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Monetary Policy during the Greenspan era: a Taylor Rule analysis through switch regimes evaluated with STR model

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Summary

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Master thesis

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Abstract

Despite the theorization of linear models to explain the monetary policy choices, as the Taylor Rule framework, evidence shows that policymakers often did not follow linear economic strategies. Here we demonstrate how Greenspan, chairman of the Federal Reserve from 1987 to 2006 followed a risk management approach and prove his preemptive choices through the switch regimes model. Through some empirical tests, we will verify how inflation and risk in financial markets influence the aggressive response to monetary policy rather than traditional macroeconomic aspects and how a better version of the model can be implemented by considering the risk factor not as a transition variable, but rather as a component of a GARCH model.

1. Introduction

Taylor Rule has been developed in 1992 by Taylor to describe monetary policy in terms of nominal interest rates and inflation and to linearize policymakers' decisions.

The rule, however, was proven to have some limitations (McCallum 2001, Orphanides 2003) for example the belief that since the future will act like the past, therefore, the rule must rely on precise information that is not provided at the time of prediction. Hence, broader versions of the rule have been implemented to better capture imperfections and give a more precise picture of the policy perspective.

Despite various implementations shown in the second chapter of the thesis, the analysis of the Taylor Rule framework pointed at divergences with the level of the economy due to the underestimation of macroeconomic factors and incorrect predictions. Even with forecasting variables, policymakers have not fully yet been able to act preemptively towards the problems of the economy.

Sources of non-linearity are thoroughly discussed as well in chapter two with the main question of whether monetary policy should be more aggressive and therefore not follow a linear pattern aimed at reducing deviations. Shock uncertainty and parameter uncertainty are displayed as the main drivers of non-linear monetary policy, as Mishkin (2008) believed linear quadratic loss function may not be a suitable measurement of good economic performances and inflation was proved to be the main source of non-linearity in the economic strategies implemented by Fed, the larger the variance of inflation the larger increases the distance from the target.

Greenspan during his period as chairman of the Federal Reserve from 1987 to 2006 adopted a new strategy that will be conferred about in depth in chapter three, the Risk Management model.

In this new framework, the focus lies on the risk components that affect the macroeconomic elements, hence the need to consider different case economic scenarios and study a policy that can act promptly to shield the economy from any of these.

Several hypotheses of risk management policy involve setting a safeguard level of inflation and adopting a state-contingent policy where the level of unemployment is relatively low.

The robust control approach ensures that the risk management policy may lead to more aggressiveness, hence a lower degree of inertia in the economy, to act pre-emptively and avoid future shocks to worsen. The risk management approach involves the creation of several strategies to maximize the probability of reaching the optimal levels of macroeconomic inputs that assure economic growth and price stability.

At the same time, robust control mechanisms such as setting a defined target for inflation may not be as desirable as it seems since it would give false predictions and increase uncertainty during policy decisions. Despite these drawbacks, Greenspan implemented target values for inflation but above all applied pre-emptive monetary policies considering the probabilistic distribution of all possible outcomes and not just certain ones.

The risk management model involves a trade-off between the Central Bank and the economy between exceeding upside levels and downside levels for inflation and output gap accounting at the same time the possibility that the target values may create predictions based on fictional optimal levels that did not occur.

During the Greenspan era, different thoughts were expressed regarding the coherence of his monetary choices and Taylor Rule framework, the purpose of this thesis is to investigate Greenspan's aggressiveness policy and to explain how to overcome shock uncertainty, data, and parameter uncertainty that compromise the analysis.

To address monetary policy studying under uncertainty we use regime-switching models where the strength of monetary response depends on the level of risks in the economy.

The aim is to investigate the dependence of the monetary policy during the Greenspan era on the risk levels of the economy and to conduct estimations of linearity through the Smooth Transition Regression Model with two transition variables in our case, outlook inflation, and VXO index.

The fourth chapter introduces the Smooth Transition Regression Model and to address risk component influence in monetary policy we conduct an empirical analysis, using inflation measures of dispersion forecast made by a Survey of professional forecasters during the Greenspan era 1987 - 2001. The measure of risk implemented by the scientists is a measure of risk in financial markets originating from stock options and we use real-time data from Greenbook since ex-posts are proven too unreliable due to the drawbacks discussed above. We then estimate regime switching models and conduct tests for linearity, evaluating non-linearities arising from the level of risk in the economy and the level of inflation and output gap.

Findings show that risk in inflation outlook and volatility influence heavily monetary policy changes rather than traditional variables such as the output gap and Federal funds rate. Evidence showed that during a period of high economic risk monetary policy responds more aggressively during a period of high economic stress hence the inertia degree tends to be lower. Based on our estimates Greenspan monetary policy did not follow a linear pattern depending on risk variables and acted pre-emptively to prevent the worsening of the economy due to political and economic distresses of the period.

Despite proving the non-linearity of monetary policy choices during Greenspan it is evident from our tables of results that the choices of initial values in the estimations of our parameters deeply influence the selection of transition variables and evaluation.

In the literature, it is possible to find other models suitable for risk management studies, that embody the measurement of volatility analysing the variance of the error by modeling the dynamic of volatility and forecasting it. The Generalized Auto-Regressive Conditional Heteroscedasticity (GARCH) set of models theorized by Engels better suits this aim and provides a solution to underestimation or overestimation of downside risks caused by the setting of faulty thresholds.

2. 2.1) Historical analysis of the Taylor Rule and Monetary Policy

In 1992 Taylor presented "Discretion versus policy rules in practice" which influenced the way monetary economists and practitioners think about the policy debate.

He described the monetary model process in terms of short-term nominal interest rates and policy in terms of inflation and economic growth, the main operational driver in monetary policy.

In his "Historical Monetary policy analysis and the Taylor Rule" work, Orphanides (2003) wants to examine the Taylor Rule model's role in describing monetary policy and analyse when the outcomes of the rule implementation were not optimal as predicted. He shows that implementation of the Taylor Rule has often led to misconduct due to false previous predictions regarding for example inflation or disinflation that leads to negative effects on the economy.

The first example presented by Taylor was already studied by participants of the Brookings conference on policy regime evaluation and studied deviations of short-term nominal interest rate from a baseline, related proportionally to deviations of a variable from its target:

$$i - i^{*} = \theta\ (z - z^{*})\ (1)$$

Subsequently, as a variable was chosen the "nominal income targeting regime", the sum of price level p and real output q; later variations used the sum of inflation, as the variation of prices and real output:

$$i - i^{*} = \theta[(\pi + q) - (\pi^{*} - q^{*})]\ (2)$$

Also written by differentiation of inflation and output response as:

$$i - i^{*} = \theta_{\pi}(\pi - \pi^{*}) + \theta_{q}(q - q^{*})\ (3)$$

Taylor adopted a variant by setting the nominal interest rate target as the sum of the equilibrium rate of interest $r^{*}$and inflation. He also used the year-over-year rate of change of the output deflator as a measure of inflation. By setting the inflation target and equilibrium interest rate as 2 and the parameter to $1/2$ he modified the rule into:

$$i = 2 + \pi + \frac{1}{2}(\pi - 2) + \frac{1}{2}(q - q^*) \ (4)$$

Some limitations to the general rule regard the "informational problem" studied by McCallum and Orphanides (2001, 2003), especially in the case of the output gap. Based on the rule policymakers should require information that they do not dispose of at the time of prediction or unreliable sources as policy indicators.

As Greenspan (1997) observed rules share the belief that the future will act like the past, though history has proven to be a fallible guide on this matter. His concerns were mainly focused on the federal funds rate and the economy's production potential, which were obtained by observing past macroeconomic behavior through data inspection or embedded in models.

Another limitation is the lack of an explicit role for forecast variables, given that they have always been important in policy decisions at the Federal reserve; preemptive monetary policy is preferred to anticipate a threat in the economy and act against it before it can be too challenging to be eradicated, to avoid destabilization.

This is also coherent with the role embodied by central banks to try to predict and anticipate emergencies hence the necessity to modify the traditional Taylor Rule with a forecast of the variables of inflation and economic activity and use the federal funds rate obtained from this application.

This broader version of the Taylor Rule better captures actual policy evolution over time since policymakers do not have to accept some given baseline benchmark: Mishkin and Bernanke (1997) discussed inflation targeting approach to policy as a "constrained discretion".

A broader interpretation of the Taylor Rule aims at adjusting interest rates and controlling for risks while respecting the objectives of price stability and economic growth.

Hence it is relevant to analyse how consistent Taylor Rule's historical interpretation has been towards the objective discussed above and for how long the policies were implemented using interest rates as the primary instrument for monetary policy.

Coherent with the two objectives of general growth and price stability, broader interpretations of the Taylor Rule help in focusing on new specifications. For example, Friedman developed a monetary growth rule by setting the nominal income $\Delta(p+q) = \pi + \Delta q$ as the target variable relying on lagged values of interest rate as instruments for policy baseline:

$$\Delta i = \theta[(\pi + \Delta q) - (\pi^* + \Delta q^*)] \quad (5)$$

It is possible to notice the relationship between money growth targeting where m is the monetary aggregate and v is the velocity:

$$\Delta m + \Delta v = \pi + \Delta q \quad (6)$$

By setting a target π^* and adjustments in a change of equilibrium velocity and potential output growth:

$$\Delta m = \pi^* + \Delta q^* - \Delta v^* \quad (7)$$

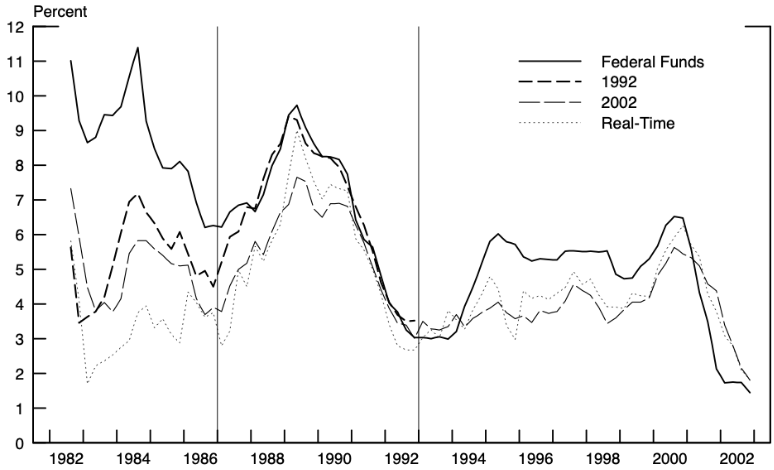
Substituting the equation in terms of velocity we obtain $\Delta v - \Delta v^* = (\pi - \pi^*) - (\Delta q - \Delta q^*)$ (8) and considering the log-linear relationship between velocity deviations from its equilibrium and interest rate we obtain: $\Delta v - \Delta v^* = a\Delta i + e$ (9), where a and e determine short-run dynamics and money demand fluctuations.

By removing short-run velocity fluctuation since it is a drawback in monetary strategies and substituting (8) and (9) to obtain: $\Delta i = \theta[(\pi - \pi^{*}) + (\Delta q - \Delta q^{*})]$ (10).

Given $\theta > 0$, $\Delta i = \theta\pi(\pi - \pi^{*}) + \theta\Delta q(\Delta q - \Delta q^{*})$ (11) for positive values of $\theta\pi$ and $\theta\Delta q$, represents natural growth targeting rule, implying that they rely on estimates of economic natural growth sensitive to an imbalance between the growth of aggregate demand and aggregate supply, not on the output gap.

The target rule could be implemented using current data and recent realization of output growth and inflation or with a sight of growth of inflation soon, given that the primary role of this rule was to raise interest rates at the rise of inflationary pressure and lower rates when recession seems to be threatening the economic system.

Figure 2. The Classic Taylor Rule



Notes: The 1992 rendition of the Taylor rule replicates the rule shown in Taylor (1993). The solid vertical lines show the beginning and end of the sample originally discussed in Taylor (1993). The 2002 rendition employs the latest data (Figure 1). The real-time rendition employs, in every quarter, data as available in that quarter (Figure 3).

Orphanides (2003) plotted different versions of the Taylor Rule from 1982 using variables of GDP deflator over four quarters, real output growth and natural growth rate, and real output growth. These estimates later subjected to revisions, remodeling, and redefinition were obtained from Commerce Department for GDP data and by Congressional Budget Office for potential output and the National Bureau of Economic Research. This was conducted to give and historical analysis of policy evaluation to explain the relationship between policy decisions and subsequent economic outcomes.

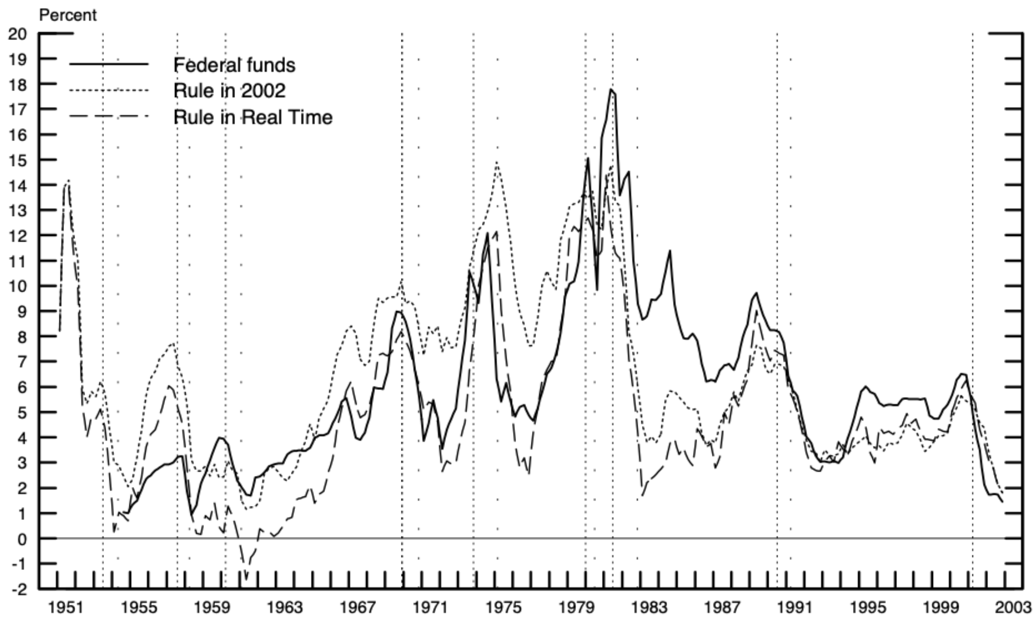
In Figure 2 the solid line in the figure shows the evolution of the federal funds rate during this period. The dark dashed line (the “1992” rule) reconstructs the Taylor rule as was originally published, replicating all of Taylor’s original assumptions. The two vertical lines mark the beginning and end of the sample over which the rule was originally examined, from the first quartile of 1987 to the end of 1992. In this period, it is possible to notice the accuracy of the match of the Taylor Rule remarkably though this cannot be seen in the earlier years.

Another rendition of the rule made with the implicit assumption that $r^* = \pi^* = 2$ does not reflect actual policy settings that policymakers could have arrived at in real-time.

The regardless difficulty of the data set was overcome by basing estimations on real-time values obtained by CBO and adopting an operational version of the classic Taylor Rule, in fact in each quarter t the output gap and inflation data inputs for the rule are the ones referred to as the $t-1$ values.

Given the lack of uniform averages generated by inflation and output gap by historical renditions, and since appropriate averages cannot be known beforehand at the time policy is made, real-time conditions of the classic Taylor Rule may result in too tight or too easy for that extended period. This issue generates a consistent problem if the aim is to evaluate the rule for real-time policy analysis, while on the other hand if the aim is to apply ex-post, then it does not concern the evaluation.

Figure 4. Real-Time and Retrospective Classic Taylor Rule



Notes: Real-time and retrospective renditions of the classic Taylor rule based on real-time and retrospective data shown in Figure 3. Dotted vertical lines denote NBER peaks and troughs.

Here are reported two versions of the classic Taylor Rule one real-time and one retrospective with the original assumption of $\pi^* = r^* = 2$ and using data shifted in one quarter to capture the quarter lag with initial estimates of actual output data, based on the output gap on Okun's and Gordon and Tobin work (1961, 1961) and Council of Economic Advisers.

Despite this output gap data are proven to be when evaluated in real-time highly imprecise its estimation is based on end-of-sample estimates of an output trend, a potential output, imprecise as said.

Statistical models employed for the estimation of potential output are continually evolving in accuracy and methodology complicating also the evaluation presented by Orphanides (2003).

If we focus the attention on three periods 1955, 1965, and 1978 based on the macroeconomic estimation of the data policy described, the graph displays the evidence of a tighter rule concerning the actual decision taken by economists.

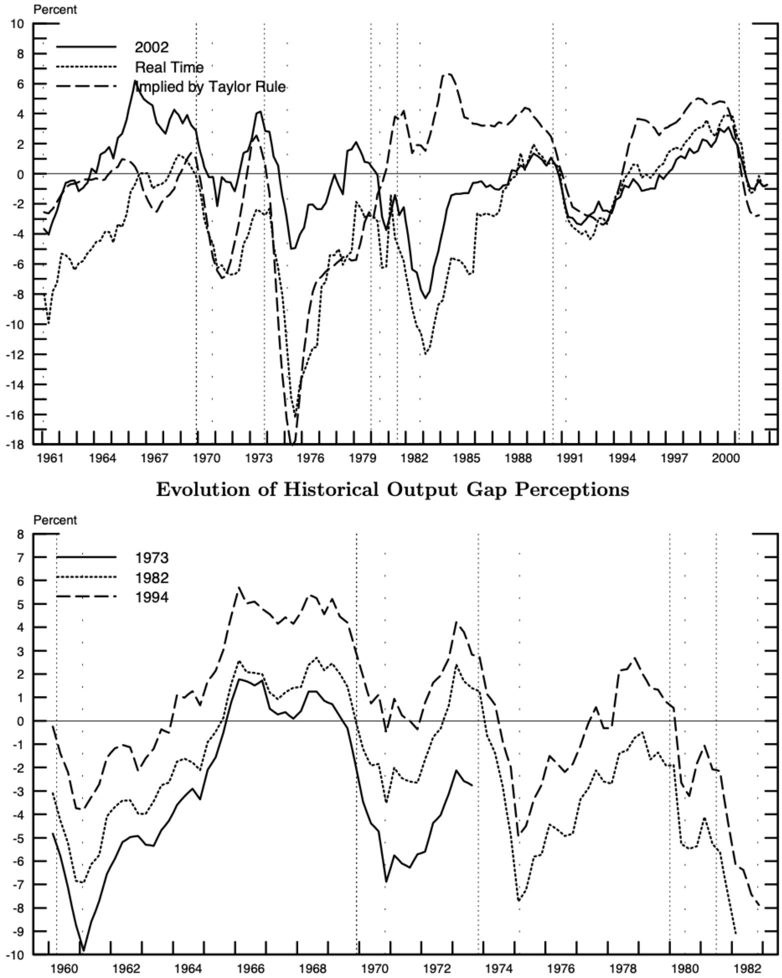
The issue stands in the perspective, in fact, at the time the so-called overheating in the economy was not perceived as later historical analysis did.

In 1955 information in real-time displayed a level of the economy reaching the full extent of its capacity, but the policy record sign of awareness of the difficulties of assessing the limits of expansion and the willingness of the Committee to act pre-emptively in the face of threats of overheating. The extent of inflation danger was not understood clearly at the time.

Despite the action of tightening monetary policy in the act by the Committee, the jump in inflation given the previous policy in the act was not predicted by Committee.

Figure 5

Output Gap: Actual and Implied by Classic Rule



In 1965, in the same way, current CBO estimates showed a severely overheated economy but again the danger was not recognized in time, even though in the past committee was split on whether to act or not.

Since gaps were wrongly examined the members of the committee were misled and when they decided to act towards tightening it by increasing the discount rate it was already late.

In Figure 5 it is possible to visualize the reason behind errors in the application of the Taylor Rule and evaluation errors by comparing real-time and ex-post renditions of the output gap with the implicit version, necessary if the Taylor Rule matched the federal funds' rates throughout history.

The gap implied by the rule was very low during the 1970s, much like the actual real-time gap estimates. By contrast policy during most of the 1980s from 1994 to 1999 would have been consistent with the real-time Taylor Rule only if the actual output gap were far greater than it was.

Orphanides recognize the role of two important elements to understand the timing, magnitude, and direction of the errors: beliefs of estimates of unemployment and productivity slowdown.

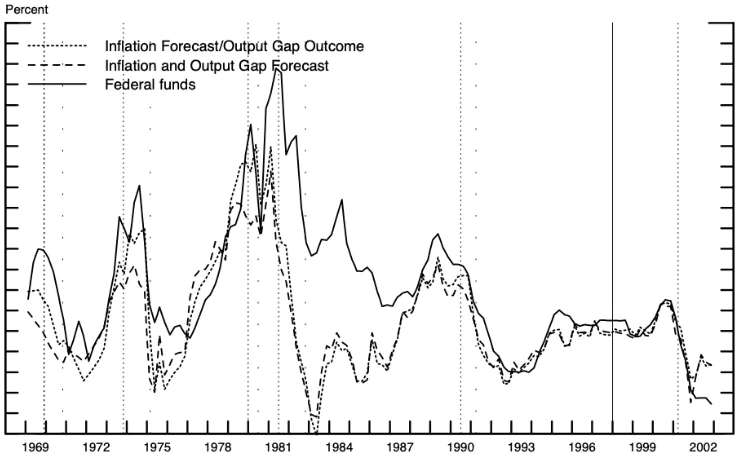
The first regards Niru or NAIRU and it is known that estimates between the 1940s to 1970s have proven to be optimistic and far from actual values. Generally, economists have failed in successfully recognizing these changes. The second one found evidence in the fact that the productivity slowdown was not predicted and embodied in estimation during the 1960s-1970s until 1994.

The pattern of output potential gap lasted for 20 years and not until 1995 did the reverse pattern start to appear. Of course, later there was evidence of the speed of the upward and downward spiral related to the output gap estimations.

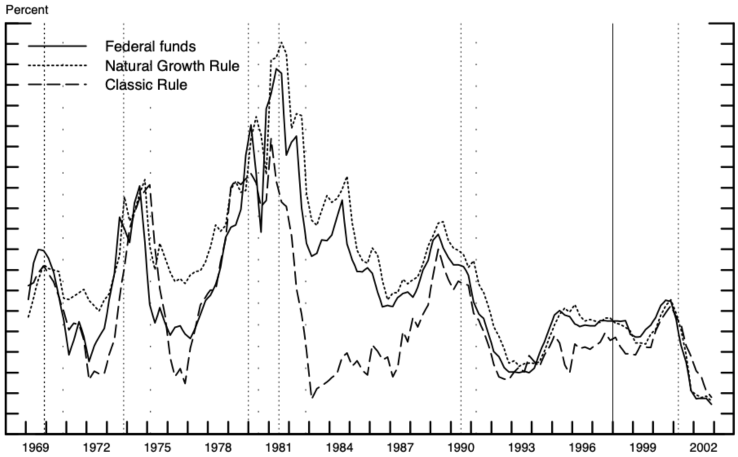
Various interpretations have led to the accuracy or less in the interpretations and it is believed that despite the optimistic choice of policymakers to dampen output gap values toward zero to reduce the magnitude of errors, it might be needed more time for example 10 years to tell festinates of the 90s were accurate or not.

Figure 6

Forecast-Based Variants of the Classic Rule



Classic and Forecast-Based Natural-Growth Rule



Notes: Forecasts are from the Greenbook until 1997Q4 and from the Survey of Professional Forecasters from 1998Q1 on. The solid vertical line indicates the break in the data source. Dotted vertical lines denote NBER peaks and troughs.

At last, we present the forecast-based variants of the classic rule, expecting that they would provide a better description of historical policy.

The graph represents the inflation forecast substituted in the Taylor Rule but keeping the original value of the output, while in the second figure, both variables are substituted with the forecast.

Using one quarter lag ahead on different periods for the output gap and Survey of Professional Forecasters, it apprises that the timing of policy reversals during the 1970s appears to be better captured with a forecast based rather than the outcome-based rule.

Also, this rule captures in 1994 did a better job than the classic Taylor Rule element of a pre-emptive strike against inflation is captured better in this forecast-based variants.

During the 1920s it is proven that policy has been highly consistent with the framework presented by Taylor Rule.

Despite several hopes presented during that period, subsequent events did not live up to that expectations, and monetary policy instruments at the time were not able to anticipate downhills and intervene pre-emptively towards the economy.

At the time policymakers did not have the information necessary to lead a successful policy and avoid losses and failures that eventually came. Orphanides (2003) asserts that despite historical data analysis the best monetary policy is still uncertain and yet to be defined.

2.2) Evaluation of the linearity of monetary policy during the Greenspan era

Nonlinearity in monetary policy according to Greenspan may be reconducted to uncertainty different sources of uncertainty are shown in imperfect knowledge of data, revision of the data used (real gap and hip deflator), and interest rates analyzed with a high margin of error due to measurement unsureness.

In the same way data uncertainty is relevant to monetary response, also parameter uncertainty plays a fundamental role in the impact of policy on the economy and the subsequent response of the economy itself.

The main question is whether the monetary response in the economy should be less or more aggressive given this parameter uncertainty in the variables considered. Different researchers are divided into two opposite opinions, either parameter uncertainty can be addressed not as the principal source of uncertainty because they contribute to negligible errors, while another line of thought believes that a more aggressive response of monetary response would be more suitable to deal with inflation persistency and relevance in shocks. As it is largely known, inflation variance increases with the distance from the target, hence aggressiveness reduces deviation for optimal level.

Another main aspect of non-linearity is represented by shock uncertainty, this belief comes from the thought of various Federal reserve governors that linear responses to shocks do not depend on the variances or distributions properties of the shocks. Mishkin (2008) believes that a linear quadratic framework that embodies a quadratic loss function may not be suitable in cases when the economic system displays poor performance.

Greenspan with other governors has tried to give a different framework approach, accounting for risk management, taking into consideration a possible optimal objective to be followed by the economy, and at the same time considering different deviations from the objective, dispersions.

Bernanke and Evans (2011) support the risk management approach asserting that authorities must build a monetary response taking into consideration forecast errors and miscalculations in different economic scenarios and the optimal economic path to be followed. In fact, when central bankers formulate monetary strategies, they must be considered the uncertainty and imperfection of information at knowledge.

The belief that monetary policies should be formulated considering and balancing risk associated with the alternative economic scenario and robustness of possible miscalculations of the correct scenario, was developed following two different perspectives.

The first perspective is structural impediments which do not seem to be reliable being based more on conjectures rather than quantitative measurements itself.

If we assume that there are structural impediments and would proceed with accommodation policy the economy would face a supply constraint that monetary strategy is not able to address, hence additional easing would increase inflation and not have an impactful change in unemployment.

The second perspective is the *"liquidity trap"* which occurs when nominal interest rates are considerably above zero and real rates of interest are positive.

If the supply of savings increased but the investment demand remained unchanged, market forces would drive down real interest rates to some natural rate of interest that equilibrates savings and investment. In case of a liquidity trap in this market, the dynamic is averted.

As people do not spend due to cautious behavior considering risk aversion or patience, the money supply savings exceed the demand for investment even at low-interest rates.

Even though liquidity trap scenarios are believed to be more compelling than structural impediments are still rare and not easy to manage.

Studies conducted by a great number of economists such as Krugman, Eggertsson, Woodford, and Werning, pointed out how the performances of economies that were stuck in liquidity traps were improved by lowering real interest rates and accommodative monetary policy in the forward-looking period. As stated previously accommodative policies are not suitable for structural impediments case scenarios.

The issue stands with the fact that policies that are optimal for the liquidity trap scenario would tend to generate higher inflation without a significant reduction in the unemployment scenario were true.

A possible solution analysed is a middle-ground robust policy in which the Fed would implement a state-contingent policy, where the federal funds rate is set at very low levels and unemployment is above the natural level. A further step in this policy would be to set a safeguard level of inflation which translate into withdrawing accommodative policy if inflation rises above a certain threshold. Recent liquidity traps studied after the Greenspan era have proven that to improve economic performance central banks are required to allow inflation to reach higher levels than their targets to reduce unemployment and restore productive economic equilibrium.

The robust control approach suggested previously ensures that the risk management approach may lead to a more aggressive response of monetary policy to macroeconomic conditions.

During a large period of shocks, the degree of inertia is very low hence the policies tend to respond more aggressively, and policymakers are advised to act pre-emptively to avoid facing more complications that the aggressiveness approach will cause.

Monetary policy needs to act aggressively when there is evidence of concern in the economy, for example, an evident shock in the market hence the strategy of the Federal Reserve must aim at providing insurance to avoid severe outcomes in the economy.

Monetary policy involves a risk management evaluation, a process that necessarily requires analysis of main sources of uncertainty and an estimate of the possible impact of these on the economy.

Risk management also implies the creation of strategies, given several risks faced by the economy, that maximize the probability of reaching certain potential objectives, among them price stability and maximum economic growth associated with it.

Greenspan recognized the importance of risk management in economic stabilization, despite the main source of criticism that has been proven to the identification of small key relationships that influence the economy's dynamics, there are the statistical models used by FED. Uncertainty remains a key factor even though strong efforts have been made to recognize these macroeconomic factors in the analysis of these statistical models.

Given the complexity and uncertainty has been necessary to introduce different hypotheses to ease parameter estimation, for example, an economic response is believed to be fixed through time and linear. Linearity assumption seems to be suited more to estimate average macroeconomic relationships, but it is not certain that models and simulations implemented by the Fed may be reasonable in the approximation of the economy's behavior during periods of idiosyncratic risk.

At the end of the Greenspan era, the first year of 2000, the introduction of new financial products modified the empirical relationship between economic activity and money as liquidity, weakening the control of monetary policy to the control of measured money stock.

The assumptions made by policymakers, linearity in the structure of the economy, slope parameters knowledge, and interest sensitivity of aggregate spending serve as instrumental functions with the purpose to give an understanding of the phenomena, even though these are not met in real life.

To understand and make previsions about the economy with low knowledge about the key influence factors policymakers need to consider alternatives to the likely future path followed by the economy considering a risk management approach accounting for possible deviations.

Different policies may exhibit different degrees of robustness concerning the true underlying structure of the economy. For example, a certain policy might be optimal to reach policymakers' objectives' but may also be seen as having relatively severe adverse consequences if the true structure of the economy turns out to be different than the one foreseen. On the other hand, there could be a less effective policy in reaching the objectives under the assumed baseline model, but it may reveal to be a suitable solution in case the structure of the economy revealed a different outcome. These considerations have inclined Federal Reserve policymakers toward policies that limit the risk of deflation even though the baseline forecasts from most conventional models would not project such an event.

Under Greenspan, important changes were conducted in the field of monetary policy response as a stronger response to unemployment through a dovish monetary policy, greater and more transparent communication of the FED to increase the power of monetary policy, and finally reduction in policy interest rate changes of 25 points due to focus on Fed rate.

The main reason the Greenspan period has been chosen for this analysis is that due to the continuity created by the chairman's leadership Fed preferences were stable, due to the absence of explicit change in preferences by the FED.

Federal reserve's main objectives interest both inflation targeting and employment factor, expressed by the general loss function also studied by Meyer (2004).

Having a defined target for inflation may not be desirable since it would give false predictions and uncertainty during prevision and policy decisions based on misleading predictions lead to unstable economies due to the misspecification of inflation.

Despite this aversion towards setting a target, this did not prevent Greenspan from setting a goal of price stability in 2003. This resistance to numerical inflation targets happens because setting a target for inflation may cause Fed to overemphasize the inflation objective and oversee the employment one. (Kohn, 2003)

Hence here we have evidence of the main issue: the announcement rather than the numerical objective itself as explained by Mankiw (2006) announcement of a change would have a more powerful role in affecting economic values rather than a substantive change.

The FOMC introduced by Greenspan, who has been Director of himself, used numerical objectives in 1996 agreeing on a 2% inflation rate consistent with price stability goals.

During the Greenspan era at Fed, there is evidence, rather more than numerical objective for inflation, of risk management policy approach to the econometric framework.

As he underlined, the traditional Taylor Rule did not serve well in capturing monetary policy, since they are predicated on symmetric and quadratic loss functions implying that conditional mean forecast is sufficient for policy analysis. The loss function of the Central Bank presented by the FOMC is stated in terms of inflation and the output gap, allowing coherence with risk management policy (Manganelli and Killian, 2008) and policymakers to reach a fixed inflation objective, to measure the relative risk of inflation and gap, and at the same time output gap objective has a certain objective.

While setting inflation numeric aims and responding aggressively according to the traditional Taylor rule, Greenspan also believed in the necessity of employing pre-emptive monetary policies, analytically this consisted in considering the probabilistic distribution of all possible outcomes and not only certain ones.

To conduct a deeper analysis of the values of the variable during the Greenspan era, we must go back to Paul Volcker's chairman policy choices, in the period 1981-1990 the Federal reserve had to fight the so-called "Great inflation". His choices reflected the new belief in conducting policies where avoidance of surprises in the markets would avoid excessive financial tightening.

The Fed funds rate has never been so high as in the 1980s, reaching spikes of 14.6 % therefore, the U.S. central bank manufactured a recession to lower prices to a more sustainable level.

Though before decaying the rates reached the highest spike of 19-20 % in December 1980, then rates began to drift down sharply till reaching a maximum level of 10% in November 1984. Instead of following the traditional path of moving gradually rates in one direction, policymakers would hike the benchmark rate, then cut it, and eventually raise it again.

During this period the Fed would also adjust rates at unscheduled meetings after which would not provide policy statements. At the same time, the target range would not be tight as it is nowadays, spanning for example at 5% points instead of a 0.25 %-point window.

During Greenspan, Fed faced a much calmer period than the previous era, the federal funds rate reached a level of 6.5 % in May 2000, the highest spike ever reached during the period, while the lowest was 3% in September 1992.

In the early 1990s, the Fed mainly adjusted rates at Federal Open market committee meetings (FOMC) a practice that is still in use by the current Fed.

The FOMC is the group of Fed given the authority of voting on whether to raise, lower, and maintain interest rates and is one of the three branches within the Federal Reserve System (FOMC board of governors and the 12 regional reserve banks). Among its duties, there is the decision-making power to change the Fed's benchmark interest rate which influences other borrowing costs throughout the financial system as credit cards, home equity lines of credit rates (HELOC), as well as yields on savings accounts, and certificates of deposits (CDs).

FOMC also conducts open market operations, buying and selling securities, a process known as large-scale asset purchases or "Quantitative easing" that can influence long-term interest rates and expand or contract the money supply.

Fed Funds rate during January 1991-December 2005 data available from FOMC (Fed)

Given the variety of Taylor rules presented in the literature, they all feature a dependence on the policy instruments (usually interest rates) from macroeconomic features such as output gap and inflation.

The relevance to control for eventual shock effect hence being able to identify the changes in the economy and act pre-emptively resulted necessarily in a better understanding and displaying the impact of the rule on the policymakers' decision.

According to Greenspan (2004) acting pre-emptively must be a preferred choice because reduces the cost of defusing the problems and at the same time since monetary policy is applied with a lag, it can anticipate problems caused by inflation for example, early rising.

In support of this Orphanides (2003) introduced the natural growth rule considering the divergence between the potential level of output and the actual level of output. The degree of inertia is the speed at which the policy interest adjusts in response to new information and inflation in the economy.

Empirically it has been shown that Fed tends to move to policy rates in a series of small moderate steps. Blinder and Reis (2005) have discussed possible reasons that may influence the degree of inertia in the economy: one reason interests the inclusion of the lagged values of the federal Funds rate in the modeled process.

3. The Risk Management approach model

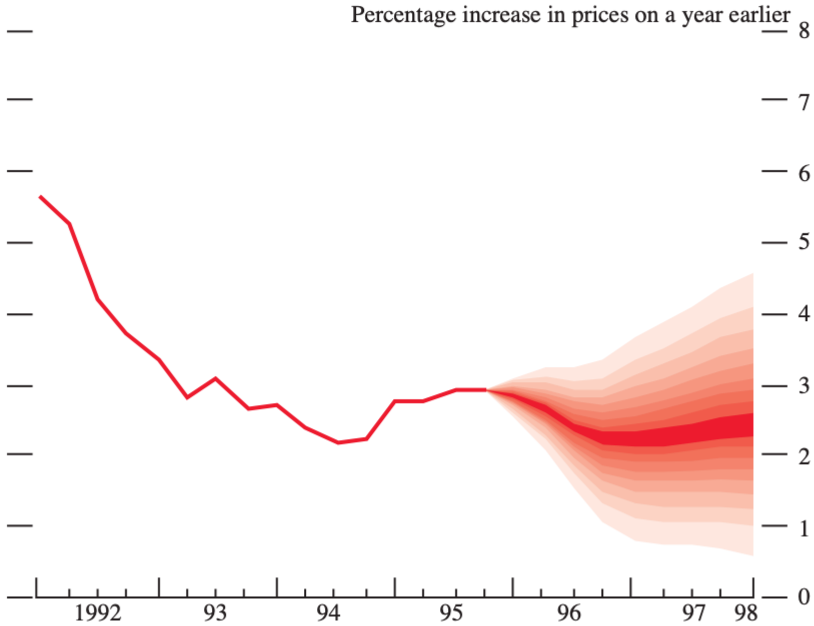
The risk management approach adopted by central banks entitles two aspects of the risk which constitute the core of uncertainty of price stability in the economy.

Upside risk refers to a random variable of interest that exceeds a certain threshold and downside risk represents the realization of the so-called variable below a certain threshold.

The central bank's objective has always been to avoid deviations of output from the potential level expressed by x and beyond an upper threshold level $\overline{x}$ and a lower threshold level $\underline{x}$.

This approach requires knowledge of the joint probability distribution of the random variables of interest to provide an exhaustive description of the uncertainty underlying the values. Hence it is relevant for bankers to define uncertainty associated with output levels and inflation outcomes in the future.

An example of a representation of the probability distribution of inflation is the "fan chart" developed by the Bank of England, graphing the 10% prediction interval as a dark band and wider intervals in lighter shades of red and the shading draws attention to the uncertainty of future inflation and output.



Bank of England, RPIX inflation projection in February 1996

The large academic literature on risk measurement defines the requirement for any measure of risk: the probability distribution of output gap and inflation, defined as F_x, F_π, must be related to the measure of risk and that usually measures of risk must be linked with preferences of agents, defined by loss functions.

These measures of risk were first proposed by Fishburn (1977) in the context of portfolio allocation in the presence of downside risk and later by Holthausen (1981) as well as Killian and Manganelli (2008).

For deflation risk DR, and risk of excessive inflation, EIR:

$$DR_\alpha = -\int_{-\infty}^{\pi} \left(\underline{\pi} - \pi\right)^\alpha dF_\pi(\pi), \quad \alpha \geq 0 \ (3)$$

$$EIR_\beta = \int_{\overline{\pi}}^{\infty} (\pi - \overline{\pi})^\beta \, dF_\pi(\pi), \qquad \beta \geq 0 \ (4)$$

In the same way the risk of negative gap NGR, and the risk of positive, PGR:

$$NGR_{\gamma} = -\int_{-\infty}^{\underline{x}} \left(\underline{x} - x\right)^{\gamma} dF_x(x), \quad \gamma \geq 0 \ \ (5)$$

$$PGR_{\delta} = \int_{\bar{x}}^{\infty} (\pi - \bar{x})^{\delta}\, dF_x(x), \qquad \delta \geq 0 \ \ (6)$$

The problem of risk management introduced requires a trade-off for the central policymakers. In the central bank model, the trade-off lies between the upside risk of inflation exceeding the upward value $\bar{\pi}$ and the downside risk of inflation falling below $\underline{\pi}$, in the same way with output values.

The risk management problem presented is displayed in a microeconomic perspective where the central banker's preferences satisfy a risk management model if and only if there is a utility function such that defined all relevant distributions F_{π} and G_{π}, F_{π} is preferred to G_{π} if and only if $U\left(DR_{\alpha}(F_{\pi}), EIR_{\beta}(F_{\pi})\right) > U\left(DR_{\alpha}(G_{\pi}), EIR_{\beta}(G_{\pi})\right)$ (7).

According FOMC definition states that the loss function of the Central bank can be explained in terms of output look and inflation, in fact by defining π_t and x_t, the loss function will be: $L_t = L(\pi_t, x_t)$ type of function.

$$L_t = \left[aI\left(\pi_t < \underline{\pi}\right)\left(\underline{\pi} - \pi_t\right)^{\alpha} + (1-\alpha)I(\pi_t > \bar{\pi})\left(\underline{\pi} - \pi_t\right)^{\beta}\right]$$

$$+ c\left[bI\left(x_t < \underline{x}\right)\left(\underline{x} - x_t\right)^{\gamma} + (1-b)I(x_t > \bar{x})\left(\underline{x} - x_t\right)^{\delta}\right]$$

$$where\ 0 \leq a, b \leq 1\ and\ c \geq 0 \ \ (8)$$

This loss function besides providing good parametrization about preferences linked to standard quadratic loss function common in macroeconomic literature (Clarida 1999, Svensson, 2002), is also congruent with the risk management model.

$$E(L_t) = \left[-aDR_\alpha + (1-a)EIR_\beta\right] + c\left[-bNGR_\gamma + (1-b)PGR_\delta\right] \quad (9)$$

The parameters of the loss function are measured as follows: c represents the relative weight of the inflation and output objective, while a, b measures the relative weight of downside risk relative to the upside risk of inflation and the gap.

This function states that the only interest of the Central Banker, in this case, is whether the realizations for inflation and output gap exceed a certain threshold ($\overline{\pi}$), hence that be this case the loss is described as a power deviation from the threshold values, where the coefficient $\alpha, \beta, \gamma, \delta \geq 0$ measure the degree of risk aversion.

Usually, Central Banks aim for setting the inflation and output rate to reach a certain optimal level inside the interval of upward and downward threshold $\underline{\pi} \leq \pi^* \leq \overline{\pi}$ and theorize policy rules based on a measure of risk such as represented to minimize the expected loss.

In the model presented, the Central Bank is risk-adverse and sets its goals by minimizing the interest rates and is risk adverse.

During the Greenspan era preferences of a central bank and authority did not shift or change consistently, therefore, this period has been used as a data sample to analyse the risk management model.

To address the model to reality of the data collected we need to underline that FED in this analysis is an authority with a dual mandate: providing for inflation and output gap target inflation targeting. The objective of inflation can be indicated as π^* even though it has been verified that during the Greenspan era, FOMC did not admit to having a target for the Federal Funds rate and did not publicly announce any changes in its target.

In 2002 Greenspan claimed how unhelpful specific numerical inflation targets would be given uncertainty and issues in measures of the general price level hence with the false prediction the federal reserve may destabilize the economy with subsequent choices through inflation.

Economists such as Bernanke and Meyer have instead favoured the adoption of an explicit numerical objective for inflation. The main resistance of the FED to numerical inflation targeting has focused on the danger that by announcing a target for inflation but not for unemployment the FED could overemphasize the inflation objective at the expense of the employment objective.

The problem lies in the public announcement rather than the numerical target the announcement would change the way FED has of communicates its objectives more than a subjective change.

Nevertheless, Greenspan as we previously stated largely expressed a preference to follow a risk management approach and how Taylor Rule was not able to capture FED's monetary policy choices during its period.

A different point of view shows how Taylor Rule was coherent with Greenspan's vision of responding aggressively when inflation departed from an anchored value.

Greenspan's policymakers were told to expand the horizons of possible outcome scenarios to the full probability rather than focusing on one outcome; this approach is coherent with risk management policy to understand different sources of risk in the outcome and quantify these different risks when it is possible.

The advantages of following Greenspan-era preferences are that they follow elements of FOMC forecasts, and they require ex-post realizations of inflation rates and output gaps, not forecasts. Several economists have provided experiments to understand the linearity or not of the Fed's monetary policy choices rather than our analysis, as they did not use Greenbook forecasts or econometric forecasts.

The method used in Killian and Manganelli's (2008) formalizing risk management approach estimates Fed's preferences through non-linear Generalized Methods of Moments which allows for ex-post realizations of inflation rates and output gaps to control output gap and inflation outcomes.

In developing the best risk management framework to quantify risks to contain deviations of inflation and output gap from their bounds, the evaluation had a further outcome, besides the quantitative output of the results: it explained how Federal Reserve communicates its objectives to the public and consequences of its announcements in the public choices.

4. 4.1) The Smooth Transition Regression model

To evaluate the risk management approach during the monetary policy in the Greenspan era, we must evaluate linearity in the monetary policy adopted.

Linear behaviour has been modelled with continuous regime switching models and discontinuous cases as Markov switching models.

The model used to evaluate risk management influence in monetary policy follows a smooth transition regression model to test the dependence of the monetary policy on macroeconomic features with risk level facing the economy.

The Smooth Transition Regression (STR) model is a non-linear regression model further developed from the switching regression model introduced by Quandt (1985). The two-regime switching regression model with an observable switching variable is a special case of the STR mode while the STAR model, univariate smooth transition autoregressive model encompasses two regime threshold autoregressive model as a special case.

Given the two switching variables considered, the VXO index and inflation outlook (using forecast given by the Survey of Professional Forecasters) and the autoregressive components displayed in the Taylor Rule the STAR panel model will be the type of STR employed in the estimation.

As developed by Teräsvirta (1994) the STR model is defined as:

$$y_t = \Phi' \, z_t + \Psi' z_t G(\gamma, c, s_t) + u_t \quad (1)$$

$$y_t = \{\Phi + \Psi G(\gamma, c, s_t)\}'^{z_t} + u_t \,, \; t = 1, \dots, T \quad (2)$$

Where $z_t = \left[1 \; \pi^e_{t|t+3} \; x^e_{t|t} \; x_{t-1}\left(x^e_{t|t+3} - x_{t-1}\right) r_{t-1} r_{t-2}\right]$ (3) represents the vector of explanatory variables and exogenous variables.

The matrix $\Psi = \left(\alpha_i^* \beta_i^* \varphi_i^* \phi_i^* \theta_i^* \ \rho_{1,i} \rho_{2,i}\right)' \left((m+1) \times 1\right)$ (4), embodies the parameters vectors and a sequence of identically distributed errors $u_t \sim i.i.d.(0, \sigma^2)$.

The transition function is displayed by $G(\gamma, c, s_t)$, is a bounded function of the continuous transition variable s_t, everywhere in the parameter space for every value of s_t, while γ is the slope parameter and $c = (c_1, \dots, c_k)'$ is the vector of the location parameters.

STR model is a linear model with stochastic time-varying coefficients $\Phi + \Psi G(\gamma, c, s_t)$, and transaction function form explains the response of monetary policy to macroeconomic conditions. They are called smooth since the transition between regimes is smooth and G is bounded between 0 and 1.

The main forms of transaction functions adopted in replica are two:

the first is

$$G(\gamma, c, s_t) = 1 - \exp(-\gamma(s_t - c)^{-1}) \quad (5)$$

and identifies the Logistic Smooth Transition model. Given that parameter, c is a vector usually the $k-th$ parameter is up to 1 or 2, hence with $K = 1$ $\Phi + \Psi G(\gamma, c, s_t)$ will change monotonically as a function of s_t from Φ to $\Phi + \Psi$.

On the other hand, if $K = 2$ it changes symmetrically around the midpoint $(c_1 + c_2)/2$ where the logistic function attains its minimum value, lying between 0 and ½.

When $\gamma \to \infty$ $\Phi + \Psi G(\gamma, c, s_t)$ is 0 and equals ½ when $c_1 = c_2$ and $\gamma < 0$, hence the LSTR1 approaches the switching regression model with two regimes that have equal variances. While in the LSTR2 model, the result is another switching regression model with three regimes, with two equal and the mid regime different from the other ones.

An alternative to the LSTR2 model studied in literature is the ESTR model

$$G(\gamma, c, s_t) = 1 - \exp(-\gamma(s_t - c)^2) \quad (6)$$

On the contrary, this function explains that the strength of the response switches when moving from intermediate to high and low values of the transition variable. This function is symmetric around threshold parameter c, monetary policy responds to macroeconomic conditions according to Φ when the transition variable is close to *c.* Instead, when the transition variable assumes high or low values, the response of monetary policy switches to $\Phi + \Psi$.

Since this function contains fewer parameters than LSTR2 it can be a useful alternative to the logistic function, though when $\gamma \rightarrow \infty$ the function becomes linear hence the transition function equals 0 when $s_t = c_1^*$ and 1 for other values. Hence ESTR model does not represent a good alternative for LSTR2 when γ is high.

The experiment aims to test if the parameters of the Taylor Rule switch with the degree of risk facing the economy, to do so we follow test specifications theorized by Teräsvirta (1998).

Following his method of specification, estimation, and evaluation, the three fundamental stages, we proceed modelling of test linearity.

When testing linearity, we must consider a formal test to decide whether the data will be adequately characterized by Taylor Rule.

Given the fact that this an empirical question, we test the hypothesis that the second term of $y_t = \Phi' z_t + \Psi' z_t G(\gamma, c, s_t) + u_t$ (7) being equal to 0, we encounter an identification problem hence we take Taylor Expansion of the transition function with $\gamma = 0$ as assumed by Teräsvirta (1998). By using tests such as the Likelihood ratio, Lagrange multiplier, and Wald test we would suffer an identification problem as well.

Hence it is possible to test the null hypothesis embodying linearity by applying an F-test, which is affected by the choice of the transition variable since the selected variable lead the switch in the monetary policy regime.

The number of degrees of freedom depends on the order of expansion and on the assumption that the transition variable belongs to the set of the regressor, this evidence is useful given the small sample of observations used to experiment.

According to the previous analysis, different statements on the dependence of monetary policy to switch regimes have been assumed. Rabanal (2004) tested the presence of asymmetries in monetary policy depending on the state of the business cycle, rather than deviations of inflation from its target value. On the contrary Dolado, Kim, and Muscatelli (2005) conveyed that the response of monetary policy to inflation and output depended on the level of equilibrium of these two variables.

Lately, the correlation between nonlinearity and instability in the economy has been overlooked by literature, letting the interest focus on new points of view.

As Mishkin observed in 2008, a new topic of interest should concern the correlation of monetary policy response to macroeconomic conditions and risk level facing the economy.

Therefore, the need to appoint as a transition variable a measure of the risk of the economy rather than inflation outlook, output gap, or other variables seems to be an alternative more suitable path to verify.

4.2) Empirical method and estimation of the Taylor Rule during the Greenspan era

To focus on errors in the application and prevision of the Taylor Rule concerning the pattern followed by monetary policy, we will replicate the estimation conducted by Gnabo and Mocero (2015) but this time using data available from Bloomberg regarding the period of reference, Greenspan era 1986-2005 and Fred of Saint Louis, North Carolina also SPF Data from Greenbook, a report by the Federal Reserve staff analysing current economic conditions and forecasting a large set of macroeconomic variables before each meeting of the Federal Open Market Committee (FOMC). The Greenbook, available from the Federal Reserve Bank of Philadelphia, allows us to reflect as closely as possible on the information set as used by policymakers in real-time.

The complete database used for the replica presented some variations to better correct the data uncertainty feature influencing the results obtained by the Gnabo and Mocero (2015).

Estimation in monetary policy reaction functions was conducted with quarterly average daily measures of Federal Funds rate, selecting four months on the annual measurements.

The same path of selection has been applied to the GDP deflator since in the Taylor Rule inflations are measured by a change in the implicit output deflator.

Output gap measures were measured considering past and present deviations from the GDP deflator between quarters in $t - 1$ and $t + 3$.

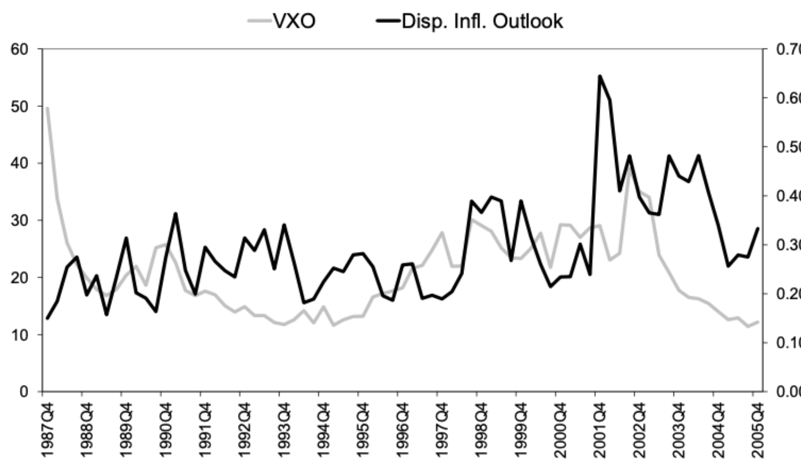
Since the relevance of the risk management approach, it is important to analyse also possible outcomes that diverge from the main path hypnotized for the economy. A possible way to proxy the distribution of outcomes is cross-sectional dispersion associated with the quarterly GDP deflator obtained by Survey Professional Forecaster.

After forecasting the quarter-on-quarter inflation rate for three quarters in the future, and using the $25th$, $50th$, and $75th$ quartiles of the forecasts for each quarter we obtained forecasts of the inflation rate between quarters $t-1$ and $t+3$. This dispersion measure is computed as the difference between the expected inflation of the $75th$ and $25th$ quartile divided by the $50th$ quartile:

$$\sigma_{\pi,t}^{2} \frac{\pi_{t-1,t+3}^{75} - \pi_{t-1,t+3}^{25}}{\pi_{t-1,t+3}^{50}} \quad (1)$$

The second measure of risk captures stress in financial markets, this measure was mentioned several times by FED governors as very important regarding financial markets' stress conduct of monetary policy, the VXO volatility index, the real-time volatility index.

Figure 1. Measures of macroeconomic risk



Source: Chicago Board Options Exchange (CBOE) and Federal Reserve Bank of Philadelphia.

If we plot a graph with the two measures of dispersion: the inflation outlook derived from the SPF and the VXO volatility index, it can be noticed that macroeconomic risk was high in three peaks: late 1980s, and the beginning of the 1990s and the end of 1990s and beginning of 2000s. When Taylor presented the variety of monetary policy feedback rules where policy instruments depend on a smaller number of macroeconomic factors, such as output gap and inflation, their application helped to understand the evolution of the effective Federal Funds rate during 1987 and 1992. Orphanides (2003) as we explained above found that estimations conducted with misspecification of values can lead to incorrect information used to estimate the responsiveness of the policies for inflation and economic activity.

According to Orphanides (2003), a more efficient approach would consist in implementing the forecast-based approach, doing so FED could act pre-emptively to changes in the economic environment.

The Taylor Rule adopted by Gnabo and Mocero (2015) in their estimation is specified as follows:

$$r_t = (1 - \rho_1 - \rho_2)\left(\alpha + \beta\pi^e_{t|t+3} + \varphi x^e_{t|t} + \phi x_{t-1} + \theta\left(x^e_{t|t+3} - x_{t-1}\right)\right) + \rho_1 r_{t-1} + p_2 r_{t-2} + \eta_t \quad (2)$$

Analysing the Taylor rule above we can divide the formula into three blocks, the last one where r_t is the Federal Funds rate at time t, describes the relationship with the AR component: the past values of the Federal funds rate. The second block refers to inflation $\pi^e_{t|t+3}$ and the expected output gap $x^e_{t|t+3}$ at time t+3, given available information at time t.

$x^e_{t|t}$ is the expected output gap for period t, also using information available until time t. expected values for the quarter output gap reflect the need for forecast-based values by authorities. At last, η_t represents the error term.

ρ_1, ρ_2 represent coefficients describing the degree of inertia in the economy, while β captures the response to inflation. If $\beta > 1$ the monetary policy is said to be proactive and the interest rate will adjust subsequently to stabilize inflation; if $\beta < 1$ monetary policy is said to be accommodative, hence the lowering of the Federal funds rate and interest rate of borrowing to stimulate the economy and reduce unemployment.

The remaining coefficients expressed by φ, θ, ϕ when positive means that policymakers conduct a counter-cyclical monetary policy, such as by increasing government spending and cutting taxes to help the recovery of the economy.

When addressing the different forms of monetary policy, also a downside of the economic effect must be recognized, for example, accommodative monetary policy may lead to excessive inflation as a repercussion in the long term.

Loosening of the money supply means an increase in money available to acquire goods and services leading to higher costs of relevant goods such as housing.

This Taylor Rule estimation has been with some variations on the variables used by Clarida (1998, 1999) and Orphanides in 2003. The main differences in their estimations were the coefficient considered.

As reported in Clarida (1999), Federal Funds rates were assumed to respond pre-emptively to inflation and output gap values, also future and past values of output gap were not accounted for, hence $\rho_2 = \theta = \phi = 0$. While in his previous work in 1998 he added an interest-smoothing component with ρ_1.

Orphanides' work (2003) diverged from Clarida (1999) as he assumed the Federal Funds rate to respond pre-emptively to inflation and lagged values of the output gap and year-ahead inflation.

When looking at the results obtained by Orphanides we see that monetary policy during Greenspan was aggressive the estimated coefficient of the output gap is positive and significant.

According to Clarida's findings, policymakers reacted only to the output gap influencing inflation expectations, the output gap had no predictive power in Taylor Rule.

In replicating the paper, we decided to follow the complete Orphanides rule and work on the available data before estimating the switching model with Smooth Transition regression panel models.

Estimates were conducted with the least square method to obtain suitable coefficients and variables relative to each component of the Taylor rule, output gap, Federal funds rate, and GDP deflator, that have been previously managed.

We evaluate the smooth transition model to the relationship between risk level variables and the strength of response to macroeconomic conditions.

We estimate models with the Non-Linear Least Square (NLS) model, based on Bekaert analysis estimated a vector of the autoregressive model to study the dynamic between risk aversion and monetary policy. Based on the findings there seems to be no contemporaneous correlation between interest policy rates and measures of risk in financial markets.

The input code is based on Fouquau, Hurlin, and Rabaud (2008) where Y represents the $T \times N, 1$ vector of values of the dependent variable and eventual missing data are to be accounted for with "Nan".

In our case, we denote as the dependent variable as r_t the Federal Funds rate value at present time and we insert data cross-unit by cross-unit as in the form of $y = (y_1', \ldots, y_N')'$.

Q is a $T \times N, 1$ vector corresponding to the threshold variable, and here we use the VXO index, one of the two transition variables mentioned above, and as previously stated the data are stacked cross-unit by cross-unit.

X is the matrix $T \times N, K$ corresponding to the values of the K explicable variables, here it represents the matrix defined in the code by "Comb" which is the matrix of variables estimated interested in the Taylor rule version we used for the analysis.

N represents the cross-section dimension.

M represents the number of location parameters, which can go from 1 to 5. When $m = 1$ the model implies that the two extreme regimes are associated with low and high values of q_{it} with a single monotonic transition of the coefficients from b_0 to $b_0 + b_1$ as q_{it} increases, where the change is centered around c_1 (the location parameter). For m $= 2$, the transition function has its minimum at $\frac{c_1 + c_2}{2}$ and attains the value 1 both at low and high values of q_{it}. In general, when $m > 1$ and gamma tends to infinity, the number of distinct regimes remains two, with the transition function switching back and forth between zero and one at $c_1 \ldots c_m$.

$Rmax$ represents the maximum number of transition functions authorized, which can range between 1 and 5. The code automatically determines the optimal number of transitions by using nonlinearity tests and tests one transition against the linearity assumption, two transition functions against one, and so on.

The function used is a version of the STAR panel presented by Hurlin (2006) to estimate the parameters pf Panel Transition Smooth Regression.

The model considered in the code in the case of one threshold parameter, so with two regimes, is represented by:

$$y_{it} = \begin{cases} \mu_i + \beta_1' x_{it} + \varepsilon_{it} & if\ q_{it} \leq \gamma \\ \mu_i + \beta_2' x_{it} + \varepsilon_{it} & if\ q_{it} > \gamma \end{cases} \quad (1)$$

Where x_{it} denotes an explicative variable associated with a parameter affected by the regimes, q_{it} is the threshold variable. As we considered lagged values for variables in the Taylor Rule as Federal funds rate at $t-1$ and $t-2$, we need to also introduce data series as inputs in the code function.

γ_j *and* β_j are estimated according to the same simple least square procedure used for the standard Smooth Transition Autoregressive model (STAR). Considering a single threshold model for a given value of the threshold parameter γ, the slope coefficients defined by β_j can be estimated by ordinary least square estimation.

Given the fact that the sum of least squared residuals depends on γ only through the indicator function, we can define it as a step function with the steps occurring at distinct values of the threshold variable. Notice that the threshold parameter is estimated by minimizing the sum of squared errors on γ, $S_1(\gamma) = \sum_{i=1}^{N} \sum_{t=1}^{T} \hat{\varepsilon}_{it}^2(\gamma)$. Therefore, the maximum number of steps of the function will be at most NT.

The minimization problem can be reconducted by searching over values of γ equalizing at the most distinct values of q_{it} in the sample.

From the estimates of the threshold variable, $\hat{\gamma}$ it is possible to estimate the coefficients of elasticity in the regime represented by slope coefficient estimations and estimation of individuals' effects.

The choice of ensuring an optimization demand theorized by Hansen (1999) has the objective to ensure the threshold regime switching with a small number of observations. Hence the minimization of $\hat{\gamma} = ArgMinS_1(\gamma)$ with $\gamma \in A$, can be solved by searching values of γ equaling at most NT distinct values of observations on q_{it} in the sample.

By sorting NT distinct values of the observations on q_{it} the constraint provides the information to eliminate the smallest and biggest values, therefore the remaining values will represent the set A among which the search for $\hat{\gamma}$ can happen.

The second part of the code deals with the testing procedure and underlines some issues in the threshold model represented, the first consists of testing the number of regimes or testing threshold specification and the second issue consists of choosing the threshold variable.

Let's reason by hypothesis to understand the issue.

If we assume to know the threshold variable, by testing the significance of the model with two regimes, the null hypothesis is represented by H_0: $\beta_1 = \beta_2$, and it corresponds to no threshold effect hypothesis.

Under H_0 the model is equivalent to a linear model and this hypothesis is usually tested by a standard test. By naming S_0 the sum of the square of a linear model, the likelihood test will approximately look line: $F_1 = \frac{S_0 - S_1(\hat{\gamma})}{\hat{\sigma}^2}$ where $\hat{\sigma}^2$ denotes a convergent estimate of σ^2.

But under the null hypothesis, the threshold parameter is not identified therefore the asymptotic distribution of F_1 is not standard and does not correspond to a chi-squared distribution.

Hansen (1996, 1999) has largely investigated the issue, and he tried to find a solution by simulating the asymptotic distribution of the statistic F_1 through bootstrap and showing that p-values obtained this way are asymptotically valid.

The same procedure can also be applied to determine the number of thresholds in general models.

If the $p - value$ associated with F_1 leads to rejecting the linear hypothesis, we will discriminate between one and two thresholds. The subsequent likelihood ratio test based on this is:

$$F_2 = \frac{S_1(\hat{\gamma}) - S_2(\widehat{\gamma_1}, \widehat{\gamma_2})}{\widehat{\sigma^2}} \quad (2)$$

Where $\widehat{\gamma_1}, \widehat{\gamma_2}$ denote the three regime thresholds estimates of the model with three regimes and $S_2(\widehat{\gamma_1}, \widehat{\gamma_2})$ is the corresponding residual sum of squares.

If F_2 is larger than the critical value of non-simulated distribution the hypothesis of one threshold is rejected in favour of two and the relative p-value can be simulated through bootstrap simulations, as shown by Hansen (1999).

5. Analysis of the results

To test for non-linearity in monetary policy function based on the real data we used for variable estimation, it is necessary to test the Taylor Rule estimated using the sets of transition variables we previously indicated as inflation dispersion measure, and the VXO index, volatility in the financial market index.

In the case of non-linearities arising, we can see that the rejection of the linear hypothesis will be way more relevant for these measures of risk hence by testing for linearity through an $F - test$ when the null hypothesis is rejected the transition function is highlighted and selected by sequential statistic tests.

From the variable estimations reported it is believed that Greenspan's monetary policy was conducted following non-linear behaviour where economic risk leads to switch transitioning regimes. This is coherent with Greenspan's belief that monetary policy strategy should go towards a wider analysis of several risks faced by the economy and their sources to prevent and foresee any sudden changes.

In our case, it is not possible to reject the null hypothesis of linearity at conventional statistical levels. From the findings, it is shown that all the variables that influence changes in monetary policy are represented by risk in inflation outlook and risk in financial markets rather than classical macroeconomic variables considered in the literature.

Analysing the data, we notice that statistical test goes towards a transition function for the risk measures implying a different approach to the response by monetary authorities facing economic risks.

In the table of results of nonlinear monetary policy reaction functions, where we consider the regime-switching models, the strength of response depends on risk levels facing the economy.

As previously stated, smooth transition regression models are estimated through variables of economic risk by the Non-Linear Least Square (NLS) function for panel smooth threshold models.

Using NLS function it is important to identify two parameters: γ, the slope of the transition function, and the vector of values previously identified with location parameters.

The issue underlying this estimation is the setting of initial values to initialize the transition function and procedure of estimation. Usually starting values are far from the actual parameter therefore number estimated is large, the existence of multiple starting suitable values does not ensure also the best fit for the model.

In fact, given that macroeconomic risk is endogenous it might be argued that a different estimation technique is required, as proven by estimating variables of the Taylor Rule through the Generalized Method of Moments. This method interests' cases of risk financial markets which react to monetary policy decisions, though as proved by Bekaert (2013) analysing the dynamic link between risk aversion in financial markets, monetary policy, and uncertainty there seems not to be a correlation between interest policy rates and measures in financial markets.

To prevent eventual mistakes a possible solution relies upon the use of a two-step identification strategy. Through the different rounds of estimation by setting each time different initial values of γ and c we would be able to estimate the parameters of the transition functions during the following rounds of estimation, restrictions of transition function are relaxed so the step of identifying an optimal value can begin.

The sensitive part of this operation is the set of values, Gnabo and Mocero's (2015) choice of incrementing by 0.1 units each time the value of γ and c was done to obtain a limit threshold set by minimum and maximum levels of the parameters, to prevent faulty values from being picked as optimal.

Results of the estimation of STR models are coherent with Taylor Rule showing that in normal times Fed implemented a counter-cyclical monetary policy, with a statistically significant coefficient for transition parameters.

Hence during the high period of risk, it is clear how Federal Reserve is responsive to the business cycle rather than the inflation outlook, when the output gap increases, then monetary policy is tightened resulting in more aggressive policy during the period of high risks.

The degree of inertia appears to be lower in a period of high economic risk, coherent with Mishkin's view of a stronger response to monetary policy during periods of high economic risk. The response of the US Fed to the output gap and degree of inertia is highlighted in the response coefficients in the case of the output gap and the case of inertia given by the two formulas:

$$\varphi_t^* = \varphi_0^* + \varphi_1^* G(\gamma, c, s_t) \;\; (1)$$

$$\rho_t = \sum_{j=1}^{J} \rho_{0j} + \sum_{j=1}^{J} \rho_{1j} G(\gamma, c, s_t) \; (2)$$

When the transition across different policy regimes is assumed to be led by risk in financial markets then it is believed that US Fed reacted more aggressively to the output gap, as a response to market crashes and high volatility periods caused by human factors (war, invasions) a natural disasters as seen at beginning of Greenspan era at Fed in 1987 and 1990.

While reacting more aggressively to the output gap for counterbalance, the degree of inertia lowered meaning that interest rates could rapidly adjust towards their principal components (fundamentals). Therefore, during the beginning of the high volatility period analysed previously transition across regimes happened fast and interest rates adjusted quickly.

The support for Gnabo and Mocero's evidence comes from Greenspan's statements elaborated in 2004 where he pointed out how during relevant economic risks times linear rules become unfit to describe processes of the economy.

Such events are represented by the stock market crashes of 1987, the crisis of 1997-1998, Russia's debt default, and the collapse of hedge funds menacing consequences towards inflation and the economy (Ferguson 2003).

The decision of the FOMC to act pre-emptively was decided as a precaution to avoid dealing with the complexity of negative shocks and zero lower bonds measures and to address the danger to future economic growth.

******************* *FINAL ESTIMATION OF PTSR MODEL*******************	
Estimation with $r = 1$ *and* $m = 1$ *by NLS*	
γ *gamma*	97.1950
c	55.2867
AIC	−2,044
BIC	−1,675

In the estimation results, we see how Akaike Information Criteria and Bayesian Information Criteria are very low in non-linear equations, assuming a negative value and RMSE is low.

The test proves how the model fits in capturing non-linear behavior by non-rejection of the null hypothesis, hence proving the outcomes to be statistically significant.

Even though general values are slightly different from Gnabo and Mocero (2015) estimation, still they prove to be coherent, however, issues can be found in the setting of initial values for

γ and c. Despite identification strategies set to avoid faulty values being selected as best choices, errors with this strategy still occur and affect the outcome of the evaluation.

Therefore, it is needed to rethink a more suitable model to improve estimation in the switch regime model and risk management procedure of evaluating monetary policy.

$H_0 = PSTR\ with\ r = 1\ vs\ H_1 = PSTR\ with\ at\ least\ r = 2$		
$Wald\ Tests\ (LM)$	$W = 10{,}695$	$Pvalue = 0{,}058$
$Fisher\ Tests\ (LMF)$	Nan	Nan
$LRT\ Tests\ (LRT)$	$LRT = 11{,}526$	$Pvalue = 0{,}042$

Since we have proven that our variables do not fit linearity models and that setting initial values for the starting process of transition variables can be faulty, alternative models can be considered to improve our fitting method and estimation.

In our analysis volatility was represented by an external component embodied in the VXO index and was the transition function of risk for the switching regime to be considered. However, we can think this time of using an instrument for risk management purposes that automatically captures volatility.

ARCH (Auto Regressive Conditional Heteroskedasticity) set of models elaborated by Engle (1982), analyses heteroskedasticity problems in time series where variance changes with time and in some periods and it demonstrates volatility clustering, such as variance small in some periods and relatively large in another period. Bollerslev (1986) to better solve estimation problems of variance from a normal distribution of ARCH model introduced GARCH (Generalized Auto Regressive Conditional Heteroscedasticity) which adjusts the variance of the error and can provide analysis by modeling the dynamic of volatility as well forecasts of it.

When we previously analysed the risk management model, presented by Killian and Manganelli considering upward and downside risk of inflation value, through GARCH type of models we can use time-varying estimates on downside risk measures such as Var (Value at Risk).

GARCH models are helpful in the analysis of conditional and unconditional but different innovation distributions of time series data lead to different shapes of downside tail, therefore downside tail risk. An appropriate distributional assumption may lead to underestimation or overestimation of downside risk therefore a faulty setting of threshold parameters to contain an optimal measure of inflation and output gap (π^*, x^*).

Hence through QMLE (Quasi Maximum Likelihood Estimates) it could be possible to estimate GARCH coefficients and innovations and Pareto Distribution could be implemented to estimate the tail shape of innovations.

Assuming x_t to be returned analytically the GARCH model appears as:

$$x_t = \varepsilon_t \sigma_t \quad (1)$$

$$\sigma_t^2 = \lambda_0 + \lambda_1 x_{t-1}^2 + \lambda_2 \sigma_{t-1}^2 \quad (2)$$

Where ε_t represents the identical and independent identically distributed (i.i.d.) errors (innovations) with zero mean and unit variance, and the parameters λ_0, λ_1, λ_2 are positive.

So hypothetically we could conduct an estimation through QML of the three parameters $\widehat{\lambda_0}$, $\widehat{\lambda_1}$, $\widehat{\lambda_2}$ on GARCH coefficients and estimate errors as:

$$\widehat{\varepsilon_t} = {x_t}/{\widehat{\sigma_t}} \quad (3)$$

$$\widehat{\sigma_t^2} = \widehat{\lambda_0} + \widehat{\lambda_1} x_{t-1}^2 + \widehat{\lambda_2} \hat{\sigma}_{t-1}^2 \quad (4)$$

In this way, we could account for volatility already and provide a better estimation of coefficients in the risk management model of monetary policy we want to test and specifically testing downside risk.

Since different assumptions in the data provide different levels of heavy tails of GARCH we expect based on different types of errors to be different tail behaviour: errors following a standard normal distribution and one following the t-students type.

Conclusion

Non-linear monetary policy choices have been proven to be usually more accurate to descript processes of the economy rather than linear ones and on this evidence, it is possible to conduct studies on the estimation of these policies. Causes of non-linearity can be searched among the distance of inflation from its actual targets, and economic uncertainty as well.

With the expression economic uncertainty, we identify the three major causes of potential non-linearity in economic history that we previously analysed: data uncertainty, shock, and parameter uncertainty.

Data uncertainty is an issue that has been tried to overcome using the operational Taylor Rule and forecasts based on present values. Forecast variables though do not provide a precise and accurate description of the future of the economy since they are based on the hypothesis that the future will act like the past, hence the need for detailed studying of possible different outcomes that might affect the economy.

The main debate among economists involves the necessity to act aggressively in a period of parameter uncertainty or implement more accommodative and cautious strategies, according to various model simulations, the debate is still on.

With respect to shock uncertainty from Greenspan's analysis, it is evident how a robust control approach, such as risk management, for monetary policy, is relevant to avoid faulty outcomes. Risk management proves to be a winning strategy if correctly implemented in trying to understand and predict future behaviors in the economy. However, the delicacy of this application comes from correctly choosing the variables leading to the forecast and not predicting erroneous choices due to faulty values and models.

From our findings lack linearity during the Greenspan era has proven that monetary policy is more affected by inflation measures rather than traditional macroeconomic outcomes and variance of the error of volatility must be modeled cautiously to give a more precise and coherent description of the economy.

As shown through regime switch models aggressiveness tends to increase during periods of high economic risk lowering the degree of inertia with respect to normal case scenarios, coherent with Mishkin view discussed previously (2008).

Further analysis can be better conducted in the future by using GARCH models (Engles 1993) to improve the description of the linearity of monetary policy and modeling variance of the shock related to volatility.

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Summary

The Taylor Rule has been developed in 1992 by Taylor to describe monetary policy in terms of nominal interest rates and inflation and to linearize policymakers' decisions.

The rule itself, however, was proven to have some limitations (McCallum 2001, Orphanides 2003) for example the belief that the future will act like the past and therefore the rule must rely on precise information that is not provided at the time of prediction. Therefore, broader versions of the rule have been implemented to better capture imperfections and give a more precise picture of the policy perspective.

Some limitations to the general rule regard the "informational problem" studied by McCallum and Orphanides (2001, 2003), especially in the case of the output gap. Based on the rule policymakers should require information that they do not dispose of at the time of prediction or unreliable sources as policy indicators. As Greenspan (1997) observed rules share the belief that the future will act like the past, though history has proven to be a fallible guide on this matter. His concerns were mainly focused on the federal funds rate and the economy's production potential, which were obtained by observing past macroeconomic behavior through data inspection or embedded in models.

A broader interpretation of the Taylor Rule aims to adjust interest rates and control risks while respecting the objectives of price stability and economic growth.

Hence it is relevant to analyse how consistent Taylor Rule's historical interpretation has been towards the objective underlined before and for how long the policies were implemented using interest rates as the primary instrument for monetary policy. The target rule could be implemented using current data and recent realization of output growth and inflation or with a sight of growth of inflation soon, given that the primary role of this rule was to raise interest rates at the rise of inflationary pressure and lower rates when recession seems to be threatening the economic system.

Nonlinearity in monetary policy according to Greenspan may be reconducted to uncertainty, different sources of uncertainty are shown in imperfect knowledge of data, revision of the data used (real gap and hip deflator), and interest rates analysed with a high margin of error due to measurement unsureness. In the same way, data uncertainty is relevant to monetary response, also parameter uncertainty plays a fundamental role in the impact of policy on the economy and the subsequent response of the economy itself.

The main question is whether the monetary response in the economy should be less or more aggressive given this parameter uncertainty in the variables considered. Researchers share two different opinions: parameter uncertainty can be addressed not as the principal source of uncertainty because it contributes to negligible errors, against the thought that a more aggressive response of monetary response would be more suitable to deal with inflation persistence and relevance in shocks. As it is largely known, inflation variance increases with the distance from the target, hence aggressiveness reduces deviation for optimal level.

Another main aspect of non-linearity is represented by shock uncertainty, coming from the thought of various Federal reserve governors that linear responses to shocks do not depend on the variances or distributions properties of the shocks and that a linear quadratic framework that embodies a quadratic loss function may not be suitable in cases when the economic system displays poor performance.

Greenspan with other governors has tried to give a different framework approach, accounting for risk management, taking into consideration a possible optimal objective to be followed by the economy, and at the same time considering different deviations from the objective, dispersions. Two perspectives play a fundamental role in risk management policy: the first is structural impediments which do not seem to be reliable being based more on conjectures rather than quantitative measurements itself.

If we assume that there are structural impediments and would proceed with accommodation policy the economy would face a supply constraint that monetary strategy is not able to address, hence additional easing would increase inflation and not have an impactful change in unemployment.

The second perspective is the *"liquidity trap"* which occurs when nominal interest rates are considerably above zero and real rates of interest are positive.

If the supply of savings increased but the investment demand remained unchanged, market forces would drive down real interest rates to some natural rate of interest that equilibrates savings and investment. In case of a liquidity trap in this market, the dynamic is averted.

The risk management approach adopted by central banks entitles two aspects of the risk which constitute the core of uncertainty of price stability in the economy.

Upside risk refers to a random variable of interest that exceeds a certain threshold and downside risk represents the realization of the so-called variable below a certain threshold.

The central bank's objective has always been to avoid deviations of output from the potential level expressed by x and beyond an upper threshold level $\overline{x}$ and a lower threshold level $\underline{x}$.

For inflation we define deflation risk DR, and risk of excessive inflation, EIR:

$$DR_\alpha = -\int_{-\infty}^{\pi} \left(\underline{\pi} - \pi\right)^\alpha dF_\pi(\pi), \quad \alpha \geq 0$$

$$EIR_\beta = \int_{\overline{\pi}}^{\infty} (\pi - \overline{\pi})^\beta \, dF_\pi(\pi), \qquad \beta \geq 0$$

The risk management requires a trade-off policy for the central bankers: the upside risk of inflation exceeding the upward value $\overline{\pi}$ and the downside risk of inflation falling below $\underline{\pi}$, in the same way with output values.

The risk management problem presented is displayed in a microeconomic perspective where the central banker's preferences satisfy a risk management model if and only if there is a utility function that defined all relevant distributions F_π and G_π, F_π is preferred to G_π if and only if $U\left(DR_\alpha(F_\pi), EIR_\beta(F_\pi)\right) > U\left(DR_\alpha(G_\pi), EIR_\beta(G_\pi)\right)$.

According FOMC definition states that the loss function of the Central bank can be explained in terms of output look and inflation, in fact by defining π_t and x_t, the loss function will be: $L_t = L(\pi_t, x_t)$ type of function.

$$L_t = \left[aI\,(\pi_t < \underline{\pi})\left(\underline{\pi} - \pi_t\right)^\alpha + (1-\alpha)I(\pi_t > \bar{\pi})\left(\underline{\pi} - \pi_t\right)^\beta\right]$$

$$+ c\,\left[bI\left(x_t < \underline{x}\right)\left(\underline{x} - x_t\right)^\gamma + (1-b)I(x_t > \bar{x})\left(\underline{x} - x_t\right)^\delta\right]$$

$$where\ 0 \leq a, b \leq 1\ and\ c \geq 0$$

To evaluate the risk management approach during the monetary policy in the Greenspan era, we must evaluate linearity in the monetary policy adopted.

The model used to evaluate risk management influence in monetary policy follows a smooth transition regression model to test the dependence of the monetary policy on macroeconomic features with risk level facing the economy.

Given the two switching variables considered, the VXO index and inflation outlook (using forecast given by the Survey of Professional Forecasters) and the autoregressive components displayed in the Taylor Rule the STAR panel model will be the type of STR employed in the estimation.

As developed by Teräsvirta (1994) the STR model is defined as:

$$y_t = \Phi'\, z_t + \Psi' z_t G(\gamma, c, s_t) + u_t$$

$$y_t = \{\Phi + \Psi G(\gamma, c, s_t)\}'^{z_t} + u_t\ ,\ \ t = 1, \ldots, T$$

Where $z_t = \left[1\ \pi^e_{t|t+3}\ x^e_{t|t}\ x_{t-1}\left(x^e_{t|t+3} - x_{t-1}\right) r_{t-1} r_{t-2}\right]$ (3) represents the vector of explanatory variables and exogenous variables.

The matrix $\Psi = \left(\alpha_i^* \beta_i^* \varphi_i^* \phi_i^* \theta_i^*\ \rho_{1,i} \rho_{2,i}\right)' \left((m+1) \times 1\right)$, embodies the parameters vectors and a sequence of identically distributed errors $u_t \sim i.i.d.(0, \sigma^2)$.

The transition function is displayed by $G(\gamma, c, s_t)$, is a bounded function of the continuous transition variable s_t, everywhere in the parameter space for every value of s_t, while γ is the slope parameter and $c = (c_1, \dots, c_k)'$ is the vector of the location parameters.

STR model is a linear model with stochastic time-varying coefficients $\Phi + \Psi G(\gamma, c, s_t)$, and transaction function form explains the response of monetary policy to macroeconomic conditions. They are called smooth since the transition between regimes is smooth and G is bounded between 0 and 1.

To focus on errors in the application and prevision of the Taylor Rule concerning the pattern followed by monetary policy during the Greenspan era, we will replicate the estimation conducted by Gnabo and Mocero (2015) but this time using data available from Bloomberg regarding the period of reference, Greenspan era 1986-2005 and Fred of Saint Louis, North Carolina also SPF Data from Greenbook, a report by the Federal Reserve staff analyzing current economic conditions and forecasting a large set of macroeconomic variables before each meeting of the Federal Open Market Committee (FOMC). The Greenbook, available from the Federal Reserve Bank of Philadelphia, allows us to reflect as closely as possible on the information set as used by policymakers in real-time.

Estimation in monetary policy reaction functions was conducted with quarterly average daily measures of Federal Funds rate, selecting four months on the annual measurements.

The same path of selection has been applied to the GDP deflator since in the Taylor Rule inflations are measured by a change in the implicit output deflator.

Output gap measures were measured considering past and present deviations from the GDP deflator between quarters in $t-1$ and $t+3$.

Since the relevance of the risk management approach, it is important to analyse also possible outcomes that diverge from the main path hypnotized for the economy. A possible way to proxy the distribution of outcomes is cross-sectional dispersion associated with the quarterly GDP deflator obtained by Survey Professional Forecaster.

After forecasting the quarter-on-quarter inflation rate for three quarters in the future, and using the 25th, 50th, and 75th quartiles of the forecasts for each quarter we obtained forecasts of the inflation rate between quarters t-1 and t+3. This dispersion measure is computed as the difference between the expected inflation of the 75th and 25th quartile divided by the 50th quartile:

$$\sigma^2_{\pi,t} \frac{\pi^{75}_{t-1,t+3} - \pi^{25}_{t-1,t+3}}{\pi^{50}_{t-1,t+3}}$$

The second measure of risk captures stress in financial markets, this measure was mentioned several times by FED governors as very important regarding financial markets' stress conduct of monetary policy, the VXO volatility index, the real-time volatility index.

The Taylor Rule adopted by Gnabo and Mocero (2015) in their estimation is specified as follows:

$$r_t = (1-\rho_1-\rho_2)\left(\alpha + \beta\pi^e_{t|t+3} + \varphi x^e_{t|t} + \phi x_{t-1} + \theta\left(x^e_{t|t+3} - x_{t-1}\right)\right) + \rho_1 r_{t-1} + p_2 r_{t-2} + \eta_t$$

Analysing the Taylor rule above we can divide the formula into three blocks, the last one where r_t is the Federal Funds rate at time t, describes the relationship with the AR component: the past values of the Federal funds rate. The second block refers to inflation $\pi^e_{t|t+3}$ and the expected output gap $x^e_{t|t+3}$ at time t+3, given available information at time t.

$x^e_{t|t}$ is the expected output gap for period t, also using information available until time t. expected values for the quarter output gap reflect the need for forecast-based values by authorities. At last, η_t represents the error term.

ρ_1, ρ_2 represent coefficients describing the degree of inertia in the economy, while β captures the response to inflation. If $\beta > 1$ the monetary policy is said to be proactive and the interest rate will adjust subsequently to stabilize inflation; if $\beta < 1$ monetary policy is said to be accommodative, hence the lowering of the Federal funds rate and interest rate of borrowing to stimulate the economy and reduce unemployment.

The remaining coefficients expressed by φ, θ, ϕ when positive means that policymakers conduct a counter-cyclical monetary policy, such as by increasing government spending and cutting taxes to help the recovery of the economy.

Estimates were conducted with the least square method to obtain suitable coefficients and variables relative to each component of the Taylor rule, output gap, Federal funds rate, and GDP deflator, that have been previously managed.

We evaluate the smooth transition model to the relationship between risk level variables and the strength of response to macroeconomic conditions.

We estimate models with the Non-Linear Least Square (NLS) model, based on Bekaert analysis estimated a vector of the autoregressive model to study the dynamic between risk aversion and monetary policy. Based on the findings there seems to be no contemporaneous correlation between interest policy rates and measures of risk in financial markets.

The input code is based on Fouquau, Hurlin, and Rabaud (2008) where Y represents the $T \times N, 1$ vector of values of the dependent variable and eventual missing data are to be accounted for with "Nan".

In our case, we denote as the dependent variable as r_t the Federal Funds rate value at present time and we insert data cross-unit by cross-unit as in the form of $y = (y_1', \dots, y_N')'$

Q is a $T \times N, 1$ vector corresponding to the threshold variable, and here we use the VXO index, one of the two transition variables mentioned above, and as previously stated the data are stacked cross-unit by cross-unit.

X is the matrix $T \times N, K$ corresponding to the values of the K explicable variables, here it represents the matrix defined in the code by "Comb" which is the matrix of variables estimated interested in the Taylor rule version we used for the analysis.

N represents the cross-section dimension.

M represents the number of location parameters, which can go from 1 to 5. When $m = 1$ the model implies that the two extreme regimes are associated with low and high values of q_{it} with a single monotonic transition of the coefficients from b_0 to $b_0 + b_1$ as q_{it} increases, where the change is centered around c_1 (the location parameter). For m $= 2$, the transition function has its minimum at $\frac{c_1 + c_2}{2}$ and attains the value 1 both at low and high values of q_{it}. In general, when $m > 1$ and gamma tends to infinity, the number of distinct regimes remains two, with the transition function switching back and forth between zero and one at $c_1 \dots c_m$.

The code automatically determines the optimal number of transitions by using nonlinearity tests and tests one transition against the linearity assumption, two transition functions against one, and so on.

The function used is a version of the STAR panel presented by Hurlin (2006) to estimate the parameters pf Panel Transition Smooth Regression.

The model considered in the code in the case of one threshold parameter, so with two regimes, is represented by:

$$y_{it} = \begin{cases} \mu_i + \beta_1' x_{it} + \varepsilon_{it} & if\ q_{it} \leq \gamma \\ \mu_i + \beta_2' x_{it} + \varepsilon_{it} & if\ q_{it} > \gamma \end{cases}$$

Where x_{it} denotes an explicative variable associated with a parameter affected by the regimes, q_{it} is the threshold variable. As we considered lagged values for variables in the Taylor Rule as Federal funds rate at $t-1$ and $t-2$, we need to also introduce data series as inputs in the code function.

$\gamma_j\ and\ \beta_j$ are estimated according to the same simple least square procedure used for the standard Smooth Transition Autoregressive model (STAR). Considering a single threshold model for a given value of the threshold parameter γ, the slope coefficients defined by β_j can be estimated by ordinary least square estimation.

Given the fact that the sum of least squared residuals depends on γ only through the indicator function, we can define it as a step function with the steps occurring at distinct values of the threshold variable. Notice that the threshold parameter is estimated by minimizing the sum of squared errors on γ, $S_1(\gamma) = \sum_{i=1}^{N} \sum_{t=1}^{T} \hat{\varepsilon}_{it}^2(\gamma)$. Therefore, the maximum number of steps of the function will be at most NT.

The minimization problem can be reconducted by searching over values of γ equalizing at the most distinct values of q_{it} in the sample.

From the estimates of the threshold variable, $\hat{\gamma}$ it is possible to estimate the coefficients of elasticity in the regime represented by slope coefficient estimations and estimation of individuals' effects.

The second part of the code deals with the testing procedure and underlines some issues in the threshold model represented. The first consists of testing the number of regimes or testing threshold specification and the second issue consists of choosing the threshold variable. Let's reason by hypothesis to understand the issue.

If we assume to know the threshold variable, by testing the significance of the model with two regimes, the null hypothesis is represented by H_0: $\beta_1 = \beta_2$, and it corresponds to no threshold effect hypothesis.

Under H_0 the model is equivalent to a linear model and this hypothesis is usually tested by a standard test. By naming S_0 the sum of the square of a linear model, the likelihood test will approximately look line: $F_1=\frac{S_0-S_1(\hat{\gamma})}{\hat{\sigma}^2}$ where $\hat{\sigma}^2$ denotes a convergent estimate of σ^2. But under the null hypothesis, the threshold parameter is not identified therefore the asymptotic distribution of F_1 is not standard and does not correspond to a chi-squared distribution.

Hansen (1996, 1999) has largely investigated the issue, and he tried to find a solution by simulating the asymptotic distribution of the statistic F_1 through bootstrap and showing that p-values obtained this way are asymptotically valid.

The same procedure can also be applied to determine the number of thresholds in general models.

If the p-value associated with F_1 leads to rejecting the linear hypothesis, we will discriminate between one and two thresholds. The subsequent likelihood ratio test based on this is:

$$F_2 = \frac{S_1(\hat{\gamma})-S_2(\widehat{\gamma_1,}\hat{\gamma_2})}{\widehat{\sigma^2}}$$

Where $\widehat{\gamma_1,}\hat{\gamma_2}$ denote the three regime thresholds estimates of the model with three regimes and $S_2(\widehat{\gamma_1,}\hat{\gamma_2})$ is the corresponding residual sum of squares.

If F_2 is larger than the critical value of non-simulated distribution the hypothesis of one threshold is rejected in favour of two and the relative p-value can be simulated through bootstrap simulations, as shown by Hansen (1999).

From the variable estimations reported it is believed that Greenspan's monetary policy was conducted following non-linear behaviour where economic risk leads to switch transitioning regimes. This is coherent with Greenspan's belief that monetary policy strategy should go towards a wider analysis of several risks faced by the economy and their sources to prevent and foresee any sudden changes.

In our case, it is not possible to reject the null hypothesis of linearity at conventional statistical levels. From the findings, it is shown that all the variables that influence changes in monetary policy are represented by risk in inflation outlook and risk in financial markets rather than classical macroeconomic variables considered in the literature.

Results of the estimation of STR models are coherent with Taylor Rule showing that in normal times Fed implemented a counter-cyclical monetary policy, with a statistically significant coefficient for transition parameters.

Hence during the high period of risk, it is clear how Federal Reserve is responsive to the business cycle rather than the inflation outlook, when the output gap increases, then monetary policy is tightened resulting in more aggressive policy during the period of high risks.

The degree of inertia appears to be lower in a period of high economic risk, coherent with Mishkin's view of a stronger response to monetary policy during periods of high economic risk.

With respect to shock uncertainty from Greenspan's analysis, it is evident how a robust control approach, such as risk management, for monetary policy, is relevant to avoid faulty outcomes. Risk management proves to be a winning strategy if correctly implemented in trying to understand and predict future behaviors in the economy. However, the delicacy of this application comes from correctly choosing the variables leading to the forecast and not predicting erroneous choices due to faulty values and models.

From our findings lack of linearity during the Greenspan era has proven that monetary policy is more affected by inflation measures rather than traditional macroeconomic outcomes and variance of the error of volatility must be modeled cautiously to give a more precise and coherent description of the economy.

As shown through regime switch models aggressiveness tends to increase during periods of high economic risk lowering the degree of inertia with respect to normal case scenarios, coherent with Mishkin view discussed previously (2008).

Further analysis can be better conducted in the future by using GARCH models (Engles 1993) to improve the description of the linearity of monetary policy and modeling variance of the shock related to volatility.