



Department of Economics and Finance

Chair of Empirical Finance

**Volatility and Liquidity
Nexus in Cryptocurrency
Markets**

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Introduction

Bitcoin and cryptocurrencies represent the most revolutionary breakthrough in the financial system. Bitcoin, introduced in 2008 by Satoshi Nakamoto, aims to overcome the shortcomings of fiat and gold-based currencies by providing alternative and digital money within a decentralised peer-to-peer system. It exploits cryptographic processes to issue currency and verify transactions, which are permanently recorded in the blockchain. Therefore, Bitcoin eliminates the need for the commitment of central or regulatory authorities. However, despite the growth in the importance of cryptocurrencies as a form of payment, they have become more of a speculative investment vehicle than an alternative currency (Yermack (2015), Baur et al. (2015), Baur et al. (2018)). Indeed, cryptocurrencies are a new financial asset that could provide higher returns (Hong (2017)) and better diversification due to the weak correlation with traditional asset classes such as bonds and equities (Baur et al. (2015)). They play a crucial role in an investor's portfolio, but they are subject to bubble behaviour and enclose idiosyncratic risks hard to hedge (Corbet et al. (2018)).

Investors can trade cryptocurrencies on exchanges that have unique characteristics. They are open 24 hours a day and 365 days a year and allow the transfer, together with clearing and settlement, of BTC against various fiat and other (crypto)currencies directly. However, these markets are highly fragmented, weakly regulated and have varying degrees of efficiency. In particular, the number of self-regulated, centralised, and decentralised trading platforms operating in tax havens with unclear legislation has exploded. The main consequence is that investors cannot easily choose a specific exchange for trading and hedging or can find restrictions to redirect orders to more convenient platforms. Indeed, this enormous fragmentation, combined with the total absence of price integration (Makarov and Schoar (2020)) and the lack of harmonised regulation, makes the cross-sectional assessment of liquidity and

volatility with high-frequency data difficult.

Actually, to characterise the time-series variation of liquidity on cryptocurrency markets, we rely on liquidity proxies derived from easy-to-access transaction data (prices). Initially, several low-frequencies measures have used to describe the liquidity dynamics of bond, commodity, foreign exchange, and equity markets(e.g., [Fong et al. \(2017\)](#), [Goyenko et al. \(2009\)](#), [Karnaukh et al. \(2015\)](#), [Johann and Theissen \(2017\)](#), [Marshall et al. \(2012\)](#), [Schestag et al. \(2016\)](#)). Then, due to the substantial difference between cryptocurrency and traditional markets, a set of high-frequency order book and transaction-based measures has been proposed (e.g., [Brauneis and Mestel \(2018\)](#), [Dimpfl \(2017\)](#), [Shi \(2017\)](#), [Dyhrberg et al. \(2018\)](#), [Hautsch et al. \(2018\)](#)). However, only [Brauneis et al. \(2021\)](#) study the efficacy of liquidity proxies respect to high-frequency order book measures (e.g., Bid-Ask Spreads, Percentage Quoted Spread, Percentage Effective Spread, Percentage cost of a roundtrip trade), understanding their relative benefits. In fact, this approach yields enormous advantages in terms of cost and process time savings. To this end, we consider a set of transaction data (OHLC prices), sampled at hourly frequency, for the cryptocurrency, BTC/USD, and two large and reliable exchanges, Bitfinex and Bitstamp, over two years. Secondly, we compute two liquidity proxies based on high, low and closing prices. In particular, we estimate [Corwin and Schultz \(2012\)](#) and [Abdi and Ranaldo \(2017\)](#) measures, aggregated at a monthly frequency, because they allow us to best capture the time-series variation in liquidity irrespective of observation frequency and trading venue ([Brauneis et al. \(2021\)](#)).

However, the main aim of our analysis is to study the nexus between liquidity and volatility in an innovative econometric framework, designed to explain empirical regularities of BTC/USD on Bitstamp and Bitfinex. Such dynamics have been already investigated with a different approach for both traditional and cryptocurrency markets ([Nguyen et al. \(2020\)](#), [Chordia et al. \(2005\)](#), [Brauneis et al. \(2021\)](#), [Corbet et al. \(2022\)](#)). Specifically, we analyse the volatility and liquidity dynamics and how they interact by introducing an innovative methodology, which embeds the logarithmic multiplicative error model (LogMEM; [Bauwens and Giot \(2000\)](#), [Nguyen et al. \(2020\)](#), [Alexander et al. \(2021\)](#)) context in mixed data sampling, or MIDAS ([Engle et al. \(2013\)](#), [Ghysels et al. \(2005\)](#), [Ghysels et al. \(2007\)](#)). Formally, the model is an extension of MEM-MIDAS introduced by [Amendola et al. \(2021\)](#).

Particularly, the dynamics of realised volatility, a positive-value variable, is decomposed into a short and long-run component. The former is the conditional mean which depends, inter alia, on the long-run evolution of volatility. The latter, obtained by smoothing our variable of interest, [Corwin and Schultz \(2012\)](#) and [Abdi and Ranaldo \(2017\)](#) estimator, in the spirit of MIDAS regression. This approach allows us to include information derived from proper economic variables, aggregated at a lower (monthly) frequency. In this way, the main drivers of long-term market volatility can be incorporated into the analysis at a higher (daily) frequency. To estimate the model, we derive a robust realised volatility estimator ([Jacod et al. \(2009\)](#)) from the BTC/USD prices on two exchanges, Bitstamp and Bitfinex, to account for the impact of microstructure noise. Furthermore, given the dramatic ups and downs in the price of cryptocurrencies, we winsorize the realised volatility values to diminish the influence of outliers.

Based on our model, we document a strong positive nexus between volatility and liquidity. Specifically, we find that high liquidity leads to high volatility. Investor seems to be attracted to the cryptocurrency market during periods when trading activity is more pronounced. This microstructure behaviour is perhaps explained by the decisions of market participants to seek higher returns. Our results are in line with the finding of several empirical analyses in the same field, such as [Brauneis et al. \(2021\)](#) and [Corbet et al. \(2022\)](#). Notwithstanding, we enrich the current literature with an extension of the well-known econometric framework, based on the multiplicative error model, including the contribution of liquidity variables through MIDAS filtering. Specifically, we incorporate easy-to-compute liquidity proxies, which are derived from easy-to-access high-frequency transaction data.

Finally, our findings have important implications for scholars, practitioners, traders, trading platforms operator and regulation authorities. Our contribution helps to understand the nexus between liquidity and volatility on cryptocurrency exchanges with a relatively inexpensive and simple process. Particularly, given the ever-increasing need to regulate the world of cryptocurrencies, authorities can use these empirical results to better understand the inherent characteristics of a new and unexplored market. Furthermore, the information, provided by our analysis, may also help the practitioners make better investment decisions.

The remainder of this paper is organized as follows. In Chapter 1 we explain our econo-

metric framework, presenting the model and the several specifications; in Chapter 2 we present our dataset and highlight the realised volatility and liquidity measure calculation for the selected exchanges; in Chapter 3 we show the empirical results and provide an economic interpretation; in the end, we present conclusions and summary.

Chapter 1

Model

Our econometric framework, designed to explain high-frequency volatility dynamics, is developed on the extensive literature on modelling high-frequency financial time series. This framework allows conveying essential intra-daily information about the micro-behaviour of volatility. In particular, this stems from the Multiplicative Error Model (MEM) introduced in [Engle and Russell \(1998\)](#) to deal with positive-valued time series, combined as the product of two non-negative random variables, representing respectively the conditionally deterministic mean (scale factor) and the error term. This model structure implicitly guarantees the non-negativity of the variables of interest and presents several advantages concerning the Standard Gaussian approach ([Engle \(2002\)](#)). However, to avoid the imposition of parameter constraints to ensure non-negativity when estimating the model, we rely on LogMEM(p,q) introduced by [Bauwens and Giot \(2000\)](#). The model, which is an alternative formulation of the original MEM, implies the application of the logarithm to the conditional mean in its specification. So, let x_i be the time series of a positive discrete-time process for the day i , our univariate model is given by

$$x_i = \mu_i \epsilon_i \quad \epsilon_i | I_{i-1} \stackrel{iid}{\sim} D^+(1, \sigma_\epsilon^2) \tag{1.0.1}$$

$$\log \mu_i = \omega + \alpha \sum_{j=1}^p \log x_{i-j} + \gamma \log x_{i-1}^- + \beta \sum_{j=1}^q \log \mu_{i-j} \tag{1.0.2}$$

where ϵ_i follows a *iid* distribution defined over a non-negative support, with unit mean and unknown variance, conditional to I_{i-1} the information set up to day $i - 1$, $\log x_{i-1}^- = \log x_{i-1}$ if the return of the respective interval is negative and zero otherwise, p and q determine the lag structure and $\Theta = (\omega, \alpha_1, \dots, \alpha_p, \gamma, \beta_1, \dots, \beta_p)$ is the parameters space.

However, although this model provides superior performance in explaining high frequency realised volatility transmission patterns in bitcoin markets (Alexander et al. (2021)), it does not allow for directly examining how economic variables may capture sources of volatility. Indeed, in our econometric framework, we suggest several new component model specifications related to economic activity in order to combine information provided by different exogenous variables. Therefore, we propose an innovative approach, which embeds the logarithmic multiplicative error model (LogMEM) context in mixed data sampling, or MIDAS, including a slowly evolving component of volatility. In particular, we present an extension of MEM-MIDAS introduced by Amendola et al. (2021). Hence, in the LogMEM-MIDAS framework, the dynamics of realised volatility, $x_{i,t}$, for the day i of any arbitrary low-frequency period t depends on short and long run component, respectively $\mu_{i,t}$ and τ_t , such that

$$x_{i,t} = \mu_{i,t} \tau_t \epsilon_{i,t} \quad \text{with } i|t = 1, \dots, N_t \quad \text{and } t = 1, \dots, T \quad (1.0.3)$$

where t may be a week, month or quarter with N_t days, depending on the frequency at which the additional variable X_t is observed¹. Moreover, $\epsilon_{i,t}$ represents the error term, assumed to follow an *iid* gamma distribution conditional on the information set up to day $i - 1$ of period t .

$$\epsilon_{i,t} | I_{i-1,t-1} \stackrel{iid}{\sim} \Gamma(1, \sigma_\epsilon^2) \quad (1.0.4)$$

Formally, the selection of the distribution of innovations does not alter the consistency properties of the Quasi-Maximum Likelihood (QML) estimator (Amendola et al. (2019)).

¹The low-frequency period t will be a choice variable, representing one of the many advantage of our model specification due to the empirical importance of its selection Engle et al. (2013)

Instead, the low-frequency component τ_t obtained by smoothing variable X_t , in the spirit of MIDAS regression (Ghysels et al. (2005), Ghysels et al. (2007)), filtering for K past lagged values of the same variables, is as follows

$$\tau_t = \exp \left(m + \theta \sum_{k=1}^K \delta_k(\omega) X_{t-k} \right) \quad (1.0.5)$$

where θ measures the effect of sum $\sum_{k=1}^K \delta_k(\omega) X_{t-k}$, obtaining thanks to weighting scheme $\delta_k(\omega)$, on volatility around an average level m . Furthermore, since the specification of τ_t make obsolete parameters constraints to guarantee non-negativity, the only restrictions is the strict stationarity of X_t .

To complete the model, we specify the Beta function as suitable weighting function:

$$\delta_k = \frac{\left(\frac{k}{K}\right)^{\omega_1-1} \left(1 - \frac{k}{K}\right)^{\omega_2-1}}{\sum_{j=1}^K \left(\frac{j}{K}\right)^{\omega_1-1} \left(\frac{1-j}{K}\right)^{\omega_2-1}} \quad (1.0.6)$$

This function is the beta lag presented in (Ghysels et al. (2007)), which is flexible to accommodate various lag structures. It allows putting different degrees of emphasis on past realizations of X_t providing $\omega_n \geq 1$ with $n = 1, 2$. In particular, it can be an equally weighting scheme when $\omega_1 = \omega_2$. It can be a monotonically increasing or decreasing scheme when respectively farther observations are weighted more, $\omega_1 > \omega_2$, or less, $\omega_1 < \omega_2$.

Nonetheless, we focus on the latter case adopting a restricted weighting lag structure, where $\omega_1 = 1$ and $\omega_2 \geq 1$. Clearly, $\sum_{k=1}^K \delta_k(\omega_2) = 1$.

Finally, the logarithmic daily conditional expectation, or short-run component $\mu_{i,t}$, contains a low-frequency component τ_t varying on t , is such that

$$\log \mu_{i,t} = \omega + \alpha \sum_{j=1}^p \log \frac{x_{i-1,t}}{\tau_t} + \gamma \log \frac{x_{i-1,t}^-}{\tau_t} + \beta \sum_{j=1}^q \log \mu_{i-1,t} \quad (1.0.7)$$

where $\log \frac{x_{i-1,t}^-}{\tau_t} = \log \frac{x_{i-1,t}}{\tau_t}$ if the return of the respective interval is negative and zero otherwise, γ represents the asymmetric parameter, which evaluates the impact of negative lagged daily returns $r_{i-1,t}$, and p and q determine the lag structure. Equations (1.0.3) to (1.0.7) form the proposed LogMEM-MIDAS model for explaining time-varying dynamics of volatility embedding information derived from proper economic variables. Furthermore, given the parameter space $\Theta = \{\omega, \alpha, \beta, \gamma, m, \theta, \omega_1, \omega_2\}$, the model presents nice peculiarities. Actually, due to the presence of a fixed number of parameters, the model is parsimonious relative to existing component volatility models and is comparable over different time spans t . Moreover, it allows varying the number K of past lagged variables without changing the number of parameters, but implying a different weighting scheme in the spirit of MIDAS (Engle et al. (2013)).

The parameter estimates are retrieved by the QML estimation. However, the initialisation value, required by this method, must be numerically optimized because the logMEM-MIDAS does not have analytically expression for the unconditional moments. Particularly, if we try to compute the unconditional mean for $x_{i,t}$ we arrive at

$$\begin{aligned} \mu &= \mathbb{E}[x_{i,t}] = \mathbb{E}[\mathbb{E}[x_{i,t} | I_{i-1,t-1}]] = \mathbb{E}[\mu_{i,t} \tau_t] \\ &= \mathbb{E}\left[\exp\left(\omega + \alpha \sum_{j=1}^p \log \frac{x_{i-1,t}}{\tau_t} + \gamma \log \frac{x_{i-1,t}^-}{\tau_t} + \beta \sum_{j=1}^q \log \mu_{i-1,t}\right) \tau_t\right] \end{aligned} \quad (1.0.8)$$

So, we decide to use as initial value in the QML method the following specification

$$\mu_{1,1} = \exp\left(\frac{\omega \log \tau_1}{\log \tau_1 (1 - \beta) - \alpha - 0.5 \gamma}\right) \quad (1.0.9)$$

1.1 How the low-frequency component incorporates liquidity information

To link the daily realized volatility of return with liquidity measure, sampled at lower frequencies, we present how the specification of the long-run component directly incorporates liquidity information. The low-frequency factor plays a crucial role in evaluating the impacts of our variables of interest on the volatility dynamics. Particularly, we use liquidity proxies derived from easy-to-access transaction data (OHLC prices) for each subinterval j , representing the frequency at which the data are available. Then, these are aggregated to estimate one liquidity measure for each interval t .

We use the following transaction-based measures

- [Corwin and Schultz \(2012\)](#) estimator (CS) for two adjacent subintervals $j, j + 1$.

$$CS_{j,j+1} = \frac{2 (\exp(\alpha) - 1)}{1 + \exp(\alpha)} \quad (1.1.1)$$

$$\alpha = \frac{\sqrt{2\beta} - \sqrt{\beta}}{3 - 2\sqrt{2}} - \sqrt{\frac{\lambda}{3 - 2\sqrt{2}}}, \quad \beta = \left[\ln\left(\frac{H_j}{L_j}\right) \right]^2 + \left[\ln\left(\frac{H_{j+1}}{L_{j+1}}\right) \right]^2, \quad \lambda = \left[\ln\left(\frac{H_{j,j+1}}{L_{j,j+1}}\right) \right]^2$$

where H_j and L_j are the high and low prices in subinterval j , $H_{j,j+1}$ and $L_{j,j+1}$ denote respectively the high and low prices over subintervals j and $j + 1$. Moreover, according to [Corwin and Schultz \(2012\)](#), we set all negative values to zero. Finally, to get CS_t , we average all estimators of adjacent subintervals, which belong to each interval t .

- [Abdi and Ranaldo \(2017\)](#) estimator (AR)

$$AR_j = \sqrt{\max[4(c_j - \bar{p}_j)(c_j - \bar{p}_{j+1})]} \quad (1.1.2)$$

$$h_j = \ln(H_j), \quad l_j = \ln(L_j), \quad c_j = \ln(C_j), \quad \bar{p}_j = \frac{(h_j + l_j)}{2}$$

where h_j , l_j , c_j are the natural logarithms of high, low and close prices, respectively, in subinterval j , while \bar{p}_j is the midpoint between the h_j and l_j . Specifically, we follow the 'two-day corrected' version of estimator which refers to two adjacent subintervals j and $j + 1$. Finally, the AR_t is defined as

$$AR_t = \frac{1}{J-1} \sum_{j=1}^{J-1} AR_{t,j} \quad (1.1.3)$$

where $AR_{t,j}$ is the estimator for adjacent subintervals j and $j + 1$ within t .

According to [Brauneis et al. \(2021\)](#), these liquidity proxies provide superior performance in terms of capturing times series variation in cryptocurrency liquidity respective to other low-frequency measures ([Roll \(1984\)](#), [Kyle and Obizhaeva \(2016\)](#), [Amihud and Noh \(2021\)](#), the number of transactions and dollar volume measures). Furthermore, [Brauneis et al. \(2021\)](#) suggest these specific low-frequency measures because they are based on easy-to-access transactions data and offer enormous savings in terms of cost and process time compared to high-frequency order book measures. Moreover, thanks to quantile dependence analysis², [Brauneis et al. \(2021\)](#) test the relative performance of liquidity proxies depending on different liquidity regimes to capture the average relationship between the well-known high-frequency and transaction-based measures. In particular, the authors verify that liquidity proxies present the same efficacy as benchmark measures (Bid-Ask Spreads, Percentage Quoted Spread, Percentage Effective Spread, Percentage cost of a roundtrip trade).

We now turn to LogMEM-MIDAS models with one-sided filters and fixed span specification, involving past liquidity variables of interest. So, τ_t process, depending on one transactions-based measure, can specify accordingly:

$$\tau_t = \exp \left(m + \theta \sum_{k=1}^K \delta_k(\omega) X_{t-k}^l \right) \quad (1.1.4)$$

where X_{t-k}^l represents the level of the liquidity proxy³. More specifically, τ_t for the single-variable models involves a parameter space composed by four parameters, $\Theta = (m, \theta, \omega_1, \omega_2)$.

²See e.g. [Duan et al. \(2021\)](#)

³The only requirement for the exogenous variables X_{t-k}^l is the strict stationarity.

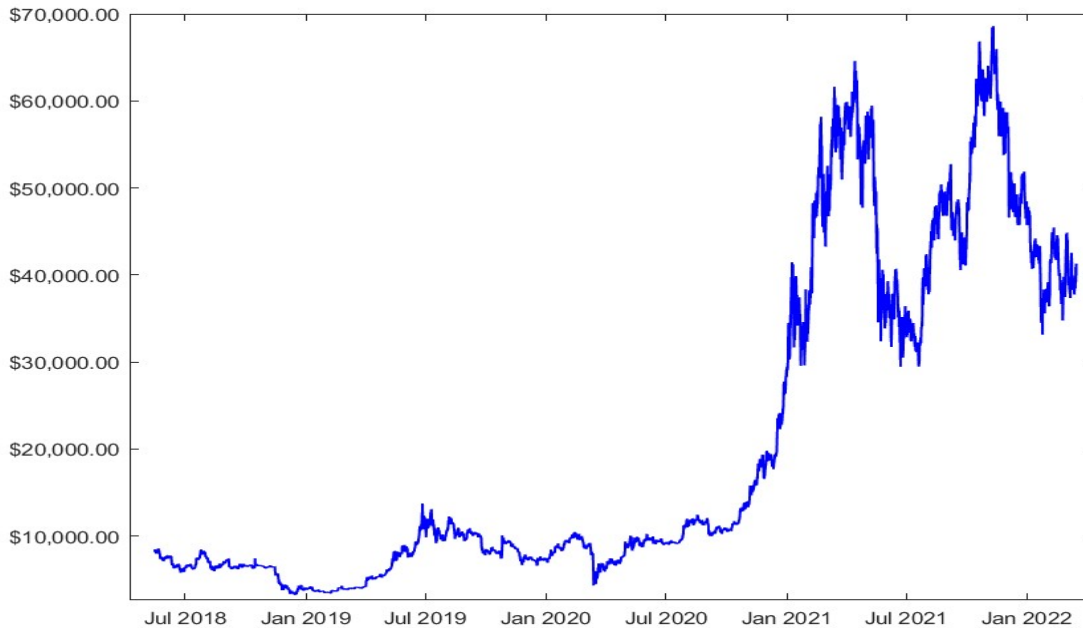
Chapter 2

Data: Prices, Realised Volatility and Liquidity Measure

The crypto market evolves rapidly and acquires growing importance for payments and investment portfolios. The cryptocurrencies are continuously traded 24 hours a day, including holidays and non-business days, on highly fragmented, and weakly regulated and interconnected exchanges. Therefore, due to the economic importance and legislative concern, we intend to analyse high-frequency liquidity together with volatility. We collect hour-by-hour historical transaction data for the Bitcoin versus US dollar (BTCUSD) pair from 2018-05-16 00:00:00 UTC to 2022-05-04 00:00:00 UTC, a total of 1401 trading days with 33620 observations. The data set consists of hourly open, high, low and close (OHLC) prices, UNIX timestamps and volume in USD. Notably, they are obtained using an algorithm to continuously access the public and freely accessible REST APIs of two exchanges, Bitstamp and Bitfinex¹. Particularly, we focus on the extended period, which includes the different phases of the economic cycle of the crypto market (e.g. the collapse in 2018, the bull market in 2021 together with the COVID-19 crisis), where the price of bitcoin has rapidly evolved presenting significant oscillations (Figure 2.1).

¹The transaction data can be purchased from data providers, as CoinAPI or Kaiko, or freely downloaded from cryptodatadownload.com.

Figure 2.1: *Bitcoin Price*



Note: The figure shows the Bitcoin price on Bitfinex from 16 May 2018 to 04 May 2022 (not shown here the price evolution of Bitfinex).

Moreover, the selection of the exchange is driven by the analysis of [Hougan et al. \(2019\)](#) and [Le Pennec et al. \(2021\)](#) which highlight the importance of reliability of the reported transactions. They analyse granular data, which is not available via the exchanges' APIs, and find that the operators tend to report not economically meaningful or fake transactions in crypto markets. However, after exploiting several identification tests, Bitstamp and Bitfinex are indicated as "real volume" and reliable exchanges.

Additionally, it may be possible to detect missing data due to technical problems, exchange-specific trading halts and missing trading activity. For this reason, when we aggregate data from hourly to daily (monthly) frequency, we require the availability of 80% of valid data for each interval (day, i , or month, t). In particular, this corresponds to 19 (576) hours for each trading day² (month³). Finally, our resulting data set approximately

²According to coinmarket.com, a trading day is defined as the time span from 00:00 UTC to 23:59 UTC.

³A month is defined as composed of 30 trading days

contains 1401 trading days and 46 monthly intervals.

Then, when we analyze empirically realized volatility in a high-frequency framework, we must consider the influence of market micro structure noise in our estimates. In particular, this is the result of “micro” frictions in the trading process (e.g. price discreteness, bid-ask bounces etc.) that subsequently have also ”macro” consequences relative to asset prices’ liquidity (Ait-Sahalia and Yu (2008), Seifoddini et al. (2017)). Therefore, literature shows that increasing the frequency of data allows for better capturing of volatility flows, but it implies higher microstructure noise (Andersen (2000)). Reasonably, we select an hourly data sampling frequency from which we compute daily realised volatility. Additionally, the selection of hour-by-hour aggregation is a more convenient alternative relative to a higher frequency (e.g. second-by-second, minute-by-minute) in terms of financial resources and data management. Indeed, to compute a noise-robust estimator of realised volatility using high-frequency data, we use the pre-averaging approach presented by Jacod et al. (2009). Here, the impact of microstructure noise is locally smoothed by applying a weighted average to a one-hour log return⁴. So, the pre-averaged log returns are given by

$$\begin{aligned} \bar{r}_j &= \sum_{l=1}^{k_n} g\left(\frac{l}{k_n}\right) r_{j+l} \\ r_j &= \log\left(\frac{C_j - H_j}{H_j}\right) \end{aligned} \tag{2.0.1}$$

where r_j is the ordinary (not pre-averaged) log returns and g is a tent-shaped weighting function⁵, $g(x) = \min(x, 1 - x)$. The k_n bandwidth of the local pre-averaging window, consistently with Hautsch and Podolskij (2013), is inversely related to the sampling frequency. Specifically, it is optimally set at $[\theta\sqrt{n}] = 2$, where n is the number of subintervals j in i (in our case $n= 24$); $\theta \in [0.3, 0.6]$ denotes the smoothing parameter and depends on the frequency of the data and the liquidity of the asset. According to Alexander et al. (2021), we choice θ as 0.4⁶, an intermediate value that allows to avoid oversmoothing effect and

⁴Andersen (2000), claims that the aggregation of intra-daily squared returns provides a more robust estimator for realized volatility relative to the sum of daily squared returns

⁵The real valued function $g : [0, 1] \rightarrow \mathbb{R}$ must be continuous, piecewise continuously differentiable with piecewise Lipschitz derivative and having $g(0)=g(1) = 0$.

⁶Hautsch and Podolskij (2013) define the optimal $\theta \approx 0.4$ (0.6) for liquid (illiquid) stocks.

negative bias in estimator. The daily pre averaging realized variance estimator is obtained by summing up squared pre-averaged hourly returns over each trading day. Finally, we compute the annualised pre-averaged realised volatility as the square root of the annualised realised variance⁷. Table 2.1 reports descriptive statistics for annualized realized volatility after considering the effect of anomalous values process. In particular, following the empirical analysis of Alexander et al. (2021), Engle et al. (2013) and Li et al. (2020), to diminish the influence of outliers, we winsorize the top 0.05 % of our realised volatility values setting all the observation greater than the 99.95%-quantile equal to this quantile.

Table 2.1: *Realised Volatility Statistics*

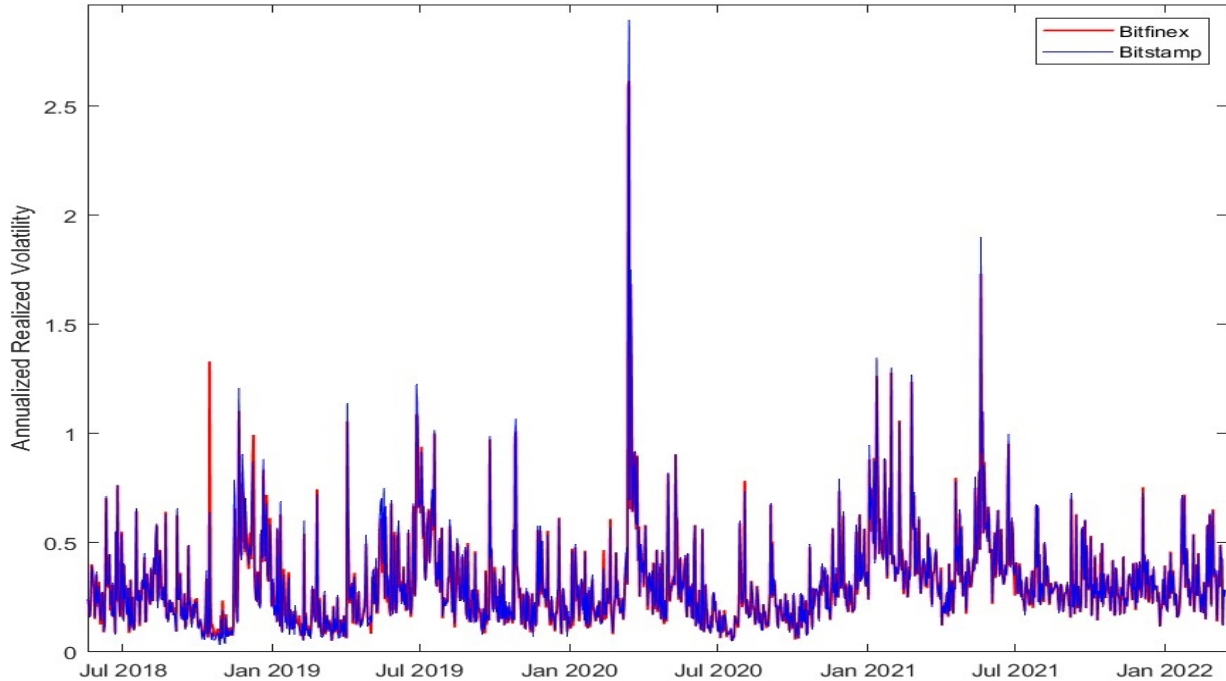
Exchange	Bitfinex	Bitstamp
Mean	0.3110	0.3130
Std	0.2113	0.2165
Min	0.0516	0.0343
25%	0.1796	0.1794
50%	0.2635	0.2676
75%	0.3794	0.3791
Max	2.6185	2.8972

Note: The table shows the descriptive statistics on the winsorized annualized realized volatility over the period from 16 May 2018 to 04 May 2022 for Bitfinex and Bitstamp.

The average levels of volatility on Bitfinex and Bitstamp are similar. The deviations across values are probably given by differences of levels of trading activity. Moreover, despite having similar standard deviation of about 20%, Bitstamp and Bitfinex display significantly diverging minimum and maximum values, respectively. Figure 2 shows the volatility dynamics for the selected sample period and the relative differences.

⁷The annualization factor is $\sqrt{(12 \times 24 \times 365)}$ because the bitcoin markets are open 365 days per year rather than 250

Figure 2.2: *Realized Volatility*



Note: The figure shows the bitcoin realized volatility on Bitfinex and Bitstamp over the period from 16 May 2018 to 04 May 2022

Spikes in the magnitude of realised volatility is registered after January 2020 as result of COVID-19 outbreak. Moreover, we notice volatility clustering in cryptocurrency markets consistently with literature stylized facts in financial markets.

Table 2.2 reports the summary statistics for [Abdi and Rinaldo \(2017\)](#) and [Corwin and Schultz \(2012\)](#) at monthly resolution. Specifically, the presented results are annualised by taking the geometric mean of monthly rates. The sample period is 16 May 2018 to 04 May 2022. The liquidity measures, which are both estimators of the bid-ask spread, show similar values. However, they indicate that Bitfinex is on average more liquid than Bitstamp even if it is subject to a larger variability. These results are probably due to the different levels of trading activity, which are subsequently connected to macroeconomic phenomena or investor sentiment. Moreover, to satisfy the requirements of strict stationarity for exogenous variables X_t in the equation ((1.1.4)), we run the Augmented Dickey-Fuller (ADF) test⁸.

⁸See for further details [Mushtaq \(2011\)](#)

Particularly, we reject the null hypothesis due to the no stationarity of liquidity measures' the time series. Therefore, to make the processes stationary, it is just required to remove trend components computing the first-order difference of the series $\nabla^1 X_t$.

Table 2.2: *Liquidity measures*

Exchange	Mean	Std	Min	25%	50%	75%	Max
Bitfinex							
CS	278.05	132.81	87.21	193.27	262.07	319.45	667.06
AR	279.60	116.07	115.44	199.61	258.53	317.23	616.53
Bitstamp							
CS	316.69	121.48	111.97	234.34	298.41	340.81	671.17
AR	287.82	116.49	102.68	209.94	268.17	329.28	632.62

Note: The table shows the descriptive data on proxy liquidity measures, [Abdi and Ranaldo \(2017\)](#) and [Corwin and Schultz \(2012\)](#), expressed in basis points for the pair BTCUSD at a monthly resolution over the period from 16 May 2018 to 04 May 2022. The results are annualised by taking the geometric mean of monthly rates.

Chapter 3

Empirical Results

To model the effect of a change in the crypto exchange’s liquidity on volatility dynamics, we estimate the LogMEM-MIDAS model with only one lag ($p = q = 1$). In particular, this choice is driven by the desire to follow parsimonious criteria for which the preferred model will be that produce the best fit but with the least number of parameters, thus reducing the randomness in the final outcome. However, according to [Alexander et al. \(2021\)](#) and [Nguyen et al. \(2020\)](#), increasing the number of covariates does not significantly impact the total persistence (the sum of all α and β), the log-likelihood and the model information criteria (BIC)¹.

Tables [3.1](#) and [3.2](#) show the parameter estimates together with the Bayesian Information Criterion (BIC) and conditional mean half-life for both model specifications. Overall, the empirical results explaining the volatility patterns are quite similar across the two exchanges. Particularly, the estimates in the tables are all statistically significant, except for the intercept, ω , and the average level of long-run component τ_t , m .

The total persistence of daily realized volatility for Bitstamp and Bitfinex ranges from 0.7542 to 0.7580 for all LogMEM-MIDAS specifications. Specifically, the sum of α and β is extremely less than one compared to conventional autoregressive models (GARCH, MEM, logMEM) because there is a partially cleaning of the series from autocorrelation. This finding implies less persistence of the short-term component of volatility, μ_t , due to the effect of τ_t .

¹We decide to implement the Bayesian information criterion (BIC) as statistical measure for comparative evaluation because it penalizes more an increase of parameters, relative to the AIC.

Furthermore, α and β , which capture short- and long-term persistence, suggest that traders are more reactive to future market conditions than their current evolution.

The half-life measures the average time, which is about 2.5 days for both exchanges, to halve the effect of a volatility shock². In particular, it indicates how traders' future expectations of volatility on the subsequent daily period are impacted. So, half-life describes the time evolution by which the logarithm of the conditional mean halves its distance to the unconditional mean (Engle and Patton (2007)).

Unsurprisingly, the estimates of γ parameter is positive and highly significant on all exchanges. This results document that volatility dynamics are asymmetric in the sense that volatility increases more after those days in which the return is negative. The strength of this effect is similar on the two exchanges at around 0.049.

To understand how much volatility is related to liquidity, we need to analyze the evolution of the long-term component, τ_t . In particular, the parameters for the single-variable models, θ_{CS} and θ_{AR} , are all positive and highly significant. This means that if the change in liquidity between t and $t-1$ is positive, τ_t increases and consequently so does the long-term volatility. However, to quantitatively proof this relation, we calculate, following Amendola et al. (2019), the Relative Marginal Effect on τ_t , RME_k^τ , of a change ΔX_{t-k} in liquidity measure X_{t-k} , as

$$RME_k^\tau = \exp\left(\hat{\theta}_l \delta_k(\omega_{2,l}) \Delta X_{t-k}^l\right) - 1 \quad (3.0.1)$$

where $\hat{\theta}$ is the estimated parameter and l identify the liquidity proxies. Finally, we can state that positive changes in liquidity in one of the two exchanges results on average in an increase in the long-term component and thus in volatility. According to the information criteria, the best performer models, meaning with the minimum BIC value, is represented by the model with Corwin and Schultz (2012) (Abdi and Rinaldo (2017)) transactions-based measure for Bitfinex (Bitstamp). Nevertheless, the values of BIC are very similar

²We compute the half-life, excluding strictly negative realised volatility values and asymmetric response, as $\log(0.5)/\log(\alpha + \beta)$.

with tenths of a gap across the two exchanges, meaning the almost equal performance of the several model specification with some deviations due to the different approximations of the two liquidity measures. Figure 3.1, 3.2, 3.3 and 3.4 present the observed realized volatility and the estimated volatility for both liquidity measure and exchanges. We can notice that our estimates describe well the variability in the data, but the fit capture partially the magnitude of the variations.

Finally, the conditional distribution of innovations, which is assumed to be a Gamma³ with an estimated parameter θ of about 6.00 on Bitstamp and Bitfinex, fits well the data despite of some outliers. This evidence is reported in the QQ-plot (3.5) for all the liquidity variables and exchanges.

³The generalised Gamma distribution for the systems of shocks may be proposed but it involves a lot more parameters, increasing the complexity and randomness in the final outcome.

Table 3.1: *Parameter estimates for Bitstamp*

Parameters	Corwin & Schultz	Abdi & Rinaldo
ω	2.4475 (1.6115)	2.3851* (1.2813)
α	0.5107*** (0.0456)	0.5084*** (0.0464)
β	0.2446*** (0.0542)	0.2458*** (0.0617)
γ	0.4893*** (0.1103)	0.4916*** (0.1199)
m	-7.6787** (3.9187)	-7.5067** (2.9337)
θ_{CS}	0.8659*** (0.0497)	- -
$\omega_{2,CS}$	3.8065*** (0.8540)	- -
θ_{AR}	- -	1.1704*** (0.3852)
$\omega_{2,AR}$	- -	3.0502*** (0.8588)
θ	6.4826*** (0.4905)	6.4851*** (0.4955)
BIC	-8.7683	-8.7709
HL	2.4707	2.4573

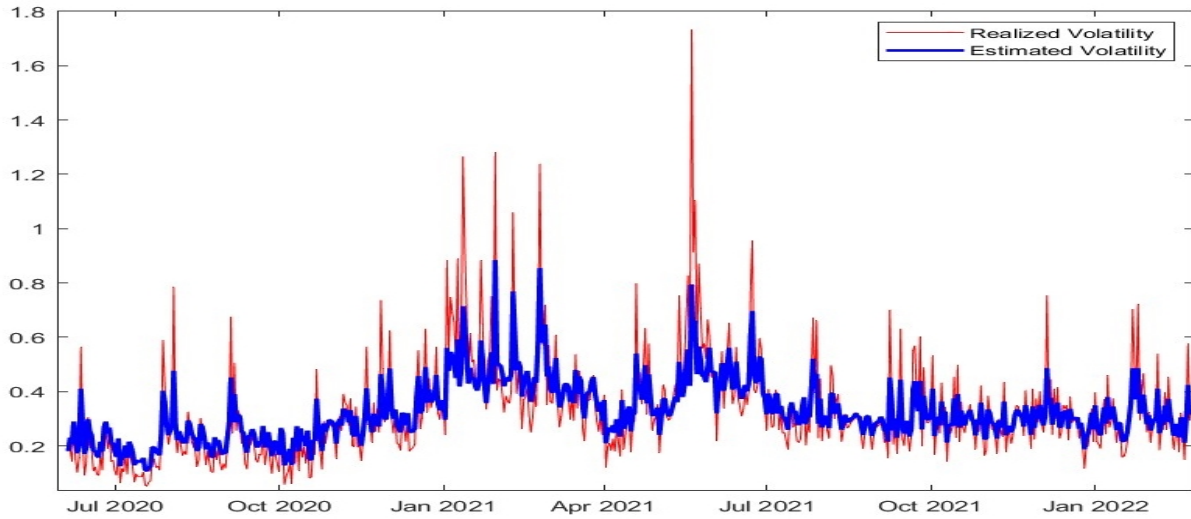
Note: The table reports the coefficients of the different LogMEM-MIDAS model specifications estimated via QMLE. *, **,*** represents the significance at 10%, 5% and 1%, respectively. BIC, rescaled by a factor 10^{-2} , is the Bayesian information and criterion. The number in parentheses are robust standard errors computed with HAC estimator. θ_{CS} , θ_{AR} and the respective standard errors are rescaled by multiplication of 10^{-2} . For liquidity variables in MIDAS filter, twenty-four lags ($K = 24$) are taken to model τ_t .

Table 3.2: *Parameter estimates for Bitfinex*

Parameters	Corwin & Schultz	Abdi & Rinaldo
ω	2.2645 (1.4009)	2.3621* (1.2450)
α	0.5104*** (0.0428)	0.5160*** (0.0430)
β	0.2448*** (0.0620)	0.2420*** (0.0632)
γ	0.4896*** (0.1241)	0.4840*** (0.1252)
m	-7.2255** (3.2381)	-7.5304*** (2.8051)
θ_{CS}	1.3047*** (0.4236)	- -
$\omega_{2,CS}$	2.4622*** (0.6228)	- -
θ_{AR}	- -	1.3475*** (0.4399)
$\omega_{2,AR}$	- -	2.6011*** (0.6783)
θ	6.6434*** (0.4759)	6.6211*** (0.4744)
BIC	-9.0069	-8.9847
HL	2.4688	2.5016

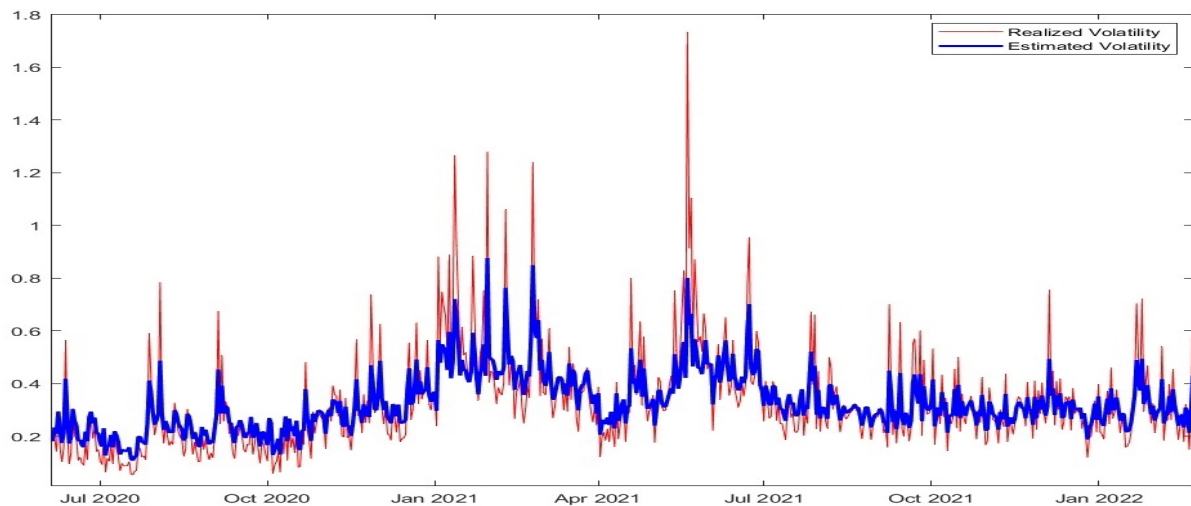
Note: The table reports the coefficients of the different LogMEM-MIDAS model specifications estimated via QMLE. *, **, *** represents the significance at 10%, 5% and 1%, respectively. BIC, rescaled by a factor 10^{-2} , is the Bayesian information and criterion. The number in parentheses are robust standard errors computed with HAC estimator. θ_{CS} , θ_{AR} and the respective standard errors are rescaled by multiplication of 10^{-2} . For liquidity variables in MIDAS filter, twenty-four lags ($K = 24$) are taken to model τ_t .

Figure 3.1: *Estimated volatility for Bitfinex*



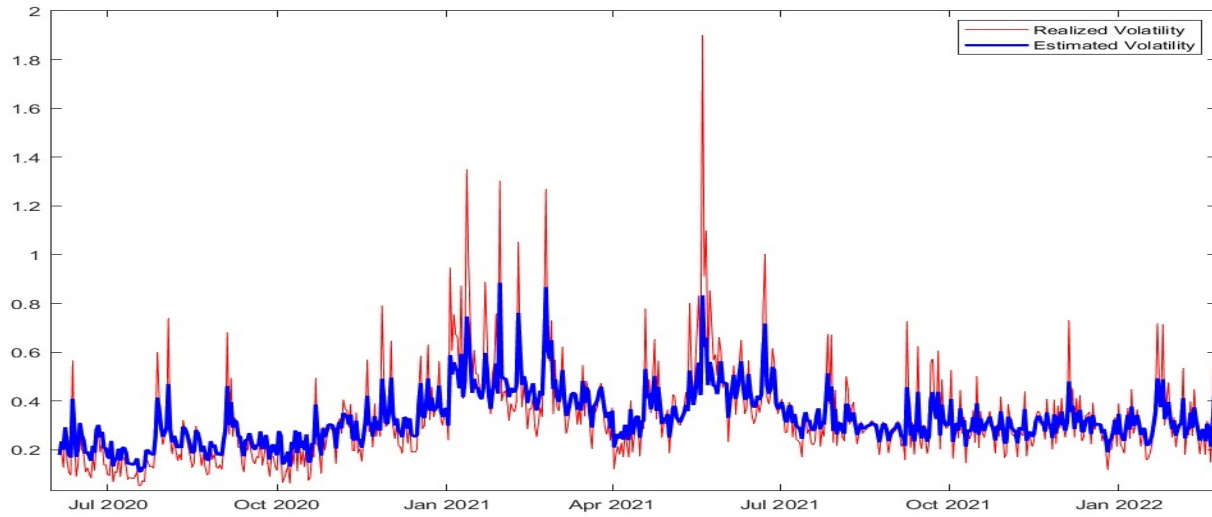
Note: The figure shows the bitcoin realized volatility (red line) and the logMEM-MIDAS estimated volatility (blue line), including [Corwin and Schultz \(2012\)](#) estimator, on Bitfinex over the period from 04 June 2020 to 23 February 2022.

Figure 3.2: *Estimated volatility for Bitfinex*



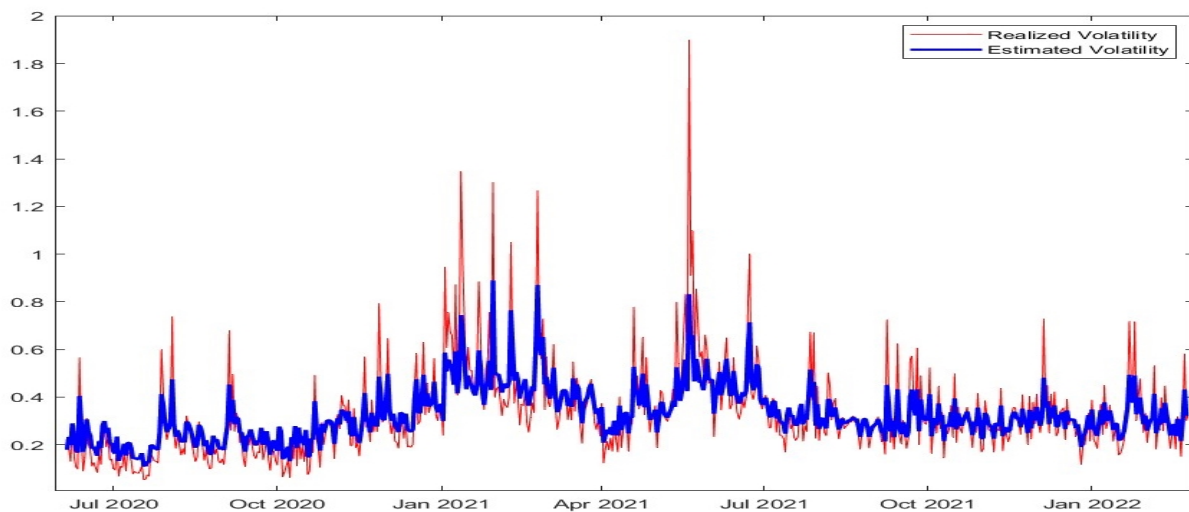
Note: The figure shows the bitcoin realized volatility (red line) and the logMEM-MIDAS estimated volatility (blue line), including [Abdi and Rinaldo \(2017\)](#) estimator, on Bitfinex over the period from 04 June 2020 to 23 February 2022.

Figure 3.3: *Estimated volatility for Bitstamp*



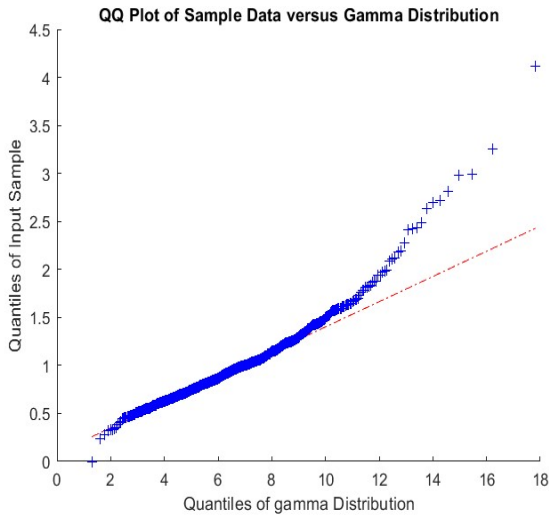
Note: The figure shows the bitcoin realized volatility (red line) and the logMEM-MIDAS estimated volatility (blue line), including [Corwin and Schultz \(2012\)](#) estimator, on Bitstamp over the period from 04 June 2020 to 23 February 2022.

Figure 3.4: *Estimated volatility for Bitstamp*

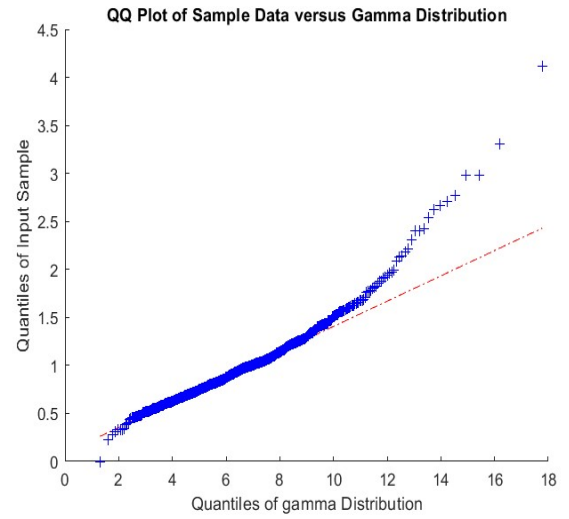


Note: The figure shows the bitcoin realized volatility (red line) and the logMEM-MIDAS estimated volatility (blue line), including [Abdi and Rinaldo \(2017\)](#) estimator, on Bitstamp over the period from 04 June 2020 to 23 February 2022.

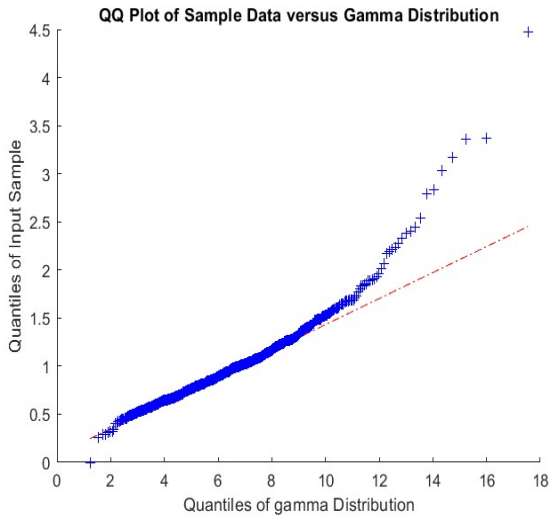
Figure 3.5: *Quantile-Quantile (QQ) plot*



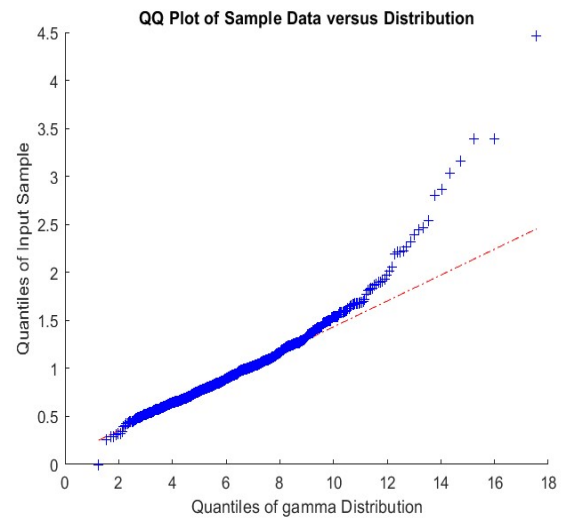
(a) QQ plot of errors from logMEM-MIDAS, including [Corwin and Schultz \(2012\)](#) estimator, for Bitfinex.



(b) QQ plot of errors from logMEM-MIDAS, including [Abdi and Rinaldo \(2017\)](#) estimator, for Bitfinex.



(c) QQ plot of errors from logMEM-MIDAS, including [Corwin and Schultz \(2012\)](#) estimator, for Bitstamp.



(d) QQ plot of errors from logMEM-MIDAS, including [Abdi and Rinaldo \(2017\)](#), for Bitstamp.

Note: QQ plot for Gamma distribution of the errors derived from the different specifications of logMEM-MIDAS models.

Conclusions

In this study, we propose an innovative and versatile class of component volatility model, logMEM-MIDAS, to address the research question regarding the nexus between liquidity and volatility in the cryptocurrency markets. For this purpose, we analyse the high-frequency realised volatility, combining the well-known MEM framework, introduced by [Engle \(2002\)](#), with MIDAS filters. We propose the logMEM-MIDAS, an extension of the MEM-MIDAS introduced by [Amendola et al. \(2021\)](#). The dynamics of realised volatility depend on short and long-run component, where the latter is a function of selected liquidity proxies, [Corwin and Schultz \(2012\)](#) and [Abdi and Rinaldo \(2017\)](#), and thus incorporate liquidity information directly. Therefore, we study how changes in our economic variables of interest, sampled at a lower frequency, influence short-run sources of high-frequency volatility.

In our analysis, we consider the main cryptocurrency traded, BTC/USD, from the largest exchanges, Bitstamp and Bitfinex, over the period from 16 May 2018 to 04 May 2022. Specifically, we retrieve transaction data, OHLC prices, by using an algorithm to continuously access the public and freely accessible REST APIs of two selected exchanges. Then, we estimate the univariate version of our LogMEM-MIDAS based on robust realised volatility and liquidity estimators.

Furthermore, we state that the proposed approach has enormous advantages in terms of cost and process time savings, and can be easily implemented. This is due to the choices as liquidity measures of [Corwin and Schultz \(2012\)](#) and [Abdi and Rinaldo \(2017\)](#) based on easy-to-access and easy-to-process transaction data. In particular, the selected transaction-based aggregate measures provide the same performance as high-frequency benchmarks in capturing the time-series variation of liquidity, irrespective of observation frequency and trading venue ([Brauneis et al. \(2021\)](#)). Thus, they provide an acceptable trade-off between

accuracy and computational workload.

Based on our model, we find a strong and positive nexus between volatility and liquidity. Specifically, we document that positive change in liquidity leads to positive variation in volatility. Investor seems to be attracted to the cryptocurrency markets during periods when trading activity is more pronounced, perhaps seeking higher returns. Moreover, this evidence supports the perception of cryptocurrencies as speculative investments.

Our results allow us to understand the main drivers of volatility and contribute significantly to the existing literature. In particular, researchers, practitioners, and central authorities can use our findings to comprehend the microstructure behaviour between liquidity and volatility and the potential effects in response to legislative or market changes.

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Summary

Bitcoin and cryptocurrencies represent the most revolutionary breakthrough in the financial system. Bitcoin, introduced in 2008 by Satoshi Nakamoto, aims to overcome the shortcomings of fiat and gold-based currencies by providing alternative and digital money within a decentralised peer-to-peer system. However, despite the growth in the importance of cryptocurrencies as a form of payment, they have become more of a speculative investment vehicle than an alternative currency (Yermack (2015), Baur et al. (2015), Baur et al. (2018)). Indeed, cryptocurrencies could provide higher returns (Hong (2017)) and better diversification due to the weak correlation with traditional asset classes (Baur et al. (2015)). They play a crucial role in an investor's portfolio, but they are subject to bubble behaviour and enclose idiosyncratic risks hard to hedge (Corbet et al. (2018)).

Investors can trade cryptocurrencies on exchanges, which open 24 hours a day and 365 days a year and have unique characteristics. They are highly fragmented, weakly regulated, and have varying degrees of efficiency. Indeed, the fragmentation, combined with the total absence of price integration (Makarov and Schoar (2020)) and the lack of harmonised regulation, makes the cross-sectional assessment of liquidity and volatility with high-frequency data difficult.

Actually, to characterise the time-series variation of liquidity on cryptocurrency markets, we rely on monthly liquidity proxies derived from easy-to-access transaction data. We compute two liquidity measures, Corwin and Schultz (2012) and Abdi and Rinaldo (2017), based on high, low, and closing prices sampled at hourly frequency. In particular, the selected transaction-based aggregate measures provide the same performance as high-frequency order book measures (e.g. Bid-Ask Spreads, Percentage Quoted Spread, Percentage Effective Spread, Percentage cost of a roundtrip trade) in capturing the time-series variation of liquidity, irrespective of observation frequency and trading venue (Brauneis et al. (2021)). Thus, they provide an acceptable trade-off between accuracy and computational workload.

However, the main aim of our analysis is to study the nexus between liquidity and volatility in an innovative econometric framework, designed to explain empirical regularities of BTC/USD. For this purpose, we analyse the high-frequency realised volatility by

introducing a new methodology, which embeds the logarithmic multiplicative error model (LogMEM; [Bauwens and Giot \(2000\)](#), [Nguyen et al. \(2020\)](#), [Alexander et al. \(2021\)](#)) context in mixed data sampling, or MIDAS ([Engle et al. \(2013\)](#), [Ghysels et al. \(2005\)](#), [Ghysels et al. \(2007\)](#)). Formally, the model is an extension of MEM-MIDAS introduced by [Amendola et al. \(2021\)](#). Particularly, the dynamics of realised volatility is decomposed into short and long-run component. The former is the conditional mean which depends, inter alia, on the long-run evolution of volatility. The latter obtained by smoothing our variable of interest, [Corwin and Schultz \(2012\)](#) and [Abdi and Rinaldo \(2017\)](#) estimator, in the spirit of MIDAS regression. In this way, we include information derived from proper economic variables aggregated at a lower (monthly) frequency. Therefore, the main drivers of long-term market volatility can be incorporated into the analysis at a higher (daily) frequency. To estimate the model, we derive a robust realised volatility estimator ([Jacod et al. \(2009\)](#)) to account for the impact of microstructure noise. Furthermore, given the dramatic ups and downs in the price of cryptocurrencies, we winsorize the realised volatility values to diminish the influence of outliers.

Based on our model, we document a positive nexus between volatility and liquidity. Specifically, we find that high liquidity leads to high volatility. Investor seems to be attracted to the cryptocurrency market during periods when trading activity is more pronounced, perhaps seeking higher returns. Moreover, this evidence supports the perception of cryptocurrencies as speculative investments. Our results are in line with the findings of several empirical analyses in the same field, such as [Brauneis et al. \(2021\)](#) and [Corbet et al. \(2022\)](#). However, we enrich the current literature with an extension of the well-known econometric framework, based on the multiplicative error model, including the contribution of easy-to-compute liquidity variables through MIDAS filtering.

Finally, our findings have important implications for scholars, practitioners, traders, trading platforms operator and regulation authorities. Our contribution helps them understand the nexus between liquidity and volatility on cryptocurrency exchanges with a relatively inexpensive and simple process.