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Extending the portfolio optimization problem with multi-objective evolutionary algorithms in Python. An application on sustainable finance.

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Abstract

Socially responsible investment, and the modern idea of sustainable finance, has prompted investors to find new ways to measure and incorporate non-financial factors in the investment decision process. Modern investors should be able to include additional criteria other than return and risk in their investment decision process to address issues such as climate impact and deploy financial resources toward a low-carbon economy. This thesis presents an extension the traditional mean-variance portfolio optimization model and implements a multi-objective evolutionary algorithm in Python to solve the multi-objective optimization model. This research differs from other studies on ESG portfolio optimization by constraining the sustainability factor to stricter requirements in contrast to synthesized measures such as ESG risk. The empirical analysis on a subset of S&P 500 securities using the expected CO_2 reduction as third criterion shows that the proposed approach is able to approximate the efficient surface in the 3D space risk-return-carbon impact and find the set of non-dominated solutions to the portfolio optimization problem. A comparison analysis shows that the proposed approach is able to find a find alternative portfolio allocations that are competitive with the traditional mean-variance and the ESG portfolio optimization model in terms of risk-return while outperforming them in terms of portfolio carbon reduction.

Contents

1 Introduction **4**

2 Literature Review **6**
2.1 Modern Portfolio Theory . . . 6
2.2 Multi-criteria portfolio optimization . . . 7
2.2.1 Approaches by exact methods . . . 7
2.2.2 Approaches by heuristic methods . . . 9

3 Methods **10**
3.1 Traditional portfolio optimization . . . 10
3.1.1 Return . . . 10
3.1.2 Risk . . . 11
3.1.3 Portfolio optimization problem . . . 12
3.2 Generalized portfolio optimization problem . . . 13
3.3 Multi-objective evolutionary algorithms to solve multi-criteria decision-making problems . . . 14
3.4 Sustainability criterion . . . 15

4 Empirical Analysis **20**
4.1 Dataset . . . 20
4.2 Optimization problem implementation in Python . . . 22
4.3 Non-dominated surface result (Pareto front) . . . 24
4.4 Comparison with traditional optimization and ESG optimization . . . 26

5 Conclusion **30**

References **32**

Appendix **35**

List of Figures

3.1 Annual change in CO_2 emissions from energy combustion and industrial processes, 1900-2021. Source *Global Energy Review* (2021) 18

4.1 Left: dataset expected return distribution. Right: dataset expected CO_2/revenue reduction distribution. 21
4.2 Assets Return CO_2 Regression 21
4.3 Comparison between traditional efficient frontier and set of results from tri-criteria AGE-MOEA-II optimization 25
4.4 Efficient surface of optimal portfolio allocations, different angle views . . . 25
4.5 Tri-criterion heuristic optimization trade-off, loss in % return 26
4.6 Optimal portfolio allocation. Comparison of three optimization methods for portfolio volatility target at 20%, 25% and 30% 29

List of Tables

4.1 Dataset descriptive statistics 20
4.2 Assets Return CO_2 Linear Regression 22
4.3 AGE-MOEA-II algorithm parameters 24
4.4 Tri-criterion heuristic optimization trade-off, loss in % return. Descriptive statistics 26
4.5 Comparison of the results of three optimization methods for portfolio volatility target at 20%, 25% and 30% 27

5.1 Dataset processed over the 21-years period 2000-2020. Source: Refinitiv . . 35

Chapter 1

Introduction

Climate change is proving to be a significant challenge for the financial sector, trying to balance purely monetary objectives in pursuit of high returns while achieving sustainability that positively impacts the environment and society.
Since the Paris Agreement on Climate Change of 2015 and the U.N. 2030 Agenda for Sustainable Development, it has become apparent that there is a clear commitment, especially in the EU, to align financial flows towards an economy that is low-carbon, resource-efficient and sustainable. The European Union launched in 2018 the Action Plan on Financing Sustainable Growth with the aim of aligning financial resources with the EU's climate and environmental objectives. The plan toward a more sustainable economy prompted the financial sector to take into consideration environmental and social objectives in the process of capital allocation.

In this regard, with the Markets in Financial Instruments Directive (MIFID II) and the Insurance Distribution Directive (IDD), investment and insurance firms now are required to ask their clients about their investment objectives also in terms of sustainability and offer financial advice considering the client's sustainability preferences.

Thus, investors should seek for additional criteria other than risk and return to account for in the investment decision-making process. This requires an extension to the traditional mean-variance portfolio optimization developed in Markowitz (1952). A review of the literature on multi-criteria portfolio optimization is found in sec. 2.

This thesis continues the research on multi-criteria portfolio optimization by extending the traditional mean-variance portfolio optimization model to include a third criterion, the carbon impact of the portfolio in sec. 3. In sec. 4 the empirical analysis of the model is conducted implementing a multi-objective evolutionary algorithm in Python to solve the multi-criteria portfolio optimization problem. Finally, in sec. 5 the results of the empirical analysis are discussed and the main findings are summarized.

Chapter 2

Literature Review

This chapter contains a brief review of the Modern Portfolio Theory history and outlines researches on the topic of multi-criteria portfolio optimization.

2.1 Modern Portfolio Theory

The famous publication Markowitz (1952) proposed the foundations of what later became known as Modern Portfolio Theory (MPT). Before that, researches on portfolio selection had used the law of large numbers formulated in Bernoulli (1954), essentially stating that all risks could be diversified. Markowitz, however, argued that the law of large numbers did not apply to a portfolio of securities, since securities' price returns are often correlated with each other. Consequently, it is not possible for diversification to eliminate all the variance in returns. As a result, there is a trade-off between return and variance. In addition, Markowitz assumes that investors are risk-averse. Thus, when making a choice between two portfolios with the same expected return, the investor will prefer the one with the lower variance. The portfolio selection process proposed in Markowitz (1952) is divided into two stages:

1. Form beliefs on future performance of available securities, based on historical observations and experience.
2. Based on the conclusions formed in the first phase, select a combination of securities to form a portfolio

Markowitz (1952) main proposal is the mean-variance rule for the second stage. It states that an investor should select one of the **efficient portfolios**, i.e. those that minimize variance for a given expected return (mean), or conversely, those that maximize expected return for a given variance. From the mean-variance rule, it is possible to identify a set of non-dominated portfolio allocations, i.e. those portfolios are not dominated by any other portfolio in the set. A portfolio is said to be dominated by another portfolio if its variance and expected return are lower than those of another portfolio. Such set is composed of Pareto efficient portfolio allocations and is commonly referred as the efficient frontier.

Decades after the publication of Markowitz (1952), the mean-variance rule and Modern Portfolio Theory has been widely acclaimed and has become the foundation of finance courses. Nevertheless, economic theory has progressed and some critics put the MPT into question, claiming that it does not correspond to the real world in several respects. This has resulted in several branching of the theory such as multi-criteria portfolio optimization.

2.2 Multi-criteria portfolio optimization

Through the years, researches have been adapting the traditional MPT model with sophisticated risk measures and additional constraints. More recently, there have been studies suggesting the idea to include additional objectives. This branch of MPT is commonly referred as multi-criteria portfolio optimization.

The main idea of multi-criteria portfolio optimization is to include one or more additional measures other than variance and expected return in the portfolio optimization process. Two main approaches can be identified in addressing the problem of extending portfolio optimization: by exact methodologies or by heuristic methodologies[1].

2.2.1 Approaches by exact methods

Exact methods are based on mathematical programming. The idea is to formulate the problem as a mathematical optimization problem and solve it using a mathematical programming solver. The main advantage of exact methods is that they are guaranteed

[1] Heuristic methodologies are techniques designed to solve a problem when classic methods are too slow or fails to find the set of solutions. This methods that are not guaranteed to find the optimal solution. However, they are computationally efficient and can be used to find a set of solutions that are close to the optimal solution.

to find the optimal solution. The main disadvantage is that they are computationally expensive and only applicable in certain circumstances.

Several authors have attempted to expand the classical two-criteria portfolio selection model to beyond expected return and variance with exact methods since the early 1970s. Hilario-Caballero et al. (2020) identifies three major groups of studies dealing with this problem. The first group, Li et al. (2006), expanded the Markowitz model by introducing additional constraints like cardinality, rounded lots or the purchase threshold. Another group of studies, Rockafellar & Uryasev (2002), proposed alternative risk measures, such as down-side risk measures and conditional value at risk.

In the 20th century, a third group of studies fostered the idea of adding additional criteria, and thus objectives, to the portfolio optimization problem. A non-dominated tri-criteria surface is found in Hirschberger et al. (2013), Utz et al. (2014), Utz et al. (2015) that employs a constrained linear programming approach by solving a quadratic-linear-linear optimization problem in which the third objective is linear. An example of three dimensional mean-variance-liquidity frontier is also constructed in Lo & Petrov (2003) by defining several liquidity measures. A general framework for computing the non-dominated surface in tri-criteria portfolio optimization problem is proposed in Hirschberger et al. (2013), which extends the Markowitz portfolio selection approach to an additional arbitrary linear criterion. The author solves a quad-lin-lin problem and provides an exact method for calculating the non-dominated surface which may outperform standard portfolio strategies for multi-criteria constrained linear program. Also, the author develops an empirical application with sustainability as the third criterion to show how to construct the non-dominated surface. In Utz et al. (2014), sustainability is integrated as a third criterion to obtain the efficient variance-expected return-sustainability frontier so as to explain how the mutual fund industry can increase its sustainability levels. The non-dominated tri-criteria surface is calculated through the Quadratic Constrained Linear Program (QCLP) methodology, and from the experimental results it can be concluded that there is room to expand sustainability levels without affecting risk and return levels.

Nevertheless, the existing approaches based on exact methods for solving tri-criteria portfolio selection problems only have limited capabilities when the third objective is non-linear. In these instances, modern heuristic techniques have been applied to solve multi-objective problems and provide fair approximations of the optimal solution set know as the Pareto front.

2.2.2 Approaches by heuristic methods

When the optimization problem involves non-convexities, discontinuities or non-integer variables, a mathematical programming solver cannot be used. In these cases, heuristic methods are used to provide good approximations of the efficient frontier. The main advantage of heuristic methods is that they are computationally efficient and can be applied to a wide range of problems. The main disadvantage is that they are not guaranteed to find the optimal solution.

Increasing complexity of multi-criteria portfolio optimization problems prompted researchers to adopt heuristic methodologies such as evolutionary multi-objective algorithms (MOEAs). First suggested in the early 1990s to study biological processes, MOEAs are now being applied in a variety of domains including finance, notably to solve portfolio selection problems Sarker (n.d.). It was in Arnone et al. (1993) that a MOEA for optimal portfolio selection was first suggested using lower partial moments as a measure of risk. Hilario-Caballero et al. (2020) finds that early attempts to propose MOEAs as an extension of the mean-variance model mainly focused on additional constraints such as cardinality, lower and upper bounds, transaction costs, transaction lots, non-negativity constraints or industry capitalization constraints Liagkouras & Metaxiotis (2018).

Other researchers have attempted to present alternative measures of risk, the most popular of which are: semivariance, value-at-risk, expected shortfall, skewness and risk parity Liagkouras (2019). While two-objective problems are the most widely implemented among the authors, three-objective problems increased in popularity in recent years. In Anagnostopoulos & Mamanis (2010), a three-objective optimization problem is presented to identify the trade-off between risk, return and number of securities in the portfolio. The authors also compare three multi-objective optimization techniques to find the best trade-off between risk, return and portfolio cardinality. In Garcia-Bernabeu et al. (2019) and Hilario-Caballero et al. (2020) a multi-objective genetic algorithm is used to solve a tri-criteria portfolio optimization and compute the non-dominated surface using carbon risk as the third objective function.

Chapter 3

Methods

3.1 Traditional portfolio optimization

This section contains some definitions and the mathematical formulation of the traditional portfolio optimization problem.

3.1.1 Return

The return of a security is the profit or loss in a defined period of time. For a given security i, the return is denoted as R_i. Moreover, the price of the security at time t is denoted as $P_i(t)$. The return of a non-dividend paying security in the period $[0, T]$ is defined as:

$$R_i = \frac{P_T - P_0}{P_0} = \frac{P_T}{P_0} - 1 \tag{3.1}$$

When a security pays dividends in the time period, denoted as Div_T, the return for the security is defined as:

$$R_i = \frac{Div_T - P_T}{P_0} + -1 \tag{3.2}$$

eq. 3.2 can be decomposed into the dividend yield component and the capital gain component:

$$R_i = \frac{Div_T}{P_0} + \frac{P_T - P_0}{P_0} \tag{3.3}$$

The return R_i over the time period $[0, T]$ is not known in advance. Thus, R_i is a random variable. The expected outcome of the random variable R_i or, more explicitly, the expected return of the security i over the time period $[0, T]$ is denoted as:

$$\mu_i = E[R_i] \tag{3.4}$$

For a portfolio of n securities, the random return is given by:

$$R_P = \sum_{i=1}^{n} x_i R_i \tag{3.5}$$

and the expected return is defined as:

$$\mu_P = \sum_{i=1}^{n} x_i \mu_i \tag{3.6}$$

where x_i is the relative allocation of the security i in the total allocation of a portfolio, commonly referred as weight of security i:

$$x_i = \frac{\text{Value of investment} i}{\text{Total value of portfolio}} \tag{3.7}$$

In vectorized form, the expected return of n securities is defined as:

$$\mu^\top = [\mu_1, \ldots, \mu_n] \tag{3.8}$$

and the relative allocation of n securities is defined as:

$$x^\top = [x_1, \ldots, x_n] \tag{3.9}$$

Thus, eq. 3.6 can be written as:

$$\mu_p = \mu^\top x \tag{3.10}$$

3.1.2 Risk

In Modern Portfolio Theory (MPT), the risk of a portfolio is measured by the variance of portfolio returns. Because of this definition, risk is also known as volatility (of portfolio returns). However, in finance, the most common way to represent risk is by avoiding a

squared measure and use the standard deviation instead.
The variance of a portfolio is defined as:

$$\sigma_P^2 = Var(R_P) = E[(R_P - E[R_P])^2] \tag{3.11}$$

and the standard deviation, or volatility, is defined as:

$$\sigma_P = StdDev(R_P) = \sqrt{Var(R_P)} \tag{3.12}$$

To calculate the variance of a portfolio, the covariance matrix of the securities is required. The covariance matrix is a $n \times n$ matrix, where n is the number of securities in the portfolio, defined as:

$$C_P = Cov(R_P) = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1n} \\ \sigma_{21} & \sigma_{22} & \cdots & \sigma_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{n1} & \sigma_{n2} & \cdots & \sigma_{nn} \end{bmatrix} \tag{3.13}$$

where σ_{ij} is the covariance between the securities i and j.
The volatility of a portfolio is given by:

$$\sigma_P = \sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j \sigma_{ij}} \tag{3.14}$$

and in vectorized form:

$$\sigma_P = \sqrt{x^\top C_P x} \tag{3.15}$$

3.1.3 Portfolio optimization problem

The assumptions of Modern Portfolio Theory can be summarized as follows:

- the investor has a set of n securities to choose from;
- the investor has a fixed sum of money to invest;
- the investment horizon, or the holding period, is fixed with a predetermined beginning and end.

Using the notation defined in the previous sections, the traditional mean–variance portfolio optimization problem for risk-averse investors can be formulated as:

$$\begin{aligned} \text{maximize} \quad & \mu_p = \mu^\top x \\ \text{minimize} \quad & \sigma_P = \sqrt{x^\top C_P x} \\ \text{subject to} \quad & \textstyle\sum_{i=1}^{n} x_i = 1 \\ & \alpha_i \leq x_i \leq \beta_i \end{aligned} \tag{3.16}$$

with α_i and β_i as the lower and upper limits of the weight of security i respectively. The most common limits are $\alpha_i = 0$ and $\beta_i = 1$, representing a portfolio without short positions.

3.2 Generalized portfolio optimization problem

The traditional mean–variance portfolio optimization problem is a special case of a more general optimization problem.

Denoting the objective functions as $f_1(x), \ldots, f_m(x)$ and the set of feasible solutions S, the general optimization problem can be formulated as:

$$\begin{aligned} \text{maximize} \quad & f_1(x) \\ & \vdots \\ \text{maximize} \quad & f_m(x) \\ \text{subject to} \quad & x \in S \end{aligned} \tag{3.17}$$

Lundström (n.d.) proposes an interesting explanation on objective functions in the realm of portfolio optimization. The author outline how objectives need to be classified as either stochastic or deterministic and, in the case of stochastic objectives, a deterministic interpretation has to be determined.

The author argue that certain measures, such as R&D, could fall into either category. They explains that R&D expenditure during the holding period is indeed stochastic but investments before the holding period may play the most important role for the investor, allowing the measure to be considered deterministic.

Other objectives, however, do not have characteristics that would allow for a similar interpretation to the one described above. In these cases, the author suggest to assign a mean-variance pair to each stochastic target, as its done for portfolio return R_P in

Modern Portfolio Theory:

$$
\begin{aligned}
\text{maximize} \quad & E[R_P] \\
\text{minimize} \quad & Var(R_P) \\
\text{subject to} \quad & x \in S
\end{aligned}
\tag{3.18}
$$

Yet, for practical reasons, whenever fluctuations of stochastic objectives are of minor amplitude or importance, the objectives might be estimated by simply considering the expected value. The author rightfully point out that this is highly convenient, since it allows to calculate only the means while reducing the effort of estimating covariances.

Denoting as $z_1, \dots, z_m$ the additional criteria to be included in portfolio optimization and disregarding the variances on these objectives, the multi-criteria portfolio optimization problem can be formulated as:

$$
\begin{aligned}
\text{maximize} \quad & E[R_P] \\
\text{minimize} \quad & Var(R_P) \\
\text{maximize} \quad & E[z_1] \\
& \vdots \\
\text{maximize} \quad & E[z_m] \\
\text{subject to} \quad & x \in S
\end{aligned}
\tag{3.19}
$$

3.3 Multi-objective evolutionary algorithms to solve multi-criteria decision-making problems

A multi-objective optimization problem (MOP) is a subclass of multi-criteria decision-making (MCDM) strategies that involves the concurrent optimization of multiple objective functions. When the conflicting degree of objectives renders it impossible to find a feasible solution that optimizes all objective functions simultaneously, a set of non-dominated solutions, known as Pareto front (or efficient frontier in portfolio optimization), exists where none of the objectives can be improved without worsening one or more of the others. Typically, a multi-objective optimization problem can be presented as follows:

$$
\begin{aligned}
\textit{minimize} \quad & f(w) = [f_1(w), f_2(w), ..., f_m(w)]^\top, \\
\textit{subject to} \quad & w \in S
\end{aligned}
$$

where the vector $w = [w_1, w_2, \dots, w_n]^\top$ is a n-parameter set included in the decision space S , and $f_i(w)$, are the objectives to be minimized at the same time.

When the objective functions of a multi-objective optimization problems are non-linear and non-convex, a mathematical programming solver cannot be used. A common approach to solve MOPs is to use multi-objective evolutionary algorithms (MOEAs). An evolutionary algorithm (EA) is a stochastic search algorithm that mimics the process of natural evolution. The algorithm starts with a population of candidate solutions, called individuals, and iteratively improves them by applying the following steps until a stopping criterion is met:

1. Evaluate the fitness of each solution in the population.
2. Select the best solutions to be used as parents for the next generation.
3. Apply crossover and mutation operators to create new solutions.
4. Replace the worst solutions in the population with the new solutions.

MOEAs have been successfully applied in studies to solve multi-objective optimization problems with conflicting objectives. The goals of these heuristic methodologies are:

- convergence: produce a set of solutions that is as close as possible to the Pareto front;
- diversity: produce a well distributed set of solutions for a complete and informed decision-making process.

The success of a multi-objective evolutionary algorithm is given by its ability to balance the goal of convergence and the goal of diversity.

For the purpose of this thesis, the multi-objective optimization problem is solved using an evolutionary algorithm called AGE-MOEA-II developed in Panichella (2022). The choice of this algorithm was driven by its performance on estimating the geometry of the non-dominated set of solution and the ease of implementation in Python.

3.4 Sustainability criterion

Previous researches including sustainability as part of multi-criteria portfolio optimization have mainly used ESG score as the sustainability criterion. In general, an ESG score is a normalized measure of the environmental, social and governance level of sustainability of a company. While ESG scores are a good comparison indicator, they are not designed to be a performance indicator.

One of the objectives of this thesis is to propose a rigorous approach in the choice of the sustainability factor that is directly related to the sustainability performance of a company. The sustainability factor is then modeled into the sustainability criterion, which is the objective function that is optimized in the multi-objective portfolio optimization problem. The goal of this approach is to provide investors with clear measures on portfolio sustainability performance (i.e. portfolio allocation A is expected to reduce carbon footprint by 7%, as opposed to portfolio allocation A has an ESG risk of 35).

An example including sustainability factor in portfolio selection is found in Nagy et al. (2013). The authors develop three different strategies to integrate ESG rating to explore how each one affects performance against a benchmark. The strategies are defined as follows:

- The first strategy, called *ESG worst-in-class exclusion*, narrows the investment universe by excluding companies with the lowest current rating. The model first weight assets by market capitalization and then over-weights securities with high ESG ratings while under-weighting those with low ratings within the universe, keeping the other portfolio allocation close the initial state.
- The second strategy, referred as *simple ESG tilt* approach, no asset is excluded from the portfolio selection. Instead, assets with higher ESG ratings are over-weighted and assets with lower ESG ratings are under-weighted. As in the first strategy, the other portfolio exposures very close the benchmark.
- The third strategy, *ESG Momentum* approach, do not exclude any asset like the second. Instead, assets that have improved their ESG rating over the previous 12 months are over-weighted and assets that have decreased their ESG rating over the same period are under-weighted.

While the first two strategies are a very common approach to sustainable portfolio optimization, the model developed in this thesis was inspired by the logic behind the third strategy. The goal is to improve the investment sustainability over time by having a **positive expected trajectory** on the sustainability factor.
In simple terms, momentum is the tendency of a variable to continue moving in the same direction, with the idea being to exploit past trends to predict future performance. In

the context of Nagy et al. (2013), the *ESG Momentum* strategy hopes to capture the tendency of a company to improve its ESG rating over time, therefore yielding a better portfolio ESG rating in the future.

However, the *ESG momentum* approach is found to be flawed as it under-weights top ESG performers. This bias happens because the *ESG momentum* at time t of a company with ESG rating equal to 100 at time $t-1$ and 100 at time t is 0. In this case, the company is not improving its ESG rating, but it is still a top performer. Being defined on a fixed scale (usually 0 to 50 or 0 to 100), the ESG rating is clearly not suitable for the purpose of sustainability performance.

In order to solve this problem and design a sustainability performance measure, this thesis proposes a conjecture on the requirements of the sustainability factor. Such factor should be designed to be:

- a continuous variable;
- influenced by the actions of the company;
- related to environmental, social or corporate governance matters.

While there are limitations in the choice of such factor, environmental impact measures are often a good candidate for the purpose.

For the empirical analysis, this thesis proposes the use of carbon dioxide emissions as the sustainability factor and the expected carbon dioxide reduction as the sustainability criterion. The rationale behind this choice is that carbon dioxide emissions are a direct result of the company operations. Moreover, there is no upper or lower limit[1] for carbon dioxide emissions, which makes it a good candidate for a continuous variable[2]. Finally, carbon dioxide emissions are a well-known and widely used measure of environmental impact, which makes it easy to find data and to compare results with other studies.

To account for company or market specific fluctuations, the carbon dioxide emissions are normalized with the company revenues. This will allow to compare the carbon dioxide emissions of a company in distressed periods such as the COVID-19 pandemic. In fact, as shown in fig. 3.1, while CO_2 emissions lowered in 2020 they quickly rebounded in 2021.

[1] Producing a negative amount of carbon dioxide emission means removing carbon dioxide from the atmosphere and thus, carbon dioxide reduction.

[2] This statement holds true for the purpose of this thesis. In the realm of physics, there is a finite amount of particles in the universe.

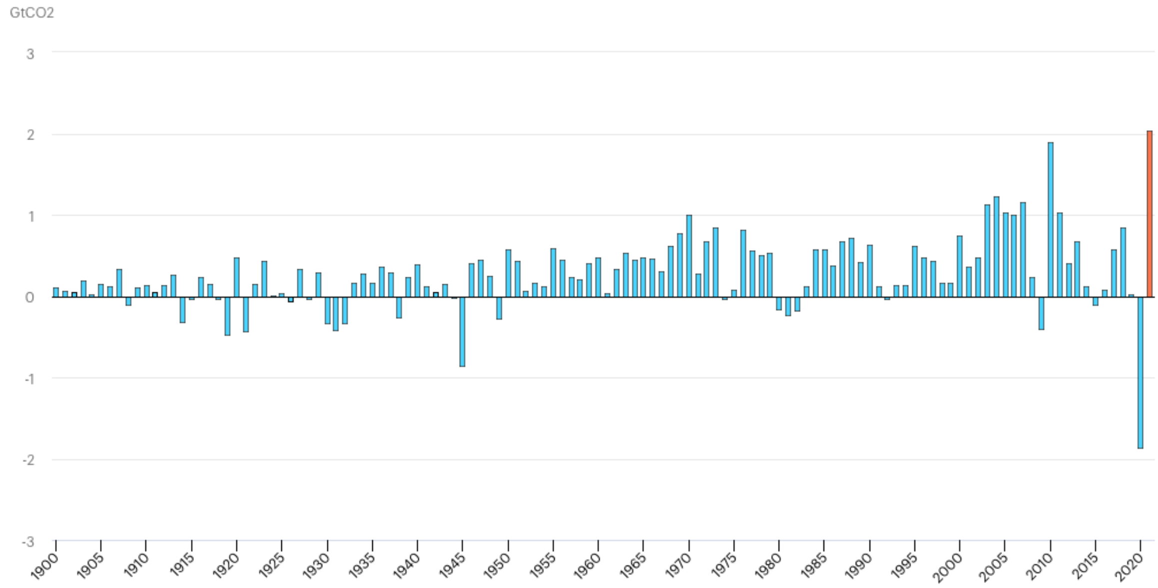

Figure 3.1: Annual change in CO_2 emissions from energy combustion and industrial processes, 1900-2021. Source *Global Energy Review* (2021)

This shows that the plunge in CO_2 emissions in 2020 was in fact due to the economic slowdown rather than improved sustainability of companies. Normalizing the carbon dioxide emissions by revenues is an attempt to neutralize this effect.

The notation used for the sustainability criterion is as follows. First, one has to choose a sustainability factor:

$$SF = \frac{tonnes\ of\ CO_2\ emission}{revenues\ in\ dollar} \tag{3.20}$$

In the interest of using the sustainability performance as a criterion in the optimization problem and to allow for an easier interpretation, the sustainability performance defined as the reduction in $\frac{tonnes\ of\ CO_2\ emission}{revenues\ in\ dollar}$. At an arbitrary time t, this is:

$$\Delta SF = -\frac{SF_t - SF_{t-1}}{SF_{t-1}} \tag{3.21}$$

Also, following the explanation in sec. 3.2, the outcome of the sustainability criterion will be assumed deterministic. Therefore, the expected sustainability performance for company i over n periods is defined as:

$$\theta_i = E[\Delta SF_i] = \frac{\sum_{t=1}^{n} \Delta SF_t}{n} \tag{3.22}$$

The portfolio expected sustainability performance, using the allocation notation x_i introduced in eq. 3.7, can then be defined as:

$$\theta_P = \sum_{i=1}^{n} \theta_i x_i \tag{3.23}$$

in vectorized form, with $\theta = [\theta_1, \dots, \theta_n]^\top$ for n companies:

$$\theta_P = \theta^\top x \tag{3.24}$$

Finally, the tri-criteria portfolio optimization problem extended from eq. 3.16 can be formulated as follows:

$$\begin{aligned} \text{maximize} \quad & \mu_p = \mu^\top x \\ \text{minimize} \quad & \sigma_P = \sqrt{x^\top C_P x} \\ \text{maximize} \quad & \theta_P = \theta^\top x \\ \text{subject to} \quad & \textstyle\sum_{i=1}^{n} x_i = 1 \\ & \alpha_i \leq x_i \leq \beta_i \end{aligned} \tag{3.25}$$

Chapter 4

Empirical Analysis

This chapter presents an application of the multi-criteria portfolio optimization model proposed in eq. 3.25.

4.1 Dataset

A dataset from Refinitiv was used to perform the analysis. To investigate the potential of multi-objective algorithms, the first analysis is performed on a large set of variables. Starting from the list of securities included in the S&P 500 Index[1], the set was narrowed down to securities with more than 10 years of valid environmental data. The resulting set of 198 securities and the related expected return and expected CO_2 reduction over the 21 years period 2000-2020 is shown in tbl. 5.1.

Descriptive statistics of expected return (μ) distribution and expected CO_2/revenue reduction (θ) distribution of the securities are illustrated in tbl. 4.1. The formula used to compute μ is described in eq. 3.4 and the formula used to compute θ is described in eq. 3.23

Table 4.1: Dataset descriptive statistics

	μ	θ
Mean	9.3744	1.4575
Standard deviation	5.5180	8.8355
Minimum	-6.3160	-44.7434

[1]As of the end of June, 2022

	μ	θ
25th percentile	6.1608	-1.4255
Median	9.1959	2.9054
75th percentile	12.8253	6.1738
Maximum	24.2802	20.6077

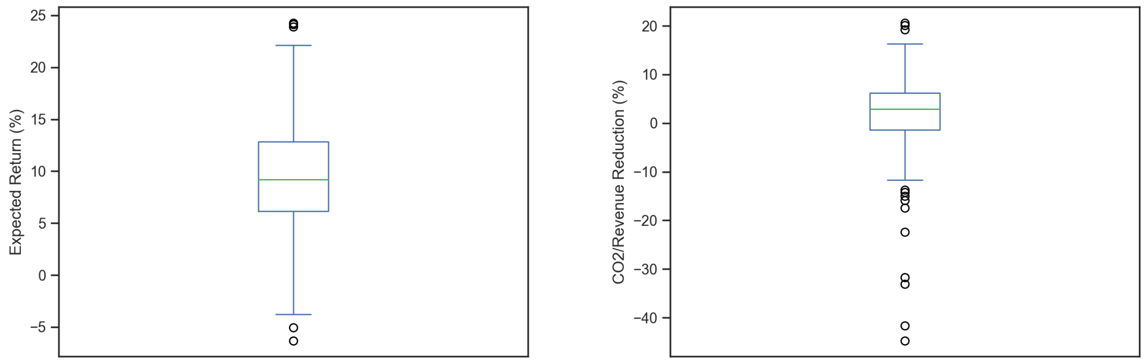

Figure 4.1: Left: dataset expected return distribution. Right: dataset expected CO_2/revenue reduction distribution.

A linear regression analysis is performed to explore a possible relationship between μ and θ. The result shown in fig. 4.2 and in tbl. 4.2.

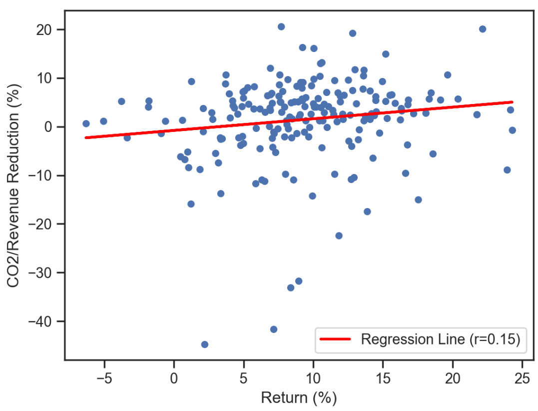

Figure 4.2: Assets Return CO_2 Regression

Table 4.2: Assets Return CO_2 Linear Regression

Slope	0.24
Intercept	-0.79
R^2	0.15
p-value	0.04
Standard error	0.11

The linear regression analysis shows that there is a positive correlation between μ and θ. The p-value is less than 0.05, which means that the correlation is statistically significant, although the R-squared value of 0.15 signifies that the linear regression model explains only 15% of the variance in θ.

4.2 Optimization problem implementation in Python

The optimization problem is implemented in Python using the *pymoo* module presented in Blank & Deb (2020). The inputs for the custom problem are:

- the list of assets;
- daily returns of the assets;
- yearly CO_2 reduction of the assets.

The parameters of the *pymoo* problem are:

- number of variables: number of assets;
- number of objectives: 3;
- lower and upper bounds of the single variables: 0 and 1.

The objective functions, denoted `f1`, `f2` and `f3` in code, are the volatility, the expected return and the CO_2 reduction. The portfolio volatility is the standard deviation of the portfolio return calculated with eq. 3.15. The expected return is the mean of the portfolio returns as per eq. 3.8. The portfolio expected CO_2 reduction per dollar revenue (eq. 3.20) is given by eq. 3.24.
Multi-objectives algorithms expect the objective functions to be minimized. Therefore, to maximize the portfolio expected return and CO_2 reduction, `f2` and `f3` objective functions

are multiplied by -1.

The `ret`, `theta` and `vol` methods of the `TriCriterionPortfolioOptimization` class contain vectorized form of the objective functions. This is done to speed up the optimization process by evaluating the algorithm population in parallel rather than element wise.

```
class TriCriterionPortfolioOptimization(pymoo.core.problem.Problem):
    def __init__(
        self,
        tickers: list[str],
        daily_log_ret: pandas.DataFrame,
        yearly_co2_change: pandas.DataFrame,
    ):

        super().__init__(
            n_var=len(tickers),
            n_obj=3,
            xl=0.0,
            xu=1.0
            )

        self.tickers: list[str] = tickers
        self.mu_vector: numpy.ndarray = (daily_log_ret.mean() * 252).values
        self.sigma_matrix: numpy.ndarray = (daily_log_ret.cov() * 252).values
        self.theta_vector: numpy.ndarray = yearly_co2_change.mean().values

    def ret(self, weights_matrix: numpy.ndarray) -> numpy.ndarray:
        return weights_matrix @ self.mu_vector

    def theta(self, weights_matrix: numpy.ndarray) -> numpy.ndarray:
        return weights_matrix @ self.theta_vector

    def vol(self, weights_matrix: numpy.ndarray) -> numpy.ndarray:
```

```
        return np.sqrt(np.diag(weights_matrix @ (self.sigma_matrix @ weights_matrix.T

    def _evaluate(self, x: numpy.matrix, out, *args, **kwargs):
        f1 = self.vol(x)
        f2 = self.ret(x) * -1
        f3 = self.theta(x) * -1

        out["F"] = [f1, f2, f3]
```

To further improve computational performance, it is possible to implement the optimization constraints with and ex-ante approach. Instead of checking the constraint satisfaction after each iteration, the code is developed to correct the input variables beforehand to satisfy the constraint in eq. 3.25, effectively making each iteration a feasible solution to the problem.

```
class PortfolioAllocationConstraint(pymoo.core.repair.Repair):

    def _do(self, problem, X: numpy.ndarray, **kwargs):
        X[X < 1e-3] = 0
        return X / X.sum(axis=1, keepdims=True)
```

Following a trial and error process, the best parameters found for AGE-MOEA-II algorithm are the following:

Table 4.3: AGE-MOEA-II algorithm parameters

Probability of mutation	0.2
Probability of crossover	0.2

4.3 Non-dominated surface result (Pareto front)

The set of results of the optimization performed is first shown in fig. 4.3 with a traditional 2D cartesian plot with the expected return on the y-axis and the portfolio volatility on the x-axis. The third objective function, the CO_2 reduction, is on the color map scale. The plot also shows in red the frontier of the mean-variance optimization.

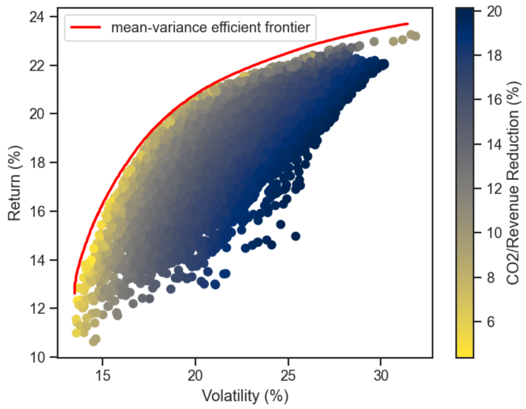

Figure 4.3: Comparison between traditional efficient frontier and set of results from tri-criteria AGE-MOEA-II optimization

The tri-criterion problem allows for the visualization of the set of non-dominated solutions in a 3D space. The set of results forms a surface in fig. 4.4 with a 3D cartesian plot having the expected return on the z-axis, the portfolio volatility on the x-axis and the expected CO_2 reduction on the y-axis.

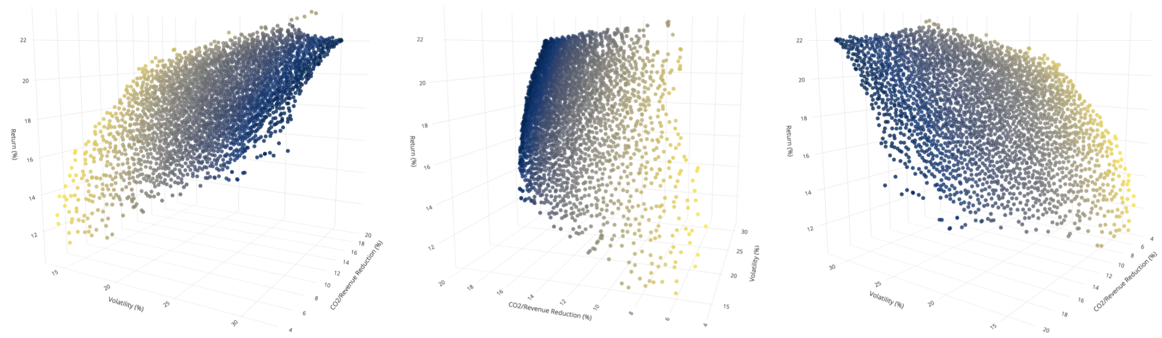

Figure 4.4: Efficient surface of optimal portfolio allocations, different angle views

As described in sec. 2.2.2, heuristic methodologies are not guaranteed to find the optimal solution to an optimization problem. An analysis is performed to show the trade-off of the approximated solution, visible in the gap between the efficient frontier and the set of solutions in fig. 4.3. This trade-off is due to computational constraints on the large number of variables to be modeled. The trade-off is expressed as loss in percentage return for any given level of portfolio volatility. The analysis is performed by computing the distance between returns on the traditional mean-variance frontier and returns on the tri-criterion frontier. Using 100 steps on the mean-variance frontier volatility range the median loss in percentage return is found to be 0.4757. This means that portfolio return on the tri-criterion solutions is averagely 0.4757% less than the optimal portfolio return on the mean-variance frontier.

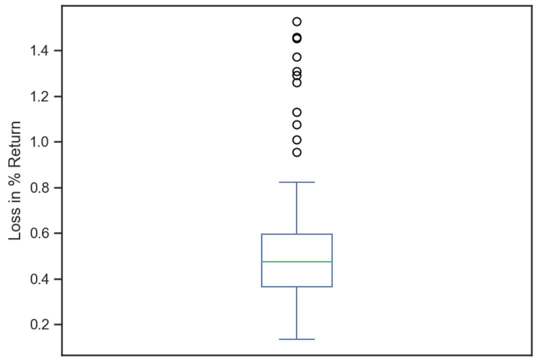

Figure 4.5: Tri-criterion heuristic optimization trade-off, loss in % return

Table 4.4: Tri-criterion heuristic optimization trade-off, loss in % return. Descriptive statistics

Mean	0.5663
Standard deviation	0.3257
Minimum	-0.1353
25th percentile	-0.3656
Median	0.4757
75th percentile	0.5940
Maximum	1.5279

4.4 Comparison with traditional optimization and ESG optimization

The comparison analysis has been performed on a smaller set of selected securities. Ten securities with highest market capitalization have been selected from tbl. 5.1. The optimization has been performed on a smaller time period than the previous analysis, from January 1, 2015 to December 31, 2020.

The analysis aims to compare the results of the traditional mean-variance portfolio optimization with the results of two multi-criteria optimization models: (i) the model developed in this thesis (hereinafter M-V-CO_2 reduction model) and (ii) the ESG optimization model (hereinafter M-V-ESG risk model) commonly applied in sustainable portfolio optimization literature.

The ESG optimization model has been structured as a general multi-criteria portfolio optimization problem in eq. 3.19. The third objective function, ESG risk of the portfolio ESG_P, is:

$$\text{minimize} \quad ESG_P = \sum_{i=1}^{n} ESG_i w_i$$

with ESG_i representing the ESG score and w_i the relative allocation of the ith security in the portfolio.

The ESG optimization model has been solved using the same algorithm described in sec. 3.3. ESG scores have been sourced from Morningstar Sustainalytics, one of the leading provider of ESG data together with MSCI ESG IVA and Refinitiv. The ESG data is in form of ESG risk, which are normalized scores between 0 and 40+, with values closer to 0 meaning negligible ESG risk and values over 40 meaning severe risk exposure to environmental, social and corporate governance matters.

To compare the models, tbl. 4.5 shows the results of the optimizations in three comparable points of each Pareto front. The first point is the optimal portfolio close to 20% volatility, the second point is the optimal portfolio close to 25% volatility and the third point is the optimal portfolio close to 30% volatility. The results are shown in terms of expected return, volatility, CO_2 reduction and ESG risk of the optimal portfolio allocation produced by the model.

Table 4.5: Comparison of the results of three optimization methods for portfolio volatility target at 20%, 25% and 30%

Optimization method	Return	CO_2 Reduction	ESG Risk	Volatility
Mean-Variance	22.51	2.53	21.35	20.14
M-V-CO_2 reduction	22.11	6.23	21.14	20.19
M-V-ESG risk	21.73	2.98	20.04	20.18

Optimization method	Return	CO_2 Reduction	ESG Risk	Volatility
Mean-Variance	31.87	0.21	18.09	25.22
M-V-CO_2 reduction	31.22	7.37	18.46	25.20
M-V-ESG risk	31.12	2.68	16.62	25.23
Mean-Variance	38.96	-0.24	15.65	30.27
M-V-CO_2 reduction	38.37	6.62	16.06	30.18
M-V-ESG risk	38.80	1.26	15.34	30.28

The first comparison with 20% volatility target shows very similar results for the tradition mean-variance optimization and both multi-criteria optimization in terms of expected return. The M-V-ESG risk model produces a portfolio with marginally better ESG risk than the other models. On the portfolio CO_2 reduction performance, the M-V-CO_2 reduction model developed in this thesis produces a portfolio with more than double the performance of the other two models.

The second comparison with 25% volatility target shows a similar trend as the first comparison. The M-V-ESG risk model portfolio allocation has a marginally better ESG risk than the other models. The M-V-CO_2 reduction model developed in this thesis outperforms the other two models by an even larger margin than the first comparison in terms of CO_2 reduction.

The third comparison with 30% volatility has the same conclusion of the second comparison. The expected return is in line for each portfolio allocation produced by the three models. A slight improvement in ESG risk is observed for the M-V-ESG risk model. The M-V-CO_2 reduction model developed in this thesis guarantees a portfolio with a CO_2 reduction by far superior to the other two models.

The conclusion of the comparison analysis is that the M-V-CO_2 reduction model developed in this thesis outperforms the traditional mean-variance model and the M-V-ESG risk model in terms of CO_2 reduction with little to no impact on the expected return of the portfolio. The M-V-ESG risk model slightly outperforms the other models in terms of ESG risk with no significant impact on the expected return. The M-V-ESG risk model also comes ahead of the traditional mean-variance model in terms of CO_2 reduction, possibly showing a relationship between lower ESG risk and positive CO_2 reduction.

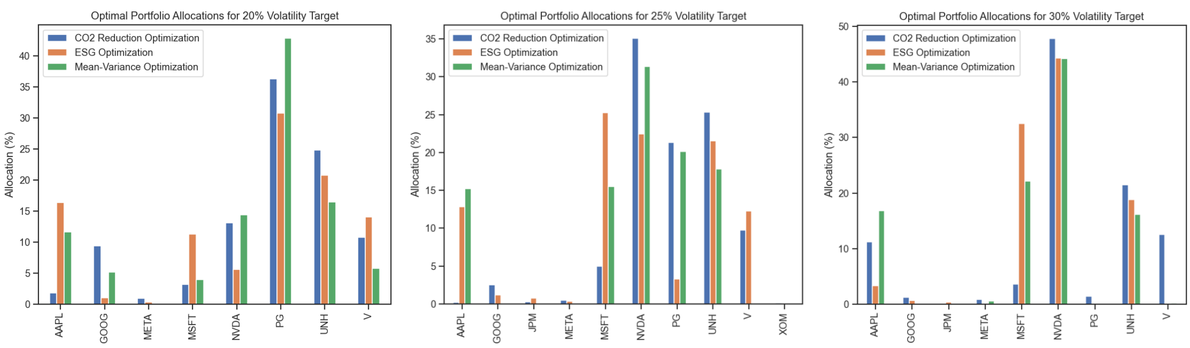

Figure 4.6: Optimal portfolio allocation. Comparison of three optimization methods for portfolio volatility target at 20%, 25% and 30%

Chapter 5

Conclusion

The traditional mean-variance model has gained popularity for its simplicity and its ability to provide risk-averse investor with the optimal portfolio allocation. However, the model is not able to incorporate criteria other than return and risk that may be concerning investors. In this thesis, I propose a model that extends the traditional mean-variance model to incorporate other criteria. In the context of sustainable finance, I propose a set of rules to select the sustainability factor as opposed to the common ESG risk found in sustainable portfolio optimization literature. Furthermore, I solve the multi-objective optimization problem using a multi-objective evolutionary algorithm implemented in Python.

The model is tested on a real-world portfolio of 198 stocks using the expected CO_2 reduction as third criterion and successfully finds the set of non-dominated portfolio allocations. The model is also compared to the traditional mean-variance model and the mean-variance-ESG risk model on a smaller set of data. The comparison shows that the model developed in this thesis outperforms the other two models in terms of CO_2 reduction with little to no impact on the expected return and volatility of the portfolio. In conclusion, the model proposed is able to find alternative allocations that may satisfy investors' sustainability concern in a more rigorous manner than the one presented in portfolio ESG risk optimization.

This model may find application in the financial industry, where asset managers are increasingly incorporating sustainability criteria in their investment strategies. The model may also be useful for companies that want to offset their carbon footprint by allocating resources in economically and sustainable efficient investments.

The main limitation on the theoretical model is assuming the sustainability criteria as a deterministic objective. On the empirical analysis, the lack of available raw ESG data plays another limiting factor in back-testing the model on different time periods, geographical regions, industries and sectors.

Further research on the model might explore the use of other(s) sustainability factor(s), fine-tune the parameters of the optimization algorithm and explore the use of other optimization algorithms.

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Appendix

Table 5.1: Dataset processed over the 21-years period 2000-2020. Source: Refinitiv

Ticker	μ	θ	Ticker	μ	θ	Ticker	μ	θ
A	4.9382	7.2041	ES	10.0422	16.1181	NEE	16.2589	1.4442
AAL	-0.9382	-1.4089	ETN	13.7274	2.2076	NEM	5.357	4.679
AAPL	24.1333	3.4388	ETR	10.4528	1.2675	NI	10.0106	4.7365
ABT	11.5354	-9.7224	EW	20.3946	5.7501	NKE	16.6087	-9.5425
ACN	16.3231	3.7962	EXC	8.0993	0.4537	NLOK	9.9377	-14.1799
ADBE	16.7378	-3.7526	EXPD	11.9196	6.2776	NOC	13.6151	11.6192
ADI	7.9257	4.1016	F	-1.8349	4.0708	NRG	8.8144	8.3747
ADSK	17.5245	-15.041	FCX	6.8372	0.0868	NSC	14.5087	3.0871
AEE	9.0482	2.5326	FDX	9.3919	4.1597	NTRS	5.0557	7.3644
AEP	9.1986	7.9758	FE	6.1542	3.6615	NVDA	24.2802	-0.7477
AES	-0.6069	1.1221	FMC	15.1769	14.9951	NWL	2.0902	3.7773
AKAM	-5.0723	1.1678	GD	10.2158	4.0763	OKE	13.871	-17.4552
ALB	15.2821	1.7027	GE	-3.7628	5.1805	OMC	3.6941	8.8481
ALL	9.8563	8.4625	GIS	8.9119	5.148	ORCL	4.6549	-2.2391
AMAT	6.5421	2.9539	GOOG	21.7498	2.5231	OXY	5.8885	-0.3447
AMCR	8.1927	-0.4472	GOOGL	21.7234	2.5231	PEG	10.0827	4.1022
AMD	9.2252	16.3041	HAL	1.8541	-8.784	PEP	8.9713	2.0026
AMGN	7.5129	6.3007	HAS	10.4725	10.3753	PFE	4.3285	3.8284
APA	1.0584	-8.4268	HBAN	1.2388	9.322	PFG	6.9239	12.0761
APD	13.2124	-2.6227	HD	9.2475	9.1198	PG	7.199	-1.3797
AWK	18.4367	6.9619	HES	6.5043	-11.2227	PLD	11.8468	-22.4021
AXP	6.6965	6.4011	HIG	2.7679	1.5646	PM	8.6482	5.1333

Ticker	μ	θ	Ticker	μ	θ	Ticker	μ	θ
BA	10.9939	-0.1275	HPQ	3.19	-7.3992	PNC	9.1931	5.6694
BAC	3.9591	6.8091	HRL	12.5285	-2.9315	PNW	9.2126	5.3993
BALL	18.3192	5.7233	HST	7.2798	-5.3064	PPG	10.6097	13.2332
BAX	9.4478	1.2565	HSY	11.4428	2.5371	PPL	9.3402	-1.114
BBWI	8.458	1.6565	HUM	19.651	10.6225	PRU	7.7322	2.9431
BDX	12.2175	6.534	HWM	10.6638	1.2131	PVH	12.9315	-10.4799
BF-B	14.3675	2.2784	IBM	2.7007	2.8678	QCOM	4.7711	-3.8599
BIIB	10.0388	9.7865	IFF	7.6415	5.122	RCL	6.3065	-10.9507
BKR	3.4577	-2.4433	INTC	3.3408	-13.7351	ROK	15.2003	5.2233
BMY	3.7375	10.6251	INTU	12.8327	19.2248	RTX	9.6719	-2.0355
BSX	5.7562	8.2537	IP	3.3647	-2.5905	SBUX	18.0591	2.7988
C	-6.316	0.6939	ITW	11.3904	3.3991	SHW	19.1396	5.5534
CAG	6.8648	3.8484	JCI	0.763	-6.7415	SJM	11.6961	1.0566
CAT	12.6474	2.3725	JNJ	8.397	4.9413	SLB	0.992	-5.1744
CB	13.5571	7.7143	JNPR	-3.376	-2.2228	SO	11.7801	6.9339
CBRE	14.0698	7.3238	JPM	7.7971	7.2728	SPG	11.5143	9.5052
CCL	0.476	-6.1573	KMB	7.0274	6.8754	SPGI	13.585	10.4414
CHD	15.6325	6.5603	KO	5.8539	-11.7299	SRE	13.0976	4.7394
CI	10.5185	12.9859	KR	7.0536	4.8526	STX	12.7634	-3.9861
CL	6.9198	3.395	LIN	13.6222	1.2154	STZ	17.1818	2.684
CLX	9.4596	5.9665	LLY	7.5698	8.5832	T	4.1884	5.3392
CMA	4.2238	4.3008	LMT	15.5023	6.4682	TAP	4.9275	-1.9509
CMI	16.8168	5.5383	LNT	10.797	2.8524	TEL	10.5338	3.9155
COF	5.2896	7.996	LUMN	-1.7928	5.3244	TGT	9.8318	2.5695
COP	7.0828	-4.2199	LUV	8.8154	-1.4311	TROW	12.991	11.7441
COST	12.1426	1.5208	LYB	18.5745	-5.6052	TSN	8.0493	1.2158
CPB	4.0403	1.7997	MAR	12.7305	-10.8586	TT	12.5125	9.627
CRM	23.9006	-8.9012	MAS	7.151	-41.6609	TXN	7.471	-0.3706
CSCO	0.6061	1.3285	MCHP	12.0889	2.6007	UAL	2.0864	-1.0197
CSX	15.8589	4.8191	MCK	10.6248	-4.2672	UNP	16.3343	3.2404
CVS	7.5823	10.7145	MDLZ	7.9332	-2.0262	UPS	7.1387	-1.0577

Ticker	μ	θ	Ticker	μ	θ	Ticker	μ	θ
CVX	7.0379	-2.3387	MDT	7.6421	5.0063	V	22.1356	20.0842
DE	14.4869	6.243	MET	8.3727	-33.074	VMC	8.1796	9.7346
DGX	14.487	5.4266	MGM	6.1804	-4.4602	VZ	4.9897	5.8963
DIS	10.7913	4.6056	MKC	14.7613	-1.2948	WAT	10.9369	7.0758
DRI	14.1709	2.6888	MMC	6.8987	6.5982	WDC	14.2841	-6.4456
DTE	11.0901	4.1484	MMM	9.0948	9.0951	WEC	14.0873	1.4877
DUK	8.5789	-10.9028	MO	14.9664	9.2658	WFC	4.9726	-3.3972
EBAY	9.7334	-0.165	MOS	2.9794	-5.4816	WHR	8.2986	2.0308
ECL	13.3056	-0.5837	MRK	4.5589	2.6337	WM	11.925	5.0835
ED	8.4376	-2.2492	MRO	2.1867	-44.7434	WMT	5.7238	3.6694
EIX	7.7014	20.6077	MSFT	8.9477	-31.737	WY	4.8841	4.1945
EL	12.8032	7.4977	MSI	1.2251	-15.8917	XEL	10.3233	5.8077
EMR	8.0065	-9.7433	MTD	16.7984	4.5392	XOM	3.3198	-2.2366