Department of Business and Management & Economics and Finance

Master's Degree in Corporate Finance

Chair of Asset Pricing

Dynamic Optimization Techniques for Crypto-Portfolios

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A conclusione di un percorso lungo cinque anni, è doveroso ringraziare le persone che più mi sono state vicine e che più hanno contribuito al raggiungimento di questo obiettivo.

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Marco

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Introduction

Last decades economic turbulences, such as the 2008 subprime mortgages crisis and the 2020 COVID-19 pandemic, have served to expose the monetary and financial system's vulnerability (Rejeb A. et Al., 2021) [1].

As a reaction to the general market instability, individuals and fund managers have began to include non-conventional investments in their portfolios, in order to mitigate the negative effects of classic asset classes downturns.

At the same time, new technologies have been developed. The blockchain is one of the most significant and disruptive innovation of them all, as it allows to collect data through a decentralized and distributed ledger maintained by all components of a certain network (Treiblmaier H., 2018) [2]. In this way, no intermediary is required to be in the middle of transactions and international wires are speed up (Bação P. et Al, 2018) [3].

The most renowned application of blockchain is linked to cryptocurrencies, a type of virtual money introduced in 2008 by Satoshi Nakamoto with Bitcoin [4] and that is now ramified into many variants with substantial differences. According to Lerer & McGarrigle (2018), one of the author's goals was to react to the financial system's manner of privatizing profits and socializing losses (Lerer M. et Al, 2018) [5].

Cryptocurrencies can offer great investment opportunities in a well diversified portfolio, as the sector continues to grow despite the high volatility; indeed, as of February 2022, the total crypto market capitalization is around $2 trillion[1].

However, whether cryptos can qualify or not as an asset class in their own right still remains a big question, as this aspect could carry important implications for fund managers, regulators and policy makers. As a result, the banking sector and financial institutions, such as the ECB, have began to be more concerned about the topic (Dashkevich et Al., 2020) [6].

The first chapter of this thesis strives to investigate the classification and the importance of cryptocurrencies in today's financial system, particularly from the asset management point of view. We will, in fact, assess if there

[1] https://coinmarketcap.com/charts/

is any room for them to be considered as an asset class and which benefits, and threats, may provide to portfolio settings.

In order to carry out the research, we will firstly introduce the cryptocurrency environment from a technical point of view.

Starting from the original work of Nakamoto (2008) [7], we will report the current views on cryptocurrencies.

In their simplest form, cryptocurrencies can be considered as digital assets built to function as a medium of exchange based on cryptographic technology, with the aim of ensuring the transactional flow as well as to control the creation of additional monetary units (Chohan W. Usman, 2017) [8].

From the ECB's point of view, cryptocurrencies are classified as a subset of virtual currencies, unregulated, issued and controlled by their developers, used and accepted among the members of a specific virtual community (ECB, 2012) [9].

Afterwards, the blockchain technology, unanimously defined as a decentralized and distributed ledger (Briere M. et Al., 2013) [10], will be presented and the functioning of crypto transactions will be outlined, according to Rejeb et Al. (2021) [1] and Hougan M. et Al. (2021) [4].

In second instance, we will investigate the crypto world starting from an institutional investment management point of view.

To do that, we will analyze the alternative investment landscape, particularly focusing on ESG issues, considered to carry superior efficiency in asset allocation by Abate G. et Al. (2021) [11], and on recent years cryptoassets' net inflows.

Moreover, despite the growth of ESG funds has faced difficulties in entering the mainstream investment strategies, in recent years they began being accepted as a distinct asset class, as noticed by Boffo R. et Al. (2020) [12], and they had a strong decade in terms of net inflows, forecasted to grow in the future as well.

Several updated reports from Fidelity, CoinShares and Grayscale will also make clear that the rising presence of cryptos on the market cannot be considered as transitory anymore, although last months net inflows suggest that institutional investors are now monetizing on 2020-2021 extreme gains.

At this stage, opportunities and threats of cryptos will be taken into account. A literature review of the available valuation methods will be proposed, starting from the total addressable market approach (Hougan M. et Al., 2021) [4], the equation of exchange (Burniske C. et Al., 2017) [13], the valuation of crypto as a network (Alabi K., 2017) [14], the cost of production approach (Hayes A., 2015) [15] and the stock-to-flow model (PlanB., 2019) [16], concluding with the reliability critics on crypto valuation made by Damodaran (2017) [17].

The chapter will be concluded with a brief empirical analysis of correlations and performances between some of main cryptocurrencies on the market (Bitcoin, Ethereum, Ripple and Cardano) and the traditional asset classes employed in portfolio management (Equity, Bond, Real Estate, Commodities and Gold).

We will find that cryptocurrencies show some key characteristics of a distinct asset class (strong internal correlations, low correlations with traditional assets, acceptable market liquidity and room for market stability improvements) that could make them suitable to diversify and boost portfolio performances, despite the legal, technical and volatility issues that still have to be improved.

Once having accepted cryptos as an asset class, clarified their opportunities in portfolio management and having understood their critical aspects, it is important to define the correct investment strategy to adopt.

This research identified quite an extensive literature on specific topics, such as portfolio optimization in general (Markowitz H., 1952 [18], Sharpe W. F., 1966 [19] and Mossin J., 1966 [20]), cryptocurrencies from a broad portfolio management point of view (Boiko V. et Al., 2021 [21] and Colombo J. et Al., 2021 [22]), crypto portfolio optimization methodologies only regarding specific coins (Bakry W. et Al., 2021 [23]) or with slightly inaccurate constraints (Gambeta et Al., 2020 [24]).

Anyway, there still a lack of reviews specifically focused on the inclusion of a cryptocurrency index, as crypto ETFs are still not widespreadly available in the investment management landscape, in a constrained portfolio optimization environment using both static and dynamic asset allocation methodologies, that will be the core research question of the study.

The cryptocurrency index employed in the optimization is the CRIX, developed by professor Wolfgang Härdle and his team of researchers from Humboldt University, Berlin, that will be assumed to be a tradable ETF. The index allows investors to track the cryptocurrency market using a small number of constituents[2], evaluated according to their market capitalization and liquidity [25]. The reallocation period is 1 month, which is the time point where coin liquidity is checked again.

Traditional asset allocation methodologies will be taken into consideration, as well as more sophisticated techniques. We will consider portfolios coming both from Modern (Markowitz H., 1952 [18], Sharpe W. F., 1966 [19], Mossin J., 1966 [20], Linter J., 1965 [26], Jensen M. et Al., 1972 [27] and Treynor J. L., 1965 [28]) and Post-Modern Portfolio Theory (Sortino F. A. et Al., 1994 [29]), based respectively on the maximization of the Sharpe and Sortino Ratio, and Risk Parity portfolios (Lee W., 2011 [30]), in their relaxed variants in order to allow the compliance to constraints (Gambeta et Al., 2020 [24]).

Portfolios will be constructed both including only equity and bonds and including cryptocurrencies too, in order to determine benefits coming from them. Moreover, all portfolios will be allocated both in a static way and considering quarterly rebalancing.

At this stage, they will be evaluated from an ex-post point of view considering main traditional and risk-adjusted statistics.

So, in the second chapter, we will focus on the empirical analysis from a methodological point of view, while in the third chapter we will report results and comments.

This study will suggests that cryptos, due to their exotic nature, unwavering appeal, and unknown set of drivers, could act as diversifiers and they might also have hedge properties, as noticed by Bakry W. et Al. (2021) too [23]. We will also find that carefully adding a basket of cryptocurrencies to traditional portfolios (in the sense that all allocations will be constrained to a maximum of 10% in crypto), as well as the quarterly

[2]As of December 2021, CRIX is constituted by Bitcoin, Ethereum, Cardano, Binance Coin, Ripple and Solana

rebalancing, leads to consistent and stable risk-adjusted outperformances with respect to static non-crypto allocations.

Empirical analyses results will support the hypothesis that a careful employment of cryptos in portfolios could be beneficial from an asset manager perspective.

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1 Cryptocurrencies as an Asset Class

1.1 Introduction

The blockchain technology is one of the most significant and disruptive innovation that has been developed in last decades, as it allows to collect data through a decentralized and distributed ledger maintained by all components of a certain network.

The most renowned application of blockchain is linked to cryptocurrencies, a type of virtual money introduced in 2008 by Bitcoin and that is now ramified into many variants with substantial differences.

In this section, the main elements characterizing the cryptoasset world will be outlined. Starting from an overview of the crypto environment, we will take a look at the various definitions of cryptocurrencies, the different technologies behind them, the functioning of transactions and the main cryptocoins existing in the market.

Subsequently, cryptocurrencies will be approached as an asset class and the market trend of cryptos will be analyzed, as well as the alternative investment management landscape and Decentralized Finance past and current trends will be taken into account.

Hence, considerations that are central in crypto investment decisions will be investigated, that is to say the valuation methods, the key characteristics in portfolio settings and the pros and cons of cryptocurrency investing.

1.2 Overview of the Cryptocurrency Environment

Blockchain and cryptoassets came out onto the the world stage in 2008 when the first cryptocurrency, the Bitcoin, was launched by Satoshi Nakamoto.

Since then, cryptocurrencies have gradually captured the attention of the whole financial and technological communities, but few people really understand what crypto is all about. An alternative currency? A revolutionary technology? A big scam?

In this section we will try to outline the main characteristics of cryptocurrencies and the underlying processes that make them so interesting, especially from a technological point of view.

1.2.1 What are Cryptocurrencies?

Virtual money has increasingly gained popularity over the last decades and today's technologies made an unknown computer programmer under the pseudonymous of "Satoshi Nakamoto" be able to create in 2008 the first popular cryptocurrency, the Bitcoin.

Since then, hundreds of cryptocurrencies have been developed and the number of people using them has significantly grown. Moreover, the cryptoasset market has gone through all the classic phases of a disruptive innovation: massive bull markets and crushing pullbacks, euphoric periods and moments of despair, FOMO (Fear Of Missing Out), and everything in between [4]. But what is it all about with the term "cryptocurrency"?

In its simplest form, a cryptocurrency can be considered as a digital asset built to function as a medium of exchange based on cryptographic technology, in order to ensure the transactional flow as well as to control the creation of additional monetary units [8].

From the ECB's point of view, cryptocurrencies are classified as a subset of virtual currencies, unregulated, issued and controlled by their developers, used and accepted among the members of a specific virtual community [9].

Having said that, the best way to understand the crypto environment is by starting talking about Bitcoins, as the developments that allowed Bitcoin to emerge are the foundation of all crypto-based projects.

Satoshi Nakamoto's vision on Bitcoin was described in his/her white paper published on October 31st, 2008 named *"Bitcoin: A peer-to-peer Electronic Cash System"*, where the author described how individuals could exchange money and other items of value without any financial intermediary in the middle [7].

Shortly after the publication of this white paper, Nakamoto released Bitcoin's software and, on January 3rd, 2009 the first token[3] was mined.

[3]Crypto tokens represents the fungible and tradable assets that resides in a cryptocurrency blockchain

To realise why the technology introduced by Nakamoto is considered so revolutionary, we need to think about a peculiarity of our society, namely that although most of our lives have migrated online, payments have remained analogue.

This aspect is often forgotten because of the various fintech apps and online banking platforms that give the perception that everything is now automated and fast, but the underlying financial system has remained archaic.

For example, wiring any amount of money or paying bills takes several days, as transferring anything valuable online is very complicated, to a greater extent than with emails, messages and photos. So, in order to allow any item to move the way messages do between two people, without any intermediary in the middle, it requires a disruptive solution.

1.2.2 The Technology behind Cryptocurrencies

Namokoto's proposal to solve the above-mentioned problem was to create a decentralised database that could be accessed by anyone willing to check balances and to make transactions at any time without any centralised entity controlling its functioning.

This system, called blockchain, is the basis of most crypto projects and it is structured to allow peer-to-peer transactions, as illustrated in figure 1 below [4].

The real innovation of the blockchain was to create timely updated and bad-actor-proof consensus copies of the decentralized ledger, in order to prevent synchronicity issues and to reflect honest transaction only [10]. Through the blockchain, Nakamoto solved the problem of trust establishment in a distributed system, as now no-one could tamper the documents without being detected [2].

In other words, the blockchain provides a distributed trust mechanism where multiple parties keep record of the transactions and every party can verify that the order and the timestamps of transactions have not

(that will be discussed more in dept in the following section). The biggest difference between cryptocurrencies and tokens is that the formers have their own blockchain, while the latters are build on an existing one

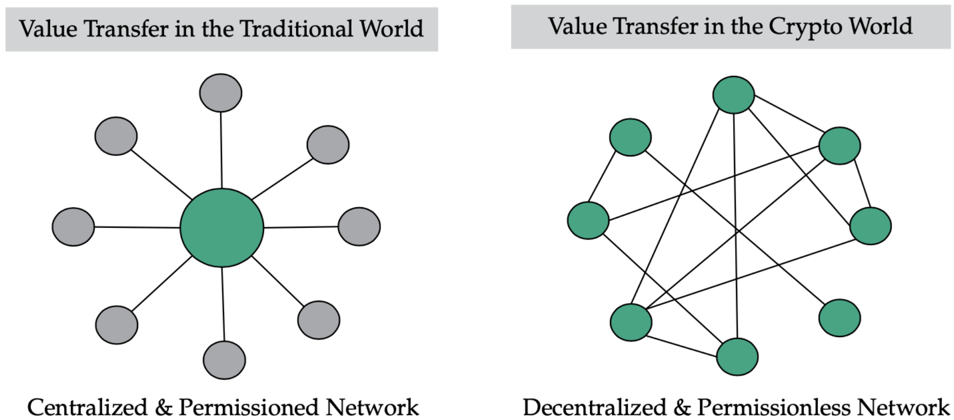

Figure 1: Value transfer in the Crypto world: the figure shows how centralized and decentralized peer-to-peer networks work with regards to value transfer

been tampered with [31]. It is possible to think about the blockchain as a three-column register, where the first column holds the transaction's timestamp, the second one stores the transaction's details and the third one stores a hash[4] of the current transaction, containing its details plus the hash of the preceding transaction.

When a new record is added to the register, the most recently computed hash is transmitted to everyone interested. As everyone knows the last hash, anyone in the blockchain network can check the validity of the data and can verify that they have not been modified, as doing that would be impossible without obtaining a new and hence incorrect hash.

The only method to keep the correct hash and manipulate the data is to find a data collision, which is computationally impossible and uneconomical.

1.2.3 How Transactions Work

In order to understand how the process of validation and adding blocks to the blockchain works, the best way is to describe a bitcoin transaction taking place between 2 counterparties.

[4] A hash is a function that converts an input into an encrypted output of fixed length. It cannot be reversely converted into the original input since a hash is a one-way function. Cryptographic hash functions, the ones that it is possible to find in a blockchain, add security features to a standard hash function and they are employed to link the blocks of the transactions

Alice wants to send Bob 2 bitcoins. In order to do that, she sends a request to all network participants[5] having a copy of the updated ledger, signing it with her private key. This key is a unique password that allows participants to effectively know that the message is coming from her and each node, using Alice's public key, verify her by identifying and checking if she has the sufficient amount of bitcoins to send.

At this stage, the proposed transaction has been placed in a sort of waiting room together with other requests waiting for confirmation, in order to make all computers of the network aware of the demands, as they all possess a copy of the database (figure 2 [4]).

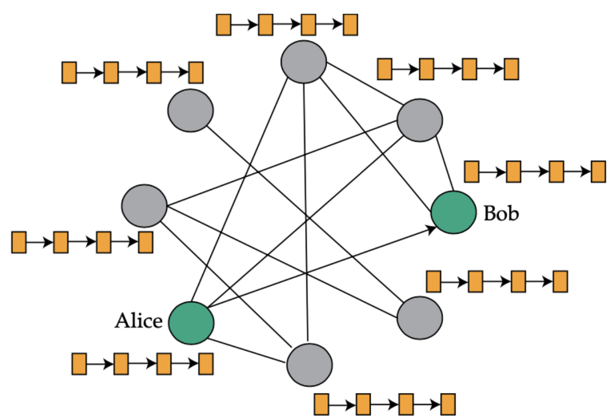

Figure 2: Network status after the proposal: the figure shows the copies of the ledger (in orange) after the transaction has been proposed. As the transaction has not been settled yet, they have not been updated

Now, miners, special participants of the network, enter the process; they are computers scattered around the world that aggregate groups of valid transactions in blocks[6] and propose them for settlement.

In order to do that, miners reprocess the data of a block through complex mathematical functions, the hashes, and the output of this process has a fixed length, making anyone unable to tell how long the input was.

Solving the hash starts with the data contained in the block header, composed by a version number, a timestamp, the hash from the previous block, the hash of the Merkle root, the nonce and the target hash [32].

[5] That are the so-called "nodes"

[6] Which is were the "block" of "blockchain" comes from

Miners focus on the nonce, a string of numbers that is appended to the hashed content from the previous block and that then it is hashed again. The new hash has to be less or equal than the target hash and if this condition is satisfied, then the new block is settled and it is added to the blockchain[7].

The miner (or pool of miners) that finds the solution first is rewarded with 6.25[8] newly minted Bitcoins [33] and, occasionally, with transaction fees that individuals append to incentivize miners to settle their pending transaction ahead of others. These recompenses push miners to verify transactions continuously and update the database, despite the fact that the mining activity requires significant computing power and burns a lot of energy.

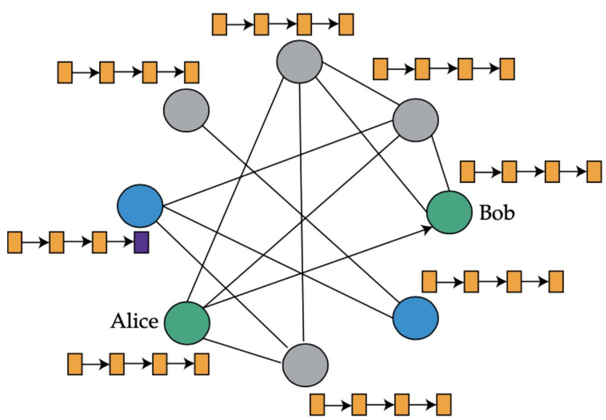

Figure 3: Network status after the approval: a miner (in blue) builds a block of transactions and updates the ledger; at the moment, only this specific miner can see the fully updated blockchain (in purple).

Once a miner posts its solution to the problem, other participants check it (figure 3 [4]) and if it is correct, they update their ledger too and Alice's transaction is settled (figure 4 [4]).

What would have happened if the miner had proposed an invalid block or Alice did not have sufficient amount of Bitcoins? Simply, the network

[7]New Bitcoin blocks are settled roughly every 10 minutes, while blockchains based on other cryptocurrencies may take significantly less time

[8]This reward used to be significantly higher, as every four years the system halves the reward for miners. When Bitcoin was launched, the reward was 50 Bitcoins for each block and now is 6.25 after the latest halving in May 2020

participants would have rejected the block.

But we have to notice that this kind of situations is very unlikely to happen, as validating transactions and checking for their validity is trivially easy, while attempting to settle them is costly. So, the incentive to even try to defraud the system is minimal and the database has never been hacked, all without a single centralized entity monitoring it.

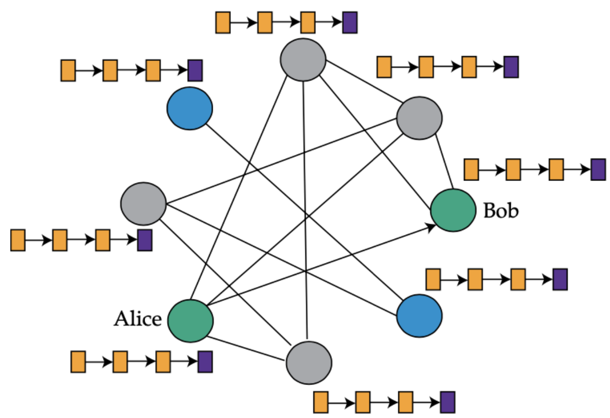

Figure 4: Network status after other participants have checked the solution: finally, the whole network validates and accepts the block.

This mechanism is the so-called "Consensus Algorithm" and it is defined as a specific (cryptographic) validation method that ensures a correct sequencing of transactions on the blockchain [34]. The consensus protocol may follow different criteria and the main ones are:

1. Proof of Work (PoW), the one that is being used by the Bitcoin's blockchain and it is based on the mining process described above.

2. Proof of Stake (PoS), where nodes must prove their stake in the cryptoasset in order to be allowed to validate a transaction. The greater the participation, the greater the likelihood that the system will not be violated. PoS mechanisms are usually applied to cryptocurrencies that have all of their tokens issued, so the only incentive to validate a transaction comes from the fees.

1.2.4 Main Cryptocurrencies

As said, the first blockchain technology was introduced by Bitcoin in 2008 and, later, many others have been developed on its basis, differing however under many aspects.

Among the main cryptocurrencies that exist today, Ethereum and Ripple have blockchains that make them two of the most interesting to analyze in the whole crypto landscape.

Paragraph: Ethereum (ETH) Ethereum is a community-run technology powering the cryptocurrency, Ether, and thousands of decentralized applications[9]. The idea behind its launch, made by Vitaliy Dmitrievič Buterin in 2015, was to expand the list of Bitcoin's capabilities. In fact, BTC's blockchain was programmed to send, receive and holding Bitcoins only, while ETH's blockchain was programmed to do anything a general computer can do thanks to its smart contracts, giving birth to the so-called "Decentralized Finance" of DeFi [35].

This translates into making people able to put in place a broad range of operations, like IPO-style fundraising, collateralized loans or automated market making softwares that are the basis of fully decentralized exchanges, increasing Ethereum's liquidity.

However, this additional flexibility comes at a a cost, because having a broader range of functionalities enlarges the attack surface of the blockchain. In other words, the simpler the software, the more secure the technology.

This is why Bitcoin is often referred to as the "digital gold", because its simplicity is what makes it secure and appeases people to put large sum of money into it.

Paragraph: Ripple (XRP) Ripple is a network for real time payments settlement created in 2012 by Ripple Labs, then OpenCoin, and its native cryptocurrency is the XRP[10]. Its blockchain differentiates from Bitcoin's

[9] https://ethereum.org/en/
[10] https://www.ripple.com

one from a complexity and centralization point of view.

As said, Bitcoin is fully decentralized and this aspect prevents any single government to disrupt or shut down its blockchain, as it is virtually maintained all around the world and this makes it a powerful vehicle to store value or to move large sum of money.

The negative side is that it is too slow for day-to-day use, as any block takes around 10 minutes to be approved.

XRP was engineered to solve this specific issue, as it possesses a more centralized system than Bitcoin. In fact, XRP's blockchain is maintained just by 36 nodes and Ripple controls 6 of these. The clear advantage is that its payment system is way faster than other blockchains and it is capable of processing tons of transactions at a significant pace.

The downside in this case is that XRP is more exposed to seizure, government oversight and censorship.

As of October 2021, other than Bitcoin, Ethereum and Ripple, the main cryptocurrencies based on market capitalization are[11] Binance Coin (BNB), Cardano (ADA), Tether (USDT), Solana (SOL), Polkadot (DOT), USD Coin (USDC) and Dogecoin (DOGE).

1.3 The Evolution of Cryptocurrency and Alternative Investments Markets

After years of steady but volatile growth, the cryptocurrency market surpassed the barrier of $1 trillion of market capitalization in January 2021, and it is now valued at over $2 trillion as of February 2022. But while many cryptocurrencies have remained on the market for many years, other projects have failed to survive in the long run.

According to data from Coinopsy.com[12], a website that tracks dead cryptocurrencies, the number of dead coins stands at 2,316 as of October 2021. The term dead coin is given to cryptos that don't exist anymore due to scams, wallet issues, low liquidity or simply their developers abandoned

[11] https://www.coinmarketcap.com

[12] https://www.coinopsy.com/dead-coins/

them and their number rose when crypto projects began being financed via ICOs[13]. Considering the number of existing coins over the years, Figure 5[14] illustrates the rising trend of cryptocurrencies on the market in the period 2013-2021.

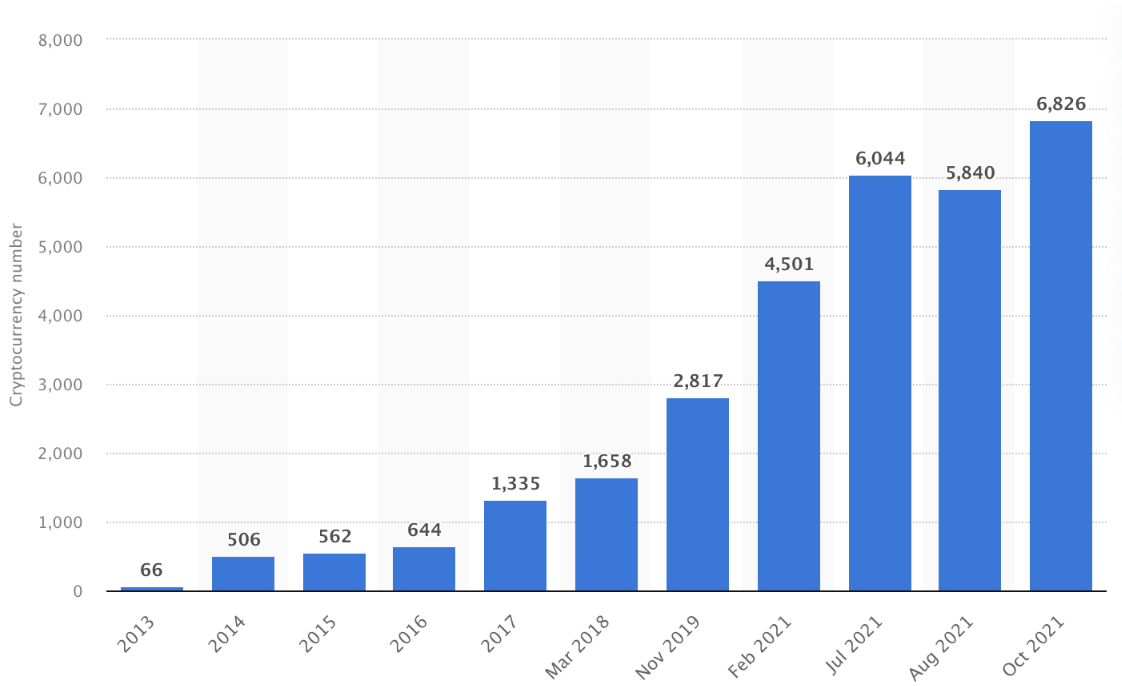

Figure 5: Number of cryptocurrencies over the years

Along with the rising number of cryptocurrencies circulating in the market, the crypto market capitalization has grown as well, as illustrated by Figure 6[15].

One important element to underline is that Bitcoin had the "first mover advantage", as it was the first and most popular currency over the years, but it steadily lost ground as other coins emerged [36].

Indeed, by looking at Figure 7[16], it is possible to notice that Bitcoin went from detain nearly the 100% of market cap in 2013 to less than 50%

[13] An "Initial Coin Offering" is the crypto equivalent of "Initial Public Offering": it is a crowdfunding mechanism used by many crypto entrepreneurs in which buyers receive tokens or a stake relative to the company launching the project

[14] https://www.statista.com/statistics/863917/number-crypto-coins-tokens/, data from CoinMarketCap

[15] https://coinmarketcap.com/charts/

[16] *Ibidem*

Figure 6: Total Cryptocurrencies Market Capitalization

in October 2021.

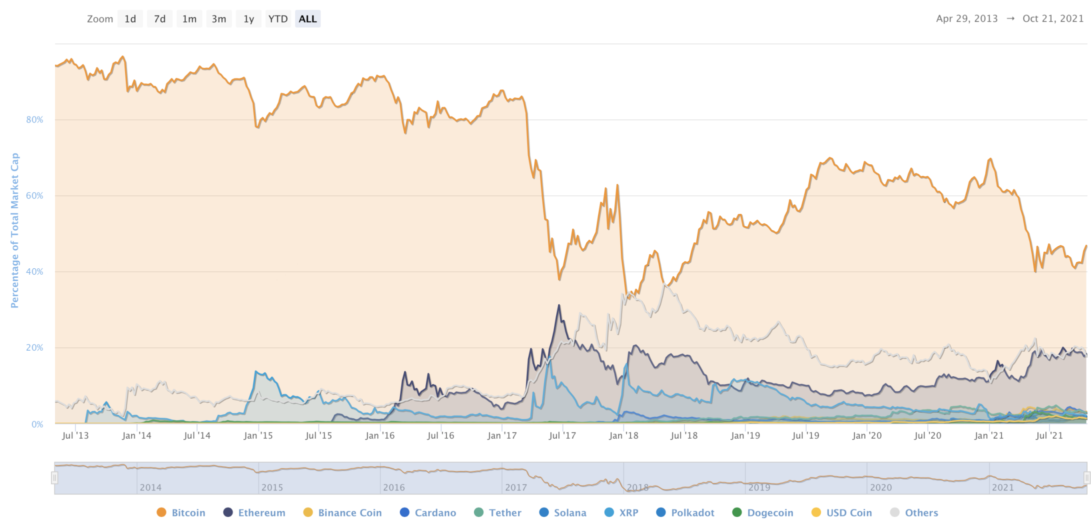

Figure 7: Cryptoassets as Percentage of Market Capitalization

These huge price growths in cryptocurrencies have attracted institutional investors' attention during last years. Especially due to COVID-19 crisis, market conditions have been a catalyst for investors with positions in classical asset classes only, and many funds have began to adopt digital assets to offset portfolios' losses.

According to a survey[17] conducted by Fidelity in September 2021[18], on a total of 1,100 total institutional investors' responses, 52% of the surveyed

[17]Pool composed by Financial Advisors, High-Net-Worth Investors, Family Offices, Pension Funds, Hedge Funds, Venture Capital Funds and Endowments & Foundations

[18]Neureuter J., *"The Institutional Investor Digital Assets Study"*, 2021. Available at https://www.fidelitydigitalassets.com/bin-public/060_www_fidelity_com/documents/FDAS/2021-digital-asset-study.pdf

pool has an investment in digital assets, with adoption rates rising each year. Moreover, European and Asian investors showed higher interest in cryptos than Americans, even if the familiarity in them is significantly growing.

An important element to highlight is that institutional investors often prefer buying digital assets in the investment product form over direct purchase. Figure 8[19] shows current adoption rates divided by investor category in the US.

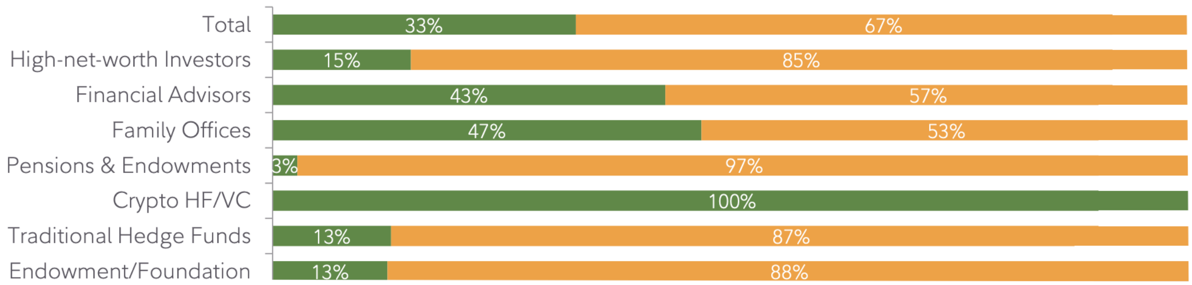

Figure 8: Current Adoption Rates and Channels to Exposure

As expected, native crypto hedge funds and venture capital have the highest rates, followed by family offices, financial advisors and high-net-worth individuals.

Nonetheless, the positive perception of digital assets has grown (figure 9[20]), as the US market increased confidence in cryptos by 11% since 2019.

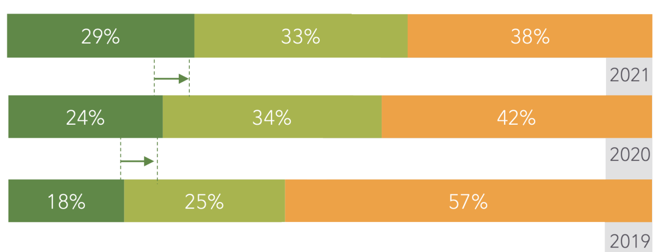

Figure 9: Perception of Digital Assets by Institutional Investors

The crypto features most sought for by institutional investors are linked to high potential upside, uncorrelation to traditional assets, as well as

[19] *Ibidem.*
[20] *Ibidem.*

macro/inflation offsetting potential, as we can see from figure 10[21].

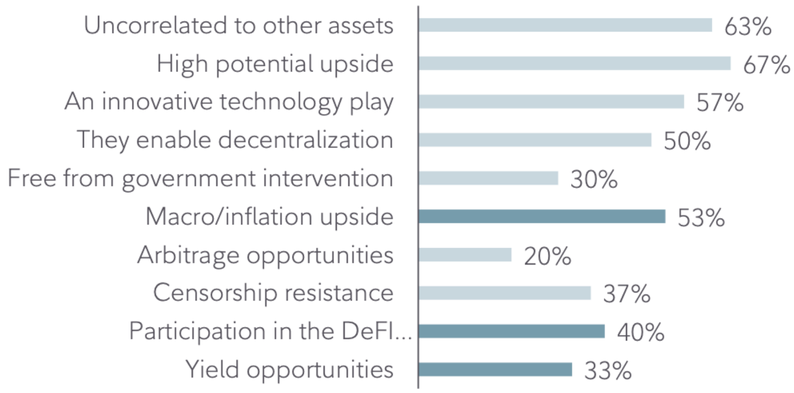

Figure 10: Appeal of Digital Assets

On the other hand, most investors answered that price volatility continues to be the most significant barrier to adoption. As this concern is pretty much widespread across the whole financial community, many coins are set in the form of stablecoins[22]

Other key concerns are represented by the lack of fundamentals to gauge coins' appropriate value[23], the security of the asset custody, regulatory classification and market manipulation.

Anyway, almost 80% of participants feel like digital assets have a place in their portfolios. In fact, generally speaking, investments in alternative assets almost doubled during last decades, growing from 6% in 2004 to 12% in 2018 and they are forecasted to be at about 18% - 24% by 2025 (figure 11[24]).

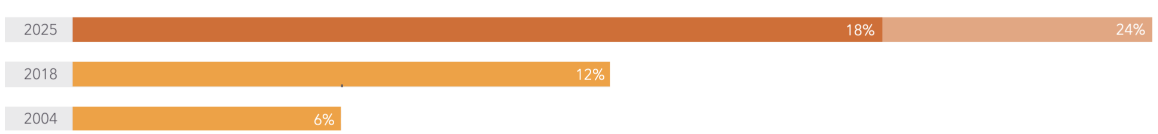

Figure 11: Alternative Assets in Institutional Investors' Portfolios

[21] *Ibidem*

[22] Class of cryptocurrencies that attempt to offer price stability and are backed by a reserve asset. In a diversified crypto portfolio, they could represent the "cash" part of the allocation.

[23] In the following section, we will try to address this concern according to recent literature.

[24] *Ibidem*

Alternative investments are mostly driven by new generations of asset classes, that, apart from cryptos, are linked to megatrends[25] and ESG[26] issues, that made institutional investors more sensible to non-conventional financial topics. The goal of these alternative investments is to assure acceptable yields while investing in sustainable assets and each year more and more funds are launched within this scope.

Despite the growth of ESG funds has faced difficulties in entering the mainstream investment strategies, in recent years they began being accepted as a distinct asset class [12] and, as we can see from figure 12[27], they had a strong decade in terms of net inflows, forecasted to grow in the future as well.

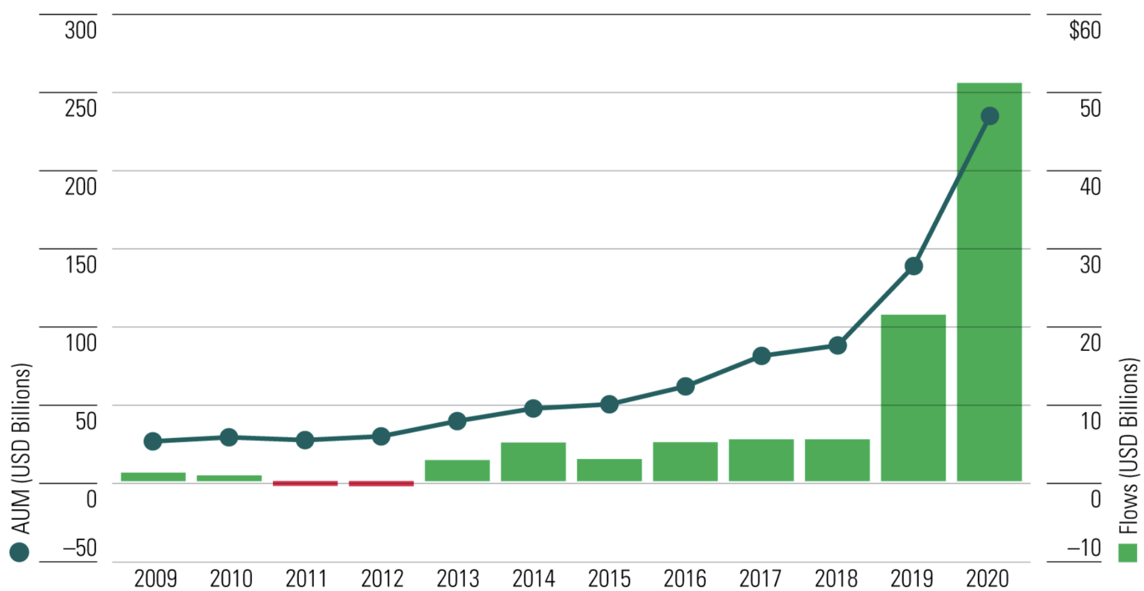

Figure 12: Sustainable Funds Annual Flows and Assets

Indeed, as observed by Seth R. et Al., 2021 [37], ESG issues are now a predominant topic and, in the near future, alternative investments will be no longer seen as an innovation but as the "new normality".

[25] Word invented by John Naisbitt in 1982 to describe the complex societal tendencies able to provoke significant structural changes on long-term economies' trends, with factors often linked to demography, innovations and environmental issues. https://www.altroconsumo.it/finanza/lexicon/m/megatrends

[26] Environmental, Social and Corporate Governance issues.

[27] https://www.morningstar.com/articles/1019195/a-broken-record-flows-for-us-sustainable-funds-again-reach-new-heights

This change in asset managers' preferences is also reflected in the cryptocurrencies' environment, as demonstrated by the number of 13F[28] filed by US funds [38] and by the exponential growth in digital ETP under management (figure 13[29]).

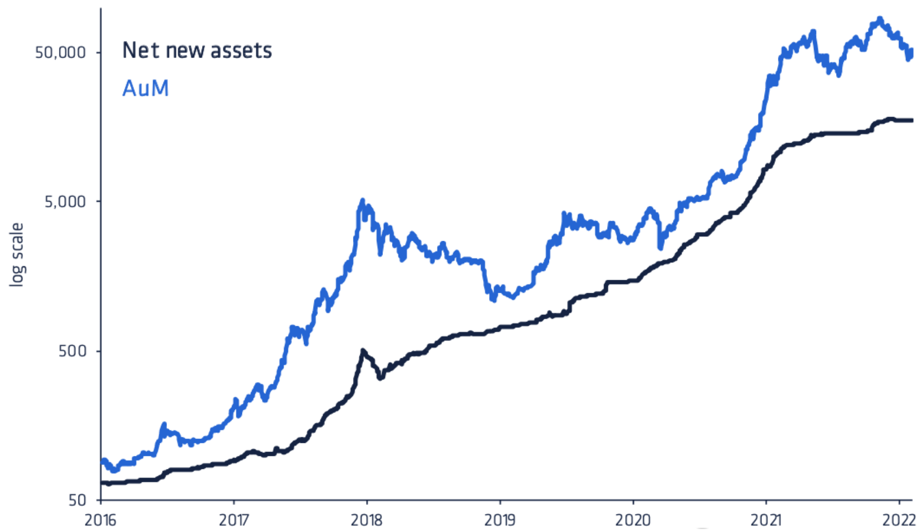

Figure 13: Digital Asset ETP & Mutual fund Net New Assets US$m

According to Coinshares' report, the total number of coins in investment product form has expanded from 9 to 15. 37 investment products were launched in 2021 versus 24 in 2020 and now total 132, indicative of the demand and popularity of digital assets[30].

Moreover, according to a recent Grayscale Investments report[31], the next phase of crypto investing will not be only driven by digital coins, but more broadly by Web 3.0 innovations, such as the Metaverse and Decentralize Finance ecosystems. In less than two years, DeFI market capitalization as percentage of S&P500 financial companies has grown from 0.11% to 2.61% (figure 14[32]).

[28] The 13F is a form required by the "Securities and Exchange Commission's" to be filled quarterly by all US institutional investment managers with at least $100 million in assets under management. It disclose their holding and can give some insights of big players' market activity. https://www.sec.gov/pdf/form13f.pdf

[29] https://coinshares.com/research/digital-asset-fund-flows

[30] *Ibidem*

[31] https://grayscale.com/learn/a-report-on-decentralized-finance-defi/

[32] *Ibidem*

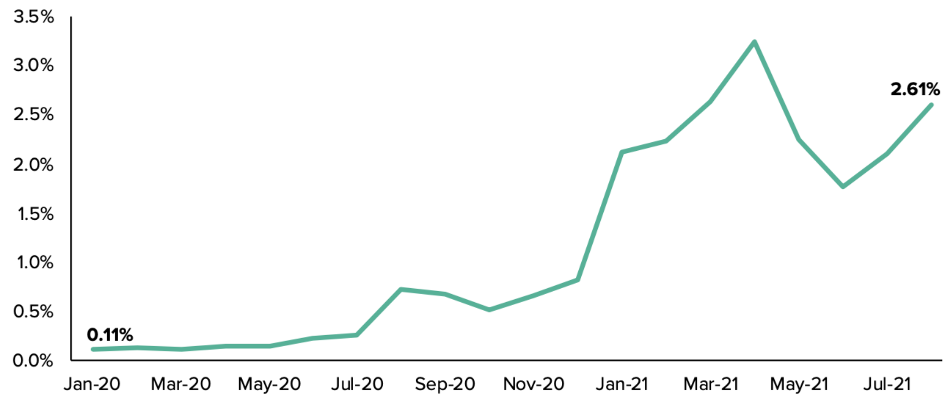

Figure 14: DeFi Market Cap as % of S&P500 Financial Services Market Cap

Having said that, according to Coinshares[33], institutional investors' most recent net inflows in crypto have been decreasing during last year, as the tendency of the sector was to take the profits realized during 2020 (figure 15[34]).

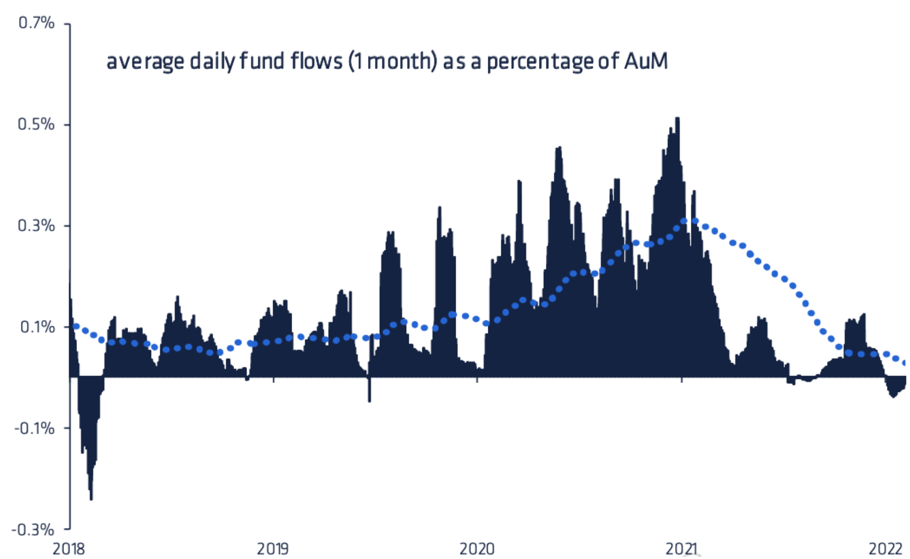

Figure 15: Crypto Asset Fund Flows as a Percentage of Fund AuM

In any case, it must be stressed that, while it is true that most institutional investors have lost interest in certain coins, they are still continuing to keep a stake in big players.

[33]https://coinshares.com/research/digital-asset-fund-flows
[34]*Ibidem*

As we can see from figure 16[35], Bitcoin and Ethereum have been the digital assets that most drove investment flows during last year and it is likely to be the case for 2022 as well, despite other cryptocurrencies now possess higher upside potential.

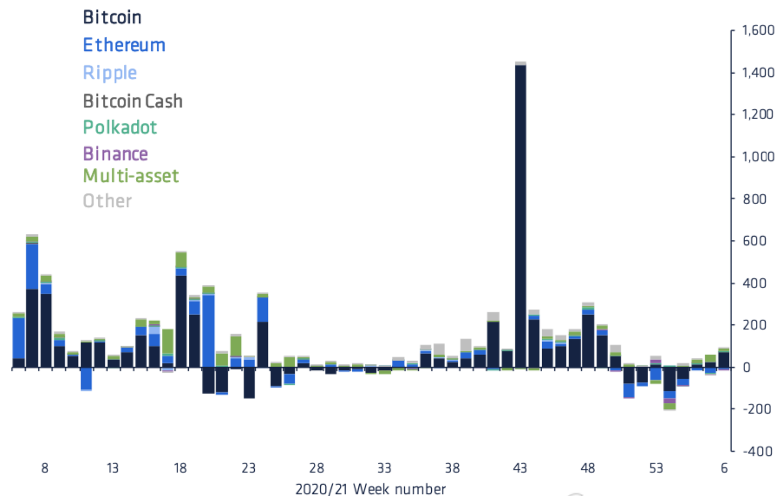

Figure 16: 2020/21 Weekly Crypto Asset Flows by Asset (US$m)

In a nutshell, we have observed that an increasing homogeneity among digital investments perception tends to characterize the investment management landscape, net of the divergences between individual cryptocoins.

As a matter of fact, the Decentralized Finance market is now expanding significantly, up to the point that the crypto sector might become a stable investment option in the years to come, even for more conservative investment funds.

1.4 Crypto as an Investment Opportunity

As stressed in the previous section, cryptocurrencies have a clear potential to be classified as a distinct asset class in the future, but many aspect remain unclear when deciding whether or not to invest in them. How to evaluate them? What are their intakes in portfolio settings? What are the benefits and risks for an investor?

In this section, we will try to answer these questions.

[35] *Ibidem*

1.4.1 Crypto Valuation Approaches

One key consideration that an investor has to make when approaching cryptoassets as an investment opportunity is how to properly evaluate them. This is of one of the most debated, challenging and disagreed aspect of the crypto world as there is not a single approach agreed by everyone.

In this section, the five most discussed approaches will be examined.

Approach 1: Total Addressable Market This is the most popular approach to evaluate the addressable market capitalization of a cryptocurrency and to compare it with its current one. For instance, considering that Bitcoin is believed to be the new non-sovereign store of value just as gold, it is possible to approach its valuation starting from the market value of the latter: the total supply of gold held above ground is estimated to be around $13 trillion at current prices, so, knowing that the total number of bitcoins that will be ever made is 21 million, each Bitcoin should be worth around $620.000 [4].

Adjusting the valuation, if Bitcoin captures 10% of the gold market, each Bitcoin should be worth $62.000, implying that, as of now, Bitcoin captures less than 8% of gold market capitalization[36].

This approach has clear advantages in terms of simplicity and easiness to understand, but it provides just rough valuation estimates as comparative metrics do not explain the true intrinsic value of a certain instruments.

Approach 2: Equation of Exchange MV = PQ Another valuation approach is based on the monetary equation of exchange traditionally used to value fiat currencies [13]:

$$MV = PQ \tag{1}$$

The equation is based on the assumption that the currency value is based on its velocity and on market's size.

[36] Data as of October 23rd 2021, source https://coinmarketcap.com/it/

For example, assuming that Bitcoin processes 100 billion transactions (Q) of $100 each (P) per year and that the average velocity (V) of a Bitcoin is 5 (i.e. one Bitcoin changes hand five times a year), then it is possible to arrive at a potential market capitalization of $2 trillion or $95,238 per Bitcoin.

The downside in this approach is the estimation of V, which is notoriously hard to do for a fiat currency, because key measures of velocity have varied significantly over time and they are likely to vary even more in the case of cryptocurrencies [4].

Approach 3: Crypto as a Network Another popular approach is based on the concept that cryptoassets should be considered as a network and their valuation should be proportional to the square of the number of participants. For instance, if the value of a network of 2 users is 4, the value of a network of 4 participant is 16 and so on.

Ken Alabi applied Metcalfe's Law to value cryptoassets using the number of daily users in their networks and he showed that the differences in valuation between certain coins can be explained through this model [14].

Two key drawbacks of this approach are that this method is appropriate only for relative valuations and that it gives equal weight to each user.

Approach 4: Cost of Production The cost of production approach is based on the fact that miners, in order to produce cryptocurrencies like Bitcoin, spend fiat money in terms of hardware and energy [15].

According to the microeconomic theory, the cost of producing each marginal token should align with the price of Bitcoin itself because, if the mining activity of a certain cryptocurrency were unprofitable, miners could just switch their investments to other coins.

Anyway, the model do not take into account the massive short-term volatility of cryptocurrencies prices and, furthermore, this approach cannot be applied to proof-of-stake based coins.

Approach 5: Stock-to-Flow Model This last approach has been developed specifically for Bitcoin and states that Bitcoin's price should reflect its scarcity, measured through the stock-to-flow ratio (extant value of Bitcoin to the amount being produced each year) [16].

This method shows that Bitcoin's price has been highly correlated over the years with the increasing scarcity expressed by the stock-to-flow ratio.

The drawbacks of this approach are that it assumes that Bitcoin's evolution is driven only by its price and it is clear that the ratio will perpetually increase over time, as new bitcoins are programmed to be mined only till a predetermined date[37].

The unfortunate reality of these approaches is that none of them is academically defensible as traditional DCFs are for stocks. Cryptoassets are still in the early stages of their development to let us know exactly how to approach their valuation.

Furthermore, NYU professor Aswath Damodaran, widely known as the "Dean of Valuation", has compared cryptocurrencies to commodities and currencies, pointing out that "cash generating assets can be both valued and priced, commodities can be priced much more easily than valued, and currencies and collectibles can only be priced" [17], implying that valuation for cryptocurrencies is almost impossible, but not their pricing.

1.4.2 Key Considerations for Crypto in Portfolio Settings

An investor looking at cryptoassets' historical returns could easily say that he/she should have allocated at least a small portion of his/her portfolio on them in the past, as these instruments have shown some characteristics often sought for during an investment decision.

Table 1 and figure 17 show respectively the correlations between main asset classes (Equity, Bond, Real Estate, Commodities and Gold) and Bitcoin, Ethereum, Ripple and Cardano[38] and their the cumulative returns.

From these data, it is possible to highlight some elements that an investor should bear in mind when deciding whether or not to invest in cryptocurrencies:

[37]Happening in 2140 [39].

[38]Data from Yahoo Finance, from 01/01/2018 to from 01/01/2022. Tickers: ĜSPC, T̂NX, GD = F, ŜP500-60, GC = F, BTC-USD, ETH-USD, XRP-USD, ADA-USD

Asset Classes Correlation Matrix									
	S&P 500	10-Year Treasury	Commodity	Real Estate	Gold	BTC	ETH	XRP	ADA
S&P 500	1								
10-Year Treasury	39.78%	1							
Commodity	40.35%	35.31%	1						
Real Estate	73.85%	27.81%	33.40%	1					
Gold	-0.40%	-33.00%	3.38%	8.69%	1				
BTC	22.84%	6.11%	6.55%	12.01%	21.99%	1			
ETH	24.22%	5.45%	17.18%	18.31%	27.08%	76.07%	1		
XRP	18.34%	5.07%	8.79%	13.18%	7.73%	42.84%	52.47%	1	
ADA	25.56%	7.39%	15.21%	18.12%	10.66%	64.99%	67.67%	56.28%	1

Table 1: Asset Classes Correlation Matrix

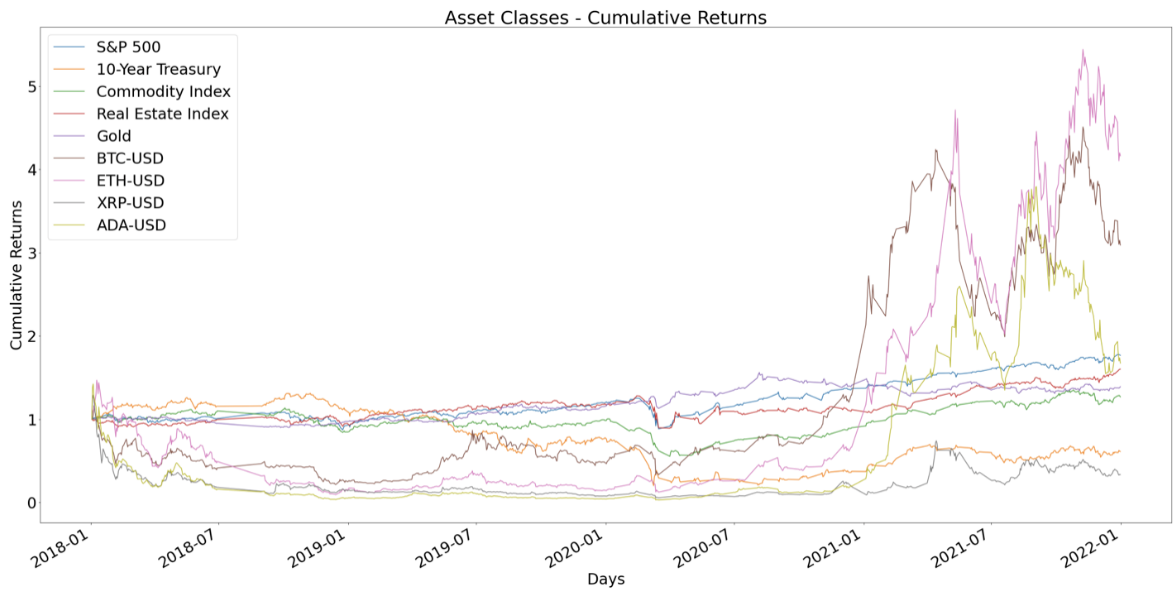

Figure 17: Asset Classes Cumulative Returns

- High Volatility: cryptoassets are still relatively in an immature phase of their evolution and their prices have always been sensible to regulations, technical hurdles and skepticism.

  This aspect has led cryptocurrencies to be historically characterized by high volatility and it is likely to continue in the future too. If we look at figure 17, it is evident that cryptos had extreme standard deviation values over the last four years, especially when we compare them to traditional asset classes.

- Rising Trading Volumes: an interesting element to underline is the increasing trading volume of cryptos, which reflects the great attention of investors in this new asset class and the good liquidity of the currencies.

However, it is important to note that surges in trading volumes suggest either strongly bullish or strongly bearish sentiment and often, especially during 2021, these fluctuations have been accompanied by price decreases in the main coins.

- Low Correlation with Traditional Assets: another key characteristic of cryptocurrencies is their high positive correlation between each other but a low correlation with traditional assets and financial markets, implying that they may offer diversification benefits to investors [40].

  In fact, by looking at table 1, we can see low-medium correlations only to the S&P500 and very modest correlations between them and other asset classes.

- High Potential Returns: although the difficulty of forecasting future prices of cryptoassets is a major downside for investors, crypto bulls argue that historical returns will persists in the next years, as crypto has just began to penetrate the mainstream and many financial institutions has began to develop blockchain-linked applications for their services.

  On the other side, crypto bears continue to argue that most cryptocurrencies are highly overvalued and that they are destined to collapse in the near future.

  Anyway, the empirical evidence shows that cryptocurrencies have survived several moments of panic even when they were seen just as bubbles, and each year they set lows higher than the preceding year. Risks will still be present, but the potential of crypto is undeniable.

1.4.3 Pros and Cons of Cryptocurrency Investing

As seen, cryptocurrencies investing has some upsides as well as downsides that will be summed up in this section.

Regarding the pros, it is possible to outline the main ones:

- Diversification: the low correlation with traditional assets makes cryptocurrencies a good instrument to increase the overall diversification of an equity-bond portfolio, and crypto may also potentially act as a hedge against periods of bear stock market.
- Potential for Appreciation: as said in the previous section, crypto prices have grown exponentially and each year their lows have been higher. If allocated in a well-balanced portfolio, cryptocurrencies have the potential to enhance returns with limited risk.
- Resilience: despite massive price fluctuations and high volatility, major cryptocurrencies have managed to remain on the market and, after an initial skepticism, they have began being understood by the investment community and financial institutions.

The main cons are:

- Volatility: huge volatility swings are common for most cryptocurrencies and speculators could benefit from them, but long-term investors see them as too dangerous. As long as cryptocurrencies remain in an immature phase, this characteristic will persist.
- Lack of Regulation: the absence of an organization monitoring for decentralized cryptocurrencies is a key element behind the crypto philosophy, but the lack of regulatory oversight is seen by some investors as a key drawback.

  As there is no governing body checking for the correct functionality of coins, if a cryptocurrency-holder has any concern or problem, in most cases he/she will be unable to complain anyone.

  Moreover, laws and taxes are often ambiguous or unclear and they differ from country to country, making the investment, in some cases, difficult to be liquidated.
- Technical Risks: cryptocurrencies are stored in digital or physical wallets whose passwords often cannot be restored if lost, and they may be sensible to file corruption.

Furthermore, even most established cryptocurrencies may have bugs that could expose them to hacks. For instance, in 2018 some researches discovered a technical bug in the Bitcoin code that could have led to potentially infinite issuance of new tokens[39].

1.5 Conclusions

The goal of this first chapter was to provide an introductory overview of cryptoassets and explain the different definitions given to them, their functioning and the key features in the underlying technology.

Cryptocurrencies are a massive step ahead in the financial industry, but they are currently a big concern for most investors, as there is no uniformity when approaching them in terms of regulations and characteristics in portfolio settings.

In fact, the valuation approaches are various and none of them is consistent with traditional literature, making crypto still an obscure topic.

Moreover, huge volatility swings, the lack of regulatory oversight and eventual technical issues that may arise, make cryptoassets still a challenging asset class to introduce in portfolios.

Nevertheless, cryptocurrencies possess some distinctive attributes often sought for by investors: low correlation with traditional assets, high potential for appreciation and diversification benefits.

This is why cryptoassets have the potential to be a huge investment opportunity in the near future, as many institutional investors have already noticed.

In the following chapters we will take a look at some applications in static and dynamic portfolio settings, starting from the methodological point of view in the second chapter and ending with the results in the last chapter.

[39] https://www.coindesk.com/markets/2018/09/21/the-latest-bitcoin-bug-was-so-bad-developers-kept-its-full-details-a-secret/

2 Theoretical Framework

2.1 Introduction

Having introduced the crypto environment, we can now move on to study cryptocurrencies as an investment opportunity. In this section, we will investigate some of the most used portfolio optimization techniques, thereby reporting the methodology behind the empirical analysis.

Firstly, we will focus on the traditional Mean-Variance Optimization introduced by Harry Markowitz and the general Modern Portfolio Theory Framework, that entails the Sharpe Ratio and the Capital Asset Pricing Model. Further, the limitations of the myopic approach will be presented.

Subsequently, we will analyze an extension of MPT, the Post-Modern Portfolio Theory: the main differences from the traditional approach will be highlighted in terms of risk-adjusted performance indicators and the optimization based on the Sortino Ratio and Volatility Skewness will be explained.

Finally, the focus will shift from the capital allocation, used up to now, to the risk allocation, implemented in the Risk Parity approach. Hence, considerations on the main differences from the pre-debated optimization methods will be made.

Throughout the research, other statistical tools have been added in order to better compare the optimization techniques and they will be presented in the last section.

2.2 Modern Portfolio Theory and Capital Asset Pricing Model

In this section, we will discuss the fundamentals of the classical portfolio optimization approach. We will introduce the main concepts of the Modern Portfolio Theory by Harry Markowitz, so the asset allocation based on the Mean-Variance optimization, and one of its extension, the Capital Asset Pricing Model, introducing concepts like the Beta, the Capital Market Line and the Security Market Line.

2.2.1 Expected Returns

Modern Portfolio Theory began with the model theorized by Nobel-prize-winner Harry Markowitz in 1952 [18], where he outlined a framework to compose and select the optimal portfolio for any given level of risk and return.

According to this theory, the starting point to construct a portfolio is to calculate its expected return, as returns can be interpreted as a non-dimensional summary of an investment opportunity.

The returns used in the development of this thesis are the continuously compounded returns, given by the natural logarithm of simple returns and represent the compounded growth rate of prices over a certain period:

$$r_{i,t} = ln(\frac{P_{i,t}}{P_{i,t-1}}) \tag{2}$$

where P represents the price of each security i at each period t and *t-1*.

Having found the continuously compounded returns, the expected returns for each security can be calculated through the arithmetic average of logarithmic returns:

$$\mathbb{E}[R_i] = \frac{1}{T}\sum_{t=1}^{T} r_{i,t} \tag{3}$$

where T represents all logarithmic returns in each period t.

At this stage, it is possible to obtain the overall expected return by weighting each security's expected return for its weight in the portfolio:

$$\mathbb{E}[R_P] = \sum_{i=1}^{N} \mathbb{E}[R_i]w_i \tag{4}$$

$$\text{s.t.} \quad \sum_{i=1}^{n} w_i = 1$$

where N represents the number of securities in the portfolio and w_i represents the weight of each security in it, with weights summing up to 1.

In matrix form, the portfolio expected return is found as:

$$\mu_P = \mu' w = \begin{bmatrix} \mu_1 & \mu_2 & \cdots & \mu_n \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix} \tag{5}$$

where μ is the expected returns vector.

2.2.2 Volatility

The estimation of risk, that is the chance that an investment actual return will be different from its expected return, for a security requires the calculation of its variance, that determines how far each return in a given period is from the mean[40]:

$$\sigma_i^2 = \frac{\sum_{t=1}^{T} (r_{i,t} - \mathbb{E}[R_i])^2}{T-1} \tag{6}$$

with T being the total number of returns in the observed period.

At this point, in order to find the security's volatility, we have to calculate the square root of the variance, that basically is its standard deviation:

$$\sigma_i = \sqrt{\sigma_i^2} \tag{7}$$

But when it comes to portfolio volatility, we cannot just calculate the weighted average of each security's standard deviation, as the diversification benefits occur and the overall riskiness of the portfolio should be lower than the sum of individual volatilities.

[40] https://www.treccani.it/vocabolario/varianza/

In order to perform it, we have to consider the correlation coefficient, that measures the degree to which two variables move in relation to each other[41]:

$$\rho_{i,j} = \frac{\sigma_{i,j}}{\sigma_i \sigma_j} \tag{8}$$

where $\sigma_{i,j}$ is the covariance between i and j, used to determine the relationship between the movement of two asset prices[42], and it is calculated as:

$$\sigma_{i,j} = \frac{\sum_{t=1}^{T}(r_{i,t} - \mathbb{E}[R_i])(r_{j,t} - \mathbb{E}[R_j])}{T-1} \tag{9}$$

Values of ρ are comprehended in the range of -1 and 1 and a ρ near 0 means that the two asset are uncorrelated, boosting the portfolio diversification.

As risk is considered to be composed by a systematic and an unsystematic component [41], where the former is inherently existing in the market and the latter is asset-specific, diversification is aimed at mitigating the unsystematic part.

In practice, if two securities move in a different way, the negative performances of an asset can be offset by the positive performance of the other one.

In this way, the overall portfolio volatility can be reduced. We can estimate portfolio variance as:

$$\begin{aligned} \sigma_P^2 &= \sum_{i=1}^{N}\sum_{j=1}^{M} w_i w_j \sigma_i \sigma_j \rho_{i,j} \\ &= \sum_{i=1}^{N}\sum_{j=1}^{M} w_i w_j \sigma_{i,j} \end{aligned} \tag{10}$$

[41] https://www.treccani.it/vocabolario/correlazione/

[42] https://www.treccani.it/enciclopedia/covarianza-%28Enciclopedia-della-Scienza-e-della-Tecnica%29/

or, in matrix form, as:

$$w'\Sigma w = \begin{bmatrix} w_1 & w_2 & \cdots & w_n \end{bmatrix} \begin{bmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1n} \\ \sigma_{21} & \sigma_{22} & \cdots & \sigma_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{n1} & \sigma_{n2} & \cdots & \sigma_{nn} \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix} \tag{11}$$

and then:

$$\sigma_P = \sqrt{\sigma_P^2} \tag{12}$$

2.2.3 Sharpe Ratio and the Capital Market Line

In order to determine the most suitable asset allocation in securities, investors choose the combination of assets that guarantees the best risk/reward relationship.

A commonly used indicator to rank portfolios in terms of return/volatility trade-off is the Sharpe Ratio [19]. For an asset i, the Sharpe Ratio is found as:

$$SR_i = \frac{\mathbb{E}[R_i] - r_F}{\sigma_i} \tag{13}$$

The ratio measures the incremental reward, in terms of expected excess return compared to the risk-free[43], for each increase in the volatility of that asset [42]; in general, the higher the ratio, the more efficient the asset.

At this stage, an investor has to decide how to allocate his/her money and he/she can simulate a large number of asset combinations in order to find the most efficient ones in terms of volatility and expected return.

[43] The risk-free rate represents the return on an investment without uncertainty associated to its cash-flows. Generally, the r_F is identified in the short-term government bonds such as T-Bills in the US. https://www.borsaitaliana.it/borsa/glossario/tasso-risk-free.html

Anyway, only certain portfolios can be considered: they are located on the Efficient Frontier, that is composed by portfolios "providing the maximum expected return for a given admissible risk or — which is the same — the minimum risk for a given desired expected return" [43].

Portfolios under the frontier are called "inefficient", as there are portfolios providing higher returns for that level of volatility, or portfolios carrying less risk for that level of return.

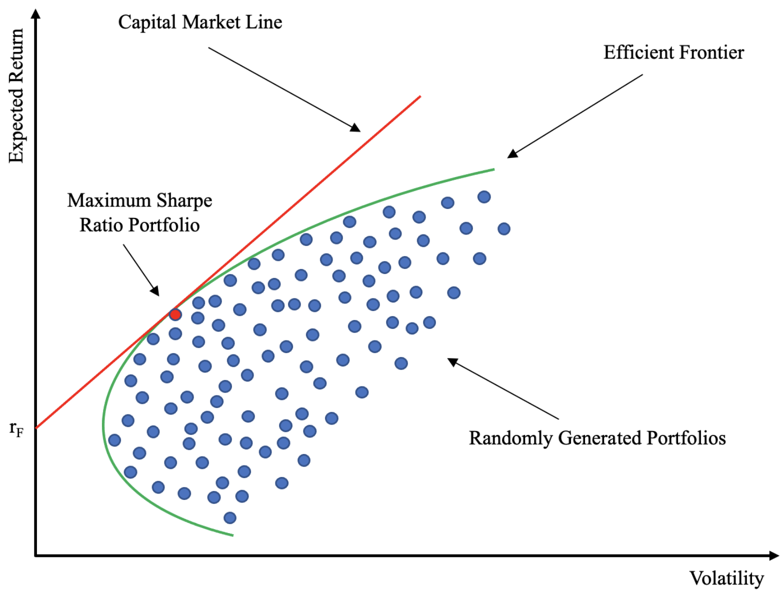

Figure 18: Efficient Frontier and Capital Market Line: the green line represents the Efficient Frontier, while the red one represents the Capital Market Line. The tangency point between the two lines corresponds to the highest Sharpe Ratio portfolio.

The next step to evaluate the best combination of assets is to draw the Capital Market Line, a tangent line starting from the risk-free asset with slope equal to the maximum Sharpe Ratio portfolio; the tangency point represents the best mix of stocks and bonds among all possible combinations [44].

Moreover, the other portfolios that lie on the CML provide the best risk/return combination too, and, according to the Tobin's Theorem [45], the more they are near to the risk-free, the more they will be selected by moderate investors, while the more they are after the tangency point, the more they will be likely to be picked by aggressive investors.

2.2.4 Capital Asset Pricing Model

As an extension to the Modern Portfolio Theory, William Sharpe [46], and subsequently Lintner [26] and Mossin [20], developed a new model in 1964, called Capital Asset Pricing Model, that measured asset returns in relation to a risk factor, the beta.

The model relies on three main assumptions about investors [47]:

- Investors can trade all securities at market prices and can borrow at the risk-free rate
- Investors only hold efficient portfolios
- Investors have homogeneous expectations on expected returns, volatilities and correlations

Given these assumptions, each investor on the market will identify and invest in the maximum Sharpe Ratio portfolio and he/she will only decide what portion of the portfolio to be composed by the r_F. Therefore, the tangent portfolio must equal the market portfolio.

For an asset i, the CAPM formula is:

$$\mathbb{E}[r_i] = r_F + \beta_i \Big(\mathbb{E}[r_M] - r_F \Big) \tag{14}$$

where:

$$\beta_i = \frac{\sigma_{i,M}}{\sigma_M^2} \tag{15}$$

In general, for any portfolio, its beta can be computed as the weighted average of its individual assets:

$$\mathbb{E}[r_P] = r_F + \beta_P \Big(\mathbb{E}[r_M] - r_F \Big) \tag{16}$$

where:

$$\beta_P = \frac{\sigma_{P,M}}{\sigma_M^2} = \sum_{i=1}^{N} w_i \beta_i \tag{17}$$

The β_i is an important element to evaluate the riskiness of a security i, as the σ_i^2 only tells us the risk associated with its own fluctuations from the mean, but not with respect to the market. For instance, if an asset i is uncorrelated with the market, then its β_i will be equal to 0 and it will mean that there is no risk, and so no reward, associated with the security.

So, the β_i can be seen as a measure of systematic risk that cannot be eliminated with diversification.

Furthermore, in practical terms, the beta corresponds to the best fitting line in the plot of a stock's excess returns versus the market excess returns and it can be obtained through a linear regression:

$$\mathbb{E}[r_{i,t}] - r_{F,t} = \alpha_i + \beta_i \Big(\mathbb{E}[r_{M,t}] - r_{F,t} \Big) + \epsilon_{i,t} \tag{18}$$

or, in matrix form as:

$$\begin{aligned}
\mathbb{E}[r_{1,t}] - r_{F,t} =& \alpha_1 + \beta_1 \Big(\mathbb{E}[r_{M,t}] - r_{F,t} \Big) + \epsilon_{1,t} \\
\mathbb{E}[r_{2,t}] - r_{F,t} =& \alpha_2 + \beta_2 \Big(\mathbb{E}[r_{M,t}] - r_{F,t} \Big) + \epsilon_{2,t} \\
&\vdots \\
\mathbb{E}[r_{N,t}] - r_{F,t} =& \alpha_N + \beta_N \Big(\mathbb{E}[r_{M,t}] - r_{F,t} \Big) + \epsilon_{N,t}
\end{aligned} \tag{19}$$

The result of the regression can be visualized in figure 9:

The last element of the regression, $\epsilon_{i,t}$, is the error or residual term and represents the deviations of the points from the best-fitting line. On average, it is equal to zero, and the aim of the simple linear regression[44] is to minimize the sum of the squared distances from the line.

[44]In this case, we are referring to the Ordinary Least Squares

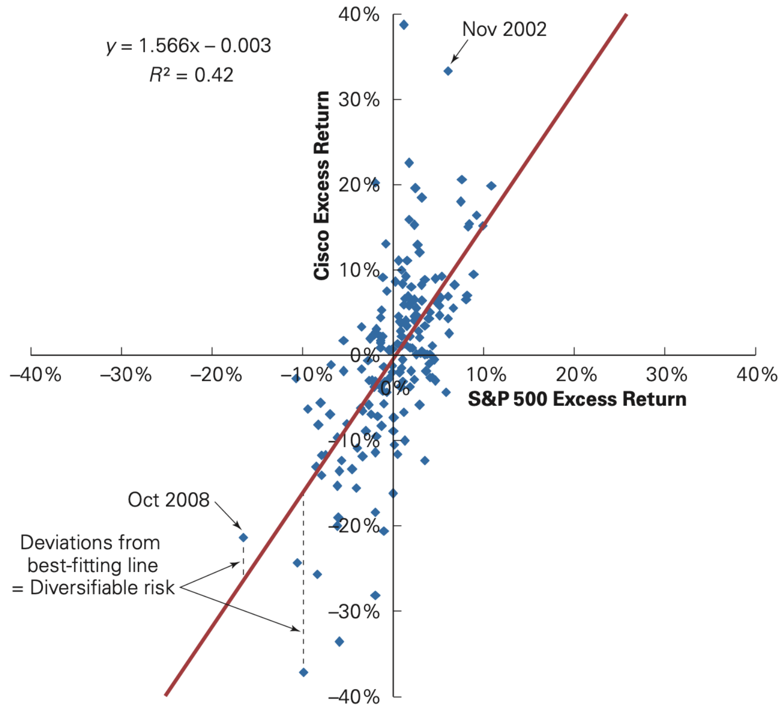

Figure 19: OLS regression: the beta corresponds to the slope of the best fitting line [47].

In order to evaluate the regression, the coefficient of determination, or simply R^2, is used. The R^2 is a statistical measure that tells us the link between the data variability and the correctness of the regression model. In finance, it tells us how closely the performance of an asset can be attributed to the performance of a selected benchmark index [48]:

$$\begin{aligned} R^2 &= 1 - \frac{RSS}{TSS} = 1 - \frac{\sum_{i=1}^{N} \epsilon_i^2}{\sum_{i=1}^{N}(y_i - \overline{y})^2} = 1 - \frac{\sum_{i=1}^{N} y_i - \hat{y}}{\sum_{i=1}^{N}(y_i - \overline{y})^2} \\ &= \frac{ESS}{TSS} = \frac{\sum_{i=1}^{N}(\hat{y} - \overline{y})}{\sum_{i=1}^{N}(y_i - \overline{y})^2} \end{aligned} \tag{20}$$

where TSS stands for Total Sum of Squares, RSS stands for Residual Sum of Squares and the ESS is the Explained Sum of Squares.

Briefly, R^2 varies from -∞ to 1 and the more the value is closer to 1, the more the model explains the data correctly.

2.2.5 The Security Market Line

The CAPM equation implies that there is a linear relation between the stock's beta and its expected return. This relation is represented by the Security Market Line, a function with slope equal to the Market Risk Premium[45] and intercept equal to the r_F.

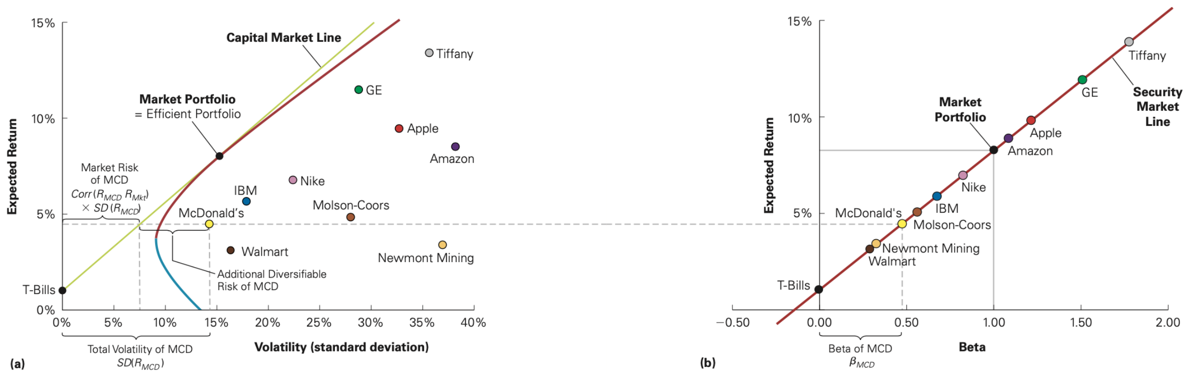

Figure 20: Security Market Line: the figure shows the relation between the CML and the SML [47].

The purpose of the SML is to determine the right rate of return of a security given a beta value and to identify those stocks not aligned with their systematic risk expectations.

2.2.6 Jensen's Alpha

Recalling equation (18), the constant α_i is the stock's Alpha and, when it is estimated in this way, it is referred as the "Jensen's Aplha" [27].

The Alpha is a ex-post indicator employed to evaluate the performances of a security (or portfolio) and it is used to determine whether that asset had or not abnormal returns over the expected return predicted by the CAPM. We can calculate it as:

$$\alpha_i = \mathbb{E}[r_i] - \left[r_F + \beta_i \Big(\mathbb{E}[r_M] - r_F \Big) \right] \tag{21}$$

[45] Given by $\beta(\mathbb{E}[r_M] - r_F)$

2.2.7 Treynor Ratio

The Treynor Ratio was introduced in 1965 by J. L. Treynor [28] and it is a similar reward-to-risk measure to the Sharpe Ratio. The main difference is that the Sharpe Ratio uses the volatility as risk measure, while the Treynor Ratio uses the beta:

For an asset i, the Treynor Ratio is found as:

$$TR_i = \frac{\mathbb{E}[R_i] - r_F}{\beta_i} \tag{22}$$

This implies that the two ratios treat risk differently because, while the Sharpe Ratio uses the total risk associated with the investment, the Treynor Ratio only uses the risk associated with the systemic component of the portfolio. This aspect could lead to very divergent results.

The interpretation of the ratio is that, the higher the value, the higher will be the portfolio capability to remunerate the exposure to systemic risk.

2.2.8 Limits of the Myopic Approach

Despite the validity of this theory, the traditional portfolio model is based on some strict assumptions that result in many limitations and weaknesses:

- The CAPM assumes risk-adverseness, rationality and realistic expectations of all market participants, but the market is not formed by rational investors only.

- The Mean-Variance optimization only uses expected returns and volatility to decide the portfolio optimization, leaving out other useful statistical measures like higher moments of the return distribution.

  Moreover, the standard deviation is a very simplistic tool to represent risk and two portfolios could exhibit the same volatility levels but for different reasons, one for small and frequent losses and the other for two or three larger declines.

- The single-period time horizon appears unrealistic as it does not consider portfolio rebalancing and cannot capture factors that vary over time.

2.3 Post-Modern Portfolio Theory

Mean and variance are important measures of the returns' distribution, but they are often not able to properly explain the data, as they are related to the first and second moment of the distribution.

These measures, as said, are useful when dealing with normally[46] distributed returns but, due to the nature of the stock market, returns are often asymmetrical (or skewed).

This huge con of MPT was handled in subsequent researches on portfolio optimization, where risk began to be treated with special focus on the downside part of volatility.

The tools introduced by Post-Modern Portfolio Theory researchers are more practitioners-world oriented and today they are industry standard. Main ones are Downside Risk, Sortino Ratio and Volatility Skewness, and they will be described in this section.

2.3.1 Higher Moments of the Distribution: Skewness

Before introducing PMPT, it is important to understand what is asymmetry in a distribution and how we can measure it through skewness.

Skewness is the third moment[49] of a distribution and measures its degree of departure from normality[50]. Left-sided distribution are defined as positive skewed, while right-sided are negative, with normal distribution having a value of skewness equal to zero and a perfectly symmetrical distribution.

Values lower than -1 and higher than 1 imply highly skewed returns, while values around zero imply moderate skewness.

Sample skewness for one asset is given by:

[46] Normal Distribution, also called Gaussian or Bell Curve, is a probability distribution in which values are usually centered around the mean

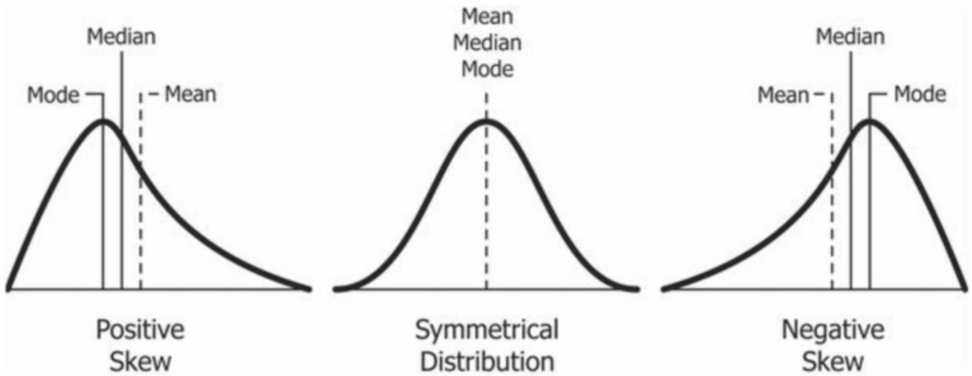

Figure 21: Skewness: A general relationship of mean and median under differently skewed unimodal distribution.

$$S_i = \frac{N}{(N-1)(N-2)} \left(\frac{\sum_{t=1}^{T} r_{i,t} - E[R_i]}{\sigma_i} \right)^3 \tag{23}$$

while portfolio skewness is given by:

$$S_P = \omega' M_3 (\omega \otimes \omega) \tag{24}$$

where M_3 is the co-skewness matrix and $\otimes$ is the Kronecker matrix operator. M_3 is found as:

$$\begin{aligned} M_3 &= \mathbb{E}[(r_{i,t} - \mathbb{E}[R_i])(r_{j,t} - \mathbb{E}[R_j])' \otimes (r_{k,t} - \mathbb{E}[R_k])'] \\ &= \{s_{i,j,k}\} \end{aligned} \tag{25}$$

with elements $s_{i,j,k}$ defined as:

$$s_{i,j,k} = \mathbb{E}[(r_{i,t} - \mathbb{E}[R_i])(r_{j,t} - \mathbb{E}[R_j])(r_{k,t} - \mathbb{E}[R_k])] \tag{26}$$

for all i,j,k = 1,...n. In matrix form, the resulting co-skewness matrix for 3 assets is given by:

$$M_3 = \left[\begin{array}{c|c|c} s_{1,11}s_{1,12}s_{1,13} & s_{2,11}s_{2,12}s_{2,13} & s_{3,11}s_{3,12}s_{3,13} \\ s_{1,21}s_{1,22}s_{1,23} & s_{2,21}s_{2,22}s_{2,23} & s_{3,21}s_{3,22}s_{3,23} \\ s_{1,31}s_{1,32}s_{1,33} & s_{2,31}s_{2,32}s_{2,33} & s_{3,31}s_{3,32}s_{3,33} \end{array}\right] = \left[S_1 \middle| S_2 \middle| S_3\right] \tag{27}$$

2.3.2 Downside Risk

Downside Risk is a measure of standard deviation of those returns that are less then the mean. It is often called semi-deviation (or square root of semi-variance) and differs from volatility as the latter assumes equivalence between upside and downside standard deviation, while the former treat them separately.

For an asset i, the downside risk is found as:

$$DSR_i = \sqrt{\frac{\sum_{t=1}^{T} \left(r_i, t - \mathbb{E}[R_i]\right)^2}{T-1}} \tag{28}$$

$$\text{s.t.} \quad r_{i,t} < \mathbb{E}[R_i]$$

Moreover, following the same approach, it is also possible to calculate the semi-covariance of n assets in order to find the optimal Sortino Ratio allocation:

$$\sigma^{-}_{i,j} = \frac{\sum_{t=1}^{T} \left(r_i, t - \mathbb{E}[R_i]\right)\left(r_j, t - \mathbb{E}[R_j]\right)}{T-1} \tag{29}$$

2.3.3 Sortino Ratio

The downside risk is useful to calculate the Sortino Ratio, a variation of the Sharpe Ratio. The two ratios differs from each other in the treatment of risk, as the Sortino Ratio uses the downside deviation instead of the standard deviation, so the focus is only on the volatility of negative asset returns [29].

For an asset i, the Sortino Ratio is found as:

$$SoR_i = \frac{\mathbb{E}[R_i] - MAR}{DSR_i} \tag{30}$$

where the MAR is the minimum acceptable return set by the investor (for instance, in case of a risk-averse individual, the MAR could be equal to the r_F).

A high Sortino index indicates that the variability of returns is not predominantly concentrated below the MAR; conversely a low Sortino index indicates that the variability is concentrated below the minimum acceptable.

The intuition behind the ratio is that investors are concerned only in case of negative fluctuations of the asset price, while they will not feel pressures in case of returns above their MAR. This is why it is useful when there are two comparable investments that show the same Sharpe ratio.

2.3.4 Volatility Skewness

Volatility skewness is another important indicator of PMPT and it measures the percentage of variance from returns above the mean to the variance from return below the mean.

Thus, if a distribution is normal, the ratio will be equal to 1, while values greater than 1 indicate positive skewness and values lower than 1 indicate negative skewness.

For an asset i, the volatility skewness is found as the ratio between upside variance and downside variance:

$$S_{\sigma_i} = \frac{\sum_{t=1}^{T} \left(r_i, t - \mathbb{E}[R_i]\right)^2 \, with \, r_{i,t} < \mathbb{E}[R_i]}{\sum_{t=1}^{T} \left(r_i, t - \mathbb{E}[R_i]\right)^2 \, with \, r_{i,t} > \mathbb{E}[R_i]} = \frac{USR_i}{DSR_i} \tag{31}$$

The more skewed a distribution, the more the traditional MPT risk measures will not be able to capture its true risk.

2.4 Risk Parity Approach

Another interesting portfolio optimization method is the Risk Parity approach, a risk-based allocation strategy firstly implemented by Bridgewater Associates in the *"All Weather Fund"* launched in 1996[47].

Risk Parity portfolios define a strategy where the focus when choosing weights is not on the capital anymore, but on the marginal risk contribution of each asset class; this approach asserts that when allocation is adjusted to the same risk level, the portfolio can achieve higher Sharpe Ratios and is more resilient to market downturns. We refer to marginal risk as the percentage of volatility that each asset brings to overall portfolio risk.

In this sense, the Risk Parity approach defines an equally-risk-weighted portfolio where risk contributions are not about having the same volatility, but they are more about contributing in the same manner to the overall portfolio risk. Furthermore, unlike Mean-Variance Optimization, no expected return is required in the allocation process.

2.4.1 Optimal Risk Parity Portfolio

In order to build a Risk Parity portfolio, we recall equation 10, 11 and 12, and we use Euler's decomposition to fractionate portfolio volatility into marginal components [24]:

$$\sigma_P = \frac{w'\Sigma w}{\sqrt{w'\Sigma w}} = \sum_{i=1}^{N} w_i \frac{[\Sigma w]_i}{\sqrt{w'\Sigma w}} = \sum_{i=1}^{N} MRC_i \tag{32}$$

$$\text{s.t.} \quad MRC_i = \frac{1}{N}\sigma_P$$

where MRC_i is the marginal risk contribution of asset i.

[47] https://www.reuters.com/article/usa-bonds-funds-idINKBN26X18W

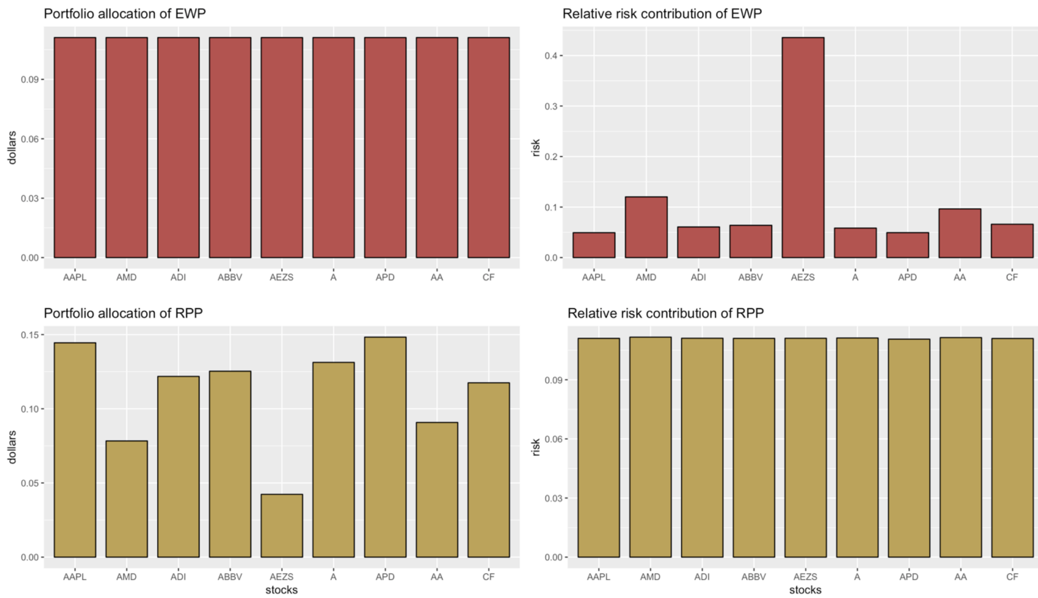

Figure 22: Equally Weighted vs Risk Parity: the figure compares the allocation based on equal weights versus the allocation based on equal risk contribution.

The optimization process has the aim to make risk contribution equal, so, in addition to the non-short selling constraint, we impose that all MRC_i are equal.

2.4.2 Relaxed Risk Parity Approach

In the development of the empirical analysis, weight constraints have been imposed to each optimization method, such that the Equity asset class has at least 50% of weight, Fixed Income has at least 20% and Crypto a maximum of 10%.

This aspect lead the Risk Parity model to violate the property of having all MRC_i equal and produce portfolios with different risk contributions.

This model can be thought of as an extended Risk Parity optimization with a risk diversification constraint that focuses on risk parity allocations, resulting in near Risk Parity portfolios [24].

2.5 Further Elements in the Analysis

Apart from the indicators contained in the Modern and Post-Modern Portfolio Theories, other statistical measures have been added to the analysis, in order to compare deeper the different asset allocation strategies.

2.5.1 Value at Risk

The Value at Risk is a Risk Management indicator that synthetically expresses the maximum potential loss that a portfolio/position can suffer given a specified time horizon and a specified confidence level during normal days or market crashes [51]. It depends on time, frequency and market value of the position and it comes with a probability that tells us how likely is the potential loss to be greater than the VaR.

The definition of a 95%, n-day, Value-at-Risk with initial value P0, is $VaR_{95\%}(P0)$ such that [52]:

$$Pr\big[(P_t - P_0) < VaR_{95\%}(P0)\big] = 0.95 \tag{33}$$

or:

$$Pr\big[(P_t - P_0) > VaR_{95\%}(P0)\big] = 0.05 \tag{34}$$

There are three main approaches for its calculation:

- Parametric Approach: it uses returns and volatility and assumes normal distribution to calculate the VaR.
- Historical Simulation: it sorts historical returns in increasing order and assumes that past returns are a benchmark for future returns.
- Monte Carlo Simulation: it develops a model to predict future price changes through multiple hypothetical trials.

During the empirical analysis, the parametric approach has been used and the general formula for its calculation is:

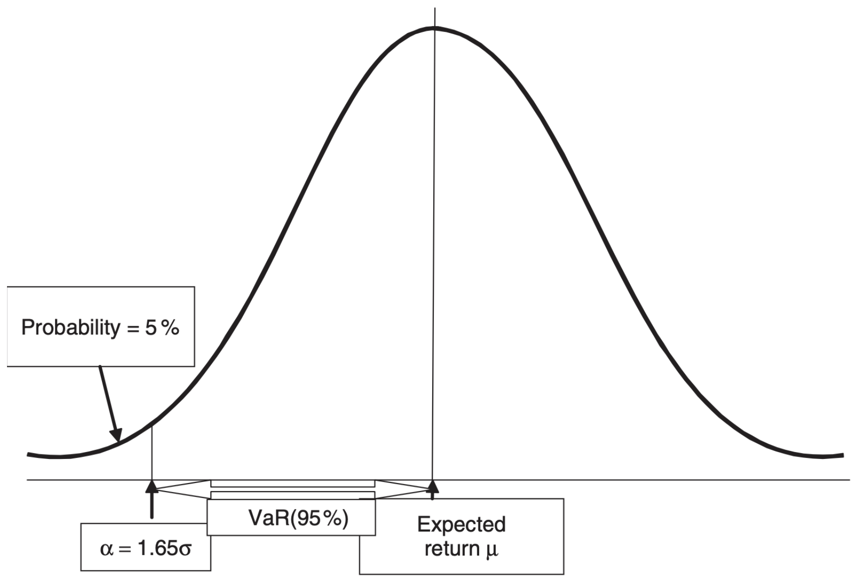

Figure 23: Value at Risk

$$VaR_{95\%,i} = \sigma_i \sqrt{t}\, \alpha_{95\%}\, \Delta \tag{35}$$

where σ_i is the asset volatility, $\sqrt{t}$ is the time horizon in year fraction, $\alpha_{95\%}$ is the value of the cumulative function given a 95% confidence interval and Δ is the sensitiveness of the underlying risk factor to market value changes.

This approach has clear advantages of simplicity, but we have to bear in mind that returns are often skewed and the assumption of normality cannot hold.

2.5.2 Expected Shortfall (Conditional Value at Risk)

An additional downside of the VaR is that it tells us what is the probability of losses exceeding the VaR, but it does not tell us the entity of the loss, (i.e. it fails to capture the "tail risk").

In order to overcome this issue, we can extend the Value at Risk model to the Conditional Value at Risk model, or Expected Shortfall; given a certain confidence level, CVaR represents the expected loss when it is greater than the value of the VaR calculated with that confidence level.

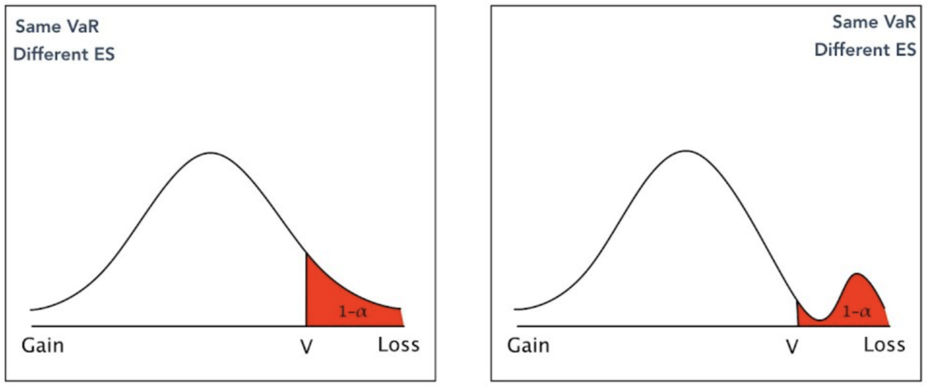

Figure 24: Conditional Value at Risk: a quick comparison between VaR and CVaR shows that the latter has greater power to capture tail risk.

We can express it as:

$$CVaR_{95\%,i} = \frac{1}{1-c}\int_{-1}^{VaR_{95\%,i}} xp(x)dx \tag{36}$$

where *p(x)* is the probability density function and c is the point where the VaR is set.

Although the ES is heavily affected by the accuracy of tail modelling, it has superior properties in portfolio management as it measures the outcome that hurt the most [53].

2.5.3 Kurtosis

In previous sections, we introduced the third moment of the distribution, but it is also important to highlight the fourth moment, the kurtosis. Kurtosis measures the height and sharpness of the central peak, relative to the ones of a standard bell curve, where values equal to zero imply mesokurtosis (normality), values greater than zero imply leptokurtosis (fatter tails) and values less than zero imply platykurtosis (thinner tails).

Sample kurtosis for one asset is given by:

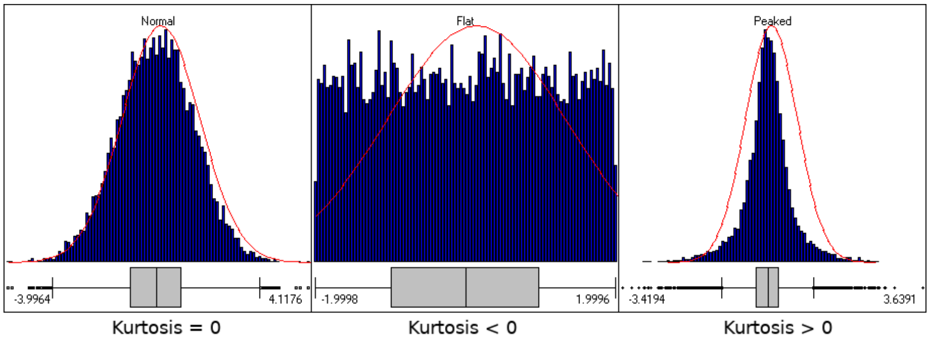

Figure 25: Kurtosis: illustration of meso, lepto and platykurtosis.

$$K_i = \left[\frac{N(N+1)}{(N-1)(N-2)(N-3)} \left(\frac{\sum_{t=1}^{T} r_{i,t} - E[R_i]}{\sigma_i} \right)^4 \right] - \frac{3(N-1)^2}{(N-2)(N-3)} \tag{37}$$

while portfolio kurtosis is given by:

$$K_P = \omega' M_4 (\omega \otimes \omega \otimes \omega) \tag{38}$$

where M_3 is the co-kurtosis matrix and it is found as:

$$\begin{aligned} M_4 &= \mathbb{E}[(r_{i,t} - \mathbb{E}[R_i])(r_{j,t} - \mathbb{E}[R_j])' \otimes (r_{k,t} - \mathbb{E}[R_k])' \otimes (r_{k,t} - \mathbb{E}[R_k])'] \\ &= \{k_{i,j,k,l}\} \end{aligned} \tag{39}$$

with elements $k_{i,j,k,l}$ defined as:

$$k_{i,j,k,l} = \mathbb{E}[(r_{i,t} - \mathbb{E}[R_i])(r_{j,t} - \mathbb{E}[R_j])(r_{k,t} - \mathbb{E}[R_k])(r_{l,t} - \mathbb{E}[R_l])] \tag{40}$$

for all i,j,k,l = 1,...n. In matrix form, the resulting Co-Kurtosis matrix for 3 assets is given by:

$$M_4 = \left[k_{11kl} k_{12kl} k_{13kl} \middle| k_{21kl} k_{22kl} k_{23kl} \middle| k_{31kl} k_{32kl} k_{33kl} \right] \tag{41}$$

2.5.4 Maximum Drawdown

The Maximum Drawdown of an investment is simply the percentage distance between the maximum peak of an investment and its minimum during a certain time frame.

It is different from a loss, as it only expresses the magnitude of a movement, but it can be used as another indicator of volatility.

For any asset i, the Maximum Drawdown is given by:

$$MD_i = \frac{P_{max} - P_{min}}{P_{max}} - 1 \tag{42}$$

2.6 Conclusions

Modern Portfolio Theory is the starting point of most optimization techniques and it has led the way of the asset allocation environment for many years.

Anyway, the model's limitations, like its strict assumptions and its myopic approach, have brought researchers to develop more advanced tools that often outperform the original methodology.

In this sense, in the second chapter we have seen that Post-Modern Portfolio Theory and Risk Parity attempt to adjust the deficiencies of MPT and introduce new risk-adjusted indicators.

In the following chapter, we will investigate the performances of portfolios constructed according to the methodologies described above and we will assess the impact of cryptocurrencies and rebalancing.

3 Analysis and Results

3.1 Introduction

At this stage, we have defined all the elements useful to conduct the empirical analysis, that is based on the application of three asset allocation methodologies outlined in the previous chapter.

We will then start with the description of the Data Sample, composed of the S&P500, the 10-Year Treasury Bond and the CRIX index. The latter is a synthetic cryptocurrency index created to track the market trend of cryptos and, for the purposes of this thesis, it has been assumed to be a tradable ETF.

Once reviewed the in-sample characteristics of the three assets, the key aspects of the portfolios constructed by optimizing the Sharpe Ratio will be outlined and they will be compared to the market index in order to perform a preliminary performance analysis.

The same process will be then performed both for Sortino Ratio and Relaxed Risk Parity portfolios.

Subsequently, all optimization techniques will be compared one with each other, both taking into account the entire out-of-sample period and considering the intertemporal statistics of the different allocations.

In conclusion, it will be possible to argue which one out of all portfolios exhibited the best performing characteristics, outlining its strengths and limitations, and, therefore, we could define the features of the optimal asset allocation methodology for crypto investing among the ones reviewed.

3.2 Settings

3.2.1 Data Sample

The data the used in the empirical analysis are divided into 3 asset classes:

- Equity: S&P500[48]

[48] Data from the GSPC ticker at https://it.finance.yahoo.com/quote/%5EGSPC?p=%5EGSPC&.tsrc=fin-srch

- Bond: 10-Year Treasury Bond[49]
- Crypto: CRIX Index[50]

The S&P500 is one the main market-capitalization-weighted indexes for the equity US market and it is constituted by the 500 largest stocks according to their market cap, while the 10-Year Treasury Bond is the benchmark for the US risk-free rate.

A deeper mention needs to be made for CRIX[51], a cryptocurrency index developed by professor Wolfgang Härdle and his team of researchers from Humboldt University, Berlin. The index allows investors to track the cryptocurrency market using a small number of constituents[52] that are evaluated according to their market capitalization and liquidity [25]. The reallocation period is 1 month, which is the time point where coin liquidity is checked again.

The collection period of the data ranges from September 30th 2014 to June 6th 2021 and the logarithmic returns have been calculated starting from October 1st 2014. For each portfolio reallocation, the look-back in-sample returns have been picked starting from the same day of the preceding year and the out-of-sample backtesting allocations have lasted for the subsequent 3 months.

3.2.2 Assumptions and Constraints

In the development of the asset allocation and during the performance evaluations, some assumptions have been made and some constraints have been imposed:

- The CRIX index has been assumed to be a tradable ETF
- The annual risk-free in the performance evaluation statistics has been assumed to be equal to 1.60% annually and 0.398% quarterly

[49] Data from the TNX ticker at https://it.finance.yahoo.com/quote/%5ETNX?p=%5ETNX&.tsrc=fin-srch

[50] Data from the official website of the index at http://data.thecrix.de/data/crix.json

[51] Known as Royalton CRIX Index after the acquisition by Royalton Partners in 2021

[52] As of December 2021, CRIX is constituted by Bitcoin, Ethereum, Cardano, Binance Coin, Ripple and Solana

- No short selling constraint
- Equity weight at least equal to 50% of the overall asset allocation
- Bond weight at least equal to 20% of the overall asset allocation
- Crypto weight at maximum equal to 10% of the overall asset allocation

Because of the last three constraints, the Risk-Parity approach has been relaxed in order to allow the compliance of these conditions.

Moreover, the allocation procedures have been set differently between the static and the dynamic portfolios:

- Each portfolio is calibrated to look back at the prior 252 trading days, with allocations starting from October 1^{st} 2015
- The rolling horizon for the out-of-sample asset allocations is set at the next 1449 trading days for static portfolios, as weights do not change, while dynamic ones are reallocated each 63 days, so the total amount of quarters is 23

Algorithm 1 shows the procedure used to set the data sample and the constraints.

3.3 Data Sample Review

Before starting with the asset allocation methodologies, it is important to analyze the data sample characteristics. Table 2 from the appendix summarizes some key metrics useful to evaluate asset classes performances over the entire in-sample and out-of-sample periods.

As we can see from figure 26, the logarithmic returns distribution of the 10-year Treasury Bond and of the CRIX index spread in a much wider way than the S&P500. This aspects resulted in high annualized standard deviation values, respectively of 53.78% for the bond and 76.49% for crypto. These volatility values were reflected in high expected returns and Sharpe Ratio only for crypto, while bond underperformed with respect to the other asset classes.

Algorithm 1 Data Set Initialization

Result: Data Selection and Constraints Setting

Initialize Data:

- Select Data Set → S&P500, 10-Year Treasury and CRIX
- Time Horizon → September 30^{th}, 2014 to June 6^{th}, 2021
- Calculate Logarithmic Returns

Constraint Settings:

- $1'\mathrm{w} = 1$
- $0.8 \geq \mathrm{w}_{\mathrm{E}} \geq 0.5$
- $0.5 \geq \mathrm{w}_{\mathrm{B}} \geq 0.2$
- $0 \leq \mathrm{w}_{\mathrm{C}} \leq 0.1$

Allocation Settings:

- Set the lookback period from ls_t to le_t → prior 252 trading days
- Set the rolling horizon from rs_t to re_t
  - Next 1449 trading days for static portfolios
  - Next 63 trading days for dynamic portfolios
- Define the number of reallocation periods
  - T = 1 for static portfolios
  - T = 23 for dynamic portfolios
- $\mathrm{r}_{\mathrm{F}} = 1.60\%$

For what regards the S&P500, its results were more stable and it showed the lowest volatility value (18.27%), with an interesting Sharpe Ratio of 54.48%.

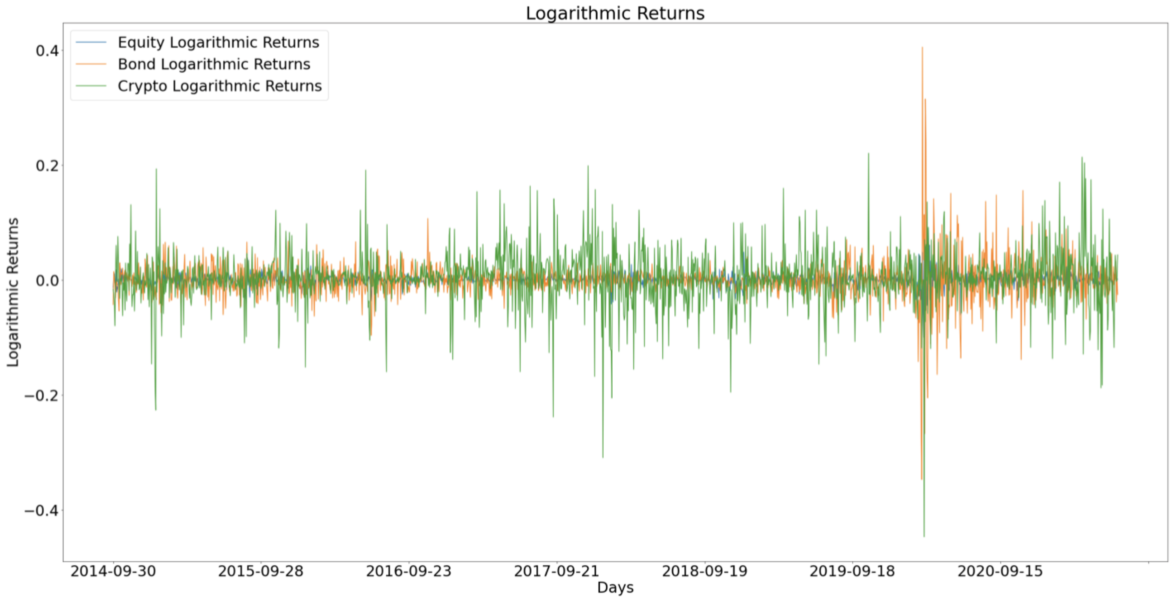

Figure 26: Logarithmic Returns (in-sample)

Skewness[53] values indicate high asymmetry in the S&P500, a fairly negative distribution for crypto and a slightly positive one for bond, while kurtosis[54] calculations tell us that all asset classes are characterized by a highly peaked platykurtosis.

These aspects imply that the classical Modern Portfolio Theory asset allocation (that in this thesis has been optimized according to the highest Sharpe Ratio) could not result in the best performing portfolio choice, as MPT is heavily based on normality assumptions.

Another important statistic to underline is the volatility skewness, telling us that, while for equity and crypto the upside variance is around 70-80% of the semivariance (or upside variance), in the case of bond it is higher by 5%, implying that the returns above the mean have been more volatile than the ones below it.

[53]Calculated over the entire sample period

[54]Calculated over the entire sample period

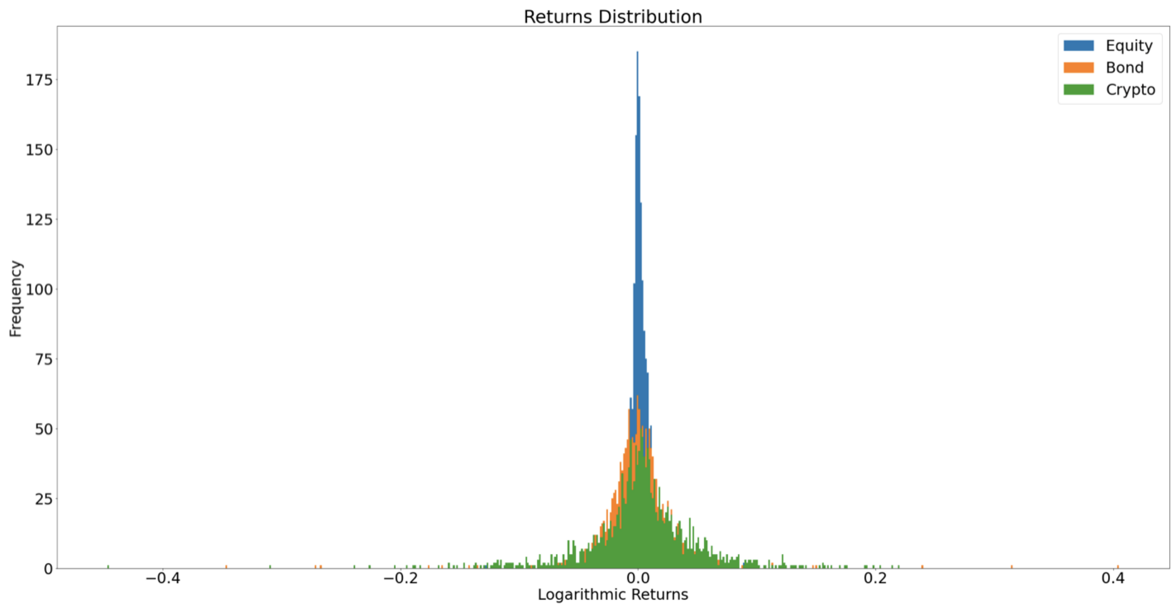

Figure 27: Returns Distribution (in-sample)

The last interesting measures to notice are the differences between Value at Risk and Expected Shortfall. In the case of equity and bond, the VaR is higher than the CVaR, meaning that the worst possible outcome is captured by the VaR. At the opposite, the crypto VaR is way less than its ES, implying that the VaR was not fully able to capture the tail risk of the distribution.

The last two graphs illustrate the cumulative returns over the entire 7-years period. Figure 29 shows the exponential growth of the cryptoasset market capitalization, especially in the first months of 2021, and a well-developed asset allocation strategy could be able to capture this opportunity.

Finally, from figure 28, it is possible to notice a common phenomenon in the market, that is the inverse correlation of equities versus bonds during certain periods. This happened in particular during 2019, when the S&P500 dropped in value during first quarters and subsequently investors regained confidence in it, selling the bonds employed to hedge against equity.

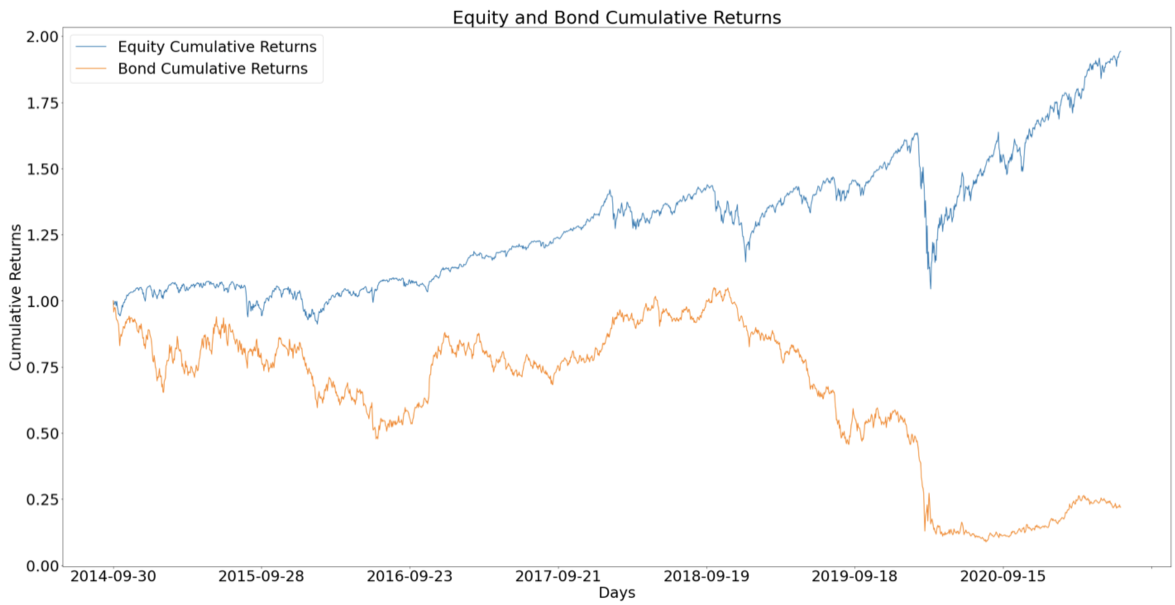

Figure 28: Equity and Bond Cumulative Returns (in-sample)

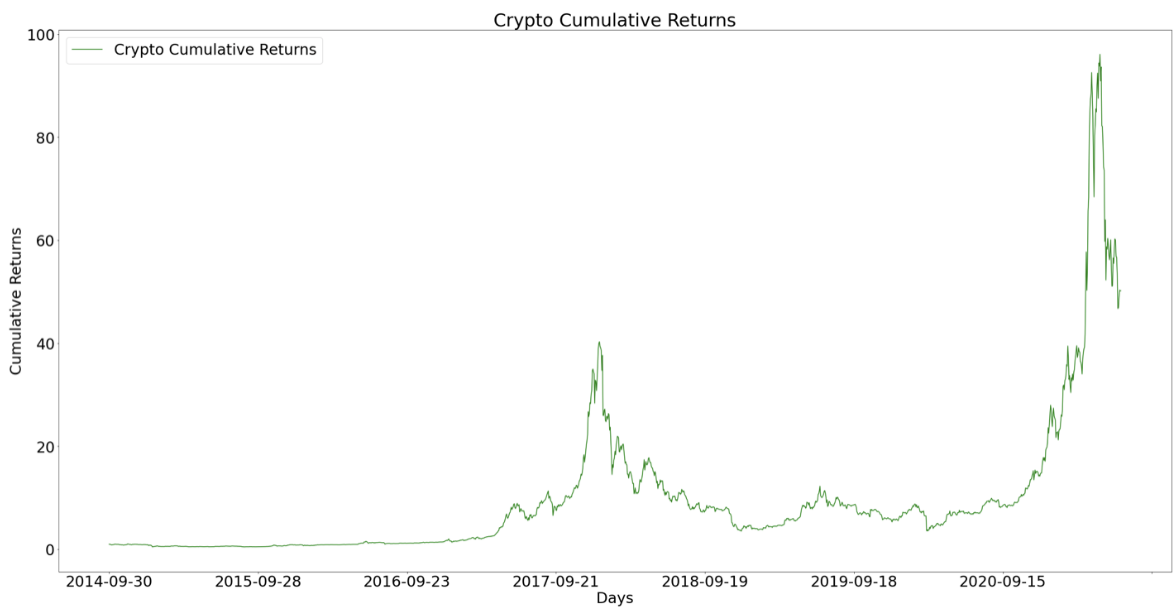

Figure 29: Crypto Cumulative Returns (in-sample)

3.4 Sharpe Ratio Optimization

In this Modern Portfolio Theory application, the asset allocation process was based upon the choice of the highest Sharpe Ratio portfolio.

Four kind of allocations have been made: static, with and without cryptos, and dynamic, with and without cryptos.

Equation 43 refers to the static asset allocation approach:

$$\begin{aligned} \max_{w} \quad & \frac{\mu' w - r_F}{\sqrt{w' \Sigma w}} \\ \text{s.t.} \quad & 1' w = 1 \\ & 0.8 \geq w_E \geq 0.5 \\ & 0.5 \geq w_B \geq 0.2 \\ & 0 \leq w_C \leq 0.1 \end{aligned} \tag{43}$$

while equation 44 refers to the dynamic strategy:

$$\begin{aligned} \max_{w} \quad & \sum_{t=1}^{T} \frac{\mu_t' w_t - r_F}{\sqrt{w_t' \Sigma_t w_t}} \\ \text{s.t.} \quad & 1' w_t = 1 \\ & 0.8 \geq w_{E,t} \geq 0.5 \\ & 0.5 \geq w_{B,t} \geq 0.2 \\ & 0 \leq w_{C,t} \leq 0.1 \end{aligned} \tag{44}$$

with T being equal to 23 quarters. The procedure was conducted as follows:

1. Looking back at the prior 252 trading days, the in-sample mean vector and the in-sample covariance matrix have been calculated

2. At this stage, a Montecarlo simulation of 1000 different weight combinations have been conducted, always in compliance with the constraints set in the previous section

3. For each random portfolio, I calculated its in-sample Sharpe Ratio

4. Subsequently, I locked the maximum Sharpe Ratio portfolio and I stored its weights vector
5. Now we have two distinct situations:
   - Static portfolios → the weights selection procedure ends here and these weights are now employed for the out-of-sample allocation, that will last for the subsequent 1449 trading days (or 23 quarters)
   - Dynamic portfolios → these weights are now employed for the out-of-sample allocation, that will last for the subsequent 63 trading days; afterwards, the whole procedure will be iterated for other 22 times

Algorithm 2 summarizes the codes used in the process.

The process was set as explained in order to ascertain if there has been a benefit in rebalancing and a positive contribution from cryptos. The portfolios were then compared to the S&P500 as a benchmark.

Starting from figure 30, we can see that portfolio returns are quite correlated, probably due to the constraint of always having at least 50% of the allocation in equity and this aspect results in a fairly comparable volatility too.

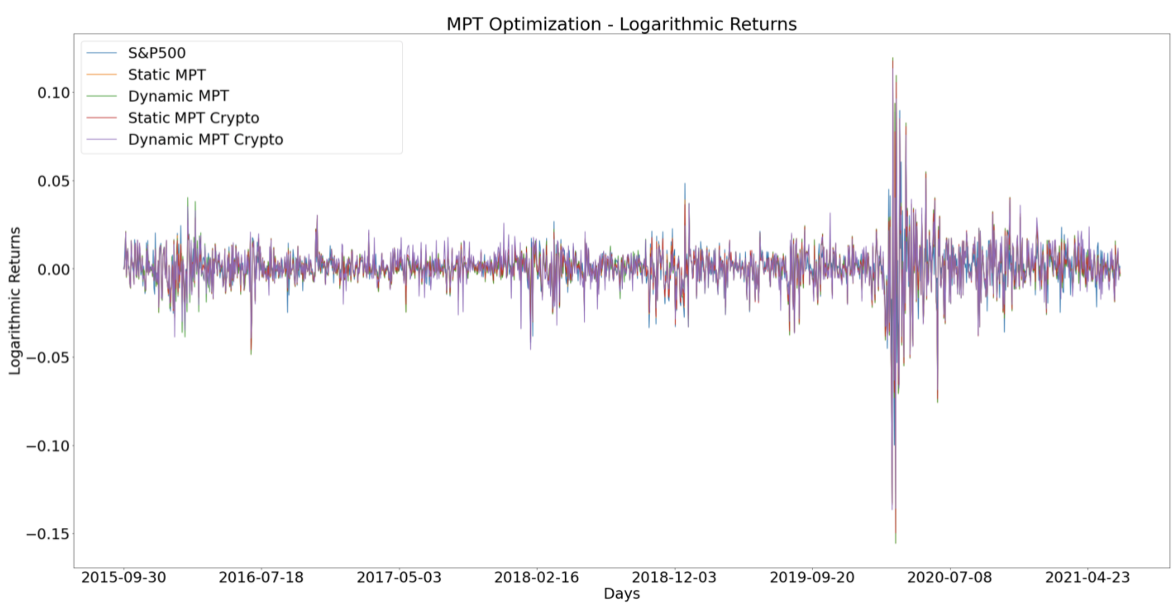

Figure 30: MPT Logarithmic Returns (out-of-sample)

Algorithm 2 Sharpe Ratio Portfolio Optimization

Stage 1: Optimal Weights

t = 0

while t $\leq$ T **do**:

- Compute the in-sample μ_{t} between ls_t and le_t
- Compute the in-sample Σ_{t} between ls_t and le_t

for i in range(1000) **do**:

- Generate 1000 random weights vectors
- Compute the in-sample SR_t (as seen in equation 43) for each weight vector

end for

- Select the maximum SR_t portfolio
- Store the optimal weights vector
- Iterate start period ls_t and end date le_t
- t = t + 1

  - T = 1 for static portfolios
  - T = 23 for dynamic portfolios

end while

Stage 2: Out-of-Sample Asset Allocation

t = 0

while t $\leq$ T **do**:

- Use optimal weights to allocate the assets in the rolling period $\rightarrow$ $w_{optimal}$' r from rs_t to re_t
- Iterate start period rs_t and end date re_t
- t = t + 1

  - T = 1 for static portfolios
  - T = 23 for dynamic portfolios

end while

Again, due to this constraint, we can observe in figure 31 how the distributions assume similar out-of-sample skewness and out-of-sample kurtosis values, all tending to negative skewness and platykurtosis.

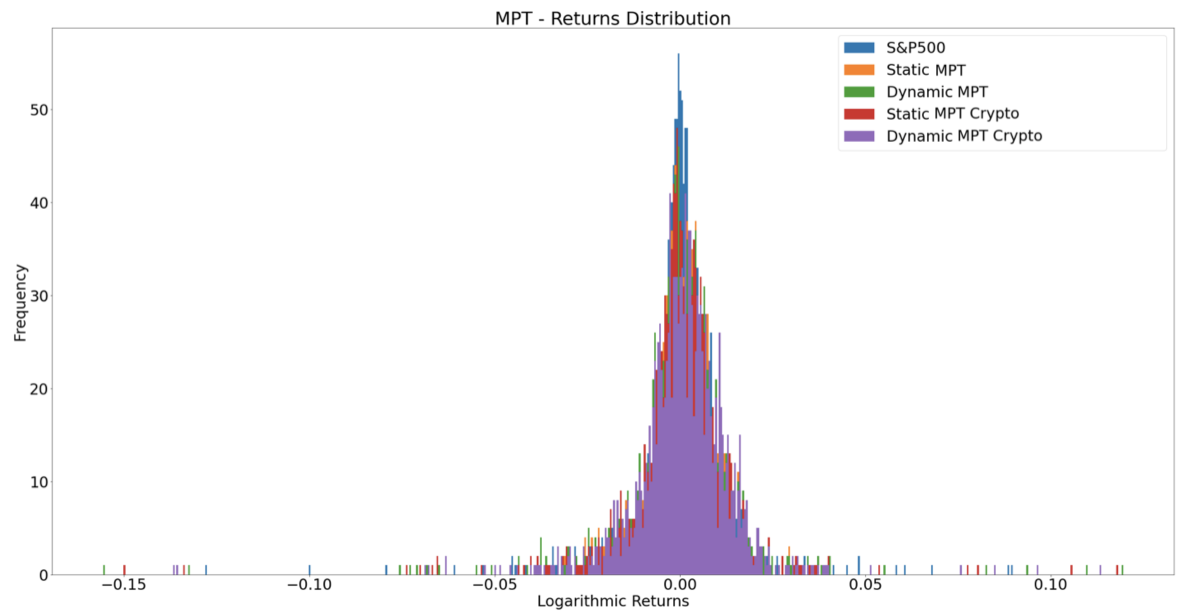

Figure 31: MPT - Returns Distribution (out-of-sample)

From the expected return point of view, only the dynamic portfolio with crypto managed to outperform the benchmark, obtaining an expected return equal to 16.29% annualized and a cumulative return of 2.22x, against 2.02x of the S&P500. Again, no other portfolio beat the market.

These conclusions were also confirmed by the Sharpe Ratio, which, in the case of the dynamic crypto portfolio, recorded a value of 67.85% versus 66.35% of the market, while other portfolios performed significantly worse.

The portfolio in question has also performed better in terms of Maximum Drawdown, recording the lowest value of -37.99%, both with respect to the -46.86% of the S&P500 and with respect to all other portfolios, all above the value of the latter.

Considering the VaR and CVaR, it can be seen from table 3 that in all distributions the first values are lower than the last ones, implying that VaR calculations have been able to capture the tail risk of the returns, but in this case no portfolio has been able to have a value lower than the market.

Finally, the Downside Risk values of all portfolios are quite aligned to the market, but only the dynamic portfolio with crypto was able to express a higher Sortino Ratio (equal to 99.60% annualized against 95.51% of the benchmark), thanks to the higher expected return.

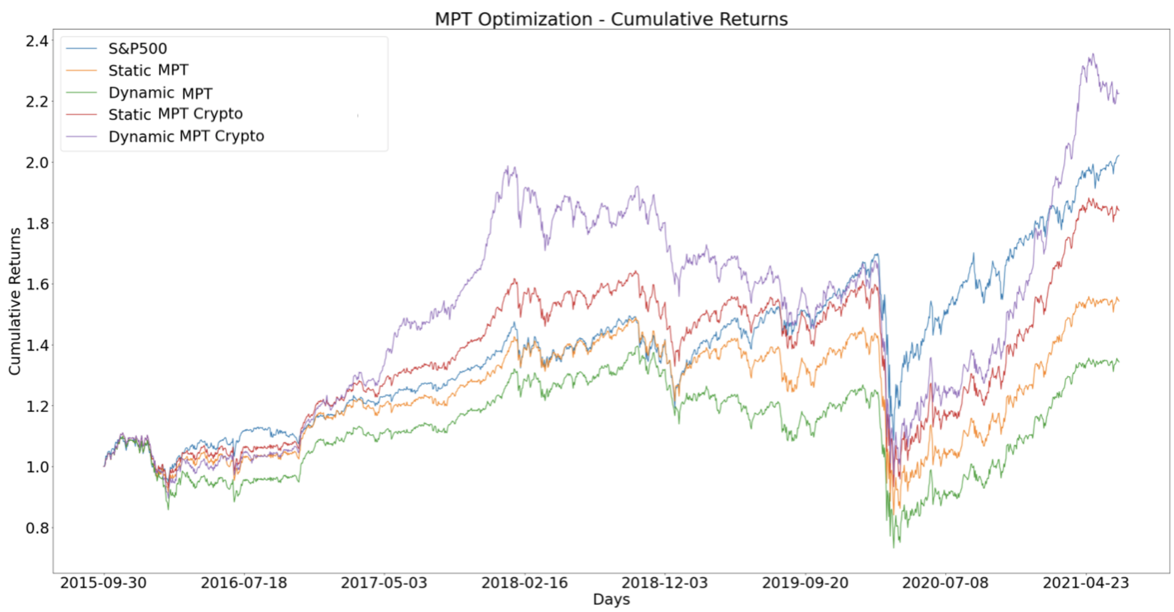

Figure 32: MPT Cumulative Returns (out-of-sample)

In conclusion, it is possible to state that, overall, the best performing portfolio has been the dynamic one with cryptos, which was the most resilient during bearish periods and has gained the most in bullish periods, benefiting both from rebalancing and from the contribution of cryptos.

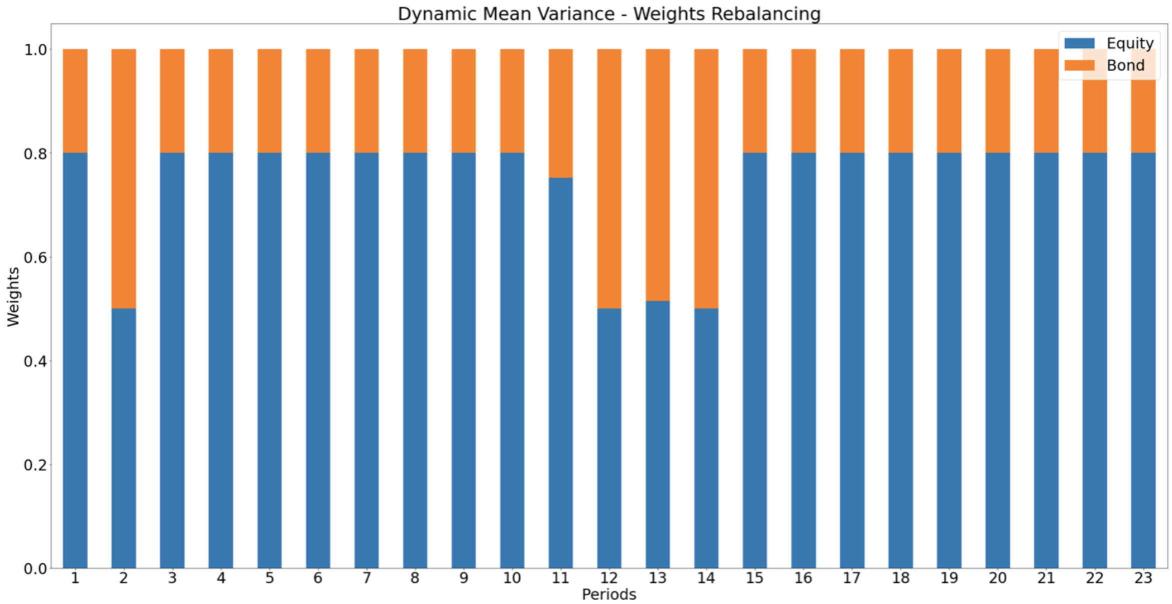

Figure 33: Weights Rebalancing - Dynamic MPT

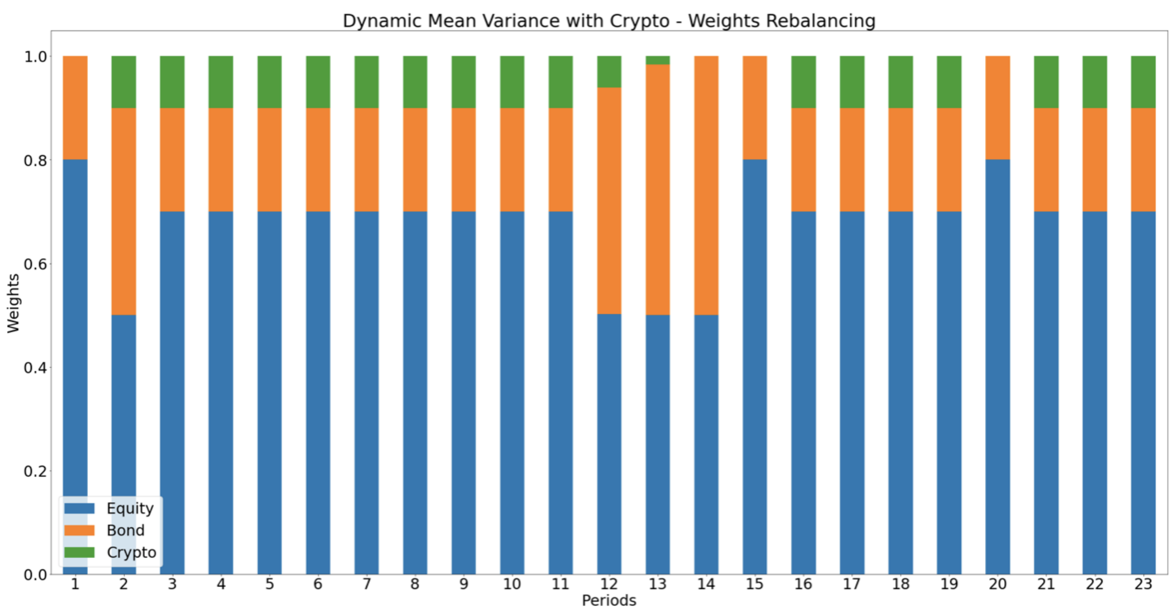

Figure 34: Weights Rebalancing - Dynamic MPT with Crypto

3.5 Sortino Ratio Optimization

The Post-Modern Portfolio Theory approach followed a similar investment strategy if compared to the Modern Portfolio Theory, but instead of the Sharpe Ratio, in this case was the Sortino Ratio the one to be maximized, so the Σ^- element in the following two equation refers to the semicovariance matrix.

Again, four portfolios have been constructed (static and dynamic, both with and without crypto) and they were then compared to the S&P500.

Equation 45 refers to the static asset allocation approach:

$$\begin{aligned} \max_{w} \quad & \frac{\mu' w - r_F}{\sqrt{w' \Sigma^- w}} \\ \text{s.t.} \quad & 1' w = 1 \\ & 0.8 \geq w_E \geq 0.5 \\ & 0.5 \geq w_B \geq 0.2 \\ & 0 \leq w_C \leq 0.1 \end{aligned} \tag{45}$$

while equation 46 refers to the dynamic strategy:

$$\begin{aligned} \max_{w} \quad & \sum_{t=1}^{T} \frac{\mu_t' w_t - r_F}{\sqrt{w_t' \Sigma_t^- w_t}} \\ \text{s.t.} \quad & 1' w_t = 1 \\ & 0.8 \geq w_{E,t} \geq 0.5 \\ & 0.5 \geq w_{B,t} \geq 0.2 \\ & 0 \leq w_{C,t} \leq 0.1 \end{aligned} \tag{46}$$

with T being equal to 23 quarters. The procedure was conducted as follows:

1. Looking back at the prior 252 trading days, the in-sample mean vector and the in-sample semicovariance matrix have been calculated

2. At this stage, a Montecarlo simulation of 1000 different weight combinations have been conducted, always in compliance with the constraints set in section 3.3
3. For each random portfolio, I calculated its in-sample Sortino Ratio
4. Subsequently, I locked the maximum Sortino Ratio portfolio and I stored its weights vector
5. Now we have two distinct situations:
   - Static portfolios → the weights selection procedure ends here and these weights are now employed for the out-of-sample allocation, that will last for the subsequent 1449 trading days (or 23 quarters)
   - Dynamic portfolios → these weights are now employed for the out-of-sample allocation, that will last for the subsequent 63 trading days; afterwards, the whole procedure will be iterated for other 22 times

Algorithm 3 summarizes the codes used in the process, that are basically the same seen in the Sharpe Ratio process (pseudocode 2) except for the Downside Risk matrix Σ^{-}.

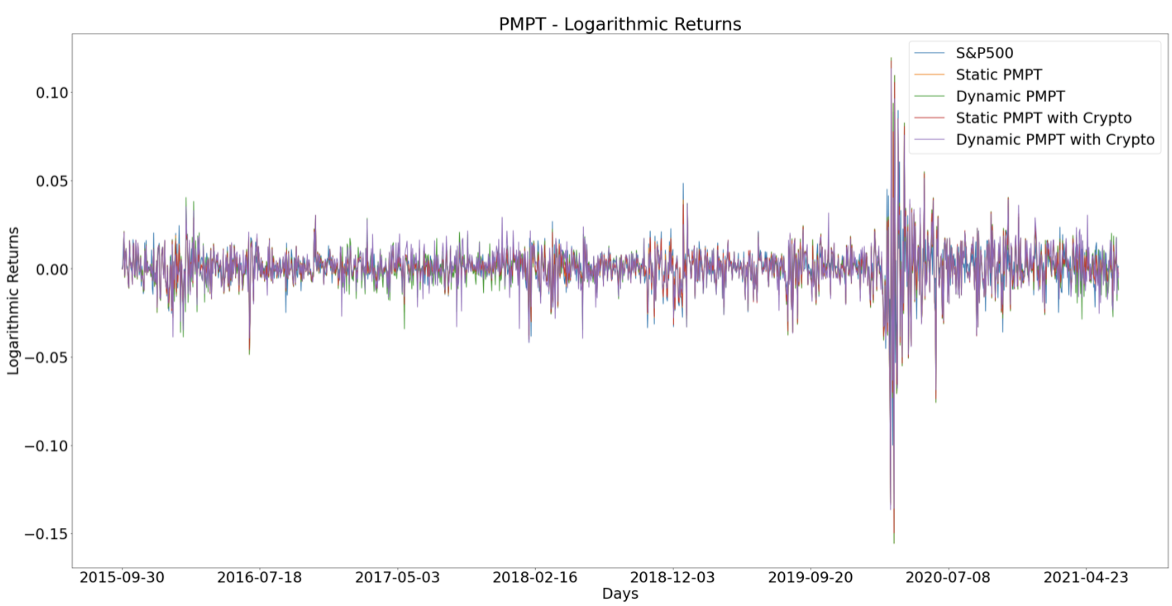

Figure 35: PMPT Logarithmic Returns (out-of-sample)

Differently from the previous strategy, the PMPT crypto portfolios had, overall, a higher allocation in the crypto asset class, as we can see comparing figure 39 to figure 34.

The logarithmic returns of all distributions are quite correlated, due to 50% in equity constraint, and exhibit a comparable volatility. None of the four allocations had a standard deviation value, all around 22%, less than the S&P500, equal to 18.72%.

Moreover, from figure 36, we can notice important negative out-of-sample skewnesses and out-of-sample platykurtosis in the returns distribution of all portfolios.

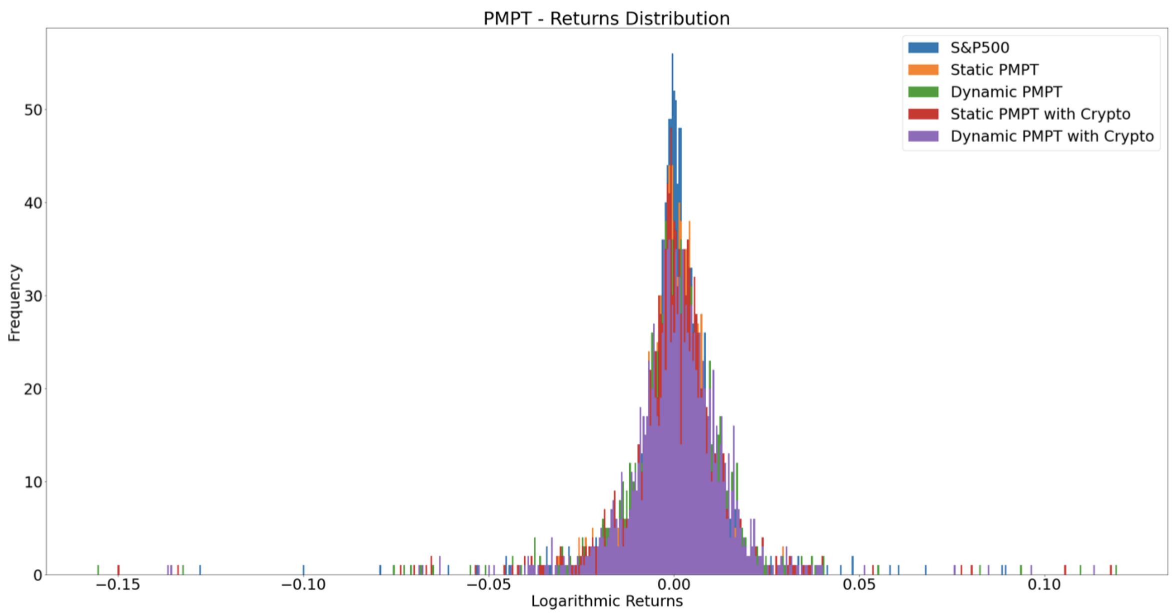

Figure 36: PMPT - Returns Distribution (out-of-sample)

Similarly to the MPT situation, only the dynamic portfolio with crypto managed to outperform the market in terms of expected return, obtaining a 15.41% annualized versus the 14.02% of the benchmark, thanks to the portion of investment in crypto (as it is possible to see from figure 39) and to the rebalancing strategy.

From the cumulative return point of view too, the dynamic crypto portfolio gained the most with respect to other allocations and it was the only one to outperform the market, obtaining a 2.10x versus 2.02x of the

Algorithm 3 Sortino Ratio Portfolio Optimization

Stage 1: Optimal Weights

t = 0

while t $\leq$ T **do**:

- Compute the in-sample μ_t between ls_t and le_t
- Compute the in-sample Σ_t^- between ls_t and le_t

for i in range(1000) **do**:

- Generate 1000 random weights vectors
- Compute the in-sample SoR_t (as seen in equation 45) for each weight vector

end for

- Select the maximum SoR_t portfolio
- Store the optimal weights vector
- Iterate start period ls_t and end date le_t
- t = t + 1
  - T = 1 for static portfolios
  - T = 23 for dynamic portfolios

end while

Stage 2: Out-of-Sample Asset Allocation

t = 0

while t $\leq$ T **do**:

- Use optimal weights to allocate the assets in the rolling period $\rightarrow$ $w_{optimal}$' r from rs_t to re_t
- Iterate start period rs_t and end date re_t
- t = t + 1
  - T = 1 for static portfolios
  - T = 23 for dynamic portfolios

end while

S&P500.

Moreover, looking at Maximum Drawdown values, the dynamic portfolio with crypto obtained the smallest value, equal to -38.96%, implying a certain resilience during bearish periods.

Anyway, these results were not confirmed in terms of Sharpe Ratio, as the dynamic crypto beat other portfolios but not the market, recording a value of 62.65% versus 66.35%.

In this sense, also from the Sortino Ratio point of view we can draw the same conclusions, where the dynamic crypto portfolio managed to do better only if compared to other PMPTs, but not with respect to the market (92.78% versus 95.51%).

Considering other risk measures, the downside volatilities of all portfolio are quite comparable, and so are the volatility skewnesses, but in terms of VaR and CVaR we can notice how the dynamic portfolio with crypto shown the biggest values, respectively equal to 34.63% and 19.41% annualized versus 27.66% and 17.67% of the benchmark.

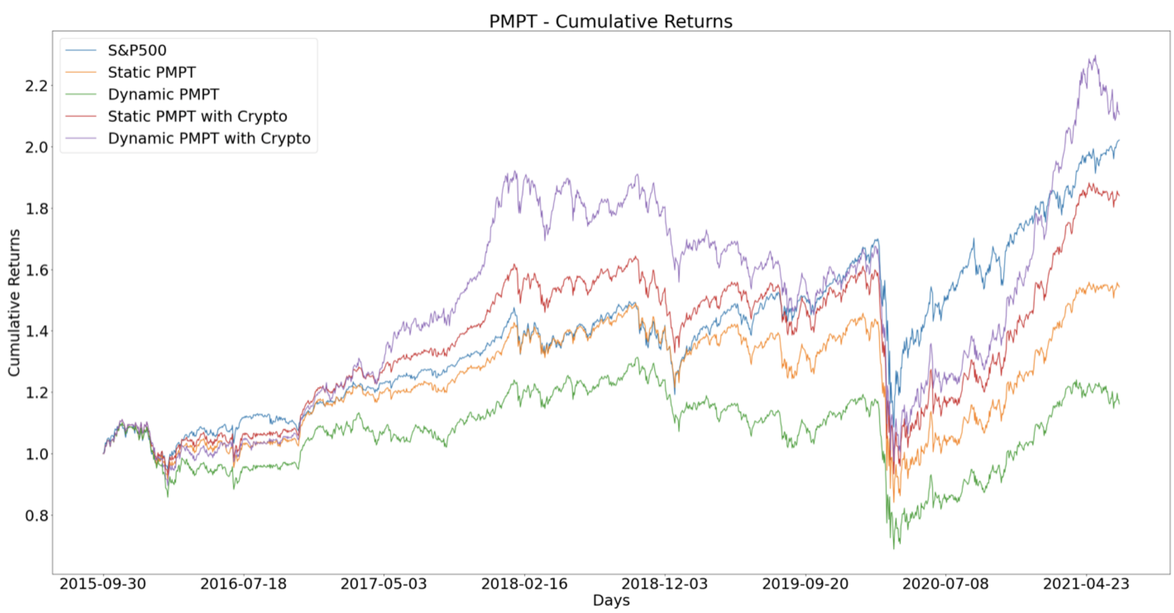

Figure 37: PMPT Cumulative Returns (out-of-sample)

In conclusion, in this allocation methodology too, the best-performing overall portfolio was the dynamic one with crypto, although it shown a certain riskiness in some alternative risk measures that make it not suitable for all investment decisions.

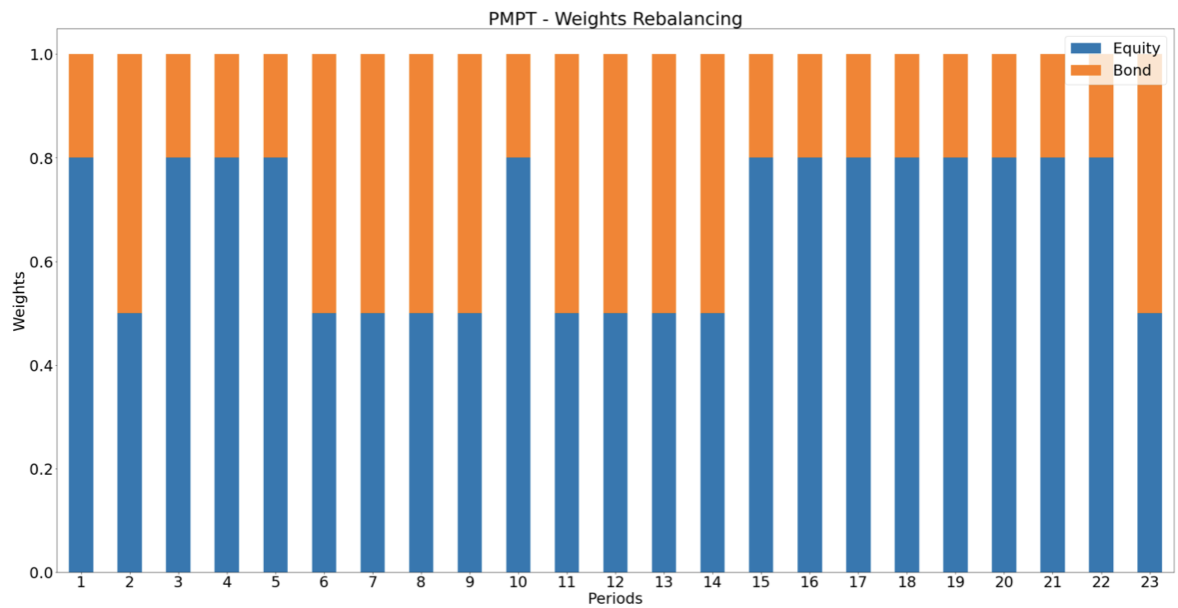

Figure 38: Weights Rebalancing - Dynamic PMPT

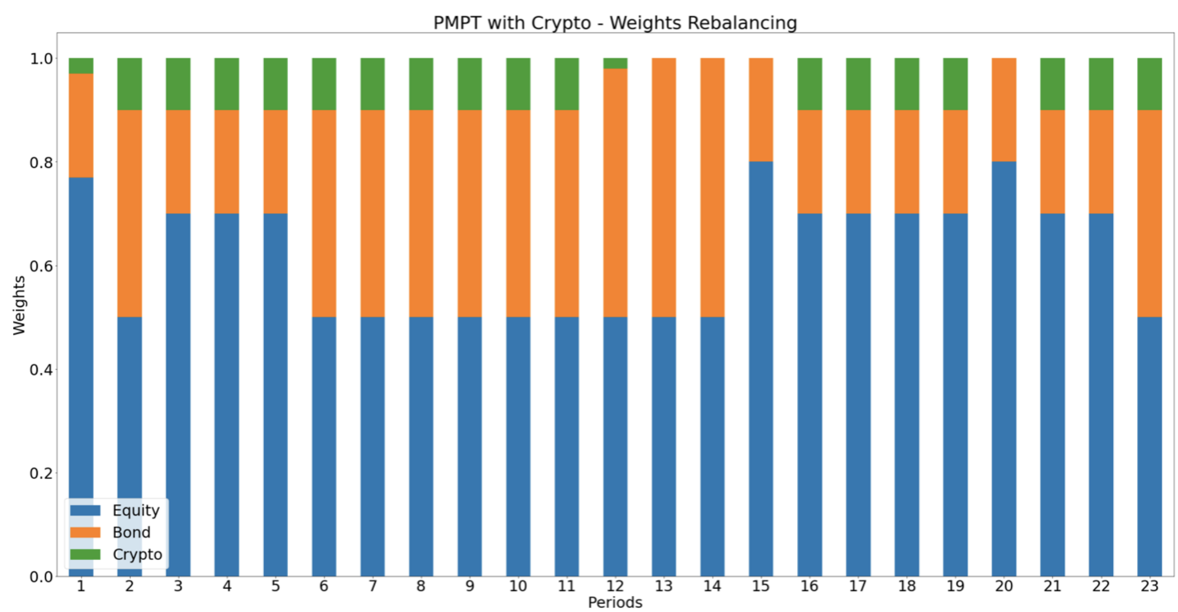

Figure 39: Weights Rebalancing - Dynamic PMPT with Crypto

3.6 Relaxed Risk-Parity Optimization

As said in chapter 2, the Relaxed Risk Parity approach is focused not on the maximization of the Sharpe or Sortino Ratio, but on the near-equal allocation of risk among all asset classes. This result can be achieved by minimizing the sum of each asset's deviation from parity.

The risk measure adopted in this approach is the volatility and the strategy used to construct the portfolios (static and dynamic, with and without crypto) resulted in very different asset allocations if compared to Modern and Post-Modern Portfolio Theory. This aspect implies heavier investments in cryptos and a smoother rebalancing during the quarters (see figure 43 and figure 44).

Equation 47 refers to the static asset allocation approach:

$$\begin{aligned} \min_{w} \quad & \sum_{i=1}^{N} \left| w_i \frac{[\Sigma w]_i}{\sqrt{w' \Sigma w}} \right| - \frac{1}{N} \\ \text{s.t.} \quad & 1'w = 1 \\ & 0.8 \geq w_E \geq 0.5 \\ & 0.5 \geq w_B \geq 0.2 \\ & 0 \leq w_C \leq 0.1 \end{aligned} \tag{47}$$

while equation 48 refers to the dynamic strategy:

$$\begin{aligned} \min_{w} \quad & \sum_{t=1}^{T} \left[\sum_{i=1}^{N} \left| w_{i,t} \frac{[\Sigma_t w_t]_i}{\sqrt{w_t' \Sigma_t w_t}} \right| - \frac{1}{N} \right] \\ \text{s.t.} \quad & 1'w_t = 1 \\ & 0.8 \geq w_{E,t} \geq 0.5 \\ & 0.5 \geq w_{B,t} \geq 0.2 \\ & 0 \leq w_{C,t} \leq 0.1 \end{aligned} \tag{48}$$

with T being equal to 23 quarters. The procedure was conducted as follows:

1. Looking back at the prior 252 trading days, the in-sample covariance matrix have been calculated

2. At this stage, a Montecarlo simulation of 1000 different weight combinations have been conducted, always in compliance with the constraints set in section 3.3

3. For each random portfolio, I calculated its in-sample sum of deviations from parity; in order to do so, the following steps have been carried out:

   (a) Recalling equation 47, through the first part of the formula, $w_i \frac{[\Sigma w]_{\mathrm{i}}}{\sqrt{w'\Sigma w}}$, I found the marginal risk contribution of asset i, while the second part, $\frac{1}{N}$, refers to the target $\mathrm{MRC_i}$

   (b) Given that the sum of all marginal risk contributions is always 1, as they are related to the total portfolio volatility, in a risk parity portfolio the ideal result of $w_i \frac{[\Sigma w]_{\mathrm{i}}}{\sqrt{w'\Sigma w}}$ should be exactly equal to $\frac{1}{N}$; so, by taking the difference between these two elements, I can find asset i's deviation from parity

   (c) By repeating the above process for each asset i and by taking the sum of all MRCs, I can find the total deviation from parity of the whole portfolio

   (d) Now, in order to obtain a near-equal risk allocation, I have to minimize the sum of these deviations by changing portfolio weights (in practical terms, in the next step I will select the portfolio that gives me the smallest total deviation from parity; in this way, if the portfolio had not had any constraint, this result would have been equal to 0, while in the case of relaxed risk parity portfolios this result is near to 0)

4. Subsequently, I locked the minimum deviation from parity portfolio and I stored its weights vector

5. Now we have two distinct situations:

- Static portfolios → the weights selection procedure ends here and these weights are now employed for the out-of-sample allocation, that will last for the subsequent 1449 trading days (or 23 quarters)
- Dynamic portfolios → these weights are now employed for the out-of-sample allocation, that will last for the subsequent 63 trading days; afterwards, the whole procedure will be iterated for other 22 times

Algorithm 4 summarizes the codes used in the process. As said, in this case the optimization follows other rules in comparison to the Sharpe and Sortino ratio portfolios, so the pseudocode will be slightly different.

Differently from other strategies, in RRP we have that both static and dynamic crypto portfolios managed to outperform the market, exhibiting annualized expected returns equal to 17.81% and 18.17% versus 14.02% of the market.

Annualized volatility values shown differences between crypto and non-crypto portfolios, with the formers around 21-22% and the latters around 24-25%, but none of them under the 18.72% of the market.

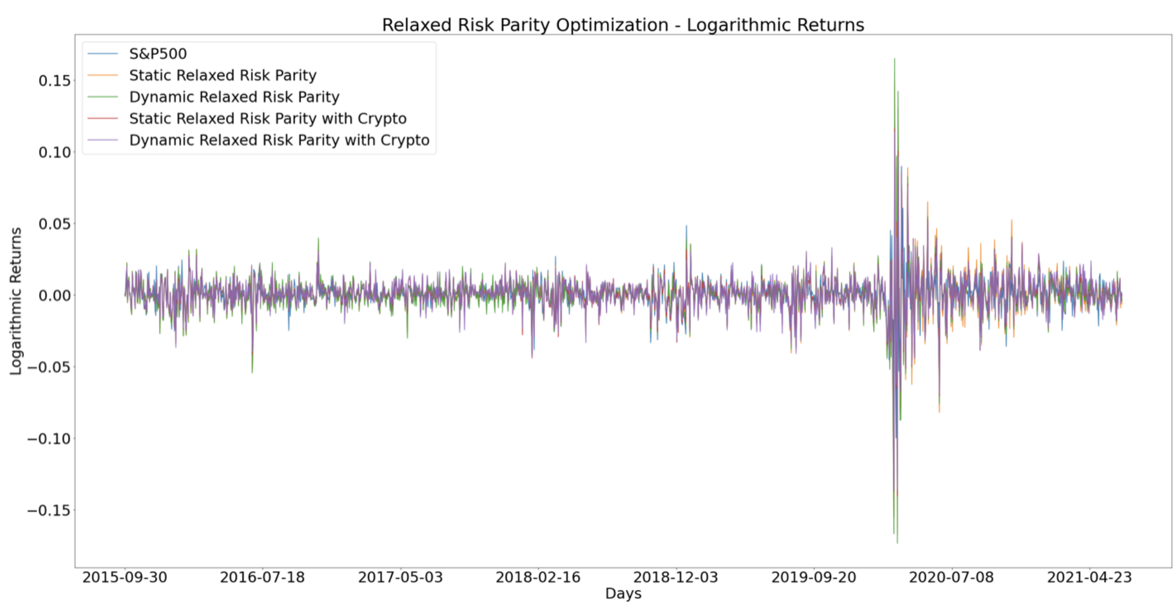

Figure 40: RRP Logarithmic Returns (out-of-sample)

Algorithm 4 Relaxed Risk Parity Portfolio Optimization

Stage 1: Optimal Weights

t = 0

while t $\leq$ T **do**:

- Compute the in-sample Σ_{t} between ls_t and le_t

for i in range(1000) **do**:

- Generate 1000 random weights vectors
- Compute the in-sample sum of deviations from parity (as seen in equation 47) for each weight vector

end for

- Select the minimum deviation from parity portfolio
- Store the optimal weights vector
- Iterate start period ls_t and end date le_t
- t = t + 1
  - T = 1 for static portfolios
  - T = 23 for dynamic portfolios

end while

Stage 2: Out-of-Sample Asset Allocation

t = 0

while t $\leq$ T **do**:

- Use optimal weights to allocate the assets in the rolling period $\rightarrow$ $w_{optimal}$' r from rs_t to re_t
- Iterate start period rs_t and end date re_t
- t = t + 1
  - T = 1 for static portfolios
  - T = 23 for dynamic portfolios

end while

In RRP too, all distributions shown high out-of-sample platykurtosis and negative out-of-sample skewnesses, as we can see from figure 41.

By looking at figure 42, it is possible to observe the extreme gains of the crypto strategies, both going over the market with cumulative returns equal to 2.43x for the static one and 2.48x for the dynamic one, versus 2.02x of the S&P500, with Maximum Drawdown values smaller than the benchmark.

These results were confirmed both in terms of Sharpe and Sortino Ratio, where the crypto strategies beat the market, and the dynamic one had the best performances overall, also if compared to all other investment techniques seen so far.

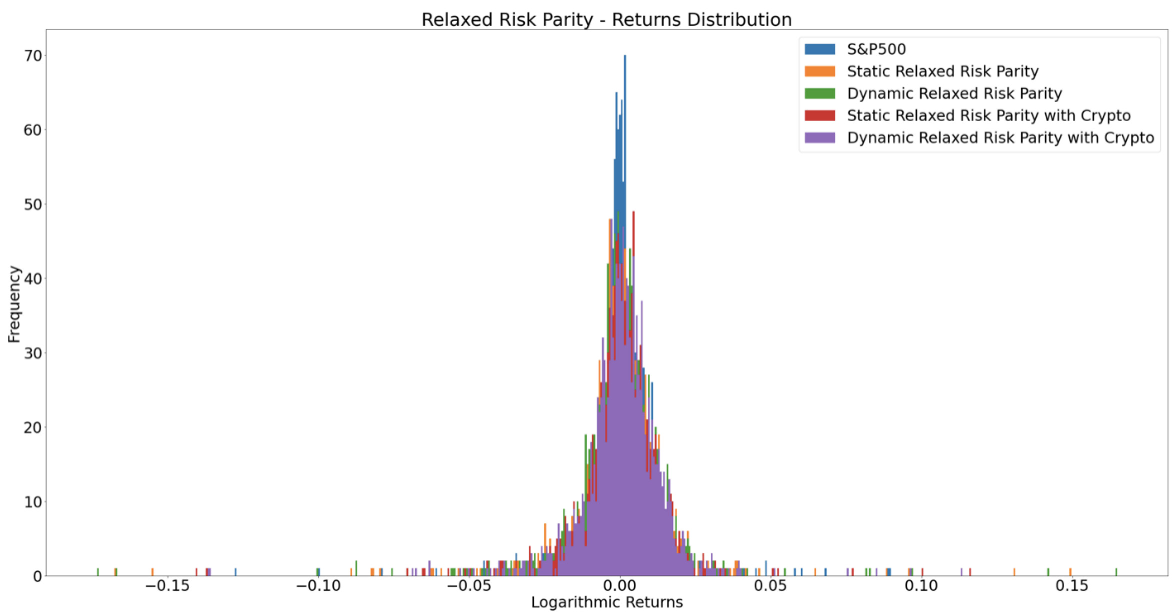

Figure 41: RRP - Returns Distribution (out-of-sample)

Crypto portfolios also shown comparable values with respect to the market in terms of VaR and, more surprisingly, the dynamic crypto portfolio had a CVaR value less than the S&P500 (13.73% versus 17.67%).

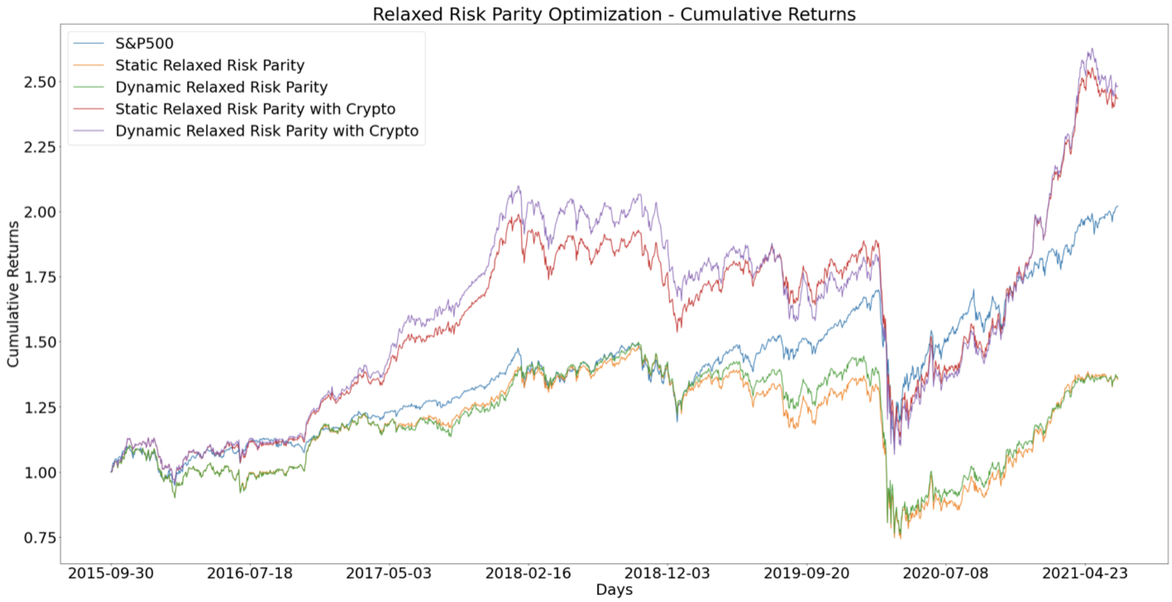

Figure 42: RRP Cumulative Returns (out-of-sample)

Figure 43: Weights Rebalancing - Dynamic RRP

Figure 44: Weights Rebalancing - Dynamic RRP with Crypto

In conclusion, the dynamic Relaxed Risk Parity approach with crypto shown the best overall statistics and it was the most interesting portfolio among all allocations made in the empirical analysis, implying that a risk-focused rebalancing technique could exploit the best out of cryptos in terms of investment properties.

3.7 Results Comparison

In this section, all portfolios will be compared and the best performing ones will be highlighted. Other intertemporal measures will be taken in consideration, such as the beta, the alpha and the Treynor Ratio, and the empirical analysis will be concluded by identifying the optimal asset allocation strategy.

Furthermore, two additional portfolios have been added as benchmarks and table 6 from the appendix highlights their statistics.

3.7.1 Performance Evaluation

From the backtesting performances seen through the analysis, we can see that the only portfolios having outperformed the market belong to the crypto category.

Only when comparing the non-crypto portfolios to the 60-40 benchmark they carry higher returns, but they are away in terms of performances from their respective allocations with crypto by a margin.

So, we can undoubtedly affirm that, despite the maximum 10% in crypto allocation constraint, cryptos had a beneficial impact on all portfolios.

Moreover, the only instances of crypto portfolios failing to exceed the cumulative return of the S&P500 involve the 50-40-10, the static MPT and the static PMPT. Leaving aside the 50-40-10, which was included as a benchmark, in the case of the other two it must be said that the respective dynamic portfolios beat the market, implying that the quarterly rebalancing has been beneficial to them.

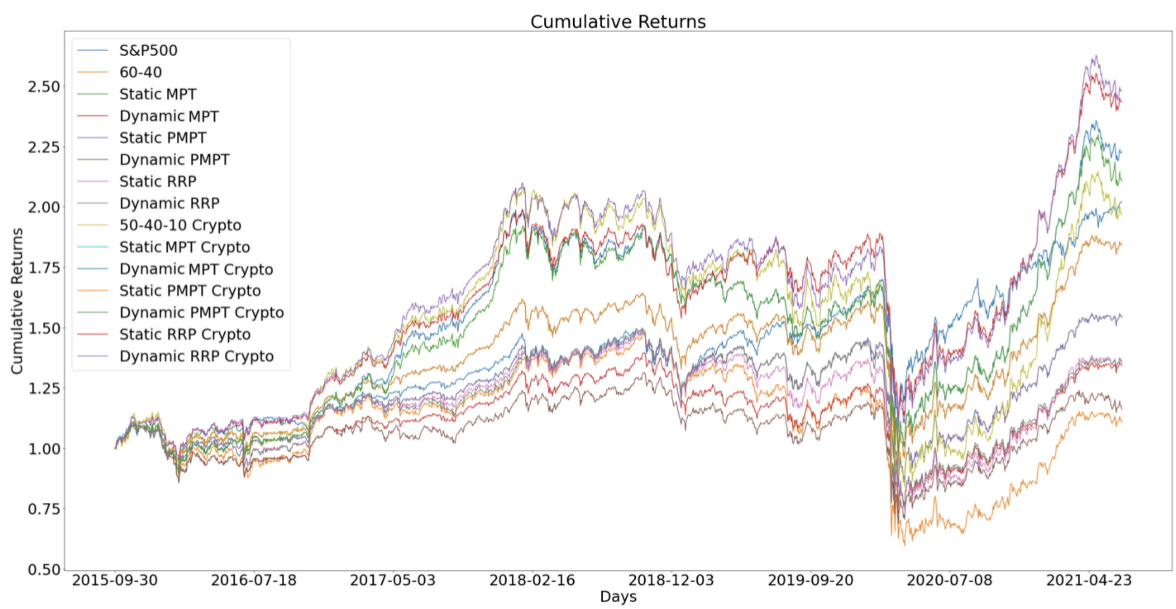

Figure 45: Cumulative Returns (out-of-sample)

From figure 45 it is possible to observe that the best performing portfolio was the dynamic crypto RRP, which obtained the best values both in terms of cumulative performance and in risk-adjusted terms, as explained deeper in previous sections.

3.7.2 Intertemporal Statistics

Apart from the entire-out-of-sample statistics, it is important to highlight some key intertemporal measures too, especially the ones that can be calculated during each quarter only (beta, alpha and Treynor Ratio, as they change during each rebalancing). These measures are expressed in quarterly values.

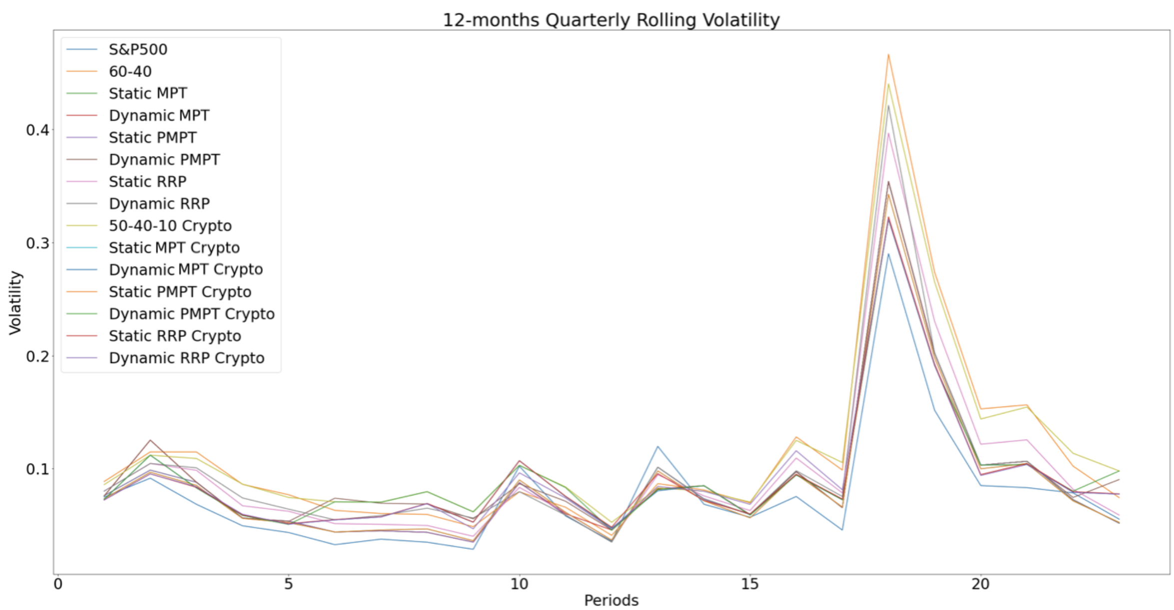

Figure 46: 12-months Quarterly Rolling Volatility (out-of-sample)

From figure 46 we can see that, with few exceptions, the most volatile portfolios have been the benchmarks 60-40 and 50-40-10. The graph also shows high rolling volatility values during first quarters of 2020, due to the huge market crash and the consequential irrationality of market participants; this was particularly true in the case of crypto portfolios, as cryptoassets were seen as shelters during COVID-19 market crisis and this resulted in extreme gains by some of them.

Another important element to outline is the high inverse correlation between Jensen's alpha and rolling volatility and their differences sharpen especially during market downturns, as we can see from figure 47.

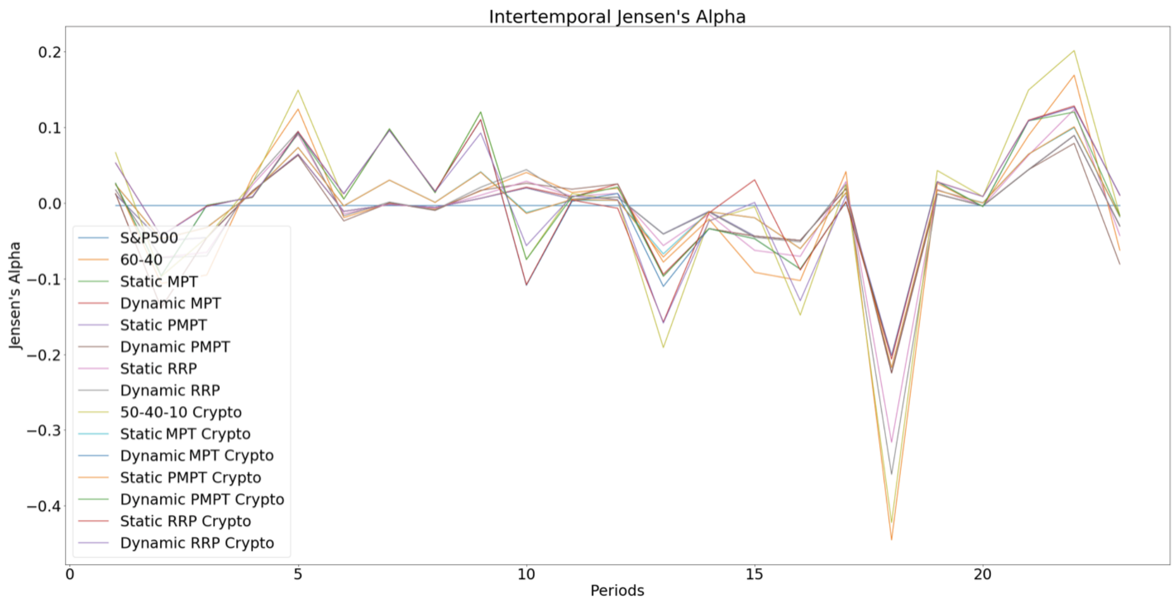

Figure 47: Intertemporal Jensen's Alpha (out-of-sample)

This aspect can be also explained through the beta, as during market crashes portfolios had a higher allocation in equity (as we can see from figure 48, where an higher beta implies higher S&P500 contribution in portfolios) that reduced the alphas (recalling equation 21).

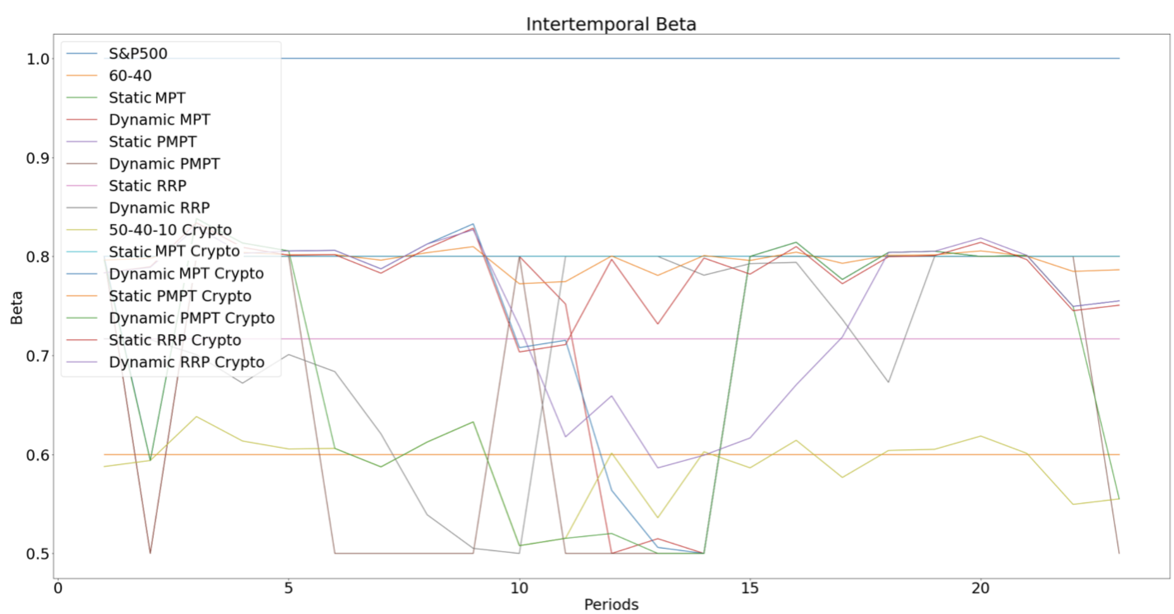

Figure 48: Intertemporal Beta (out-of-sample)

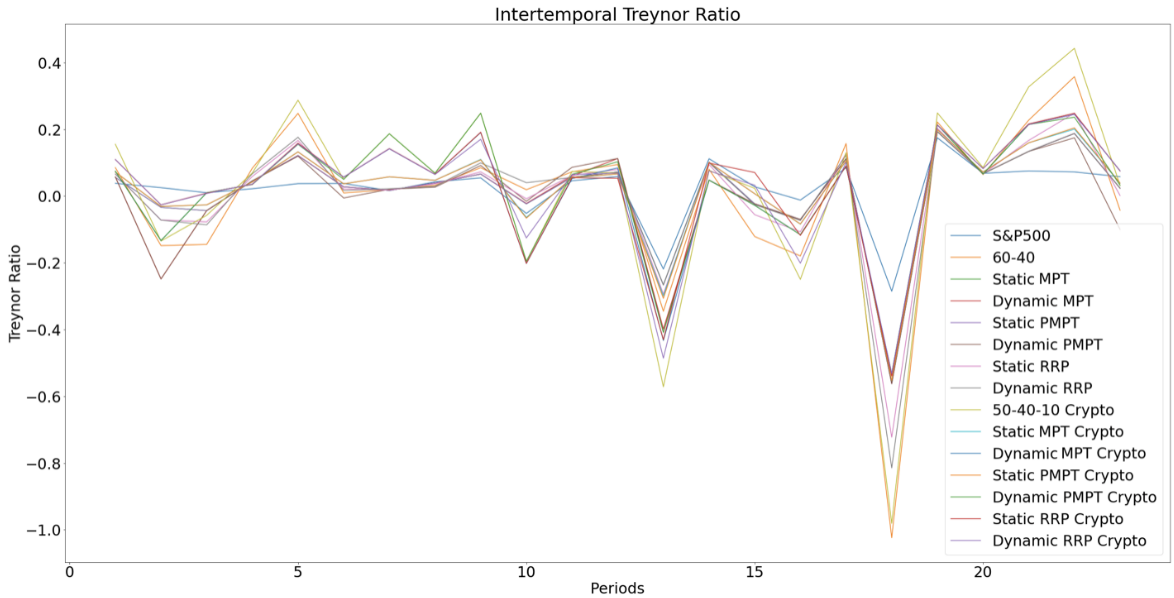

Figure 49: Intertemporal Treynor Ratio (out-of-sample)

The same logic can be applied to the Treynor Ratio, as lower exptected returns and higher betas during crises had a huge downward effect on the indicator, while the Sharpe Ratio was affected by the huge increase in volatility.

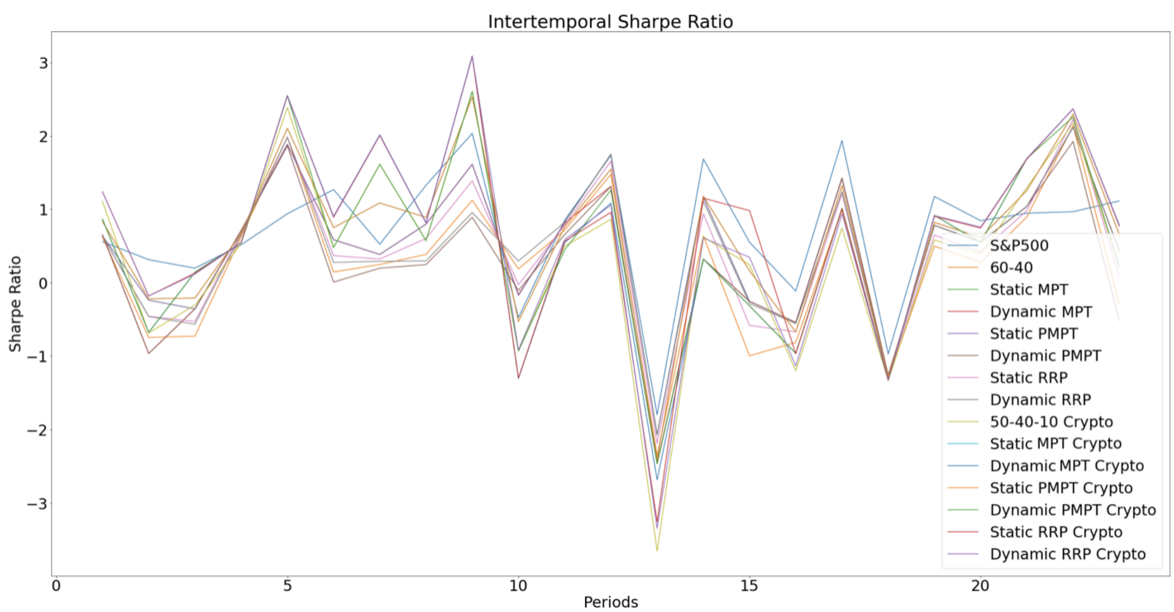

Figure 50: Intertemporal Sharpe Ratio (out-of-sample)

Despite the dynamic crypto RRP has not been the best performing portfolio during intertemporal performances, as it did not show any highest Sharpe or Treynor Ratio value over the quarters, it has been one of the most consistent and its performances have always been superior to most portfolios.

This is not surprising because Risk Parity portfolios have a conservative nature that do not let them have extreme gains; anyway, this feature enables them avoid excessive drawdowns and be more stable than other methodologies.

The clearest conclusion is that a long-term vision in portfolio allocation, as seen in Risk Parity models, can led to superior performances and in the next section we will outline why a dynamic Relaxed Risk Parity approach could be one of the best portfolio choice for cryptos.

3.7.3 Optimal Asset Allocation

Through figure 51 and 52 we can visualize the Capital Allocation Line with respect to the volatility and downside risk and the graphs clarify why the dynamic Relaxed Risk Parity with crypto had the best risk-return combination.

In fact, the portfolio has the highest annualized Sharpe Ratio, equal to 76.73%, and the highest annualized Sortino Ratio, equal to 112.52%, making both its Capital Allocation Lines equal to the Sharpe and Sortino Ratio slope.

At this stage, we can outline the main winning elements of the strategy.

The risk parity approach was developed in order to avoid the uncertainty linked to expected returns. In fact, while traditional techniques rely on too strong risk/return forecasting assumptions, risk models are only based on volatility and correlation, which are often easier to predict and may led to more robust out-of-sample performances.

This characteristic helps the portfolio prevent disproportionate concentrations in few assets, thanks to an enhanced forced diversification; indeed, the dynamic RRP with crypto portfolio was the only one to include a portion of the investment in cryptos that lasted for the whole out-of-sample period and this was reflected in quarterly expected returns often superior to other portfolios.

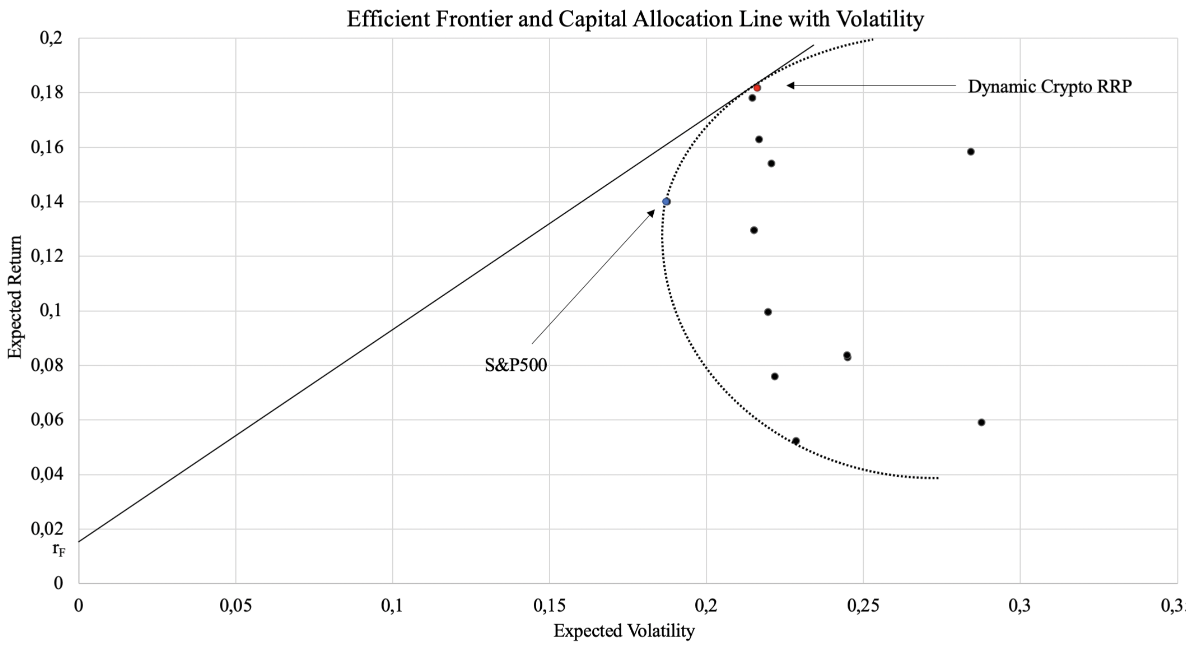

Figure 51: Efficient Frontier and Capital Allocation Line with Volatility

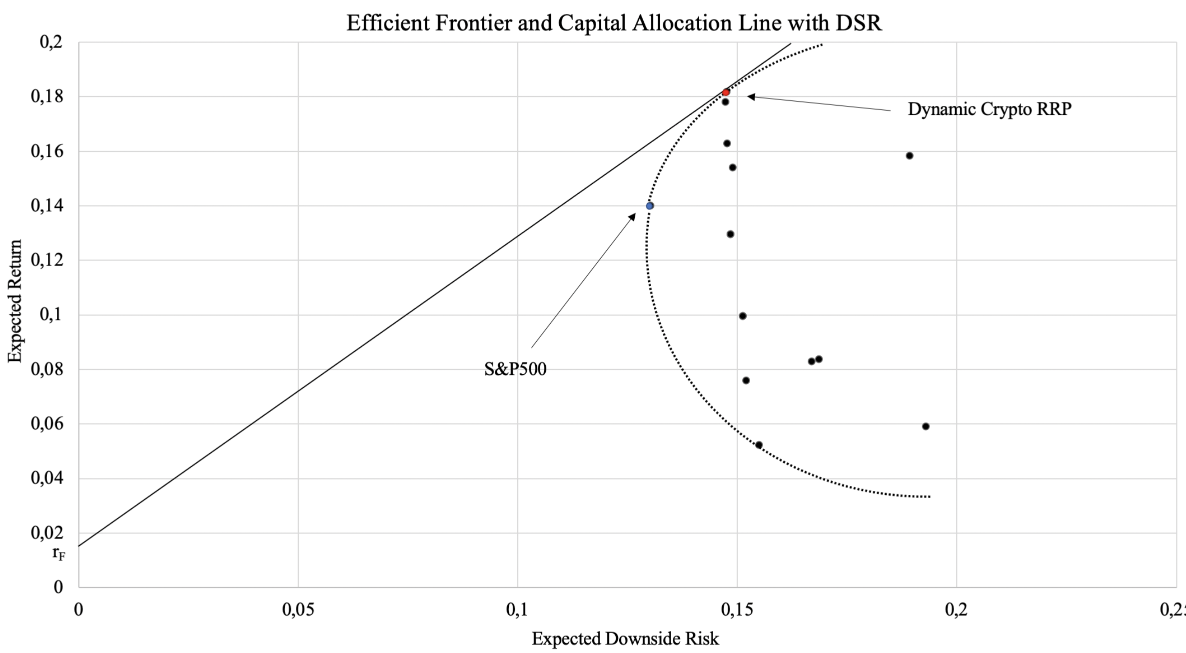

Figure 52: Efficient Frontier and Capital Allocation Line with DSR

Anyway, given the conservative nature of risk parity, it is not strange to find intertemporal Sharpe and Sortino Ratio values between other portfolios, since this aspect allows the model to avoid important drawdowns.

In fact, as noticed by Qian (2005) [54], Maillard et Al. (2010) [55] and Lee (2011) [30], risk parity techniques protect portfolios from bearish trends thanks to superior diversification capabilities and, especially in the case of the relaxed risk parity approach adopted in this thesis, the parameters' relaxations allow the portfolio to take advantage of early gains during bullish periods, without having to dramatically raise the risk.

Regarding the diversification, differently from the approaches that aim at maximize a certain risk/reward ratio (that may lead to non-properly diversified portfolios), risk parity models delete the uncertainty linked to non-reliable expected return estimations and the outcome is more robust and stabler realized returns.

The key to this result relies in the fact that each asset class in portfolio possess a certain degree of risk and, to this extent, it is not possible to completely eradicate it from the allocation, as occurred in MPT and PMPT strategies.

Therefore, excessive concentrations in certain assets are limited, resulting in a portfolio with more decorrelated elements that are able to capture the benefits of bull markets more quickly than other strategies.

Moreover, the dynamic feature of the strategy allows the portfolio to adjust the positioning dynamically in response to changes in volatility of each asset class and in the whole portfolio. This means that, when the decline of an asset is accompanied by an increase in volatility, the dynamic RRP strategy decreases the allocation in that particular asset and may allow the investor to avoid extreme losses if the asset continues to decline, maintaining a certain potential for the subsequent rebound.

For example, during the bearish market caused by COVID-19 pandemic, the losses were sustained and accompanied by sharp increases in volatility. This resulted in significant shift in allocations from the S&P500 and CRIX to the safer 10-Year Treasury. The reallocation process allowed the portfolio to sidestep subsequent losses in stocks, thus facing smaller drawdowns than other strategies.

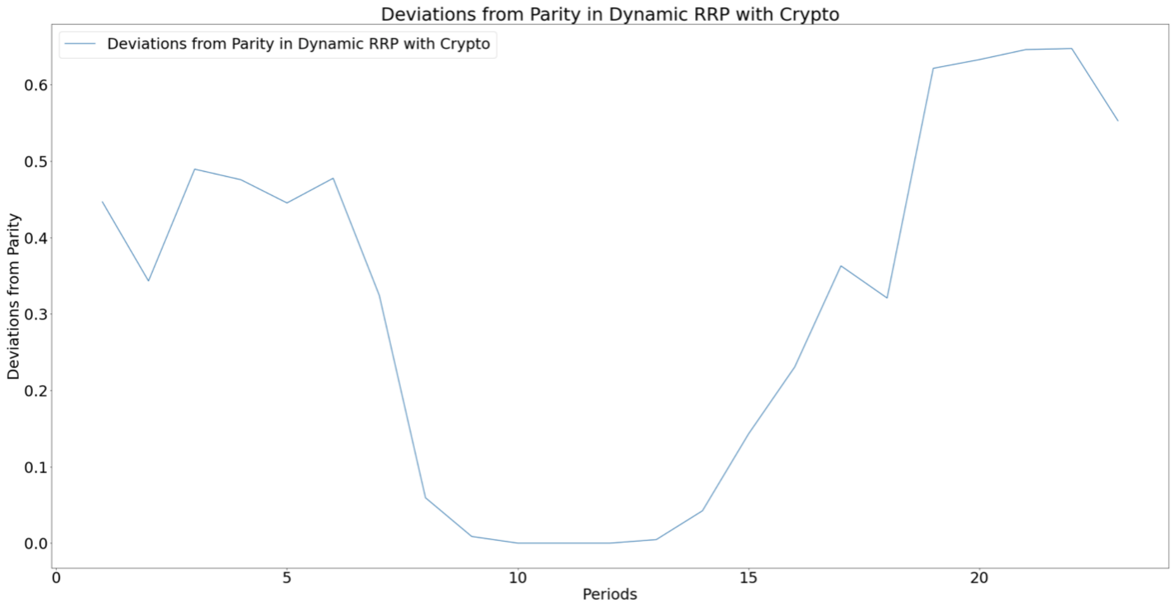

Figure 53: Sum of Deviations from Parity in Dynamic RRP with Crypto

As we said, the relaxation of parameters allows the portfolio to take advantage of market rebounds after bearish periods. This aspect clearly implies a certain distance to the risk parity condition, that is depicted in figure 53.

As the RRP model of this thesis is based on the research from Gambeta et Al., 2020 [24], in which the limitation in response to market rebounds was due to the long lookback epoch of the reallocations, I reduced the training period of the data from three years to one year. This adjustment resulted in more reactive portfolio responses to market changes and so the distance to risk parity properly followed the volatility patterns of the asset classes.

In fact, if we compare figure 53 to figure 26, we can see how the highest volatility period of the three assets coincides with the highest distance from risk parity, occurred during last quarters of 2020, right after COVID-19 market crash.

This recalibration let the dynamic RRP with crypto portfolio deliver better risk-adjusted out-of-sample performances (in fact, the more the distance from risk parity, the higher are the improvements in Sharpe and Treynor Ratio) and to go back to pre-pandemic levels in a shorter time than other portfolios, as we can see from figure 45.

Furthermore, as explained in Gambeta et Al., 2020 [24], the relaxation usually comes at a cost, so higher portfolio volatility after the reallocation. But the main difference between my research and the one in the paper is that I only included three asset classes instead of ten, so the increase in volatility is mainly due to general market conditions generated by COVID-19.

Other key elements of the winning strategy reside in the portfolio's low intertemporal betas, low tail risk and, obviously, in the enhancements coming from the CRIX index.

CRIX allowed all crypto portfolios to gain superior returns with respect to their non-crypto reciprocals, thanks to its extreme gains, especially during the second half of the observed in-sample period, and to the rebalancing strategy inherent in the index.

Indeed, as said in section 3.2.1, CRIX rebalances the crypto portfolio composition each month so it does not have any concentration polarized in one specific coin and this aspect makes it relatively diversified, as long as all cryptos are quire correlated between each other.

To conclude, the out-of sample performance analysis demonstrates the model's success by delivering higher returns while staying near risk parity. Further researches could extend the model by adding more constraints or by adjusting the magnitude of the deviations from parity to match with different degrees of risk tolerance.

3.8 Conclusions

The final part of the thesis was focused on the evaluation of the allocation methodologies seen in the previous chapter, that is to say the Sharpe Ratio, Sortino Ratio and Relaxed Risk Parity optimizations.

Results showed sensible differences between the static and dynamic asset allocation techniques and a general positive impact from the CRIX index.

Overall, this outcome has been confirmed both in the entire out-of-sample period and in the intertemporal performance evaluation, although some considerations have to be made for the dynamic Relaxed Risk Parity with crypto portfolio.

Indeed, it has been the best performing portfolio in the out-of-sample horizon, reported the highest cumulative return among all techniques and was the most reactive during the COVID-19 market crash.

Anyway, it did not show any particularly high-performing statistics in the intertemporal evaluation, given its conservative nature that do not let it earn excessive returns in the short term.

On the other side, this aspect has been quite beneficial to its performances, as it protected the portfolio from extreme drawdowns and let it be more stable during the rolling horizon, also thanks to its forced diversification feature.

The conclusion is that this approach, among the ones analyzed, could be a interesting portfolio choice when deciding whether or not to include a small portion of the allocation in cryptos.

Conclusions

In this thesis, we investigated whether cryptocurrencies can qualify or not as an asset class in their own right, and if their introduction in diversified asset allocations can enhance portfolio performances, along with dynamic optimization techniques.

We found that cryptos show strong internal correlations, low correlations with traditional assets, acceptable market liquidity and room for market stability improvements. These characteristics allow us to identify them as a distinct asset class, as they have the potential to provide diversification benefits and superior portfolio returns, despite the legal, technical and volatility issues that still have to be improved.

In fact, during each optimization technique, crypto portfolios managed to outperform their respective equity-bond only reciprocals, both in their static and dynamic specifications.

Moreover, we found that Relaxed Risk Parity portfolios have the ability to exploit the full potential out of cryptos.

Indeed, while traditional techniques rely on too strong risk/return forecasting assumptions, risk models are only based on volatility and correlation, which are often easier to predict and this aspect led the RRP portfolios to more robust out-of-sample performances.

We also found that this characteristic helps the portfolios to prevent disproportionate concentrations in few assets, thanks to an enhanced forced diversification; as a matter of fact, the dynamic RRP with crypto portfolio was the only dynamic allocation to include a portion of the investment in cryptos that lasted for the whole out-of-sample period and this was reflected in quarterly expected returns often superior to other portfolios.

Therefore, excessive concentrations in certain assets are limited, resulting in crypto portfolios with more decorrelated elements that are able to capture the benefits of bull markets more quickly than other strategies.

Anyway, given the conservative nature of risk parity, it is not strange to find intertemporal Sharpe and Sortino Ratio values between other portfolios, since this aspect allowed RRP models to avoid important drawdowns as well as take advantage from early gains during bullish rebounds.

An important implication of the dynamic strategies, especially in the case of Risk Parity ones, is that their weights recalibration feature allows the portfolios to adjust the positioning dynamically in response to changes in volatility of each asset class, permitting them to sidestep losses in stocks during COVID-19 crisis, thus facing smaller drawdowns than the other strategies.

Referring particularly to the dynamic RRP with crypto portfolio, we also found that the highest volatility period of the three assets coincides with the highest distance from risk parity (occurred during last quarters of 2020, right after COVID-19 market crash) and that the more the distance from risk parity, the higher are the improvements in Sharpe and Treynor Ratio.

To conclude, the CRIX index allowed all crypto portfolios to gain superior returns with respect to their non-crypto reciprocals, thanks to its extreme gains, especially during the second half of the observed in-sample period, and to the rebalancing strategy inherent in the index.

Out-of sample performance analysis demonstrates the dynamic crypto RRP model's success by delivering higher returns while staying near risk parity. Further researches could extend the model by adding more constraints or by adjusting the magnitude of the deviations from parity to match with different degrees of risk tolerance.

The conclusion is that this approach, among the ones analyzed, could be a interesting portfolio choice when deciding whether or not to include a small portion of the allocation in cryptos.

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Appendix

Data Sample Summary Statistics			
	Equity	Bond	Crypto
Expected Return	11.55%	-8.20%	89.06%
Expected Excess Return	9.95%	-9.80%	87.46%
Variance	3.34%	28.93%	58.50%
Volatility	18.27%	53.78%	76.49%
Sharpe Ratio	54.48%	-18.22%	114.35%
Maximum Drawdown	-46.93%	-8.44%	-0.48%
Skewness	-1.03	0.34	-0.73
Kurtosis	20.78	31.05	8.48
Beta	1	0	0.98
R^2	1	/	0.27
Jensen's Alpha	0.00%	-9.80%	77.70%
Treynor Ratio	9.95%	/	89.16%
Upside Variance	0.01%	0.05%	0.08%
Downside Variance	0.01%	0.05%	0.10%
Upside Volatility	0.67%	2.20%	2.87%
Downside Volatility	0.79%	2.14%	3.18%
Volatility Skewness	72.14%	105.04%	81.42%
Sortino Ratio	12.57	-4.57	27.51
1-day 95% VaR	1.66%	2.10%	0.29%
252-day 95% VaR	26.33%	33.23%	4.58%
1-day 95% CVaR (ES)	0.92%	0.65%	7.07%
252-day 95% CVaR (ES)	14.50%	10.29%	111.76%

Table 2: Data Sample Summary Statistics

Asset Classes Correlation Matrix									
	S&P 500	10-Year Treasury	Commodity	Real Estate	Gold	BTC	ETH	XRP	ADA
S&P 500	1								
10-Year Treasury	39.78%	1							
Commodity	40.35%	35.31%	1						
Real Estate	73.85%	27.81%	33.40%	1					
Gold	-0.40%	-33.00%	3.38%	8.69%	1				
BTC	22.84%	6.11%	6.55%	12.01%	21.99%	1			
ETH	24.22%	5.45%	17.18%	18.31%	27.08%	76.07%	1		
XRP	18.34%	5.07%	8.79%	13.18%	7.73%	42.84%	52.47%	1	
ADA	25.56%	7.39%	15.21%	18.12%	10.66%	64.99%	67.67%	56.28%	1

Table 1: Asset Classes Correlation Matrix

MPT Summary Statistics					
	S&P500	Static MPT	Dynamic MPT	Static Crypto MPT	Dynamic Crypto MPT
Expected Return	14.02%	9.98%	7.61%	12.95%	16.29%
Expected Excess Return	12.42%	8.38%	6.01%	11.35%	14.69%
Cumulative Return	2.02	1.54	1.34	1.84	2.22
Variance	3.51%	4.82%	4.91%	4.62%	4.68%
Volatility	18.72%	21.95%	22.15%	21.48%	21.64%
Sharpe Ratio	66.35%	38.17%	27.12%	52.83%	67.85%
Maximum Drawdown	-46.86%	-53.94%	-52.42%	-49.42%	-37.99%
Skewness	-1.11	-0.97	-0.94	-1.07	-1.03
Kurtosis	21.93	28.63	27.59	27.92	21.75
Upside Variance	1.20%	1.72%	1.75%	1.61%	1.57%
Downside Variance	1.69%	2.28%	2.31%	2.20%	2.17%
Upside Volatility	10.94%	13.11%	13.22%	12.70%	12.54%
Downside Volatility	13.01%	15.11%	15.19%	14.83%	14.75%
Volatility Skewness	84.10%	86.74%	87.02%	85.66%	85.03%
Sortino Ratio	95.51%	55.45%	39.56%	76.55%	99.60%
1-day 95% VaR	1.74%	1.80%	1.80%	1.90%	2.12%
252-day 95% VaR	27.66%	28.56%	28.56%	30.11%	33.73%
1-day 95% CVaR (ES)	1.11%	0.79%	0.60%	1.03%	1.29%
252-day 95% CVaR (ES)	17.67%	12.57%	9.58%	16.31%	20.52%

Table 3: MPT Summary Statistics

PMPT Summary Statistics					
	S&P500	Static PMPT	Dynamic PMPT	Static Crypto PMPT	Dynamic Crypto PMPT
Expected Return	14.02%	9.98%	5.24%	12.95%	15.41%
Expected Excess Return	12.42%	8.38%	3.64%	11.35%	13.81%
Cumulative Return	2.02	1.54	1.16	1.84	2.10
Variance	3.51%	4.82%	5.21%	4.62%	4.86%
Volatility	18.72%	21.95%	22.83%	21.48%	22.04%
Sharpe Ratio	66.35%	38.17%	15.93%	52.83%	62.65%
Maximum Drawdown	-46.86%	-53.94%	-52.39%	-49.42%	-38.96%
Skewness	-1.11	-0.97	-0.86	-1.07	-0.96
Kurtosis	21.93	28.63	24.22	27.92	20.09
Upside Variance	1.20%	1.72%	1.82%	1.61%	1.63%
Downside Variance	1.69%	2.28%	2.39%	2.20%	2.21%
Upside Volatility	10.94%	13.11%	13.48%	12.70%	12.76%
Downside Volatility	13.01%	15.11%	15.47%	14.83%	14.88%
Volatility Skewness	84.10%	86.74%	87.17%	85.66%	85.71%
Sortino Ratio	95.51%	55.45%	23.51%	76.55%	92.78%
1-day 95% VaR	1.74%	1.80%	1.80%	1.90%	2.18%
252-day 95% VaR	27.66%	28.56%	28.56%	30.11%	34.63%
1-day 95% CVaR (ES)	1.11%	0.79%	0.42%	1.03%	1.22%
252-day 95% CVaR (ES)	17.67%	12.57%	6.60%	16.31%	19.41%

Table 4: PMPT Summary Statistics

RRP Summary Statistics					
	S&P500	Static RRP	Dynamic RRP	Static Crypto RRP	Dynamic Crypto RRP
Expected Return	14.02%	8.30%	8.39%	17.81%	18.17%
Expected Excess Return	12.42%	6.70%	6.79%	16.21%	16.57%
Cumulative Return	2.02	1.35	1.36	2.43	2.48
Variance	3.51%	5.99%	5.99%	4.60%	4.67%
Volatility	18.72%	24.47%	24.47%	21.44%	21.60%
Sharpe Ratio	66.35%	27.37%	27.76%	75.61%	76.73%
Maximum Drawdown	-46.86%	-50.06%	-50.74%	-37.46%	-36.44%
Skewness	-1.11	-0.77	-0.85	-1.09	-1.05
Kurtosis	21.93	30.57	38.94	24.65	21.92
Upside Variance	1.20%	2.20%	2.17%	1.56%	1.56%
Downside Variance	1.69%	2.78%	2.84%	2.17%	2.17%
Upside Volatility	10.94%	14.84%	14.72%	12.48%	12.50%
Downside Volatility	13.01%	16.68%	16.84%	14.72%	14.73%
Volatility Skewness	84.10%	88.95%	87.42%	84.78%	84.88%
Sortino Ratio	95.51%	40.16%	40.33%	110.17%	112.52%
1-day 95% VaR	1.74%	1.82%	1.88%	1.79%	2.07%
252-day 95% VaR	27.66%	28.93%	29.91%	28.41%	32.90%
1-day 95% CVaR (ES)	1.11%	0.66%	0.67%	1.29%	0.87%
252-day 95% CVaR (ES)	17.67%	10.45%	10.57%	20.41%	13.73%

Table 5: RRP Summary Statistics

Benchmarks Summary Statistics			
	S&P500	60-40	50-40-10
Expected Return	14.02%	5.93%	15.84%
Expected Excess Return	12.42%	4.33%	14.24%
Cumulative Return	2.02	1.11	1.97
Variance	3.51%	8.26%	8.06%
Volatility	18.72%	28.73%	28.39%
Sharpe Ratio	66.35%	15.09%	50.14%
Maximum Drawdown	-46.86%	-40.44%	-39.18%
Skewness	-1.11	-0.48	-0.55
Kurtosis	21.93	31.94	27.79
Upside Variance	1.20%	3.16%	2.99%
Downside Variance	1.69%	3.72%	3.57%
Upside Volatility	10.94%	17.78%	17.28%
Downside Volatility	13.01%	19.29%	18.91%
Volatility Skewness	84.10%	92.18%	91.40%
Sortino Ratio	95.51%	22.47%	75.31%
1-day 95% VaR	1.74%	1.86%	2.18%
252-day 95% VaR	27.66%	29.46%	34.63%
1-day 95% CVaR (ES)	1.11%	0.47%	1.26%
252-day 95% CVaR (ES)	17.67%	7.48%	19.95%

Table 6: Benchmarks Summary Statistics

12-months Quarterly Rolling Volatility															
Period	S&P 500	60-40	Static MPT	Dynamic MPT	Static PMPT	Dynamic PMPT	Static RRP	Dynamic RRP	50-40-10	Static MPT Crypto	Dynamic MPT Crypto	Static PMPT Crypto	Dynamic PMPT Crypto	Static RRP Crypto	Dynamic RRP Crypto
1	7,47%	8,85%	7,60%	7,57%	7,60%	7,57%	8,00%	8,00%	8,57%	7,32%	7,28%	7,32%	7,28%	7,21%	7,21%
2	9,15%	11,47%	9,86%	12,52%	9,86%	12,52%	10,44%	10,45%	11,18%	9,68%	11,19%	9,68%	11,19%	9,52%	9,53%
3	6,84%	11,47%	8,79%	8,79%	8,79%	8,79%	9,84%	10,06%	10,89%	8,51%	8,29%	8,51%	8,29%	8,34%	8,39%
4	4,93%	8,60%	5,62%	5,62%	5,62%	5,62%	6,70%	7,40%	8,62%	5,59%	5,85%	5,59%	5,85%	5,89%	5,95%
5	4,34%	7,68%	5,34%	5,34%	5,34%	5,33%	6,22%	6,40%	7,44%	5,21%	5,08%	5,21%	5,05%	5,12%	5,08%
6	3,26%	6,30%	4,38%	4,38%	4,38%	7,38%	5,13%	5,46%	7,06%	4,39%	5,45%	4,39%	7,06%	5,48%	5,45%
7	3,75%	6,02%	4,49%	4,49%	4,49%	6,93%	5,06%	5,84%	7,02%	4,56%	5,71%	4,56%	7,02%	5,73%	5,71%
8	3,48%	5,94%	4,36%	4,36%	4,36%	6,86%	4,96%	6,49%	7,95%	4,66%	6,90%	4,66%	7,95%	6,92%	6,89%
9	2,86%	4,86%	3,49%	3,49%	3,49%	5,55%	4,00%	5,62%	6,17%	3,61%	5,25%	3,61%	6,17%	5,26%	4,67%
10	10,20%	7,94%	8,76%	8,74%	8,76%	8,65%	8,33%	7,96%	10,28%	8,99%	10,70%	8,99%	10,28%	10,69%	9,62%
11	5,81%	6,58%	5,88%	6,00%	5,88%	7,11%	6,10%	5,88%	8,37%	6,13%	7,51%	6,13%	8,32%	7,55%	7,39%
12	3,58%	4,09%	3,50%	4,56%	3,50%	4,56%	3,67%	3,50%	5,24%	3,67%	4,73%	3,67%	4,53%	4,81%	4,63%
13	11,96%	8,66%	10,12%	8,21%	10,12%	8,14%	9,46%	10,12%	8,29%	9,79%	8,03%	9,79%	8,14%	9,55%	8,41%
14	6,84%	8,11%	7,28%	8,48%	7,28%	8,48%	7,58%	7,34%	8,00%	7,15%	8,48%	7,15%	8,48%	7,17%	7,99%
15	5,69%	6,96%	5,93%	5,93%	5,93%	5,93%	6,28%	5,96%	7,03%	5,64%	5,89%	5,64%	5,89%	5,99%	6,83%
16	7,53%	12,80%	9,72%	9,72%	9,72%	9,72%	10,93%	9,81%	12,50%	9,50%	9,43%	9,50%	9,43%	9,49%	11,58%
17	4,55%	9,86%	6,54%	6,54%	6,54%	6,54%	7,84%	7,52%	10,53%	6,60%	7,23%	6,60%	7,23%	7,30%	8,12%
18	29,02%	46,66%	35,40%	35,40%	35,40%	35,40%	39,69%	42,14%	44,05%	34,27%	32,05%	34,27%	32,05%	32,28%	32,05%
19	15,15%	27,37%	20,20%	20,20%	20,20%	20,20%	23,02%	20,20%	26,50%	19,81%	19,11%	19,81%	19,11%	19,25%	19,10%
20	8,49%	15,28%	10,30%	10,30%	10,30%	10,30%	12,15%	10,30%	14,38%	9,98%	10,30%	9,98%	10,30%	9,47%	9,38%
21	8,31%	15,64%	10,64%	10,64%	10,64%	10,64%	12,54%	10,64%	15,44%	10,44%	10,35%	10,44%	10,35%	10,45%	10,35%
22	7,85%	10,21%	7,25%	7,25%	7,25%	7,44%	8,16%	7,25%	11,35%	7,14%	7,90%	7,14%	8,00%	7,95%	7,90%
23	5,52%	7,43%	5,16%	5,16%	5,16%	9,03%	5,88%	5,16%	9,79%	5,23%	7,73%	5,23%	9,79%	7,76%	7,73%

Table 7: 12-months Quarterly Rolling Volatility

Intertemporal Jensen's Alpha															
Period	S&P 500	60-40	Static MPT	Dynamic MPT	Static PMPT	Dynamic PMPT	Static RRP	Dynamic RRP	50-40-10	Static MPT Crypto	Dynamic MPT Crypto	Static PMPT Crypto	Dynamic PMPT Crypto	Static RRP Crypto	Dynamic RRP Crypto
1	-0,36%	2,60%	1,13%	1,25%	1,13%	1,25%	1,74%	1,74%	6,67%	2,40%	2,60%	2,42%	2,61%	5,30%	5,30%
2	-0,36%	-10,63%	-5,00%	-13,86%	-5,00%	-13,86%	-7,19%	-7,22%	-9,71%	-4,77%	-9,63%	-4,76%	-9,63%	-4,34%	-4,35%
3	-0,36%	-9,51%	-4,57%	-4,57%	-4,57%	-4,57%	-6,52%	-6,99%	-4,55%	-3,22%	-0,31%	-3,23%	-0,31%	-0,40%	-0,49%
4	-0,36%	3,48%	1,59%	1,59%	1,59%	1,59%	2,38%	2,78%	2,68%	1,35%	0,76%	1,34%	0,76%	0,80%	0,85%
5	-0,36%	12,42%	6,45%	6,45%	6,45%	6,30%	9,02%	9,49%	14,92%	7,34%	9,30%	7,33%	9,21%	9,43%	9,30%
6	-0,36%	-1,92%	-1,10%	-1,10%	-1,10%	-2,36%	-1,43%	-1,68%	0,52%	-0,38%	1,23%	-0,39%	0,52%	1,21%	1,23%
7	-0,36%	-0,04%	-0,20%	-0,20%	-0,20%	0,03%	-0,14%	0,13%	9,81%	3,02%	9,59%	3,03%	9,81%	9,60%	9,59%
8	-0,36%	-0,83%	-0,58%	-0,58%	-0,58%	-0,98%	-0,68%	-0,92%	1,37%	0,09%	1,55%	0,08%	1,37%	1,55%	1,49%
9	-0,36%	1,61%	0,63%	0,63%	0,63%	1,68%	1,03%	2,08%	12,03%	4,13%	11,00%	4,07%	12,03%	11,03%	9,25%
10	-0,36%	4,04%	1,99%	2,09%	1,99%	2,61%	2,88%	4,41%	-7,47%	-1,24%	-10,89%	-1,38%	-7,47%	-10,80%	-5,63%
11	-0,36%	1,39%	0,52%	0,79%	0,52%	1,83%	0,88%	0,52%	1,25%	0,35%	0,21%	0,48%	0,81%	0,40%	0,93%
12	-0,36%	1,94%	0,79%	2,52%	0,79%	2,52%	1,27%	0,79%	0,43%	0,35%	1,26%	0,35%	2,12%	-0,69%	0,44%
13	-0,36%	-7,81%	-4,08%	-9,43%	-4,08%	-9,68%	-5,63%	-4,12%	-19,11%	-6,73%	-11,02%	-7,14%	-9,68%	-15,74%	-15,87%
14	-0,36%	-2,12%	-1,10%	-3,39%	-1,10%	-3,39%	-1,49%	-1,20%	-2,32%	-1,14%	-3,39%	-1,15%	-3,39%	-1,30%	-2,40%
15	-0,36%	-9,16%	-4,38%	-4,38%	-4,38%	-4,38%	-6,26%	-4,54%	-0,49%	-1,96%	-4,72%	-1,94%	-4,72%	3,07%	0,07%
16	-0,36%	-10,25%	-4,95%	-4,95%	-4,95%	-4,95%	-7,05%	-5,14%	-14,81%	-6,04%	-8,74%	-6,03%	-8,74%	-8,86%	-12,90%
17	-0,36%	4,15%	1,90%	1,90%	1,90%	1,90%	2,83%	2,48%	2,41%	1,32%	0,13%	1,38%	0,13%	0,18%	0,92%
18	-0,36%	-44,53%	-22,45%	-22,45%	-22,45%	-22,45%	-31,62%	-35,86%	-42,23%	-21,79%	-20,15%	-21,76%	-20,15%	-20,63%	-20,15%
19	-0,36%	2,62%	1,19%	1,19%	1,19%	1,19%	1,80%	1,19%	4,30%	1,70%	2,83%	1,67%	2,83%	2,80%	2,77%
20	-0,36%	-0,45%	-0,37%	-0,37%	-0,37%	-0,37%	-0,40%	-0,37%	0,84%	0,04%	-0,46%	0,00%	-0,46%	0,86%	0,86%
21	-0,36%	8,92%	4,41%	4,41%	4,41%	4,41%	6,31%	4,41%	14,93%	6,46%	10,86%	6,46%	10,86%	10,95%	10,86%
22	-0,36%	16,90%	8,93%	8,93%	8,93%	7,89%	12,37%	8,93%	20,15%	9,96%	12,66%	10,08%	12,02%	12,83%	12,66%
23	-0,36%	-6,24%	-3,04%	-3,04%	-3,04%	-8,08%	-4,30%	-3,04%	-1,63%	-1,85%	1,06%	-1,77%	-1,63%	1,01%	1,06%

Table 8: Intertemporal Jensen's Alpha

Intertemporal Beta														
Period	S&P 500	60-40	Static MPT	Dynamic MPT	Static PMPT	Dynamic PMPT	Static RRP	Dynamic RRP	50-40-10	Static MPT Crypto	Dynamic MPT Crypto	Static PMPT Crypto	Dynamic PMPT Crypto	Static RRP Crypto	Dynamic RRP Crypto
1	1,0000	0,6000	0,8000	0,8000	0,8000	0,8000	0,7169	0,7169	0,5878	0,8000	0,8000	0,7963	0,7963	0,7834	0,7834
2	1,0000	0,6000	0,8000	0,5000	0,8000	0,5000	0,7169	0,7160	0,5939	0,8000	0,5939	0,7982	0,5939	0,7895	0,7889
3	1,0000	0,6000	0,8000	0,8000	0,8000	0,8000	0,7169	0,7008	0,6383	0,8000	0,8383	0,8115	0,8383	0,8339	0,8295
4	1,0000	0,6000	0,8000	0,8000	0,8000	0,8000	0,7169	0,6720	0,6135	0,8000	0,8135	0,8041	0,8135	0,8092	0,8030
5	1,0000	0,6000	0,8000	0,8000	0,8000	0,8000	0,7169	0,7009	0,6055	0,8000	0,8055	0,8017	0,8055	0,8011	0,8055
6	1,0000	0,6000	0,8000	0,8000	0,8000	0,5000	0,7169	0,6836	0,6061	0,8000	0,8061	0,8018	0,6061	0,8017	0,8061
7	1,0000	0,6000	0,8000	0,8000	0,8000	0,5000	0,7169	0,6208	0,5874	0,8000	0,7874	0,7962	0,5874	0,7830	0,7874
8	1,0000	0,6000	0,8000	0,8000	0,8000	0,5000	0,7169	0,5391	0,6125	0,8000	0,8125	0,8037	0,6125	0,8081	0,8125
9	1,0000	0,6000	0,8000	0,8000	0,8000	0,5000	0,7169	0,5051	0,6329	0,8000	0,8329	0,8099	0,6329	0,8285	0,8269
10	1,0000	0,6000	0,8000	0,8000	0,8000	0,8000	0,7169	0,5000	0,5079	0,8000	0,7079	0,7724	0,5079	0,7035	0,7295
11	1,0000	0,6000	0,8000	0,7519	0,8000	0,5000	0,7169	0,8000	0,5153	0,8000	0,7153	0,7746	0,5153	0,7109	0,6177
12	1,0000	0,6000	0,8000	0,5000	0,8000	0,5000	0,7169	0,8000	0,6015	0,8000	0,5638	0,8004	0,5202	0,7971	0,6592
13	1,0000	0,6000	0,8000	0,5149	0,8000	0,5000	0,7169	0,8000	0,5360	0,8000	0,5061	0,7808	0,5000	0,7317	0,5864
14	1,0000	0,6000	0,8000	0,5000	0,8000	0,5000	0,7169	0,7810	0,6027	0,8000	0,5000	0,8008	0,5000	0,7984	0,5992
15	1,0000	0,6000	0,8000	0,8000	0,8000	0,8000	0,7169	0,7927	0,5864	0,8000	0,8000	0,7959	0,8000	0,7821	0,6166
16	1,0000	0,6000	0,8000	0,8000	0,8000	0,8000	0,7169	0,7940	0,6143	0,8000	0,8143	0,8043	0,8143	0,8099	0,6703
17	1,0000	0,6000	0,8000	0,8000	0,8000	0,8000	0,7169	0,7364	0,5767	0,8000	0,7767	0,7930	0,7767	0,7724	0,7186
18	1,0000	0,6000	0,8000	0,8000	0,8000	0,8000	0,7169	0,6728	0,6040	0,8000	0,8040	0,8012	0,8040	0,7997	0,8040
19	1,0000	0,6000	0,8000	0,8000	0,8000	0,8000	0,7169	0,8000	0,6052	0,8000	0,8052	0,8016	0,8052	0,8008	0,8052
20	1,0000	0,6000	0,8000	0,8000	0,8000	0,8000	0,7169	0,8000	0,6186	0,8000	0,8000	0,8056	0,8000	0,8142	0,8186
21	1,0000	0,6000	0,8000	0,8000	0,8000	0,8000	0,7169	0,8000	0,6010	0,8000	0,8010	0,8003	0,8010	0,7967	0,8011
22	1,0000	0,6000	0,8000	0,8000	0,8000	0,8000	0,7169	0,8000	0,5495	0,8000	0,7495	0,7849	0,7495	0,7452	0,7495
23	1,0000	0,6000	0,8000	0,8000	0,8000	0,5000	0,7169	0,8000	0,5551	0,8000	0,7551	0,7865	0,5551	0,7507	0,7551

Table 9: Intertemporal Beta

Intertemporal Treynor Ratio															
Period	S&P 500	60-40	Static MPT	Dynamic MPT	Static PMPT	Dynamic PMPT	Static RRP	Dynamic RRP	50-40-10	Static MPT Crypto	Dynamic MPT Crypto	Static PMPT Crypto	Dynamic PMPT Crypto	Static RRP Crypto	Dynamic RRP Crypto
1	3,82%	8,52%	5,59%	5,74%	5,59%	5,74%	6,61%	6,61%	15,53%	7,19%	7,43%	7,22%	7,46%	10,94%	10,94%
2	2,51%	-14,85%	-3,37%	-24,85%	-3,37%	-24,85%	-7,16%	-7,22%	-13,47%	-3,09%	-13,34%	-3,10%	-13,34%	-2,62%	-2,65%
3	1,00%	-14,50%	-4,36%	-4,36%	-4,36%	-4,36%	-7,74%	-8,63%	-5,77%	-2,67%	0,98%	-2,63%	0,98%	0,88%	0,76%
4	2,19%	8,35%	4,54%	4,54%	4,54%	4,54%	5,87%	6,68%	6,92%	4,24%	3,48%	4,22%	3,48%	3,54%	3,60%
5	3,72%	24,77%	12,14%	12,14%	12,14%	11,95%	16,65%	17,61%	28,71%	13,25%	15,62%	13,22%	15,51%	15,85%	15,62%
6	3,78%	0,94%	2,76%	2,76%	2,76%	-0,58%	2,14%	1,69%	5,00%	3,67%	5,66%	3,66%	5,00%	5,65%	5,66%
7	1,61%	1,89%	1,72%	1,72%	1,72%	2,03%	1,78%	2,17%	18,66%	5,75%	14,15%	5,78%	18,66%	14,23%	14,15%
8	4,26%	3,23%	3,90%	3,90%	3,90%	2,66%	3,67%	2,91%	6,85%	4,74%	6,53%	4,71%	6,85%	6,54%	6,45%
9	5,45%	8,49%	6,59%	6,59%	6,59%	9,17%	7,25%	9,93%	24,82%	10,97%	19,02%	10,83%	24,82%	19,12%	17,00%
10	-5,19%	1,90%	-2,34%	-2,22%	-2,34%	-1,57%	-0,82%	3,99%	-19,54%	-6,39%	-20,21%	-6,62%	-19,54%	-20,19%	-12,56%
11	4,59%	7,27%	5,59%	6,00%	5,59%	8,61%	6,18%	5,59%	7,38%	5,39%	5,25%	5,56%	6,51%	5,51%	6,46%
12	5,85%	9,45%	7,20%	11,26%	7,20%	11,25%	7,98%	7,20%	6,92%	6,64%	8,44%	6,64%	10,28%	5,34%	6,88%
13	-21,87%	-34,52%	-26,61%	-39,82%	-26,61%	-40,87%	-29,36%	-26,66%	-57,15%	-29,92%	-43,29%	-30,65%	-40,87%	-43,02%	-48,57%
14	11,17%	8,00%	10,16%	4,76%	10,16%	4,76%	9,46%	9,99%	7,68%	10,10%	4,76%	10,09%	4,76%	9,90%	7,53%
15	2,78%	-12,13%	-2,33%	-2,33%	-2,33%	-2,33%	-5,59%	-2,59%	2,31%	0,69%	-2,76%	0,69%	-2,76%	7,06%	3,26%
16	-1,21%	-17,94%	-7,03%	-7,03%	-7,03%	-7,03%	-10,68%	-7,32%	-24,96%	-8,40%	-11,58%	-8,36%	-11,58%	-11,79%	-20,10%
17	8,45%	15,73%	11,18%	11,18%	11,18%	11,18%	12,77%	12,18%	12,99%	10,46%	8,98%	10,55%	8,98%	9,05%	10,10%
18	-28,51%	-102,37%	-56,21%	-56,21%	-56,21%	-56,21%	-72,26%	-81,45%	-98,07%	-55,39%	-53,21%	-55,30%	-53,21%	-53,95%	-53,21%
19	17,44%	22,17%	19,28%	19,28%	19,28%	19,28%	20,31%	19,28%	24,90%	19,92%	21,32%	19,88%	21,32%	21,30%	21,24%
20	6,82%	6,42%	6,71%	6,71%	6,71%	6,71%	6,62%	6,71%	8,54%	7,23%	6,60%	7,18%	6,60%	8,23%	8,23%
21	7,51%	22,74%	13,38%	13,38%	13,38%	13,38%	16,67%	13,38%	32,71%	15,95%	21,42%	15,94%	21,42%	21,61%	21,42%
22	7,23%	35,76%	18,75%	18,75%	18,75%	17,45%	24,84%	18,75%	44,25%	20,04%	24,47%	20,43%	23,63%	24,81%	24,47%
23	5,79%	-4,25%	2,34%	2,34%	2,34%	-10,02%	0,15%	2,34%	3,21%	3,84%	7,56%	3,90%	3,21%	7,49%	7,56%

Table 10: Intertemporal Treynor Ratio

Intertemporal Sharpe Ratio															
Period	S&P 500	60-40	Static MPT	Dynamic MPT	Static PMPT	Dynamic PMPT	Static RRP	Dynamic RRP	50-40-10	Static MPT Crypto	Dynamic MPT Crypto	Static PMPT Crypto	Dynamic PMPT Crypto	Static RRP Crypto	Dynamic RRP Crypto
1	55,97%	61,80%	63,59%	65,39%	63,59%	65,39%	63,72%	63,72%	110,65%	83,43%	86,56%	83,43%	86,56%	123,82%	123,83%
2	31,38%	-74,56%	-23,74%	-96,41%	-23,74%	-96,41%	-45,74%	-46,03%	-68,34%	-21,85%	-67,63%	-21,85%	-67,63%	-17,98%	-18,15%
3	19,79%	-72,71%	-35,62%	-35,62%	-35,62%	-35,62%	-52,73%	-56,51%	-30,55%	-20,87%	14,25%	-20,87%	14,25%	13,06%	11,81%
4	51,64%	62,43%	70,92%	70,92%	70,92%	70,92%	68,14%	65,57%	53,41%	67,12%	54,49%	67,12%	54,49%	54,63%	54,62%
5	93,89%	198,18%	188,47%	188,47%	188,47%	186,17%	197,72%	198,30%	238,46%	210,23%	254,74%	210,23%	254,36%	254,85%	254,74%
6	126,79%	14,62%	58,57%	58,57%	58,57%	0,90%	36,93%	27,65%	48,01%	75,04%	90,26%	75,04%	48,01%	89,20%	90,26%
7	52,46%	24,83%	38,56%	38,56%	38,56%	19,85%	32,26%	29,20%	161,23%	108,63%	201,38%	108,63%	161,23%	200,52%	201,38%
8	132,69%	38,60%	79,75%	79,75%	79,75%	24,61%	60,24%	29,71%	57,29%	89,01%	82,11%	89,01%	57,29%	81,56%	81,29%
9	203,41%	112,28%	161,32%	161,32%	161,32%	89,12%	138,81%	95,62%	260,50%	252,95%	308,85%	252,95%	260,50%	307,95%	308,87%
10	-47,36%	18,85%	-17,30%	-16,19%	-17,30%	-10,36%	-2,77%	29,61%	-93,04%	-52,86%	-130,33%	-52,86%	-93,04%	-129,56%	-91,47%
11	85,19%	71,76%	82,22%	81,16%	82,22%	65,57%	78,46%	82,22%	49,71%	76,10%	54,70%	76,10%	44,64%	56,60%	58,81%
12	173,52%	147,47%	174,88%	131,22%	174,88%	131,08%	165,58%	174,88%	86,27%	154,75%	108,25%	154,75%	125,99%	95,89%	105,66%
13	-179,81%	-235,05%	-206,78%	-245,50%	-206,78%	-246,62%	-218,78%	-207,19%	-365,38%	-240,81%	-268,34%	-240,81%	-246,62%	-325,72%	-334,27%
14	168,52%	63,61%	116,58%	32,26%	116,58%	32,26%	94,18%	111,24%	62,36%	118,10%	32,26%	118,10%	32,26%	115,22%	60,97%
15	55,14%	-99,44%	-25,42%	-25,42%	-25,42%	-25,42%	-58,09%	-28,40%	24,35%	16,17%	-31,45%	16,17%	-31,45%	98,18%	34,64%
16	-11,32%	-81,30%	-54,20%	-54,20%	-54,20%	-54,20%	-66,79%	-55,60%	-119,85%	-66,96%	-96,18%	-66,96%	-96,18%	-96,81%	-113,29%
17	193,50%	99,28%	142,35%	142,35%	142,35%	142,35%	121,30%	123,99%	74,54%	132,26%	101,37%	132,26%	101,37%	100,64%	93,72%
18	-97,01%	-130,87%	-126,00%	-126,00%	-126,00%	-126,00%	-129,63%	-129,20%	-133,67%	-128,26%	-132,35%	-128,26%	-132,35%	-132,55%	-132,35%
19	117,48%	49,92%	78,16%	78,16%	78,16%	78,16%	64,81%	78,16%	58,22%	82,26%	91,71%	82,26%	91,71%	90,47%	91,40%
20	84,49%	27,56%	55,58%	55,58%	55,58%	55,58%	42,00%	55,58%	39,21%	61,53%	54,74%	61,53%	54,74%	74,58%	75,58%
21	94,72%	89,56%	103,95%	103,95%	103,95%	103,95%	98,19%	103,95%	129,63%	125,60%	169,22%	125,60%	169,22%	168,24%	169,22%
22	96,71%	213,56%	211,94%	211,94%	211,94%	192,37%	222,56%	211,94%	217,42%	229,68%	236,72%	229,68%	225,76%	236,98%	236,72%
23	111,40%	-29,54%	43,24%	43,24%	43,24%	-51,53%	7,89%	43,24%	21,86%	65,50%	78,42%	65,50%	21,86%	77,09%	78,42%

Table 11: Intertemporal Sharpe Ratio

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Summary

Introduction

Last decades economic turbulences, such as the 2008 subprime mortgages crisis and 2020 COVID-19 pandemic, have served to expose the monetary and financial system's vulnerability.

As a reaction to the general market instability, individuals and fund managers have began to include non-conventional investments in their portfolios, in order to mitigate the negative effects of classic asset classes downturns.

At the same time, new technologies have been developed during last decades and cryptocurrencies are one of the most disruptive of them all. Cryptocurrencies can offer great investment opportunities in a well diversified portfolio, as the sector continues to grow despite the high volatility.

However, whether cryptos can qualify or not as an asset class in their own right still remains a big question; for this reason, the first research question of this thesis has been to ascertain if there is any room for them to be considered as an asset class.

Once having accepted cryptos as an asset class useful to diversify and boost portfolio performances, it is important to define the correct investment strategy to adopt.

Indeed, the second research question has been focused on the analysis of main dynamic portfolio optimization techniques, in order to ascertain what could be the optimal one when including the CRIX index (assumed to be a tradable ETF) in the allocation.

Traditional asset allocation methodologies has been taken into consideration, as well as more sophisticated techniques. We considered portfolios coming both from Modern and Post-Modern Portfolio Theory, based respectively on the maximization of the Sharpe and Sortino Ratio, and Risk Parity portfolios, in their relaxed variants in order to allow the compliance to constraints.

Portfolios have been constructed both including only equity and bonds and including cryptocurrencies too, in order to determine benefits coming from them. Moreover, all portfolios have been allocated both in a static way and considering quarterly rebalancing.

At this stage, they have been evaluated from an ex-post point of view considering main traditional and risk-adjusted statistics.

Cryptocurrencies as an Asset Class

What are Cryptocurrencies?

Virtual money has increasingly gained popularity over the last decades and today's technologies made an unknown computer programmer under the pseudonymous of "Satoshi Nakamoto" be able to create in 2008 the first popular cryptocurrency, the Bitcoin.

Since then, hundreds of cryptocurrencies have been developed and the number of people using them has significantly grown.

In its simplest form, a cryptocurrency can be considered as a digital asset built to function as a medium of exchange based on cryptographic technology, in order to ensure the transactional flow as well as to control the creation of additional monetary units [8].

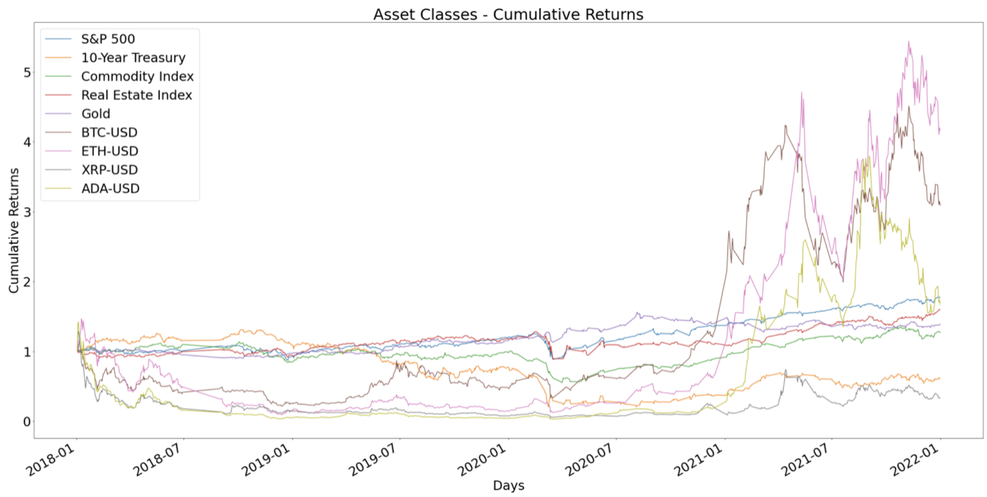
The best way to understand the crypto environment is by starting talking about Bitcoins, as the developments that allowed Bitcoin to emerge are the foundation of all crypto-based projects.

Satoshi Nakamoto's vision on Bitcoin was described in his/her white paper published on October 31st, 2008 named *"Bitcoin: A peer-to-peer Electronic Cash System"*, where the author described how individuals could exchange money and other items of value without any financial intermediary in the middle [7].

Shortly after the publication of this white paper, Nakamoto released Bitcoin's software and on January 3rd, 2009 the first token was mined.

Crypto as an Investment Opportunity

Cryptocurrencies have a clear potential to be an important asset class in the future, but some considerations have to be made when deciding whether or not to invest in them.



Asset Classes Cumulative Returns

Figure "Asset Classes Cumulative Returns" and table "Asset Classes Correlation Matrix" show respectively the cumulative returns and the correlations between main asset classes and main cryptocurrencies.

Asset Classes Correlation Matrix									
	S&P 500	10-Year Treasury	Commodity	Real Estate	Gold	BTC	ETH	XRP	ADA
S&P 500	1								
10-Year Treasury	39.78%	1							
Commodity	40.35%	35.31%	1						
Real Estate	73.85%	27.81%	33.40%	1					
Gold	-0.40%	-33.00%	3.38%	8.69%	1				
BTC	22.84%	6.11%	6.55%	12.01%	21.99%	1			
ETH	24.22%	5.45%	17.18%	18.31%	27.08%	76.07%	1		
XRP	18.34%	5.07%	8.79%	13.18%	7.73%	42.84%	52.47%	1	
ADA	25.56%	7.39%	15.21%	18.12%	10.66%	64.99%	67.67%	56.28%	1

Asset Classes Correlation Matrix

From these data, we can highlight some elements:

- High Volatility: cryptoassets are still relatively in an immature phase of their evolution and their prices have always been sensible to regulations, technical hurdles and skepticism.

  This aspect has led cryptocurrencies to be historically characterized by high volatility and it is likely to continue in the future too. If we look at the figure above, it is evident that cryptos had extreme standard deviation values over the last four years, especially when we compare them to traditional asset classes.

- Rising Trading Volumes: an interesting element to underline is the increasing trading volume of cryptos, which reflects the great attention of investors in this new asset class and the good liquidity of the currencies.

  However, it is important to note that surges in trading volumes suggest either strongly bullish or strongly bearish sentiment and often, especially during 2021, these fluctuations have been accompanied by price decreases in the main coins.

- Low Correlation with Traditional Assets: another key characteristic of cryptocurrencies is their high positive correlation between each other but a low correlation with traditional assets and financial markets, implying that they may offer diversification benefits to investors [40].

  In fact, by looking at the table above, we can see low-medium correlations only to the S&P500 and very modest correlations between them and other asset classes.

- High Potential Returns: although the difficulty of forecasting future prices of cryptoassets is a major downside for investors, crypto bulls argue that historical returns will persists in the next years, as crypto has just began to penetrate the mainstream and many financial institutions has begun to develop blockchain-linked applications for their services.

  On the other side, crypto bears continue to argue that most cryptocurrencies are highly overvalued and that they are destined to collapse in the near future.

  Anyway, the empirical evidence shows that cryptocurrencies have survived several moments of panic even when they were seen just as bubbles, and each year they set lows higher than the preceding year. Risks will still be present, but the potential of crypto is undeniable.

Empirical Analysis

The assets employed in the asset allocation processes have been:

- Equity: S&P500
- Bond: 10-Year Treasury Bond
- Crypto: CRIX Index

Some constraint and assumptions have been made:

- The CRIX index has been assumed to be a tradable ETF
- The annual risk-free in the performance evaluation statistics has been assumed to be equal to 1.60% annually and 0.398% quarterly
- No short selling constraint
- Equity weight at least equal to 50% of the overall asset allocation
- Bond weight at least equal to 20% of the overall asset allocation
- Crypto weight at maximum equal to 10% of the overall asset allocation

For all allocation methodologies, four kind of allocations have been made: static, with and without cryptos, and dynamic, with and without cryptos. Moreover, the allocation procedures have been set differently between the static and the dynamic portfolios:

- Each portfolio is calibrated to look back at the prior 252 trading days, with allocations starting from October 1st 2015
- The rolling horizon for the out-of-sample asset allocations is set at the next 1449 trading days for static portfolios, as weights do not change, while dynamic ones are reallocated each 63 days, so the total amount of quarters is 23

The processes has been set as follows in order to ascertain if there has been a benefit in rebalancing and a positive contribution from cryptos. The portfolios were then compared to the S&P500, 60-40 and 50-40-10 portfolios as benchmarks.

Sharpe Ratio Optimization

Equation 49 refers to the static asset allocation approach:

$$
\begin{aligned}
\max_{w} \quad & \frac{\mu' w - r_F}{\sqrt{w'\Sigma w}} \\
\text{s.t.} \quad & 1'w = 1 \\
& 0.8 \geq w_E \geq 0.5 \\
& 0.5 \geq w_B \geq 0.2 \\
& 0 \leq w_C \leq 0.1
\end{aligned}
\tag{49}
$$

while equation 50 refers to the dynamic strategy:

$$
\begin{aligned}
\max_{w} \quad & \sum_{t=1}^{T} \frac{\mu_t' w_t - r_F}{\sqrt{w_t'\Sigma_t w_t}} \\
\text{s.t.} \quad & 1'w_t = 1 \\
& 0.8 \geq w_{E,t} \geq 0.5 \\
& 0.5 \geq w_{B,t} \geq 0.2 \\
& 0 \leq w_{C,t} \leq 0.1
\end{aligned}
\tag{50}
$$

with T being equal to 23 quarters. The procedure was conducted as follows:

1. Looking back at the prior 252 trading days, the in-sample mean vector and the in-sample covariance matrix have been calculated
2. At this stage, a Montecarlo simulation of 1000 different weight combinations have been conducted, always in compliance with the constraints set in the previous section
3. For each random portfolio, I calculated its in-sample Sharpe Ratio
4. Subsequently, I locked the maximum Sharpe Ratio portfolio and I stored its weights vector
5. Now we have two distinct situations:

- Static portfolios → the weights selection procedure ends here and these weights are now employed for the out-of-sample allocation, that will last for the subsequent 1449 trading days (or 23 quarters)
- Dynamic portfolios → these weights are now employed for the out-of-sample allocation, that will last for the subsequent 63 trading days; afterwards, the whole procedure will be iterated for other 22 times

Sortino Ratio Optimization

In this case, it was the Sortino Ratio the one to be maximized, so the Σ^- element in the following two equation refers to the semicovariance matrix.

Equation 51 refers to the static asset allocation approach:

$$\begin{aligned} \max_{w} \quad & \frac{\mu' w - r_F}{\sqrt{w' \Sigma^{-} w}} \\ \text{s.t.} \quad & 1' w = 1 \\ & 0.8 \geq w_E \geq 0.5 \\ & 0.5 \geq w_B \geq 0.2 \\ & 0 \leq w_C \leq 0.1 \end{aligned} \tag{51}$$

while equation 52 refers to the dynamic strategy:

$$\begin{aligned} \max_{w} \quad & \sum_{t=1}^{T} \frac{\mu_t' w_t - r_F}{\sqrt{w_t' \Sigma_t^{-} w_t}} \\ \text{s.t.} \quad & 1' w_t = 1 \\ & 0.8 \geq w_{E,t} \geq 0.5 \\ & 0.5 \geq w_{B,t} \geq 0.2 \\ & 0 \leq w_{C,t} \leq 0.1 \end{aligned} \tag{52}$$

with T being equal to 23 quarters. The procedure was conducted as follows:

1. Looking back at the prior 252 trading days, the in-sample mean vector and the in-sample semicovariance matrix have been calculated
2. At this stage, a Montecarlo simulation of 1000 different weight combinations have been conducted, always in compliance with the constraints set in section 3.3
3. For each random portfolio, I calculated its in-sample Sortino Ratio
4. Subsequently, I locked the maximum Sortino Ratio portfolio and I stored its weights vector
5. Now we have two distinct situations:
   - Static portfolios $\rightarrow$ the weights selection procedure ends here and these weights are now employed for the out-of-sample allocation, that will last for the subsequent 1449 trading days (or 23 quarters)
   - Dynamic portfolios $\rightarrow$ these weights are now employed for the out-of-sample allocation, that will last for the subsequent 63 trading days; afterwards, the whole procedure will be iterated for other 22 times

Relaxed Risk Parity Optimization

The risk measure adopted in this approach is the volatility and the strategy used to construct the portfolios resulted in very different asset allocations if compared to MPT and PMPT. This aspect implies heavier investments in cryptos and a smoother rebalancing during the quarters.

Equation 53 refers to the static asset allocation approach:

$$\begin{aligned} \min_{w} \quad & \sum_{i=1}^{N} \left| w_i \frac{[\Sigma w]_i}{\sqrt{w'\Sigma w}} \right| - \frac{1}{N} \\ \text{s.t.} \quad & 1'w = 1 \\ & 0.8 \geq w_E \geq 0.5 \\ & 0.5 \geq w_B \geq 0.2 \\ & 0 \leq w_C \leq 0.1 \end{aligned} \tag{53}$$

while equation 54 refers to the dynamic strategy:

$$\begin{aligned}
\min_{w} \quad & \sum_{t=1}^{T}\left[\sum_{i=1}^{N}\left|w_{i,t}\frac{[\Sigma_t w_t]_i}{\sqrt{w_t'\Sigma_t w_t}}\right|-\frac{1}{N}\right] \\
\text{s.t.} \quad & 1'w_t = 1 \\
& 0.8 \geq w_{E,t} \geq 0.5 \\
& 0.5 \geq w_{B,t} \geq 0.2 \\
& 0 \leq w_{C,t} \leq 0.1
\end{aligned} \tag{54}$$

with T being equal to 23 quarters. The procedure was conducted as follows:

1. Looking back at the prior 252 trading days, the in-sample covariance matrix have been calculated
2. At this stage, a Montecarlo simulation of 1000 different weight combinations have been conducted, always in compliance with the constraints set in section 3.3
3. For each random portfolio, I calculated its in-sample sum of deviations from parity; in order to do so, the following steps have been carried out:
   (a) Recalling equation 53, through the first part of the formula, $w_i\frac{[\Sigma w]_\mathrm{i}}{\sqrt{w'\Sigma w}}$, I found the marginal risk contribution of asset i, while the second part, $\frac{1}{N}$, refers to the target $\mathrm{MRC_i}$
   (b) Given that the sum of all marginal risk contributions is always 1, as they are related to the total portfolio volatility, in a risk parity portfolio the ideal result of $w_i\frac{[\Sigma w]_\mathrm{i}}{\sqrt{w'\Sigma w}}$ should be exactly equal to $\frac{1}{N}$; so, by taking the difference between these two elements, I can find asset i's deviation from parity
   (c) By repeating the above process for each asset i and by taking the sum of all MRCs, I can find the total deviation from parity of the whole portfolio

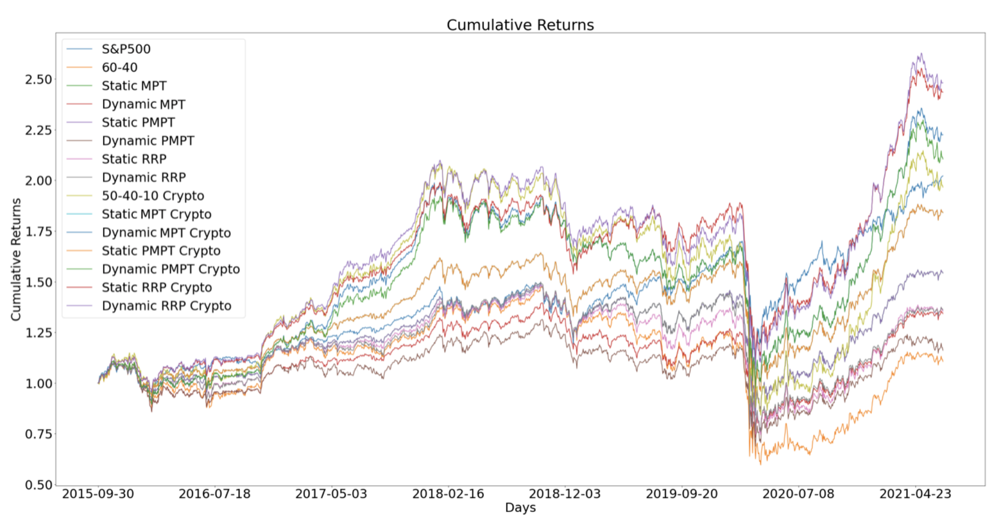
(d) Now, in order to obtain a near-equal risk allocation, I have to minimize the sum of these deviations by changing portfolio weights (in practical terms, in the next step I will select the portfolio that gives me the smallest total deviation from parity; in this way, if the portfolio had not had any constraint, this result would have been equal to 0, while in the case of relaxed risk parity portfolios this result is near to 0)

4. Subsequently, I locked the minimum deviation from parity portfolio and I stored its weights vector

5. Now we have two distinct situations:

   - Static portfolios → the weights selection procedure ends here and these weights are now employed for the out-of-sample allocation, that will last for the subsequent 1449 trading days (or 23 quarters)
   - Dynamic portfolios → these weights are now employed for the out-of-sample allocation, that will last for the subsequent 63 trading days; afterwards, the whole procedure will be iterated for other 22 times

Comparison



Cumulative Returns (out-of-sample)

From the backtesting performances, the only portfolios having outperformed the market belong to the crypto category.

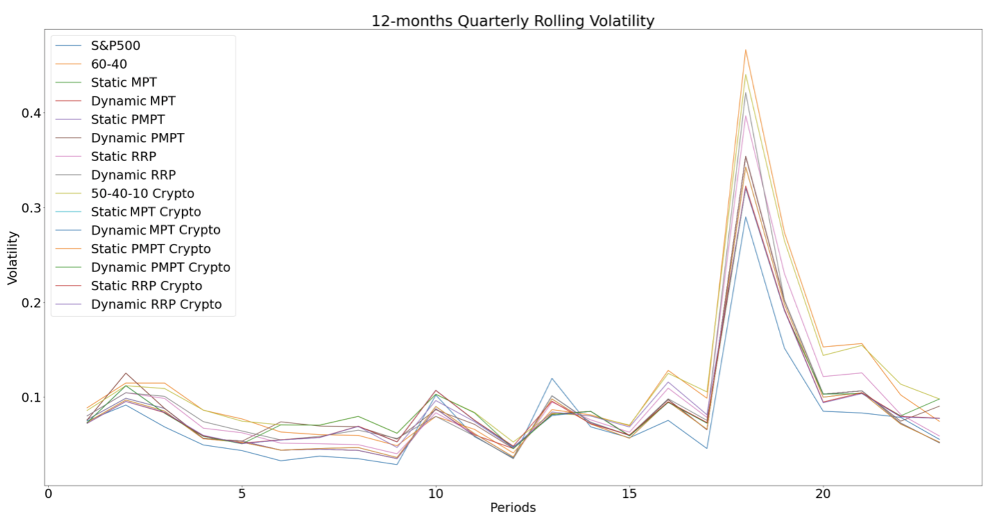
Only when comparing the non-crypto portfolios to the 60-40 benchmark they carry higher returns, but they are away in terms of performances from their respective allocations with crypto by a margin.

So, we can undoubtedly affirm that, despite the maximum 10% in crypto allocation constraint, cryptos had a beneficial impact on all portfolios.

Moreover, the only instances of crypto portfolios failing to exceed the cumulative return of the S&P500 involve the 50-40-10, the static MPT and the static PMPT. Leaving aside the 50-40-10, which was included as a benchmark, in the case of the other two it must be said that the respective dynamic portfolios beat the market, implying that the quarterly rebalancing has been beneficial to them.

From figure "Cumulative Returns (out-of-sample)" it is possible to observe that the best performing portfolio was the dynamic crypto RRP, which obtained the best values both in terms of cumulative performance and in risk-adjusted terms.

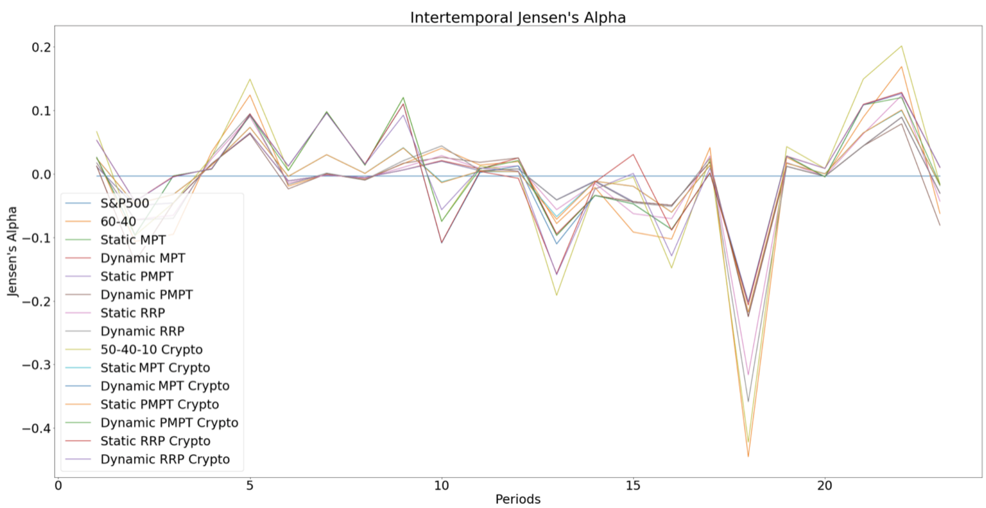
Apart from the entire-out-of-sample statistics, it is important to highlight some key intertemporal measures too, expressed in quarterly values.



12-months Quarterly Rolling Volatility (out-of-sample)

From figure "12-months Quarterly Rolling Volatility (out-of-sample)" we can see that, with few exceptions, the most volatile portfolios have been the benchmarks 60-40 and 50-40-10. The graph also shows high rolling volatility values during first quarters of 2020, due to the huge market crash and the consequential irrationality of market participants; this was particularly true in the case of crypto portfolios, as cryptoassets were seen as shelters during COVID-19 market crisis and this resulted in extreme gains by some of them.

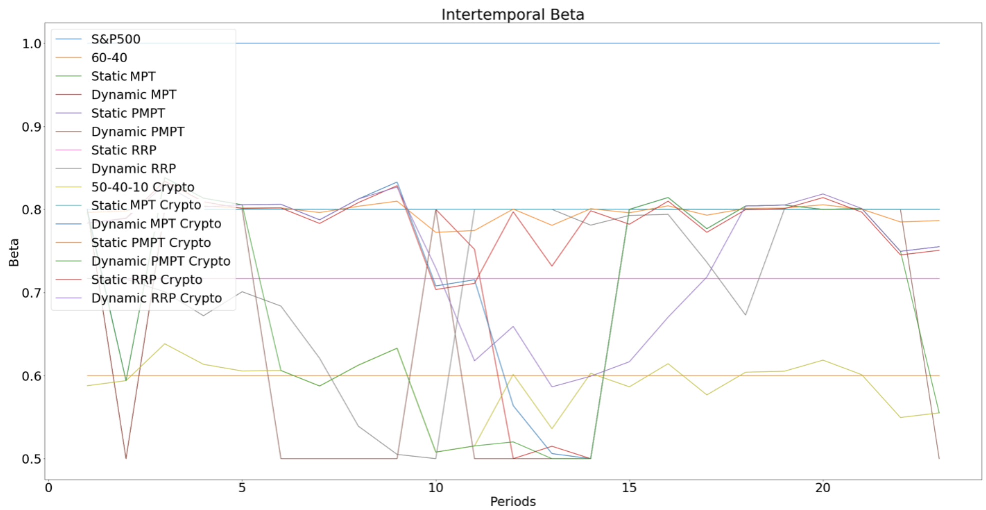
Another important element to outline is the high inverse correlation between Jensen's alpha and rolling volatility and their differences sharpen especially during market downturns, as we can see from figure "Intertemporal Jensen's Alpha (out-of-sample)".



Intertemporal Jensen's Alpha (out-of-sample)

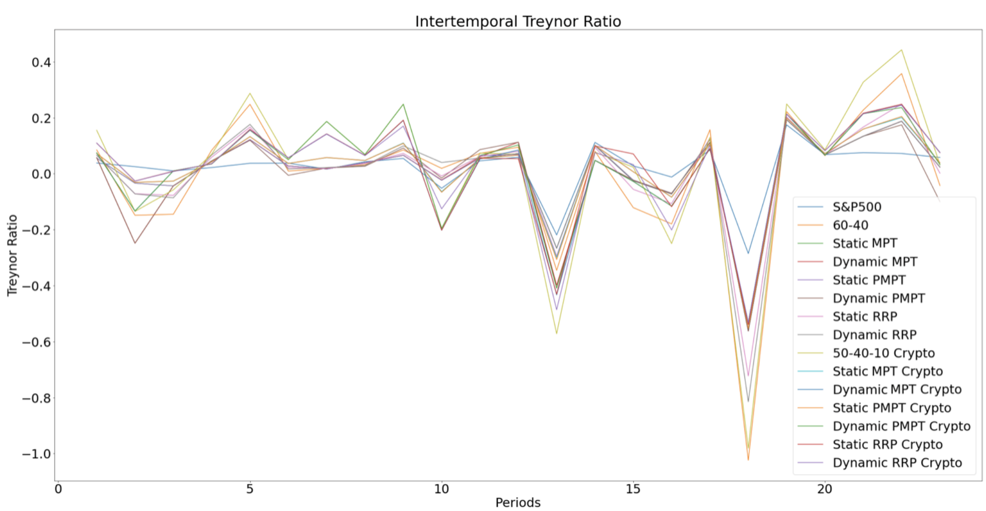
This aspect can be also explained through the beta, as during market crashes portfolios had a higher allocation in equity (as we can see from figure "Intertemporal Beta (out-of-sample)", where an higher beta implies higher S&P500 contribution in portfolios) that reduced the alphas (recalling equation 21).

The same logic can be applied to the Treynor Ratio, as lower exptected returns and higher betas during crises had a huge downward effect on the indicator, while the Sharpe Ratio was affected by the huge increase in volatility.



Intertemporal Beta (out-of-sample)

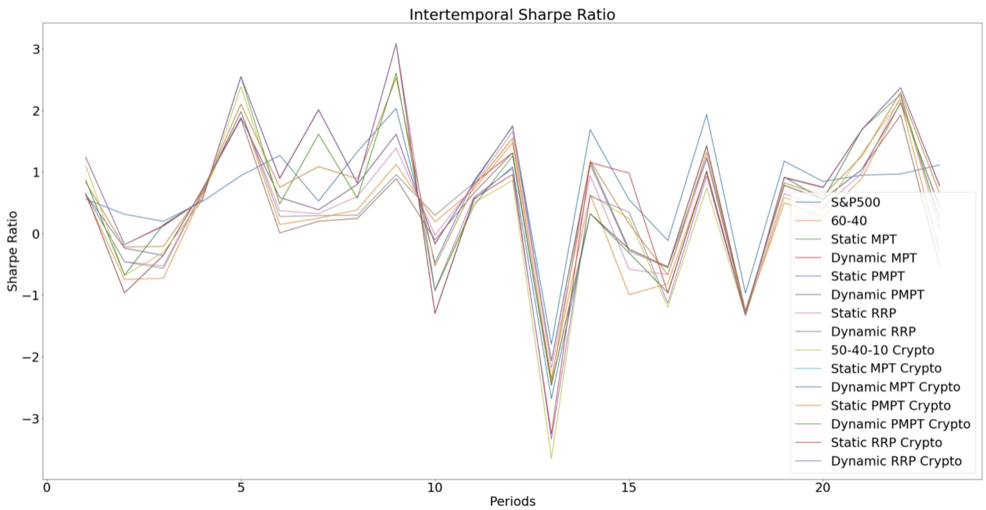
Despite the dynamic crypto RRP has not been the best performing portfolio during intertemporal performances, as it did not show any highest Sharpe or Treynor Ratio value over the quarters, it has been one of the most consistent and its performances have always been superior to most portfolios.



Intertemporal Treynor Ratio (out-of-sample)

This is not surprising because Risk Parity portfolios have a conservative nature that do not let them have extreme gains; anyway, this feature enables them avoid excessive drawdowns and be more stable than other methodologies.

The clearest conclusion is that a long-term vision in portfolio allocation, as seen in Risk Parity models, can led to superior performances and in the next section we will outline why a dynamic Relaxed Risk Parity approach could be one of the best portfolio choice for cryptos.



Intertemporal Sharpe Ratio (out-of-sample)

Optimal Asset Allocation

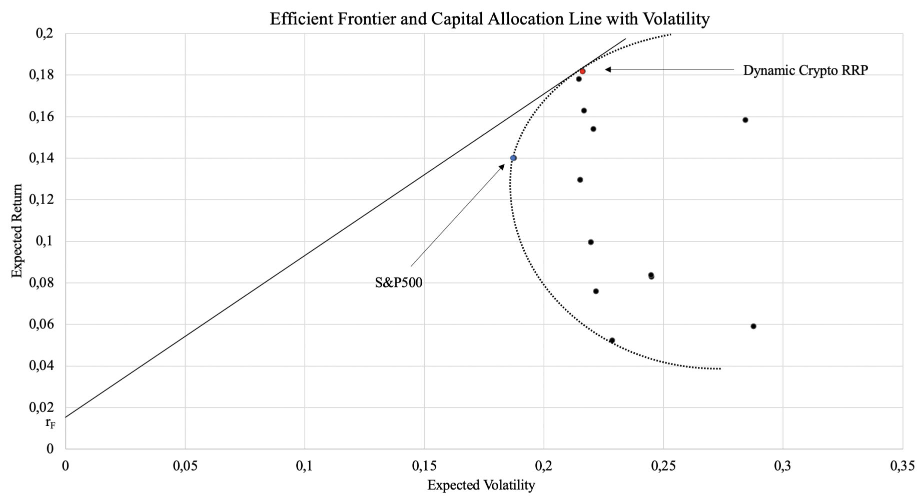
Through figures relative to the Efficient Frontier and CML below, we can visualize the Capital Allocation Lines with respect to the volatility and downside risk and the graphs clarify why the dynamic Relaxed Risk Parity with crypto had the best risk-return combination.

In fact, the portfolio has the highest annualized Sharpe Ratio, equal to 76.73%, and the highest annualized Sortino Ratio, equal to 112.52%, making both its Capital Allocation Lines equal to the Sharpe and Sortino Ratio slope.

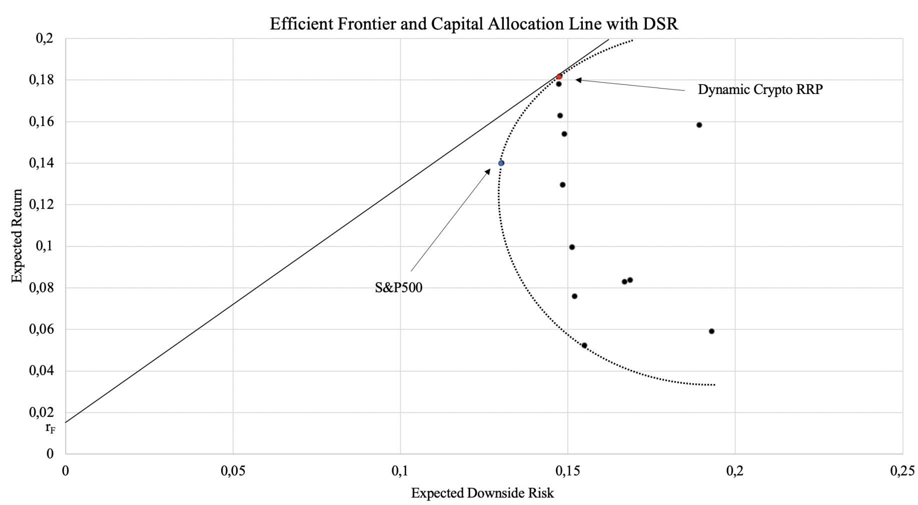
At this stage, we can outline the main winning elements of the strategy.

The risk parity approach was developed in order to avoid the uncertainty linked to expected returns. In fact, while traditional techniques rely on too strong risk/return forecasting assumptions, risk models are only based on volatility and correlation, which are often easier to predict and may led to more robust out-of-sample performances.

This characteristic helps the portfolio prevent disproportionate concentrations in few assets, thanks to an enhanced forced diversification; indeed, the dynamic RRP with crypto portfolio was the only one to include a portion of the investment in cryptos that lasted for the whole out-of-sample period and this was reflected in quarterly expected returns often superior to other portfolios.



Efficient Frontier and Capital Allocation Line with Volatility



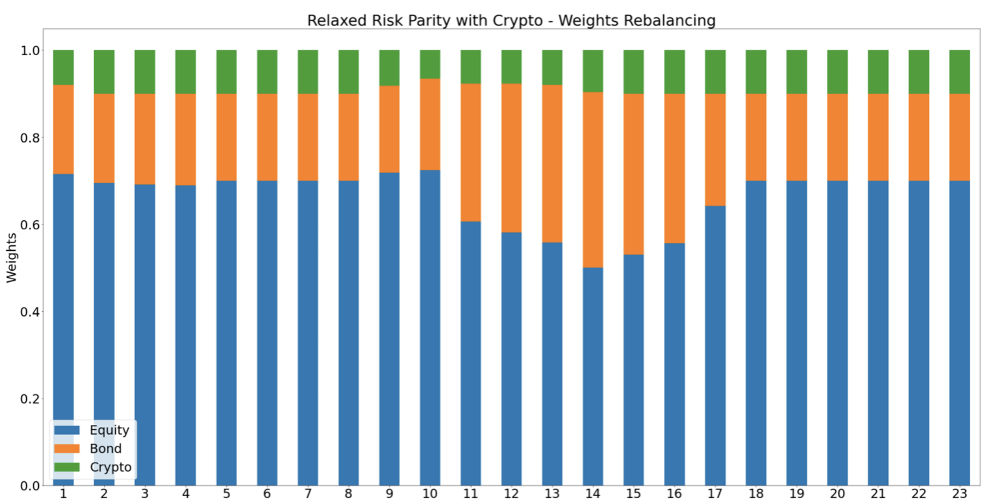
Efficient Frontier and Capital Allocation Line with DSR

Anyway, given the conservative nature of risk parity, it is not strange to find intertemporal Sharpe and Sortino Ratio values between other portfolios, since this aspect allows the model to avoid important drawdowns.

In fact, as noticed by Qian (2005) [54], Maillard et Al. (2010) [55] and Lee (2011) [30], risk parity techniques protect portfolios from bearish trends thanks to superior diversification capabilities and, especially in the case of the relaxed risk parity approach adopted in this thesis, the parameters' relaxations allow the portfolio to take advantage of early gains during bullish periods, without having to dramatically raise the risk.

Regarding the diversification, differently from the approaches that aim at maximize a certain risk/reward ratio (that may lead to non-properly diversified portfolios), risk parity models delete the uncertainty linked to non-reliable expected return estimations and the outcome is more robust and stabler realized returns.

The key to this result relies in the fact that each asset class in portfolio possess a certain degree of risk and, to this extent, it is not possible to completely eradicate it from the allocation, as occurred in MPT and PMPT strategies.



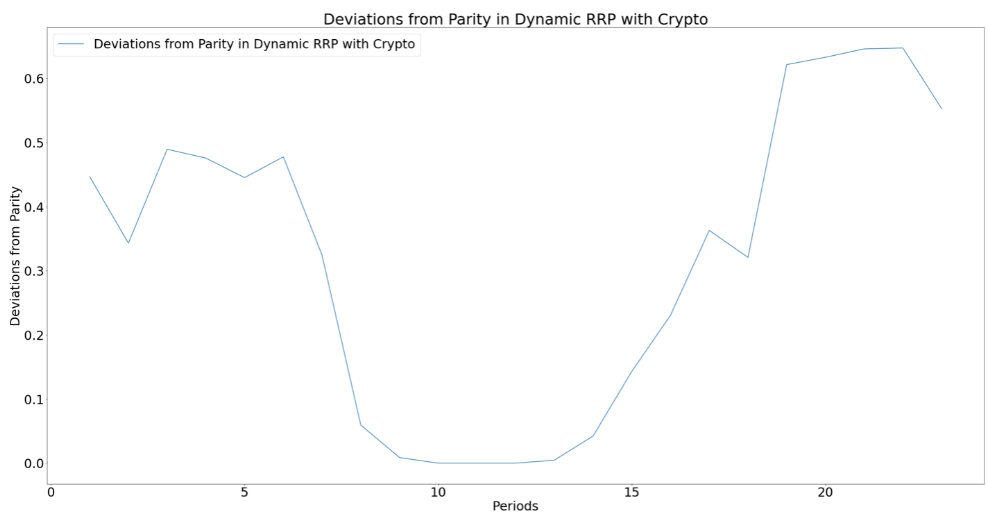
Weights Rebalancing - Dynamic RRP with Crypto

In fact, as we can see from figure above, the dynamic crypto RRP was the only allocation to include a portion of CRIX that lasted for the whole out-of-sample period.

Therefore, excessive concentrations in certain assets are limited, resulting in a portfolio with more decorrelated elements that are able to capture the benefits of bull markets more quickly than other strategies.

Moreover, the dynamic feature of the strategy allows the portfolio to adjust the positioning dynamically in response to changes in volatility of each asset class and in the whole portfolio. This means that, when the decline of an asset is accompanied by an increase in volatility, the dynamic RRP strategy decreases the allocation in that particular asset and may allow the investor to avoid extreme losses if the asset continues to decline, maintaining a certain potential for the subsequent rebound.

For example, during the bearish market caused by COVID-19 pandemic, the losses were sustained and accompanied by sharp increases in volatility. This resulted in significant shift in allocations from the S&P500 and CRIX to the safer 10-Year Treasury. The reallocation process allowed the portfolio to sidestep subsequent losses in stocks, thus facing smaller drawdowns than other strategies.



Sum of Deviations from Parity in Dynamic RRP with Crypto

As we said, the relaxation of parameters allows the portfolio to take advantage of market rebounds after bearish periods. This aspect clearly implies a certain distance to the risk parity condition, that is depicted in figure "Sum of Deviations from Parity in Dynamic RRP with Crypto".

As the RRP model of this thesis is based on the research from Gambeta et Al., 2020 [24], in which the limitation in response to market rebounds was due to the long lookback epoch of the reallocations, I reduced the training period of the data from three years to one year. This adjustment resulted in more reactive portfolio responses to market changes and so the distance to risk parity properly followed the volatility patterns of the asset classes.

In fact, if we compare figure "Sum of Deviations from Parity in Dynamic RRP with Crypto" to figure "Logarithmic Returns (in-sample)" (page 67), we can see how the highest volatility period of the three assets coincides with the highest distance from risk parity, occurred during last quarters of 2020, right after COVID-19 market crash.

This recalibration let the dynamic RRP with crypto portfolio deliver better risk-adjusted out-of-sample performances (in fact, the more the distance from risk parity, the higher are the improvements in Sharpe and Treynor Ratio) and to go back to pre-pandemic levels in a shorter time than other portfolios, as we can see from figure "Cumulative Returns (out-of-sample)".

Furthermore, as explained in Gambeta et Al., 2020 [24], the relaxation usually comes at a cost, so higher portfolio volatility after the reallocation. But the main difference between my research and the one in the paper is that I only included three asset classes instead of ten, so the increase in volatility is mainly due to general market conditions generated by COVID-19.

Other key elements of the winning strategy reside in the portfolio's low intertemporal betas, low tail risk and, obviously, in the enhancements coming from the CRIX index.

CRIX allowed all crypto portfolios to gain superior returns with respect to their non-crypto reciprocals, thanks to its extreme gains, especially during the second half of the observed in-sample period, and to the rebalancing strategy inherent in the index.

Indeed, as said in section 3.2.1, CRIX rebalances the crypto portfolio composition each month so it does not have any concentration polarized in one specific coin and this aspect makes it relatively diversified, as long as all cryptos are quire correlated between each other.

To conclude, the out-of sample performance analysis demonstrates the model's success by delivering higher returns while staying near risk parity. Further researches could extend the model by adding more constraints or by adjusting the magnitude of the deviations from parity to match with different degrees of risk tolerance.

Conclusions

In this thesis we investigated whether cryptocurrencies can qualify or not as an asset class in their own right, and if their introduction in diversified asset allocations can enhance portfolio performances, along with dynamic optimization techniques.

We found that cryptos show strong internal correlations, low correlations with traditional assets, acceptable market liquidity and room for market stability improvements. These characteristics allow us to identify them as a distinct asset class, as they have the potential to provide diversification benefits and superior portfolio returns.

In fact, during each optimization technique, crypto portfolios managed to outperform their respective equity-bond only reciprocals, both in their static and dynamic specifications.

Moreover, we found that Relaxed Risk Parity portfolios have the ability to exploit the full potential out of cryptos.

This approach, among the ones analyzed, could be a interesting portfolio choice when deciding whether or not to include a small portion of the allocation in cryptos.

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