1% of its maximum value. So, for $R_0 = 3$ the epidemic runs from day 59 to day 182, for a duration of 124 days. For $R_0 = 2{,}5, 2$ and 1,5 the durations are 146, 189 and 307 days respectively.

Figure 6 shows some important characteristics of the model and their implications for social distancing policies. When R_0 and thus β are lowered, the epidemic spreads later, evolves more slowly and lasts longer[21].

2.5.1 Parameter calibration to the Model

Since public data on the β parameter was not provided, I could get this value starting from data about the R_0 in Italy as of November 2020. To start, I will assume a R_0 equal to 1,5[28] and then

[27] Suppose $R_0 = 3$, so $\beta = 0{,}214$ (the dark blue line in each panel). Then for the Italian population of about 60 million people, on Day 1 there will be about 17 new infections. On day 250 there are about 5.000 infected and on day 350 about 10 infected.

[28] Ministero della Salute, *Weekly Monitoring Covid-19*

I will analyse results with changing parameters. Remind that $R_0 = \frac{\beta}{\gamma}$. Assuming that the average life of the virus is 14 days, I assume the removal rate γ to be equal to 1/14 (≈ 0,07). This leads to a β equal to 0,107, which represents the external infection rate. As regarding the recovery rate γ_r, from calculations made above I will assume it to be equal to 0,07 in the inactive regime (γ_{r2}) and equal to 0,01 in the active regime (γ_{r1}), to reflect the fact that the recovery rate should be higher when workers are separated from each other. The value of internal transmission rate α is set to be equal to 1 and the volatility coefficient c is set at 0,1, as suggested by Cobb (1998). The fatality rate δ, as previously calculated, is set at 4%. The values of other parameters are set as follows[29]:

- Discount rate:[30] $\rho = 0{,}05$
- Productivity of an infective: $\xi = 0{,}5$
- Total number of employees: $N = 547$
- Price of product: $P = 10{,}95$
- Variable costs: $VC = 4{,}98$
- Fixed costs: $FC = 48181$
- Penalty cost: $E = 10$
- Mothballing cost: $C_{12} = 1500$
- Reactivation cost: $C_{21} = 1500$

2.6 Covid crisis: most and least affected sectors

The outbreak of Covid-19 pandemic transformed the pattern of the world economy. Once the spring lockdown had passed, economic production and the labour market began a delicate process of adaptation to the new conditions. In this situation, the various sectors have been affected in a heterogeneous manner: it may therefore be useful to analyse the differences in an attempt to define their development prospects.

[29] Data about Fixed, Variable Costs and Number of Employees will be shown in the next Paragraphs

[30] Dynamic programming assumes a constant discount rate, given exogenously. Miranda and Fackler (2002) assume a discount rate equal to 0,05. The same assumption is made here.

At the European level, medical, pharmaceutical, and cosmetic goods sold on average the same in 2020 as they did in 2019, but with major dissimilarities across countries. Comparison of the March 2020 and 2019 retail purchase index shows negative changes for Italy (-12) and the United Kingdom (-1) while Germany and France showed positive values (+5 and +16, respectively).[31]

The pandemic also severely affected tourism, which experienced a uniform slowdown in Europe across all member states. Compared to the same period in 2019, arrivals at tourist facilities fell by 44% year-over-year, averaged over the month. April, May, November, and December all had reductions of more than 70% in tourist arrivals compared to the previous year. In Italy, tourism declined 98,8% in April compared to the same month in 2019, and the only positive figure in 2020 was a 0,2% increase in August. This slump has negatively impacted workers in the tourism sector: in fact, provisional data from Istat shows that there has been a decrease of more than 300,000 employees in the first six months of 2020.

Retail has been one of the sectors that has struggled most, albeit with important differences between the various sectors. For example, the Eurostat index measuring food and beverage retail sales decreased by just one point in 2020 compared to the 2019 average, despite the fact that restaurants and bars were subject to restrictions and closures. In contrast, the overall retail spending index that excludes food and fuel consumption shows a sharp decline in turnover in many countries: in Italy, the difference between 2020 and 2019 is 13 points, in France 6, and in the United Kingdom 4.

The clothing and textile industry has been penalized even more: the index calculating its average turnover at the European level has decreased by 26 points between 2019 and 2020. This large difference is due both to the many months in which stores in the sector were forced to work at a reduced pace and to the changes in lifestyle and needs that the pandemic introduced.

2.7 The impact of Covid-19 on the Textile Sector

2.7.1 The international scenario

[31] https://www.lavoce.info/

Covid-19 has had a great impact on all social and economic sectors, as we have seen before the textiles, clothing, leather and footwear industries have been struggling. Quarantine measures, closure of retail stores, illness, and salary reductions have suppressed consumer demand.[32] This sector is also struggling due to sever supply-side disruption, since supply chains grind to a halt and factories close. The economic impact on industries has affected the livelihoods of employees, in addition to health risks posed by the virus, because factors and retail closures led to the dismissal of workers.

a. Sales

More and more shops were forced to closure, and consumers were instructed to stay home, due to government restrictions, which caused a substantial drop in sales worldwide. Major brands have been forced to close stores in several countries, let us look at some examples:

- Ralph Lauren warned that global sales could drop by as much as US$ 70 million[33];
- Gap expects to experience a first quarter global sales hit of around US$ 100 million[34];
- Inditex has closed 3.785 stores in 39 markets – over 50 per cent of its stores – with combined store and online sales falling by 24,1% in the first half of March 2020.[35]

Retailers have employed some tactics to compensate the drop in sales, such as free shipping and heavily discounted products to encourage consumers to buy online. Nevertheless, rising unemployment and growing uncertainty led clothing no longer being a priority to several consumers.

b. Production

At the height of the epidemic in China, shortages of raw materials were the primary concern to apparel manufacturers, causing production disruptions, especially in the T&C industries in Asia. Later, as the epidemic shifted to Europe and then Unite States and rest of the world, many factories were forced to close. In Mexico, for example, maquila industries, which include textile manufacturing and whose industry employs more than 2.1 million workers[36], have halted

[32] Just-Style. "Timeline: Timeline – How coronavirus is impacting the global apparel industry".
[33] Economic Times. "Ralph Lauren: 4Q sales hit of up to $70M from coronavirus".
[34] Just-Style. "Gap expects coronavirus to hurt Q1 sales by $100m", 13 March 2020.
[35]Financial Times. "Zara owner to write off nearly €300m of inventory", 18 March 2020.
[36] Mexico 2020 Population Census.

production following a government order to close all dispensable economic activities for no less than 30 days. Companies in China are facing challenges to increase production, such as higher costs and continued shortages of raw materials. The Bangladesh Garment Manufacturers and Exporters Association (BGMEA) has reported a series of order cancellations, even for garments already in production or completed, which has caused most affected factories to close.[37] According to the BGMEA, this equates to lost revenue of approximately $3 billion and affects about 2.17 million workers.[38] These effects were felt throughout the supply chain. Cotton prices plunged and hit their lowest since the 2008 financial crisis.

c. Trade

Falling demand from major economies has been the main limiting factor for trade. In Central America, Nicaragua has forecasted a full year of declining exports, and Guatemala has announced that its shipments will be delayed. The medium-term impact of the pandemic will see the major importing countries in key market emerge from the wort of the crisis. However , in the long run, the pandemic ought to affect the composition of global textile, apparel, leather, and trade, and fasten the offshoring of production.

d. Employment and working conditions

The decline in production and sales has had a knock-on effect on workers, both in terms of employment and working conditions:

- An estimated 200 factories in Cambodia have suspended or reduced production and at least 5,000 workers have lost their jobs.
- In Myanmar, the lack of raw materials from China has led to the closure of at least 20 factories and the loss of 10,000 jobs and at the same time, the number of orders has plummeted.[39]

---

[37] Anner, M. 2020. "Abandoned? The impact of Covid-19 on workers and businesses at the bottom of global garment supply chains", (Penn State Center for Global Workers' Rights).
[38] Bangladesh Garment Manufacturers and Exporters Association. Impact of COVID-19.
[39] Myanmar Times. "More woes for Myanmar garment industry as EU cancels orders".

- In Bangladesh, as many as 2.17 million workers have been affected by the crisis, and many are facing unemployment, as orders are cancelled and production declines dramatically.

Non-payment of wages and the closure of factories is especially hard for employees in countries with very vulnerable social protection structures.

2.7.2 The European scenario

The European textile and clothing sector comprehends about 170.000 companies, of which 99,8% are microenterprises and SMEs. Collectively, they generate an annual turnover of approximately 180 billion euros, employing 1,7 million people[40]. The sector has been transforming recently, since the production of mass consumption items has been reduced in order to integrate the industry towards higher value-added products, like technical and industrial textiles[41]. The EU Textile Strategy is under development, the goal is to achieve a green, digital and resilient economy, additionally to the latest emerging needs triggered by the pandemic.

Figure 7 shows data in the textile and clothing industry up to December 2020 in the European context. The dramatic contraction in demand and production of textile and clothing items, caused by the Covid-19 pandemic can be clearly observed.

[40] EURATEX. 2020: https://EURATEX.eu/wp-content/uploads/Post-Corona-Strategy-Final.pdf.
[41] European Commission. 2020. *Textiles, Fashion and Creative Industries*

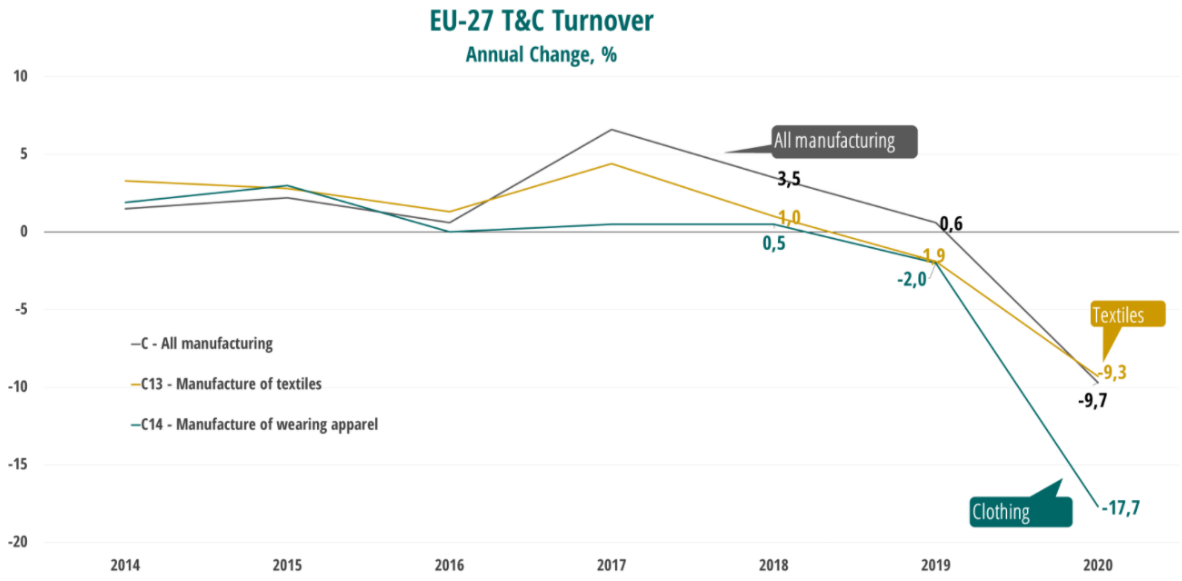


Figure 7: Textile and Clothing Turnover in Europe

Source: EURATEX

In the early 2020s, global health crisis caused a drop in production and disruption to the textile supply chains. With China being a critical global supplier of textile inputs, the trade impact on production consequently extended to the entire global market, including the EU. Production in Europe fell by more than 10% in Q1 compared to 2019, which a 38% and 57% difference in textiles and apparel subsectors respectively in April 2020. Regarding employment, the labour market for T&C suffered a relatively small setback in the first few months of the crisis, with a decline of 1,5% in textiles and 4,9% in apparel across the EU compared to 2019, in part due to short-term measures taken at the national level to support employment.

Demand has declined significantly, with retail sales dropping by 18,8% in Europe during Q1. 60% of European textile companies interviewed in a survey conducted between March and April 2020 expected sales to fall by more than half; 7 in 10 faced serious financial difficulties and 8 in 10 said they had reduced, at least temporarily their workforce[42].

Sales though online channels reached historic highs in some European countries, showing a shift in consumer behaviour towards e-commerce that has continued through the entire 2020. However, this shift toward online shopping failed to offset the overall decline in sales for the entire sector.

[42] EURATEX, CORONAVIRUS survey 2020

The industry began to recover in the third quarter of 2020, albeit at a small pace. Production experienced a rebound from Q2 of 25% in textiles and 33% in the apparel sub-sectors. Sales figures also improved, with overall retail sales recovering 62% from Q2.

Despite this rebound, the full-year figures are negative compared to 2019. Production and retail sales declined 15% and 9,4% for apparel and 7% and 9,7% for textiles, due to lower interest in purchasing clothing. Employment faced declines by 2,9% and 7,5% for the respective subsectors in Q3 2020.

Despite this series of not reassuring information, the overall industry sales are expected to rebound by approximately 15% in 2021 (with a potential recovery in consumer spending) but is not expected to return to pre-recession levels until Q3 2023, assuming a gradual easing of the health emergency and substantial measures to support the economy. Estimates for total industry employment in the T&C labour market could decline by approximately 8% (about 158,000 jobs) by the end of 2021[42]. The number of businesses, moreover, is expected to decline by 6% (about 13,000 firms) in the same year. Based on these forecasts, the recovery scenario for the industry is likely to be U-shaped, as shown in Figure 8[43].

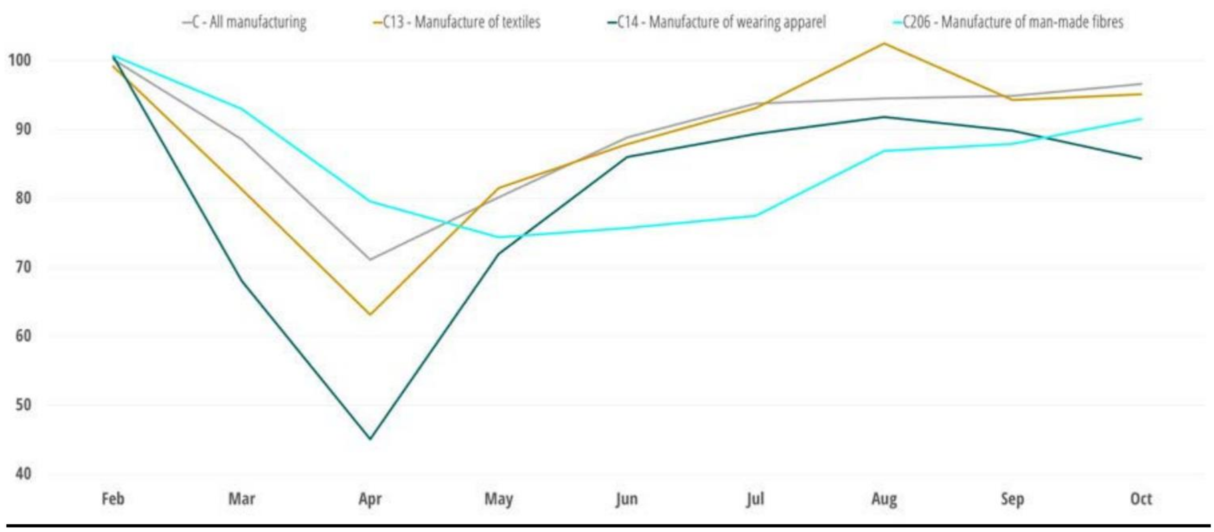


Figure 8: Monthly output index of EU T&C sectors

[43] De Vet J.M. et al. 2021. *Impacts of the Covid-19 pandemic on EU industries*

Source: EURATEX

2.7.3 The Italian Scenario

The textile and clothing industry represents one of the most important sectors of the manufacturing industry in Italy. It is a sector that boasts an ancient tradition in our country. With over 500.000 employees, this sector employs 12% of all workers in the manufacturing sector. Its turnover represents 9% of the turnover of the manufacturing sector. Moreover, the Italian textile industry exports represent 77,8% of total European exports[44].

Being the textile sector so fundamental in Italy, I considered it would be interesting to focus on it on my thesis.

As we have seen, the T&C sector is among the most exposed to the effects of the Covid-19 crisis, second only to the hospitality and tourism sectors. The production of textiles, clothing, leather and accessories collapsed by 81% year-on-year in April 2020[45]. The almost total closure of commercial channels, with the exception of online, has led to a contraction in retail sales of clothing and leather goods in April were down by more than 83%[46] compared to the same month of the previous year.

The lockdown period has led to the blocking of all commercial activities of clothing and accessories stores, affecting around 300 thousand employees. E-commerce, by guaranteeing the existence of a minimum turnover for companies active in online sales, has been one of the main factors of resilience in the sector, but, at the same time, is one of the main risk factors for retail employment.

The Italian clothing and accessories sector bases its solid foundations on historical experience and success. To recover the lost ground and return to record the positive trends of the last few years the sector needs an articulated plan to restart, which includes the definition of policies to create long-term value and innovation in business models[47].

[44] Sistema moda Confindustria. 2019. Lo stato della moda 2019
[45] ISTAT. 2020. Produzione industriale, 11th June 2020
[46] ISTAT. 2020. Commercio al dettaglio, 5th June 2020
[47] Cassa Depositi e Prestiti, Ernst&Young and Luiss Business School. 2020. *Settore Moda e Covid-19*

2.8 Ratti S.p.A.

"*Living silk is an art, today as yesterday*" Antonio Ratti

Ratti S.p.A. was founded in 1945 by Antonio Ratti, who opened in Como his "Tessitura Serica Antonio Ratti", for the creation and marketing of silk fabrics and accessories.

In 1958 the Guanzate plant for the integrated cycle production process of silk was inaugurated, from yarn to finished product through the phases of weaving, dyeing, photoengraving, printing and finishing. Guanzate is still the main production plant in Italy.

Few years after opening new offices in Wall Street, the Company became listed on the Milan Stock Exchange in 1989. With the contribution of the Antonio Ratti Foundation, in 1995 the Metropolitan Museum of New York opened the Antonio Ratti Textile Centre, one of the largest and most technologically advanced facilities for the study and conservation of textiles. In the early 2000s, the Company opened new plants several countries, among which Tunisia and Romania. In 2010 Marzotto Textile Group and Faber Five Srl entered into the shareholding of Ratti S.p.A., holding its control. Donatella Ratti, daughter of the founder, is currently President of the Ratti Group.

The harmonious growth of the Group has led Ratti over the years to become a member of Associations, networks and bodies involved in the promotion and development of the textile industry. The Group adheres to Confindustria (Unindustria Como and Sistema Moda Italia) and Centro Tessile Serico; moreover, it participates in the activities of the European Technological Platform of Textile Clothing and is a contributor to the ZDHC program. The Company also adheres to the BCI (Better Cotton Initiative), an association formed by producers, intermediaries and non-profit groups to promote environmentally friendly cotton cultivation in respect of the environment.[48]

The Ratti Group consists of the parent company Ratti S.p.A. (Italy), the subsidiaries Textrom Srl (Romania), Creomoda Sarl (Tunisia), La Maison des Accessoires Sarl (Tunisia), Ratti International Trading Co Ltd (China), Ratti USA Inc. (United States), Second Life Fibers Srl (Italy) and a stake in Marielle Srl (Italy). Marielle Srl (Italy).

[48] Gruppo Ratti, Sustainability Financial Statements 2020

The Group is today one of the major players in the production of printed, plain, yarn-dyed, and jacquard fabrics for clothing, ties, shirts, beachwear, and furnishings. It also creates men's and women's accessories such as ties, scarves, and foulards.

The structures, machinery and equipment are the most modern and specialized in the field of printing, managed by highly qualified personnel. Ratti preserves its tradition of craftsmanship, aimed at the continuous search for excellence expressed in all its forms.

Ratti manages and controls the entire production cycle: from the creative idea that develops around a design, to the design of the fabric up to the finishing phase. A production that tells all the quality and luxury of an all-Italian excellence, to define, through each single step, a fabric appreciated and requested by the luxury and pret-à-porter maisons of the world.

I will focus in my thesis on the Parent Company Ratti S.p.A., in order to make the data smoother, necessary to obtain more truthful results.

2.8.1 Ratti S.p.A. and Covid-19

To handle the inevitable impact caused by the spread of the Covid-19 virus, Ratti has implemented a form of resilience as a strategy to adapt and transform to the changed marketplace. This system has allowed the company to react in the face of the difficulties of the period, proposing new business models and transforming external stimuli into concrete action and new forms of innovation. Despite the strong reaction of the Company to the crisis, the drop in demand and the lockdown measures have inevitably impacted on performance.

Some data from Ratti’s Individual Financial Statements as of 31st December 2020 are here presented.

The Parent Company closed the 2020 financial year with revenues from the sales of goods and services of €71,1 million (-€45,0 million compared to 2019) and a gross margin (EBITDA) of €5,2 million (-€14,1 million compared to 2019). Profit before taxes and profit for the year amounted to €0,8 million.

Revenues from the sale of goods and services are broken down as follows:

a. by product type:

€/000	2020	2019	Delta
Ratti Luxe	34.526	55.067	-20.541
Collection	15.926	30.601	-14.675
Carnet	6.985	10.493	-3.508
Fast fashion	2.129	4.636	-2.507
Ratti Studio	7.563	10.299	-2.736
Furnishing	3.264	4.779	-1.515
Other	756	313	443
Total	**71.149**	**116.188**	**-45.039**

Table 4: Revenues by product type

Source: Ratti's Individual Financial Statements 2020

b. by Geographical area:

€/000	2020	2019	Delta
Italy	30.319	49.917	-19.598
Europe EU	24.589	37.458	-12.869
USA	3.075	6.698	-3.623
Japan	2.075	2.805	-730
Other	11.091	19.310	-8.219
Total	**71.149**	**116.188**	**-45.039**

Table 5: Revenues by Geographical area

Source: Ratti's Individual Financial Statements 2020

Following the significant restrictions on the market, the fall in sales affected all areas of business. With reference to the larger hubs, the Luxe Hub reported a drop in sales of €20,5 million (down 37,3%), whilst the Collections Hub reported a decrease of €14,7 million (down 48,0%).

Sales by geographical area showed a widespread reduction in all the main outlet markets. In particular, sales on the US market fell by €3,6 million (-54,1%) and revenues on the domestic market by 19,6 million euros (-39,3%).

Costs also have been deeply affected by the pandemic: the cost of raw materials, ancillary materials, consumables, and goods for resale decreased overall compared to FY 2019 by €17 million. The decrease is directly related to the decline in sales volumes[49].

2.8.2 Ratti S.p.A: Cost analysis

The analysis of costs, in particular the allocation of fixed and variable costs, is fundamental for the subsequent MATLAB implementation.

a. **Cost of raw materials, ancillary materials, consumables, and goods for resale:** this Financial Statement Line Item (FSLI) amounts to €18,9 million as of December 2020 and to €35,9 million as of December 2019. As already seen, the sharp decrease in costs is mainly due to the decline in sales volumes. I classified this FSLI in the variable costs group.

b. **Cost of services:** the total FSLI amounts to €16,5 million and €27,4 million in 2020 and 2019, respectively. I underlined the costs classified as variable costs, while the others are fixed. This item is broken down as follows:

€/000	**2020**	**2019**	**Delta**
<u>Outsourcing to third parties</u>	<u>3.783</u>	<u>8.319</u>	-4.536
<u>Outsourcing of subsidiary undertakings</u>	<u>1.690</u>	<u>2.348</u>	-658
Utilities	1.693	2.549	-856
<u>Commissions' payable</u>	<u>1.256</u>	<u>2.455</u>	-1.199
Maintenance	1.205	1.834	-629
<u>Transport</u>	<u>1.178</u>	<u>1.629</u>	-451
Consultancy	1.090	1.509	-419
Cleaning, waste disposal/cleaning, surveillance	1.005	1.115	-110
Sampling and creation expenses	722	1.123	-401
Advertising and promotion	534	935	-401
Insurance	382	475	-93
<u>Services in outsourcing</u>	<u>276</u>	<u>282</u>	-6
<u>Customs charges on purchases</u>	<u>258</u>	<u>585</u>	-327
<u>Travel and accommodation expenses</u>	<u>247</u>	<u>920</u>	-673

[49] Ratti S.p.A. Financial Statements 2020

Charges for services from related parties	192	250	-58
Bank charges	62	81	-19
Fees for supervisory bodies	36	36	0
Other costs	927	939	-12
Total Cost of Services	**16.536**	**27.384**	**-10.848**

Table 6: Cost of services

Source: Ratti's Individual Financial Statements 2020

Costs of services decreased by €10,8 million compared to the previous year, primarily due to the decrease in the cost of external processing, commissions and travel and accommodation expenses. It should be noted that operating costs include in total extraordinary costs for the management of the Covid-19 emergency amounting to €0,5 million, including costs for protective equipment, compliance with protocols and donations made to local associations and bodies involved in the health emergency management[49].

c. **Costs for use of third-party assets:** in 2020, it was equal to €868 million and in 2019 to €979 million. I classified the entire FSLI, which includes royalties, rentals, and leases among the fixed costs.

d. **Personnel costs:** these expenses amount to €24,5 million and €31,6 million in 2020 and 2019 respectively and they have been classified among the fixed costs. The decrease in payroll costs, amounting to €7,1 million, is primarily due to use of the redundancy fund and use of vacation days accrued during 2020[49].
Changes in the number of staff during the year, broken down by category, are shown below:

	2020	**2019**	**Delta**
Managers	9	12	-3
Executives	42	44	-2
Employees	264	277	-13
Intermediaries	55	57	-2
Workmen	143	157	-14
Total	**513**	**547**	**-34**

Table 7: Staff Numbers

Source: Ratti's Individual Financial Statements 2020

As of December 31, 2020, there were 34 fewer employees than of December 31, 2019. The social and economic emergency situation created by the Covid-19 pandemic has obliged the Company to use the forms of wage supplementation adopted by the government. In this context of a contraction in orders and volumes and, consequently, the need to reduce personnel costs, the Company has decided not to replace staff leaving due to retirement or resignation and not to confirm certain apprenticeship contracts[49].

e. **Other operating expenses:** these expenses amount to €1,8 million and €1,7 million in 2020 and 2019 respectively. As per Cost of Services, the underlined expenses have been classified as variable, while the others as fixed. The breakdown of this FSLI is the following:

€/000	**2020**	**2019**	**Delta**
<u>Consumables, stationery, fuels</u>	<u>634</u>	<u>655</u>	-21
Liberal donations	255	0	255
IMU	303	302	1
Purchase of paintings and samples	131	233	-102
<u>Contingencies and non-existent liabilities</u>	<u>100</u>	<u>108</u>	-8
Membership fees	92	92	0
Purchase of drawings	11	53	-42
<u>Deductible taxes</u>	<u>31</u>	<u>41</u>	-10
Waste tax	36	40	-4
Representation expenses	9	28	-19
<u>Capital losses on disposals</u>	<u>132</u>	<u>21</u>	111
Other expenses	51	95	-44
Total Other Expenses	**1.785**	**1.668**	**117**

Table 8: Other Expenses

Source: Ratti's Individual Financial Statements 2020

f. **Amortization, depreciation, provisions and write-downs:** the total of these items amounts to €4,9 in 2020 and €4,3 in 2019. These expenses have been classified among the fixed costs.

On the basis of the classification made in the preceding paragraphs, total fixed costs amount to €38,8 million as of December 2020 and to €48,2 million as of December 2019, determining a decrease of €9,4 million, mainly attributable to personnel reduction.

Concerning variable expenses, in 2020 they are equal to €28,8 million, while in 2019 they amounted to €53,6 million. This deep difference is due to the decline in sales volume attributable to the sharp drop in demand, as previously analysed.

Table 9 shows a recap of fixed costs (FC) and variable costs (VC) breakdown in 2019 and 2020:

FC		
2020	**2019**	**Delta**
38.750	48.181	-9.431

VC		
2020	**2019**	**Delta**
28.738	53.620	-24.882

Table 9: Fixed and variable costs breakdown

Source: Own calculations

In order to get the variable cost needed for one meter of produced fabric, I made the following calculations[50]:

- I took from Amazon the price of fabric per meter: on average, fabrics are sold at €59,99 for 6 yards (5,48 mt), thus €10,95 per meter.
- The value of production resulted to be equal to €118 million as of 2019[49].
- From simple calculations, it resulted that in 2019, 10,8 million meters of fabric were produced.

[50] I consider the values of a company in normal conditions, so I will take the values from the Balance Sheet at 2019, because they are not compromised by the current crisis.

- Dividing the total variable costs of 2019 by the meters of fabric produced, the variable cost to produce one single meter of fabric resulted being equal to €4,98.

It should be noted that I considered financial statement data as of 2019, because the Company was not too significantly affected by the pandemic yet. Taking data from 2020 could have led to compromised results.

2.9 The Stationary Probability Distribution of the Epidemic Process

People are concerned what might be the threshold above which the epidemic is likely to spread. To answer this question, it is necessary to examine the distribution of I_t. Generally, suppose we are working with the variable x, which evolves according to a stochastic differential equation[3]:

$$dx_t = \mu(x)dt + \sigma(x)dW_t.$$

The probability density function of the random variable $f(x,t)$ depends on the random variable itself and on time t. The evolution of the density function is shown here in form of the Kolmogorov equation[51]:

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial x}\big(\mu(x)f(x,t)\big) + \frac{\partial^2}{\partial x^2}\big(\sigma^2(x)f(x,t)\big).$$

I will consider the equation when the process reaches the equilibrium (i.e., $\frac{\partial f}{\partial t} = 0$). The solution to the stationary probability function $f(x)$ has been developed by Wright:

$$f(x) = \frac{\psi}{6^2(x)}\exp\left\{\int_{-\infty}^{\infty}\frac{\mu(y)}{6^2(y)}dy\right\}$$

[51] Cobb (1998).

In which ψ is a constant so that $\int_{-\infty}^{\infty} f(x)dx = 1$. Here, we need the stationary distribution of I_t, in which $\mu(I) = \beta I(1-I) - \gamma I + \alpha(1-I) - \delta I$ and $\sigma^2(I) = cI(1-I)$. By Wright's formula, we get:

$$f(I) = \frac{\psi}{I(1-I)} exp\left\{\int_0^I \frac{\beta\gamma(1-y) - y\gamma + \alpha(1-y) - \delta y}{cy(1-y)} dy\right\}$$

$$= \psi I^{-1+\frac{\alpha}{c}}(1-I)^{-1+(\gamma+\delta)/c} e^{\beta I/c}.$$

The antimode of the equation represents the boundary beyond which the disease has higher probability to spread. Given the parameter values, we get an antimode equal to $I_1 = 45{,}3\%$. This means that the ailment is unlikely to proliferate unless more than 45,3% of employees get contaminated.[52]

Now that all parameters have been calculated, in the next Chapter I will show the solutions to the regime-switching model, based on MATLAB implementation using the Optimal Switching Solver in the CompEcon Toolbox proposed by Fackler in 2004.

[52] By solving the equation $f'(I) = 0$, we get the antimode $I_1 = d - \sqrt{d^2 - \frac{c-\alpha}{\beta}}$ where $d = \frac{\beta-\gamma-\delta-\alpha+2c}{2\beta}$

Chapter III

MATLAB implementation of the Regime-Switching Model on Ratti S.p.A

3.1 Parameters Recap

According to what mentioned at the end of Chapter II, I will start by assuming that R_0 in November was approximately equal 1,5. The β will result to be equal to 0,107[53], assuming that the average life of the virus is 14 days[54].

Let us recap the parameters set in the previous Chapter:

- Discount rate: $\rho = 0{,}05$
- Productivity of an infective: $\xi = 0{,}5$
- Total number of employees: $N = 547$
- Price of product: $P = 10{,}95$
- Variable costs: $VC = 4{,}98$
- Fixed costs: $FC = 48181$
- Penalty cost: $E = 10$
- Mothballing cost: $C_{12} = 1500$
- Reactivation cost: $C_{21} = 1500$
- Recovery rate in the active regime: $\gamma_{r1} = 0{,}01$
- Recovery rate in the inactive regime: $\gamma_{r2} = 0{,}07$
- Internal transmission rate: $\alpha = 1$
- Volatility coefficient: $c = 0{,}1$
- Fatality rate: $\delta = 0{,}04$

I will solve the regime-switching problem by dynamic programming, through the Optimal Switching Solver through the CompEcon Toolbox proposed by Fackler in 2004.

[53] Remind that $R_0 = \frac{\beta}{\gamma}$

[54] Leading to a removal rate equal to 1/14 (≈ 0,07)

3.2 Numerical Results on Optimal Regime-Switching Thresholds

3.2.1 Stochastic vs deterministic results

Based on the above parameters, it is optimal for the company the active regime to temporarily shut down operations when the fraction of infected workers reaches the threshold of 51%. On the other hand, when the company is already in the idle regime, it is optimal to resume operations when the fraction of infectives falls below 4%. The visual result from MATLAB is shown in Figure 9. The highlighted numbers (4% and 51%) are the only ones we are interested in and represent the reactivation and suspension thresholds, respectively.

```
xindex =

        0.04           2      3283.4           1           0           0
           0           0           0        0.51           1     -36.442
```

Figure 9: MATLAB results of the regime-switching problem: the Stochastic Case

Source: Own calculations

In case uncertainty regarding the behaviour of the virus is removed (i.e., the volatility coefficient c is equal to 0), the model becomes deterministic, and the verges become 5% and 51% for reactivation and suspension, respectively, as shown the Figure 10.

```
xindex =

        0.05           2       19297           1           0           0
           0           0           0        0.51           1     -4402.5
```

Figure 10: MATLAB results of the regime-switching problem: the Deterministic Case

Source: Own calculations

This examination suggests that companies are a little more cautious in settling the suspension-reactivation decisions when they consider unpredictability, given that the business will not be resuming operations until the fraction of contagious individuals reaches one point below 5%

(the reactivation verge in case volatility is removed). It is the real options that make enterprises act differently.

3.2.2 The effect of Switching Costs on Switching Thresholds

I will now explore the effect of commuting the suspension and reactivation costs on the switching verges. As shown in Table 10, the mothballing verge I_H rises to 52%, while the reactivation one I_L falls to 2% when the mothballing cost C_{12} surges to 2000 and the reactivation cost is kept fixed at 1500. By definition, when the suspension cost increases, the company will have to pay more to suspend operations. This leads the manager to be more disinclined in the decision of temporarily shutting down operations, thus he will wait until the proportion of infected workers reach a higher level. The intuition on the mothballing threshold is clear, but additional inspection should be made on the effect on the reactivation verge I_L. The increase in the suspension cost leads to a decrease in the reactivation threshold due to a similar outcome: the enterprise could reactivate the business with less enthusiasm if it has to pay a large amount of lump-sum costs.

Initial parameters $C_{12} = C_{21} = 1500$	*Mothballing threshold* $I_H = 51\%$	*Reactivation threshold* $I_L = 4\%$	*Conclusions*
$C_{12} = 2000$ $C_{21} = 1500$	52%	2%	▪ I_L decreases with C_{12} ▪ I_H increases with C_{21}
$C_{12} = 1500$ $C_{21} = 2500$	52%	3%	▪ I_L decreases with C_{12} ▪ I_H increases with C_{21}

Table 10: Effect of changing switching costs

Source: Own calculations

The change in the reactivation cost C_{21} has similar effects to the change already analysed. The reactivation threshold I_L slightly decreases to 3%, meaning that the firm is more grudging in resuming production as the reactivation cost increases. The mothballing threshold increases with the surge in reactivation cost to 52%, because the company suspends the business with some demurral to lose its option value. Considering the possibility that the fraction of infectives could drop soon, the firm might avoid paying the reactivation cost again by remaining productive. Therefore, the higher the reactivation cost, the higher the option value, the lower inclined decision makers are in suspending production.

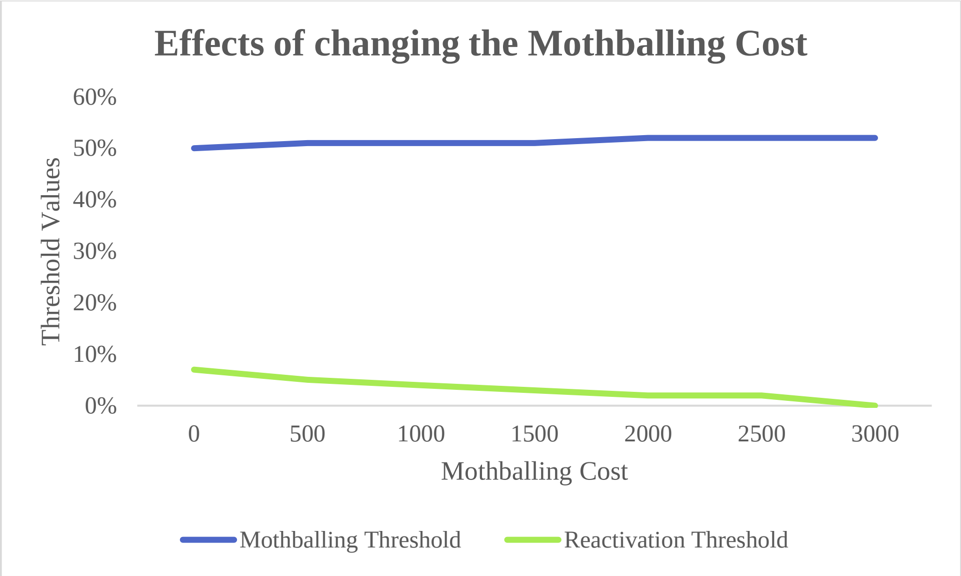


Figure 11: Effect of changing the Mothballing Cost C_{12}

Source: Own calculations

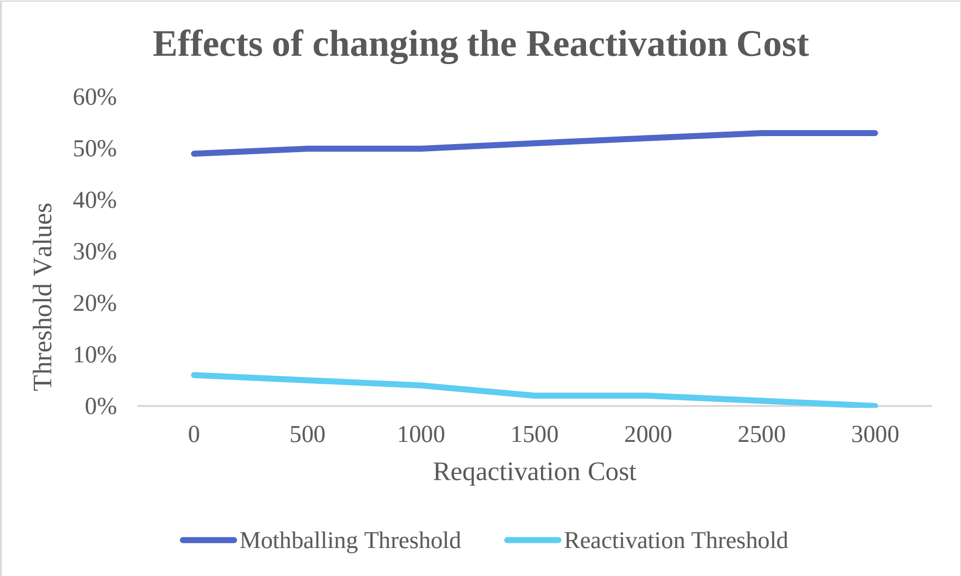


Figure 12: Effect of changing the Reactivation Cost C_{21}

Source: Own calculations

Figures 11 and 12 above help us to observe the comparative results more clearly when changing switching costs. We can see from Figures that the two costs have very similar outcomes on analysed thresholds.

3.3 The effect of disease control strategies

The main goal leading to the modelling of epidemic outbreaks is to determine a rational basis for regulations drawn to control the unfold of a virus[3]. Whenever possible and in case the government does not impose specific rules on management, each company can adopt different plans to alter the spread of the virus internally to the firm. These strategies include for instance the utilization of *smart working*, the screening of workers showing symptoms, periodical tests for all employees or immunization of some or part of workers through vaccination, whenever available. All these actions aim at:

- Decreasing the internal transmission rate α
- Increasing the recovery rate in the active regime γ_{r1}
- Decreasing the external transmission rate β

Companies can benefit notably from preserving health of their employees through the administration of disease control strategies. In the next paragraphs, each parameter and the consequences of its changes are analysed.

3.3.1 Changing the Internal Infection Rate α

As previously described, the internal infection rate represents the rate at which the virus spreads among the firm when it is open. In the base case I assumed it was equal to 1 because it is presumable that employees work closely together, thus causing the infection rate to be very high, as what happens in very high-density population places**.** Considering that, it is reasonable to assume that the external infection rate β is lower than the internal infection rate α, because the population density is assumed to be higher within the workplace.

The goal of firms is to keep the internal infection rate under control, whatever the external rate is. The aim now is not to determine the difference between these two rates, but to study how the switching verges change as the internal infection rate changes. I will study the behaviour of thresholds by decreasing the infection rate from 1 to 0,1. Figure 13 shows that both mothballing and reactivation thresholds decrease as this parameter increases, as I was expecting.

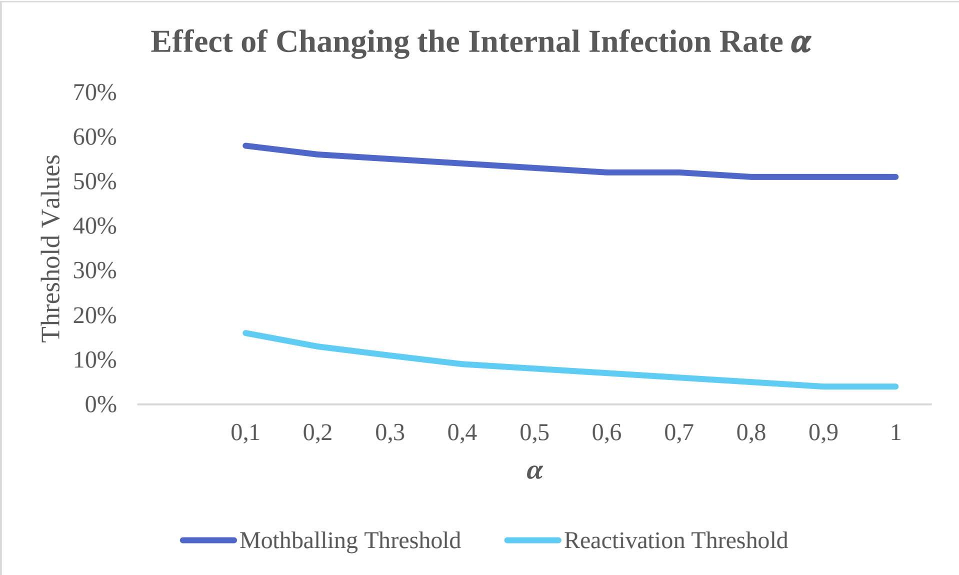


Figure 13: Effect of changing the infection rate α

Source: Own calculations

Intuitively, both thresholds decrease as α increases. As the internal transmission rate increases, the firm will be more prudent and wait for less workers to get infected before temporarily shutting down operations. The same prudent effect is shown by the reactivation threshold, it is clear that with increasing infection rate, the firm will prefer waiting for less infected workers before reopening operations.

If the internal infection rate α is kept at low levels, employees would work in a safer workplace and would run less risks of infection. Before closing, the company would be willing to wait for more employees to get infected. The same reasoning happens before reopening: directors will be less cautious and call back workers when the fraction of infectious reaches a higher threshold than in the base case, where $\alpha = 1$.

3.3.2 Changing the Internal Recovery Rate in the Active Regime γ_{r1}

The parameter γ_{r1} represents the recovery rate in the active regime. Intuitively this will be lower than the recovery rate in the inactive regime γ_{r2}, because workers are forced to stay home. Obviously, if the infection is detected soon, the worker will be forced to go home and rest. These two parameters are actually different only when the worker does not show symptoms and his disease is not detected, thus he will keep working, which will cause the protraction of his recovery and above all, he will be a potential risk for his co-workers. That is why firms have implemented strong disease detection such as a mandatory and full-paid Covid-19 tests every week or month. Considering that, the internal recovery rate can be kept under firms' control through disease detection and other less costly safety measures, such as providing masks, sanitisers, and temperature measurement.

As we can see in Figure 14, the mothballing and reactivation thresholds increase as the recovery rate increases. This means that as the ailment is kept under control, thus the internal recovery rate increases, the firm will be willing to wait for more workers to get infected before temporarily shutting down operations and will be restarting the business with more infected workers.

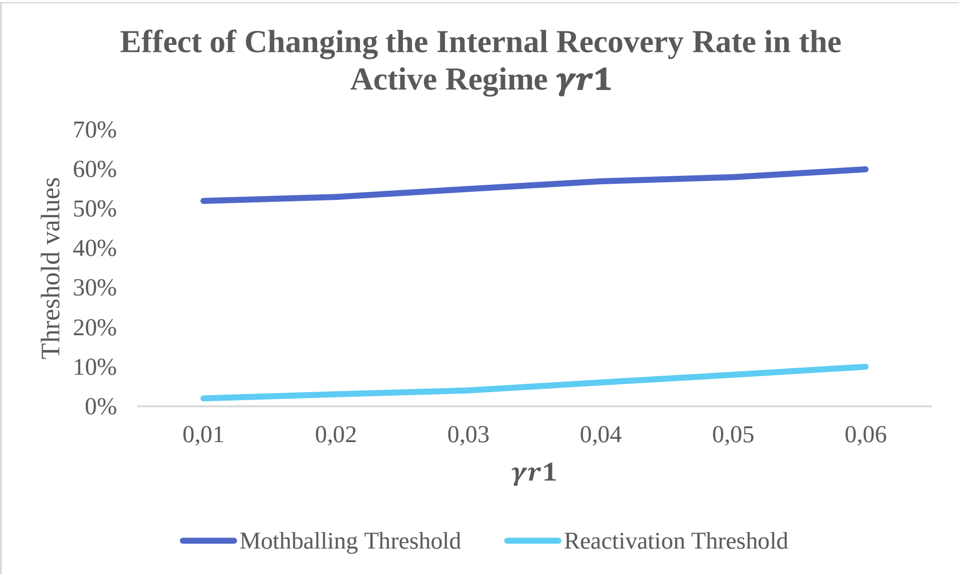


Figure 14: Effect of changing the Recovery Rate in the active regime γ_{r1}

Source: Own calculations

3.3.3 Changing the External Infection Rate β

The external infection rate β actually cannot be classified as the parameters that the firm can keep under control, because it depends on government strategies. In reality, what the firm can control is the internal infection rate.

But it would be interesting to see how the switching thresholds behave as the external infection rate β, and thus R_0, change. This is actually the most important parameter to analyse because most of the strategies to keep infection rate low within companies depend on government policies regarding the mandatory shutting down of operations. Moreover, the internal infection rate is assumed to be positively correlated to the external one, thus a deeper analysis on this parameter should be done.

Table 10 shows how the values of β react to changes in the reproduction number R_0, all other parameters fixed.

$\boldsymbol{\beta}$	$\boldsymbol{R_0}$
0	0
0,036	0,5
0,071	1
0,107	1,5
0,143	2
0,179	2,5
0,214	3
0,250	3,5
0,286	4
0,321	4,5
0,357	5
0,393	5,5
0,429	6
0,464	6,5
0,500	7
0,536	7,5
0,571	8
0,607	8,5
0,643	9
0,679	9,5
0,714	10

Table 11: How β changes as R_0 changes

Source: Own calculations

Table 11 shows that the external infection rate increases as the reproduction number increases. An increase in these parameters is most of the times caused by less stringent infection measures.

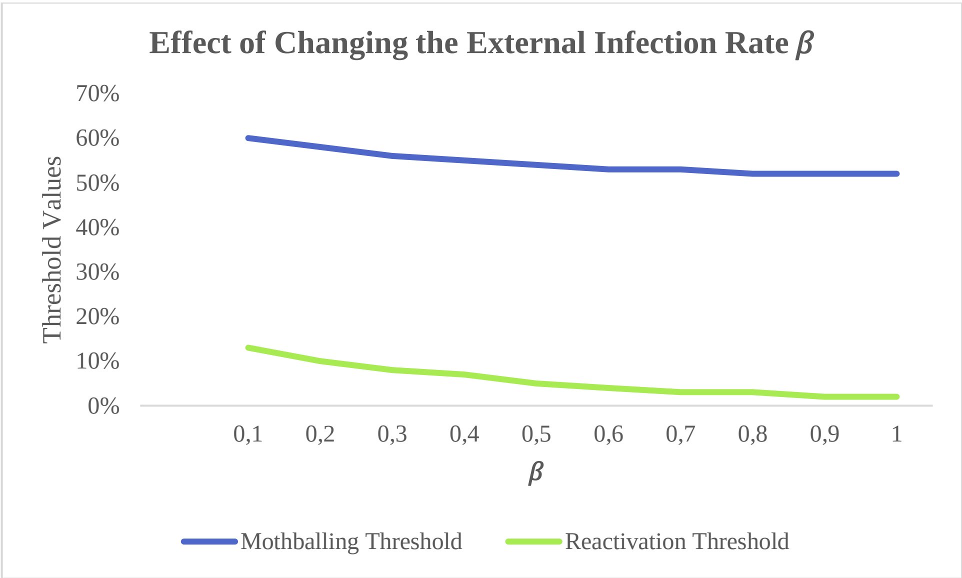


Figure 15: The effect of changing the External Infection Rate β

Source: Own calculations

We can see that the regime-switching thresholds behave exactly as they did for the internal infection rate case. This effect shows the prudential behaviour of managers when facing greater risks for their workers. As the external infection rate increases, managers will be less willing to reactivate production when the risk of contagion is high and will be more prudent in their shutting down.

3.4 Results on the Value Function

3.4.1 Changing the disease control strategies

The next three Figures (16,17 and 18) represent how the value of the firm in both the active and inactive regime changes as the diseases control parameters change. First of all, it is important to note that the value of the company decreases as the fraction of infectives increases, ascertaining the conclusion that the value of the firm is compromised by the presence of infected workers. Moreover, notice that the value of the firm in the active regime becomes lower than the value in the inactive regime as the fraction of infectives gets higher, showing that as the fraction of infectives increases, firms would be better off when closed.

Figure 16 below shows how the value functions behave as the internal infection rate α changes. As we can clearly see, the value function has a greater value when the internal infection rate decreases.

The opposite effect is verified when considering an increase in the internal recovery rate γ_{r1}: as the internal recovery rate increases, so does the value of the firm, as shown in Figure 17.

Finally, I assumed the external infection rate to take values equal to 0,179, 0,107 and 0,036 which correspond to a reproduction number equal to 2,5, 1,5 and 0,5 respectively[55]. Not surprisingly, the value of the enterprise increases as the external infection rate decreases. Actually, this parameter cannot be directly controlled by managers, but assuming that the government imposes a general lockdown leaving firms open, the external infection rate would have an impact on the internal one. The impact will result in a decreasing value of the firm if the optimal policy is not implemented.

These three effects certify that managers have actually a great responsibility over their business, because the value of the firm greatly depends on how they manage to keep the disease parameters under control and on how they correctly manage the suspension-reactivation strategies. It is clear that the aim of managers should be that of increasing the value of their firm. The only way to reach this goal is to keep the disease controlling parameters under constant check, to make sure the degree of infection within the firm does not reach too high levels, in order to guarantee the correct continuation of operations. This is not possible when security-measures are weak, and the safety of employees takes second place.

[55] Look at Table 11.

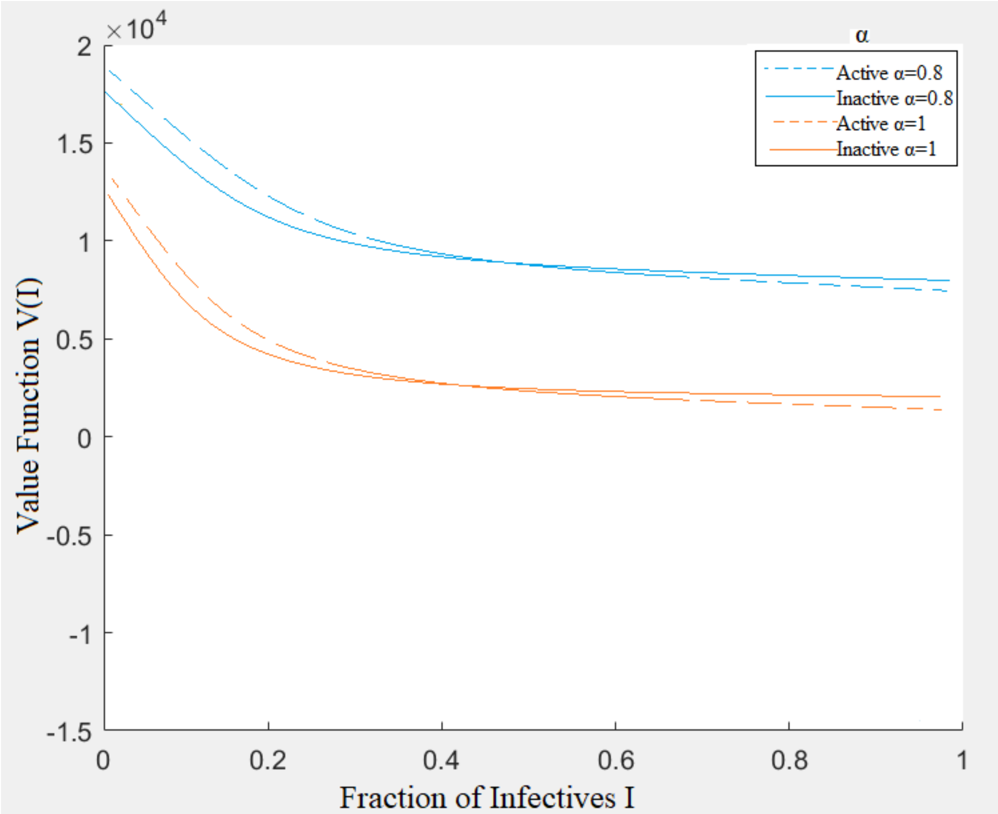


Figure 16: The effect of changing the Internal Infection Rate α on the Value Function

Source: Own calculations

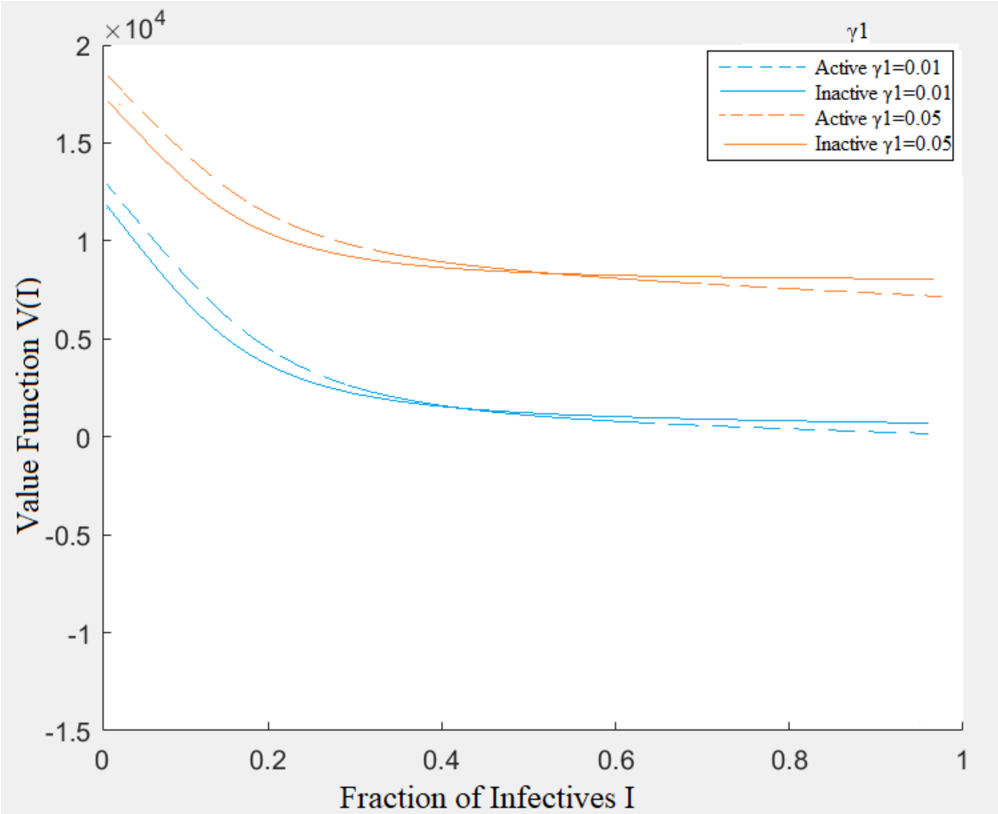


Figure 17: The effect of changing the Internal Recovery Rate in the Active Regime γ_{r1} on the Value Function

Source: Own calculations

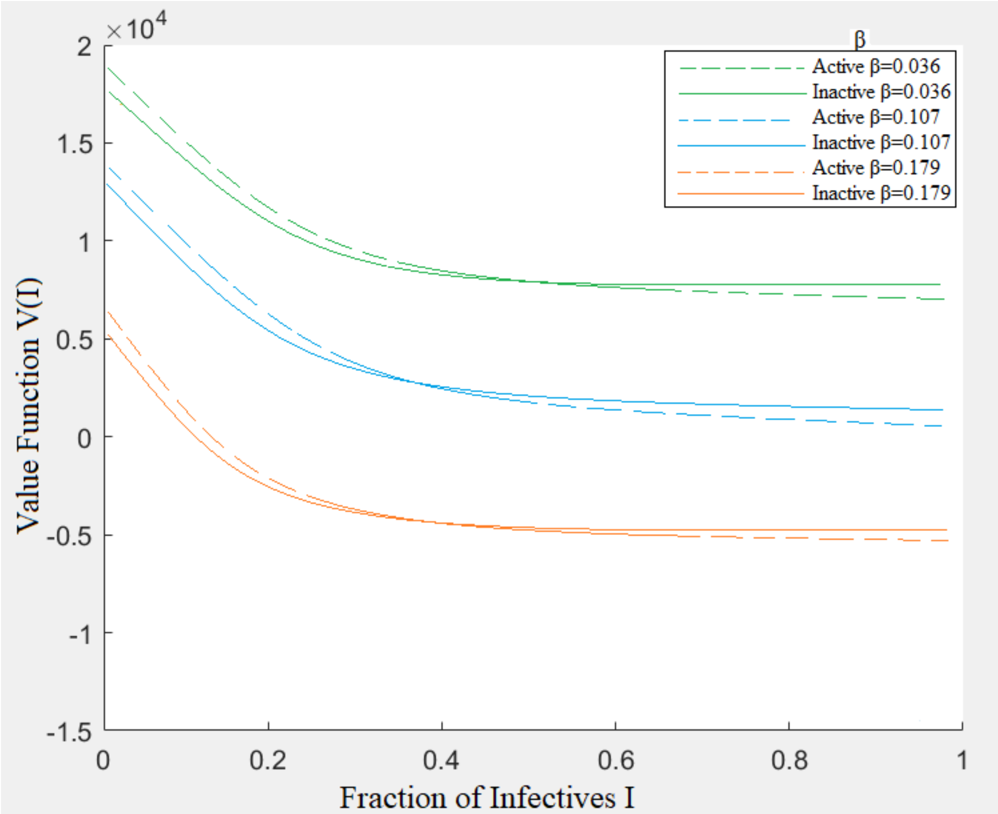


Figure 18: The effect of changing the External Infection Rate β on the Value Function

Source: Own calculations

3.4.2 Changing the Volatility Coefficient

Figure 19 represents the value functions in both the deterministic and stochastic case. The fact that the value of the firm in the deterministic case $(c = 0)$ is much lower than the stochastic case $(c = 0{,}1)$ demonstrates that future uncertainty makes real options valuable because management has more flexibility to change the course of the project in a more favourable direction.

The finding is in line with the theory of real option valuation: a company's value is usually underestimated under the conventional NPV approach.

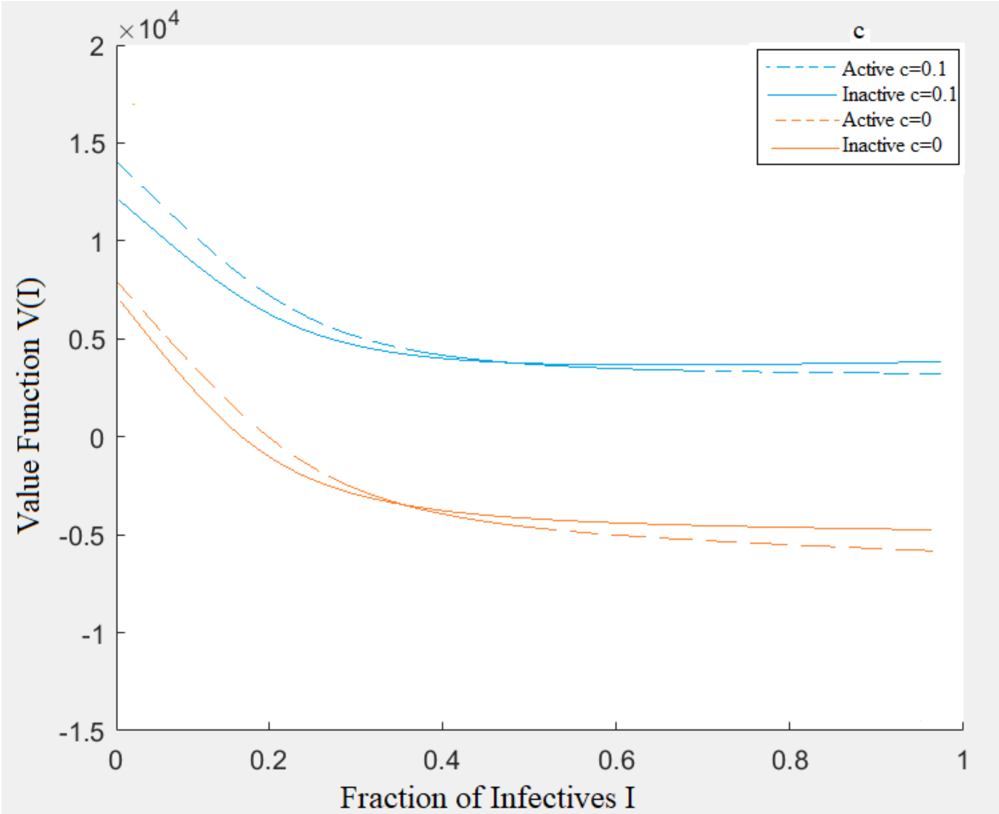


Figure 19: The effect of changing the Volatility Coefficient *c* on the Value Function

Source: Own calculations

3.4.3 Effect on the Value of the Firm without Optimal Regimes

As seen so far, the ultimate goal of implementing optimal switching threshold is increasing the value of the firm, thus keeping the level of production at satisfactory levels but at the same time safeguarding employees. All of these considerations have an economic sense when comparing how the firm would value in case managers do not consider the possibility of shutting down temporarily and reopening.

In order to do this, I propose again the value function presented in Chapter II, which represents how the regime choice is made.

$$max_r V(I,r) = E\left[\int_0^{\infty} e^{-\rho t}\pi(I_t, r)dt - \sum_k \sum_{i=1}^{2}\sum_{j=1}^{2} e^{-\rho t_k^{ij}} C_{ij}\right]$$

The first part of the equation represents the expected discount cash inflows, while the second part represents the switching costs (mothballing and reactivation) incurred over an infinite time horizon. By removing the second part of the maximization function, we obtain the simple Net Present Value of the future cash inflows to the firm.

Through a simple MATLAB integration, I found out that the value of the firm resulted to be equal to 116[56]. As we can see from the Value Functions in the Figures above, the value of the firm when taking into account optimal regimes, oscillates between €15 and 5 million, depending on the fraction of infectives.

This finding corroborates the theory of real option valuation which states that the value of the firm is usually underestimated with the NPV approach, but also validates the basic idea of my thesis: implementing optimal switching strategies not only safeguards operativity and employees, but also contributes to the maximization of the firm's value.

[56] Expressed in thousand euros

Conclusions and Discussion

Extensive studies concerning human epidemics have been developed, but it is evident that very few of them scrutinize epidemic risks for private companies.

Epidemic models have shown fundamental contribution in dispensing appreciations regarding the effectiveness of safety measures, such as lockdowns, immunization campaigns, or periodic screening. The optimal rules for the implementation of these possible strategies are given by modelling.

In this work, I developed a two-stage model to examine how enterprises can deal with the threats associated to pandemics which can seriously harm their workforce. Within the first stage of the model, I showed a simple regime-dependent epidemic model allowing for external contagion and deaths from the virus.

In the second stage I proposed an optimal suspension-reactivation strategy, applied to a real-world firm.

This two-stage model provides realistic insights for large firms to set up verges for activating and temporarily shutting down operations in the events of pandemics.

Dynamic programming has been discussed in this thesis and implemented to find optimal switching strategies obtained by real-world parameters. The firm should mothball operations when the fraction of infected workers rises above the suspension threshold and reactivate operations when the portion of infected workers falls below the reactivation threshold. These triggers for mothballing and reactivation are company specific because they rely on many factors which characterise each specific firm, such as the number of employees, the fixed or variable costs.

I decided to analyse a manufacturing company because its business cannot be simply transferred into smart working, as it could work for other sectors, and I thought the model could fit better in this sense. However, the economic rationale can be applied everywhere.

Final results showed that it is optimal for the active regime to suspend operations when the fraction of contagions reaches the threshold of 51%, while when the company is already in the inactive regime, it is optimal to reactivate operations when the fraction of infectives drops below 4%, thus leading to a mothballing threshold equal to 51% and a reactivation threshold equal to 4%. Further analysis was developed by observing the behaviour of regime-switching thresholds as certain parameters changed.

Changing the model from stochastic to deterministic (thus assigning the volatility coefficient a value equal to 0), resulted in the mothballing threshold to remain unchanged to 51% but the reactivation threshold to increase from 4% to 5%. This shows that when there is no uncertainty regarding the dynamics of the virus, firms are less conservative in making the suspension-reactivation strategy, because businesses will be put back in operation with a slightly higher fraction of infectives.

Then, I analysed the behaviour of thresholds by changing switching costs and some basic parameters: the external infection rate, the internal infection rate, and the recovery rate in the active regime.

Concerning the changing switching costs, results showed that by increasing one of the two costs at a time, the reactivation threshold decreases, and the mothballing threshold increases, meaning that managers will be more reluctant in the decision of temporarily shutting down the business when the suspension cost increases, and will be less willing to reopen when the reactivation cost increases. This conclusion is quite straight and in line with my expectations.

For what concerns the effect of diseases control strategies, results showed that with increasing both internal and external infection rates at a time, both thresholds were decreasing, meaning that firms will are more prudent and wait for less workers to get infected before both temporarily suspending and reopening operations.

The opposite effect was given by changing the recovery rate in the active regime, because both thresholds increased with the recovery rate. This is quite intuitive, because as the internal recovery rate increases, managers will be less prudent and wait for more workers to get infected before suspending the business and will reopen with more infected workers.

Final analysis was made on the actual firm value maximization. Last graphs showed that the value of the firm is positively impacted by implementing correct disease control strategies aiming at reducing infection rate or increasing recovery rate.

Concerning the value of the firm with respect to volatility, results clearly shows that when there is future uncertainty, the existence of real options makes firms more valuable. At the same time, firms are more conservative about the decisions of suspension and reactivation.

These strategies aim at controlling the epidemic, but at the same time they increase the firm's value. Disease control and value maximization can be obtained simultaneously following this model.

Firms are motivated by the goal of profit maximization. An infected employee not only has lower productivity, but also transmits the virus to other people in the company. Firms need to implement different strategies to decrease internal infection rates and increase the recovery rate.

Results show that managers have actually a great responsibility over their businesses because they can control the main contagion parameters within their firm. It is on them that the decision-making power regarding the optimal suspension-reactivation triggers resides. And it is upon them that these thresholds change depending on how they keep internal contagion under control. Their aim should be that of increasing the value of their firm, but this is not possible when security-measures are weak, and the safety of employees takes second place.

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Synthesis

Introduction

Nowadays, most theorists and practitioners believe that real options should be considered when analysing corporate decisions. Throughout my thesis I have built on this consideration by employing a regime-switching model, which arises from real options valuation, to study firms' optimal strategic decisions in the event of pandemics. In such circumstances, businesses play a special role in protecting employees' health and, at the same time, in minimizing economic losses. This work is motivated by the concern of a new pandemic, now that the problems it can give rise to are real and tangible.

In details, I conducted a MATLAB implementation, through the CompEcon Toolbox proposed by Fackler in 2004, to determine the optimal suspension-reactivation triggers, where reactivation decision is viewed as a call option and the suspension as a put option, using a dynamic programming method to obtain optimal switching thresholds applied to a real-world firm.

Chapter I. An Overview of Epidemiological Models and Real Options

In the course of Chapter I, I have analysed the importance of developing epidemiological models in the context of corporate finance and I have described the dynamics of three basic models: the SI, SIR and $SEIR$ models. Finally, the importance of adding stochasticity to the models and their connection with the world of real options has been examined through a theoretical example.

To date, 32 pandemics have occurred in the past 500 years of which three in the past century. Historic data reveals that influenza pandemics occur with frightening regularity every 30 to 50 years.

Given this pattern, it is reasonable to develop economic models in order to be better prepared in the event of other outbreaks. Impacts on almost all kind of business organizations are staggering: businesses would have to shut down for quarantine, firm's earnings would plunge and leading to default on corporate debts. Moreover, consumers' confidence might crash, thus further deteriorating financial distress. Without any improvement in our techniques for fighting this invisible war, sacrifices by households and businesses will be startling.

According to searches carried out by Area Studi Legacoop, Italian economy due to the pandemic has lost €150 billion in 2020, with a collapse of 8,9%. The lost surplus from market activity, while massive, understates the true costs of sacrifices that households and businesses are making. This is why better techniques for fighting the war are highly beneficial.

Mathematical models can provide precious tools to public health authorities for the management of epidemics, potentially contributing to limiting the portion of infected people and victims. These models can be used to reap long and short-term forecasts, which allow decision-makers to optimize accessible control policies, such as lockdowns and vaccination campaigns. Models are also very beneficial in other duties which include the estimation of transmission parameters, analysing the contagion mechanisms and simulation of different epidemic scenarios.
However, these models only evaluate effectiveness of epidemic control strategies based on country-wide needs and lead cost-benefit analysis from the macroeconomic standpoint. They do not deliver directives for large corporations to prepare for pandemics nor provide instructions about the implementation of optimal control policies.

All epidemic models have a common feature, which dividing the population into two different health states: susceptibles (S) and infectives (I) and studying the disease transmission among these classes. Beside the simple SI model, other more elaborated models incorporate other groups, of which the most used is the recovered (R).
The group S represents the group of people who are healthy but vulnerable to the disease. The class I denotes the individuals who have been infected and can contaminate others. The class R represents people who have recovered from the infection and have acquired immunity.
Scientists use a basic measure to track the infectiousness of a disease called the basic reproduction number (o reproduction ratio) — also known as R_0, that indicates how contagious an infectious disease is. It tells the average number of people who will contract a contagious disease from one person with that disease. The purpose of policymakers is actually to maintain this term below 1, because in this case every existing infected individual causes less than one new contamination, thus the ailment will decline and ultimately die out.

Chapter II. Regime-Switching Models and Calibration to Covid-19

In Chapter II, I provided a rough calibration to Covid-19 using a modified SIR model allowing for deaths, in order to obtain parameters that I used in the subsequent part of the Chapter.
Then I proposed a two-stage model to address the following research questions:

- In the event of an epidemic, should the firm continue operating considering the loss of productivity of its employees or temporarily suspend operations to circumvent contagion?
- Is the company's objective to maximize value in conflict with its desire to control the disease?
- What are the optimal triggers for businesses to implement the suspension-reactivation strategy?

Stage I

Within the first stage, I adapted an epidemic model to explain the stochastic dynamics of an ailment that spreads within a given organization, taking into consideration external contagion and deaths from the virus.

This work assumes that the productivity of a worker decreases when he gets infected. With the spread of the ailment, the proportion of infective workers tends to increase progressively, thus harming the firm's productivity and, by consequence, its revenues.
I defined the "mothballing threshold" as the percentage of infected employees over which it is better to temporarily shut down the business and dismiss employees whether they are infected or not. When instead the percentage of infected employees drops below a certain low threshold (the "reactivation threshold"), the business can be resumed, and employees can be called back to work.
Strategic decisions that managers make have an impact on the evolution of the disease. Such decisions might be used to reduce the spread of the contagion, thus altering the parameters α and γ, which represent the internal infection rate and the removal rate, respectively.

The company may adopt disease control programs to lower the infection rate between infectives and susceptibles, for example screening the suspected infectives and ordering a full paid leave or testing for the disease all employees every week or month.

I denoted r as the regime of the firm, thus $r = 1$ if the firm is active and $r = 2$ if the firm is inactive. γ_1 represents the recovery rate when the firm is active. The corresponding epidemic model resulted to be:

$$dI_t = [\alpha I_t(1 - I_t) - \gamma_1 I_t + \beta(1 - I_t) - \delta I_t]dt + \sqrt{cI_t\,(1 - I_t)}\,dW_t \qquad if\ r = 1$$

Where β is defined as the external infection rate.

In the inactive regime, the internal transmission of the disease is cut off, thus $\alpha = 0$. Infectives will recover at a greater recovery rate $\gamma_2 > \gamma_1$ and the contagion will be kept under control. Assuming that the external infection rate β and the death rate δ remain unaltered, the dynamics of the disease when the company is idle resulted to be:

$$dI_t = [-\gamma_2 I_t + \beta(1 - I_t) - \delta I_t]dt + \sqrt{cI_t\,(1 - I_t)}\,dW_t \qquad if\ r = 2$$

Finally, the epidemic model was expressed in the following manner:

$$dI_t = \mu(I_t, r)\,dt + \sigma(I_t, r)\,dW_t$$

In which,

$$\mu(I_t, r) = \begin{cases} \alpha I_t(1 - I_t) - \gamma_1 I_t + \beta(1 - I_t) - \delta I_t & if\ r = 1 \\ -\gamma_2 I_t + \beta(1 - I_t) - \delta I_t & if\ r = 2 \end{cases}$$

And

$$\sigma(I_t, r) = \sqrt{cI_t\,(1 - I_t)}$$

Stage II

Within Stage II, I used a regime-switching model through MATLAB to find the optimal thresholds, based on the theory of real option valuation. This model has been applied to a real-world company operating in the textile sector, the Ratti S.p.A. Later in the Chapter, I described how this particular sector has reacted to Covid-19 and the reasons that led me to choose this company.

Suppose the manager can use some methods to estimate the fraction of infectives I_t, because it is too costly to tell if an individual employee is actually infected or not. For instance, he could use public daily released data of disease cases or the hospitalization levels in his region. His goal is to determine two optimal thresholds: the mothballing I_H and the reactivation I_L. The mothballing threshold tells the manager what is the percentage of I_t above which it is better to suspend operations temporarily and offer full paid leave to all workers, independently of whether they are infected or not. If, on the other hand, the fraction of infectives I_t is lower than the reactivation threshold I_L, the manager can call back all workers and resume operations.

I assumed that the productivity of a worker drops to a certain level ξ $(0 < \xi < 1)$ once he gets the disease. The total number of employees in the firm was denoted by N. The variable and fixed costs were denoted by VC and FC, respectively. Among the fixed costs, employees' wages were included because I assumed a full paid leave. Additionally, there is a penalty cost E for every infected worker when the firm is active. This cost may include the firm's reputational damage or employees' unwillingness to work.

The cash flow function was defined as:

$$\pi(I_t, r) = [\xi * I_t * N + (1 - I_t) * N] * (P - VC) - FC - E * I_t * N \qquad \text{if r = 1}$$

And $\pi(I_t, r) = -FC$ if r = 2

The above equations tell that the cash flow to the firm $\pi(I_t, r)$ depends both on the fraction of infectives and the regime variable. Clearly, if the company is inactive, the cash flows are only represented by an outflow due to fixed costs (wages, insurance etc.).

Once the model was defined, I needed to determine the main parameters to be inserted.

I denoted the *removal rate* by $\gamma \equiv \gamma_r + \gamma_d$, which represents the rate at which people leave the pool of infectives either by recovering ($\gamma_r I_t$) or dying ($\gamma_d I_t$). The ratio $\rho = \gamma/\beta$ is the *relative removal rate* and $1/\rho$ is the *reproduction rate* and is denoted by R_0.

To start, I selected values for β, γ_d and γ_r based on information provided by the Italian Ministry of Health, as of November 2020, because much more information was available compared to the very beginning of the virus outbreak.

I assumed that the initial number of infectives was $I_0 = 2 * 10^{-6}$. Based on ISTAT data, Italian population was of about 60 million people, corresponding to 121 infected individuals at the outset. The initial number of susceptibles was $S_0 = 1 - I_0$.

The time interval Δt is one day. I set the removal rate γ at 0,07, based on the assumption that the average duration of the virus is 14 days. Assuming that the average illness duration is the same whether the patient recovers or dies, γ_d depends only on the fraction of patients that die. Based on data from Italian Ministry of Health as of November 2020, I estimated fraction of deaths to be equal 4,43%, which I rounded to 4% assuming an underestimation of the actual number of cases caused by a high level of asymptomatic infected individuals.

Since $\gamma_d = \delta\gamma$, we got $\gamma_d = 0{,}04 * 0{,}07 = 0{,}29\%$.

Thus, γ_r resulted to be equal to 6,86% considering that $\gamma \equiv \gamma_r + \gamma_d$.

Given γ and its components, I was left with the contact rate β or equivalently, with the reproduction number $R_0 = \beta/\gamma$ which is a function of the social distancing policy implemented.

Since my study is based on November data, I assumed that in that period the reproduction number was around 1,5, considering that the social distancing policies were strongly applied all over Italy.

As regarding the recovery rate γ_r, from calculations made above I assumed it to be equal to 0,07 in the inactive regime (γ_{r2}) and equal to 0,01 in the active regime (γ_{r1}), to reflect the fact that the recovery rate should be higher when workers are separated from each other. The value of internal transmission rate α was set to be equal to 1 and the volatility coefficient c was set at 0,1. The fatality rate δ, as previously calculated, was set at 4%.

After the parameter calibration, I provided an overview of the textile sector and how it reacted to Covid-19, at the global, European, and Italian levels. Here, only the Italian overview has been summarized.

The textile and clothing industry represents one of the most important sectors of the manufacturing industry in Italy. It is a sector that boasts an ancient tradition in our country. With over 500.000 employees, this sector employs 12% of all workers in the manufacturing sector. Moreover, the Italian textile industry exports represent 77,8% of total European exports.

Being the textile sector so fundamental in Italy, I considered it could be interesting to focus on it on my thesis.

The T&C sector is among the most exposed to the effects of the Covid-19 crisis, second only to the hospitality and tourism sectors. The production of textiles, clothing, leather, and accessories collapsed by 81% year-on-year in April 2020.

The lockdown period has led to the blocking of all commercial activities of clothing and accessories stores, affecting around 300 thousand employees. E-commerce, by guaranteeing the existence of a minimum turnover for companies active in online sales, has been one of the main factors of resilience in the sector, but, at the same time, was one of the main risk factors for retail employment.

Then, an overview of Ratti S.p.A. was provided. Ratti S.p.A. was founded in 1945 by Antonio Ratti, who opened in Como his “Tessitura Serica Antonio Ratti”, for the creation and marketing of silk fabrics and accessories.

In 1958 the Guanzate plant for the integrated cycle production process of silk was inaugurated, from yarn to finished product through the phases of weaving, dyeing, photoengraving, printing, and finishing. Guanzate is still the main production plant in Italy. Few years after opening new offices in Wall Street, the Company became listed on the Milan Stock Exchange in 1989. In the early 2000s, the Company opened new plants several countries, among which Tunisia and Romania. In 2010 Marzotto Textile Group and Faber Five Srl entered into the shareholding of Ratti S.p.A., holding its control.

The harmonious growth of the Group has led Ratti over the years to become a member of Associations, networks and bodies involved in the promotion and development of the textile industry.

The Group is today one of the major players in the production of printed, plain, yarn-dyed, and jacquard fabrics for clothing, ties, shirts, beachwear, and furnishings. It also creates men's and women's accessories such as ties, scarves, and foulards.

I have focused on the Parent Company Ratti S.p.A., in order to make the data smoother, necessary to obtain more truthful results.

To handle the inevitable impact caused by the spread of the Covid-19 virus, Ratti has implemented a form of resilience as a strategy to adapt and transform to the changed marketplace. This system has allowed the company to react in the face of the difficulties of the period, proposing new business models and transforming external stimuli into concrete action and new forms of innovation. Despite the strong reaction of the Company to the crisis, the drop in demand and the lockdown measures have inevitably impacted on performance.

The Parent Company closed the 2020 financial year with revenues from the sales of goods and services of €71,1 million (-€45,0 million compared to 2019) and a gross margin (EBITDA) of €5,2 million (-€14,1 million compared to 2019). Profit before taxes and profit for the year amounted to €0,8 million.

Following the significant restrictions on the market, the fall in sales affected all areas of business. With reference to the larger hubs, the Luxe Hub reported a drop in sales of €20,5 million (down 37,3%), whilst the Collections Hub reported a decrease of €14,7 million (down 48,0%).

Sales by geographical area showed a widespread reduction in all the main outlet markets. In particular, sales on the US market fell by €3,6 million (-54,1%) and revenues on the domestic market by 19,6 million euros (-39,3%).

Finally, I conducted the cost analysis, in particular the allocation of fixed and variable costs, fundamental for the subsequent MATLAB implementation. The analysis was the following:

- **Cost of raw materials, ancillary materials, consumables, and goods for resale:** this Financial Statement Line Item (FSLI) amounts to €18,9 million as of December 2020 and to €35,9 million as of December 2019. The sharp decrease in costs is mainly due to the decline in sales volumes. I classified this FSLI in the variable costs group.
- **Cost of services:** the total FSLI amounts to €16,5 million and €27,4 million in 2020 and 2019, respectively. Costs of services decreased by €10,8 million compared to the previous year, primarily due to the decrease in the cost of external processing, commissions and travel and accommodation expenses. I classified the single items among the fixed or variable costs depending on their nature.
- **Costs for use of third-party assets:** in 2020, it was equal to €868 million and in 2019 to €979 million. I classified the entire FSLI, which includes royalties, rentals, and leases among the fixed costs.
- **Personnel costs:** these expenses amount to €24,5 million and €31,6 million in 2020 and 2019 respectively and they have been classified among the fixed costs. The decrease in payroll costs, amounting to €7,1 million, is primarily due to use of the redundancy fund and use of vacation days accrued during 2020. As of December 31, 2020, there were 34 fewer employees than of December 31, 2019. The social and economic emergency situation created by the Covid-19 pandemic has obliged the Company to use the forms of wage supplementation adopted by the government.
- **Other operating expenses:** these expenses amount to €1,8 million and €1,7 million in 2020 and 2019, respectively. I classified the single items among the fixed or variable costs depending on their nature.
- **Amortization, depreciation, provisions, and write-downs:** the total of these items amounts to €4,9 in 2020 and €4,3 in 2019. These expenses have been classified among the fixed costs.

On the basis of the classification provided, total fixed costs amounted to €38,8 million as of December 2020 and to €48,2 million as of December 2019, determining a decrease of €9,4 million, mainly attributable to personnel reduction.

Concerning variable expenses, in 2020 they were equal to €28,8 million, while in 2019 they amounted to €53,6 million. This deep difference is due to the decline in sales volume attributable to the sharp drop in demand.

Then, I determined the variable cost needed to produce one meter of fabric in the following way:

- I took from Amazon the price of fabric per meter: on average, fabrics are sold at €59,99 for 6 yards (5,48 mt), thus €10,95 per meter.
- The value of production resulted to be equal to €118 million as of 2019.
- From simple calculations, it resulted that in 2019, 10,8 million meters of fabric were produced.
- Dividing the total variable costs of 2019 by the meters of fabric produced, the variable cost to produce one single meter of fabric resulted being equal to €4,98.

It should be noted that I considered financial statement data as of 2019, because the Company was not too significantly affected by the pandemic yet. Taking data from 2020 could have led to compromised results.

Chapter III. MATLAB Implementation of the Regime-Switching Model on Ratti S.p.A.

In Chapter III, analysis of results and graphs have been provided.
Results showed that managers have actually a great responsibility over their businesses because they can control the main contagion parameters within their firm. It is on them that the decision-making power regarding the optimal suspension-reactivation triggers resides. And it is upon them that these thresholds change depending on how they keep internal contagion under control. Their aim should be that of increasing the value of their firm, which is guaranteed by keeping operations open but at the same time by taking care of workers' health, especially because their productivity is damaged when they are infected. This is not possible when security-measures are weak, and the safety of employees takes second place.

After making my assumptions and considerations about the model parameters, I implemented the MATLAB codes that returned the regime-switching results as output.

I found out that it is optimal for the company the active regime to temporarily shut down operations when the fraction of infected workers reaches the threshold of 51%. Instead, when the company is already in the idle regime, it is optimal to resume operations when the fraction of infectives falls below 4%.

In case uncertainty regarding the behaviour of the virus is removed (i.e., the volatility coefficient c is equal to 0), the model becomes deterministic, and the verges become 5% and 51% for reactivation and suspension, respectively.

This examination suggests that companies are a little more cautious in settling the suspension-reactivation decisions when they consider unpredictability, given that the business will not be resuming operations until the fraction of contagious individuals reaches one point below 5% (the reactivation verge in case volatility is removed). It is the real options that make enterprises act differently.

I implemented the value function on MATLAB to get the value of the company depending on the regime it is currently in. Firstly, I noted that the firm's value decreases as the fraction of infectives increases, ascertaining the conclusion that the value of the firm is compromised by the presence of infected workers. Moreover, I noticed that the value of the firm in the active regime becomes lower than the value in the inactive regime as the fraction of infectives gets higher, showing that as the fraction of infectives increases, firms would be better off when closed.

Then, I analysed the behaviour of the company's value as the internal infection rate α changes. Results showed that the value function has a greater value when the internal infection rate decreases.

The opposite effect was verified when considering an increase in the internal recovery rate γ_{r1}: as the internal recovery rate increases, so does the value of the firm.

Finally, I assumed the external infection rate β to take values equal to 0,179, 0,107 and 0,036 which correspond to reproduction numbers equal to 2,5, 1,5 and 0,5 respectively. Not surprisingly, the value of the enterprise increases as the external infection rate decreases. Actually, this parameter cannot be directly controlled by managers, but assuming that the government imposes a partial lockdown leaving firms open, the external infection rate would have an impact on the internal one. The impact resulted in a decreasing value of the firm if the optimal policy is not implemented.

These three effects certified that managers have actually a great responsibility over their business, because the value of the firm greatly depends on how they manage to keep the disease parameters under control and on how they correctly manage the suspension-reactivation strategies.

All of these considerations have an economic sense when the firm actually values more when it implements optimal regimes with respect to the case it does not contemplate temporary shutdowns and reopening, that is, assuming the value of the firm only equals the traditional NPV.

Through a simple MATLAB integration, I found out that the value of the firm resulted to be equal to 116 thousand euros in that case. Instead, when taking into account optimal regimes, the value was oscillating between €15 and 5 million, depending on the fraction of infectives.

This finding corroborates the theory of real option valuation which states that the value of the firm is usually underestimated with the NPV approach, but also validates the basic idea of my thesis: implementing optimal switching strategies not only safeguards operativity and employees, but also contributes to the maximization of the firm's value.

Conclusions and Discussion

Extensive studies concerning human epidemics have been developed, but it is evident that very few of them scrutinize epidemic risks for private companies.

Epidemic models have shown fundamental contribution in dispensing appreciations regarding the effectiveness of safety measures, such as lockdowns, immunization campaigns, or periodic screening. The optimal rules for the implementation of these possible strategies are given by modelling.

In this work, I developed a two-stage model to examine how enterprises can deal with the threats associated to pandemics which can seriously harm their workforce. Within the first stage of the model, I showed a simple regime-dependent epidemic model allowing for external contagion and deaths from the virus.

In the second stage I proposed an optimal suspension-reactivation strategy, applied to a real-world firm.

This two-stage model provides realistic insights for large firms to set up verges for activating and temporarily shutting down operations in the events of pandemics.

Dynamic programming has been discussed in this thesis and implemented to find optimal switching strategies obtained by real-world parameters. The firm should mothball operations when the fraction of infected workers rises above the suspension threshold and reactivate operations when the portion of infected workers falls below the reactivation threshold. These triggers for mothballing and reactivation are company specific because they rely on many factors which characterise each specific firm, such as the number of employees, the fixed or variable costs.

I decided to analyse a manufacturing company because its business cannot be simply transferred into smart working, as it could work for other sectors, and I thought the model could fit better in this sense. However, the economic rationale can be applied everywhere.

Final results showed that it is optimal for the active regime to suspend operations when the fraction of contagions reaches the threshold of 51%, while when the company is already in the inactive regime, it is optimal to reactivate operations when the fraction of infectives drops below 4%, thus leading to a mothballing threshold equal to 51% and a reactivation threshold equal to 4%. Further analysis was developed by observing the behaviour of regime-switching thresholds as certain parameters changed.

Changing the model from stochastic to deterministic (thus assigning the volatility coefficient a value equal to 0), resulted in the mothballing threshold to remain unchanged to 51% but the reactivation threshold to increase from 4% to 5%. This shows that when there is no uncertainty regarding the dynamics of the virus, firms are less conservative in making the suspension-reactivation strategy, because businesses will be put back in operation with a slightly higher fraction of infectives.

Then, I analysed the behaviour of thresholds by changing switching costs and some basic parameters: the external infection rate, the internal infection rate, and the recovery rate in the active regime.

Concerning the changing switching costs, results showed that by increasing one of the two costs at a time, the reactivation threshold decreases, and the mothballing threshold increases, meaning that managers will be more reluctant in the decision of temporarily shutting down the business when the suspension cost increases, and will be less willing to reopen when the reactivation cost increases. This conclusion is quite straight and in line with my expectations.

For what concerns the effect of diseases control strategies, results showed that with increasing both internal and external infection rates at a time, both thresholds were decreasing, meaning that firms will are more prudent and wait for less workers to get infected before both temporarily suspending and reopening operations.

The opposite effect was given by changing the recovery rate in the active regime, because both thresholds increased with the recovery rate. This is quite intuitive, because as the internal recovery rate increases, managers will be less prudent and wait for more workers to get infected before suspending the business and will reopen with more infected workers.

Final analysis was made on the actual firm value maximization. Results showed that the value of the firm is positively impacted by implementing correct disease control strategies aiming at reducing infection rate or increasing recovery rate.

Concerning the value of the firm with respect to volatility, results clearly showed that when there is future uncertainty, the existence of real options makes firms more valuable. At the same time, firms are more conservative about the decisions of suspension and reactivation.

These strategies aim at controlling the epidemic, but at the same time they increase the firm's value. Disease control and value maximization can be obtained simultaneously following this model.

Firms are motivated by the goal of profit maximization. An infected employee not only has lower productivity, but also transmits the virus to other people in the company. Firms need to implement different strategies to decrease internal infection rates and increase the recovery rate.

Based on above findings, I could conclude that managers have actually a great responsibility over their businesses because they can control the main contagion parameters within their firm. It is on them that the decision-making power regarding the optimal suspension-reactivation triggers resides. And it is upon them that these thresholds change depending on how they keep internal contagion under control.

Their aim should be that of increasing the value of their firm, but this is not possible when security-measures are weak, and the safety of employees takes second place.