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How options have affected short squeeze phenomenon.

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INTRODUCTION

During the past year, options trading hit a record with some 7,47 billion contracts traded, according to the Options Clearing Corporation. That was 45 percent higher than the previous record, set in 2018. Much of this money has come from small-time traders hoping to make fast gains by buying call options.

Retail investor segment operating in U.S. market switched to represent 10% of transactions to 20%, reason why, even during the deep uncertainty that stunned the market at the start of the pandemic, individual investors have been pouring into the market.

This situation led them to be capable to influence the stock market structure, investing in the same direction.

Regarding options contracts, the skew is evident in something called the “put-call ratio”, which shows how many contracts are betting on gains compared with those betting on losses through put options. At the end of 2020, the 50-day moving average of that ratio was 0,42, near the lowest level in two decades. Speculation has reached a frenetic level not seen since the tail end of the dot-com boom two decades ago.

As if that were not enough, last January was the highest volume month ever for the U.S equity options, up 61,7% compared to the January 2020, in addition, the weekly volume of call options showed that small lots of account for 43% of the total volume of call options.

The new stock market dynamic has caused a boost in short squeeze phenomenon because not only the investments funds had to cover the short position, but also the market maker had to hedge their short call position.

This phenomenon occurred, to a great magnitude, within the Game Stop case, from which this treatise wants to take cue.

The wild price action that hit Game Stop, was a textbook example of how the options related flow can accelerate volatility in underlying asset, forcing, short position investors, to buy the asset, lengthening the short squeeze window.

Purpose of treatise is to study the aspects that led Game Stop price to record such a large uptick, focusing on options contracts that have been the drivers of this process.

First part of treatise it will outline the history of option trading to better understand the usefulness of this financial device to the world economy and their relation to the term market efficiency.

Modern day terminology and technical details it will be covered, and instructive examples will serve to understand the mechanics of options trading, the leverage effect as well as some of the risks involved.

Thereafter it will discuss the main models involved to price options. Purpose here is to implement the different models of option pricing, starting from “Binomial Pricing Model”, which allows to calculate in a relatively easy way the price of options through a dynamical development. Following it will be analyzed the “Black and Scholes model”, which assumes the log-normal distribution of the stock price, and how apply this model to American options. Last model researched will be “Monte Carlo Simulation Model”.

Assumptions and simplifications related to the models will be explained and the respective equations will be stated to capture differences and peculiarities between three methods of evaluation.

In conclusion, of the first part, it will be covered the “Greeks” to describe the different dimensions of the risk involved in the options, which will be subject of the analysis in the following section.

In the second part, it will be introduced the concept of short squeeze phenomenon and it will be treated the value of short interest ratio on Game Stop stocks, focusing the analysis on the number of open interest positions in out-of-the money call options to understand whether it was possible foresee the short squeeze.

Then it will test the correlation between short interest and difference in implied volatility between put option and call option on Game Stop and the correlation between put-call ratio and GME price.

Subsequently it will be argued of “Game Stop case”, explaining what happened and studying the relation between option transactions and total float of the stock, during the phenomenon.

Finally, starting from a dataset composed by options contract traded during January and February on Game Stop stock, will be investigated the “Gamma squeeze” phenomenon, through the concept of negative gamma and will be analyzed an anomaly related the put option contract.

1 STOCK OPTIONS AND TRADING STRATEGIES INVOLVING OPTIONS

1.1 MECHANISM OF OPTIONS MARKET

Options are a form of derivative financial instrument in which two parties, contractually agree to transact an asset at specified price before a future date. An option gives its owner the right to either buy or sell an asset at the exercise or strike price, but the owner is not obligated to exercise the option. When expiration date is reached without exercise the option, it is rendered useless with no value.

The objectives of the option traders are as varied as there are option strategies. Options can be used to take positions that are extremely bullish, extremely bearish, or in between and are used to limit risk, hedge against loss, or speculate.

There are two types of options: call and put. Call options allow the holder to purchase an asset at a specified price, before or at a particular time. Put options are opposites of calls in that, they allow the holder to sell an asset at a specified price, before or at a particular time.

The holder of a call relies on that, the value of the underlying asset will move above the exercise price before expiry. Conversely, a holder of a put option assumes that the value of the underlying asset will move below the exercise price before expiry.

People cyclically think that options are just gambling vehicles, but the options have some basic purposes. First one is "theoretical". Following the idea of Kenneth Arrow, an economic theorist, who wrote a classic paper[1], in which he argued that, unless we have prices for all states of nature, there

[1] Kenneth J. Arrow "General Economic Equilibrium: purpose, analytic techniques, collective choice" 1972.

is a sense in which economic system is inefficient. For him, people really need the price of everything, including the price of some possibility.

On this way, Stephen A. Ross in "Quarterly Journal of Economics" wrote a classical paper about options, showing that, in a sense, they complete the state space. They create prices for everything that affects decision-making. This paper argues that in an uncertain world, options written on existing assets can improve efficiency by permitting an expansion of the contingencies that are covered by the market[2].

Second one is "behavioral" purpose of options. From this point it is observable how many different aspects of human behavior tie into options, something to do with "attention anomalies" and "salience". Psychologists talk about this, saying that people that make mistakes very commonly in what they pay attention to, what strikes the fancy of their imagination. Salience is something psychologists also talk about, (salient events are events that tend to attract attention) tend to be remembered.

A lot of options are called "incentive" options. An example of "incentive" options is "warrant**"** sold to an employer of firm. In this case is not very expensive for the company offers the option to buy the share, but in this way the employer starts to pay attention about the value of company, and it becomes salient for him and he starts hoping that the price of the company will go up, because he has option to buy it, at strike price. Employer hopes that the company's price per share goes above the strike price, because then his option is worth something. In that case, he is earning money. So, it may change his motivation and his morale at work, or sense of identity with the company.

In real life a typical example related to behavioral purpose of options is insurance, in the sense that when someone buy insurance on his house, it is like buying a put option on the house although it may be not directly connected to the home's value. If the house burns down, the owner collects the insurance policy. Well, the price of house drops to zero. It is like have an option to sell at a high price on something that is now worthless. Insurance is a sort of options, and insurance gives to holder peace of mind.

The serious study of options, in the financial literature, began with the long-neglected thesis by Bachelier[3] and was revived in the 1960's by a number of authors who focused on the pricing

[2] Stephen A. Ross "Options and Efficiency" on Quarterly Journal of Economics 1976.

[3] Louis Bachelier "Théorie de la Spéculation" 1900.

problem, i.e., the problem of determining the equilibrium relationship between the value of an option and the value of the stock, it is written on.

The first options exchange was the Chicago Board Options Exchange (CBOE) which was established in 1973, before that, options were traded solely through brokers and they had not the same relevance.

Before 1973 there was no great interest in the subject of option valuation. As Cox, Ross and Rubinstein put it in their 1979 paper: “options have been traded for centuries, but they remained relatively obscure financial instruments until the introduction of a listed options exchange in 1973[4]”. Indeed, “optionality” in various aspects is a widespread feature of financial markets, and how to value it, is a key component of asset valuation in general, but this point was not widely appreciated before the massive expansion of financial markets activity, in latter third of the 20th century. Furthermore, effective management of option risk depends on having a “deep” market and trading on a sufficiently fast time scale. Before the modern era of massive computational power, none of this was feasible. In the middle of 20th, markets were too illiquid, costs too high and information to scanty.

Currently option contracts are traded either:

- ➔ On a public stock exchange, also known as ETO’s (Exchange Traded Options), and the most relevant of these, remains the Chicago Board Options Exchange; In ETO’s are traded standardized options in a way that the specifications that make up the option contracts are set by the stock exchange and cannot be changed.
- ➔ Implicitly agreed between two parties, knows as OTC’s (Over The counter options). In this type of market traders from large institutions can create and trade non-standard option derivatives.

Most options exchange use market makers to facilitate trading.

Market maker for a certain option is an individual who, when asked to do so, will quote both a bid and an offer price on the option. The bid is the price at which the market maker is prepared to buy, and the offer or asked is the price at which the market maker is prepared to sell. The existence of market maker ensures that buy and sell orders can always be executed at some price without any

[4] John Cox, Stephen A. Ross, Mark Rubinstein “Option pricing: a simplified approach” Journal of Financial Economics 1979.

delays. Market makers therefore add liquidity to the market. The market makers themselves make their profits from the bid-ask spread.

The option's transactions are guaranteed by Options Clearing Corporation (OCC), it guarantees that options writers will fulfill their obligations under the terms of options contract and keeps a record of all long and short positions.

Must be differentiate the various types of options, based on whether the underlying asset is a real asset (commodity options) or a financial asset (financial options).

This treatise will concentrate on the second one. These types do not exhaust the field of underlying activities; indeed, alongside them, there are also: stock options, currency options, interest rate options, bond options, swap options, futures options, credit spread options, index options and ETF fund options.

In the last chapter stock options will be analyzed, through the case of Game Stop, but is important observe that the latest burst of options market growth, began with the advent of the Index and ETF fund options. These combine the diversification benefits of ETFs with the flexibility of options. On the other hand, the listing of options on various market indexes allowed many traders, for the first time, to trade a broad segment of the financial market with one transaction. About this, is important to observe how CBOE offers listed options on over 50 domestic, foreign, sector and volatility-based index.

The main difference between these options is that index options are "European" style options and settle in cash, while options on ETFs are "American" type and are settled in shares of the underlying security. Subsequently it will be explained the differences between European and American options, but for the moment is important specify this aspect.

There are two factors that characterize all options: one of these is the current market price of an option, also called premium. It is the income received by the seller (writer) of an option contract to another party. Other one is strike price, the set price at which an underlying can be bought or sold when the option is exercised.

For call options, the strike price is the amount at which the security can be bought by the option holder; for put options, the strike price is the price at which the security can be sold. Essentially it tells the investor, which price the underlying must reach before the option is in the money. Strike price are established when a contract is first written.

Components, that structure the price of every options, are called intrinsic value and extrinsic or time value. The intrinsic value is given by the difference between current market price of underlying and the strike price for a call option. For the put option is the opposite, in this case, the intrinsic value results from the difference between strike price and actual market price. The intrinsic value cannot be negative and represents the minimum value for option, indeed the benefit it can provide the owner at that moment is the minimum that the contract should be worth.

The option’s extrinsic value is the portion of the option’s price that exceeds its intrinsic value. An option’s extrinsic/time value as the part of its value associated with the potential for the option to become more valuable before it reaches expiration in the future, through share price movements. Options with larger time left to expiration have higher extrinsic value, compared to same exact option with less time until expiration because there is more time left for the underlying’s value to move. For this, even if an option has intrinsic value equal zero, the premium can be higher in total by extrinsic or time value. Related to this, is the meaning of “time decay”, since as the option gets closer and closer to expiration, the time value will decay out of the option’s price. Hence a call or put option’s owner will lose money if the price does not move.

There are three terms that describe the options strike price relative to the stock price, which essentially if or not they have intrinsic value:

- ➔ In the money (ITM) call options with strike price below the stock price and in the money put options with strike price above the stock price.
- ➔ Out of the money (OTM) call options with strike price above the stock price and out of the money put option with strike price below the stock price. In this case the premium of option is given entirely from extrinsic value.
- ➔ At the money (ATM) call or put options with a strike price equals or near to stock price.

1.2 DIRECTIONAL OPTION STRATEGIES

Directional option trade is a strategy whereby investors use to make money betting on the direction of the market. It will be showed the main directional strategies, starting from four basic options existing in the market: long call, short call, long put and short put.

For calls, an option contract on Game Stop shares, available on the market on January 21st and expiring on February 19th, will be examined. The strike price of this contract was 41$. It will be token in exam a lot size of options of 100[5].

LONG CALL is a strategy that must be devised when the investor is bullish on the market direction moving up in the short term. Buying or going long on a call option is the simplest way to benefit if the investor believes that the market will make an upward move. In this case the buyer will benefit if the underlying price increase.

Investor View: Bullish on the Stock.

Reward: Unlimited.

Risk: Limited to the premium paid.

Breakeven: Strike price + premium paid.

Strategy	Stock	Type	Strike	Premium Outflow
Buy call	GME	Buy call	41	8,6

[5] Bloomberg Data

Market price of stock	Net payoff
0	-860
10	-860
20	-860
30	-860
40,59	-860
41	-860
49,6	0
60	1040
70	2040
80	3040
90	4040
100	5040
200	15040
347,52	29792

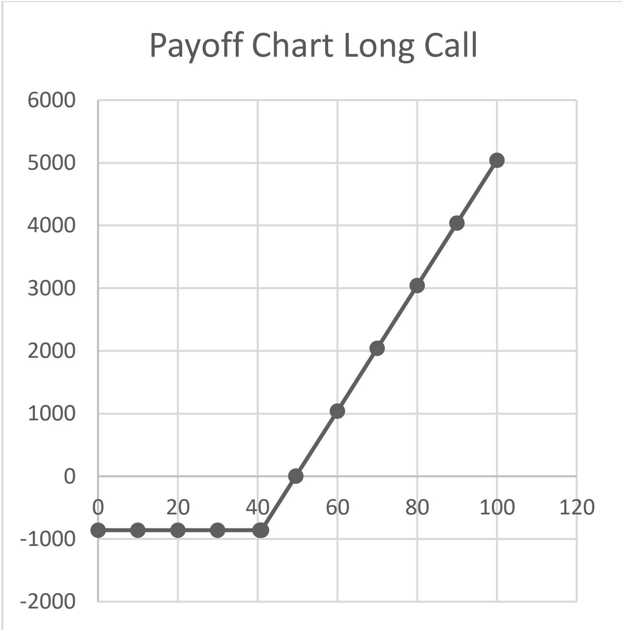


Chart 1.1[6]

SHORT CALL is a strategy that must be formulated when the investor is bearish on the market. On a selling a call, the investor earns a premium (from the buyer of the call). This position offers limited profit potential and the possibility of large losses on big advances in underlying prices. Although easy to execute, it is a risky strategy since the seller of the call is exposed to unlimited risk.

Investor view: Very bearish on the stock.

Reward: Limited to the premium received.

Risk: unlimited.

[6] In the above chart, the breakeven happens the moment GME crosses 49,6 and risk is limited to a maximum of -8,6*100 (premium paid*lot size), because if market price of GME will be lower than strike price, the option's buyer will not exercise the option.

Breakeven: Strike price + premium received.

Strategy	Stock	Type	Strike	Premium Inflow
Sell call	GME	Sell call	41	8,6

Market price of stock	Net payoff
0	860
10	860
20	860
30	860
40,59	860
41	860
49,6	0
60	-1040
70	-2040
80	-3040
90	-4040
100	-5040
200	-15040
347,52	-29792

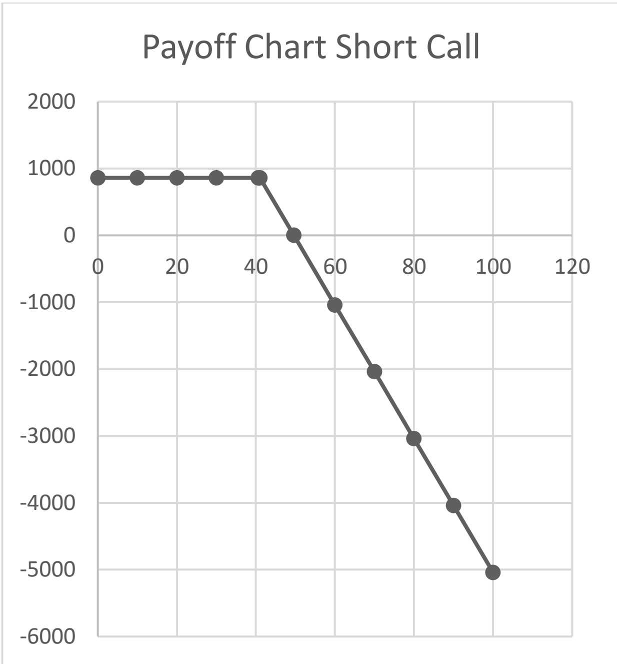


Chart 1.2[7]

[7] In the above chart, the breakeven happens the moment GME crosses 49,6 and risk is unlimited. It is important to note that regardless of how much the market falls, the reward is limited to 8,6*100 only.

For puts, an option contract on Game Stop shares, available on the market on January 21st and expiring on February 19th, will be examined. The strike price of this contract was 41$, but the premium paid was not equal than calls one. Lot options size of 100 will be analyzed.

LONG PUT is a strategy that must be implemented when the investor is bearish on the market direction going down in short-term. Buying put option gives the buyer a right to sell the stock (to the put seller) at a pre-specified price and thereby limit his risk. “Being Long” on a put option means the investor will benefit if the underlying stock/index drops. However, the risk is limited on the upside if the underlying stock/index rallies.

Investor view: bearish on the stock/index.

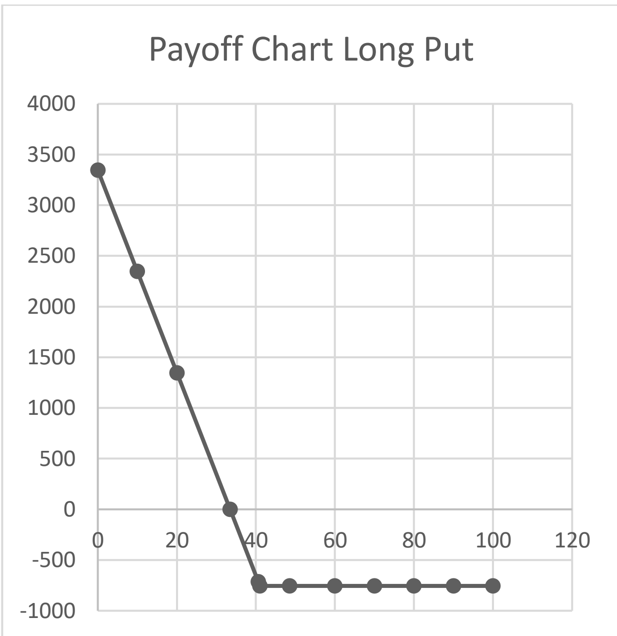
Reward: limited to strike price-premium paid.

Risk: limited to the premium paid.

Breakeven: strike price-premium paid.

Strategy	Stock	Type	Strike	Premium Outflow
Buy put	GME	Buy put	41	7,55

Market price of stock	Net payoff
0	3345
10	2345
20	1345
33,45	0
40,59	-714
41	-755
48,55	-755



60	-755
70	-755
80	-755
90	-755
100	-755
200	-755
347,52	-755

Chart 1.3[8]

SHORT PUT is a strategy that must be developed when the investor is bullish on the market direction and expects the stock price to rise or stay sideways at the minimum. When investor sells a put, he earns a premium (from the buyer of the put). If the underlying price increases beyond the strike price, the short put position will make a profit for the seller by the amount of the premium. But, if the price decreases below the strike price, by more than the amount of the premium, the put seller will lose money.

Investor View: Very bearish on the stock.

Reward: Limited to the premium received.

Risk: Limited to strike price-premium received.

Breakeven: Strike price + premium received.

Strategy	Stock	Type	Strike	Premium Inflow
Sell put	GME	Sell put	41	7,55

[8]In the above chart, the breakeven happens the moment GME crosses 33,5 and risk is limited to 755. It is important to note that the maximum reward is 3345.

Market price of stock	Net payoff
0	-3345
10	-2345
20	-1345
33,45	0
40,59	714
41	755
48,55	755
60	755
70	755
80	755
90	755
100	755
200	755
347,52	755

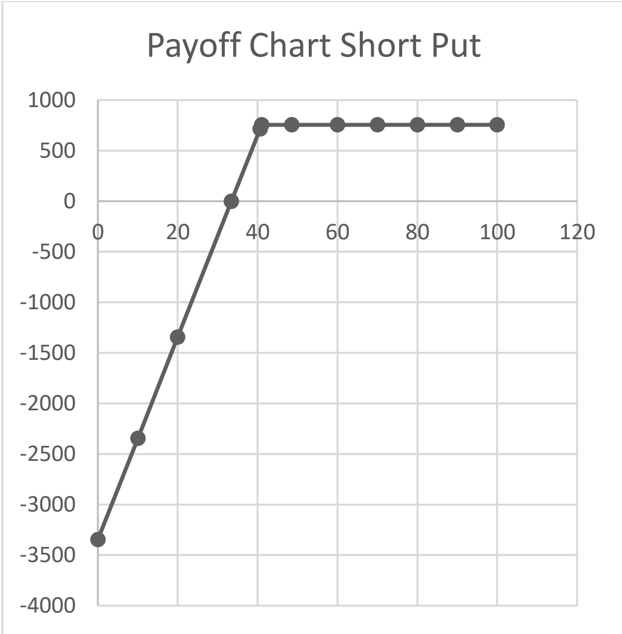


Chart 1.4[9]

It is possible develop directional strategies using more than one option contract, as in the case of “bull strategy” and “bear strategy”. Showing these strategies: a call option on Game Stop shares, in the money, with strike price of 41$ and a call option on Game stop shares, out of the money, with

[9] In the above chart, the breakeven happens the moment GME crosses 33,45 and risk is limited to the difference between strike price and premium received. It is important to note that irrespective of how much the market gains, the reward is limited to 7,55*100 only.

strike price of 45$, will be analyzed. The expiry of these two calls is the same and the market price of Game Stop on January 21st was 43,03$. A lot size of 100 options is considered for both options.

BULL SPREAD is a strategy that must be devised when the investor is moderately bullish on the market direction going up in the short-term. A Bull Call Spread is formed by buying an "In-the-Money Call option" (lower strike) and selling an "out of the-money Call option" (higher strike). Both the call options must have the same underlying security and expiration month.

The net effect of the strategy is to bring down the cost and breakeven on a buy Call (long Call) strategy. The investor will benefit if the stock rallies. However, the risk is limited on the downside if the underlying stock make a correction.

Investor view: moderately bullish on the stock.

Reward: limited.

Risk: limited to the net premium paid.

Breakeven: Strike price of Buy Call + net debt (1,6*100).

Strategy	Stock	Type	Strike	Premium Outflow	Premium Inflow
Bull Call	GME	Long call (In the money)	41	8,6	
spread	GME	Short Call (Out of the money)	45		7

Market price of stock	Net payoff
0	-160
10	-160
20	-160
30	-160
40,59	-160
41	-160
42,6	-160
45	240
60	240
80	240
90	240
100	240
200	240
347,52	240

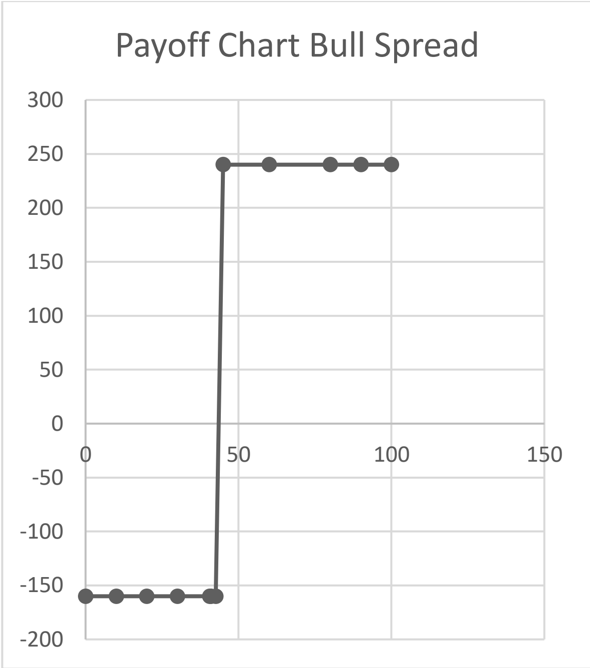


Chart 1.5[10]

[10] In this chart, the breakeven verifies when GME crosses 42,6 and risk is limited to a maximum of net debt*size. The maximum profit of a bull call spread is the difference between the spread's maximum potential value and what the investor pays for it (Net debt).

Bull spread strategy can be formed even buying an “out of the money put option” (lower strike) and selling an “in the money put option” (higher strike). Both put options must have the same underlying security and expiration date.

The concept is to protect the downside of a put sold by buying a lower strike put, which acts as insurance for the put sold. The chart of net profit is the same of payoff bull call spread.

BEAR SPREAD strategy is implemented when the investor is moderately bearish on the market direction and is expecting the underlying to fall in the short-term. A bear call spread is formed by buying an “out of the money call option” (higher strike) and selling an “in the money call option” (lower strike). Both call options must have the same underlying security and expiration date. The investor receives a net credit because the call bought is of a higher strike price than the call sold. So, here the concept is to protect the downside of a call sold by buying a call of a higher strike price to secure the call sold.

Investor view: moderately bearish on stock.

Reward: limited to the net premium received.

Risk: limited.

Breakeven: strike price of short call + net premium received.

Strategy	Stock	Type	Strike	Premium Outflow	Premium Inflow
Bear Call	GME	Short call (In the money)	41		8,6
spread	GME	Long call (Out of the money)	45	7	

Market price of stock	Net payoff
0	160
10	160
20	160
30	160
40,59	160
41	160
42,6	0
45	-240
60	-240
80	-240
90	-240
100	-240
200	-240
347,52	-240

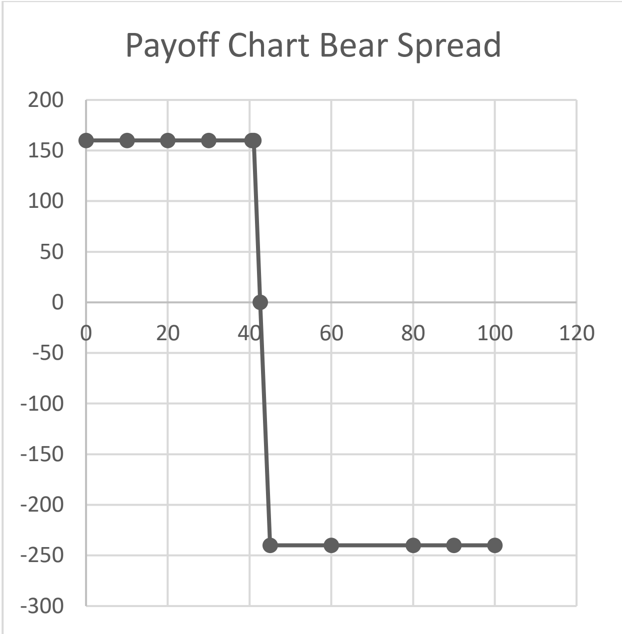


Chart 1.6[11]

Bear spread strategy can be developed even buying an "in the money put option" (higher strike) and selling an "out of the money put option" (higher strike). Both put options must have the same underlying security and expiration date. The investor will pay a net premium because the put bought has higher strike price than the put sold. The net effect of the strategy is to bring down the cost and

[11] In this chart, the breakeven happens the moment GME crosses 42,6 and risk is limited to a maximum of 2,4*100.

raise the breakeven on buying a put (long put). The chart of net profit is the same of payoff bear call spread.

1.3 NON-DIRECTIONAL OPTIONS STARTEGIES

Non directional strategy means a combination of options capable of making a pay-off that is indifferent to which direction the underlying is going to go.

Non-directional strategies are bets about the volatility of the underlying asset. Long Straddle and Long Strangle strategies, non-directional options type, are used by investors if they believe there will be high volatility with asset prices. However, if they believe there will be low volatility, then they will create a Butterfly Spread or Short Straddle/Strangle.

All strategies are created by using either call or put options. Initially, it will be analyzed the Straddle and Strangle strategy, through use of two specific call and put contracts. Over straddle options, for call, an option contract on Game Stop shares, available on the market on January 26th and expiring on February 19th, will be examined. The strike price of this contract was 150$. For put options the contract analyzed is the same, with different premium. It will be token in exam lot size of options size of 100[12].

LONG STRADDLE

Straddle strategy involves buying a call and put option with the same strike price and expiration date. A straddle is appropriate when an investor is expecting a large move in a stock price but does not know in which direction the move will be. Profits can be made in either direction if the underlying shows volatility to cover the cost of the trade. Loss is limited to the premium paid in buying options.

Investor View: Neutral direction but expecting significant volatility in underlying movement.

Reward: Unlimited

Risk: Limited to the premium paid.

[12] Bloomberg Data (The option contract under analysis has been changed as the underlying volatility was higher on January 26th than January 21st.

Lower Breakeven: Strike Price – net premium paid.

Higher Breakeven: Strike Price + net premium paid.

Strategy	Stock	Type	Strike	Premium Outflow
Long	GME	Long Call	150	54
Straddle	GME	Long Put	150	69,9

Current Market Price	Long Straddle payoff
0	2610
10	1610
26,1	0
40,59	-1449
100	-7390
150	-12390
200	-7390
250	-2390
273,9	0
300	2610
347,52	7362

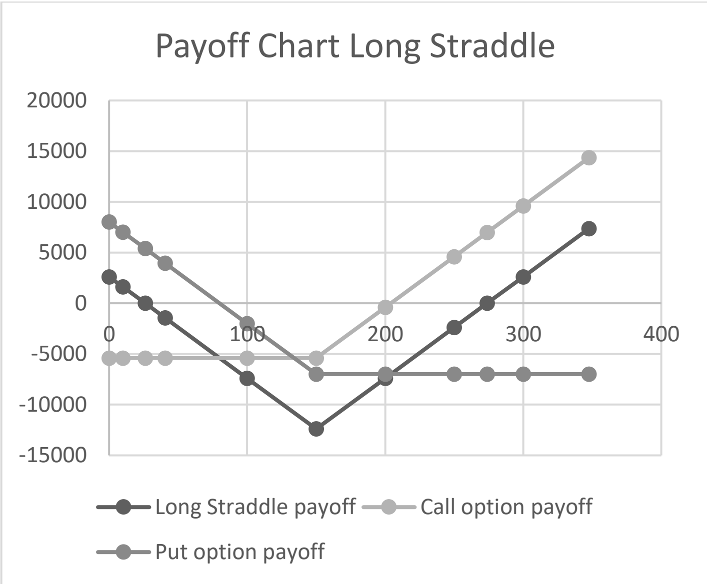


Chart 1.7[13]

[13] In this chart, the breakeven happens the moment GME crosses 26,1 or 273,9 and risk is limited to 12390 (premium paid*lot size) when the current market price is equal to exercise price. Here it is important to note that the premium is calculated as the sum of premium paid for the call and put option.

SHORT STRADDLE

Sell straddle is the opposite of buy straddle. It is used when the investor is expecting underlying to show no large movement. Investor expects the underlying to show little volatility upside or downside.

This strategy involves selling a call as well put on the same underlying for the same maturity and strike price. It creates a net income for the investor. If the underlying does not move much in either direction, the investor retains the premium as neither the call not the put will be exercised. However, in case the underlying moves in either direction, up or down significantly, the investor's loss can be unlimited.

Investor View: neutral direction but expecting little volatility in underlying movement.

Reward: limited to the premium received.

Risk: unlimited.

Lower breakeven: strike price – net premium received.

Higher breakeven: strike price + net premium received.

Strategy	Stock	Type	Strike	Premium Inflow
Short	GME	Short Call	150	54
Straddle	GME	Short Put	150	69,9

Current market price	Short straddle payoff
0	-2610
10	-1610
26,1	0
40,59	1449
100	7390
150	12390
200	7390
250	2390
273,9	0
300	-2610
347,52	-7362

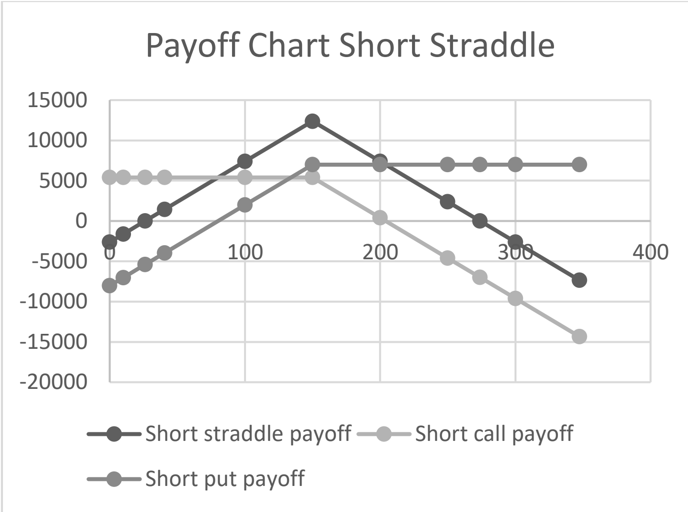


Chart 1.8[14]

LONG STRANGLE

A strangle is similar strategy to a straddle. The investor is betting that there will be a large move in price but is uncertain whether it will be an increase or a decrease.

In this case an investor buys a put and call with the same expiration date and different strike price (higher for call option), in such a way that they are both out of the money. The profit pattern obtained with a strangle depends on how close together the strike prices are. The farther they are apart, the less the downside risk and the farther the stock price should move for a profit to be realized.

[14] In this chart, the breakeven happens the moment GME crosses 26,1 and 273,9 and reward is limited to a maximum of 12390 (premium paid*lot size). Here it is important to note that the premium is calculated as the sum of premium for the call and put option. The risk in such strategy is unlimited.

In this case, for call, an option contract on Game Stop shares, available on the market on January 26th and expiring on February 19th, will be examined. The strike price of this contract was 160$. For put options the contract analyzed is the same, with different premium and different strike price, equal to 140$. It will be considered a lot size of options of 100[15].

Investor view: neutral on direction but bullish on volatility of the stock.

Reward: unlimited.

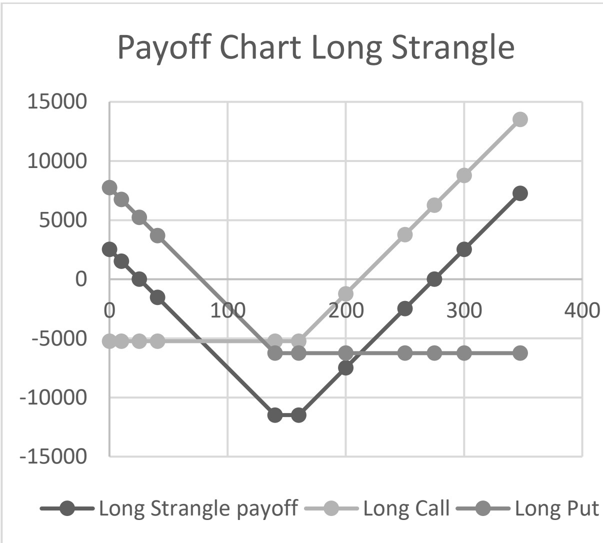
Risk: limited to premium paid.

Upper breakeven: buy call strike price + net premium paid.

Lower breakeven: buy put strike price – net premium paid.

Strategy	Stock	Type	Strike	Premium Outflow
Long	GME	Long Call	160	52,35
Strangle	GME	Long Put	140	62,5

Current Market Price	Long Strangle payoff
0	2515
10	1515
25,15	0
40,59	-1544
140	-11485
160	-11485
200	-7485
250	-2485
274,85	0



15 Bloomberg Data

300	2515
347,52	7267

Chart 1.9[16]

Comparing chart of long strangle with long straddle, it is possible to note how the stock price should move farther in a strangle than in a straddle for the investor, to make a profit. However, the downside risk, if the stock price ends up at a central value, is less with a strangle.

SHORT STRANGLE

Short strangle is a strategy to be used when the investor is neutral on the market direction and bearish on volatility, expecting markets to trade in a narrow range.

This strategy involves selling an out of the money call option and selling an out of the money put option. Both options must have the same underlying security and expiration date. Short strangle is a slight modification to the short straddle. The profit payoff region is much wider as compared to short straddle.

Investor View: neutral on direction and bearish on volatility of the stock.

Reward: limited to net premium received.

Risk: unlimited.

Upper breakeven: short call strike price + net premium received.

Lower breakeven short put strike price – net premium received.

Strategy	Stock	Type	Strike	Premium Inflow
Short	GME	Short Call	160	52,35
Strangle	GME	Short Put	140	62,5

[16] In this chart, the breakeven happens the moment GME crosses 25,15 and 274,85 and risk is limited to 11485 (calculated as lot size* premium paid). Here it is important to note that the premium is calculated as the sum of premium paid for a call and for a put option.

Current Market Price	Short Strangle payoff
0	-2515
10	-1515
25,15	0
40,59	1544
140	11485
160	11485
200	7485
250	2485
274,85	0
300	-2515
347,52	-7267

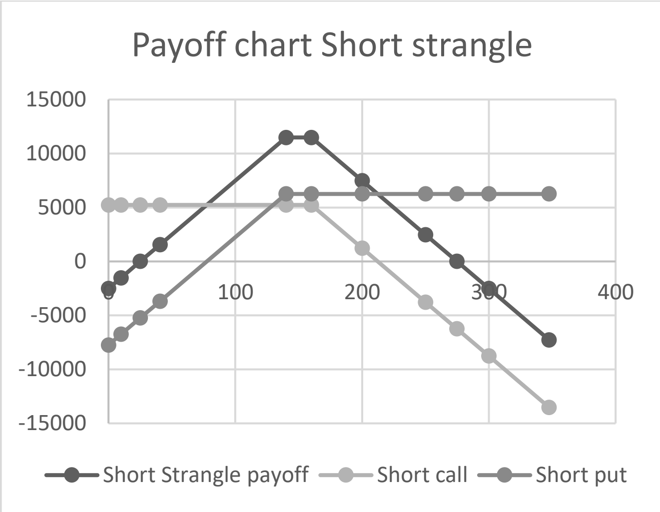


Chart 1.10[17]

LONG CALL BUTTERFLY is a strategy that must be devised when the investor is neutral on the market direction and expects volatility to be less in the market.

A long call butterfly strategy is formed by selling two at-the-money call options, buying on out of the money call option and one in the money call option. This strategy is like to short straddle except that here the investor's losses are limited. The investor will benefit if the underlying stock remains at the middle strike at expiration.

Investor view: neutral on direction and bearish on stock volatility

Reward: limited

[17] In this chart, the breakeven happens the moment GME crosses 25,15 and 274,85. Here it is important to note that the premium is calculated as the sum of premium received for the call and put option. The risk in such strategy is unlimited.

Risk: limited to the premium paid.

Lower breakeven: strike price of lower strike long call + net premium paid.

Higher breakeven: strike price of higher strike long call – net premium paid.

Strategy	Stock	Type	Strike	Premium outflow	Premium Inflow
LONG	GME	Long call	140	57,6	
CALL	GME	Short call (Lot*2)	150		54
BUTTERFLY	GME	Long call	160	52,35	

Current market price	Long butterfly playoff
0	-195
10	-195
40,59	-195
100	-195
140	-195
141,95	0
150	805
158,05	0
160	-195
200	-195
250	-195
300	-195
347,52	-195

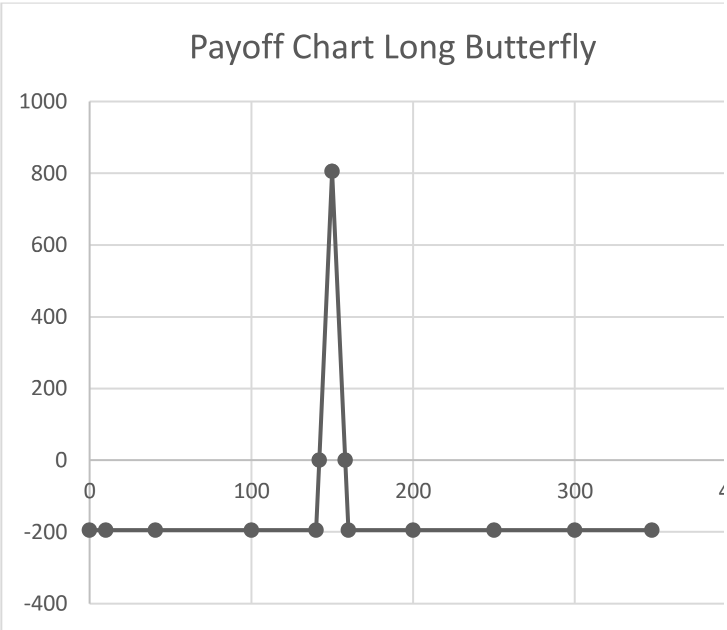


Chart 1.11[18]

Same strategy con be constructed using put options selling 2 put options in the money and buying one put option in the money e one put option out of the money.

[18] In this chart, the breakeven happens the moment GME crosses 141,95 and 158,05. The reward is limited to 805 and risk is limited to 195.

1.4 AMERICAN VERSUS EUROPEAN OPTIONS

In the first paragraph it was stated how options can be of two different styles: European and American.

Now is important to analyze, firstly the main differences and then, the distinct pricing models based on options style. Formally European options are usually traded over the counter (OTC) and settle in cash instead American Option settle through stock delivery.

A European option can be exercised only at the expiration date, whereas the American option can be exercised always before the expiration date, when the option holder desires. This added flexibility of American options increases their value over European options, in certain situations. Thus, American options value is equal to European options value plus a premium, where the premium is greater or equal to zero.

Nonetheless American call options are usually exercised solely when they are deep in the money with asset's price very much higher than the strike price. This happens because, for standard American call options without dividends, there are several reasons why the call should never be exercised before the expiration date.

Firstly, there is an intrinsic time value of the option that would be lost by exercising the option prior the expiration date. The call option, however, has the added benefit of protecting against the risk of a downward price movement below the strike price.

Additionally, because of the time value of the money, it costs more to exercise the option today at fixed strike price K than in the future at K.

Finally, for a given movement in an underlying asset, the profit from holding an in the money call is equivalent to the profit from holding the underlying asset.

On the other hand, the price of an American call option on an underlying asset that pays dividends, however, may diverge from its European counterpart. For an American call embedded with dividends it may be beneficial to exercise the option prior the expiration. By exercising the call, the owner of the call will be entitled to dividend payments that they would not have otherwise received.

This means that prior to dividend announcement American options must include a premium based on some distribution of an expected dividend. Once the dividend is announced, the premium must be adjusted again to account for this revised information.

Account for the premium added by an American option, it is necessary to consider time as a discrete variable. At one time step before maturity *(T-1*), the option will be exercised if *h(T-1) > f(T-1)*, where h is the value of exercising the option and f is the value of holding the option until the maturity. Hence, the price of the option is max *(h(T-1), f(T-1))*.

Since American option can be exercised at any time, the risk is higher, whereas a European option that can only be exercised on a particular future date has less risk and formulating a hedging strategy in this case is easier because the option holder can exercise the contract only at a pre-determined date.

1.5 FACTORS AFFECTING OPTIONS

Stock option price is affected by six factors: current stock price (St), strike price (K), time to expiration (T), volatility of the stock price (σ), risk-free rate (r), and dividends that are expected to be paid. It is possible to observe what happens to GME option when there is a change to one of these factors, with all the other remaining fixed.

Variable	*European Call*	*European Put*	*American Call*	*American Put*
Current stock price	+	-	+	-
Strike price	-	+	-	+
Time to expiration	?	?	+	+
Volatility	+	+	+	+
Risk-free rate	+	-	+	-
Future Dividends	-	+	-	+

Table 1.1[19]

➔ STOCK PRICE AND STRIKE PRICE

Call options become more valuable as the stock price increases and less valuable as the strike price decreases. Picking two GME call options valued on January 22nd equal, with just different strike price, it is observable that first one with K=55 is priced 20,75$, and second one with K=75 is priced 13,8$. Put options become more valuable as the stock price decreases and more valuable as the

[19] + indicates that an increase in the variable causes the option price to increase or remains equal.
- indicates that an increase in the variable causes the option price to decrease or remains equal.
? indicates that the relationship is uncertain.

strike price increases. Considering two GME put options valued on January 22nd equal with just different strike price. First one with K=55 is priced 14$ and second one with K=75 is priced 28,14$.

➔ TIME TO EXPIRATION

In this case both put and call American options become more valuable (or at least do not decrease in value) as time to expiration increases.

Consider two GME options, valued on January 22nd that differ only as far as the expiration date is concerned, one expires on February 19th and other one expires on 16th July. In case of call options, first one is priced 20,75$ and second one 27,5$. The same effect occurs in put options case, first one is valued 14$ and other one 24,55$. This is given by the fact that the owner of the long-life option has all the exercise opportunities open to the owner of short-life option.

In case of European options, it is not possible say that higher time left before the expire affects positively the option price. Indeed, considering two European call options on a stock: one with an expiration date in 1 month, the other with an expiration date in 1 year. Suppose that is expected a very large dividend in 6 months. The dividend will cause the stock price to decline, so that the short-life option could be worth more than the long-life option.

➔ VOLATILITY

As volatility increases, the possibilities that the stock will increase or decrease a lot, increases. For the owner of the stock, these two outcomes tend to offset each other. However, this is not so for the owner of a call or put. The owner of a put benefits from price decrease but has limited downside risk in the event of price increases because the maximum amount that owner can lose is the price of the option.

Similarly, the owner of a call benefits from price increases but has limited downside risk in the event of price decreases. This because the “Option value”, concept associated with the limit to the downside risk to the investor who owns the option. Hence the values of both calls and puts therefore increases as volatility increases.

➔ RISK FREE RATE

The risk-free rate affects the price of call option positively and price of put option negatively. This happens for two reasons; first, as interest rates in the economy increase, the expected return required by investors from the stock tends to increase.

Second reason is the following: assuming the natural hedging for call options, in this case owner hedges position selling the stock and, if interest rate is high, he earns more putting money in the bank.

On the other hand, if interest rate goes down, option's owner earns less hedging the call so becomes less valuable. Instead, in put options case, if interest rate goes up option's owner pays more hedging his position and put becomes less valuable. The opposite if interest rate goes down.

➔ DIVIDENDS

Dividends affect option prices through their effect on the underlying stock price. Because the stock price is expected to drop by the amount of the dividend on the ex-dividend rate. Hence the value of the option is negatively related to the size of the dividend if the option is a call and positively related to the size of the dividend if the option is a put.

1.6 PUT-CALL PARITY

Every option price, on no dividend-paying stock, has a lower and upper bound.

Whether option price is above the upper bound or below the lower bound, then there are profitable opportunities for arbitrage.

An American call option gives the holder the right to buy one share of a stock for a certain price. Regardless the stock movements, the option can never be worth more than the stock. Hence, the stock price is an upper bound to the option price: $C \leq S$, if relationship is not true, an arbitrageur could easily make a riskless profit by buying the stock and selling the call option.

Same concept is related also to put option price. An American put option gives the holder the right to sell one share of a stock for K (strike price). No matter how low the stock price becomes, the option can never be worth more than K. Hence $P \leq K$.

For the European call option, the upper bound is the same of American call option, so, also in this case $C \leq S$, instead for the European put option the upper bound is different from American option. In this case at maturity the option cannot be worth more than K, hence it cannot be worth more than the present value of K today: $P \leq Ke^{-r(T-t)}$.

If not, an arbitrageur could make a riskless profit by writing the option and investing the proceeds of the sale at the risk-free interest rate.

On the other hand, lower bound, in absence of arbitrage opportunities, is $C \geq S - Ke^{-r(T-t)}$ related to European call option and $P \geq Ke^{-r(T-t)} - S$ for European put option.

Starting from this assumption, it is possible to analyze the value of two different portfolios:

- Portfolio A: one European call option, plus a zero-coupon bond worth K at time T.
- Portfolio B: one European put option plus one share of the stock.

Assuming also that options in portfolios are no dividend-paying options and that call and put have the same strike price K and the same time to maturity T.

- If $S(T) > K$, both portfolios are worth S(T) at time T.

- if $S(T) < K$, both portfolios are worth K at time T.

  These option contracts are European, so they cannot be exercised prior to time T. Since the portfolios have identical values at time T, they must have identical values today.

  If it is not the case, an arbitrageur could buy the less expensive portfolio and sell the more expensive one. The components of portfolio A are worth C and $Ke^{-r(T-t)}$ today, and the components of portfolio C are worth P and S today. Hence:

  $$c + Ke^{-rT} = p + S_0 .$$

  This relationship is called Put-Call Parity and holds only for European options. It is possible to derive some results also for American option prices. When there are options contract on no dividends stock, difference between call option and put option price has the range:

  $$S_0 - K \leqslant C - P \leqslant S_0 - Ke^{-rT} .$$

  These equations give the possibility to construct long and short positions holding just options contract in the portfolio.

1.7 SYNTHETIC POSITIONS

All corporate securities could be interpreted as portfolios of puts and calls on the assets of the firm. For example, consider a firm with a single liability of a homogeneous class of pure discount bonds. The stockholders then have a call on the assets of the firm which they can choose to exercise at the maturity date of the debt by paying its principal to the bondholders.

In turn, the bonds can be interpreted as a portfolio containing a default-free loan with the same face value as the bonds and short position in a put on the assets of the firm. This is an example of synthetic position, namely a trading option used to simulate the features of another comparable position. More specifically, a synthetic position is created to pretend a similar reward or risk profile as that of a comparable position. A synthetic position is obtained combining different contracts or options to match a short or long position on the stock. Following it will show how construct synthetic positions using Game Stop options analyzed in advance. In the following positions will not include transaction costs (commission, margin interest, fees) or tax implications.

- SYNTHETIC LONG POSITION

Synthetic long stock is constructed buying at-the-money strike calls and selling the same expiration at-the-money strike puts. However, different strikes may be used as well.

Market price of stock	Payoff Synthetic Long Position
0	-4205
10	-3205
20	-2205
33,45	-860
40,59	-146
41	-105
49,6	755
60	1795
70	2795
80	3795
90	4795
100	5795
200	15795
347,52	30547

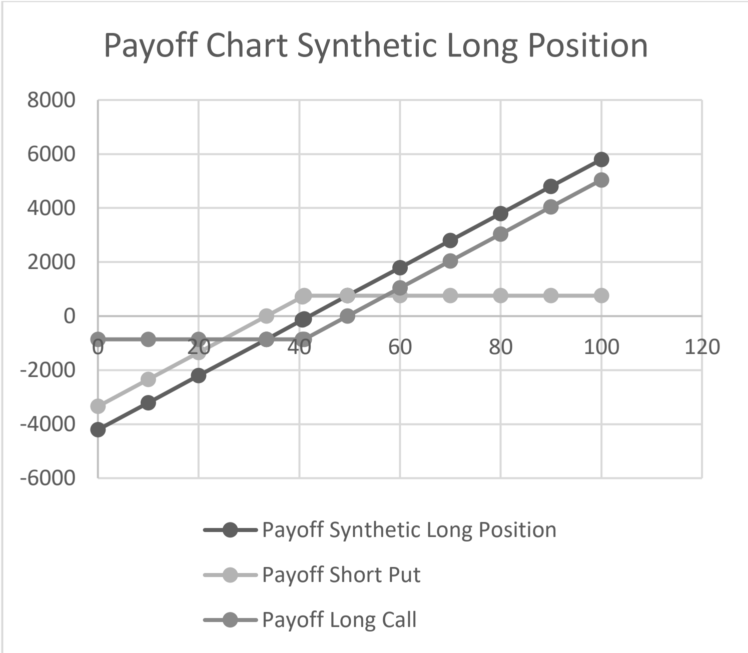


Chart 1.12

- SYNTHETIC SHORT POSITION

Synthetic short position is constructed shorting at the money call option and buying at the money put option with same strike price. Once again, different strikes may be used.

Market price of stock	Payoff Short Synthetic position
0	4205
10	3205
20	2205
33,45	860
40,59	146
41	105
49,6	-755
60	-1795
70	-2795
80	-3795
90	-4795
100	-5795
200	-15795
347,52	-30547

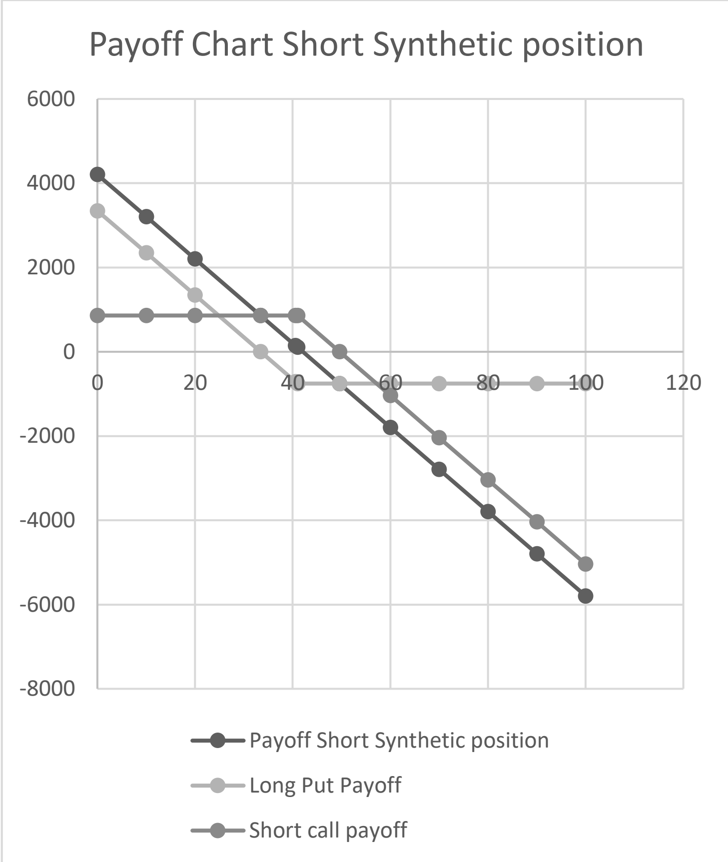


Chart 1.13

In addition, synthetic position can use a series of options stock or contracts to simulate a standard options trading approach. Hence it is possible construct a long call without holds it directly from market maker.

SYNTHETIC LONG CALL: An investor or trader holding long stock can hedge against downside risk by buying a put on the same stock. This position is also called “protective put” and is typically used when an investor is still bullish on a stock but wishes to hedge against potential losses and uncertainty. The protective put sets a known floor price below which the investor will not continue to lose any added money even as the underlying asset’s price continues to fall.

SYNTHETIC SHORT PUT: An investor, holding a long position in an asset, after sells call options on the same asset to generate an income stream. This strategy is also called “covered call” because the investor’s long position in the asset is the “cover” because it means the seller of stock can deliver the shares if the buyer of the call option chooses to exercise.

SYNTHETIC LONG PUT: An investor holding a short position in an asset, can hedge against a really in the stock price buy buying a call. In this case the investor sets a floor price above which the investor does not continue to lose any added money even if the underlying asset continues to raise.

SYNTHETIC SHORT CALL: An investor, holding a short stock position then shorts put option on the same asset to generate an income stream. The idea is to reduce the losses if the price of the stock goes up.

2 OPTION PRICING MODELS

Option pricing theory estimates the value of an options contract by assigning a price, also called premium, based on the calculated probability that the contract will finish in the money at expiration. Essentially, option pricing theory provides an evaluation of an option's fair value, which traders incorporate into their strategies.

Models used to price options account for variables such as current market price, strike price, volatility, interest rate, and time to expiration to theoretically value an option. Commonly used models to value options are Black-Scholes-Merton, Binomial Option Pricing and Monte-Carlo simulation.

Option pricing theory has a long and illustrious history, but it also underwent a revolutionary change in 1973. At that time, Fischer Black and Myron Scholes presented the first completely satisfactory equilibrium option pricing model. In the same year, Robert Merton extended their model in several important way. The problem was that the mathematical tools employed in the Black-Scholes and Merton articles are quite advanced and have tended to obscure the underlying economics[20]. Later, thanks to a suggestion by William Sharpe, it was possible to derive the same results using only elementary mathematics.

[20] John Cox, Stephen A. Ross, Mark Rubinstein "Option pricing: a simplified approach" Journal of Financial Economics 1979.

2.1 BINOMIAL OPTION PRICING MODEL

Binomial option pricing model present a simple discrete-time option pricing formula by which the model follows an iterative method to evaluate each period, considering either an up or down movement and the respective probabilities.

The model creates a binomial distribution of possible stock prices. Model assumes no-arbitrage, meaning there is no buying or selling at a higher price. Having no-arbitrage ensures the value of the asset remains the same, which is a requirement for the Binomial Pricing model to work. This model is flexible, so it is possible to adapt it also to American style options and to options related to discrete time dividend stocks respect other pricing option models, like Black-Scholes and Merton, that is more restricted respect its assumptions.

When setting up a binomial pricing model, it is fundamental to be aware of the underlying assumptions, to also understand the limitations of this approach better:

- At every point in time, the price can go to only two possible new prices, one up and one down.
- The interest rate is a constant throughout the period.
- The market is frictionless, and there are no transaction costs and no taxes.
- Investors are risk-neutral, indifferent to risk.
- The risk-free rate remains constant.

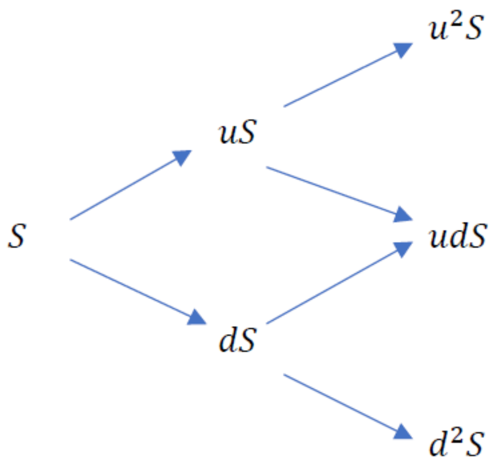
Particularly, the risk-neutral assumption is based on the probability that the stock price would rise in a risk-neutral world. Hence in the model as will see also in the Black-Scholes-Merton model, the equations do not involve any variables that are affected by the risk preferences of investors. This concept presumes that current value of an asset is equal to its expected payoff discounted at the risk-free rate and that there are no arbitrage opportunities in the market.

The price follows a multiplicative binomial process over discrete time. The rate of return on the stock over each period can have two possible values: μ-1, where $\mu=exp(\sigma\sqrt{\Delta t})$, with probability p[21], or d-1, where $d=1/\mu$ with probability 1-p.

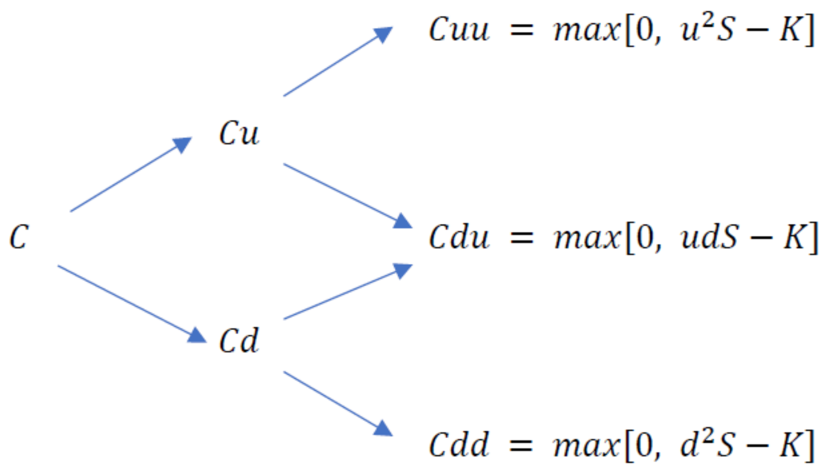
To circumvent arbitrage opportunities between risk free rate asset and risky one, a necessary and sufficient relation must hold: $d < R < \mu$.

Where $R = e^{Rf\,\Delta T}$.

At this point, if the current stock price is S, the stock price at the end of the period will be either μS or dS.



Then the evolution of option call price is given by:



[21] Probability $p=(R-d)/(\mu-d)$

Binomial Model has numerous notable futures; even if different investors have different subjective probabilities about an upward or downward movement in the stock, they could still agree on the relationship of C to S, μ, d and r. Second, the value of the call does not depend on investors' attitudes toward the risk and lastly, the only random variable on which the call value depends is the stock price itself.

In the chart below is illustrated a valuation, through binomial pricing model, of Game Stop call and put options, either with maturity on 19th of February, strike price equals to 55$ and time expiration of 28 days. The same characterized Game Stop options will be used in the following valuation models.

		STOCK PRICE EVOLUTION							
Binomial pricing model		0	1	2	3				
					78,3789176				
Stock Price(S)	43,03			64,17855824					
Strike/Exercise Price (K)	55		52,55095966		52,5509597				
Time to maturity (T)	0,076712	43,03		43,03					
Risk free rate (rf)	0,01		35,23400737		35,2340074				
Volatility (σ)	1,25			28,85045957					
Number of steps (n)	3				23,6234558				
Option type (Call or Put)									
		EUROPEAN CALL PRICE				AMERICAN CALL PRICE			
		0	1	2	3	0	1	2	3
Up factor (u)	1,221263				23,3789176				23,3789176
Down factor (d)	0,818824			11				11	
			5		0		5		0
dt(incremental time step)	0,025571	2,141309		0		2,141309		0	
			0		0		0		0
Risk neutral probability (p)	0,450893			0				0	
1-p	0,549107				0				0
		EUROPEAN PUT PRICE				AMERICAN PUT OPTION			
		0	1	2	3	0	1	2	3
					0				0
				1,344406256				1,344406256	
			7,168480697		2,44904034		7,17697197		2,44904034
		14,06492		11,95453186		14,08573		11,97	
			19,73506069		19,7659926		19,76599263		19,7659926
				26,13407228				26,14954043	
					31,3765442				31,3765442

It is possible compute the options price also through MATLAB.

```
NumPeriods = 3
Settle = "22-January-2021"
Maturity = "19-Febryary-2021"
vol = 1.25
S = 43.03
T=28/365
dt=T/NumPeriods
St = S*exp(vol*sqrt(dt))
r = 0.011
K = 55
CRRTimeSpec = crrtimespec(Settle,Maturity,NumPeriods)
StockSpec = stockspec(vol,S)
RateSpec = intenvset("Rate",r)
CRRTree = crrtree(StockSpec,RateSpec,CRRTimeSpec)
StockSpecATM=stockspec(vol,St)
CRRTreeATM = crrtree(StockSpecATM,RateSpec,CRRTimeSpec)
PriceCRRCall = optstockbycrr(CRRTree,"CALL",K,Settle,Maturity,0)
PriceCRRPut = optstockbycrr(CRRTree,"PUT",K,Settle,Maturity,0)

PriceCRRCall =

    2.1274


PriceCRRPut =

   14.0565
```

To switch this model to treat American options, it is necessary adapt just one adjustment respect the case of European option.

This adjustment is related to the treatment of all interim nodes: they are equal to the maximum expected discounted value or intrinsic value. The logic behind is given by the fact that the option can be exercised after one period and if the intrinsic value is greater than the expected discounted value, then rational agent would exercise the option.

The price of American calls will be always equal to European calls, as explained in the first chapter. Hence, in calls case, the expected discounted value of the options will be always greater than intrinsic value in the interim nodes.

While, in the put option case, the American put is more expensive than European one because we are pricing the possibility of early exercise of the option. In puts option case is possible to observe if interest rate increase, price of put option will decrease while the difference between value of American put option and European put option will be larger.

Finally, it is important to specify that whenever stock price movements conform to a discrete binomial process, or to a limiting form of a such process, options can be priced solely based on arbitrage considerations.

2.2 GREEKS

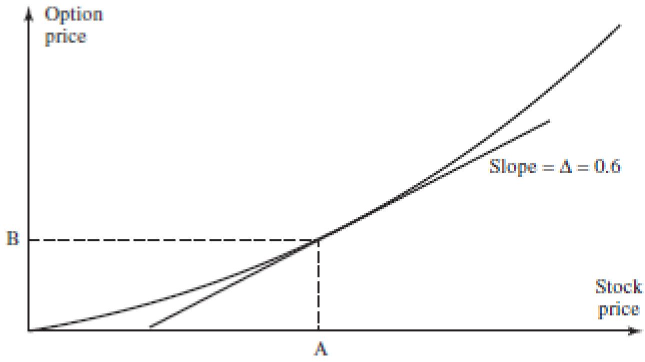
Operating in the option market a financial institution or a market maker, that sells an option, is faced with the problem of managing its risk. If the option happens to be the same as one that is traded on an exchange, the market maker can neutralize its exposure by buying on the exchange the same option as it has sold. But when the option has been tailored to the needs of a client, hedging the exposure is far more difficult.

"Greeks" represent an alternative approach to this problem and measure different dimensions to the risk in an option position. Follow it will be showed the different "Greeks" related to option's portfolio.

DELTA

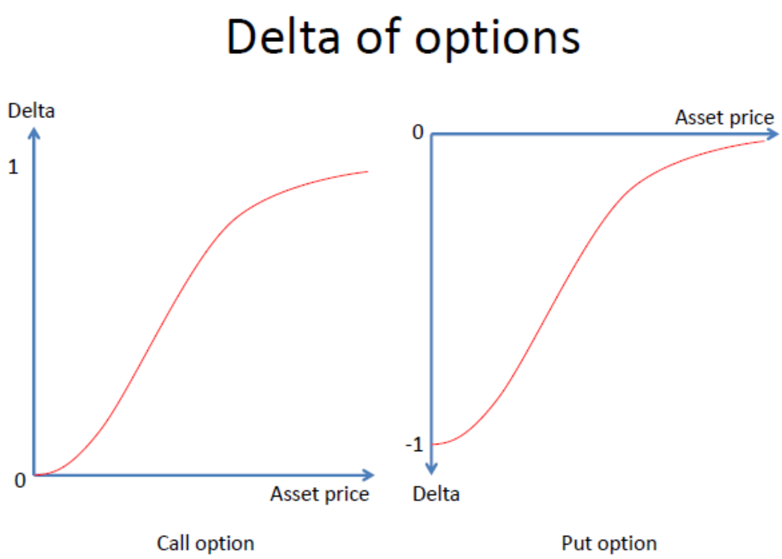
The delta (Δ) of an option's portfolio is defined as the rate of change of the portfolio value with respect to the price of the underlying asset: $\partial \Pi / \partial S$. Substantially it is the slope of the curve that relates the option price to the underlying asset price.

Suppose that the delta of a call option is 0,6. This means that when the stock price changes by a small amount, the option price changes by about 60% of that amount, so $\Delta = \partial c / \partial S$. Consider, now, a portfolio of 100 call options with Δ of 0,6. This imply that if the market price of the underlying asset falls by 1$ per unit, the overall decline in the value of portfolio would be 60$. Suppose that the investor who owns the portfolio shorts, so without owning it, 60 units of underlying asset. In this case the 60$ loss in the value of the call option is completely offset by shorting 60 units of the underlying asset, when 60 equals Δ times 100, the number of call options in the portfolio. This portfolio hedging procedure is also called "delta hedging".



Conversely, for a put option, the payoff either decreases as the market price of the underlying asset increases, or it remains unchanged. Hence, for a given change in the market price of the asset, the change in the price of a put option should either be zero or negative.

The concept is related also to concept of “Option Value” of the options, introduced in the first chapter of the thesis. Indeed, consider that the market price of the underlying asset increases by 1$. In that case, the intrinsic value of an in the money (American) call option increases by 1$ as well. However, since the market price of the asset already exceeds the strike price and has gone up further, there is a lower probability that this price would be below the strike price on or below the strike date. Hence, the option value of the call option decreases. The net impact of the 1$ change in the market price of the call option, therefore, is less than 1$, and so the upper limit of a call option’s delta is 1. Similarly, the lower limit of a put option’s delta is -1.



GAMMA

The gamma (Γ) of a portfolio of options on an underlying asset is the rate of change of the portfolio's delta with respect to the price of the underlying asset. It is the second partial derivative of the portfolio respect to asset price: $\Gamma=\partial^2\Pi/\partial S^2$. If gamma is small, delta changes slowly, and the adjustments to keep a portfolio delta neutral need to be made only relatively infrequently. However, if gamma is highly positive, delta is very sensitive to the price of the underlying asset. Then is relatively risky to leave a delta-neutral portfolio unchanged for any length of time.

The gamma of an option is large if the option is at the money, and it is close to expiration. For such an option, even a small change in the market price of the underlying asset can result in the option expiring in the money. Hence, the change in its delta on account of a change in the market price of the asset (its gamma) is large.

THETA

An option's theta (θ) is the rate of change of an option's portfolio value price respect to time, when everything else is unchanged: $\theta=\partial\Pi/\partial t$. It is known as the rate of time decay of the option. Presume that an option has a θ equal to -25, it means that if 0,01 years (approximately 2,5 trading days) pass without a change in the market price and volatility of the underlying asset, then the value of the option will decline by 0,25. An option's theta is generally negative because an option becomes less valuable as the expiration date approaches.

VEGA

An option's portfolio Vega ($\mathcal{V}$) is the change of its value with respect to the volatility in the price of the underlying asset: $\mathcal{V}=\partial\Pi/\partial\sigma$. In practice, volatilities change over time. This means that the value of a derivative is liable to change because of movements in volatility as well as because of changes in the asset price and the passage of time. $\mathcal{V}$ is positive for both call and put options; an option becomes more valuable when the volatility of the market price of the underlying asset increases. Moreover, other things being the same, the Vega of an option is lower when it is close to expiration and it is higher for option when the market price of the underlying asset is close to the strike price.

RHO

An option's portfolio ρ is the rate of change of the value of the portfolio with respect to the interest rate: $\partial\Pi/\partial r$. It measures the sensitivity of the value of portfolio to a change in the interest rate when all else remains the same. Considers rho equal -50, then 100 basis point (1%) increase in the risk-free interest rate reduces the value of the option by 0,5.

2.3 IMPLIED VOLATILITY

Implied volatility is one of deciding factors in the pricing of options. Buying options contracts lets the holder buy or sell an asset at a specified price during a pre-determined period.

Implied volatility is the expected magnitude of a stock's future price changes, as implied by the stock's option prices. It is substantially a way to compare an option prices to another option prices. More expensive options have higher implied volatility. If market options are expensive, consequently implied volatility is large and market is expecting significant moves on that stock. It is important to note that whereas historical volatilities are backward looking, implied volatility is forward looking.

In practice, implied volatility represents a one standard deviation change in the stock price. In statistics, one standard deviation encompasses about 68% of the occurrences around the average (the current stock price of Game Stop inside the models). One period expected standard deviation range formula is given by stock price ± (stock price x implied volatility) x (days period/365).

Just as with the market as whole, implied volatility is not constant and it is subject to unpredictable changes. Supply and demand are major determining factors for implied volatility. When an asset is in high demand, the price tends to rise, so does the implied volatility, which leads to a higher option premium due to the risky nature of the option. The opposite is also true. When there is plenty of supply but not enough market demand, the implied volatility falls, and the option price becomes cheaper.

An option's factor that influences the size of implied volatility is the amount of time until the option expires. A short-dated option often results in low implied volatility, whereas a long-dated option tends to result in high implied volatility. The difference lays in the amount of time left before the expiration of the contract. Since there is a lengthier time, the price has an extended period to move into a favorable price level in comparison to the strike price.

Implied is calculated by using the market price of the option in the Black and Scholes model and solving for volatility via an iterative and inverse process.

Finally, implied volatility defines the relative value of options premium while historical volatility reading serves as the baseline. When the two measures represent similar values, option premiums are generally considered to be valued fairly.

Within valuation models used implied volatility is considered given by Bloomberg data.

2. 4 THE BLACK – SCHOLES - MERTON OPTION PRICING MODEL

The Black-Scholes was first invented by Fischer Black and Myron Scholes in the "Pricing of Options and Corporate Liabilities" which was published in the Journal of Political Economy in 1973. The publishing of the Black-Scholes model roughly coincides with the start of option trading at the newly opened Chicago Board Options Exchange – two events which continued to reinforce on another's importance in the years that followed.

Almost at the same time as Black and Scholes, Robert Merton presented his own contributions to the model in a paper titled "*Theory of Rational Option Pricing*". In the paper he suggested several extensions to the model. The ability to account for dividends is the most widely known one, but even more importantly, Merton provided an alternative derivation of Black-Scholes formula, valid under weaker assumptions and therefore more widely usable.

The historical context of the time when the Black-Scholes model was published was the end of Vietnam war, collapse of the Bretton-Woods system, the 1973 oil crisis, stock market crash high inflation, high interest rates, and the longest recession since World War II. Highly volatility and uncertainty contributed to increase need for risk management tools and innovations in the derivatives universe. Trading expanded both on and off exchanges.

In 1976, Fischer Black proposed a way to apply the Black-Scholes model to options on forwards and futures in "*The Pricing of Commodity Contracts"*. The Black 1976 model, as it is now known, is mathematically identical to the Black-Scholes model (the Merton's extension), with the different being the use of (discounted) futures price as underlying in place of spot price. The method is also applicable to other kinds of derivatives, such as bond options, swap options, or interest rate caps or floors.

In 1997, 24 years later the Black-Scholes model was first published, Scholes and Merton were awarded the Nobel Prize in Economics "for a new method to determine the value of derivatives".

Ever since its initial proposal, the Black-Scholes has proved to be a powerful instrument in estimating the price of European style options.

In the first version of the model, Black and Scholes, assumed "ideal conditions" in the market for stock and for the option:

- The short-term interest rate is known and is constant through time.
- The stock price follows a "random walk" in continuous time with a variance rate proportional to the square of the stock price. Consequently, the distribution of possible stock prices at the end of any finite interval is lognormal. The variance rate of return on the stock is constant.
- The stock pays no dividend or other distributions.
- The option is "European", that is, it can only be exercised at maturity.
- There are no transaction costs in buying or selling the stock or the option.
- It is possible to borrow any fraction of the price of a security to buy it or to hold it, at the short-term interest rate[22].
- Do not exist riskless arbitrage opportunities.

Under these assumptions, the value of the option depends only on the price of the stock and time and on variables that are taken constants.

There is one important difference between the Black-Scholes (the Merton's extension) analysis and previous one using a binomial model. In Black-Scholes-Merton, the position in the stock and the derivative is riskless for only a very short period. To remain riskless, it must be adjusted frequently the relationship between ΔC and ΔS, hence the model is based on continuously delta hedging process.

Mathematically the model is based on Black-Scholes-Merton differential equation:

$$\frac{\partial f}{\partial t}+rS\frac{\partial f}{\partial S}+\tfrac{1}{2}\sigma^2S^2\frac{\partial^2 f}{\partial S^2}=rf$$

This equation has many solutions, corresponding to all the different derivatives that can be defined with S as the underlying variable. The specific derivative that is obtained when the equation is solved depends on the boundary conditions for the European options.

Hence given the boundary values, seen in the first chapter, for European call and put option the solutions to the differential equation are the Black-Scholes-Merton formulas for the prices of European call and put options. These formulas are:

$$c = S_0N(d_1) - Ke^{-rT}N(d_2)$$

[22] Fischer Black and Myron Scholes "The Pricing of Options and Corporate Liabilities" on Chicago Journal 1973.

And

$$p = Ke^{-rT}N(-d_2) - S_0N(-d_1)$$

Where:

$$d_1 = \frac{\ln(S_0/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{\ln(S_0/K) + (r - \sigma^2/2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

The function *N(x)* is the cumulative probability distribution function for a variable with a standard normal distribution. The probability that a variable with a standard normal distribution will be less than *x.*

The term $N(d_2)$ in the call option equation represents the probability that a call option will be exercised in a risk-neutral world.

Moreover, the expression $S_0N(d_1)e^{rT}$ is the expected stock price at time T in a risk-neutral world when stock prices less than the strike price are counted as zero. The strike price is only paid if the stock price is greater than K and this has probability $N(d_2)$. Hence the expected payoff in a risk-neutral world is therefore:

$$S_0N(d_1)e^{rT} - KN(d_2).$$

Discounting this formula from time T to time zero gives the Black-Scholes-Merton equation for a European call option.

Black-Scholes-Merton model						
Price (S)	43,03					
Maturity	19-feb					
Volatility (σ)	1,25			CALL PRICE	2,359062	
Strike (K)	55			PUT PRICE	14,28267	
Interest rate (r)	0,011					
Time to maturity (T)	0,076712329					
d1	-0,533372881					
d2	-0,879585264					
N(d1)	0,296887743					
N(d2)	0,1895420					
N(-d1)	0,703112257					
N(-d2)	0,810457988					

It is possible also to use MATLAB to computation:

```
S = 43.03
K = 55
vol = 1.25
r = 0.011
T = 28/365
[Call,Put] = blsprice(S,K,r,T,vol)

Call =

    2.3591


Put =

   14.2827
```

The results in this case are the same.

2.4.1 LIMITITATIONS BLACK AND SCHOLES MODEL

Despite their popularity and widespread use, the model is built on some non-real-life assumptions about the market. These assumptions are stated and challenged below:

- Volatility is constant over time. While volatility can be relatively constant in very short term, it is never constant in longer term. Indeed, large price changes tend to be followed by large price changes in the future, leading to a property called “volatility clustering”. But measures of volatility are negatively correlated with asset price returns while trading volumes are positively correlated. This imply that volatility cannot be constant over the time.

- Assumes stock prices to follow lognormal pattern, e.g., a random walk process. This assumption ignores large price swings that are observed more frequently in the real world. Asset returns have a finite variance and semi-heavy tails contrary to stable distributions like log normal with infinite variance and heavy tails. As the time scale, over which return asset are calculated, increases, the distribution of asset prices looks more like a normal distribution with heavy tails even though autocorrelation of asset prices is often insignificant.

- Interest rates are constant and known. It uses the risk-free rate to represent this constant and known rate. In the real world, there is no such thing as a risk-free rate, but it is possible to use the U.S. Government Treasury Bills 30-day rate. However, these treasury rates can change in times of increased volatility.

- The underlying stock does not pay dividends during the option’s life. In the real world, most companies pay dividends to their shareholders. The basic Black-Scholes model was later adjusted for dividends, so there is a workaround for this. A common way of adjusting the Black-Scholes model for dividends is to subtract the discounted value of future dividend from the stock price.

- No commissions and transaction costs. The model assumes that there are no fees for buying and selling options and stocks and no barriers to trading. Usually, it is not true as stockbrokers charge rates based on spreads and other criteria.

- Assumes European-style options which can only be exercised at the expiration date. American-style options can be exercised at any time during the life of the option, making American options more valuable due their greater flexibility.

- Markets are perfectly liquid, and it is possible to purchase or sell any amount of stock or options or their fractions at any given time. This again is not plausible as investor are limited by the amount of the money they can invest, policies of their companies and by the wish of sellers to sell. It may not be possible to sell fractions of options as well.

Since most of these limitations relate to fundamental aspects of the market, it is necessary to come up with models that will take into consideration some of the assumptions not addressed by Black-Scholes models. Many models have been proposed over time, all attempting to mimic the characteristics of the market fully. Every aspect of the market cannot be considered in any given model, as every factor affecting the price of a financial security cannot be captured mathematically.

2.5 A MONTE CARLO APPROACH TO OPTION PRICES

Monte Carlo method was developed in the late 1940s by Enrico Fermi, John von Neumann, and Stanislaw Marcin Ulam. The name has been invented by Nicholas Constantine Metropolis with a reference to Monte Carlo famous Casino.

Monte Carlo methods are a class of computational algorithms that are based on repeated computation and random sampling. Monte Carlo simulation is used in finance to value and analyze instruments, portfolios, and investments by simulating the sources of uncertainty that affect their value.

Also, options can be priced by Monte Carlo simulation. First, the price of the underlying asset is simulated by random generation for several paths. Then, the value of the option is found by calculating the average of discounted returns over all paths. Since the option is priced under risk-neutral measure, the discount rate is the risk-free interest rate.

To get a good estimate from simulation, the variance of the estimator should go to zero and thus the number of samples should go to infinity, which is computationally not feasible. Therefore, variance reduction techniques such as antithetic variates and control variates help us to obtain a better estimate in simulation.

```
S = 43.03
K = 55
r = 0.011
vol = 1.25
time = 28/365
N = 2500
BS = blsprice(S,K,r,time,vol)
for i = 1:N
Z(i) = normrnd(0,1)
Y(i) = S*exp((r-0.5*vol^2)*time+vol*sqrt(time)*Z(i))
C(i) = max(Y(i)-K,0)
D(i) = max((Y(i)-K)^2,0)
end
callprice = sum(C)/N
standarderror = sqrt((1/(N*(N-1)))*(sum(D)-N*callprice^2))
```

```
callprice =

    2.1490


standarderror =

    0.3781
```

It is possible to note that increasing the number of paths N, the result from Monte Carlo Simulation converges towards the option price of call option found by Black – Scholes – Merton model.

To test this convergence, it will construct a t-test.

To confirm or rejects it, it will be tested the following hypothesis:

- H0 = mean of Monte-Carlo Simulation = Black-Scholes price.
- H1 = mean of Monte-Carlo Simulation ≠ Black-Scholes price.

$$t = \frac{C_{MCS} - C_{BS}}{SE}$$

The null hypothesis is rejected if the t statistics is greater than 1,96 which is the critical value under a 95% confidence interval.

	BS	MCS N=1000	MSC N=1500	MSC N=2500
Call price	**2,3591**	3,7837	2,6052	2,1490
Standard error		0,9502	0,4979	0,3781
T-test value		1,499	0,4942	0,55

The hypothesis is not rejected; hence the test confirms that MCS call price converge to BS one.

Is possible also to check the convergence with CRR model option call price.

	CRR	MSC N=1000	MSC N=1500	MSC N=2500
Call price	**2,1274**	3,7837	2,6052	2,1490
Standard error		0,9502	0,4979	0,3781

T-test value		1,74	0,63	0,06

3 HOW OPTIONS TRADING INFLUENCED SHORT SQUEEZE PHENOMENON

3.1 THE SHORT SQUEEZE

Short squeeze is a term used to describe a phenomenon in financial markets where a sharp rise in the price of an asset forces traders who previously sold short to close out their positions. The strong buying pressure “squeezes” the short sellers out.

A short squeeze often feeds on itself, sending the asset’s trading price even higher and forcing more short sellers to cover their positions.

As traders who previously sold short the asset must buy to cover their positions, the closing out of their positions, the closing out of their short trades simply adds more buying pressure to the market, thus further fueling a rise in the asset’s price.

The first major “short squeeze” was employed by a well-known 19th century entrepreneur, named Vanderbilt.

Indeed, in winter 1862, Vanderbilt became fascinated by the business potential he saw in the New York and Harlem Railroad as knows as simply “the Harlem”. This 131 – mile long line had been unprofitable, but it was the only railroad line that entered New York City.

He began buying the stock of the Harlem, at 8$ per share. When he had bought enough shares to gain control of the company, Vanderbilt started to make improvements to this railroad line.

The Harlem stock gradually rose to 30$ per share, and then slowly to 50$. On April 23, 1863, the city council of New York City approved Vanderbilt’s request to build a new line along Broadway to the Battery. The day after, the stock surged to 75$, and soon to 100$ in a few days. This authorization made the Harlem, not only the one railroad entering New York City, but also the only line running the length of Manhattan Island.

At that point, Daniel Drew, an old rival of Vanderbilt in boat business, started to talk to the city council. Drew told the members at the city council that, if this franchise were abolished, the Harlem stock would collapse.

Hence if they were going to short the stock, they would make a lot of money. Hence all of them borrowed as much money as possible and began shorting the Harlem's stock. After Vanderbilt heard about their scheme, he began buying bravely.

On June 25, 1863, the city council repealed the franchise as planned and they issued an order to prohibit the construction of the new Harlem line along Broadway to the Battery. The stock fell from 110$ to 72$ on that day. Unexpectedly, on the next day, the stock started to rise. Vanderbilt had bought every outstanding share of the Harlem. By the end of the day, the stock rebounded to 97$. It reached 106$ the day after. The stock continued to rise and there was not an upper limit for losses of short sellers. Their money was bleeding but there were no available shares for them to buyback and cover their short position.

By the end of summer 1863, Vanderbilt allowed Drew and the members of the city council to buy the stock back from him at 180$ per share. This marked the end of the short squeeze of the Harlem, which made Vanderbilt 5$ million richer.

The Harlem case was an effective example of short squeeze phenomenon even if not so immediate. This because, at the time of Vanderbilt, option contracts did not yet exist. These, as it will be seen later, have represented the gasoline that has accelerated the process of short squeeze.

3.1.1 SHORT SQUEEZE PHASES

A short squeeze is divided into 3 well-defined phases:

- ➔ The price is subject to a sharp decline, which causes distrust among investors and leads many to think that the company in question is close to collapse.
- ➔ A triggering event occurs, which violently impacts dominant beliefs and sows panic among investors. This impact causes prices to rise quickly and the greatest losses: on this occasion, short contracts are executed, or hedging is used to stem losses. Hedging favors the crazy upward race of the stock, leading the company's shares to touch even higher peaks in the next future.
- ➔ The market tends to reset itself, through progressive declines that bring the quotations back to their value.

The first step of short squeeze is characterized by a progressive price decline following a particularly significant triggering event. This can be related to:

- Financial crises such as that of 2008/2009 or the March 2020 crash. It is not important that the crises lead to catastrophic events for the world economy: the triggering event of a short squeeze must have strong psychological impact on investors, bringing the prices of an asset to levels that suggest to investors that the stock itself is not capable of recovery.
- Sector crises, as in the case of the automotive sector in 2008, or that of retail sales in the "March crash". In this case, companies that were in a precarious situation will tend to record the worst damage and, consequently, be the object of short transactions.
- Corporate data: if a company shows sharply declining fundamentals, hedge funds will begin to investigate the possibility of shorting its shares over the long term. Their availability of liquidity will favor the creation of conditions for a short squeeze.

The second phase of a short squeeze is the most interesting for investors. During this phase, indeed, prices see the most significant changes and profits, for those who know how to act, accumulate. The event that triggers this second phase must have a strong psychological value: it is often an announcement by groups of investors or shareholders of the shorted security. This is one of the basic principles of market psychology: when a stock falls, investor insecurity increases exponentially. Tension levels skyrocket as everyone is waiting for the right moment to close their short positions or to buy the stock at bargain prices. This collective neurosis finds its release valve in the news of the moment.

This step of short squeeze is structured as follows:

- A first day violent upward moment. A first violent upward moment. During this phase, prices literally skyrocket, rising more than 100/200% in the space of a few hours. This phase usually lasts one day and is often anticipated by the fact that the pre-market phases are extremely chaotic. Through a technical analysis is observable that the candles that anticipate the beginning of a short squeeze usually open in a considerable gap-up.
- The first day candle is followed by other candles (bullish or bearish in a gap-up), characterized by a reduced body. In this phase other highs can be touched.
- The volumes in correspondence of these candles are high, which excites even more the volatility of the stock.

The last phase of short squeeze is also called the settlement phase. The volatility of the security stays high but the volumes are clearly decreasing, a symptom that the short squeeze has now come to an end. Prices show a clear bearish trend, bringing the stock back to its real price i.e. price levels that reflect the real economic and financial capabilities of the company.

During this phase short positions, that are still open, are closed, whether they are at a loss or at breakeven. Hardly a short squeeze allows short sales to close in profit, also because throughout the duration of the previous violent upturns hedge funds had to pay premiums to financial institutions to secure their positions.

3.2 PREDATOR TRADING: GAME STOP

Predator trading is a trading that induces and/or exploits the need of other investors to reduce their position[23].

A paper written by Markus Brunnermeier and Lasse Pedersen, published in the Journal of Finance, highlights the key features of financial shark attack. They have assumed illiquid market (i.e., large trades are able to move prices in the short run) and that traders must have limited capacity, meaning that they cannot sustain losses beyond a certain point.

The model pictures a world in which there are only two type of investors – fast and slow. The slow-money investors enter in the market when shares prices look excessively cheap or dear. Instead, the fast money crowd are hedge funds, that trade every day.

Consider an example where there are just two hedge funds and each of them has ten shares, with expected value, in the long run, of 150$. If the stock falls to 100$ per share, low enough to force one of the hedge funds to sell its entire holding. Market is not very liquid, and the slow-money traders will buy two shares per day, but the price must get cheaper by 2$ a day to induce them to trade. Other hedge fund knows that the prey is wounded so it becomes a predator and start to join the selling. With only a few buyers, it now takes the distressed seller ten days, rather than five, to get rid of its share. The predator is then free to buy back shares at a lower price than he sold them for.

The authors draw out some implications: the more illiquid the market, the more scope for predators to profit because it takes longer for the prey to escape their positions, so the price falls by more. The quicker the distressed trader sells, the fewer losses it makes. Any delay allows the predator to trade front-run the prey. The more predators there are, the less profitable predation is.

The Game Stop episode fits well with this template. In this case predatory trading is given by the fact that retail investors have forced or tricked hedge funds to buy Game Stop shares. It is possible assume just one predator because retail investors acted in concert.

[23] Markus K. Brunnermeier and Lasse Heje Pedersen, “Predator Trading” Journal of Finance 2005.

Market was populated by retail investors that was price-takers and by hedge funds. Consider that Game Stop stock started the year at 19$ per share hitting an intra-day high of 483$ on January 28 and assume that retail investors bought the stocks at the average price around 220$.

Retail investors have pushed it up, forcing the short sellers to buy the shares because they could not sustain the risk.

The trouble was that when liquidity was needed by the distressed trader (hedge fund in this case), however, the liquidity was lower due the fact that the market became “one-sided” since predator (retail investors) was buying as well. In this manner, hedge funds have suffered more losses to cover their position.

Short sellers, in turn, pushed up the price giving the chance to retail investors to buy stocks between 19$ and 220$ and then to exit the trade, selling between 220$ and 420$ and the price collapsed afterwards. For short sellers, possibility to move the price represented a source of trading costs, moreover, their positions were often common knowledge.

For this reason, a lot of hedge funds, usually, put their trades through different brokers in attempt to mask them.

3.3 VOLKSWAGEN SHORT SQUEEZE

The starting point for the short squeeze at that time was Porsche's desire to accumulate more voting rights in Volkswagen. It did so by buying up VW shares to gain a greater foothold in the company, even if Porsche was making it clear, at that time, it was not attempting to take over Volkswagen.

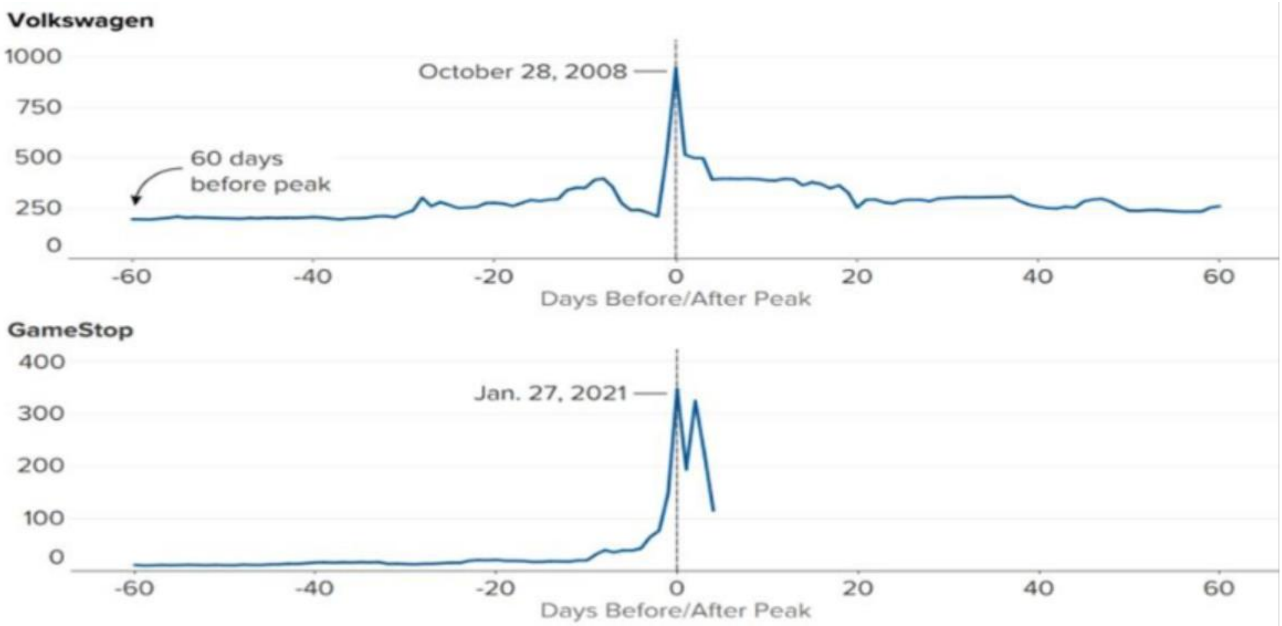
As Porsche started buying up VW shares, seeking more voting rights, VW stock price continued to increase through 2006 and 2007, going from about 30€ in 2005 to over 150€ by 2007, seemingly absent any outside reason. The stock began to appear massively overvalued, and hedge funds took notice and began shorting the stock, betting that it would go down eventually.

The squeeze itself happened in late 2008. By the time VW became the most valuable automaker on the planet thanks to its stock price having skyrocketed, while the short positions were 12% above outstanding shares.

So, Porsche owned 43% of VW shares and 32% in share options. The German government, however, owned another 20,2%. This left very little that could be purchased by anybody else. This disparity caused short sellers to rush to buy more stock to cover their positions, driving the stock price further still through the month of October 2008, with VW stock price now soaring just above 900€, and at one point exceeding 1000€ in intraday trading.

Result was that hedge funds that had been shorting VW stock had lost some 30$ billion in the process. The result for Porsche was that it had been able to make billions in just a few weeks at a time when the auto industry was doing exceptionally badly in car sales.

Even if, after the short squeeze the VW share price returned to previous levels, this, as can be seen from the following chart, did not happen with Game Stop. Indeed, the funds that have shorted VW shares and have waited for the end of the short squeeze did not suffer such large losses.



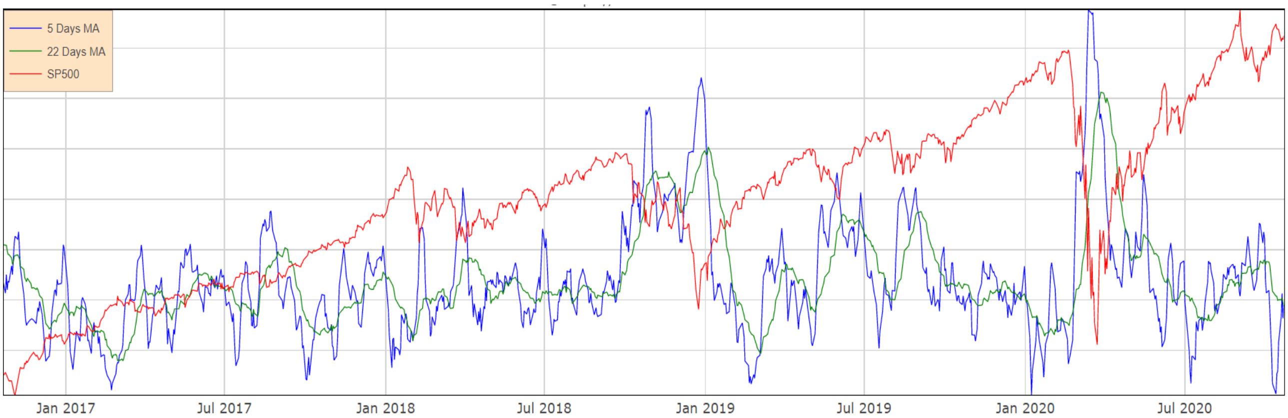
[24] Chart 3.1

[24] FACTSET: "VW and GME prices". In the fall of 2008, shares of Volkswagen more than quadrupled in a matter of two days. The stock dropped 58% in four days after its peak. Chart shows share price for 60 days before and after the stock's peak.

3.4 GAME STOP CASE

In January 2021, a short squeeze of the stock of the American video game retailer took place, causing major financial consequences for certain hedge funds and large losses for short sellers.

Before going deeply in the Game Stop case, it is important to make some clarifications regarding the development of market structure.



[25] Chart 3.2

With the beginning of pandemic, the value of insiders buy/sell ratio is decreased and then the market is collapsed.

In most of the cases, market follows the transactions from the insiders, if they are bullish, market starts to increase in value.

In this case, after the collapse, S&P 500 is started to boost not due to insiders buying but to retail investors buying.

After the Covid-19 the difference between Buy/Sell ratio of insiders and value of S&P 500 increased sharply. Hence the market even if insiders were bearish, market increased in value.

An unprecedent market volatility and Covid-19 lockdowns created a unique opportunity for retail investors to play the stock market's surprising comeback.

[25] SECform4 data: "GME buy/sell insiders ratio".

During the 2020 the brokerage industry added roughly 10 million new clients[26]. The boom continued in 2021, more than 7.8[27] million of retail clients entered in the market in January and February.

This also occurred thanks to the money distributed by the U.S. government to citizens, to face the Covid-19, and that have been invested in the stock market.

Reason why number of retail investors is doubled in the last year and for the first time, influencing the market structure.

Game Stop is a company operating in specialty electronic game and PC entertainment software stores. The company stores sell new and used video game hardware and software, as well as accessories. The core business of the company is the sale of physical video games.

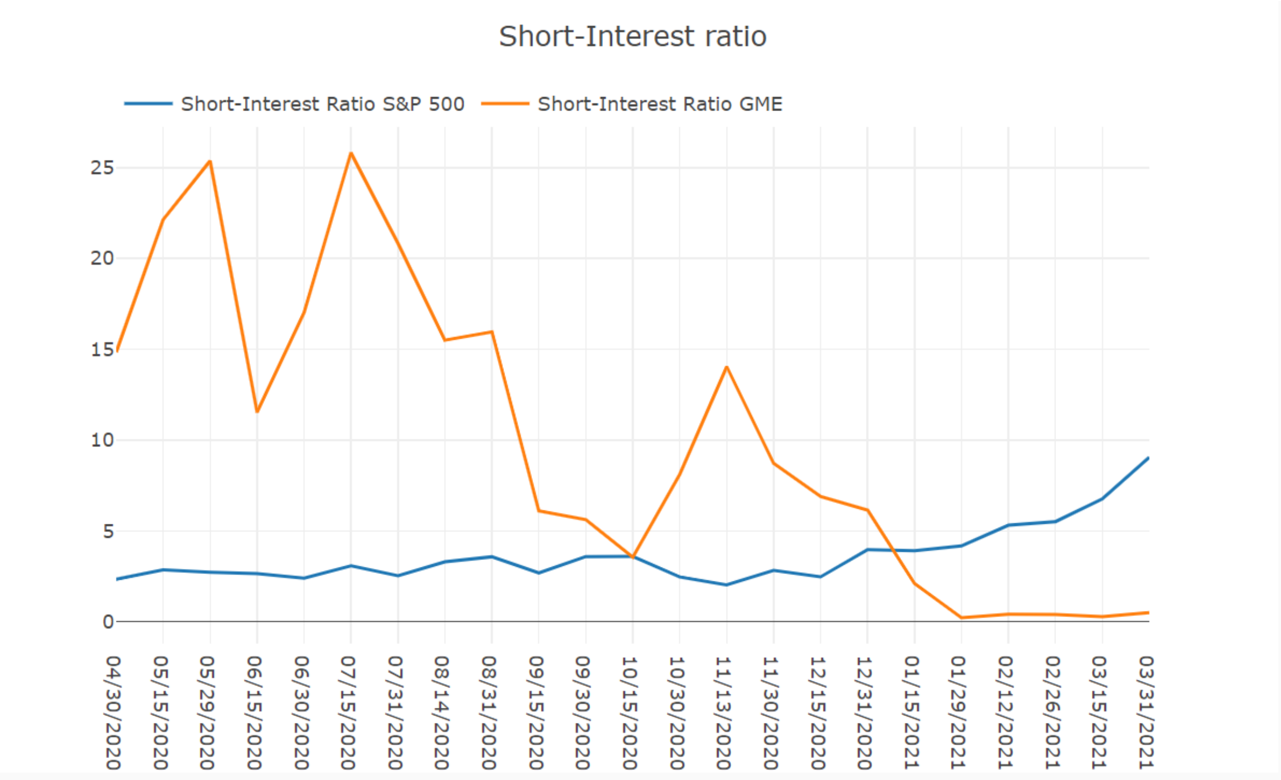
Consoles, PC or smartphones, video games are and will increasingly be a digital product to download online. The physical medium, the diskette, although still alive and well, is destined to play an increasingly ancillary role.

Revenue from physical sold video games market is decreased from 15$ million in 2017 to 11$ million in 2021[28]. This led several investment funds to short sell game stop stocks, driving the price down to as low as 3$ per share during the last summer.

Before to introduce the short squeeze on Game Stop, is important analyze two indicators: first one is called "short interest ratio" and it can be found by taking the number of shares sold short, known as short interest, and dividing by average daily volume.

This is often called the "days – to – cover ratio" because it determines, based on the stock's average trading volume, how many days it will take short sellers to cover their positions if positive news about the company lifts the price.

[26] JMP securities data.
[27] JMP securities data.
[28] Statista data.



[29]Chart 3.2

There is no accurate threshold describing what would be high short interest ratio. Generally, a days-to-cover ratio of over 10 is seen as high.

Short-interest GME, during July 2020, reached the value of 25 respect the mean of S&P 500 equals to 4 at such time.

The second indicator comes from the ratio of operations to floaters: the value of the stocks circulating on the market.

One of the tactics that investment funds use to secure the greatest possible profit from a short contract is to sell a greater quantity of shares than those floating, by repeatedly shorting their own positions.

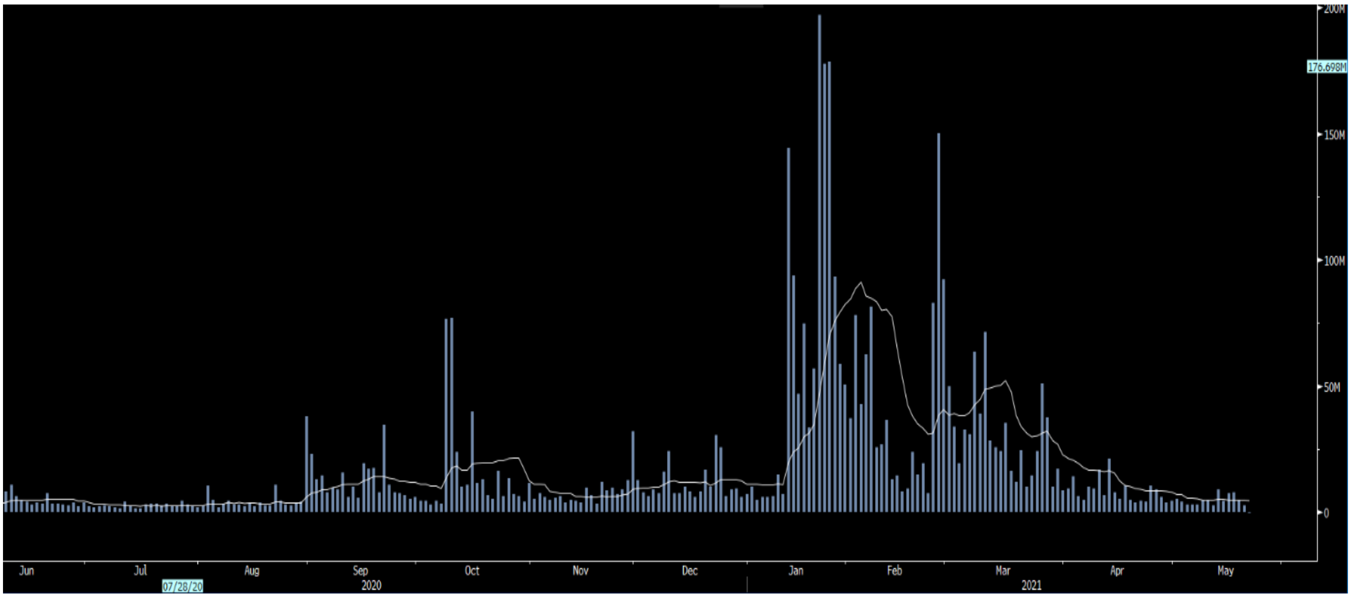
This implies that market valuations are "drugged" by the volumes of the shorts and that the stock cannot exceed certain price levels. It is a dangerous technique but potentially very profitable. Dangerous because when the squeeze occurs it is more difficult cover previous positions by hedging and buying shares.

[29] GME short interest ratio.

Before the Game Stop stock rose, the ratio of shorts to free float was over 140%, despite that “only” 93,96% of the free float was committed to short contracts.

The short squeeze tends to occur more often in smaller-cap stocks, which have a very small float. Such stocks generally are more volatile than a stock with a large float. This occurs because, with fewer shares available, it may be harder to find a buyer or seller. With lower trading volumes than more high-profile stocks, bigger price swings are inherent in the small-cap world.

This also means that fewer shares can push a small-cap company up much higher during a short squeeze. Traders, typically consider a float between 10 million and 20 million shares as low.



[30] Chart 3.3

As it is observable by the chart, the volume of GME shares traded, up to January 12th reached the threshold of 5 million on average.

These data reveal how Game Stop, at that time was the perfect “victim” for a short squeeze process.

During 2020 some analysts and research firms believed GameStop as a business was on its downturn and would soon go out of business. They encouraged investors to short the stock.

[30] Bloomberg data: “GME volume”.

On December 8th, 2020, Game Stop shares tanked, after the company missed Wall Street estimates for quarterly revenue as pandemic-led store closures and intense competition from digital-game sellers hit sales.

On January 11th, 2021, Game stop appointed "Chewy.com" founder and two other e-commerce veterans to its board, as it would double down on digital sales. The shares were traded at 19,94$.

On January 22nd approximately 140% of the Game Stop's public float had been sold short. As said before this means, in more concrete terms, that people who had "shorted" Game stop's stock by betting on the value of it falling, were borrowing shares that were already borrowed by other short sellers. The stock surged 50% and closed at 65,01$.

On January 25th stock opened at 96,73$.

On January 27th Game Stop reached 347,51$ at the end of the day and an intraday price of 483$. Some hedge funds confirmed on Social Media that they had bought the shares, exited the position, and admitted heavy losses.

All this happened because several retail investors got in touch through the social network named "Reddit", especially the web page named "Wall Street Bets", opened many long positions on the shares of GME. The pool of retail investors could be distinguished into three categories:

- Experts: these have been the driving force behind the short squeeze process, as, thanks to their financial knowledge, they judged Game Stop as the ideal firm to do this type of operation and incentivized other retail operators to buy the stock.
  They implied a strategy like "Pump and dump" one. Indeed, they tried to profit by first buying an asset and then pretended to have a fanatic bullish view on the asset to create a wave of buying by his followers, leading to a price increase. If these investors then sold despite talking up the asset, they engaged in "pump and dump", that is an illegal form of market manipulation.
- Knowledgeable: these enjoy low to medium knowledge of the finance world, able to understand the idea behind action by the experts.
- Unknowledgeable: this last category represented most of retail investors who opened long positions on GME. These have bought the stock either because they are affectionate to the brand or because they have been pushed to do so, without understanding the potential of the short squeeze.

These retail investors were also joined by private investors.

Most of the transactions on the stock took place on “Robinhood”, the American investment platform most used by retail investors.

“Robinhood” suddenly found itself having to increase the concentration of securities on its platform and consequently increase collateral. The collateral required from the entire broker industry increased from 26 billion to 33 billion. Robinhood bears a business risk, given the lack of liquidity to offer as collateral, and consequently had to block certain transactions to reduce the concentration of GME stock on the platform. It had to choose between giving a disservice or being in a serious lack of liquidity. All this was caused by the fact that Robinhood gave the investor the possibility to operate with leverage.

On January 28th some brokers, such as Robinhood, halted the buying of Game Stop, citing the next day their inability to post sufficient at clearing houses to execute their clients’ orders. This caused a big dip in the share price. It closed at 193,60$.

Melvin Capital, an investment fund that heavily shorted GameStop, had lost 30 percent of its value since the start of 2021, and by the end of January had suffered a loss of 53% of its investments.

3.5 USE OF OPTION CONTRACTS INSIDE THE GAME STOP SHORT SQUEEZE

Within our case study, which is the short squeeze on Game Stop, there is one element that was the gasoline on the fire and accelerated the short squeeze process, which is the purchase of Game Stop's stocks, through option contracts.

This represented the element of distinction respect the cases of short squeeze analyzed in the past. In this case the phenomenon of short squeeze is not revealed exclusively through the purchase of the security via investors and via the Hedge Funds, that had shorted the security, forced to cover their position.

If a trader buys a call option on a stock which price is significantly inflated from its current market value, he or she can buy it cheaply.

For example: on January 22nd, a Game Stop option with a strike price of 60$ could be purchased for 18$. The market maker who sold the option usually buys some amount of the stock to cover his position, which again pushes the price higher.

The more options are purchased, the more amount of the stock the dealer or market maker must purchase and higher the price goes.

Hence, in this case, the short squeeze phenomenon was influenced by purchasing of Game Stop security from Hedge Fund, from retail investor and from market makers, at same time.

At this point it is relevant to analyze how the option contracts affected the market on the Game Stop stock and what factors influenced the value of the options during the Short Squeeze process. In way to understand, if through the study of the option contracts, it was possible to predict the short squeeze.

Previously it has been stated how one of the factors, that acted as a prelude to the short squeeze, was the short interest on the Game Stop stock.

In the June of 1993 Stephen Figlewski and Gwendolyn P. Webb with their paper, published on the "Journal of Finance"[31], tested whether short interest was higher for stock that have exchange traded options than for stocks with no options.

They showed that the average relative short interest annually from 1973 to 1983 for stocks with and without options. The relative short interest is significantly higher for stocks that have traded options than for those without options.

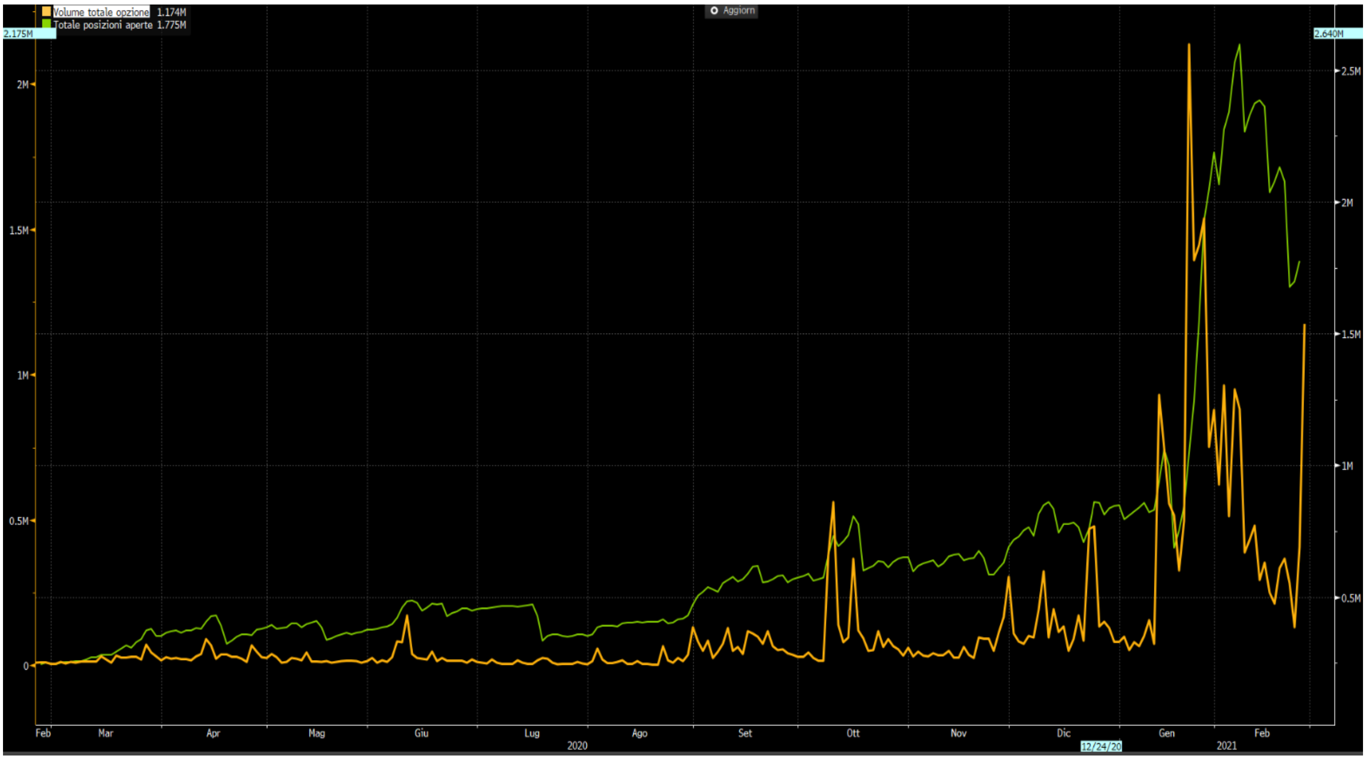
In a cross-sectional regression analysis of short interest, a significantly positive coefficient was estimated for a dummy variable representing the existence of options.

This occur because, in this case, is easier short the position, through a put option. When a put is purchased from an options market maker, he normally hedges by shorting short, and perhaps buying a call also to turn the position into a reverse conversion arbitrage. The put buyer's desire to sell the stock is transformed, through options market, into an actual short sale by a market professional who faces the lowest cost and fewest constraints.

Hence, possibility to trade the stock with the options contract increase the short interest amount on this stock, factor that has influenced the short squeeze phenomenon.

Is interesting to observe the volume of option position open respect the total volume of the stock.

[31] Stephen Fliglewaki and Gwendolyn P.Webb "Options, Short Sales and Market Completeness" published on Journal of Finance 1993.



32 Chart 3.4

As can be seen from the chart the total volume of options exceeds the total volume of transactions in mid-January.

Amount of active options contract is also called "open interest" and gives information regarding the liquidity of the option.

When options have a significant open interest, it means there are many buyers and sellers in the market.

In addition, exceptionally large volume, compared to historical averages, is one reason for which options market activity can be considered unusual.

Another indicator of unusual options activity is the trading of a contract with an expiration date in the distant future. Additional time until a contract expires generally increases the potential for it to grow its time value and reach its strike price.

[32] Bloomberg data: "GME total options volume and floating"

Contracts that are “out of the money” are also indicative of unusual options activity. These trades are made with the expectation that the value of the underlying asset is going to change dramatically in the future, and buyers and sellers will benefit from a greater profit margin.

Particularly Game Stop at the end of 2020 and first days of 2021 was a company with large open interest in out-of-the money call options.

Specifically in mid-December 2020 several retail investors bought out-of-the money call options as the price of Game Stop was falling. The purchase of these options can be considered one of the common factors among companies that have experienced the short squeeze phenomenon.

3.5.1 SHORT INTEREST RATIO AND DIFFERENCE BETWEEN PUT OPTIONS IMPLIED VOLATILITY AND CALL OPTIONS IMPLIED VOLATILITY.

Previously it has been stated a strong connection between short interest and the existence of option trading. Now will be useful investigate the relation between short interest and call/put option implied volatilities, to study whether short interest, in Game Stop case, had a measurable effect on option prices.

Assuming that for many investors, buying puts or writing calls is preferable than selling the stock. Transaction costs are lower, there are fewer impediments like the uptick rule, and buying puts also has the virtue of limited liability.

High short interest indicates a large investor demand for short position, and this is revealed by larger difference between put and call implied volatility.

The hypothesis tested was that when high short interest indicates a large investor demand for short positions, put prices will also be high relative to call prices depressed by pressure from pessimistic investors, should be observable a positive correlation between short interest and the difference between put and call implied volatilities.

There is a problem with linking the short interest data, which are published once a month, with option price data, for which daily closing prices and even transactions data are available.

Since not always are available option prices as of the same points in time that short sales were being made, will be used option prices on the last day relevant to the date of the short interest.

Short interest rate on fifteenth of the months relates to stock trades that have settled by that date, so it will take the options price from five days earlier.

Regression Statistics	
Multiple R	0,202219169
R Square	0,040892592
Adjusted R Square	-0,004779189
Standard Error	0,178158316
Observations	23

ANOVA

	df	SS	MS	F	Significance F
Regression	1	0,028419007	0,028419007	0,89535795	0,354791044
Residual	21	0,666548099	0,031740386		
Total	22	0,694967106			

	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%	Lower 95,0%	Upper 95,0%
Intercept	0,086954233	0,054674356	1,590402505	0,126686633	-0,026747315	0,200655781	-0,026747315	0,200655781
X Variable 1	0,0044013	0,004651388	0,94623356	0,354791044	-0,005271792	0,014074391	-0,005271792	0,014074391

[33] Table 3.1

This regression statistic gives a small level of R square implying that the linear model does not fit the data well. It will test the following hypothesis:

- H0 = β1=0
- H1 = β1≠0

Comparing the F value with the critical value of F determines from Fischer table (with degrees of freedom of 1,21), it is observable how F value is lower and consequently the null hypothesis is not rejected with a significance level of 0,05.

In this case large value of F indicates that there is not a relationship between the Put-Call implied volatility difference and short interest of GME.

Findings show how, in the last year, particularly in the last few months, a high level of short interest on the Game Stop stock did not result in a high difference between the price of the put option and the price of the call option.

This happened because the level of implied volatility has always been significantly high for call options, even where the level of short interest was high.

The elevated volume of transactions made during the short squeeze have influenced the prices of the options, making them not moldable.

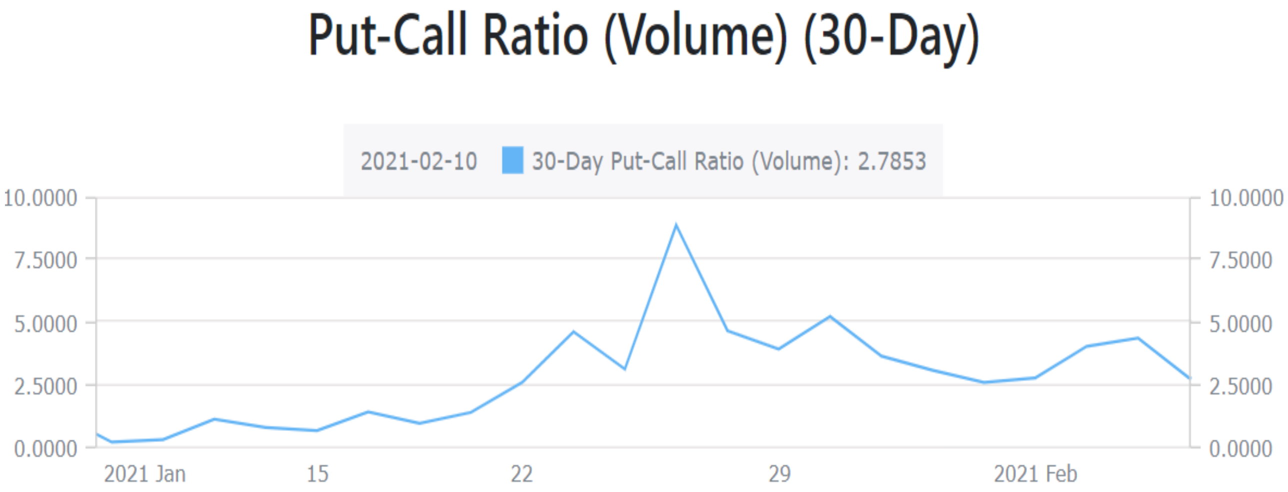
[33] Regression statistics

The valuation models of the options seen in the second chapter, did not give a correct estimate of the price of the options, because the level of demand and supply in the phases of pre-market determined the price of the options, creating anomalies and detachments from the valuation models.

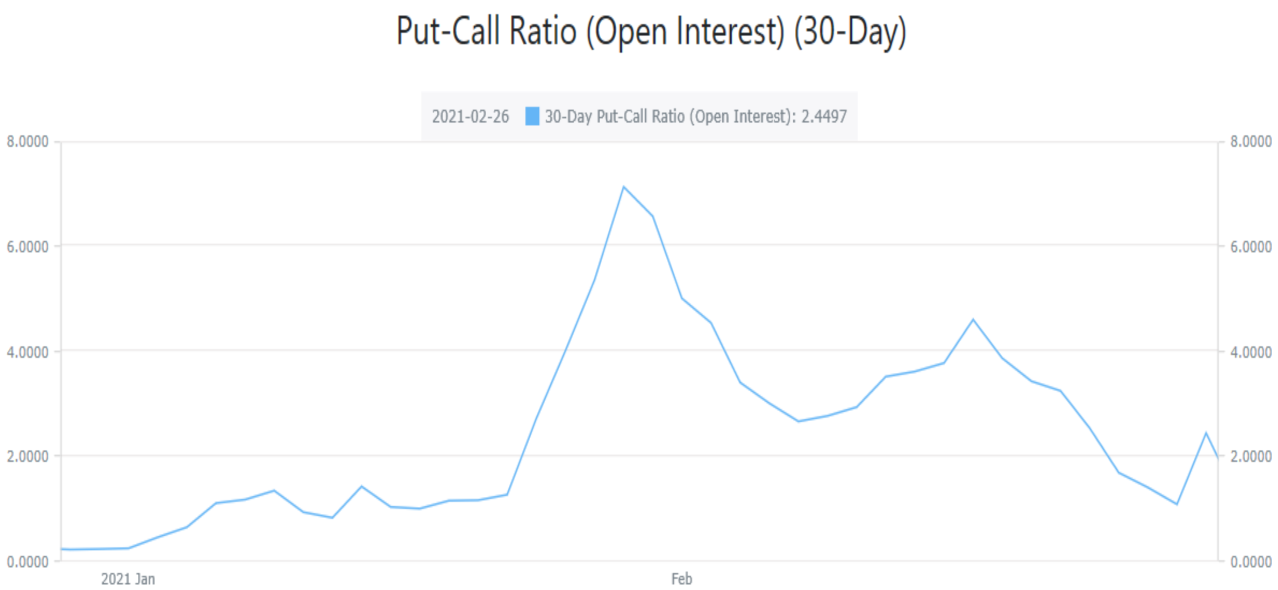
This was also due to the large level of "monthly implied volatility" which reached 587,234% on January 28th and created anomalies on the price of put options.

3.5.2 PUT-CALL OPTIONS RATIO

Options contracts were the main driver of the short squeeze on Game Stop. At this point the ratio between the volume of put options and call options can be interpreted as a leading indicator.



[34] Chart 3.5



[35]Chart

[34] Alpha Query data: "GME Put-call ratio (Volume)"
[35] Alpha Query data: "GME Put-Call ratio (Open interest)

On 27thJanuary the put-call ratio is increased sharply, this means that volume of put options transaction amplified respect the call ones.

Setting Put-Call ratio (volume) and Put-Call ratio (open interest) as dependent variables is possible to construct a linear regression verifying if the GME price is influenced by these factors.

Regression Statistics								
Multiple R	0,835211173							
R Square	0,697577703							
Adjusted R Square	0,667335474							
Standard Error	52,76191762							
Observations	23							
ANOVA								
	df	*SS*	*MS*	*F*	*Significance F*			
Regression	2	128425,103	64212,55148	23,06635	0,000006399			
Residual	20	55676,39902	2783,819951					
Total	22	184101,502						
	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95,0%*	*Upper 95,0%*
Intercept	-38,46928485	22,57061242	-1,704397033	0,103794	-85,55075734	8,61218764	-85,55075734	8,61218764
X Variable 1	18,25673629	8,965253105	2,036388273	0,05517	-0,444453981	36,95792656	-0,444453981	36,95792656
X Variable 2	25,80986733	9,300866519	2,774995993	0,011685	6,408599738	45,21113491	6,408599738	45,21113491

[36] Table 3.3

This regression statistic displays an elevated level of R square implying that the linear model fits the data well.

With the aim to verify if R square is significant, it will test the following hypothesis:

- H0 = β1=β2=0
- H1 = β1≠β2≠0

Comparing the F value with the critical value of F determines from Fischer table (with degrees of freedom of 2,20), it is observable how F value is bigger and consequently the null hypothesis is rejected with a significance level of 0,05.

[36] Regression statistics 2 (model does not assuming multicollinearity)

In this case large value of F indicates that there is a relationship between Put-Call ratio (volume), Put-Call ratio (open interest) and GME price and value of R square indicates that this relationship is very strong.

This means that the system globally is significant, so now is possible test each variable in the model. From t-test and associated p-value just the second variable is significant associated to dependent variable.

The p-value associated to put-call ratio open interest is lower than 0,05.

Practically talking, the price of GME from January 12th to February 12th it was influenced exclusively by Put-Call ratio open interest (ratio between number of put and call contracts active in a certain period but not yet settled).

3.5.3 SHORT SQUEEZE AND GAMMA SQUEEZE

Greeks were introduced in the second chapter of this paper, in particular gamma (the sensitivity of an option's delta to a movement in the underlying security). Gamma affected trading behavior as well as whether there are opportunities for individual investors in Game Stop options.

It was noted how a large proportion of retail investors, in the early of January, purchased out-of-the money call option. Consequently, market makers, who sold the options, must hedged the trade against movement in the underlying by buying shares to offset the delta of the short position.

It has seen also how if the stock were to move a large amount, the delta of the options changes and the market maker who wants to remain delta neutral will have to buy or sell shares to adjust the position.

For example: if the stock rallies to the strike of the call option, the delta will be very close to 0,50 and the market maker will need to buy 50 shares to bring the total position delta back to zero.

If the stock continues to rally until the now in-the money option has a 75 delta, the market maker would need to buy 25 more shares, and another 25 if the option's delta went to 100.

That is the curse of having a negative or short gamma – the readjustment to the hedge involves buying when the stock rallies and selling when the stock falls.

This is exactly what happens during the Game Stop short squeeze, gamma value led market makers to buy the Game Stop stocks to hedge their positions, and the amounted shares to buy increased as the delta become larger.

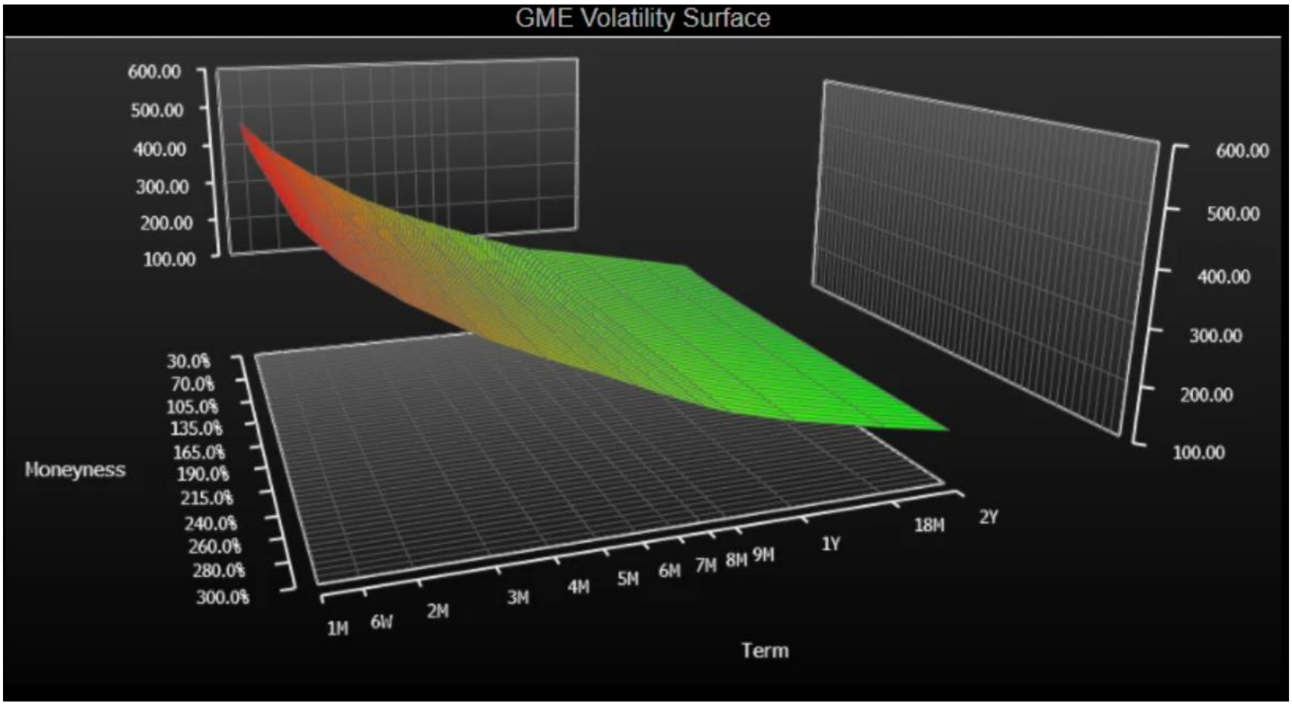
Hence when market makers are short gamma, hedging is done in the direction of movement of the underlying stock, acting as an accelerator on a stock's price.

Hence "momentum" is incredibly impactful during gamma squeezes as rising prices create a feedback loop of continual hedging and re-hedging.

In this case a bullish behavior of retail investors, through call option, has caused a short squeeze and short gamma to occur simultaneously.

Analyzing in a deeper way the gamma, all else being equal, options with less time remaining to expiration have more gamma than options with relatively more time left and options with a lower implied volatility have more gamma than options with higher implied volatilities.

In the second case, if a stock barely ever moves and the options are trading at a low implied volatility, it is relatively unlikely that out-of-the money options will wind up in the money, so they have low deltas. That means if the stock does make a move, the option deltas will change quickly. Those options have high gamma.



[37] Chart 3.6

Conversely, during January, Game Stop options has been incredibly high intraday implied volatility between 300% and 600% implying a high value of delta and low gamma.

[37] Bloomberg data: "GME volatility surface"

3.5.4 HIGHER-ODER SENSITIVITES

In the second chapter has been introduced the “Greeks” i.e. sensitivities of option prices respect certain parameters, moreover there are also higher-order sensitivities that represented indicators in the short squeeze.

Dealers with short call positions (market makers) have negative “Vanna”. Vanna is a second-order sensitivity and has two meanings: the change in Vega, given a change in the underlying ($\partial Vega/\partial Spot$) and the change in delta with respect to the change in volatility ($\partial\Delta/\partial S$).

It can be thought of as a measure of exposure to skew. Trader will lose money if they are short the skew as volatility rises and the market moves towards the strike.

It is more important when managing a book of exotic derivatives to monitor the change in Vega (sensitivity of option price to volatility) to the underlying.

Hence in the Game Stop short squeeze, experienced a large, in absolute value, Vanna that has implied a delta rising as volatility increased.

Hence market makers had to buy more of the underlying to hedge their positions.

3.5.5 PUT ANOMALY

Call						Put			
Strike	Ticker	Bid	Ask	IVM	Volm	Strike	Bid	Ask	Volm
19-Feb-21 (22g); VolC 100									
185	GME 2/19/21 C185	110,3000031	117,75	587,1376343	19	185	91,1499939	95,90000916	44
190	GME 2/19/21 C190	106,4499969	119,3500061	588,3100586	43	190	94,8500061	99,65000916	28
195	GME 2/19/21 C195	107,6999969	115,1500092	586,9472656	45	195	98,55000305	102,5	45
200	GME 2/19/21 C200	106,4499969	113,8500061	586,7283325	896	200	102,3000031	105	981
205	GME 2/19/21 C205	105,25	112,6500092	586,8182373	16	205	106,0500031	111	44

[38]Table 3.2

Setting this option as example is observable how price of put options of Game Stop was high despite it was out-of-the money.

The second put, in the table, has strike price of 190$, and the previous closing price of Game Stop stock was 347,51$, this means that put option to became in the money, the price of the underlying would have to drop more than 45%.

Following the traditional criteria for options valuation the price of put option and the call option, given the high price level, should be much further apart, instead they are very similar. This depends both on the high level of volatility, around 500% on January 28, which makes it difficult to model and creates distortions, and above all on the activity of traders in the pre-market.

Indeed, during the pre-market on January 28th, retail investors decided to sell their call options or buy their put options to hedge their position, totally influencing the price of the options, as the transaction volumes were very high.

This has meant that the models analyzed in the second chapter: "Black and Scholes model", "Binomial option pricing model" and Monte Carlo simulation. They were not able to correctly evaluate the price of options, in particular way of the put.

This happened, primarily because these models offer a static vision while during the short squeeze the level of dynamism given by the large volumes, was very high.

[38] Call and Put options valued available on the market on January 28th and expiring on February 19th.

Especially because market recorded a high volatility when the price was rising. Indeed, for the first time in the market, high volatility corresponds to a bullish and not bearish market.

CONCLUSION

This phenomenon was born from a sort of "social anger" that has its roots in 2010. When the big world banks carried out an expansive policy inflating the market and creating an inflationary bubble. Those who, at that time, were able to invest capital enjoyed this expansive policy, while those who could not, due to lack of funds to invest, became even more impoverished due to rising inflation.

Retail investors, who increased their presence within the stock market over the past year, got their revenge on the large hedge funds.

During January and February, Game Stop was not the only one that experienced the short squeeze, also other stocks such as AMC, Blackberry. Not level of short interest but the call-option volume was the best indicator of popularity of these companies on Reddit and made them subject of the short squeeze. Indeed, after the overshoot of the price, the AMC drop due to increase in number of outstanding shares, not reduced borrowing. On the other hand, BlackBerry shares have rallied, even without enough short interest for a "short squeeze".

In these events the traditional short squeeze was being super-charged by what happened in the options market. From the study carried out, it has been noticed that the options market, on the Game Stop stocks particularly, during the short squeeze, had anomalous behaviors.

Stated that stocks which allow options trading usually, guarantee a higher level of short interest ratio which is the main indicator of short squeeze chance, a great amount of out-of-the money call options were open during the month of December on Game Stop. In addition, during the squeeze, number of total options contracts trading exceed the total volume of transactions.

Treatise showed as there is not any dependence between the level of short interest and the difference between the put options implied volatility and call ones, because the level of GME implied volatility for calls remained very high even when the short interest was elevated.

Strongly dependence was experienced, through linear regression, between Put/Call ratio open interest and GME price, from January 12th to February 12th. This implied that a larger value of the ratio corresponded to a high price for GME.

The main phenomenon, which brought the price level of Game Stop to very high levels was the gamma squeeze, meaning the market makers need to buy more stock as it rises. This aspect of the options market adds yet another buyer of stock as it rises. Hence market makers bought increasing amounts of stock to cover the put-of-the money calls (as volatility increases and as time expiration approaches, game stop options experienced larger delta and lower gamma).

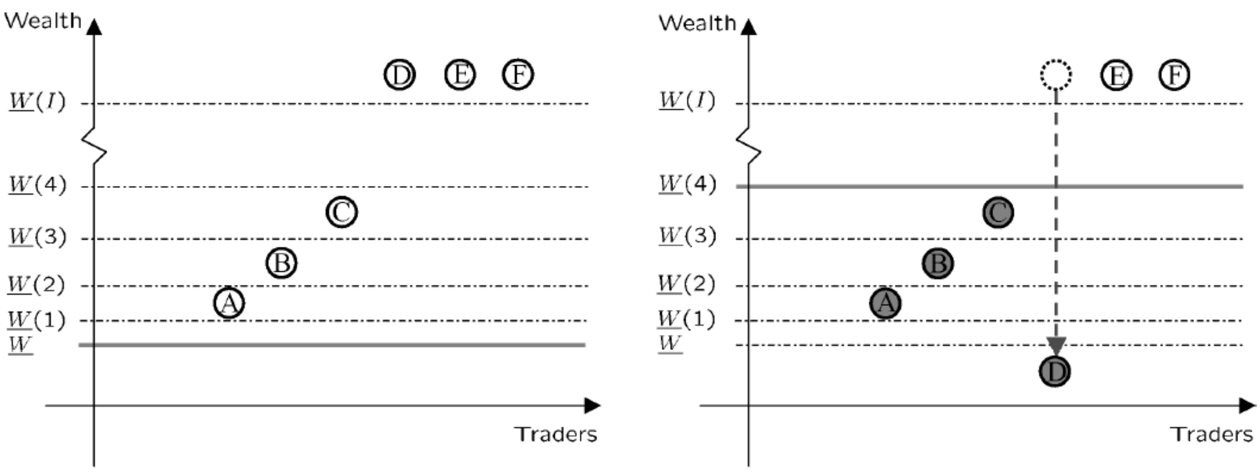
Finally, the high level of volatility experienced made the options not moldable with classic option pricing models and created an anomaly mainly in puts price. For the first time in the market high volatility level corresponded to a bearish market.

Option contracts played a dominant role within the short squeeze, increasing the effectiveness of this phenomenon. Indeed, compared to previous short squeeze cases, the price of Game Stop, even after months remained relatively high, more than before the squeeze.

Using options, retail investors have been able to challenge hedge funds and influence partially the market. The problem with hedge fund, which are based on fundamentals or value analysis, is that short positions are often balanced by long positions. Closing short positions also implies closing long positions, placing on the market valid securities with good fundamentals.

This means if retail investor were able to create the same phenomenon on more capitalized stocks than Game Stop or AMC, they could completely influence the current market structure.

Ultimately, this phenomenon can create a sort of "spillover effect".



[39] Chart

[39] Markus K. Brunnermeier and Lasse Heje Pedersen, "Predator Trading" Journal of Finance 2005.

Indeed, assumed these six hedge funds within the chart and W (i=1,2….I) the wealth necessary to survive if the market believes that hedge funds will be in distress.

Now if one of hedge fund runs out of capital, it moves down (like hedge fund D in the chart) and the line pushed up. This leads to predatory trading, which can drag traders A, B and C in distress to. If the price is pushed up, one hedge fund goes out of its business, and it liquidates. In this way it pushes the price up more, bringing the other three hedge funds into liquidation. Hence in this case market face a systemic risk of fragility where there are multiple equilibria.

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DATASET:

Bloomberg data

FACTSET data

Secform4 data

JMP securities data

Statista data

Alpha Query data