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ASSET MANAGEMENT: THE INTRODUCTION OF CRYPTOCURRENCIES IN A MULTI-ASSET PORTFOLIO

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Introduction

During the Covid-19 pandemic crisis, the interest regarding cryptocurrencies has been subject to a relevant increment. Thanks also to social networks, many people begin to look at this alternative typology of currency like the new frontier of investments or as an easy new source of financing. In that context, the idea of structuring an analysis about cryptocurrencies for this master's degree thesis was the most appropriate one.

Here, the analysis has the aim of investigating the effect on the performance of a portfolio composed by traditional asset classes, after including the cryptocurrencies, as a new one. The main theoretical source of this analysis will be the Modern Portfolio theory established by Markowitz. In addition, the results obtained will be regressed in two typology of analysis with a market benchmark, in order to see their significance.

The first chapter is a sort of overview of the cryptocurrencies theme, and it will be divided in three paragraphs.

In the first one a short focus to the evolution of the cryptocurrencies market will be given, from the creation of the first Bitcoin back in 2008 to the current time with the boom after Covid-19 crisis. In addition, the main characteristics of this alternative typology of payment will also be explained.

The core of the second paragraph will be the discussion of whether considering cryptocurrencies as a real currency or as an asset class. Both classification have their own constrains. The fitting of cryptocurrencies will be analysed for each type of requirements, in order to see the best matching. Moreover, there will be also a section about the point of view about it of the academics and the main financial institutions.

The last paragraph of the first chapter is focused on the relationship between cryptocurrencies and European investors. This choice was made because the average European investor will be the main actor of this thesis.

Even the second chapter will be divided in three paragraphs, and it will be structured around the theoretical assumptions for the analysis.

The beginning of the first section will be about a brief description of the modern portfolio theory. After that, all the relevant literature regarding cryptocurrencies in the landscape of investment

analysis will be catalogued. The criterion chosen is time, so the first paper in the section is dated back in 2014, in order to see the evolution of the studies.

The second paragraph is the one that contains all the notional aspects of the thesis, that will be used in the estimation process of the results. The decision was to start from listing the descriptive statistics and explaining the three optimization problems, with the relative constraints: maximization of the expected return and Sharpe ratio, and minimization of the standard deviation. After that, it will be explained the typologies of Value-at-Risk approach used, the historical and the parametric. The last part of the paragraph concerns about the regression analysis, which will give the results of the analysis.

The third part of this chapter will be about the composition of the reference portfolio. There will be a part about the reasons for the choices of the asset classes, and a second one with a brief description of each of the twelve indexes of the asset classes.

In the third chapter, all the results obtained in the analysis will be discussed and commented in three paragraphs.

The starting point is the analysis of the descriptive data table and the correlation matrix. The second step concerns the reference portfolio with its efficient frontier. After that, there will be a section with the result of optimization problems after the introduction of the two crypto indexes, and the consequent movements of the efficient frontier.

There will be also a small case regarding the evolution of the expected return from the beginning of the sample time window. In addition, the results of the VaR estimation will be given in order to see which is the portfolio with the highest potential loss with a fixed confidence interval. Lastly, the final part of this thesis will focus on the results of the two typologies of regression analysis: the alphas with their relative T-stat will be commented, in order to see if the values fit the model.

Chapter 1: Overview

1.1 Features and evolution of the Cryptocurrency market

Since the creation of the first peer-to-peer version of electronic cash called “Bitcoin”, by the unknown Satoshi Nakamoto, almost twelve years ago, the Cryptocurrencies market has increased its volume exponentially.

While at the beginning Cryptocurrency was just another typology of digital currency, now it is considered as the most important. One of the reasons for this success, and a common feature of Cryptos is the use of the Blockchain technology, which is the great innovation behind this new world between technology and finance. In few words, a Blockchain is an open and distributed ledger with the task of recording all transactions, which are called blocks. Those units are sequentially linked and secured in a verifiable and permanent way using cryptography. Each block contains a link to a previous block, that is, a hash pointer, a time stamp to identify the timing of the transaction, and the transaction data (Bianchi, 2020). Through this mechanism, the requirement for a third party as a guarantee for the transaction is no more mandatory. In addition, the nodes of these blocks are accessible to all the participant of the Crypto network (Nakamoto, 2008). Another feature is the absence of a central entity. This means no circulation within the boundaries of a community or geographical location, but literally in every place where there is internet connection. This decentralization consequence allows the Blockchain technology to perform increased volume, better security, and faster settlement and some of these features are at the top of the list of shortcomings of traditional financial systems (Lee, Kuo and Wang, 2017).

In June 2021, the global crypto market cap is $1.57 trillion according to the website CoinMarketCap, a useful online service which gives its users the possibility to stay updated on the values and charts of all the Cryptocurrencies. This website collects material from more than 150 major exchanges and offers daily data on prices (opening, closing, highest and lowest ones), volume and market capitalization (in dollars) for the majority of the Cryptocurrencies. For each of them, algorithm of CoinMarketCap estimates their price by taking the volume weighted average of all prices reported at each market. The global market includes more than 4500 different coins in circulation (CoinMarketCap, 2021), with each of them that belongs to different categories of Cryptos. The first distinction to made is between limited and unlimited coins: the formers are deflationary because they have a value trends similar to the one of restricted goods,

while the latter depend exclusively on their utility because are limitless (once there is no more mining to do, the price of them is set with the law of demand and supply). For the second classification, Cryptocurrencies are grouped based on their main features:

1. Digital Cash Coins: this category includes currencies to which belong most of the market cap and are used as a digital alternative for cash. Examples are Bitcoin and the other Altcoins (alternative Crypto that are not bitcoin like Ethereum, Ripple, Iota…). In this market segment the greatest capital gains/losses can be realized, due to the high volatility that affects the performance of returns.

2. Crypto Tokens: these items are offered during the ICOs. These are "Initial Coin Offering", something similar to the IPOs in the physical Stock Exchanges, without being subjected to the taxation and regulations of the latter ones. Then they are distributed and offered on the exchange websites for Cryptos and Tokens. They own symbols and market values independent from the anchor coins. The increasing interest on them came from the simpler procedure of creation: it is easier when it is based on an already existing coin (the one subject of the ICO) than create it from zero.

3. Stable Coins: these are Cryptocurrencies which have a much-secured price than Bitcoin and the other Altcoins. The reason is that they are pegged to a stable medium of exchange (such as fiat money or exchange-traded commodities) outside the Crypto market space, reducing the financial risk. Volatility and risk associated of these backed stable coins are the same associated with the backing asset.

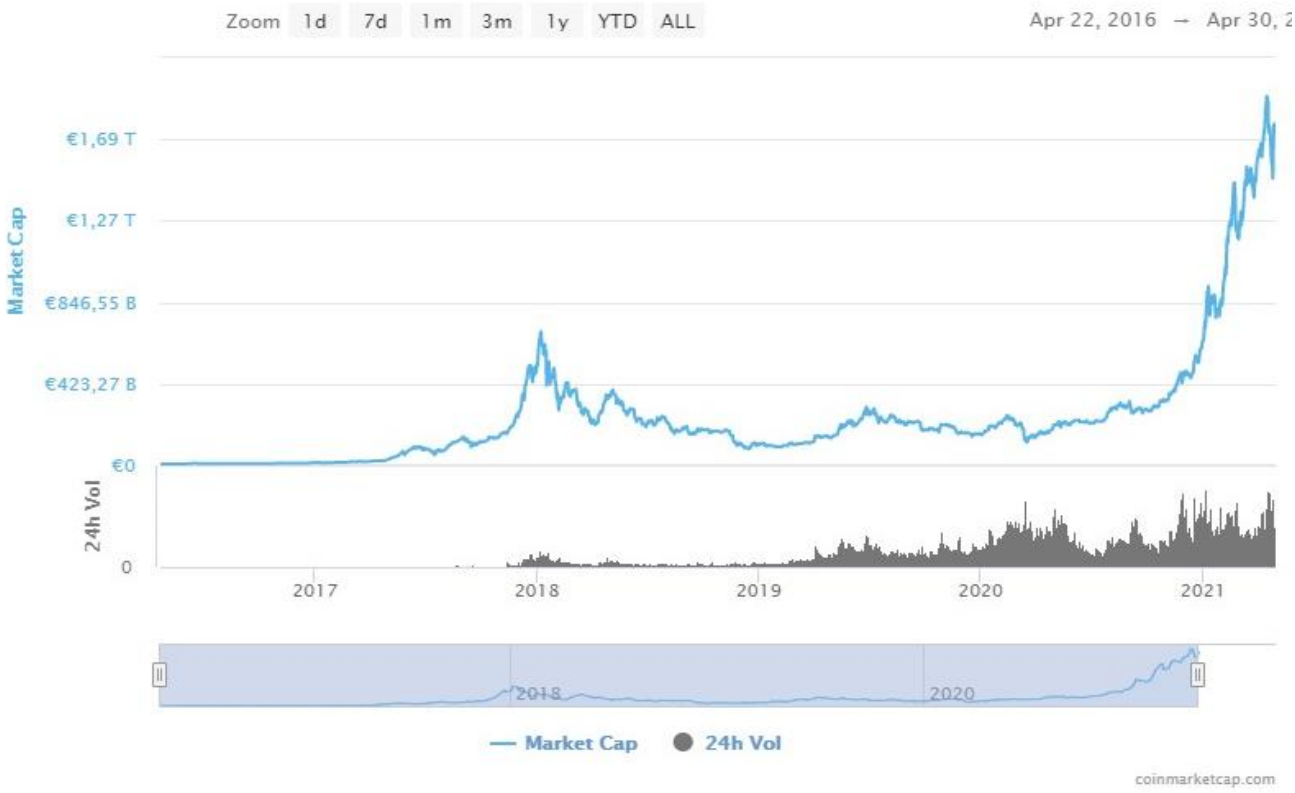
The last feature of Cryptocurrencies concerns the fact of being listed. In order to acknowledge this characteristic, there are requirements that have to be met. Two of them are the followings: the possibility of being negotiated on a public exchange with an API (Application Programming Interface) which must reports the last traded price and the last 24-hour trading volume and having a non-zero trading volume on at least one supported exchange so that a price can be known at any moment. For a generic investor there are several possibilities to get exposure to the Cryptocurrencies market. They go from the primary market, with the participation of the above-mentioned Initial Coin Offerings, to the secondary market. Here, the investor has multiple choice:

- directly trading the cryptocurrencies,

- negotiating the derivatives with Cryptos as underlying asset (e.g., Chicago Board Option Exchange and CME Group both offer cash-settled Bitcoin futures),
- venture capital funding into start-ups operative in the Crypto sector,
- significant equity investment in public firms with important related investment (for example both IBM and Microsoft decided to invest a huge amount of funds into blockchain technology and associated businesses).

Moving to the evolution of the Crypto market, as shown in Figure 1 (exported from CoinMarketCap, from the first, 22/04/16, to the final observation date, 30/04/21, of the sample under exam), it has developed at an high growth rate since the creation of the first Cryptocurrencies.

Figure 1: The overall market capitalization of cryptocurrencies in the window of the analysis



It has to be highlighted that at the beginning there was a phase of strong scepticism towards these tools which were still to be explored. The suspicions about Cryptos have not disappeared completely. This can be noticed in one of the two current views about the Cryptocurrency market (Liu, Tsyvinski, Wu, 2019). The former and negative one is that most and perhaps all of the Cryptos could lead to bubbles and fraud. The latter one is that the blockchain technology embodied in these peer-to-peer currencies may become an important innovation to exploit, even

for alternative use. In addition, at least some of these coins may be assets that represent a stake in the future of this technology. Just to give an example, Jamie Dimon, actual CEO of JP Morgan, stated that Bitcoin and its coin sisters are a mere fraud and cannot be compared to fiat money. However, during the Axios conference set in Los Angeles in 2019, he also said that the Blockchain could became a very useful and important technology for multiple uses in the future.

Another important aspect to be clarified is the role of Bitcoin, displayed in Figure 2. Besides the "reward" of being the first Cryptocurrency to see life, it is also the one that had the highest percentage of the total market capitalization over these years. In the period of this thesis analysis (2016-2021), Bitcoin went from a maximum of 87,62% of the Crypto market cap at the beginning of 2017, to a still considerable 32,84% for the minimum value in January of 2018 (nowadays the percentage has settled to a range between 50% and 55%). The cryptocurrency with the second highest market cap, Ethereum, reached as a maximum value of only 31.17% in the middle of the 2017 when there was the first "boom and bust" of the Cryptos. It is clear that every decision made over Bitcoin in the past or in the present has a direct impact on all the other Cryptos.

Figure 2: Top cryptocurrencies by % of the total market capitalization

Anyway, the first signs of significant growth are dated back in 2017. When Japan gave the authorization for payments through Bitcoin and the other well-known Altcoins (and, as a consequence, made them legal) in April, the overall market capitalization reached almost 250

billion dollars, triggering the first event of a "boom and bust" cycle. This latter one describes different phases of economic expansion and regression typically found in modern capitalist economies, due, in particular, to the psychological element of investors that affect their decisions. In fact, during the second half of the year there was the first steep climb, leading the total market capitalization nearly to one trillion of dollars. There are multiple reasons for this huge and unpredictable rise of the cryptos. In December of 2017, for the first time ever futures contracts with Bitcoin's trend as underlying asset were negotiated on the CBOE. However, the main cause of this speedy growth has to be found behind an exchange platform called Bitfinex and a stable coin named Tether. In the key moments when the price of Bitcoin was declining, Tether was used to buy Bitcoin on this exchange in order to stabilize and manipulate its price, with an increase of nearly 50% of the value back in March 2017 (Griffin, Shams, 2019). After some negative statements made by the SEC and the justice minister of South Korea (the third country in the world for trading volume of cryptos at that time) at the beginning of 2018, the market capitalization experienced a huge loss, and the value went back to the level of early 2017. A decline at the beginning of the year is an event that happens quite every year since 2015, when the crypto market cap has undergone the highest decrease of 30%.

In the following two years, there were not significant highlights. The overall market capitalization began to go down after May of 2018, when the value was nearly 350 billion of dollars. After that date and till September of 2020, the range of the market cap was between 100 and 300 billion of dollar, with only once under 100 billion among 2018 and 2019 during the usual decline in January.

During the second semester of 2020, it has been recorded the highest increase in market capitalization of cryptocurrencies, which had all the feature to be catalogued as one of the most unpredictable events. There are multiple reasons for this scenario that happens just after the first Covid-19 pandemic wave (from March to June), so some of them are going to be discussed here:

- *Accessibility and hedging*: cryptocurrencies can be traded from everywhere around the world without any limitations, the only requirement is an internet connection and an electronic device. During the Coronavirus crisis investors began to fear that central banks or important political players could interfere in "traditional" exchange markets with liquidity constraints or restricted trading activities, so they moved their investments to a decentralized market with no central authority and no control like the cryptos one. This

characteristic made possible for investor to hedge their richness from some types of risk, for example the political one.

- *Inflation*: lots of countries decided to boost money printing during the pandemic, in order to try avoiding a collapse in the consumptions of their families and corporations. This decision leads to an unavoidable increase of the inflation rates. As a consequence, investors decided to transfer money from the unstable stock market (high influenced by inflation) to deflationary assets. Cryptocurrencies are included in this category so, together with the feature of limited supply in the majority of cryptos, this volatile situation leads to an increase of investments.

- *Source of income*: during the quarantine period, which affected most of the developed countries, many people were unable to work with the smart working method and started losing their jobs. These workers started investing as a means of extra income, and during the pandemic those earnings passed from side to principal source of income. Cryptocurrencies ere one of the most exchanged assets, because of their high returns in the short run. There were also lots of new professions correlated to Cryptos that had the birth due to lesser jobs in the traditional job market. Some examples are crypto traders in larger exchange platform, technical financial analysts of the new corporations or crypto influencers, due to the success of social networks

- *Social media*: it is probably the most influential vehicle for the spread of Cryptocurrencies popularity. First of all, they allowed the average user to learn and comprehend the crypto world. Words like altcoin or token and mechanism like were known only by few experts and after the Covid-19 crisis, they began to be understood. Be locked in for months leads a lot of people to tutorial on YouTube about the features of these alternative investments. The other important tool was the publication of posts or tweets on social network by influential people. Elon Musk could be awarded as the most famous crypto influencer. In fact, from October 2020 to April 2021 he published some tweets that altered the price of the principal Cryptocurrencies (Bitcoin and Ethereum). In addition, he also created his own Crypto, named “Dogecoin”, and he pumped it from a price of 0.006 in January to 0.56 at the beginning of May, just by posting enigmatic quotes with the emoticon of a dog and the slogan #tothemoon. Musk is part of a category that leaded many investor to move their money to the crypto market. The absence of

regulations and laws of market abuse for what concerns the activity of social media made these activities possible.

As I wrote above, since October 2020, the growth of the Cryptos market was quick and deep, with Bitcoin that was the main agent of this bull run. The value of the market cap. passed from 400 billion of dollar to nearly 1.9 trillion in April 2021. Facing the extreme period passed and the relative economic breakdowns, Cryptocurrencies have shown a remarkable resilience. In addition to that, an increasing number of countries decided to start a normalization process for this alternative asset class. Just to give some examples, from September 2021 in El Salvador the bitcoin will become a legal tender (which is any kind of payment recognized by a central authority, used to extinguish debts or financial obligations). Furthermore, a few weeks ago Costa Rica made the announcement about its employees that will get rightfully paid in Cryptos. It has to be said that most of these countries are emerging realities from the third world, but also developed countries want to issue new laws and regulation about this topic.

1.2 Cryptocurrency as a possible new Asset Class

Here the analysis is focused on the possible classification of cryptocurrencies, whether it is a matter of currency or of asset class. Both categories are going to be examined in order to find the most appropriate identity for Bitcoin and its coin sisters.

Based on the standards set by Central Banks, there are three definitions for traditional currency (or three functions it should theoretically fulfil):

1. *Store of value*, as a way to keep the value "untouched", in order to retain the purchasing power in the future
2. *Medium of exchange*, as an item for payments in the sale and purchase process and in commercial transactions
3. *Unit of account*, as a standard unit of measurement of goods or services in order to allow the comparison among each other

The high capitalization cryptos, Bitcoin and the most important Altcoins, could have the possibility to meet all the three above constraints. The problem is that the Cryptos universe is composed by thousands of currencies and the large majority has serious difficulties to meet just one of the aforesaid conditions.

For what concerns the first requirement (store of value), the currency under review should have a respectable degree of predictability of its future value. As it is written in the previous paragraph, Cryptos are characterized by high volatility, a feature that could make the forecast of future value more difficult. In theory, both gold and digital coins are good investments in order to store the value because they are not linked to fiat money. In fact, they offer a sort of safe zone during currency or inflationary crises. The problem is that only gold holds these characteristics in the long run, because crypto market is not stable. To make an example, in 2018 the top Cryptocurrencies (Bitcoin, Ethereum and Litecoin) prices were even higher than the inflation rate of lots of countries that were hitting by recession crisis (e.g., South Africa or Mexico). This means that at that time, it was way much riskier to hold those cryptos than the Mexican Peso, the currency of a country affected by economic depression. As I stated above, till the market will not be stabilised, the categorization of cryptocurrencies as a reliable store of value is doubtful. This evaluation is due also to two features of cryptos, like the high amount of volatility and the potential hacking attacks that may cryptos exchange suffer during their short life.

Regarding the second function, medium of exchange, it is necessary to be an economic instrument that is broadly recognized and exchangeable for all the existing goods and services. So, basically the instrument has to behave like a physical intermediary in order to prevent the restrictions of barter transactions. Unfortunately, the majority of cryptos does not meet this condition, because they are not easily available for standard payments. Only a few of them (Bitcoin, Litecoin, Ethereum and the US dollar Tether) give access to the other elements of the cryptos market and exercise the function of medium of exchange between fiat money, like euro or dollar, and cryptocurrencies. In conclusion, this requirement is at the stage of development, because is observable just in the top crypto coins, while is missing for most of the components of this class.

Finally for the third definition of currency (units of account), the overall situation is worse than the previous two functions. Cryptocurrencies are theoretically constituted of equal, specific, and measurable units of account. However, in practice, this third requirement is easily reached till the currency under exam is liquid on the market, because its value can be determined, and be comparable as a consequence (Kim, Sarin, & Virdi, 2018). Most of the cryptocurrencies appear and disappear from the exchanges within a few market days, making the comparison very difficult.

For what concerns the evaluation of the suitability of the currency classification, cryptos are far away to be recognized as it. As it is stated before the analysis, only a few of them is able to reach the needed requirements, and it is thought that these obstacles can be passed only with a future appropriate regulation. The problem lies behind the fact that cryptos with strong regulation will not be cryptos, so the conclusion is that currency is not the appropriate categorization for them.

After this digression based on the pure definition of currency, the analysis will continue with the point of view of the main banking authorities. It has to be clarified that there is not a common opinion within these institutions on whether cryptocurrencies conform to the canons of regular currency. In 2019 The Bank of England imposed a sharp refuse regarding the categorization of cryptos as standard money. The European Central Bank had the same point of view, but it has concluded also that the blockchain technology behind cryptocurrencies might quickly have a great influence on the economy. In addition, it has to be stated that virtual money has to be constantly monitored in order to see the future developments. In terms of national central banks, the majority of them does not consider cryptocurrencies as a unit of account and some central banks have an even harsher point of view. In fact, in France cryptos are not even considered financial instruments. There are also some exceptions in Europe. The Italian authorities (Agenzia delle Entrate and the Ministry of Economy and Finance) decided to validate these virtual currencies as a regular means of exchange. Furthermore, in 2020 the German federal financial supervisory authority recognized Bitcoin and other top cryptos as a unit of account similar to a foreign exchange, even if Bitcoin and the Altcoins do not meet the standards to be a legal tender. This validation has to be considered only as a kind of private means of payments. In the same time window, the EBA (European Banking Authority) refuses the word currency regarding the world of crypto assets. It also states that due to considerable technological risks cryptos have to be separated from normal payments activities. The ESA (European Supervisory Authority) is of the same advice of the EBA. In fact, it published a warning for retail investors regarding the risks of buying and holding cryptocurrencies. In the rest of the world there are different point of views. For example, in the United States, these digital currencies are considered both currencies and securities. The national authority has not declared them illegal, but, at the same time, cryptos are not legal tender. Chinese government authority banned the mining industry within their boards in 2014, due to financial stability prospects.

In the previous years, the vast majority of the researchers agreed that all the items of the Crypto category (tokens, coins, etc…) cannot be catalogued as “currency”. In fact, they are more likely to be classified as financial instruments for speculative purpose (Demertzis and Wolff, 2018).

Other studies stated that the term “currency” is a contradiction for Bitcoin and its sisters, with the classification of “crypto asset” that sounds more appropriate (Yermack, 2015).

Further analysis of the “classification” issue was conducted during the last five years, with different conclusions. For some authors, crypto assets would have led to speculative bubbles (Kreuser and Sornette, 2018). There were assembled many financial model that forecasted possible futures with cryptocurrencies bubble, predicting their early bust (Brown, 2018).

However, the new point of view about cryptos tends to see them as a gradually evolving asset class. Due to the uncorrelated nature of cryptos many economists are optimistic for what concern the future of this asset class. They support the idea that cryptocurrencies will be part of a future asset class with huge growth potential. At the moment, it is at the stage of development, and it is gaining the primary features of a separate class (Sontakke and Ghaisas, 2017), (Bianchi, 2018), (Trautman and Dorman, 2018).

In addition, there is a group of academics who support the idea that cryptocurrencies are already showing the essential features to be classified as an asset class, regardless of existing restrictions and risks. The names within this list include Elendner, Trimborn, Ong and Lee (2018), Burniske and White (2017), Ankenbrand and Bieri (2018), Kim et al. (2018) and Krueckeberg and Scholz (2018). The arguments which support this idea of the new emerging asset class are the following: internal correlation between crypto assets, absence of/very low correlation with external groups of assets, growing liquidity from both institutional and retail investors, increasing interest of public authorities and reglementary organization, and finally the implementation into multiple different industries.

In the meantime, other researchers have concluded that there are many doubts concerning the readiness of cryptos to become a separate asset class. The names of these academics are the following: Härdle, Chen and Overbeck (2017), Baur, Hong and Lee (2018) and Kurka (2019). In their papers, they have demonstrated a clear dependence of the crypto market on shocks, speculations, hacking attacks and regulation variations. These events are expected to define the future of crypto assets, seen that we are entering into a financial volatile era.

If the broadest description regarding asset classes is taking under consideration, William Sharpe has to be mentioned with his Asset Class Factor Model (1992). At the root of this, three conditions were suggested: mutual distinctiveness between other classes, completeness within the class itself and significant variations in returns related to other typologies of asset. In other words, the three requirements mentioned above leads to the following conclusion: nearly every

kind of asset may have the possibility to be included within one asset class “x”. This asset class should be capable of containing the highest number possible needed of assets, with similar characteristics. In addition, the returns of the asset under exam within asset class “x” have both a really small correlation and a distinct level of volatility with other classes. Another definition which it is considered more advanced with respect of Sharpe, and it also comprises both traditional and alternative assets, was proposed by Kinlaw, Kritzman, Turkington, and Markowitz (2017). According to their book “A Practitioner's Guide to Asset Allocation”, the following explanation is given: “an asset class is a stable aggregation of investable units that is internally homogeneous and externally heterogeneous, that when added to a portfolio raises its expected utility without benefit of selection skill, and which can be accessed cost effectively in size”.

Furthermore, in their book, there are also seven criteria that an asset class has to meet in order to be classified as it, and so it should also work for cryptocurrencies. Here is the “list”:

1. *Stable Aggregation*, which implies the stability of the class composition. In order to be considered as an asset class, a mandatory condition for the structure of the cryptocurrency market concerns a volatility with a low value in terms of the nature of its constituents. However, it has to be written that continuous rebalancing, misclassifications, and supervision of the new components may be excessively expensive. Market capitalisation of specific assets may be variable as a result of price movements, while the characteristics, statistical properties, aim of the usage and so on should theoretically remain the same. If the asset composition is based on external factors characterized by an high volatility over the time, the assets would be classified as unstable and, as a consequence, it would not be qualified as a class. For what concerns cryptocurrencies, this criterion should be checked via qualitative analysis.

2. *Investability*, that means the asset should be directly investible. In order to check this condition of cryptos, the required proof regards the accessibility to channels of direct investing for this asset category in an easy way. For example, let’s talk about the exposure of an asset performance. If random retail investor has to generate a replicating portfolio for that, it cannot be handled as an asset class. Replication of portfolios causes additional costs to keep an appropriate structure and is sensible to external events. For this reason, it cannot truly imitate the performance of the underlying asset.

3. *Internal homogeneity*, because it is presumed that all components of the asset class have similar characteristics from the investor perspective. Internal homogeneity signifies comparability inside the class. There could be a possibility that several groups may have different features within asset class "x", but, together, they all must have the same characteristics related to other asset classes. The requirement of internal homogeneity of an asset class can be demonstrated when assets are positively correlated. Therefore, it is expected a correlation coefficient to be positive from 0 to 1 (Krueckeberg and Scholz, 2018).

4. *External homogeneity*. In contrast to what it is explained above about the internally homogeneous structure of the asset class, on the external side, assets have to be heterogeneous. The reason behind is that considerable differences with other asset classes are valuable for a random retail investor. In the opposite situation, the asset class may be simply redundant on the market. A rightful comparison between two or more asset classes should be based on their representation as a whole. Thus, in order to test the heterogeneity, the optimal method involves using proxies, namely indices, which give the best representation of the overall performance of the asset class. This analysis consists of three steps: an examination of statistical properties of the asset classes, comparison of their financial profiles and correlation matrix evaluation. Statistical profiles comprise weekly mean, standard deviation, minimum, maximum, skewness, kurtosis, but also less known trimmed mean, median, median absolute deviation (MAD), and standard error; these above-mentioned profiles talk about the asset under exam by showing how its returns are distributed. To fulfil the external heterogeneity criterion, the statistical properties of the new asset class have to be different from already existing classes. The correlation matrix is computed on the basis of Spearman's coefficient, the r_s coefficient, that can assume both negative and positive values. This coefficient fits the cryptocurrencies' properties the most, as it is not limited to linear relation only as the Pearson's coefficient does. In statistical terms, heterogeneity signifies absence of correlation with other asset classes.

5. *Expected utility*. In portfolio management, when a new asset is included into an investment portfolio, the expected aim is to bring with its presence an higher expected utility of the portfolio. This outcome can mean either to increase the return or decrease the risk. It is possible to reach it in two cases: when relatively high return and/or low risk belong to the main features of the asset under exam; or when the latter is extremely

heterogeneous, and so it is uncorrelated with other asset classes. In other words, the desired aim of this analysis is to get a diversification benefit from the inclusion of this asset. The growth of the expected utility occasionally depends on the overall market conditions and may occur in periods of economic crises. On the other hand, this rise is not observed during a period of economic growth.

6. *Selection skill*, which can be tested by using analysis of current indices and also by looking at the internal homogeneity. However, the introduction of those indices usually reduces the necessity for selection. It has to be said that a random retail investor is not supposed to have any particular abilities to pick and choose a suitable unit from an "x" asset class, in order to add an expected utility to his/her portfolio. The "selection skill" requirement is supported by the help of internal homogeneity of the asset class, as I showed above. As a consequence, any unit within the class brings relatively similar exposure.

7. *Cost-effective access*, because in the mind of every kind of investor transaction fees, spread, opportunity costs and liquidity level play a key role when determining whether to invest or not his/her finances. The expected utility of inclusion of the unit from a "x" asset class to the portfolio also depends on these factors. Therefore, the asset class under exam should be accessible at reasonable costs. Due to the need of constant rebalancing of the portfolio weights, the above stated trading costs should not weaken both profitability and liquidity of the investor portfolio (Frazzini, Israel, & Moskowitz, 2018).

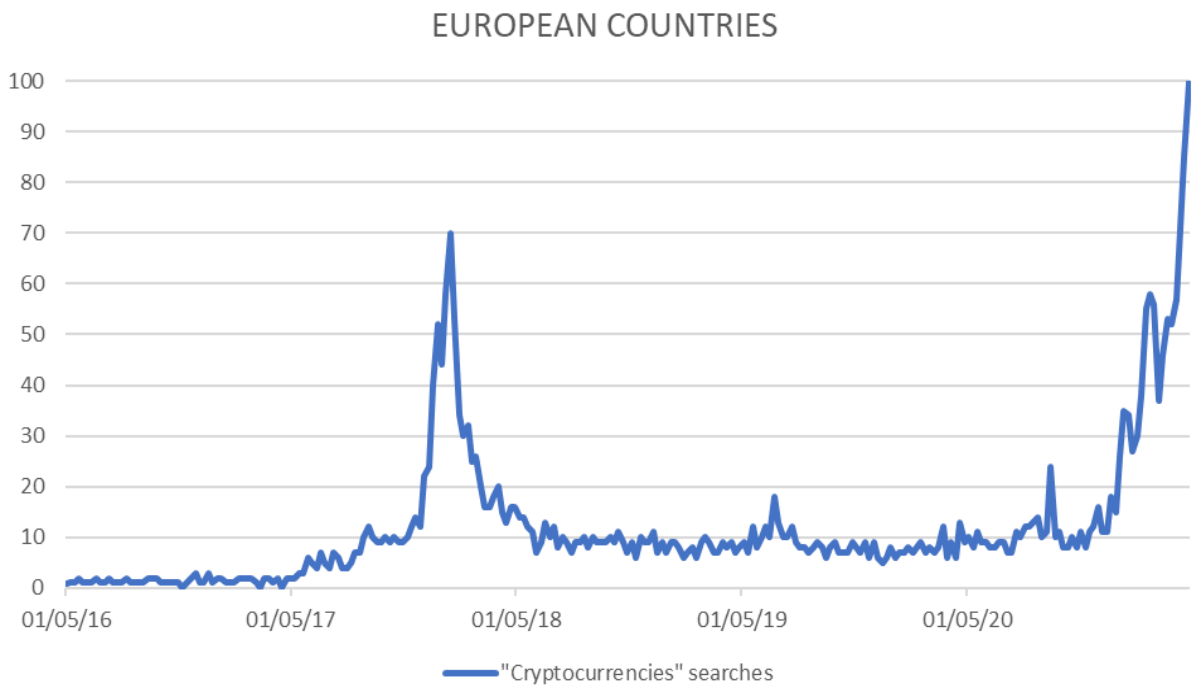
The overall final judgement of cryptocurrencies through the seven requirements and the thoughts of the academics is quite positive. For sure, it is much more positive than the analysis regarding the currency evaluation, but it is not sure and defined as the latter. Cryptocurrencies are still not too stable, and for the expected utility there are some doubts yet. On the other hand, the investability, together with the homogeneities and costs, brings the balance to the positive side.

1.3 European investors relationship with Cryptocurrencies

The point of view of a European retail investor and its relationship with the universe of cryptocurrencies will be clarified, due to the fact that he/she is the main actor of this thesis analysis. In addition, the majority of the economic data under exam will be expressed in euros, in order to avoid misunderstandings.

Chainalysis, an important blockchain data platform and also a crypto analyst firm, released some statements and data at the mass media (Bloomberg and Al Jazeera, 2021) about the overall investors' horizon. First of all, cryptos adoption in the world has increased by more than 881% with respect to the previous year (2020) and it's the result of half year. Furthermore, "In emerging markets, many turn to cryptocurrency to preserve their savings in the face of currency devaluation, send and receive remittances, and carry out business transactions," Chainalysis wrote in the report, "while adoption in North America, Western Europe, and Eastern Asia over the last year has been powered largely by institutional investment". They also tested a new factor during the analysis of the countries' adoption of cryptos: number of deposits by country weighted by number of internet users. Due to it, they came up with a ranking where the first countries were the ones with relatively more decentralized finance, also known as DeFi, users. Those countries are Vietnam, Ukraine, India and similar, because in those environments there is an higher concentration of daily transactions and individual savings, rather than speculative trading and massive investments from institutional firms.

Figure 3: Trend of the searches on Google of the world "cryptocurrencies"



Therefore, this relationship is something not been experimented or tested yet. It is important to see how much is correlated the increase in market cap of cryptos and the number of times that the word "cryptocurrencies" appeared on the Google search-bar. The values of the graph, on a scale of 1 to 100, do not indicate the number of searches made by users for these keywords, otherwise the tool would be redundant with the one present in Google AdWords, but rather offer

us a percentage index of the popularity of the term (or terms in this case) over time.As it is possible to see from the graph in Figure 3, exported from GoogleTrends.com, the two highest peaks reached are at same time of the two “boom and bust” period of Bitcoin and the other Altcoins. in the 3 months between the 2017 and 2018, “cryptocurrencies” reached a 70% of popularity among European Google users. A percentage that was exceeded, after the outbreak of the Coronavirus Crisis, in the second half of 2020 and the first months of 2021.

The exponential interest on cryptos from European retail investors could be testified also by the data published by eToro. The famous trading online platform had 200.000 new users just in January and February of 2021, with an increase in Bitcoin holders of 61% and in Ethereum holders of 49% (source CNBC). Revolut, another well-known trading platform, has acquired a lot of praise and notoriety after the GameStop scandal, that is considered the first act of retail investors unity. However, in that case, investors from all over the globe took part of the action against the banking institutions. Just to remain with the focus on Europe and U.K., the British trading platform saw the entrance of 175.000 crypto users. The universe of cryptocurrencies is going towards a mainstream dimension. The rally of the last six months in their prices has happened in a distinct landscape from the one between 2017 and 2018. Institutional Corporations are slowly but surely making their way into the crypto environment. Their presence has brought much-needed liquidity to crypto exchanges, making them less vulnerable to violent price shifts due to minor trades. These numbers may be justified with the rising phenomenon of the “fear of missing out”, which affected Europe after the success in the United States and countries from the Eastern Asia.

However, for eToro and other crypto trading platforms as Revolut, most of these European retail investors tends to build portfolios with a high degree of volatility, composed mainly by cryptocurrencies with great correlation index. Some platforms give the possibility to replicate portfolios of high reward seekers, and many retail investors follow them, without taking in consideration that the funds availability are not the same. On the contrary, here the target is to build a portfolio for every retail investor, with a controlled volatility and a relative return. This outcome aims to avoid huge losses in case of cryptos downturn in the future, as it happens in the previous years.

Chapter 2: Portfolio Theory

2.1 Related literature

The conventional mean-variance (MV) portfolio selection framework developed by Markowitz (1952) is the basis theory of this analysis. The Modern Portfolio Theory (MPT) is the most appropriate method to evaluate the introduction of cryptocurrencies in a multi-asset portfolio, and its effects on the risk-return characteristics.

Considering the finance literature, Markowitz gave a huge contribution to its development with the creation of Modern Portfolio Theory. In a few words, for the MPT, a random investor is able to develop a multi-asset portfolio which maximizes its return for a determined threshold of standard deviation, given his/her risk aversion level. Many academics decided to utilize the assumptions of this theory for their studies. For the general research, it is fundamental to exploit the benefit of the portfolio diversification. This outcome is reached by including many distinct asset classes to the portfolio under exam. The traditional asset classes were "equity", "bond" and "cash and its equivalent", but nowadays there are new typologies to be classified as it. Commodities, real estate, or private equity are some examples of the new entries, and also the category of Cryptocurrencies is going to that direction. In addition to the MPT, there is also the Sharpe ratio, developed by the economist in 1966. It is an supplementary measure, very useful to evaluate the performance of a portfolio/single asset, adjusted for its risk.

The presence of articles and studies about cryptocurrencies in the finance literature is strictly connected and correlated with their increasing notoriety and reputation. It has to be said that the focus was principally on Bitcoin, for its first-role importance in the category. The majority of the researchers' papers in the years following the creation of cryptos highlighted the economic, regulatory, and technical aspects of Bitcoin. On the contrary, for what concerns the opportunities coming from the investment in cryptocurrencies, there was not an equal number of scientific studies.

One of the first financial papers regarding the consequence for the introduction of cryptos in an investor's portfolio was published in 2014. Wu and Pandey (2014) designed a portfolio composed of both traditional asset classes (equity and bonds) and alternative investments (private equity, commodities, and cryptocurrencies). The outcome of the study was the following: having a reduced weight of just Bitcoin leaded to an increase in the risk/return trade

off. The same result was obtained in a similar research in the next year: Brière, Oosterlinck and Szafarz (2015) designed a time window from the creation of Bitcoin to 2013, and there was a change in the typologies of the asset classes. In this case, the authors added cash in traditional assets, while they substituted private equity with hedge funds for what concerns the alternative investments. In the same year, Eisl, Weinmayer and Gasser (2015) focused on the introduction of cryptos in a multi asset portfolio, but this time the approach adopted was the Conditional Value-at-Risk (CVaR) framework, instead of the MP theory. Their decision of changing the methodology was justified from one of the features of Bitcoin: it is not normally distributed. The authors decided to include the first altcoin in the efficient portfolio, contributing to reduce the very low correlation with the other assets. This outcome has to be taken with care because in the early stage of life, Bitcoin and other cryptos did not show the roller-coaster performances of the last years.

In the following year, a new study pointed the attention on central banks. Moore and Stephen (2016) questioned whether cryptocurrencies could be included in an external asset portfolio (without currency) held by national banking institution. Two approaches were used, Montecarlo and counterfactual, and both gave the same effect: cryptos were too volatile to be kept in a national bank's portfolio. During the same year, Elendner, Trimborn, Ong and Lee (2016) published the first research study on the features and performance of the Cryptocurrencies Index (CRIX). The paper stressed the fact that CRIX had a nearly zero correlation with the other asset classes, and, at the same time, its average return on a daily basis was high. Furthermore, the risk/return of CRIX was much better than any single of the liquid cryptocurrency. Its addition into a multi asset portfolio leaded to an increase of the overall mean variance ratio. At the end of the paper, there is a section concerning the risks and complication derived from cryptos. So, before taking the decision of investing in this category, the investor should actively follow the developments of this market.

Cryptocurrencies have shown low daily trading volumes during those years and their market was considered particularly illiquid. Trimborn, Li and Hardle (2017) showed a new approach to solve this problem with the utilization of a liquidity constrained investment. The analysis was conducted with the factors of the German and Portuguese market, and the overall outcome was of a higher Sharpe Ratio than the one with the traditional approach. Writing about this alternative method, called Liquidity Bounded Risk-return Optimization (LIBRO), it is a combination of the Markowitz framework with certain liquidity constraints, which modify with a notable impact the original theory.

A higher number of papers regarding investment strategies with cryptocurrencies became available from 2018. In fact, Veldmeijer (2018) examined the effects of a basket of cryptos together with Bitcoin alone, from a Dutch pension fund's point of view. The strategic asset allocation began with the constraint of fixed weights for the starting portfolio, data available on the website of De Nederlandsche Bank. In this case, regulation and risk management issues gain more importance than the previous examples of literature.

Andrianto and Diputra (2018) designed with intensive asset allocation four portfolios, that included both Bitcoin and smaller cryptocurrencies. All the reference portfolios had different weight allocations. The final result was that the investors obtain a positive effect on portfolio performance, by adding cryptocurrency assets. Renkes (2018) was the first to investigate another typology of investment: the factor investing regarding the cryptocurrency universe. This strategy is based on the Fama French model, also known as three (1993) or five (2015) factors model, which is in contrast with the Capital Asset Pricing Model, where the return is based on only one factor. The already known factors "momentum", "value" and "low volatility" are evaluated by applying various performance measures. In addition to those three factors, the research investigates two new factors pertinent to the cryptocurrency market: Google query volumes and hash rates. His conclusion was that all the multi factor portfolios outperform the relative benchmarks.

If I exclude the research conducted by Veldmeijer, the majority of the studies published embrace the point of view of an average American investor. For this reason, Kajtazi and Moro (2018) were the first academics to study the effect of Bitcoin in portfolios of not only US, but also European and Chinese assets. The study emphasized the fact that Bitcoin generates a positive outcome. At the same time, there is no important distinction among the three geographical portfolios. However, as happened in the research of Elendner et al. (2016), the academics advise the investors for what concerns the linked complexity and threat of cryptocurrencies as an investment product. One of the reason was the declared experimental stage of the asset class.

The papers of the last two years have extended the scope of the analysis regarding investment strategy. Perrin and Roncalli (2019) focus the core of their study on regression analysis with the machine learning optimization. For the researchers, most academic portfolio optimization models, proposed as alternatives to Markowitz's MPT are not applicable to real portfolios, even if they have strong theoretical characteristics. The authors analysed the large-scale optimization algorithms and shows how they can be implemented in portfolio optimization problems.

Holovatiuk (2020) verified whether cryptocurrencies can be classified as an asset class and what kind of profits they may carry to the investor's portfolio. The final results were the followings:

1. Cryptocurrencies quite totally fulfilled the seven asset class requirements, theorized by Kinlaw, Kritzman, Turkington, and Markowitz in 2017.
2. Portfolio optimisation with the Modern Portfolio Theory showed an increase in the Sharpe ratio of tangency portfolios with the inclusion of CRIX.

Kruckeberg and Scholz (2020) investigated whether cryptos are a distinct asset class or can be included in a broader one. The outcome was that cryptocurrencies show attributes of a distinct asset class for many factors including solid internal correlation and absence of correlation with any traditional asset class, among the others. A negative aspect was the market stability, which has to be improved. Bianchi (2020) wrote an empirical assessment, which focus on 300 types of cryptos, their characteristics and the relationship with macroeconomic factors. He underlines that, even if there is a moderate correlation among returns on cryptocurrencies and commodities, with precious metals in particular, this weak correlation does not translate into volatility spill over effect.

The aim of this thesis is to extend the current literature by trying to mix the previous studies regarding investment strategies with cryptocurrencies. Instead of analysing a specific individual cryptocurrency, I decided to examine the effect of an index of cryptocurrencies (CRIX), in combination with a crypto that can be considered as an index of the category, due to its percentage of market cap over the total market cap of cryptocurrencies (Bitcoin). It addresses the view of an average European retail investor, so without any particular form constraints, except for the currency that is fully in Euro. Following Campbell & Viceira (2002), the strategic asset allocation has a long run vision. The reference portfolio, based on data from 2016 to 2021, is structured with different asset allocations weights. The distinct optimal crypto weights are included to the reference portfolio using the mean-variance analysis of Markowitz. Once the data are collected the four portfolios' performances are evaluated with a regression analysis of the portfolio returns on a market benchmark to assess the portfolio alphas. In order to verify the statistical significance of higher returns an in-sample and out-of-sample analysis is conducted.

2.2 Methodology and analysis

2.2.1 Econometric analysis

One of the most important assumptions of the modern portfolio theory regards the fact that every kind of investor is risk adverse. Given a situation where there are two portfolios with equal standard deviation, investors will always choose the portfolio with a higher return. While in the opposite situation, so with equal return, they will prefer the portfolio with a lower risk.

Two significant properties that are very helpful to reduce risks associated with the performance of a portfolio are the diversification and the correlations between assets within it. An investor can build up a portfolio of multiple assets within the same class or coming from different classes and maximize the return for his/her desired level of risk. Likewise, as I wrote above, he/she can minimize risk, given his/her desired level of return.

The general formula for the expected return of the portfolio with k-risky assets is the following:

$$E(u_p) = w\,'\mu \quad (2.1)$$

where *w* is the vector of weights assigned to the specific assets (1 through k) and μ is the vector of expected returns produced by the individual assets (1 through k) within the portfolio.

The second formula presented in this section is the one for the portfolio standard deviation with k-risky assets, which is calculated as:

$$\sigma_p = \sqrt{w'\Sigma w} \quad (2.2)$$

again, w is the vector of weights of the single assets (1 through k) and Σ is the variance – covariance matrix (k x k), also known as dispersion matrix.

The last formula for the problems of maximization and minimization is the Sharpe ratio, and it is estimated as:

$$SR_p = (E(r_p) - r_f)/\sigma_p \quad (2.3)$$

where the $E(r_p)$ is the expected return of the portfolio, r_f is the return of an investment that carries zero risk, and σ_p, as it is stated above, is the measure of the standard deviation. The definition of his reward-to-volatility ratio, given by Sharpe in 1966, states that it calculates the excess return per unit of risk. When there is an inclusion of new assets within the investor's portfolio, the Sharpe ratio is a fundamental item for the comparison in the portfolio's overall risk-return

characteristics, before and after the addition. The general rule states that the higher the Sharpe ratio, the better is the overall effect for the portfolio. For the estimation of the risk-free rate used in the Sharpe ratio formula there were many options available. Considering that the investor of this analysis is from Europe, the two main solutions were the Euribor (Euro Inter Bank Offered Rate) and the Eonia (Euro Over Night Index Average). The former one is a reference rate published by EMMI (European Money Markets Institute), estimated on the average interest rates at which banks within the Eurozone offer to lend unsecured funds to other banks in the interbank market. On the other side, the latter one represents the weighted average of all the overnight rates applied on the unsecured financing transactions, undertaken in the European Union and the EFTA (European Free Trade Transaction). I decided to pick the second option, because the Eonia is considered more solid than the Euribor for what concerns the Euro currency. The rate used in the analysis is on a weekly basis like the data of the asset classes' indexes. It is estimated by taking the average of all weekly Eonia rates at each observation date of the sample, from the number 1 to 262.

Together with the reference portfolio, there are three other portfolios to be examined:

1. Reference Portfolio with the introduction of Bitcoin
2. Reference Portfolio with the addition of the Cryptocurrencies Index (CRIX)
3. Reference Portfolio with the inclusion of both Bitcoin and CRIX

After the estimation of the return and standard deviation of the reference portfolio, all the four portfolios listed above (reference + three cryptos) are maximized in terms of the former and minimized in terms of the latter, in order to be able to compare results. It has also been included a third variable constrain, which regards the maximization of the Sharpe ratio. The following formulas are needed to solve these problems:

1. $Max_{\{w_i \mu_i\}} E(r_p) = w'\mu$
2. $Min_{\{w_i \sigma_i\}} \sigma_p = \sqrt{w' \sum w}$ with $\sum w$ that is the matrix between the weights transpose and the standard deviations of each asset class
3. $Max_{\{w_i\}} SR(r_p) = \frac{E(r_p) - r_f}{\sigma_p}$

I decided also to add two fixed constraints in all the four portfolios along with the three variables (maximization of π, minimisation of σ, maximization of SR). The first one is that short sales are not allowed, while the second one regards the fact that all the investor's richness is used. The above-mentioned restrictions are added in the following way to the optimization strategies.

1) $wi \geq 0 \; \forall \; i = 1, \ldots, k$

2) $w'1 = 1$

Basically, each individual asset weight i can't assume a negative value and the sum of the weights must be equal to 1 (or 100%).

Furthermore, it has been determined the efficient frontier for every portfolio. In order to construct the frontier, it has been estimated the minimum variance portfolio and other eight portfolios with given increasing levels of return and their relative minimized standard deviations. With the purpose of establishing the set of portfolios on the efficient frontier, the following problem is solved:

$$\text{Minimize } \sigma_p^2 = w'\Sigma w \quad (2.4)$$

Subject to:

$$E(r_p) = w'\mu \text{ (several target returns chosen)}$$

2.2.3 VaR approaches

In order to see the quality and safety of the four portfolios, I decided to calculate the VaR (Value at Risk) for each of the portfolios maximized in terms of return, using two of the three possible approach for the estimation of the Var. The chosen two are the historical and the parametric approach, while the Montecarlo simulation was not suited due to its random nature. In fact, at the bottom of the Montecarlo method, there is the assumption of a generation of a high number of scenarios, while, in this analysis, the information is already available thanks to the historical data.

Before the explanation of the models, I am going to explain this risk measure.

The VaR determines the highest possible variation in the value of a portfolio of financial instruments, over a pre-set horizon and a specified confidence level. In other words, VaR offers a statistical estimate of the probable loss when markets are behaving 'normally'. In order to be estimated, this risk value needs statistical distribution assumption over the relevant risk factors that affect the underlying portfolio.

The problem of this measure is that it cannot tell the investor how big the loss will be in the moments where the market is affected by extraordinary events and with which likelihood.

Another important aspect regards the treatment with different distributions. Indeed, given the same confidence level and the same holding period, VaR calculated according to a "fat-tails" distribution is higher than VaR calculated according to standard normal assumptions. The last weakness is that VaR models based on historical data of risk factors provide just limited insight into risks that are not shown within the model's 'time window'.

The first approach I'm going to explain is the historical approach. In a historical simulation model, possible market factor variations are expected to be suitably represented by their historical empirical distribution. In this way, their previous behaviour is a consistent guidance to forecast their potential future developments. The risk factor changes recorded in the past are transformed into possible future values of the bank's portfolio through full revaluation. The historical approach is theoretically very similar to the Montecarlo approach. However, the former uses historical data/time series instead of ones that are generated through the use of estimated stochastic processes. The problem of this approach is that it is completely dependent on the dataset. Therefore, if there are extreme events such as market crashes, they will either lie outside the data set and be ignored or lie within the data set and (for some purposes) act to distort it.

In the analysis it has been followed the fair value-based method. It is one of the two options of the historical simulation, with the other that is called sensitivity-based method. The former begins with the estimation of the fair value and the decision of the confidence level. After that the percentile of the potential loss is calculated over all the dataset available, and lastly the value at risk for the fair value. The formula below synthetizes the last passage:

$$VaR_p = FV * potential\ loss \quad (2.5)$$

The second approach I used in this analysis is the parametric approach. It also referred to as variance covariance approach, and it is the most widespread among financial institutions. The two main reasons for this popularity are its simplicity and the fact that it was the first VaR model developed (the quickest to spread among the Anglo-Saxon banks). There are also some disadvantages regarding this approach, which concerns in particular two aspects:

1. Market factors' returns distribution, that for this approach are assumed to be normal distributed.
2. Sensitivity of portfolio positions to variations in market factors.

The variance-covariance approach is based upon the assumption that market factor return volatilities and correlation coefficients can be estimated correctly. As for the historical approach,

there are two methods for this accurate calculation. The first one, here in the model, is based on the assumption that historical volatility and correlation data are able to predict future volatility and correlations. Past models had the assumption of a fixed correlation and volatility for all the dataset, while I used the most recent, where the two measures are the average of the five-year time.

The first passage in this approach is the estimation of the VaR for each asset class. The fair values of them are calculated considering the last observation (n=263), with the weights of the portfolios maximized for the return. Consequently, the following formula is applied for all the four portfolios, as shown below:

$$VaR_p = \sqrt{V \cdot C \cdot V'} \quad (2.6)$$

where V stands for the row vector of VaR of each individual asset class, V' is the transpose of matrix V, and lastly C is the correlation matrix.

2.2.3 Regression Analysis

The last part of this analysis is focused on the regression analysis in order to examine the incidence of the four developed portfolios with a market index.

Two typologies of analysis are carried out for the determination of the performances with respect to the alphas and their t-statistics: an in-sample and an out-of-sample analysis. The primary distinction among the two is the evaluation time frame used to compute the optimal portfolio weights and returns. Indeed, in the in-sample analysis, all the observations from t_0 to T (n=1, …, 262) are used as the estimation window. On the contrary, in the out-of-sample analysis the dataset is split in two parts. So, in this second case the estimation window is made only of observations from t_0 to t (n=1, …, 130).

The whole process at the basis of this analysis starts from the construction of the reference portfolios and the other portfolios with the fixed weights. After the estimation of expected return and standard deviation for each asset class, the in-sample analysis focus on the solution at the problem of the maximization of the expected return. It is fundamental to keep the standard deviation fixed, in order to have an appropriate evaluation among the different outcomes in the expected returns. Once the measure of expected return is obtained, the four portfolios are ready to be regressed.

In the out-of-sample analysis there are other steps to be made, in order to observe the forecast for the second part of the dataset. In fact, the aim of this analysis is to try to predict in the most accurate way the values of the second half, by regressing the first half data with the external benchmark. In this case, the optimal weights are not the same of the in-sample, because they are estimated only for the observation till week t=130. Then, the weights obtained are multiplied with the return from week t+1 (n=131) to week T (n=262). In this way, the alphas of the out-of-sample are calculated with the portion of observations that is not part of the computation of the weights. In conclusion for the elements of the two analysis, they produce a different risk-return characteristic of the four portfolios, and as a consequence also a distinct set of optimal weights.

The core point of regression analysis of this type is the extra return produced by the built portfolios, compared to an overall market benchmark. Considering that the portfolio contains many different asset classes, the choice of the benchmark has been very complex, because you need to find a suitable index, that is the most representative possible of the overall market. The final decision went to the MSCI All Country World Index. It is constructed to represent performance of the full opportunity set of large- and mid-cap stocks across 23 developed countries and 27 emerging markets (msci.com/acwi). MSCI ACW Index is built using the building block approach (the other possible option is the total factor productivity approach). The methodology applied is reliable, rules-based, and transparent. In addition, the utilization of this index as a support to build and compare portfolios can help to prevent unplanned bets and risks. The last aspect of the MSCI ACWI regards its structure and the fact that it allows investors to quantify their portfolio's exposure to the world of equity, by only using one world framework.

The outcome of the both analysis is recorded after the running of this bivariate ordinary least-squares regression:

$$y_i = \alpha + \beta \cdot x_i + \varepsilon_i \quad (2.7)$$

In the above equation, y_i is the dependent variables that indicates the returns of the four different portfolios (reference one and the three with cryptos). Then, there is the return of the selected market benchmark, the MSCI ACWI, represented by the independent variable x_i. the remaining three terms of the equation are described as it follows:

1. α = vertical intercept of the regression line
2. β = slope of the regression line
3. ε = error term of the regression

and lastly the $\alpha + \beta \cdot x_i$ is the regression line.

The focus is on the value of the alpha of the four portfolios. In case the value is positive, the portfolio is performing better than the market benchmark. On the contrary, with a negative value, the portfolio is underperforming compared to the MSCI ACWI. It is important also the value assumed by the T-statistic and its relative p-value. The former is a measure obtained by dividing the coefficient of the alpha with its standard error, and it is considered significant if its value is higher than 1.96, with a confidence level of 95%.

2.3 Portfolio construction

2.3.1 Reference Portfolio

The multi asset reference portfolio is built up taking into consideration the principal asset classes available on the market, which can be truly represented by world indexes. Here below in Table 1, there is the full list of the asset classes, with the relative weights.

Table 1: Traditional asset classes and their sub-divisions

Investment Category	***Investment Product***	***Weights (%)***
Equity	Developed Markets	10.00%
	Emerging Markets	10.00%
Debt Securities	Sovereign Bonds	10.00%
	Corporate Bonds	10.00%
	High Yields Bonds	10.00%
Hedge Funds	Hedge Funds	10.00%
Real Estate	Real Estate	10.00%
Alternative Investments	Private Equity	10.00%
	Commodities	10.00%
	Infrastructure	10.00%
Total:		100.00%

This analysis begins in a situation of an equal weighted portfolio, so the overall richness is equally divided into the ten investment products. It could be questioned that this is not the most correct way to split the weights for each class, because every retail investor from Europe may have his/her preferences, so whether to distribute the total wealth on equity rather than alternative investments or bonds. However, in absence of objective data, it is the best method possible to begin the analysis, before the introduction of the asset class of cryptocurrencies. In addition, following the data available on Bloomberg.com, not always an equal weighted portfolio

is a poor idea: in fact, the S&P 500 Equal Weighted Index outperform the traditional S&P 500, which is a market-cap-weighted index in the last fifteen years.

The list of the selectable asset classes is updating with daily frequency, in particular the category of the alternative investments, so I decided to pick the most used ones in similar studies and analysis, written in the literature paragraph. The five-asset class selected, shown in the table above, and their relative ten investment products, give a greater degree of diversification to the average European retail investor than investing in a single asset class. Diversifying through a wide range of investment strategies, styles, sectors, and geographical regions can help mitigate the occasional shocks that can affect a single asset class. In this case, considering the nature of cryptocurrencies, the diversification effect is much more helpful, and it fully counterbalances the high volatility of them. Diversifying also improves the potential for investing in a better performing asset class, while spreading the risk of investing in lower performing asset classes. On the other hand, it must be written that the diversification does not fully cover the investors' portfolio from the market risk. This one, also known as systemic risk, implies the threat of a breakdown of an entire system rather than only the failure of individual parts. In a financial framework, it means the risk of a collapse in the financial sector, due to connections within the financial system, resulting in a harsh economic recession.

In order to represent the five asset classes, the indexes shown in the table below have been selected. The weekly prices have been downloaded on Bloomberg platform for a time horizon of five years, from April 2016 to April 2021. The reason of this choice is justified from the fact that ten years ago cryptocurrencies were just introduced in the financial market (some of these coins included in the CRIX were not yet), while a less time horizon was not significant to see the effect of the crypto introduction. In Table 2 displayed below, there are also written the tickers of the indexes findable on Bloomberg platform.

Table 2: Selected indexes for each traditional asset class

Asset class	***Index***	***Mnemonic***
Equity (Developed)	MSCI Developed Markets	MXWO
Equity (Emerging)	MSCI Emerging Markets	MSDEEEMN
Debt (Sovereign)	FTSE World Government Bond	SBWGU
Debt (Corporate)	Bloomberg Barclays Global Aggregate Corporate	LGCPTRGH
Debt (High Yields)	Bloomberg Barclays Global High Yield	LG30TRUU
Hedge Funds	HFRX Global Hedge Fund	HFRXGL
Real Estate	MSCI World Real Estate	MXWO0RE
Private Equity	S&P Listed Private Equity	SPLPETRE
Commodities	Bloomberg Global Commodity	BCOM
Infrastructure	S&P Global Infrastructure	SPGTIND

2.3.2 Indexes of the asset classes

All the indexes that will represent the several asset classes, aim to offer a global coverage of the markets. Their prices are expressed in euros, and here it will be given a brief description of each index:

1. *MSCI Developed Markets Index*: it is developed by using MSCI (Morgan Stanley Capital International)'s Global Investable Market Index (GIMI) methodology, which is established to consider changes showing different conditions across regions, market cap segments, sectors and styles. Whitin this index there are two countries from the Americas, sixteen from the European and Middle East region, and five from the Pacific area.
2. *MSCI Emerging Markets Index*: it was launched in 1988. At that time, it included 10 countries with a weight of about 0.9% in the MSCI ACWI Index, while, at the moment, it captures 26 countries across the globe and has a weight of 12% in the MSCI ACWI Index. Whit respect to the MSCI DM Index, there are more countries from the Middle East and from Asia within this index. Built up according to the MSCI GIMI Methodology, the MSCI Emerging Market Index is designed to quickly replicate the growth of the emerging markets opportunity set. Another positive aspect of this index is that it helps investors meet global and regional asset allocation needs through an higher presence of different countries. The increasing economic volume and technological

impact of emerging markets are among the biggest influences shaping the world economic and financial market landscape. The continuing capital market liberalization and growing market availability in emerging markets are the reason of a probable reconsideration of the future of equity investing.

3. *FTSE World Government Bond Index (WGBI)*: it is a market capitalization weighted bond index composed by the government bond markets of the multiple countries. Country admissibility is determined based upon market capitalization and investability criteria. At the moment these requirements allow only European and advanced countries from the rest of the world (Japan, USA, Singapore…). The index comprises all fixed-rate bonds with a residual maturity of one year or longer and with amounts outstanding of no less than twenty-five millions of US dollars. Sovereign securities normally prohibit floating or variable rate bonds, US/Canadian savings bonds, and private placements.
4. *Bloomberg Barclays Global Aggregate-Corporate Index*: it is a leading measure of worldwide investment grade, fixed-rate corporate securities. This multi-currency benchmark includes obligations from developed and emerging markets issuers within the industrial, utility, and financial sectors.
5. *Bloomberg Global High Yield Index*: it is a flagship measure of the global high yield debt market with many currencies within it. The benchmark represents the sum of the US High Yield, the Pan-European High Yield, and Emerging Markets (EM) Hard Currency High Yield Indices. The high yield and emerging markets sub-components are mutually exclusive.
6. *HFRX Global Hedge Fund Index*: it is planned to be representative of the overall structure of the hedge fund world. It included all the qualified hedge fund with strategies that fall within four principal categories: equity hedge, event driven, macro/CTA (commodity trading advisor), and relative value arbitrage. The underlying components are asset weighted based on the distribution of assets in the overall hedge fund industry.
7. *MSCI World Real Estate Index*: it is a free float-adjusted market capitalization benchmark that contains large and mid-cap equity through twenty-three Developed Markets countries. All securities in the index are ranked in the Real Estate Sector according to the Global Industry Classification Standard (GICS). The GICS establishes a foundation for the creation of replicable, custom-tailored portfolios and allows meaningful comparisons of sectors and industries worldwide.

8. *S&P Listed Private Equity Index*: it tracks the performances of the most important listed private equity firms that encounter specific size, liquidity, exposure, and activity conditions. The benchmark is designed to offer tradable exposure to the top publicly listed companies that are active in the PE universe.
9. *Bloomberg Commodity Index*: it is part of a family of financial indexes meant to deliver liquid and diversified exposure through futures contracts to normal commodities, findable in everyday life. The main potential advantages of including commodities in a diversified financial portfolio include positive returns with the passage of time and more important, low correlation with equities and fixed income. These commodity index offers broad-based exposure to commodities, and there is another unique characteristics: there is not a predominant category or firm of commodities within the index. Rather than being pushed by micro-economic outcomes affecting one commodity market or sector, the diversified commodity exposure of this benchmark potentially lowers the overall risk in comparison with non-diversified commodity investments.
10. *S&P Global Infrastructure Index*: it is designed to track 75 companies from around the world selected to represent the listed infrastructure industry. At the same time, it also maintains liquidity and tradability. In order to create a well-diversified exposure, the index comprises three different infrastructure clusters: energy, transportation, and utilities.

In order to study the consequences of the introduction of cryptocurrencies in the reference portfolio, two additional "indexes" have been selected as a representation of the new asset class. The first one is Bitcoin, denominated in euro currency (BTCEUR). The reason stands behind the fact that it dominates the cryptocurrency exchanges and, as the graph in chapter one shows, it is the market leader in terms of market capitalization, with 55% of the total market cap, and trading volume, with 31 billion of euro out of 114 of the total market trading volume (Coinmarketcap platform). For this two indicators, in my opinion Bitcoin is the best proxy among the single currencies. The second index is a real one, the Cryptocurrencies Index (CRIX). It is a benchmark for the cryptocurrency market and was introduced at the end of July in 2014, so it is the only index that tracks data for a time horizon bigger than the one of this thesis (five years, 2016-2021). It was developed by the "Ladislaus von Bortkiewicz" Chair of Statistics at the Humboldt University of Berlin, Germany. In addition to this academic institution, the development of the price dataset was a joint effort organized with "SKBI" at the Singapore Management University and CoinGecko. The latter is the world's largest independent cryptocurrencies data aggregator, and it still provide the data for the computation of CRIX. The benchmark is a market

capitalization weighted index with real-time computation and a dynamic reallocation of the number of constituents. Currently, the CRIX contains 20 index crypto members. The top five since 2014 consists of Bitcoin, Link, Stellar, Ethereum and Compound Ether. All data are available on the Humboldt University website. It has to be specified that the CRIX is an academic initiative and so it is not tradable. Although there are this features, from a theoretical point of view, it is the best financial instrument to represents the crypto market and is considered as a true benchmark among both academics and traders (Trimborn, Li and Hardle, 2017). Moreover, it is adapted to the aspects of the crypto market. Among them, the most important are a very dynamic internal structure, the possibility of frequently vanishing and emerging coins and tokens, high volatility, necessity of constant monitoring, recalculation and so on.

The index is calculated following the Laspeyres derivation with regular rebalancing. The construction formula for the adjusted Laspeyres index is presented below in Equation 1:

Equation 1: Estimation of the Cryptocurrencies Index

$$\mathrm{CRIX}_t(k,\beta) = \frac{\sum_{i=1}^{k} \beta_{i,t_l^-} P_{it} Q_{i,t_l^-}}{Divisor(k)_{t_l^-}},$$

$$Divisor(k,\beta)_0 = \frac{\sum_{i=1}^{k} \beta_{i0} P_{i0} Q_{i0}}{\text{starting value}},$$

where P_{it} is the price of the asset i at time t, Q_{it} is the quantity of the asset i at time t, $\beta_{i,\,t}$ is the i-th asset's adjustment factor at time t, l is the adjustment factor and $t-l$ is the last time point of update (Trimborn and Härdle, 2018).

So, as it was in Table 2, the last two indexes for the cryptocurrencies are added and displayed below in Table 3:

Table 3: The Crypto asset class and its indexes

Asset class	***Index***	***Mnemonic***
Cryptocurrencies	Bitcoin Cryptocurrencies Index	BTCEUR CRIX

With a final number of twelve indexes, the cryptos portfolios have a relevant size and can be representative of the whole market.

Chapter 3: Empirical Analysis

3.1 Starting Data

3.1.1 Descriptive Data of the Asset Classes

The dataset for the calculation of the returns for each of the twelve indexes was quite entirely downloaded from Bloomberg.com. The only exception was for the Cryptocurrencies Index, whose data were collected from the website of the Humboldt University of Berlin. The prices under exam are the closing prices of total return price indexes for each of the asset classes. The majority of the calculation were made on Microsoft Excel, excluding the returns of CRIX estimated with Python and the kurtosis test made with R.

The descriptive statistics of each index are shown in Table 1. The total estimation time window consists of 262 weeks, with 261 return observations. In this way, the research has a significant number of observations for a five-year time (daily would have led to a huge dataset, while monthly would have been not significant). The sample period spans from April 22nd of 2016 to April 30th of 2021.

Table 4: Descriptive statistics of weekly returns for traditional and crypto asset classes, estimation window 2016-2021

	Obs	*Mean*	*Std. Dev.*	*Median*	*Min.*	*Max.*	*Skew.*	*Kurt.*
Developed	261	0.0021	0.0222	0.0035	-0.1205	0.0934	-1.1297	7.0232
Emerging	261	0.0022	0.0224	0.0032	-0.1000	0.0606	-0.6902	2.4274
Sovereign	261	0.0002	0.0074	0.0005	-0.0228	0.0213	-0.2329	0.4728
Corporate	261	0.0007	0.0080	0.0011	-0.0761	0.0420	-4.3233	44.9064
High Yields	261	0.0013	0.0118	0.0016	-0.1067	0.0519	-4.0492	40.6603
Private Equity	261	0.0034	0.0320	0.0035	-0.2064	0.2069	-0.7470	16.8582
Hedge Funds	261	0.0001	0.0130	0.0011	-0.1298	0.0350	-3.8412	37.3016
Real Estate	261	0.0008	0.0262	0.0011	-0.1813	0.1650	-0.7865	16.3287
Commodities	261	0.0001	0.0168	0.0005	-0.0812	0.0401	-0.5645	1.9436
Infrastructure	261	0.0006	0.0244	0.0013	-0.1905	0.1227	-2.1068	20.0117
Bitcoin	261	0.0236	0.1070	0.0126	-0.3930	0.4337	0.2185	1.5453
Crix Index	261	0.0291	0.1085	0.0181	-0.2700	0.5632	0.8401	3.1150

If we exclude the crypto indexes, the private equity index shows the highest expected return (0.0034), but, at the same time the greatest standard deviation (0.032). On the contrary, the lowest mean value was obtained for the hedge funds and commodities indexes (both values are

0.0001). The notable aspect is that both do not have the lowest standard deviation. In fact, sovereign bonds gain the title of most riskless index ($\sigma = 0.0074$), followed by the other bond securities: corporate in the second position (0.008) and high yields (0.0118) as the third, with an expected return (0.0013) more than six times higher than sovereign (0.0002). The real estate index, one of the elements of the alternative investments category, shows a relatively low mean value (0.0008), compared with the risk value that is quite high (0.0262).

Writing about the asset class in general, the equity class shows the best performance behind the cryptocurrencies, with a more than acceptable return/risk of both developed markets (DM) and emerging markets (EM). Both have an expected return (0.0021 DM and 0.0022 EM) lower than only private equity, with risk values (0.0222 DM and 0.0224 EM) that is nearly in the average of the standard deviations (0.019).

For what concerns the last asset class to be analysed, it has been showed that cryptocurrencies overcome with a wide margin the other classes in terms of expected return. Both Bitcoin and CRIX have a mean value nearly ten times the first of the traditional asset classes, private equity. This great performance in term of expected return is justified by the high standard deviation relative to the other classes. However, this measure is only three time the one of private equity, so the overall performance of cryptocurrencies is much better than the traditional asset classes.

Regarding the third and the fourth moment, the overall situation is more complex. For what concerns the skewness, the value of a symmetric normal distribution should be equal to 0. Furthermore, the kurtosis index of the same kind of distribution should have a value of 3. Nevertheless, is quite difficult to obtain these exact value, considering that the values under analysis belong to real financial indexes.

As it can be seen in Table 1, all the traditional classes present a negative value in the skewness column, so they all have their distribution moved to the left side. However, they all have statistically significant values. It is curious to see that within the debt securities asset class, there is both the most and the less symmetric distribution of the traditional. In fact, the former is the sovereign bond index, with a value of -0.23, while the latter is the corporate bond index with a skewness of -4.32. Theoretically writing there are three different ranges

1. if the value is between 0.5 and -0.5 the distribution is approximately symmetric
2. if the value is among +/-0.5 and +/-1 the distribution is relatively skewed
3. if the skewness is more than 1 or less than -1, the distribution in highly skewed.

Considering the other indexes, most of their skewness fall in the second group, so with a distribution that is moderately skewed. The exceptions are infrastructure, developed markets, hedge funds and high yields bonds which have all less than -1 as a skewness value.

On the other hand, cryptocurrencies indexes present a positive value, so it means that their distribution is towards the right side. In particular, Bitcoin has the most symmetric value of the basket of indexes with a value of 0.22 and, consequently, it falls within the "approximately symmetric" group.

For what concerns the kurtosis, it is clearly viewable that most of the values in its column are far from the one of a normal distribution, which is 3. This number comes from the following formula:

$$t_k = k - 3 \quad (3.1)$$

with t_k that is the kurtosis coefficient, while k is the kurtosis showed in Table 1. On the basis of Equation (3.1) the distribution of a sample data is divided, as the skewness, in three groups:

1. if the kurtosis coefficient is higher than 0, the distribution is leptokurtic (highly concentrated around the average value)
2. if it is equal to 0, the distribution is mesokurtic
3. if it is lower than 0, the distribution is platykurtic (lowly concentrated around the mean value)

From Table 1, there are only three indexes of the traditional classes with a platykurtic distribution, respectively emerging markets, sovereign bonds, and commodities. All the others fall into the first group, with hedge funds, corporate and high yields bond that have values very far from the mesokurtic threshold. Regarding the cryptocurrencies' indexes, Bitcoin has a platykurtic distribution while CRIX is the index nearest to the normal value.

Due to the distance from the mesokurtic threshold, a test for normality is conducted on R, in order to see if the indexes can be utilized in the research. The formula, that was run on the software, is the following:

$$t_k = \frac{k-3}{\sqrt{\frac{24}{T}}} \quad (3.2)$$

with t_k and k that represent the same of Equation 3.1, while T is the number of the total return observations, in this case 261. Fortunately, the result obtained was positive, because all the eleven indexes tested are statistically significant, so they are able to be within the research.

In Table 2, the correlations among the asset classes are displayed. In order to be the perfect diversification instrument, the value of the indexes under exam should be equal to 0. More than the threshold the value is considered positively correlated, while if it is under, then it is negatively correlated.

Sovereign Bond is the index with the best result in terms of correlation. In fact, it has a series of positive values close to 0, with the exception of the equity class indexes and the other bond indexes, which assumes negative values. In addition, only the result with the hedge fund index reaches the normal correlation threshold (from +/- 0.3 to +/- 0.7).

Table 5: Correlation matrix of the weekly returns of all the asset classes, estimation window 2016-2021

	Devel.	*Emerg.*	*Sover.*	*Corp.*	*High Yields*	*Private Equity*	*Hedge Funds*	*Real Estate*	*Comm.*	*Infras.*	*BTC*	*CRIX*
Developed	1.0000											
Emerging	0.7498	1.0000										
Sovereign	-0.0719	-0.0810	1.0000									
Corporate	0.3707	0.2959	0.2480	1.0000								
High Yields	0.6338	0.5510	-0.2450	0.7770	1.0000							
Private Equity	0.8766	0.6334	-0.1970	0.4896	0.7594	1.0000						
Hedge Funds	0.4855	0.4094	0.3316	0.0002	0.0217	0.3018	1.0000					
Real Estate	0.7701	0.5468	0.0589	0.5897	0.6827	0.8302	0.2413	1.0000				
Commodities	0.5163	0.4998	0.0784	0.0776	0.2632	0.3963	0.4428	0.3854	1.0000			
Infrastructure	0.8361	0.6223	0.0139	0.5521	0.7288	0.8470	0.3263	0.8579	0.4689	1.0000		
Bitcoin	0.1426	0.1232	0.0055	0.0694	0.1380	0.1218	0.0399	0.0523	0.1404	0.1691	1.0000	
Crix Index	0.1892	0.1763	-0.1329	0.1535	0.2535	0.2099	-0.0212	0.1325	0.1450	0.2050	0.7660	1.0000

On the other hand, for the worst diversification instrument, there are three indexes that showed bad performances. These three are developed market, private equity, and commodities. All of them have four values out of eleven that reach the threshold of strong correlation (among +/-0.7 and +/-1), with some of the remaining seven values of these three indexes that do not fall within this classification for a few units. The other indexes show weak or normal correlation values with the other indexes, but a strong one with the indexes within the same class. The only exception is, again, the sovereign bond index, which has a weak correlation with the other indexes within the debt securities asset class. In theory, within the alternative investments, there is not a strong correlation among private equity and commodities. However, the value is still in the normal threshold, therefore it is not considered as an exception.

As a huge surprise, Bitcoin and the cryptocurrency index show brilliant performance, since the correlation values with the other indexes are all in the weak correlation threshold (between 0 and +/- 0.3). Even if both the components of the crypto asset class show an high standard deviation, and, as a consequence, be considered a risky stand-alone investment, they could be a great diversification vehicle. As for the traditional asset classes, the correlation between Bitcoin and CRIX is very high. This is quite normal, also because Bitcoin is one of the components of CRIX.

3.1.2 Reference portfolio analysis

Table 3 shows the reference portfolio (mentioned in Chapter 2, Paragraph 2.3) and its relative weights, expected returns and standard deviations. The four column represent the initial stage with equal weights, the maximization and minimization problems' weights.

The equal-weighted reference portfolio has an expected return of 0.117% and a standard deviation of 1.424%. If these data are compared with the average of the single indexes' mean and risk values, the former is the same, but the latter is lower. This is a consequence of the diversification. In fact, the formula of the standard deviation of the portfolio takes in consideration also the covariance among the single asset class. This last measure is presented in the dispersion matrix, within the Appendix A.

Table 6: output of the mean-variance analysis of the reference portfolio without cryptocurrencies

	Ref. Port.	***Max μ***	***Min σ***	***Max SR***
Developed	10.00%	0.00%	0.00%	0.00%
Emerging	10.00%	23.12%	12.59%	9.01%
Sovereign	10.00%	24.25%	42.52%	48.74%
Corporate	10.00%	0.00%	0.00%	0.00%
High Yields	10.00%	28.01%	35.33%	37.83%
Private Equity	10.00%	24.62%	9.55%	4.41%
Hedge Funds	10.00%	0.00%	0.00%	0.00%
Real Estate	10.00%	0.00%	0.00%	0.00%
Commodities	10.00%	0.00%	0.00%	0.00%
Infrastructure	10.00%	0.00%	0.00%	0.00%
Σ Ref. Port. Wi	100%	100%	100%	100%
Bitcoin	-	-	-	-
Crix Index	-	-	-	-
Σ weights	*100%*	100%	100%	100%
Exp. Return	0.117%	0.178%	0.117%	0.096%
Std. Deviat.	1.424%	1.424%	0.872%	0.714%
Sharpe ratio	0.08749	0.1304	0.1429	0.1454

A better mean-variance result is obtained in all the other three distributions of weights. One aspect that is clearly viewable regard the asset allocation. Indeed, it is quite extreme because 6 indexes out of ten are left out in the maximization of expected return and Sharpe ratio, and in the minimization of the standard deviation. Only emerging markets, sovereign and high yield bonds, and, lastly, private equity are taken into consideration for the estimation of the portfolios. Considering the correlation values in Table 2, the only problem in terms of diversification is the relationship between high yield bonds and private equity, because they are highly correlated (0.7594).

The first of the portfolios built for the problems of constrained optimization (max μ, min σ and max SR) is the most balanced. All the four indexes have a weight within a range between 23.12% to 28.01%. In this case, the same standard deviation of the equal-weighted portfolio is added to the other two fixed constraints (total wealth invested and prohibition of short sales).

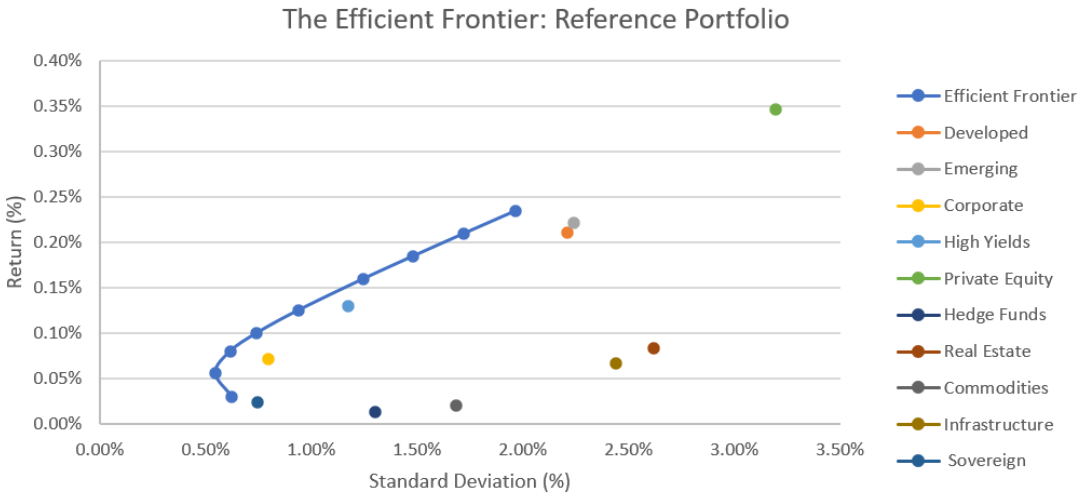
In the other two optimized portfolios, the weights of private equity (PE) and emerging markets (EM) decrease, while the opposite situation happens to sovereign and high yield bonds. The reason is quite obvious, and it comes from Table 1. In fact, both PE and EM have two of the highest expected returns, matched with related high risk values. On the contrary, the two bond indexes have two of the best standard deviation values.

It is interesting to see that in the minimized risk portfolio, the Sharpe ratio is higher than the one of the maximized expected return portfolio. This outcome is obtained due to the fact that in the two ratio, the first has higher number (0.178% and 1.424% against 0.117% and 0.872%). Even in this case, the weights are subject to the same mean value of the reference portfolios and the two fixed constraints.

The third portfolio, subject to the maximization of the Sharpe ratio, does not have any additional constraints to the wealth management and the short selling ones. In this case, the Sharpe ratio obtained is just a little bit higher than the one found in the previous portfolio. The expected return is the smallest of the four portfolios (0.096%), but, at the same time, also the standard deviation is the lowest value obtained (0.714%). Here there is the maximum disparity in the asset allocation for what concerns these four portfolios: emerging markets and private equity only have 13.42% of the total weights, with the remaining 86.58% shared between the two debt securities indexes.

In Figure 1, shown below, the efficient frontier for the reference portfolio is displayed, together with the combinations of risk/return of the single asset classes.

Figure 1: the efficient frontier of the reference portfolio with the single asset classes



For the plotting of the line, nine additional portfolios were constructed, which are the nine blue dots on the frontier. The starting point of the frontier is the minimum variance portfolio, MVP, (the dot that stands on the extreme left on Figure 1), obtained with the minimization of the standard deviation, subjected to only the two fixed constraints. The other eight portfolios are constructed with the same optimization problem, but for each portfolio there is a certain already given expected returns. The calculations are presented in Appendix A.

The private equity index is the only one that is quite distant from the other elements of the graph, because its descriptive statistics put it on the far right. It is also the nearest index to the frontier, followed by high yield and corporate bonds.

3.2 Development of the Analysis

3.2.1 Portfolio optimizations

The outcomes of the asset allocation process with the adjunct of bitcoin and the cryptocurrencies index are displayed in Table 4. Starting from the equal weight reference portfolio with ten indexes, the result is the nine portfolios shown below, with six of them composed by eleven indexes and three of them with twelve indexes. All the portfolios are subject to the two fixed constraints (overall wealth used and block of short selling), with the addition of a third restriction for each of the three optimization problems.

As it was in Table 3, the first thing that grabs the attention is the distribution of the weights. Here as well the asset allocation is very harsh: the minimum number of index with a percentage higher

than zero is four out of eleven indexes, while the maximum is just six in the portfolios with twelve indexes. Sovereign and high yield bonds are the only two elements present in all the nine portfolios, due to their optimal characteristics, also in terms of diversification.

For what concerns the maximization of the expected return subject to a fixed standard deviation, an outcome better than the reference portfolio (0.117%) is obtained in all the three solutions (0.347% and 0.407%). Therefore, it can be written that the introduction of both bitcoin and CRIX (alone or together) increases the performance of a traditional portfolio. The sum of the traditional weights decreases after every new introduction: from just Bitcoin (88.89%) to both the crypto indexes (87.99%). The four traditional indexes are the same of the optimization problem for the reference portfolio. However, this time the weights are much more concentrated in the bond asset class, in particular when there is the addition of both bitcoin and CRIX with the overall weights of EM and PE is only 9.60%. To conclude, it seems that second (CRIX) and third (BTC + CRIX) option shows the same expected return, but due to the different Sharpe ratios, it can be stated that the third option has a higher mean value.

Table 7: Results of the asset allocation after the addition of cryptocurrencies to the reference portfolio

	Max μ BTC	***Max μ CRIX***	***Max μ BTC+CRIX***	***Min σ BTC***	***Min σ CRIX***	***Min σ BTC+CRIX***	***Max SR BTC***	***Max SR CRIX***	***Max SR BTC+ CRIX***
Developed	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Emerging	7.25%	5.29%	5.71%	0.03%	0.00%	0.00%	6.70%	5.87%	5.77%
Sovereign	46.10%	62.39%	61.11%	60.25%	63.54%	63.54%	47.30%	61.98%	61.09%
Corporate	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
High Yields	30.13%	16.01%	17.27%	35.45%	31.14%	31.14%	31.30%	16.50%	17.03%
Private Equity	5.41%	4.49%	3.89%	0.00%	0.00%	0.00%	4.46%	3.86%	3.98%
Hedge Funds	0.00%	0.00%	0.00%	1.89%	3.23%	3.23%	0.00%	0.00%	0.00%
Real Estate	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Commodities	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Infrastructure	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Σ Ref. Port. Wi	88.89%	88.19%	87.99%	97.63%	97.91%	97.91%	89.76%	88.22%	87.86%
Bitcoin	11.11%	-	1.02%	2.37%	-	-	10.24%	-	1.03%
Crix Index	-	11.81%	10.99%	-	2.09%	2.09%	-	11.78%	11.11%
Σ weights	100%	100%	100%	100%	100%	100%	100%	100%	100%
Exp. Return	0.347%	0.407%	0.407%	0.117%	0.117%	0.117%	0.324%	0.406%	0.411%
Std. Deviat.	1.424%	1.424%	1.424%	0.622%	0.593%	0.593%	1.330%	1.420%	1.436%
Sharpe ratio	0.2494	0.2910	0.2914	0.2003	0.2099	0.2099	0.2495	0.2910	0.2914

Regarding the minimization of the risk with a fixed expected return of 0.117%, the first thing that comes to the eyes is the sum of the weights of the traditional indexes: nearly the total of the wealth invested. The reason for this is that the indexes of the cryptocurrencies class have an high standard deviation compared to the other classes, so only around 2% is allowed in order to keep

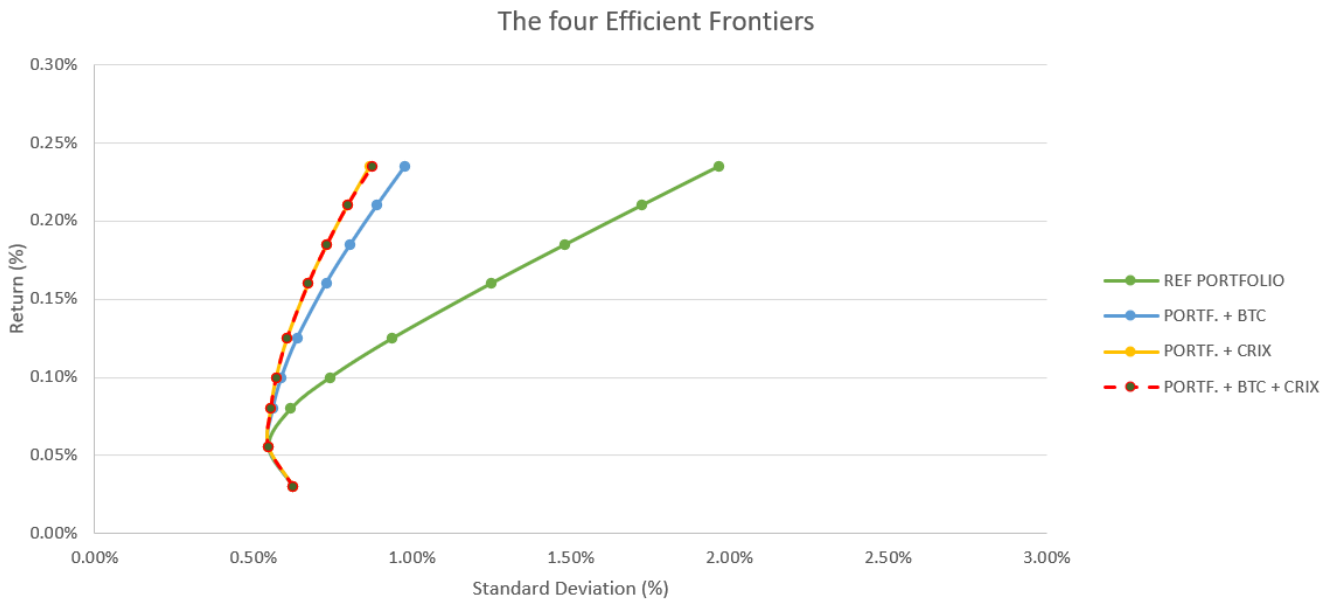
the expected return to the level of reference portfolio. In this optimization problems, the debt securities class holds more than the 97%, due to their relatively low risk characteristics, considering that the small amount of cryptos offset the mean value of the portfolios. This outcome leads to a better performance in terms of risk compared to the reference portfolio (the worst is 0.622% against the original 0.872%). For what concerns the other indexes part of this asset allocation, only hedge funds have been used, in a quite small percentage that goes from 1.89% with bitcoin to 3.23% in the other two cases (there is also the emerging markets index in the first case, but the percentage is pretty irrelevant, 0.03%). It has to be said that bitcoin does not have any particular risk benefit when there is the possibility of adding both it and CRIX. In fact, the crypto share remains the same with only CRIX or with either of the two (2.09%). In this way the standard deviation obtained (0.593%) is a better result than the one with only bitcoin (0.622%).

Lastly there are the three portfolios subjected to the maximization of the Sharpe ratio. As it was for the reference portfolio in Table 3, there are not additional constraints apart from the original fixed two. The indexes used are the same of the expected return problem, but in this case, with the addition of both bitcoin and CRIX, the highest percentage of cryptocurrencies within the portfolio is reached (12.14%). By making a comparison with the other optimization problems, it can be seen that the result of this third typology is pretty equal to the Sharpe ratio obtained in the maximization of the expected return (0.29104 vs 0.29103 with CRIX and 0.29141 vs 0.29139 with both BTC and CRIX). So, this means that the founded value in the third problem is a valid alternative also for the solution of the first problem, even if in that case there is also the constraint of the fixed standard deviation.

To summarize, the best performances are obtained when both bitcoin and the cryptocurrencies index are included in the reference portfolios for every optimization. In particular, CRIX overcomes bitcoin. Even if the latter has a better correlation relationship with the other indexes, the former has a better risk/return profile.

In Figure 2 the efficient frontiers for each of the four cases of the reference portfolios are displayed. The calculations for the lines obtained after the introduction of cryptocurrencies are available in the Appendix A, while their graphs are in Appendix B. The dots on the lines are the same of the frontier in Figure 1.

Figure 2: The efficient frontiers of the reference portfolio with and without cryptocurrencies

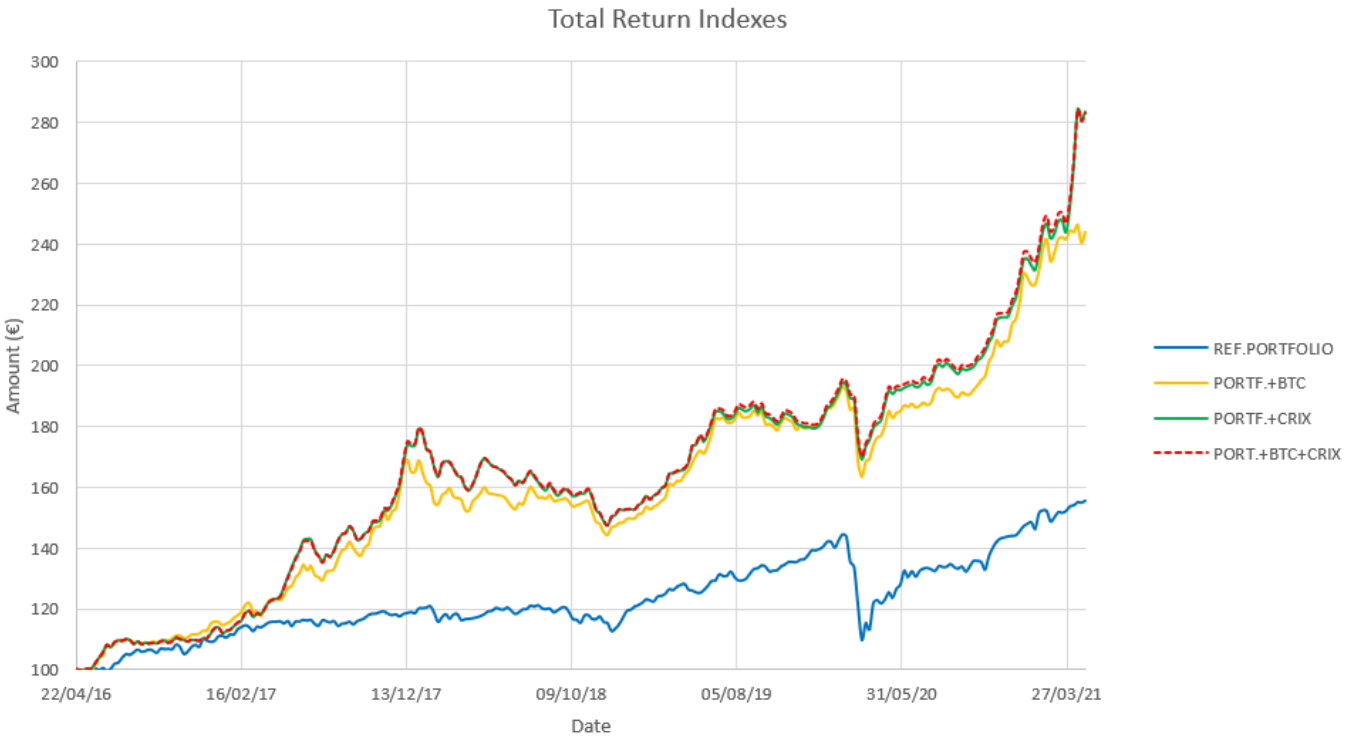


The returns are fixed while the standard deviations are estimated trying to solve a problem of minimization. The minimum variance portfolio is the same for all the frontiers, due to the characteristics of the crypto indexes, which add nothing to the already available MVP. However, the inclusion of both kind of cryptocurrencies generates a movement of the efficient frontier to the left and upwards. So, this means that a higher return is reached at every given level of portfolio standard deviation.

Overall, the best performance is obtained when both BTC and CRIX are added. Nevertheless, the difference with the solely CRIX addition is quite irrelevant, as it can be seen among the lines.

Figure 3 shows a small case I wanted to develop in order to see in practice the effect of cryptocurrencies. The hypothesis at the basis of the experiment is that at the beginning of my sample period (April 22nd of 2016) the subject of this analysis (average European retail investor) has an overall wealth of 100 euros. Four typologies of portfolios multi asset class are submitted to him/her, one with only traditional assets and the others with the cryptocurrencies. The graph shows the performances of the four portfolios and the wealth he/she could have gained at the end of the sample period (April 30th of 2021).

Figure 3: Total return indexes with and without cryptocurrencies



The total return obtained by the four portfolios are the followings: +9.22% for the reference portfolio, +19.55% with the addition of bitcoin, +23.18% with the introduction of CRIX, and +23.22% when both the crypto indexes are included.

It is interesting to see that the portfolio solutions which considers also the cryptocurrencies have for most of the time window the same trend in the performances. The only periods where an important difference is clearly viable are two. The first one is after the first half of 2017: the reason is due to the "boom and bust" event after the Bitfinex fraud and the introduction of features for the cryptos. The second one occurred at the end of 2020, when the notoriety of cryptocurrencies, together with the major development of many online trading platforms, increase the value of their market capitalization. There is also a point where all the portfolios suffer a considerable loss: the outbreak of the Coronavirus pandemic. From Figure 1, it can be stated that the addition of cryptocurrencies contained the damages. An hypothesis may be that the traditional assets reflect the consequences on the real economy, while the cryptocurrencies are not affected. During a period in which the world production stopped and many industries suffered huge losses, cryptos were a sort of "safe heaven".

The last aspect regards the relationship between the portfolio with bitcoin and the two subject to the introduction of CRIX. They maintained the same performance for most of the time window,

but in 2021, a notable difference occurred between the two types. The cause is the composition of CRIX: it means that, starting from 2021, the other cryptocurrencies gained popularity and helped the index to achieve better visible returns than the bitcoin alone.

3.2.2 VaR results

Table 5 shows the output of the historical approach, one of the two typologies used for the estimation of the Value-at-Risk. The chosen fair values are the last observations of the sampling period, as for the assumptions of this approach. The confidence interval is 99%, so the alpha is 2.326. The portfolios used for the VaR estimation are the one subject to the maximization of the expected return, in order to have the same standard deviation for all.

A remarkable aspect regards the fact that the reference portfolio, composed by traditional index characterized by low volatility, has the highest potential loss percentage (7.05%). This former measure is the result of the division between the VaR and the fair value considered. In addition to this, the VaR of the first portfolio is the greatest value of the four, even if the fair value is nearly the half of the others.

Table 8: Results of the VaR estimation with the Historical Approach

	Ref. Portf.	***Portf.+ BTC***	***Portf.+ CRIX***	***Portf.+ BTC+CRIX***
Fair Value	155.41	244.22	283.54	284.01
Confid. Int.	0.99	0.99	0.99	0.99
Potential Loss	7.05%	3.25%	3.21%	3.18%
Value at Risk	10.96	7.94	9.10	9.02

For what concerns the portfolios with cryptos, the percentages of the potential loss are quite the half of the traditional portfolios. Even in this case, CRIX shows that it brings more advantages in term of risk, compared to the bitcoin. In fact, the percentages of it (3.21% with only CRIX, 3.18% by adding both) are slightly lower than the one of bitcoin (3.25%). Since that the fair values are not so different like in the case of the reference portfolio, this CRIX benefit is not observed in the VaR too (the portfolio with CRIX alone register the highest value with 9.10).

In addition, the number of observations when the return obtained by each portfolio is worse than the potential loss is estimated for the back testing procedure. In all the four portfolio, this scenario happened twice. For the reference portfolio and the two solutions after the inclusion of

CRIX, it occurred at the beginning of the Covid-19 pandemic (weekly returns of 13/04/20 and 20/04/20). Instead, the portfolio with the addition of Bitcoin shared only the observation of 13/04/20 with the others, while the second time it exceeds the VaR was observed in 02/02/18, back to the first crypto crisis described in Chapter 1, Paragraph 1.1.

In the second approach, the parametric one, the differences seen in the previous table are not remarkable. Table 6 shows the outcome of the Value-at-Risk calculation, while the other estimated data are available in Appendix A. Even in this case, the portfolios considered are the ones that solves the return optimization problem and the confidence interval is 99%.

The VaRs of the single asset classes were the first step of the process. The decision of which moment was the most suited for the calculations went to the last observation. There were two reasons for this choice. In order to make a comparison of results with the historical approach, the same observation was the right way. In addition, these values are the highest encountered in the five-year time window, so it is the highest amount to which deduct the highest possible loss in a 99% confidence interval.

Table 9: Results of the VaR estimation with the Parametric Approach

	Ref. Portf.	***Portf.+ BTC***	***Portf.+ CRIX***	***Portf.+ BTC+CRIX***
Fair Value	155.41	244.22	283.54	284.01
Confid. Int.	0.99	0.99	0.99	0.99
Potential Loss	3.3179%	3.3176%	3.3180%	3.3175%
Value at Risk	5.16	8.10	9.41	9.42

After that, the following passage was the product between these VaRs obtained, the correlation matrix and the transposed vector of VaR, in order to complete the members of the formula explained in Chapter 2.

The highest Value-at-Risk is the one of the reference portfolio after the addition of bitcoin and CRIX (9.42), while the lowest is obtained by the traditional asset classes with quite the half of the value (5.16). However, it is all in line with the starting fair values, so the difference among the portfolios is proportional to them. In fact, this time, the percentages of the potential losses are nearly the same in all the four scenarios. It was necessary to increase the digits, in order to establish which portfolio solution had the highest one.

The difference is quite irrelevant but the best portfolio for the parametric approach is the one with BTC and CRIX. However, compared to the historical approach results, this time bitcoin has

the role of asset which reduces the potential loss. Indeed, the portfolio with CRIX alone is the worst one, while the one subject to the introduction of bitcoin is the second best portfolio.

The reason why the results obtained differs from one approach to the other could be due to the assumptions of the parametric approach, mentioned in Chapter 2, Paragraph 2.2. In fact, as already shown in the descriptive data section of this chapter, all the indexes used in this analysis do not show a normal distribution, but a fat-tails one, because are all real indexes. This conclusion makes the results of the parametric approach not so significant for the analysis. On the contrary, for the historical approach, the kind of distribution has not the same relevance for the output obtained.

3.3 Performance of the Portfolios

The two typologies of regression analysis are used in order to study whether the constructed portfolio leads to a positive or negative result. The four portfolios' returns are regressed with the MSCI All Country World Index, which was chosen as the market benchmark for the reference portfolio. In the in-sample analysis the returns of this index used are 261, while in the out-of-sample one just 132 returns are considered. The focal point of this paragraph is on the alphas obtained in the analysis, which are the interception on the vertical axis of the regression line. The analysis is conducted in order to find if the alphas are positive or negative. In the first case, it means that the reference portfolio has outperformed the market, while in the second case it has underperformed the benchmark. Additional importance must be given also to other two statistical measure: T-stat and p value. In fact, both tell if the results obtained are near or far from the values which would give significance to the model. In the optimization process for the estimation of the optimal weights, the risk values are fixed to the ones obtained by the reference portfolios (both analysis have its own reference portfolio since the time window is different, and they are available in Appendix A) to make fair comparison between the portfolio expected returns.

The results are shown in Table 7, divided between the in-sample and the out-of-sample analysis, and there are some aspects to be discussed after reading the table.

Starting from the in-sample one, all the alphas of the four portfolios have a value higher than 0. The highest one was obtained by the reference portfolio with the addition of CRIX (0.00356), while the lowest was the one composed by only the traditional asset classes (0.00055). This outcome means that in the selected time period the inclusion of cryptocurrencies indexes leads to

a better performance than only traditional, compared to the market. In addition, the alpha of the reference portfolio without cryptos is the only one that is not statistically significant, considering that the T-stat (1.51) is under the thresholds with a confidence interval of 95% (1.96) and 90% (1.64). This means that its data are not functional for the model.

Table 10: Results of the two regression analysis of the reference portfolio with or without cryptocurrencies

Type of analysis	*Perf. Measure*	*Ref. Portf.*	*Ref. Por.+ BTC*	*Ref. Por.+ CRIX*	*Ref. Por.+ BTC + CRIX*
IN-SAMPLE ANALYSIS	*Alpha*	0.00055	0.00300	0.00356	0.00355
	T-stat	1.50949	3.58789	4.36347	4.36115
	P value	0.13239	0.00040	0.00002	0.00002
	Beta	0.60084	0.22740	0.25684	0.25807
	Observations	261	261	261	261
	Adj. R squared	82.75%	11.42%	14.81%	14.95%
OUT-OF-SAMPLE ANALYSIS	*Alpha*	0.00052	0.00160	0.00212	0.00201
	T-stat	0.42768	1.42370	1.73774	1.70584
	P value	0.66959	0.15693	0.08462	0.09043
	Beta	0.90616	0.63216	0.62354	0.62335
	Observations	132	132	132	132
	Adj. R squared	74.34%	62.53%	57.95%	59.61%

The adjusted R^2 percentages are a more accurate version of the normal R^2, because the formers take in consideration more variables than the latter, so it is a. However, they both shows the correlation between the value of the model and the selected benchmark. Regarding the value obtained, only the reference portfolio (82.75%) has the returns nearly aligned with the ones of the MSCI ACW Index. All the portfolios including cryptos have a low correlation with the market benchmark, because the value obtained are 11.42% with bitcoin, 14.81% with CRIX and 14.95% with both of them.

Regarding the forecasted typology of analysis, the out-of-sample one, the results obtained are worse than the previous case. There are two possible reasons for this outcome: the first one is the number of observation that is lower than the in-sample and thus could be not enough for the model, while the second is that the data used are only a forecast of the future. The alphas are still all higher than 0, so it means that also the predicted values outperform the benchmark of the market. Even in this case the first value is obtained by the portfolio subject to the introduction of CRIX (0.00212), and the last one is achieved by the basic reference portfolio (0.00052). The problem arises for the values obtained in the row of the T-stat. in fact, every typology of reference portfolio does not reach the threshold of 95% confidence interval: the traditional has the lowest value with 0.43, followed by the one with bitcoin (1.42). However, the other two

portfolios are at least statistically significant at a confidence interval of 90%: the reference portfolio with CRIX has a T-stat of 1.73, while the last solution achieved a value of 1.71.

Conclusion

The results obtained led to some conclusion, regarding the initial question of this thesis. In the time window of this analysis the reference portfolio after the introduction of the crypto indexes showed better performances than the one build with only the traditional asset classes. In details, a portfolio composed by the five traditional classes, well-represented by the indexes chosen, and both Bitcoin and the Cryptocurrencies Index is the best solution possible.

In the optimization problems, the crypto portfolios obtained the following outcomes, with respect to the traditional portfolio:

1. A higher expected return, given a fixed level of standard deviation. This result is due to the fact that both the crypto elements showed returns ten times higher than traditional indexes. Therefore, a small contribution from cryptos by fixing the sigma value leads to a better output.
2. A lower standard deviation, given a fixed level of the mean value. Even if crypto indexes show a high risk value with respect to the other traditional asset classes, it was possible to obtain this result too. The reason is that the above mentioned expected return of cryptos allows the distribution of the majority of weights to the debt securities indexes (that are the safest) and, at the same time, maintain the given level of expected return.
3. A higher Sharpe ratio, with no constraints about the mean and the risk values, obtained thanks to the reasons explained in the previous two points.

For what concerns the efficient frontiers, as showed in Chapter 3, the inclusion of a small percentage of both Bitcoin and CRIX allows the reference portfolios to have lower standard deviations for seven out of nine fixed level of expected return (the minimum variance portfolio and the first portfolio gave the same results).

Even the results of the Value-at-Risk analysis are in favour of the introduction of cryptocurrencies. In fact, the historical approach showed that the biggest VaR was obtained by the traditional portfolio, when the highest value of each portfolio is reached within the time window of the analysis. This output was achieved also in term of percentage with the measure of potential loss. However, in the parametric approach the differences between the portfolios were quite slight, but even in this case the portfolio with CRIX and Bitcoin obtained better results than the traditional one.

Lastly, the regression analysis confirmed the results of the previous steps. In fact, the alphas of the crypto portfolios obtained an higher value than the one of the traditional one, in both the in-sample and the out-of-sample analysis. Moreover, the combinations containing CRIX are the only statistically significant to a minimum confidence interval of 90% (in the in-sample also at a 95% confidence interval). On the contrary, the traditional portfolio does not reach the thresholds in any kind of regression analysis.

In conclusion, for the future of cryptocurrencies there are two possible scenarios. It is clear that, in order to prosper and reach the first scenario, this asset class needs a real regulation, and it cannot continue to be anchored to the law of demand and supply, as it should be in Nakamoto's plan. More and more financial institutions would be willing to invest in cryptocurrencies, but, at the same time, they are frightened by the possibility of losing their funds in just a day. The time period of this analysis is quite significant for the results achieved because it comprises a period of crisis for cryptocurrencies (2017-18) and a pandemic that affected all the world, and consequently the real economy. These results tell that a small amount of cryptocurrencies would improve the performances of a multi asset portfolio, and it would be an excellent diversification instrument. The second scenario would be a future disappearance of cryptos, due to a decreasing appeal. The fact that one day an investor can obtain a +25% and the day after a -15% will exhaust the market in the future.

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Appendix

Appendix A

Table 1: Dispersion matrix of the weekly returns of all the asset classes, estimation window 2016-2021

	Devel.	*Emerg.*	*Sover.*	*Corp.*	*High Yields*	*Private Equity*	*Hedge Funds*	*Real Estate*	*Comm.*	*Infras.*	*BTC*	*CRIX*
Developed	0.00049											
Emerging	0.00037	0.00050										
Sovereign	-0.00001	-0.00001	0.00006									
Corporate	0.00007	0.00005	0.00001	0.00006								
High Yields	0.00016	0.00015	-0.00002	0.00007	0.00014							
Private Equity	0.00062	0.00045	-0.00005	0.00012	0.00029	0.00102						
Hedge Funds	0.00014	0.00012	0.00003	0.00000	0.00000	0.00013	0.00017					
Real Estate	0.00045	0.00032	0.00001	0.00012	0.00021	0.00070	0.00008	0.00069				
Commodities	0.00019	0.00019	0.00001	0.00001	0.00005	0.00021	0.00010	0.00017	0.00028			
Infrastructure	0.00045	0.00034	0.00000	0.00011	0.00021	0.00066	0.00010	0.00055	0.00019	0.00059		
Bitcoin	0.00034	0.00029	0.00000	0.00006	0.00017	0.00042	0.00006	0.00015	0.00025	0.00044	0.01142	
Crix Index	0.00045	0.00043	-0.00011	0.00013	0.00032	0.00073	-0.00003	0.00038	0.00026	0.00054	0.00887	0.01173

Table 2: Minimization of the standard deviations at different target returns to construct the efficient frontier of the Ref. Portfolio

	N°1	***Min Var Port***	***N°2***	***N°3***	***N°4***	***N°5***	***N°6***	***N°7***	***N°8***
Developed	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Emerging	0.00%	0.00%	6.23%	9.67%	13.98%	20.02%	24.33%	28.64%	32.94%
Sovereign	63.21%	63.52%	53.57%	47.59%	40.12%	29.65%	22.18%	14.71%	7.23%
Corporate	17.20%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
High Yields	0.00%	30.87%	39.78%	37.37%	34.36%	30.14%	27.13%	24.12%	21.11%
Private Equity	0.00%	0.00%	0.43%	5.37%	11.54%	20.19%	26.36%	32.53%	38.71%
Hedge Funds	8.76%	4.31%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Real Estate	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Commodities	10.83%	1.29%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Infrastructure	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Σ Ref. Por. Wi	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
Bitcoin	-	-	-	-	-	-	-	-	-
Crix Index	-	-	-	-	-	-	-	-	-
Σ weights	100%	100%	100%	100%	100%	100%	100%	100%	100%
μ	0.0300%	0.0561%	0.0800%	0.1000%	0.1250%	0.1600%	0.1850%	0.2100%	0.2350%
σ	0.6229%	0.5470%	0.6152%	0.7407%	0.9387%	1.2532%	1.4908%	1.7343%	1.9814%
Sharpe ratio	0.0604	0.1165	0.1424	0.1453	0.1413	0.1337	0.1292	0.1255	0.1224

Table 3: Minimization of the standard deviations at different target returns to construct the efficient frontier of the Reference Portfolio with Bitcoin

	N°1	Min Var Port	N°2	N°3	N°4	N°5	N°6	N°7	N°8
Developed	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.17%	0.00%	0.00%
Emerging	0.00%	0.00%	0.00%	0.00%	0.37%	2.10%	3.06%	4.03%	4.96%
Sovereign	63.21%	63.55%	62.17%	61.02%	59.92%	58.08%	55.52%	53.77%	52.17%
Corporate	17.20%	0.00%	0.00%	0.00%	0.00%	0.00%	0.04%	0.00%	0.00%
High Yields	0.00%	30.88%	33.21%	34.51%	35.68%	35.81%	35.93%	36.22%	35.17%
Private Equity	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.29%	0.01%	0.81%
Hedge Funds	8.76%	4.30%	3.71%	2.77%	1.36%	0.00%	0.00%	0.00%	0.00%
Real Estate	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.05%	0.00%	0.00%
Commodities	10.83%	1.28%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Infrastructure	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Σ Ref. Por. Wi	100.00%	100.00%	99.10%	98.30%	97.32%	95.99%	95.06%	94.04%	93.11%
Bitcoin	-	-	0.90%	1.70%	2.68%	4.01%	4.94%	5.96%	6.89%
Crix Index	-	-	-	-	-	-	-	-	-
Σ weights	100%	100%	100%	100%	100%	100%	100%	100%	100%
μ	0.0300%	0.0561%	0.0800%	0.1000%	0.1250%	0.1600%	0.1850%	0.2100%	0.2350%
σ	0.6229%	0.5470%	0.5621%	0.5893%	0.6395%	0.7319%	0.8106%	0.8947%	0.9848%
Sharpe ratio	0.0604	0.1165	0.1559	0.1826	0.2073	0.2290	0.2376	0.2432	0.2464

Table 4: Minimization of the standard deviations at different target returns to construct the efficient frontier of the Reference Portfolio with the Cryptocurrencies Index

	N°1	Min Var Port	N°2	N°3	N°4	N°5	N°6	N°7	N°8
Developed	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Emerging	0.00%	0.00%	0.00%	0.00%	0.00%	0.02%	1.18%	2.02%	2.71%
Sovereign	63.24%	63.55%	63.55%	63.54%	63.47%	63.39%	63.99%	63.81%	63.25%
Corporate	17.19%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
High Yields	0.00%	30.88%	31.30%	31.22%	31.08%	30.95%	29.88%	28.93%	27.95%
Private Equity	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Hedge Funds	8.78%	4.29%	4.34%	3.74%	3.08%	2.07%	0.55%	0.00%	0.00%
Real Estate	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Commodities	10.79%	1.29%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Infrastructure	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Σ Ref. Por. Wi	100.00%	100.00%	99.19%	98.50%	97.63%	96.42%	95.60%	94.76%	93.90%
Bitcoin	-	-	-	-	-	-	-	-	-
Crix Index	-	-	0.81%	1.50%	2.37%	3.58%	4.40%	5.24%	6.10%
Σ weights	100%	100%	100%	100%	100%	100%	100%	100%	100%
μ	0.0300%	0.0561%	0.0800%	0.1000%	0.1250%	0.1600%	0.1850%	0.2100%	0.2350%
σ	0.6230%	0.5471%	0.5546%	0.5717%	0.6057%	0.6729%	0.7315%	0.7966%	0.8670%
Sharpe ratio	0.0604	0.1165	0.1579	0.1882	0.2189	0.2491	0.2633	0.2732	0.2798

Table 5: Minimization of the standard deviations at different target returns to construct the efficient frontier of the Reference Portfolio with Bitcoin and the Cryptocurrencies Index

	N°1	Min Var Port	N°2	N°3	N°4	N°5	N°6	N°7	N°8
Developed	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Emerging	0.00%	0.00%	0.00%	0.00%	0.00%	0.05%	1.17%	2.01%	2.84%
Sovereign	63.21%	63.52%	63.56%	63.55%	63.47%	63.38%	63.90%	63.49%	63.05%
Corporate	17.20%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
High Yields	0.00%	30.87%	31.30%	31.22%	31.08%	30.95%	29.98%	29.15%	27.95%
Private Equity	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.02%
Hedge Funds	8.76%	4.31%	4.33%	3.74%	3.08%	2.04%	0.54%	0.06%	0.00%
Real Estate	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Commodities	10.83%	1.29%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Infrastructure	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Σ Ref. Por. Wi	100.00%	100.00%	99.19%	98.50%	97.63%	96.41%	95.58%	94.72%	93.87%
Bitcoin	-	-	-	-	-	0.04%	0.11%	0.26%	0.26%
Crix Index	-	-	0.81%	1.50%	2.37%	3.54%	4.31%	5.03%	5.87%
Σ weights	100%	100%	100%	100%	100%	100%	100%	100%	100%
μ	0.0300%	0.0561%	0.0800%	0.1000%	0.1250%	0.1600%	0.1850%	0.2100%	0.2350%
σ	0.6229%	0.5470%	0.5546%	0.5717%	0.6057%	0.6730%	0.7316%	0.7967%	0.8669%
Sharpe ratio	0.0604	0.1165	0.1580	0.1882	0.2189	0.2490	0.2633	0.2731	0.2798

Table 6: VaR estimation of the reference portfolio with the parametric approach

	Values at 30/04/2021	VaR'	VaR*C
Developed	0.00	0.00	4.61
Emerging	35.93	1.88	4.29
Sovereign	37.69	0.65	-0.35
Corporate	0.00	0.00	3.04
High Yields	43.53	1.19	4.23
Private Equity	38.27	2.85	4.82
Hedge Funds	0.00	0.00	1.87
Real Estate	0.00	0.00	4.25
Commodities	0.00	0.00	2.43
Infrastructure	0.00	0.00	4.46
Bitcoin	-	-	-
Crix Index	-	-	-
Fair Value (Σ)	155.41		
VaR Portfolio	5.16		
Potential Loss	3.318%		

Table 7: VaR estimation of the reference portfolio + Bitcoin with the parametric approach

	Values at 30/04/2021	***VaR'***	***VaR*C***
Developed	0.00	0.00	*3.66*
Emerging	17.72	0.92	*3.33*
Sovereign	112.59	1.95	*1.22*
Corporate	0.00	0.00	*3.27*
High Yields	73.58	2.02	*3.73*
Private Equity	13.21	0.98	*3.54*
Hedge Funds	0.00	0.00	*1.63*
Real Estate	0.00	0.00	*3.17*
Commodities	0.00	0.00	*2.48*
Infrastructure	0.00	0.00	4.05
Bitcoin	27.13	6.76	7.28
Crix Index	-	-	-
Fair Value (Σ)	244.22		
VaR Portfolio	8.10		
Potential Loss	3.318%		

Table 8: VaR estimation of the reference portfolio + Cryptocurrencies Index with the parametric approach

	Values at 30/04/2021	***VaR'***	***VaR*C***
Developed	0.00	0.00	3.59
Emerging	15.00	0.78	3.31
Sovereign	176.90	3.06	1.38
Corporate	0.00	0.00	3.72
High Yields	45.40	1.24	3.79
Private Equity	12.74	0.95	3.56
Hedge Funds	0.00	0.00	1.47
Real Estate	0.00	0.00	3.37
Commodities	0.00	0.00	2.56
Infrastructure	0.00	0.00	3.97
Bitcoin	-	-	-
Crix Index	33.50	8.46	8.70
Fair Value (Σ)	283.54		
VaR Portfolio	9.41		
Potential Loss	3.318%		

Table 9: VaR estimation of the reference portfolio + Bitcoin + Cryptocurrencies Index with the parametric approach

	Values at 30/04/2021	***VaR'***	***VaR*C***
Developed	0.00	0.00	3.59
Emerging	16.22	0.85	3.34
Sovereign	173.56	3.00	1.40
Corporate	0.00	0.00	3.70
High Yields	49.06	1.34	3.80
Private Equity	11.06	0.82	3.53
Hedge Funds	0.00	0.00	1.48
Real Estate	0.00	0.00	3.32
Commodities	0.00	0.00	2.58
Infrastructure	0.00	0.00	3.98
Bitcoin	2.90	0.72	7.17
Crix Index	31.22	7.88	8.70
Fair Value (Σ)	284.01		
VaR Portfolio	9.42		
Potential Loss	3.317%		

Table 10: Maximization of the expected return for the four portfolios within out-of-sample analysis

	Initial weights	***Reference Portfolio***	***Ref. Port + BTC***	***Ref. Port + CRIX***	***Ref. Port + BTC + CRIX***
Developed	10.00%	0.00%	0.00%	0.00%	0.00%
Emerging	10.00%	0.00%	0.00%	0.00%	0.00%
Sovereign	10.00%	0.00%	0.00%	0.00%	0.00%
Corporate	10.00%	0.00%	0.00%	0.00%	0.00%
High Yields	10.00%	50.92%	76.14%	78.56%	77.91%
Private Equity	10.00%	49.08%	17.22%	14.80%	15.12%
Hedge Funds	10.00%	0.00%	0.00%	0.00%	0.00%
Real Estate	10.00%	0.00%	0.00%	0.00%	0.00%
Commodities	10.00%	0.00%	0.00%	0.00%	0.00%
Infrastructure	10.00%	0.00%	0.00%	0.00%	0.00%
Σ Ref. Por. Wi	100.00%	100.00%	93.36%	93.36%	93.03%
Bitcoin	-	-	6.64%	-	2.50%
Crix Index	-	-	-	6.64%	4.47%
Σ weights	100%	100%	100%	100%	100%
μ	0.0646%	0.1762%	0.3065%	0.3162%	0.3208%
σ	0.9206%	0.9206%	0.9206%	0.9206%	0.9206%

Appendix B

Figure 1

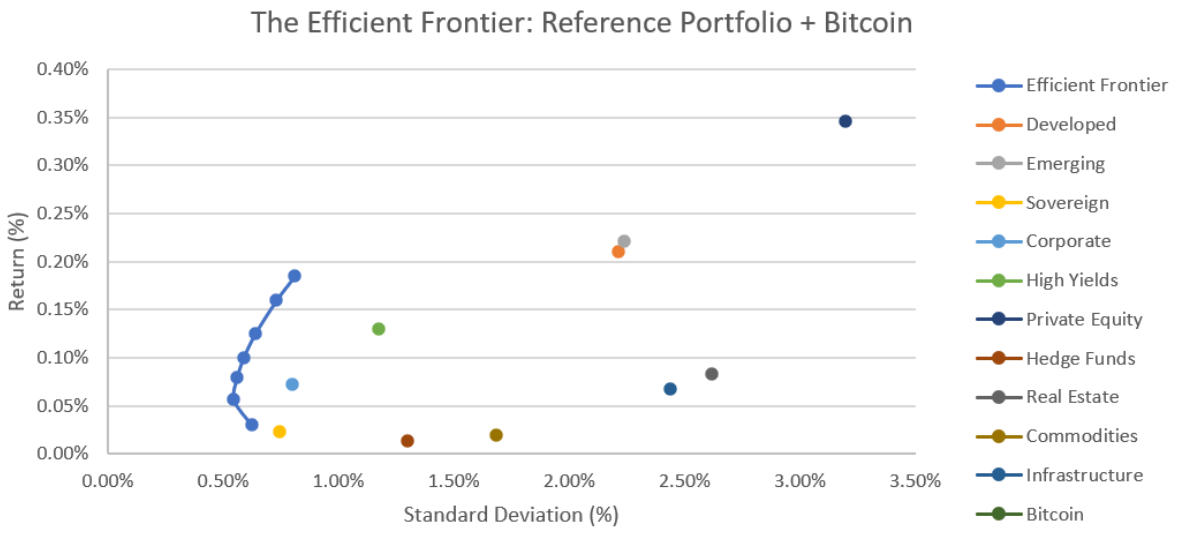


Figure 2

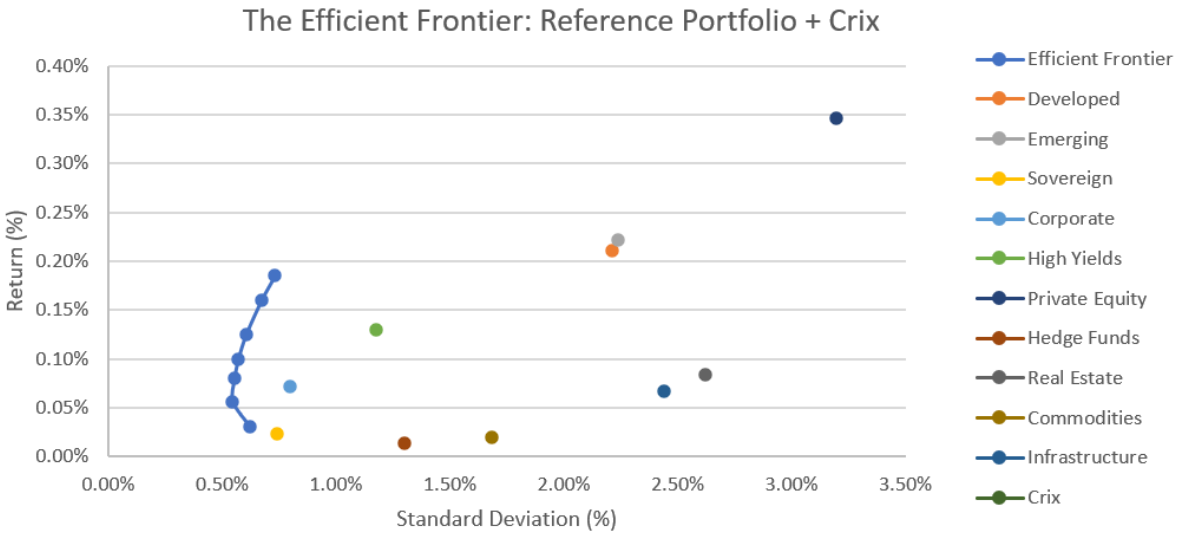
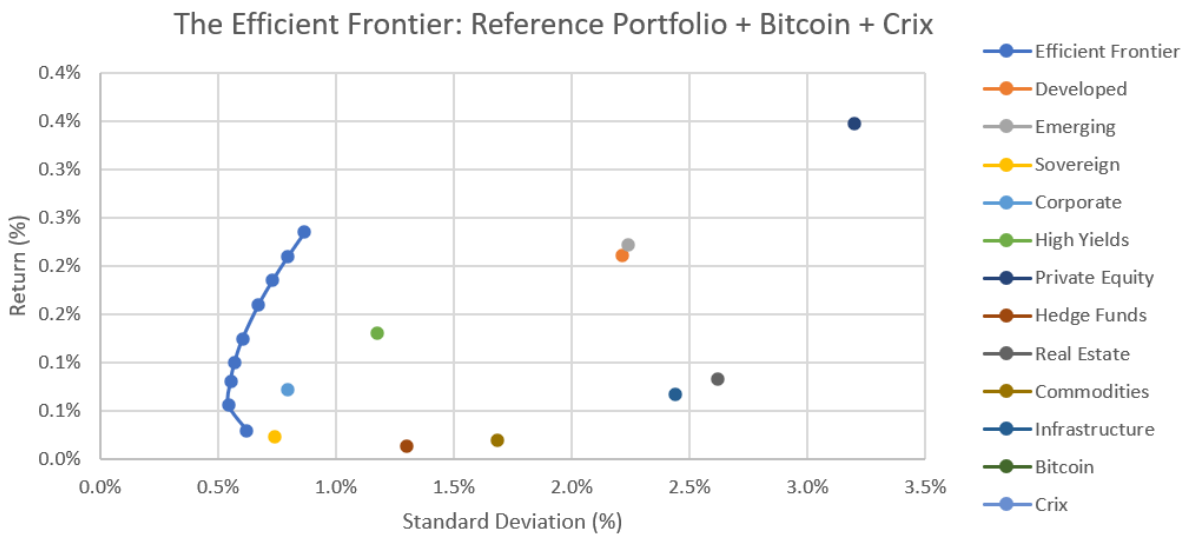


Figure 3


The Efficient Frontier: Reference Portfolio + Bitcoin + Crix
Return (%)
0.4%
0.4%
0.3%
0.3%
0.2%
0.2%
0.1%
0.1%
0.0%
0.0%
0.5%
1.0%
1.5%
2.0%
2.5%
3.0%
3.5%
Standard Deviation (%)
Efficient Frontier
Developed
Emerging
Sovereign
Corporate
High Yields
Private Equity
Hedge Funds
Real Estate
Commodities
Infrastructure
Bitcoin
Crix

Summary

Chapter 1

This analysis has the aim of investigating the effect on the performance of a portfolio composed by traditional asset classes, after including the cryptocurrencies, as a new one.

In June 2021, the global crypto market cap is $1.57 trillion according to the website CoinMarketCap, a useful online service which gives its users the possibility to stay updated on the values and charts of all the Cryptocurrencies. While at the beginning they were just another typology of digital currency, now they are considered as the most important. One of the reasons for this success, and a common feature of Cryptos, is the use of the Blockchain technology, which is the great innovation behind this new world between technology and finance. A Blockchain is an open and distributed ledger with the task of recording all transactions, which are called blocks. There are two kind of classification for cryptocurrencies:

1. For the usage time: limited vs unlimited
2. For the typology: digital cash coins, crypto tokens, and stable coins

For what concerns the evaluation of the suitability of the three constraints (store of value, medium of exchange, and unit of account) for the currency classification, cryptos are far away to be recognized as it. In fact, only a few of them is able to reach the needed requirements, and it is thought that these obstacles can be passed only with a future appropriate regulation. The problem lies behind the fact that cryptos with strong regulation will not be cryptos, so the conclusion is that currency is not the appropriate categorization for them.

Regarding the asset class classification, the final judgement of cryptocurrencies through the seven requirements (stable aggregation, investability, internal and external heterogeneity, expected utility, selection skill and cost-effective access) and the thoughts of the academics is quite positive. For sure, the result is much more positive than the analysis regarding the currency evaluation, but it is not sure and defined as the latter. Cryptocurrencies are still not too stable, and for the expected utility there are some doubts yet. On the other hand, the investability, together with the homogeneities and costs, brings the balance to the positive side.

The last part of this chapter is focused on the relationship between cryptocurrencies and the European retail investors. The exponential interest on cryptos from them could be testified by the data published by eToro. The famous trading online platform had 200.000 new users just in January and February of 2021, with an increase in Bitcoin holders of 61% and in Ethereum

holders of 49% (source CNBC). Even, the trading platform Revolut saw the entrance of 175.000 crypto users

However, for eToro, Revolut, and other crypto trading platforms, most of these European retail investors tends to build portfolios with a high degree of volatility, composed mainly by cryptocurrencies with great correlation index. Some platforms give the possibility to replicate portfolios of high reward seekers, and many retail investors follow them, without taking in consideration that the funds availability are not the same. On the contrary, here the target is to build a portfolio for every retail investor, with a controlled volatility and a relative return. This outcome aims to avoid huge losses in case of cryptos downturn in the future, as it happens in the previous years.

Chapter 2

The conventional mean-variance (MV) portfolio selection framework developed by Markowitz (1952) is the basis theory of this analysis. The Modern Portfolio Theory (MPT) is the most appropriate method to evaluate the introduction of cryptocurrencies in a multi-asset portfolio, and its effects on the risk-return characteristics. In details, for the MPT, a random investor is able to develop a multi-asset portfolio which maximizes its return for a determined threshold of standard deviation, given his/her risk aversion level.

The presence of studies about cryptocurrencies is strictly connected and correlated with their increasing notoriety and reputation. It has to be written that the focus was principally on Bitcoin, for its first-role importance in the category. The majority of the researchers' papers in the years following the creation of cryptos highlighted the economic, regulatory, and technical aspects of Bitcoin.

Wu and Pandey (2014) designed a portfolio composed of both traditional asset classes (equity and bonds) and alternative investments (private equity, commodities, and cryptocurrencies). Their study was one of the first with cryptos within portfolio strategies. Eisl, Weinmayer and Gasser (2015) focused on the introduction of cryptos in a multi asset portfolio, but this time the approach adopted was the Conditional Value-at-Risk (CVaR) framework, instead of the MP theory. Elendner, Trimborn, Ong and Lee (2016) published the first research study on the features and performance of the Cryptocurrencies Index (CRIX).

A higher number of papers regarding investment strategies with cryptocurrencies became available from 2018. Veldmeijer (2018) examined the effects of a basket of cryptos together with Bitcoin alone, from a Dutch pension fund's point of view. Andrianto and Diputra (2018) designed with intensive asset allocation four portfolios, that included both Bitcoin and smaller cryptocurrencies. However, the majority of the studies published embrace the point of view of an average American investor. Holovatiuk (2020) verified whether cryptocurrencies can be classified as an asset class and what kind of profits they may carry to the investor's portfolio.

One of the aims of this thesis is to extend the current literature by trying to mix the previous studies regarding investment strategies with cryptocurrencies. Instead of analysing a specific individual cryptocurrency, the decision was to examine the effect of an index of cryptocurrencies (CRIX), in combination with a crypto that can be considered as an index of the category, due to its percentage of market cap over the total market cap of cryptocurrencies (Bitcoin). It addresses the view of an average European retail investor, so without any particular form constraints, except for the currency that is fully in Euro. Following Campbell & Viceira (2002), the strategic asset allocation has a long run vision. The reference portfolio, based on data from 2016 to 2021, is structured with different asset allocations weights. The methodology of the analysis and the results obtained will be discussed in Chapter 3.

The second part of this chapter concerns the econometric aspect of the analysis and the theoretical assumptions. The general formula for the expected return of the portfolio with k-risky assets is the following:

$$E(u_p) = w'\mu \quad (2.1)$$

where w is the vector of weights assigned to the specific assets (1 through k) and μ is the vector of expected returns produced by the individual assets (1 through k) within the portfolio.

The second formula presented in this section is the one for the portfolio standard deviation with k-risky assets, which is calculated as:

$$\sigma_p = \sqrt{w'\Sigma w} \quad (2.2)$$

again, w is the vector of weights of the single assets (1 through k) and Σ is the variance – covariance matrix (k x k), also known as dispersion matrix.

The last formula for the problems of maximization and minimization is the Sharpe ratio, and it is estimated as:

$$SR_p = (E(r_p) - r_f)/\sigma_p \quad (2.3)$$

where the $E(r_p)$ is the expected return of the portfolio, r_f is the return of an investment that carries zero risk, and σ_p, as it is stated above, is the measure of the standard deviation.

Together with the reference portfolio, there are three other portfolios to be examined:

4. Reference Portfolio with the introduction of Bitcoin
5. Reference Portfolio with the addition of the Cryptocurrencies Index (CRIX)
6. Reference Portfolio with the inclusion of both Bitcoin and CRIX

The following formulas are needed to solve the three optimization problems:

4. $Max_{\{w_i \mu_i\}} E(r_p) = w'\mu$ (Expected return)
5. $Min_{\{w_i \sigma_i\}} \sigma_p = \sqrt{w' \sum w}$ with $\sum w$ that is the matrix between the weights transpose and the standard deviations of each asset class (Standard deviation)
6. $Max_{\{w_i\}} SR(r_p) = \frac{E(r_p) - r_f}{\sigma_p}$ (Sharpe ratio)

A further decision was to add two fixed constraints in all the four portfolios along with the three variables (maximization of π, minimisation of σ, maximization of SR). The first one is that short sales are not allowed, while the second one regards the fact that all the investor's richness is used. The above-mentioned restrictions are added in the following way to the optimization strategies.

$$3) \quad wi \geq 0 \ \forall \ i = 1, \ldots, k$$

$$4) \quad w'1 = 1$$

Basically, each individual asset weight i can't assume a negative value and the sum of the weights must be equal to 1 (or 100%).

In order to see the quality and safety of the four portfolios, I decided to calculate the VaR (Value at Risk) for each of the portfolios maximized in terms of return, using two of the three possible approach for the estimation of the Var. The chosen two are the historical and the parametric approach, while the Montecarlo simulation was not suited due to its random nature.

In the historical approach, it has been followed the fair value-based method, which begins with the estimation of the fair value and the decision of the confidence level. After that the percentile

of the potential loss is calculated over all the dataset available, and lastly the value at risk for the fair value. The formula below synthetizes the last passage:

$$VaR_p = FV * potential\ loss \quad (2.4)$$

The second approach I used in this analysis is the parametric approach. The first passage in this approach is the estimation of the VaR for each asset class. The fair values of them are calculated considering the last observation (n=263), with the weights of the portfolios maximized for the return. Consequently, the following formula is applied for all the four portfolios, as shown below:

$$VaR_p = \sqrt{V \cdot C \cdot V'} \quad (2.5)$$

where V stands for the row vector of VaR of each individual asset class, V' is the transpose of matrix V, and lastly C is the correlation matrix.

The last part is focused on the regression analysis in order to examine the incidence of the four developed portfolios with a market index, through two typologies: an in-sample and an out-of-sample analysis. The primary distinction among the two is the evaluation time frame used to compute the optimal portfolio weights and returns. Indeed, in the in-sample analysis, all the observations from t_0 to T (n=1, …, 262) are used as the estimation window. On the contrary, in the out-of-sample analysis the dataset is split in two parts. So, in this second case the estimation window is made only of observations from t_0 to t (n=1, …, 130).

Differently from the in-sample, in the out-of-sample analysis, the aim is to try to predict in the most accurate way the values of the second half, by regressing the first half data with the external benchmark. The elements of the two analysis produce a different risk-return characteristic of the four portfolios, and as a consequence also a distinct set of optimal weights

The core point of regression analysis of this type is the extra return produced by the built portfolios, compared to an overall market benchmark, the MSCI All Country World Index.

The outcome of the both analysis is recorded after the running of this bivariate ordinary least-squares regression:

$$y_i = \alpha + \beta \cdot x_i + \varepsilon_i \quad (2.7)$$

where y_i is the dependent variables that indicates the returns of the four different portfolios (reference one and the three with cryptos). Then, there is the return of the selected market

benchmark, the MSCI ACWI, represented by the independent variable x_i. The remaining three terms of the equation are described as it follows:

4. α = vertical intercept of the regression line
5. β = slope of the regression line
6. ε = error term of the regression

and lastly the $\alpha + \beta \cdot x_i$ is the regression line.

In this analysis, the focus is on the value of the alpha of the four portfolios. In case the value is positive, the portfolio is performing better than the market benchmark. On the contrary, with a negative value, the portfolio is underperforming compared to the MSCI ACWI. It is important also the value assumed by the T-statistic and its relative p-value. The former is a measure obtained by dividing the coefficient of the alpha with its standard error, and it is considered significant if its value is higher than 1.96, with a confidence level of 95%.

The last part of this chapter concerns the description of the multi asset reference portfolio. It is built up taking into consideration the principal asset classes available on the market, which can be truly represented by world indexes. This analysis begins in a situation of an equal weighted portfolio, so the overall richness is equally divided into the ten investment products.

The list of the selectable asset classes is updating with daily frequency (in particular the category of the alternative investments), so the most used ones in similar studies and analysis were picked: equity, debt securities, hedge funds, real estate, and alternative investments. The five-asset class selected, and their relative ten investment products, give a greater degree of diversification to the average European retail investor than investing in a single asset class.

The weekly prices have been downloaded on Bloomberg platform for a time horizon of five years, from April 2016 to April 2021. The reason of this choice is justified from the fact that ten years ago cryptocurrencies were just introduced in the financial market (some of these coins included in the CRIX were not yet), while a less time horizon was not significant to see the effect of the crypto introduction.

All the indexes that will represent the several asset classes, aim to offer a global coverage of the markets. Their prices are expressed in euros, and here it will be given the full list:

11. MSCI Developed Markets Index

12. MSCI Emerging Markets Index
13. FTSE World Government Bond Index (WGBI)
14. Bloomberg Barclays Global Aggregate-Corporate Index
15. Bloomberg Global High Yield Index.
16. HFRX Global Hedge Fund Index
17. MSCI World Real Estate Index
18. S&P Listed Private Equity Index
19. Bloomberg Commodity Index
20. S&P Global Infrastructure Index

In order to study the consequences of the introduction of cryptocurrencies in the reference portfolio, two additional "indexes" have been selected as a representation of the new asset class. The first one is Bitcoin, denominated in euro currency. The second index is a real one, the Cryptocurrencies Index (CRIX). It is a benchmark for the cryptocurrency market and was introduced at the end of July in 2014, so it is the only index that tracks data for a time horizon bigger than the one of this thesis (five years, 2016-2021). It was developed by the "Ladislaus von Bortkiewicz" Chair of Statistics at the Humboldt University of Berlin, Germany.

Chapter 3

Here, all the results obtained in the analysis are shown and discussed, in order to reach a conclusion for this thesis.

The first step regards the descriptive data of the asset classes. If we exclude the crypto indexes, the private equity index shows the highest expected return (0.0034), but, at the same time the greatest standard deviation (0.032). On the contrary, the lowest mean value was obtained for the hedge funds and commodities indexes (both values are 0.0001). The notable aspect is that both do not have the lowest standard deviation. In fact, sovereign bonds gain the title of most riskless index (σ = 0.0074), followed by the other bond securities: corporate in the second position (0.008) and high yields (0.0118) as the third, with an expected return (0.0013) more than six times higher than sovereign (0.0002).

Writing about the asset class in general, the equity class shows the best performance behind the cryptocurrencies, with a more than acceptable return/risk of both developed markets (DM) and emerging markets (EM). Both have an expected return (0.0021 DM and 0.0022 EM) lower than only private equity, with risk values (0.0222 DM and 0.0224 EM) that is nearly in the average of the standard deviations (0.019).

For what concerns the last asset class to be analysed, it has been showed that cryptocurrencies overcome with a wide margin the other classes in terms of expected return. Both Bitcoin and CRIX have a mean value nearly ten times the first of the traditional asset classes, private equity. This great performance in term of expected return is justified by the high standard deviation relative to the other classes. However, this measure is only three time the one of private equity, so the overall performance of cryptocurrencies is much better than the traditional asset classes.

Regarding the third momentum, all the traditional classes present a negative value in the skewness column, so they all have their distribution moved to the left side. However, they all have statistically significant values. It is curious to see that within the debt securities asset class, there is both the most and the less symmetric distribution of the traditional. In fact, the former is the sovereign bond index, with a value of -0.23, while the latter is the corporate bond index with a skewness of -4.32.

On the other hand, cryptocurrencies indexes present a positive skewness value, so it means that their distribution is towards the right side. In particular, Bitcoin has the most symmetric value of the basket of indexes with a value of 0.22 and, consequently, it falls within the “approximately symmetric” group.

Debating on the fourth momentum, most of the values are far from the kurtosis of a normal distribution, which is 3. In details, there are only three indexes of the traditional classes with a platykurtic distribution (kurtosis < 3) respectively emerging markets, sovereign bonds, and commodities. All the others fall into the first group of leptokurtic (kurtosis > 3), with hedge funds, corporate and high yields bond that have values very far from the mesokurtic threshold (kurtosis = 3). Regarding the cryptocurrencies’ indexes, Bitcoin has a platykurtic distribution (1.5453) while CRIX is the index nearest to the normal value (3.115).

Next passage is about the correlation matrix. Sovereign Bond is the index with the best result in terms of correlation. In fact, it has a series of positive values close to 0, with the exception of the equity class indexes and the other bond indexes, which assumes negative values.

On the other hand, for the worst diversification instrument, there are three indexes that showed bad performances. These three are developed market, private equity, and commodities. All of them have four values out of eleven that reach the threshold of strong correlation (among +/-0.7 and +/-1), with some of the remaining seven values of these three indexes that do not fall within this classification for a few units.

Bitcoin and the cryptocurrency index show brilliant performance, since the correlation values with the other indexes are all in the weak correlation threshold (between 0 and +/- 0.3). Even if both the components of the crypto asset class show an high standard deviation, and, as a consequence, be considered a risky stand-alone investment, they could be a great diversification vehicle.

The second step regards the optimization problems for the equal-weighted reference portfolio, which has an expected return of 0.117% and a standard deviation of 1.424%. A better mean-variance result is obtained in all the other three distributions of weights.

The asset allocation is quite extreme because six indexes out of ten are left out in the maximization of expected return and Sharpe ratio, and in the minimization of the standard deviation. Only emerging markets (EM), sovereign and high yield bonds (SB and HYB), and, lastly, private equity (PE)are taken into consideration for the estimation of the portfolios.

The portfolio built for the maximisation of the expected return under the two fixed constraints of Chapter 2 is the most balanced. All the four indexes have a weight within a range between 23.12% to 28.01%. In the other two optimized portfolios, the weights of private equity and emerging markets decrease, while the opposite situation happens to sovereign and high yield bonds. These outcomes are justified by the descriptive characteristics of the indexes.

The third step concerns the outcomes of the asset allocation process with the inclusion of Bitcoin and the Cryptocurrencies Index. An outcome better than the reference portfolio (0.117%) is obtained in all the three solutions (0.347% and 0.407%) of expected return maximized problem. Therefore, it can be written that the introduction of both bitcoin and CRIX (alone or together) increases the performance of a traditional portfolio. The sum of the traditional weights decreases after every new introduction: from just Bitcoin (88.89%) to both the crypto indexes (87.99%). The four traditional indexes are the same of the optimization problems for the reference portfolio (EM, SB, HYB and PE).

Regarding the minimization of the risk with a fixed expected return of 0.117%, the first thing that comes to the eyes is the sum of the weights of the traditional indexes: nearly the total of the wealth invested. The reason for this is that the indexes of the cryptocurrencies class have an high standard deviation compared to the other classes, so only around 2% is allowed in order to keep the expected return to the level of reference portfolio. This outcome leads to a better performance in terms of risk compared to the reference portfolio (the worst is the one including Bitcoin with 0.622% against the original 0.872%). In this optimization problem, the debt securities class holds more than the 97%, due to their relatively low risk characteristics. Lastly, Bitcoin does not have any particular risk benefit when there is the possibility of adding both it and CRIX (the standard deviation remains the same with both or with only CRIX).

Lastly there are the three portfolios subjected to the maximization of the Sharpe ratio. The indexes used are the same of the expected return problem, but in this case, with the addition of both bitcoin and CRIX, the highest percentage of cryptocurrencies within the portfolio is reached (12.14%).

To summarize, the best performances are obtained when both bitcoin and the cryptocurrencies index are included in the reference portfolios for every optimization. In particular, CRIX overcomes Bitcoin.

For what concerns the efficient frontiers, the inclusion of both kind of cryptocurrencies generates a movement of the curve of the traditional indexes to the left and upwards. So, this means that a higher return is reached at every given level of portfolio standard deviation. The best performance is obtained when both BTC and CRIX are added. Nevertheless, the difference with the solely CRIX addition is quite irrelevant because the lines are nearly the same.

In addition, there is a practical test at the end of this section. The initial hypothesis is that at the beginning of my sample period (22/04/16) the subject of this analysis (European retail investor) has an overall wealth of 100 euros. The total return obtained by the four portfolios are the followings: +9.22% for the reference portfolio, +19.55% with the addition of bitcoin, +23.18% with the introduction of CRIX, and +23.22% when both the crypto indexes are included. The portfolio with bitcoin and the two subjects to the introduction of CRIX maintained the same performance for most of the time window, but in 2021, a notable difference occurred between the two types. The cause is the composition of CRIX: it means that, starting from 2021, the other cryptocurrencies gained popularity and helped the index to achieve better visible returns than the bitcoin alone.

The fourth step is about the VaR calculations. For both the approaches used the inputs are the same. In fact, the chosen fair values are the last observations of the sampling period. The confidence interval is 99%, so the alpha is 2.326. The portfolios used for the VaR estimation are the one subject to the maximization of the expected return, in order to have the same standard deviation for all.

Writing about the first approach, the historical one, the portfolios with cryptos shows percentages of potential loss that are quite the half of the traditional portfolio (7.05%). Even in this case, CRIX shows that it brings more advantages in term of risk, compared to the Bitcoin. In fact, the percentages of it (3.21% with only CRIX, 3.18% by adding both) are slightly lower than the one of bitcoin (3.25%). Since that the fair values are not so different like in the case of the reference portfolio, this CRIX benefit is not observed in the VaR too (the portfolio with CRIX alone register the highest value with 9.10). In addition, the number of observations when the return obtained by each portfolio is worse than the potential loss is twice.

In the second approach, the parametric one, the differences seen in the previous table are not remarkable. The highest Value-at-Risk is the one of the reference portfolio after the addition of bitcoin and CRIX (9.42), while the lowest is obtained by the traditional asset classes with quite the half of the value (5.16). However, it is all in line with the starting fair values, so the difference among the portfolios is proportional to them. As mentioned above, the difference is quite irrelevant but the best portfolio for the parametric approach is the one with BTC and CRIX (3.3175%). However, compared to the historical approach results, this time Bitcoin has the role of asset which reduces the potential loss. Indeed, the portfolio with CRIX alone is the worst one (3.318%), while the one subject to the introduction of bitcoin is the second best portfolio (3.3176%).

The last step of this analysis is the evaluation of the portfolios' performances, through the regression analysis. The four portfolios' returns are regressed with the MSCI All Country World Index, which was chosen as the market benchmark for the reference portfolio. In the in-sample analysis the returns of this index used are 261, while in the out-of-sample one just 132 returns are considered.

Starting from the in-sample one, all the alphas of the four portfolios have a value higher than 0. The highest one was obtained by the reference portfolio with the addition of CRIX (0.00356), while the lowest was the one composed by only the traditional asset classes (0.00055). This outcome means that in the selected time period the inclusion of cryptocurrencies indexes leads to

a better performance than only traditional, compared to the market. In addition, the alpha of the reference portfolio without cryptos is the only one that is not statistically significant, considering that the T-stat (1.51) is under the thresholds with a confidence interval of 95% (1.96) and 90% (1.64). This means that its data are not functional for the model.

Regarding the forecasted typology of analysis, the out-of-sample one, the results obtained are worse than the previous case. Even in this case the first value is obtained by the portfolio subject to the introduction of CRIX (0.00212), and the last one is achieved by the basic reference portfolio (0.00052). The problem arises for the values obtained in the row of the T-stat. in fact, every typology of reference portfolio does not reach the threshold of 95% confidence interval: the traditional has the lowest value with 0.43, followed by the one with bitcoin (1.42). However, the other two portfolios are at least statistically significant at a confidence interval of 90%: the reference portfolio with CRIX has a T-stat of 1.73, while the last solution achieved a value of 1.71.