Earnings Inequality and Minimum Wages in Italy

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Abstract

This paper exploits theory and empirical evidence to analyse the distributional consequences of the minimum wage. The main contribution of this work is to quantify the effects on earnings inequality of a fall of 16 percent in the real minimum wage of the manufacturing sector in Italy between 1981-1997. To this end, I develop and estimate an equilibrium search model of the labor market. I used the estimated model to simulate the decrease in real minimum wage observed in Italy over this period, and I find that it induces a notable rise in inequality. Minimum wage affects the bottom part of the earnings distribution primarily, but there are spillover effects until the 90th percentile. Indirect effects, due to equilibrium responses of firms and workers, account for most of the increase in wage dispersion.

1 Introduction

In the last decades, several OECD nations have experienced a dramatic increase in earnings inequality. Most of the literature has stated that technological progress is primarily responsible for the increase in wage dispersion (e.g. Katz and Murphy, 1992 [18] and Autor et al., 2008 [2]). However, some authors have proposed a different hypothesis to explain this secular trend. Di Nardo (1996) [9], Lee (1999) [20] and Autor et al. (2016) [7] state that changes in labor market institutions, such us declining minimum wage and trade union strength, play a fundamental role in rationalizing the secular rise in wage differentials. In light of these findings, understanding the impact of minimum wage on labor market outcomes is increasingly important.

This paper attempts to unify theory and empirical evidence to analyze the distributional consequences of the minimum wage. The main contribution of this work is to quantify the effects on earnings inequality of a fall of 16 percent in the real minimum wage of the manufacturing sector in Italy between 1981-1997. To this end, I develop and estimate an equilibrium search model of the labor market. I used the estimated model to simulate the decrease in real minimum wage observed in Italy over this period, and I find that it induces a notable rise in inequality.

My analysis proceeds in three steps. In the first step, I show empirical evidence of the correlation between minimum wage and inequality in Italy. To illustrate this relation, I exploit variation in the effective bindingness of the minimum wage across Italian regions over time. The descriptive analysis of the data shows a negative correlation between minimum wage and inequality, measured as log percentile ratios. Additionally, regression estimates imply that the bottom part of the distribution is the most affected by minimum wage changes. However, there are spillover effects until the 90th percentile. This suggests that minimum wage shapes earnings inequality directly through pay policies of binding firms and indirectly through equilibrium responses of firms and workers. I analyse this mechanism by introducing an equilibrium search model.

In the second step, I follow Engbom and Moser (2018) [12] in developing an equilibrium model of the labor market in the style of Burnett and Mortensen (1998) [5] with firm heterogeneity. In the model, there is only one ability type for workers, while firms differ in productivity. Workers can be employed or unemployed and search for job in both states. Firms decide their wage policy by weighing two forces. On the one hand, a lower wage can increase per-worker profit, but on the other hand, a higher wage might increase recruitment by improving their ranking position in the job offer distribution. Therefore, firms' equilibrium choices are interdependent. Minimum wage induces all firms in the distribution to adjust. As a result, minimum wage leads to spillovers higher up the wage distribution.
In the third step, I bring the theory to the data in order to simulate the decrease in 16 percent of real minimum wage and look at the steady-state effects in the wage distribution. To this end, I estimate the model based on data from 1981-1982 when the minimum wage has not decreased yet. Then, I use the estimated model to assess the impact of a 16 percent decrease in the real minimum wage. The minimum wage fall increases earnings variance by 3.6 log points, which is 66 percent of the overall empirical increase in Italy over this period. Most of the total impact is due to equilibrium effects.
Even if wage decrease is most pronounced in the bottom part of the distribution, spillovers reach up the 90th percentile of the distribution.
The investigation was motivated by the lack of research on the impact of contractual minimum wages in Italy on labor market outcomes. Most of the existing literature focuses on employment effects, while distributional effects are less studied. This paper provides insights into how minimum wage shapes worker's wage distribution and shows that firm heterogeneity and job-to-job search are the keys determinant of minimum wage effects.

Related Literature. This paper contributes to three strands of literature. The first is concerned with collective bargaining in Italy. In Italy, there is not a state minimum wage. It is determined by national contracts signed between trade unions and associations of employers. Since there is no law regulating it, the legal enforcement of the minimum wage is questionable and, it is not clear if it is applied erga omnes. Research on the effects of contractual minimum wage on labor outcomes could raise the discussion about introducing a legal state minimum wage. Few studies analyse the effects of contractual minimum wage in Italy; an example is Fanfani (2019) [13] that studies the wage and employment effects of contractual minimum wages. For what concerns the relation between collective bargaining and wage inequality in Italy, Manacorda (2004) [22] investigates the role the Scala Mobile [1] played in the initial fall and subsequent rise in earnings inequality in Italy between 1977 and 1993. Devicienti et al. (2019) [8] studies the evolution of Italian male wage inequality between 1982-2001, assessing the role played by collective bargaining in shaping the wage distribution. This paper contributes to this literature quantifying the effect of minimum wage in explaining the increase in wage inequality observed between 1981-1997.

The second literature is concerned with reduced-form estimates of the impact of the minimum wage. Most of the literature has focused on employment effects, some examples are illustrated in Card and Krueger (1995) [6], and Neumark and Wascher (2008) [24]. Fewer studies examine the effects on wage inequality, some examples are Grossman (1983) [15], DiNardo et al. (1996) [9], Machin et al. (2003) [21], and Brochu at al. (2018) [4]. In order to show evidence of a negative correlation between inequality and minimum wage in Italy, I follow Lee (1999) [20] and Autor et al. (2016) [7], exploiting variation in the effective bindingness of the minimum wage across regions and over time.

The third literature provides a structural assessment of minimum wage effects in frictional labor markets. Eckstein and Wolpin (1990) [10] estimate a generalization of the Albrecht and Axell (1984) [1] model assessing welfare effects of a minimum wage. Koning et al. (1995) [19] and van den Berg and Ridder (1998) [25] develop a wage posting model with on the job search and homogeneous firms competing in segmented labor markets in order to study the effects of minimum wage on unemployment. Burnett and Mortensen (1998) [5] and Bon-

[1]Scala Mobile is a cost-of-living allowance added to the bargained contractual minimum wage. It was introduced in Italy in 1970 and formally abolished in 1991.

temps et al. (1999) [3] develop an equilibrium search model in which minimum wage have spillover effects because firms are interdependent when deciding their wage policy. Finally, Engbom and Moser (2018) [12] develop a model with heterogeneity of both firms and workers. In this paper, I introduce a simplified version of the latter with only heterogeneity in firms' productivity and exogenous search parameters.

Outline. The paper proceeds as follows. Section 2 presents motivating facts related to inequality and real minimum wage in Italy. Section 3 illustrates the institutional framework of minimum wage in Italy. Section 4 presents the data used in this paper. Section 5 provides empirical evidence of the relation between inequality and real minimum wage . Section 6 develops our equilibrium search model and characterizes the effects of the minimum wage in this environment. Section 7 estimates the model, which I use in Section 8 to quantify the equilibrium effects of the minimum wage. Finally, Section 9 concludes.

2 Motivating facts

Figure 1 represents the evolution of real minimum wage and inequality in the manufacturing sector in Italy between 1981 to 1997. The decreasing solid line is the real minimum wage, while the increasing dashed line is the inequality measured as the standard deviation of wages. It is evident the negative co-movement between the two variables. The first main objective of this paper will be to understand the mechanism behind this negative relation, i.e., why changes in the real minimum wage should affect inequality. Furthermore, I want to quantify this effect to see how much of the increase in inequality is due to a decrease in the real minimum wage. To answer these questions, I will follow the strategy of Engbom and Moser (2018) [12], exploiting an equilibrium search model, a version of Burnett and Mortensen, 1998 [5] model with firm heterogeneity.

Figure 1: Evolution of wage inequality and real minimum wage in Italy, 1981-1997

Notes: Statistics are from a sample of blue-collar workers of manufacturing sector regulated by the same national contract (C011, metal workers). Real minimum wage is hourly and in 1995 euros. Source: INVIND, INPS workers survey and CNEL.

3 The Minimum Wage in Italy

In Italy, the National Collective Labor Agreement (CCNL) is the source of legislation through which workers' trade unions and employers' associations unanimously define the employment relationship rules.

The contracts and their subsequent amendments are collected and stored in the National Council of Economic and Labor national archives (CNEL).

Collective bargaining takes place at different levels. The first level consists of agreements signed by all trade unions and all employer associations; the objective in this first stage is to define general rules that affect all workers regardless of the production sector they belong to. Then collective bargaining takes place at industry level and finally at local and corporate level. The contracts with the greatest practical relevance are the national collective bargaining agreements (CCNL), concluded at industry level.

The primary purpose of the collective agreements is to determine the actual content of individual employment contracts in a specific industry, both from the pay policy point of view (minimum wage, seniority treatments) and from the regulatory point of view (timetable, qualifications, and duties, etc.). Such agreements are typically renewed every two years at dates that are not coordinated across different agreements.

As well explained in Devicienti et al. (2019) [8], CNEL classifies the CCNLs in industry groups. Then, for each industry group, there are several contracts for each industry. Each industry-wide collective contract regulates specific job titles ("livelli di inquadramento") and the contractual minimum wages that must be applied for each of them. Such job titles are job classifications defined by collective bargaining agreements and based on the complexity of workers' tasks and, in some circumstances, also on qualifications and seniority levels. Depending on the sector of activity, the same type of job could be classified in more than one "livello di inquadramento". For each of these groups, a specific minimum wage applies. The reasons for such differentiation lie in historical and organizational reasons. In general, more than one minimum wage can coexist within the same industry and the same occupation.

It is crucial to notice that there are no opting-out clauses in the Italian system. Firms cannot decide to resort to firm-level contractual agreements derogating to the wage standards settled at the industrial level. There are two main channels to enforce minimum contractual pay levels. First, the National Social Security Institute (INPS) is in charge of sending officers to firms, which are asked to check, among other infractions, whether wages adhere to the relevant collective contract. Second, employees can sue employers either directly or through the local trade union, in which case a judge is asked to check whether wages adhere to the sector-wide minimum contractual standards. In case of a violation, employers are not only asked to cover any difference in social security contributions between what they have paid and what they should have paid applying the correct contractual wage level, but they also incur in the potential loss of several fiscal benefits and incentives, as these tax exemptions typically include firms' adherence to collective bargaining standards as an eligibility rule.

For what concerns the historical evolution of national contracts in Italy, they were introduced during the fascist period. Then, after the Second World War, with the abolition of corporations, new collective agreements were drawn up based on Constitution (January 1, 1948) Art. 39. In 1970, an institution Scala Mobile introduced a cost-of-living allowance added quarterly to the bargained contractual minimum wage. As well explained in Manacorda (2004) [22], The Scala Mobile implied a flat increase in nominal wages for each point increase in a special quarterly consumer price index. Since, at least in its original formulation, the SM awarded the same absolute wage increase (as opposed to the same percentage wage increase) to both high- and low-wage workers, this had the potential to compress wage differentials. Most of the potential equalizing effect of the SM took place from the late 1970s to the early 1980s. This wage-adjustment mechanism had been particularly disadvantageous for more qualified white-collars and skilled workers. Partly as a consequence of this growing dissatisfaction, in 1983, the government lowered the value of the SM point. Since then, the real minimum wage started to decrease gradually, and the inequality contemporaneously increased. Then, in 1985 a proposition (referendum) called by the Communist wing of the unions (CGIL) to recoup that lost increments of SM was defeated. This marked the demise of the Scala mobile and an additional impulse to the decline in the real minimum wage.

4 Data

4.1 Data Description

In describing the dataset, I follow Iranzo et al. (2008) [16]. The data used in this paper were constructed from the Bank of Italy's annual INVIND survey of manufacturing firms. INVIND is a panel of around 1200 firms per year representative of manufacturing firms with at least 50 employees. It contains detailed information on firms' characteristics. The Social Security Institute (Inps) was asked to provide the complete works histories of all workers that ever transited in an INVIND firm for 1981-1997. including spells of employment in which they were employed at firms not listed in the INVIND survey. I have information on about a million workers per year, more than half of whom are employed in INVIND firms in any year. The rest are employed in 100,000 other firms of which we only know the fiscal identifier.

The data on workers include age, gender, area where the employee works, occupational status, annual gross earnings, number of weeks worked and the firm identifier. The dataset has been cleaned by eliminating the records with missing entries on either the firms or worker identifier, those of workers younger than 15 and older than 65, those who had worked less than 4 weeks in a year and those in the first and the last percentiles of the earnings distribution.[2] After this cleaning procedure, I am left with a total of 21,189,495 records and 1,749,061 individuals.

Attrition in INVIND firms is sizable: on average 10% of workers enter and 12% exit the sample from one year to the next. Overall, approximately 80% of workers in an INVIND

[2] Extreme values of the earnings distribution could be due to exceptional events (illness and the like) or to measurement error. Given that measures of dispersion can be very sensitive to such values, I decided to drop them from the analysis altogether.

firm in 1981 had dropped out of the sample by 1997, and 72% of the workers in the 1997 sample had not been present in 1981. This implies that in principle, the skill distribution could have changed significantly due to turnover.

The INVIND survey gives an extensive list of firm characteristics, including the industrial sector, geographical region, the average number of employees during the year.

4.2 Minimum Wage

INVIND survey provides information about the industrial sector of firms, and the INPS survey on workers provides the job title of the workers. With these two pieces of information together, from the national archive CNEL, I can identify the specific national contract that regulates each employment relationship of the sample. Once I have linked each worker to his specific contract, I can also identify the minimum wage assigned. In order to find the proper contract, I used the tables provided by ISTAT that link the code of the industry (ATECO code) with the code of the contract that regulates the employment relationship.

In the sample period (1981-1997), the minimum wage is formed by two main parts: the contractual minimum wage and the "contingenza". The latter comes from the institution Scala Mobile explained in Section 3, it is a flexible component of the wage that increases accordingly to a special quarterly consumer price index. When Scala mobile was formally abolished in 1991, the component called "contingenza" remained fixed at the level reached that year, and it was introduced another element called "Elemento Distinto della Retribuzione" (EDR) equal to 10.33 euros per month. All these elements together constitute the contractual minimum wage.

4.3 Sample Construction

The total sample of more than 21 million records comes from the manufacturing sector but involves many industries, so it involves different contracts. Since each contract has a specific minimum wage, I need to focus on one subsample regulated by the same contract. I choose the group of metal workers with the lowest qualification that is the biggest subsample at my disposal. This group of workers is about the 50% of the total sample and counts more than 8 million records. I use this sample for all the following analyses of this paper. Furthermore, as a robustness check, I also do the same for all the other contracts in the total sample, obtaining similar results. It is worth noting that the metal industry is well representative of many types of products. Indeed, it goes from the production of cars to the production of medical implants to the production of furniture. Additionally, I focus on the group of workers with the lowest qualification. It is the one that involves the biggest share of workers in the sample, and that is more affected by changes in minimum wages.

Table 7 in Appendix A gives the statistics on workers' characteristics for the total sample and the metal workers sample. For the total sample, average gross hourly earnings at 1995 constant prices are 8,44 euros; the average age is 37 years; almost 80% of the observations pertain to males. The subsample of metal workers consists of almost 8.6 million observations. The descriptive statistics are quite similar to those of the total sample.

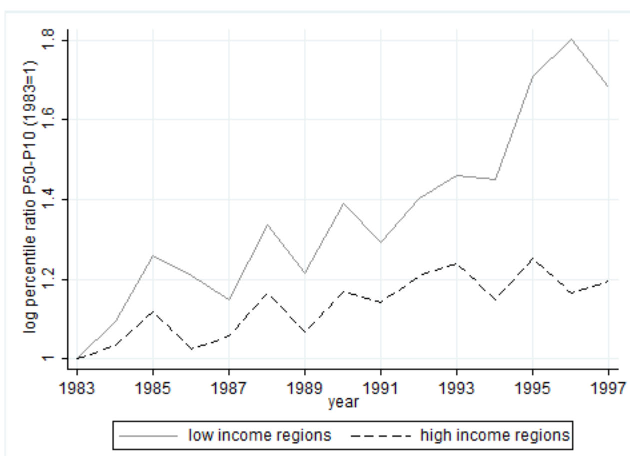
Table 8 in Appendix A reports summary statistics for the firm data. On average, firms employ 600 workers. The average size is substantially smaller, as the survey over-samples large firms. Additionally, firms hold a capital stock of 45 million euros, and most are located in the North of Italy. By sector, our data confirm the specialization of Italian manufacturing in industries with low technological content.

5 Empirical Evidence of the Relation between Inequality and Minimum Wage

Before introducing the equilibrium model, let me show some empirical evidence from the data of the negative relation between real minimum wage and inequality.

I have already illustrated the negative co-movement between real minimum wage and inequality through time. However, I may also exploit variation across Italian regions. Indeed, high-income regions are less binding in the minimum wage, while in the low-income regions, a larger share of workers earns a wage equal to the minimum wage. Furthermore, richer regions have more dispersion in wages than the other regions. In fact, from the data, it can be seen that the more is binding the minimum wage, the higher is inequality, and this relation can be seen by using variation through time and across regions. Figure 2 plots normalized wage inequality between 1983 and 1997 [3]. It shows clearly that the lower-tail inequality increases significantly for low-income regions, with the percentile ratio P50-P10 increasing by 80 percent, but markedly less for initially high-income regions in which it increases by 20 percent.

Figure 2: Evolution of wage inequality across regions

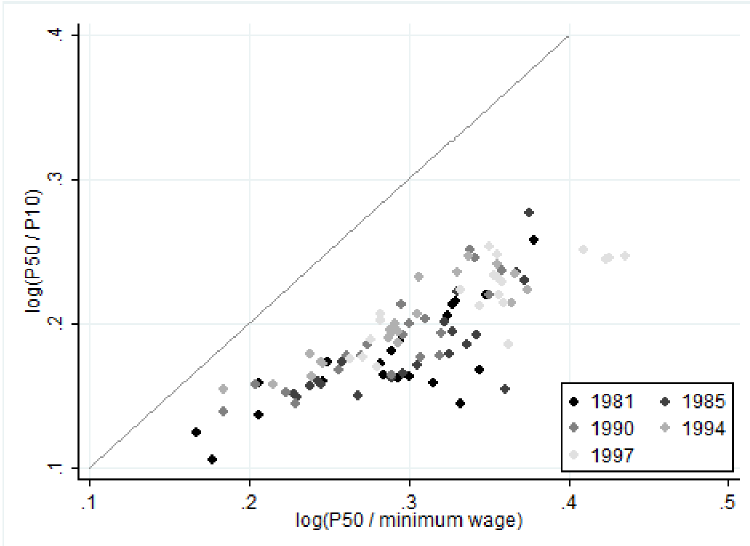


Notes: The figure groups "low income" and "high income" the two lowest and the two highest regions ranked by mean log wage in 1983, and plot between 1983 and 1997 lower-tail log percentile ratio P50-P10 normalized to 1.0 in 1983. Source: INVIND, INPS.

[3] I exclude the first two years of the sample because, in 1981-1982, the real minimum wage had not started to decrease yet.

These patterns lead us to ask to what extent the decrease in real minimum wage can rationalize the observed heterogeneity in wage inequality in the cross-section and over time. In order to identify this relation we follow Lee (1999) [20], Autor et al.(2016) [7] and Engobom and Moser (2018) [12] in exploiting variation across regions and across time. To this purpose we introduce the Kaitz index as measure of bindingness in minimum wage, $kaitz_{rt} = \log w^{50}_{rt} - \log w^{min}_{t}$. It is the log difference between the minimum wage at time t, w^{min}_{t} and the median wage in region r at time t, w^{50}_{rt}. The lower is this index, the higher is the bindingness in the minimum wage. Suppose that the index is equal to zero; this implies that fifty percent of the workers earn a wage equal to the minimum wage. However, when the index increases and is higher than zero, the minimum wage is binding for a lower share of the population of workers. Figure 3 plots the relation between wage inequality and the Kaitz index for the data. The x-axis shows the Kaitz index, while the y-axis shows lower-tail inequality measured as the log percentile ratio between the fifth and tenth percentile. This log percentile ratio represents a measure of the dispersion of the bottom part of the wage distribution. The higher is its value, the higher is the inequality. Indeed, suppose that the index is equal to zero, this means that there is no difference between the tenth percentile and the fifth percentile, while if the index is higher, we have more dispersion. Each marker in the scatter represents one region-year combination. The 45-degree line marks points in which the minimum wage is binding for the lower 10 percent of workers. Since all markers are below this line, we have that the minimum wage is binding for less than ten percent of workers for all regions and the entire period in the data. From the scatter, it is evident a positive relationship between the P50-P10 and the Kaitz index. This implies that when the minimum wage becomes less binding (due to a decrease in the real minimum wage), the Kaitz index increases, and also inequality increases. This result is obtained by exploiting the variation across regions and across time.

Figure 3: Wage percentile ratios across Italian regions over time, 1981-1997



Notes: Each marker represents one region-year combination for each of Italian's 20 regions in the data. The 45 degree line represents points in which the Kaitz index (x axis) is equal to the log percentile ratio P50-P10 (y axis). Source: INVIND, INPS

We can see this negative correlation not only graphically but also from reduced-form evidence. For this purpose, following Lee (1999) [20] and Autor et al. (2016) [7] I regress log percentile ratios for different percentile p of region r in year t on the Kaitz index and controls.

$$w_{rt}(50) - w_{rt}(p) = \beta(p)[w_{rt}(50) - w_t^{min}] + \sigma_t + \gamma_r(p)t + \varepsilon_{rt}(p) \tag{1}$$

where $\gamma_r(p)t$ denotes region-specific trend and σ_t denotes year dummies. I want to remark that this regression has no causal purpose. It is only part of the descriptive analysis to better see the negative correlation between inequality and minimum wage data. Table 1 shows results from estimating regression (1). In specification (3) and (4) I consider a quadratic model and the estimates reported in Table 1 are the marginal effect $\rho(p) = \beta_1(p) + \beta_2(p)kaitz_{rt}$ evaluated at the worker-weighted mean region-year. I find that the coefficients are significant until the 90th percentile of the wage distribution. This implies that there might be spillover effects. However, as I expected, minimum wage affects more the bottom part of the distribution; in fact, when I include region-specific trend, the coefficients are strictly decreasing in percentile p (specification (2) and (4)). In other words, the correlation between the P50-P10 and the real minimum wage is higher in absolute terms than the correlation with the P50-P90 ratio.

Table 1: Correlation between inequality and various wage percentile ratios relative to the median

Percentile	(1)	(2)	(3)	(4)
$p = 10$	0.484***	0.571***	0.478***	0.590***
	(0.0001)	(0.0002)	(0.0001)	(0.0002)
$p = 20$	0.251***	0.283***	0.246***	0.296***
	(0.0001)	(0.0002)	(0.0001)	(0.0002)
$p = 30$	0.127***	0.129***	0.124***	0.135***
	(0.0001)	(0.0002)	(0.0002)	(0.0002)
$p = 40$	0.052***	0.046***	0.052***	0.047***
	(0.0001)	(0.0001)	(0.0001)	(0.0001)
$p = 60$	−0.046***	−0.028***	−0.047***	−0.029***
	(0.0001)	(0.0001)	(0.0001)	(0.0001)
$p = 70$	−0.101***	−0.053***	−0.101***	−0.033***
	(0.0001)	(0.0002)	(0.0002)	(0.0002)
$p = 90$	−0.213***	−0.004***	−0.218***	−0.016***
	(0.0002)	(0.0003)	(0.0002)	(0.0003)

Notes: Table shows correlation between inequality and real minimum wage. $*$ = significant at 10% level, $**$ = 5%, $***$ = 1%. Underlying regressions are variants of equation (1). Specification (1) includes year effects. Specification (2) includes additional linear region-year trends. Specification (3) is a quadratic model with year effects. Specification (4) includes also region specific trends. For the last two specifications the reported value is the marginal effect evaluated at the worker-weighted mean across years 1981-1997.

6 Equilibrium model

This section develops a version of the Burdett and Mortensen (1998) [5] equilibrium model with firm heterogeneity that we use to assess the effects of a minimum wage decrease. The Burnett and Mortensen model is widely used to study wage determination, and our exposition closely follows that of Engbom and Moser (2018) [12] , Bontemps et al. (1999,2000) [3], Mortensen (2003) [23], and Jolivet et al. (2006) [17].

6.1 Environment

I study a stationary economy in continuous time that consists of a mass M_w of infinitely-lived workers and a mass M_f of firms who meet in a frictional market.

6.1.1 Workers

Workers are identical, there is only one ability type. They can be employed or unemployed, and in both states, workers search for job. Let λ^u denote the job offer arrival rate for the unemployed and λ^e, the arrival rate for employed. A job offer entails a wage draw $w \sim F(.)$ over support $[\underline{w}, \bar{w}]$. Although workers take wage offer distribution as given, it is determined endogenously through the firm's wage posting decisions, subject to a minimum wage. Matches dissolve exogenously at rate δ. As employed workers search for higher-paying jobs through on-the-job search, the realized wage distribution G first-order stochastically dominates the wage offer distribution F.

6.1.2 Firms

Firms differ in productivity p distributed continuously over $[p_0, \bar{p}]$ with cdf Γ. They use a linear production technology with labor ℓ as unique input to produce output y:

$$y(p, \ell) = p\ell$$

In contrast with Engbom and Moser (2018) [12] and Flinn (2006) [14], but in line with van den Berg and Ridder (1998) [25] and Bontemps et al. (1999,2000) [3] I consider contract rates as exogenous. Each firm has one job to fill. The wage equilibrium posting decision determines the wage offer distribution F. The wage policy of a firm with productivity p, together with the ranking position in the wage offer distribution, jointly determine the firm's employment level $\ell(w)$. A productivity p firm's problem coincides with the following profit maximization:

$$\max_{w \geq w^{min}} (p - w)\ell(w)$$

A firm makes positive profits only if it posts a wage between minimum wage w^{min} and its productivity p. Suppose that the lower bound $p_0 > w^{min}$ for the entire period that I consider. Then, since each firm has one vacancy to fill, the wage offer distribution is equal to the distribution of firms' productivity, $F(w(p)) = \Gamma(p)$.

6.2 Steady state

We will see that changes in the minimum wage affect the distribution of wages through the wage posting policy of firms. However, before that, I want to describe the labor flows of this economy that are driven by optimal decisions of workers and firms. By understanding these flows in the economy, we can understand how the wage distribution $G(w)$ is formed. We analyse the economy at steady-state, therefore, even when I change the minimum wage, I will consider only the newly reached steady-state economy, disregarding the transitional economy. The steady-state is characterized by a triple (u, G, ℓ), the unemployment rate u, the distribution function of wages across workers G, and steady-state size of firm ℓ.

Steady state unemployment rate u. The outflow is given by individuals that receive offers with an arrival rate λ^u. The inflow is given by workers that lose their jobs with destruction rate δ. In order to find the steady-state unemployment rate u, I equalize inflow and outflow.

$$\lambda^u u = \delta(1 - u) \quad \Longleftrightarrow \quad u = \frac{\delta}{\delta + \lambda^u}$$

Steady state wage distribution G. The function G(w) represents the proportion of employed workers that earn a wage lower than w. The inflow is given by unemployed individuals that receive an offer lower than w ($F(w)$ represents the probability that the offer is lower than w). The outflow is given by workers who lose their job with a destruction rate δ and workers who receive a better offer and decide to quit voluntarily. $G(w)(1-u)$ represents the proportion of workers that earn a wage lower than w, λ^e denotes the offer arrival rate when the worker is employed, and $(1 - F(w))$ is the probability that the offer is higher than w. Then, we can find steady-state distribution G as follows.

$$\lambda^u u F(w) = [\delta + \lambda^e(1 - F(w))]G(w)(1-u) \quad \Longleftrightarrow \quad G(w) = \frac{\delta F(w)}{\delta + \lambda^e(1 - F(w))} \tag{2}$$

As employed workers search for higher-paying jobs, the realized wage distribution G first-order stochastically dominates the wage offer distribution F.

Steady state size of firm $\ell(w)$. Given wage w, the function $\ell(w)$ represents the workforce of firms paying w. In order to obtain the size of firms at steady-state, I equalize inflow and outflow. Unemployed workers accept all wage offers in equilibrium, while only a fraction $G(w)$ of employed workers accepts an offer of wage w. Out of a firm's workforce $\ell(w)$, a fraction δ exits to unemployment and a fraction $\lambda^e(1 - F(w))$ quits to higher-paying firms.

$$\begin{aligned} &[\lambda^u u + \lambda^e G(w)(1-u)]M_w \mathrm{f(w)} = [\delta + \lambda^e(1 - F(w))]\ell(w)\mathrm{f(w)}M_f \\ \Longleftrightarrow \quad &\ell(w) = \frac{M_w}{M_f} \frac{\delta\lambda^u(\delta + \lambda^e)}{(\delta + \lambda^u)(\delta + \lambda^e(1 - F(w)))^2} \end{aligned} \tag{3}$$

Where (3) is obtained by substituting the steady state unemployment level u and the wage distribution G found above.

6.3 Effects of the minimum wage

Once I obtain the steady state size of firm $\ell(w)$, I can substitute it in the productivity p firm's maximization problem:

$$\max_{w \geq w^{min}} (p - w)\ell(w)$$

By solving the F.O.C. I obtain the wage policy $w(p)$[4] that is increasing in p, $w'(p) > 0$, i.e., more productive firms pay higher wages, leading to an equilibrium wage dispersion for identical workers. This can be explained by the fact that in choosing a wage, firms face a trade-off. On the one hand, lower wage increases per-worker profit. On the other hand, a higher wage rank raises steady-state recruitment through increased poaching and decreased voluntary quits. In choosing their wage policy, firms are affected by the wage posting decisions of other firms. Therefore, changes in the minimum wage lead to spillovers between employers, even if only a subset of firms is directly affected. Concretely, let me consider the effects of a decrease in minimum wage between steady states, that is what we observe in Italy between 1981 to 1997. A set of firms that were binding at the previous minimum wage will comply with the new wage floor. However, some of them will decide to stop before reaching the new minimum wage in order to obtain a better ranking position in the wage offer distribution. This will eventually lead higher-paying firms to decrease their wage less and less through the wage distribution to retain their pay rank. Therefore, the wage will decrease more for lowest paying firms than for higher-paying firms, implying that wage dispersion increases.

How does the minimum wage affect inequality in this equilibrium framework?

Proposition. *A marginal decrease in the minimum wage*

1. *decreases wages at all firms:* $\partial w(p; w^{min})/\partial w^{min} > 0$

[4] For details see Appendix B.1

2. increases the productivity pay premium across firms: $\partial[\partial w(p; w^{min})/\partial p]/\partial w^{min} < 0$

Proof. See Appendix B.2.

The proposition is interpreted as follows. Part 1 states that wage decreases for all firms. However, Part 2 states that this decrease is not the same for all firms across the wage distribution. Indeed, wage decrease at the initially lowest-productivity firms is one-for-one with the minimum wage but gradually declines for higher-paying firms, leading to a steeper firm productivity-pay gradient.

It is interesting to analyse two extreme cases of the model. That of perfectly competitive labor market, in which workers are paid their marginal product ($\lambda^e/\delta \to +\infty$), and the monopsony outcome ($\lambda^e/\delta = 0$). In both cases, there is no wage dispersion caused by frictions of the market. In these extreme cases, the minimum wage changes induce no spillovers. For the intermediate range, the parameterization of the model determines the strength of equilibrium spillovers. Hence, the model's predictions for minimum wage effects depend crucially on estimates of the labor market parameters.

7 Estimation of the model

This section estimates the model based on INPS matched employer-employee data from 1981-1982 when there is no decrease in real minimum wage yet. We want to use the estimated model to analyse the effects on the wage distribution of a 16 percent decrease in Italy's real hourly minimum wage.

7.1 Methodology

The model described in Section 6 requires to determine two sets of parameters. On one side, we need to calibrate the so-called search parameters that determine the flows in the labor market, more precisely the destruction rate, δ, the arrival rates for unemployed and employed workers, λ^u, λ^e and the ratio between the mass of workers and the mass of firms, $m = M_w/M_f$. On the other side, I need to estimate parameters that shape the distribution of productivity of firms. In this regard, I assume that log firm productivity follows a Gamma distribution with shape parameter γ_1 and scale parameter γ_2.

I start by setting some parameters based on values in the literature. Following Elsby et al. (2008) [11], I set the destruction rate δ and the unemployed arrival rate λ^u. Then, I estimate the remaining parameters by indirect inference with two distinct procedures for search and distribution parameters.

Search Parameters. From equation (3), it can be seen that the number of workers per firm in the stationary equilibrium is determined by the search parameters $\delta, \lambda^u, \lambda^e$ and the ratio between the mass of workers and the mass of firms, m. As I have the destruction rate δ and the offer arrival rate for unemployed workers λ^u from the literature, I need to estimate λ^e and m. In order to do so, I estimate the parameters targeting the median and the highest value of the size of firms from the data. See Appendix C.1 for more details on the estimation procedure.

Distribution Parameters. I assume that log firm productivity follows a Gamma distribution with shape parameter γ_1 and scale parameter γ_2. Since each firm has just one vacancy to fill, for any value of productivity p, the value of the job offer distribution $F(w(p))$ should correspond exactly with the value of the productivity distribution, $\Gamma(p)$. Thus, once I obtain the offer distribution for each level of wage $F(w)$, and the wage policy $w(p)$, I can estimate the Gamma distribution parameters γ_1 and γ_2 such that $F(w(p)) = \Gamma(p)$. See Appendix C.2 for more details.

7.2 Parameter estimates and model fit

Following Section 7.1, I determine a set of search parameters that I reported in Table 2.

Table 2: Search Parameters Estimates

Description	Parameter	Value
Destruction rate	δ	0.004
Job offer arrival rate when unemployed	λ^u	0.04
Job offer arrival rate when employed	λ^e	0.017
ratio workers firms	m	175.55

Notes: Table shows estimated search parameters. λ^u and δ are set following Elsby et al. (2008) [11]. The other parameters λ^e and m are obtained by indirect inference following the procedure in Appendix C.1.

As expected, the estimated value of offer arrival rate for employed workers λ^e is less than half of the arrival rate for unemployed workers because there is less search intensity. Following Elsby et al.(2008) [11], the reported destruction rate δ for Italy is the lowest among OECD countries. A low δ implies a high rate at which workers move up the firm ladder given by the ratio between the offer arrival rate and the destruction rate λ^e/δ. Indeed, since the probability of losing the job is low, Italian workers can gradually climb the firms' ladder through on-the-job mobility.

In order to estimate the distribution parameters γ_1 and γ_2 of firms' productivity distribution, I follow the strategy described in Appendix C.2. In Tables 3 and 4, I reported respectively the estimates of the parameters of the Gamma distribution and the targeted moments.

Table 3: Distribution Parameters estimates

Description	Parameter	Value
Shape	γ_1	2.92
Scale	γ_2	0.10

Notes: Table shows estimates of shape parameter γ_1 and scale parameter γ_2 of the Gamma distribution for log productivity of firms.

Table 4: Targeted moments

Moment	Data	Model
Mass at the minimum wage	1%	2%
proportion below 1.5minimum wage	95.2%	89.3%

Notes: Table shows moments of the job offer distribution from data and of Gamma distribution from the model. These targeted moments are used to estimate the distribution parameters γ_1 and γ_2.

Table 4 compares targeted moments used in the indirect inference procedure in the model and the data. The model replicates the empirical job offer distribution, suggesting that the parametric assumption on the firm productivity distribution is sufficiently flexible to capture the data distribution.

8 Effects of a lower minimum wage

I use the estimated model to simulate the following policy experiment: what are the steady state effects of A 16 percent decrease of hourly minimum wage between 1981 to 1997 on the wage distribution, holding fixed all structural parameters ?

8.1 Effect on the distribution of wages

The 16 percent decrease in real minimum wage increases the wage inequality, as summarized in Table 5.

Table 5: Data vs. model: Inequality, 1981-1982 and 1997

	1980-1981		1997		Change	
	Data	Model	Data	Model	Data	Model
Panel A: Wage Variance						
Standard Deviation	0.1302	0.1459	0.1832	0.1815	0.053	0.036
Panel B : Wage percentiles						
P50-P10	0.17	0.26	0.25	0.32	0.08	0.06
P50-P25	0.09	0.15	0.14	0.18	0.05	0.03
P75-P50	0.10	0.29	0.13	0.31	0.03	0.03
P90-P50	0.21	0.36	0.23	0.39	0.02	0.03

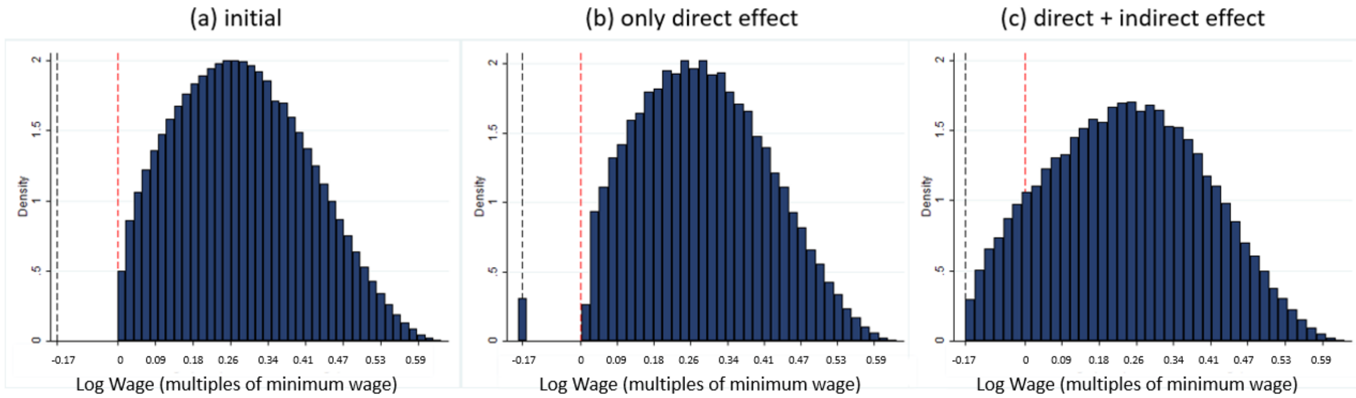
Notes: Table shows the evolution through time of two measures of wage dispersion: the standard deviation of log wages, and the log percentile ratio. In the table I reported the values before and after the change in minimum wage, both from data and from simulations of the model. Source: INPS survey and simulations.

The model predicts an increase in the standard deviation of log wage by 3.6 log points, compared to an increase of 5.3 log points from data in Italy between 1981 to 1997. Hence, the decline in minimum wage accounts for 66 percent of the overall increase in wage inequality experienced by Italy between this period. As in the data, the rise in inequality in response to the real minimum wage fall is concentrated at the bottom of the wage distribution. The log 50-10 percentile ratio of earning increases by 5 log points, while the log 90-50 ratio increases by 3 log points. Furthermore, I can state that the minimum wage accounts for a similar fraction of the empirical increase in earnings inequality throughout the earnings distribution in Italy. It is worth noting that the proportion of workers that earn the minimum wage is far below the 10 percent throughout this period, suggesting extensive spillover effects of the minimum wage.

In order to understand better how the minimum wage affects inequality in this equilibrium framework, let me define the "direct effect" as firms move to the new wage floor and the "indirect effect" as the adjustment due to workers' and firms' equilibrium responses.

Figure 4 shows this two-step decomposition for the simulations of the model. In panel (a), you can find the wage distribution with the initial level of minimum wage obtained from the model simulations. It matches well the empirical earnings distribution form data. In the x-axis, there are the log wages in terms of multiples of the initial minimum wage marked by a vertical red line. Then, the minimum wage decreases by 16 percent, reaching the grey vertical line. Panel (b) illustrates the "direct effect", in which some firms that were binding at the previous minimum wage remain binding also at the new level. By looking at this first step, we can conclude that we have a mechanical increase in the wage distribution, even without considering the response of all the other firms that are not directly affected by the minimum wage. The second step of the decomposition, going from panel (b) to panel (c) of the figure, allows for the equilibrium adjustment of firm wage offers and worker mobility in the new steady state of the model economy. Panel (c) illustrates wage distribution obtained from the model simulations with the lower level of minimum wage. Note that the result is not a complete shift of the distribution to the new wage floor. Indeed, the dispersion is markedly increased, and the mass of workers binding at the minimum wage is decreased. Let me illustrate the mechanism behind this result. As explained in Section 6.3, in choosing their wage policy, firms face a trade-off. A lower wage can increase the per-worker profit, but a higher wage can improve the rank position of the firm in the job offer distribution. Thus, when the wage floor decreases, not all firms that were binding before decide to reach the new lower level. Some of them might stop before to improve their ranking position and increase the probability of filling their vacancy. This implies that upper in the distribution firms will decrease their wage offer to retain their pay rank. Hence, wages decrease more for lowest-paying firms than for highest-paying firms, resulting in an increase in wage dispersion.

Figure 4: Model: Direct and Indirect effects on wages



Notes: decomposition of the overall increase in dispersion due to the minimum wage in the estimated model into a "direct effect" and "indirect effect". The red vertical line indicates the initial level of the minimum wage, normalized to zero. The grey dashed line corresponds to the final level of the minimum wage. Panel (a) shows the initial wage distribution in the estimated model. Panel (b) shows the "direct effect" in which only firms that were binding before are affected. Panel (c) shows the final equilibrium wage distribution. Source: simulations.

Table 6 illustrates the decomposition of the total increase in the variance of wages in the model and in the data, both showing that most of the increase in inequality is due to indirect effect of the minimum wage. In fact, only 14-17 percent of the total increase in the variance of wages is explained by the direct effect, while more than the 80 percent is due to equilibrium responses of workers and firms. Therefore, the results from the estimated model suggest that spillover effects lead to a sizable propagation of the minimum wage through all the wage distribution.

Table 6: Data vs model: Decomposition into direct and indirect effects

	Data		Model	
	SD	$\Delta(\%total)$	SD	$\Delta(\%total)$
Initial Period (1981-1982)	0.1302	-	0.1459	-
+ Direct effect	0.1403	0.009 (17%)	0.1505	0.005 (14%)
+ Indirect effect	0.1832	0.053 (83%)	0.1815	0.036 (86%)

Notes: Table shows decomposition results into "direct" and "indirect" effects of the minimum wage in the model and in the data. Source: INPS survey and simulations

9 Conclusion

Between 1981-1997 in Italy, the real minimum wage declined while inequality increased. The reduced-form analysis of data shows a negative relationship between inequality and minimum wage, exploiting variation through time and across regions. Additionally, regression estimates imply spillover effects of minimum wage until the 90th percentile. In order to quantify the effect of minimum wage in shaping the wage distribution, I introduced a model in the style of Burnett and Mortensen (1998) [5] with firm heterogeneity. I find that changes in the wage floor had significant effects throughout the distribution. In fact, the fall in minimum wage induces 3 log points increase in the variance of wages that corresponds to 66 percent of the overall rise in inequality in Italy over this period. Both data and the model show that most of the increase in inequality is due to indirect effects determined by equilibrium responses of firms and workers.

This paper shows that minimum wage plays a sizable role in shaping inequality in Italy. So, suppose the government is interested in containing inequality. In that case, these findings may raise the debate about increasing legal enforcement of national collective contracts in Italy or introducing a state minimum wage centrally regulated.

These insights point to fruitful avenues for future research. First, it could be interesting to extend these findings to other sectors or other job qualification levels of workers. Second, since I based the research only on the wage policy margin, this could also be the start for analysing the vacancy policy margin. Indeed, in this paper, firms have only one vacancy to fill, and minimum wage does not affect their vacancy policy. Even if the wage policy margin already explains a lot of the impact of minimum wage on inequality, there could be additional effects through the vacancy channel. Finally, It could be interesting to complete this research by analysing the effect of minimum wage on other labor outcomes like employment, output, and labor productivity.

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A Data Appendix

Table 7: Workers' characteristics

	Whole sample		Metal workers sample (C011 contract)	
	Mean	St.Dev.	Mean	St.Dev.
Hourly Wage (1995 Euros)	8.44	3.51	8.58	2.51
Age	37.13	10.34	38.11	9.72
Share of males	.78		.84	
Share of blue-collar workers	.63		1	
No. of observations	21,189,495		8,326,520	

Notes: Tables shows some workers' characteristics for the whole sample and for the subsample of metal workers.

Table 8: Firms' characteristics

	INVIND firms	
	Mean	Std.Dev.
Value added	26.6	118.1
Capital stock	46.4	242.0
No.of workers	624	1611
No.of low qual. workers	327	699
Sectorial shares		
Low-tech	.38	
Medium-low	.25	
Medium-low	.30	
High	.07	
Geographical shares		
North-West	.44	
Medium-low	.25	
Medium-high	.20	
High	.11	
Share of single-plant firms	.63	
No. of observations	9,790	

Notes: Table shows firms' characteristics from the INVIND whole sample. Value added and capital stock are in millions of 1995 euros. See Table 9 for the sectoral classification in terms of technological content.

Table 9: OECD Technology Classification system

CLASSIFICATION
HIGH-TECHNOLOGY MANUFACTURES
Pharmaceuticals
Office, Accounting and Computing Machinery
Radio, Television and Communication Equipment
MEDIUM-HIGH TECHNOLOGY MANUFACTURES
Chemicals Excluding Pharmaceuticals
Machinery and Equipment, N. E. C.
Electrical Machinery and Optical Instruments
Motor vehicles, Trailers and Semi-Trailers
Railroad Equipment and Transport Equipment N. E.C.
MEDIUM-LOW TECHNOLOGY MANUFACTURES
Coke, Refined Petroleum Products and Nuclear Fuel
Rubber and Plastics Products
Other Non-Metallic Mineral Products
Basic Metals and Fabbricated Metal products
Building and Repairing of Ships and Boats
Manufacturing N.E.C.
LOW-TECHNOLOGY MANUFACTURES
Food Products and Beverages
Tobacco Products
Textiles
Wearing Apparel, Dressing and Dyeing of Fur
Leather, Leather Products and Footwear
Wood and Products of Wood and Cork
Pulp, Paper, Paper Products, Printing and Publishing
Forniture

Notes: This list assigns each sector to a specific technology group. Source: OECD

B Theory Appendix

This appendix provides details on the model outlined in section 6, including subsections on the equilibrium characterization (Appendix B.1), and proof of the proposition (Appendix B.2).

B.1 Equilibrium Characterization

Once I obtained the steady state size of firm $\ell(w)$, I can substitute it in the productivity p firm problem.

$$\max_{w \geq w^{min}} (p - w) \frac{M_w}{M_f} \frac{\delta \lambda^u (\delta + \lambda^e)}{(\delta + \lambda^u)(\delta + \lambda^e (1 - F(w)))^2}$$

The first-order condition with respect to the wage w is

$$1 = (p - w(p)) \frac{2\lambda^e f(w(p))}{\delta + \lambda^e (1 - F(w(p)))} \tag{4}$$

Let $\Gamma(p)$ denote the proportion of employers with productivity no greater than p, with $p \in [p_0, \bar{p}]$. As all wage offers must be at least as great as the minimum wage, w^{min}, only employers with productivity $p \geq w^{min}$ can make a profit and hence will participate. Hence, without loss of generality I assume $p_0 = w^{min}$. All employers with the same productivity must offer the same wage in the case of a continuum of productivity types. The wage policy that maps productivity to the wage offer is a function $w(p)$ that satisfies the first-order profit maximization condition (4), so that

$$F(w(p)) = \Gamma(p) \quad \text{for all p} \in [p_0, \bar{p}]$$

From (4) since $F'(w(p))w'(p) = \Gamma'(p)$, we have

$$w'(p) = \frac{(p - w(p))(\lambda^e/\delta)\Gamma'(p)}{1 + (\lambda^e/\delta)(1 - \Gamma(p))} \quad \text{with} \quad w(p_0) = w^{min}$$

By solving the first order differential equation, I obtain the wage policy for firm with productivity p

$$w(p) = p - \int_{p_0}^{p} \left[\frac{1 + (\lambda^e/\delta)(1 - \Gamma(p))}{1 + (\lambda^e/\delta)(1 - \Gamma(x))}\right]^2 dx \tag{5}$$

By differentiation

$$w'(p) = 2(\lambda^e/\delta)\Gamma'(p) \int_{p_0}^{p} \frac{1 + (\lambda^e/\delta)(1 - \Gamma(p))}{[1 + (\lambda^e/\delta)(1 - \Gamma(x))]^2} dx > 0 \quad \text{for all } p > p_0$$

Thus, higher productivity implies higher wages.

B.2 Proof of the proposition

Part 1. I want to show that a decrease in the minimum wage decreases all wages for all firms. Differentiating equation (5) with respect to minimum wage gives that

$$\frac{\partial w(p)}{\partial w^{min}} = \left[\frac{1 + (\lambda^e/\delta)(1 - \Gamma(p))}{1 + (\lambda^e/\delta)(1 - \Gamma(w^{min}))}\right]^2 > 0,$$

which establishes the first part of the proposition.

Pert 2. Differentiating equation (5) with respect to productivity gives that the productivity-pay gradient is given by

$$\frac{\partial w(p)}{\partial p} = 2(\lambda^e/\delta)\gamma(p)[1 + (\lambda^e/\delta)] \int_{p_0}^{p} \left(\frac{1}{1 + (\lambda^e/\delta)(1 - \Gamma(x))}\right)^2 dx$$

Differentiating this equation with respect to the minimum wage gives that

$$\frac{\partial(\frac{\partial w(p)}{\partial w^{min}})}{\partial w^{min}} = 2(\lambda^e/\delta)\gamma(p)[1+(\lambda^e/\delta)(1-\Gamma(p))]\left(\frac{1}{1+(\lambda^e/\delta)(1-\Gamma(w^{min}))}\right)^2 < 0$$

Hence, the firm productivity pay gradient increases when minimum wage falls. This leads to an increase in wage dispersion.

C Estimation Appendix

This appendix provides details on the estimation procedure of Section 7, including subsection on the algorithm that I use to estimates the model search parameters (Appendix C.1), and distribution parameters (Appendix C.2).

C.1 Search parameters

From the literature, I have the destruction rate δ, and the offer arrival rate for unemployed workers λ^u. I need to estimate the offer arrival rate for employed workers λ^e and the ratio between mass of workers and firms m. From equation (3) it is possible to see that the workforce for firm, $\ell(w)$ is a function of the two unknowns λ^e and m. In order to estimate them, I take the median and the highest level of size of firm from data and I solve the system with two equations and two unknowns.

$$\ell(w)^{median} = m\frac{\delta\lambda^u(\delta+\lambda^e)}{(\delta+\lambda^u)(\delta+\lambda^e(1-F^{0.5}))^2}$$
$$\ell(w)^{highest} = m\frac{\delta\lambda^u(\delta+\lambda^e)}{(\delta+\lambda^u)(\delta+\lambda^e(1-F^{1}))^2}$$

For what concerns the value of F in the formula of $\ell(w)$, I found the value of $F^{0.5}$ by inverting equation (2) with $G(w) = 0.5$, while for F^1 it is equal to 1 as the G function. From this procedure I obtained the estimates of all the search parameters in the model.

C.2 Distribution parameters

From the data of period 1981-1982 I have the wage distribution $G(w)$. As I showed in Appendix C.1, I have the estimates of all the search parameters $\delta, \lambda^u, \lambda^e, m$. Thus, By inverting equation (2) and interpolating it is possible to obtain the job offer distribution $F(w)$ as continuous function of w.

$$G(w) = \frac{\delta F(w)}{\delta + \lambda^e(1 - F(w))} \quad \iff \quad F(w) = \frac{G(w)(\delta + \lambda^e)}{\delta + \lambda^e G(w)}$$

As shown in Appendix B.1, I obtain that the wage policy $w(p)$ is a function of productivity distribution $\Gamma(p)$. I normalize the initial minimum wage to be the numeraire. Then, I estimate parameters γ_1 and γ_2 such that the resulting wage policy gives $F(w(p)) = \Gamma(p)$. In order to do that I use as target moments the mass of firms binding at minimum wage and the value of wage distribution at a wage level far from the minimum wage in order to capture the dispersion of the wage offer distribution.