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The role of climate change on inflation forecasting using time-varying parameters model

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Introduction

Euro area annual inflation rate is expected to continue increasing and to be above 8% in 2022 and decrease to 4.6% next year. If we look at the main components of euro area inflation, energy will have the highest annual rate, reaching 38.3% in August 2022. This inflation surge can partly be explained by some pandemic-driven shocks such as the shift in consumption patterns, bottlenecks in global shipping, and shortages in the labor market. Moreover, Russia's invasion of Ukraine has impacted global energy and food supply, driving even upwards their prices, with consequences all over the world. Largely ignored as an inflationary driver, however, is climate change. There are different channels through which climate change affects the economy. For instance, droughts or flooding could destroy crops and affect agricultural products' prices, major weather disasters imply that housing could become more expensive, insurance companies could raise premiums, transportation costs might rise, and many others.

According to the latest research from the WMO, greenhouse gas concentrations, sea level rise, ocean heat and ocean acidification hit new records in 2021. Extreme weather led to hundreds of billions in economic losses and triggered food and water security shocks. Conflicts, abnormal weather conditions and economic shocks exacerbated by the Covid-19 pandemic caused a worsening in humanitarian crises, increased internal displacement and accelerated the degradation of ecosystems[1].

In a longer-term perspective, inflation in the euro area is likely to be influenced by some structural changes. Climate mitigation policies will impact energy prices, and changes in relative prices affect the overall inflation path. By the end of this decade, even if there could be upward pressures on consumer energy prices and increased price volatility, such pressures might ease as the share of renewable sources for energy will grow faster[2].

[1] World Meteorological Organization (May 2022).

[2] Nickel et al. (2022).

The research question of this thesis is inspired by the fact that climate-related supply shocks are much more frequent with respect to the past, and hence, it could be useful to understand which is the role of climate risks on inflation.
The methodology used is the Dynamic Model Averaging (DMA) forecasts. This choice is because there is strong empirical evidence of time variation in the inflation process and also since there is no unique measure of overall economic activity that provides accurate forecasts in all time periods. Using DMA allows us to consider time-varying parameters for inflation in each forecasting model and also to choose which is the best model for forecasting at each point in time. The algorithm used for estimation is the same as in Koop and Korobilis (2012), with the addition of a climate-related predictor, which is CO2 emissions growth. Results show that CO2 emissions have predictive power for inflation, particularly in medium-term forecasts and after the Great financial crisis. Hence, climate shocks are relevant for the conduct of monetary policy and, in the end, there is hope that also Central Banks will contribute to the transition to a more environmentally sustainable steady state.
Furthermore, the increasing frequency of physical risks stemming from climate change might cause short-term fluctuations in prices that amplify inflation volatility. Therefore the linkage between inflation risk and climate-related risk should be analyzed more in detail. To this end, an extension of the previous research is proposed using the quantile regression model with time-varying parameters presented by Korobilis et al. (2021).
The rest of this thesis is organized as follows. Chapter 1 includes an overview of the causes for the current surge in inflation rates and analyzes the role of climate change on the economy. Then, some empirical evidence and summary statistics are provided. Chapter 2 deals with the model used for estimation. Chapter 3 presents the results and reports which is the role of CO2 emissions in inflation forecasting. Moreover, some policy implications are discussed. In Chapter 4 is addressed the extension for the forecast of inflation risk. The last section concludes the thesis.

Chapter 1

Inflation and Climate change

1.1 Inflation

In modern economies, forecasting how the price level will evolve is an essential part of private sector decision-making. The existence of long-term nominal commitments such as mortgages and labor contracts has raised the prominence of inflation forecasting in policymaking. Moreover, starting from the 1980s the aim of Central Banks is to keep inflation stable, thus optimal policy will depend on optimal forecasts and policy will be most effective when it is well understood by the public[1]. The main feature of this revolution in central banking is the practice of central bankers to announce forecasts of inflation and other key economic variables to boost transparency. In the euro area, the ECB decides on monetary policy every six weeks to keep inflation under control, being price stabilization the primary mandate of our Central Bank, and then the President and the Vice-President explain them in detail in a press conference. The measurement of inflation in the euro area includes all goods and services that households consume, namely everyday items, durable goods, and services. All goods and services consumed by households during a year are represented by a "basket" of items. Every item in the basket has a price and the impact of price changes of the item on the basket depends on a weight that is assigned based on how much households spend, on average, on that product. The annual rate of inflation, thus, is the price of the total basket in a given month compared with its price in the same month one year before[2].

July 2021, for the first time since 2003, the ECB updated its monetary policy strategy.

[1] Faust and Wright (2013).

[2] ECB, "What is inflation?", ecb.europa.eu.

The former inflation target was to set the annual increase of the HICP (Harmonized Index of Consumer Prices) "below but close to 2%", whereas today the target is set at exactly 2% of the annual increase of the HICP over the medium run. This new definition implies that the ECB is willing to tolerate a transitory period of moderate inflation overshoot to push inflation upwards after a period in which the inflation rate in the euro area was close to zero. It is important to stress that, in the aftermath of the financial crisis, the ECB deployed unprecedented measures to push inflation near its target, such as negative policy rates and quantitative easing[3]. Notwithstanding these measures, since 2013 the inflation rate deviated from the ideal path of the 2% annual increase.

Therefore, considering also that the increase in inflation last year was supposed to be temporary, the ECB was not willing to change its current monetary policy. High inflation in the euro area, it was argued, reflects shocks generated abroad. If we decompose HICP inflation into energy and food and items with high and low import content, it can be shown that inflation for items characterized by low import content, which is mainly driven by domestic price pressures, had remained stable[4]. Indeed, the thesis of Phillip Lane, chief economist of the ECB, was that the high inflation rates we have observed over the last year were driven by a pandemic cycle that had generated global bottlenecks for manufactured goods and by the surge in energy prices, and both were thought to be temporary shocks that did not affect long-term inflation.

After the peak of the Covid-19 pandemic, the fast recovery in demand over 2020 was accompanied by a slow recovery of production capacity in some sectors that have generated supply shortages. Sanitary restrictions around the world to reduce the spread of the virus have contributed to the persistent supply disruptions that are restraining the economic recovery and, additionally, Covid-19 outbreaks have led to the closure of key ports, creating bottlenecks in global shipping. On the other hand, there are significant signs of shortages in labor markets that reflect both the change in the location of activities and the skills required from workers and the fact that the pandemic led some people to withdraw from the labor force. Such shortages are more evident in countries and sectors that rely on cross-border inflow into the labor force, which declined by 30%

[3]Hennecke (2021).

[4]Nickel et al. (2022).

last year. As a consequence, wages are subject to upward pressure due to the need to re-hiring workers. In addition, the number of disruptions related to climate change are increasing in recent years and the economic impact of such shocks has increased correspondingly. For instance, in 2020 there have been huge wildfires (Siberia, California and Turkey), heatwaves and droughts (North America), extreme cold weather events and destructive floods (Germany and Belgium), and many others. Therefore, supply disruptions related to the pandemic and other events have contributed to the rise in commodity prices.

Some of those supply disruptions associated with the pandemic were supposed to ease facilitating the global economic recovery and reducing inflationary pressures. The projections implied that output in the advanced economies was likely to converge on the pre-pandemic path[5]. However, longer-term inflation expectations did not remain anchored in advanced economies. During 2020, the ECB Survey of Professional Forecasters computed long-term inflation expectations at 1.64%, and in 2021 they have been moving up to close to 2%, indicating that longer-term expectations have re-anchored at the ECB's target[6].

For what concerns the impact of high energy prices on inflation dynamics, we should consider both the direct effect through the energy component in the HICP, and the indirect effect since energy is an input to produce many other commodities. Also, we should take into account the potential effect on wages and a revision in inflation expectations. According to Lane, the shift in medium-term inflation expectation depends on the intensity and duration of the current burst of inflation, its underlying drivers, and the credibility of the central bank in keeping an inflation rate consistent with the target level. In the euro area, there is evidence that the low inflation environment that prevailed after the financial crisis will not reoccur when the pandemic cycle is over. The first factor indicating a re-anchoring in expectations in line with the 2% level is the huge monetary and fiscal response to the pandemic demonstrating the commitment to stabilize the economy, including the SURE and the NGEU initiative. Secondly, there are some revisions to beliefs about the operation of the world economy such as the trend in rising wages and the transition to a green economy that will reduce the cost of energy

[5]OECD (2021).

[6]Nickel et. al (2022).

in the long run. Hence, monetary policy needs to take into account both near-term and medium-term forces driving inflation dynamics. Accordingly, with its new strategy, the ECB aims to deliver its 2% target over the medium term, meaning that monetary policy tightening is required if current high inflation threatens to persist above the target level over the medium term.

In recent times, just when some of the supply-chain problems caused by the pandemic started to fade, the war in Ukraine created a new negative supply shock. Russia and Ukraine have a limited role in the global economy, as they account for just 2% of global GDP and almost 2% of total global trade, and they have limited financial linkages with other countries as well. However, they are major suppliers in several commodity markets. The two countries together account for about 30% of exports of wheat, 20% of corn, mineral fertilizers, and natural gas, and 11% of oil globally. Exports of metal and other commodities are crucial in the functioning of supply chains around the world. The economic impact of the war is highly uncertain in magnitude, and it depends on the duration of the war and the policy responses, however, there will be surely even stronger inflationary pressures. Simulations run by the OECD show that global inflation could raise by about 2.5% in the first year after the beginning of the conflict under the assumption that the conflict lasts for at least one year and there is a deep recession in the Russian economy. In those simulation studies, European economies will be affected the most due to the increase in gas prices and the strengths of businesses and energy linkages with Russia. Another risk factor is the complete cessation of energy exports from Russia, which will cause abrupt peaks in energy prices given the limited possibility to substitute supplies from other markets in the short term and the low level of gas reserves[7].

Given this scenario, the ECB Governing Council decided to raise the interest rate on the main refinancing operations, the interest rates on the marginal lending facility and the deposit facility by 50 basis points, with effect from the 27th of July 2022.

[7] OECD (2022).

1.2 Climate Change

Climate change, as defined by the Intergovernmental Panel on Climate Change (IPCC), refers to changes in climate due to natural variability or human activity. Scientists showed that changes in the atmospheric presence of greenhouse gases and aerosols, in solar radiation and on land surfaces alter the energy balance of the climate system. In particular, the concentration of carbon dioxide, methane and nitrous oxide (usually referred to as greenhouse gases or GHG) in the atmosphere has increased sharply from the first half of the XVIII century as a result of anthropogenic activities, mainly fossil fuel use and agriculture. Carbon Dioxide (Co2) is the most present anthropogenic greenhouse gas in the atmosphere, accounting for 66% of the radioactive forcing, that is the warming effect on the climate. It is estimated that atmospheric Co2 reached 148% of the preindustrial level in 2019, mostly because of fossil fuel and cement production. Moreover, of the total Co2 emissions from anthropogenic sources from 2008 to 2018 about 44% accumulated in the atmosphere, 23% in oceans and 29% on land. Since most of the Co2 already present in the atmosphere will remain there for several centuries, even when net emissions of Co2 will approach zero the climate will continue to warm. Methane, instead, accounts for about 16% of the radioactive forcing. Almost half of Methane emissions come from natural sources, and the remaining part is due to anthropogenic activities such as rice agriculture, landfills, biomass burning and fossil fuel exploitation. The latter, Nitrous Oxide, is the third most important contributor to global warming, accounting for about the 7% of radioactive forcing. It is emitted from both natural sources (60%) and anthropogenic ones (40%) such as fertilizer use and various industrial processes[8].

In the last decades, there is evidence of increases in global average air and ocean temperatures, increased melting of snow and ice, and rising average sea level, which indicates a warming of the climate system that very likely is due to human activities. Moreover, there have been observed many long-term changes in climate including changes in precipitation amounts, wind patterns and extreme weather events such as droughts, tropical cyclones, and heat waves. According to the environmental analyst of the IPCC (2007), projections show that we will experience a warming of about 0.2 °C per decade in the

[8] WMO (2020).

next 20 years, which will cause further warming and induce many changes in the global climate system that would be very likely greater than those observed during the previous century.

On the other hand, climate change could have an impact on the economy both through gradual warming and through the increased frequency of extreme weather events. Therefore, it is important to understand which are the channels through which climate change can affect the economic system and the possible policy responses. We can distinguish between two different risks that arise from climate change that impact macroeconomy and price stability. The former is the so-called physical risk that pertains to those risks that are due to the vulnerability of exposure of human and natural systems to climate-related hazards. They include both risks from gradual global warming and extreme weather events. Transition risk, instead, is defined as that risk that originates from the transition to a low-carbon economy[9]. To this end, in 2015 the Paris Agreement was adopted by 196 countries with the aim of reducing the global emissions of GHG to limit global warming below 2 degrees Celsius, compared to the pre-industrial level.

The transition to a more sustainable economy will contribute to affecting the outlook on output and price stability through changes in employment, interest rates, investments and productivity, financial stability, and transmission of monetary policy.

There are several linkages between climate and economy that are evolving over time. On one hand, the technological innovations enabling the large-scale use of fossil fuels boosted economic growth, causing, on the other hand, higher GHG emissions. As a consequence of this self-reinforcing mechanism, there is an attempt to try to adjust to climate change through adaptation, such as building stronger sea defenses to avoid damage from rising sea levels. Such activities involve significant investment, implying an impact on the economy. The other response to climate change is the implementation of policies aimed at climate mitigation. One way of doing that is to change the technology that leads to GHG emissions while maintaining a sustained level of economic growth. The transition to a low-emission economy pertains to this kind of policy, however, it implies significant expenditures, investments and changes in relative prices that would have a great economic effect. The last channel to consider regards the interaction of climate change with the financial systems. In recent years, there has been growing

[9]Batten et al. (2020).

recognition that climate issues might be relevant for price and financial stability. Physical risks may affect the banking and insurance sectors and transition risks might lead to some businesses becoming unfeasible. Furthermore, climate change risk accounts for the alteration of the risk profile of the asset held on the balance sheet of the Eurosystem, causing the increase of climate-related financial risks[10].

To sum up, climate change can be interpreted as an adverse permanent shock to the supply potential of the economy. Likely, such shock would push output below its level and lower future potential growth. At the same time, climate change in the short-run could lead to changes in demand conditions, through a reduction in consumption by households and in investment by firms due to greater uncertainty about future economic growth and income prospects.

According to the purpose of this thesis, the focus will be on the linkages between inflation and climate change. In this regard, the agricultural sector is widely exposed to the effect of climate change, thus there is a potential for lasting effects on the prices of agricultural products. Clearly, agricultural yields may rise in some regions of the world and fall in others, hence the final impact is a function of the location of a country and its agricultural imports. Recent studies point out that storms and floods are positively related to increases in inflation in developing countries in the short term and, on the contrary, that droughts have a more persistent effect. Moreover, rising inflation rates may be related to lower supply potential in the economy after climate-related shocks.

It is essential to also consider the contribution of changes in energy prices to inflation. One of the main pillars of climate policy mitigation is, indeed, the transition to sustainable energy sources for production that could potentially reduce the overall expenditure for energy consumption and reduce the weight that energy has in the computation of the consumer price index. However, the downward pressure on inflation might be offset by a "rebound effect" if the income gains emerging from lower expenditure on energy lead households to increase their demand[11].

At the same time, when considering the implication of climate change on inflation it is important to evaluate how durable the impact is, and we still know relatively little about the impact on medium-term inflationary pressure. There is empirical evidence

[10] ECB Press Release (2021).

[11] Andersson et al. (2020).

that over the medium term there are negative inflation dynamics when we experience a rise in the temperatures, especially in emerging economies. This result suggests that the short-run supply turmoil in agriculture can result in longer-lasting lowering pressures on demand[12].

Therefore, it is clear that climate change is a critical issue that needs to be addressed by central banks in the future, consistently with their mandate of price stabilization.

Indeed, the ECB drafted its climate action plan in 2021 as a result of the review of monetary policy strategy. The plan objectives are, among others, accelerating the development of new models and new indicators covering relevant green financial instruments, and introducing climate stress tests to assess the exposure of the Eurosystem to climate change risks[13].

1.2.1 Existing literature about economics of climate change

From an economic point of view, climate change is a public good or externality. The costs or benefits of such activities are not captured by market prices and the effects spill outside the market. In this framework positive spillovers of climate change are new technologies, while the negative spillover is pollution. Most of the research about the economic effects of climate change focuses on the negative externality, that is GHG emissions and global warming[14].

GHG emissions are an externality that differs from the classical examples in that they are global in their impact, their effects are long-lasting, there is high uncertainty in the steps of the scientific path that produces global warming and most likely their effect is irreversible. Therefore, the economic analysis must take into account the economic implication of risk and uncertainty, the policy trade-offs, and the role of international economic policy.

The Stern Review (2006) addresses these issues starting from a crucial assumption which is the choice of a reduction of the stock target for GHG emissions. The first step in this analysis is to define the major risks related to climate change and, hence, to set the targets for stock and flow of emissions and the related costs to mitigate such

[12]Faccia et al. (2021).

[13]ECB Press Release (8th July 2021)

[14]Nordhaus (2019).

risks. The first challenge is to provide estimates of GHG damages, that are highly sensitive to assumptions about the future and highly uncertain. Then, the price mechanism to be put in place in order to reach the targets needs to be defined. Assuming that there will be a cut in global emissions of at least 30% in 2050, given the avoided risk, N. Stern, states that the cost in world GDP is around 1% per annum. Furthermore, the carbon price required to achieve the target may be 30$ per ton of Co2.

For what it concerns the implication of risks arising from climate change for the conduct of monetary policy there are few empirical studies to look at. For instance, a recent study by the Deutsch Central Bank (2022) points out that there is a statistically significant negative correlation between climate concern and expected inflation. They measured climate concern through households' perception of the gravity of the implication of climate change and found out that individuals with higher inflation expectations are those that consider climate change a less serious concern. Although their research design does not allow for any causal statement, it is useful in understanding expectations about inflation that play a key role in the transmission of monetary policy.

Other contributions are those related to the concept of "Environmental Phillips Curve". The Phillips curve states that there is a trade-off between national inflation and unemployment, meaning that when a country can tolerate high inflation, it can reach a low unemployment target. Unemployment and production are related by Okun's law, according to which if unemployment increases, actual GDP will be lower than the country's potential. Given that output and pollution are highly positively correlated, there will be a positive relationship between employment and pollution. Therefore, in this theoretical framework when pollution increases there will be also less unemployment and higher inflation[15].

1.3 Empirical evidence

The aim of this section is to better understand our data, and which could be the relationship between inflation and climate change, before going into the details of the model. During the Covid-19 pandemic, the World Meteorological Organization (WMO), estimated a decrease of about 5.6% of Co2 emissions in 2020. It is predicted, though, that

[15] Kashem and Rahman (2020).

at the global scale, a reduction of this magnitude will not cause atmospheric Co2 to decrease. Notwithstanding the temporary decrease in growth rates of new emissions, the increase from 2019-2020 was still larger than the average annual growth rate over the last decade[16]. Data also show that levels of Co2 emissions continued to grow in 2021.
At the same time, we could observe considerable volatility of inflation worldwide. The pandemic shock required lockdowns which caused a shutdown of businesses, an increase in the prices of certain products, and, in the meantime, there was an unprecedented policy response to the supply shock. In the euro area headline inflation, as measured by HICP, was equal to 1.2% in 2019, decreased to 0.3% in 2020 and became negative in the second half of 2020 before increasing again to 2.6% in 2021. Recently, inflation has reached a historical high of 8.6% in June 2022[17].
This data suggests that there could be a negative relationship between inflation and climate change.
In the following sections it is presented the methodology for the selection of the dataset, the transformations applied to the series, and then a preliminary analysis of data with some summary statistics and correlations.

1.3.1 Data selection

For what concerns inflation, two indicators have been used, the Consumer Price Index (CPI) and the GDP Deflator (GDPDEFL).
The former measures changes over time in the general level of prices for goods and services that households acquire. The latter, instead, measures the ratio of the value at current prices of goods and services an economy produces in a particular year and the value at the prices that prevailed during the base year. It shows how much the change in GDP depends on changes in the price level. Therefore, the GDP price deflator is not based on a fixed basket of items consumed by households, and it captures also changes in consumption patterns. In this analysis, the time series used are, respectively, the CPI for all urban consumers: all Items in U.S. city average, and the US GDP deflator with base year 2015, from "Federal Reserve economic data" dataset. The data is downloaded on a yearly basis choosing a time window from 1959 to 2021 and it is seasonally ad-

[16]WMO (2021).
[17]Nickel et al. (2022).

justed in order to get rid of the effect of changes in prices that normally occur at the same and in the same magnitude every year, such as the effect of holidays. The first step is to compute the inflation rate π:

$$\pi_t = \frac{p_t - p_{t-1}}{p_{t-1}} - 1$$

Then, the first-difference log transformation is applied to the series since log-returns have better mathematical properties. The first introductory step to my analysis is plotting the data to see the general behavior of the series over time to conduct a first graphical analysis.

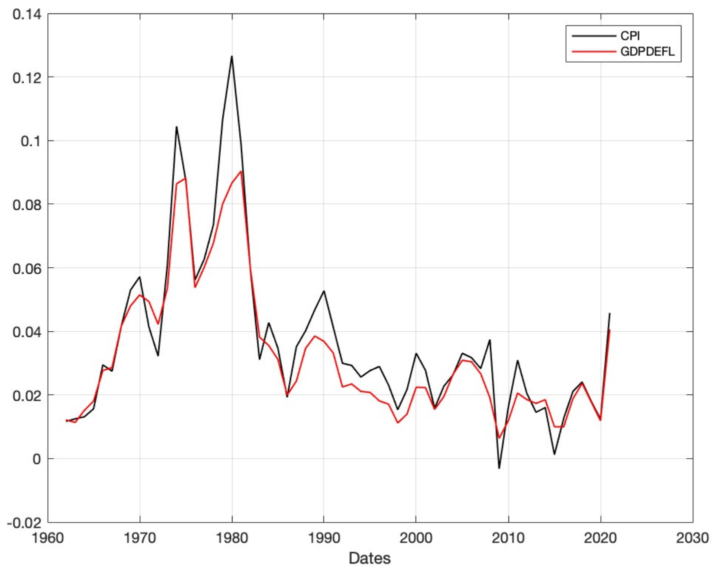


Figure 1.1: CPI and GDP Deflator inflation time series

The plot of both time series does not show the presence of a trend or a seasonal component, hence the process appears to be stationary and, therefore, there is homogeneity in the probabilistic structure of the series. Then, the stationarity of the series is verified by running the Augmented Dickey-Fuller test. It tests the null hypothesis of the presence of unit root in the process of the time series. From the results, the null hypothesis is rejected and hence the process is stationary.

Moreover, in figure 1.2 and 1.3 the autocorrelation and the partial autocorrelation functions of CPI and GDPDELF time series are plotted. In our case the ACF decays to zero after the first few lags and the PACF decays to zero after the first lag, showing small memory in the process.

The earth's changing climate can be measured using different indicators such as global average surface temperature or the level of ocean acidification. However, scientists

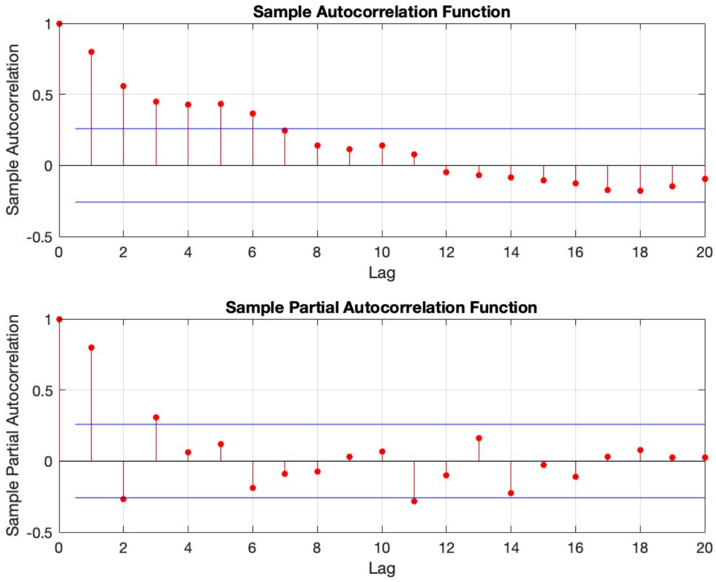


Figure 1.2: ACF and PACF of CPI inflation

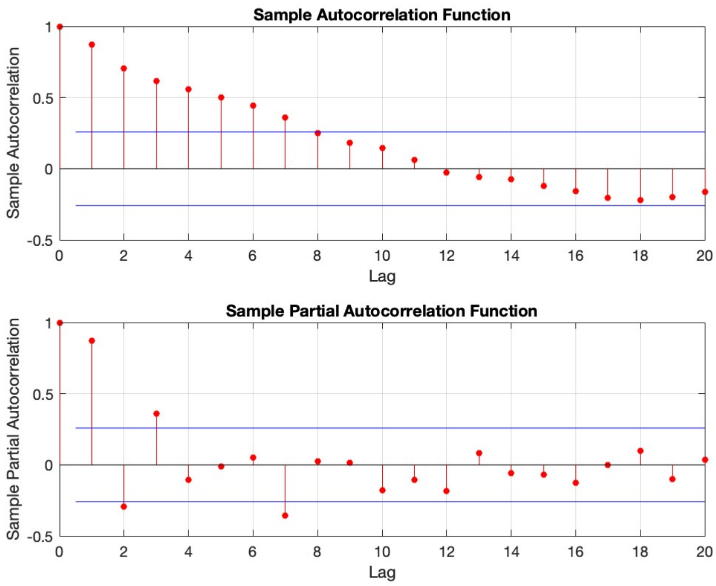


Figure 1.3: ACF and PACF of GDP Deflator inflation

have maintained a long-running observational record of the levels of CO2 since 1950. This is a rough indicator of human-induced climate change since as previously stated it is the most abundant greenhouse gas in the atmosphere and its sources are mainly anthropogenic. In order to model climate change, hence, it is used the time series of CO2 global emissions in millions of tons units from 1961 to 2021. From the series of yearly observations, it is then computed the annual growth rate of carbon dioxide emissions that is easier to interpret. The transformed series is then plotted to perform a graphical analysis.

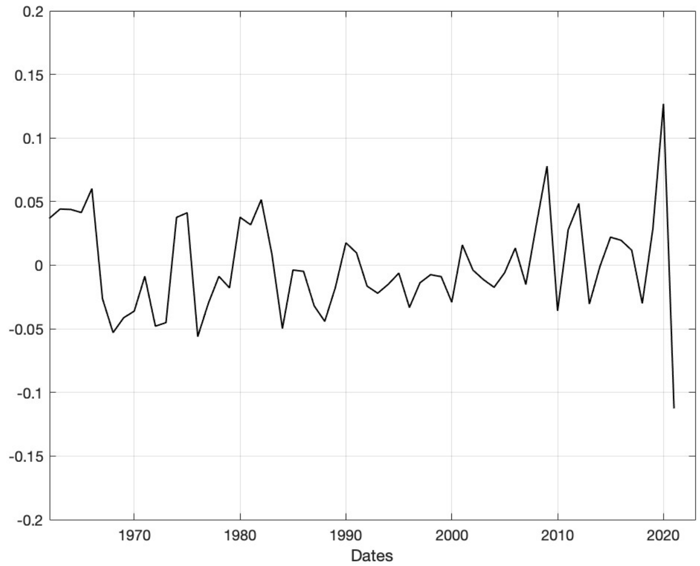


Figure 1.4: Co2 emission growth time series

From figure 1.4 it is possible to guess that the series is stationary, and the result is also confirmed by the ADF stationarity test. Moreover, the plot of ACF and PACF of the time series shows no serial autocorrelation.

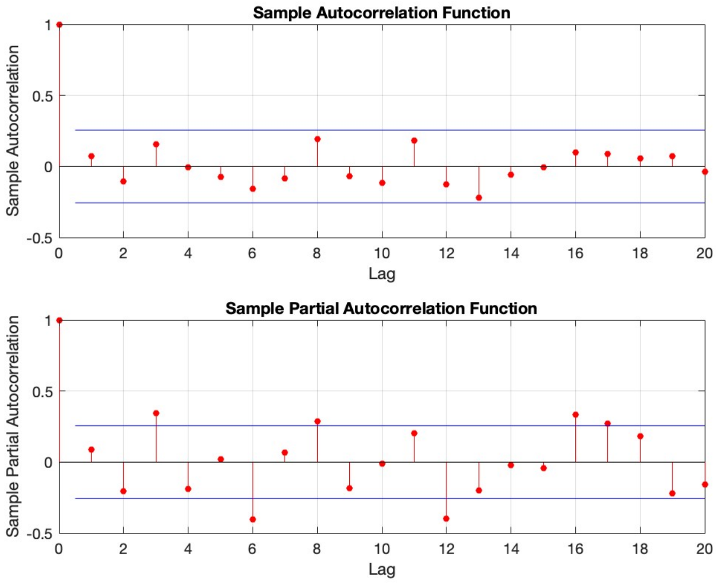


Figure 1.5: ACF and PACF of Co2 emission growth

1.3.2 Summary statistics

The table below shows summary statistics.

	Cpi Inflation	Deflator Inflation	CO2 Growth
Minimum	-0.0032	0.0063	-0.11258
Maximum	0.1266	0.0903	0.12686
Mean	0.0367	0.0325	-0.0008
Variance	0.0006	0.0004	0.0014
Standard deviation	0.02605	0.02157	0.03845
Skewness	1.5393	1.3021	0.37431
Kurtosis	5.3376	3.8982	4.33419

Table 1.1: Summary statistics

The table shows an average value for inflation of about 3% both for the CPI indicator and the GDP Deflator. The growth in CO2 emissions shows a negative average value of -0.09%. The minimum value for the inflation rate was reached in 2009, in correspondence with the world financial crisis. Instead, the lower value for CO2 emission growth was registered in 2020 due to restrictions and shutdowns imposed to contain Covid-19 virus. The highest values for inflation were reached in the decade' 70-'80, and the maximum was hit in 1974. For what it concerns climate change, the highest growth in emissions is found in 2019.

The data shows also a right-skewed distribution for inflation, which implies a fatter tail on the right side of the distribution, and excessive kurtosis for the CPI-based inflation. Lastly, the correlation between inflation and CO_2 emission growth is negative and equal to -0.09 and -0.10, for inflation computed with CPI and GDP Deflator, respectively. The latter result is consistent with our guess of a negative relationship between price levels and climate change.

From now on, the analysis will be carried out with the aid of more complex and statistically robust models to assess whether this result is confirmed or rejected.

Chapter 2

Inflation Forecast

2.1 Phillips curve inflation forecast

The most successful approaches to forecasting inflation are those based on extensions of the Phillips Curve. Therefore, the first step in understanding the model used to perform the empirical analysis is to review some of the most influential papers addressing this topic. Following Stock and Watson (2008), the focus will be on models that conduct a true or pseudo out-of-sample forecast evaluation. With this methodology it is possible to estimate the model based on data up to date t, construct a forecast for the values at time $t+h$ and compare the result of the forecast with the actual values registered in $t+h$, then moving forward to date $t+1$ and repeat the procedure through the sample.

The review of literature begins in 1958 when A. W. Phillips found empirical evidence of a negative relationship between inflation and unemployment. Two years later, the economists Solow and Samuelson repeated Phillips's exercise using data for inflation in the US from 1900 to 1960 and found the same results. They called this negative relationship "Phillips' Curve". This suggested that policymakers could choose between different combinations of the unemployment rate and inflation, and in the following years the economic debate focused mainly on which point in the Phillips curve was best for a country's economy. However, during the 70s the relationship of the Phillips Curve seemed to disappear. Indeed, when the Organization of Petroleum Exporting Countries was created in 1973 many countries experienced a period of high inflation, driven upwards by the rise in oil prices, and growing unemployment. It became clear that when non-labor input prices change dramatically, the relationship implied by the

Phillips curve does not exist anymore in the economy[1].

Consequently, between 1980 and 1990, the approach to forecasting inflation changed with respect to "non-accelerationist" Phillips curves due to a mismatch in the classical relationship between unemployment and price levels[2]. Many economists argued that the most performing models in terms of forecast were those built around the Gordon triangle model (1982) and its variants.

In the triangle model inflation is specified by the following equation:

$$\pi_{t+1} = \mu + \alpha^G(L)\pi_t + \beta(L)u_{t+1} + \gamma(L)z_t + v_{t+1}, \tag{2.1}$$

where u_t is the unemployment rate, z_t represent supply shock variables, and $\alpha(L)$, $\beta(L)$, $\gamma(L)$ are polinomials in the lag operator L.

In eq. (2.1) inflation depends on inflation already built into the economy, contemporaneous and previous lags of the unemployment gap, which represents demand-pull inflation, and energy and food price shocks that affect aggregate supply, that is the cost-push inflation. The unemployment gap is defined as the difference between the unemployment rate and the NAIRU (non-accelerating inflation rate of unemployment), which is the unemployment rate at which inflation remains constant over time. The main idea is that when unemployment is below the NAIRU, inflation tends to rise over time, and vice versa when the unemployment gap is positive. Modern specifications of the Phillips curve that use the unemployment gap are also called "NAIRU Phillips curves". In those models, unemployment and the level of economic activity are used to forecast future inflation rather than the inflation rate itself and they were introduced after the period of high volatility in inflation of the 1970s when the negative relationship between inflation and unemployment seemed to disappear. In those years, the inflation rate became more persistent and, the Triangle model reflects this shift in inflation behavior.

Later, there were some studies about the deterioration of the Phillips curve forecast. Most of the research during the 1990s documented that activity-based inflation forecasts had a small advantage relative to an AR benchmark after 1985 with respect to before.

Moreover, Stock and Watson (1999) provided evidence that the US Phillips curve is not stable over time and that the performance of the forecast can be improved by adding

[1]Kashem and Rahman (2020).

[2]Stock and Watson (2008).

alternative measures of aggregate activity other than unemployment. The specification of inflation in this model is:

$$\pi_{t+h}^{h} - \pi_t = \phi + \beta(L)u_t + \gamma(L)\delta\pi_t + \varepsilon_{t+h},$$

where π_t^h is the h-period inflation in the price level P_t, reported at an annual rate; π_t is monthly inflation at the annual rate and u_t is the unemployment rate. In the latter equation inflation is integrated of order one, and the constant term in the regression captures the NAIRU rate that is assumed to be constant.

The first result of Stock and Watson's analysis is that there is statistical evidence of time-changing coefficients of the Phillips curve. Time variation could be due to the natural level of unemployment, which is a function of all the political and economic variables that determine the level of wages; hence, it is not surprising that the NAIRU is different across time and countries. Amongst the different causes that could bring a variation in the NAIRU, there are government subsidies, employment protection framework, the existence of minimum wage, and the monopolistic power of firms.

Surprisingly, according to Stock and Watson, instability stems from changes in the contribution of lags of inflation in the Phillips curve and not from a shift in the NAIRU. However, there is evidence that in the US there has been a decrease in the NAIRU in the 90s due to several factors, such as increased competition between US firms and the rest of the world, and the aging population [3]. Indeed, more recent studies have challenged the results of Stock and Watson, since in 1999 the instability registered in the mid of the 1990s was too recent to have clear results given the limited amount of data.

The second important contribution of this research is to have shown that including alternative measures of real economic activity, namely housing starts or the growth of trade sales, can improve the forecasting performance of the model.

In 2001 another fundamental piece of evidence was published by Atkenson and Ohanian. Their research aimed to prove the usefulness of NAIRU Phillips curves in forecasting inflation. The methodology consists in comparing the accuracy of forecasts made with the classical specification of the Phillips curve used in Stock and Watson (1999), with a naïve model that predicts that the inflation rate follows a random walk process.

Over the period between 1984 and 1999 the AO forecast seems to outperform the Phillips curve forecast. Therefore, the problem with Phillips curve models was that they

[3] Blanchard et al. (2008).

performed poorly starting from the 90s, a period of strong, but at the time unmeasured, productivity growth that pressured inflation downwards.

After the publication of Atkenson and Ohanian paper, it was clear that there was no unique measure of overall economic activity to be used in the forecasting model. This suggested modeling the huge number of activity indicators using a dynamic factor model. Those models are flexible in that the observed dependent variable is a linear function of exogenous covariates and unobserved factors that have an autoregressive structure. In this way, economists were able to estimate the common latent factor, that is the economic activity, and use that estimation as the activity variable in the forecast carried out with Phillips curve models.

Other statistical methods that were proposed to overcome the problems of dealing with a very large number of predictors are model combination or model averaging, such as Bayesian Model Averaging (BMA) or bagging. Another approach is to model all series simultaneously using high-dimensional VARs with strong parameter restrictions. However, the gains obtained performing forecasts with many predictors are not systematic and they seem to not overcome Atkenson and Ohanian's results. Moreover, according to Stock and Watson (2008), who reviewed and tested the forecasting accuracy of all previously stated models, the performance of the Phillips curve model is episodic in relation to univariate forecasts. Their result suggests that when the unemployment rate is close to the NAIRU, is better to use a univariate forecast, whereas when the unemployment gap is big, it should be included in the covariates. To sum up, all this evidence points out that there is strong empirical evidence of time variation in the inflation process. In this thesis, I follow the approach of Koop and Korobilis (2012) to address the problem of time variation in the model relevant for forecasting.

2.2 Bayesian approach to time-varying parameters models (DMA)

This econometric methodology aims to address three different issues that arise when estimating future inflation: firstly, the slope of the Phillips curve changes over time and there is empirical evidence of structural breaks in the time series; secondly, the number of potential exogenous variables can be very large; and, lastly, the best model for forecasting might also be changing over time. This leaves us with a very complex problem in terms of the number of data that needs to be processed. For instance, let's suppose that the set of models is defined by whether a given potential predictor is included or not in the estimation. If there is a set of m predictors, the forecaster has 2^m models to evaluate. If we assume also, that at any point in time a different forecasting model needs to be used, at time τ the number of models to be estimated in order to have a forecast is $2^{m\tau}$, that is an incredibly huge amount of data to be evaluated. Koop and Korobilis (2012) consider in their research a strategy that is called dynamic model averaging (DMA), which allows for the forecasting model to be time-varying and the coefficients in each model to also change over time.

The starting point is the following generalized Phillips curve:

$$y_t = \phi + x'_{t-1}\beta + \sum_{j=1}^{p} \gamma_j y_{t-j} + \varepsilon_t \tag{2.2}$$

where y_t stands for inflation and x_t is a vector of predictors. This representation allows to conduct forecasts h-steps ahead directly, simply replacing y_t and ε_t with y_{t+h-1} and ε_{t+h-1} in (2.2).

Then, the problem of time variation in parameters needs to be addressed. Usually, models that include time-varying parameters (TVP) are estimated using state space methods such as the Kalman filter.

2.2.1 State space filters for TVP models

Kalman filter is an algorithm that computes estimates based on linear dynamical systems in state space form. In general, there is a process model which defines the evolution of the state over time, and a measurement model that defines the relationship between the state and the observations at current time. The specification of the model in our framework becomes:

$$y_t = z_t \omega_t + \varepsilon_t$$

$$\omega_t = \omega_{t-1} + \eta_t$$

With y_t that is inflation, z_t is a $mx1$ vector of predictors of inflation including: a constant, the vector of exogenous variables x_{t-1}, and previous p lags of inflation. ω_t is a $mx1$ vector of coefficients, such that $\omega_t = [\phi_{t-1}, \beta_{t-1}, \gamma_{t-1}, \dots, \gamma_{t-p}]$. Finally, the errors, ε_t and η_t, are assumed to be mutually independent at all leads and lags, and i.i.d. normally distributed with zero mean and variance equal to, respectively, H_t and Q_t. Let's define also and $y^t = (y_1, \dots, y_t)'$.

Usually, the variance-covariance matrices of errors are used as tuning parameters since the true statistics of the noises are not known.

The algorithm usually works in two stages: prediction and update. In general, starting from an initial condition, the first step is to define the prior distribution in order to project the value in the following time period. Then, the prior values are used in the measurement part, so to obtain the posterior values and update the estimates and the error covariance. For given values of H_t and Q_t, the predictive distribution is:

$$y_t|y^{t-1} \sim N\left(z_t\hat{\theta}_{t-1}, H_t + z_t\Sigma_{t|t-1}z_t'\right).$$

Then, it proceeds using

$$\theta_t|y^{t-1} \sim \left(\hat{\theta}_{t-1}, \Sigma_{t|t-1}\right),$$

and the update in Kalman filtering is

$$\theta_t|y^t \sim N(\hat{\theta}_t, \Sigma_{t|t}).$$

Where,

$$\hat{\theta}_t = \hat{\theta}_{t-1} + \Sigma_{t|t-1}z_t(H_t + z_t\Sigma_{t|t-1}z_t')^{-1}(y_t - z_t\hat{\theta}_{t-1}) \tag{2.3}$$

and

$$\Sigma_{t|t} = +\Sigma_{t|t-1} - \Sigma_{t|t-1}z_t(H_t + z_t\Sigma_{t|t-1}z_t')^{-1}z_t + \Sigma_{t|t-1}, \tag{2.4}$$

$$\Sigma_{t|t-1} = \Sigma_{t-1|t-1} + Q_t. \tag{2.5}$$

This latter equation can be simplified using the approximation in Raftery et al. (2007) according to which $Q_t = (1-\lambda^{-1})\Sigma_{t-1|t-1}$, with $0 < \lambda \leq 1$. The parameter λ will be called "forgetting factor" because it implies that observations that are t periods in the past have weight λ^i. Hence, the only unknown to be estimated or simulated is H_t.

2.2.2 Dynamic Model Averaging

The model specified in the previous section, however, has the drawback that it considers the same set of explanatory variables to be always relevant. Hence, when the number of predictors m is very large, the model could over-fit in-sample and forecast poorly. To overcome this issue, Koop and Korobilis propose an application of the classical Kalman filter algorithm to allow for averaging across different kinds of models at each time and choose the best one in terms of forecast. Let's define, therefore, the DMA model equations as:

$$y_t = z_t^{(k)}\theta_t^{(k)} + \varepsilon_t^{(k)}$$

$$\theta_{t+1}^{(k)} = \theta_t(k) + \eta_t^k,$$

with $k = 1, \ldots, K$, $\varepsilon^{(k)} \sim N(0, H_t^{(k)})$ and $\eta_t^{(k)} \sim N(0, Q_t^{(k)})$. In this specification, there is a set of K models characterized by having different subsets of predictors in z_t. Moreover, let's define $L_t \in 1, 2, \ldots, K$ as the model that applies at time t, and the vector $\Omega_t = (\theta_t^{(1)\prime}, \ldots, \theta_t^{(K)\prime})'$.

DMA prescribes to compute the probability that a given model applies in time t given variable information through time *t-1*, that is:

$$\mathbb{P}(L_t = k | y^{t-1})$$

for k=1, ...,K, and averaging forecasts across models using those probabilities. The model, therefore, should be completed by a transition matrix that specifies how predictors enter or leave the model in real time. The elements of this transition matrix, P, would be $p_{ij} = \mathbb{P}(L_t = i | L_{t-1} = j)$ for $i, j = 1, \ldots K$.

Of course, the estimation can take a long time when the number of predictors is large using this methodology, hence, they use an approximation that allows for the use of Kalman filter. Such approximation involves the definition of two forgetting factors, λ, as defined in section 2.2.1, and α.

To better understand the role of these parameters let's rewrite the Kalman algorithm for the multi-model case:

$$\Theta_{t-1} | L_{t-1} = k, y^{t-1} \sim N\left(\hat{\theta}_{t-1}^{(k)}, \Sigma_{t-1|t-1}^{(k)}\right) \tag{2.6}$$

$$\Theta_t | L_t = k, y^{t-1} \sim N\left(\hat{\theta}_{t-1}^{(k)}, \Sigma_{t|t-1}^{(k)}\right) \tag{2.7}$$

and,

$$\Theta_t | L_t = k, y^t \sim N\left(\hat{\theta}_t^{(k)}, \Sigma_{t|t}^{(k)}\right). \tag{2.8}$$

Also here, we can compute $\hat{\theta}_t^{(k)}$, $\Sigma_{t|t}^{(k)}$, and $\Sigma_{t|t-1}^{(k)}$ using equations (2.3), (2.4) and (2.5), considering that the k superscript means that we are computing the state conditional on a certain model k. To be clearer, equation (2.6) only gives information about $\theta_t^{(k)}$, since it is conditional on a given model at time t.

However, we need to derive unconditional predictions. The straightforward way is to compute the transition matrix P as defined above and then use Markov chain Monte Carlo (MCMC) methods to retrieve unconditional results. MCMC methods are used when direct sampling from a target posterior distribution is difficult, and they provide a way to approximate that distribution of interest. Intuitively, MCMC starts with generating some random values, subject to a rule that specifies whether the parameter value is good or bad. For each set of parameter values is possible to compute which is better by assessing how likely that parameter can explain our data. Hence, if the last value estimated is better than the previous one it is added to the chain of parameter values with a given probability. After a period of time, the random samples tend to converge in the region of highest probability for the parameter of interest and the algorithm can start sampling from the estimated posterior distribution. Gibbs sampling is one of the most common MCMC method for obtaining sampling from multivariate probability distributions.

In this framework we need to evaluate the posterior:

$$p(\Theta_{t-1}, L_{t-1}|y^{t-1}) = \sum_{k=1}^{K} p(\theta_{t-1}^{(k)}|L_{t-1} = k, y^{t-1}) Pr(L_{t-1} = k|y^{t-1}),$$

where $p(\theta_{t-1}^{(k)}|L_{t-1} = k, y^{t-1})$ is given by eq. (2.6) and we denote $Pr(L_{t-1} = k|y^{t-1})$ as $\pi_{t-1|t-1,k}$.

Using the elements of the transition matrix, $p_{ij} = \mathbb{P}(L_t = i|L_{t-1} = j)$ for $i, j = 1, \dots K$, the model predicting equation is:

$$\pi_{t|t-1,k} = \sum_{l=1}^{K} \pi_{t-1|t-1,l} p_{k,l}. \tag{2.9}$$

As we know this is computationally infeasible and, thus, following again Koop and Korobilis (2012), we can define:

$$\pi_{t|t-1,k} = \frac{\pi_{t-1|t-1,k}^{\alpha}}{\sum_{l=1}^{K} \pi_{t-1|t-1,l}^{\alpha}}, \tag{2.10}$$

with $0 < \alpha \leq 1$, that is a forgetting factor. Using that approximation, the updating equation of the algorithm becomes:

$$\pi_{t|t,k} = \frac{\pi_{t|t-1,k} p_k(y_t|y^{t-1})}{\sum_{l=1}^{K} \pi_{t|t-1,l} p_l(y_t|y^{t-1})},$$

with,

$$y_t|y^{t-1} \sim N\left(z_t^{(k)} \hat{\theta}_{t-1}^{(k)}, H_t^{(k)} + z_t^{(k)} \Sigma_{t|t-1}^{(k)} z_t^{(k)\prime}\right),$$

that is the prior density defined in section 2.2.1 but for the multimodel case.

At this point, the DMA forecast is given by averaging over the predictive results given by any model using (2.10). Let's notice that,

$$\pi_{t|t-1,k} \propto [\pi_{t-1|t-2,k} p_k(y_{t-1}|y^{t-2})]^{\alpha} = \prod_{i=1}^{t-1} [p_k(y_{t-1}|y^{t-i-1})]^{\alpha^i}.$$

Therefore, if the forecast performance in the recent past, which is measured by the predictive density $p_k(y_{t-1}|y^{t-i-1})$ is high, the model k will receive more weight at time t. The forgetting factor α controls how much weight is associated to forecast in previous periods. For instance, using $\alpha = 0.95$, the forecast performance five years ago, that is 20 trimesters ago, receives $0.95^{20} = 35\%$ as much weight as the forecast performance last period.

2.3 The model

The model used for forecasting is specified by the following equations:

$$y_t = z_t^{(k)} \theta_t^{(k)} + \varepsilon_t^{(k)}$$

$$\theta_{t+1}^{(k)} = \theta_t^{(k)} + \eta_t^k,$$

with $k = 1, \dots, K$, $\varepsilon^{(k)} \sim N(0, H_t^{(k)})$ and $\eta_t^{(k)} \sim N(0, Q_t^{(k)})$.
In this model we need to set priors for $\theta_0^k \sim N\left(\hat{\theta}_0^{(k)}, \Sigma_0\right)$, and $\pi_{0|0,k}$ as initial conditions. In the empirical analysis we assume a data based prior for $\theta_0^{(k)}$, instead $\pi_{0|0,k} = \frac{1}{k}$, for $k = 1, \dots, K$. Moreover, following Raftery et al., $\lambda = \alpha = 0.99$, suggesting a gradual evolution of coefficients. The last parameter to be set is $H_t^{(k)}$.
Clearly, when forecasting inflation, the error variance is also time-varying, hence, we could use a stochastic volatility or ARCH specification. In order to simplify computation Koop and Korobilis (2012) use an alternative simpler plug-in approach:

$$\tilde{H}_t^{(k)} = \frac{1}{20} \sum_{j=t-20+1}^{t} \left[\left(y_t - z_t^{(k)} \hat{\theta}_{t-1}^{(k)} \right)^2 - z_t^{(k)} \Sigma_{t|t-1}^{(k)} z_t^{(k)\prime} \right],$$

and,

$$\hat{H}_t^{(k)} = \begin{cases} \tilde{H}_t^{(k)} & \text{if } \tilde{H}_t^{(k)} > 0 \\ \hat{H}_{t-1}^{(k)} & \text{otherwise} \end{cases}.$$

Then the algorithm follows the Kalman filter procedure set out in section 2.2.1, but with the additional prediction about the probability update explained in section 2.2.2. Once all parameters are estimated, forecasting is done recursively. Thus, DMA forecasts are given by:

$$\mathbb{E}(y_t|y^{t-1}) = \sum_{k=1}^{K} \pi_{t|t-1,k} z_t^{(k)} \hat{\theta}_{t-1}^{(k)}.$$

Chapter 3

Results

3.1 Data and transformations

To estimate the conditional distribution of inflation I have used two different indicators for inflation and fourteen other indicators, distinguishing between monetary measures for inflation and other relevant data. Inflation is computed using the US Consumer Price Index (CPI) and the US Gross Domestic Product deflator (GDPDEFL) time series, using quarterly observation from 1959Q1 to 2021Q3. The inflation rate is the annualized quarter-on-quarter growth rate over the whole sample size. Then, the log transformation is applied to the time series to work with stationary data. The distribution of inflation measured with both CPI and GDPDEFL shows mean around 3% and variance close to zero, it is lightly right skewed and has high kurtosis. The empirical quantiles at the 5th and 95th percentiles correspond to quarterly inflation rates at -0.14% and 10.2% for CPI inflation, and 0.7% and 8.1% for GDP Deflator inflation. These statistics are summarized in table 4.1 below, which includes also descriptive statistics for the time series of inflation before and after the great financial crisis and the Covid-19 pandemic.

		1959-2021	1959-2008	1959-2019
Cpi inflation	Mean	0.0361	0.0414	0.0360
	Variance	9.14e-04	8.66e-04	9.17e-04
	Standard deviation	0.0302	0.0294	0.0303
	Skewness	0.8103	1.2660	0.8177
	Kurtosis	5.9347	4.8322	5.9724
	5th quantile	-0.0014	0.0076	-0.0014
	Median	0.0312	0.0351	0.0312
	95th quantile	0.1014	0.1097	0.1022
GDP Deflator inflation	Mean	0.0320	0.0360	0.0319
	Variance	5.34e-04	5.56e-04	5.36e-04
	Standard deviation	0.0231	0.0236	0.0231
	Skewness	1.3112	1.2416	1.3261
	Kurtosis	4.6289	4.0643	4.6633
	5th quantile	0.0069	0.0093	0.0069
	Median	0.0247	0.0292	0.0246
	95th quantile	0.0816	0.0862	0.0819

Table 3.1: Summary statistics

Finally, the Bai-Perron test is performed to assess whether there are structural breaks in the series, which rejects the hypothesis of the presence of structural breaks in the inflation time series.
The monetary measures for inflation include:

- 3-month Treasury Bill (TBILL) secondary market rate, quarterly observations from 1959Q1to 2021Q3. The TBILL series is not stationary, hence it is applied the first difference transformation.

- Spread between the 10-year T-Bond yield and the 3-month T-Bill, i.e. GS10-TR3M, (SPREAD), quarterly observations from 1959Q1to 2021Q3. It is not applied any transformation.

- Real M1 Money Stock (M1), billions of 1982-84 dollars, quarterly observations for the whole sample (1959Q1-2021Q3). Notice that, in May 2020 there is a structural break in the time series that is due to a change in the composition of the Real M1 index by the Federal Reserve. Until April 2020 M1 consisted of currency, demand deposits, and other highly liquid accounts called "other checkable deposits" (OCDs), whereas now it includes also savings accounts. In order to avoid this structural break, data from May 2020 to July 2021 is retrieved by subtracting the new time series for M1 from M2 Money stock, that is a rough indicator of what would have been M1 without the change introduced by the FED[1]. Then, in order to make the series stationary the log first difference transformation is applied.

The non-monetary measures are:

- US Civilian Unemployment rate (UNEMP), quarterly observations from 1959Q1 to 2021Q3, the first difference is applied.
- Real personal consumption expenditures per capita, quarterly, seasonally adjusted (CONS), the first difference transformation is applied.
- Private Residential Fixed Investment, quarterly seasonally adjusted annual rate (INV). Then, I applied the log first difference transformation.
- US Real gross domestic product (GDP), quarterly observations, seasonally adjusted annual rate from 1959Q1 to 2021Q3, with log first difference transformation.
- New privately owned housing units started (HSTARTS), quarterly data, seasonally adjusted, from 1959Q1 to 2021Q3, with first difference transformation.
- US all employees in total private industries (EMPLOY), quarterly data from 1959Q1 to 2021Q3, first difference transformation.
- ISM Manufacturing, composite Purchasing Managers' Index (PMI), quarterly data from 1959Q1 to 2021Q3, first difference transformation.
- Average hourly earnings of US manufacturing employees in dollars per hour (WAGE), quarterly from 1959Q1 to 2021Q3, seasonally adjusted and log-first difference transformation applied.

[1] https://fred.stlouisfed.org/series/M1SL

- Dow Jones Industrial Average (DIJA), quarterly data from 1959Q1 to 2021Q3, with log-first difference transformation.
- Inflation expectations in the US, University of Michigan survey of consumers, (INFEXP), quarterly data from 1978Q1 to 2021Q3, first difference transformation.

All the data is downloaded from the FRED Dataset, except for the Dow Jones industrial Average that is sourced from Bloomberg.
For what it concerns US CO2 emissions percentage growth, data from 1960 up to 1990 is sourced from Carbon Dioxide Information Analysis Center, data from 1990 are Climate Watch data. Carbon dioxide emissions are those stemming from the burning of fossil fuels and the manufacture of cement. They include carbon dioxide produced during consumption of solid, liquid, and gas fuels and gas flaring[2]. The data is only available on a yearly basis; hence it is assumed that through each quarter of the year CO2 emissions growth remains constant. The time series is stationary, thus no further transformation is applied. Some descriptive statistics are provided in table 1.1.

3.2 Results from DMA

In this section are presented the results from DMA forecasting for the two proxies of inflation for the short-term (h=1), medium-term (h=4), and long-term (h=8) forecast horizons. The sample period used to start producing forecasts is 30 years, thus, after the last quarter of 1990 forecasts are computed recursively. The model includes all fourteen predictors plus an intercept and two lags of the dependent variable, therefore the total number of models to be considered at each time t is $K = 2^{14}$. The main improvement of DMA with respect to other forecasting procedures is that it allows the model relevant for the forecast to change over time.
To assess the forecasting performance of DMA I have used three indicators: the mean squared forecast error (MSFE) the mean absolute forecast error (MAFE) and the bias. I present forecasting results for the DMA model and three alternative forecasting models for comparison:

[2]Definition of variable from World Bank dataset.

1. Forecasts using DMA, but the coefficients do not vary over time (a special case of DMA where $\lambda = 1$ and $\alpha = 0.99$).

2. Forecasts using Bayesian model averaging (BMA: a special case of DMA where $\alpha = \lambda = 1$).

3. Forecasts using the random walk (RW).

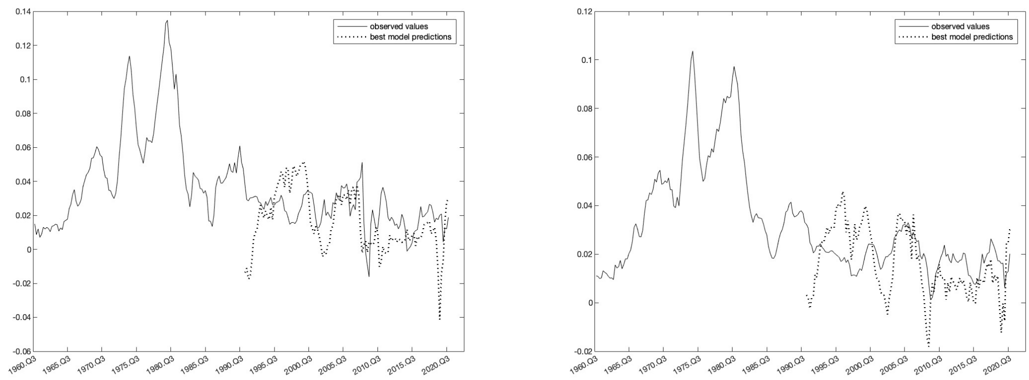
The table below presents results for my forecasting exercise for the two different measures of inflation. DMA in most cases performs better than the other three models, and in no case much worse than the best alternative method, consistently with Koop and Korobilis's results[3]. The benchmark model (random walk) does well for longer-term horizons forecasts. For what it concerns DMA with constant parameters model, we can see that it forecasts better with respect to the benchmark and also with respect to BMA. This latter result suggests that allowing for the model to change over time is at least as important as having time-varying parameters.

[3] Koop, Korobilis (2012).

		MSFE	MAFE	Bias
		DMA		
CPI	h=1	1.7719	0.2468	0.4065
	h=4	1.5100	0.1779	0.9497
	h=8	1.4376	0.1823	0.4215
GDP Deflator	h=1	1.0430	0.1231	0.1818
	h=4	0.9932	0.1119	0.1628
	h=8	2.1242	0.2517	0.4589
		DMA with $\lambda = 1$ and $\alpha = 0.99$		
CPI	h=1	2.0000	0.2420	0.4304
	h=4	2.0916	0.2386	1.1781
	h=8	2.1394	0.2398	1.1286
GDP Deflator	h=1	1.5426	0.1997	0.4654
	h=4	1.7433	0.2002	0.2984
	h=8	2.8517	0.3455	0.2421
		BMA		
CPI	h=1	2.3436	0.2828	0.7436
	h=4	2.2107	0.2458	1.1467
	h=8	2.0835	0.2350	0.6524
GDP Deflator	h=1	1.3550	0.1750	0.4933
	h=4	1.6082	0.1830	0.3431
	h=8	2.2508	0.2971	0.6524
		Random Walk		
CPI	h=1	3.1548	0.3214	2.7102
	h=4	2.5867	0.2609	2.5294
	h=8	2.3349	0.2386	2.3307
GDP Deflator	h=1	2.4021	0.2361	2.3397
	h=4	2.1587	0.2121	2.1587
	h=8	1.9994	0.2012	1.9994

Table 3.2: Forecast performances

In addition, graph 3.1 plots the actual values for inflation and the forecasted values with the best-selected model, for the medium-term forecast (h=4) that is the best in terms of performance.



(a) Cpi

(b) GDP Deflator

Figure 3.1: Actual values vs forecasted values for inflation

The next result provided deals with the expected number of predictors when forecasting with DMA. Although I have used fourteen potential predictors, higher probability is attached to more parsimonious models. Let $Size_k$ indicate the number of predictors included in model k, thus:

$$\mathbb{E}(Size_t) = \sum_{k=1}^{K} \pi_{t|t-1} Size_k,$$

is the average number of independent variables used in DMA at time t, not including the intercept and the two lags of inflation that are common to each model. In the plot below is shown that the shrinkage of the DMA in each forecasting exercise is very effective for the short-term forecast with a maximum value of two predictors for the GDP Deflator, and roughly four for CPI in the last years of the sample. The highest expected number of predictors can be seen for the longer-term forecast horizon (CPI H=4, GDPDEFL H=8 and GDPDEFL H=4); however, it does not exceed six, and, overall, we can assess that DMA chooses parsimonious models.

The plot below, however, does not provide information about which predictor is more relevant in each period. To assess whether a predictor has explanatory power in a time period we can look at the weight attached to models which include one predictor used by DMA, that is the posterior inclusion probability. A predictor is considered relevant if its posterior inclusion probability is above 0.5 for at least one point in time. The plots

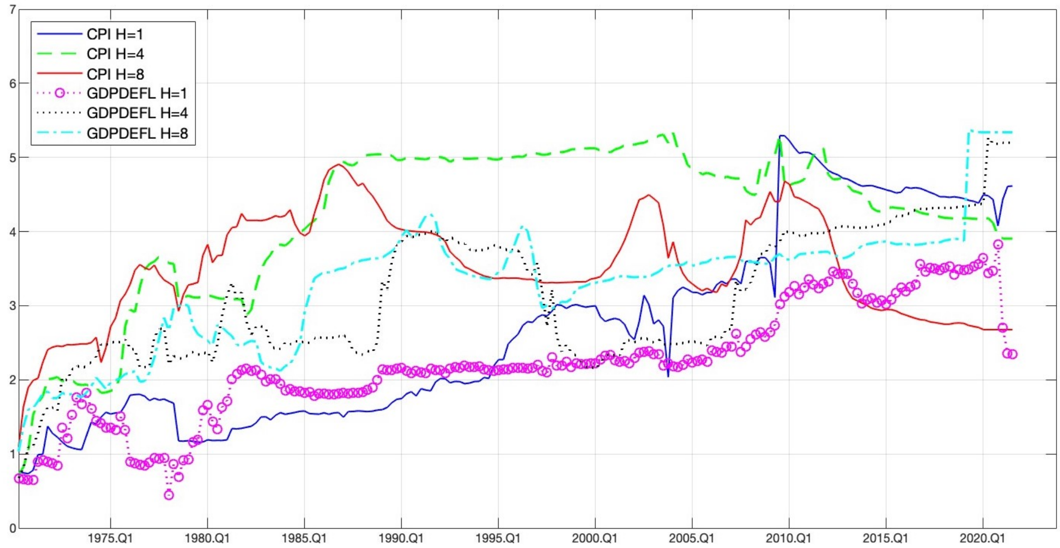


Figure 3.2: Expected number of predictors in the forecasting model used by DMA

are provided in the Appendix in figures A.1 to A.6. The results show strong evidence of time-varying models with predictors that change over time. For the short-term horizon forecast, the average number of predictors is low, and the most relevant ones are, for CPI, Money stock from 1975 and, T-bill and Inflation Expectations from 2000. For what it concerns the GDP Deflator, Wage and Money Stock have the greatest weight attached between 1970 and 1980, whereas from 2000 the most relevant is CO2 Emissions. For H=4 and H=8, Inflation Expectation is a useful predictor for inflation starting from the 1980s. Another useful predictor for both proxies of inflation in longer-term horizon forecasts are Wage and Employment which are included in almost the whole sample. Spread and Money stock are other two variables that show strong predictive power between 1975 and the early 1990s. Consumption, GDP and Investments also often are relevant predictors for inflation, especially after the 1990s. Housing starts has significant predictive power for the H=8 horizon of the forecast of inflation with CPI. Finally, it is important to notice that the DIJA and the PMI are never included in the analysis.

3.2.1 Predictive power of Co2 emissions

In this thesis, we are mainly interested in studying the predictive power of CO2 emissions with respect to inflation. In figure 3.3 below it is shown the posterior inclusion probability of CO2 predictor for all the different specifications of the model studied.

The case for CPI with H=1 is excluded because the posterior probability of inclusion is never above 0.5.

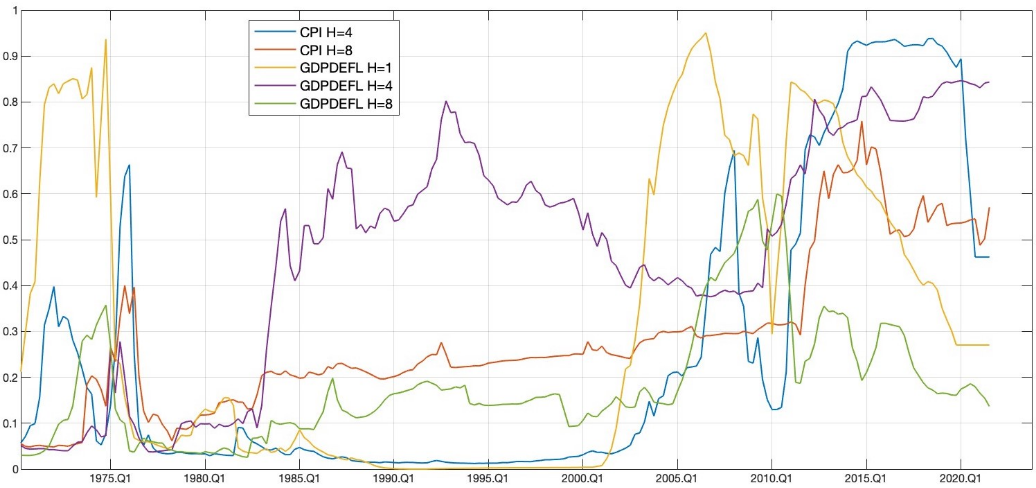


Figure 3.3: Probability of inclusion of CO2 emission growth in the forecasting model

The graph highlights that the CO2 emissions predictor has stronger predictive power for the medium-term horizon forecast, and it is more useful after the great financial crisis of 2008-09.
Finally, table 4.3 shows the value of the coefficient β that measures the effect of CO2 emissions growth on inflation over time. In our model, when the probability of inclusion of the predictor CO2 in the forecasting model is below 0.5, β is shrunk to zero. From the table, Carbon Dioxide emissions have a negative relationship with inflation in all time periods in which the indicator has predictive power.
These results are consistent with the empirical evidence presented in section 1.3. However, they are surprising, given that most of the literature about climate change's effect on inflation suggests that it is likely to periodically push up inflation and add to economic instability. According to DMA forecasts, instead, climate change seems to have a negative correlation with inflation in the long run. Decreasing inflation can often be associated with economic growth, at least when inflation is not already close to zero. On the other hand, rising inflation implies higher prices of products and services. Thus, consumer demands for goods will fall, and by decreasing production, CO2 emissions will decrease[4].

[4]Shpak et al. (2022).

	year	h=1	h=4	h=8
CPI	1960-1970	0	0	0
	1970-1973	0	-0.065%	-12%
	1974-2004	0	0	0
	2004-2012	0	-0.065%	-7%
	2012-2019	0	0	0
	2019-2020	0	-1.8%	0
GDP Deflator	1960-1983	0	0	0
	1983-1985	0	0.05%	0
	1985-2003	0	0	0
	2003-2011	0	-1.5%	-0.5%
	2011-2015	0	-0.5%	0
	2015-2021	-2%	-0.5%	0

Table 3.3: Time-varying coefficients of CO2

In conclusion, two additional remarks could be relevant to understand what will be the relationship between climate change and inflation in the next years. The first consideration is that climate-related shocks are likely to affect supply chain production, especially in the agricultural sector, pushing up food prices. This is why such shocks have a nonnegligible effect on medium-term inflation, in particular in developing economies, where food is more relevant in the basket of goods used to compute inflation and they are less resilient to natural hazards. This suggests that the increased presence of climate shocks can in the future bring the same effect also on developed countries even if we have not yet experienced them.

The second aspect to consider is that while we will experience temporary inflation surges from climate change, there will be price increases related to actions to mitigate climate change. In most scenarios, the green transition will imply sizeable changes in the level and volatility of energy prices, accelerated obsolescence of the existing capital stock, significant reallocation of labor and capital, and strong acceleration in investment. Such developments will have major implications for monetary policy, especially in a disorderly transition[5].

Climate change, hence, affects the conduct and transmission of monetary policy. Regardless of the sign of the effect on inflation the main challenge is that higher prices, especially for commodities such as fuel and food could reduce production and consumption. Therefore, the conventional monetary policy measure of increasing interest rates to reduce inflation will likely make these negative impacts of inflation on demand and job creation worse and increase inequality. Moreover, rising interest rates for the entire economy would harm investments, including the ones in renewable energies and new green technologies.

A growing literature has emerged on the policy options available to central banks to respond to climate change. These measures include outright exclusion of certain assets deemed more polluting from monetary policy operations (e.g. from eligibility as collateral for refinancing operations or for central bank asset purchases in so-called quantitative easing programs). On the other hand, such measures involve complex trade-offs that have to be assessed on a case-by-case basis, depending on the central bank's mandate, the institutional setting, and societal preferences[6]. The second policy action consists of an upgrading of the macroeconomic models including climate change variables, promoting the creation of more comprehensive climate-related datasets, and supporting ongoing initiatives aimed at identifying taxonomies of sustainable and polluting activities.

[5]Boneva, Ferrucci (2022)

[6]Boneva, Ferrucci (2022)

Chapter 4

Further extension: inflation risk forecast

4.1 Inflation risk forecast with Bayesian model averaging quantile regression models (BMA-QR)

This expansion of the previous analysis deals with the prediction of inflation risk, that is the risk of extreme values for inflation that lie on the tails of its distribution. The aim is to bring the analysis carried out with the model in the previous chapter one step forward, with some modeling extensions that could improve the forecast of the distribution of inflation, with a focus on its left and right tails.

Following Korobilis et al. (2021)[1], the econometric setting is a time-series quantile regression that is enhanced including time-varying parameters (TVP-QR), considering a Bayesian framework in which parameter and error have a flexible parametric distribution to capture the behavior of extreme quantiles.

Recall that, the cumulative distribution function (F_Y) is the function that, for each value of y, gives the probability for which the random variable Y is smaller than y, i. e. $\mathbb{P}(Y \leq y)$. Given a sample from a distribution $y_1, y_2, \ldots, y_n$, the τ^{th} quantile Q^{τ} of the CDF is the smallest y such that $F(y) \geq \tau$, or in other terms:

$$Q^{(\tau)}(\hat{F}) = \hat{F_y}^{-1}(y \geq q_{\tau}).$$

A very basic formulation of QR model is the following linear regression with only one covariate:

$$y_i = \beta_0^{(\tau)} + \beta_1^{(\tau)} x_i + \varepsilon_i^{(\tau)},$$

[1] Korobilis, Landau, Musso and Phella (2021)

and,

$$Q^{(\tau)}(y_i|x_i) = \beta_0^{(\tau)} + \beta_1^{(\tau)} x_i,$$

where τ represents the quantile. The quantile regression estimates for $\hat{\beta_0}^{(\tau)}$ and $\hat{\beta_1}^{(\tau)}$ are the values that minimize the weighted sum of residuals $y_i - \hat{y}_i$, where the weight is $1-\tau$ if the fitted value overpredicts the observed value, and τ otherwise[2].

In this thesis I use the approach of Korobilis et al., which is a bayesian approach to inference. For the quantile regression formulation, there are no available standard conjugate prior distributions but MCMC methods can be applied to estimate the posterior distributions of parameters. This allows for the use of any prior distribution.

Given the observations $y = (y_1, \ldots, y_n)$, the posterior distribution of $\beta^{(\tau)} = (\beta_0^{(\tau)}, \beta_1^{(\tau)})$ is:

$$p(\beta^{(\tau)}|y) \propto L(y|\beta^{(\tau)})p(\beta^{(\tau)}).$$

In this expression $p(\beta^{(\tau)})$ is the prior distribution of the parameter and $L(y|\beta^{(\tau)})$ is the likelihood function. In Yu et al.[3] is shown that the minimization problem described above is equivalent to the maximization of a likelihood function assuming asymmetric Laplace density. Whatever the actual distribution of the data, hence, Bayesian inference for quantile regression proceeds by forming the likelihood function under the assumption that the prior distribution of y is asymmetric Laplace.

Now, let π_t be the observation of inflation at time t and x_t a p-dimensional vector that includes an intercept, previous lags of inflation, and a set of independent predictors. The inference aims to estimate the full distribution of inflation by specifying:

$$\pi_t = Q_\tau(\pi_t|x_t) + \varepsilon_t, \tag{4.1}$$

where Q_τ is the conditional quantile function of the τ-th quantile, and,

$$Q_\tau(\pi_t|x_t) = x_t\beta(\tau). \tag{4.2}$$

By assumption $\tau = 0.05, 0.10, \ldots, 0.90, 0.95$.

The estimation of $\beta(\tau)$ is the solution to the following minimization problem:

$$min \sum_{t=1}^{T} \rho_\tau(\varepsilon_t),$$

[2]Hao (2020).

[3]Yu, Moyeed (2001).

with $\rho_\tau(u) = \tau(\tau - \mathbb{I}(u < 0))$ that is a loss function.
We know that the estimates from the minimization problem can also be retrieved by maximizing a likelihood function under the asymmetric Laplace error distribution.
The asymmetric Laplace distribution has different mixture representations. According to Kozumi et al.[4], to develop the Gibbs sampler for the quantile regression model is most efficient to utilize a mixture based on exponential and normal distributions for the error, that takes the following form:

$$\varepsilon_t = \theta z_t + \tau\sqrt{z_t}u_t,$$

where $z_t \sim Exponential(1)$, and $u_t \sim N(0,1)$. Substituting into eq.(4.1) the quantile regression takes the form:

$$\pi_t = x_t'\beta_{(\tau)} + \theta z_t + \tau\sqrt{z_t}u_t.$$

Thus, the conditional density of inflation is normal, and the conditional parameter posterior will be identical to the standard expression for linear Gaussian regression models[5].

4.2 Treating time variation in the parameters

The second step in building the model is treating time variation in the parameters. If we consider time-varying parameters, the model's equations become

$$\pi_t = Q_\tau(\pi_t|x_t) + \varepsilon_t,$$

$$Q_\tau(\pi_t|x_t) = x_t\beta_t(\tau),$$

$$\beta_t(\tau) = \beta_{t-1}(\tau) + v_t,$$

with $v_t \sim N(0, V(\tau))$ that is a state error with covariance matrix $V(\tau)$. Under this assumption, the parameters follow a random walk process.
Given that in the previous section we have assumed that ε_t follows an ALD distribution it is possible to treat the system as a linear conditionally Gaussian state-space model.

[4]Kozumi and Kobayashi (2009).
[5]Korobilis (2017).

Under this representation it is possible to write:

$$\begin{aligned}
\pi_t &= x_t\beta_t(\tau) + \varepsilon_t \\
&= x_t\Delta\beta_t(\tau) + x_t\beta_{t-1}(\tau) + \varepsilon_t \\
&= x_t\Delta\beta_t(\tau) + x_t\Delta\beta_{t-1}(\tau) + x_t\beta_{t-2}(\tau) + \varepsilon_t \\
&\dots \\
&= x_t\Delta\beta_t(\tau) + x_t\Delta\beta_{t-1}(\tau) + \dots + x_t\Delta\beta_2(\tau) + x_t\beta_1(\tau) + \varepsilon_t.
\end{aligned}$$

This suggests that $\beta_t(\tau)$ is the cumulative sum of changes over the previous periods, thus, if we stack for all observations t, the equations above can be rewritten as:

$$Q_t(\boldsymbol{\pi}|\boldsymbol{\chi}) = \boldsymbol{\chi}\boldsymbol{\beta}^{\Delta}(\tau), \tag{4.3}$$

$$\boldsymbol{\beta}^{\Delta}(\tau) = \boldsymbol{v}, \tag{4.4}$$

where, $\boldsymbol{\pi} = [\pi_1, \dots, \pi_T]'$, $\boldsymbol{v} = [v_1', \dots, v_T']'$,

$$\boldsymbol{\chi} = \underbrace{\begin{bmatrix} x_1 & 0 & \dots & 0 & 0 \\ x_2 & x_2 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots \\ x_{T-1} & x_{T-1} & \dots & x_{T-1} & 0 \\ x_T & x_T & \dots & x_{T-1} & x_T \end{bmatrix}}_{TxT_p}, \boldsymbol{\beta}^{\Delta}(\tau) = \underbrace{\begin{bmatrix} \beta_1(\tau) \\ \Delta\beta_2(\tau) \\ \dots \\ \Delta\beta_{T-1}(\tau) \\ \Delta\beta_T(\tau) \end{bmatrix}}_{T_p x1}{}^{6}.$$

This is clearly a high-dimensional setting since the lower triangular matrix χ has more covariates than observations.

Usually, equation (4.4) is considered as a prior for $\beta^{\Delta}(\tau)$, such that eq. (4.3) can be treated as a regression in state-space form model and inference can be carried out using MCMC methods.

However, existing algorithms used for simulating posterior distributions are inefficient when dealing with very high dimensions. Hence, borrowing ideas from Korobilis (2019), inference will be done in this thesis without the conditional prior assumption outlined above, but rather using an algorithm with hierarchical data-based shrinkage priors due to the high-dimensional setting.

[6] Korobilis, Landau, Musso and Phella (2021).

4.3 Bayesian time-varying parameter quantile regression

Putting together those pieces, the model has the following form:

$$\pi = \chi\beta^{\Delta}(\tau) + \varepsilon$$

$$\beta^{\Delta}(\tau) = \boldsymbol{v}.$$

The first assumption from section 4.1 is that ε follows an Asymmetric Laplace distribution, that can be represented by a Gaussian-Exponential scale distribution:

$$(\varepsilon_t|u_t, z_t) \sim \theta(\tau)z_t + \sqrt{\sigma(\tau)^2 k(\tau)^2 z_t(\tau)}u_t,$$

where, $\sigma(\tau)$ is a scale parameter that takes the standard inverse Gamma prior, i.e, $\sigma(\tau) \sim IG(\rho_1, \rho_2)$, $u_t \sim N(0,1)$, $z_t(\tau) \sim exp(\sigma(\tau)^2)$, $\theta(\tau) = \frac{1-2\tau}{\tau(1-\tau)}$, and $k(\tau)^2 = \frac{2}{\tau(1-\tau)}$. This implies that the likelihood function of π_t takes the form:

$$\prod_{t=1}^{T} \frac{1}{\sqrt{2\pi z_t(\tau)\sigma(\tau)^2 k(\tau)^2}} exp\left\{ -\frac{(\pi_t - \chi_t\beta^{\Delta}(\tau) - \theta(\tau)z_t(\tau))^2}{2z_t(\tau)\sigma(\tau)^2 k(\tau)^2} \right\} exp\left\{ -\frac{z_t(\tau)}{\sigma(\tau)^2} \right\}.$$

Under this mixture the model can be rewritten as:

$$\pi = \chi\beta^{\Delta}(\tau) + \theta(\tau)z(\tau) + \tilde{S}u,$$

with $\tilde{S}$ that is a TxT diagonal matrix with diagonal elements $\sigma(\tau)k(\tau)\sqrt{z_t(\tau)}$.
To complete the model we need to set a prior for $\beta^{\Delta}(\tau)$. In quantile regression, shrinkage is necessary because we are going to estimate a Tp-dimensional vector for each quantile level $\tau = 0.05, 0.10, \ldots, 0.90, 0.95$. Therefore, it is used a Horseshoe prior from Carvalho et al.[7] that allows for shrinkage of the time-varying parameters without any dependence on prior hyperparameters. It is defined as,

$$\beta^{\Delta}(\tau)|\lambda(\tau)^2, \psi_i(\tau)^{2}{}_{i=1}^{Tp} \sim N(0, V(\tau)),$$

$$V_{i,i} = \sigma(\tau)\lambda(\tau)^2\psi_i(\tau)^2,$$

$$\lambda(\tau) \sim Cauchy^{+}(0,1),$$

$$\psi_i(\tau) \sim Cauchy^{+}(0,1).$$

[7] Carvalho, Polson and Scott (2010).

Estimation is done with a Gibbs sampler, derived from an extension of the sampler proposed by Kozumi et al.[8]. In particular, from the prior and the likelihood above, the posterior distribution of $\beta^{\Delta}(\tau)$ has the form:

$$\beta^{\Delta}(\tau)|\bullet \sim N\Big((\chi' U\chi + V(\tau)^{-1})^{-1}(\chi' U[\pi - \theta(\tau)z(\tau)]), (\chi' U\chi + V(\tau)^{-1})^{-1}\Big)^{9}.$$

Due to the high-dimensional setting, sampling from the posterior is done using the following sampling scheme proposed by Bhattacharya et al.[10]:

$$\begin{aligned}
&\text{Step 1: Sample } \eta \sim N(0, V(\tau)) \text{ and } \delta \sim N(0, I_T) \\
&\text{Step 2: Set } v = \tilde{\chi}\eta + \delta && (4.5)\\
&\text{Step 3: Set } w = (\tilde{\chi}V\tilde{\chi}' + T_t)^{-1}[\pi - \theta(\tau)z(\tau) - v] && (4.6)\\
&\text{Step 4: Set } \beta^{\Delta}(\tau) = \eta + V\tilde{\chi}' w && (4.7)
\end{aligned}$$

In the algorithm above, $\tilde{\chi} = \chi U^{-1/2}$,where $U^{1/2}$ is a diagonal matrix with diagonal elements $(\sqrt{z_t(\tau)}\sigma(\tau)k(\tau))^{-1}$, and $\sigma^2(\tau)$ and $z_t(\tau)$ are updated using the posteriors:

$$\sigma(\tau)|\bullet \sim Inverse-Gamma\Big(\rho_1 + \frac{3T}{2}, \rho_2 + \sum_{t=1}^{T}\frac{(\pi_t - \chi_t\beta^{\Delta}(\tau) - \theta(\tau)z_t(\tau))^2}{2z_t(\tau)k(\tau)^2)} + \sum_{t=1}^{T} z_t(\tau)\Big),$$

$$z_t(\tau)|\bullet \sim IG\Big(\frac{\sqrt{\theta(\tau)^2 + 2k(\tau)^2}}{|\pi_t - \chi_t\beta^{\Delta}(\tau)|}, \frac{\theta(\tau)^2 + 2k(\tau)^2}{\sigma^2(\tau)k(\tau)^2}\Big).$$

The variance-covariance matrix is updated following the scheme below,

$$V_{i,i}(\tau) = \sigma(\tau)^2\lambda(\tau)^2\psi_i(\tau)^2 \qquad i = 1, \dots, Tp,$$

with the following horseshoe prior for λ and ψ_i:

$$\begin{aligned}
\lambda(\tau)^2|\xi(\tau) &\sim Inverse-Gamma(1/2, 1/\xi(\tau)),\\
\xi(\tau) &\sim IG(1/2, 1),\\
\psi_i(\tau)|\zeta_i(\tau) &\sim Inverse-Gamma(1/2, 1/\zeta_i(\tau),\\
\zeta_i(\tau) &\sim Inverse-Gamma(1/2, 1).
\end{aligned}$$

From Makalic and Schmidt (2016) I use the exact conditional posterior distributions under this prior formulation for the global and local hypervariaces:

$$\lambda(\tau)^2|\bullet \sim Inverse-Gamma\Big(\frac{Tp+1}{2}, \frac{1}{\xi(\tau)} + \frac{1}{2\sigma^2(\tau)}\sum_{i=1}^{Tp}\frac{\beta_i^2(\tau)}{\psi_i^2(\tau)}\Big),$$

[8]Kozumi and Kobayashi (2009).

[9]The bullet stands for "conditional on other parameters and variables".

[10]Bhattacharya, Chakraborty and Mallick (2016).

$$\psi_i^2(\tau)|\bullet \sim Inverse-Gamma\left(1, \frac{1}{\zeta_i(\tau)} + \frac{\beta_i^2(\tau)}{2\lambda^2(\tau)\sigma^2(\tau)}\right),$$

$i = 1,\ldots,Tp$. Finally, the conditional posteriors for the auxiliary variables are also of Inverse Gamma type:

$$\zeta_i(\tau)|\bullet \sim Inverse-Gamma\left(1, \frac{1}{\psi_i^2(\tau)}\right),$$

$i = 1,\ldots,Tp$, and,

$$\xi(\tau)|\bullet \sim Inverse-Gamma\left(1, 1+\frac{1}{\lambda^2(\tau)}\right)^{11}.$$

The algorithm requires for each MCMC estimation to iterate through equations (4.5)-(4.7) for each quantile, however, using MATLAB computation can be sped up sampling for all values of τ simultaneously.
To sum up, the full Bayesian time-varying parameter quantile regression model is:

$$\begin{aligned}
\pi &= \chi\beta^{\Delta}(\tau) + \theta(\tau)z(\tau) + \tilde{S}u,\\
\beta^{\Delta}(\tau) &\sim N(0, V(\tau))\\
V_{i,i}(\tau) &= \sigma^2(\tau)\lambda^2(\tau)\psi_i^2(\tau), \qquad i = 1,\ldots,Tp,\\
\lambda(\tau) &\sim Cauchy(0,1)^+,\\
\psi_i(\tau) &\sim Cauchy(0,1)^+,\\
\sigma(\tau) &\sim Inverse-Gamma(\rho_1, \rho_2),\\
z_t(\tau) &\sim \exp(\sigma(\tau)), \qquad t = 1,\ldots,T,
\end{aligned}$$

with $\tilde{S} = \sigma(\tau)\kappa(\tau)\sqrt{z_t(\tau)}$.

[11] Makalic and Schimdt (2016).

4.4 Results

This section presents some initial results from the algorithm described above to implement the TVP-QR model.

The performance of each different specification of the model is assessed by comparing the quantile score. In particular, I have computed the average of the quantile scores across all forecasting periods. For each competing model j the average quantile score for each quantile τ and forecast horizon h is:

$$QS_h^j(\tau) = \frac{1}{R_h}\sum_{t=1}^{R_h}[\pi_{t+h} - \hat{Q}_\tau(\pi_{t+h}|\boldsymbol{x_t})][\mathbb{I}(\pi_{t+h} \leq \hat{Q}_\tau(\pi_{t+h}|\boldsymbol{x_t})],$$

where R_h is the length of the period in which the forecast is produced.

The benchmark model is a time-varying parameter quantile AR model with two lags of inflation.

Here the focus is on understanding the role of CO2 emissions on the forecast for tail inflation risk, hence, it would be interesting to look at the average quantile scores for the 5th and the 95th quantile of a model that includes two lags of inflation and only CO2 emissions as the exogenous variable. However, the Gibbs sampler is incomplete and does not provide accurate results.

		Qscore05	Qscore95
Quantile AR(2) with TVP			
CPI	h=4	-0.3202	-0.7157
GDPDEFL	h=4	-0.3237	-0.4290
Quantile AR(2) with TVP including CO2			
CPI	h=4	-1.4774	-0.6486
GDPDEFL	h=4	-1.0345	-0.7327

Table 4.1: Average quantile scores from TVP-QR

The exercise could be repeated with other predictors to understand which are the best models.

Moreover, as suggested in Korobilis et al. (2021)., it can be interesting to compare the evolution over time of the upper and lower quantile scores, as a measure of tail risk forecast errors, for different models. Comparing the evolution of quantile scores of different models that are specified with or without a certain indicator could help understand the importance of including that specific additional indicator when forecasting inflation tail risk.

Conclusion

Through this work it is exposed the role of climate change on inflation using dynamic model averaging. In a nutshell, given the evidence of instability of Phillips curve forecasts over time, and the fact that the economic activity predictors for inflation are not constant, I have used the methodology of Koop and Korobilis to allow for the set of predictors to be time-varying. DMA, in particular, combines the forecasts produced by a large number of different models to predict the future value of a time series.

I have used as the dependent variable US inflation, and a set of fourteen different predictors, that are the same used by Koop and Korobilis in their research, plus a predictor for climate change that is CO2 emissions.

I have observed that there is empirical evidence of the time-varying nature of inflation and the best performance of DMA has been confirmed with a new and more recent dataset. Secondly, I have found evidence of the predictive power that climate change has on forecasting inflation, in particular over the medium term, and that the sign of their relationship is negative.

To sum up, climate change affects the conduct and transmission of monetary policy and it should be included in the models used by Central Banks when forecasting macroeconomic variables. To this end, there is some research on the policy measures that could be put in place to dampen the effects of climate shocks. The measures available include, among others, upgrading the climate-related datasets to produce more precise forecasts, and excluding certain assets that are considered more polluting from monetary policy operations.

It is uncertain how this relationship will evolve, and which implications will have the transition to a more sustainable economy. In the next years, a greater incidence of physical risk could cause short-term fluctuations that amplify longer-term macroeconomic volatility, therefore the linkage between inflation risk and climate-related risk should be analyzed more in detail.

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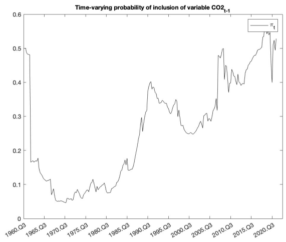
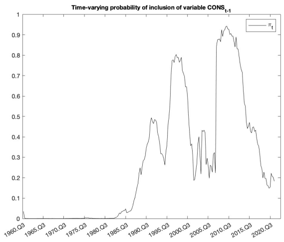
Appendix

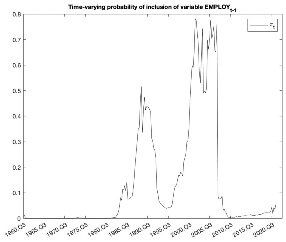
Figure A.1: GDP Deflator; Probability of inclusion of predictors (h=1)



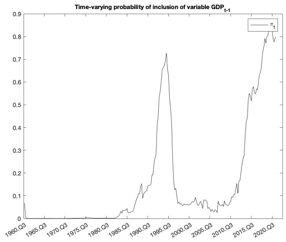
(a) CO2



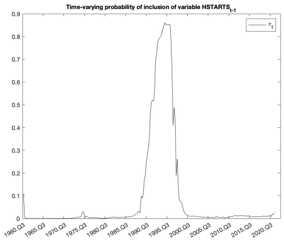
(b) Consumption



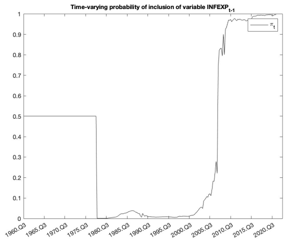
(c) Employment



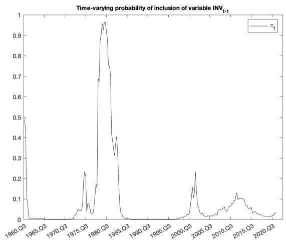
(d) GDP



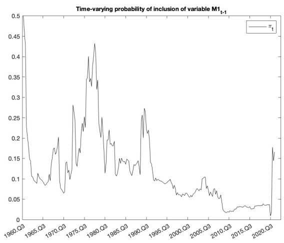
(e) Housing Starts



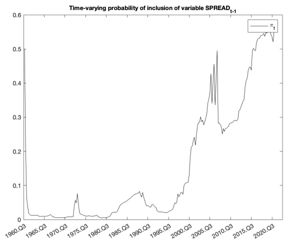
(f) Inflation Expectations



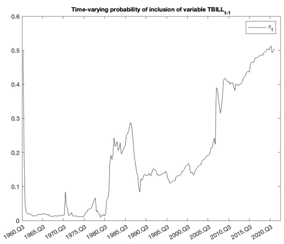
(g) Investments



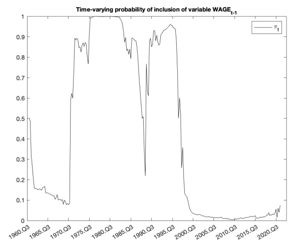
(h) Money Stock



(i) Spread

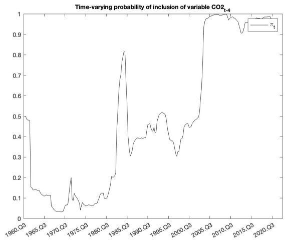


(j) T-bill

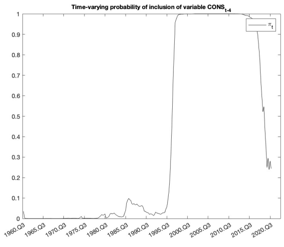


(k) Wage

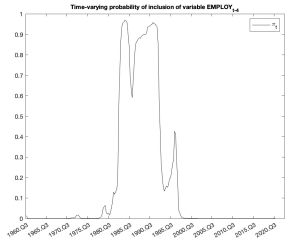
Figure A.2: GDP Deflator; Probability of inclusion of predictors (h=4)



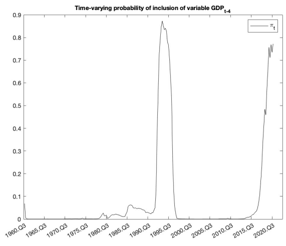
(a) CO2



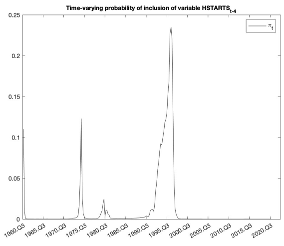
(b) Consumption



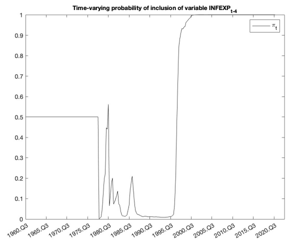
(c) Employment



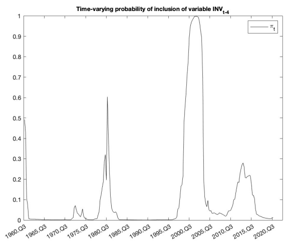
(d) GDP



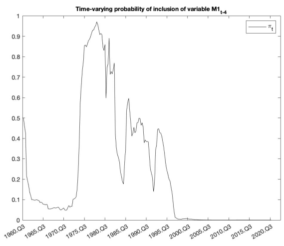
(e) Housing Starts



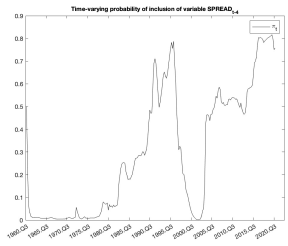
(f) Inflation Expectations



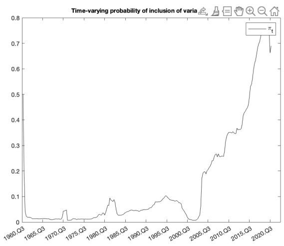
(g) Investments



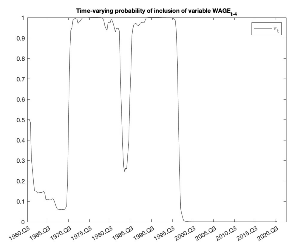
(h) Money Stock



(i) Spread

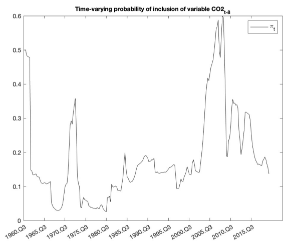


(j) T-bill

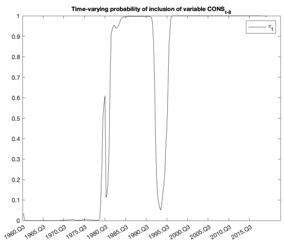


(k) Wage

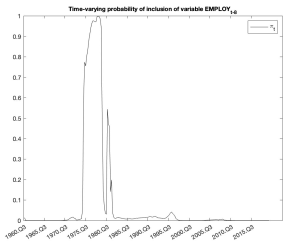
Figure A.3: GDP Deflator; Probability of inclusion of predictors (h=8)



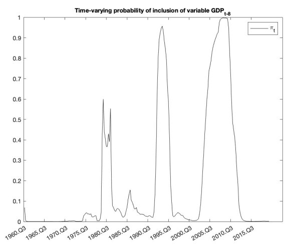
(a) CO2



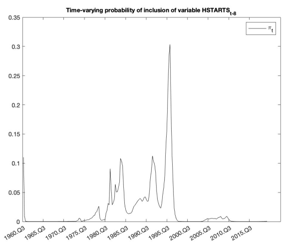
(b) Consumption



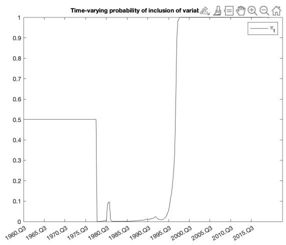
(c) Employment



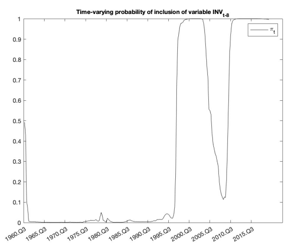
(d) GDP



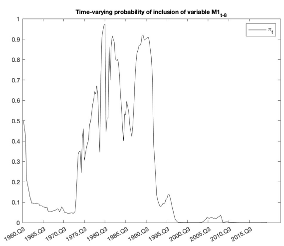
(e) Housing Starts



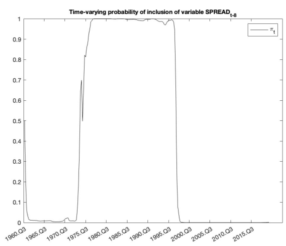
(f) Inflation Expectations



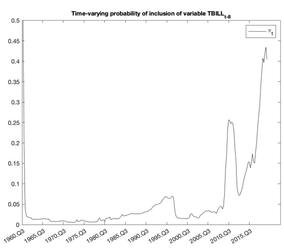
(g) Investments



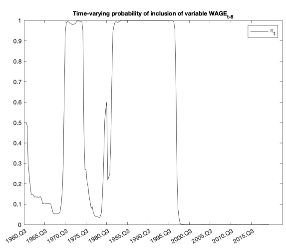
(h) Money Stock



(i) Spread

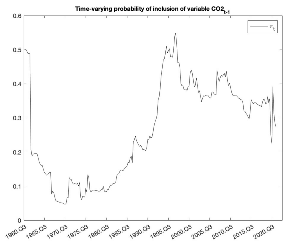


(j) T-bill

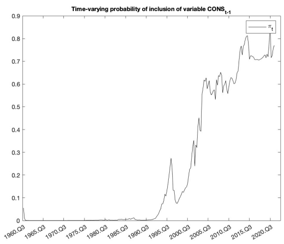


(k) Wage

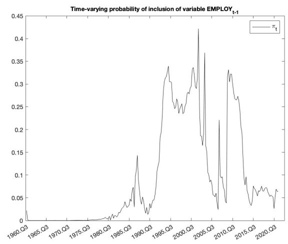
Figure A.4: CPI; Probability of inclusion of predictors (h=1)



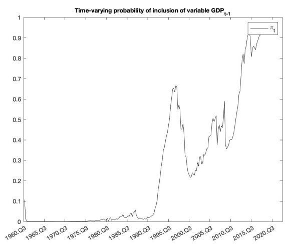
(a) CO2



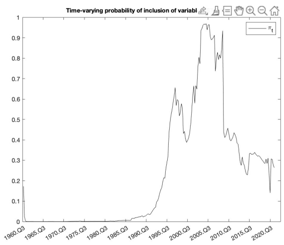
(b) Consumption



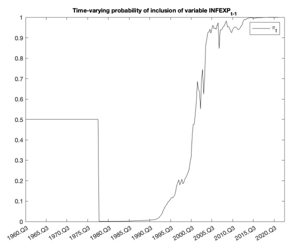
(c) Employment



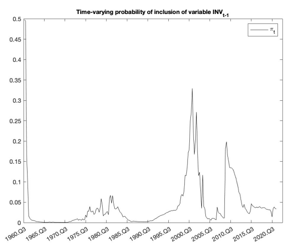
(d) GDP



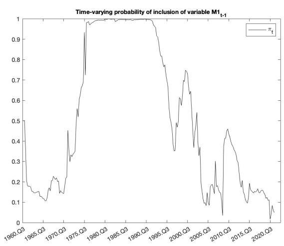
(e) Housing Starts



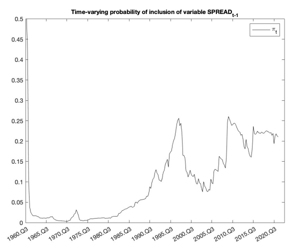
(f) Inflation Expectations



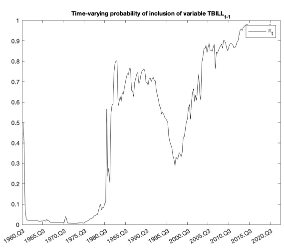
(g) Investments



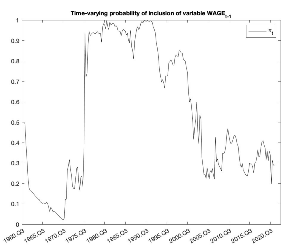
(h) Money Stock



(i) Spread

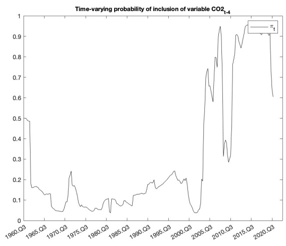


(j) T-bill

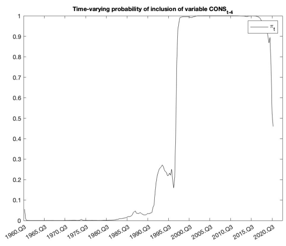


(k) Wage

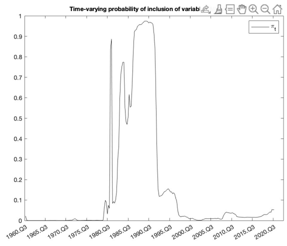
Figure A.5: CPI; Probability of inclusion of predictors (h=4)



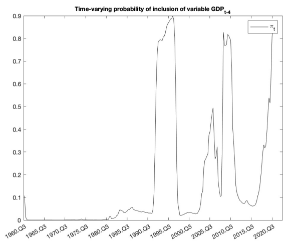
(a) CO2



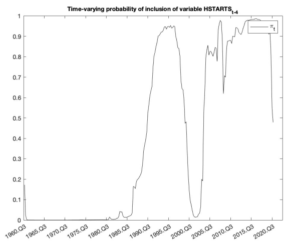
(b) Consumption



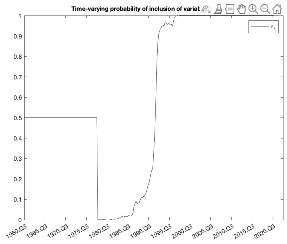
(c) Employment



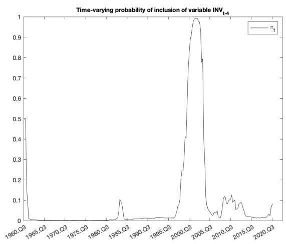
(d) GDP



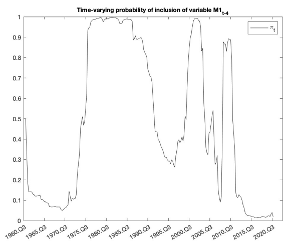
(e) Housing Starts



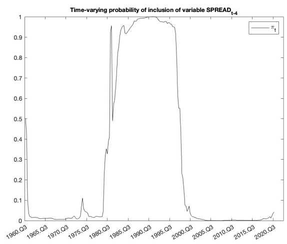
(f) Inflation Expectations



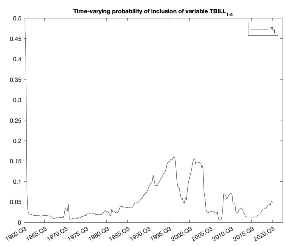
(g) Investments



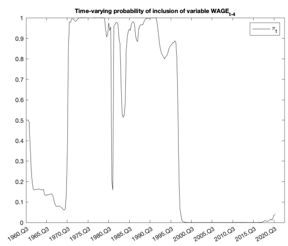
(h) Money Stock



(i) Spread

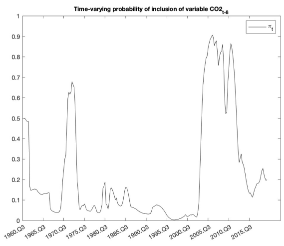


(j) T-bill

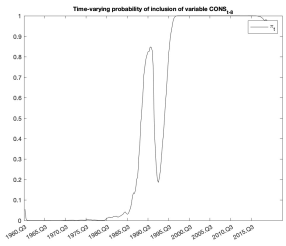


(k) Wage

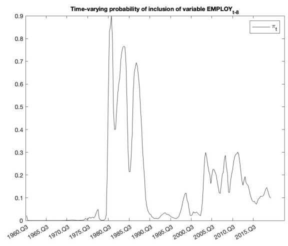
Figure A.6: CPI; Probability of inclusion of predictors (h=8)



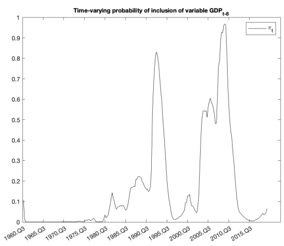
(a) CO2



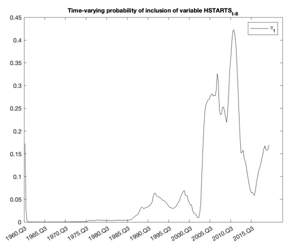
(b) Consumption



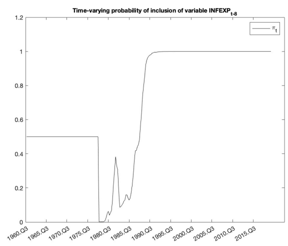
(c) Employment



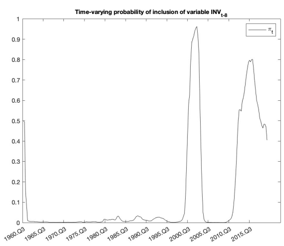
(d) GDP



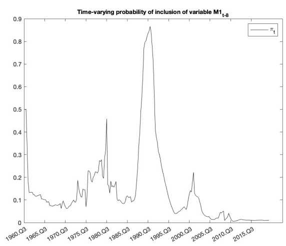
(e) Housing Starts



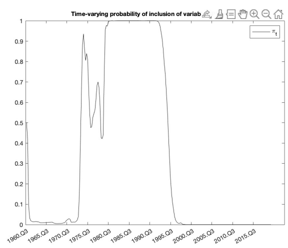
(f) Inflation Expectations



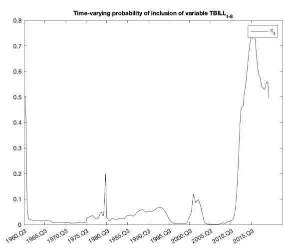
(g) Investments



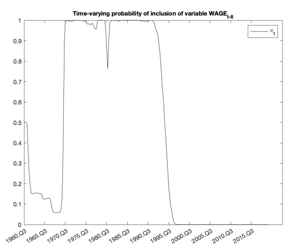
(h) Money Stock



(i) Spread



(j) T-bill



(k) Wage

Summary

Abstract

This thesis aims to understand the role of climate change on inflation. The model used is the Dynamic Model Averaging developed by Koop and Korobilis (2012). The DMA is a time-varying parameter model that allows changing the predictors for inflation in each forecasting model and also choosing which is the best model for forecasting at each point in time. The algorithm used for estimation is the same as in Koop and Korobilis (2012), with the addition of a climate-related predictor, which is CO2 emissions growth. Results show that CO2 emissions have predictive power for inflation, particularly in medium-term forecasts and after the Great financial crisis. Hence, climate change is relevant to the conduct of monetary policy. In addition, an extension to analyze the impact of climate change on inflation risk with a TVP-Quantile Regression model is proposed for further investigations.

Chapter I

My thesis deals with inflation forecasting, which is an essential part of private and public sector decision-making. Starting from the 1980s, the aim of Central Banks is to keep inflation stable, thus optimal policy will depend on optimal forecasts and policy will be most effective when it is well understood by the public[1].
The first chapter of this thesis is an overview of the main topics that inspired the research question and that will be addressed in the next chapters from an econometric standpoint. Here I will focus on the recent developments in price levels and their causes and implications for the future. In the euro area headline inflation, as measured by HICP, was

[1] Faust J., Wright J. H., "Forecasting Inflation", Handbook of Economic Forecasting, Editor(s): Graham Elliott, Allan Timmermann, Elsevier, 2013, Volume 2, Part A, pp. 2-56.

equal to 1.2% in 2019, decreased to 0.3% in 2020, and became negative in the second half of 2020 before increasing again to 2.6% in 2021[2]. In July 2021, the ECB updated its monetary policy strategy. The former inflation target was to set the annual increase of the HICP (Harmonized Index of Consumer Prices) “below but close to 2%”, whereas today the target is set at exactly 2% of the annual increase of the HICP over the medium run. This new definition implies that the ECB is willing to tolerate a transitory period of moderate inflation overshoot to push inflation upwards after a period in which the inflation rate in the euro area was close to zero[3]. However, recently, inflation has reached a historical high of 8.6% in June 2022.

The increase in inflation last year was supposed to be temporary. According to Philip Lane, chief economist of the ECB, high inflation rates that were observed during 2020-2021 were driven by a pandemic cycle that had generated global bottlenecks for manufactured goods and by the surge in energy prices, and both were thought to be temporary shocks that did not affect long-term inflation.

After the peak of the Covid-19 pandemic, the fast recovery in demand over 2020 was accompanied by a slow recovery of production capacity in some sectors that have generated supply shortages. Sanitary restrictions around the world to reduce the spread of the virus have contributed to the persistent supply disruptions that restrained the economic recovery and, additionally, Covid-19 outbreaks have led to the closure of key ports, creating bottlenecks in global shipping. On the other hand, there are significant signs of shortages in labor markets that reflect both the change in the location of activities and the skills required from workers, and the fact that the pandemic led some people to withdraw from the labor force[4]. For what it concerns the impact of high energy prices on inflation dynamics, we should consider both the direct effect through the energy component in the HICP, and the indirect effect since energy is an input to produce many other commodities. Therefore, supply disruptions related to the pandemic and other events have contributed to the rise in commodity prices. Some of those supply disruptions

[2]Nickel C., Koester G., Lis E., “ Inflation developments in the Euro Area since the onset of the pandemic”, The challenges of Inflation in Europe and the US, Intereconomics, Review of European economic policy, 2022, Vol. 57, No. 2, pp. 69-75.

[3]Hennecke P., "The ECB’s new monetary policy strategy", published in Review of European economic policy, 2021, Vol. 2021, No. 5.

[4]OECD, "Economic Outlook", 2021, Vol. 2021, Issue 2.

associated with the pandemic were supposed to ease facilitating the global economic recovery and reducing inflationary pressures. The projections implied that output in the advanced economies was likely to converge on the pre-pandemic path. However, longer-term inflation expectations did not remain anchored in advanced economies. In 2020, the ECB Survey of Professional Forecasters computed long-term inflation expectations at 1.64%, and in 2021 they have been moving up to close to 2%. The first factor indicating a re-anchoring in expectations in line with the 2% level is the huge monetary and fiscal response to the pandemic demonstrating the commitment to stabilize the economy, including the SURE and the NGEU initiative. Secondly, there are some revisions to beliefs about the operation of the world economy.

In recent times, just when some of the supply-chain problems caused by the pandemic started to fade, the war in Ukraine has created a new negative supply shock. Simulations run by the OECD show that global inflation could raise by about 2.5% in the first year after the beginning of the conflict under the assumptions that the conflict lasts for at least one year and there is a deep recession in the Russian economy[5]. Given this scenario, the ECB Governing Council decided to raise the interest rate on the main refinancing operations, the interest rates on the marginal lending facility and the deposit facility by 50 basis points, with effect from the 27th of July 2022.

Moreover, the number of disruptions related to climate change is increasing in recent years and the economic impact of such shocks has increased correspondingly. For instance, in 2020 there have been huge wildfires (Siberia, California and Turkey), heatwaves and droughts (North America), extreme cold weather events and destructive floods (Germany and Belgium), and many others, and hence, it is crucial to take into account climate shocks when making forecasts on the price levels[6].

In a nutshell, from an economic point of view climate change can be interpreted as an adverse permanent shock to the supply potential of the economy and likely such shock will push output below its level and will lower future potential growth. At the same time, climate change in the short-run could lead to changes in demand conditions, through a reduction in consumption by households and in investment by firms due to greater uncertainty about future economic growth and income prospects.

[5]OECD, "Economic Outlook, Interim Report March 2022: Economic and Social Impacts and Policy Implications of the War in Ukraine", 2022, Vol. 2022.

[6]OECD (2021).

According to the purpose of this thesis, the focus will be on the linkages between inflation and climate change. In this regard, the agricultural sector is widely exposed to the effect of climate change, thus there is a potential for lasting effects on the prices of agricultural products. Clearly, agricultural yields may rise in some regions of the world and fall in others, hence the final impact is a function of the location of a country and its agricultural imports. Recent studies point out that storms and floods are positively related to increases in inflation in developing countries in the short term and, on the contrary, that droughts have a more persistent effect. Moreover, rising inflation rates may be related to lower supply potential in the economy after climate-related shocks. It is essential to also consider the contribution of changes in energy prices to inflation. One of the main pillars of climate policy mitigation is, indeed, the transition to sustainable energy sources for production that could potentially reduce the overall expenditure for energy consumption and reduce the weight that energy has in the computation of the consumer price index[7].

For what it concerns the implication of risks arising from climate change for the conduct of monetary policy there are few empirical studies to look at. Therefore, the next step in this analysis is to understand the data, and which could be the relationship between inflation and climate change. For what concerns inflation, two indicators have been used, the Consumer Price Index (CPI) and the GDP Deflator (GDPDEFL). The earth's changing climate can be measured using different indicators such as global average surface temperature or the level of ocean acidification. However, scientists have maintained a long-running observational record of the levels of CO2 since 1950. This is a rough indicator of human-induced climate change since it is the most abundant greenhouse gas in the atmosphere and its sources are mainly anthropogenic.

Carbon Dioxide (Co2) is the most present anthropogenic greenhouse gas in the atmosphere, accounting for 66% of the radioactive forcing, that is the warming effect on the climate. It is estimated that atmospheric Co2 reached 148% of the preindustrial level in 2019, mostly because of fossil fuel and cement production. Moreover, of the total Co2 emissions from anthropogenic sources from 2008 to 2018 about 44% accumulated in the atmosphere, 23% in oceans and 29% on land. Since most of the Co2 already present

[7] Andersson M., Baccianti C., Morgan J.,"Climate change and the macroeconomy", ECB, Occasional Paper Series, 2020, No 243.

in the atmosphere will remain there for several centuries, even when net emissions of Co2 will approach zero the climate will continue to warm[8].

During the Covid-19 pandemic, the World Meteorological Organization (WMO), estimated a decrease of about 5.6% of Co2 emissions in 2020. It is predicted, though, that at the global scale, a reduction of this magnitude will not cause atmospheric Co2 to decrease. This data suggests that there could be a negative relationship between inflation and climate change.

For all the details about the time series analysis and the summary statistics you can look at section 1.3.2. The main result from this preliminary analysis is that the correlation between inflation and CO2 emission growth is negative and equal to -0.10, for inflation computed with CPI and GDP Deflator. This result is consistent with my guess of a negative relationship between price levels and climate change.

Chapter II

In the second chapter is included a brief overview of the existing literature about inflation forecasting and the rationale for choosing a time-varying parameter model. Then, the model is presented in detail.

The most successful approaches to forecasting inflation are those based on extensions of the Phillips Curve. The review of literature begins in 1958 when A. W. Phillips found empirical evidence of a negative relationship between inflation and unemployment. This suggested that policymakers could choose between different combinations of the unemployment rate and inflation, and in the following years the economic debate focused mainly on which point in the Phillips curve was best for a country's economy. However, during the 70s the relationship of the Phillips Curve seemed to disappear. Consequently, between 1980 and 1990, the approach to forecasting inflation changed with respect to "non-accelerationist" Phillips curves due to a mismatch in the classical relationship between unemployment and price levels. Economists argued that the most performing models in terms of forecast were those built around Gordon triangle model and its variants. In such models, inflation depends, among others, on the unemployment gap, defined as the difference between the unemployment rate and the NAIRU (non-

[8] WMO, "GHG Bulleting, The state of GHG in the atmosphere based on global observation through 2019",2020, Vol. 16.

accelerating inflation rate of unemployment), which is the unemployment rate at which inflation remains constant over time. The main idea is that when unemployment is below the NAIRU, inflation tends to rise over time, and vice versa when the unemployment gap is positive. Later, there were some studies about the deterioration of Phillips curve forecasts, especially in the 1990s. To sum up, all this evidence points to the fact that the Phillips curve is not stable over time and there is statistical evidence of time-changing coefficients. Moreover, it seems that, including alternative measures of real economic activity, namely housing starts or the growth of trade sales, can improve the forecasting performance of the model[9]. Given that there is no unique measure of overall economic activity to be used in the forecasting model, economists suggested modeling the huge number of activity indicators using a dynamic factor model. Other statistical methods that were proposed to overcome the problems of dealing with a very large number of predictors are model combination or model averaging, such as Bayesian Model Averaging (BMA) or bagging.

In this thesis, I follow the approach of Koop and Korobilis (2012) to address the problem of time variation in the model relevant for forecasting. They considered in their research a strategy that is called dynamic model averaging (DMA), which allows for the forecasting model to be time-varying and the coefficients in each model to also change over time.

Usually, time-varying parameter models are estimated using state space methods such as the Kalman filter, which is an algorithm that computes estimates based on linear dynamical systems in state-space form. Koop and Korobilis propose an application of the classical Kalman filter algorithm to allow for averaging across different kinds of models at each time and choose the best one in terms of forecast.

The starting point is the following generalized Phillips curve:

$$y_t = \phi + x'_{t-1}\beta + \sum_{j=1}^{p} \gamma_j y_{t-j} + \varepsilon_t.$$

Here, y_t is inflation, that is dependent on previous p lags of inflation and a set of predictor variables x_t.

Let's assume that we have m predictors, which gives a total of $K = 2^m$ models to estimate. If we assume also, that at any point in time a different forecasting model needs

[9] Stock J. H., Watson M. W., "Phillips curve inflation forecast", National bureau of economic research, working paper, 2008, No. 14322.

to be used, at time t the number of models to be estimated in order to have a forecast is 2^{mt}, that is an incredibly huge amount of data to be evaluated[10].

Hence, the state-space form of the model is specified by the following equations:

$$y_t = z_t^{(k)} \theta_t^{(k)} + \varepsilon_t^{(k)}$$

$$\theta_t^{(k)} = \theta_{t-1}^{(k)} + \eta_t^{(k)},$$

where z_t is the vector of predictors, y_t is inflation, $\theta_t = [\phi, \beta_{t-1}, \gamma_{t-1}, \dots, \gamma_{t-p}]$, with,$L_t = k$, and L_t is the model that applies at time t. Moreover, $\varepsilon^{(k)} \sim N(0, H_t^{(k)})$, and $\eta^{(k)} \sim N(0, Q_t^{(k)})$.

To complete the model specification, transitions between models are indexed from a matrix P with elements (p_{ij}) that represent the probability of using model i at time t, given that in $t-1$ the model relevant for forecasting was model j.

Subsequently, to switch between models we need to estimate the posterior probability distribution:

$$p(\Theta_t, L_t | y^t) = \sum_{k=1}^{K} p(\theta_t^{(k)} | L_t = k, y^t) p(L_t = k | y^t),$$

with $\Theta_t = (\theta_t^1, \dots, \theta_t^K)$.

The estimation is carried out with a Kalman filter with forgetting factors. Details about the algorithm used can be found in equations (2.6)-(2.10). In section 2.3 are presented all the initial conditions and assumptions needed for estimations. In the end, once all parameters are estimated forecasting is done in a recursive manner:

$$\mathbb{E}(y_t | y^{t-1}) = \sum_{k=1}^{K} \pi_{t|t-1,k} z_t^{(k)} \hat{\theta}_{t-1}^{(k)}.$$

Chapter III

In this final chapter are included details about the dataset used to carry out estimation and the results retrieved. To estimate the conditional distribution of inflation I have used two different indicators for inflation and fourteen other indicators, distinguishing between monetary measures for inflation and other relevant data. Inflation is computed using the US Consumer Price Index (CPI) and the US Gross Domestic Product deflator (GDPDEFL) time series, using quarterly observation from 1959Q1 to 2021Q3. The

[10] Koop G., Korobilis D., "Forecasting inflation using dynamic model averaging", International Economic Review, 2012, Vol. 53, pp. 867-886.

inflation rate is the annualized quarter-on-quarter growth rate over the whole sample size. Then, the log transformation is applied to the time series to work with stationary data. Finally, the Bai-Perron test rejects the presence of structural breaks in inflation time series.

The monetary measures of inflation include: 3-month Treasury Bill (TBILL), the Spread between the 10-year T-Bond yield and the TBILL (SPREAD), and the real M1 money stock (M1).

The non-monetary measures are: US Civilian Unemployment rate (UNEMP), real personal consumption expenditures per capita (CONS), private real fixed investments (INV), US real gross domestic product (GDP), new privately owned housing units started (HSTARTS), US all employees in total private industries (EMPLOY), PMI Index (PMI), average hourly earnings of US manufacturing employees in dollars per hours (WAGE), the Dow Jones Industrial Average (DIJA), CO2 emissions growth (CO2), and inflation expectations in the US (INFEXP).

Then, the results from DMA forecasting for the two proxy of inflation for the short-term (h=1), medium-term (h=4), and long-term (h=8) forecast horizons are presented. The sample period used to start producing forecasts is 30 years, thus, after the last quarter of 1990 forecasts are computed recursively. To assess the performance of DMA I have used the MSFE, MAFE and the bias. Moreover, I present forecasting results for the DMA model and three alternative forecasting models for comparison, that are:

- Forecasts using DMA, but the coefficients do not vary over time (a special case of DMA where $\lambda = 1$ and $\alpha = 0.99$).
- Forecasts using Bayesian model averaging (BMA: a special case of DMA where $\alpha = \lambda = 1$).
- Forecasts using the random walk (RW).

DMA in most cases performs better than the other three models, and in no case much worse than the best alternative method. The benchmark model (random walk) does well for longer-term horizon forecasts. For what concerns DMA with constant parameters model, we can see that it forecasts better with respect to the benchmark and also with respect to BMA. This latter result suggests that allowing for the model to change over time is at least as important as having time-varying parameters[11].

DMA, hence, forecasts well with respect to other models. In addition, although there have been used 14 predictors, the higher probability is attached to more parsimonious models. The highest expected number of predictors is six and it can be seen for longer-term forecast horizons.

This result, however, does not provide information about which predictors are more relevant in each period. As a consequence, we can look at the weight attached to models which include one predictor used by DMA, which is the posterior inclusion probability. I assumed that a predictor is considered relevant if its posterior inclusion probability is above 0.5 for at least one point in time.

For the short-term horizon forecast the average number of predictors is low, and the most relevant ones are, for CPI, Money stock from 1975 and, T-bill and Inflation Expectations from 2000. For what it concerns the GDP Deflator, Wage and Money Stock have the greatest weight attached between 1970 and 1980, whereas from 2000 the most relevant is CO2 Emissions. For H=4 and H=8, Inflation Expectation is a useful predictor for inflation starting from the 1980s. Another useful predictor for both proxies of inflation in longer-term horizon forecasts are Wage and Employment which are included in almost the whole sample. Spread and Money stock are other two variables that show strong predictive power between 1975 and the early 1990s. Consumption, GDP and Investments also often are relevant predictors for inflation, especially after the 1990s. Housing starts has significant predictive power for the H=8 horizon of forecast of inflation with CPI. Finally, it is important to notice that the DIJA and the PMI are never included in the analysis.

In this thesis the focus is on studying the predictive power of CO2 emissions with respect to inflation. Results show that the CO2 emissions predictor has stronger predictive power for the medium-term horizon forecast, and it is more useful after the great financial crisis of 2008.

Moreover, we can look at the β coefficients that measure the effect of CO2 emissions on inflation over time. From table 3.3, Carbon Dioxide emissions have a negative relationship with inflation in all time periods in which the indicator has predictive power. These

[11] Koop G., Korobilis D., "Forecasting inflation using dynamic model averaging", International Economic Review, 2012, Vol. 53, pp. 867-886.

results are consistent with the empirical evidence presented in section 1.3. According to DMA forecasts, climate change seems to have a negative correlation with inflation in the long run. Decreasing inflation can often be associated with economic growth, at least when inflation is not already close to zero. On the other hand, rising inflation implies higher prices of products and services. Thus, consumer demands for goods will fall, and by decreasing production, CO2 emissions will decrease[12].

In conclusion, there is evidence that climate change affects the conduct and transmission of monetary policy.

Chapter IV

The aim of this chapter is to bring the analysis carried out with the model in the previous chapter one step forward, with some modeling extensions that could improve the forecast of the distribution of inflation, with a focus on its left and right tails. Following Korobilis et al., the econometric setting is a time-series quantile regression that is enhanced by including time-varying parameters (TVP-QR), considering a Bayesian framework in which parameter and error have a flexible parametric distribution to capture the behavior of extreme quantiles.

Let π_t be the scalar observation of inflation and x_t a p-dimensional vector of predetermined variables that includes an intercept, lags of inflation, and exogenous predictors. In order to model the full distribution of π_t, for each of its quantiles τ, we specify the following model:

$$\pi_t = Q_\tau(\pi_t|x_t) + \varepsilon_t,$$

$$Q_\tau(\pi_t|x_t) = x_t\beta_t(\tau),$$

$$\beta_t(\tau) = \beta_{t-1}(\tau) + v_t,$$

where Q_τ denotes the conditional quantile function of the $\tau-$th quantile of inflation and $v_t \sim N_p(0, V(\tau))$ is a state error with covariance matrix $V(\tau)$.

In a quantile regression framework the estimates $\hat{\beta}(\tau)$ are the result of the minimization

[12] Shpak N., et al., "CO2 Emissions and Macroeconomic Indicators: Analysis of the Most Polluted Regions in the World", Energies, 2022, Vol. 15, No. 2928.

problem:

$$\hat{\beta}(\tau) = \min_{\beta(\tau)} \mathbb{E} \sum_{t=1}^{T} \rho_{\tau)}(\varepsilon_t),$$

with $\rho_\tau(u) = (\tau - \mathbb{I}(u < 0))u$ that is a loss function.

Following Yu and Moyeed (2001) the minimizer of the above problem is equivalent to maximizing an asymmetric Laplace likelihood. Again, following Kozumi and Kobayashi (2011), the asymmetric Laplace distribution can be written as a Gaussian-Exponential scale mixture. Hence, the first assumption is that the error distribution follows a mixture of Normals specifications and, therefore, the conditional parameter posteriors will resemble standard expressions from linear Gaussian regression models.

Then, we need to address the problem of time variation in the parameters. Borrowing ideas from Korobilis (2021), the model specified above is rewritten as:

$$Q_\tau(\boldsymbol{\pi}|\boldsymbol{\chi}) = \boldsymbol{\chi}\boldsymbol{\beta}^{\Delta}(\tau),$$

$$\boldsymbol{\beta}^{\Delta}(\tau) = \boldsymbol{v},$$

where, $\boldsymbol{\pi} = [\pi_1, \ldots, \pi_T]'$, $\boldsymbol{v} = [v_1', \ldots, v_T']'$,

$$\boldsymbol{\chi} = \underset{TxT_p}{\begin{bmatrix} x_1 & 0 & \ldots & 0 & 0 \\ x_2 & x_2 & \ldots & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ x_{T-1} & x_{T-1} & \ldots & x_{T-1} & 0 \\ x_T & x_T & \ldots & x_{T-1} & x_T \end{bmatrix}}, \boldsymbol{\beta}^{\Delta}(\tau) = \underset{T_p x1}{\begin{bmatrix} \beta_1(\tau) \\ \Delta\beta_2(\tau) \\ \ldots \\ \Delta\beta_{T-1}(\tau) \\ \Delta\beta_T(\tau) \end{bmatrix}}.$$

Putting together those pieces, the time-varying parameter quantile regression model, considering the distributional assumption on the error term, takes the following form:

$$\boldsymbol{\pi} = \boldsymbol{\chi}\boldsymbol{\beta}^{\Delta}(\boldsymbol{\tau}) + \theta(\tau)\boldsymbol{z}(\tau) + \tilde{\boldsymbol{S}}\boldsymbol{u}$$

where $\tilde{\boldsymbol{S}}$ is a TxT diagonal matrix with $t-$th diagonal element $\sqrt{\sigma(\tau)^2 k(\tau) z_t(\tau)}$. This model is completed by considering the following priors:

$$\sigma(\tau) \sim Inverse - Gamma(\rho_1, \rho_2),$$
$$z_t(\tau) \sim \exp(\sigma(\tau)), \qquad t = 1, \ldots, T.$$

Multiplication of these priors with the conditional likelihood gives the conditional posteriors:

$$\beta^{\Delta}(\tau)|\bullet \sim N\Big((\chi'U\chi+V(\tau)^{-1})^{-1}(\chi'U[\pi-\theta(\tau)z(\tau)]),(\chi'U\chi+V(\tau)^{-1})^{-1}\Big)$$

$$\sigma(\tau)|\bullet \sim Inverse-Gamma\Big(\rho_1+\frac{3T}{2},\rho_2+\sum_{t=1}^{T}\frac{(\pi_t-\chi_t\beta^{\Delta}(\tau)-\theta(\tau)z_t(\tau))^2}{2z_t(\tau)k(\tau)^2)}+\sum_{t=1}^{T}z_t(\tau)\Big)$$

$$z_t(\tau)|\bullet \sim IG\Big(\frac{\sqrt{\theta(\tau)^2+2k(\tau)^2}}{|\pi_t-\chi_t\beta^{\Delta}(\tau)|},\frac{\theta(\tau)^2+2k(\tau)^2}{\sigma^2(\tau)k(\tau)^2}\Big)^{13}.$$

The algorithm used for estimation is a Gibbs sampler and all the information about implementation is included in section 4.3. Lastly, let's notice that this is a model with more predictors than observations, thus, as a prior for the high-dimensional vector $\beta^{\Delta}(\tau)$ I use the horseshoe prior of Carvalho et al. (2010):

$$\beta^{\Delta}(\tau)|\lambda(\tau)^2,\psi_i(\tau)^{2}{}_{i=1}^{Tp} \sim N(0,V(\tau)),$$

$$V_{i,i}=\sigma(\tau)\lambda(\tau)^2\psi_i(\tau)^2,$$

$$\lambda(\tau)\sim Cauchy^{+}(0,1),$$

$$\psi_i(\tau)\sim Cauchy^{+}(0,1).$$

Conclusions

Through this work, it is exposed the role of climate change on inflation using dynamic model averaging. I have used as the dependent variable US inflation, and a set of fourteen different predictors, that are the same used by Koop and Korobilis in their research, plus a predictor for climate change that is CO2 emissions. I have observed that there is empirical evidence of the time-varying nature of inflation and the best performance of DMA has been confirmed with a more recent dataset. Secondly, I have found evidence of the predictive power that climate change has on forecasting inflation, in particular over the medium term, and that the sign of their relationship is negative. To sum up, climate change affects the conduct and transmission of monetary policy and it should be included in the models used by Central Banks when forecasting macroeconomic variables.

[13] The bullet stands for "conditional on other parameters and variables".