Department of Economics and Finance

Chair of International Finance

The Fiscal Theory of the Price Level: Theoretical Foundations and Empirical Analysis

Supervisor:
Prof. Guido Traficante

Candidate:
Sara D'Ecclesiis
715441

Co-supervisor:
Prof. Giorgio Di Giorgio

Academic Year 2020 - 2021

Contents

List of Figures and Tables

Introduction

When the COVID-19 pandemic started spreading around the world in 2020, no one could imagine the extent of the consequences on the economy. Immediately, the health crisis turned into a real economy crisis. Due to restrictions, consumption and all small businesses suffered a setback: retailing, tourism, catering industry, etc., recorded unprecedented losses. Governments were forced to take extraordinary measures to face this crisis and support financially families and businesses. Moreover, in addition to these expenses, other significant costs were born to purchase vaccines, organize their distribution, create infrastructures for their administrations, and to reopen public spaces such as schools with safety.

United States had to tolerate many expenses as all the other countries but what has risen the interest of many economists is the extent of the measures laid down by the American Rescue Plan Act of 2021, proposed by the newly elected President Joe Biden. His unprecedented fiscal stimulus opened a debate among notorious economists over the concern of inflation risk and the validity of the Fiscal Theory of the Price Level. This theory is relatively modern and it investigates the dynamics of the inflation related to fiscal policy. This approach differs from other theories, such as the monetarist one: while the latter relates changes in price level to changes in the quantity of money, fiscal theory claims that prices adjust in order to make the nominal debt of the government equal to the present value of primary surpluses.

In this thesis I will analyze Fiscal theory, its critiques and developments, and I will also carry out an empirical analysis to test whether it is valid. In the first chapter I will discuss about Biden's fiscal stimulus, by analysing the main points of the American Rescue Plan Act. Then, I will report the interviews and articles of economists such as Lawrence Summers, Paul Krugman, Olivier Blanchard, Ricardo Reis and Jean-Pierre Landau over their opinion on the U.S. fiscal policy and the risk of inflation. On one hand, Summers, Blanchard and Landau are those who expresse more concern over this risk, claiming that the measures taken by the U.S. government could lead to an overheated economy. On the other hand, Krugman and Reis are convinced that these government expenses are necessary, and that inflation would rise only in the short-run. Given this general framework, I will analyse the roots of the Fiscal Theory starting from its precursor, i.e., Sargent and

Wallace. In 1981 they published a paper where they analysed the interaction of monetary and fiscal policy and the effects on price level determination. After this introductory analysis, I will present the contribution of Leeper, who showed how ‘active’ fiscal policy can uniquely determine price level. Then I will go through the contribution of (Sims, 1994) and (Woodford, 1995) who claimed the importance of fiscal policy in price determination. Finally, I will analyse the main parts of (Cochrane, 2021), one of the most recent and harmonised works on the Fiscal Theory of the Price Level. This thesis would not be complete without the inclusion of critiques over this theory. For this reason, in the second chapter I will talk about main scepticism addressed by some economists, such as (Buiter, 1999) and (Niepelt, 2004). Then I will go through new recent contributions to the Fiscal Theory. The first one is (Berentsen & Waller, 2018) which elaborates a new version of the Fiscal theory based on the assumption that market value of government debt includes a liquidity premium which reflects not only the claim on the stream of future surpluses, but also its value for trading; the second is (Bassetto & Cui, 2017) which analyses the Fiscal Theory in a context of low interest rates. In the third and last chapter I will give my contribution to the empirical testing of the Fiscal Theory. Specifically, following the approach of (Canzoneri, et al., 2001), (Bajo-Rubio, et al., 2009), and (Afonso & Tovar Jalles, Revisiting fiscal sustainability, 2012), I will try to establish if the empirical evidence would support the assumptions of the Fiscal Theory of the Price Level for the case of G7 countries over the period 1980-2020 (except for U.S. data which cover the period 1960-2019).
Hence, my main contribution with respect to the past works is taking in consideration the most advanced countries over a period which includes some crucial economics events such as the global financial crisis of 2007-2008, the European sovereign debt crisis of 2010 and the COVID-19 pandemic of 2020. I will investigate in which way the fiscal solvency is achieved: by endogenously adjusting the primary budget surplus (i.e., in the “monetary dominant regime”) or by endogenously adjusting the price level (i.e., in the “fiscal dominant regime”). For the purpose of this analysis, I will use two approaches: a backward-looking one and a forward-looking one. In the backward-looking approach I will firstly run Phillips-Perron tests and Dickey-Fuller tests to verify the stationarity of the time series of

primary surplus and debt. Then, I will compute a dynamic ordinary least squares regression of primary surplus on lagged debt, both as percentage of GDP. A positive coefficient in that regression would mean that the analyzed government is solvent, i.e., it satisfies its present-value budget constraint, while a coefficient equal to or smaller than zero would show that government is not solvent and that a Fiscal regime dominates. This last result would support Fiscal Theory of the Price Level. After the regression, I will run cointegration tests to assess whether the two time series are cointegrated, i.e., if they move with a common trend. Granger causality tests will follow to study the 'predictive' causality of the two variables. Finally, for the forward-looking approach I will use the VAR model to compute the response functions of debt to innovations in the primary surplus for all the countries under investigation.

Chapter 1 | The Fiscal Theory of the Price Level

1.1 American Rescue Plan Act of 2021

On March 11th, 2021, Joe Biden, the president of the U.S. signed into law the American Rescue Plan Act of 2021. The $1.9 trillion dollars stimulus is the largest package of measures in the history of U.S.

It aims to rescue the economy from the crisis generated by the COVID-19 pandemic, to recover through massive investments, and to invest in racial justice.

The plan contains three main points:

- The organization of a vaccination program at a national level, together with additional measures to contain COVID-19, and the reopening of schools
- A significative economic aid to working families
- Support to the communities which were more affected by the pandemic.

1.1.1 National vaccination program, COVID-19' s containment, reopening of schools

The first objective encompasses a national vaccination program with the collaboration of states, localities, tribes and territories through the allocation of $50 billion to the Federal Emergency Management Agency. But vaccination is just one tool in the fight against the COVID-19[1]: in fact, the rescue plan provides $47.8 billion for a massive expansion of testing, tracing and monitoring[2]. In addition, Biden's plan provides funding in order to expand the health workforce, to address health disparities, to purchase supplies and protective gears, as well as treatments for COVID-19, and to deploy National Guard.

Another crucial point is safe reopening of schools and the expansion of the Higher Education Emergency Relief Fund: in fact, a total of almost $123 billions have been allocated for primary and secondary schools to be reopened within 100 days with

[1] One Hundred Seventeenth Congress of the United States of America, SEC. 4005. FEDERAL EMERGENCY MANAGEMENT AGENCY APPROPRIATION.

[2] One Hundred Seventeenth Congress of the United States of America, Subtitle E, SEC. 2401. FUNDING FOR COVID–19 TESTING, CONTACT TRACING, AND MITIGATION ACTIVITIES.

the necessary equipment and staff in order to ensure social distancing[3]; moreover, almost $40 billion have been assigned to colleges and universities, especially for emergency grants to students to remain available through September 30, 2023[4].

1.1.2 Relief to working families suffering the crisis

One third of households and half of Black and Latino households encounter difficulties paying bills, mortgages, rents or buying groceries. In order to make up for these difficulties, President Biden's plan gives working families a monthly $1,400 per-person check.

Furthermore, in addition to the extension of the unemployment insurance program, the plan provides a $300 supplement per-week through September 6, 2021.

Another important aspect is the tax provisions. The plan broadens the child tax credit[5]: for the 2021 tax year, a qualified family can compensate $3,000 per child who has not attained age 17 and $3,600 per child under age 6. Moreover, the Act expands and increases the child and dependent care credit by making it refundable[6].

Aside from these measures, the rescue plan contains some provisions about funding for housing program to help families to cover their housing expenses.

The emergency assistance programs provide states and local governments with $21,55 billion which in turn will give grants to eligible grantees[7].

The Housing Choice Voucher Program allocates $5 billion to homeless or at risk of homeless people, as well as people who are fleeing from domestic violence, sexual assault, stalking, or human trafficking[8].

Other measures are:

---

[3] One Hundred Seventeenth Congress of the United States of America, SEC. 2001. ELEMENTARY AND SECONDARY SCHOOL EMERGENCY RELIEF FUND.

[4] One Hundred Seventeenth Congress of the United States of America, SEC. 2003. HIGHER EDUCATION EMERGENCY RELIEF FUND.

[5] One Hundred Seventeenth Congress of the United States of America, Part 2 Child Tax Credit, SEC. 9611. CHILD TAX CREDIT IMPROVEMENTS FOR 2021.

[6] One Hundred Seventeenth Congress of the United States of America, Part 4 Dependent Care Assistance, SEC. 9631. REFUNDABILITY AND ENHANCEMENT OF CHILD AND DEPENDENT CARE TAX CREDIT.

[7] One Hundred Seventeenth Congress of the United States of America, - Housing provision, SEC. 3201. EMERGENCY RENTAL ASSISTANCE

[8] One Hundred Seventeenth Congress of the United States of America, - Housing provision, SEC. 3202. EMERGENCY HOUSING VOUCHERS

- the allocation of $100 million for rural housing assistance programs[9]
- the same amount for housing counseling services addressed to low-income workers or people facing housing problems[10]
- the establishment of a fund for homeowner assistance[11].

1.1.3 Critical support to struggling communities

One of the most hit communities during the pandemic is the one of the small businesses' owners. For this reason, the rescue plan contains many provisions which provide for to this problem.

The section dedicated to this aspect is the Title V of the Rescue plan. However, compared with other provisions, these aids are less significative[12].

The amount of $28.6 billion has been directed to the so-called "Restaurant Revitalization Fund" in the form of grants.

Moreover, $15 billion have been allocated in targeted aid for businesses seeking Economic Injury Disaster Loan Advance grants (EIDL) and $1,25 billion for the shuttered venue operators.

In the end, there is the extension of the Paycheck Protection Program (PPP) with the addition of a $7,25 billion grant, and also of the Employee Retention Credit, a refundable tax credit which allows employers who can't run their activities due to restrictions to continue paying salaries.

1.1.4 Emergency funding to upgrade federal information technology

Apart from the provisions addressing vaccinations, schools and small businesses, the plan also includes a section dedicated to the improvement of cybersecurity and, in general, the federal information technology.

Firstly, $650 million, available until September 30, 2024, have been allocated for the Cybersecurity and Infrastructure Security Agency in order to mitigate cybersecurity risk.

---

[9] One Hundred Seventeenth Congress of the United States of America, - Housing provision, SEC. 3203. EMERGENCY ASSISTANCE FOR RURAL HOUSING.

[10] One Hundred Seventeenth Congress of the United States of America, - Housing provision, SEC. 3204. HOUSING COUNSELING

[11] One Hundred Seventeenth Congress of the United States of America, - Housing provision, SEC. 3206. HOMEOWNER ASSISTANCE FUND

[12] One Hundred Seventeenth Congress of the United States of America, TITLE V – COMMITTEE ON SMALL BUSINESS & ENTREPRENEURS

Secondly, the plan provides the U.S. Digital Services with $200 million[13] and makes available $1 billion to the Technology Modernization Fund to allow it launching major new IT and cybersecurity shared services[14].

1.2 Current debate on Biden's stimulus

After the announcement of the new fiscal stimulus proposed by Biden to recovery the economy and emerge from the crisis, some notorious economists have launched a debate over the concern of inflation risk.

Some of them are concerned about this risk argue that the $1,9 trillion stimulus would widen the output gap in an unprecedented way, increasing the probability of higher inflation. The other group of economists agree with them on the fact that there could be an increase in inflation, but they are convinced that this increment would be temporary.

Before going on with the overview of different economists' opinions, it is important to understand the meaning of output gap and how it is computed.

The Output Gap

The Output Gap is the difference between the effective and the potential output, which is the maximum value that can be obtained by efficiently using the production factors of an economic system under stable inflation conditions. Potential output is, however, an unobservable and highly uncertain variable; empirical measures vary greatly depending on the econometric approach adopted, the specification chosen for the data-generating process and the series used, both in history and in forecasting.

In the United States, the Congressional Budget Office (CBO) has recently documented the evolution of its model, which is based on the production function. CBO's estimates mostly rely on the framework of Solow model and on Okun's law. The former has its focus on the supply side of the economy (labor and productive

[13] One Hundred Seventeenth Congress of the United States of America, TITLE IV – COMMITTEE ON HOMELAND SECURITY AND GOVERNMENTAL AFFAIRS, SEC. 4010. APPROPRIATION FOR THE UNITED STATES DIGITAL SERVICES

[14] One Hundred Seventeenth Congress of the United States of America, TITLE IV – COMMITTEE ON HOMELAND SECURITY AND GOVERNMENTAL AFFAIRS, SEC. 4011. APPROPRIATION FOR THE TECHNOLOGY MODERNIZATION FUND

services provided by capital), the latter associates changes in output to those in unemployment.

1.3 An ambitious but risky plan: opinion by Lawrence Summers and debate with Paul Krugman

The first economist to question Biden's fiscal policy was Lawrence Summers. In an article on the Washington Post of February 4th, 2021, he recognizes the importance of a recovery plan for providing relief to people hit by pandemic but at the same time he is concerned about the extent of these measures[15]. For this purpose, Summers makes a comparison between the fiscal stimulus implemented in 2009 by the Obama's administration in response of the Great Recession, and the one proposed by Biden. On one hand, Obama's stimulus added an incremental $30 billion to $40 billion a month to the already existing output gap of $80 billion of 2008, that means half the difference between effective and potential output.

On the other hand, Biden's measures will be three times the existing output gap. Obviously, as said in the previous paragraph, these kinds of calculations are very approximative and we should take everything with a grain of salt. Moreover, work conditions, monetary policy and demand of the two periods are different: while in the pandemic scenario unemployment is falling, in 2009 it was increasing; monetary policy was tighter in 2009 than today; furthermore, today consumers are willing to spend all the money they could not spend for the restrictions due to lockdowns.

Another aspect to be analyzed in order to understand the scope of these measures is comparing the loss of wages and salaries with the benefits earned thanks to the rescue plan through tax credits and direct payments.

According to Summers, while wages and salaries are $30 billion a month below the pre-covid projections, the increase in direct payments and tax credits will be around $150 billion, i.e. 5 times the losses. The risk is that due to these benefits a person would opt not to work.

[15] "Opinion: The Biden stimulus is admirably ambitious. But it brings some big risks, too", https://www.washingtonpost.com/opinions/2021/02/04/larry-summers-biden-covid-stimulus/

In the end, the economist argues that there are two important issues to be considered: the first is the risk of higher inflation that could be avoided with the coordination between fiscal and monetary policy. In this case, the problem is that Fed is committed to keep the same level of inflation and, in addition, the expectations of inflation could increase if the government is not willing to cut spending or increase taxes in the eventuality of an overheating economy.

The second issue is related to the fact that the measures taken in the Rescue plan for new investments infrastructures and to reduce social inequalities are a small proportion of the fiscal stimulus.

The opinion of the Nobel winning economist Paul Krugman is different: on February 12th, 2021, in a joint interview with Summers, Krugman claimed that he did not share Summers' concern over the inflation risk[16]. In fact, according to him the recession that the United States (and the world in general) is experiencing is unconventional and we are still in a partial lockdown due to restrictions but also to people's voluntary decision to stay at home. This phenomenon implies a "suppressed output" which is not well-described by the concept of output gap. Consequently, as this is a temporary situation, a budget deficit is appropriate. He has just one criticism to make about it, that is related to the inclusion of generalized checks.

In addition, according to Krugman the expectations about inflation are the most important issue. Inflation is a succession of long-term regimes driven by expectations. These regimes are very resilient, expectations are well-anchored so, it is quite unlikely that they would change now only for Biden's stimulus.

1.3.1 Blanchard's support toward Summer's concerns

The economist Olivier Blanchard shares Summers' concerns and is skeptic about the fact that the output gap created through Biden's stimulus would be completely filled by the end of 2021 by an increase in demand[17]. In fact, this increase depends on multipliers which in turn are influenced by consumers' attitude and liquidity constraints: the more optimist consumers are, the higher the multipliers. An issue

[16] "A conversation with Lawrence Summer and Paul Krugman" https://www.youtube.com/watch?v=EbZ3_LZxs54&t=1s

[17] "In defense of concerns over the $1.9 trillion relief plan" by Olivier Blanchard, feb 18, 2021

about these indexes is that they are not well-estimated, and they influence the marginal rates of consumption in a non-linear way.
As far as the risk of inflation, Blanchard is worried about the consequences of an overheated economy: expectations could change, and inflation could rise significantly. In this prospective, the Fed could react in two ways with undesirable implications. In the first case, it could leave inflation to rise. It would mean that expectations on inflation would change, throwing away the monetary policy's achievements gained over the last 20 years and making the instruments of monetary policy more difficult to use in the future. In the second case, it could implement a tighter monetary policy. This would imply higher interest rates with dramatic consequences in the financial markets.

1.4 The need of a revitalized approach of central bank independence: opinion by Jean-Pierre Landau

In an [18]article of February 8th, 2021, Jean-Pierre Landau gave his opinion over the debate opened by Summers. In Landau's vision, there are two important issues: how a shock would impact the economy in the short term, and if it would influence the long-run dynamics. According to him, the risk is that the unprecedented increase of the public debt and the high output gap would create not only an instantaneous and temporary inflation, but also a change in the anchored expectations on future inflation i.e. a change of regime. In this prospective, Landaus argues that there may be the need of a new central banking approach where the central bank is less committed in solving the time-inconsistency problem and more in keeping its ability to take any necessary decision and avoiding fiscal dominance[19].

1.5 Interview to Ricardo Reis: fiscal sustainability

Inflation has never been such a controversial theme as today. Ricardo Reis, economist and professor at the London School of Economics, in an interview done

[18] "Inflation and the Biden stimulus", https://voxeu.org/article/inflation-and-biden-stimulus

[19] Fiscal dominance is the situation in which expansionary fiscal policies are accommodated by monetary policy in order to unburden the debt

by David Beckworth of the Mercatus Center, stated that he expects inflation to remain at the current level but, anyway, he is really uncertain about future dynamics[20].

Actually, according to Reis one can look at the situation from different points of view or, in other words, through four camps of macroeconomic theories, all consistent among them.

The first one is the “interest rate view”. At the moment, it is unthinkable that interest rates will go much lower. Besides this, given the actual amount of debt, the increase in interest rates would impact negatively on the Treasury. In addition, the Fed has committed to keep interest rates at these levels. This type of situation where we cannot rise nor lower interest rates is known as the “interest rate peg” from the studies of Friedman. According to Friedman, monetary policy cannot peg interest rates for more than very limited periods (Friedman, 1968). In fact, in order to keep the interest rates down the Fed starts purchasing securities, raising their prices and lowering their yields. A further implication is that the quantity of reserves held by banks increases, and the amount of credit extended by banks increases as well. Hence, in total the amount of money in circulation raises. Initially, the faster rate of monetary growth makes the interest rates lower than the level they would have otherwise had. After this initial effect, the larger quantity of available money, stimulates private spending. One’s more spending corresponds to another one’s income and, consequently, more demand for loans. All these effects lead to a reversal of the downward pressure on interest rates in less than one year. Moreover, it is likely that interest rates will get to an even higher level than the initial one. Since the higher rates lead to higher prices, public expectations change resulting in rising prices expectations. That’s why in subsequent periods every attempt to keep interest rates low forces the monetary authority to be involved into larger and larger open markets operations. In such a scenario, there is inflation indeterminacy.

The second point view is the monetarist one. Right now, we are witnessing a significant increase in the size of banks’ balance sheets. Two alternatives could

[20] “Ricardo Reis on Central Bank Swap Lines, Fiscal Sustainability, and Outlooks for Inflation”, https://www.mercatus.org/bridge/podcasts/02152021/ricardo-reis-central-bank-swap-lines-fiscal-sustainability-and-outlooks

happen: people could ask for more currency; then banks would exchange reserves for currency with the Fed; this increase in money supply would lead to an increase in the price level. On the other hand, people could decide not to spend the checks received through the Rescue plan and to deposit them in banks. In turn, banks would deposit currency at Fed which would use it to buy government bonds; in this case, the inflation would be too low.

The third point of view is looking at the Phillips Curve: also in this case we don't have any certainty. We are at a point where it could move up or down.

The fourth perspective is the fiscal theory. In fact, according to it the large amount of outstanding debt could mean a higher inflation. Reis is not really concerned about this risk. In his recent paper (Reis, 2021) he analyses the implications for government debt of having $r < g < m$ where r is the long-term real interest rate on U.S. government debt, g defines the growth rate of output and m is the marginal product of capital[21]. Reis observed that in the last century, and even more in the last decade (with the exception of 2020 characterized by the global pandemic), the real interest rate r has always been lower than the output growth rate g. Moreover, U.S. data show that the marginal product of capital, has remained constant and always above g. In such a scenario, where despite the bubble ($r < g$) the economy is dynamically efficient ($r < m$), he finds a well-defined budget constraint on government debt and a limit to spending. The novelty of this paper is the definition of a "bubble premium" on the debt which is the difference between the return that private agents can earn on the marginal unit of capital and the one earned on government bonds (m-r). The presence of this premium allows the government to run permanent deficits, of a small size, even with a positive outstanding debt. Hence, in the end Reis highlights the importance of the bubble m-g for the fiscal sustainability and spending limit. The smaller this premium, the more sustainable the debt is. Consequently, Reis is not concerned about the risk of being forced to inflate the debt since today this premium is not so large.

Anyway, only time will tell us what will happen.

[21] The marginal product of capital gives the private return of investing in production as opposed to in the government debt (Reis, 2021)

1.6 A theoretical approach: the origin of Fiscal Theory of Price Level

The concerns over Biden's fiscal stimulus have reopened a debate over the Fiscal Theory of Price Level which investigates the dynamics of the inflation related to fiscal policy. Even if this theory is still discussed by some economists such as John Cochrane, it has its roots in the past and precisely, in the 80's.

The determination of price level has always been a central theme in the economic theory. There are three main theories: the first is no more applicable today and it was based on the gold standard. In such a monetary system the value of money is given by the gold reserves hold by central banks.

The second theory is the quantity theory of money. It relates changes in price level to changes in the quantity of money in circulation.

This relation is described by the following formula:

$$MV = PY$$

Where:

- M is the money in circulation over a certain period
- V is the velocity of circulating money
- P is the price level
- Y is the national real income.

By the way this theory was overcome since now central banks do not follow money supply targets.

The third theory is based on Taylor's principle and it argues that stable levels of inflation can be achieved only through an interest rate targeting from the central bank.

The fiscal theory of price level claims that prices adjust in order to make the nominal debt of the government equal to the present value of primary surpluses (Cochrane, 2021).

Before going through this theory, it is worth investigating its origins.

1.6.1 Sargent and Wallace: the precursors of the Fiscal Theory of the Price Level

The first two economists who brought to light the interweaving of monetary and fiscal policy were Sargent and Wallace in their paper "Some Unpleasant Monetarist Arithmetic" published in 1981.

They wanted to demonstrate that in an economy which satisfies the monetarist assumptions, where the monetary authority implements open markets operations, then this authority cannot control inflation (Sargent & Wallace, 1981). The authors claimed that the control by the monetary authority could happen only in certain circumstances, for example when monetary and fiscal policies are coordinated in a certain way and the public's demand for interest bearing government debt has a certain form.

According to them, the demand for government bonds put two constraints on government:

1) Setting an upper limit on real stock of government bonds relative to the size of the economy
2) by affecting the interest rate that the government must pay on bonds.

Sargent and Wallace show two forms of coordination. The first one is characterized by the dominance of monetary policy over the fiscal policy. In fact, the monetary authority independently decides about monetary policy. Consequently, the fiscal authority has to manage its current and future deficits financing them through a combination of the seigniorage chosen by the monetary authority and the sale of government bonds.

Under this scenario, the monetary authority can keep inflation at the desired level.

The second type of coordination is the one in which the fiscal policy prevails. The fiscal authority sets the current and future deficits or surpluses. Only after this decision, the monetary authority can set the amount of reserves. If the sale of new bonds can't finance all the deficit, then the monetary authority is forced to tolerate higher inflation. In this type of scenario, the form of the demand of government bonds determines whether or not the monetary authority can control the inflation. In fact, it can control inflation keeping low the growth of base money and allowing the fiscal authority to continue selling bonds. The problem arises when the interest rates on government bonds become higher than the growth rate of the economy: at that point the growth of real stock of bonds will be faster than the growth of the size of the economy.

In order to show these two schemes, the authors make two crucial assumptions:

i. the real rate of interest is greater than the growth rate of economy

ii. fiscal authority sets its path of fiscal policy D(t) which is not influenced by current or future monetary policy

The paper analyses the situation where the fiscal authority moves first and the monetary one has to adapt to its decisions about the annual deficit or surplus. It is proved that if the D(t) sequence is too big for too long, then the central bank must generate some seigniorage revenue to pay the debt off. Moreover, since the central bank is free to decide when to print more money, the paper shows that a tighter monetary policy and less inflation now will require a looser monetary policy and more inflation later.

As far as this last point, it was a clear reference to U.S. situation, characterized by many annual deficits and a monetary contraction to fight inflation (Cochrane, 2021). In fact, Sargent and Wallace were concerned about the risk that a fiscal policy with deficits together with a monetary contraction, would have led to a temporary decline of inflation and to more severe inflation in subsequent periods. However, as history showed, their predictions did not come true and by 90's U.S. returned to fiscal surpluses.

In conclusion, the main contribution of Sargent and Wallace's paper was the study of fiscal and monetary policy interactions[22].

1.6.2 Leeper's contribution to the birth of Fiscal Theory of Price Level

The first important contribution to the Fiscal Theory of Price Level (FTPL) was given by Eric M. Leeper in 1991 in his paper "Equilibria under 'active' and 'passive' monetary and fiscal policies". In fact, he demonstrated that under a 'passive' monetary policy and 'active' fiscal policy regime the price level equilibrium can be uniquely determined through the active fiscal policy.

The author categorizes equilibrium policies as representing 'active' or 'passive' behavior (Leeper, 1991).

The active authority has a budget constraint which holds through an appropriate change in prices, and therefore its decision rules can be based on past, current, or expected future variables. On the other hand, the passive authority must take into account government debt shocks and its decisions are tied to the active authority's actions and to private optimization.

[22] See "The Fiscal Theory of the Price Level" by John H. Cochrane, 2021, p. 557

Another assumption is that the monetary authority sets the interest rate as a function of the current inflation rate while the fiscal authority sets lump-sum taxes as a function of the current level of debt.

After these definitions, the author investigates four regions of the policy parameter space.

The first region is characterized by an active monetary policy and passive fiscal policy. Under this scenario, the monetary authority's primary goal is price stability and so it strongly reacts to inflation. On the other side, the fiscal authority passively determines the level of taxes to balance the budget.

In the second region, fiscal policy dominates on the monetary one. Fiscal authority takes decisions over its budget as it sees fit. In turn, monetary authority adjusts its money supply to accommodate deficit stocks.

In the third region, both policies are passive. In such a case, price-level indetermination occurs, which is algebraically represented by a system with no unstable roots.

On the opposite, in the fourth region both authorities act actively. This dynamic ends up with two unstable roots that means no equilibria.

In the end, the aforementioned second scenario laid the foundation for the subsequent studies on the fiscal theory of price level.

1.6.3 The evolution of subsequent studies over the fiscal theory

Many important studies followed Leeper's model. One example is Sims's contribution in 1994 where he shows the importance of fiscal policy on the determination of price level, as well as the price indetermination in the case of a fixed money supply and the eventual existence of a unique, stable price level under a scenario of pegged nominal interest rates (Sims, 1994).

Woodford extended this analysis the following year in the paper "Price-level determinacy without control of a monetary aggregate". This was the first time that the term "fiscal theory of the price level" was used.

The author shows how price level determinacy is still possible in two cases where the money supply is endogenously defined (Woodford, 1995):

i. when the central banks opt for an interest rate peg
ii. in a "free-banking" regime

His model is characterized by infinite-lived households that want to maximize their lifetime utility defined as:

$$\sum_{t=0}^{\infty} \beta^t U(c_t, M_t/p_t) \tag{1.1}$$

where:

- $0 < \beta < 1$ is a constant discount factor and U(c, m) is a concave and increasing function in both arguments
- M_t is the nominal money held by households at time t. Its presence in the utility function must be interpreted as the convenience value of carrying out transactions using money
- c_t is the consumption in period t
- p_t is the money price of goods.

Households choose their level of consumption, the money holdings M_t, and end-of-period bond holdings, B_t under the following budget constraint:

$$p_t c_t + M_t + B_t \leq W_t + p_t y_t - T_t \tag{1.2}$$

where:

- W_t represent the nominal value of beginning-of-period wealth
- y_t is the real income (in terms of consumption goods owned by the household)
- T_t is the lump-sum taxes paid in period t.

Another assumption of this model is that the nominal wealth of the next period is given by:

$$W_{t+1} = M_t R_t^m + B_t R_t^b \tag{1.3}$$

where:

- R_t^m is the gross nominal return on monetary base from t to t+1
- R_t^b is the gross nominal return on one-period bonds from t to t+1

The level of consumption and of money holdings must always be non-negative; borrowing is allowed but there is a lower bound upon the value of W_{t+1} represented by the following constraint:

$$W_t \geq -\sum_{j=0}^{\infty} \frac{p_{t+j} y_{t+j} - T_{t+j}}{\prod_{s=0}^{\infty} R_{t+s}^b} \tag{1.4}$$

This bound is necessary to avoid Ponzi schemes.

Then, according to Woodford, we have a perfect foresight equilibrium if:

- the money demanded by the household equals the money supplied by the government
- the bonds demanded equal the amount of bonds issued by the government
- And, $c_t + g_t = y_t$ (1.5)

where g_t is defined as the government purchases of the good in period t.

On the other hand, in each period the government must satisfy the following financing constraint:

$$p_t g_t = T_t + (M_t - M_{t-1}R_{t-1}^m) + (B_t - B_{t-1}R_{t-1}^b) \tag{1.6}$$

The government can set at most three among the variables R_t^m, R_t^b, M_t, B_t.

This is due to the fact that if it sets the quantity of bonds supplied, then the market will determine their price; if instead it sets their relative price (i.e. by fixing R_t^m and R_t^b), then the households will decide the number of bonds that they want to buy at that price.

Now, let's analyze the requirements for the equilibrium[23].

Firstly, the budget constraint (1.6) can be rewritten in another form which explicitly represents the fact that the lifetime consumption and money-holding of the household must be non-negative for all $t \geq 0$ and can be written as:

$$\sum_{t=0}^{\infty} \frac{p_t c_t + \Delta_t M_t}{\prod_{s=0}^{t-1} R_s^b} \leq \sum_{t=0}^{\infty} \frac{p_t y_t - T_t}{\prod_{s=0}^{t-1} R_s^b} + W_0 \tag{1.7}$$

where:

- W_0 is the given initial wealth
- $\Delta_t = (R_t^b - R_t^m)/R_t^b$ is the interest rate differential which defines the "price" of holding money[24]

The optimal combination of consumption and holding-money is obtained if the following first-order conditions hold:

$$\frac{u_m(c_t, m_t)}{u_c(c_t, m_t)} = \Delta_t \tag{1.8}$$

$$u_c(c_t, m_t) = \beta(1 + r_t^b)u_c(c_{t+1}, m_{t+1}) \tag{1.9}$$

for all $t \geq 0$, where:

---

[23] The following model and conclusions are taken from Woodford's paper, "Price-level determinacy without control of a monetary aggregate"

[24] Because holding money the household does not earn interests

- $m_t = M_t/p_t$ are the real balances
- $r_t^b = R_t^b(p_t/p_{t+1})$ is the real rate of return on bonds

Moreover, the budget-constraint (1.7) must hold with equality.[25]

Since c_t and m_t are normal goods, (1.8) can be inverted and rewritten to obtain:

$$m_t = L(c_t, \Delta_t) \tag{1.10}$$

with L increasing in c and decreasing in Δ.

If we substitute (1.5) into (1.8), then the equilibrium condition becomes:

$$m_t = L(y_t - g_t, \Delta_t) \tag{1.11}$$

This equation is a money demand equation where the demand depends only on private purchases.

Replacing (1.5) into (1.9) results in:

$$\lambda(y_t - g_t, \Delta_t) = \beta(1 + r_t^b)\lambda(y_{t+1} - g_{t+1}, \Delta_{t+1}) \tag{1.12}$$

with $\lambda(c, \Delta) = u_c(c, L(c, \Delta))$.

Finally, we can obtain the equality budget-constraint[26]

$$\frac{W_0}{p_0} = \sum_{t=0}^{\infty} \frac{(\tau_t - g_t) + \Delta_t m_t}{\prod_{s=0}^{t-1}(1 + r_s^b)} \tag{1.13}$$

where $\tau_t = T_t/p_t$ is the real tax revenue for each period.

In words, the present value of future primary government budget surpluses must equal the amount of current net government liabilities.

Consequently, in each period t, government liabilities must satisfy

$$\frac{W_t}{p_t} = \sum_{s=t}^{\infty} \frac{(\tau_s - g_s) + \Delta_s m_s}{\prod_{j=t}^{s-1}(1 + r_j^b)} \tag{1.14}$$

with:

$$W_t = R_t^b[W_t + p_t(g_t - \tau_t - \Delta_t m_t)], \tag{1.15}$$

that is the law of motion of W_t.

So, in contrast with the traditional quantitative monetary view, Woodford focuses more on the last equilibrium condition than on condition (1.11) because he considers these fiscal variables fundamental to price level determination (it being understood that the formula (1.11) must be also satisfied).

From (1.14) it is clear that if there is an increase in nominal government debt or in future deficits, then households perceive these changes as an expansion of their

---

[25] For condition of existence, left- and right-side of (7) must be finite

[26] Substituting (5) into (7)

budget. Consequently, their consumption demand raises with the effect of an excess of demand for goods. This ~~dynamic inevitably~~ pushes up prices[27] in order to allow households to buy all goods that the market can offer.

Therefore, this model innovates by focusing on expectations over future government budgets and on the nominal value of government debt which are responsible for price determination.

Woodford then shows how it is possible to still have price determination in two typical cases where there is no control over money supply: interest rate-peg and free-banking.

Interest rate-peg

In this scenario, government policy exogenously fixes four variables, i.e. the sequence $\{R_t, R_t^b, g_t, \tau_t\}$, leaving the nominal debt free to vary on the base of households' demand and determined by the equation (1.6).

Then, substituting (1.11) and (1.12) into (1.14) we obtain

$$\frac{W_t}{p_t} = \frac{1}{\lambda(y_t - g_t, \Delta_t)} \sum_{s=t}^{\infty} \beta^{s-t} \lambda(y_s - g_s, \Delta_s)[(\tau_s - g_s) + \Delta_s L(y_s - g_s, \Delta_s)] \qquad (1.16)$$

Again, this equation relates price level at time t to the net government liabilities W_t and to the projected future policy variables.

Consequently, the equation (1.16) can be solved for a unique price level at time t[28] and it demonstrates that price determination is still possible in the case of an interest rate-peg.

Free banking

Woodford analyses also the case in which private intermediaries are allowed to create substitutes for government issued money (Woodford, 1995).

Firstly, he assumes that households want to maximize

$$\sum_{t=0}^{\infty} \beta^t U(c_t, \frac{M_t + D_t}{p_t}) \qquad (1.17)$$

In this formula, D_t represents the nominal value of households' deposits with an intermediary. Constraint (1.2) is still valid, but D_t must be added to the left-hand side while B_t is substituted by B_t^h that is the household bond holding.

---

[27] We must assume $W_t > 0$

[28] Assuming that in each period $R_s^m < R_s^b, g_s < y_s$

Also equation (1.3) must be modified adding the term $D_t R_t^d$ to the right-hand side, with R_t^d defining the gross return on deposits. The intermediary purchases government bonds using a fraction $1-\rho$ of D_t. The remaining fraction $0<\rho<1$ is used to cover the costs of intermediation.

Woodford claims that in order to have a competition among intermediaries with free entry, the following equation must hold

$$R_t^d = (1-\rho)R_t^b \tag{1.18}$$

Moreover, the market clearing in the bond market needs that

$$B_t^h + (1-\rho)D_t = B_t \tag{1.19}$$

On the other hand, in order to reach the equilibrium in the goods market,

$$c_t + g_t + \rho\frac{D_t}{p_t} = y_t \tag{1.20}$$

must be satisfied.

Households would hold money if $R_t^m \geq R_t^d$ and, vice versa, would hold deposits if $R_t^d \geq R_t^m$.

In this case, the demand for money-like assets would be

$$\frac{M_t+D_t}{p_t} = L(c_t, \Delta_t) \tag{1.21}$$

where the interest rate differential is now defined as

$$\Delta_t = \frac{R_t^b - max(R_t^m, R_t^d)}{R_t^b} \tag{1.22}$$

An alternative is to consider c*(e, Δ) and m*(e, Δ) as the solution to maximizing u(c, m) under the constraint $c + \Delta m \leq e$, where e is the household "total expenditure" written as:

$$e_t = c_t + \Delta_t\left(\frac{M_t+D_t}{p_t}\right) \tag{1.23}$$

In equilibrium, Δ_t must equal ρ; in fact, if $\Delta_t \geq \rho$ households will hold only deposits and if $\Delta_t \leq \rho$ they will hold only money.

So, the total supply of money is infinitely elastic at the nominal interest rate spread of ρ and this scenario is comparable with a regime of interest-rate peg[29].

[29] An interest rate peg is an interest rate that is constant over time and does not respond systematically to other variables (Cochrane, The Fiscal Theory of the Price Level, 2021)

At this point, Woodford shows how it possible to have price determination also in this context. He takes as example a policy regime where $\{M_t, R_t^m, \tau_t\}$are exogenously fixed[30], and where government purchases are determined by $g_t = z_t + \Delta_t m_t$, with $\{z_t\}$being an exogenous non-negative sequence; again, $\{R_t^b\}$ is determined in the bond market and $\{B_t\}$by the financial constraint (1.6).
Now, the equilibrium condition (1.14) takes the form

$$\frac{W_t}{p_t} = \frac{1}{\lambda * (y_t - z_t, \rho)} \sum_{s=t}^{\infty} \beta^{s-t} \lambda * (y_s - z_s, \rho)[\tau_s - z_s] \qquad (1.24)$$

with $\lambda * (e, \Delta) = u_c(c * (e, \Delta), m * (e, \Delta))$.
Then, also in this kind of situation, given (1.24) and an initial wealth W_t, a unique price level can be obtained.

In conclusion, Woodford showed that price determination is possible not only when money supply is exogenously defined but also in the case when it is perfectly elastic at a given short-term interest rate. This is the Fiscal Theory of Price Level which gives importance to the stability of expectations over the future government policy in order to ensure a price level stability.

1.7 Application of the Fiscal theory of the price level: Cochrane's contribution

One of the most important and recent contribution to Fiscal Theory was given by the economist John Cochrane. He got involved in these studies in the mid-1990s with the publication of "A frictionless View of U.S. Inflation" (Cochrane, 1998). Here, he gave a radical view of U.S. inflation analysing it with the assumption that money and monetary frictions are irrelevant. According to him, money is not necessary for price determination while nominal bonds are essential. In this paper he tried to compare fiscal theory with U.S. data but obtaining poor results. Moreover, he presented a model with long-term bonds. However, his work was not complete, and he developed a deeper long-term debt analysis in a new paper (Cochrane, 2001). Finally, he developed an organic and detailed book, entitled "The Fiscal Theory of the Price Level" published for the first time in 2020. Cochrane has always focused his studies on the asset pricing field and, as he says in the preface

[30] M_t and R_t^m are always greater than 0

of his book, they proved to be useful also in the Fiscal Theory research. According to him, the central equilibrium equation of Fiscal Theory can be easily associated with the valuation equation for stock prices. As stock prices are computed as the present value of future discounted dividends, in the same way surplus forecasts determine the price level. These two types of valuations have a common problem: discounting at a constant interest rate is not realistic and the result would be different from actual stock price (in asset pricing) and from actual price level (in the Fiscal Theory). By the way, all theories have always needed a bit of generality in order to be presented.

The Fiscal Theory of the Price Level

The first chapters of the book are dedicated to the analysis of the fiscal theory[31]. Cochrane begins showing two models, the first one that lasts one period and the other one that is intertemporal. In both models there are flexible prices, constant interest rates, and no risk premiums. These elements are added later to make the model more realistic.

The one-period model

The equilibrium equation of the one-period model is

$$\frac{B_{T-1}}{P_T} = s_T \tag{1.25}$$

In words, at the beginning of day T, bondholders own B_{T-1} one-period zero-coupon government bonds coming due on day T[32]. The government prints new cash in order to pay bondholders. The latter will use money for buying and selling goods and at the end of the day they will pay taxes $P_T s_T$ to the government where

P_T is the price level. In this way, money is absorbed. In fact, at the end of the day no one wants to hold cash or bonds and, for this reason, in equilibrium

$$B_{T-1} = P_T s_T \tag{1.26}$$

must hold (and can be rewritten as in (1.25)).

In this equation, B_{T-1} and s_T are predetermined; the only variable which adjusts to the other ones, is the price level P_T.

[31] The contents of the following part are taken from chapter 2 of "The fiscal Theory of the Price Level", by John H. Cochrane, February 27, 2021

[32] Each bond promises to pay $1

If prices are too low, it means that too much money has been printed and people have an excess on cash to spend more than they need to pay taxes. Consequently, they start spending it in more goods with the effect of increasing the aggregate demand more than the aggregate supply. So, in other words, too much government debt, relative to surpluses, creates a "wealth effect of government bonds" which raises the aggregate demand.

Therefore, the main difference between the monetary and fiscal view, is in the source of inflation. In the former, inflation is determined by a higher amount of money in the economy than needed to settle transactions, or than desired to maintain in liquid form or in assets, etc. In the latter, it is due to more money in the economy than needed to pay net taxes (Cochrane, 2021).

The intertemporal model

The simplest equation for an intertemporal model is

$$\frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j} \tag{1.27}$$

According to it, the price level adjusts so that the real value of government debt is equal to the present value of the expected future surpluses.

In the intertemporal model the government issues new debt at time t-1 and pays it off at time t. At time t, it issues new bonds $\{B_t\}$at price Q_t and collect taxes $\{s_t\}$, absorbing money from the economy.

The government has the following budget constraint:

$$M_{t-1} + B_{t-1} = P_t s_t + M_t + Q_t B_t \tag{1.28}$$

where:

- M_{t-1} is the money held overnight from t-1 to t. Holding it does not encompass any interest gain
- P_t is the price level at time t
- $Q_t = 1/(1 + i_t)$ is the price of the nominal bond and i_t is the nominal interest rate[33] .

The surplus is represented by the real "primary surplus", i.e. tax collected minus expenses without including interest payments.

[33] i_t is paid overnight between the end of day t and the beginning of day t+1

As far as households, they maximize

$$max\ E\sum_{t=0}^{\infty}\beta^t u(c_t) \tag{1.29}$$

and pay a flat proportion of their income *y*:

$$P_t s_t = \tau_t P_t c_t \tag{1.30}$$

Also each household has a budget constraint that is specular to (1.28).

Firstly, bonds and money holding must be non-negative.

Given the consumer's first order conditions and given that in equilibrium $c_t = y$, then the gross real interest rate is $R = 1/\beta$ and the bond price Q_t is equal to:

$$Q_t = \frac{1}{(1+i_t)} = \frac{1}{R}E_t(\frac{P_t}{P_{t+1}}) = \beta E_t(\frac{P_t}{P_{t+1}}) \tag{1.31}$$

If $i_t > 0$, each household does not hold money M_t at the end of the day[34], as he/she obviously prefers to hold bonds bearing interest at i_t; if $i_t = 0$, he/she can indifferently hold money or bonds since they are perfect substitutes: then, in following formulas B_t will be the sum of bonds and money.

Since in this model the interest rate cannot be less than zero, then we can eliminate money from (1.28) and the flow equilibrium condition becomes:

$$B_{t-1} = P_t s_t + Q_t B_t \tag{1.32}$$

Cochrane then substitutes the bond price Q_t into (1.32) and divides both sides by P_t getting:

$$\frac{B_{t-1}}{P_t} = s_t + \beta B_t E_t(\frac{1}{P_{t-1}}) \tag{1.33}$$

Moreover, also the following transversality condition[35] must hold:

$$\lim_{T\to\infty} E_t(\beta^T \frac{B_{T-1}}{P_T}) = 0 \tag{1.34}$$

Iterating (1.33), we get

$$\frac{B_{t-1}}{P_t} = E_t\sum_{j=0}^{\infty}\beta^j s_{t+j} \tag{1.35}$$

[34] During the day t, people go about their business and use the cash printed by the government at the beginning of the day to buy and sell goods. But this is not relevant to the Fiscal Theory because it analyses households' behavior at the end of the day.

[35] "The transversality condition for an infinite horizon dynamic optimization problem is the boundary condition determining a solution to the problem's first-order conditions together with the initial condition. The transversality condition requires the present value of the state variables to converge to zero as the planning horizon recedes towards infinity. The first-order and transversality conditions are sufficient to identify an optimum in a concave optimization problem. Given an optimal path, the necessity of the transversality condition reflects the impossibility of finding an alternative feasible path for which each state variable deviates from the optimum at each time and increases discounted utility" (Macmillan, s.d.)

Looking at equation (1.35) we have that B_{t-1} is predetermined, s_t does not respond to price level by assumption, and the same happens for the real interest rate.
The only variable that must adjust so that (1.35) holds is the price level P_t.
Cochrane's intuition is that price level adjusts as a stock price. In fact, as in asset pricing the asset price adjust to make the price per share equal to the present value of expected future dividends, in the fiscal theory the price level adjusts to make the price per share $1/P_t$ times the nominal debt B_{t-1} equal to the present value of future primary surpluses[36].
The author sees the nominal government debt as a "stock in the government".
If the present value of expected future surpluses decreases, then the price level rises to keep the real debt in equilibrium with these expectations. This is the same dynamic of stock prices which fall when the expected present value of dividends decreases.
In the case of a recession, when there are lower surpluses or even deficits, in order to soak up the money in excess, the government can sell more debt with the promise of achieving higher surpluses in the future to compensate it. Otherwise, without this promise, the only effect would be the lowering of Q_t, i.e. a higher inflation.

Out of equilibrium behavior

Cochrane highlights that price level is pushed to its equilibrium when one of the three consumers' optimality conditions are violated, i.e. zero money demand, intertemporal optimization and the transversality condition.
In the case of the violation of the zero-money demand, there is a situation in which price level is too low because current surplus and the revenues from bond sales do not soak up all the money. Another case occurs when prices are too low because debt sales absorb too much money (i.e. revenues are greater than the present value of future surpluses). In such an eventuality, consumers either violate their intertemporal first-order condition (buying too many bonds now and dis-saving later) or their transversality condition (buying bonds and holding them forever).

[36] In the Fiscal Theory surpluses are not exogenous since they are determined by tax and spending policies

A more realistic model

Since in the real world governments do not announce nominal debt sequences and surpluses sequences, Cochrane describes some more realistic policies which are closer to reality[37]. In particular, he focuses on interest rate targeting.

Moreover, he makes a distinction between "monetary" policy and "fiscal" policy. The former concerns change in debt B_t (with no change in surpluses) through interest rate targeting. The latter includes modification in future surpluses. Cochrane's definition of "monetary policy" is quite different from the traditional one: while the first one is related to interest targeting, the traditional one refers to money supply. However, this kind of "fiscal theory of monetary policy" is very similar to standard new-Keynesian models.

In order to better understand the different dynamics of monetary and fiscal policy, Cochrane splits the present value relation into expected and unexpected components. Starting from

$$\frac{B_t}{P_t}\Delta E_{t+1}(\frac{P_t}{P_{t+1}}) = \Delta E_{t+1}\sum_{j=0}^{\infty}\beta^j s_{t+j}$$

and manipulating it

$$\frac{B_t}{P_t}\frac{1}{1+i_t} = \frac{B_t}{P_t}\frac{1}{R}E_t(\frac{P_t}{P_{t+1}}) = E_t\sum_{j=1}^{\infty}\beta^j s_{t+j}$$

he gets

$$\frac{B_{t-1}}{P_t} = s_t + \frac{B_t}{P_t}\frac{1}{1+i_t} = E_t\sum_{j=0}^{\infty}\beta^j s_{t+j}$$

Then, he moves forward the time index in order to describe expected variables with *t* and unexpected with *t+1*:

$$\frac{B_t}{P_{t+1}} = E_{t+1}\sum_{j=0}^{\infty}\beta^j s_{t+1+j} \tag{1.36}$$

In the following paragraphs I will develop Cochrane's intuition that any change in future surpluses (i.e. in fiscal policy) results in unexpected inflation and that monetary policy only affects expected inflation.

[37] This part is taken from chapter 3 of "Fiscal Theory of the Price Level", by John H. Cochrane

Fiscal policy and unexpected inflation

As far as fiscal policy let's consider when people expect lower surpluses in the future. The natural consequence of this expectation is that the value of real debt must fall. Since government debt consists of one-period bonds, then people cannot get rid of it and the only alternative is buying goods and services. This causes in turn an increase in price level until the real value of debt equals the present value of future expected surpluses[38]. We can observe this dynamic in the following equation

$$\frac{B_t}{P_t}\Delta E_{t+1}\left(\frac{P_t}{P_{t+1}}\right) = \Delta E_{t+1}\sum_{j=0}^{\infty}\beta^j s_{t+1+j} \tag{1.37}$$

where B_t and P_t are predetermined.

Monetary policy and expected inflation

In order to analyze the effects of monetary policy over the inflation, Cochrane multiplies and divides (1.36) by P_t, then multiplies both sides by β and obtains

$$\frac{B_t}{P_t}\frac{1}{1+i_t} = \frac{B_t}{P_t}\frac{1}{R}E_t\left(\frac{P_t}{P_{t+1}}\right) = E_t\sum_{j=1}^{\infty}\beta^j s_{t+j} \tag{1.38}$$

where the first term is the revenue from bond sale at the end of time t; the right-side term represents the equivalence between the present value of future surpluses with real value of debt.

If we look at the case in which the government increases β_t without changing the future surpluses, then the variables which must vary are the bond price (Q_t)[39], the interest rate, and expected future inflation $E_t\left(\frac{P_t}{P_{t+1}}\right)$. More precisely, they must move one for one with the nominal debt B_t.

According to Cochrane, the bonds' sale without any change in future surpluses can be compared to a share split. In fact, in a share split (for example when each owner of one old share receives two new shares) there is an increase in the number of shares and investors expect no future increase in dividends. Consequently, the price drops. In the same way, the additional government bonds with the expectation of no future higher surpluses, lead to an instant change in the price level.

[38] In this model, inflation is affected for one period only.

[39] The bond price is given by $Q_t = \frac{1}{1+i_t} = \beta E_t\left(\frac{P_t}{P_{t+1}}\right)$

Another instrument of monetary policy could be the interest rate targeting. In such a situation, the government sets the interest rate on government bonds and their price. Investors in turn are free to choose the amount of bonds they are willing to buy. Also in this case, future surpluses remain unchanged. Again, expected inflation is determined.

Applying the Fiscal theory to new-Keynesian DSGE models

Cochrane shows that it is possible to apply fiscal theory via a "fiscal theory of monetary policy", i.e. incorporating standard new-Keynesian DSGE models into fiscal theory. In fact, very often central banks set an interest rate target and it is really interesting to study how shocks in interest rates affect the economy.

In order to better understand the connection with standard models, the author linearizes previous equations. For example, the interest rate target i_t is defined as

$$i_t \approx r + E_t \pi_{t+1} \tag{1.39}$$

As said before, (1.37) defines unexpected inflation at time t. Then, writing the real debt as $v_t \equiv B_t / P_t$, Cochrane linearizes (1.37) at time t+1 in this way

$$\Delta E_{t+1} \pi_{t+1} = -\Delta E_{t+1} \sum_{j=0}^{\infty} \beta^j \frac{s_{t+1+j}}{v_t} \tag{1.40}$$

It can be rewritten as:

$$\Delta E_{t+1} \pi_{t+1} = -\varepsilon_{t+1}^{s} \tag{1.41}$$

with ε_{t+1}^{s} representing the shock to the present value of surpluses, scaled by the value of debt.

Summing up the expected future inflation at time t with the unexpected future inflation

$$\pi_{t+1} = E_t \pi_{t+1} + \Delta E_{t+1} \pi_{t+1} \tag{1.42}$$

Cochrane obtains the full solution of the model[40] where the path of inflation is a function of monetary and fiscal shocks:

$$\pi_{t+1} = i_t - \varepsilon_{t+1}^{s} \tag{1.43}$$

In the figure 1.1, the first graph represents the response of the model when a permanent interest rate shock occurs at time 1 with no fiscal shock ($\varepsilon_1^s = 0$), while

[40] Cochrane drops r from relation (1.39)

the second shows the response to a fiscal shock ($\varepsilon_1^s = -1$) at time 1 with no interest rate change.

In the former case, inflation increases one period later and remains at that level; in the latter, there is a one-period inflation (inflation goes back to the previous state at time 2). When the fiscal shock is announced in advance, the inflation happens at the moment of the announcement (pink line).

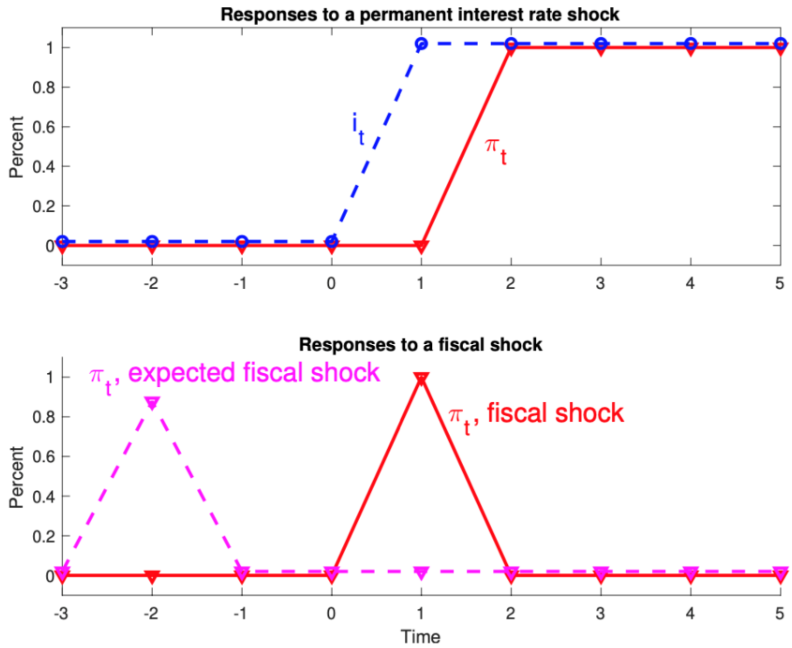

Figure 1.1 - Inflation response functions (Source: The Fiscal Theory of the Price Level, John H. Cochrane, Chapter 3 p. 30

According to Cochrane, these responses are not realistic but consistent with a frictionless model. Specifically, they are very similar to a "Fisherian" monetary policy response where an interest rate rise brings inflation to a higher level, one period later.

Changes in government debt and fiscal policy

As said before, fiscal policy concerns change in future surpluses. Obviously, any decrease in future surpluses must be compensated by an increase in nominal debt, so a change in B_t. If government raises debt without increasing the expected surpluses, then the interest rate rises, bonds' price falls and inflation increases, keeping the real value of debt unchanged.

If on the contrary the government sells more debt promising an increase in expected future surpluses, then the real value of debt rises. The extra money gained through the sale can finance a deficit or a lower surplus with no unexpected inflation.
So, while the former case can be seen as a share split, the latter has the characteristics of an equity issue which promises to increase future dividends (i.e. future surpluses in the fiscal theory).

In both "monetary policy" debt sale and "fiscal policy" debt sale the visible government action is that it sells more bonds. While in the former the bonds' sale does not imply any change in future surpluses but only in interest rates and expected inflation, in the latter there is the expectation that future surpluses will rise in order to balance the larger debt. In the last case there is no change in prices and interest rates. It follows that a central issue is how the government creates these expectations on future surpluses.
A key-role in the creation of expectations is played by institutions and their communication with investors.
In the U.S. context, when more debt is sold, there is not an explicit promise to pay it off creating higher future surpluses. It follows that the expectations are based on the reputation that the Treasury and the Congress have earned over the centuries.
On the other hand, the Federal Reserve, as a central bank cannot directly interfere with fiscal policy, but its actions can have indirect fiscal effects (for example, through the purchase of risky assets or government bonds on the open market, always ensuring price transparency).

Conclusion

The topics covered in the previous paragraphs retrace the first chapters of Cochrane's book which give a general idea of the Fiscal Theory of the Price Level in a frictionless scenario. For the purpose of this thesis, the further studies which develop Cochrane's model with more realistic assumptions (such as sticky prices, long-term debt and surplus process with an s-shaped moving average representation) are not mentioned but in the following chapters they will be considered in order to apply them to an empirical research.

1.8 Final considerations

As also remarked by Cochrane in his book[41], the weakness of the Fiscal Theory lies in the difficulty of testing it with real data and most of the applications of the last two decades have the form of model-building.

Nevertheless, it is interesting to take this theory and to apply it to the current events, especially to U.S. fiscal policy.

Currently, with the American Rescue Plan, U.S. has increased its debt in an unprecedented way. According to the Fiscal Theory, these measures could have different effects on the economy and, of course, on inflation, depending on the how further debt will be paid off (with higher future surpluses or with an increase in price level). As far as the expectations on future surpluses, they are strongly affected by the decisions the Congress will take about taxes and spending cut (the first being hindered by Republicans and the second by Democrats, especially during COVID-pandemic).

[41] "The Fiscal Theory of the Price Level", by John Cochrane, Feb. 2021, p. 571

Chapter 2 | Critiques and developments

2.1 Some critics to the Fiscal theory of the price level

Over the years, there has been no shortage of critics of the Fiscal Theory of the Price Level. Indeed, each economic theory has its weaknesses and one of the most critical points of this theory is the "intertemporal budget constraint".

A very strong critique came from the economist Willem Buiter in his paper "The Fallacy of the Fiscal Theory of the Price Level" where he claims that the Fiscal Theory is totally "flawed" (Buiter, 1999). According to Buiter, the main mistake of the proponents of this theory is stating that the intertemporal budget constraint is not a constraint that must be satisfied for all values of the endogenous variables. In fact, this wrong assumption leads to incorrect mathematical and logical properties which are at the basis of the model such as:

- Overdetermined equilibria
- Pricing of things that do not exist
- Arbitrary restrictions on the exogenous and predetermined variables in the government's budget constraint
- Price sequences with anomalous dynamics

The author distinguishes between Ricardian and non-Ricardian fiscal rules. The former demand that for all possible sequences of endogenous variables the government's solvency constraint must hold, i.e., not just in equilibrium. The latter allow to run overdetermined fiscal-financial-monetary programmes[42]. In fact, as opposed to Ricardian rules, non-Ricardian ones impose that the solvency constraint must hold only in equilibrium.

Buiter presents a fully specified general equilibrium model, where the government also offers index-linked one-period government bonds and there is a non-unitary equilibrium value for the default discount factor of government bonds. Such a discount factor is defined as the ratio of the present discounted value of the resources that will actually be available for servicing the public debt to the notional or contractual value of the public debt (Buiter, 1999). Given that in the model

[42] In linear algebra overdetermination occurs when in a linear system there are more equations than there are variables (Gentle, 1998)

described by Buiter there is no uncertainty, the author specifies that unanticipated default could occur only in the initial period. In fact, at time $t = 1$ there is certainty about the future payments associated with the non-monetary debt instruments. Therefore, the analysis is restricted to the case in which only the period 1 default discount factor can differ from 1. As far as the default discount factor, whilst the fiscal approach associates it to the role played by the price level in the intertemporal budget constraint, the author claims that this last assumption is incorrect. For this reason, he shows that, by adding an endogenous public debt default discount factor into the model, the overdetermined fiscal-financial monetary programme only defines the effective value of the initial public debt, not the notional one which, in turn, will be different[43]. Furthermore, in contrast to the fiscal theory, the discount factor not only produces logically consistent and economically sensible outcomes, but it also works both in the scenario of flexible prices and in that of the rigid prices. Another critique to the Fiscal Theory came from McCallum who defined it a theory of dubious validity (McCallum, 2001). In his view, the problem of fiscalists is their belief that price level P_t is fiscally determined even when the monetary authority does not accommodate and keeps the money supply at the same level, regardless of fiscal policy. Moreover, he points out that this issue does not depend upon the assumption that monetary base is kept constant. In fact, even with a growing monetary base, the problem of an explosion of price level relative to money supply could arise.

However, Buiter's and McCallum's critiques were addressed by Cochrane in the paper "Money as a Stock" firstly (2005), and in his book "The Fiscal Theory of the Price Level" later (2021). Cochrane asserts that the Fiscal Theory equation is not a budget constraint – as claimed by Buiter – but it is a valuation equation. Actually, government is not forced to adjust its future primary surpluses when there is an

[43] "The effective bond prices are the market prices that actually prevail, if there is (a risk of) partial or complete default and the notional bond prices are the prices that will prevail if there is no default, that is, if the contractual payments are made with certainty." (Buiter, 1999)

unexpected "off-equilibrium" deflation. Furthermore, Cochrane clarifies that there is not a supply curve of government bonds along which the government can observe the price per share before deciding the number of issued bonds; otherwise (i.e. if such a curve existed) their amount would be too elevated. Moreover, in (Cochrane, 2021) he specifies that government's budget constraint is instead

$$B_{T-1} = P_T s_T + M_T \tag{2.1}$$

where M_T represents the sum of the remaining money at the end of the day and any debt that investors did not redeem. Anyway, the government may decide to leave the money outstanding without absorbing it through taxes since no budget constraint forces it to act in that way. The reason why $M_T = 0$ is consumer demand. In fact, since people do not get any utility or tax-paying ability by holding money, then they do not want to possess it at the end of the day. From (2.1) and consumer demand ($M_T = 0$), Cochrane obtains the equilibrium condition

$$B_{T-1}/P_T = s_T \tag{2.2}$$

which can be considered a market-clearing condition, or a supply = demand condition that comes from consumer optimization and budget constraints. Hence, since it is an equilibrium condition, prices adjust to make it hold.

The initial government debt

Another type of critique came from Niepelt who referred to the Fiscal Theory as a "fiscal myth" (Niepelt, 2004). According to him, the fiscal price level determination is inconsistent with an equilibrium where all asset holdings are the results of households' optimal choices. He stresses that the Fiscal Theory of the Price Level is based on two assumptions:

a. There is a predetermined nominal debt at the beginning of time 0 ($W_0 \neq 0$).
b. In period 0 non-Ricardian fiscal policy does not move with price level and so it is independent of W_0/p_0.

So, a non-Ricardian fiscal policy does not influence the price level when $W_0 = 0$. Niepelt claims that Fiscal Theory is therefore inconsistent since the assumptions aforementioned are such that there is inconsistence of investors' formation of expectations before time 0 with the investors' formation of expectations at time 0, and that government bond market does not clear.

However, Daniel answered Niepelt's critique demonstrating that the Fiscal Theory of the Price Level does work in a standard dynamic macroeconomic model as long as the government issues a restricted set of securities (Daniel, 2007).

A two-period economy

Firstly, he defines equilibrium price sequences under Ricardian and non-Ricardian fiscal policy in a two-period economy.

Daniel lets the economy begin at time $t = 0$: at this time, a government consisting of a monetary authority and a fiscal one is created, and there is the birth of a representative agent. The latter gets an endowment of y at time t = 0 and t = 1, while the government spends g in both periods. After period 1, the economy ends.

At the beginning of the first period there is no outstanding debt and new government issues nominal debt. On one hand, the fiscal authority decides the fixed amount of spending for each period and the financing, which is a combination of a lump-sum tax (τ_0) and one-period nominal bonds (B_0). On the other hand, the monetary authority purchases government bonds and offers in turn money (M_0), setting a fixed interest rate i[44]. In the following period, the government pays interests and principles on its debt, finances new expenses by collecting taxes and issuing new debt. The monetary and fiscal authority are subject to two flow budget constraints

$$M_0 + B_0 = P_0(g - \tau_0) \quad (2.3)$$

$$M_1 + B_1 = (1 + i^*)(M_0 + B_0) + P_1(g - \tau_1) - i^* M_0$$

where:

- P_t represents the price of a single good at time t
- $M_0 + B_0 > 0$ since the government issues debt in the initial period

Then, Daniel defines the following government's real primary surplus which includes seigniorage revenue

$$s_t = \tau_t - g + (\frac{i^*}{1+i^*}) \frac{M_t}{P_t} \quad (2.4)$$

Denoting with D_t the debt inclusive of interests at the end of period t,

$$D_t = (1 + i^*)B_t + M_t \ , \quad (2.5)$$

then the government budget constraints become

$$D_0 = -(1 + i^*)(s_0 P_0), \quad (2.6)$$

[44] This is the same assumption made by Niepelt in his paper.

$D_1 = (1 + i^*)(D_0 - s_1 P_1)$ (2.7)

In the end, the combination of equations (2.6) and (2.7) and setting $D_1 = 0$, results in

$0 = s_0 + \frac{P_1}{P_0}(\frac{1}{1+i^*})s_1$ (2.8)

In this way, the present value of surpluses over period 1 and 2 equals the value of outstanding debt at time 0, i.e., it equals 0.

Since the government spending g is fixed and since the monetary authority pegs the nominal interest rate, what characterizes the fiscal policy is the setting of taxes.

Then, following (Phelan & Kocherlakota, 1999), Daniel defines $P = \{P_t\}_{t=0}^{1}$ as a stochastic process of positive prices with $\mathbb{P}$ denoting the space of these processes. He also defines $(\tau, D, M) = \{\tau_t, D_t, M_t\}_{t=0}^{1} \in \mathbb{T} \times \mathbb{D} \times \mathbb{M}$ as the fiscal policy processes.

According to the author, a fiscal policy is a function $\Omega_j: \mathbb{P} \rightarrow \mathbb{T} \times \mathbb{D} \times \mathbb{M}$ such that for all price sequences $\{P_t\}_{t=0}^{1}$, the corresponding policy sequence $\{\tau_t, D_t, M_t\}_{t=0}^{1}$ satisfies the government's flow budget constraints, given by equations (2.6) and (2.7) (Daniel, 2007). However, such a fiscal policy can be Ricardian or non-Ricardian. The former implies that the budget constraint is satisfied for any price level; the latter claims that there is at least one price sequence for which the government's intertemporal budget constraint does not hold.

As far as the representative agent, he/she wants to maximize a certain utility function, is subject to flow budget constraints and to a borrowing constraint.

The expected utility is given by

$E_t \sum_{t=0}^{1} \beta^t \left[u(c_t) + v(\frac{M_t}{P_t})\right],$ (2.9)

with

- c_t denoting real consumption at time *t*
- *u* and *v* being twice differentiable, strictly increasing and concave, with $u' = \infty$.
- E_t representing the expectation conditioned on all variables dated t or earlier.

The flow budget constraints are

$M_0 + P_0 \leq P_0(y - \tau_0 - c_0)$ (2.10)

$M_1 + B_1 \leq (1+i^*)(M_0 + B_0) + P_1(y - \tau_1 - c_1) - i^* M_0$

(2.11)

Then, using equation (2.5) for the definition of D_t, the flow budget constraint can be written as

$$D_t \leq (1+i^*)\left[D_{t-1} + P_t(y - \tau_t - c_t - \frac{i^*}{1+i^*}\frac{M_t}{P_t})\right], t \in (0,1) \tag{2.12}$$

where $D_{t-1} = 0$.

The borrowing constraint is such that the agent cannot borrow more than the expected present value of future income

$$\frac{D_0}{P_0} \geq -E_0\left(\frac{P_1(y-\tau_1)}{P_0}\right),$$

and

$$\frac{D_1}{P_1} \geq 0$$

Combining the flow budget constraint with the budget constraints Daniel obtains the following intertemporal budget constraint

$$\sum_{t=0}^{1}(c_t + \frac{i^*}{1+i^*}\frac{M_t}{P_t})\frac{P_t}{P_0}(\frac{1}{1+i^*})^t \leq \sum_{t=0}^{1}(y - \tau_t)\frac{P_t}{P_0}(\frac{1}{1+i^*})^t \tag{2.13}$$

Finally, first order conditions for the agent provide the Euler equation

$$E_0\left[\frac{\beta(1+i^*)P_0 u\prime(c_1)}{P_1 u\prime(c_0)}\right] = 1 \tag{2.14}$$

and an implicit money demand equation

$$\frac{i^*}{1+i^*} = \frac{v\prime(\frac{M_t}{P_t})}{u\prime(c_t)}, t \in (0,1) \tag{2.15}$$

Given equation (2.14), then (2.12) and (2.13) hold with equality along the optimal path.

Equilibrium, price in equilibrium and future price in equilibrium

Given the nominal debt, the real surpluses, a fiscal policy Ω_j, and a monetary policy which fixes $i_t = i^*$, Daniel gives the definition of equilibrium that is a collection of sequences for real consumption, real taxes, nominal asset quantities and prices where goods market equilibrium holds ($c = y - g$), flow and intertemporal constraints as well, expectations are rational, prices are positive, and realizations of nominal money are non-negative.

Since the paper assumes that spending g and endowment y are constant, then equilibrium consumption is constant at y-g. Hence, the Euler equation (2.14) becomes

$$E_0\left[\frac{\beta(1+i^*)}{P_1}\right] = 1 \tag{2.16}$$

In addition, equilibrium real money balances are fixed at

$$\frac{M_t}{P_t} = v'^{-1}\left(\frac{i^*}{1+i^*}u'(y-g)\right), t \in (0,1) \tag{2.17}$$

Equilibrium is reached when money demand equals money supply. Consequently, (2.17) determines the equilibrium quantity of nominal money.

As far as price in equilibrium, when the outstanding debt is zero, then the intertemporal constraint holds for any positive value for the initial price level P_0. In fact, if equation (2.8) holds, and since it must hold in equilibrium, then (2.8) only affects the ratio $\frac{P_1}{P_0}$. Moreover, fiscal policy does not put any constraint on the equilibrium value of the initial price level (P_0) when D_0 is endogenous[45].

Until this point, Daniel's results are the same of (Niepelt, 2004): when outstanding debt is zero, the present value of future surpluses must be zero leaving the prices indetermined.

As far as future price in equilibrium, Daniel wants to demonstrate that the future prices can be determined by a fiscal non-Ricardian policy as the Fiscal Theory claims even if the price is indetermined at time 0, i.e., when the outstanding debt is zero.

He starts claiming that, in a Ricardian fiscal policy, any positive price sequence satisfying the Euler equation (2.16) is an equilibrium sequence. In equilibrium, the future price (P_1) can be a stochastic variable and its expected value is determined by the value of price in the previous period and the interest rate peg chosen by the monetary authority. In the same way, also the expectation of inflation is a stochastic variable.

[45] At the beginning it was assumed that $D_0 > 0$ which implies that $s_0 < 0$. D_0 can adjust to any positive value of P_0 which in turn is not restricted by equation (2.8). Consequently, fiscal policy does not constrain P_0.

After this definition, the author describes two types of non-Ricardian policies, a non-stochastic one (Ω_n) and a stochastic one (Ω_s).
The non-stochastic policy consists of a fixed value for $s_0 = \bar{s}_0 < 0$ and a stochastic observable value $s_1 = \bar{s}_1 = -\frac{\bar{s}_0}{\beta}$. The stochastic non-Ricardian one consists of a fixed value for $s_0 = \bar{s}_0 < 0$ and a stochastic observable value for $s_1 = \bar{s}_1 + \epsilon$, with $s_1 = -\frac{\bar{s}_0}{\beta}$, and ϵ being a stochastic with mean zero restricted such that $s_1 > 0$.
Given a set of equilibrium price level sequences P^* of a Ricardian policy, then the set of price level sequences of the non-Ricardian policies Ω_n or Ω_s is a proper subset of P^*. Since P^* is the set of price level sequences in a Ricardian policy, then it contains all price sequences which satisfy Eq. (2.16).
As far as the set of equilibrium price sequences under the two non-Ricardian policies, Daniel combines equations (2.8) and (2.6) obtaining

$$P_1 s_1 = D_0 = -P_0 s_0 (1 + i^*) \tag{2.18}$$

At time 1, D_0 and P_0 are predetermined and consistent with equation (2.6). Moreover, a value for s_1 is set. Given these variables, $P_1 = \hat{P}_1$ is the only value which can solve equation (2.18). It is important to highlight that $\hat{P}_1$ must satisfy equation (2.16) as well. For this reason, the author solves the first part of equation (2.18) for s_1 and takes its time expectation using equation (2.16):

$$E_0 s_1 = D_0 E_0 (\frac{1}{P_1}) = \frac{D_0}{\beta(1+i^*)P_0}$$

It is to be noted that both non-Ricardian fiscal policies meet $E_0 s_1 = -\frac{\bar{s}_0}{\beta}$.
Hence, this means that $\hat{P}_1$ satisfies (2.16) in both non-Ricardian fiscal policies, i.e., they both lead to the definition of a unique equilibrium value for the price level.
Even if there is nothing that identifies the price level when the initial debt is zero, however in the following periods, given the initial data and the realized value for s_1, non-Ricardian policy determines the set of equilibrium price sequences by setting the equilibrium value for the price as a function of the aforementioned initial values[46]. Therefore, Daniel has demonstrated that fiscal policy can determine the equilibrium price level in the subsequent periods after the first issuing of nominal debt.

[46] Daniel shows that the same results can be achieved also in an infinite horizon economy.

2.2 Liquidity premiums on government debt in the context of the Fiscal Theory of the Price Level

So far discussions over the Fiscal Theory of the Price Level have always hinged on the assumption that government debt has the only function to reallocate taxes over subsequent periods of time. In (Berentsen & Waller, 2018), there is a different opinion over this point: the authors assume that government debt serves a second function: in fact, it is also used as a collateral for secured lending in financial markets. Hence, its market value includes a liquidity premium which reflects not only the claim on the stream of future surpluses, but also its value for trading. This claim changes the typical price level determination of the Fiscal Theory because it suggests that price level dynamics may be affected by changes in the liquidity value of government debt. Berentsen and Waller build a model which considers this new assumption, and they show that there is no liquidity premium on the debt and the price level is determined in the same way as in the Fiscal Theory, if the real value of government debt is high enough; if instead it is sufficiently low, then there are the collateral constraints, and the market value of debt reflects a liquidity premium. This means that the market value of government debt is higher than its "fundamental value", i.e., the present value of future surpluses. The inclusion of the collateral value of government debt leads to some results that are new in the literature. On one hand, any time that the real value of government debt increases, the collateral constraint is eased and there is the expansion of economic activity in the secured lending sector. An example is cutting expenses or increasing taxes with the effect of raising the fundamental value of government debt. Therefore, the economic activity expands through an increase in secured lending. In such a scenario, cutting expenses and raising taxes have a "stimulative" effect.

On the other hand, liquidity premium assumption implies that movements in the price level can be generated also when there are no expected changes in current or future fiscal policies.

Description of the model

The framework is characterized by an infinite horizon, infinitely lived agents with perfect foresight who consume perishable goods and discount at rate $\beta =$

$1/(1+r)$ in each period[47]. In each period agents are involved in two sequential rounds of trade, the decentralized market (DM) and centralized market (CM).
In the decentralized market half of the agents are consumer and the other half consists of producers. The trade is bilateral, and the probability of a meeting is σ. Consumers get a utility u(q) from the consumption of $q > 0$ units of DM goods and such a utility function is characterized by $u'(q) > 0, u''(q) < 0, u'(0) = +\infty$, and $u'(\infty) = 0$. Producers bear a utility cost $c(q) \geq 0$ to produce q units of DM goods. The derivatives of this function are $c'(q) < 0$, $c''(q) \geq 0$ and $c'(0) = 0$. The equilibrium value of q is reached when $u'(q^*) = c'(q^*)$.
In the centralized market CM, agents can both consume and produce. The agents' quasilinear preference is given by $U(x) - h$, with

- $U(x)$ representing the utility from the consumption of x units of the CM good
- h defining the disutility from working h units in the centralized market
- $U'(x) > 0, U''(x) \leq 0, U'(0) = +\infty \; and \; U'(\infty) = 0$.

It is necessary one unit of labor to produce one unit of the CM good. Hence, the real wage is 1. While DM consumers make their price offer to producers, in the centralized market prices are determined in competitive Walrasian markets. The numeraire price of a unit of CM goods is *P*, also defined as the price level. The number of units of CM goods that an agent gives up for a unite of numeraire is represented by $\phi = 1/P$.
In this model, the government issues one-period nominal bonds with a payout of one unit of an arbitrary numeraire[48]. Moreover, there is an initial positive bond stock $B_0 > 0$. Government collects taxes *T* in the form of lump-sum in the centralized market and buys G units of CM goods. In period *t*, $S \equiv T - G$ is the real government surplus. Government is subject to the following real budget constraint:

$$\rho\phi B_{t+1} = \phi B - S \tag{2.19}$$

where

- $\rho = 1/(1+i)$ is the price of bond at time *t* with maturity at time *t+1*

[47] The framework is the same described in (Lagos & Wright, 2005)
[48] In this model bonds are not medium of exchange, but they are used as collaterals against the loans made by DM producers.

- i is the nominal return on the bond.

Then, the authors assume that the stream of surpluses is constant in each period in order to allow readers to better understand change in the real value of government debt due to changes in liquidity needs instead of those in primary surplus.

In the decentralized market private unsecured credit arrangements are not allowed but they can be made to finance DM consumption. In fact, DM consumers can use part of their bond holdings (a fraction ϑ) to finance purchases of DM goods. Hence, DM producers can grant loans to DM producers. These loans are then repaid in the next CM with either units of numeraire or CM goods.

<u>*CM trade*</u>

During the centralized market the agents decide on how much to consume and work, and on the number of bonds they want to hold until the next period. Moreover, all the pledged repayments of secured loans from the previous DM must be settled.

Then, the authors define the value function $W(b, \ell)$ for an agent entering the CM market where b are the units of bonds and ℓ are those of his/her outstanding secured loans. If $\ell > 0$, the agent is a borrower, if $\ell < 0$ he/she is a creditor.

The agent type is identified by $j = c, p$ with c standing for consumer and p for producer.

The value function of a type j agent entering the next period DM market with b_{t+1} units of bonds is $V_j(b_{t+1})$.

An agent in the centralized market wants to maximize

$$W(b, \ell) = max\{U(x) - h + \beta V_j(b_{t+1})\} \tag{2.20}$$

under the constraint

$$x + \phi\rho b_{t+1} = h + \phi b - \phi\ell - \tau \; and \; b_{t+1} \geq 0 \tag{2.21}$$

The first order conditions result in

$$U'(x) = 1 \tag{2.22}$$

$$\beta V_j^b(b_{t+1}) \leq \phi\rho \quad (= \phi\rho \; if \;\; b_{t+1} \geq 0) \tag{2.23}$$

In the last formula, V_j^b denotes the partial derivative of the value function $V_j(b_{t+1})$ with respect to b_{t+1}.

According to the envelope conditions, $W^b(b,\ell) = \phi$ and $W^\ell(b,\ell) = -\phi$, where $W^b(b,\ell)$ stands for the partial derivative of the value function with respect to b and $W^\ell(b,\ell)$ is the partial derivative of the value function with respect to ℓ.

This means that any additional unit of nominal bonds corresponds to one unit of numeraire, i.e., ϕ units of CM consumption goods. On the contrary, as far as the loans, the marginal value of holding one additional unit of numeraire is -ϕ, where ϕ are the units that the agent must give up repaying the debt.

<u>DM trade</u>

As far as DM value functions, they are given by

$V_p(b) = (1-\sigma)W(b,0) + \sigma[-c(q) + W(b,\ell)]$

$V_c(b) =) = (1-\sigma)W(b,0) + \sigma[u(q) + W(b,\ell)]$.

In the above formula, $1-\sigma$ represents the agent's probability of not being matched with a trading partner and continuing into the next CM. With probability σ he/she is paired with another one. Each time a DM consumer is paired with a DM seller, the latter provides a loan of $\ell > 0$ which is to be repaid with CM goods or units of numeraire in the next CM.

In addition, the consumer asks for q units of the DM good. It is essential for the producer that payoff from producing and selling the DM good is higher than simply walking away and entering the centralized market with $\ell = 0$. Moreover, the value of the future repayment minus the cost of producing the good must be higher than the value of walking away. Hence, it arises the following maximization problem

$max_{q,\ell} u(q) + W(b,\ell) - W(b,0)$

such that

$\phi\ell \leq \phi\vartheta b \;\; and \;\; W(\tilde{b}, -\ell) - c(q) \geq W(\tilde{b}, 0)$

where

- $\tilde{b}$ represents the bond holdings of a producer encountered in the market,
- the first constraint is the secured lending constraint which implies that the loan must be less than the seizable amount of government bonds,
- the second one is the producer's participation constraint.

Under the "take-it-or-leave-it" offer, the two constraints can be combined resulting in:

$max_u u(q) - c(q)$ such that $c(q) \leq \theta\phi b$ (2.24)

At this point, the authors use the Lagrangian multiplier (λ) to find the first order condition that in this case is $u'(q) - (1+\lambda)c'(q) = 0$. There are different implications which depend on the value of λ:

- if $\lambda = 0$, then the consumer owns even more seizable bonds than it is necessary to get the first optimal allocation, i.e., $q = q^*$ and the quantity of available seizable bonds is higher than the optimal loan requested ($\ell^* \leq \theta b$).
- if $\lambda > 0$, there is the opposite situation, i.e., the consumer has less seizable bonds than he needs to obtain the first best; consequently, we have $q < q^*$ and ($\ell^* = \theta b$); in such a case, inefficient allocations occur in the decentralized market.

From these results and the DM value functions, it follows that

$V_p^b(b) = \phi$ (for the producer) (2.25)

$V_c^b(b) = \phi + \sigma\phi\left[u'(q)\frac{\partial q}{\partial b} - \frac{\partial \ell}{\partial b}\right] (for\ the\ consumer)$ (2.26)

As far as producers, they use bonds just to store value. On the other hand, as far as consumers, it happens the same if $\lambda = 0$, otherwise, if $\lambda > 0$, then $\partial q/\partial b = \phi\ell/c'(q)$ and $\partial\ell/\partial b = \theta$. By substituting them in (2.26), we get

$V_c^b(b) = \phi + \sigma\phi\left[\frac{u\prime(q)}{c\prime(q)} - 1\right]$

Then the authors put $V_p^b(b) = \phi$ in (2.21) lagged one period and obtain

$\rho \geq \beta(\phi_{t+1}/\phi_t)$ (2.27)

which is the same result that we get by putting $V_c^b(b) = \phi$ in (2.21) when $\lambda = 0$.

If instead $\lambda > 0$, putting $V_c^b(b) = \phi + \sigma\phi\left[\frac{u\prime(q)}{c\prime(q)} - 1\right]$ in (2.21) yields

$\rho = \beta(\phi_{t+1}/\phi_t)\,[1 + L(q_{t+1})]$ (2.28)

where

$L(q_{t+1}) \equiv \sigma\vartheta[\frac{u\prime(q_{t+1})}{c\prime(q_{t+1})} - 1] \geq 0$ (2.29)

is the liquidity premium on government debt.

Such a premium is equal to zero when $\lambda = 0$ and from (2.27) we get $\rho = \beta(\phi_{t+1}/\phi_t)$.

Given that $\phi_{t+1}/\phi_t) = P_t/P_{t+1} = 1/(1+\pi)$, then we have

$1 + i = (1 + r)(1 + \pi)$[49].

This is the standard Fisher equation which states that the nominal return on the government bond must compensates investors not only for the time cost of holding the bond, but also for the inflation.

When $\lambda > 0$, the liquidity premium is greater than zero ($L(q_{t+1}) > 0$, and $\rho > \beta(\phi_{t+1}/\phi_t)$. Consequently, DM sellers are not willing to buy government bonds since we have that $1 + i = \frac{(1+r)(1+\pi)}{1+L(q_{t+1})}$, which means that the price of the bonds is too high while the nominal return is too low. On the other side, DM consumers will buy anyway government bonds because they need them to finance DM consumption.

At this point, Berentsen and Waller discuss about the equilibrium of the model.

As said before, the consumer's budget constraint in the decentralized market is $c(q) \leq \theta\phi b$. Moreover, in any equilibrium the number of outstanding bonds (b) is equal to the outstanding nominal debt (B). Hence, for $\lambda = 0$, the real value of the government debt satisfies $\phi B > c(q^*)/\theta$ while for $\lambda > 0$, we have

$$\phi B = c(q^*)/\theta \tag{2.30}$$

By assuming that we can use (2.30) for all *k*, then we rewrite the government budget constraint (2.19) as

$$\rho = (\frac{\phi_{t+1}}{\phi_t})[\frac{c(q_t)-\theta S}{c(q_{t+1})}]. \tag{2.31}$$

Then, the authors combine (2.28) and (2.31) and obtain the following dynamic equation in q

$$c(q_{t+1})\beta[1 + L(q_{t+1})] = c(q_t) - \theta S \tag{2.32}$$

In this model, the equilibrium is the sequence of $\{q_t\}_{t=0}^{\infty}$ which satisfies (2.32) with $q_t \in (0, q^*)$ for all *t*. Then, assuming that the surplus (S) is constant for all *t*, a steady equilibrium is a $\bar{q}$ that solves

$$c(\bar{q})\{1 - \beta[1 + L(\bar{q})]\} = \theta S \tag{2.33}$$

and the steady state real value of debt satisfies

$$\phi B = c(\bar{q})/\theta \tag{2.34}$$

[49] Remember that $\beta = \frac{1}{1+r}$

Such an equilibrium is unique, if $0 \leq \theta S \leq c(q^*)(1-\beta)$ with $\frac{\partial \bar{q}}{\partial \theta S} > 0$.

When we are in a steady state, q is kept constant. Hence, the inflation rate is equal to the growth rate of bonds, since price level and bonds increase at the same rate. From (2.34), it follows that

$$\frac{\phi_{t+1}}{\phi_t} = \frac{B_{t+1}}{B_t} = 1 + \pi \ .$$

An interesting result which is implied by $\frac{\partial \bar{q}}{\partial \theta S} > 0$ is that the steady state value q raises with the increase in current or future primary surpluses. This means that raising non-distortionary taxes or cutting government spending stimulate economic activity. In fact, these actions increase the fundamental value of government debt and, consequently, the real value of current debt with the effect of loosening the collateral constraints in the decentralized market. This implies an increase in secured lending and consumption of DM goods.

Berentsen and Waller's analysis goes even further and tries to link their model to the traditional Fiscal Theory of the Price Level.

They assume that $c(q) = q$ and rewrite (2.33) as follows

$$\overline{\phi B} = \frac{S}{1-\beta[1+\sigma\theta L(\theta\overline{\phi B})]} \tag{2.35}$$

Then, they set $\bar{\beta} \equiv \beta[1+\sigma\theta L(\theta\overline{\phi B})] \leq 1$ and $\bar{r} \equiv (1-\bar{\beta})/\bar{\beta}$ [50]. Now, it is possible to rewrite (2.35) as

$$\overline{\phi B} = \sum_{k=0}^{\infty} \left(\frac{1}{1+\bar{r}}\right)^k S \tag{2.36}$$

The first significative result implied by (2.36) is that if the liquidity premium on government bonds is zero ($L(\theta\overline{\phi B}) = 0$), then $\bar{r} = r$ and we obtain the intertemporal government budget constraint

$$\widehat{\phi B} = \frac{S}{1-\beta} = \sum_{k=0}^{\infty} \left(\frac{1}{1+r}\right)^k S \tag{2.37}$$

which is the standard Cochrane's valuation equation. Here $\widehat{\phi B}$ is to be intended as the 'fundamental value' of the government debt.

The second result is that the fiscal theory of the price level is still valid even if the liquidity premium on nominal government debt is greater than zero. In fact, given

[50] $L(\theta\overline{\phi B}) > 0$ implies $\bar{\beta} > \beta$ and $\bar{r} > r$.

that S and $\overline{\phi B}$ are constant, for a given nominal bond stock B, $\phi = 1/P$ adjusts to satisfy (2.36). As in the fiscal theory, the price level adjusts in response to any change in future taxes or spending.

The third key point is that if $\bar{r} < r$, then $\overline{\phi B} > \widehat{\phi B}$, which implies that the presence of the collateral constraint makes the real market value of the outstanding nominal bond stock greater than its fundamental value. This means that investors also consider the marginal collateral value when evaluating the bonds. In the Fiscal theory, a 'mispricing' of the debt would mean a violation of the budget constrain and an adjustment of the price level. By the way, it does not happen in this model because budget constraint is not violated here. What it is important to stress is that there can be a "bubble" in the value of nominal government debt.

The fourth point is that in contrast to Fiscal Theory, inflation is due to the growth rate of bonds and not to changes in the present value of future surpluses.

Fifth, here the inflation is "costless" in that it does not affect the allocation of bonds but only the nominal interest rate. In fact, rewriting (2.28) as

$$\frac{1+\pi}{1+i} = \frac{1+\sigma\theta L(\bar{q})}{1+r} \tag{2.38}$$

we can clearly see that, given the constant values on the right-side, then any change in inflation π is followed by a one-to-one change in the nominal interest rate i.

Lastly, the real value of the government debt can still be positive even if future surpluses are equal to zero for all t. In fact, looking at the surpluses as they were dividends on a government claim, because of its collateral value, such a claim is valued in equilibrium even if the dividend is zero.

2.3 The Fiscal Theory of the Price Level in an economy of low interest rates

The Fiscal Theory of the Price Level (FTPL) originated many controversies and one of its weaknesses arises when the interest rate is persistently below the growth rate of the economy. (Bassetto & Cui, 2017) analyzes this situation considering different causes of low interest rates, such as dynamic inefficiency, the liquidity premium of government debt, and its favorable risk profile.

In their analysis, the authors confirm the validity of the Fiscal Theory when surpluses are always positive, but they adopt more sophisticated strategies when there is the need of some deficits to finance government's spending. Moreover, they claim that, as in the FTPL, government sets sequences of taxes independently of the realization of the price level, but they doubt the uniqueness of the equilibrium value of price level when interests rate are permanently lower than the growth rate of the economy. In fact, in such a case, the infinite sum of the present value of future surpluses may diverge.

Before illustrating the models characterized by low interest rates and their implications for the FTPL, Bassetto and Cui investigate the consequences of low interest rates just by analyzing the government budget constraint.

Starting with a one-period model, they firstly define the variables: B_t are the promised nominal debt repayments due by the government at the beginning of period *t*, R_t is the nominal interest rate, P_t stands for the price level, and τ_t for the real taxes. The government budget constraint is

$$\frac{B_{t+1}}{1+R_t} = B_t - P_t\tau_t \tag{2.39}$$

and it does not depend on government spending. The authors rescale debt and taxes by real output y_t to reflect the fact that the tax base is related to the size of the economy. Setting $x_t := \tau_t/y_t$, they rewrite (2.39) as

$$\frac{B_{t+1}}{P_{t+1}y_{t+1}} = \frac{(1+R_t)P_t y_t}{P_{t+1}y_{t+1}}\left(\frac{B_t}{P_t y_t} - x_t\right) \tag{2.40}$$

The growth rate of the economy is $1 + g_{t+1} = y_{t+1}/y_t$ and the (gross) real return on government debt is $1 + r_{t+1} = (1 + R_t)\, P_t/P_{t+1}$. In order to ensure that the real return is bounded away from the growth rate, in this deterministic economy it is required that

$$\frac{1+r_{t+1}}{1+g_{t+1}} < \alpha, where\ \alpha < 1 \tag{2.41}$$

Assuming there is a positive outstanding initial debt, and iterating (2.40) we obtain

$$\begin{aligned}\frac{B_t}{P_t y_t} &= \frac{B_0}{P_0 y_0}\prod_{s=1}^{t}\left(\frac{1+r_s}{1+g_s}\right) \\ &\quad -\sum_{s=0}^{t-1} x_s \prod_{v=s+1}^{t}\left(\frac{1+r_v}{1+g_v}\right) < \alpha^t \frac{B_0}{P_0 y_0} - \sum_{s=0}^{t-1} x_s \prod_{v=s+1}^{t}\left(\frac{1+r_v}{1+g_v}\right)\end{aligned} \tag{2.42}$$

Since taxes converge asymptotically to some value $\bar{x}$ by assumption, and given that the debt must remain positive, in an economy with $r < g$ the debt/GDP ratio decreases toward zero and to avoid it vanishing or becoming negative, continuing primary deficits are needed.

The authors go further analyzing what happens in a stochastic environment when the expected return is low, i.e.,

$$E_t\left[\frac{(1+R_t)P_t y_t}{P_{t+1}y_{t+1}}\right] < \alpha < 1 \tag{2.43}$$

Then, (2.42) becomes

$$E_0\frac{B_t}{P_t y_t} = E_0\left\{\frac{B_0}{P_0 y_0}\prod_{s=1}^{t}\left(\frac{1+r_s}{1+g_s}\right) - \sum_{s=0}^{t-1} x_s \prod_{v=s+1}^{t}\left(\frac{1+r_v}{1+g_v}\right)\right\} <$$

$$\alpha^t\frac{B_0}{P_0 y_0}E_0\left[\sum_{s=0}^{t-1} x_s \prod_{v=s+1}^{t}\left(\frac{1+r_v}{1+g_v}\right)\right] \tag{2.44}$$

Since we want that the expected debt/GDP ratio remain bounded away from zero in the limit, then it is needed that

$$\lim_{t\to\infty} E_0\left[\sum_{s=0}^{t-1} x_s \prod_{v=s+1}^{t}\left(\frac{1+r_v}{1+g_v}\right)\right] < 0 \tag{2.45}$$

These calculations imply that an economy with low interest rates needs a government running recurrent primary deficits.

2.3.1 Risk premia

Firstly, the authors analyze the situation where households are willing to buy government's bonds even if the expected interest rate is lower than the expected growth of the economy for precautionary reasons. Similarly to the one analyzed by (Cochrane, 2005), the pure-exchange economy described by Bassetto and Cui is featured by the same infinitely lived agents and the government.

Private agents invest in one-period government bonds B_t but there is a lower bound on their holdings $\left(\frac{B_t}{P_t y_t}\right) \geq -\underline{B}$. The nominal interest rate *R* on bonds is fixed by the government and it remains constant over time[51].

The representative consumer maximizes the following utility function

$$max\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\frac{c_t^{1-\gamma}-1}{1-\gamma}$$

under the constraint

$$P_t c_t + \frac{B_{t+1}}{(1+R)} + P_t\tau_t = P_t y_t + B_t$$

where:

[51] As in Cochrane's analysis, also in this case the authors abstract from any role for money because the presence of cash is not essential for the results.

- $\beta \in (0,1)$ is the discount factor of the utility function
- τ_t are the lump-sum taxes paid by the consumer and used by the government to repay its existing obligations every period
- the agent chooses a sequence $\{c_t, B_t\}_{t=0}^{\infty}$ to solve the optimization problem, taking as given the sequence $\{y, \tau_t, R_t, p_t\}_{t=0}^{\infty}$ and the initial bond holding B_0.

Given the gross inflation $\pi_{t+1} = P_{t+1}/P_t$ from t to $t+1$, and the real stochastic discount factor $z_t := \beta^t c_t^{-\gamma}$, then the first-order condition for the consumer becomes

$$\mathbb{E}_t\left[\frac{1+R}{\pi_{t+1}}\frac{z_{t+1}}{z_t}\right] = 1 \tag{2.45}$$

together with the transversality condition

$$\lim_{s\to\infty} \mathbb{E}_t\left[\frac{B_{t+s}z_{t+s}}{P_{t+s}}\right] = 0 \tag{2.46}$$

From the iteration of the budget constraint (2.39) until an arbitrary period *T*, taking the limits as $T \to \infty$ and using (2.46), the authors obtain the intertemporal budget constraint, i.e., the typical evaluation equation of the Fiscal Theory:

$$\frac{B_t}{P_t} = \mathbb{E}_t\left[\sum_{s=0}^{\infty}\frac{z_{t+s}}{z_t}\tau_{t+s}\right]. \tag{2.47}$$

However, this economy differs from the FTPL since the equation (2.47) does not exclude the possibility that equations (2.45) and (2.46) are satisfied as well. Hence, this means that the expected value of future taxes can be negative in all periods even if the present value must be positive[52].

2.3.2 The Fiscal Theory of the Price Level in dynamically inefficient economies

Bassetto and Cui analyze the case of an overlapping generations (OLG) economy with people living for two periods. They consider a pure exchange economy where the overlapping generations are of constant size, normalized to 1, and they also assume the endowment remains the same over time. The two generations have a different income based on their age: w^y when young and w^o when old. Households' preferences are given by

$$U(c_t^y, c_{t+1}^0) \tag{2.48}$$

[52] (Bassetto & Cui, 2017) proves it in the Appendix A through a specific endowment process and fiscal policy. In such a scenario, government debt is risk free in real terms, equation (2.43) holds, and expected taxes are always negative.

Where c_t^y is consumption by the young in period t, and c_{t+1}^0 is the one by old in period $t+1$. Consumption when young and old are gross substitutes. Such a function is assumed to be strictly increasing in both arguments, strictly quasiconcave, continuously differentiable, and Inada conditions apply[53]. There are the same assumptions described at the beginning of paragraph 2.3, i.e., the presence of only one asset (one-period government bond), a positive initial government debt B_0, a constant nominal interest rate R[54], a real amount of taxes τ_t. The generation born in period t is characterized by the following budget constraint:

$$P_t c_t^y + \frac{B_{t+1}}{1+R} \leq P_t w^y \tag{2.49}$$

$$P_{t+1} c_{t+1}^o \leq P_{t+1}(w^o - \tau_{t+1}) + B_{t+1} \tag{2.50}$$

In period 0, the people who are part of the old group consume their savings and all the endowment left after the payment of taxes:

$$P_0 c_0^o = P_0(w^o - \tau_0) + B_0 \tag{2.51}$$

From the maximization of the utility function (2.48) under the constraints (2.49) and (2.50), a (real) saving function $f(1 + r_{t+1})$ is obtained. Given the assumptions over the utility function, f is strictly increasing in the real interest rate. This allows the inversion of f in order to define the equilibrium real interest rate as a function of savings by the young: $r_{t+1} = r(s_{t+1})$, where $s_{t+1} = w^y - c_t^y$.

Bassetto and Cui are interested to investigate the situation in which $r_0 < 0$: in fact, young households need to save for their old age so much that they are willing to do so even if the real interest rate is zero.

In this economy the equilibrium is given by a sequence $\{c_t^y, c_t^o, P_t, r_t, B_{t+1}\}_{t=0}^{\infty}$ such that households maximize their utility function under the aforementioned constraints, the government budget constraint (2.39) holds in each period t, the definition of real rate given in the footnote 55 applies, and markets clear. This is described by:

---

[53] A continuously differentiable function f(x) satisfies Inada conditions if:

i. The value of the function in x=0 is zero, i.e., $f(0)=0$
ii. The function is concave in its domain
iii. the limit of the first derivative is positive infinity as x_i approaches zero
iv. the limit of the first derivative is zero as x_i approaches positive infinite

[54] On the other hand, the real interest rate in the economy between t and $t+1$ is $1 + r_{t+1} = (1 + R)\frac{P_t}{P_{t+1}}$

$P_t f(1 + r_{t+1}) = B_{t+1}/(1 + R)$ (2.52)

Then, combining equations (2.39), the definition of real interest rate, and (2.52), the following difference equation must be satisfied in equilibrium:

$f(1 + r_{t+1}) = (1 + r_t)f(1 + r_t) - \tau_t$ [55] (2.53)

with the initial condition

$f(1 + r_1) = \frac{B_0}{P_0} - \tau_0$ (2.54)

Instead of writing $f(1 + r_{t+1})$ it is possible to analyze equation (2.53) in terms of savings by the young, i.e.,

$s_{t+1} = (1 + r(s_t))s_t - \tau_t$ with $t \geq 1$ (2.55)

and

$s_1 = \frac{B_0}{P_0} - \tau_0$ (2.56)

Solving the difference equation (2.55) to find the steady states[56], Bassetto and Cui explain that:

- when $\tau > 0$, two steady states are found, one with positive and one with negative savings,
- if $\tau = 0$, the steady states are two: one is zero and the other one is positive,
- in the case of $\tau < 0$, there exists an even number of steady states, all characterized by positive saving; when τ is close to 0, there are two steady states; if τ is sufficiently negative, there are no steady states.

In the following paragraphs I will describe the evolution of the economy in these 3 cases.

---

[55] The authors assume that taxes are constant $(\tau_t = \tau)$, $t \geq 0$ to make the difference equation time invariant and to allow clearer results at an analytical level.

[56] In such a Difference Equation, the steady states $\bar{s}$ are such that $r(\bar{s})\,\bar{s} = \tau$. Because of Inada conditions, $\lim_{s \to w^y} r(s) = \infty$ and $\lim_{s \to -\infty} r(s) = -1$, which in turn implies that $\lim_{s \to w^y} r(s)\, s = \lim_{s \to -\infty} r(s)\, s = \infty$. Moreover, $\frac{d[r(s)s]}{ds}$, meaning that $sr(s)$ is monotonically increasing when both s and r(s) are positive, and decreasing when they are both negative. In addition, $sr(s) = 0$ when either s=0 or r(s)=0 (Bassetto & Cui, 2017).

First case: $\tau > 0$

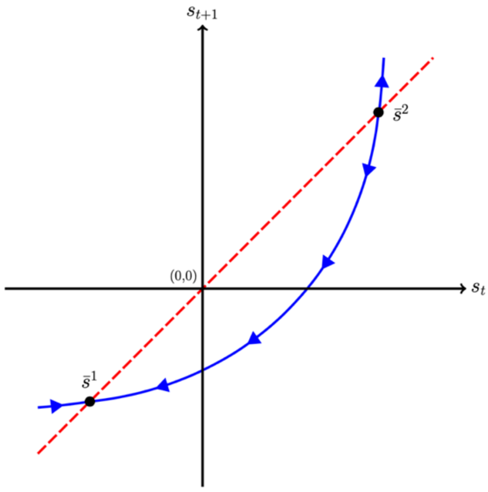

Figure 2.1- Source: Bassetto & Cui, 2017, p.16

As said before, when $\tau > 0$ there are two steady states. The steady state with positive savings ($\bar{s}^2$) is comparable to the standard solution obtained in the standard treatment of the Fiscal Theory of the Price Level. In this steady state the interest rate is positive. When household saving is equal to the steady value ($s = \bar{s}^2$), taxes are sufficient to pay interest on government debt. If $s > \bar{s}^2$, savings are too high (and consequently also government debt is too high) and taxes are insufficient to pay interest on government debt. On the opposite, if $s < \bar{s}^2$, savings are lower, taxes are even higher than the amount needed to repay interests, and government debt decreases. In this economy, the interest rate falls as s_t becomes lower. In the steady state $\bar{s}^1$ young households borrow at a negative interest rate, repaying to the government a lower amount when they become old. In the end, any value $s_1 \in (-\infty, \bar{s}^2]$ allows a stable evolution going forward, making the economy always converge to $\bar{s}^1$ (unless the economy begins and stays at $\bar{s}^2$).

Second case: $\tau = 0$

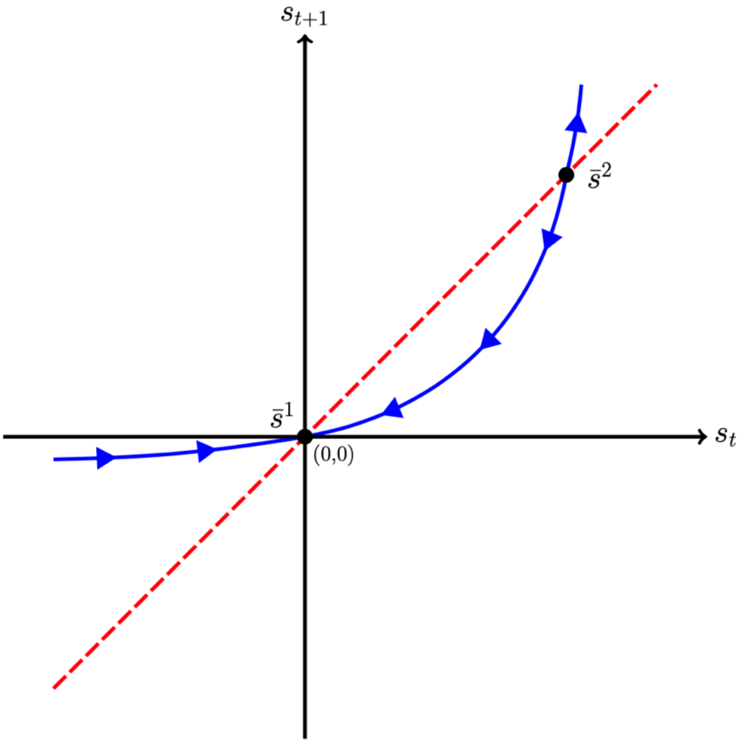

Figure 2.2 - Source: Bassetto & Cui, 2017, p. 17

When $\tau = 0$ the solid line shifts toward upper left, passing through the origin. In the higher steady state, $\bar{s}^2$, the interest rate is zero, but government debt is still used as a substitute for fiat money, and it is passed from a generation to another at a constant price. If the value of savings is higher than $\bar{s}^2$, then the debt would explode with a positive interest rate and zero taxes. This scenario would be inconsistent with the equilibrium since household saving is bounded above by w^y. If instead we observe a value of savings which is lower than $\bar{s}^2$, savings go toward zero over time since it is the second steady state $\bar{s}^1$.

<u>Third case: $\tau < 0$</u>

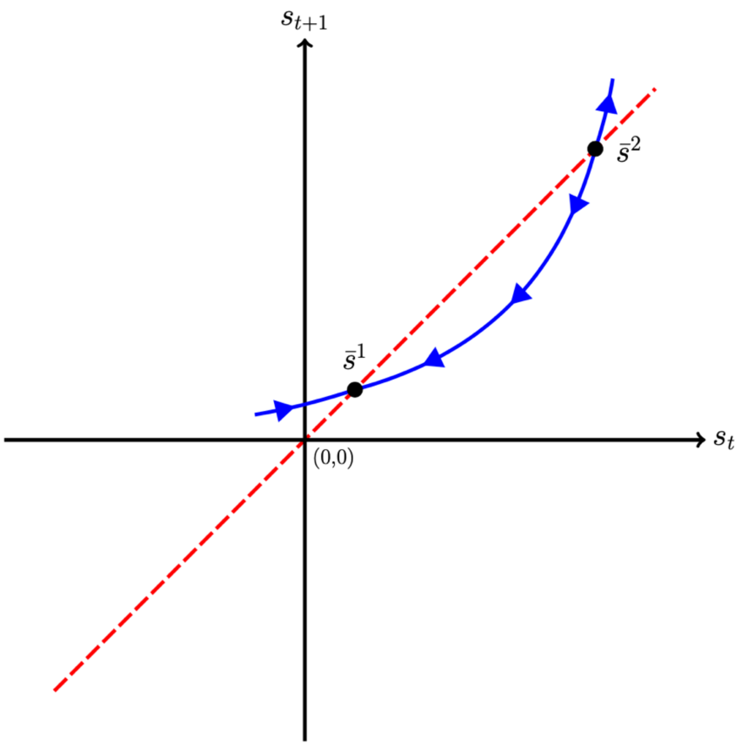

Figure 2.3- Source: Bassetto & Cui, p. 18

The last case is the situation where taxes are negative. Both steady states are characterized by positive savings and negative interest rate, meaning that government does not repay its debt in full, but it compensates this shortfall with negative taxes, i.e., with transfers. By the way, if τ is too negative, there is no equilibrium, and the debt explodes.

At this point, the authors link the initial level of saving s_1 to the initial price level P_0 in order to define the set of competitive equilibria of the economy. They get:

$\frac{B_0}{P_0} = \tau + s_1$.

Given that by assumption the government starts with a positive outstanding debt, and given that $\bar{s}^{MAX} + \tau > 0$[57], then the specification $s_1 \in (-\infty, \bar{s}^{MAX}]$ allows the presence of a continuum of equilibria indexed by the initial price level, with

$$P_0 \in [\bar{s}^{MAX} + \tau)/B_0, \infty). \quad (2.57)$$

[57] In fact, in all the figures $\bar{s}^2 + \tau > 0$

The last equation (2.57) shows that even in a scenario of fixed nominal interest rate and fixed taxes, the price cannot be uniquely defined since a continuum of possible price levels emerges. This is clearly in contrast to the Fiscal Theory, and it demonstrates that in OLG economies with dynamic inefficiency and where government debt itself is similar to money, tight predictions on the inflationary consequences of lowering τ are not possible. At least equation (2.57) imposes a lower bound on prices.

2.3.3 Liquidity function of debt

The last scenario analyzed by Bassetto and Cui is a dynamically efficient economy in which private assets yield a rate of return which is higher than the growth rate of the economy (which is normalized to 0). On the other hand, government debt pays a lower interest rate, but it has a special liquidity role since it allows certain transactions to be settled that would not be possible through private assets.

As in the previous section, government debt has itself the characteristics of the money.

The authors consider an economy in which there are infinitely lived households and government, and where each period is split into two subperiods.

The first subperiod is the "morning" and it features households involved in bilateral anonymous markets. Households buy the goods that they like with probability $\chi \in (0,1)$ and produce goods that are bought from another counterparty with the same probability χ. Hence, in this market there are double-coincidence meetings where private credit and privately issued assets cannot be recorded or recognized. Government debt is used as means of payment in these transactions. Another assumption is that buyers offer a "take-it-or-leave-it" price[58].

The second subperiod takes place in the "evening". This time households make transactions in a centralized market where a record-keeping technology is present. The evening good, privately issued claims, as well as government debt are traded here. Government sets taxes on the base of an exogenous real sequence $\{\tau_t\}_{t=0}^{\infty}$, settles debt at maturity, and issues new bonds at a fixed interest rate R[59].

[58] These assumptions are similar to those contained in (Berentsen & Waller, 2018)

[59] In Appendix B of (Bassetto & Cui, 2017) the authors add money paying no interest rates. In that case R is the opportunity cost of holding money vs. government debt.

Households have some preferences which can be represented by $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t [u(q_t) - n_t + c_t - y_t]$, where

- q_t is the quantity consumed of the morning good and n_t is its production function
- The utility function u is strictly increasing, strictly concave, and with $u(0) = 0$
- c_t represents the consumption of the evening good and y_t its production function

Production functions are of negative sign since they require labor effort. It is assumed that there exists $q^* \in (0, +\infty)$ such that $u'(q^*) = 1$.

Characteristics of the economy

There are two value functions: $W_t(b, a)$, that is the value for an agent entering the centralized market owing b units of nominal bonds and a units of private claims in nominal claims, and $V_t(b, a)$, that represents the same value when entering the decentralized market. Then, we can write

$$W_t(b, a) = max_{c,y,b\prime,a\prime} \{c - y + \beta \mathbb{E}_t V_{t+1}(b', a')\} \quad (2.58)$$

such that

$$P_t c + \frac{a\prime}{1+R_t^p} + \frac{b\prime}{1+R} \leq P_t(y - \tau_t) + a + b, \quad (2.59)$$

$$a' \leq -\underline{a} P_t, \quad (2.60)$$

$$b' \geq 0. \quad (2.61)$$

In these equations, P_t is the price of goods in terms of nominal claims in the centralized market, while R_t^p is the nominal interest rate on private claims between t and $t+1$. Equation (2.60) is a borrowing constraint for households to make them not to engage Ponzi schemes. Equation (2.61) is a kind of nonnegativity constraint on money holdings in monetary models: the only claims recognized in the decentralized markets are those issued by government.

The solution of the maximization problem is

$$W_t(b, a) = \widehat{W}_t + \frac{a+b}{P_t} \quad (2.62)$$

Where $\widehat{W}_t$ does not depend on current state but only on subsequent choices. In the decentralized market meetings between sellers and buyers happen and the latter exchange $\tilde{b}$ units of government bonds for $\tilde{q}$ units of goods. From equation (2.62) it is implied that sellers have the following participation constraint

$$-\tilde{q} + \tilde{b}/P_t \geq 0. \quad (2.63)$$

This last equation can be seen as a "bonds-in-advance" constraint.

In such a market, buyers are those who gain the surplus. Given that, and using equation (2.63), the value function at the beginning of the period is given by

$$V_t(b,a) = W_t(b,a) + \chi max_q[u(q) - q] = \widehat{W}_t + \frac{a+b}{P_t} + \chi max_q[u(q) - q]^{60},$$

(2.64)

under the constraint

$$P_t q \leq b \quad (2.65)$$

At this point, the authors write the first order conditions of the maximization problem

$$\frac{\beta(1+R_t^p)}{P_{t+1}} = 1 \quad (2.66)$$

and

$$\frac{P_{t+1}}{\beta(1+R_t^p)} - 1 \geq \chi\left[max\left\{u'\left(\frac{B_{t+1}}{P_{t+1}}\right), 1\right\} - 1\right]^{61}$$

(2.67)

Keeping aside the case in which households are satisfied with their liquidity ($B_{t+1}/P_{t+1} \geq q^*$), the rate of return on government bonds will be lower than the one on private assets or even negative.

2.3.4 The Fiscal Theory of the Price Level when debt provides liquidity

Since Bassetto and Cui are interested in equilibria where the real rate on government debt is lower than the growth rate of the economy, the authors firstly define $\underline{r}$ as the lowest rate level at which the nonnegativity constraint on government debt holdings is not binding. Then they invert equation (2.67) to write the demand for real bonds as a function of the real interest rate on government debt. The result is the same obtained in equation (2.52) in the OLG economy, with f determined as

---

[60] One can easily verify that $\frac{\partial V_t(b,a)}{\partial b} = \frac{1}{P_t}$ for any $b > P_t q^*$ and $\frac{\partial V_t(b,a)}{\partial b} = \chi u'(b/P_t) + (1 - \chi)(1/P_t)$ for any $b < P_t q^*$, and that $\frac{\partial V_t(b,a)}{\partial a} = \frac{1}{P_t}$ in any case.

[61] It holds with equality if $B_{t+1} > 0$.

$$
f(1+r_{t+1}):\begin{cases} = 0 & for\ r_{t+1} \leq \underline{r} \\ \frac{1+R}{1+r_{t+1}}(u')^{-1}\left[\frac{\frac{1}{\beta(1+r_{t+1})}-(1-\chi)}{\chi}\right] & for\ \underline{r} \leq r_{t+1} < 1/\beta - 1 \\ \in [\beta(1+R)q^*, \infty) & for\ r_{t+1} = 1/\beta - 1 \end{cases}
$$

(2.68)

In order to make households be willing to hold government debt even when it yields a zero-interest rate, the liquidity demand must be sufficient.

The authors assume *f* being increasing in *r*. Moreover, if $r_{t+1} > 1/\beta - 1$, it is not possible to reach an equilibrium since households would prefer to continue saving indefinitely at that interest rate.

The definition of *f* is such that an equilibrium is defined by the same difference equation as in the OLG economy, which is (2.53) and (2.54) in terms of interest rate and (2.55) and (2.56) in terms of real purchases of government bonds. Even if the difference equation is the same in the two economies and the results as well, it is important to highlight some differences:

- s_t is defined on a different domain: differently from the OLG economy where $s_t \in (-\infty, w^y)$, in the liquidity economy s_t takes values in $(0, \infty)$
- in the OLG economy $\lim_{s_t \to w^y} r(s_t) = \infty$ while here $r(s_t)$ remains constant at $1/\beta - 1$ if $s_t \geq \beta(1+R)q^*$
- at $s_t = 0$, in the OLG $r(s_t)$ is finite while in the liquidity economy it may approach *-1*

Once these differences have been explained, it is possible to analyze the steady states of the difference equation (2.55). The steady states are the same obtained in the previous section for the OLG economy[62].

The authors found that in the OLG economy a continuum of initial values s_1 is consistent with an equilibrium while for the liquidity economy this happens only when $\tau \leq 0$. In fact, when $\tau \leq 0$, Figures (2.2) and (2.3) are still valid[63], the Fiscal Theory of the Price Level is not applicable, and a continuum of values of $s_1 \in [0, \bar{s}^{MAX}]$ is consistent with an equilibrium. Except for the case in which the

[62] For convergence properties, look at page 25 of (Bassetto & Cui, 2017).
[63] It being understood that the focus is only on the upper-right quadrant.

economy begins at the highest steady state $\bar{s}^{MAX}$, it converges to a lower value of s, which implies positive debt when $\tau < 0$, and no debt at all if $\tau = 0$.

When $\tau > 0$, the dynamics of the OLG economy and the liquidity are different.

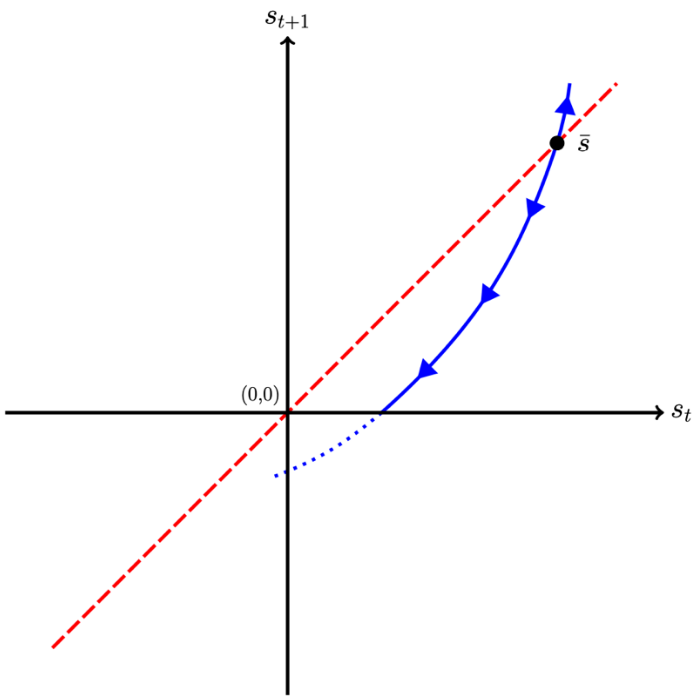

Figure 2.4- Source: (Bassetto & Cui, 2017), p.28

As shown in Figure (2.4), unless the economy starts at $s_1 = \bar{s}$, in all the other cases the nonnegativity constraint or the transversality condition are violated[64]. Consequently, we can obtain the price level uniquely from the condition

$$\frac{B_0}{P_0} = \tau + s_1 \tag{2.69}$$

In such a steady state, $\bar{s}r(\bar{s}) > 0$, which in turn implies $r(\bar{s}) > 0$. This means that the real interest rate on government bond is positive and greater than the growth rate of the economy. This result is consistent with the Fiscal Theory but not with the premises of the paper.

In conclusion, Bassetto and Cui demonstrated that the Fiscal Theory of Price Level cannot be considered a robust equilibrium selection criterion in an environment of low interest rates. In fact, the validity of this theory in such a scenario depends on

[64] In the graph, the phenomenon is represented with the arrows pointing to opposite direction in the point from the steady state

the specific economic cause of low returns. For this reason and given that policymakers are not completely able to estimate the true stochastic process of interest rates at a secular frequency, the authors suggest caution in relying on fiscal surprises to manage inflation.

2.4 Conclusions

The Fiscal Theory of the Price Level is without any doubt an appealing theory which has its pros and cons. Whilst it has the merit of giving a crucial role to the fiscal policy in determining the price level, it also has some weaknesses that I have analyzed in the previous paragraphs. As far as (Bassetto & Cui, 2017), this paper is a starting point for further studies over the analysis of interest-growth rate differential and the implications for the economy and the debt sustainability. However, (Cochrane, 2021) considers the debate over $r-g$ "irrelevant to current U.S. fiscal policy issues", and hence the related question of the current danger of fiscal inflation as well. Looking at the evolution of the debt-to-GDP ratio, with $r < g$, the potential problem is indeterminacy (i.e. the presence of multiple equilibria for the price level). In such a scenario, government runs a sequence of large primary deficits $s_t < 0$, which increase the debt; then, it keeps rollovering the debt without increasing the surpluses. With a debt growing at r, GDP growing at g and the debt-to-GDP declining at r-g, the debt-to-GDP ratio evolves as

$$\frac{d}{dt}\left(\frac{b_t}{y_t}\right) = (r_t - g_t)\frac{b_t}{y_t} - \frac{s_t}{y_t} \tag{2.70}$$

In the case where $r-g$, the debt-to-GDP ratio converges to its own.

In fact, for any initial price level the corresponding debt-to-GDP ratio will fade, even with no primary surpluses. According to Cochrane, such an analysis would suggest that there are no fiscal limits, and such a thing is not plausible. In fact, if a government could borrow without limits and without repaing its debts, there would not be the need for households of paying taxes, working, repaying mortgages, etc. But we all know that it is unrealist.

Apart from this view, it would be interesting to empirically analyze the differential between the interest rate and the growth rate in different countries in order to make some observations.

Chapter 3 | Empirical evidence

3.1 Introduction

In the last two decades some economists tried to test the Fiscal Theory of the Price Level through econometric tools. The approaches used in these tests were mainly two:

- the backward-looking approach used, for instance, in (Bohn, 1998). According to it, in a Ricardian regime (or "monetary dominant" regime) an increase in the level of debt in the previous period would be followed by a larger primary surplus in the next period (i.e., $\Delta b_{t-1} \rightarrow \Delta s_t$);
- the forward-looking approach, as the one implemented in (Canzoneri, et al., 2001). In this case, in a Ricardian regime, a larger primary surplus today would result in a lower level of debt in the future (i.e., $\Delta s_t \rightarrow \nabla b_{t+1}$).

In this chapter I will run an econometric analysis based on (Bajo-Rubio, et al., 2009). This paper aimed to analyze the empirical evidence in support of the assumptions of the Fiscal Theory of the Price Level for the 11 EU member states which were part of the Monetary Union. Specifically, it investigated in which way the fiscal solvency is achieved: by endogenously adjusting the primary budget surplus (i.e., in the "monetary dominant regime") or by endogenously adjusting the price level (i.e., in the "fiscal dominant regime").

3.2 Data and econometric methodology

Following the approach of (Bajo-Rubio, et al., 2009), I estimated the cointegration relationships between the primary surplus and the (lagged) level of debt, both as ratios to GDP:

$$\mathrm{s_t} = \alpha + \beta \mathrm{b_{t-1}} + \mathrm{v_t} \tag{3.1}$$

where $\mathrm{v_t}$ is an error term. The estimation of β allows us to understand if in the investigated country there is a monetary dominant regime or a fiscal dominant regime. In fact, when $\beta > 0$ it means that the present value of the government budget constraint is satisfied and, consequently, there is fiscal solvency. If instead $\beta \leq 0$, the present value of the government budget constraint is not satisfied, and this would mean that the government is not solvent. This event would coincide with the presence of a non-Ricardian or Fiscal dominant regime which allows fiscal policy to set primary balances and to follow an arbitrary budget process, not necessarily compatible with solvency (Afonso & Tovar Jalles, 2012). The problem arises when the estimated β is significantly greater than zero: this case would be compatible with both the monetary dominant regime and the fiscal dominant one. In fact, in a monetary dominant regime an increase in debt in period t would be followed by a larger primary surplus in the next period (i.e., $\Delta \mathrm{b_t} \rightarrow \Delta \mathrm{s_{t+1}}$, which implies an estimated $\beta > 0$). Moreover, in a fiscal dominant regime, a decrease in the expected primary surplus would imply an increase in price level which in turn would decrease the current debt ratio (i.e., $\nabla \mathrm{E_t s_{t+1}} \rightarrow \nabla \mathrm{b_t}$ which implies an estimated $\beta > 0$ also in this case).

3.2.1 Data

I used data on primary budget surplus and general government consolidated gross debt, both as percentage of GDP, for the G7 countries. I chose this group of countries to compare the results among the largest advanced economies in the world. The data source is ECB 'Statistical Data Warehouse' for the three European member states, U.K and U.S., 'Statistics Canada' for Canada, and the 'International Monetary Fund' for Japan. As far as primary surplus, I used the variable "Government primary deficit (-) or surplus (+) (as % of GDP)", also defined as "Net lending/net borrowing excluding interest payable". As far as debt, I used the variable "Government debt (consolidated) (as % of GDP)". This variable refers to the so called "Maastricht Debt" and it was defined in ESA 2010 as the total consolidated gross debt at face value in the following categories of government liabilities: currency and deposits, debt securities and loans[65].

The range of time of data differs across countries:

- except for Germany, for which data go from 2002, for France, Italy and U.K. I used quarterly data which go from 1999 to 2020;
- for Japan I used annual data from 1980 to 2020;
- for Canada I used annual data from 1981 to 2020;

[65] Source: "https://ec.europa.eu/eurostat/documents/3859598/5925693/KS-02-13-269-EN.PDF/44cd9d01-bc64-40e5-bd40-d17df0c69334"

- U.S. annual data go from 1960 to 2019[66]. In fact, data on 2020 primary surplus/deficit will not be available until autumn 2021.

I chose quarterly data to have a longer dataset. However, for Japan, Canada and U.S. only annual data were available.

Hence, differently from (Bajo-Rubio, et al., 2009), which studied the period 1970 through 2005, I used more recent data which cover also the 2007-2008 global financial crisis, the European sovereign debt, and the beginning of COVID pandemic in 2020 (apart from U.S.).

3.2.2 Stationarity

Variables such as surplus and debt are usually not stationary and consequently cannot be analyzed in the standard way. Very often it is possible to consider the first difference of the time series to make it stationary. If taking a first difference of a time series y_t produces a stationary process, then such a series is said to be integrated of order one, denoted $I(1)$. Graphs of $I(1)$ series will typically wander about with no tendency to revert to a fixed mean (Greene, 2012).

To visualize the path of the time series, on MATLAB I plotted the graphs of primary surplus (%GDP) and of debt (%GDP) and their mean for each country, using as source of data ECB 'Statistical Data Warehouse', 'Statistics Canada' and the 'International Monetary Fund'.

[66] Data for 2020 are still not available

France

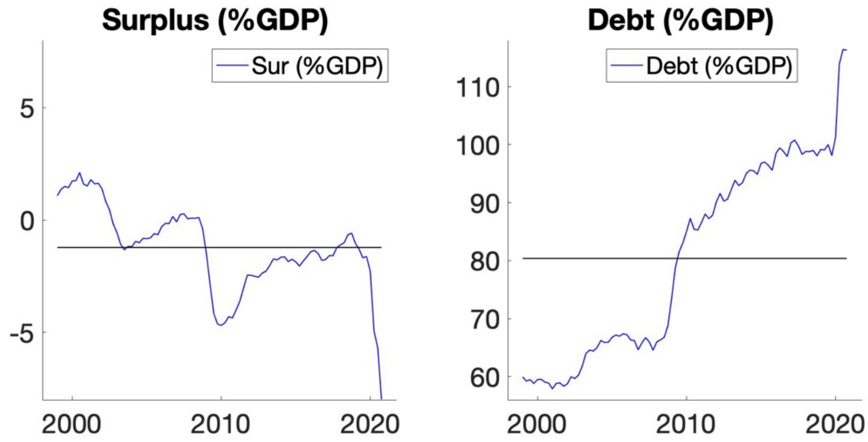

Figure 3.1 – France primary surplus and debt

From the graphs it is possible to observe the path of the time series of primary surplus (%GDP) and debt (%GDP) around their mean (black line). As far as primary surplus, it is characterized by various peaks, with the highest surplus in 2000 and the highest deficits in 2010 (in the correspondence of the period of the European sovereign debt crisis) and in 2020 (during the period of COVID-19 pandemic). As far as debt, it seems to have an increasing trend and it reached its highest value (116,406% of GDP) in the third quarter of 2020. All in all, the two time series would seem to be non-stationary.

Germany

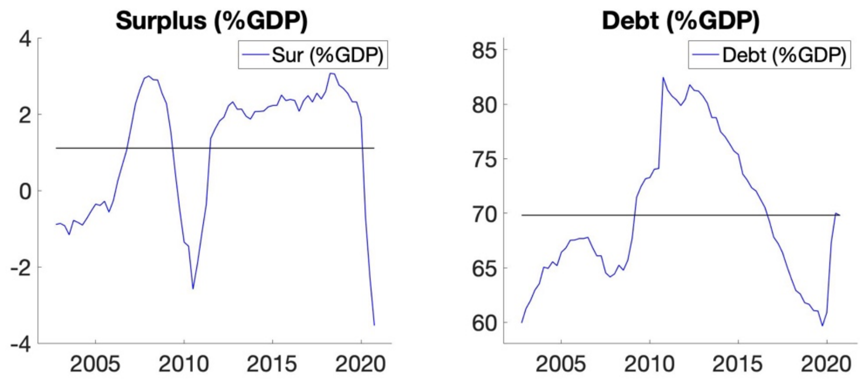

Figure 3.2 – Germany primary surplus and debt

Also in this case it is possible to observe a non-stationary trend and the time series are featured by some peaks. As far as primary surplus, from 2006 through 2008 it

is possible to observe an increasing trend up to a maximum of 3% of GDP which converts in a decreasing trend through 2010 characterized also by deficits. This last period overlaps with the financial crisis of 2008. From 2010, when the first peak of deficit in order of time occurs, there is a decrease in deficits and from 2011 surpluses keep being positive and over the mean value up to 2020, when there is a negative peak (higher than the 2010 one) coinciding with the COVID-19 pandemic. As far as debt, it shows an increasing trend through all the period under investigation, but it has never been above the 80% of GDP.
However, the two time series would not seem to be stationary.

Italy

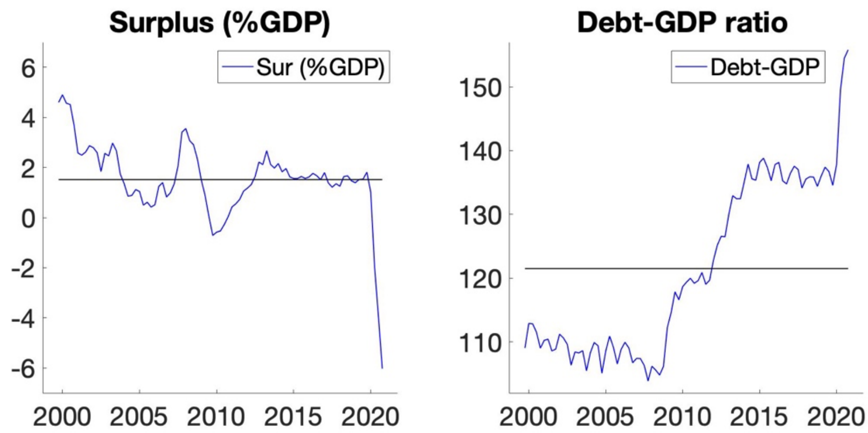

Figure 3.3 – Italy primary surplus and debt

As far as the path of primary surplus, under the period of investigation Italy has had just two periods of deficit, 2010 and 2020, during the sovereign debt crisis and the COVID-19 pandemic respectively. As far as debt, its level has always been over the 100% of GDP. The time series seems to be stationary just over two periods of time, 2000-2008 and 2015-2019. Over the other periods it shows an increasing trend. Moreover, there is a peak in 2020 which corresponds to the high deficit due to pandemic.

U.K.

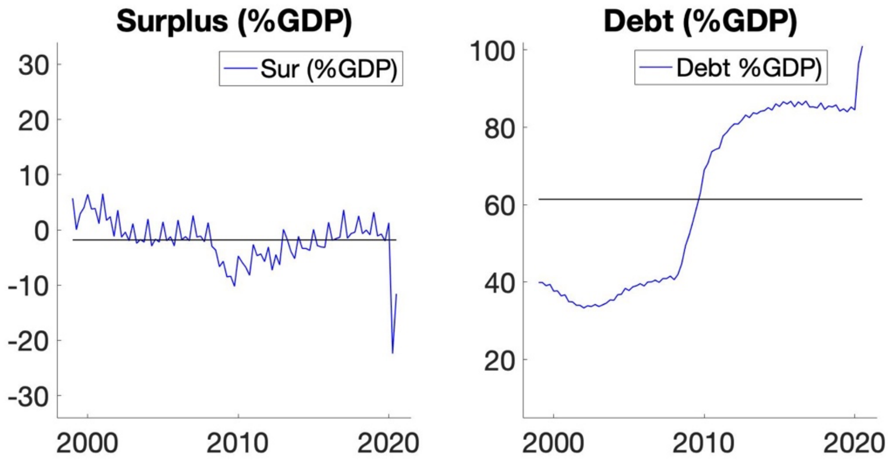

Figure 3.4 – U.K. primary surplus and debt

Under the period of investigation, U.K. would seem to have a more stationary path for surpluses than the other analyzed countries. However, also in this case it is possible to observe two significant negative peaks (-10,23% and -22,41% of GDP) in correspondence of 2010 and 2020 as in the other European countries. As far as debt, it has been under the 100% of GDP until 2020 and except for the periods 2008-2013 and 2020, it does not show an increasing path.

Canada

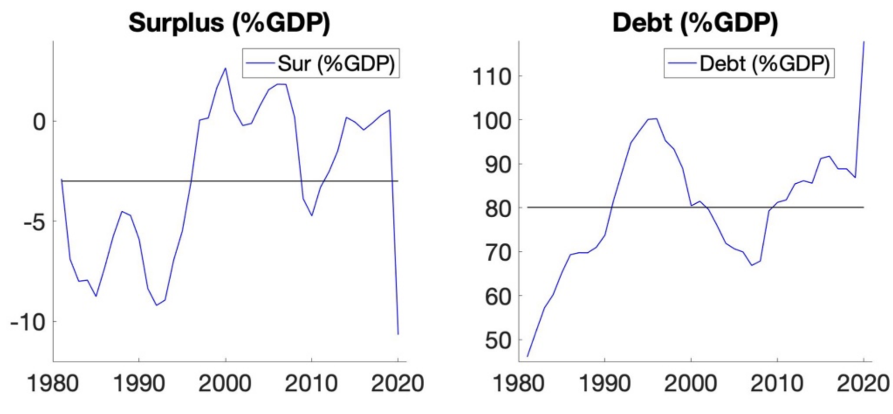

Figure 3.5 – Canada primary surplus and debt

As far as primary surplus, it does not seem to be stationary and it shows many peaks (positive and negative). The time series path is characterized by the prevailing presence of deficits, especially until 1993 and it does not seem to be stationary. The debt shows an increasing trend from 1981 to 1996 which converts in a decreasing one from 1997 to 2007, and then again turns into an increasing trend with a peak in 2020. Despite the frequent deficits, debt has been above the 100% of GDP only from 2020. Also this time series looks not-stationary.

Japan

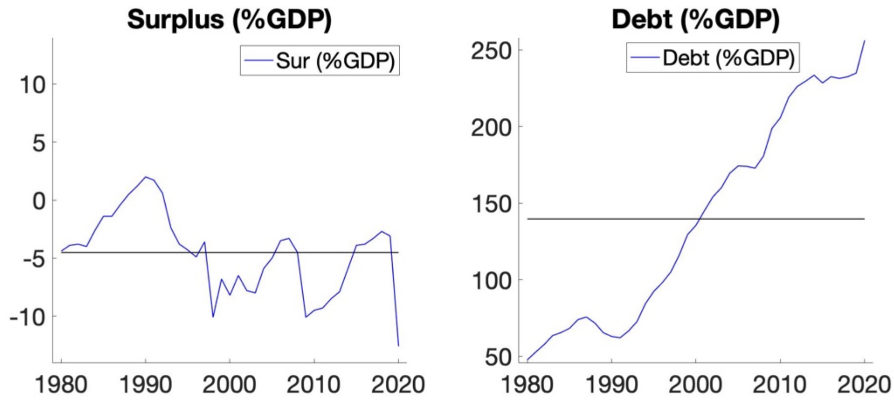

Figure 3.6 – Japan primary surplus and debt

Japan has run continuous deficits over the whole period of investigation (except for the period 1989-1991), with a mean value of -4,5% of GDP. The path looks not-stationarity. As far as debt, it shows an increasing trend and its current level is the highest among the other analyzed countries (256,2% of GDP).

U.S.

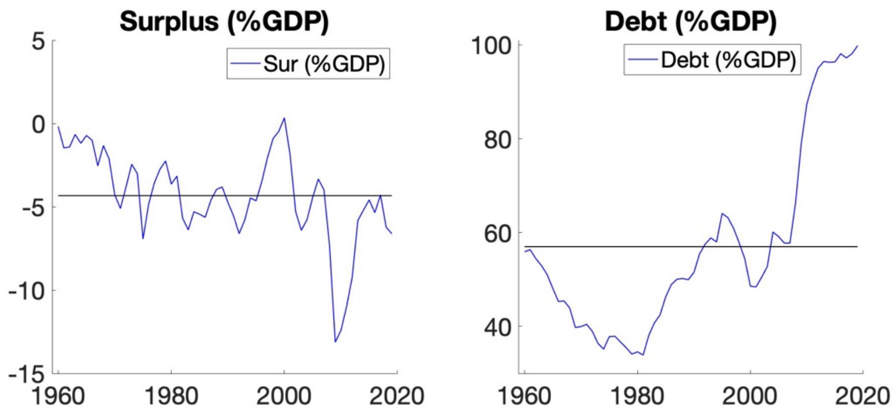

Figure 3.7 – U.S. primary surplus and debt

U.S. data unfortunately do not include the 2020 but if they did, they would show a high negative peak in primary surplus due to COVID-pandemic measures.

Both surplus and debt do not look stationary. A significant deficit is observed in 2009 due to the measures taken to tackle the financial crisis.

The debt shows a tendentially increasing debt and its current level is of 99,82% of GDP.

3.2.3 Unit root tests

Of course, looking at the time series of primary surplus and debt is not enough to say whether they are stationary or not. For this reason, I used two unit root tests to analyze stationarity, the Phillips-Perron test and the Augmented Dickey-Fuller test. These tests assess the presence of a unit root in the time series[67].

Phillips-Perron test

Phillips-Perron tests (PP tests) assess the null hypothesis of a unit root in a univariate time series y (Mathworks). The model used in all tests is:

$$y_t = c + \delta t + a y_{t-1} + e(t) \tag{3.2}$$

The null hypothesis puts a restriction on 'a' that must be equal to 1.

In my analysis, I run this test for both primary surplus and debt, using some variants of the simple Phillips-Perron test. Firstly, I used the basic PP test. Then I ran it specifying other three model variants: the trend stationary (TS), the autoregressive with drift (ARD) and the trend stationary alternative with 0, 1, and 2 lags.

The trend stationary one tests the null model

$$y_t = c + y_{t-1} + e(t) \tag{3.3}$$

against the alternative model

$$y_t = c + \delta t + a y_{t-1} + e(t) \tag{3.4}$$

where c is the drift coefficient, δ is the deterministic trend coefficient, and AR (1) coefficient must satisfy $a < 1$.

The PP test for the Autoregressive model tests the null model

$$y_t = y_{t-1} + e(t) \tag{3.5}$$

against the alternative model

$$y_t = a y_{t-1} + e(t) \tag{3.6}$$

with AR (1) coefficient $a < 1$.

As far as the PP test for the Autoregressive with drift, the autoregressive null model (3.5) is tested against the alternative model

[67] A linear stochastic process has a unit root if 1 is a root of the process's characteristic equation.

$$y_t = c + a y_{t-1} + e(t) \tag{3.7}$$

where 'c' is the drift coefficient and AR(1) coefficient $a < 1$.

On MATLAB the output of these tests is 'h' which takes the value 0 when the null hypothesis is accepted (i.e., when the time series contains unit roots and it is $I(1)$), and a value equal to 1 when the null hypothesis is not accepted (i.e., when the time series has no unit roots and it is $I(0)$).

Augmented Dickey-Fuller test

The Augmented Dickey-Fuller assesses the null hypothesis of a unit root using the model

$$y_t = c + \delta t + \phi y_{t-1} + \beta_1 \Delta y_{t-1} + \ldots + \beta_p \Delta y_{t-p} + e(t) \tag{3.8}$$

where

- Δ stands for the differencing operator, such that $\Delta y_t = y_t - y_{t-1}$.
- The number of lagged difference terms, p, is arbitrarily identified.
- ε_t is a mean zero innovation process.

The null hypothesis of a unit root is $H_0\colon \phi = 1$ while the alternative one is $H_1\colon \phi < 1$. The model can have different growth characteristics: if $\delta = 0$, the model has no trend component, while if $c = 0$ and $\delta = 0$, then it has no drift or trend. If the null hypothesis is accepted, then the possibility of a unit root is rejected.

On MATLAB, the function 'adftest' performs ordinary least square (OLS) regression to estimate coefficients in the alternative model. Under the null hypothesis Dickey-Fuller statistics do not have standard distributions.

In my analysis I firstly carried out a Dickey-Fuller test without augmentation, then an augmented Dickey-Fuller test against a trend-stationary alternative augmented with 0, 1, and 2 lagged difference terms. The three lag choices are treated as separate tests (i.e., there are three outputs).

Both the Phillips-Perron and the Dickey-Fuller tests – run on MATLAB – have 'h' as output, which can take value 0 if the null hypothesis is accepted (i.e., there is a unit root and the time series is $I(1)$), or value 1 if the null hypothesis cannot be accepted and the time series is stationary (i.e., $I(0)$).

Unit root test results

PP Test						
Values of h						
	PP	PP$_{TS}$	PP$_{ARD}$	PP$_{TSlag}$		
A) Government gross debt						
France	0	0	0	0	0	0
Germany	0	0	0	0	0	0
Italy	0	0	0	0	0	0
Canada	0	0	0	0	0	0
USA	0	0	0	0	0	0
UK	0	0	0	0	0	0
Japan	0	0	0	0	0	0

	PP	PP$_{TS}$	PP$_{ARD}$	PP$_{TSlag}$		
B) Primary budget surplus						
France	0	0	0	0	0	0
Germany	0	0	0	0	0	0
Italy	0	0	0	0	0	0
Canada	1	1	1	1	1	1
USA	0	0	0	0	0	0
UK	1	1	1	1	1	1
Japan	0	0	0	0	0	0

Table 3.1 – Results Phillips-Perron test

DF Test				
Result h				
	Simple ADF	AgDF$_{Lag\,0}$	AgDF$_{Lag1}$	AgDF$_{Lag\,2}$
A) Government gross debt				
France	0	0	0	0
Germany	0	0	0	0
Italy	0	0	0	0
Canada	0	0	0	0
USA	0	0	0	0
UK	0	0	0	0
Japan	0	0	0	0

	Simple ADF	AgDF$_{Lag\,0}$	AgDF$_{Lag1}$	AgDF$_{Lag\,2}$
B) Primary budget surplus				
France	0	0	0	0
Germany	0	0	0	0
Italy	0	0	0	0
Canada	0	1	1	0
USA	0	0	1	1
UK	1	1	0	0
Japan	0	0	0	0

Table 3.2 – Results Dickey-Fuller Test

As far as Phillips-Perron test for the government debt, it accepts the null hypothesis for all the analyzed countries implying non-stationarity for all the time series and for all the variants of the test at 5% confidence level.

As far as the primary surplus, the PP test does not accept the null hypothesis for Canada and UK in all variants of the model.

On the other hand, the Dickey-Fuller test shows the same results of the PP test for the debt. As far as primary surplus, the DF does not accept the null hypothesis in all cases. In fact, Canada time series are $I(1)$ according to the simple DF test and the augmented DF test with two lags, while they are $I(0)$ according to the augmented DF test with 0 lags and 1 lag.

U.K. surplus is $I(0)$ according to the simple DF test and the the augmented DF test with lag 1; in the tests with one and two lags it is $I(1)$.

All in all, we can consider all time series non-stationary as confirmed by the unit root tests and by literature as well.

When a time series is not stationary it is still possible to transform them in a stationary one through first order differencing, i.e., by taking Δy_t.

On MATLAB I computed these differentials (Delta Surplus and Delta Debt) and then I ran again the Phillips-Perron test and the Dickey-Fuller test on these new time-series to confirm their stationarity.

3.2.4 DOLS regression

Once established the order of integration of the series, I estimated the parameter β in the equation (3.1). I used the Dynamic Ordinary Least Squares method used by (Stock & Watson, 1993) which provides a robust correction to the possible presence of endogeneity in the explanatory variables, as well as of serial correlation in the error terms of the OLS estimation (Bajo-Rubio, et al., 2009). For this reason, I estimated the long-run dynamic equation which includes leads and lags of the first difference of the explanatory variable in equation (3.1):

$$s_t = \alpha + \beta b_{t-1} + \sum_{j=-q}^{q} \varphi_j \Delta b_{t-1-j} + v_t \qquad (3.9)$$

where the index q is computed as $q = INT\left(T^{1/3}\right)$, i.e., the cubic root of the number of observations, rounded to an integer. Given my data, q was equal to 4 for the time series with more observations, and equal to 3 for those with less observations.

I will take as example Italy to have a clear idea on the method of calculation of each regressor.

Italy has a time series made of 85 quarterly observations, from 31/12/99 to 31/12/20. Hence, $q = INT\left(85^{1/3}\right) = 4$.

Once q is identified, I can make equation (3.9) explicit:

$$s_t = \alpha + \beta b_{t-1} + \varphi_{-4}\Delta b_{t+3} + \varphi_{-3}\Delta b_{t+2} + \varphi_{-2}\Delta b_{t+1} + \varphi_{-1}\Delta b_t + \varphi_0 \Delta b_{t-1} + \varphi_1 \Delta b_{t-2} + \varphi_2 \Delta b_{t-3} + \varphi_3 \Delta b_{t-4} + \varphi_4 \Delta b_{t-5} + v_t \quad (3.10)$$

Then, on Excel I computed the 10 regressors, i.e., the debt at *t-1* (b_{t-1}), and the 9 differentials of the debt which can be written as:

$$\Delta b_t = b_t - b_{t-1}$$

$$\Delta b_{t-1} = b_{t-1} - b_{t-2}$$

$$\Delta b_{t-2} = b_{t-2} - b_{t-3}$$

$$\Delta b_{t-3} = b_{t-3} - b_{t-4}$$

$$\Delta b_{t-4} = b_{t-4} - b_{t-5}$$

$$\Delta b_{t-5} = b_{t-5} - b_{t-6}$$

$$\Delta b_{t+1} = b_{t+1} - b_t$$

$$\Delta b_{t+2} = b_{t+2} - b_{t+1}$$

$$\Delta b_{t+3} = b_{t+3} - b_{t+2}$$

Period\Unit:	Sur/GDP	Debt/GDP	T-1	T-2	T-3	T-4	T-5	T-6	Dt	Dt-1	Dt-2	Dt-3	Dt-4	Dt-5
31/12/99	4,598	109,054	NA	NA	NA	NA	NA	NA		NA				
31/03/00	4,9	112,914	109,054	NA	NA	NA	NA	NA	=C3-E3	NA				
30/06/00	4,567	112,841	112,914	109,054					-0,073	3,86				
30/09/00	4,512	111,521	112,841	112,914	109,054				-1,32	-0,073	3,86			
31/12/00	3,69	109,026	111,521	112,841	112,914	109,054			-2,495	-1,32	-0,073	3,86		
31/03/01	2,579	110,245	109,026	111,521	112,841	112,914	109,054		1,219	-2,495	-1,32	-0,073	3,86	
30/06/01	2,493	110,443	110,245	109,026	111,521	112,841	112,914	109,054	0,198	1,219	-2,495	-1,32	-0,073	3,86
30/09/01	2,613	108,589	110,443	110,245	109,026	111,521	112,841	112,914	-1,854	0,198	1,219	-2,495	-1,32	-0,073
31/12/01	2,868	108,886	108,589	110,443	110,245	109,026	111,521	112,841	0,297	-1,854	0,198	1,219	-2,495	-1,32

T+1	T+2	T+3		Dt+2	Dt+3
112,91	112,84	111,52	=R2-C2	-0,073	-1,32
112,84	111,52	109,03	-0,073	-1,32	-2,495
111,52	109,03	110,25	-1,32	-2,495	1,219
109,03	110,25	110,44	-2,495	1,219	0,198
110,25	110,44	108,59	1,219	0,198	-1,854
110,44	108,59	108,89	0,198	-1,854	0,297

Figure 3.8 – Examples File Excel for computation of regressors

Because of the lags and leads, the time series loses the first 6 and the last 3 observations. Therefore, the remaining observations to use in the regression are 76.

Before performing the DOLS method, I ran a simple OLS regression like equation (3.1) taking as independent variable the observations from the first to second last of the debt (%GDP) and as dependent variable the primary surplus from the second observation to the last one. Then I ran the Dynamic OLS and I observed the differences.

Table 3.3 – Betas OLS and Betas DOLS

	βOLS	βDOLS
France	-0,077	-0,045
Germany	0,012	0,046
Italy	-0,044	0,002
Canada	0,105	0,055
USA	-0,051	-0,019
UK	-0,057	-0,024
Japan	-0,028	-0,018

As far as OLS betas, they are mostly negative except for Germany and Canada. DOLS betas are all negative apart from Germany, Italy, and Canada. However, only the beta of Germany is significantly different from 0 according to the T-Test at 5% level. Therefore, a preliminary conclusion could be that only in Germany fiscal policy would have been sustainable and a Ricardian or Monetary dominant regime would have prevailed. In the other countries, there is no evidence of fiscal sustainability, and a Fiscal dominant regime could be compatible.

3.2.5 Cointegration test

Cointegration tests are implemented to observe if two time series with stochastic trends move together in a similar way in the long run.

The first method that I used was running an augmented Dickey-Fuller test on the residuals of the Dynamic OLS regression. If the output of the test is h=0, then the null hypothesis is accepted, and the residuals are non-stationary. If h=1, the null hypothesis cannot be accepted, and the residuals are stationary. If residuals are stationary, then the cointegration is verified.

The second method was Engle-Granger test to assess the null hypothesis of no cointegration among the time series in Y. On MATLAB, the function 'egcitest' regresses Y(:,1) on Y(:,2:end), then tests the residuals for a unit root.

Table 3.4 Results cointegration test

	Cvalue
France	-6,582
Germany	-6,710
Italy	-6,596
Canada	-6,622
USA	-6,839
UK	-6,587
Japan	-6,597

The table shows the critical values of the Engel-Granger test for each country. As opposed to (Bajo-Rubio, et al., 2009), the Engel-Granger test does not show cointegration. However, this result is consistent with the one found in (Afonso & Tovar Jalles, 2012), a working paper of the ECB of 2012 which assessed the sustainability of public finances in OECD countries over the period 1970-2010. The absence of cointegration would foster the hypothesis of fiscal policy unsustainability.

3.2.6 Granger Causality Test

After the cointegration test, I ran a Granger Causality test to assess if each of the two variables is useful for forecasting the other one.

Since the variables under consideration are not stationary, the test is run taking in consideration first or higher differences. The regression that must be tested is the one proposed by (Sims, 1972):

$$X_t = \alpha_0 + \delta_1 X_{t-1} + \gamma_1 (X_{t-1} - \beta Y_{t-1}) + \sum_{i=1}^{m} \alpha_{1i} \Delta X_{t-1} + \sum_{i=1}^{n} \alpha_{2i} \Delta Y_{t-1} + \varepsilon_t$$

(3.11)

where $Z_{t-1} = X_{t-1} - \beta Y_{t-1}$ stands for an error correction model.

To assess the direction of the causality, the test must be run using the primary surplus at time t (s_t) and the debt at time *t-1* (b_{t-1}) alternatively as dependent variables and it must include up to three lags of the first difference of each of these variables.

The explicit equations for s_t and b_t would be:

$s_t = \alpha_0 + \delta_1 s_{t-1} + \gamma_1 Z_{t-1} + \alpha_{11}\Delta s_{t-1} + \alpha_{12}\Delta s_{t-2} + \alpha_{13}\Delta s_{t-3} + \alpha_{21}\Delta b_{t-1} +$
$\alpha_{22}\Delta b_{t-2} + \alpha_{23}\Delta b_{t-3} + \varepsilon_t$ (3.12)

when s_t is the dependent variable, and

$b_t = \alpha_0 + \delta_1 b_{t-1} + \gamma_1 Z_{t-1} + \alpha_{11}\Delta b_{t-1} + \alpha_{12}\Delta b_{t-2} + \alpha_{13}\Delta b_{t-3} + \alpha_{21}\Delta s_{t-1} +$
$\alpha_{22}\Delta s_{t-2} + \alpha_{23}\Delta s_{t-3} + \varepsilon_t$ (3.13)

when b_t is the dependent variable.

I used the standard F test to test for Granger Causality: the null hypothesis is $\gamma_1 = 0$ for the absence of a long-run causality, and $\alpha_{2i} = 0$ for the absence of a short-run causality.

I started with s_t as dependent variable. On MATLAB I regressed s_t on b_{t-1} according to the equation

$s_t = \beta b_{t-1} + z_t$ (3.14)

After the estimation of β, I could compute the regressor Z_{t-1} and I constructed all the other regressor vectors. The matrix of independent variable is made of 9 regressors, included the intercept. As in the Dynamic OLS, some observations are cut due to the lags. Then, I ran the regression (3.12) and I computed the covariance matrix under homoscedasticity assumption. In the Granger causality test I verified if the error correction regressor, and the differentials regressors $\Delta b_{t-1}, \Delta b_{t-2}$, and Δb_{t-3}, are equal to zero.

```
c = [0, 0, 0, 0];
H = [0, 0, 1, 0, 0, 0, 0, 0, 0;
     0, 0, 0, 0, 0, 0, 1, 0, 0;
     0, 0, 0, 0, 0, 0, 0, 1, 0;
     0, 0, 0, 0, 0, 0, 0, 0, 1];

p_value = linhyptest(beta_ECM1,V_betaECM,c,H,9);
```

Figure 3.9 – Granger Causality Test

The function 'linhyptest' returns the p-value of a hypothesis test on the vector of parameters. The objects of the function are the betas, the covariance matrix, the vector 'c', 'H', and the degrees of freedom. 'c' and 'H' specify the null hypothesis in the form H*b = c, where b is the vector of unknown parameters estimated by beta. If the obtained p-value is greater than 0.05, then the null hypothesis is accepted and the dependent variable is well predicted by the other one, i.e., there is Granger causality.

Then I ran the same procedure taking b_t as dependent variable.

Table 3.5 – Granger Causality results

P-value results

	$B_{t-1} \rightarrow S$	yes/no	$S_{t-1} \rightarrow B$	yes/no
France	0,00047400	no	0,00872052	no
Germany	0,00628568	no	0,0081213	no
Italy	0,00621587	no	0,0403249	no
Canada	0,00197245	no	0,0567454	yes
USA	0,03625410	no	0,81249261	yes
UK	0,02110369	no	0,00098377	no
Japan	0,00032277	no	0,79439869	yes

From the Granger causality test it is clear that debt at time *t-1* does not have significant predictive power on primary surplus at time *t* for all countries under investigation. Moreover, the same result is obtained when the surplus is the independent variable except for Canada and USA, which have a p-value > 0,05.

3.2.7 Forward-looking approach

As last step I computed the impulse-response functions of the debt-GDP ratio to innovations in the primary surplus-GDP ratio from an estimated VAR in these two variables following the approach applied in (Canzoneri, et al., 2001).

The Vector Autoregressive model (VAR) is a multivariate stochastic process, i.e., with random multiple variables. VAR can be of different orders depending on the lag operator.

Taking for example, a VAR(1) , the matrix form of the VAR model for surplus and debt would be:

$$\begin{bmatrix} s_t \\ b_t \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} + \begin{bmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{bmatrix} \begin{bmatrix} s_{t-1} \\ b_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_t^s \\ \varepsilon_t^d \end{bmatrix} \tag{3.15}$$

where:

- c_1 and c_2 are the intercepts
- β_{ij} are the estimators
- s_{t-1} and b_{t-1} are the regressors
- ε_t^s and ε_t^d represent respectively a surplus shock and a debt shock.

As said before, the number of lags can vary. To choose the optimal lag order I used the Schwarz information criterion or Bayesian information criterion (BIC): in fact, the optimal VAR model is the one with the lowest BIC.
On MATLAB I computed the VAR model for 1, 2, 3, and 4 lags for all countries and then by using the Schwarz information criterion I selected the appropriate one to go ahead with the analysis.

Table 3.6 – B.I.C. results

B.I.C.

	VAR (1)	VAR (2)	VAR (3)	VAR(4)
France	463,48	445,645	338,397	387,78
Germany	392,998	365,843	363,13	374,02
Italy	541,084	517,315	509,701	519,476
Canada	362,916	356,03	361,045	356,281
USA	456,236	455,396	462,981	470,458
UK	722,244	718,788	715,734	694,234
Japan	401,979	387,733	391,059	395,279

The Bayesian Information Criterion was computed through a maximum likelihood estimation.
As shown in the table, the highlighted cells are the results of the B.I.C. criterion. Hence, for France and UK I chose a VAR (4), for Germany and Italy a VAR (3), for Canada, USA, and Japan a VAR (2).
As next step I computed and plotted the Impulse Response Function for each country. This function returns the dynamic response to a one-standard-deviation to primary surplus in a VAR (p) model. Hence it traces the effects of an innovation shock to the primary surplus on the response of debt.
For example, with two variables there will be 2 shocks and 4 vectors of impulse functions:

$$\begin{bmatrix} \frac{\delta s}{\delta \varepsilon_t^s} & \frac{\delta s}{\delta \varepsilon_t^b} \\ \frac{\delta b}{\delta \varepsilon_t^s} & \frac{\delta b}{\delta \varepsilon_t^b} \end{bmatrix} \tag{3.16}$$

On MATLAB I took the vector $\frac{\delta b}{\delta \varepsilon_t^s}$ to analyze the effects of an innovation shock to the primary surplus on the response of debt over a 10-year horizon with two confidence intervals at 95%

France – VAR (4)

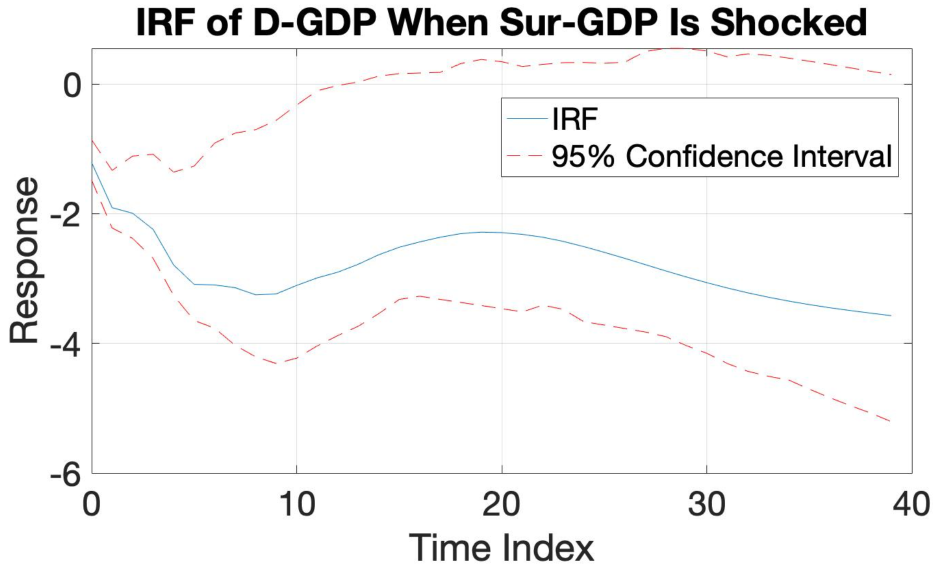

Figure 3.10 – France Impulse Response Function

In this graph the time represents the number of quarters over an horizon of ten years. After a positive shock of the primary surplus at time 0, the debt shows a negative response for two years (i.e., 8 quarters), then it is increasing for almost 2 years and becomes stable later. It can be observed that the response function is not always statistically significant given the confidence intervals.

Germany VAR (3)

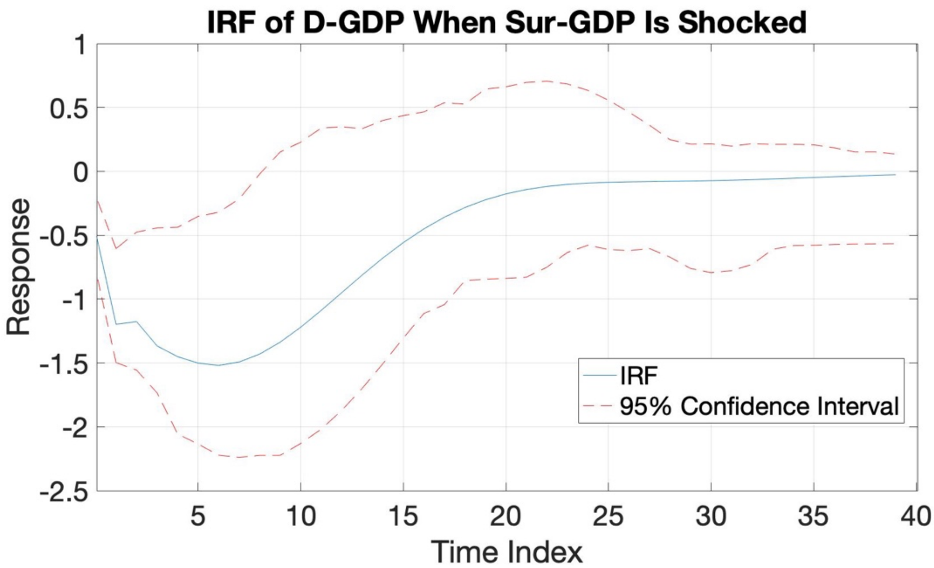

Figure 3.11 – Germany Impulse Response Function

Also in this case, time is expressed in quarters, over a 10-year horizon. The debt has a negative response following an innovation in the surplus (%GDP) for the first 6 quarters (less than 2 years). Then, it increases again up to the 25th quarter when it becomes stable. Except for the final quarters, the response function does not seem statistically significant.

Italy – VAR (3)

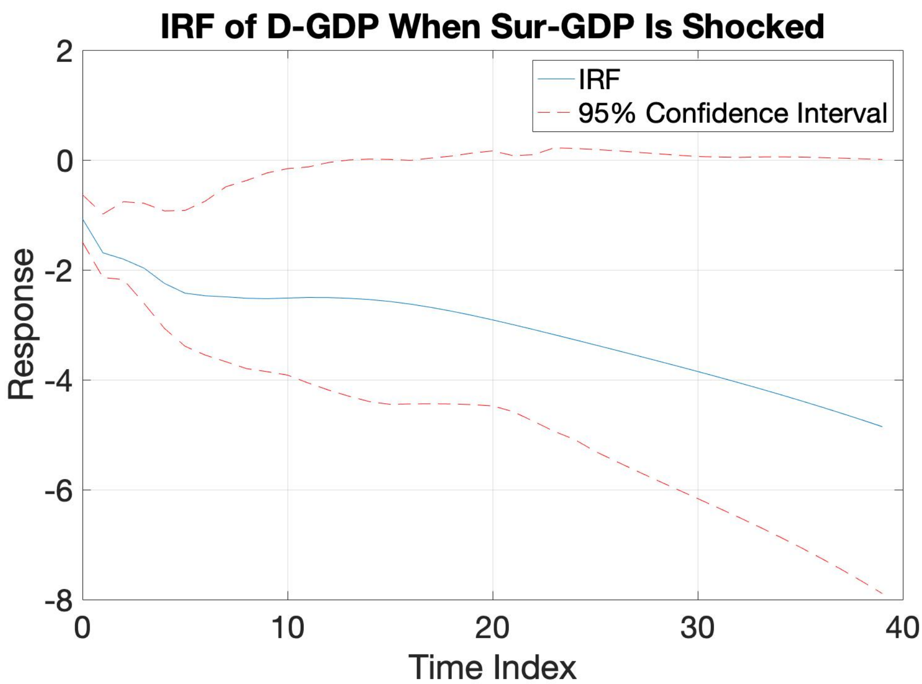

Figure 3.12 – Italy Impulse Response Function

Time is represented by quarters. As far as Italy, the positive innovation of primary surplus is followed by a negative response of debt which keeps decreasing through the following next ten years. The impulse response function does not seem statistically significant.

Canada VAR (2)

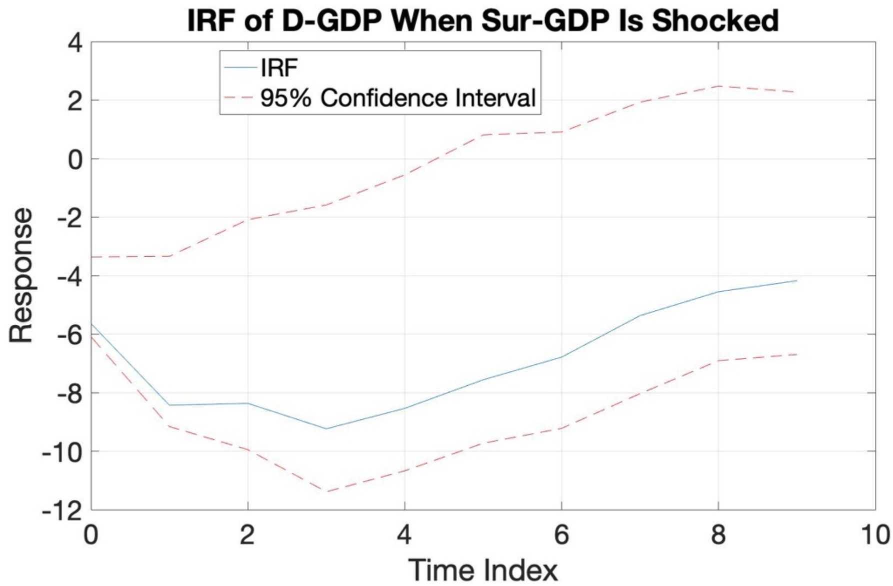

Figure 3.13 – Canada Impulse Response Function

In this case, time is expressed in years. The positive innovation of primary surplus is followed by a negative response of debt until year 1. Then, after a period of stability, from year 2 up to year 3 the debt decreases and then starts increasing over the following years. However, impulse functions look not statistically significant.

USA – VAR (2)

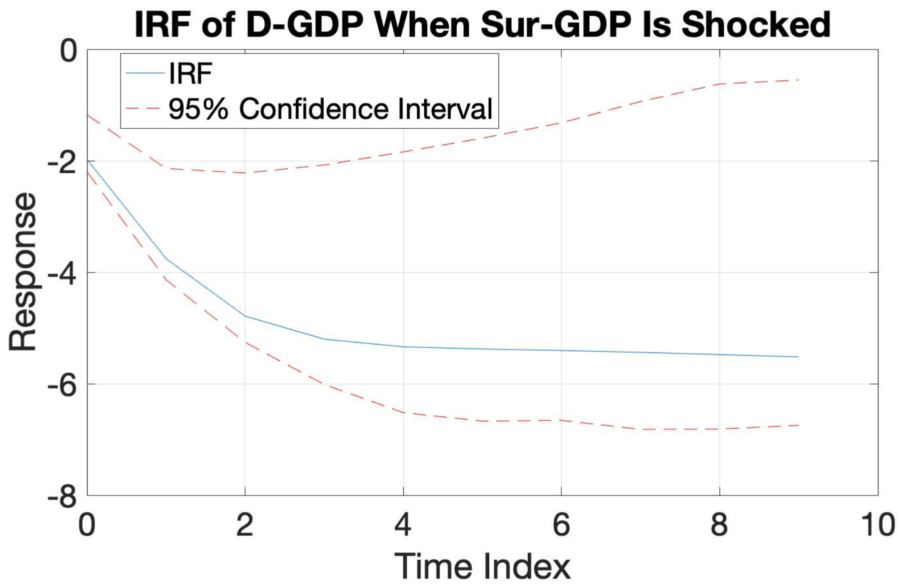

Figure 3.14 – USA Impulse Response Function

The time index indicates the years. In this case, the positive innovation in primary surplus is followed by a negative response of debt for the first three years. After this period, it becomes stable. The impulse response function does not seem statistically significant.

UK – VAR (4)

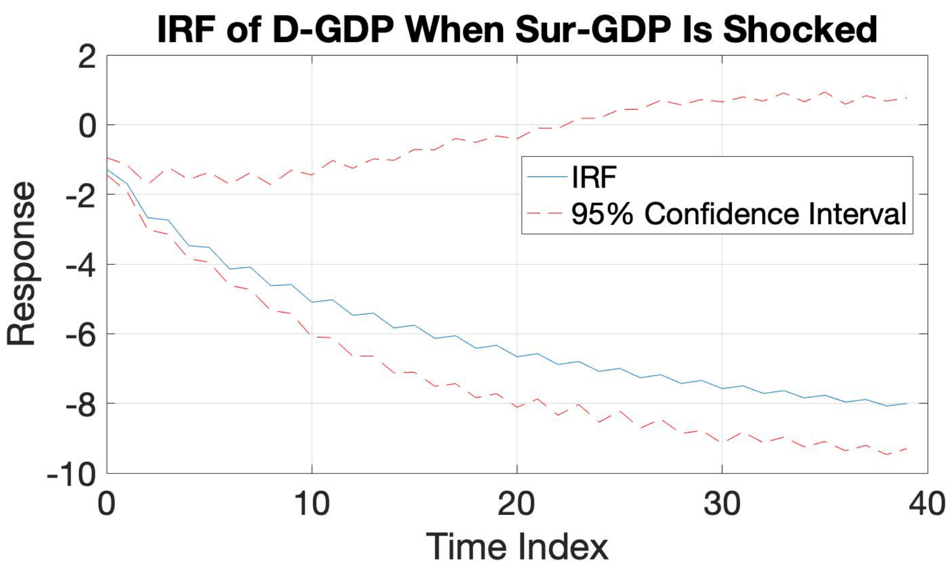

Figure 3.15 – UK Impulse Response Function

The time index represents the quarters. After the primary surplus shock at time 0, the debt shows a swinging and decreasing trend through the next 10 years. The impulse response function does not seem statistically significant.

Japan - VAR (2)

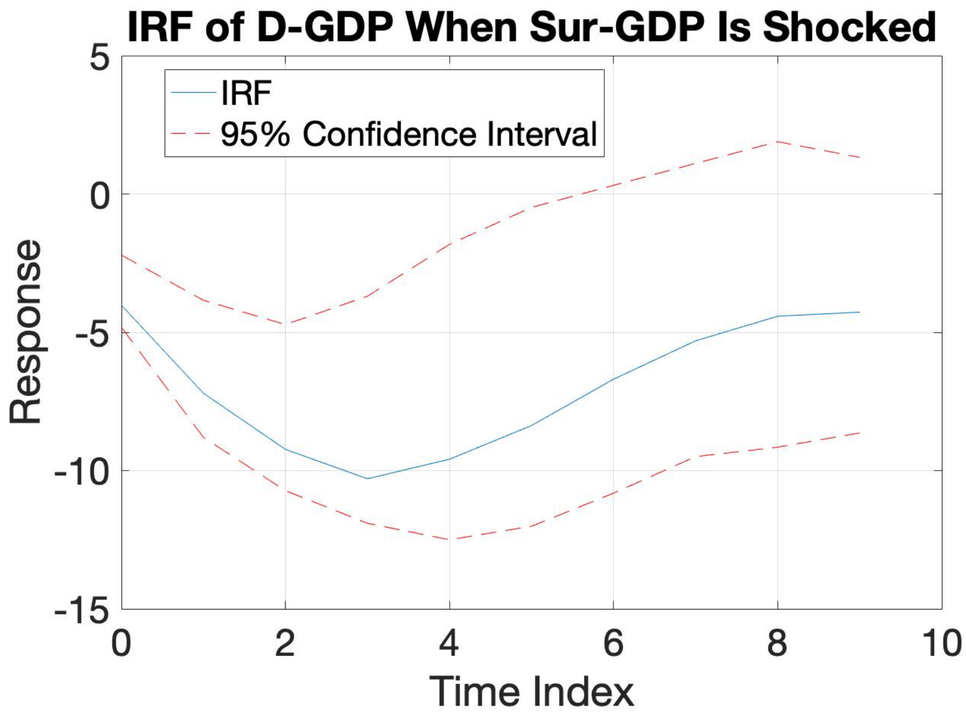

Figure 3.16 – Japan Impulse Response Function

The time index represents the years. The positive innovation of primary surplus at time 0 is followed by a negative response of debt up to year 3. From the fourth year it starts increasing to become more stable from year 8. The impulse response function does not seem statistically significant.

The general pattern that appears from these graphs is a negative response of debt (%GDP) following an innovation in the surplus (%GDP) which becomes increasing for a number of years depending on the country, and stable later on. This result is similar to the one showed in (Bajo-Rubio, et al., 2009).

There are a few exceptions like Italy and UK where the debt keeps decreasing. However, for all the countries the impulse response function does not seem statistically significant.

3.2 Final considerations

In this analysis I tried to establish if the empirical evidence would support the assumptions of the Fiscal Theory of the Price Level for the case of G7 countries over the period 1980-2020 (except for U.S. data which cover the period 1960-2019). So, in addition to previous studies, this analysis also covers the period of the global financial crisis of 2007-2008, the European sovereign debt crisis in 2010 and the COVID-19 pandemic of 2020. Hence, I estimated solvency equations for each country, by regressing the primary surplus on the lagged government gross debt, both as percentage of GDP. A positive coefficient in that regression would mean that the analyzed government is solvent, i.e., it satisfies its present-value budget constraint. Results showed that cointegration was absent and that in all cases the estimated coefficient was always negative and not significantly different from zero, with the only exception of Germany and Italy. Therefore, in all the G7 countries, but Germany and Italy, fiscal policy would have been not sustainable over the whole period, with the primary surplus responding negatively to the debt, which would indicate the prevalence of a non-Ricardian or Fiscal Dominant regime. The exception of Germany and Italy could be due to the continues surpluses that their government have run in the last decade, especially after the Pact for Stability and Growth.

I also ran Granger-causality tests which did not allow me to give any firm conclusion about the presence of a Monetary or a Fiscal Dominant regime in those countries which had a positive beta.

Finally, I provided additional results using the VAR methodology. In fact, I computed the impulse response functions of the debt to innovations in the primary surplus. All in all, debt showed a decreasing path followed by a period of stability. However, the impulse response function was not statistically significant for all countries.

In the end these results could be compatible with the Fiscal Theory of the Price Level: in fact, the primary surplus seems to be set exogenously by the governments, regardless of the level of the public debt. Consequently, the price level would adjust in order to satisfy the intertemporal budget constraint.

Nowadays, all the measures taken by governments to address the pandemic have widened primary deficits as never before with consequences on the price level.

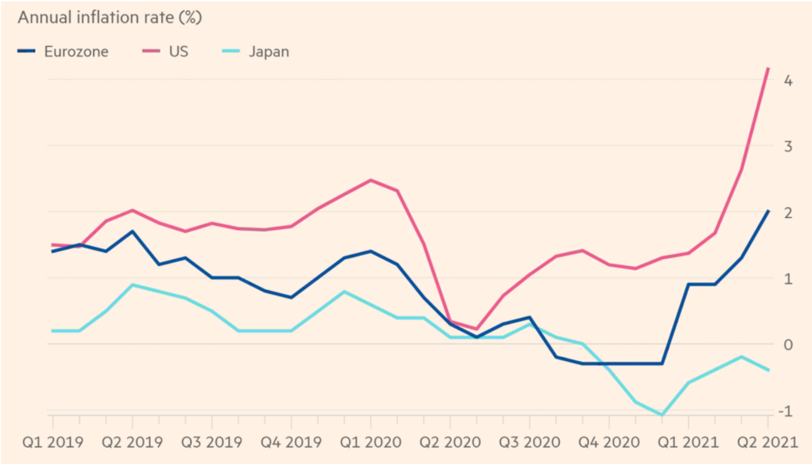

Figure 3.17 – Annual inflation rate, source Financial Times

Over the 2021 inflation has started increasing, and while for Japan an inversion of trend is expected, for Eurozone countries and USA this trend is not expected to stop.

This could be a point in favor of the FTPL: in fact, given the increased deficits, the level of prices would rise to make the valuation equation hold.

To conclude, fiscal policy seems to play an important role in achieving macroeconomic stability; shocks like the one we are living today put in trouble central banks which have to take a decision over the level of interest rates in the future.

Conclusion

COVID-19 pandemic represented a shock for the real economy and for our lives in general. Governments all around the world had to take significant measures to address the crisis, incurring in unprecedented deficits. In particular, U.S. fiscal policy caused concern among several economists worried about the risk of inflation. In light of these events, a theory over the price level determination re-emerged, i.e., the Fiscal Theory of the Price Level.
The purpose of this thesis was the analysis of the Fiscal Theory of the Price Level through both a theoretical and an empirical approach. This theory is an alternative to the more 'classic' ones on inflation and it claims that prices adjust in order to make the nominal debt of the government equal to the present value of primary surpluses.

The first chapter focused on the analysis of Fiscal Theory, with a particular attention to its precursors and supporters. Before investigating the theoretical aspects of this theory, I gave a general idea of the American Rescue Plan Act, outlying the most important measures and their extent. Specifically, I highlighted three main points: the organization of a vaccination program at a national level, the economic aids to working families, and the support to the communities which were more affected by the pandemic. With the announcement of these special measures, some economists launched a debate over the concern of inflation risk. Lawrence Summers (supported by Jean-Pierre Landau) pointed out the fact that the measures taken by Biden would widen the output gap (i.e., the difference between the effective output and the potential output), and, consequently, they would lead to an overheated economy characterized by inflation. Olivier Blanchard agreed with this view and he also proposed two perspectives for the future: the first one is a change in expectations over inflation while the second is a tighter monetary policy to address inflation. The Nobel prize Paul Krugman took a different opinion: according to him, as expectations over inflation are well-anchored, it is unlikely that they would change now. This view was shared also by Ricardo Reis.
After this general overview over the current situation and debates, I moved on to the analysis of the developments of the Fiscal Theory of the Price Level (FTPL). It

has its roots in the 80's and it is an alternative to the quantity theory of money which relates changes in price level to changes in the quantity of money in circulation. On the other hand, Fiscal theory claims that prices adjust in order to make the nominal debt of the government equal to the present value of primary surpluses.
Firstly, I considered the precursors of this theory, Sargent and Wallace. In (Sargent & Wallace, 1981) they studied the coordination between monetary and fiscal policy and, specifically, the situation where the fiscal authority moves first and the monetary one has to adapt to its decisions about the annual deficit or surplus. The authors found that if the D(t) sequence of deficits is too big for too long, then the central bank must generate some seigniorage revenue to pay the debt off. Moreover, since the central bank is free to decide when to print more money, the paper showed that a tighter monetary policy and less inflation now require a looser monetary policy and more inflation later.
After this analysis I passed to (Leeper, 1991) which was the first important contribution to the birth of the Fiscal Theory. He categorized equilibrium policies as representing 'active' or 'passive' behaviour and analysed the alternative scenarios in which the monetary authority and the fiscal one assumed one of the two behaviours. The scenario with an active fiscal authority which sets its variables exogenously letting the passive monetary authority adjust endogenously its money supply, laid the foundation for the subsequent studies on the Fiscal Theory of the Price Level.
There were several studies which followed Leeper's contribution and in my analysis I focused on (Sims, 1994), (Woodford, 1995) and (Cochrane, 2021). Sims showed the importance of fiscal policy on the determination of price level and Woodford extended this analysis. Finally, in the last part of the first chapter I focused on (Cochrane, 2021): I considered both the simple and the intertemporal government. Cochrane showed that the price level endogenously adjusts to the make the valuation equation of government real debt equal to the present value of expected future surpluses.

In the first part of the second chapter I went through the critiques addressed to the FTPL over the years. One example was (Buiter, 1999), which states that the main mistake of the proponents of this theory is stating that the intertemporal budget constraint is not a constraint that must be satisfied for all values of the endogenous variables. However, (Cochrane, 2021) responded to this critique specifying that Fiscal Theory is based on a 'valuation' equation, not a budget constraint. Another critique came from (Niepelt, 2004). This paper defined the FTPL 'inconsistent' given the fact that at time $t = 0$ it is not possible to uniquely define the price level. By the way, (Daniel, 2007) demonstrated that even if at time 0 price level cannot assume a unique value, the Fiscal Theory is still valid.

In the second part of the second chapter, I took into consideration two revisitations of the typical FTPL models. (Berentsen & Waller, 2018) described a model based on the further assumption that debt serves as collateral for secured lending in financial markets. Hence, its market value includes a liquidity premium which reflects not only the claim on the stream of future surpluses, but also its value for trading. This claim changes the typical price level determination of the Fiscal Theory because it suggests that price level dynamics may be affected by changes in the liquidity value of government debt.

In the end, I went through the analysis of (Bassetto & Cui, 2017). This paper focused the attention on FTPL in a context of low interest rates. Specifically, it considered different causes of low interest rates, such as dynamic inefficiency, the liquidity premium of government debt, and its favorable risk profile. In the end, the authors confirmed the validity of the FTPL but they doubted the uniqueness of the equilibrium value of price level when interest rates are permanently lower than the growth rate of the economy.

Given this theoretical background, in the third chapter I gave my personal contribution carrying out an econometric analysis to assess the validity of the assumptions of the Fiscal Theory.

Following the approach of (Canzoneri, et al., 2001), (Bajo-Rubio, et al., 2009), and (Afonso & Tovar Jalles, Revisiting fiscal sustainability, 2012), I tried to establish if the empirical evidence would support the assumptions of the Fiscal Theory of the Price Level for the case of G7 countries over the period 1980-2020 (except for U.S. data which cover the period 1960-2019).

Hence, my main contribution with respect to the past works was taking in consideration the most advanced countries over a period which includes some crucial economics events such as the global financial crisis of 2007-2008, the European sovereign debt crisis of 2010 and the COVID-19 pandemic of 2020. I investigated in which way the fiscal solvency is achieved: by endogenously adjusting the primary budget surplus (i.e., in the "monetary dominant regime") or by endogenously adjusting the price level (i.e., in the "fiscal dominant regime"). For the purpose of this analysis, I used two approaches: a backward-looking one and a forward-looking one. In the backward-looking approach I firstly run Phillips-Perron tests and Dickey-Fuller tests to verify the stationarity of the time series of primary surplus and debt. Then, I computed a dynamic ordinary least squares regression of primary surplus on lagged debt, both as percentage of GDP. A positive coefficient in that regression would mean that the analyzed government is solvent, i.e., it satisfies its present-value budget constraint, while a coefficient equal to or smaller than zero would show that government is not solvent and that a Fiscal regime dominates. This last result would support Fiscal Theory of the Price Level. After the regression, I ran cointegration tests to assess whether the two time series are cointegrated, i.e., if they move with a common trend. Granger causality tests followed to study the 'predictive' causality of the two variables. Finally, for the forward-looking approach I used the VAR model to compute the response functions of debt to innovations in the primary surplus for all the countries under investigation. As far as final results, I found that primary surplus and debt, both as percentage of GDP, are not stationary. However, the cointegration tests showed that there is no cointegration between them. From the dynamic OLS I found that betas were all negative and not significantly different from zero, except for Italy (with a positive but not significantly different from zero) and Germany, which had a positive beta, significantly different from zero. Therefore, in all the G7 countries, but Germany and Italy, fiscal policy would have been not sustainable over the whole period, with the primary surplus responding negatively to the debt, which would indicate the prevalence of a non-Ricardian or Fiscal Dominant regime. The exception of Germany and Italy could be due to the continues surpluses that their government have run in the last decade, especially after the Pact for Stability and

Growth. All in all, the results of this analysis are in favour of the Fiscal Theory of the Price Level. In fact, primary surplus seem to be set exougenously by government regardless of the level of debt. Consequently, the price level would adjust in order to satisfy the intertemporal budget constraint.

Hence, I would confirm the validity of the Fiscal Theory especially in the current situation where governments are forced to take straordinary measures to address the pandemic widening primary deficits as never before with consequences on the price level. At the moment there are expectations of increasing inflation, especially, over the 2021 in the U.S. and in the Eurozone and it is clear the correlation with the deficits caused by the pandemic. The question is if this inflation will be temporary or permanent but this depends also on the expectation about future surpluses of governments. Maybe the newly global corporate tax approved by the G7 countries could contribute to making expectations on surpluses more positive.

Bibliography

- Afonso, A.; Tovar Jalles, J. (2012). Revisiting Fiscal Sustainability. *ECB Working papers series.*
- Bajo-Rubio, O., Diaz-Roldan, C., Esteve, V. (2009). Deficit Sustainability and inflation in EMU: an analysis from the Fiscal Theory of the Price Level. *European Journal of Political Economy*. pp. 525-539
- Barret, P. (2018). Interest-Growth Differentials and Debt Limits in Advanced Economics. *IMF Working Paper*.
- Bassetto, M., Cui, W. (2017). The Fiscal Theory of the Price Level in a World of Low Interest Rates.
- Berentsen, A., Waller, C. (2018). Liquidity premiums on government debt and the fiscal theory of the price level. *Journal of Economic Dynamics and Control*. pp. 173-182
- Bianchi, F. and Melosi, L. (2019). The dire effects of the lack of monetary and fiscal coordination. *Journal of Monetary Economics*. Vol. 104. pp. 1–22
- Bohn, H. (1998). The behavior of U.S. public debt and deficits. *Quarterly Journal of Economics*. pp. 459-480
- Buiter, W. H. (2002). The Fiscal Theory of the Price Level: A Critique. *The Economic Journal*. Vol. 112. pp. 459-480
- Canzoneri, Matthew, B., Robert E. Cumby, and Behzad T. Diba. (2001). Is the Price Level Determined by the Needs of Fiscal Solvency? *American Economic Review*. Vol. 91, pp. 1221-1238
- Cochrane, J. H. (1998). A Frictionless View of U.S Inflation.

- Cochrane, J. H. (2001). Long-Term Debt and Optimal Policy in the Fiscal Theory of the Price Level. *Econometrica*.
- Cochrane, J. H. (2021). The Fiscal Theory of the Price Level.
- Daniel, Betty C. (2007). The fiscal theory of the price level and initial government debt. *Review of Economic Dynamics*. Vol. 10., pp. 193-206
- Friedman, M. (1968). The role of monetary policy. *The American Economic Review.*
- Geerolf, F. (2018). Reassessing Dynamic Efficiency.
- Gomme, P., Ravikumar, B. and Rupert, P. (2010). The return to capital and the business cycle. *Review of Economic Dynamics*. Vol. 14. pp. 262-278.
- Greene, W. H. (2012). *Econometric Analysis*. Pearson. pp. 982-983.
- Leeper, E. M. (1991). Equilibria under 'active' and 'passive' monetary and fiscal policies. *Journal of Monetary Economics* , pp. 129-147.
- Hansen, B. E. (1992). Testing for Parameter Instability in Linear Models. *Journal of Policy Modeling*. Vol. 14, pp. 517-533.
- Loyo, E. (1999). Tight money paradox on the loose: a fiscalist hyperinflation.
- Macmillan, P. (n.d.). *The New Palgrave Dictionary of Economics*.
- McCallum, Bennett T., (2001). Indeterminacy, bubbles, and the fiscal theory of price level determination. *Journal of Monetary Economics.* pp. 19-30.
- Ng, S. and Perron, P. (2001). Lag Length Selection and the Construction of Unit Root Tests with Good Size and Power. *Econometrica*. Vol. 69, No. 6, pp. 1519-1554.
- Niepelt, D. (2004). The Fiscal Mith of the Price Level. *The Quarterly Journal of Economics.* pp. 277-300.

- Phelan, C., Kocherlakota, N. (1999). Explaining the Fiscal Theory of the Price Level.
- Phillips, P. C. B., Hansen, B. E. (1990). Statistical inference in Instrumental Variables Regression with I(1) Processes. *The review of Economic Studies*. Vol. 57, No. 1, pp. 99-125.
- Reis, R. (2021). The constraint on public debt when r<g but g<m.
- Sargent, T. J., & Wallace, N. (1981). *Some Unpleasant Monetarist Arithmetic.*
- Shin, Y. (1994). A Residual-Based Test of the Null of Cointegration against the Alternative of No Cointegration. *Econometric Theory.* Vol. 10., N. 1, pp. 91-115
- Sims, C. A. (1994). A simple model for study of the determination of the price level and the interaction of monetary and fiscal policy. *Economic Theory*.
- Stock, J. H. and Watson, M. W. (1993). A simple estimator of cointegrating vectors in higher order integrated systems. *Econometrica.* Vol. 61, No. 4, pp 783-820.
- Woodford, M. (1994). Monetary Policy and price level determinacy in a cash-in-advance economy. *Economic Theory*, Vol. 4, No.3
- Woodford, M. (1995). Price-level determinacy without control of a monetary aggregate.
- Woodford, M. (1995). Price-level determinacy without control of a monetary aggregate. *Carnegie-Rochester Conference Series on Public Policy*
- Woodford, M. (1996). Control of the Public Debt: a requirement for price stability?
- Woodford, M. (1998). Price Debt and The Price Level. *Princeton University*

Sitography

- "President Biden Announces American Rescue Plan", https://www.whitehouse.gov/briefing-room/legislation/2021/01/20/president-biden-announces-american-rescue-plan/
- "One Hundred Seventeenth Congress of the United States of America", https://www.congress.gov/117/bills/hr1319/BILLS-117hr1319enr.pdf
- "Estimating and Projecting Potential Output Using CBO's Forecasting Growth Model", by Robert Shackleton, Working Paper 2018-03
- "Opinion: The Biden stimulus is admirably ambitious. But it brings some big risks, too", https://www.washingtonpost.com/opinions/2021/02/04/larry-summers-biden-covid-stimulus/
- "Inflation and the Biden stimulus", https://voxeu.org/article/inflation-and-biden-stimulus
- "A conversation with Lawrence Summer and Paul Krugman" https://www.youtube.com/watch?v=EbZ3_LZxs54&t=1s
- "In defense of concerns over the $1.9 trillion relief plan" by Olivier Blanchard, feb 18, 2021
- Becker R.A. (2008) Transversality Condition. In: Palgrave Macmillan (eds) The New Palgrave Dictionary of Economics. Palgrave Macmillan, London. https://doi.org/10.1057/978-1-349-95121-5_2158-1

- “Ricardo Reis on Central Bank Swap Lines, Fiscal Sustainability, and Outlooks for Inflation”, https://www.mercatus.org/bridge/podcasts/02152021/ricardo-reis-central-bank-swap-lines-fiscal-sustainability-and-outlooks

Appendix A

MATLAB code

France

```
clear; clc;
format long g
%% FRANCE 1999:Q1 - 2020:Q4
%%%%%%%%%%%%%%%%%%%%%%%%%%%

FRA_data = readtable('FRA_Q.xlsx');
Sur_GDP = table2array(FRA_data(:,2));
Debt_GDP = table2array(FRA_data(:,3));
T = (1999.0:0.25:2020.75)';
t = length(T)

%% PLOT
figure(1)
subplot(1,2,1); hold on;  box on ;
               plot(T, Sur_GDP, 'LineWidth',1,'Color','blue');
               plot(T, mean(Sur_GDP)*ones(t,1),'-
k','LineWidth',1);
               xlim([T(1)-1 T(end)+1]); ylim([-8 8]);
               title('Surplus (%GDP) ');
               legend('Sur (%GDP)')

               set(gca,'FontSize',25)
               hold off; box off;
subplot(1,2,2); hold on;  box on ;
               plot(T, Debt_GDP, 'LineWidth',1,'Color','blue');
               plot(T, mean(Debt_GDP)*ones(t,1),'-
k','LineWidth',1);
               xlim([T(1)-1 T(end)+1]);  ylim([56 118]);
               title('Debt (%GDP) ');
               legend('Debt (%GDP)')

               set(gca,'FontSize',25)
               hold off; box off;
%% UNIT ROOT TESTS:
   %% 1. PHILLIPS-PERRON TEST OF UNIT ROOT:
   % INPUT: the time series under investigation
% Sur_GDP test
[PP,pValue_0,stat_0,cValue_0,reg_0] = pptest(Sur_GDP)
[PP_ts,pValue_1,stat_1,cValue_1,reg_1] =
pptest(Sur_GDP,'model','TS')
[PP_ard,pValue_2,stat_2,cValue_2,reg_2] =
pptest(Sur_GDP,'model','ARD')
[PP_lags,pValue_3,stat_3,cValue_3,reg_3] =
pptest(Sur_GDP,'model','TS','lags',0:2)
% We want h=1 for the first difference:
Delta_SGDP = diff(Sur_GDP)
[PPD1_d0,pValue_d0,stat_d0,cValue_d0,reg_d0] = pptest(Delta_SGDP);
[PPD1_ts,pValue_d1,stat_ts,cValue_d1,reg_d1] =
pptest(Delta_SGDP,'model','TS');
[PPD1_ard,pValue_d2,stat_ard,cValue_d2,reg_d2] =
pptest(Delta_SGDP,'model','ARD');
```

```
[PPD1_lags,pValue_d3,stat_lags,cValue_d3,reg_d3] =
pptest(Delta_SGDP,'model','TS','lags',0:2);
%% Debt_GDP test
[h,pValue_4,stat_4,cValue_4,reg_4] = pptest(Debt_GDP)
[PP_ts_debt,pValue_5,stat_5,cValue_5,reg_5] =
pptest(Debt_GDP,'model','TS')
[PP_ard_debt,pValue_6,stat_6,cValue_6,reg_6] =
pptest(Debt_GDP,'model','ARD')
[PP_lag_debt,pValue_7,stat_7,cValue_7,reg_7] =
pptest(Debt_GDP,'model','TS','lags',0:2)
% We want h=1 for the first difference:
Delta_DGDP = diff(Debt_GDP)
[Delta_DGDP_ts,pValue_d4,stat_d4,cValue_d4,reg_d4] =
pptest(Delta_DGDP,'model','TS');
[Delta_DGDP_ard,pValue_d5,stat_d5,cValue_d5,reg_d5] =
pptest(Delta_DGDP,'model','ARD');
[Delta_DGDP_lag,pValue_d6,stat_d6,cValue_d6,reg_d6] =
pptest(Delta_DGDP,'model','TS','lags',0:2);
%% 2.DICKEY-FULLER TEST STATISTICS:
  % 1. Simple DF:
  s = adftest(Sur_GDP)

  d = adftest(Debt_GDP)

  % 2.Test for a unit root against a trend-stationary alternative,
augmenting the model with 0, 1, and 2 lagged difference terms.
  s_TS = adftest(Sur_GDP,'model','TS','lags',0:2)

  d_TS = adftest(Debt_GDP, 'model','TS','lags',0:2)
  % 3. Test for a unit root using three different choices for the
number of lagged difference terms. Return the regression
statistics for each alternative model.
  [h,~,~,~,reg] = adftest(Sur_GDP,'model','TS','lags',0:2);
  [h1,~,~,~,reg1] = adftest(Debt_GDP,'model','TS','lags',0:2);
  % The output shows which terms are included in the three
alternative models.
% The first model has no added difference terms, the second model
has one difference term (b1),
% and the third model has two difference terms (b1 and b2).
reg.names;
reg.BIC;
reg1.names;
reg1.BIC;
% Repeat the analysis with the differences of the variables to
verify they
% are I(0)
 % 1. Simple DF:
  s_delta = adftest(Delta_SGDP)

  d_delta = adftest(Delta_DGDP)
% 2.Test for a unit root against a trend-stationary alternative,
augmenting the model with 0, 1, and 2 lagged difference terms.
 s_TS_delta = adftest(Delta_SGDP,'model','TS','lags',0:2);

  d_TS_delta = adftest(Delta_DGDP, 'model','TS','lags',0:2);
% 3. Test for a unit root using three different choices for the
number of lagged difference terms. Return the regression
statistics for each alternative model.
[h_d,~,~,~,reg_d] = adftest(Delta_SGDP,'model','TS','lags',0:2);
```

```
[h1_d,~,~,~,reg1_d] = adftest(Delta_DGDP,'model','TS','lags',0:2);
%% DOLS: DYNAMIC OLS AS IN STOCK AND WATSON (1993).

%% 1. FIRST OLS REGRESSION BETWEEN I(1) SERIES:
y = Sur_GDP(2:end);
X = [ones(t-1,1), Debt_GDP(1:end-1)];
B = (X'*X)\(X'*y)

e = y - X*B;
RSS = e'*e;
sigma_2 = RSS/(t-3)

[beta,Sigma] = mvregress(X, y)
%% 2. DYNAMIC OLS:
q = round(t^(1/3));

NA = NaN
% DEFINE THE REGRESSORS:
D_b_0 = [NA; Debt_GDP(2:end)-Debt_GDP(1:end-1)];     % 88 x 1

D_b_1 = [NA; D_b_0(1:end-1)];                        % 88 x 1
D_b_2 = [NA; NA; D_b_0(1:end-2)];                    % 88 x 1
D_b_3 = [NA; NA; NA; D_b_0(1:end-3)];                % 88 x 1
D_b_4 = [NA; NA; NA; NA; D_b_0(1:end-4)];            % 88 x 1
D_b_5 = [NA; NA; NA; NA; NA; D_b_0(1:end-5)];       % 88 x 1

D_b_p1 = [D_b_0(2:end); NA];                        % 88 x 1
D_b_p2 = [D_b_0(3:end); NA; NA];                   % 88 x 1
D_b_p3 = [D_b_0(4:end); NA; NA; NA];              % 88 x 1

b_t_1 = [NA; Debt_GDP(1:end-1)];
N = length(D_b_0); % N=88

XD = [ones(N,1), b_t_1, D_b_0, D_b_1, D_b_2, D_b_3, D_b_4, D_b_5,
D_b_p1, D_b_p2, D_b_p3];

%% CUT THE MATRICES FOR THE CORRECT SIZE:
X_d = XD(7:85,:);
Y_d = y(7:85);

k = size(X_d, 2);
beta_hat = zeros(k,1);

beta_hat = (X_d'*X_d)\(X_d'*Y_d);
e_d = Y_d - X_d*beta_hat;
[beta_d,Sigma_d] = mvregress(X_d, Y_d);

%% t test
n=size(X_d,1);
V_beta=Sigma_d*inv(X_d'*X_d);
% compute the observed statistic
t_obs=(beta_d(2)-0)/sqrt(V_beta(2,2));
% Define statistic test
t_teo=tinv(0.95,n-11);
% if t_obs>t_-teo I reject H_0
%% ENGLE-GRANGER COINTEGRATION TEST. NULL HYPOTHESIS OF NO-
COINTEGRATION
figure(2)
```

```
subplot(1,2,1)
plot(e)
subplot(1,2,2)
plot(e_d)
% 1. First method. SIMPLE DICKEY-FULLER test on residuals, without
constant:
C01 = adftest(e)     % if 0, accept non-stationariety
CO2 = adftest(e_d)  % if 0, accept non-stazionariety

% 2. Second method:
[h,pValue,stat,cValue_en,reg1,reg2] = egcitest([y,X])

[h_delta,pValue_egc,stat_egc,cValue_en_d,reg1,reg2] =
egcitest([Y_d, X_d])

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%
% TEST DI GRANGER CAUSALITY:

% 1. Rewrite the model in ECM.
   % FIRST EQUATION. SURPLUS_G VS DEBT_G
   yec1 = Sur_GDP(2:end);
   Xec1 = [Debt_GDP(1:end-1)];
   Bec1 = (Xec1'*Xec1)\(Xec1'*yec1);

   Zec1 = yec1 - Xec1*Bec1;  % ECM1

   St_1 = [NA; Sur_GDP(1:end-1)];                      % 88 OSS
   ZEC1 = [NA; NA; Zec1(1:end-1)];                     % 88 OSS

   DST_1 =[NA; NA; Delta_SGDP(1:end-1)];
   DST_2 =[NA; NA; NA; Delta_SGDP(1:end-2)];
   DST_3 =[NA; NA; NA; NA; Delta_SGDP(1:end-3)];

   DDGT_1 =[NA; NA; Delta_DGDP(1:end-1)];
   DDGT_2 =[NA; NA; NA; Delta_DGDP(1:end-2)];
   DDGT_3 =[NA; NA; NA; NA; Delta_DGDP(1:end-3)];

n_ecm1 = size(St_1,1);
%                 1        2      3     4      5       6      7
8       9
X_ECM1 = [ones(n_ecm1,1), St_1, ZEC1, DST_1, DST_2, DST_3, DDGT_1,
DDGT_2, DDGT_3];

X_ECM11 = X_ECM1(5:end,:);
Y_ECM1 =  Sur_GDP(5:end);

[beta_ECM1,Sigma_ECM1] = mvregress(Y_ECM1, X_ECM11);
beta_ECM1;
V_betaECM = Sigma_ECM1*inv(X_ECM11'*X_ECM11);
% GRANGER CAUSALITY TEST IS AN F-TEST OF JOINT SIGNIFICATIVITY ON
THE
% FOLLOWING PARAMETHERS:
c = [0, 0, 0, 0];
H = [0, 0, 1, 0, 0, 0, 0, 0, 0;
     0, 0, 0, 0, 0, 0, 1, 0, 0;
     0, 0, 0, 0, 0, 0, 0, 1, 0;
     0, 0, 0, 0, 0, 0, 0, 0, 1];
```

```
p_value_granger_s = linhyptest(beta_ECM1,V_betaECM,c,H,9);
%p_value > 0.05 accept null hypothesis and granger causality
% y=debt
yec2 = Debt_GDP(2:end);
Xec2 = [Sur_GDP(1:end-1)];
Bec2 = (Xec2'*Xec2)\(Xec2'*yec2);

Zec2 = yec2-Xec2*Bec2; %ECM2

Dt_1=[NA; Debt_GDP(1:end-1)];
ZEC2= [NA; NA; Zec2(1:end-1)];

DDT_1 =[NA; NA; Delta_DGDP(1:end-1)];
DDT_2 =[NA; NA; NA; Delta_DGDP(1:end-2)];
DDT_3 =[NA; NA; NA; NA; Delta_DGDP(1:end-3)];

DSGT_1 =[NA; NA; Delta_SGDP(1:end-1)];
DSGT_2 =[NA; NA; NA; Delta_SGDP(1:end-2)];
DSGT_3 =[NA; NA; NA; NA; Delta_SGDP(1:end-3)];

n_ecm2 = size(Dt_1,1);
%               1          2     3     4      5     6      7      8
9
X_ECM2=[ones(n_ecm2,1), Dt_1, ZEC2, DDT_1, DDT_2, DDT_3, DSGT_1,
DSGT_2, DSGT_3];

X_ECM22 = X_ECM2(5:end,:);
Y_ECM2 = Debt_GDP(5:end);

[beta_ECM2, Sigma_ECM2] = mvregress(Y_ECM2, X_ECM22);
beta_ECM2;
V_betaECM2 = Sigma_ECM2*inv(X_ECM22'*X_ECM22);
% parameters
c1 = [0, 0, 0, 0];
H1 = [0, 0, 1, 0, 0, 0, 0, 0, 0;
      0, 0, 0, 0, 0, 0, 1, 0, 0;
      0, 0, 0, 0, 0, 0, 0, 1, 0;
      0, 0, 0, 0, 0, 0, 0, 0, 1];

 p_value_gr_d = linhyptest(beta_ECM2,V_betaECM2,c1,H1,9);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%
% THE FORWARD LOOKINF APPROACH: VAR(1) - VAR(4)
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%
% VAR(1) MODEL WITH ONE LAG
Mdl = varm(2,1)
EstMdl = estimate(Mdl,[Sur_GDP, Debt_GDP]);
summarize(EstMdl)

% VAR(2) MODEL WITH TWO LAGS
Md2 = varm(2,2);
EstMd2 = estimate(Md2,[Sur_GDP, Debt_GDP]);
summarize(EstMd2)

% VAR(3) MODEL WITH TWO LAGS
```

```
Md3 = varm(2,3);
EstMd3 = estimate(Md3,[Sur_GDP, Debt_GDP]);
summarize(EstMd3)

% VAR(4) MODEL WITH TWO LAGS
Md4 = varm(2,4);
EstMd4 = estimate(Md4,[Sur_GDP, Debt_GDP]);
summarize(EstMd4)
% BIC1 = 463.48
% BIC2 = 445.645
% BIC3 = 338.397
% BIC4 = 397.78
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%
% VAR(3) - IRF
summarize(EstMd3)
Response1 = irf(EstMd3) % 20 OBSERVATIONS HORIZON

% 10 YEAR HOTIZON: 4x10=40
[Response,Lower,Upper] = irf(EstMd3,'NumObs',40)

% IRF OF DEBITO_GDP ON SURPLUS SHOCK:
irfshock1resp2 = Response(:,1,2);
IRFCIShock1Resp2 = [Lower(:,1,2) Upper(:,1,2)];

figure(3);
h1 = plot(0:39,irfshock1resp2);
hold on
h2 = plot(0:39,IRFCIShock1Resp2,'r--');
legend([h1 h2(1)],["IRF" "95% Confidence Interval"])
set(gca,'FontSize',25)
xlabel("Time Index");
ylabel("Response");
title("IRF of D-GDP When Sur-GDP Is Shocked");
grid on
hold off
```

Germany

```
clear; clc;
format long g
%% Germany 2002:Q3 - 2020:Q4
%%%%%%%%%%%%%%%%%%%%%%%%%%%

GER_data = readtable('GER_Q.xlsx');
Sur_GDP = table2array(GER_data(:,2));
Debt_GDP = table2array(GER_data(:,3));
T = (2002.75:0.25:2020.75)';
t = length(T)
%% PLOT
figure(1)
subplot(1,2,1); hold on;  box on ;
               plot(T, Sur_GDP, 'LineWidth',1,'Color','blue');
               plot(T, mean(Sur_GDP)*ones(t,1),'-
k','LineWidth',1);
               xlim([T(1)-1 T(end)+1]); ylim([-4 4]);
               title('Surplus (%GDP) ');
               legend('Sur (%GDP)')

               set(gca,'FontSize',25)
```

```
                hold off; box off;
subplot(1,2,2); hold on;  box on ;
                plot(T, Debt_GDP, 'LineWidth',1,'Color','blue');
                plot(T, mean(Debt_GDP)*ones(t,1),'-
k','LineWidth',1);
                xlim([T(1)-1 T(end)+1]);  ylim([58 86]);
                title('Debt (%GDP)');
                legend('Debt (%GDP)')

                set(gca,'FontSize',25)
                hold off; box off;
%% UNIT ROOT TESTS:
   %% 1. PHILLIPS-PERRON TEST OF UNIT ROOT:
   % INPUT: the time series under investigation
% Sur_GDP test
[PP,pValue_0,stat_0,cValue_0,reg_0] = pptest(Sur_GDP)
[PP_ts,pValue_1,stat_1,cValue_1,reg_1] =
pptest(Sur_GDP,'model','TS')
[PP_ard,pValue_2,stat_2,cValue_2,reg_2] =
pptest(Sur_GDP,'model','ARD')
[PP_lags,pValue_3,stat_3,cValue_3,reg_3] =
pptest(Sur_GDP,'model','TS','lags',0:2)
% We want h=1 for the first difference:
Delta_SGDP = diff(Sur_GDP)
[PPD1_d0,pValue_d0,stat_d0,cValue_d0,reg_d0] = pptest(Delta_SGDP);
[PPD1_ts,pValue_d1,stat_ts,cValue_d1,reg_d1] =
pptest(Delta_SGDP,'model','TS');
[PPD1_ard,pValue_d2,stat_ard,cValue_d2,reg_d2] =
pptest(Delta_SGDP,'model','ARD');
[PPD1_lags,pValue_d3,stat_lags,cValue_d3,reg_d3] =
pptest(Delta_SGDP,'model','TS','lags',0:2);
%% Debt_GDP test
[h,pValue_4,stat_4,cValue_4,reg_4] = pptest(Debt_GDP)
[PP_ts_debt,pValue_5,stat_5,cValue_5,reg_5] =
pptest(Debt_GDP,'model','TS')
[PP_ard_debt,pValue_6,stat_6,cValue_6,reg_6] =
pptest(Debt_GDP,'model','ARD')
[PP_lag_debt,pValue_7,stat_7,cValue_7,reg_7] =
pptest(Debt_GDP,'model','TS','lags',0:2)
% We want h=1 for the first difference:
Delta_DGDP = diff(Debt_GDP)
[Delta_DGDP_ts,pValue_d4,stat_d4,cValue_d4,reg_d4] =
pptest(Delta_DGDP,'model','TS');
[Delta_DGDP_ard,pValue_d5,stat_d5,cValue_d5,reg_d5] =
pptest(Delta_DGDP,'model','ARD');
[Delta_DGDP_lag,pValue_d6,stat_d6,cValue_d6,reg_d6] =
pptest(Delta_DGDP,'model','TS','lags',0:2);
%% 2.DICKEY-FULLER TEST STATISTICS:
  % 1. Simple DF:
  s = adftest(Sur_GDP)

  d = adftest(Debt_GDP)

  % 2.Test for a unit root against a trend-stationary alternative,
augmenting the model with 0, 1, and 2 lagged difference terms.
  s_TS = adftest(Sur_GDP,'model','TS','lags',0:2)

  d_TS = adftest(Debt_GDP, 'model','TS','lags',0:2)
```

```
  % 3. Test for a unit root using three different choices for the
number of lagged difference terms. Return the regression
statistics for each alternative model.
  [h,~,~,~,reg] = adftest(Sur_GDP,'model','TS','lags',0:2);
  [h1,~,~,~,reg1] = adftest(Debt_GDP,'model','TS','lags',0:2);
% The output shows which terms are included in the three
alternative models.
% The first model has no added difference terms, the second model
has one difference term (b1),
% and the third model has two difference terms (b1 and b2).
reg.names;
reg.BIC;
reg1.names;
reg1.BIC;
% Repeat the analysis with the differences of the variables to
verify they
% are I(0)
 % 1. Simple DF:
  s_delta = adftest(Delta_SGDP)

  d_delta = adftest(Delta_DGDP)
% 2.Test for a unit root against a trend-stationary alternative,
augmenting the model with 0, 1, and 2 lagged difference terms.
 s_TS_delta = adftest(Delta_SGDP,'model','TS','lags',0:2);

  d_TS_delta = adftest(Delta_DGDP, 'model','TS','lags',0:2);
% 3. Test for a unit root using three different choices for the
number of lagged difference terms. Return the regression
statistics for each alternative model.
[h_d,~,~,~,reg_d] = adftest(Delta_SGDP,'model','TS','lags',0:2);
[h1_d,~,~,~,reg1_d] = adftest(Delta_DGDP,'model','TS','lags',0:2);
%% DOLS: DYNAMIC OLS AS IN STOCK AND WATSON (1993).

%% 1. FIRST OLS REGRESSION BETWEEN I(1) SERIES:
y = Sur_GDP(2:end);
X = [ones(t-1,1), Debt_GDP(1:end-1)];
B = (X'*X)\(X'*y)

e = y - X*B;
RSS = e'*e;
sigma_2 = RSS/(t-3)

[beta,Sigma] = mvregress(X, y)
%% 2. DYNAMIC OLS:
q = round(t^(1/3));

NA = NaN
% DEFINE THE REGRESSORS:
D_b_0 = [NA; Debt_GDP(2:end)-Debt_GDP(1:end-1)];     % 73 x 1

D_b_1 = [NA; D_b_0(1:end-1)];                        % 73 x 1
D_b_2 = [NA; NA; D_b_0(1:end-2)];                     % 73 x 1
D_b_3 = [NA; NA; NA; D_b_0(1:end-3)];                % 73 x 1
D_b_4 = [NA; NA; NA; NA; D_b_0(1:end-4)];             % 73 x 1
D_b_5 = [NA; NA; NA; NA; NA; D_b_0(1:end-5)];       % 73 x 1

D_b_p1 = [D_b_0(2:end); NA];                         % 73 x 1
D_b_p2 = [D_b_0(3:end); NA; NA];                   % 73 x 1
D_b_p3 = [D_b_0(4:end); NA; NA; NA];             % 73 x 1
```

```
b_t_1 = [NA; Debt_GDP(1:end-1)];
N = length(D_b_0); % N=73

XD = [ones(N,1), b_t_1, D_b_0, D_b_1, D_b_2, D_b_3, D_b_4, D_b_5,
D_b_p1, D_b_p2, D_b_p3];

%% CUT THE MATRICES FOR THE CORRECT SIZE:
X_d = XD(7:70,:);
Y_d = y(7:70);

k = size(X_d, 2);
beta_hat = zeros(k,1);

beta_hat = (X_d'*X_d)\(X_d'*Y_d);
e_d = Y_d - X_d*beta_hat;
[beta_d,Sigma_d] = mvregress(X_d, Y_d);

%% t test
n=size(X_d,1);
V_beta=Sigma_d*inv(X_d'*X_d);
%% compute the observed statistic
t_obs=(beta_d(2)-0)/sqrt(V_beta(2,2));
%% Define statistic test
t_teo=tinv(0.95,n-11);

% if t_obs>t_-teo I reject H_0
%% ENGLE-GRANGER COINTEGRATION TEST
figure(2)
subplot(1,2,1)
plot(e)
subplot(1,2,2)
plot(e_d)
% 1. FIRST METHOD. SIMPLE DICKEY-FULLER test on residuals
C01 = adftest(e)
CO2 = adftest(e_d)

% 2. SECOND METHOD:
[h,pValue,stat,cValue_en,reg1,reg2] = egcitest([y,X])

[h_delta,pValue,stat,cValue_en_d,reg1,reg2] = egcitest([Y_d, X_d])
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%
% GRANGER CAUSALITY TESTS:

% 1. model in ECM.
   % FIRST EQUATION. SURPLUS_G VS DEBT_G
   yec1 = Sur_GDP(2:end);
   Xec1 = [Debt_GDP(1:end-1)];
   Bec1 = (Xec1'*Xec1)\(Xec1'*yec1);

   Zec1 = yec1 - Xec1*Bec1;  % ECM1

   St_1 = [NA; Sur_GDP(1:end-1)];                     % 73 OSS
   ZEC1 = [NA; NA; Zec1(1:end-1)];                    % 73 OSS

   DST_1 =[NA; NA; Delta_SGDP(1:end-1)];
   DST_2 =[NA; NA; NA; Delta_SGDP(1:end-2)];
```

```
   DST_3 =[NA; NA; NA; NA; Delta_SGDP(1:end-3)];

   DDGT_1 =[NA; NA; Delta_DGDP(1:end-1)];
   DDGT_2 =[NA; NA; NA; Delta_DGDP(1:end-2)];
   DDGT_3 =[NA; NA; NA; NA; Delta_DGDP(1:end-3)];

n_ecm1 = size(St_1,1);
%                 1        2      3     4      5       6      7
8       9
X_ECM1 = [ones(n_ecm1,1), St_1, ZEC1, DST_1, DST_2, DST_3, DDGT_1,
DDGT_2, DDGT_3];

X_ECM11 = X_ECM1(5:end,:);
Y_ECM1 =  Sur_GDP(5:end);

[beta_ECM1,Sigma_ECM1] = mvregress(Y_ECM1, X_ECM11);
beta_ECM1;
V_betaECM = Sigma_ECM1*inv(X_ECM11'*X_ECM11);
% GRANGER CAUSALITY TEST:
c = [0, 0, 0, 0];
H = [0, 0, 1, 0, 0, 0, 0, 0, 0;
     0, 0, 0, 0, 0, 0, 1, 0, 0;
     0, 0, 0, 0, 0, 0, 0, 1, 0;
     0, 0, 0, 0, 0, 0, 0, 0, 1];

p_value_granger_s = linhyptest(beta_ECM1,V_betaECM,c,H,9);
%p_value > 0.05 accept null hypothesis and granger causality
%% y=debt
yec2 = Debt_GDP(2:end);
Xec2 = [Sur_GDP(1:end-1)];
Bec2 = (Xec2'*Xec2)\(Xec2'*yec2);

Zec2 = yec2-Xec2*Bec2; %ECM2

Dt_1=[NA; Debt_GDP(1:end-1)];
ZEC2= [NA; NA; Zec2(1:end-1)];

DDT_1 =[NA; NA; Delta_DGDP(1:end-1)];
DDT_2 =[NA; NA; NA; Delta_DGDP(1:end-2)];
DDT_3 =[NA; NA; NA; NA; Delta_DGDP(1:end-3)];

DSGT_1 =[NA; NA; Delta_SGDP(1:end-1)];
DSGT_2 =[NA; NA; NA; Delta_SGDP(1:end-2)];
DSGT_3 =[NA; NA; NA; NA; Delta_SGDP(1:end-3)];

n_ecm2 = size(Dt_1,1);
%              1          2     3     4      5     6      7      8
9
X_ECM2=[ones(n_ecm2,1), Dt_1, ZEC2, DDT_1, DDT_2, DDT_3, DSGT_1,
DSGT_2, DSGT_3];

X_ECM22 = X_ECM2(5:end,:);
Y_ECM2 = Debt_GDP(5:end);

[beta_ECM2, Sigma_ECM2] = mvregress(Y_ECM2, X_ECM22);
beta_ECM2;
V_betaECM2 = Sigma_ECM2*inv(X_ECM22'*X_ECM22);
% parameters
```

```
c1 = [0, 0, 0, 0];
H1 = [0, 0, 1, 0, 0, 0, 0, 0, 0;
      0, 0, 0, 0, 0, 0, 1, 0, 0;
      0, 0, 0, 0, 0, 0, 0, 1, 0;
      0, 0, 0, 0, 0, 0, 0, 0, 1];

 p_value_gr_d = linhyptest(beta_ECM2,V_betaECM2,c1,H1,9);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%
% VAR(1) MODEL WITH ONE LAG
Mdl = varm(2,1)
EstMdl = estimate(Mdl,[Sur_GDP, Debt_GDP]);
summarize(EstMdl)

% VAR(2) MODEL WITH TWO LAGS
Md2 = varm(2,2);
EstMd2 = estimate(Md2,[Sur_GDP, Debt_GDP]);
summarize(EstMd2)

% VAR(3) MODEL WITH TWO LAGS
Md3 = varm(2,3);
EstMd3 = estimate(Md3,[Sur_GDP, Debt_GDP]);
summarize(EstMd3)

% VAR(4) MODEL WITH TWO LAGS
Md4 = varm(2,4);
EstMd4 = estimate(Md4,[Sur_GDP, Debt_GDP]);
summarize(EstMd4)
% BIC1 = 392.998
% BIC2 = 365.843
% BIC3 = 363.13
% BIC4 = 374.02
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%
% VAR(3) - IRF
summarize(EstMd3)
Response1 = irf(EstMd3) % 20 OBSERVATIONS HORIZON

% 10 YEAR HOTIZON: 4x10=40
[Response,Lower,Upper] = irf(EstMd3,'NumObs',40)

% IRF of DEBT_GDP on SURPLUS SHOCK:
irfshock1resp2 = Response(:,1,2);
IRFCIShock1Resp2 = [Lower(:,1,2) Upper(:,1,2)];

figure(3);
h1 = plot(0:39,irfshock1resp2);
hold on
h2 = plot(0:39,IRFCIShock1Resp2,'r--');
legend([h1 h2(1)],["IRF" "95% Confidence Interval"])
set(gca,'FontSize',25)
xlabel("Time Index");
ylabel("Response");
title("IRF of D-GDP When Sur-GDP Is Shocked");
grid on
hold off
```

Italy

```
clear; clc;
format long g
%% Italy 1999:Q3 - 2020:Q4
%%%%%%%%%%%%%%%%%%%%%%%%%%%

IT_data = readtable('IT_Q.xlsx');
Sur_GDP = table2array(IT_data(:,2));
Debt_GDP = table2array(IT_data(:,3));
T = (1999.75:0.25:2020.75)';
t = length(T)
%% PLOT
figure(1)
subplot(1,2,1); hold on;  box on ;
               plot(T, Sur_GDP, 'LineWidth',1,'Color','blue');
               plot(T, mean(Sur_GDP)*ones(t,1),'-
k','LineWidth',1);
               xlim([T(1)-1 T(end)+1]); ylim([-7 7]);
               title('Surplus (%GDP)');
               legend('Sur (%GDP)')

               set(gca,'FontSize',25)
               hold off; box off;
subplot(1,2,2); hold on;  box on ;
               plot(T, Debt_GDP, 'LineWidth',1,'Color','blue');
               plot(T, mean(Debt_GDP)*ones(t,1),'-
k','LineWidth',1);
               xlim([T(1)-1 T(end)+1]);  ylim([102 157]);
               title('Debt (%GDP)');
               legend('Debt (%GDP)')

               set(gca,'FontSize',25)
               hold off; box off;
%% UNIT ROOT TESTS:
   %% 1. PHILLIPS-PERRON TEST OF UNIT ROOT:
   % INPUT: the time series under investigation
% Sur_GDP test
[PP,pValue_0,stat_0,cValue_0,reg_0] = pptest(Sur_GDP)
[PP_ts,pValue_1,stat_1,cValue_1,reg_1] =
pptest(Sur_GDP,'model','TS')
[PP_ard,pValue_2,stat_2,cValue_2,reg_2] =
pptest(Sur_GDP,'model','ARD')
[PP_lags,pValue_3,stat_3,cValue_3,reg_3] =
pptest(Sur_GDP,'model','TS','lags',0:2)

% We want h=1 for the first difference:
Delta_SGDP = diff(Sur_GDP)
[PPD1_d0,pValue_d0,stat_d0,cValue_d0,reg_d0] = pptest(Delta_SGDP);
[PPD1_ts,pValue_d1,stat_ts,cValue_d1,reg_d1] =
pptest(Delta_SGDP,'model','TS');
[PPD1_ard,pValue_d2,stat_ard,cValue_d2,reg_d2] =
pptest(Delta_SGDP,'model','ARD');
[PPD1_lags,pValue_d3,stat_lags,cValue_d3,reg_d3] =
pptest(Delta_SGDP,'model','TS','lags',0:2);
%% Debt_GDP test
[h,pValue_4,stat_4,cValue_4,reg_4] = pptest(Debt_GDP)
[PP_ts_debt,pValue_5,stat_5,cValue_5,reg_5] =
pptest(Debt_GDP,'model','TS')
[PP_ard_debt,pValue_6,stat_6,cValue_6,reg_6] =
pptest(Debt_GDP,'model','ARD')
```

```
[PP_lag_debt,pValue_7,stat_7,cValue_7,reg_7] =
pptest(Debt_GDP,'model','TS','lags',0:2)

% We want h=1 for the first difference:
Delta_DGDP = diff(Debt_GDP)
[Delta_DGDP_ts,pValue_d4,stat_d4,cValue_d4,reg_d4] =
pptest(Delta_DGDP,'model','TS');
[Delta_DGDP_ard,pValue_d5,stat_d5,cValue_d5,reg_d5] =
pptest(Delta_DGDP,'model','ARD');
[Delta_DGDP_lag,pValue_d6,stat_d6,cValue_d6,reg_d6] =
pptest(Delta_DGDP,'model','TS','lags',0:2);


%% 2.DICKEY-FULLER TEST STATISTICS:
  % 1. Simple DF:
  s = adftest(Sur_GDP)

  d = adftest(Debt_GDP)
% 2.Test for a unit root against a trend-stationary alternative,
augmenting the model with 0, 1, and 2 lagged difference terms.
  s_TS = adftest(Sur_GDP,'model','TS','lags',0:2);

  d_TS = adftest(Debt_GDP, 'model','TS','lags',0:2);
% 3. Test for a unit root using three different choices for the
number of lagged difference terms. Return the regression
statistics for each alternative model.
  [h,~,~,~,reg] = adftest(Sur_GDP,'model','TS','lags',0:2);
  [h1,~,~,~,reg1] = adftest(Debt_GDP,'model','TS','lags',0:2);
% The output shows which terms are included in the three
alternative models.
% The first model has no added difference terms, the second model
has one difference term (b1),
% and the third model has two difference terms (b1 and b2).
reg.names;
reg.BIC;
reg1.names;
reg1.BIC;
% Repeat the analysis with the differences of the variables to
verify they
% are I(0)
 % 1. Simple DF:
  s_delta = adftest(Delta_SGDP)

  d_delta = adftest(Delta_DGDP)
% 2.Test for a unit root against a trend-stationary alternative,
augmenting the model with 0, 1, and 2 lagged difference terms.
 s_TS_delta = adftest(Delta_SGDP,'model','TS','lags',0:2);

  d_TS_delta = adftest(Delta_DGDP, 'model','TS','lags',0:2);
% 3. Test for a unit root using three different choices for the
number of lagged difference terms. Return the regression
statistics for each alternative model.
[h_d,~,~,~,reg_d] = adftest(Delta_SGDP,'model','TS','lags',0:2);
[h1_d,~,~,~,reg1_d] = adftest(Delta_DGDP,'model','TS','lags',0:2);

%% DOLS: DYNAMIC OLS AS IN STOCK AND WATSON (1993).

%% 1. FIRST OLS REGRESSION BETWEEN I(1) SERIES:
y = Sur_GDP(2:end);
```

```
X = [ones(t-1,1), Debt_GDP(1:end-1)];
B = (X'*X)\(X'*y)

e = y - X*B;
RSS = e'*e;
sigma_2 = RSS/(t-3)

[beta,Sigma] = mvregress(X, y)
%% 2. DYNAMIC OLS:
q = round(t^(1/3));

NA = NaN
% DEFINE THE REGRESSORS:
D_b_0 = [NA; Debt_GDP(2:end)-Debt_GDP(1:end-1)];     % 85 x 1

D_b_1 = [NA; D_b_0(1:end-1)];                        % 85 x 1
D_b_2 = [NA; NA; D_b_0(1:end-2)];                    % 85 x 1
D_b_3 = [NA; NA; NA; D_b_0(1:end-3)];                % 85 x 1
D_b_4 = [NA; NA; NA; NA; D_b_0(1:end-4)];            % 85 x 1
D_b_5 = [NA; NA; NA; NA; NA; D_b_0(1:end-5)];       % 85 x 1

D_b_p1 = [D_b_0(2:end); NA];                        % 85 x 1
D_b_p2 = [D_b_0(3:end); NA; NA];                   % 85 x 1
D_b_p3 = [D_b_0(4:end); NA; NA; NA];              % 85 x 1

b_t_1 = [NA; Debt_GDP(1:end-1)];

N = length(D_b_0); % N=85

XD = [ones(N,1), b_t_1, D_b_0, D_b_1, D_b_2, D_b_3, D_b_4, D_b_5,
D_b_p1, D_b_p2, D_b_p3];

% CUT THE MATRICES FOR THE CORRECT SIZE:
X_d = XD(7:82,:);
Y_d = y(7:82);

k = size(X_d, 2);

beta_hat = zeros(k,1)

beta_hat = (X_d'*X_d)\(X_d'*Y_d)
e_d = Y_d - X_d*beta_hat

[beta_d,Sigma_d] = mvregress(X_d, Y_d)

%% t test
n=size(X_d,1);
V_beta=Sigma_d*inv(X_d'*X_d);
%% compute the observed statistic
t_obs=(beta_d(2)-0)/sqrt(V_beta(2,2));
%% Define statistic test
t_teo=tinv(0.95,n-11);

t_obs>t_teo % WE DO NOT REJECT H0.
% if t_obs>t_teo I reject H_0
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%

%% ENGLE-GRANGER COINTEGRATION TEST

figure(2)
subplot(1,2,1)
plot(e)
subplot(1,2,2)
plot(e_d)
% 1. FIRST METHOD. SIMPLE DICKEY-FULLER test on residuals:
C01 = adftest(e)
CO2 = adftest(e_d)

% 2. SECOND METHOD:
[h,pValue,stat,cValue_en,reg1,reg2] = egcitest([y,X])

[h_delta,pValue,stat,cValue_en_d,reg1,reg2] = egcitest([Y_d, X_d])
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%
% GRANGER CAUSALITY TEST:
% y=primary surplus
% 1. MODEL IN ECM.
   % FIRST EQUATION. SURPLUS_G VS DEBT_G
   yec1 = Sur_GDP(2:end);
   Xec1 = [Debt_GDP(1:end-1)];
   Bec1 = (Xec1'*Xec1)\(Xec1'*yec1);

   Zec1 = yec1 - Xec1*Bec1;  % ECM1

   St_1 = [NA; Sur_GDP(1:end-1)];                  % 85 OSS
   ZEC1 = [NA; NA; Zec1(1:end-1)];                 % 85 OSS

   DST_1 =[NA; NA; Delta_SGDP(1:end-1)];
   DST_2 =[NA; NA; NA; Delta_SGDP(1:end-2)];
   DST_3 =[NA; NA; NA; NA; Delta_SGDP(1:end-3)];

   DDGT_1 =[NA; NA; Delta_DGDP(1:end-1)];
   DDGT_2 =[NA; NA; NA; Delta_DGDP(1:end-2)];
   DDGT_3 =[NA; NA; NA; NA; Delta_DGDP(1:end-3)];

n_ecm1 = size(St_1,1);
%                  1        2      3     4      5       6      7
8       9
X_ECM1 = [ones(n_ecm1,1), St_1, ZEC1, DST_1, DST_2, DST_3, DDGT_1,
DDGT_2, DDGT_3];

X_ECM11 = X_ECM1(5:end,:);
Y_ECM1 =  Sur_GDP(5:end);

[beta_ECM1,Sigma_ECM1] = mvregress(Y_ECM1, X_ECM11);
beta_ECM1;
V_betaECM = Sigma_ECM1*inv(X_ECM11'*X_ECM11);
% TEST DI GRANGER CAUSALITY IS AN F-TEST ON THE FOLLOWING
PARAMETHERS:
c = [0, 0, 0, 0];
H = [0, 0, 1, 0, 0, 0, 0, 0, 0;
     0, 0, 0, 0, 0, 0, 1, 0, 0;
```

```
     0, 0, 0, 0, 0, 0, 0, 1, 0;
     0, 0, 0, 0, 0, 0, 0, 0, 1];

p_value_granger_s = linhyptest(beta_ECM1,V_betaECM,c,H,9);
%p_value > 0.05
%% y=debt
yec2 = Debt_GDP(2:end);
Xec2 = [Sur_GDP(1:end-1)];
Bec2 = (Xec2'*Xec2)\(Xec2'*yec2);

Zec2 = yec2-Xec2*Bec2; %ECM2

Dt_1=[NA; Debt_GDP(1:end-1)];
ZEC2= [NA; NA; Zec2(1:end-1)];

DDT_1 =[NA; NA; Delta_DGDP(1:end-1)];
DDT_2 =[NA; NA; NA; Delta_DGDP(1:end-2)];
DDT_3 =[NA; NA; NA; NA; Delta_DGDP(1:end-3)];

DSGT_1 =[NA; NA; Delta_SGDP(1:end-1)];
DSGT_2 =[NA; NA; NA; Delta_SGDP(1:end-2)];
DSGT_3 =[NA; NA; NA; NA; Delta_SGDP(1:end-3)];

n_ecm2 = size(Dt_1,1);
%              1          2     3     4      5     6      7      8
9
X_ECM2=[ones(n_ecm2,1), Dt_1, ZEC2, DDT_1, DDT_2, DDT_3, DSGT_1,
DSGT_2, DSGT_3];

X_ECM22 = X_ECM2(5:end,:);
Y_ECM2 = Debt_GDP(5:end);

[beta_ECM2, Sigma_ECM2] = mvregress(Y_ECM2, X_ECM22);
beta_ECM2;
V_betaECM2 = Sigma_ECM2*inv(X_ECM22'*X_ECM22);
% parameters
c1 = [0, 0, 0, 0];
H1 = [0, 0, 1, 0, 0, 0, 0, 0, 0;
      0, 0, 0, 0, 0, 0, 1, 0, 0;
      0, 0, 0, 0, 0, 0, 0, 1, 0;
      0, 0, 0, 0, 0, 0, 0, 0, 1];

 p_value_gr_d = linhyptest(beta_ECM2,V_betaECM2,c1,H1,9);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%
% THE FORWARD LOOKINF APPROACH: VAR(1) - VAR(4)
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%
% VAR(1) MODEL WITH ONE LAG
Mdl = varm(2,1)
EstMdl = estimate(Mdl,[Sur_GDP, Debt_GDP]);
summarize(EstMdl)

% VAR(2) MODEL WITH TWO LAGS
Md2 = varm(2,2);
EstMd2 = estimate(Md2,[Sur_GDP, Debt_GDP]);
summarize(EstMd2)
```

```
% VAR(3) MODEL WITH TWO LAGS
Md3 = varm(2,3);
EstMd3 = estimate(Md3,[Sur_GDP, Debt_GDP]);
summarize(EstMd3)

% VAR(4) MODEL WITH TWO LAGS
Md4 = varm(2,4);
EstMd4 = estimate(Md4,[Sur_GDP, Debt_GDP]);
summarize(EstMd4)

% BIC1 = 541.084
% BIC2 = 517.315
% BIC3 = 509.701
% BIC4 = 519.476

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%
% VAR(3) - IRF
summarize(EstMd3)
Response1 = irf(EstMd3) % 20 OBSERVATIONS HORIZON

% 10 YEAR HOTIZON: 4x10=40
[Response,Lower,Upper] = irf(EstMd3,'NumObs',40)

% IRF OF DEBT_GDP ON SURPLUS SHOCK:
irfshock1resp2 = Response(:,1,2);
IRFCIShock1Resp2 = [Lower(:,1,2) Upper(:,1,2)];

figure(3);
h1 = plot(0:39,irfshock1resp2);
hold on
h2 = plot(0:39,IRFCIShock1Resp2,'r--');
legend([h1 h2(1)],["IRF" "95% Confidence Interval"])
set(gca,'FontSize',25)
xlabel("Time Index");
ylabel("Response");
title("IRF of D-GDP When Sur-GDP Is Shocked");
grid on
hold off
```

Canada

```
clear; clc;
format long g
%% CANADA 1981 - 2020
%%%%%%%%%%%%%%%%%%%%%%%%%%%

CAN_data = readtable('CAN_A.xlsx');
Sur_GDP = table2array(CAN_data(:,2));
Debt_GDP = table2array(CAN_data(:,3));
T = (1981:1:2020)';
t = length(T)

%% PLOT
figure(1)
subplot(1,2,1); hold on;  box on ;
               plot(T, Sur_GDP, 'LineWidth',1,'Color','blue');
```

```
                plot(T, mean(Sur_GDP)*ones(t,1),'-
k','LineWidth',1);
                xlim([T(1)-1 T(end)+1]); ylim([-12 4]);
                title('Surplus (%GDP)');
                legend('Sur (%GDP)')

                set(gca,'FontSize',25)
                hold off; box off;
subplot(1,2,2); hold on;  box on ;
                plot(T, Debt_GDP, 'LineWidth',1,'Color','blue');
                plot(T, mean(Debt_GDP)*ones(t,1),'-
k','LineWidth',1);
                xlim([T(1)-1 T(end)+1]);  ylim([45 118]);
                title('Debt (%GDP)');
                legend('Debt (%GDP)')

                set(gca,'FontSize',25)
                hold off; box off;
%% UNIT ROOT TESTS:
   %% 1. PHILLIPS-PERRON TEST OF UNIT ROOT:
   % INPUT: the time series under investigation
% Sur_GDP test
[PP,pValue_0,stat_0,cValue_0,reg_0] = pptest(Sur_GDP)
[PP_ts,pValue_1,stat_1,cValue_1,reg_1] =
pptest(Sur_GDP,'model','TS')
[PP_ard,pValue_2,stat_2,cValue_2,reg_2] =
pptest(Sur_GDP,'model','ARD')
[PP_lags,pValue_3,stat_3,cValue_3,reg_3] =
pptest(Sur_GDP,'model','TS','lags',0:2)
% We want h=1 for the first difference:
Delta_SGDP = diff(Sur_GDP)
[PPD1_d0,pValue_d0,stat_d0,cValue_d0,reg_d0] = pptest(Delta_SGDP);
[PPD1_ts,pValue_d1,stat_ts,cValue_d1,reg_d1] =
pptest(Delta_SGDP,'model','TS');
[PPD1_ard,pValue_d2,stat_ard,cValue_d2,reg_d2] =
pptest(Delta_SGDP,'model','ARD');
[PPD1_lags,pValue_d3,stat_lags,cValue_d3,reg_d3] =
pptest(Delta_SGDP,'model','TS','lags',0:2);
%% Debt_GDP test
[h,pValue_4,stat_4,cValue_4,reg_4] = pptest(Debt_GDP)
[PP_ts_debt,pValue_5,stat_5,cValue_5,reg_5] =
pptest(Debt_GDP,'model','TS')
[PP_ard_debt,pValue_6,stat_6,cValue_6,reg_6] =
pptest(Debt_GDP,'model','ARD')
[PP_lag_debt,pValue_7,stat_7,cValue_7,reg_7] =
pptest(Debt_GDP,'model','TS','lags',0:2)
% We want h=1 for the first difference:
Delta_DGDP = diff(Debt_GDP)
[Delta_DGDP_ts,pValue_d4,stat_d4,cValue_d4,reg_d4] =
pptest(Delta_DGDP,'model','TS');
[Delta_DGDP_ard,pValue_d5,stat_d5,cValue_d5,reg_d5] =
pptest(Delta_DGDP,'model','ARD');
[Delta_DGDP_lag,pValue_d6,stat_d6,cValue_d6,reg_d6] =
pptest(Delta_DGDP,'model','TS','lags',0:2);
%% 2.DICKEY-FULLER TEST STATISTICS:
  % 1. Simple DF:
  s = adftest(Sur_GDP)

  d = adftest(Debt_GDP)
```

```
  % 2.Test for a unit root against a trend-stationary alternative,
augmenting the model with 0, 1, and 2 lagged difference terms.
  s_TS = adftest(Sur_GDP,'model','TS','lags',0:2)

  d_TS = adftest(Debt_GDP, 'model','TS','lags',0:2)
  % 3. Test for a unit root using three different choices for the
number of lagged difference terms. Return the regression
statistics for each alternative model.
  [h,~,~,~,reg] = adftest(Sur_GDP,'model','TS','lags',0:2);
  [h1,~,~,~,reg1] = adftest(Debt_GDP,'model','TS','lags',0:2);
  % The output shows which terms are included in the three
alternative models.
% The first model has no added difference terms, the second model
has one difference term (b1),
% and the third model has two difference terms (b1 and b2).
reg.names;
reg.BIC;
reg1.names;
reg1.BIC;
% Repeat the analysis with the differences of the variables to
verify they
% are I(0)
 % 1. Simple DF:
  s_delta = adftest(Delta_SGDP)

  d_delta = adftest(Delta_DGDP)
% 2.Test for a unit root against a trend-stationary alternative,
augmenting the model with 0, 1, and 2 lagged difference terms.
 s_TS_delta = adftest(Delta_SGDP,'model','TS','lags',0:2);

  d_TS_delta = adftest(Delta_DGDP, 'model','TS','lags',0:2);
% 3. Test for a unit root using three different choices for the
number of lagged difference terms. Return the regression
statistics for each alternative model.
[h_d,~,~,~,reg_d] = adftest(Delta_SGDP,'model','TS','lags',0:2);
[h1_d,~,~,~,reg1_d] = adftest(Delta_DGDP,'model','TS','lags',0:2);
%% DOLS: DYNAMIC OLS AS IN STOCK AND WATSON (1993).

%% 1. FIRST OLS REGRESSION BETWEEN I(1) SERIES:
y = Sur_GDP(2:end);
X = [ones(t-1,1), Debt_GDP(1:end-1)];
B = (X'*X)\(X'*y)

e = y - X*B;
RSS = e'*e;
sigma_2 = RSS/(t-3)

[beta,Sigma] = mvregress(X, y)
%% 2. DYNAMIC OLS:
q = round(t^(1/3));

NA = NaN
%% DEFINE THE REGRESSORS:
D_b_0 = [NA; Debt_GDP(2:end)-Debt_GDP(1:end-1)];     % 41 x 1

D_b_1 = [NA; D_b_0(1:end-1)];                         % 41 x 1
D_b_2 = [NA; NA; D_b_0(1:end-2)];                    % 41 x 1
D_b_3 = [NA; NA; NA; D_b_0(1:end-3)];                % 41 x 1
D_b_4 = [NA; NA; NA; NA; D_b_0(1:end-4)];           % 41 x 1
```

```
D_b_p1 = [D_b_0(2:end); NA];                              % 41 x 1
D_b_p2 = [D_b_0(3:end); NA; NA];                         % 41 x 1

b_t_1 = [NA; Debt_GDP(1:end-1)];
N = length(D_b_0); % N=41

XD = [ones(N,1), b_t_1, D_b_0, D_b_1, D_b_2, D_b_3, D_b_4, D_b_p1,
D_b_p2];

%% CUT THE MATRICES FOR THE CORRECT SIZE:
X_d = XD(6:38,:);
Y_d = y(6:38);

k = size(X_d, 2);
beta_hat = zeros(k,1);

beta_hat = (X_d'*X_d)\(X_d'*Y_d);
e_d = Y_d - X_d*beta_hat;
[beta_d,Sigma_d] = mvregress(X_d, Y_d);
%% t test
n=size(X_d,1);
V_beta=Sigma_d*inv(X_d'*X_d);
% compute the observed statistic
t_obs=(beta_d(2)-0)/sqrt(V_beta(2,2));
% Define statistic test
t_teo=tinv(0.95,n-9);
% if t_obs>t_-teo I reject H_0
%% ENGLE-GRANGER COINTEGRATION TEST

figure(2)
subplot(1,2,1)
plot(e)
subplot(1,2,2)
plot(e_d)
% 1. FIRST METHOD. SIMPLE DICKEY-FULLER test on residuals:
C01 = adftest(e)
CO2 = adftest(e_d)
% 2. SECOND METHOD:
[h,pValue,stat,cValue_en,reg1,reg2] = egcitest([y,X])

[h_delta,pValue,stat,cValue_en_d,reg1,reg2] = egcitest([Y_d, X_d])

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%
% TEST DI GRANGER CAUSALITY:

% 1. MODEL IN ECM.
   % FIRST EQUATION. SURPLUS_G VS DEBT_G
   yec1 = Sur_GDP(2:end);
   Xec1 = [Debt_GDP(1:end-1)];
   Bec1 = (Xec1'*Xec1)\(Xec1'*yec1);

   Zec1 = yec1 - Xec1*Bec1;  % ECM1

   St_1 = [NA; Sur_GDP(1:end-1)];                         % 40 OSS
   ZEC1 = [NA; NA; Zec1(1:end-1)];                        % 40 OSS
```

```
   DST_1 =[NA; NA; Delta_SGDP(1:end-1)];
   DST_2 =[NA; NA; NA; Delta_SGDP(1:end-2)];
   DST_3 =[NA; NA; NA; NA; Delta_SGDP(1:end-3)];

   DDGT_1 =[NA; NA; Delta_DGDP(1:end-1)];
   DDGT_2 =[NA; NA; NA; Delta_DGDP(1:end-2)];
   DDGT_3 =[NA; NA; NA; NA; Delta_DGDP(1:end-3)];

n_ecm1 = size(St_1,1);
%                  1        2      3     4      5       6      7
8       9
X_ECM1 = [ones(n_ecm1,1), St_1, ZEC1, DST_1, DST_2, DST_3, DDGT_1,
DDGT_2, DDGT_3];

X_ECM11 = X_ECM1(5:end,:);
Y_ECM1 =  Sur_GDP(5:end);

[beta_ECM1,Sigma_ECM1] = mvregress(Y_ECM1, X_ECM11);
beta_ECM1;
V_betaECM = Sigma_ECM1*inv(X_ECM11'*X_ECM11);
% GRANGER CAUSALITY TEST on the following parameters:
c = [0, 0, 0, 0];
H = [0, 0, 1, 0, 0, 0, 0, 0, 0;
     0, 0, 0, 0, 0, 0, 1, 0, 0;
     0, 0, 0, 0, 0, 0, 0, 1, 0;
     0, 0, 0, 0, 0, 0, 0, 0, 1];

p_value_granger_s = linhyptest(beta_ECM1,V_betaECM,c,H,9);
%p_value > 0.05
%% y=debt
yec2 = Debt_GDP(2:end);
Xec2 = [Sur_GDP(1:end-1)];
Bec2 = (Xec2'*Xec2)\(Xec2'*yec2);

Zec2 = yec2-Xec2*Bec2; %ECM2

Dt_1=[NA; Debt_GDP(1:end-1)];
ZEC2= [NA; NA; Zec2(1:end-1)];

DDT_1 =[NA; NA; Delta_DGDP(1:end-1)];
DDT_2 =[NA; NA; NA; Delta_DGDP(1:end-2)];
DDT_3 =[NA; NA; NA; NA; Delta_DGDP(1:end-3)];

DSGT_1 =[NA; NA; Delta_SGDP(1:end-1)];
DSGT_2 =[NA; NA; NA; Delta_SGDP(1:end-2)];
DSGT_3 =[NA; NA; NA; NA; Delta_SGDP(1:end-3)];

n_ecm2 = size(Dt_1,1);
%               1          2      3     4      5      6      7      8
9
X_ECM2=[ones(n_ecm2,1), Dt_1, ZEC2, DDT_1, DDT_2, DDT_3, DSGT_1,
DSGT_2, DSGT_3];

X_ECM22 = X_ECM2(5:end,:);
Y_ECM2 = Debt_GDP(5:end);

[beta_ECM2, Sigma_ECM2] = mvregress(Y_ECM2, X_ECM22);
```

```
beta_ECM2;
V_betaECM2 = Sigma_ECM2*inv(X_ECM22'*X_ECM22);
% parameters
c1 = [0, 0, 0, 0];
H1 = [0, 0, 1, 0, 0, 0, 0, 0, 0;
      0, 0, 0, 0, 0, 0, 1, 0, 0;
      0, 0, 0, 0, 0, 0, 0, 1, 0;
      0, 0, 0, 0, 0, 0, 0, 0, 1];

 p_value_gr_d = linhyptest(beta_ECM2,V_betaECM2,c1,H1,9);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%
% VAR(1) MODEL WITH ONE LAG
Mdl = varm(2,1)
EstMdl = estimate(Mdl,[Sur_GDP, Debt_GDP]);
summarize(EstMdl)

% VAR(2) MODEL WITH TWO LAGS
Md2 = varm(2,2);
EstMd2 = estimate(Md2,[Sur_GDP, Debt_GDP]);
summarize(EstMd2)

% VAR(3) MODEL WITH TWO LAGS
Md3 = varm(2,3);
EstMd3 = estimate(Md3,[Sur_GDP, Debt_GDP]);
summarize(EstMd3)

% VAR(4) MODEL WITH TWO LAGS
Md4 = varm(2,4);
EstMd4 = estimate(Md4,[Sur_GDP, Debt_GDP]);
summarize(EstMd4)
% BIC1 = 362.916
% BIC2 = 356.03
% BIC3 = 361.045
% BIC4 = 356.281
% VAR(2) - IRF
summarize(EstMd2)
Response1 = irf(EstMd2)
% 10 YEAR HOTIZON: 10
[Response,Lower,Upper] = irf(EstMd4,'NumObs',10)

% IRF of DEBT_GDP ON SURPLUS SHOCK:
irfshock1resp2 = Response(:,1,2);
IRFCIShock1Resp2 = [Lower(:,1,2) Upper(:,1,2)];

figure(3);
h1 = plot(0:9,irfshock1resp2);
hold on
h2 = plot(0:9,IRFCIShock1Resp2,'r--');
legend([h1 h2(1)],["IRF" "95% Confidence Interval"])
set(gca,'FontSize',25)
xlabel("Time Index");
ylabel("Response");
title("IRF of D-GDP When Sur-GDP Is Shocked");
grid on
hold off
```

USA

```
clear; clc;
format long g
%% USA 1960 - 2019
%%%%%%%%%%%%%%%%%%%%%%%%%%%

US_data = readtable('USA_A.xlsx');
Sur_GDP = table2array(US_data(:,2));
Debt_GDP = table2array(US_data(:,3));
T = (1960:1:2019)';
t = length(T)
%% PLOT
figure(1)
subplot(1,2,1); hold on;  box on ;
               plot(T, Sur_GDP, 'LineWidth',1,'Color','blue');
               plot(T, mean(Sur_GDP)*ones(t,1),'-
k','LineWidth',1);
               xlim([T(1)-1 T(end)+1]); ylim([-15 5]);
               title('Surplus (%GDP)');
               legend('Sur (%GDP)')

               set(gca,'FontSize',25)
               hold off; box off;
subplot(1,2,2); hold on;  box on ;
               plot(T, Debt_GDP, 'LineWidth',1,'Color','blue');
               plot(T, mean(Debt_GDP)*ones(t,1),'-
k','LineWidth',1);
               xlim([T(1)-1 T(end)+1]);  ylim([30 101]);
               title('Debt (%GDP)');
               legend('Debt (%GDP)')

               set(gca,'FontSize',25)
               hold off; box off;
%% UNIT ROOT TESTS:
   %% 1. PHILLIPS-PERRON TEST OF UNIT ROOT:
   % INPUT: the time series under investigation
% Sur_GDP test
[PP,pValue_0,stat_0,cValue_0,reg_0] = pptest(Sur_GDP)
[PP_ts,pValue_1,stat_1,cValue_1,reg_1] =
pptest(Sur_GDP,'model','TS')
[PP_ard,pValue_2,stat_2,cValue_2,reg_2] =
pptest(Sur_GDP,'model','ARD')
[PP_lags,pValue_3,stat_3,cValue_3,reg_3] =
pptest(Sur_GDP,'model','TS','lags',0:2)
% We want h=1 for the first difference:
Delta_SGDP = diff(Sur_GDP)
[PPD1_d0,pValue_d0,stat_d0,cValue_d0,reg_d0] = pptest(Delta_SGDP);
[PPD1_ts,pValue_d1,stat_ts,cValue_d1,reg_d1] =
pptest(Delta_SGDP,'model','TS');
[PPD1_ard,pValue_d2,stat_ard,cValue_d2,reg_d2] =
pptest(Delta_SGDP,'model','ARD');
[PPD1_lags,pValue_d3,stat_lags,cValue_d3,reg_d3] =
pptest(Delta_SGDP,'model','TS','lags',0:2);
%% Debt_GDP test
[h,pValue_4,stat_4,cValue_4,reg_4] = pptest(Debt_GDP)
[PP_ts_debt,pValue_5,stat_5,cValue_5,reg_5] =
pptest(Debt_GDP,'model','TS')
[PP_ard_debt,pValue_6,stat_6,cValue_6,reg_6] =
pptest(Debt_GDP,'model','ARD')
[PP_lag_debt,pValue_7,stat_7,cValue_7,reg_7] =
pptest(Debt_GDP,'model','TS','lags',0:2)
```

```
% We want h=1 for the first difference:
Delta_DGDP = diff(Debt_GDP)
[Delta_DGDP_ts,pValue_d4,stat_d4,cValue_d4,reg_d4] =
pptest(Delta_DGDP,'model','TS');
[Delta_DGDP_ard,pValue_d5,stat_d5,cValue_d5,reg_d5] =
pptest(Delta_DGDP,'model','ARD');
[Delta_DGDP_lag,pValue_d6,stat_d6,cValue_d6,reg_d6] =
pptest(Delta_DGDP,'model','TS','lags',0:2);
%% 2.DICKEY-FULLER TEST STATISTICS:
  % 1. Simple DF:
  s = adftest(Sur_GDP)

  d = adftest(Debt_GDP)

  % 2.Test for a unit root against a trend-stationary alternative,
augmenting the model with 0, 1, and 2 lagged difference terms.
  s_TS = adftest(Sur_GDP,'model','TS','lags',0:2)

  d_TS = adftest(Debt_GDP, 'model','TS','lags',0:2)
  % 3. Test for a unit root using three different choices for the
number of lagged difference terms. Return the regression
statistics for each alternative model.
  [h,~,~,~,reg] = adftest(Sur_GDP,'model','TS','lags',0:2);
  [h1,~,~,~,reg1] = adftest(Debt_GDP,'model','TS','lags',0:2);
  % The output shows which terms are included in the three
alternative models.
% The first model has no added difference terms, the second model
has one difference term (b1),
% and the third model has two difference terms (b1 and b2).
reg.names;
reg.BIC;
reg1.names;
reg1.BIC;
% Repeat the analysis with the differences of the variables to
verify they
% are I(0)
 % 1. Simple DF:
  s_delta = adftest(Delta_SGDP)

  d_delta = adftest(Delta_DGDP)
% 2.Test for a unit root against a trend-stationary alternative,
augmenting the model with 0, 1, and 2 lagged difference terms.
 s_TS_delta = adftest(Delta_SGDP,'model','TS','lags',0:2);

  d_TS_delta = adftest(Delta_DGDP, 'model','TS','lags',0:2);
% 3. Test for a unit root using three different choices for the
number of lagged difference terms. Return the regression
statistics for each alternative model.
[h_d,~,~,~,reg_d] = adftest(Delta_SGDP,'model','TS','lags',0:2);
[h1_d,~,~,~,reg1_d] = adftest(Delta_DGDP,'model','TS','lags',0:2);
%% DOLS: DYNAMIC OLS AS IN STOCK AND WATSON (1993).

%% 1. FIRST OLS REGRESSION BETWEEN I(1) SERIES:
y = Sur_GDP(2:end);
X = [ones(t-1,1), Debt_GDP(1:end-1)];
B = (X'*X)\(X'*y)

e = y - X*B;
RSS = e'*e;
```

```
sigma_2 = RSS/(t-3)

[beta,Sigma] = mvregress(X, y)
%% 2. DYNAMIC OLS:
q = round(t^(1/3));

NA = NaN
%% DEFINE THE REGRESSORS:
D_b_0 = [NA; Debt_GDP(2:end)-Debt_GDP(1:end-1)];     % 60 x 1

D_b_1 = [NA; D_b_0(1:end-1)];                          % 60 x 1
D_b_2 = [NA; NA; D_b_0(1:end-2)];                     % 60 x 1
D_b_3 = [NA; NA; NA; D_b_0(1:end-3)];                  % 60 x 1
D_b_4 = [NA; NA; NA; NA; D_b_0(1:end-4)];             % 60 x 1
D_b_5 = [NA; NA; NA; NA; NA; D_b_0(1:end-5)];         %60 x 1

D_b_p1 = [D_b_0(2:end); NA];                          % 60 x 1
D_b_p2 = [D_b_0(3:end); NA; NA];                     % 60 x 1
D_b_p3 = [D_b_0(4:end); NA; NA; NA];                % 60 x 1

b_t_1 = [NA; Debt_GDP(1:end-1)];
N = length(D_b_0); % N=60

XD = [ones(N,1), b_t_1, D_b_0, D_b_1, D_b_2, D_b_3, D_b_4, D_b_5,
D_b_p1, D_b_p2, D_b_p3];

%% CUT THE MATRICES FOR THE CORRECT SIZE:
X_d = XD(7:57,:);
Y_d = y(7:57);

k = size(X_d, 2);
beta_hat = zeros(k,1);

beta_hat = (X_d'*X_d)\(X_d'*Y_d);
e_d = Y_d - X_d*beta_hat;
[beta_d,Sigma_d] = mvregress(X_d, Y_d);
%% t test
n=size(X_d,1);
V_beta=Sigma_d*inv(X_d'*X_d);
% compute the observed statistic
t_obs=(beta_d(2)-0)/sqrt(V_beta(2,2));
% Define statistic test
t_teo=tinv(0.95,n-11);
% if t_obs>t_-teo I reject H_0
%% ENGLE-GRANGER COINTEGRATION TEST.
figure(2)
subplot(1,2,1)
plot(e)
subplot(1,2,2)
plot(e_d)
% 1. FIRST METHOD. SIMPLE DICKEY-FULLER test on residuals
C01 = adftest(e)     % se 0, accetto la non-stazionariet√†
CO2 = adftest(e_d)  % se 0, accetto la non-stazionariet√†

% 2. SECOND METHOD:
[h,pValue,stat,cValue_en,reg1,reg2] = egcitest([y,X])

[h_delta,pValue,stat,cValue_en_d,reg1,reg2] = egcitest([Y_d, X_d])
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%
% TEST DI GRANGER CAUSALITY:

% 1. MODEL IN ECM.
   % FIRST EQUATION. SURPLUS_G VS DEBT_G
   yec1 = Sur_GDP(2:end);
   Xec1 = [Debt_GDP(1:end-1)];
   Bec1 = (Xec1'*Xec1)\(Xec1'*yec1);

   Zec1 = yec1 - Xec1*Bec1;  % ECM1

   St_1 = [NA; Sur_GDP(1:end-1)];                        % 60 OSS
   ZEC1 = [NA; NA; Zec1(1:end-1)];                       % 60 OSS

   DST_1 =[NA; NA; Delta_SGDP(1:end-1)];
   DST_2 =[NA; NA; NA; Delta_SGDP(1:end-2)];
   DST_3 =[NA; NA; NA; NA; Delta_SGDP(1:end-3)];

   DDGT_1 =[NA; NA; Delta_DGDP(1:end-1)];
   DDGT_2 =[NA; NA; NA; Delta_DGDP(1:end-2)];
   DDGT_3 =[NA; NA; NA; NA; Delta_DGDP(1:end-3)];

n_ecm1 = size(St_1,1);
%                 1        2      3     4      5       6      7
8       9
X_ECM1 = [ones(n_ecm1,1), St_1, ZEC1, DST_1, DST_2, DST_3, DDGT_1,
DDGT_2, DDGT_3];

X_ECM11 = X_ECM1(5:end,:);
Y_ECM1 =  Sur_GDP(5:end);

[beta_ECM1,Sigma_ECM1] = mvregress(Y_ECM1, X_ECM11);
beta_ECM1;
V_betaECM = Sigma_ECM1*inv(X_ECM11'*X_ECM11);
% GRANGER CAUSALITY TEST on the following parameters:
c = [0, 0, 0, 0];
H = [0, 0, 1, 0, 0, 0, 0, 0, 0;
     0, 0, 0, 0, 0, 0, 1, 0, 0;
     0, 0, 0, 0, 0, 0, 0, 1, 0;
     0, 0, 0, 0, 0, 0, 0, 0, 1];

p_value_granger_s = linhyptest(beta_ECM1,V_betaECM,c,H,9);
%p_value > 0.05
%% y=debt
yec2 = Debt_GDP(2:end);
Xec2 = [Sur_GDP(1:end-1)];
Bec2 = (Xec2'*Xec2)\(Xec2'*yec2);

Zec2 = yec2-Xec2*Bec2; %ECM2

Dt_1=[NA; Debt_GDP(1:end-1)];
ZEC2= [NA; NA; Zec2(1:end-1)];

DDT_1 =[NA; NA; Delta_DGDP(1:end-1)];
DDT_2 =[NA; NA; NA; Delta_DGDP(1:end-2)];
DDT_3 =[NA; NA; NA; NA; Delta_DGDP(1:end-3)];
```

```
DSGT_1 =[NA; NA; Delta_SGDP(1:end-1)];
DSGT_2 =[NA; NA; NA; Delta_SGDP(1:end-2)];
DSGT_3 =[NA; NA; NA; NA; Delta_SGDP(1:end-3)];

n_ecm2 = size(Dt_1,1);
%              1          2     3     4      5     6      7      8
9
X_ECM2=[ones(n_ecm2,1), Dt_1, ZEC2, DDT_1, DDT_2, DDT_3, DSGT_1,
DSGT_2, DSGT_3];

X_ECM22 = X_ECM2(5:end,:);
Y_ECM2 = Debt_GDP(5:end);

[beta_ECM2, Sigma_ECM2] = mvregress(Y_ECM2, X_ECM22);
beta_ECM2;
V_betaECM2 = Sigma_ECM2*inv(X_ECM22'*X_ECM22);
% parameters
c1 = [0, 0, 0, 0];
H1 = [0, 0, 1, 0, 0, 0, 0, 0, 0;
      0, 0, 0, 0, 0, 0, 1, 0, 0;
      0, 0, 0, 0, 0, 0, 0, 1, 0;
      0, 0, 0, 0, 0, 0, 0, 0, 1];

 p_value_gr_d = linhyptest(beta_ECM2,V_betaECM2,c1,H1,9);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%
% VAR(1) MODEL WITH ONE LAG
Mdl = varm(2,1)
EstMdl = estimate(Mdl,[Sur_GDP, Debt_GDP]);
summarize(EstMdl)

% VAR(2) MODEL WITH TWO LAGS
Md2 = varm(2,2);
EstMd2 = estimate(Md2,[Sur_GDP, Debt_GDP]);
summarize(EstMd2)

% VAR(3) MODEL WITH TWO LAGS
Md3 = varm(2,3);
EstMd3 = estimate(Md3,[Sur_GDP, Debt_GDP]);
summarize(EstMd3)

% VAR(4) MODEL WITH TWO LAGS
Md4 = varm(2,4);
EstMd4 = estimate(Md4,[Sur_GDP, Debt_GDP]);
summarize(EstMd4)
% BIC1 = 456.236
% BIC2 = 455.396
% BIC3 = 462.981
% BIC4= 470.458
% VAR(2) - IRF
summarize(EstMd2)
Response1 = irf(EstMd2)
% 10 YEAR HOTIZON: 10
[Response,Lower,Upper] = irf(EstMd4,'NumObs',10)

% IRF OF DEBT_GDP ON SURPLUSSHOCK:
irfshock1resp2 = Response(:,1,2);
```

```
IRFCIShock1Resp2 = [Lower(:,1,2) Upper(:,1,2)];

figure(3);
h1 = plot(0:9,irfshock1resp2);
hold on
h2 = plot(0:9,IRFCIShock1Resp2,'r--');
legend([h1 h2(1)],["IRF" "95% Confidence Interval"])
set(gca,'FontSize',25)
xlabel("Time Index");
ylabel("Response");
title("IRF of D-GDP When Sur-GDP Is Shocked");
grid on
hold off
```

UK

```
clear; clc;
format long g
%% UK 1999:Q1 - 2020:Q3
%%%%%%%%%%%%%%%%%%%%%%%%%%%

UK_data = readtable('UK_Q.xlsx');
Sur_GDP = table2array(UK_data(:,2));
Debt_GDP = table2array(UK_data(:,3));
T = (1999.0:0.25:2020.50)';
t = length(T)
%% PLOT
figure(1)
subplot(1,2,1); hold on;  box on ;
               plot(T, Sur_GDP, 'LineWidth',1,'Color','blue');
               plot(T, mean(Sur_GDP)*ones(t,1),'-
k','LineWidth',1);
               xlim([T(1)-1 T(end)+1]); ylim([-34 34]);
               title('Surplus (%GDP)');
               legend('Sur (%GDP)')

               set(gca,'FontSize',25)
               hold off; box off;
subplot(1,2,2); hold on;  box on ;
               plot(T, Debt_GDP, 'LineWidth',1,'Color','blue');
               plot(T, mean(Debt_GDP)*ones(t,1),'-
k','LineWidth',1);
               xlim([T(1)-1 T(end)+1]);  ylim([5 102]);
               title('Debt (%GDP)');
               legend('Debt %GDP)')

               set(gca,'FontSize',25)
               hold off; box off;
%% UNIT ROOT TESTS:
   %% 1. PHILLIPS-PERRON TEST OF UNIT ROOT:
   % INPUT: the time series under investigation
% Sur_GDP test
[PP,pValue_0,stat_0,cValue_0,reg_0] = pptest(Sur_GDP)
[PP_ts,pValue_1,stat_1,cValue_1,reg_1] =
pptest(Sur_GDP,'model','TS')
[PP_ard,pValue_2,stat_2,cValue_2,reg_2] =
pptest(Sur_GDP,'model','ARD')
% We want h=1 for the first difference:
Delta_SGDP = diff(Sur_GDP)
[PPD1_d0,pValue_d0,stat_d0,cValue_d0,reg_d0] = pptest(Delta_SGDP);
```

```
[PPD1_ts,pValue_d1,stat_ts,cValue_d1,reg_d1] =
pptest(Delta_SGDP,'model','TS');
[PPD1_ard,pValue_d2,stat_ard,cValue_d2,reg_d2] =
pptest(Delta_SGDP,'model','ARD');
[PPD1_lags,pValue_d3,stat_lags,cValue_d3,reg_d3] =
pptest(Delta_SGDP,'model','TS','lags',0:2);
%% Debt_GDP test
[h,pValue_4,stat_4,cValue_4,reg_4] = pptest(Debt_GDP)
[PP_ts_debt,pValue_5,stat_5,cValue_5,reg_5] =
pptest(Debt_GDP,'model','TS')
[PP_ard_debt,pValue_6,stat_6,cValue_6,reg_6] =
pptest(Debt_GDP,'model','ARD')
[PP_lag_debt,pValue_7,stat_7,cValue_7,reg_7] =
pptest(Debt_GDP,'model','TS','lags',0:2)
% We want h=1 for the first difference:
Delta_DGDP = diff(Debt_GDP)
[Delta_DGDP_ts,pValue_d4,stat_d4,cValue_d4,reg_d4] =
pptest(Delta_DGDP,'model','TS');
[Delta_DGDP_ard,pValue_d5,stat_d5,cValue_d5,reg_d5] =
pptest(Delta_DGDP,'model','ARD');
[Delta_DGDP_lag,pValue_d6,stat_d6,cValue_d6,reg_d6] =
pptest(Delta_DGDP,'model','TS','lags',0:2);
%% 2.DICKEY-FULLER TEST STATISTICS:
  % 1. Simple DF:
  s = adftest(Sur_GDP)

  d = adftest(Debt_GDP)

  % 2.Test for a unit root against a trend-stationary alternative,
augmenting the model with 0, 1, and 2 lagged difference terms.
  s_TS = adftest(Sur_GDP,'model','TS','lags',0:2)

  d_TS = adftest(Debt_GDP, 'model','TS','lags',0:2)
  % 3. Test for a unit root using three different choices for the
number of lagged difference terms. Return the regression
statistics for each alternative model.
  [h,~,~,~,reg] = adftest(Sur_GDP,'model','TS','lags',0:2);
  [h1,~,~,~,reg1] = adftest(Debt_GDP,'model','TS','lags',0:2);
  % The output shows which terms are included in the three
alternative models.
% The first model has no added difference terms, the second model
has one difference term (b1),
% and the third model has two difference terms (b1 and b2).
reg.names;
reg.BIC;
reg1.names;
reg1.BIC;
% Repeat the analysis with the differences of the variables to
verify they
% are I(0)
 % 1. Simple DF:
  s_delta = adftest(Delta_SGDP)

  d_delta = adftest(Delta_DGDP)
% 2.Test for a unit root against a trend-stationary alternative,
augmenting the model with 0, 1, and 2 lagged difference terms.
 s_TS_delta = adftest(Delta_SGDP,'model','TS','lags',0:2);

  d_TS_delta = adftest(Delta_DGDP, 'model','TS','lags',0:2);
```

```
% 3. Test for a unit root using three different choices for the
number of lagged difference terms. Return the regression
statistics for each alternative model.
[h_d,~,~,~,reg_d] = adftest(Delta_SGDP,'model','TS','lags',0:2);
[h1_d,~,~,~,reg1_d] = adftest(Delta_DGDP,'model','TS','lags',0:2);
%% DOLS: DYNAMIC OLS AS IN STOCK AND WATSON (1993).

%% 1. FIRST OLS REGRESSION BETWEEN I(1) SERIES:
y = Sur_GDP(2:end);
X = [ones(t-1,1), Debt_GDP(1:end-1)];
B = (X'*X)\(X'*y)

e = y - X*B;
RSS = e'*e;
sigma_2 = RSS/(t-3)

[beta,Sigma] = mvregress(X, y)
%% 2. DYNAMIC OLS:
q = round(t^(1/3));

NA = NaN
%% DEFINE THE REGRESSORS:
D_b_0 = [NA; Debt_GDP(2:end)-Debt_GDP(1:end-1)];     % 87 x 1

D_b_1 = [NA; D_b_0(1:end-1)];                          % 87 x 1
D_b_2 = [NA; NA; D_b_0(1:end-2)];                     % 87 x 1
D_b_3 = [NA; NA; NA; D_b_0(1:end-3)];                % 87 x 1
D_b_4 = [NA; NA; NA; NA; D_b_0(1:end-4)];              % 87 x 1
D_b_5 = [NA; NA; NA; NA; NA; D_b_0(1:end-5)];      % 87 x 1

D_b_p1 = [D_b_0(2:end); NA];                          % 87 x 1
D_b_p2 = [D_b_0(3:end); NA; NA];                 % 87 x 1
D_b_p3 = [D_b_0(4:end); NA; NA; NA];                % 87 x 1

b_t_1 = [NA; Debt_GDP(1:end-1)];
N = length(D_b_0); % N=88

XD = [ones(N,1), b_t_1, D_b_0, D_b_1, D_b_2, D_b_3, D_b_4, D_b_5,
D_b_p1, D_b_p2, D_b_p3];

%% CUT THE MATRICES FOR THE CORRECT SIZE:
X_d = XD(7:84,:);
Y_d= y(7:84);

k = size(X_d, 2);
beta_hat = zeros(k,1);

beta_hat = (X_d'*X_d)\(X_d'*Y_d);
e_d = Y_d - X_d*beta_hat;
[beta_d,Sigma_d] = mvregress(X_d, Y_d);
%% t test
n=size(X_d,1);
V_beta=Sigma_d*inv(X_d'*X_d);
% compute the observed statistic
t_obs=(beta_d(2)-0)/sqrt(V_beta(2,2));
% Define statistic test
t_teo=tinv(0.95,n-11);
% if t_obs>t_-teo I reject H_0
```

```
%% ENGLE-GRANGER COINTEGRATION TEST.
figure(2)
subplot(1,2,1)
plot(e)
subplot(1,2,2)
plot(e_d)
% 1. FIRST METHOD. SIMPLE DICKEY-FULLER test on residuals
C01 = adftest(e)
CO2 = adftest(e_d)

% 2.SECOND METHOD:
[h,pValue,stat,cValue_en,reg1,reg2] = egcitest([y,X])

[h_delta,pValue,stat,cValue_en_d,reg1,reg2] = egcitest([Y_d, X_d])
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%
% TEST DI GRANGER CAUSALITY:

% 1. MODEL IN ECM.
   % FIRST EQUATION. SURPLUS_G VS DEBT_G
   yec1 = Sur_GDP(2:end);
   Xec1 = [Debt_GDP(1:end-1)];
   Bec1 = (Xec1'*Xec1)\(Xec1'*yec1);

   Zec1 = yec1 - Xec1*Bec1;  % ECM1

   St_1 = [NA; Sur_GDP(1:end-1)];                       % 87 OSS
   ZEC1 = [NA; NA; Zec1(1:end-1)];                      % 87 OSS

   DST_1 =[NA; NA; Delta_SGDP(1:end-1)];
   DST_2 =[NA; NA; NA; Delta_SGDP(1:end-2)];
   DST_3 =[NA; NA; NA; NA; Delta_SGDP(1:end-3)];

   DDGT_1 =[NA; NA; Delta_DGDP(1:end-1)];
   DDGT_2 =[NA; NA; NA; Delta_DGDP(1:end-2)];
   DDGT_3 =[NA; NA; NA; NA; Delta_DGDP(1:end-3)];

n_ecm1 = size(St_1,1);
%                  1        2      3     4      5       6      7
8       9
X_ECM1 = [ones(n_ecm1,1), St_1, ZEC1, DST_1, DST_2, DST_3, DDGT_1,
DDGT_2, DDGT_3];

X_ECM11 = X_ECM1(5:end,:);
Y_ECM1 =  Sur_GDP(5:end);

[beta_ECM1,Sigma_ECM1] = mvregress(Y_ECM1, X_ECM11);
beta_ECM1;
V_betaECM = Sigma_ECM1*inv(X_ECM11'*X_ECM11);
% GRANGER CAUSALITY TEST ON:
c = [0, 0, 0, 0];
H = [0, 0, 1, 0, 0, 0, 0, 0, 0;
     0, 0, 0, 0, 0, 0, 1, 0, 0;
     0, 0, 0, 0, 0, 0, 0, 1, 0;
     0, 0, 0, 0, 0, 0, 0, 0, 1];

p_value_granger_s = linhyptest(beta_ECM1,V_betaECM,c,H,9);
% p_value > 0.05
```

```
% y=debt
yec2 = Debt_GDP(2:end);
Xec2 = [Sur_GDP(1:end-1)];
Bec2 = (Xec2'*Xec2)\(Xec2'*yec2);

Zec2 = yec2-Xec2*Bec2; %ECM2

Dt_1=[NA; Debt_GDP(1:end-1)];
ZEC2= [NA; NA; Zec2(1:end-1)];

DDT_1 =[NA; NA; Delta_DGDP(1:end-1)];
DDT_2 =[NA; NA; NA; Delta_DGDP(1:end-2)];
DDT_3 =[NA; NA; NA; NA; Delta_DGDP(1:end-3)];

DSGT_1 =[NA; NA; Delta_SGDP(1:end-1)];
DSGT_2 =[NA; NA; NA; Delta_SGDP(1:end-2)];
DSGT_3 =[NA; NA; NA; NA; Delta_SGDP(1:end-3)];

n_ecm2 = size(Dt_1,1);
%               1          2      3      4       5      6       7       8
9
X_ECM2=[ones(n_ecm2,1), Dt_1, ZEC2, DDT_1, DDT_2, DDT_3, DSGT_1,
DSGT_2, DSGT_3];

X_ECM22 = X_ECM2(5:end,:);
Y_ECM2 = Debt_GDP(5:end);

[beta_ECM2, Sigma_ECM2] = mvregress(Y_ECM2, X_ECM22);
beta_ECM2;
V_betaECM2 = Sigma_ECM2*inv(X_ECM22'*X_ECM22);
% parameters
c1 = [0, 0, 0, 0];
H1 = [0, 0, 1, 0, 0, 0, 0, 0, 0;
      0, 0, 0, 0, 0, 0, 1, 0, 0;
      0, 0, 0, 0, 0, 0, 0, 1, 0;
      0, 0, 0, 0, 0, 0, 0, 0, 1];

 p_value_gr_d = linhyptest(beta_ECM2,V_betaECM2,c1,H1,9);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%
% VAR(1) MODEL WITH ONE LAG
Mdl = varm(2,1)
EstMdl = estimate(Mdl,[Sur_GDP, Debt_GDP]);
summarize(EstMdl)

% VAR(2) MODEL WITH TWO LAGS
Md2 = varm(2,2);
EstMd2 = estimate(Md2,[Sur_GDP, Debt_GDP]);
summarize(EstMd2)

% VAR(3) MODEL WITH TWO LAGS
Md3 = varm(2,3);
EstMd3 = estimate(Md3,[Sur_GDP, Debt_GDP]);
summarize(EstMd3)

% VAR(4) MODEL WITH TWO LAGS
Md4 = varm(2,4);
```

```
EstMd4 = estimate(Md4,[Sur_GDP, Debt_GDP]);
summarize(EstMd4)
% BIC1 = 722.244
% BIC2 = 718.788
% BIC3 = 715.734
% BIC4 = 694.234
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% VAR(4) - IRF
summarize(EstMd4)
Response1 = irf(EstMd4)
% 10 YEAR HOTIZON: 4x10=40
[Response,Lower,Upper] = irf(EstMd4,'NumObs',40)

% IRF OF DEBT_GDP ON SURPLUS SHOCK:
irfshock1resp2 = Response(:,1,2);
IRFCIShock1Resp2 = [Lower(:,1,2) Upper(:,1,2)];

figure(3);
h1 = plot(0:39,irfshock1resp2);
hold on
h2 = plot(0:39,IRFCIShock1Resp2,'r--');
legend([h1 h2(1)],["IRF" "95% Confidence Interval"])
set(gca,'FontSize',25)
xlabel("Time Index");
ylabel("Response");
title("IRF of D-GDP When Sur-GDP Is Shocked");
grid on
hold off
```

Japan

```
clear; clc;
format long g
%% JAPAN 1980 - 2020
%%%%%%%%%%%%%%%%%%%%%%%%%%%

JAP_data = readtable('JAPAN_A.xlsx');
Sur_GDP = table2array(JAP_data(:,2));
Debt_GDP = table2array(JAP_data(:,3));
T = (1980:1:2020)';
t = length(T)
%% PLOT
figure(1)
subplot(1,2,1); hold on;  box on ;
               plot(T, Sur_GDP, 'LineWidth',1,'Color','blue');
               plot(T, mean(Sur_GDP)*ones(t,1),'-
k','LineWidth',1);
               xlim([T(1)-1 T(end)+1]); ylim([-14 14]);
               title('Surplus (%GDP)');
               legend('Sur (%GDP)')

               set(gca,'FontSize',25)
               hold off; box off;
subplot(1,2,2); hold on;  box on ;
               plot(T, Debt_GDP, 'LineWidth',1,'Color','blue');
               plot(T, mean(Debt_GDP)*ones(t,1),'-
k','LineWidth',1);
               xlim([T(1)-1 T(end)+1]);  ylim([46 258]);
               title('Debt (%GDP)');
               legend('Debt (%GDP)')
```

```
                set(gca,'FontSize',25)
                hold off; box off;
%% UNIT ROOT TESTS:
   %% 1. PHILLIPS-PERRON TEST OF UNIT ROOT:
   % INPUT: the time series under investigation
% Sur_GDP test
[PP,pValue_0,stat_0,cValue_0,reg_0] = pptest(Sur_GDP)
[PP_ts,pValue_1,stat_1,cValue_1,reg_1] =
pptest(Sur_GDP,'model','TS')
[PP_ard,pValue_2,stat_2,cValue_2,reg_2] =
pptest(Sur_GDP,'model','ARD')
[PP_lags,pValue_3,stat_3,cValue_3,reg_3] =
pptest(Sur_GDP,'model','TS','lags',0:2)
% We want h=1 for the first difference:
Delta_SGDP = diff(Sur_GDP)
[PPD1_d0,pValue_d0,stat_d0,cValue_d0,reg_d0] = pptest(Delta_SGDP);
[PPD1_ts,pValue_d1,stat_ts,cValue_d1,reg_d1] =
pptest(Delta_SGDP,'model','TS');
[PPD1_ard,pValue_d2,stat_ard,cValue_d2,reg_d2] =
pptest(Delta_SGDP,'model','ARD');
[PPD1_lags,pValue_d3,stat_lags,cValue_d3,reg_d3] =
pptest(Delta_SGDP,'model','TS','lags',0:2);
%% Debt_GDP test
[h,pValue_4,stat_4,cValue_4,reg_4] = pptest(Debt_GDP)
[PP_ts_debt,pValue_5,stat_5,cValue_5,reg_5] =
pptest(Debt_GDP,'model','TS')
[PP_ard_debt,pValue_6,stat_6,cValue_6,reg_6] =
pptest(Debt_GDP,'model','ARD')
[PP_lag_debt,pValue_7,stat_7,cValue_7,reg_7] =
pptest(Debt_GDP,'model','TS','lags',0:2)
% We want h=1 for the first difference:
Delta_DGDP = diff(Debt_GDP)
[Delta_DGDP_ts,pValue_d4,stat_d4,cValue_d4,reg_d4] =
pptest(Delta_DGDP,'model','TS');
[Delta_DGDP_ard,pValue_d5,stat_d5,cValue_d5,reg_d5] =
pptest(Delta_DGDP,'model','ARD');
[Delta_DGDP_lag,pValue_d6,stat_d6,cValue_d6,reg_d6] =
pptest(Delta_DGDP,'model','TS','lags',0:2);
%% 2.DICKEY-FULLER TEST STATISTICS:
  % 1. Simple DF:
  s = adftest(Sur_GDP)

  d = adftest(Debt_GDP)

  % 2.Test for a unit root against a trend-stationary alternative,
augmenting the model with 0, 1, and 2 lagged difference terms.
  s_TS = adftest(Sur_GDP,'model','TS','lags',0:2)

  d_TS = adftest(Debt_GDP, 'model','TS','lags',0:2)
  % 3. Test for a unit root using three different choices for the
number of lagged difference terms. Return the regression
statistics for each alternative model.
  [h,~,~,~,reg] = adftest(Sur_GDP,'model','TS','lags',0:2);
  [h1,~,~,~,reg1] = adftest(Debt_GDP,'model','TS','lags',0:2);
  % The output shows which terms are included in the three
alternative models.
% The first model has no added difference terms, the second model
has one difference term (b1),
% and the third model has two difference terms (b1 and b2).
```

```
reg.names;
reg.BIC;
reg1.names;
reg1.BIC;
% Repeat the analysis with the differences of the variables to
verify they
% are I(0)
 % 1. Simple DF:
  s_delta = adftest(Delta_SGDP)

  d_delta = adftest(Delta_DGDP)
% 2.Test for a unit root against a trend-stationary alternative,
augmenting the model with 0, 1, and 2 lagged difference terms.
 s_TS_delta = adftest(Delta_SGDP,'model','TS','lags',0:2);

  d_TS_delta = adftest(Delta_DGDP, 'model','TS','lags',0:2);
% 3. Test for a unit root using three different choices for the
number of lagged difference terms. Return the regression
statistics for each alternative model.
[h_d,~,~,~,reg_d] = adftest(Delta_SGDP,'model','TS','lags',0:2);
[h1_d,~,~,~,reg1_d] = adftest(Delta_DGDP,'model','TS','lags',0:2);
%% DOLS: DYNAMIC OLS AS IN STOCK AND WATSON (1993).

%% 1. FIRST OLS REGRESSION BETWEEN I(1) SERIES:
y = Sur_GDP(2:end);
X = [ones(t-1,1), Debt_GDP(1:end-1)];
B = (X'*X)\(X'*y)

e = y - X*B;
RSS = e'*e;
sigma_2 = RSS/(t-3)

[beta,Sigma] = mvregress(X, y)
%% 2. DYNAMIC OLS:
q = round(t^(1/3));

NA = NaN
%% DEFINE THE REGRESSORS:
D_b_0 = [NA; Debt_GDP(2:end)-Debt_GDP(1:end-1)];     % 40 x 1

D_b_1 = [NA; D_b_0(1:end-1)];                         % 40 x 1
D_b_2 = [NA; NA; D_b_0(1:end-2)];                    % 40 x 1
D_b_3 = [NA; NA; NA; D_b_0(1:end-3)];                % 41 x 1
D_b_4 = [NA; NA; NA; NA; D_b_0(1:end-4)];           % 40 x 1


D_b_p1 = [D_b_0(2:end); NA];                        % 40 x 1
D_b_p2 = [D_b_0(3:end); NA; NA];                    % 40 x 1

b_t_1 = [NA; Debt_GDP(1:end-1)];
N = length(D_b_0); % N=40

XD = [ones(N,1), b_t_1, D_b_0, D_b_1, D_b_2, D_b_3, D_b_4, D_b_p1,
D_b_p2];

%% CUT THE MATRICES FOR THE CORRECT SIZE:
X_d = XD(6:39,:);
Y_d = y(6:39);
```

```
k = size(X_d, 2);
beta_hat = zeros(k,1);

beta_hat = (X_d'*X_d)\(X_d'*Y_d);
e_d = Y_d - X_d*beta_hat;
[beta_d,Sigma_d] = mvregress(X_d, Y_d);
%% t test
n=size(X_d,1);
V_beta=Sigma_d*inv(X_d'*X_d);
% compute the observed statistic
t_obs=(beta_d(2)-0)/sqrt(V_beta(2,2));
% Define statistic test
t_teo=tinv(0.95,n-9);
% if t_obs>t_-teo I reject H_0
%% ENGLE-GRANGER COINTEGRATION TEST.
figure(2)
subplot(1,2,1)
plot(e)
subplot(1,2,2)
plot(e_d)
% 1. FIRST METHOD. SIMPLE DICKEY-FULLER test on residuals
C01 = adftest(e)
CO2 = adftest(e_d)
% 2. SECOND METHOD:
[h,pValue,stat,cValue_en,reg1,reg2] = egcitest([y,X])

[h_delta,pValue,stat,cValue_en_d,reg1,reg2] = egcitest([Y_d, X_d])

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%
% TEST DI GRANGER CAUSALITY:

% 1. DOBBIAMO RISCRIVERE IL MODELLO IN ECM.
   % PRIMA EQUAZIONE. SURPLUS_G VS DEBT_G
   yec1 = Sur_GDP(2:end);
   Xec1 = [Debt_GDP(1:end-1)];
   Bec1 = (Xec1'*Xec1)\(Xec1'*yec1);

   Zec1 = yec1 - Xec1*Bec1;  % ECM1

   St_1 = [NA; Sur_GDP(1:end-1)];                         % 41 OSS
   ZEC1 = [NA; NA; Zec1(1:end-1)];                        % 41 OSS

   DST_1 =[NA; NA; Delta_SGDP(1:end-1)];
   DST_2 =[NA; NA; NA; Delta_SGDP(1:end-2)];
   DST_3 =[NA; NA; NA; NA; Delta_SGDP(1:end-3)];

   DDGT_1 =[NA; NA; Delta_DGDP(1:end-1)];
   DDGT_2 =[NA; NA; NA; Delta_DGDP(1:end-2)];
   DDGT_3 =[NA; NA; NA; NA; Delta_DGDP(1:end-3)];

n_ecm1 = size(St_1,1);
%                  1        2      3     4      5       6      7
8       9
X_ECM1 = [ones(n_ecm1,1), St_1, ZEC1, DST_1, DST_2, DST_3, DDGT_1,
DDGT_2, DDGT_3];
```

```
X_ECM11 = X_ECM1(5:end,:);
Y_ECM1 =  Sur_GDP(5:end);

[beta_ECM1,Sigma_ECM1] = mvregress(Y_ECM1, X_ECM11);
beta_ECM1;
V_betaECM = Sigma_ECM1*inv(X_ECM11'*X_ECM11);
% GRANGER CAUSALITY TEST ON:
c = [0, 0, 0, 0];
H = [0, 0, 1, 0, 0, 0, 0, 0, 0;
     0, 0, 0, 0, 0, 0, 1, 0, 0;
     0, 0, 0, 0, 0, 0, 0, 1, 0;
     0, 0, 0, 0, 0, 0, 0, 0, 1];

p_value_granger_s = linhyptest(beta_ECM1,V_betaECM,c,H,9);
%p_value > 0.05
% y=debt
yec2 = Debt_GDP(2:end);
Xec2 = [Sur_GDP(1:end-1)];
Bec2 = (Xec2'*Xec2)\(Xec2'*yec2);

Zec2 = yec2-Xec2*Bec2; %ECM2

Dt_1=[NA; Debt_GDP(1:end-1)];
ZEC2= [NA; NA; Zec2(1:end-1)];

DDT_1 =[NA; NA; Delta_DGDP(1:end-1)];
DDT_2 =[NA; NA; NA; Delta_DGDP(1:end-2)];
DDT_3 =[NA; NA; NA; NA; Delta_DGDP(1:end-3)];

DSGT_1 =[NA; NA; Delta_SGDP(1:end-1)];
DSGT_2 =[NA; NA; NA; Delta_SGDP(1:end-2)];
DSGT_3 =[NA; NA; NA; NA; Delta_SGDP(1:end-3)];

n_ecm2 = size(Dt_1,1);
%               1          2     3     4      5     6      7      8
9
X_ECM2=[ones(n_ecm2,1), Dt_1, ZEC2, DDT_1, DDT_2, DDT_3, DSGT_1,
DSGT_2, DSGT_3];

X_ECM22 = X_ECM2(5:end,:);
Y_ECM2 = Debt_GDP(5:end);

[beta_ECM2, Sigma_ECM2] = mvregress(Y_ECM2, X_ECM22);
beta_ECM2;
V_betaECM2 = Sigma_ECM2*inv(X_ECM22'*X_ECM22);
% parameters
c1 = [0, 0, 0, 0];
H1 = [0, 0, 1, 0, 0, 0, 0, 0, 0;
      0, 0, 0, 0, 0, 0, 1, 0, 0;
      0, 0, 0, 0, 0, 0, 0, 1, 0;
      0, 0, 0, 0, 0, 0, 0, 0, 1];

 p_value_gr_d = linhyptest(beta_ECM2,V_betaECM2,c1,H1,9);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%
% VAR(1) MODEL WITH ONE LAG
Mdl = varm(2,1)
```

```
EstMdl = estimate(Mdl,[Sur_GDP, Debt_GDP]);
summarize(EstMdl)

% VAR(2) MODEL WITH TWO LAGS
Md2 = varm(2,2);
EstMd2 = estimate(Md2,[Sur_GDP, Debt_GDP]);
summarize(EstMd2)

% VAR(3) MODEL WITH TWO LAGS
Md3 = varm(2,3);
EstMd3 = estimate(Md3,[Sur_GDP, Debt_GDP]);
summarize(EstMd3)

% VAR(4) MODEL WITH TWO LAGS
Md4 = varm(2,4);
EstMd4 = estimate(Md4,[Sur_GDP, Debt_GDP]);
summarize(EstMd4)
% BIC1 = 401.979
% BIC2 = 387.733
% BIC3 = 391.059
% BIC4= 395.279
% VAR(2) - IRF
summarize(EstMd2)
Response1 = irf(EstMd2)
% 10 YEAR HOTIZON: 10
[Response,Lower,Upper] = irf(EstMd4,'NumObs',10)

% IRF OF DEBT_GDP ON SURPLUS SHOCK:
irfshock1resp2 = Response(:,1,2);
IRFCIShock1Resp2 = [Lower(:,1,2) Upper(:,1,2)];

figure(3);
h1 = plot(0:9,irfshock1resp2);
hold on
h2 = plot(0:9,IRFCIShock1Resp2,'r--');
legend([h1 h2(1)],["IRF" "95% Confidence Interval"])
set(gca,'FontSize',25)
xlabel("Time Index");
ylabel("Response");
title("IRF of D-GDP When Sur-GDP Is Shocked");
grid on
hold off
```

Appendix B

Summary

Introduction

When the COVID-19 pandemic started spreading around the world in 2020, no one could imagine the extent of the consequences on the economy. Immediately, the health crisis turned into a real economy crisis. Due to restrictions, consumption and all small businesses suffered a setback: retailing, tourism, catering industry, etc., recorded unprecedented losses. Governments were forced to take extraordinary measures to face this crisis and support financially families and businesses. Moreover, in addition to these expenses, other significant costs were born to purchase vaccines, organize their distribution, create infrastructures for their administrations, and to safely reopen public spaces such as schools.

United States had to tolerate many expenses as all the other countries but what has risen the interest of many economists is the extent of the measures laid down by the American Rescue Plan Act of 2021, proposed by the newly elected President Joe Biden. His unprecedented fiscal stimulus opened a debate among notorious economists over the concern of inflation risk and the validity of the Fiscal Theory of the Price Level (FTPL). This theory is relatively modern and it investigates the dynamics of the inflation related to fiscal policy. This approach differs from other theories, such as the monetarist one: while the latter relates changes in price level to changes in the quantity of money, fiscal theory claims that prices adjust in order to make the nominal debt of the government equal to the present value of primary surpluses. The purpose of this thesis is the analysis of the Fiscal Theory of the Price Level through both a theoretical and an empirical approach. Chapter 1 in the first part focuses on the debate about Biden's fiscal stimulus and the consequences on inflation, while in the second part it analyses the major contributions to the Fiscal Theory of the Price Level. Chapter 2 goes through the main critiques addressed to the Fiscal Theory of the Price Level and the new developments of this theory. Chapter 3 is an econometric analysis to assess the validity of the assumptions of the FTPL based on data over the G7 countries.

Chapter 1

The first chapter focuses on the analysis of Fiscal Theory, with a particular attention to its precursors and supporters. Before investigating the theoretical aspects of this theory, I give a general idea of the American Rescue Plan Act, outlying the most important measures and their extent. Specifically, I highlight three main points: the organization of a vaccination program at a national level, the economic aids to working families, and the support to the communities which were more affected by the pandemic. With the announcement of these special measures, some economists launched a debate over the concern of inflation risk. Lawrence Summers (supported by Jean-Pierre Landau) pointed out the fact that the measures taken by Biden would widen the output gap (i.e., the difference between the effective output and the potential output), and, consequently, they would lead to an overheated economy characterized by inflation. Olivier Blanchard agreed with this view and he also proposed two perspectives for the future: the first one is a change in expectations over inflation while the second is a tighter monetary policy to address inflation. The Nobel prize Paul Krugman took a different opinion: according to him, as expectations over inflation are well-anchored, it is unlikely that they would change now. This view was shared also by Ricardo Reis.

After this general overview over the current situation and debates, I move on to the analysis of the developments of the Fiscal Theory of the Price Level (FTPL). It has its roots in the 80's and it is an alternative to the quantity theory of money which relates changes in price level to changes in the quantity of money in circulation. On the other hand, Fiscal theory claims that prices adjust in order to make the nominal debt of the government equal to the present value of primary surpluses.

Firstly, I consider the precursors of this theory, Sargent and Wallace. In (Sargent & Wallace, 1981) they studied the coordination between monetary and fiscal policy and, specifically, the situation where the fiscal authority moves first and the monetary one has to adapt to its decisions about the annual deficit or surplus. The authors found that if the D(t) sequence of deficits is too big for too long, then the central bank must generate some seigniorage revenue to pay the debt off. Moreover, since the central bank is free to decide when to print more money, the paper showed that a tighter monetary policy and less inflation now require a looser monetary policy and more inflation later.

After this analysis I pass to (Leeper, 1991) which was the first important contribution to the birth of the Fiscal Theory. He categorized equilibrium policies as representing ‘active’ or ‘passive’ behaviour and analysed the alternative scenarios in which the monetary authority and the fiscal one assumed one of the two behaviours. The scenario with an active fiscal authority which sets its variables exogenously letting the passive monetary authority adjust endogenously its money supply, laid the foundation for the subsequent studies on the Fiscal Theory of the Price Level.

There were several studies which followed Leeper’s contribution and in my analysis I focus on (Sims, 1994), (Woodford, 1995) and (Cochrane, 2021). Sims showed the importance of fiscal policy on the determination of price level and Woodford extended this analysis. Finally, in the last part of the first chapter I focus on (Cochrane, 2021): I consider both the simple and the intertemporal government budget constraints. Cochrane showed that the price level endogenously adjusts to the make the valuation equation of government real debt equal to the present value of expected future surpluses. Hence, Cochrane’s valuation equation is:

$$\frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j}$$

It looks like a valuation equation for stock prices. As stock prices are computed as the present value of future discounted dividends, in the same way surplus forecasts determine the price level.

Chapter 2

In the first part of the second chapter I go through the critiques addressed to the FTPL over the years. One example is (Buiter, 1999), which states that the main mistake of the proponents of this theory is stating that the intertemporal budget constraint is not a constraint that must be satisfied for all values of the endogenous variables. However, (Cochrane, 2021) responded to this critique specifying that Fiscal Theory is based on a ‘valuation’ equation, not a budget constraint. Another critique came from (Niepelt, 2004). This paper defined the FTPL ‘inconsistent’ given the fact that at time $t = 0$ it is not possible to uniquely define the price level. By the way, (Daniel, 2007) demonstrated that even if at time 0 price level cannot assume a unique value, the Fiscal Theory is still valid.

In the second part of the second chapter, I take into consideration two revisitations of the typical FTPL models. (Berentsen & Waller, 2018) described a model based on the further assumption that debt serves as collateral for secured lending in financial markets. Hence, its market value includes a liquidity premium which reflects not only the claim on the stream of future surpluses, but also its value for trading. This claim changes the typical price level determination of the Fiscal Theory because it suggests that price level dynamics may be affected by changes in the liquidity value of government debt. Consequently, investors also consider the marginal collateral value when evaluating the bonds. Hence, the model of Berentsen and Waller, differs from the 'classic' FTPL because:

- while in the Fiscal theory, a 'mispricing' of the debt would mean a violation of the budget constrain and an adjustment of the price level, in this model budget constraint is not violated. What it is important to stress is that there can be a "bubble" in the value of nominal government debt.
- In contrast to Fiscal Theory, inflation is due to the growth rate of bonds and not to changes in the present value of future surpluses.
- Inflation is "costless" in that it does not affect the allocation of bonds but only the nominal interest rate.

Last part of Chapter 2 analyses (Bassetto & Cui, 2017). This paper focused the attention on FTPL in a context of low interest rates. Specifically, it considered different causes of low interest rates, such as dynamic inefficiency, the liquidity premium of government debt, and its favorable risk profile. As far as the the case in which investors buy government bonds for precautionary reasons (even when interest rates are lower than the growth rate of the economy), Bassetto ad Cui found that in the tipical valuation equation of the FTPL, expected value of future taxes can be negative in all periods even if the present value must be positive. In the case of a dynamic inefficient economy (i.e., in an overlapping generations economy), they showed that even in a scenario of fixed nominal interest rate and fixed taxes, the price cannot be uniquely defined since a continuum of possible price levels emerges. This is clearly in contrast to the Fiscal Theory, and it demonstrates that in OLG economies with dynamic inefficiency and where government debt itself is similar to money, tight predictions on the inflationary consequences of lowering taxes are not possible. The last scenario analyzed by Bassetto and Cui is a dynamically efficient economy in which private assets yield a rate of return which is higher than the growth rate of the economy (which is normalized to 0). On the other hand, government debt pays a lower interest rate, but it has a special liquidity role since it allows certain transactions to be settled even when this would not be possible through private assets.

In the end, the authors confirmed the validity of the FTPL but they doubted the uniqueness of the equilibrium value of price level when interest rates are permanently lower than the growth rate of the economy.

Chapter 3

In the third chapter I try to establish if the empirical evidence would support the assumptions of the Fiscal Theory of the Price Level for the case of G7 countries. I chose this group of countries to compare the results among the largest advanced economies in the world. My analysis is mainly based on (Bajo-Rubio, Diaz-Roldan, & Esteve, 2009), (Canzoneri, Cumby, & Diba, 2001) and (Afonso & Tovar Jalles, 2012). The analysis consists in estimating the cointegration relationships between the primary surplus and the (lagged) level of debt, both as ratios to GDP:

$$\mathrm{s_t} = \alpha + \beta \mathrm{b_{t-1}} + \mathrm{v_t}$$

where $\mathrm{v_t}$ is an error term. The estimation of β allows us to understand if in the investigated country there is a monetary dominant regime or a fiscal dominant regime. In fact, when $\beta > 0$ it means that the present value of the government budget constraint is satisfied and, consequently, there is fiscal solvency. If instead $\beta \leq 0$, the present value of the government budget constraint is not satisfied, and this would mean that the government is not solvent. This event would coincide with the presence of a non-Ricardian or Fiscal dominant regime which allows fiscal policy to set primary balances and to follow an arbitrary budget process, not necessarily compatible with solvency (Afonso & Tovar Jalles, 2012). The problem arises when the estimated β is significantly greater than zero: this case would be compatible with both the monetary dominant regime and the fiscal dominant one. In fact, in a monetary dominant regime an increase in debt in period t would be followed by a larger primary surplus in the next period (i.e., $\Delta b_t \rightarrow \Delta s_{t+1}$, which implies an estimated $\beta > 0$). Moreover, in a fiscal dominant regime, a decrease in the expected primary surplus would imply an increase in price level which in turn would decrease the current debt ratio (i.e., $\nabla E_t s_{t+1} \rightarrow \nabla b_t$ which implies an estimated $\beta > 0$ also in this case).

Data used in the analysis are those on primary budget surplus and general government consolidated gross debt, both as percentage of GDP. The data source is ECB 'Statistical Data Warehouse' for the three European member states, U.K and U.S., 'Statistics Canada' for Canada, and the 'International Monetary Fund' for Japan. As far as primary surplus, I used the variable "Government primary deficit (-) or surplus (+) (as % of GDP)", also defined as "Net lending/net borrowing excluding interest payable". As far as debt, I used the variable "Government debt (consolidated) (as % of GDP)". This variable refers to the so called "Maastricht Debt" and it was defined in ESA 2010 as the total consolidated gross debt at face value in the following categories of government liabilities: currency and deposits, debt securities and loans[68].

The range of time of data differs across countries:

- except for Germany, for which data go from 2002, for France, Italy and U.K. I used quarterly data which go from 1999 to 2020;
- for Japan I use annual data from 1980 to 2020;
- for Canada I use annual data from 1981 to 2020;
- U.S. annual data go from 1960 to 2019[69]. In fact, data on 2020 primary surplus/deficit will not be available until autumn 2021.

I use quarterly data to have a longer dataset. However, for Japan, Canada and U.S. only annual data are available.

Hence, differently from (Bajo-Rubio, et al., 2009), which studied the period 1970 through 2005, I use more recent data which cover also the 2007-2008 global financial

[68] Source: "https://ec.europa.eu/eurostat/documents/3859598/5925693/KS-02-13-269-EN.PDF/44cd9d01-bc64-40e5-bd40-d17df0c69334"

[69] Data for 2020 are still not available

crisis, the European sovereign debt crisis, and the beginning of COVID pandemic in 2020 (apart from U.S.).

Firstly, I use two unit root tests to analyze stationarity, the Phillips-Perron test and the Augmented Dickey-Fuller test. These tests assess the presence of a unit root in the time series of primary surplus and debt, both as percentage of GDP. For the Phillips-Perron (PP) Test I run the basic PP test, and then other three model variants: the trend stationary (TS), the autoregressive with drift (ARD) and the trend stationary alternative with 0, 1, and 2 lags. For the Dickey-Fuller (DF) test I use the simple test and then the augmented one with 0, 1 and 2 lags. As far as Phillips-Perron test for the government debt, it accepts the null hypothesis for all the analyzed countries implying non-stationarity for all the time series and for all the variants of the test at 95% confidence level.

As far as the primary surplus, the PP test does not accept the null hypothesis for Canada and UK in all variants of the model.

On the other hand, the Dickey-Fuller test shows the same results of the PP test for the debt. As far as primary surplus, the DF does not accept the null hypothesis in all cases. In fact, Canada time series are $I(1)$ according to the simple DF test and the augmented DF test with two lags, while they are $I(0)$ according to the augmented DF test with 0 lags and 1 lag.

U.K. surplus is $I(0)$ according to the simple DF test and the the augmented DF test with lag 1; in the tests with one and two lags it is $I(1)$.

All in all, we can consider all time series non-stationary as confirmed by the unit root tests and by literature as well.

When time series are not stationary it is still possible to transform them in stationary ones through first order differencing, i.e., by taking Δy_t.

On MATLAB I compute these differentials (Delta Surplus and Delta Debt) and then I run again the Phillips-Perron test and the Dickey-Fuller test on these new time-series to confirm their stationarity.

Once established the order of integration of the series, I estimate the parameter β in the equation $\mathrm{s_t} = \alpha + \beta \mathrm{b_{t-1}} + \mathrm{v_t}$. I use the Dynamic Ordinary Least Squares method implemented by (Stock & Watson, 1993) which provides a robust correction to the possible presence of endogeneity in the explanatory variables, as well as of serial correlation in the error terms of the OLS estimation (Bajo-Rubio, et al., 2009). For this reason, I estimate the long-run dynamic equation which includes leads and lags of the first difference of the explanatory variable, i.e., of b_{t-1} :

$$s_t = \alpha + \beta b_{t-1} + \sum_{j=-q}^{q} \varphi_j \Delta b_{t-1-j} + v_t$$

where the index q is computed as $q = INT\left(T^{1/3}\right)$, i.e., the cubic root of the number of observations, rounded to an integer. Given my data, q is equal to 4 for time series with more observations, and equal to 3 for those with less observations. As far as OLS betas, they are mostly negative except for Germany and Canada.

DOLS betas are all negative apart from Germany, Italy, and Canada. However, only the beta of Germany is significantly different from 0 according to the T-Test at 5% level. Therefore, a preliminary conclusion could be that only in Germany fiscal policy would have been sustainable and a Ricardian or Monetary dominant regime would have prevailed. In the other countries, there is no evidence of fiscal sustainability, and a Fiscal dominant regime could be compatible. As opposed to (Bajo-Rubio, et al., 2009), the Engel-Granger test does not show cointegration. However, this result is consistent with the one found in (Afonso & Tovar Jalles, 2012), a working paper of the ECB of 2012 which assessed the sustainability of public finances in OECD countries over the period 1970-2010. The absence of cointegration would foster the hypothesis of fiscal policy unsustainability. After the cointegration test, I can run a Granger Causality test to assess if each of the two variables is useful for forecasting the other one.

Since the variables under consideration are not stationary, the test is run taking in consideration first or higher differences. The regression that must be tested is the one proposed by (Sims, 1972):

$$X_t = \alpha_0 + \delta_1 X_{t-1} + \gamma_1 (X_{t-1} - \beta Y_{t-1}) + \sum_{i=1}^{m} \alpha_{1i} \Delta X_{t-1} + \sum_{i=1}^{n} \alpha_{2i} \Delta Y_{t-1} + \varepsilon_t$$

where $Z_{t-1} = X_{t-1} - \beta Y_{t-1}$ stands for an error correction model.

To assess the direction of the causality, the test must be run using the primary surplus at time *t* (s_t) and the debt at time *t-1* (b_{t-1}) alternatively as dependent variables and it must include up to three lags of the first difference of each of these variables. From the Granger causality test it is clear that debt at time *t-1* does not have significant predictive power on primary surplus at time *t* for all countries under investigation. Moreover, the same result is obtained when the surplus is the independent variable except for Canada and USA, which have a p-value > 0,05. Finally, for a check of robustness, I compute the impulse-response functions of the debt-GDP ratio to innovations in the primary surplus-GDP ratio from an estimated VAR in these two variables following the approach applied in (Canzoneri, et al., 2001). All in all, debt shows a decreasing path followed by a period of stability. However, the impulse response function is not statistically significant for all countries.

Conclusion

In the end these results could be compatible with the Fiscal Theory of the Price Level: in fact, the primary surplus seems to be set exogenously by the governments, regardless of the level of the public debt. Consequently, the price level would adjust in order to satisfy the intertemporal budget constraint.

Nowadays, all the measures taken by governments to address the pandemic have widened primary deficits as never before with consequences on the price level.

Over the 2021 inflation has started increasing, and while for Japan an inversion of trend is expected, for Eurozone countries and USA this trend is not expected to stop. This could be a point in favor of the FTPL: in fact, given the increased deficits, the level of prices would rise to make Cochrane's valuation equation hold.

To conclude, fiscal policy seems to play an important role in achieving macroeconomic stability; shocks like the one we are living today put in trouble central banks which have to take a decision over the level of interest rates in the future.