Department of Business and Management

International Financial Economics' course

An empirical study on asset pricing models for the Italian stock market: a Capital Asset Pricing Model and Fama-French Three-Factor Model comparison

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Abstract

The thesis aims to compare performances of the Capital Asset Pricing Model and the Fama-French Three-Factor Model in the Italian stock market using the FTSE MIB index as the market proxy. The work will review the main discoveries, developments, and dilemmas crossed by the evolution of asset pricing models over the years to date, will briefly address the domestic-international application of the models, and finally present an empirical study on the constituents of FTSE MIB stocks from August 2001 to December 2020. The thesis demonstrates that the Fama-French Three-Factor model performs better than Capital Asset Pricing Model but still carries some imperfections which left partially unexplained asset returns.

1. Introduction

Everyday trillions of dollars, euros, and other major currencies are traded in the main financial markets of the world, based on parameters, and indices, that in the end, turn out to be quite simple.

The methods of calculation, developments, the empirical evidence, and, above all, the brilliant ideas and theories that have led to these simple parameters, have not been straightforward at all, but have required many efforts, errors, and attempts.

Every single day thousands of capital budgeting problems, risk-return estimations, accounting and company evaluations, portfolio or stock return predictions analysis are carried out, guiding the allocation of huge amounts of money, every single day.

For most of the activities presented above, there is one single key factor estimation that controls and influences almost all the decisions undertaken in those above scenarios: The asset-specific required rate of return estimation, or, most commonly known as, The cost of capital estimation.

Here asset pricing models kick in, representing the modern solution for the asset expected return estimations. The investigation on how those models work and how they account for the time value of money and compensate for risks undertaken is something crucial for growth and improvements in practical and theoretical finance

This study objective is to analyze and compare two famous asset pricing models, the Capital Asset Pricing Model and the Fama and French three-factor model. Through OLS regressions and predictive tests, these models are tested and studied outside the context of the American market in which they were originally built and designed.

The paper aims to highlight the differences and empirical results of the two models without questioning the brilliant insights underlying them. Over the years many researchers' contributions have led to fine-tuning of predictive precision, improved the information produced and usable from various models of asset pricing, and above all have allowed through debate and criticism to mature brilliant and progressive literature to grow day by day the power of the tools available for the world of finance.

Guided mainly by curiosity, the student wants to investigate the actual "usability" of the two models in a market very different from the American one (NYSE, AMEX, NASDAQ), for the number of companies, volume of transactions, generic size, and market capitalization of components.

In particular, the work focuses on the constituents of FTSE MIB, the representative market index of the principal Italian stock market for the period 2001-2020.

In few words, this thesis tries to answer two main research questions:

1- Are empirical asset pricing models able to capture the variation in average stock returns in the Italian stock market through risk factors related to Size, Value, and the FTSE MIB as a market portfolio?

2- Does the Fama-French tree-factor model outperform the CAPM for the last 20 years in the Italian stock market?

Starting from the main works of literature at the base of the evolution of asset pricing models and going through the main historical moments that led to the CAPM and its implementations, modifications, and alternatives. The paper aims to present a times series - OLS regression following the method of building the classical variables and portfolios proposed by Fama and French in their paper of 1992 (similar method used by Black Jensen and Scholes in 1972) furthermore will be presented a brief digression on the main differences and results brought in the literature between national markets and a single global international market.

The thesis chapter structure is as follows: after the introductory Chapter 1, Chapter 2 defines the basic financial concepts used in this work and continues with the description of the main evolutionary processes of asset pricing; starting from Markovitz, passing through Treynor, Sharpe, Lintner, Mossin, as well as Ross, Roll, Fama and French and others who have made contributions till nowadays, mainly focusing on the CAPM and Fama and French 3 factors.

Chapter 3 will show a review of the literature related to a global/international asset pricing model and the domestic Italian market, together with a brief description of where to locate the contributions of this work compared with previous research in the Italian market.

Finally in Chapter 5 will be explained the methodologies and data used in this study, and empirical analysis will be conducted to compare CAPM and Fama-French 3 factor. Chapter 6 will summarize the results obtained, final opinions, and discuss possible future works.

2. Theoretical framework

Before explaining the evolution of the models, I will first present the basic theoretical framework which is needed for understanding the empirical part of the thesis.

Immediately after, starting from the Portfolio Theory and the brilliant formulations of Markovitz, the paper traces the literature on asset pricing, giving credit and importance to the contributions of the various economists who have enhanced and enriched the wealth of knowledge related to the subject under analysis.

The basic concept of risk diversification, the principle of mean- variance, separation theorem, the concept of market portfolio and risk-free asset, the first criticisms and weaknesses of the assumptions and hypothesis of the CAPM and the subsequent attempts to explain the anomalies of the model or to derive theories with different assumptions, allowed more and more for a clear understanding of the functioning of trade and markets.

Knowledge that requires a continuous evolution in analysis tools to keep the pace with the growing complexity of international markets and the performance of securities and indices. In short, the following pages propose to derive a logical, coherent, and rational path in the history of the main contributions to the subject, trying to derive a "portfolio" of sufficient knowledge to critically address the contemporary debate on return forecasting models.

2.1. Stock risk-return

The following is an account of statistical mathematics used in the empirical analysis, most of the formulas are mainly based on the first formulations of H.Markovitz.

Let’s start by defining the fundamental notion of return on a financial asset: defined as the relationship between the initial capital and the profits generated by investment or sale transactions over a specified period of time. Risk can be defined as the degree of uncertainty that the market expresses on the actual realization of the expected returns. Both return and risk can be measured ex ante or ex-post.

The return on a share, measured ex post over a period T, may be expressed as:

$$R_{i,t} = \ln(\frac{P_{i,t}}{P_{i,t-1}})$$

(Formula 2.1.1)

The standard approach considers Rt, evaluated ex ante, as a random variable characterized by a mean value (μ), which measures the expected yield on the stock and a level of variance (σ^2) taken as a reliable measure of uncertainty that the expected level of return is pursued and a probability distribution, that statistically identifies the price-generating process.

In formula the expected return is defined as:

$$\mu = \mathrm{E}[\mathrm{R}_T] = \sum_{i=1}^{n} R_i * p(R_i)$$

(Formula 2.1.2)

Here p(Ri) represent the probability that the expected return for the i-th title is R_i. Probability is defined as the limit of the ratio between the number of favorable events (Ni) and the total number of observations (N):

$$p(R_i) = \lim_{n \to \infty} \frac{Ni}{N}$$

(Formula 2.1.3)

The (2.1.2) requires, in order to be calculated, the estimate of p(Ri) obtainable from the models of analysis of the historical series or from the fundamental analysis. With a sample of N observations available in the form of historical returns series, the arithmetic mean of the observations can therefore be considered as a reliable estimator of the expected yield μ.

The expected return is therefore in this scenario defined by the equation:

$$\mu = \mathrm{E}[\mathrm{R}_T] = \frac{1}{N} \sum_{j=1}^{N} R_i(j)$$

(Formula 2.1.4)

In modern portfolio theory, a financial asset is considered to be “more risky” the higher the probability that future returns will disperse relative to the estimated average value. A valid statistical measure of this effect derived for the first time from the works of Markovitz is represented by the variance, defined as the sum of the deviations from the mean to square weighed for the respective probabilities of realization and expressed by the relation.

$$\sigma^2 = \mathrm{Var}\ [\mathrm{R}] = \sum_{i=1}^{N} (R_i - \mu)^2 * p(R_i)$$

(Formula 2.1.5)

The function σ^2, taken as a reliable estimate of the financial risk of the security, is evaluated through mathematical-statistical models. Considering the sample variance as a reliable estimate of the variance of the entire population and indicated by N the number of observations available, the risk of a financial asset is calculated using the following formula.

$$\sigma^2 = \mathrm{Var}[\mathrm{R}] = \frac{1}{N-1} \sum_{j=1}^{N} [R_i(j) - \mu]^2$$

Formula 2.1.6

A fundamental assumption of the works and of the Markovitz model, concerns the distribution of the probabilities on which the mechanism of formation of the yields is based, which is assumed to be of Gaussian type. This means considering that prices are generated by a random process that expresses an expected average value equal to μ and a variance equal to σ^2, this assumption is key since the distributed random variables are usually described in a complete way only by the average functions and variance.

For completeness the formulas concerning variance, covariance, and correlation:

$$\sigma_A = \sqrt{\sum_{i=1}^{n} \frac{(r_{A,i} - \bar{r})}{(n-1)}}$$

Formula 2.1.7

$$Cov_{A,B} = \sum_{i=1}^{n} \frac{[(r_{A,i} - \bar{r_A})(r_{B,i} - \bar{r_B})]}{(n-1)}$$

Formula 2.1.8

$$\rho_{A,B} = \frac{Cov_{A,B}}{\sigma_A \sigma_B}$$

Formula 2.1.9

2.2. Portfolio return-variance

To calculate the risk and the return on a portfolio of N securities, it is necessary to refer to the correlation between the securities and the ratio of the proportion of wealth invested on each security to the total portfolio.

$$w_j = \frac{W_j}{W}$$

Formula 2.2.1

It can be demonstrated that for a portfolio of n risky assets the expressions of ex-ante yield and variance are the following:

$$\mu_p = \sum_{i=1}^{n} w_i * \mu_i$$

$$\sigma_y^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i * w_j * C_{ij} = \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_i \sigma_j \rho_{ij}$$

Formula 2.2.2 – 2.2.3

where wi is the ratio of the amount of wealth invested on the i-th stock to the total one available to the investor, Cij and ρij are respectively the covariance and the coefficient of correlation between the i-th stock and the j-th stock.

It is important to note that the expected returns and variances for the individual securities that make up the portfolio are random variables, governed by a conditional probability distribution that takes into account the link between a security and the remaining part of the market.

It should be emphasized the concept of covariance and correlation, which effect can be noted explicitly and in a simple way in a hypothetical portfolio composed of 2 securities, where you will have:

$$\sigma_p^2 = (X_A\sigma_A)^2 + (X_B\sigma_B)^2 + 2X_AX_B\sigma_A\sigma_B\rho_{A,B}$$

$$\sigma_p = \sqrt{(X_A\sigma_A)^2 + (X_B\sigma_B)^2 + 2X_AX_B\sigma_A\sigma_B\rho_{A,B}}$$

$$\sigma_p^2 = (X_A\sigma_A)^2 + (X_B\sigma_B)^2 + 2X_AX_BCov_{A,B}$$

$$\sigma_p = \sqrt{(X_A\sigma_A)^2 + (X_B\sigma_B)^2 + 2X_AX_BCov_{A,B}}$$

$$E_{(r_p)} = \sum_{i=1}^{n} X_i \, E_{(r_i)}$$

Formula 2.2.4[1] – 2.2.5 – 2.2.6 – 2.2.7 – 2.2.8

Correlation that becomes source of risk of a portfolio; It is recognized that, if ρ is null the variance of the portfolio is equal to the weighted average of the variances of the single Stocks, weighed from the percentage of wealth in they invested.

To sum it up in simpler terms:

Return on a portfolio consisting of N stocks:

$$R_p = \sum_{i=1}^{N} x_i \, \bar{R}_i$$

Formula 2.2.9

Expected return on a portfolio consisting of N stocks:

$$\hat{E}(R_p) = \hat{E}\left(\sum_{i=1}^{N} x_i \, \bar{R}_i\right) = \sum_{i=1}^{N} x_i \, \bar{R}_i$$

Formula 2.2.10

Portfolio variance:

$$v\hat{a}r(R_p) = v\hat{a}r(\textstyle\sum_{i=1}^{N} x_i R_i) = \sum_{i=1}^{N}\sum_{j=1}^{N} x_i x_j \hat{\sigma}_{ij} = \sum_{i=1}^{N}\sum_{j=1}^{N} x_i x_j \hat{\rho}_{ij} \hat{\sigma}_i \hat{\sigma}_j$$

Formula 2.2.11

with the aim of being as clear as possible on what are the formulas used later in the thesis, even if introduced years after Markovitz work, it deserves a particular mention the Sharpe ratio.

When comparing performances between stock or portfolios, we can use the Sharpe Ratio, which is defined as a ratio of the excess return of an asset divided by its risk:

$$Sharpe\ ratio = \frac{\bar{R}_i - R_f}{\hat{\sigma}_i}$$

Formula 2.2.12

where R_f is risk-free rate of return[2]. This measure helps compare performances of investments by adjusting for their risk.

[1] where X is the percentage weights of securities and E(ri) is the expected return on i-th

[2] The risk-free rate of return refers to the theoretical rate of return of an investment with zero risk. In practice, the risk-free rate of return does not truly exist, as every investment carries at least a small amount of risk. (Source Investopedia)

2.3. Modern portfolio theory

Studying and deepening the behavior of share prices, but above all, understanding which are the underlying factors influencing and determining the evolution of the price of a share, is a matter of significant interest since the financial markets exist with their attached public shares.

To retrace the origins of asset pricing models and analyze the basics, we need to go back to the exact middle of the twentieth century.

The year is 1952. About 4.2% of the US population invests in the stock market[3]. Most of the country believes that investing is a way in which wealthy individuals can play with their broad media. The Fed raised the interest rate cap a year earlier and shares are outperforming bonds by 25 basis points[4].

Investment in indices has yet to regain popularity since the 1929 crash and international investment is non-existent.

And it's right here that 25-year-old Harry Markovitz is putting together his doctoral thesis, "Portfolio Selection"[5], the work that will eventually become a classic in finance papers will make him earn a Nobel Prize and mark the beginning of what is now called Modern Portfolio Theory.

In a nutshell, Modern Portfolio Theory or Mean-Variance Analysis is a model that explains how rational investors can use diversification to optimize their returns at a given level of risk, so it is possible to construct an Efficient Frontier which represents the set of all portfolios of which expected returns (E (Rp)) reach the maximum given a certain level of risk (Var(Rp)).

The critical insight is that stocks should not be evaluated based on itself; instead, it should be evaluated based on how it contributes to the risk and return of a portfolio.

In his paper, Efficient portfolio Frontier Theory relies on the following assumptions:

• Investors are risk-averse and rational, so they aim to maximize final wealth: they are evaluating portfolios solely on expected return and standard deviation of return.

• Transaction costs and taxes shall be zero, assets shall be perfectly divisible

• The investment horizon is single-period (one month, one year, three years).

• The market is efficient which means that a stock price accurately reflects its value.

• Asset returns are distributed by the normal distribution

• The market shall be fully competitive

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[3] Stocks Then And Now: The 1950s And 1970s. (2008, October 01). Retrieved March 23, 2018, from https://www.investopedia.com/articles/stocks/09/stocks-1950s-1970s.asp.

[4] Lauricella, T. (2012, February 19). 'Retro' Investing-Look Back to Get Ahead. Retrieved March 25, 2018, from https://www.wsj.com/articles/SB10001424052970204880404577227043217645160

[5] Markowitz H M. Portfolio selection: efficient diversification of investments. "This Week's Citation Classic", CC/NUMBER 36 SEPTEMBER 7, 1992

As stated above, it's possible to provide a graphical representation of the Portfolio Selection of Markowitz that allows you to define the frontier of efficient portfolios, or even said dominant, as necessarily preferable to all other portfolios. It is important to remember that all portfolios available are different combinations in weight of the securities included, and the dominant/efficient frontier result in the best combinations of those securities.

Suppose there are n risky assets, and let X = $(x_1, \dots, x_n)^T$, where xi denotes the proportion of wealth invested in asset i, and Pn i=1 xi = 1. If we plot every possible asset combination of n-assets (3 in the graphical example below) with expected return on the y-axis and portfolio risk (standard deviation) on the x-axis, then the collection of all such possible portfolios, represented by the crosses in the graph below, defines a region and the boundary of the cloud of points is called portfolio possibilities curve (AFB curve). Efficient Frontier is the upper half of the portfolio possibilities curve from point F to point B.

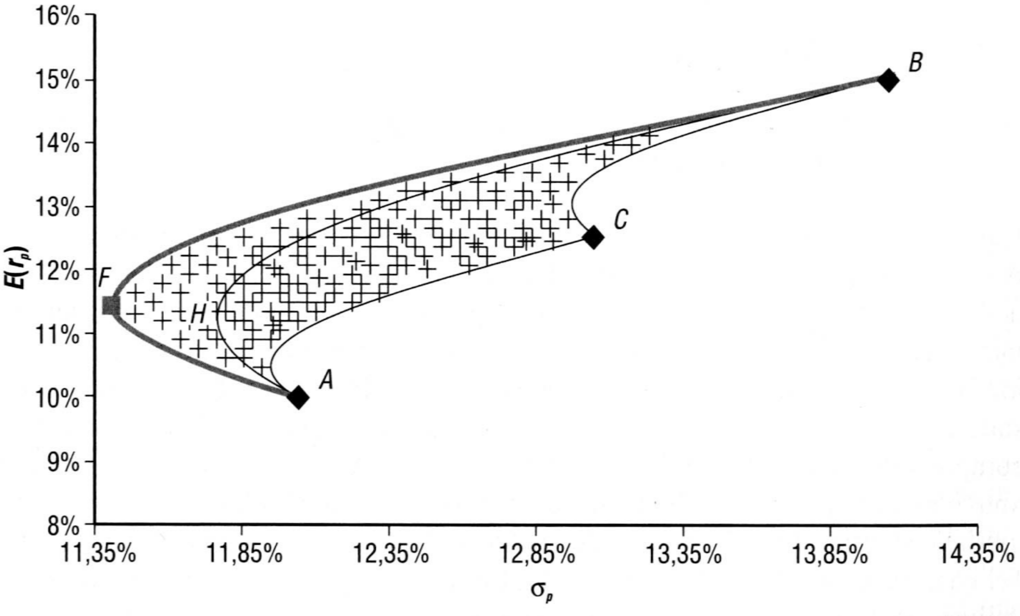

Figure 2.3.1, Efficient portfolio frontier

SOURCE: FABRIZI, P.L., "ECONOMIA DEL MERCATO IMMOBILIARE", EGEA, MILANO 2003 EFFICIENT PORTFOLIO FRONTIER WITH 3 ASSETS

As can be seen from Figure 2.3.1, the FB section will be the efficient frontier with F minimum variance point. Portfolios along the Efficient Frontier (FB curves) offer the least risk for a given level of return, so they are the best possible portfolio combinations. So not choosing a portfolio on the efficient frontier would be sub-optimal because there would exist another portfolio with the same expected return but with a lower risk.

Later on, mainly thanks to James Tobin's (Separation theorem - 1958) contributions[6], a riskless asset was included, which implied that investors could create a portfolio of a riskless asset and n risky assets.

[6] Only after 1958 Separation theorem, Lintner and Treynor directly include in their works the possibility of borrowing the risk-free asset also to negative extent, Tobin remain in a risk-free borrow positivity constraint and Sharpe and Mossin do not explicitly consent leverage

The combination of risk-free and risky assets results in a new mean-variance efficient portfolio "curve" which will be a straight line starting from the risk-free return in the y-axis (intercept in figure 2.3.2), the new Mean-Variance-efficient frontier will be tangent to the previous efficient frontier, and the tangency portfolio T will be the one with the highest Sharpe ratio.

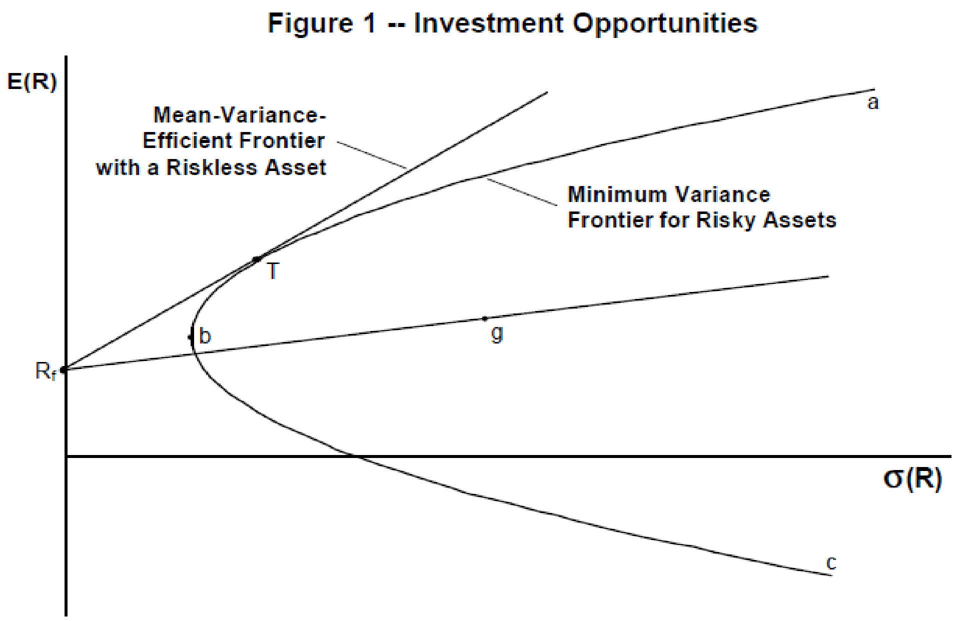

Figure 2.3.2, Efficient portfolio frontier with a risk-free asset

(THE CURVE ABC, THAT IT IS CALLED THE FRONTIER OF THE MINIMAL VARIANCE, TRACES COMBINATIONS OF EXPECTED RETURN AND RISK FOR THE PORTFOLIOS OF RISKY ASSETS THAT REDUCE TO THE MINIMAL RELATIONSHIP RETURN-VARIANCE TO VARIOUS LEVELS OF EXPECTED RETURN. THESE PORTFOLIOS DO NOT INCLUDE THE RISK-FREE RATE)

(SOURCE: E. F. FAMA AND K. R. FRENCH, "THE CAPITAL ASSET PRICING MODEL…", P.27)

Investors can invest all their wealth in the tangency portfolio or invest some of their wealth in the tangency portfolio and the remaining in Rf (the point at the left of T) or borrow money to invest in the tangency portfolio (the point at the right of T). The intercept on the y-axis represents a portfolio composed of 100% risk-free assets, the tangency portfolio represents holdings of 0% risk-free assets and 100% of the portfolio at the tangency point, and portfolios above the tangency point represent a negative holding of the risk-free asset (Leverage T holdings formally introduced by Treynor and Mossin works[5]).

Considering the same rates apply for every investor in the market, all those investors will end up choosing the same tangent portfolio, therefore such portfolio represents the market portfolio! The first step in the formulation of the CML (Capital Market Line) and the Capital Asset Pricing Model.

2.4. CAPM

The Capital Asset Pricing Model is a model of equilibrium of the financial markets, that determines a relationship between the yield of a Stock and its riskiness, measured through a single "factor of risk", called beta.

The CAPM is based on the portfolio choice model developed by Harry Markowitz (1959) and Tobin (1958).

In the Markowitz model, an investor selects a portfolio at time t-1 that produces a stochastic return at time t. The model assumes that investors are risk-averse, and when choosing portfolios, only care about the average and variance of return on investment over a single period. As a result, investors choose "mean-variance-efficient" portfolios, in the sense that portfolios:

1) minimize the variance in portfolio performance, based on expected return

2) maximize the expected return, given the variance. Therefore, Markowitz's approach is often called the "Mean-Variance model".

On one hand, therefore, in the "Portfolio Model", an algebraic formulation is provided on the weights of assets that are part of the efficient portfolios according to the principle of mean-variance. As an improvement, the CAPM transforms this algebraic statement into a verifiable forecast regarding the relationship between risk and expected return, identifying a portfolio that is efficient if asset prices remain consistent with the market.

Treynor (1961), Sharpe (1964), Lintner (1965), and Mossin (1966) are the four economists who contribute to the realization of the first Capital Asset Pricing Model; Through their work, they contributed to the transition from the theories and laws identified by Tobin and Markovitz to the relationships of linearity between expected returns and covariance with a "market portfolio".

Their intuitions and formulations lead to a first CAPM that through improvements and changes over time have made it become that model of equilibrium of the markets that is still studied in every course of economics and finance of our day, and which also represents the conceptual basis of almost all the financial models made in the following years to date.

The CAPM considers 4 assumptions additional to those presented by Markovitz:

1. The single-period horizon adopted by each investor shall be the same for all investors.

2. Each investor may invest or borrow - without limitation - at a risk-free rate, which is the same for all investors.

3. Investors have homogeneous expectations about expected returns and expected variances and covariances of returns compared to all risky assets in which they may invest.

4. There are no taxes, transaction costs or other market imperfections.

With full agreement on the distribution of returns, all investors have access to the same set of opportunities (Figure 2.3.2) and combine the same risky, T tangent portfolio[7] with risk-free loans or borrowed funds.

As all investors hold the same portfolio of risky assets, that portfolio should contain values weighted by their market value in relation to the total value of the risky assets.

Specifically, the weight of each risky asset in the tangent portfolio, which we now call M (for the "Market" by Fama), shall be the total market value of all outstanding units of the asset divided by the total market value of all risky assets included in the "market portfolio".

In short, the CAPM assumptions imply that the M-market portfolio must be at the frontier of minimum variance if the asset market is balanced.

This means that the algebraic relationship that applies to any portfolio of minimum variance must apply to the market portfolio. If there are N risky assets:

$$E(R_i) = E(R_{zM}) + [E(R_M) - E(R_{zM})]\beta_{iM}, \qquad i = 1, \ldots, N$$

(Figure: 2.4.1)

In this equation, $E(R_i)$ is the expected return on asset i and $ß_{iM}$, the beta of the market for asset i, which represents the covariance of its return with the market return divided by the variance of market returns:

$$\beta_{iM} = \frac{cov(R_i, R_M)}{\sigma^2(R_M)}$$

(Figure: 2.4.2, Market Beta)

The first term on the right side of the minimum variance condition (2.4.1), E(Rzm), is the expected return on assets that have zero market beta, which means that their earnings are unrelated to the market return.

The second term is a risk premium - the beta of the asset i, ßiM, multiplies the premium per beta unit, which is the expected return on the market, E(RM), minus E(Rzm). Since the beta of the asset i is also the slope in the regression of its yields,

$$\sigma^2(R_M) = Cov(R_M, R_M) = Cov(\sum_{i=1}^{N} x_{iM} R_i, R_M) = \sum_{i=1}^{N} x_{iM} Cov(R_i, R_M)$$

(Figure: 2.4.3)

a common (and correct) interpretation of beta is that it represents and measures the sensitivity of an asset’s performance to the variation in market performance.

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[7] the same value-weighted proportion inside the market portfolio

But there is another interpretation of the beta more coherent with the underlying principles of the portfolio model on which CAPM relies.

The risk of the market portfolio, measured by the variance of its yield (the ßiM denominator), is the weighted average of the covariance risks of assets in M (the ßiM numerators for different resources). Thus, ßiM also represents the covariance risk of asset i in M, measured against the average of the covariance risk of "all" assets, which is nothing more than the variance of the market return.

In economic terms, ßiM is proportional to the risk that every dollar/euro invested in assets contributes to the market portfolio.

The last step in the development of the Treynor - Sharpe - Lintner - Mossin model is to use the hypothesis of risk-free loans to identify E(Rzm), that is the expected return on a zero-beta asset.

The return on a risky asset is not related to the market return when the average of the covariances of the asset with the returns on other assets simply compensates for the variance of the returns (returns) of the asset (if its beta is zero). Such risky activity is risk-free in the market portfolio in the sense that it does not contribute to the change in the return on the market.

Where risk-free loans exist, the expected return on assets that are unrelated to the market return, E(Rzm), shall be equal to the risk-free rate, Rf. The relationship between expected returns and beta then becomes the CAPM equation,

$$E(R_i) = R_f + [E(R_M) - R_f)]\beta_{iM}, \qquad i = 1, \dots, N$$

(Figure: 2.4.4)

Transforming the formula into words, the expected return on any asset i is the risk-free interest rate, Rf, plus a risk premium, which is the beta of the asset market, ßiM, multiplied by the beta risk unit premium, [E(RM) - Rf]. It remains the fact that obtaining funds and lending them at a risk-free rate is an unrealistic assumption.

So, in simpler words, the CAPM as we know it today classifies risk into two macro-categories: systematic and unsystematic risk.

The unsystematic risk, also called idiosyncratic/specific risk, is the "specific" risk of a particular action or industry. According to the CAPM formulations this type of risk can be completely eliminated by the diversification of the portfolios held and therefore is not worthy of compensation.

The systematic risk, on the other hand, which we can consider as market risk, is the risk related to the overall market, since this risk cannot be eliminated by the diversification of a rational investor, it is the only one that deserves compensation in the form of an excess return.

Below, the graphic representation of the CAPM. The Security Market Line (SML) that, as it can be noticed, introduces the expected rate of return like a function of the beta (β) and so of the systematic risk.

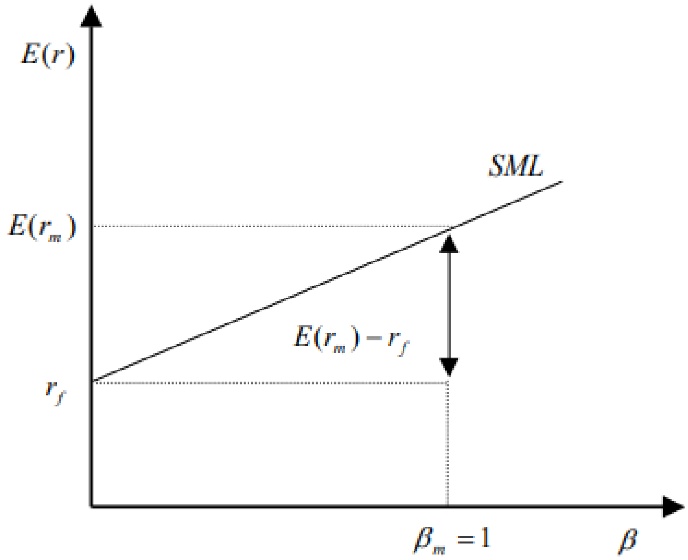

Figure 2.4.1, (SML) Security Market Line

In short, the CAPM familiar equation that reports the expected returns of assets to their market beta is only an application to the market portfolio of the relationship between expected return and beta that exists in any efficient portfolio in a mean-variance perspective. The efficiency of the market portfolio is based on many unrealistic assumptions, including a complete and unrestricted homogeneity of expectations, risk-free activities (in reality there are no activities that are actually risk-free and with absolute certainty of returns) or short selling without restrictions of risky activities (Black - 1972).

But, as many other works also state, all interesting models involve unrealistic simplifications, which is why they must be tested and verified against empirical data.

2.5. CAPM criticism and empirical testing

Much criticism has been directed towards the CAPM, which is still a widely used model and on which all the literature and the evolution of asset pricing models from about 1970 onwards is based.

Report all the criticisms, together with qualitative and quantitative observations that the model has received over the years is out of the scope of this elaboration, the student thinks however that are worthy of a brief quotation: the criticism of the market portfolio of Rolle, the empirical tests, and criticisms of Black and the APT of Ross, before arriving at the analysis of the Fama and French models. Those three contributions have inevitably influenced the direction taken by the majority of works published after the CAPM to date.

CAPM early empirical tests and Black works

The first CAPM verification tests (2.5.4 and 2.5.1) were carried out by Sharpe (1966) and Jensen (1967, 1969) on the relationship between expected return and systematic risk of a large sample of mutual funds obtaining positive results. However, Douglas (1969), Lintner (1965), and many others such as Miller and Scholes (1972) suggest that the model does not provide such a complete description of the structure of the stock expected return.

The work done by Miller and Scholes suggests that the α of individual assets are systematically dependent on their β: assets with high beta tend to have negative alpha, while those with low beta have positive alpha trends. Hence the real need to verify, if there are, "anomalies" relevant from the statistical point of view of the basic CAPM. Such “verification” is conducted mainly through two methods that aim to confirm or not the key results of the CAPM.

CAPM tests are based on three implications of the relationship between expected return and market beta implied in the model.

First, that the expected returns on all activities have linear relationships to their beta and no other variable has an explanatory (marginal) power.

Second, the beta premium is positive, which means that the expected return on the market portfolio exceeds the expected return on assets whose returns are not related to the market return.

Third, in the Treynor-Sharpe-Lintner-Mossin model, non-market-related assets have projected yields equal to the risk-free interest rate, and the beta premium is the expected market return minus the risk-free rate.

Most tests of these predictions use cross-section regressions or time-series regressions. Both approaches date back to early model testing. The first cross-section regression tests focus on the predictions of the Treynor-Sharpe-Lintner-Mossin model on intercept and slope, in the relationship between expected return and market beta.

The approach is to conduct an analysis of the cross-section of average asset returns on asset beta estimates. The model provides that the intercept in these regressions is the risk-free interest rate, Rf and the beta coefficient is the expected return on the market above the risk-free rate, E(RM) - Rf.

Two problems in these tests quickly became apparent. First, beta estimates for individual assets are inaccurate, creating a measurement error problem when used to explain average returns. Second, the deviations (residuals) of the regression have common sources of variation, such as effects related to average returns in an industrial sector.

Positive correlation in discards produces downward "anomalies" in the usual estimates of ordinary least squares of standard errors in the slope of cross-section regression.

To improve the accuracy of estimated betas, researchers such as Blume (1970), Friend and Blume (1970), and Black, Jensen, and Scholes (1972) work with portfolios, rather than with individual titles. As the expected returns and market betas combine in the same way as portfolios, if the CAPM explains the returns on securities, it also explains the returns on portfolios.

Beta estimates for diversified portfolios are more accurate than estimates for individual securities. Thus, the use of portfolios in cross-section regression of average beta returns reduces critical errors in the variable problem.

Grouping, however, reduces the range of betas and reduces the significance of statistics. To mitigate this problem, researchers order stocks versus betas when they form portfolios; the first portfolio contains stocks with the lowest betas, and so on, up to the last portfolio with the highest beta assets. This methodology and these procedures are now standard in empirical testing.

In the empirical analysis carried out by Black Jensen and Scholes (1972), the equation:

$$\bar{R}_j = \gamma_0 + \gamma_1 \hat{\beta}_j + \tilde{u}_j$$

Formula 2.5.1

Has been used to conduct estimates in their work "The Capital Asset Pricing Model: Some Empirical Tests" and contrary to theory, γ0, seems to be significantly different from zero and, γ1, significantly different from, (Rm-Rf), that is the slope of the market line previewed from the model.

The outcome of their work lead to results and conclusions that indicate that the "classic" formulation of the asset pricing model given by (2.4.4) does not give an accurate description of the structure of securities returns.

Tests confirm the occurrence of positive and negative differentials of parameter α. Their analysis leads them to say that the data analyzed with their techniques can best represent the expected returns of the stocks, thanks to a two-factor model such as:

$$E(\tilde{r}_j) = E(\tilde{r}_z)(1 - \beta_j) + E(\tilde{r}_M)\beta_j$$

Formula 2.5.2

Where, r, indicates total returns and E(rz) is the expected return of a second factor, which is called a "beta-factor" since its coefficient is a function of the β of the asset.

Observed this phenomenon, Black (1970) was able to demonstrate that eliminating the assumption of the possibility of lending and borrowing to riskless assets, generates an asset pricing model that allows finding the expected return on an asset, in equilibrium, through equation (2.4.4). Its results provide an explicit definition of the beta-factor, rz, as the return of a portfolio that has zero covariance with the return of the market portfolio rm, such model is also often called zero-beta CAPM.

The major influence in the work of Black Jensen and Scholes however aims to test the traditional form of the CAPM ($\gamma 0$ is different from zero?).

Another important contribution in the field of empirical tests carried out on the CAPM and to demonstrate the existence of important "anomalies" coming from the forecasts of the CAPM is the work of Fisher Black "Beta and Return".

The research work carried out by Black examined the period 1931-1991, retracing the methodologies of analysis of the work of Black-Jensen-Scholes (in fact it uses the two-factor model for its analysis), Black uses monthly data and the method of data collection in portfolios, particularly effective for long-term data analysis as stocks under analysis undergoes many changes.

Briefly, Black considers 10 different portfolios containing assets from the smallest to the largest systematic risk, portfolio 10 (1) contains assets with the least (greatest) systematic risk. For each portfolio Black estimates the excess return and beta from historical data from 1931 until 1991.

Black thus emphasizes how in the 25 years passed from 1966 to 1991 the returns remain practically the same between high beta and low beta stocks maintaining however the overperformance of low-beta stocks. This shows how the slope of the CAPM reaches a practically flat level, which means that even the related risk premium turns out to be not relevant and very close to zero.

These observations give rise to divergent views and opinions about the model, which however strongly question the effectiveness and reliability of the CAPM. Below are the tables and graphics to highlight the results of the work of Fisher Black:

EXHIBIT 2
Monthly Regressions: 1931 to 1965

	Black-Jensen-Scholes Study Portfolio Number										
Item	1	2	3	4	5	6	7	8	9	10	M
1. β	1.56	1.38	1.25	1.16	1.06	0.92	0.85	0.75	0.63	0.50	1.00
2. α	-0.01	-0.02	-0.01	0.00	-0.01	0.00	10.01	0.01	0.02	0.02	
3. $t(\alpha)$	-0.43	-1.99	-0.76	-0.25	-0.89	0.79	0.71	1.18	2.31	1.87	
4. $\rho(\tilde{R}, \tilde{R}_m)$	0.96	0.99	0.99	0.99	0.99	1.98	0.99	0.98	0.96	0.90	
5. $\rho(\tilde{e}_t, \tilde{e}_{t-1})$	0.05	-0.06	0.04	-0.01	-0.07	-0.12	0.13	0.10	0.04	0.10	
6. $\sigma(\tilde{e})$	0.14	0.07	0.06	0.05	0.04	0.05	0.05	0.05	0.06	0.08	
7. μ	0.26	0.21	0.21	0.20	0.17	0.16	0.15	0.14	0.13	0.11	0.17
8. σ	0.50	0.43	0.39	0.36	0.33	0.29	0.25	0.24	0.20	0.17	0.31

	Current Study Portfolio Number										
Item	1	2	3	4	5	6	7	8	9	10	M
1. β	1.53	1.36	1.24	1.17	1.06	0.92	0.84	0.76	0.63	0.48	1.00
2. α	-0.02	-0.02	-0.01	0.00	-0.01	0.00	0.01	0.01	0.02	0.03	
3. $t(\alpha)$	-0.78	-2.12	-1.30	-0.54	-1.38	0.55	0.72	1.64	1.74	2.21	
4. $\rho(\tilde{R}, \tilde{R}_m)$	0.97	0.99	0.99	0.99	0.99	0.99	0.98	0.98	0.96	0.90	
5. $\rho(\tilde{e}_t, \tilde{e}_{t-1})$	0.05	-0.06	0.00	-0.13	-0.11	-0.07	0.10	0.06	0.11	0.15	
6. $\sigma(\tilde{e})$	0.12	0.06	0.06	0.05	0.04	0.05	0.05	0.05	0.06	0.07	
7. μ	0.26	0.22	0.21	0.21	0.18	0.17	0.16	0.15	0.13	0.12	0.18
8. σ	0.49	0.43	0.39	0.37	0.33	0.29	0.27	0.24	0.20	0.17	0.31

Table 2.5.1, Black regression table 1931-65

(SOURCE: FISCHER BLACK, "BETA AND RETURN", PP.80)

EXHIBIT 4
Monthly Regressions: 1966 through 1991

	Portfolio Number										
Item	1	2	3	4	5	6	7	8	9	10	M
1. β	1.50	1.30	1.17	1.09	1.03	0.95	0.87	0.78	0.67	0.51	1.00
2. α	0.00	-0.01	0.00	0.00	0.00	0.01	0.00	0.01	0.01	0.03	
3. $t(\alpha)$	-3.24	-0.93	-1.02	-0.24	-0.57	1.31	0.63	0.81	0.94	1.79	
4. $\rho(\tilde{R}, \tilde{R}_m)$	0.96	0.98	0.99	0.99	0.99	0.99	0.98	0.97	0.93	0.82	
5. $\rho(\tilde{e}_t, \tilde{e}_{t-1})$	-0.02	-0.02	0.00	0.04	0.06	0.02	-0.03	-0.02	0.09	0.12	
6. $\sigma(\tilde{e})$	0.08	0.05	0.04	0.03	0.03	0.03	0.03	0.04	0.05	0.08	
7. μ	0.06	0.08	0.08	0.08	0.08	0.08	0.07	0.07	0.07	0.06	0.08
8. σ	0.31	0.26	0.24	0.22	0.21	0.19	0.18	0.16	0.14	0.12	0.20

Table 2.5.2, Black regression table 1966-91

(SOURCE: FISCHER BLACK, "BETA AND RETURN", PP.81)

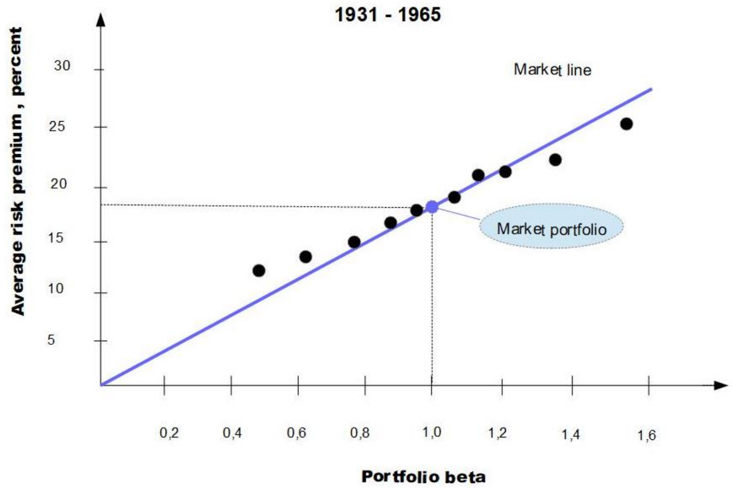

Figure 2.5.1, SML Black studies 1931-1965

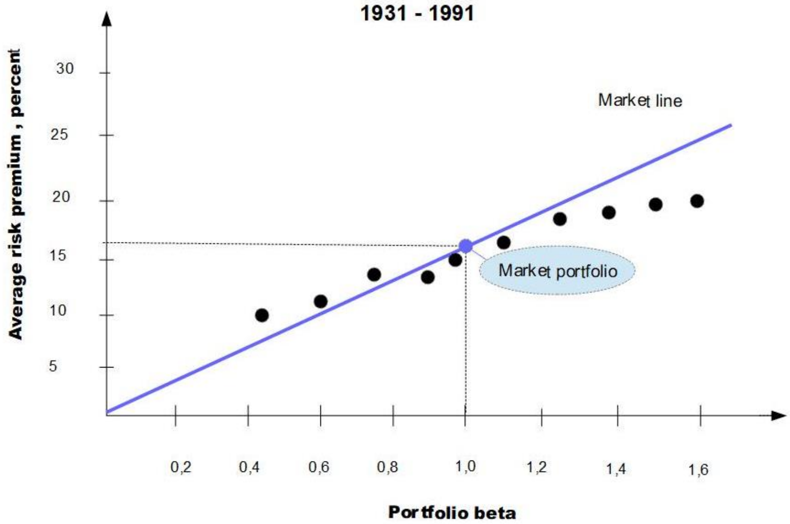

Figure 2.5.2, SML Black studies 1966-1991

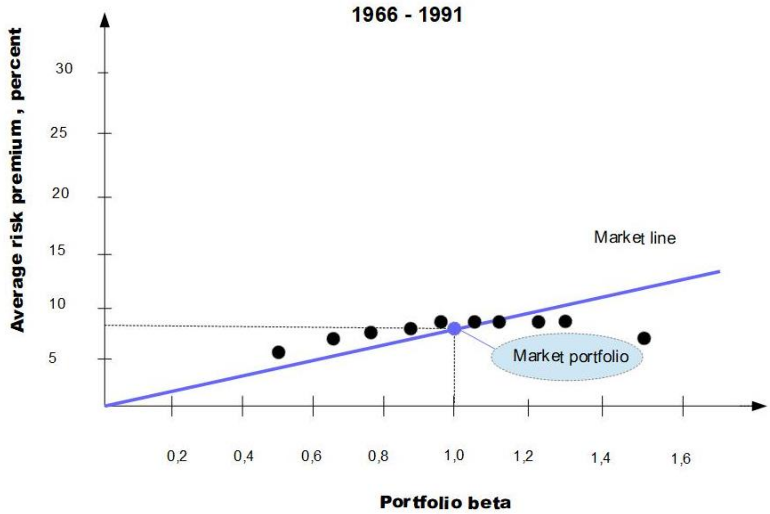

Figure 2.5.3, SML Black studies 1931-1991

"A Critique of the Asset's Pricing Theory's Tests: Part I"

Roll in 1977 questioned the possibility of effectively testing CAPM and suggested that empirical evidence is inconclusive. His argument is based on two fundamental considerations:

1. A CAPM validity test based on Black's two-factor model, admits only one testable hypothesis: that the market portfolio is medium-efficient variance.

2. The market portfolio is not observable, as it should include every single asset in a market.

Therefore, in empirical studies researchers are forced to use "market proxies" and consequently these tests are inconclusive/not conclusive.

Therefore, the tests of the validity of the CAPM, as already said, try to verify the existence of a linear relationship between the expected returns of the activities and their beta with the proxy of the market. These tests are based on "time-series" and "cross-section" methods we have already discussed.

As the market portfolio and market proxy generally do not coincide, CAPM tests can lead to two different statistical errors:

1) Type I error: The test rejects the CAPM when it is valid (i.e., when a linear relationship exists), because the market portfolio is efficient, while the market proxy is not.

2) Type II error: The test accepts the CAPM when it is not valid, as the market portfolio is not efficient, but the market proxy is.

APT Arbitrage Pricing Theory

In the years following the CAPM formulations, there have been numerous attempts to produce and implement alternative theories to explain asset pricing and the balance of financial markets. A theorization that has achieved this goal and that has also opened the doors to new contemporary models is that of Ross (1976) in "The Arbitrage Theory of Capital Asset Pricing".

Ross in his work wants to propose an alternative model to that of Capital Asset Pricing introduced by Treynor-Sharpe-Lintner-Mossin that becomes in those years and up to the present day the most used analysis tool to explain the phenomena observed in the capital markets of risky assets.

The Arbitrage Pricing Theory (APT) develops based on the insights of the CAPM, adding to the single factor "correlation to market risk", a multiplicity of other factors, which are relevant in the context of determining the value of financial assets.

The APT assumes that investors take advantage of all arbitrage opportunities: if two financial assets (or two portfolios) have the same risk exposure but different expected returns, Investors buy the asset with the higher expected return, increasing its price (thus decreasing the expected return) and restoring the equilibrium.

The multifactorial model (APT) also makes a distinction, like the CAPM, between the diversifiable risk and the non-diversifiable risk, but differs in the approach to non-diversifiable (or market) risk measurement: the CAPM assumes that market risk is only related to the market portfolio, while the APT accepts multiple sources of (systematic) market risk, represented by unexpected changes in key macroeconomic variables such as changes in the interest rate structure, exchange rate changes, changes in real GDP growth rates, changes in inflation rate... These phenomena called "factors" measure the sensitivity of the investment to each variation of these variables with a specific beta coefficient.

Thus, the performance of activity i is a linear relationship between n factors, each of which is a random variable with zero mean. In algebraic terms:

$$\tilde{x}_i = E_i + \beta_i \tilde{\delta} + \tilde{\varepsilon}_i$$

(Figure 1.5.9, Source: The Arbitrage Theory of Capital Asset Pricing)

Where εi is the diversifiable risk of the asset, and βiδ is the set of key factors with the respective beta and represent the diversifiable (market) risk expressed by: βiδ = β1f1 + β2f2 + ...+ βnfn.

With βi representing the sensitivity of the investment to unexpected changes in factor i; and fi representing unexpected changes in factor i.

The reference portfolios (benchmarks) in the APT are portfolios of well diversified factors built to have a beta of 1 on one of the factors and a beta of zero on any other factor. We can think of each factor portfolio as a tracking portfolio. The returns of this portfolio track the evolution of a particular source of macroeconomic risk but are not correlated with other sources of risk

2.6. Fama & French three factor Model

The APT shows us how multiple risk factors allow us to realize a multi-factor SML. But how can we identify the most likely/important sources of systematic risk?

One approach, which is now the most widely used, concerns feature derived from solid empirical bases, to procure proxies to measure exposure to systematic risk. The factors chosen as sources of risk are variables that, based on past evidence, predicted average returns well and which could therefore capture the main risk premiums. An example of this approach is the three-factor model of Fama and French and its variants, which have dominated in recent years empirical research in securities yields.

$$R_{i,t} = \alpha_i + \beta_{i,M} R_{M,t} + \beta_{i,SMB} SMB_t + \beta_{i,HML} HML_t + e_{it}$$

(Formula 2.6.1)

where:

SMB = Small Minus Big (the return on a portfolio of small securities (size) in excess of the return on a portfolio of large securities).

HML = High Minus Low (the return on a portfolio of securities with a high book-to-market ratio in excess of the return on a portfolio of securities with a low book-to-market ratio).

This means that Rmt, Smbt, Hmlt, represent the premiums related to the factors considered, and βiM, βiSMB, βiHML, represent the correlation with that premium.

Note that in this model the market index (βim) captures the systematic risk arising from macroeconomic factors.

The other two extra-market factors are chosen based on long-standing observations that the size of the company, measured by market capitalization (market value of outstanding shares, market equity) and book-to-market ratio (book value per share divided by the market price of the shares) provide deviations of the average returns of the shares from levels consistent with the CAPM. Fama and French justify this model on an empirical basis; SMB and HML are not obvious candidates as relevant risk factors, but the analyses carried out in their work lead us to think that these variables can be excellent proxies of fundamental variables much more difficult to measure.

Considering the equation on Fama and French's official paper.

$$R_i - R_f = \alpha_i + b_i\left(R_M - R_f\right) + s_i SMB + h_i HML + \varepsilon_i$$

(Formula 2.6.2)

In this equation, one can observe the expected premiums and sensitivity factors or loadings bi, si, and hi, which represent the gradients in the time-series regression.

Fama and French in 1995 demonstrate how the book-to-market equity and the relative slope to HML make from indicator of approximation (proxy) for periods of "relative difficulty" of the companies/stocks (relative distress). As will be explained in the following lines, this period of "relative distress", translates into an important situation, for companies and related shares, which alter future returns, therefore also the expected returns from the models under consideration.

The dependence of returns on the book-to-market ratio is independent of the market beta, suggesting that companies with a high book-to-market ratio are relatively depreciated or that the book-to-market ratio act as a proxy for a risk factor that influences expected equilibrium returns. Fama and French found that, after controlling the size and the book-to-market effect, the market beta seemed to have no power to explain the average securities returns. This discovery is a major challenge to the notion of rational markets because it seems to imply that a factor that should affect returns, systematic risk, seems to have no importance, while a factor that should not matter, the book-to-market relationship, seems capable of predicting future returns. In particular, "weak" companies with low revenues tend to have low BE/ME and positive HML (si) gradients, while the opposite happens with "strong" companies.

The work of Fama and French, therefore wants to underline, also recalling the work of Chan and Chen (1991)[8], how exists a covariance of the returns connected with the negative periods and difficulties of companies/stocks (relative distress) which is not captured by the return of the market portfolio but is only compensated by average returns.

In short, companies with low E/P (earnings/price), low C/P (cash flow/price), and high sales/revenue growth forecasts, are usually "strong" companies that have a negative slope compared to HML, given that the average HML performance is strongly positive, these negative loadings (low E/P and low C/P), like the HML slope for low BE/ME stocks, imply lower expected returns in equation 2.6.2.

On the contrary, as shares with high BE/ME, shares with high E/P, high C/P, or growth expectations on sales low, tend to have positive HML gradients/loadings, they are therefore in a "relative period of difficulty" (relative distress), and have higher average returns.

The following are the first lines of the document written by the two economists "Multifactor Explanations of Asset Pricing Anomalies" which aim precisely to deepen these anomalies as empirical bases on which a good part of the model develops:

"*Researchers have identified many patterns in average stock returns, for example, DeBondt and Thaler (1985) find a reversal in long-term returns; stocks with low long-term past returns tend to have higher future returns. In contrast, Jegadeesh and Titman (1993) find that short-term returns tend to continue; stocks with higher returns in the previous twelve months tend to have higher future returns. Others show that a firm's average stock return is related to its size (ME, stock price times numbers of shares), book-to-market equity (BE/ME, the ratio of the book value of common equity to its market value), earnings/price (E/P), cash flow/price (C/P), and past sales growth. (Banz (1981), Basu (1983), Rosenberg, Reid, and Lanstein (1985), and Lakonishok, Shleifer and Vishny (1994).) Because these patterns in average stock returns are not explained by the CAPM, they are typically called anomalies.*"

(Source:" Multifactor Explanations of Asset Pricing Anomalies" pp.55)

[8] "Structural and Return Characteristics of Small and Large Firms", K. C. Chan and Nai-Fu Chen

These "anomalies" represent the true starting point of the multifactorial models and their attempt to explain variations in returns not calculated by the CAPM.

These frequently cited anomalies are easily available statistics in the market that apparently serve to predict adjusted returns for "abnormal" risk or significant risk premiums. Examples of this are studies by Basu[9] (1977) which show how portfolios with a low price/earnings ratio over time have generated higher returns than higher P/E ratios, or the "small-firm effect" of Banz[10] (1981) showing that by dividing NYSE's shares into ten portfolios by company size (from the smallest to the largest), average returns were consistently higher in the portfolios of small companies from 1926 to 2015.

Fama and French argue that these effects can be explained as demonstrations/evidence of risk premiums. Using their three-factor model, they show that higher beta stocks (also known as factor loadings in this context) in size or book-to-market ratio have higher average returns; Fama and French interpret these returns as evidence of a risk premium associated with the factor considered (e.g., book-to-market ratio).

The three-factor model, therefore, does a better job than the CAPM, which uses only one factor, to explain securities yields more accurately. While book-to-market size or ratios for themselves are not risk factors, they could act as a proxy for "more fundamental" risk determinants as mentioned above.

Fama and French argue that these return models can therefore be consistent with an efficient market where returns are consistent with risk.

The opposite interpretation is offered by Lakonishok, Shleifer and Vishny[11] (1996), who argue that these phenomena are evidence of inefficient markets, more specifically, of systematic errors in the forecasts of stock analysts. Such analysts project past performance too far into the future, and thus overestimate companies with recent good performance and underestimate companies with recent poor performance. Eventually, when market participants recognize their mistakes, prices "reverse" (price reverse phenomenon). This explanation is consistent with the "reversal effect" of De Bondt and Thaler[12] and, to a certain extent, with the effects of small enterprises (size effect) and book-to-market effect because companies with strong price drops may tend to be small or with high book-to-market ratios.

If Lakonishok, Shleifer, and Vishny and De Bondt and Thaler, are right, we should find that analysts, are systematically wrong when they predict the latest returns from "winning" companies compared to "losers". A study by La Porta[13] (1996) is consistent with this model. He finds that the equity of companies for which analysts predict low rates of return growth, performs better than those with higher growth expectations. Analysts seem overly pessimistic about companies with low growth

[9] "Investment performance of common stocks in relation to their price-earnings ratios: a test of the efficient market hypothesis", S. Basu (1977)

[10] "The relationship between return and market value of common stocks", Rolf W. Banz

[11] "Contrarian investment, extrapolation, and risk." The journal of finance 49, no. 5 (1994): 1541-1578.

[12] "Does the Stock Market Overreact?", Werner F. M. De bondt, Richard Thaler, 1985

[13] "Expectations and the Cross-Section of Stock Returns", Rafael La Porta, 1996

prospects and overly optimistic about companies with high growth prospects. When these two extreme expectations are "corrected", low-growth companies exceed high-growth ones.

The document "Multifactor Explanations of Asset Pricing Anomalies" explains, after the introductory lines mentioned above, how the previous equation (2.6.1), written by Fama and French collects and captures many of the anomalies of the CAPM also previously mentioned. Before presenting all the empirical analyses and evidence gathered by the model to demonstrate how the three-factor Fama and French collects most of the average returns of the CAPM anomalies, the two authors discuss how their interpretation of the tests conducted may have some basic problems or errors.

The main problems and criticisms that arise against this model are:

1. According to the two authors, the "reasonable skepticism" is caused by the "premium for distress" or the average return on the High Minus Low portfolio. Kothari, Shanken, and Sloan[14] (1995) argue that much of this award is generated by survivorship bias[15] in the number of companies with a high book-to-market ratio.
2. Another point of view is that the identification of such premium distress is only due to data snooping or data mining bias[16].
3. The third point of view is that such premium distress is real but irrational; the result of over-reaction leading to a depreciation of distressed stock/holdings (stocks).

[14] "Another Look at the Cross-section of Expected Stock Returns", S. P. Kothari, Jay Shanken, Richard G. Sloan

[15] error that arises in focusing on people or things who have gone through a selection process and neglecting those who have not, generally because of their lack of visibility. This can lead to false conclusions in several ways. The inclination to survival can lead to overly optimistic beliefs because failures are ignored, for example, when companies that no longer exist are excluded from the analysis of financial benefits.

[16] researchers tend to seek and correct results that demonstrate the importance of some variables but are only effective in their examples

3. Literature review

In this chapter, the student wants to briefly address an important dilemma and discussion which is taking more and more attention regarding the asset pricing model debate. Such discussions rely mainly on one big question: are the financial national market integrated due to globalization? and if so, is it better to use a global or local asset pricing model?

Those kinds of topics and discussions open the doors to a huge diversity of approaches, points of view, and literature, which will not be discussed here, being out of the scope of this study.

However, to present coherently the result obtained with a study conducted in a national market outside the US and with high structural differences, it is fair to introduce this sort of dilemma.

This chapter will shortly describe the main model's development and implementations within an international application span. We will shortly present international and global CAPM, and other asset pricing models developed after the Fama-French 3 factor with their international counterpart, summing up the literature behind those works.

After that, the literature and previous work specifically conducted in the Italian stock market will be presented and summed up. Lastly a short comparison between previous work in the Italian market and my contribution with the analysis developed in this work.

3.1. International asset pricing literature overview

The need for an asset pricing model which could efficiently operate in a range wider than a single nation is something wanted and researched by both practitioners and theorists for capital budgeting problems and in particular cost of capital formulations in international and worldwide contexts.

As already said, summing up all the literature in international asset pricing is out of the scope of this study. However, the following chapter aims to present the main contributions and models adopted and tested in international environments.

Since the first published formulation of CAPM many research and analysis have been conducted in the worldwide application of the model and in tests conducted outside the US.

This, outside of the US application of the model, was supported by the idea of a deeper questioning of the validity of its underlying assumptions and practical functionality.

The idea of a single market portfolio as a unique risk factor premium, to predict expected return, recall immediately a simple question: which market portfolio should be used? The one of a single nation? A global portfolio comprising every tradable asset worldwide?

The first attempts to answer these questions started from a not directly stated assumption of the CAPM, which is that the output of the classic CAPM is based on expected returns and risk of a single currency.

Since the ’70 the trend of globalization has increased the volumes of foreign investments by international companies and retailer investors who operate on a world scale. Consequently, in presence of different currencies, the efficiency of classic CAPM[17] was criticized.

It is in this period that Solnik (1974) Sercu (1980) Stulz (1981) and Adler-Dumas (1983) develop and fine-tune an international CAPM which included an additional risk factor that compensates for exchange rate risk.

“The model contains risk premia which are based on the covariance with exchange rates, in addition to the traditional premium based on the covariance with the market portfolio. These new premia are present because of deviation from Purchasing Power Parity[18]. In consuming their capital income, investors of different countries have access to goods at unequal prices, across different nations. Therefore, they view differently the returns on the same assets. The Japanese grant a premium on an asset that protects their real purchasing power; this is a different premium from the one granted by the US investors. Hence the 2 premia show up as separate terms in the aggregate model.”

(Dumas and Sornik 1993 p.1)

Here the model presented in modern Terms:

$$E\left[R_{i,t}\right] = DR_f + \beta\left(R_{m,t} - R_f\right) + \beta(FCRP)$$

Formula 3.1.1

Where DRf represent a domestic risk-free rate, β are the 2 betas corresponding to the risk factors, E[Ri,t] is the expected return on asset i, Rm the expected return of a market portfolio, Rm – Rf the premium for global market risk measured in domestic currency and FCRP the foreign currency risk premium.

In few words, International CAPM allows investors to add a currency effects to classic CAPM, to account for the sensitivity to changes in foreign currency when investors hold an asset. This sensitivity accounts for changes in a currency that directly and indirectly affects profitability and, thus, returns.

The key characteristic of this model is that do not rely completely on all the assumption of the classic CAPM. The international CAPM relies on the fully integrated international capital markets assumption but at the same time relaxes some other classic CAPM assumptions.

In the meantime, much other research and analysis were carried out trying to learn more from international capital asset pricing applications.

Another simpler model used in worldwide asset pricing is the global CAPM, a single factor version of the CAPM that concentrates all its pricing power in a single global asset portfolio.

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[17] Ri = rf + β(rm-rf)

[18] purchase power parity is a measurement of prices in different countries that uses the prices of specific goods to compare the absolute purchasing power of countries' currencies and so allow for the possibility of comparing pricing between different currencies (Ex. Big Mac Index)

The Global CAPM is a single-factor variation that does not capture foreign currency risk. The model applied by Stulz (1995)[19] has the same structure as the domestic CAPM, but with the global market index replacing the domestic market index, as shown in equation (3.1.2):

$$E(R_G) = R_f + \beta_G[E(R_G) - R_f]$$

Formula 3.1.2

"Where RG denotes the required expected return on a stock when markets are global; Rf is still the local country risk-free rate; bG is the global beta of the company in question; and RG denotes the return of the global market portfolio. In the equation above, one would use a proxy for the global market portfolio like the Morgan-Stanley International (MSCI) index."

(globalization of capital markets… P.35-36)

Such a model assumes global supply and demand for capital, and it is typically implemented by corporations operating all around the globe, heavily diversified investors, or large institutional funds.

Although the Global CAPM should be a better valuation tool than the domestic CAPM, especially for internationally traded assets, there is a body of empirical evidence that systematic exposure to exchange rate changes may be "priced" in financial markets, which supports the conceptual superiority of the International CAPM over the Global CAPM.

However, the global CAPM is easier to apply than international CAPM, therefore practitioners still use the global version in situations in which estimates will not fall too far from the more complex (more complex estimation process) international version.

Fama and French (1998), after the general acceptance of their 3-factor model, expanded the debate between growth and value stocks to thirteen major capital markets around the world. They find that in twelve markets - Italy is the exception - there is a value premium. Their tests showed that the international CAPM model fails to capture the value premium in international returns, but a two-factor APT that attempts to explain stock returns using a market return factor and a relative distress factor does a better job, both, on a country level and a global level.

$$R - F = a + b[M - F] + c[H - LB/M] + e$$

Formula 3.1.3[20]

Griffin (2003) tests country-specific and global versions of the Fama and French three-factor model on stocks in 4 different developed countries: the US, Canada, UK, and Japan, for the period from 1981-1995. Regressions for individual stocks and portfolios show that country-specific versions of the three-factor model perform better than global versions in terms of having a higher explanatory

[19] "Globalization of capital markets and the cost of capital: the case of Nestlé"

[20] Where M is the dollar global market return, F is the US treasury bill, H-LB/M is an international version of the HML (distress factor) in the 3-factor model and R is the portfolio or asset dollar return

power and lower pricing errors. The findings in this paper do not support the notion that there are benefits to extending the Fama and French three-factor model to a global context. So, the paper says that country-specific three-factor models are more useful in explaining average stock returns than are world and international versions. These findings have important implications. Cost-of-capital calculations, performance measurement, and risk analysis using Fama French-style models are best done on a within-country basis.

On the wave of models described above and with the experimentation and deepening of multi-factorial models based on empirical evidence, many new models and research have come to light.

Local, international, emerging, and single global market models have undergone changes and developments in recent years.

Many studies have been based on Carhart's four-factor model, which develops from the Fama-French 3-factor by adding a momentum factor.

$$r_i - r_f = \alpha_i + \beta_{i,m}(r_m - r_f) + \beta_{i,SMB} SMB + \beta_{i,HML} HML + \beta_{i,MOM} MOM + \varepsilon_i$$

Formula 3.1.4

MOM (Monthly Momentum factor) or WML(winner minus losers): consists of a portfolio whose premium is obtained by the difference between returns on shares with high past returns minus returns on shares with low past returns.

From this model Chui et al (2010) examine the extent to which momentum pattern is due to behavioral biases.

"*The evidence in this paper indicates that culture can have an important effect on stock return patterns, which is consistent with the idea that investors in different cultures interpret information in different ways and are subject to different biases. One interpretation of our results on the relation between momentum profits and cultural differences is that in less individualistic cultures investors put less weight on information that they come up with on their own and more weight on the consensus of their peers*."

(Chui et Al., Individualism and momentum around the World, P. 389)

Their findings support the notion that culture affects the patterns of stock returns differently, in different countries, because individuals are subject to different "individualistic behavioural biases" and therefore, they interpret information differently.

Fama French (2012) explore size value and momentum risk factor in 23 developed markets. They test both F-F 3-factor and Carhart 4-factor model using two different versions, a local and a global adaptation, to value if asset pricing is integrated or segmented across regions. They do not find much support for the integrated asset pricing approach and the performance of local models explaining variations of portfolios formed on size and value is satisfactory enough according to the authors.

In 2015 F-F introduced their 5-factor model by adding two new factors to the 3-factor: profitability and investment decisions and together with this many other models with different combinations of factors are tested and put into practice to try to give more and more answers extrapolated from empirical research.

The difficulty of finding satisfactory answers from global and international asset pricing models remains constant, although many variables with explanatory power are documented and added in new tests on multi-factorial models. Most evidence from many works suggests that developed and emerging markets have a segmented and not yet globally integrated structure.

3.2. Studies conducted on the Italian stock market

This section introduces past literature and analysis of asset pricing models that have been conducted in the Italian stock market. While there is extensive empirical evidence of the performance of the Fama-French and CAPM model in the US and in other major stock markets like the UK Canada and Japan, few papers have investigated it on the Italian stock market.

Cavaliere and Costa (1999), are the firsts to research for a size effect in the Italian market, Aleati et al. (2000), Cagnetti (2002) search for multifactor risk premum consistency, the former studying HML and SMB and the latter testing APT macroeconomic and empirical factors, Bruni Campisi and Rossi (2006), Rossi (2012), Federico Giovanni Rega (2017) compare the CAPM and Fama-French 3 factor, Brighi and D’addona (2008, 2010, 2013, 2015), extend the comparison also to the Charhart 4-factor model running different type of regressions. Canegrati (2008), De Chiara and Puopolo (2015), investigate only the CAPM with different methodologies, Veltri and Silvestri (2011), focus mainly on the Fama-French model and CAPM literature comparison. Pirogova and Roma (2019) Gagliolo and Cardullo (2020) are the 2 most recent studies where the former compare CAPM and F-F 3-factor and the latter focus on the existence of a value premium.

Trying to maintain a chronological order, all the previous work available has been briefly reviewed and in the following line, I will try to report all the major insight and evidence highlighted in those works. It must be said, generally speaking, there is high heterogeneity across all the studies in the Italian stock market, regarding the sample period, models, econometric method, portfolios construction, and other minor differences.

Cavaliere and Costa's (1999) research for a relationship between firm size and asset returns after Banz's (1981) publication. The following is their main motivation behind the size effect:

“One of the most persuasive explanations of this phenomenon, which is compatible with the efficient market hypothesis, is that operators have a less consistent and less accurate flow of information when small firms are considered; financial markets transfer this higher degree of uncertainty into a higher risk and, consequently, into a higher expected return”

Firm size and the Italian Stock Exchange, p. 729

They use a sample of 178 firms, analyzed from 1986 to 1995 by referring to monthly returns of the corresponding ordinary shares and capitalizations on 31 December of each year. They adopt a 2-factor version of the CAPM with a market portfolio and a size factor. Their result confirmed that firm size represents a relevant component of the return generating process in the Italian market and consequently suggests that a 2-factor model overperforms a single factor CAPM-like model.

Aleati et al. (2000) studied the sample period 1981-1993, investigating the relationship between common risk factors and average returns for Italian stocks. They studied the explanatory power of HML and SMB for average stock returns compared to macroeconomic factors[21].

They applied tests on the returns of single stocks, and, as they wrote in their conclusion paragraph:

"We found that changes in the market index, changes in oil prices, a default premium, changes in interest rates, and the SMB and HML factors represent a good summary of the risks captured by the cross-section regression of average Italian returns. Our findings revealed that the economic risk premium associated with the size and book-to-market equity variables are priced even in combination with macroeconomic factors.

Results for the Italian market reveal that both macroeconomic variables and equity risk factors are relevant for pricing stock returns. This is not surprising for an open economy such as the Italian one, but our paper provides evidence that differs substantially from the well-known results for the US stock market."

The pricing of Italian equity returns p. 169

Cagnetti (2002) carry out an empirical study for the period 1990-2001 showing weak results of the CAPM and better performances showed by an APT model.

"Five factors have been found relevant in the APT model, with the first factor explaining nearly 40% of the total variance. The significant macroeconomic variables in the Italian stock market overlap considerably with factors found relevant on other countries (market portfolio, fixed income securities, inflation, imports), with the interesting relevance of the factor representing people's expectation, introduced for the first time in the APT studies of Chen (1995)."

"Capital Asset Pricing Model and Arbitrage Pricing … " p.26

The study focused on showing a relevant percentage of normally distributed shares return in the Italian market, comparing CAPM and APT with overperformance of the latter, and a final focus on factor construction for APT models[22].

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[21] The research compared explanatory power of equity related variables and macroeconomic factors such as: (MPS) monthly change in industrial production, (UI) unanticipated inflation calculated from the consumer price index, Foreign exchange risk premia like the Italian lira volatility compared to the cross rate lira/Deutsche Mark in European central parity, change in oil prices and others (P.157)

[22] The main results presented 8 major factors underlying Italian economy: first a market portfolio factor which included MIBTEL and FTEU (financial times european index), second fixed income italian yields, third and fourth factors representing the impact of foreign variables on the italian economy, fifth a production and monetary GDP related factor, sixth inflation, seventh industrial production, eight a trends indicator index which the author interpreted as a people expectation factor (P. 19, 20, 22)

Bruni Campisi and Rossi (2006) are probably the first researchers which present a paper comparing CAPM and Fama-French 3 factor in the Italian market, they carried out time-series tests on size and value sorted portfolio (following Fama French method) for the 1973-1986 period. Their study Underlined CAPM empirical weaknesses and accepted the better empirical result of the Fama French model even with the not so strong significance of the HML factor.

Rossi (2012) shows a similar result in a Fama-French and CAPM comparison for the 1989-2004 period, highlighting a size-effect[23]. Also, in this case the results obtained seem to confirm the existence of additional factors that can better explain returns, so the result wants to show pieces of evidence of CAPM Weakness and Fama-French relative empirical superiority.

Brighi and d'Addona (2008, 2010), probably for the first time in an official paper in the Italian market, implement an empirical comparison of CAPM, Fama-French 3-factor, and the Carhart 4-factor model.

In 2008 they carried test with OLS and GMM time-series regressions on 598 assets of the Italian Stock market for the period 1986-2002. They found that the size premium is confirmed, the value premium is weak, and the momentum effect[24](WML) shows no evidence of significance in their studies.

In 2010 they focus on the 4-factor model carrying Generalized method of moments (GMM) regressions and receiving similar results of 2008 analysis: size effect confirmed, value effect weakly confirmed and momentum effect substantially absent in the Italian market.

With an opposite view relative to other similar studies of that period De Chiara Puopolo (2015), fully support the CAPM efficiency and present results in favor of the full capacity of the market portfolio to explain the expected returns of stocks in the Italian market in a medium-long run. Their work focuses on the period 1983-2013, using log return and monthly frequencies to conduct a time series-regression OLS on individual stocks and not using portfolios.

Pirogova Roma (2019) test performance of portfolio sorted on size and BE/ME for the period 2000-2018.

They focus a consistent share of their attention on the inaccuracy of the Datastream database which was the main database used for most of the previous works. They recall the presence of errors in some values reports so they use *Borsa Italiana* and *Mediobanca* databases (*indici e dati*).

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[23] The size effect in finance literature refers to the observation that smaller firms have higher returns than larger firms, on average over long horizons. It also describes the contribution that firm size has in explaining stock returns. Discovered by Banz (1981) in testing the Capital Asset Pricing Model. Michael A. Crain, "A Literature Review of the Size Effect"

[24] The momentum bias suggests that traders can take advantage of price movements by going long on winners (last 12-month good performing stocks) and shorting the losers (last 12 month bad performing stocks). One of the popular explanations for the momentum effect is that markets do not immediately price in new information but do so more gradually.

Their results show CAPM failure in explanation of cross-section of returns, while the Fama-French three-factor model provides a better fit, their results show the significance of all the three-factor used in the F-F model.

Gagliolo Cardullo (2020) work on a study that aimed to verify the existence of the value premium and its evolution from 2001-2018. They showed that value shares outperform growth ones only in the 2001-2006 period, then, after the outbreak of the financial crisis in 2008, the value premium disappeared (mainly negative and very close to zero) confirming the evidence documented also in the US market, according to their results.

3.3. The Contribution of this thesis

The objective of this thesis is to add material and results to the empirical research that have been conducted in the Italian stock market.

Furthermore, this work is different from the studies that have been previously conducted on the Italian Market in several ways. Firstly, it uses a relative smaller dataset, compared to the average of the works presented above. This work focuses mainly on the FTSE MIB[25] index and its constituents, using all the stock that has been included in such index for the period 2001-2020 and so the ones that have been in the 40 most capitalized stock in the Italian market for the chosen period, represent 80% of the total Italian market capitalization and the most traded and liquid stocks available in the public market.

The student chooses the FTSE MIB since it's the worldwide index which characterizes Italian stock market, represent most of the public traded company in Italy and so the most "mature" and "reliable" index, from an international investor point of view.

The student knows the bias and implication of not building an all-asset available market portfolio but still thinks that empirical testing is always conducted in approximated samples and so, the choice ends up with the most characteristic and straightforward index, which could also, more probably, receive those kinds of analysis from a practical/practitioner standpoint.

This procedure represents a strong approximation but still should bring out its piece of empirical insight for a small and less structured market compared to the US one.

Secondly, a prediction test activity has been carried out to investigate and compare in alternative ways the models under analysis.

[25] The FTSE MIB is a benchmark index for the Italian equity markets. Capturing approximately 80% of the domestic market capitalization, the index is comprised of highly liquid companies in Italy. The FTSE MIB Index measures the performance of 40 shares listed on Borsa Italiana and seeks to replicate the broad sector weights of the Italian stock market. The index is derived from the universe of stocks trading on the Borsa Italiana (BIt) MTA and MIV markets. The index has been created to be suitable for futures and options trading, as a benchmark index for Exchange Traded Funds (ETFs), and for tracking large capitalisation stocks in the Italian market. The FTSE MIB Index does not take account of ESG factors in its index design.
(FTSE Russell | FTSE MIB Index, v4.3, March 2021)

Third, in the regression analysis, a 3x3 LHS-dependent variable portfolio construction has been carried out, and, also in this case, it kind of represents an approximation compared to a 5x5 or 4x4 portfolio structure. The choice was made due to the restricted number of available stocks and to maintain diversified portfolios.

Fourth, the work does not focus completely on the analysis but wants also to offer a critical point of view of many arguments related to the asset pricing subject from the literature development to the international and national scope of the model's discussion.

4. Empirical studies

In the wake of the literature analysis, the study attempts to compare the performances of the two models introduced above, CAPM and Fama-French 3 factor model using Italian market stock data.

CAPM

$$R_{i,t} - r_{f,t} = \alpha_i + \beta_i\left(R_{m,t} - r_{f,t}\right) + \varepsilon$$

FAMA-FRENCH 3-factor

$$R_{i,t} - r_{f,t} = \alpha_i + b_i\left(R_{m,t} - r_{f,t}\right) + s_i SMB + h_i HML + \varepsilon_i$$

Where Rit represent the return of asset i in period t, rf is the risk-free rate in period t, Rm is the return on the market portfolio, SMBt represent the size factor and consist of the return of a group of diversified portfolios of small stock minus a group of diversified portfolios of big stocks, HMLt represent the value factor and consist of the returns of a group of diversified portfolios of high BE/ME ratio minus the returns of a group of diversified portfolios of low BE/ME ratio, bi, si, hi are regression slope coefficient.

4.1. Data

This study is based on the Italian stock market from august 2001 to December 2020.

The data sample used in the thesis was obtained Mainly from Datastream except for the Historical FTSE MIB monthly prices (used as a proxy for the market return) and the 3-month BOT (*Buoni ordinari del tesoro*) historical rates, used as risk-free rates, downloaded from Bloomberg.

I've worked with monthly log returns calculated from the historical monthly prices downloaded. Regarding the stock universe, to avoid Survivorship Bias all the stock that had been included in the FTSE MIB Portfolio for at least 1 year: dead, active, or suspended, had been included in the analysis.

The following financial figures were downloaded for each stock: Historical close prices, Market Capitalization, Book value of equity (Common equity).

Variable's definition

Size: As a measure of size have been used the market capitalization of each stock. In similar studies performed on other markets Price (P) has been used multiplied by the number of outstanding shares. Since it gives the same results and removes one calculation step, Market Capitalization has been used in this study (WC08001)

Book-to-Market Equity: the book-to-market variable is calculated as common equity divided by market capitalization. Book Equity was downloaded from Datastream using common equity function (WC03501).

Cleaning the data

The initial data set contained multiple companies which presented data errors, or that did not contain the financial figures required for the whole period.

In the cleaning process the following companies or data were removed:

1) Companies that do not contain any data for the period
2) Data points classified as outliers
3) Companies that do not have available the necessary financial figures for at least one year

After the cleaning process, the universe of initial companies has been reduced and Table 4.1.1 shows the 75 remaining ones included in the actual analysis.

It's important to notice that the companies were actively included in the calculations only if data for all 12 months in a year were present. If, as an example, a company were listed in august 2007, and so didn't have closing price data for the previous month available, that company with its relative prices would have been included in portfolio calculation only in the successive year.

A2A SPA	EXOR NV	POSTE ITALIANE SPA
ALITALIA - LINEE AEREE ITALI	FASTWEB SPA	PRYSMIAN SPA
ALLEANZA TORO SPA	FERRARI NV	RCS MEDIAGROUP SPA
AMPLIFON SPA	FIAT SPA (STELLANTIS)	RECORDATI INDUSTRIA CHIMICA
ANSALDO STS SPA	FINECOBANK SPA	SAIPEM SPA
ARNOLDO MONDADORI EDITORE	GEDI GRUPPO EDITORIALE SPA	SALVATORE FERRAGAMO SPA
ASSICURAZIONI GENERALI	GEOX SPA	SNAM SPA
ATLANTIA S.P.A.	GTECH SPA	STMICROELECTRONICS NV
AUTOGRILL SPA	HERA SPA	TELECOM ITALIA SPA
AZIMUT HOLDING SPA	INFRASTRUTTURE WIRELESS ITAL	TENARIS SA
BANCA GENERALI SPA	INTERPUMP GROUP SPA	TERNA-RETE ELETTRICA NAZIONA
BANCA MONTE DEI PASCHI SIENA	INTESA SANPAOLO	TIM SPA
BANCA POPOLARE DI MILANO	ITALCEMENTI SPA	TISCALI SPA
BENETTON GROUP SRL	ITALGAS SPA	TOD'S SPA
BPER BANCA	ITALIAONLINE	UBI BANCA SPA
BREMBO SPA	JUVENTUS FOOTBALL CLUB SPA	UNICREDIT SPA
BULGARI SPA	LEONARDO SPA	UNIPOL GRUPPO SPA
BUZZI UNICEM SPA	LUXOTTICA GROUP SPA	UNIPOLSAI ASSICURAZIONI SPA
CAPITALIA SPA	MEDIASET SPA	WEBUILD SPA
CIR SPA	MEDIOBANCA SPA	WORLD DUTY FREE SPA
CNH INDUSTRIAL NV	MEDIOLANUM SPA	YOOX NET-A-PORTER GROUP
EDISON SPA	MONCLER SPA	UNICREDIT SPA
ENEL GREEN POWER SPA	NEXI SPA	UNIPOL GRUPPO SPA
ENEL SPA	PARMALAT FINANZIARIA SPA	UNIPOLSAI ASSICURAZIONI SPA
ENI SPA	PIRELLI SPA	WEBUILD SPA

Table 4.1.1, shares sample

The stocks in table 4.1.1 are the ones used for constructing the LHS and RHS portfolios (factors) used for the regression and the test.

Portfolio construction

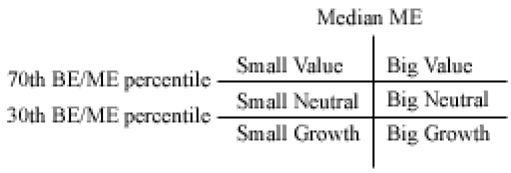
The following two tables show the number of stocks included in each portfolio created for the Right-Hand side factor construction and the Left-Hand Side ones.

Constructing the RHS factors

The CAPM has only one RHS portfolio; the excess return of the market portfolio (market return minus risk-free rate).

The F&F 3 factors, except for the excess return on the market portfolio that is the same as above, can be constructed using different sets of breakpoints. On the Kenneth French official website, the formula factors (SMB, HML for the 3-factor model) are constructed using 2x3, 5x5, or 10x10 portfolios.

In this work, the RHS portfolios are constructed using the 2x3 method (see figure 4.1.1 for an explanation)



$$SMB = 1/3\ (Small\ Value + Small\ Neutral + Small\ Growth) - 1/3\ (Big\ Value + Big\ Neutral + Big\ Growth).$$

$$HML = 1/2\ (Small\ Value + Big\ Value) - 1/2\ (Small\ Growth + Big\ Growth).$$

Figure 4.1.1, RHS factor construction methodologies

SOURCE KENNET FRENCH DATA LIBRARY

SMB and HML factors are calculated as follows: each year, stocks are sorted ascendingly according to their market capitalization, then using the median market capitalization as a breakpoint, stocks are allocated to 2 size groups: Big and Small. After that, stocks are independently sorted in ascending order regarding their BE/ME ratio. Stocks whose BE/ME ratio is below the 30th percentile are labeled growth stocks, the ones whose BE/ME ratio is above the 70th percentile are labeled Value, and stocks between the 33th percentile and the 67th percentile are labeled Neutral. At the intersection of the two size groups and the three BE/ME groups, six portfolios are constructed: SG, SN, SV, BG, BN, and BV.

For each one of those portfolios, monthly equally weighted returns are calculated each year.

Finally, to construct the SMB and HML factor the procedure represented in the figures above have been used (figure 4.1.2): I've calculated the arithmetic mean of the three small stocks portfolios minus the arithmetic mean of the three Big stocks portfolios, and to construct the HML factor, I've calculated the arithmetic mean of the two Value (high BE/ME) stock portfolios minus the arithmetic mean of the two Growth (low BE/ME) stock portfolios.

Down below the table 4.1.2 represent the number of shares included in each portfolio for the RHS factor construction. It is noteworthy that the values are expressed in thousand euros (€ '000), and that in the building process, if the number of the total shares in a year is odd, the median value and therefore, the related company, are excluded from the calculation.

RHS size-BM portfolios

Year	SG	SN	SV	BG	BN	BV	Total	Size median
2001	6	8	6	6	8	6	40	2.611.644
2002	7	8	7	7	8	7	45	2.078.857
2003	7	8	7	7	8	7	45	2.815.471
2004	8	8	8	8	8	8	49	3.173.859
2005	9	8	9	9	8	9	52	4.108.681
2006	9	8	9	9	8	9	53	4.872.746
2007	9	9	9	9	9	9	54	3.984.199
2008	9	9	9	9	9	9	55	2.376.860
2009	9	9	9	9	9	9	55	2.366.429
2010	9	9	9	9	9	9	54	2.303.226
2011	9	9	9	9	9	9	54	1.964.239
2012	9	9	9	9	9	9	54	2.570.097
2013	9	9	9	9	9	9	54	3.341.347
2014	10	9	10	10	9	10	58	3.089.395
2015	10	8	10	10	8	10	57	4.335.919
2016	10	8	10	10	8	10	57	4.119.805
2017	10	9	10	10	9	10	59	4.634.591
2018	10	9	10	10	9	10	59	4.042.441
2019	10	8	10	10	8	10	56	5.597.810
2020	9	9	9	9	9	9	54	6.895.349

Table 4.1.2, RHS size-BM portfolios

LHS 3x3 portfolios

For the Left Hand Side (LHS), I have built 9 different portfolios, so for the dependent variable in the regression, I have divided my set of companies into 9 different groups (SG, SN, SV, MG, MN, MV, BG, BN, BV) after sorting for 3 groups of sizes Small, Medium, Big, and 3 groups of Book-to-Market ratio for each size group dividing each size group for Growth stocks (low B-to-M), Neutral stocks, Value Stocks (high B-to-M).

The equally weighted return of each portfolio is then calculated each year, afterward, the excess returns over the risk-free rate of each one of the portfolios are calculated and included in the regression.

Table 4.1.3 down below show the number of stocks in each portfolio every year.

LHS Size-BM portfolios

Year	SG	SN	SV	MG	MN	MV	BG	BN	BV	Total
2001	4	5	4	5	4	5	4	5	4	40
2002	5	5	5	5	5	5	5	5	5	45
2003	4	5	4	5	4	5	4	5	4	45
2004	5	6	5	6	5	6	5	6	5	49
2005	6	5	6	6	6	6	6	5	6	52
2006	6	6	6	6	5	6	6	6	6	53
2007	6	6	6	6	6	6	6	6	6	54
2008	6	6	6	6	7	6	6	6	6	55
2009	6	6	6	6	7	6	6	6	6	55
2010	6	6	6	6	6	6	6	6	6	54
2011	6	6	6	6	6	6	6	6	6	54
2012	6	6	6	6	6	6	6	6	6	54
2013	6	6	6	6	6	6	6	6	6	54
2014	6	7	6	7	6	7	6	7	6	58
2015	6	7	6	6	7	6	6	7	6	57
2016	6	7	6	6	7	6	6	7	6	57
2017	7	6	7	6	7	6	7	6	7	59
2018	7	6	7	6	7	6	7	6	7	59
2019	6	7	6	6	6	6	6	7	6	56
2020	6	6	6	6	6	6	6	6	6	54

Table 4.1.3, LHS Size-BM portfolios

Finally, in table 4.1.4 a descriptive statistic summary for the LHS portfolios (dependent variables) constructed using size-Book-to-Market dimensions is presented, showing average returns, standard deviations, and number of stocks for the period under analysis.

It can be easily seen in table 4.1.4 that the growth portfolios outperform the value ones, presenting a negative relationship between Book-to-Market and returns, so a reverse value effect is being observed. On the other hand, we have a less pronounced reverse size effect where bigger stocks "outperform" smaller ones.

Excess Return				**Standard deviation**				**Number of stocks**			
(%)	**Growth**	**Neutral**	**Value**	**(%)**	**G**	**N**	**V**	53.2	**G**	**N**	**V**
Small	0.512	-0.549	-1.997	**S**	7.08	6.56	10.67	**S**	5.8	6	5.8
Medium	0.853	0.084	-0.898	**M**	6.70	6.05	8.51	**M**	5.9	5.95	5.9
Big	0.600	0.012	-0.825	**B**	5.95	5.72	7.94	**B**	5.8	6	5.8

Table 4.1.4, LHS average excess returns, st. dev., n° of stock

4.2. Findings and interpretations

This section aims to provide an overview of the data that has been used in the analysis.

After a summary descriptive statistic, 2 charts showing the values of the 3-month BOT and the FTSE MIB will be presented to give an overview of the Italian market's main trends during the period under analysis.

Lastly, the correlation matrix of the factors used will be presented and briefly discussed.

Monthly return (%)	*rf*	*rm*	*Rm-rf*	*SMB*	*HML*
Average return	0.09	-0.21	-0.30	-0.54	-2.075
Max	0.403	20.660	20.70	7.67	12.20
Min	-0.054	-25.412	-25.41	-8.60	-18.11
Standard Dev.	0.108	6.253	6.27	2.749	5.220

Table 4.2.1, RHS summary descriptive statistics

The return on the market factor is negative together with the SMB and the HML, meaning that in the Italian market, for the period and sample under analysis, the stocks overperforming are the big ones over smaller ones and the ones with a higher BE/ME ratio over growth stocks, exactly the opposite of US market evidence.

Below are the 2 Figures representing the Italian market's main tendencies for the period under analysis, as can be seen in 2008 a violent drop in the FTSE MIB capitalization together with the government bond rates affected the market. The American crisis of subprime mortgage from the mid-'70s started spreading all around the globe, hitting hard in Italy in the 4th quarter of 2007 and 2008 and putting in trouble an economy already precarious because of its high public debt[26].

During 2008 stock volatility increased tremendously and in subsequent years, bad economic management and the heavy sovereign debt crisis influenced deeply the market performances and the overall Italian economic recovery.

Furthermore, for the 2020 period, the COVID-19 economic crisis is still ongoing even if its results for the FTSE MIB market are not so marked and detectable from this analysis and sample.

These events must be considered in the analysis and have the most direct and marked effects on volatility and market returns.

[26] The second-highest sovereign debt in Europe in 2010 and more than 100% of its GDP

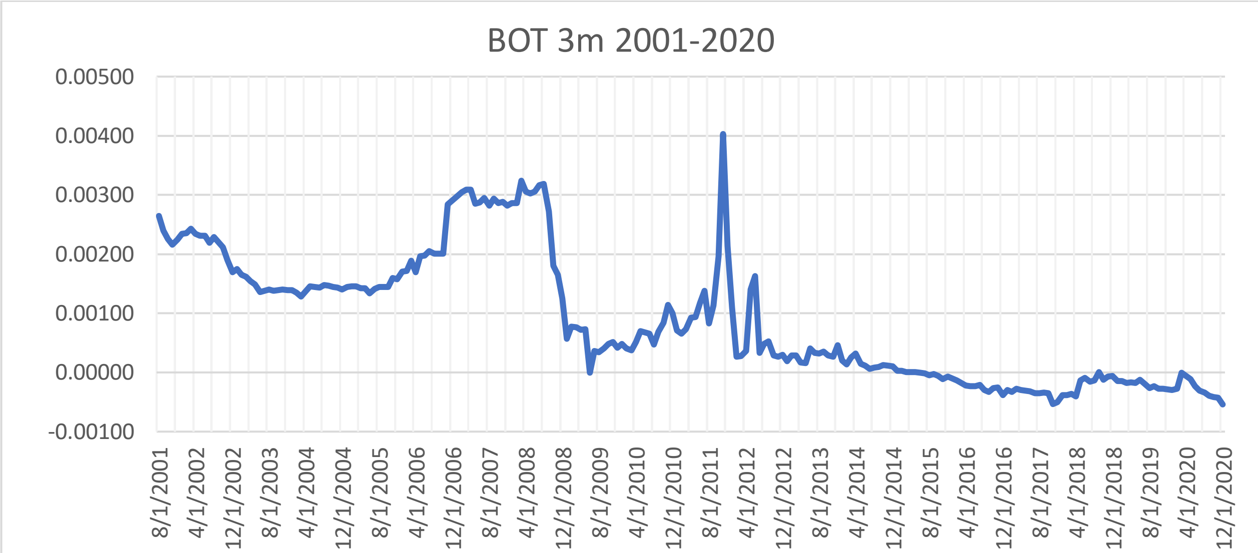


Figure 4.2.1, BOT 3m 2001-2020

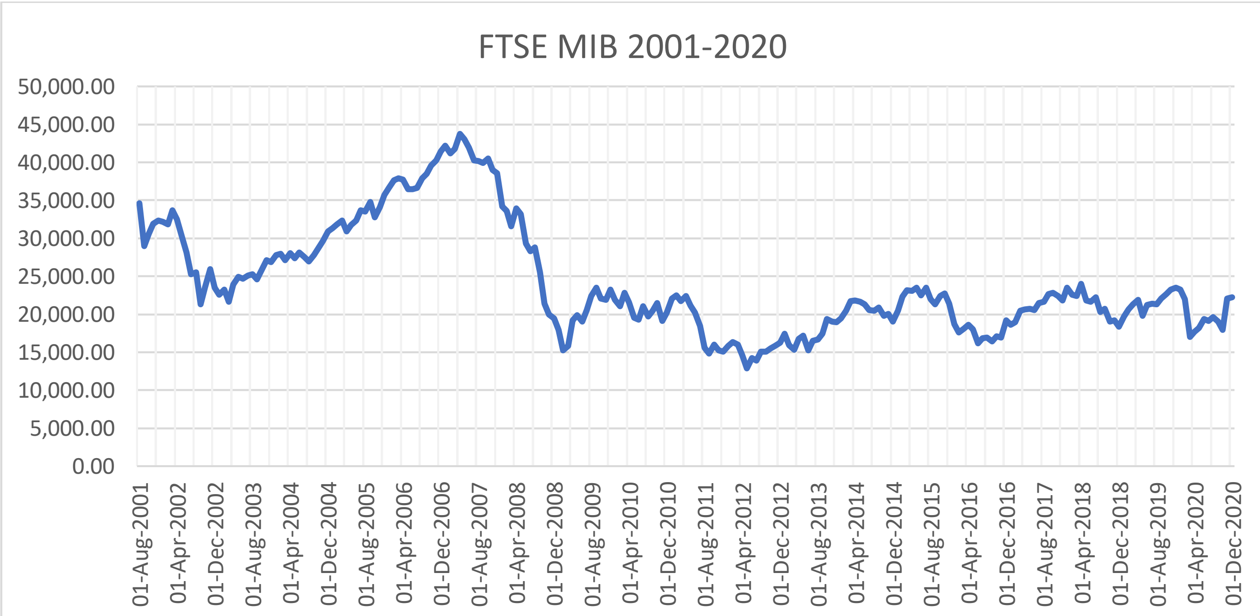


Figure 4.2.2, FTSE MIB 2001-2020

Regarding the correlation matrix (Table 4.2.2), a quite strong correlation emerges between HML-SMB and most importantly HML-market portfolio. Those results could question the appropriate diversification of the portfolio used for the analysis or could logically mean a strong relationship of the market proxy toward the value stocks which mimic quite precisely the market portfolio patterns. This, however, rise a multicollinearity problem which could weaken our estimated coefficient and, as we are going to see below, such multicollinearity will influence the p-value significance.

Corr. matrix	rmrf	SMB	HML
rmrf	1.00	0.08	0.51
SMB		1.00	0.26
HML			1.00

Table 4.2.2, Correlation Matrix

4.3. Regression Analysis Results

This section will explain how the pricing models were tested. Recalling our purpose, our objective was to find empirical evidence for the two pricing models' effectiveness in the Italian stock market. 18 different time-series regressions have been conducted on the LHS portfolios created earlier (9 with CAPM and 9 with F-F). In the following tables the results of the regressions are presented: alpha and betas together with their t-statistics and p-values, and a comparison of adjusted R^2 result will be presented, explained, and analyzed.

To increase the comparison presented in the analysis and so, to effectively compare the predictive ability of CAPM and Fama-French 3 factor model I have also tested, using the predict() function in R, how the two models would have predicted excess returns for 2020 using the period 2001-2019 as the training set.

All the results have been gathered through R using the linear model function and analyzing the result of the summary of the regression. For the sake of clarity in the following line are described the main descriptive data used in the following analysis.

Coefficients: in linear regressions, the coefficients are unknown constant which represents an intercept (first coefficient) and several slopes depending on the variables included in the regression (3 in our F-F case). The analysis aims to find an intercept and a slope(slopes) such that the resulting fitted line is as close as possible to the data points included in the dataset.

t – statistic: this coefficient is a measure of how many standard deviations our coefficient estimate is far away from zero. Ideally, the results are far away from zero so that we can reject the null hypothesis, which in our case is that the intercept (α) is zero so that we can declare that a relationship between the variables exists.

Pr(>t) or p-value: indicate the probability of observing any value equal to or larger than the t statistic. A small p-value states that it is unlikely we will observe a relationship between the dependent and independent variables due to chance. A 5% p-value or less is generally a good breakpoint. A small p-value indicates we can reject the null hypothesis (so a relationship between the variables exists, otherwise the null hypothesis is not rejected) on the contrary a high p-value means there is insufficient evidence to confirm the presence of an "effect".

Adjusted R^2: The R-squared statistic provides a measure of how well the model is fitting the dependent variable. It takes the form of a proportion of variance and always lies between 0 and 1. For example, an R^2 of 0.48 or 48% means that 48% of the variance found in the dependent variable can be explained by the set of independent variables. Since adding variables (multivariate regressions), naturally increase R^2, we use the adjusted R^2 measure is preferred as it adjusts for the number of variables considered.

The first performance model comparison will be examining the adjusted coefficient goodness of fit (adjusted R^2). Table 4.3.1 shows the statistic for CAPM and F-F 3 factor.

Our results in table 4.3.1 show that the Fama-French 3 factor performs slightly better than CAPM from an R^2 point of view, on average as can be seen from the table below the adjusted R squared increase slightly from the CAPM to F-F for each portfolio of roughly 10 percentage points, except for the SG portfolio where F-F perform better of almost 20 percentage points.

F-F adjusted R^2				**CAPM adjusted R^2**			
	G	**N**	**V**		**G**	**N**	**V**
S	0.78	0.69	0.76	**S**	0.58	0.62	0.56
M	0.71	0.74	0.79	**M**	0.59	0.73	0.71
B	0.80	0.85	0.90	**B**	0.68	0.85	0.86
Average: 0.78				**Average: 0.68**			

Table 4.3.1, Adj. R^2 comparison

Regression analysis using the OLS method has been done to test both models. If one of the two models is correct the expected value of return should be fully explained by their risk premium. Hence, if CAPM or F-F is correct, the Beta/s coefficient/s should explain all the expected value of returns, and the alpha coefficient (α) for all portfolios should be zero and the null Hypothesis NOT rejected (H0: intercept/coefficient equal to zero, H1: intercept/coefficient is not equal to zero).

Table 4.3.2 summarizes the regression estimates for CAPM. **The p-values above the 5% confidence interval are marked in bold**. As the table shows the CAPM result indicates that 5 out of 9 alpha coefficients (αi) are significantly different from zero (5% level). This insight indicates that 5 out of 9 portfolios ARE NOT fully explained by the market beta coefficient (βi,m), thus the CAPM have issues in empirically predict risk premium. It is also important to notice that the market beta coefficient is significant at a 5% level for all 9 portfolios, meaning that the market beta as an independent variable do a good job in explaining stock returns in the tested model.

CAPM: $R_{i,t} - r_{f,t} = \alpha_i + \beta_{i,m}(R_{m,t} - r_{f,t}) + \varepsilon_{i,t}$											
Alpha (α)				**T – value of α**				**P – value of α**			
	Growth	**Neutral**	**Value**		**G**	**N**	**V**		**G**	**N**	**V**
Small	0.0077	-0.0029	-0.0161	**S**	2.602	-1.132	-3.469	**S**	0.009	**0.259**	0.000
Medium	0.0110	0.0033	-0.0055	**M**	3.954	1.633	-1.835	**M**	0.000	**0.104**	**0.067**
Big	0.0083	0.0026	-0.0046	**B**	3.826	1.837	-2.472	**B**	0.000	**0.067**	0.014
Market Beta ($\beta_{i,m}$)				**T – value of $\beta_{i,m}$**				**P – value of $\beta_{i,m}$**			
	Growth	**Neutral**	**Value**		**G**	**N**	**V**	53.2	**G**	**N**	**V**
Small	0.860	0.829	1.275	**S**	18.15	19.73	17.20	**S**	0.000	0.000	0.000
Medium	0.827	0.827	1.145	**M**	18.56	25.26	23.87	**M**	0.000	0.000	0.000
Big	0.787	0.842	1.181	**B**	22.49	36.31	39.19	**B**	0.000	0.000	0.000

Table 4.3.2, CAPM regression results

Table 4.3.3 summarizes the regression estimates for the Fama-French 3 factor model.

The result shows that no alpha is significantly different from zero, meaning that all the LHS portfolios are fully explained by the risk factors of the model ($\beta_{i,m}$, $\beta_{i,SMB}$, $\beta_{i,HML}$). This insight suggests that when strictly comparing the alpha coefficients, the Fama-French model performs much better than the CAPM

Regarding the significance of the betas: all the $\beta_{i,m}$ risk factors are significantly different from zero, SMB betas are not fully significantly different from zero, in particular, when used in big stock portfolios analysis they show weaknesses (significant in 7 out of 9 portfolios) also HML betas are significantly different from zero in 8 out of 9 portfolios showing robustness in the explanation of the neutral portfolios.

In Addition is worth mentioning that the $\beta_{i,HML}$ increase monotonically from the lowest BE/ME portfolio to the highest (increase from growth to value portfolios) exactly as in Fama-French works, on the other hand the SMB effect is not observed.

F-F: $R_{i,t} - r_{f,t} = \alpha_i + \beta_{i,m}(R_{m,t} - r_{f,t}) + \beta_{i,HML}HML + \beta_{i,SMB}SMB + \varepsilon_{i,t}$

Alpha (α)				**T – value of α**				**P – value of α**		
	Growth	**Neutral**	**Value**		**G**	**N**	**V**	53.2	**G**	**N**	**V**
Small	0.0032	-0.0020	-0.0010	**S**	1.416	-0.788	-0.290	**S**	**0.158**	**0.431**	**0.772**
Medium	0.0023	0.0025	0.0050	**M**	0.921	1.142	1.815	**M**	**0.357**	**0.254**	**0.070**
Big	-0.0006	0.0019	0.0006	**B**	-0.364	1.195	0.350	**B**	**0.716**	**0.233**	**0.726**

Market Beta ($\beta_{i,m}$)				**T – value of $\beta_{i,m}$**				**P – value of $\beta_{i,m}$**			
	Growth	**Neutral**	**Value**		**G**	**N**	**V**	53.2	**G**	**N**	**V**
Small	1.044	0.863	1.069	**S**	26.31	19.45	16.97	**S**	0.000	0.000	0.000
Medium	1.046	0.863	0.943	**M**	23.86	22.91	19.85	**M**	0.000	0.000	0.000
Big	0.988	0.857	1.052	**B**	30.84	31.55	33.61	**B**	0.000	0.000	0.000

SMB Beta ($\beta_{i,SMB}$)				**T – value of $\beta_{i,SMB}$**				**P – value of $\beta_{i,SMB}$**			
	Growth	**Neutral**	**Value**		**G**	**N**	**V**	53.2	**G**	**N**	**V**
Small	1.047	0.665	1.545	**S**	13.02	7.408	12.14	**S**	0.000	0.000	0.000
Medium	0.327	0.215	0.394	**M**	3.668	2.827	4.092	**M**	0.000	0.005	0.000
Big	0.011	-0.014	-0.144	**B**	0.178	-0.263	-2.278	**B**	**0.859**	**0.792**	0.023

HML Beta ($\beta_{i,HML}$)				**T – value of $\beta_{i,HML}$**				**P – value of $\beta_{i,HML}$**			
	Growth	**Neutral**	**Value**		**G**	**N**	**V**	53.2	**G**	**N**	**V**
Small	-0.515	-0.134	0.349	**S**	-10.45	-2.442	4.467	**S**	0.000	0.015	0.000
Medium	-0.536	-0.101	0.435	**M**	-9.842	-2.179	7.369	**M**	0.000	0.030	0.000
Big	-0.469	-0.035	0.312	**B**	-11.79	-1.050	8.041	**B**	0.000	**0.295**	0.000

Table 4.3.3 F-F 3-factor regression results

Overall, Fama-French Beta results show small beta significance weaknesses coming from the additional risk factors (SMB, HML) when compared with the CAPM. However, the comparison state that Fama-French 3 factor performs better, for every analyzed portfolio, in predicting expected returns and intercept coefficients.

4.4. Testing models and Predictive Power Comparison

To further compare the predictive power of the two models, the student have pursued the analysis into using the predict() function in R to use the model to try to predict expected excess returns of the LHS portfolios for the year 2020, using as a training set for the model coefficients the 2001 – 2019 period.

The results are a comparison of prediction errors and the correlation among them.

The results were 12 predicted results (1 for each month) for the 9 portfolios tested. Thus, the sum of squared errors corresponds to the sum of the 12 values (for the 12 months) obtained for each portfolio from the difference between the actual LHS portfolios returns and the predicted ones, squared (EX. January 2020 Small Growth excess return – Jan. 2020 Predicted SG excess return).

The mean corresponds to the arithmetical mean of the squared differences, and the average range corresponds to the arithmetical mean of the differences between max and min errors for each portfolio.

Also, in this case Fama-French performs slightly better than the CAPM in terms of predicting more precisely expected excess returns.

Error summary	CAPM	Fama-French Three-Factor
Sum of squared errors	0.245422	0.186982
Mean of squared errors	0.027269	0.020776
Average range of squared errors	0.009070	0.007592

Table 4.4.1, TEST errors comparison

Average Correlation between Actual and predicted excess returns	
CAPM	0.9204
Fama-French Three factor	0.9416

Table 4.4.2, TEST average correlation comparison

As can be seen, also in this circumstance from Table 4.4.1 and 4.4.2, the two models perform similarly with the Fama-French result being better than the CAPM ones. Both models have very high correlation results, those shreds of evidence could mean that Fama French SMB and HML risk factors do not have, relatively speaking, a great impact in increasing the predictive ability of F-F compared to CAPM.

It is worth noticing the high correlation shared by the two models between the predictions and the actual results. This observation could also mean that the market risk factor plays a crucial role in the prediction and empirical implementation of the 2 models, meaning that the training coefficients did a good job in carrying on the relationship between risk factors and predicted returns.

5. Conclusions

This thesis aimed to offer further results and empirical research regarding asset pricing models in the Italian market.

As already discussed, the main difference between this work and other Italian one is the sample choice. This empirical analysis wanted to focus its attention on what must be considered the Italian market portfolio and so its most traded and analyzed shares. The sample choice ends up with the FTSE MIB to bring under analysis shares of the Italian market which are more consolidated and mature, even remembering the Italian weaker and smaller market structure compared to the one of the US.

To test the predictive ability of the expected returns of the two models examined (CAPM and Fama-French 3 factors) the study wanted to conduct OLS time-series regressions using 9 portfolios (18 regressions has been carried out) as dependent variables for the period 2001-2020 critically following the construction of the models and their implementation, analyzing their compatibility with the Italian domestic market, and taking into account their substantial differences.

The results overall show better performance from the Fama-French 3 factor empirically speaking. However, the two additional risk factors of the Fama-French model make a minimal additional contribution to the overall predictive capacity of the model and if the high correlation between the risk factors is considered, the independence of the factors within them is seriously questioned and could mean that the additional risk premiums of the F-F price twice the same sources of risk.

In addition, the premiums of the three portfolios RHS (market beta, SMB, HML) are all 3 negatives, demonstrating that small firms and Value stock relative to the FTSE MIB, do not outperform their counterparties Big and Growth, rather is the opposite.

Surely the F-F model explains through empirical factors some risk components not covered by the CAPM as demonstrated by the significance of alpha and adjusted R^2, but this is not enough to perfectly describe the expected returns of the market or to unquestionably prefer the F-F to the CAPM.

Regarding the prediction test for the 2020 period using 2001-2019 period as training set, the high correlations of the predicted values and the real ones show a strong capability of both the 2 models in predicting market returns from a more practical application standpoint.

About the regression and the prediction, play a relevant part of the results obtained the "simplistic" methodologies of the OLS and equally weighted portfolios, which should be further investigated with different econometric models, approach, and samples to be completely sure of the correctness of the results presented.

In conclusion, the student, after the analysis results, thinks that the brilliant intuition of the market portfolio is still the most valid and performing tool available in the field of asset pricing, surely the deepening of multifactorial models is a very attractive and functional compromise solution, especially from an empirical and practical point of view.

There is still strong criticism of the CAPM regarding the impossibility of observing a true market portfolio and the increasingly weak predictive capacity of the CAPM especially in times of high market volatility, exposure to a crisis, and index collapse.

From this point onward we should investigate further and so, open the doors, to all multifactorial models and factors proposed to describe “additional” risk premiums, without forgetting the growing "behavioral" component of globalized markets which are increasingly driven by the "sensations" and "feelings" of retail investors who inevitably distort the assumption of the rational investor guided by cold logic.

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Thesis Summary

This study objective is to analyze and compare two famous asset pricing models, the Capital Asset Pricing Model, and the Fama-French three-factor model. In the following line will be presented a smaller, condensed version of the above thesis using the same shrunken structure. The First part will be a theoretical framework review presenting the basic mathematics behind all the work and the underlying theory developed through the years until the F-F 3 factor main discoveries. The second part will comprehend 3 main sections: a focus on the literature, about the international application of asset pricing models, a literature review of the Italian national application in the past years, and a third part summarizing the main contributions of this work. Finally, the last part empirically analyzes through OLS regressions and predictive tests the Italian market. The Fama-French 3 factor and the CAPM are so studied outside the context of the American market in which they were originally built and designed.

the work focuses on the constituents of FTSE MIB, the representative market index of the principal Italian stock market for the period 2001-2020.

In few words, this thesis tries to answer two main research questions:

1- Are empirical asset pricing models able to capture the variation in average stock returns in the Italian stock market through risk factors related to Size, Value, and the FTSE MIB as a market portfolio?
2- Does the Fama-French tree-factor model outperform the CAPM for the last 20 years in the Italian stock market?

Theoretical framework

The following is an account of statistical mathematics used in the empirical analysis, most of the formulas are mainly based on the first formulations of H.Markovitz.

The return on a share, measured ex post over a period T, may be expressed as:

$$R_{i,t} = \ln(\frac{P_{i,t}}{P_{i,t-1}})$$

In modern portfolio theory, a financial asset is considered to be "more risky" the higher the probability that future returns will disperse relative to the estimated average value. A valid statistical measure of this effect derived for the first time from the works of Markovitz is represented by the variance, defined as the sum of the deviations from the mean to square weighed for the respective probabilities of realization and expressed by the relation.

$$\sigma^2 = \mathrm{Var}\,[\mathrm{R}] = \sum_{i=1}^{N}(R_i - \mu)^2 * p(R_i) = \frac{1}{N-1}\sum_{j=1}^{N}[R_i(j) - \mu]^2$$

Where,

$$p(R_i) = \lim_{n\to\infty}\frac{Ni}{N}$$

$$\mu = \mathrm{E}[\mathrm{R}_T] = \frac{1}{N}\sum_{j=1}^{N}R_i(j) = \sum_{i=1}^{n}R_i * p(R_i)$$

p(Ri) is obtainable from the models of analysis of the historical series or from the fundamental analysis and a sample of N observations available.

The standard approach considers Rt, evaluated ex ante, as a random variable characterized by a mean value (μ), which measures the expected yield on the stock and a level of variance (σ^2) taken as a reliable measure of uncertainty that the expected level of return is pursued and a probability distribution, that statistically identifies the price-generating process.

For completeness the formulas concerning variance, covariance, and correlation:

$$\sigma_A = \sqrt{\sum_{i=1}^{n}\frac{(r_{A,i} - \bar{r})}{(n-1)}}$$

$$Cov_{A,B} = \sum_{i=1}^{n}\frac{[(r_{A,i} - \bar{r_A})(r_{B,i} - \bar{r_B})]}{(n-1)}$$

$$\rho_{A,B} = \frac{Cov_{A,B}}{\sigma_A\sigma_B}$$

Another key element is the portfolio composed by N security which represented one of the main discoveries in the “portfolio slection” paper of H. Markovitz.

$$w_j = \frac{W_j}{W}$$

It can be demonstrated that for a portfolio of n risky assets the expressions of ex-ante yield and variance are the following:

$$\mu_p = \sum_{i=1}^{n} w_i * \mu_i$$

Or

$$R_p = \sum_{i=1}^{N} x_i \bar{R}_i$$

$$\sigma_y^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i * w_j * C_{ij} = \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_i \sigma_j \rho_{ij}$$

Or

$$v\hat{a}r\left(R_p\right) = v\hat{a}r(\sum_{i=1}^{N} x_i R_i) = \sum_{i=1}^{N}\sum_{j=1}^{N} x_i x_j \hat{\sigma}_{ij} = \sum_{i=1}^{N}\sum_{j=1}^{N} x_i x_j \hat{\rho}_{ij} \hat{\sigma}_i \hat{\sigma}_j$$

It is important to note that the expected returns and variances for the individual securities that make up the portfolio are random variables, governed by a conditional probability distribution that takes into account the link between a security and the remaining part of the market.

It should be emphasized the concept of covariance and correlation, which effect can be noted explicitly and in a simple way in a hypothetical portfolio composed of 2 securities, where you will have:

$$\sigma_p^2 = (X_A \sigma_A)^2 + (X_B \sigma_B)^2 + 2 X_A X_B \sigma_A \sigma_B \rho_{A,B}$$

$$\sigma_p = \sqrt{(X_A \sigma_A)^2 + (X_B \sigma_B)^2 + 2 X_A X_B \sigma_A \sigma_B \rho_{A,B}}$$

$$\sigma_p^2 = (X_A \sigma_A)^2 + (X_B \sigma_B)^2 + 2 X_A X_B Cov_{A,B}$$

$$\sigma_p = \sqrt{(X_A \sigma_A)^2 + (X_B \sigma_B)^2 + 2 X_A X_B Cov_{A,B}}$$

$$E_{(r_p)} = \sum_{i=1}^{n} X_i \, E_{(r_i)}$$

Correlation that becomes source of risk of a portfolio; It is recognized that, if ρ is null the variance of the portfolio is equal to the weighted average of the variances of the single Stocks, weighed from the percentage of wealth in they invested.

Modern portfolio theory-CAPM-Fama-French three factor

In a nutshell, Modern Portfolio Theory or Mean-Variance Analysis, modernized and developed from H. Markovitz works, is a model that explains how rational investors can use diversification to optimize their returns at a given level of risk, so it is possible to construct an Efficient Frontier which represents the set of all portfolios of which expected returns (E (Rp)) reach the maximum given a certain level of risk (Var(Rp)).

The critical insight is that stocks should not be evaluated based on itself; instead, it should be evaluated based on how it contributes to the risk and return of a portfolio.

In his paper, Efficient portfolio Frontier Theory relies on the following assumptions:

• Investors are risk-averse and rational, so they aim to maximize final wealth: they are evaluating portfolios solely on expected return and standard deviation of return.

• Transaction costs and taxes shall be zero, assets shall be perfectly divisible

• The investment horizon is single-period (one month, one year, three years).

• The market is efficient which means that a stock price accurately reflects its value.

• Asset returns are distributed by the normal distribution

• The market shall be fully competitive

it's possible to provide a graphical representation of the Portfolio Selection of Markowitz that allows you to define the frontier of efficient portfolios

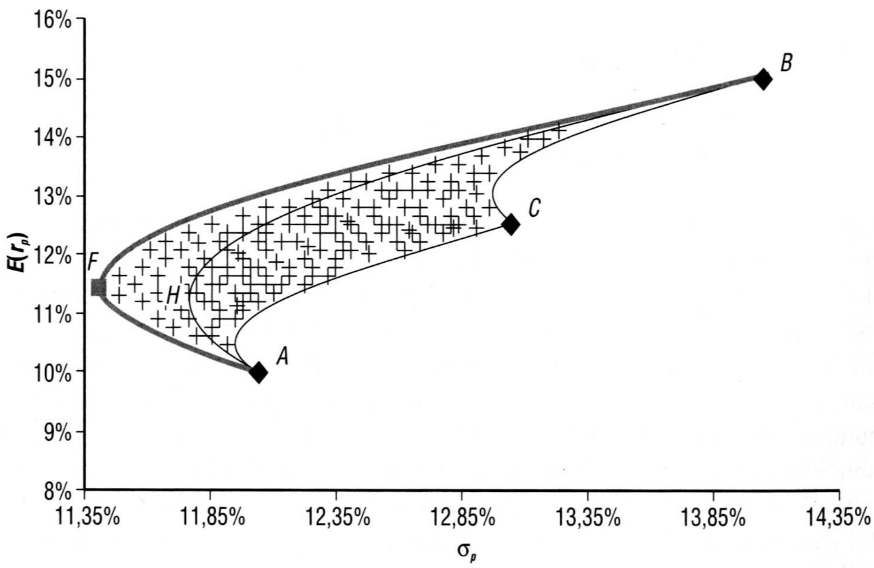


figure 2.3.1

As can be seen from Figure 2.3.1, the FB section will be the efficient frontier with F minimum variance point. Portfolios along the Efficient Frontier (FB curves) offer the least risk for a given level of return

The combination of risk-free and risky assets results in a new mean-variance efficient portfolio "curve" which will be a straight line starting from the risk-free return in the y-axis (intercept in figure 2.3.2), the new Mean-Variance-efficient frontier will be tangent to the previous efficient frontier, and the tangency portfolio T will be the one with the highest Sharpe ratio.

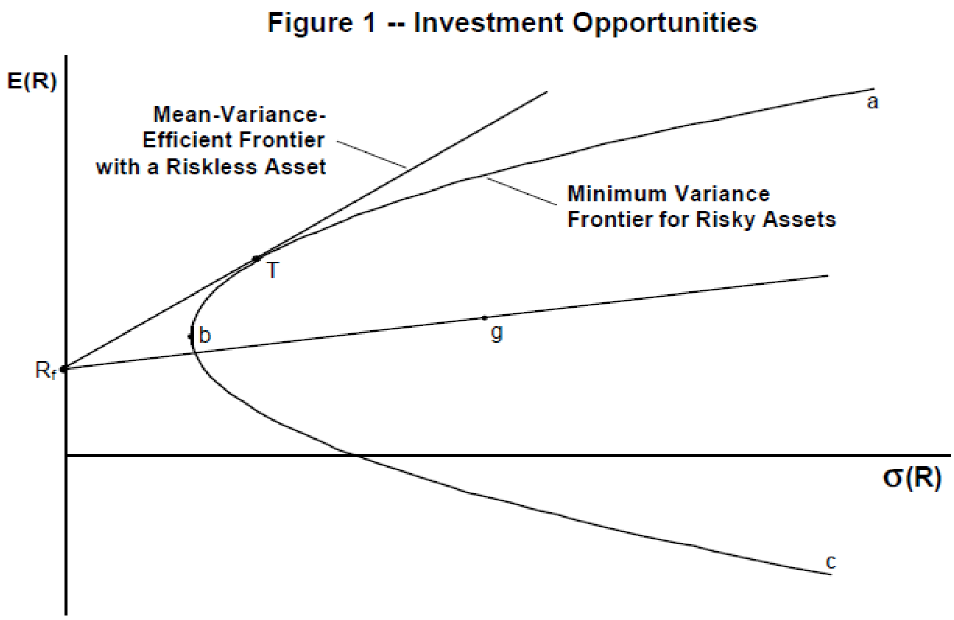


figure 2.3.2

Considering the same rates apply for every investor in the market, all those investors will end up choosing the same tangent portfolio, therefore such portfolio represents the market portfolio! The first step in the formulation of the CML (Capital Market Line) and the Capital Asset Pricing Model.

From here we can speak about the CAPM.

The Capital Asset Pricing Model is a model of equilibrium of the financial markets, that determines a relationship between the yield of a Stock and its riskiness, measured through a single "factor of risk", called beta.

The CAPM is based on the portfolio choice model developed by Harry Markowitz (1959) and Tobin (1958).

In the Markowitz model, an investor selects a portfolio at time t-1 that produces a stochastic return at time t. The model assumes that investors are risk-averse, and when choosing portfolios, only care about the average and variance of return on investment over a single period. As a result, investors choose "mean-variance-efficient" portfolios, in the sense that portfolios:

1) minimize the variance in portfolio performance, based on expected return

2) maximize the expected return, given the variance. Therefore, Markowitz's approach is often called the "Mean-Variance model".

Treynor (1961), Sharpe (1964), Lintner (1965), and Mossin (1966) are the four economists who contribute to the realization of the first Capital Asset Pricing Model; Through their work, they contributed to the transition from the theories and laws identified by Tobin and Markovitz to the relationships of linearity between expected returns and covariance with a "market portfolio".

The CAPM considers 4 assumptions additional to those presented by Markovitz:

1. The single-period horizon adopted by each investor shall be the same for all investors.

2. Each investor may invest or borrow - without limitation - at a risk-free rate, which is the same for all investors.

3. Investors have homogeneous expectations about expected returns and expected variances and covariances of returns compared to all risky assets in which they may invest.

4. There are no taxes, transaction costs or other market imperfections.

As all investors hold the same portfolio of risky assets, that portfolio should contain values weighted by their market value in relation to the total value of the risky assets.

This means that the algebraic relationship that applies to any portfolio of minimum variance must apply to the market portfolio. If there are N risky assets:

$$E(R_i) = E(R_{zM}) + [E(R_M) - E(R_{zM})]\beta_{iM}, \qquad i = 1, \dots, N$$

With,

$$\beta_{iM} = \frac{cov(R_i, R_M)}{\sigma^2(R_M)}$$

E(Rzm), is the expected return on assets that have zero market beta, which means that their earnings are unrelated to the market return.

The second term is a risk premium - the beta of the asset i, ßiM, multiplies the premium per beta unit, which is the expected return on the market, E(RM), minus E(Rzm). Since the beta of the asset i is also the slope in the regression of its yields. A common (and correct) interpretation of beta is that it represents and measures the sensitivity of an asset's performance to the variation in market performance.

The last step in the development of the Treynor - Sharpe - Lintner - Mossin model is to use the hypothesis of risk-free loans to identify E(Rzm), that is the expected return on a zero-beta asset.

The return on a risky asset is not related to the market return when the average of the covariances of the asset with the returns on other assets simply compensates for the variance of the returns (returns) of the asset (if its beta is zero). Such risky activity is risk-free in the market portfolio in the sense that it does not contribute to the change in the return on the market.

Where risk-free loans exist, the expected return on assets that are unrelated to the market return, E(Rzm), shall be equal to the risk-free rate, Rf. The relationship between expected returns and beta then becomes the CAPM equation

$$E(R_i) = R_f + [E(R_M) - R_f)]\beta_{iM}, \qquad i = 1, \dots, N$$

in simpler words, the CAPM as we know it today classifies risk into two macro-categories: systematic and unsystematic risk.

The unsystematic risk, also called idiosyncratic/specific risk, is the "specific" risk of a particular action or industry. According to the CAPM formulations this type of risk can be completely eliminated by the diversification of the portfolios held and therefore is not worthy of compensation.

The systematic risk, on the other hand, which we can consider as market risk, is the risk related to the overall market, since this risk cannot be eliminated by the diversification of a rational investor, it is the only one that deserves compensation in the form of an excess return.

Much criticism has been directed towards the CAPM, which is still a widely used model and on which all the literature and the evolution of asset pricing models from about 1970 onwards is based.

In the work presented above 3 main work that summarized the main development and criticism of the model are briefly described and presented: the criticism of the market portfolio of Rolle, the empirical tests, and criticisms of Black and the APT of Ross.

Criticisms of Black

The research work carried out by Black examined the period 1931-1991, retracing the methodologies of analysis of the work of Black-Jensen-Scholes (in fact it uses the two-factor model for its analysis), Black uses monthly data and the method of data collection in portfolios, particularly effective for long-term data analysis as stocks under analysis undergoes many changes.

Briefly, Black considers 10 different portfolios containing assets from the smallest to the largest systematic risk, portfolio 10 (1) contains assets with the least (greatest) systematic risk. For each portfolio Black estimates the excess return and beta from historical data from 1931 until 1991.

Black thus emphasizes how in the 25 years passed from 1966 to 1991 the returns remain practically the same between high beta and low beta stocks maintaining however the overperformance of low-beta stocks. This shows how the slope of the CAPM reaches a practically flat level, which means that even the related risk premium turns out to be not relevant and very close to zero.

Criticism of the market portfolio of Rolle

Roll in 1977 questioned the possibility of effectively testing CAPM and suggested that empirical evidence is inconclusive. His argument is based on two fundamental considerations:

1. A CAPM validity test based on Black's two-factor model, admits only one testable hypothesis: that the market portfolio is medium-efficient variance.

2. The market portfolio is not observable, as it should include every single asset in a market.

Therefore, in empirical studies researchers are forced to use "market proxies" and consequently these tests are inconclusive/not conclusive.

As the market portfolio and market proxy generally do not coincide, CAPM tests can lead to two different statistical errors:

1) Type I error: The test rejects the CAPM when it is valid (i.e., when a linear relationship exists), because the market portfolio is efficient, while the market proxy is not.

2) Type II error: The test accepts the CAPM when it is not valid, as the market portfolio is not efficient, but the market proxy is.

The APT of Ross

The Arbitrage Pricing Theory (APT) develops based on the insights of the CAPM, adding to the single factor "correlation to market risk", a multiplicity of other factors, which are relevant in the context of determining the value of financial assets.

The multifactorial model (APT) also makes a distinction, like the CAPM, between the diversifiable risk and the non-diversifiable risk, but differs in the approach to non-diversifiable (or market) risk measurement: the CAPM assumes that market risk is only related to the market portfolio, while the APT accepts multiple sources of (systematic) market risk, represented by unexpected changes in key macroeconomic variables such as changes in the interest rate structure, exchange rate changes, changes in real GDP growth rates, changes in inflation rate... These phenomena called "factors" measure the sensitivity of the investment to each variation of these variables with a specific beta coefficient.

The reference portfolios (benchmarks) in the APT are portfolios of well diversified factors built to have a beta of 1 on one of the factors and a beta of zero on any other factor. We can think of each factor portfolio as a tracking portfolio. The returns of this portfolio track the evolution of a particular source of macroeconomic risk but are not correlated with other sources of risk

Fama & French three factor Model

The APT shows us how multiple risk factors allow us to realize a multi-factor SML. But how can we identify the most likely/important sources of systematic risk?

One approach, which is now the most widely used, concerns feature derived from solid empirical bases, to procure proxies to measure exposure to systematic risk. The factors chosen as sources of risk are variables that, based on past evidence, predicted average returns well and which could therefore capture the main risk premiums. An example of this approach is the three-factor model of Fama and French and its variants, which have dominated in recent years empirical research in securities yields.

Note that in this model the market index (βim) captures the systematic risk arising from macroeconomic factors.

The other two extra-market factors are chosen based on long-standing observations that the size of the company, measured by market capitalization (market value of outstanding shares, market equity) and book-to-market ratio (book value per share divided by the market price of the shares) provide deviations of the average returns of the shares from levels consistent with the CAPM. Fama and French justify this model on an empirical basis; SMB and HML are not obvious candidates as relevant risk factors, but the analyses carried out in their work lead us to think that these variables can be excellent proxies of fundamental variables much more difficult to measure

$$R_i - R_f = \alpha_i + b_i\left(R_M - R_f\right) + s_i SMB + h_i HML + \varepsilon_i$$

The following are the first lines of the document written by the two economists "Multifactor Explanations of Asset Pricing Anomalies" which aim precisely to deepen these anomalies as empirical bases on which a good part of the model develops:

"*Researchers have identified many patterns in average stock returns, for example, DeBondt and Thaler (1985) find a reversal in long-term returns; stocks with low long-term past returns tend to have higher future returns. In contrast, Jegadeesh and Titman (1993) find that short-term returns tend to continue; stocks with higher returns in the previous twelve months tend to have higher future returns. Others show that a firm's average stock return is related to its size (ME, stock price times numbers of shares), book-to-market equity (BE/ME, the ratio of the book value of common equity to its market value), earnings/price (E/P), cash flow/price (C/P), and past sales growth. (Banz (1981), Basu (1983), Rosenberg, Reid, and Lanstein (1985), and Lakonishok, Shleifer and Vishny (1994).) Because these patterns in average stock returns are not explained by the CAPM, they are typically called anomalies.*"

(Source:" Multifactor Explanations of Asset Pricing Anomalies" pp.55)

These "anomalies" represent the true starting point of the multifactorial models and their attempt to explain variations in returns not calculated by the CAPM.

These frequently cited anomalies are easily available statistics in the market that apparently serve to predict adjusted returns for "abnormal" risk or significant risk premiums.

The three-factor model, therefore, does a better job than the CAPM, which uses only one factor, to explain securities yields more accurately. While book-to-market size or ratios for themselves are not risk factors, they could act as a proxy for "more fundamental" risk determinants as mentioned above.

Fama and French argue that these return models can therefore be consistent with an efficient market where returns are consistent with risk.

The main problems and criticisms that arise against this model are:

7. According to the two authors, the "reasonable skepticism" is caused by the "premium for distress" or the average return on the High Minus Low portfolio. Kothari, Shanken, and Sloan (1995) argue that much of this award is generated by survivorship bias in the number of companies with a high book-to-market ratio.
8. Another point of view is that the identification of such premium distress is only due to data snooping or data mining bias.
9. The third point of view is that such premium distress is real but irrational; the result of over-reaction leading to a depreciation of distressed stock/holdings (stocks).

Literature Review

In this chapter, the student wants to briefly address the literature regarding applications of asset pricing models outside of a domestic national market, in a global or international scale, then present the past literature involving tests of asset pricing models in the Italian stock market and finally briefly discuss the actual contribution of this thesis for the Italian stock market empirical analysis.

International asset pricing literature overview

Since the ’70 the trend of globalization has increased the volumes of foreign investments by international companies and retailer investors who operate on a world scale. Consequently, in presence of different currencies, the efficiency of classic CAPM was criticized.

It is in this period that Solnik (1974) Sercu (1980) Stulz (1981) and Adler-Dumas (1983) develop and fine-tune an international CAPM which included an additional risk factor that compensates for exchange rate risk.

Here the model presented in modern Terms:

$$E\left[R_{i,t}\right] = DR_f + \beta\left(R_{m,t} - R_f\right) + \beta(FCRP)$$

In few words, International CAPM allows investors to add a currency effects to classic CAPM, to account for the sensitivity to changes in foreign currency when investors hold an asset. This sensitivity accounts for changes in a currency that directly and indirectly affects profitability and, thus, returns.

Another simpler model used in worldwide asset pricing is the global CAPM, a single factor version of the CAPM that concentrates all its pricing power in a single global asset portfolio.

The Global CAPM is a single-factor variation that does not capture foreign currency risk. The model applied by Stulz (1995) has the same structure as the domestic CAPM, but with the global market index replacing the domestic market index, as shown in below equation :

$$E(R_G) = R_f + \beta_G[E(R_G) - R_f]$$

Such a model assumes global supply and demand for capital, and it is typically implemented by corporations operating all around the globe, heavily diversified investors, or large institutional funds.

Fama and French (1998), after the general acceptance of their 3-factor model, expanded the debate between growth and value stocks to thirteen major capital markets around the world. They find that in twelve markets - Italy is the exception - there is a value premium. Their tests showed that the international CAPM model fails to capture the value premium in international returns, but a two-factor APT that attempts to explain stock returns using a market return factor and a relative distress factor does a better job, both, on a country level and a global level.

$$R - F = a + b[M - F] + c[H - LB/M] + e$$

Griffin (2003) tests country-specific and global versions of the Fama and French three-factor model on stocks in 4 different developed countries: the US, Canada, UK, and Japan, for the period from 1981-1995. Regressions for individual stocks and portfolios show that country-specific versions of the three-factor model perform better than global versions in terms of having a higher explanatory power and lower pricing errors. The findings in this paper do not support the notion that there are benefits to extending the Fama and French three-factor model to a global context.

Many studies have also been carried out using augmented models of the Fama-French 3-factor, in particular using the Carhart 4-factor model and the Fama-French 5 factor model, still, most evidence from many works suggests that developed and emerging markets have a segmented and not yet globally integrated structure, meaning local version of above cited asset pricing models perform better.

Studies conducted on the Italian stock market

This section introduces past literature and analysis of asset pricing models that have been conducted in the Italian stock market. While there is extensive empirical evidence of the performance of the Fama-French and CAPM model in the US and in other major stock markets like the UK Canada and Japan, few papers have investigated it on the Italian stock market.

Cavaliere and Costa (1999), are the firsts to research for a size effect in the Italian market, Aleati et al. (2000), Cagnetti (2002) search for multifactor risk premum consistency, the former studying HML and SMB and the latter testing APT macroeconomic and empirical factors, Bruni Campisi and Rossi (2006), Rossi (2012), Federico Giovanni Rega (2017) compare the CAPM and Fama-French 3 factor, Brighi and D'addona (2008, 2010, 2013, 2015), extend the comparison also to the Charhart 4-factor model running different type of regressions. Canegrati (2008), De Chiara and Puopolo (2015), investigate only the CAPM with different methodologies, Veltri and Silvestri (2011), focus mainly on the Fama-French model and CAPM literature comparison. Pirogova and Roma (2019) Gagliolo and Cardullo (2020) are the 2 most recent studies where the former compare CAPM and F-F 3-factor and the latter focus on the existence of a value premium.

Generally speaking, there is high heterogeneity across all the studies in the Italian stock market, regarding the sample period, models, econometric method, portfolios construction, and other minor differences.

The contribution of this thesis

The objective of this thesis is to add material and results to the empirical research that have been conducted in the Italian stock market.

Below the main differences and characteristics of this work compared to the previous Italian ones:

Firstly, it uses a relative smaller dataset, compared to the average of the works presented above. This work focuses mainly on the FTSE MIB index and its constituents, using all the stock that has been included in such index for the period 2001-2020 and so the ones that have been in the 40 most capitalized stock in the Italian market for the chosen period, represent 80% of the total Italian market capitalization and the most traded and liquid stocks available in the public market.

The student chooses the FTSE MIB since it's the worldwide index which characterizes Italian stock market, represent most of the public traded company in Italy and so the most "mature" and "reliable" index, from an international investor point of view.

The student knows the bias and implication of not building an all-asset available market portfolio but still thinks that empirical testing is always conducted in approximated samples and so, the choice ends up with the most characteristic and straightforward index, which could also, more probably, receive those kinds of analysis from a practical/practitioner standpoint.

Secondly, a prediction test activity has been carried out to investigate and compare in alternative ways the models under analysis.

Third, in the regression analysis, a 3x3 LHS-dependent variable portfolio construction has been carried out, and, also in this case, it kind of represents an approximation compared to a 5x5 or 4x4 portfolio structure. The choice was made due to the restricted number of available stocks and to maintain diversified portfolios.

Fourth, the work does not focus completely on the analysis but wants also to offer a critical point of view of many arguments related to the asset pricing subject from the literature development to the international and national scope of the model's discussion.

Empirical Studies

In the wake of the literature analysis, the study attempts to compare the performances of the two models introduced above, CAPM and Fama-French 3 factor model using Italian market stock data.

CAPM

$$R_{i,t} - r_{f,t} = \alpha_i + \beta_i(R_{m,t} - r_{f,t}) + \varepsilon$$

FAMA-FRENCH 3-factor

$$R_{i,t} - r_{f,t} = \alpha_i + b_i(R_{m,t} - r_{f,t}) + s_i SMB + h_i HML + \varepsilon_i$$

Data

This study is based on the Italian stock market from august 2001 to December 2020.

The data sample used in the thesis was obtained Mainly from Datastream except for the Historical FTSE MIB monthly prices (used as a proxy for the market return) and the 3-month BOT (*Buoni ordinari del tesoro*) historical rates, used as risk-free rates, downloaded from Bloomberg.

I've worked with monthly log returns calculated from the historical monthly prices downloaded. Regarding the stock universe, to avoid Survivorship Bias all the stock that had been included in the FTSE MIB Portfolio for at least 1 year: dead, active, or suspended, had been included in the analysis.

All the results have been gathered through R using the linear model function and analyzing the result of the summary of the regression

The following financial figures were downloaded for each stock: Historical close prices, Market Capitalization, Book value of equity (Common equity).

The initial data set contained multiple companies which presented data errors, or that did not contain the financial figures required for the whole period.

In the cleaning process the following companies or data were removed:

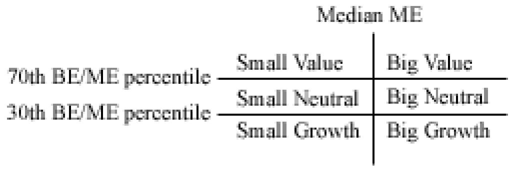
4) Companies that do not contain any data for the period
5) Data points classified as outliers
6) Companies that do not have available the necessary financial figures for at least one year

After the cleaning process, the universe of initial companies has been reduced and Table 4.1.1 shows the 75 remaining ones included in the actual analysis (P.41 for the full list of stock included).

Portfolio construction

The CAPM has only one RHS portfolio; the excess return of the market portfolio (market return minus risk-free rate).

The F&F 3 factors, except for the excess return on the market portfolio that is the same as above, can be constructed using different sets of breakpoints. On the Kenneth French official website, the formula factors (SMB, HML for the 3-factor model) are constructed using 2x3, 5x5, or 10x10 portfolios.



SMB =1/3 (Small Value + Small Neutral + Small Growth)
- 1/3 (Big Value + Big Neutral + Big Growth).

HML =1/2 (Small Value + Big Value)
- 1/2 (Small Growth + Big Growth).

In this work, the RHS portfolios are constructed using the 2x3 method, shortly, all the available stock for each year are sorted first for size, divided in 2 group of small and big stocks, and then the single group are independently sorted for Book-to-Market equity into growth (low BE/ME) neutral and value portfolios. (see P.42-43 for a full explanation)

For the Left Hand Side (LHS), I have built 9 different portfolios, so for the dependent variable in the regression, I have divided my set of companies into 9 different groups (SG, SN, SV, MG, MN, MV, BG, BN, BV) after sorting for 3 groups of sizes Small, Medium, Big, and 3 groups of Book-to-Market ratio for each size group dividing each size group for Growth stocks (low B-to-M), Neutral stocks, Value Stocks (high B-to-M).

The equally weighted return of each portfolio is then calculated each year, afterward, the excess returns over the risk-free rate of each one of the portfolios are calculated and included in the regression.

Empirical results

Monthly return (%)	*rf*	*rm*	*Rm-rf*	*SMB*	*HML*
Average return	0.09	-0.21	-0.30	-0.54	-2.075
Max	0.403	20.660	20.70	7.67	12.20
Min	-0.054	-25.412	-25.41	-8.60	-18.11
Standard Dev.	0.108	6.253	6.27	2.749	5.220

The return on the market factor is negative together with the SMB and the HML, meaning that in the Italian market, for the period and sample under analysis, the stocks overperforming are the big ones over smaller ones and the ones with a higher BE/ME ratio over growth stocks, exactly the opposite of US market evidence.

In 2008 a violent drop in the FTSE MIB capitalization together with the government bond rates affected the market. The American crisis of subprime mortgage from the mid-’70s started spreading all around the globe, hitting hard in Italy in the 4th quarter of 2007 and 2008 and putting in trouble an economy already precarious because of its high public debt.

Furthermore, for the 2020 period, the COVID-19 economic crisis is still ongoing even if its results for the FTSE MIB market are not so marked and detectable from this analysis and sample.

These events must be considered in the analysis and have the most direct and marked effects on volatility and market returns.

Regarding the correlation matrix, a quite strong correlation emerges between HML-SMB and most importantly HML-market portfolio. Those results could question the appropriate diversification of the portfolio used for the analysis or could logically mean a strong relationship of the market proxy toward the value stocks which mimic quite precisely the market portfolio patterns. This, however, rise a multicollinearity problem which could weaken our estimated coefficient and, as we are going to see below, such multicollinearity will influence the p-value significance.

Corr. matrix	rmrf	SMB	HML
rmrf	1.00	0.08	0.051
SMB		1.00	0.26
HML			1.00

The following tables will present an adjusted R^2 comparison, the CAPM regression results, and the F-F 3 factor regression result. the number in bold indicate p-values above the 5% confidence interval

F-F adjusted R^2				**CAPM adjusted R^2**			
	G	**N**	**V**		**G**	**N**	**V**
S	0.78	0.69	0.76	**S**	0.58	0.62	0.56
M	0.71	0.74	0.79	**M**	0.59	0.73	0.71
B	0.80	0.85	0.90	**B**	0.68	0.85	0.86
Average: 0.78				**Average: 0.68**			

CAPM: $R_{i,t} - r_{f,t} = \alpha_i + \beta_{i,m}(R_{m,t} - r_{f,t}) + \varepsilon_{i,t}$											
Alpha (α)				**T – value of α**				**P – value of α**			
	Growth	**Neutral**	**Value**		**G**	**N**	**V**		**G**	**N**	**V**
Small	0.0077	-0.0029	-0.0161	**S**	2.602	-1.132	-3.469	**S**	0.009	**0.259**	0.000
Medium	0.0110	0.0033	-0.0055	**M**	3.954	1.633	-1.835	**M**	0.000	**0.104**	**0.067**
Big	0.0083	0.0026	-0.0046	**B**	3.826	1.837	-2.472	**B**	0.000	**0.067**	0.014

Market Beta ($\beta_{i,m}$)				T – value of $\beta_{i,m}$			P – value of $\beta_{i,m}$			
	Growth	Neutral	Value		G	N	V	53.2	G	N	V
Small	0.860	0.829	1.275	S	18.15	19.73	17.20	S	0.000	0.000	0.000
Medium	0.827	0.827	1.145	M	18.56	25.26	23.87	M	0.000	0.000	0.000
Big	0.787	0.842	1.181	B	22.49	36.31	39.19	B	0.000	0.000	0.000

F-F: $R_{i,t} - r_{f,t} = \alpha_i + \beta_{i,m}(R_{m,t} - r_{f,t}) + \beta_{i,HML}HML + \beta_{i,SMB}SMB + \varepsilon_{i,t}$

Alpha (α)				T – value of α				P – value of α			
	Growth	Neutral	Value		G	N	V	53.2	G	N	V
Small	0.0032	-0.0020	-0.0010	S	1.416	-0.788	-0.290	S	**0.158**	**0.431**	**0.772**
Medium	0.0023	0.0025	0.0050	M	0.921	1.142	1.815	M	**0.357**	**0.254**	**0.070**
Big	-0.0006	0.0019	0.0006	B	-0.364	1.195	0.350	B	**0.716**	**0.233**	**0.726**

Market Beta ($\beta_{i,m}$)				T – value of $\beta_{i,m}$				P – value of $\beta_{i,m}$			
	Growth	Neutral	Value		G	N	V	53.2	G	N	V
Small	1.044	0.863	1.069	S	26.31	19.45	16.97	S	0.000	0.000	0.000
Medium	1.046	0.863	0.943	M	23.86	22.91	19.85	M	0.000	0.000	0.000
Big	0.988	0.857	1.052	B	30.84	31.55	33.61	B	0.000	0.000	0.000

SMB Beta ($\beta_{i,SMB}$)				T – value of $\beta_{i,SMB}$				P – value of $\beta_{i,SMB}$			
	Growth	Neutral	Value		G	N	V	53.2	G	N	V
Small	1.047	0.665	1.545	S	13.02	7.408	12.14	S	0.000	0.000	0.000
Medium	0.327	0.215	0.394	M	3.668	2.827	4.092	M	0.000	0.005	0.000
Big	0.011	-0.014	-0.144	B	0.178	-0.263	-2.278	B	**0.859**	**0.792**	0.023

HML Beta ($\beta_{i,HML}$)				T – value of $\beta_{i,HML}$				P – value of $\beta_{i,HML}$			
	Growth	Neutral	Value		G	N	V	53.2	G	N	V
Small	-0.515	-0.134	0.349	S	-10.45	-2.442	4.467	S	0.000	0.015	0.000
Medium	-0.536	-0.101	0.435	M	-9.842	-2.179	7.369	M	0.000	0.030	0.000
Big	-0.469	-0.035	0.312	B	-11.79	-1.050	8.041	B	0.000	**0.295**	0.000

Overall, Fama-French Beta results show some beta significance weaknesses coming from the additional risk factors (SMB, HML) when compared with the CAPM. However, the overall comparison state that F-F 3 factor performs better (due to R^2 and α consistency), for every analyzed portfolio, in predicting expected returns and intercept coefficients.

To further compare the predictive power of the two models, the student have pursued the analysis into using the predict() function in R to use the model to try to predict expected excess returns of the LHS portfolios for the year 2020, using as a training set for the model coefficients the 2001 – 2019 period.

The results are a comparison of prediction errors and the correlation among them

Error summary	CAPM	Fama-French Three-Factor
Sum of squared errors	0.245422	0.186982
Mean of squared errors	0.027269	0.020776
Average range of squared errors	0.009070	0.007592

Average Correlation between Actual and predicted excess returns	
CAPM	0.9204
Fama-French Three factor	0.9416

As can be seen, also in this circumstance from Table 4.4.1 and 4.4.2, the two models perform similarly with the Fama-French result being just a bit better than the CAPM ones but substantially the same, both models have very high correlation results, but those shreds of evidence could mean that Fama French SMB and HML risk factors do not have a great impact in increasing the predictive ability of F-F compared to CAPM.

The results overall show better performance from the Fama-French 3 factor empirically speaking. However, the two additional risk factors of the Fama-French model make a minimal additional contribution to the overall predictive capacity of the model and if the high correlation between the risk factors is considered, the independence of the factors within them is seriously questioned and could mean that the additional risk premiums of the F-F price twice the same sources of risk.

In addition, the premiums of the three portfolios RHS (market beta, SMB, HML) are all 3 negatives, demonstrating that small firms and Value stock relative to the FTSE MIB, do not outperform their counterparties Big and Growth, rather is the opposite.

Surely the F-F model explains through empirical factors some risk components not covered by the CAPM as demonstrated by the significance of alpha and adjusted R^2, but this is not enough to perfectly describe the expected returns of the market or to unquestionably prefer the F-F to the CAPM.

From this point onward we should investigate further and so, open the doors, to all multifactorial models and factors proposed to describe “additional” risk premiums, without forgetting the growing "behavioral" component of globalized markets which are increasingly driven by the "sensations" and "feelings" of retail investors who inevitably distort the assumption of the rational investor guided by cold logic.