MSc in Corporate Finance

Chair of Business Valuation

Premiums and Discounts in Firm Valuation

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TABLE OF CONTENTS

1. Introduction

This work starts with a comprehensive literature review of premiums and discounts, then focuses on the recent developments in the literature around the objective of bringing together the most important factors that drive premiums and discounts.
It will be possible to see that different frameworks have been proposed, the latest and most comprehensive one being the Popularity Asset Pricing Model, first proposed by the CFA Institute in 2018.
Even though it is possible to find earlier theories that hint at something similar, as we will see, the PAPM is the only mathematized framework presented to date that captures company characteristics that are specifically not part of a risk-return approach, thereby allowing not only for a theoretical explanation but for practical application application as well. Contributions so far have assessed the framework, but, as of 2023, other than some theoretical examples from the authors of the main paper, there are no practical real-world applications in terms of valuing a company's cost of equity using this framework.
The research question will then be: is it possible to employ the PAPM to build a new valuation tool, one that can comprehensively consider premiums and discounts while computing a company's cost of equity, ultimately impacting the valuation of the firm?
Before exploring the research question, it is essential to acknowledge that the choice of valuation methods impacts the considerations regarding premiums and discounts.
Therefore, this study will begin with an overview of the main valuation methods.
In order to discuss premiums and discounts, we will rely on the categorization illustrated in the book "Business Valuation Discounts and Premiums" by Shannon P. Pratt which states that discounts and premiums fall into two categories:
-*entity level* ones which affect all shareholders as they affect a company as a whole, an example being the size premium
-*shareholder level* ones which affect one shareholder or a group of shareholders that share a characteristic, for example: being "minority shareholders".

2. Literature Review

2.1 Overview of relevant valuation methods

Let's start by looking at the definitions of premiums and discounts, starting from the definition of "fair value" as premiums and discounts are applied to an already defined value (base value), they are company-specific and are meant to be applied only when there are enough reasons to think that the base value doesn't accurately capture the value of a certain company.
In a hostile takeover, for example, the two parties more easily come to an agreement on the base value but where they differ is exactly on company-specific factors.
Fair market value is a concept of value in exchange. It is defined as 'the net amount that a willing purchaser, whether an individual or a corporation, would pay for the interest to a willing seller, neither being under any compulsion to buy or to sell and both having reasonable knowledge of relevant facts[1].'
Let's now look at the definition of premiums and discounts as presented by the American Society of Appraisers (ASA):
"A discount or premium quantifies an adjustment to account for differences in characteristics affecting the value of the subject interest relative to the base value to which it is compared. Premiums and discounts have no meaning until the conceptual basis underlying the base value to which it is applied is defined. A discount or premium is warranted when characteristics affecting the value of the subject interest differ sufficiently from those inherent in the base value to which the discount or premium is applied[2]."
A recurring reference to *base value* is made in the definitions of premiums and discounts in the literature, it is then useful to provide an overview of the main valuation methods that enable an appraiser to arrive at a base value.
Valuation methods can be divided into two broad categories: market methods and financial methods, the latter refer solely to the financial statements of the company.
Market methods arrive at a company valuation by looking at comparable companies and applying then multiplicative factors to relate specific multiples of comparables with the ones of the company being valued. This approach therefore provides an assessment of the relative value of a company, based on the comparison of its financial performance to those of similar firms. The analyst calculates the average multiples for the comparable companies and, using these values, arrives at an estimate of the value of the company.
These methods are useful to perform a sanity check of the fundamental valuation obtained with other methods, by giving an important indication on whether the more precise valuation methods are producing reasonable results, unexpected and large discrepancies between valuation methods may indicate that further investigation is needed.

[1] Shannon P. Pratt. (2009). Business Valuation Discounts and Premiums. John Wiley & Sons. p.10

[2] American Society of Appraisers Business Valuation Standards. (2002).

In the case of premiums, comparable companies can be used as a stand-alone method for the Size premium but, again, since it is not possible to find companies that are identical to the one being valued, this approach will always produce an approximation that will only be useful as a check.
Under the second category there are many different methods, such as: DCF, Adjusted Present Value, Net Income, Economic Value Added, Mixed and methods based on value creation using Option Theory.
The ones we will look at more closely are those more relevant when discussing premiums and discounts, such as: cash flow discounting (DCF) method, asset methods and options methods.
It is not the purpose of this dissertation to extensively cover every valuation method, but it is important to understand the rationale behind the ones that will be discussed.
In the Discounted Cash Flow (DCF) method, the company is considered a generator of cash flows and its value obtained by the appropriate discounting of them.
The discount rate reflects the risk of the cash flow, as perceived by the investors. The discount rate will also depend upon the cash flows we decide to utilize for the valuation, if Free Cash Flows are used i.e. "the operating cash flow, that is, the cash flow generated by operations, without taking into account borrowing (financial debt), after tax" then the appropriate discount rate is the Weighted Average Cost of Capital (WACC), which allows us to take into account the required return on both equity and debt, according to their proportion in the financial structure of the company. If the model relied on Equity cash flows, than the appropriate discount rate to use would be the cost of equity and we would arrive at Equity Value, adding the value of debt separately we would obtain Firm Value.
The main formula of the DCF methods is:

$$V = \frac{FCF_1}{(1+WACC)} + \frac{FCF_2}{(1+WACC)^2} + \frac{FCF_3}{(1+WACC)^3} + \cdots + \frac{FCF_n + VR}{(1+WACC)^n}$$

VR stands for residual value and is calculated using the Gordon's formula, assuming constant infinite growth.[3]

This method is very useful when dealing with premiums and discounts as it allows us to include them in two ways: through the discounting factor by using the one derived through PAPM and through a direct reduction or increment to cash flows, making sure to not make the mistake of counting the same factor twice.

The other method useful for our purposes is the Asset Method, as for the DCF, there are different variations of it, what matters is understanding the rationale behind it since this is the way of reasoning that then must be followed to correctly account for premiums and

[3] Pablo Fernandez. (2001). Company Valuation Methods.

discounts within this method.

In this category fall all the methods that value a company starting from finding the market value of all the assets net of liabilities, on a going concern basis. The more articulated methods include the valuation of intangibles, this aspect will be investigated in more details in the discussion of brand value in the following chapter.

2.2 Premiums and Discounts

As mentioned in the introduction, premiums and discounts concern two different levels: entity level and shareholder level. Let's review the literature concerning the ones affecting a company at the entity level first.

*2.2.*1 Value

The *value factor* can be found in empirical data going back over a century and has been studied academically for at least 4 decades, the debate over value really concerns its definition and its possible persistence in the future but not its validity.

The problem of the definition relates to the multiples used to capture it. Value can be captured using different metrics, since the focus later will be on the PAPM, that builds on the work of Fama&French, amongst others, the definition of value here will be the one used in the work of Fama&French.

In their model, the value factor is captured in the portfolio called HML that stands for "high value minus low value" i.e. going long on value stocks and short on growth stocks[4].

This portfolio is based on what has been called the "pure value" definition of value as stated in the paper of Asness, Frazzini, Isreal, Moskowitz: " Fact,Fiction,and Value Investing (2015)", where by pure value it is intended the version of Fama and French that is characterized for relying on P/B as the sorting variable. The HML factor therefore represents the excess return of a portfolio composed of high price-to-book (P/B) ratio firms relative to a portfolio of low P/B firms.

For portfolio optimization it is strongly recommended to capture value using different metrics together (such as profitability/quality measures) in order to capture the signal better and reduce the noise generated by the use of a single metric, but this is beyond the scope of this dissertation.

[4] Fama, E. F., & French, K. R. (2013). A Four-Factor Model for the Size, Value, and Profitability Patterns in Stock Returns.

Using this definition of value, the premium over the market in the period that goes from 1926 to 2014 is of 3.6%. From 1963 to 1981 the premium was of 6.3%, this is interesting to notice because it is around the eighties that the value factor was discovered and presented in different academic studies, probably because due to its magnitude in that period, it caught the attention of different researchers. The discussion over its future persistence is essential for the focus of this work because the difference between premium and mispricing is exactly this: a premium refers to a characteristic that is here to stay, a mispricing is a temporary mismatch of supply and demand that will adjust in the short term. Value is not easily explained through the lens of Classical Finance that views the market as efficient and investors as solely differentiated by their risk tolerance, value firms usually have a lower β thus should have a weaker return since investors do not have to be compensated for bearing more risk. To fit the model the definition of risk is broadened, it does not coincide with β but refers to "distress risk" as proposed by Fama & French since value firms undergo through long periods of underperformance thus, they bear some systematic risk and have a high β when measured against market-wide distress factors.

Behavioral Finance conceptualize investors as irrational and the market as inefficient and explains the value-factor as an overreaction of investors that have previously neglected these companies due to various biases that led them to focus on more popular stocks.

The Popularity Framework gives us a precious bridge between these two views and explains value without having to bend any other definition in the framework, every premium will be briefly restated and explained through PAPM when it will be presented in chapter 3. It is a common misconception that if a factor is related to risk it will be permanent whereas if it is related to behaviors is transient, that's why during the tech boom of the late 90s it was thought that investors were arbitraging away from value but that was not the case.[5] There is no consensus on the theoretical explanation of value but it will persist unless risk appetites and investor behavior change substantially in the future or as commented by Eugene Fama: "Value stocks tend to be companies that have few investment opportunities and aren't very profitable. Maybe people just don't like that type of company. That to me has more appeal than a mispricing story, because mispricing, at least in the standard economic framework,

[5] Fact, Fiction, and Value Investing by Clifford S. Asness, Andrea Frazzini, Ronen Israel, Tobias J. Moskowitz:: SSRN

should eventually correct itself, whereas taste can go on forever", (Fama and Thaler 2016). Despite a larger amount of research is available for the US, when investigating the value premium, across the world, its presence is confirmed. In the work presented by David Blitz and Pim Van Vliet in 2008, the value premium was present across the world, specifically, when the authors examined the performance of value and growth portfolios, they estimated the value premium to be equal to 4.4% annually across all nations, a result that is consistent with earlier research that have looked into the value element in the US market. The average value premium was positive over the course of the period analyzed in the research that goes from February 1986 through September 2007, bringing further evidence on its persistence over time[6]. To conclude, I quote the rather enthusiastic conclusion on the evidence of value presented in the paper 'Fact, Fiction, and Value Investing' (2015) by Asness, Frazzini, Isreal, and Moskowitz:

"it is evident in 87 years of U.S. equity data, in over 30 years of out-of-sample evidence from the original studies, in 40 other countries, in more than a dozen other asset classes, and even dating back to Victorian age England!"

2.2.2 Size

The second entity-level factor in analysis is *Size*.

In the book "Discounts and Premiums" by Shannon P. Pratt we find the following definition: "the size effect is the concept that the smaller the company the greater the risk and thus the higher the companies' cost of capital"[7], as evidence from 1926 to 2006 shows:

[6] David Blitz and Pim Van Vliet (2008). Global Tactical Cross-Asset Allocation: Applying Value and Momentum Across Asset Classes

[7] Shannon P. Pratt. (2009). Business Valuation Discounts and Premiums. John Wiley & Sons.

Long-Term Returns in Excess of CAPM Estimation for Decile Portfolios of the NYSE/AMEX/NASDAQ with 10th Decile Split: 1926–2006

Decile	Beta*	Arithmetic Mean Return	Realized Return in Excess of Riskless Rate**	Estimated Return in Excess of Riskless Rate†	Size Premium (Return in Excess of CAPM)
1—Largest	0.91	11.35%	6.13%	6.49%	–0.36%
2	1.04	13.25%	8.04%	7.39%	0.65%
3	1.10	13.85%	8.64%	7.82%	0.81%
4	1.13	14.28%	9.07%	8.04%	1.03%
5	1.16	14.92%	9.71%	8.26%	1.45%
6	1.18	15.33%	10.11%	8.45%	1.67%
7	1.23	15.63%	10.42%	8.80%	1.62%
8	1.28	16.61%	11.39%	9.12%	2.28%
9	1.34	17.48%	12.27%	9.57%	2.70%
10—Smallest	1.41	21.57%	16.36%	10.09%	6.27%
Mid-Cap, 3–5	1.12	14.15%	8.94%	7.97%	0.97%
Low-Cap, 6–8	1.22	15.67%	10.46%	8.70%	1.76%
Micro-Cap, 9–10	1.36	18.77%	13.56%	9.68%	3.88%

Source: Stocks, Bonds, Bills, and Inflation Valuation Edition 2007 Yearbook. Copyright © 2007 Morningstar, Inc. All rights reserved. Used with permission. To purchase copies of the *Valuation Edition Yearbook*, or for more information on other Morningstar publications, please visit global.morningstar.com/DataPublications. Calculated (or derived) based on CRSP data, © 2007 Center for Research in Security Prices (CRSP), Graduate School of Business, the University of Chicago.

*Betas are estimated from monthly portfolio total returns in excess of the 30-day U.S. Treasury bill total return versus the S&P 500 total returns in excess of the 30-day U.S. Treasury bill, January 1926–December 2006.

**Historical riskless rate is measured by the 81-year arithmetic mean income return component of 20-year government bonds (5.21 percent).

†Calculated in the context of the CAPM by multiplying the equity risk premium by beta. The equity risk premium is estimated by the arithmetic mean total return of the S&P 500 (12.34 percent) minus the arithmetic mean income return component of 20-year government bonds (5.21 percent) from 1926 to 2006.

The last column shows a size premium % that can be used in an expanded form of CAPM to account for the size premium:

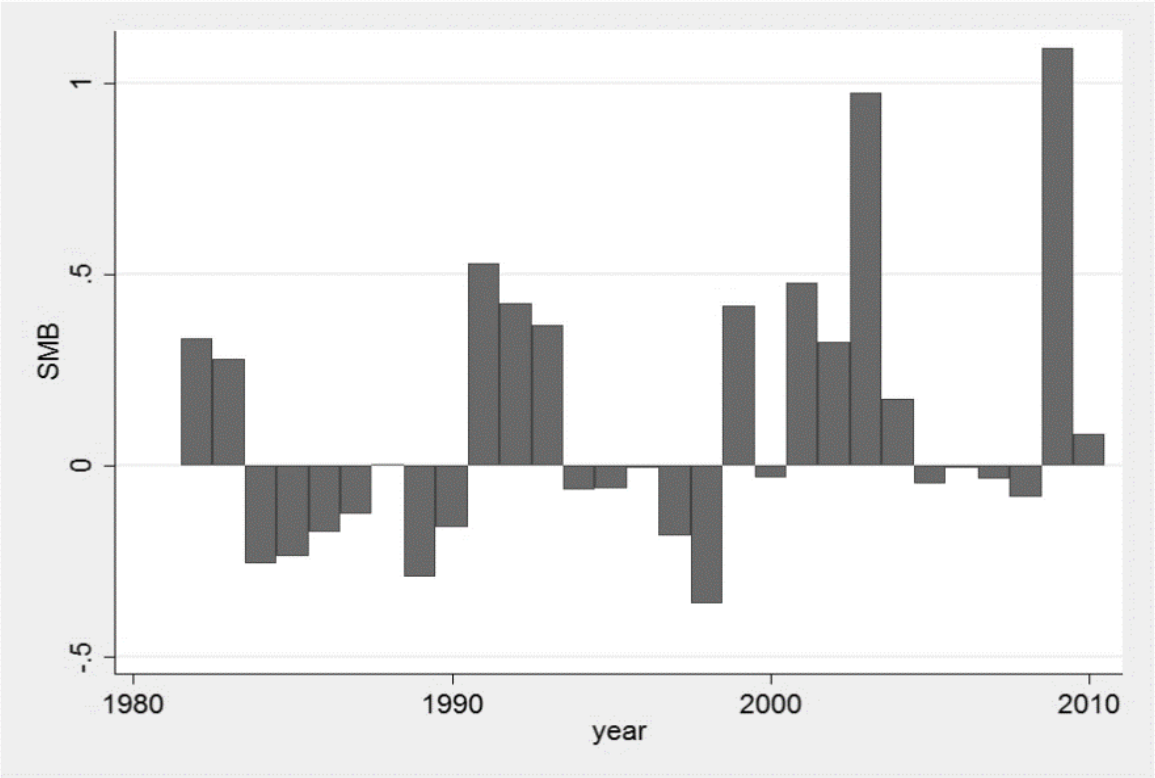
$E(R) = R_f + B(RP_m) + RP_s + RP_u$ where RP_s stands for Risk Premium for size and RP_u stands for Risk Premium of the specific company. Within this framework it would be sufficient to locate in which decile the company stands and adjust for the appropriate premium.

The PAPM framework builds on Fama&French, therefore it's based on their approach which captures this premium in a factor portfolio called SMB (small minus big) in which the return on small firms is subtracted from the return on big firms, if positive, this represents a premium.[8]

The following is empirical evidence building an SMB portfolio based on the returns of the

[8] Fama, Eugene F. and French, Kenneth R. (2014). A Five-Factor Asset Pricing Model

smallest and largest decile, using data from the Center for Research in Security Prices (CRSP) of the University of Chicago's Graduate School of Business:



9

The size effect doesn't struggle to be ex'lained within the e'ficient market hypothesis, smaller companies do have a higher β, size could in fact be thought as a proxy for other risks inherent in a company being small, whether size is a factor or a proxy for other aspects remains controversial, as was noted already by Rolf W. Banz who was the first academic to recognize this aspect in 1981.

Some of the most relevant alternative explanations, beyond the one coherent with CAPM are mainly those relating to behavioral factors and market frictions. The explanations derived through behavioral observations concern

a) the anchoring bias i.e. the tendency of investors to rely mainly on past information, when

[9] Micheal A. Crain. (2010). A Literature Review of the Size Effect.

making investment decisions.This can lead to a persistence of mispricing in small-cap companies as investors overlook new information

b) herding behavior, which describes the tendency of investors to follow the investment decisions of others thus enabling persisting premiums on small companies.[10]

Market frictions explanations focus on

a) the possible lack of liquididity of these companies compared to large companies, making it more difficult and expensive for investors to invest in these companies thus a higher expected return.

b) information asymmetry, due to smaller companies receiving a limited analyst coverage,.c) agency costs, which refer to the costs associated with aligning the interests of all the different parties involved in the capital structure of the company, e.g. the higher risk of insider trading, as the concentration of information in smaller companies may give executives incentives to engage in insider trading.[11]

The evidence on the size effect has also been recognized multiple times in court cases.[12]

2.2.3 Distress

The third entity-level factor is distress, its definition can be derived by looking at what the models implied to capture distress look for. Distress comprises two types of distress: financial and operational. Financial distress refers to the capital structure of the company and stems from the difficulties that a company is facing in meeting its financial obligations. Operational distress can be a consequence of financial distress or can be more directly related to specific characteristics of the company such as low level of capacity utilization, high employee turnover, negative margins or it can be related to the broader market conditions. Understanding where distress originates and its degree of severity is the first part of the valuation of this aspect.[13]

This aspect can't be captured in the PAPM through the discount factor, but it can still be accounted for in a DCF model with the following approach:

1) calculate the normalized situation, estimating the amount of the cash flows once the

[10] Daniel, K., Hirshleifer, D., & Subrahmanyam, A. (1998). Investor psychology and security market under- and overreactions. The Journal of Finance

[11] Keim, D. B., & Stambaugh, R. F. (1986). Predicting returns in the stock and bond markets. Journal of Financial Economics, 17(2), 357-390

[12] Shannon P. Pratt. (2009). Business Valuation Discounts and Premiums. John Wiley & Sons, pp. 329-333

[13] Vulpiani Marco. (2014). Special Cases of Business Valuation. McGraw-Hill, pp.245-247

distressed situation will be resolved, this will be the terminal value

2) calculate the years preceding the normalized situation backwards, through an iterative approach.

This approach is called "changing capital structure method".[14]

By looking at the literature and at literature reviews performed by different academics what is evident is that one valuation method is the most widely studied to account for distress i.e. the use of the Altman Z-score[15], this is why the focus will be on this approach, since internal ratings-based approach was suggested in the Basel 2, this model has become the most widely used also in that context.

The model proposed by Altman is based on the statistical technique called multiple discriminant analysis (MDA), he selected the financial ratios that best predicted bankruptcy, considering the predictive power of each of them and their correlations to arrive at the mix of financial ratios that yielded the most accurate result, his estimation of the discriminant function is the following: $Z= 1.2X_1+ 14X_2 + 3.3X_3 + 0.6X_4 + 1X_5$, where

$X_{1=}$ working capital/Total assets , X_2= retained earnings/Total assets, X_3 = EBIT/Total assets, X_4=Market value of equity/Book value of total liabilities, X_5= Sales/Total assets.

The first factor is a measure of liquidity, the second one is a proxy for profitability over time, the third is a measure of productivity, the fourth is a more direct measure of the solvency capacity of the firm and finally the fifth factor measures capital turnover. In the case of private firms, the variables become four to reduce a potential industry effect and market value of equity was substituted by book value of equity, since the former clearly is not available for private companies.

In both cases, the third factor made the highest contribution. The author suggested to use the most relevant group of companies possible both in terms of comparability and timeliness of data, in every estimation. This hypothesis was tested in different reviews and the accuracy of the model was only slightly improved by the re-estimation of the coefficients in most cases, except for certain countries, this proves that the original model is

[14] Vulpiani Marco. (2014). Special Cases of Business Valuation. McGraw-Hill.

[15] Distressed Firm and Bankruptcy Prediction in an International Context: A Review. E I. Altman, M. Iwanicz-Drozdowska, E.K. Laitinen, A. Suvas (2014)

still valid but using a more recent sample can still be a useful way to improve the model. Adding other relevant variables also does not yield significant results but the main difference is observed among countries, the accuracy varies between 75% and 90% using the original model, it is thus recommended to use country-specific sampling when using the Altman Z-score model. Similarly, an adjustment is suggested when using the z-score on non-manufacturing firms, this model was originally developed for manufacturing firms, it can be used for other industries but the sampling of companies has to be industry-specific for better accuracy[16].

In conclusion, the Z-score is still valid and widely used by many professionals as the literature shows, the relevant question for our purpose is: how is it implemented in premiums and discounts for business valuation? A possible answer is given by the use of Duff&Phelps report.

In their report, a section is dedicated to high-risk companies, referring specifically to distressed companies and the use of the Z-score. The result of the Z-score gives a value that is then used in relation to cut-off scores i.e., thresholds that indicate if a company is in the safe-zone, grey-zone or distress zone thus helping in the estimation of the severity of the distress.

The Duff&Phelps report suggests a way to arrive at a cost of capital depending on where the firm lies in terms of distress as established by the Z-score. The methodology is the following: a large number of firms is tested for the Z-score, a portfolio is then created with companies within a certain cut-off score and their β is calculated via the Sum-Beta approach. The analyst then assigns the company under valuation to the appropriate group among those proposed in the report and uses the cost of capital recommended for the valuation.[17]

A brief recap of the three entity-level factors discussed here will be given when introducing the PAPM framework alongside with a discussion on other entity-level premiums and discounts included in the PAPM.

[16] Distressed Firm and Bankruptcy Prediction in an International Context: A Review. E I. Altman, M. Iwanicz-Drozdowska, E.K. Laitinen, A. Suvas (2014)

[17] Duff&Phelps. (2013). Risk Premium Report.

2.2.4 Shareholder level analysis

Entity-level considerations must be integrated with a shareholder-level analysis during a valuation.

Shareholder-level analysis refers to the fact that it is possible to own fractions of a firm in the forms of shares and that a shareholder can hold different types of shares or, more importantly, different amounts of said shares, with the particularly relevant case of shareholders owning more than 50% of a company shares. As the literature shows, there are overarching considerations, theories and a lively debate on which framework is better capable at describing and predicting the realized premiums and discounts, but every company must be analyzed on its own, looking not only at financial statements but at the specific by-laws and the laws of the country in which it operates, the reasons for this need will be soon made clear.

The shareholder level analysis starts with one key observation: there are differences in the level of ownerships and these differences must be reflected in the valuation. This is the reason why there are 2 concepts that are essential at this level of analysis: minority discount and control premium.

Quoting from the latest version (as of 2022) of the International Glossary of Business Valuation Terms by AICPA:

-Control Premium: an amount or a percentage by which the pro rata value of a controlling interest exceeds the pro rata value of a noncontrolling interest in a business enterprise that reflects the power of control.

-Minority Discount: a discount for lack of control applicable to a minority interest.

-Discount for Lack of Control: an amount or percentage deducted from the pro rata share of value of 100% of an equity interest in a business to reflect the absence of some or all of the powers of control

-Minority interest: an ownership interest less than 50% of the voting interest in a business enterprise

-Majority interest: an ownership interest greater than 50% of the voting interest in a business enterprise.

-Discount for Lack of Marketability: an amount or percentage deducted from the value of an

ownership interest to reflect the relative absence of marketability.[18]

When analyzing an individual company, it is then important to review the relevant statutes in order to understand the amount of shares needed to qualify for swing vote potential, the prerogatives of control defined in contractual arrangements specific to the company and the amount of shares necessary to qualify for "blocking power". These aspects help us realize that proposing a simple black and white distinction between premium or discount is misleading, first of all because an ownership greater than 50%, say 51%, may not be sufficient to trigger substantial changes in the organization of a company that are usually those that an acquirer is seeking, this issue is called "supermajority requirements" i.e. there are situations, either due to the laws of the country in which the company operates or due to clauses in the company's by-laws, that require a shareholder to own the supermajority in order to be granted the powers to meaningfully alter the structure of the company.

Secondly, realizing that the issue of ownership is not on-off but is on a continuum will help the analyst better evaluate each specific situation.

This refinement to the previously accepted conceptualization was proposed by Michael Bolotsky in a meeting at the American Society of Appraisers in 1992.

Before delving deeper into the practical implications of these observations, as noticed for the entity level analysis, it is important to keep in mind that the valuation methodology used to account for these premiums and discounts, affects the outcome of the analysis.

There are 3 approaches: income approach, asset approach and market approach.

The first 2 methods mentioned, focus more closely on the differences that the acquirer can bring to the target companies, be it in terms of increased cash flows (income approach) or better utilization of assets (asset approach), the market approach focuses instead on realized transactions to estimate the likely premium or discount that will be applied.

For example, if the income or asset approach is used, the starting point will be the financial statements of a company and the adjustment upward in terms of cash flows can be practically achieved by re-estimating the financial structure, for example by projecting an increase in long term debt leading to a reduction in cost of capital, if the adjustments is instead to asset value, it can be achieved by considering possible divestments, the possibility to sell individual units of the company etc.

[18] International Glossary of Business Valuation Terms, AICPA (2022)

If the starting point is instead the deal price observed in other relevant transactions in the market, the premium (if any) is already reflected in the transactions, it will then be important to consider if the transaction is of only equity interests, in that case it's necessary to subtract the value of debt in order to apply a discount as this is referring only to common equity, not to the entire capital structure[19].

It is on this approach that the work of Damodaran on premiums and discounts is based, he clearly states that: "the difference between the optimal and the status quo values can be considered the value of controlling the business" whereby "optimal value" stands for the hypothetical value that a business would have if it was optimally managed[20]. Expected value of control is therefore the product of the difference between status quo and optimal value and the probability that this change will occur. Damodaran therefore focuses on the determinants of firm value to project what the optimal value could be and then on the determinants of management change to estimate the probability that a change of management will occur. The determinants of firm value are all these components of value well known among analyst so: cash flow, capital structure, utilization of assets and discount rate. Clearly the value of management change will be the difference between optimal firm value and status quo value. The determinants of management change are institutional constraints and firm specific constraints. It is possible to estimate the probability of change in management by comparing the characteristics of companies in which this occurred with the company under valuation. The characteristics are low stock price and earnings performance, small board of directors, non-concentrated ownership structure and whether or not the firm operates in a competitive industry. This is the rationale behind an approach based on financial methods. The discussion around the market approach on shareholder level premiums and discounts, has to begin by looking at the difference fair value and investment value, for the sake of consistency the definitions will be quoted again from the International Glossary of Business Valuation Terms.

Standard of Value: the identification of the type of value being utilized in a specific engagement; for example, fair market value, fair value, investment value

Fair Market Value: the price, expressed in terms of cash equivalents, at which property would change hands between a hypothetical willing and able buyer and a hypothetical

[19] Shannon P. Pratt. (2009). Business Valuation Discounts and Premiums. John Wiley & Sons, p.27

[20] Aswath Damodaran. (2005). The Value of Control: Implications for Control Premia, Minority Discounts and Voting Share Differentials

willing and able seller, acting at arm's length in an open and unrestricted market, when neither is under compulsion to buy or sell and when both have reasonable knowledge of the relevant facts.

Investment Value: the value to a particular investor based on individual investment requirements and expectations[21].

When looking at prices of shares trading in the stock market then, what are we looking at? This question has long been answered as "we are looking at minority interests, thus a control premium is due to evaluate the acquisition of a majority interest" but this approach has been put under scrutiny both among academics and in practice, also because negative premiums have been observed. What follows is a summary of the debate around this question and its practical implications. In 1990 Eric Nath argued that most public companies trade at or near their controlling interest values: "as blood attracts sharks, a significant difference between the current price of a stock and its value to a controlling owner should trigger some form of takeover attack".

Nath pointed out also that many takeovers are strategic in nature thus making the control premiums data biased, an observation that is shared by many others and can be thought as part of the general consensus today, as analysts try to filter out the contribution of synergies in control premium to try and arrive at a less biased data. This aspect will be explained in more detail when looking at historical control premium data. Micheal J. Bolotsky replied to E. Nath that, although this can be true for some public companies, it's incorrect to deduct a minority interest discount from what essentially are already minority interests (shares) and also proposed that there exists a disparity in access to information between a minority and majority owner, furthermore he pointed out that the price of a stock represent the consensus for a "minority block". The debate continued and Chris Mercer intervened in support of the classical Levels of Value model as it is closer to the financial reality observed in business valuation by professionals in their day-to-day work. M. Bolotsky tried to bridge the gap with his "Two Attribute Model." A. Damodaran also explored the significance of stock prices and reached an interesting conclusion: "while markets may not use sophisticated models to make the assessments, they do try to value and price in control"[22].

[21] International Glossary of Business Valuation Terms, AICPA (2022)

[22] Aswath Damodaran. (2005). The Value of Control: Implications for Control Premia, Minority Discounts and Voting Share Differentials

The price on the stock market does already reflect the value of control as he defines it, to prove the existence of this "already built in expectations", Damodaran supports his claims with empirical evidence, showing that, changes in the frequency of hostile takeovers in an industry affects the price of all the peer group and that changes in the laws of a country that make acquisitions more or less easy to carry on, also do affect the price of companies. These are additional aspects to take into account when considering applying a premium to a publicly traded company: was this acquisition expected in that industry in this moment in the market?

The approach suggested by the debate around the meaning of stock prices, is that of considering the conditions of the market. In an oversaturated market, the observation of Nath that stocks are trading already at a "control premium" seems valid and is supported by the existence of negative premiums as well, which, as the review of market data will soon show, is not as rare as the classical conception might suggest. On the other hand, in the 1980s there were a lot of takeovers with the goal of selling a firm or parts of it for a premium and the Level of Value framework did apply. It depends on the state of the market, the analyst should arrive at an intrinsic value, compare it to the price at which a company currently is trading and apply the framework that is more appropriate, lack of marketability (LOMD) is a discount that can also be applied without the need to apply a minority discount but based solely on the issue of liquidity. To summarize and reconcile these different perspectives on the meaning of stock prices, it can be said that if there's an M&A market for a company i.e. the acquirer thinks that it can create added value, than the acquisition value exceeds fair value (market value) therefore there will be a premium. The absence of such market entails that, most likely, no premium is due. In emerging thus risky industries, on the contrary, the Market Value of equity as the sum of all minority interests, exceeds that of the company itself because a diversified portfolio is preferrable, investors value more the value of a single share or a minority block, than the whole company because it is characterized by a higher risk.[23]

A reliable and recognized source of control premiums is provided by Business Valuation Review (BVR) in their quarterly updated Mergerstat/BVR Control Premium Study, here premiums are expressed as a % of the unaffected minority price at various periods prior to

[23] Shannon P. Pratt. (2009). Business Valuation Discounts and Premiums. John Wiley & Sons, p.36

the announcement of an acquisition. Transactions are classified in: Horizontal Integration, Vertical Integration, Conglomerate and Financial. The data collected by BVR is compelling and goes back over 20 years, includes different financial ratios and the possibility to filter out certain industries or type of transactions. A relevant example would be the following: an analyst is valuing a deal of $200 million, so the analyst can first filter for size, then for industry and can also select the time frame deemed more relevant. The data shows that there is specific industry control premium i.e., certain industries usually provide higher or lower premiums. The ones providing higher premiums are those that are rapidly expanding and that can therefore give more opportunity to the buyer and a higher value creation potential thus, coherently with the theory explored before, will lead to a higher premium, an obvious example being technology-based industries. It is a widespread convention to exclude transactions classified as Financial in order to arrive at a less biased premium. The rationale behind this practice is that these transactions lack industry specific synergies and can thus give a less biased value. The value that we are looking for is, in fact, a control premium not an acquisition premium that, as for the definition before, describes the situation of a specific deal not a broader value to be taken for reference.

If data from 1993 to 2008 was examined only with the classical Levels of Value framework in mind, without considering the previously discussed debate, it would be surprising to observe that 15% of transactions had negative premiums, with a peak of 30% in the 3rd quarter of 1998. The existence of negative premiums and the high dispersion of data serve as a cautionary tale for the use of these resources acritically, these guidelines are helpful only if used carefully, considering the specific market conditions and the specific company otherwise they may prove misleading.

The formula used to arrive at a minority discount, starting with a control premium is the following:

$$Minority\ Discount = 1 - \left(\frac{1}{1+Control\ Premium}\right)$$ [24].

Closely related to minority discounts and controlling premiums is the concept of lack of marketability discount (LOMD), which is defined as: “an amount or percentage deducted from the value of an ownership interest to reflect the relative absence of marketability”[25].

[24] Shannon P. Pratt. (2009). Business Valuation Discounts and Premiums. John Wiley & Sons, p.17

[25] International Glossary of Business Valuation Terms, AICPA (2022)

It is important to notice that this factor applies not only to minority discounts but also to controlling premiums in privately held companies, "the concept of marketability refers to how quickly an asset can be converted to cash without the owner incurring substantial transaction costs or price concessions[26]".

In order to estimate the magnitude of this discount there are 2 main types of studies available in the literature: restricted stock studies and pre-IPO studies.

The first type of studies looks at the difference between the price of the same stock of a company, one is the price of the publicly traded stock, the other price is the price for the same stock that is unavailable on the public market for different reasons, many companies do in fact keep a certain percentage of their equity private, for example to sell it privately in order to raise capital without the registering expenses or because lock-up agreements were put in place so that the selling of the shares was restricted for a certain period of time due to worries about investors depressing the market. From the 1966 to 1990, the average discounts amounted at 35%[27]. The year 1990 is not arbitrary nor its due to lack of more recent data but it is chosen as a watershed between before Rule 144A and after. In 1990, the SEC issued the Rule 144A which still to this day regulates the market for unregistered securities in the USA, albeit it has been modified twice and this was reflected in the magnitude of the discount, providing further support to the concept of LOMD.

In the period between 1990 to 1997, due to the newly introduced possibility for institutional investors to trade among themselves unregistered securities without the need to file registration statements, the discount went from 35% to around 20%, reflecting the fact that a quicker selling process increased marketability. In 1997 the required holding period was reduced from 2 years to 1 year and the discounts further lowered to around 15%[28]. In 2008 a last amendment was introduced that reduced the holding period to 6 months, as a result, holders of restricted stocks could access a bigger market more quickly and naturally also their exposure in time to volatility was reduced, both contributed to the lowering of discounts, again as result of an increase in marketability. The pre-IPO studies compare the price of stocks in privately held companies to the price of these companies after IPO and

[26] M.Bajaj, D. J. Denis, S.P. Ferris, A. Sarin. (2001). Firm Value and Marketability Discounts

[27] Shannon P. Pratt. (2009). Business Valuation Discounts and Premiums. John Wiley & Sons, p.88

[28] Shannon P. Pratt. (2009). Business Valuation Discounts and Premiums. John Wiley & Sons, p.90

confirm the existence of strong LOMD discounts[29].

A less common and possibly more susceptible to bias way of estimating LOMD discounts, is the "Acquisition Approach". Under this approach, the acquisition prices for public companies are compared to those of private companies deemed comparable enough, one of the main studies was conducted by Koeplin, Sarin and Shapiro, looking at hundreds of acquisitions from 1984 and 1998 yielded the result that the average discount for private transactions is 20.4% when calculated using the EV/EBITDA ratio as a proxy for valuation.[30]

The issue of all these 3 methods is that the marketability discount is considered to account for the entire difference in value between public and private securities, to try and isolate the influence of the LOMD from other factors that might be at play, a paper proposes the use of multivariate regression using the OLS method. Cross sectional determinants are in this way isolated and the relevance of each factors is shown by the individual coefficients and informs us on the influence that each aspect has, the model takes the following form:

$$Discount = a + b_1 \times Fraction\ of\ Shares\ Issued + b_2 \times Zscore + b_3 \times STDEV\ Returns + b_4 \times Registration\ Indicator$$

The factors were selected based on theoretical considerations and data assessment. The "Registration Indicator" is a dummy variable that takes the value of 1 if the security is publicly registered and 0 if it is not. Isolating this factor in a sample of 88 private transactions going from 1990 to 1995, the researchers found a statistically significant value of 7.23%[31]. This value was then considered to be appropriately attributable to the issue of marketability only.

Accounting for other factors therefore does lower the value of discount for marketability, showing that considering the difference between a public and private security being due to solely lack of marketability is incorrect. The literature review thus shows that LOMD is significant and must be taken into account, the magnitude of it varies depending on the approach used to value it and, as usual, specific considerations have to be made for every situation.

The main entity level and shareholder level discounts have been reviewed, premiums and discounts do impact the valuation of a firm in every context, but the analysis around

[29] Shannon P. Pratt. (2009). Business Valuation Discounts and Premiums. John Wiley & Sons, p.91

[30] M.Bajaj, D. J. Denis, S.P. Ferris, A. Sarin. (2001). Firm Value and Marketability Discounts

[31] M.Bajaj, D. J. Denis, S.P. Ferris, A. Sarin. (2001). Firm Value and Marketability Discounts, p.26

premiums and discounts would not be comprehensive if it didn't include an analysis of Mergers and Acquisitions (M&A). The analysis in this context builds upon the factors previously considered but the final result is Acquisition Value, not what we might consider a more comprehensive Fair Value. This means that the analysis has to start from an objective value to arrive then at a subjective value (Acquisition Value) due to considerations on premiums and discounts, the analyst has also to add considerations on synergies that depend both on the characteristics of the buyer and of the seller. The overall premium paid in a M&A transaction is called Deal Premium, Purchase Premium or Acquisition Premium and refers to the difference between the offered share price and the price at which the target company was trading before deal announcement or before any rumors of the deal leaked out. The unaffected price is often a source of dispute and has to be established specifically for every situation, this is done usually by looking at volumes of trading as their change can signal the leaking out of rumors, not always the unaffected price is simply the price the day before the announcement. The first objective is therefore to understand how to measure this premium in order to better isolate the unaffected price to which it is correct comparing the offered price, the second objective is to understand the factors that contribute to the premium.

The use of fixed windows (e.g. 20 trading days prior to announcement) to determine the unaffected stock price can be misleading and produce biases, despite its widespread use in literature as a means of standardizing and comparing M&A transactions. The issue is not only with the use of a standard window for different transactions but, even when this is deemed as a viable approximation to analyze a large quantity of data, the fact that often, guidelines used today are based on samples gathered in the 1980s, the most cited work being the one presented by Schwert (1996). In the 1980s the negotiation time was much shorter than the average one occurring now, therefore it is necessary to rely on more recent evidence as these sample are no longer representative of the average time that deal negotiation takes now. By looking at samples from the 1990s to the 2010s, the evidence shows that the "runup" starts, on average, 105 trading days prior to public announcement and that using a standard fixed window based on earlier research underestimates premiums

by up to 8%[32].

There are additional premiums observed in M&A transactions (deal premiums) and they are measured in comparison to the unaffected share price, the recommended window according to the latest research available is of 105 trading days, but why do these premiums occur? The answer is given primarily by the occurrence of synergies and by other considerations such as the career stage of the CEO, particularly whether the CEO is near retirement or not, whether the merger was initiated by the target or by the acquirer, by the state of the economy, by laws and regulation affecting M&As in specific countries and by the specific industry, particularly its competitiveness. Sometimes the premium is affected by psychological factors such as the overconfidence of the CEO of the acquiring company, something called the "hubris factor" borrowing a term from Ancient Greece, a fault punished back then by the Gods, today by shareholders or perhaps inflicted upon them due to the wealth destroying nature of such operations.

A valuable resource for understanding M&A premiums can be found in the paper by J.H. Mulherin, Jeffry M. Netter, and Annette B. Poulsen (2017), which presents a comprehensive literature review of M&A transactions, investigating the nature of the premiums, the wealth gains, if any, and the different factors influencing M&A from the 1980s to 2017, reviewing over 120 articles focused on empirical M&A data. The evidence concerning the existence of wealth gains for the target has been supported already in some of the first available research on M&A, the causes were, and sometimes still are, not clear, the existence of gains for the bidder though were not observed.

The idea that an overconfident CEO merely causes a the transfer of wealth from one company to another, known as the hubris hypothesis, was widely accepted until Bradlety et al.'s 1988 study, which showed a statistically significant increase in returns for both the bidder and the target. In the 1990s the rise in M&A transactions seen in the 1980s slowed down due to macroeconomic factors, the research shows that gains for the target companies continued, the bidders were breaking even and the combined return were positive. This evidence was confirmed in the work of Weston et al. (2004), reviewing transactions from 1980s to 1990s, the research finds that: target shareholders experience a

[32] Gregory W. Eaton, Tingting Liu, Micah S. Officer. (2019). Rethinking Measures of Mergers & Acquisitions Deal Premiums, pp. 1,2

clear gain, bidders break even, lose or gain but not substantially, the combined gains are positive. The literature identifies two main sources of gains: operating synergies and restructuring. In the case of similar firms an important role is played by the increase in product differentiation. Despite the wealth gains coming primarily from synergies, Ismail (2011) finds that there is not a strict relation between premiums and management's expected synergies. Other research does find a correlation between management's predicted synergies and a higher or lower premium but the premium is related also to other factors, such as: a) takeover strategies b) the existence of parachutes for the CEOs, this makes a CEO, on average, less likely to conflate their personal interests with a takeover thus resulting in lower premiums c) the presence of investment bankers on the board of the bidder, this aspect leads to a lower premiums thus benefiting the bidder d) transactions involving financial conglomerates, also called "dual holders" since they have significant equity and bond holdings and they can accept a lower equity premium if there are offsetting gains for their bond holdings quota.

With all the caveats explored so far, knowing that it's misleading to simply apply a premium based on average ones found in past transactions, still it is worth to mention that, as shown by Dimopoulos and Sacchetto (2014), from 1988 to 2006, 86% of takeovers averaged premiums of 50%[33].

3. Merging the main factors: the Popularity Framework

3.1 Classical and Behavioral Finance, overview and differences

After having reviewed the meaning of premiums and discounts, which factors drive them, how to account for them and what has been their quantifiable historical impact on Firm Valuation, it is now the objective of this dissertation to bring them together, by using a very recent framework called Popularity Asset Pricing Model (PAPM).

The PAPM has been proposed by the CFA Institute in 2018 and it is their publication "Popularity, a bridge between Classical and Behavioral Finance" that will guide us, as the main source, into the description and finally the implementation of the model.

[33] Mulherin, J. H., Netter, J. M., & Poulsen, A. B. (2017). The Evidence on Mergers and Acquisitions: A Historical and Modern Report. (pp. 235–290).

The PAPM proposes a fully mathematized asset pricing model that aims at reconciliating the differences between the classical and the behavioral approach under a single framework. Briefly, an overview of what classical and behavioral mean in this context.

The necessity to distinguish between the two approaches, points to the fact that, to this day, there's not a unified approach capable of putting together in a single coherent framework both the classical and the behavioral view, this differentiation is not merely necessary when describing the development of asset pricing theories. The need to specify the school of thought that one is coming from, when discussing valuations, may slowly be overcome by a paradigm that is capable of taking into account the wisdom of Classical and Behavioral Finance, there are increasingly more attempts at moving beyond this dichotomy, the PAPM represents the most recent one.

According to the perspective of Classical Finance, investors are rational agents who make decisions based on two principles: internal consistency and the pursuit of self-interest. The first refers to the fact that investors reliably take rational decisions across time and update their expectations hence their strategy according to the most recent information available, in accordance with Bayes' rule that prior probabilities combined with new information is what determines future probabilities. The latter, pursue of self-interest, implies that the marginal utility is always positive but decreasing, meaning that the investor is insatiable (more profit is always more profit, hence the utility curve stays positive) but also risk adverse.[34]

In the 1970s, the simultaneous development of psychological research applied to economics and the increasing research unveiling new exceptions to the predictions of the classical model, gave rise to the desire to explain these incongruences, hence many economists turned to psychology for answers. The approach here is almost the opposite of the classical one, in the classical approach aggregate market preferences reveal individual actions, in Behavioral Finance, the movement is the opposite, it's inductive. This means that behaviors not congruent with the classical view are observed at an individual level and then they are generalized and their impact is observed, this is called "aggregation issue", understanding these psychological factors at the individual level is important in order to understand how they may impact market level trends, even with an ever more precise understanding of

[34] Van der Sar, N. L. (2004). Behavioral finance: How matters stand. Journal of Economic Psychology, 25(3), 425–444. pp.425-427.

individual behavior, generalizing observation made at the individual level behavior to the market level, poses a challenge.

Barber and Odean (2000) investigated the common investment performance of individual investors and found that investors tend to make decisions that deviate from those that would be coherent with the rational expectations theory of classical finance : "Investors as a group tend to act as if they are overconfident in their ability to predict security returns and they trade too much" (p. 774).[35] A rather famous exception documented in the literature is the one pertaining the weather, with investors being more pessimistic on cloudy days and more confident on sunny days, leading to deviations that can be measured at the market level.[36]

Systematical individual exceptions to the expected behaviors lead to an effect in the market and some of the first models proposed were the "behavioral portfolio theory" (BPT) and the "safety-first portfolio model". What comes out of these initial attempts is that human decision lacks the internal consistency suggested by the classical view, and that the heuristics used by the common investors fail to take into account all information, which are not the same for every investor, and is subjected to many different biases such as the disposition effect, representativeness, mental accounting, myopic forecasting and asymmetry between gains and losses.

The low predictive power of the first behavioral models, did not discredit the importance of their findings, rather it motivated scholars to refine more and more theories and models. Anomalies are neither sporadic nor isolated, equilibrium premia are determined by fundamental risk and by an additional risk determined by irrational decision making, finally, arbitrageurs do not always immediately restore equilibrium. These are three of the most significant conclusions obtained by the behavioral approach from 1970 to 2000[37].

Following this overview of the theories behind the two approaches, classical and behavioral, that PAPM aims at merging, it is now important to see the research preceding PAPM, that is, the previous attempts at moving beyond CAPM because these are the building blocks that ultimately led to this newer framework.

[35] Barber, B. M., & Odean, T. (2000). Trading is hazardous to your wealth: The common stock investment performance of individual investors. The Journal of Finance, 55(2), 773-806

[36] Hirshleifer, D., & Shumway, T. (2003). Good day sunshine: Stock returns and the weather. The Journal of Finance, 58(3), 1009-1032.

[37] Van der Sar, N. L. (2004). Behavioral finance: How matters stand. Journal of Economic Psychology, 25(3), 425–444. pp.428-444.

The most widespread and well known framework stemming from Classical Finance used to capture expected returns is the Capital Asset Pricing Model (CAPM), firstly proposed by William Sharpe and John Lintner between 1964 and 1965, their work builds on the portfolio choice model developed by H. Markowitz in 1959[38] which is built on Classical Finance assumptions i.e., on the notion that investors are rational and risk-averse, interested in maximizing the returns on their investments whilst minimizing risks. Markowitz (1959) developed a formula to find the asset weights for mean-variance-efficient portfolios. The CAPM adds two assumptions: the first one is called "complete agreement" which says that investors agree on the distribution of asset returns from t-1 to t, the second one is the assumption of "borrowing and lending at a risk-free rate". Given that investors are rational and risk averse, have the same predictions on returns distributions because they have the same information on companies and on the market (1st assumption) and they can all borrow at the same risk-free rate (2nd assumption), the efficient portfolio will be the aggregate portfolio of all investors called "market portfolio".

The Sharpe-Lintner CAPM equation is:

$E(R_i) = R_f + \left[E(R_M) - R_f\right)\beta_{iM},\ i = 1,\ldots,N.$

The expected return of an asset i is equal to the risk-free rate, R_f, plus the equity risk premium, which is found by multiplying $E(R_M) - R_f$ with β_{iM} that represents the Beta i.e. the covariance of the returns of the risky asset with the returns of the overall market:

$\beta_{iM} = \frac{cov(R_i, R_M)}{\sigma^2(R_M)}$.

The problem is twofold: theoretically, as per the reasoning reported above, the view of the investor as a rational agent is now, if not disputed, at least in need of being integrated with behavioral considerations, secondly, empirically, the observed relation between β and returns is, more often than not, not the one predicted by the CAPM[39].

It is implicit in the data reported in the previous chapter on premiums and discounts that, the considerations on firm valuation, can't be exhaustively summarized with an estimate of the degree of risk i.e. volatility of returns of an asset relative to those of the market. Furthermore, what is meant by "market" and the possible proxies used to account for it,

[38] Fama, E. F., & French, K. R. (2004). The Capital Asset Pricing Model: Theory and Evidence. Journal of Economic Perspectives, 18(3), 25–46.

[39] Fama, E. F., & French, K. R. (2004). The Capital Asset Pricing Model: Theory and Evidence. Journal of Economic Perspectives, 18(3), 25–46.

would open a discussion that would require an entire dissertation in itself.

The most noteworthy and recognized approach that moves beyond the CAPM and does so with a strong consistency over long periods of time, is the one proposed by Fama and French in 1992 as the 3-factor model and refined and updated throughout the years. In 1997 with the contribution of Mark Carhart a 4-factor model is presented and finally, in 2014 Fama and French further revisited their original model presented in 1992 and updated it to a 5-factor model. This is a multifactor approach which decomposes risk in different factors, the sensitivity (β) of a stock's returns to each factor informs on the exposure of that company to a certain factor, the first of these factors is in fact the market portfolio, which is the only factor in the CAPM, the other 4 factors are therefore building on the initial framework presented by Sharpe and Lintner.

The equation of the 5-factor model is:

$$R_{it} - R_{Ft} = a_i + b_i(R_{Mt} - R_{Ft}) + s_i(SMB_t) + h_i(HML_t) + r_i(RMW_t) + c_i(CMA_t) + e_{it}$$

It is an extension of the Fama & French initial three-factor model, which was composed by the market beta, the size (SMB) and the value factors(HML) and it now includes two additional factors: profitability represented by RMW and investment, CMA.

The first factor, market beta, is the factor that has been traditionally used to measure a stock's systematic risk and is the factor presented above with the CAPM. It measures the sensitivity of a stock's returns to the overall market's returns.

The second factor, size, measures the risk associated with small-cap stocks. Small-cap stocks have been found to have higher returns than large-cap stocks, on average. According to Fama and French, "The size factor is the excess return on a portfolio of small stocks minus the excess return on a portfolio of big stocks". The formula for the Size Factor is: SMB = R(Small) - R(Big), where SMB is the size factor, R(Small) is the return on a portfolio of small stocks, and R(Big) is the return on a portfolio of big stocks. The third factor, Value, measures the risk associated with value stocks. The fourth factor, Profitability, measures the risk associated with stocks of companies that have high profits relative to their assets, it is calculated using the market capitalization weighted average of the individual stock returns for a portfolio of profitable firms minus those for a portfolio of unprofitable firms, where for profitability it is meant operating profitability and RMW stands for robust minus weak profitability.

The formula for the profitability factor is: RMW = R(Profitable) - R(Unprofitable), where RMW is the profitability factor, R(Profitable) is the return on a portfolio of profitable firms and R(Unprofitable) is the return on a portfolio of unprofitable firms. The fifth factor, Investment, measures the risk associated with stocks of companies that have high investments relative to their assets calculated using the market capitalization weighted average of the individual stock returns for a portfolio of firms with high investment and a portfolio of firms with low relative investments.

The formula for the Investment Factor is: CMA = R(High Inv) - R(Low Inv) , where CMA is the Investment Factor, R(High Inv) is the return on a portfolio of firms with high investment and R(Low Inv) is the return on a portfolio of firms with low investment[40].

Since our aim is to capture the factors that drive premiums and discounts, it is useful to notice that in one of the most recent study [41]that compares the Fama-French Five-Factor Model to the CAPM, the authors review the empirical evidence on the Fama-French Five-Factor Model and the CAPM using a sample of US stocks over the period 1963-2014.

The authors find that the Fama-French Five-Factor Model significantly improves upon the predictions of the CAPM, "the Fama-French Five-Factor Model is a significant improvement over the CAPM in explaining the cross-section of average returns." Specifically, the R-squared of the Fama-French Five-Factor Model is 0.56, while the R-squared of the CAPM is 0.32. This means that the Fama-French Five-Factor Model explains 56% of the variation in average returns, while the CAPM explains 32% of the variation. The R-squared of the Fama-French Five-Factor Model is significantly higher than that of the CAPM, meaning that the Five-Factor Model explains a much larger portion of the cross-sectional variation in average returns.

The models proposed by Fama and French are the most widely used alternatives to the CAPM, depending on the context and the available tools, one of their 3 models can be used, sometimes alongside the CAPM, to better understand where the differences in expected returns are coming from. The link between previous models and the PAPM, is not only represented by models presented by Fama & French, but, the model that more closely shows this gradual transition from a more strictly Classical Finance approach to an approach that takes into account behavioral evidence, is the model offered by the "New Equilibrium

[40] Fama, Eugene F. and French, Kenneth R. (2014). A Five-Factor Asset Pricing Model

[41] Brennan, M. J., & Xia, Y. (2016). The Fama-French Five-Factor Model: A Critical Review. Journal of Economic Perspectives

Theory" (NET) which was first presented as an article in the Financial Analysts Journal published in 1984 by Ibbotson, Diermeier and Siegel, this is the main theoretical link because it includes non-risk characteristics even though the NET remains anchored to a classical approach when it comes to describing investors, it does not include behavioral evidence on the way that an investor act, but nonetheless it evidently goes beyond a risk-only approach in the explanation of investors choices. The non-risk characteristics included relate mainly to taxation and to considerations that go under the umbrella of "market frictions" i.e., marketability and information costs. These issues are explored within a supply and demand framework wherein supply is relatively fixed and demand is the aggregate demand across investors. The key aspect of interest for the purpose of this dissertation is that NET includes characteristics not related to risk as intended by the CAPM and that it expects that investors, that here remain rational, will preferer these characteristics. It is important to note that Ibbotson, Diermeier and Siegel are building their framework with a rational investor in mind, a rational investor will prefer more marketable companies to less marketable ones etc., thus NET implements Classical Finance when explaining decision making. [42]The result of balancing supply and demand in financial markets is that the aggregate preferences of investors will make certain companies more popular than others, this means that the companies that are overlooked because they lack these characteristics i.e. companies that are unpopular, will end up having return premiums, a way of reasoning that will soon become familiar when presenting the evidence around the PAPM even though it may sound counter intuitive at first. Within an equilibrium of demand and supply, prices are the result of the aggregation of investor's preferences, the nature of these preferences, their predictability and their consistency over time makes them important factors to be included when valuing a company because, as noted before, these properties, namely, the predictability and consistency overtime of investors preferences, render the observed differences in expected returns from the ones predicted by the CAPM, not merely a result of mispricing or market failure, but premiums and discounts that can be taken into account during valuations.

3.1 The Popularity Asset Pricing Model (PAPM): theory

42 Ibbotson, R. G., Idzorek, T., Kaplan, P. D., & Xiong, J. X. (2018). Popularity: A Bridge between Classical and Behavioral Finance, pp: 5-6, 42-50.

There are already many different well established alternatives to the CAPM and the most recognized and widespread have been described previously, therefore, a new model, in order to be relevant, has to be not only consistent in capturing already well known premiums, but it has to offer additional value, that is, it has to be able to capture premiums that have not been considered before as priced characteristics and that have thus not been captured by other models. The capacity of the PAPM to be coherent with Classical Finance and to capture aggregate irrational behaviors described by Behavioral Finance, is what makes this framework a bridge between the two approaches, a model that improves the on the explanatory potential of previously available models. The rationale behind this model and its usefulness in valuation is the given by the observation that an asset characteristic that cannot be easily replaced or securitized, will result in a premium or in a discount, this will be reflected in the demand of investors, making this characteristic popular or unpopular. Popularity can be thought as a lens through which demand can be interpreted and calculated, investors demand expected returns and popular characteristics, companies thus supply earnings and popular characteristics, in order to capture that demand.

The only characteristic priced in the CAPM is risk, even though this is the only one considered in the model, it does not seem to be captured effectively. Empirical data does not robustly support the positive relationship between a company returns and its risk (beta), this is known as the “low-beta anomaly”, the explanation given by the PAPM is that the possibility of additional return promised by the higher beta, make that characteristic a popular one, thus, due to the law of supply and demand, the price of the companies offering that characteristic will go up and this will ultimately reduce the expected returns for this companies. This is an example that gives an introduction into the framework proposed by the PAPM. Another notable exception to the risk-return paradigm is the one captured by the Value Factor. Premiums are sometimes based on risk characteristics and can be explained within that model, but these are nonetheless always associated with investor preferences thus with popularity.

Unattractive i.e. unpopular companies will have lower valuations, thus a higher cost of capital compared to popular companies, this aspect impacts the expected discounted value of their cash flows. Disliked characteristics will result in long-term higher expected returns and this can be predicted over time using this model. Going beyond the risk-return paradigm and maintaining the scientific rigor that many behavioral models lack, is the

achievement of the PAPM.

The dimensions of popularity that classify as premiums due to their consistency over time are:

size, value, liquidity, severe downside risk, low volatility, low beta, momentum, ESG, competitive advantage, brand and reputation. Clearly, once we start thinking of demand in terms of popularity, more characteristics offered by companies that can classify as popular, come to mind or can be extracted as a result of data mining, the hurdle rate for accepting a characteristic as an actual premium has to be set high and cross correlations need to be considered carefully, it would be redundant to include characteristics that are not only less consistent over time but that also appear to be strongly correlated with others, especially within certain time periods. When a long-term view is taken into account, it is important to capture premiums that are consistent over time and to do so in a framework that also explains why premiums exists.

Popularity-based explanations of premiums:

Factors	*PAPM explanation*
Size	Smaller companies receive less media coverage than larger companies
Value	Value companies tend to operate in less publicly exposed industries
Momentum	News causes an unsustainable mispricing cycle
Low volatily/Beta	Investors seek high beta in an attempt to outperform the market.
ESG	Companies with high ESG ratings tend to attract more capital.
Competitive advantage, brand, reputation	Herd behavior and emotional decision making make investors attracted to companies beyond what would be considered reasonable in an intrinsic company valuation

Each characteristic is subjected to the law of demand and supply, high demand relative to supply leads to a higher price due to the aggregation of investor preferences. It is important to notice that a single company can change and sometimes also quickly in its popularity but this is not the case for the characteristics themselves overtime, when the methodology to capture these factors will be presented, the companies offering certain characteristics will be allowed to change over-time, it is the aggregate approach towards the characteristics that will instead kept stable over time by the model[43]. In the words of Eugene Fama and Thaler (2016), when discussing value in an interview: “Maybe people just don’t like that type

[43] Ibbotson, R. G., Idzorek, T., Kaplan, P. D., & Xiong, J. X. (2018). Popularity: A Bridge between Classical and Behavioral Finance, chapter 2.

of company. That to me has more appeal than a mispricing story, because mispricing, at least in the standard economic framework, should eventually correct itself, whereas taste can go on forever"[44]. Markets in the PAPM are intended as *beyond efficient*, meaning that prices reflect both the relevant information and the "irrelevant information", as it would be called in the efficient market view. Markets that are beyond efficient thus have prices that reflect also the irrational behavior of investors, but this is not a characteristic that renders the markets inefficient. This change of view is important and coherent with a framework that is able to describe and capture several of the irrational behaviors of investors. The theory driving the PAPM and the way in which popularity explains premiums, equip an analyst with an understanding of the tendency of a characteristic to cause a premium or a discount in a company over time, thus providing a useful heuristic to better take into account premiums and discounts.

3.2 The Popularity Asset Pricing Model (PAPM): a tool for valuation

The PAPM formula is introduced in this section alongside a practical explanation of how the formula can be implemented in the valuation of a company.

The following table presents the PAPM formula together with other more commonly used models used to explain market returns, this allows us to understand the scope of the PAPM and how it compares with the more familiar models:

CAPM:	$E[R_i] = \mathbf{R_f} + \boldsymbol{\beta} * \mathbf{E}[\mathbf{R_{MKT}} - \mathbf{R_f}]$
F&F:	$E[R_i] = \mathbf{R_f} + \boldsymbol{\beta} * \mathbf{E}[\mathbf{R_{MKT}} - \mathbf{R_f}] + \boldsymbol{\beta}_{i2}\mathbf{SMB} + \boldsymbol{\beta}_{i3}\mathbf{HML}$
NET:	$E[R_i] = \mathbf{R_f} + \boldsymbol{\beta} * \mathbf{E}[\mathbf{R_{MKT}} - \mathbf{R_f}] + \boldsymbol{\beta}_{i2}\mathbf{SMB} + \boldsymbol{\beta}_{i3}\mathbf{HML} + \boldsymbol{\beta}_{i4}LIQ + B_{i5}RISKA$
PAPM:	$E[R_i] = \mathbf{R_f} + \boldsymbol{\beta} * \mathbf{E}[\mathbf{R_{MKT}} - \mathbf{R_f}] + \boldsymbol{\beta}_{i2}\mathbf{SMB} + \boldsymbol{\beta}_{i3}\mathbf{HML} + \boldsymbol{\beta}_{i4}LIQ + B_{i5}RISKA + \boldsymbol{\beta}_{i6}\mathbf{ESG} + \boldsymbol{\beta}_{i7}\mathbf{CABR+}\ \boldsymbol{\beta}_{i8}\mathbf{MOM}$

R_f stands for risk free rate
RISKA stands for risk anomalies: low beta anomaly and severe downside risk
CABR stands for competitive advantage, brand and reputation
MOM stands for momentum[45]

The PAPM is thus an expansion of the CAPM and the CABR (Competitive advantage, Brand and Reputation) factor is that which characterizes this framework the most and that offers the highest contribution within the focus of this dissertation.

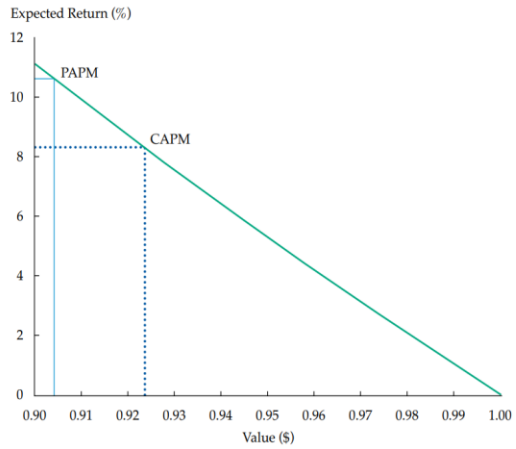
[44] (Fama and Thaler 2016).

[45] Ibbotson, R. G., Idzorek, T., Kaplan, P. D., & Xiong, J. X. (2018). Popularity: A Bridge between Classical and Behavioral Finance, p. 39

It has been discussed so far how PAPM uses the notion of Popularity to combine Classical and Behavioral Finance and its methodology and empirical evidence will be shortly presented, but what exactly makes PAPM a tool for valuation and, more specifically, a tool for valuations that require a particular attention to premiums and discounts which are the focus of this dissertation? The PAPM formula for expected returns i.e., $E[R_i] = \mathrm{R_f} + \beta * \mathrm{E[R_{MKT} - R_f]} + \beta_{i2}\mathrm{SMB} + \beta_{i3}\mathrm{HML} + \beta_{i4}LIQ + B_{i5}RISKA + \beta_{i6}\mathrm{ESG} + \beta_{i7}\mathrm{CABR} + \beta_{i8}\mathrm{MOM}$ becomes a tool for valuation, especially of companies with relevant premiums and discounts, when included in a DCF model. Briefly, let's recall the DCF formula presented before: $V = \frac{FCF_1}{(1+WACC)} + \frac{FCF_2}{(1+WACC)^2} + \frac{FCF_3}{(1+WACC)^3} + \cdots + \frac{FCF_n+VR}{(1+WACC)^n}$, WACC stands for weighted average cost of capital : $K_e * W_e + K_d * W_d * (1-t)$, where $K_d * W_d * (1-t)$ represents the cost of debt net of taxes (t) multiplied by its weight in the capital structure of the company under valuation, K_e represents the cost of equity and is usually estimated by applying the CAPM formula, it is considered equal to the expected return as calculated by the CAPM, the rationale behind it is that investors invest their capital expecting a certain return that the company is required to satisfy in order to continue to access this form of financing. The required expected return is thus one of the two components of the discount factor, together with the cost of debt i.e., the WACC in the denominator of DCF model, the weighted average cost of capital is then raised to a number that represents the periods of time from one cash flow (FCF) to the next, to account for time-value of money.

It is argued in this dissertation that, instead of using the CAPM to capture the cost of equity, it is valuable to use the PAPM, especially when valuing companies that have a high sensitivity to premiums and discounts and for which the use of the CAPM would produce an oversimplification. The use of CAPM is certainly easier, due to its widespread use and the availability of the tools to compute it also makes it less work-intensive to use in a daily professional setting, thus it is argued here that the use of PAPM, since it requires additional efforts to gather data and requires additional considerations, is not meant to substitute the CAPM in every valuation, but it is considered as particularly useful, and possibly time-saving when dealing with companies that are expected to show a higher degree of sensitivity than normal to these factors. It's efficiency is granted by the fact that it allows an analyst to still maintain the valuation within a DCF model yet, thanks to the use of a different discount factor, it allows the analyst to include considerations on premiums and discounts without

further adjustments as it is instead needed in methods that start from a base value that does not account for premiums and discounts and considers these factors only with later adjustments. Going back to the WACC, the cost of equity is dictated by the expectations of investors and they cannot be considered rational, according to the evidence presented so far and the direction in which Finance as a field seems to be moving, aided by an ever-growing data availability on investors behavior that shows their (irrational) preferences. Finally, a needed clarification on how the PAPM is used in valuations and the meaning of the words premiums and discounts. Companies that are unpopular, as will be clearly shown in the following paragraph, have higher expected returns in the long-run and this is the (un)popularity premium. This means that, as shown in the literature review, intangible assets do provide an increase in the valuation thus a premium and this is reflected, in a DCF model, with a lower discount factor, which implies a lower expected return over the long run. As exemplified by the intuitive graph that follows:



value and expected return (discount factor) are inversely related and this is more so in the case of a company that is particularly sensitive to the characteristics captured in the PAPM, either because it happens to be on the higher end of the spectrum of popularity or on the lower one.

This clarification is needed because the use of the terms premiums and discounts made in "Popularity: a bridge between Classical and Behavioral Finance" (2018) can create confusion if one does not consider that, within a DCF model, a higher expected return, a premium, translates as a lower valuation today but a better expected return for the investor tomorrow, whilst a "discount" actually means a higher valuation today, when the valuation

is taking place, because the discount is in the cost of equity for the company and that means that its financial structure will tend to impact less negatively the value of the firm[46].

3.3 PAPM: the methodology

The methodology of the PAPM, more specifically, the methodology behind the calculation of the CABR (competitive advantage, brand value, reputation) factor is presented in the following section, along with the evidence of the model's consistency in capturing premiums over time.

The way in which the CABR is constructed, relies on the use of widely recognized and consistent methodologies that capture a company's competitive advantage, brand value, and reputation. The specific tools used by the authors of the paper for each of the three characteristic composing CABR are now presented, with the acknowledgment that it is possible to use other tools as long as they show the same consistency and attention to detail as the ones implemented by the proponents of the framework.

Morningstar's Economic Moat, is used to individuate whether or not a company has a sustainable competitive advantage and to which extent. The Economic Moat is calculated based on five criteria: the network effect, intangible assets, cost advantage, switching costs, and efficient scale. The network effect refers to the process that makes the value of a product or service increase as the number of its users rises, this is common especially in digital and social media companies. The growing popularity of the product begins attracting more users, the value of the product which is strictly related to the amount of people using it goes up, this creates a positive feedback loop[47]. Intangible assets like trademarks, patents, and copyrights make up the second criteria, intangibles are difficult to recreate or replace in a short amount of time, this characteristic allows a company to build a sustainable competitive advantage over time. The third criteria is cost advantage which is the capacity of a company to produce goods or services at a lower cost compared to its competitors, this allows the company to offer its products at a better prices. The fourth criteria is that related to switching costs, which are the costs associated with changing from on product to another, when this aspect is present in any industry in which customers have made significant investments, whether in terms of time or money, in a particular product. The

[46] Ibbotson, R. G., Idzorek, T., Kaplan, P. D., &rsio Xiong, J. X. (2018). Popularity: A Bridge between Classical and Behavioral Finance, pp. 1-12; 70.

[47] Shapiro, C., & Varian, H. R. (1999). Information Rules: A Strategic Guide to the Network Economy. Harvard Business Review Press

higher the investment in the previous product the higher the switching costs, this aspect concerns primary new competitors in their ability to gain market share. Companies that offer a different platform for other companies to conduct their business in, face resistance in terms of the costs that a company implementing their new platform has to undergo in terms of retraining employees and exporting data, to name a few. This costs are not only strictly related to financial expenditures but are also psychological, for example, in service industries, over time there is a relationship between the service providers and the customers, and this creates a "lock-in effect" that makes it harder for customers to switch to another service.[48] Every criteria considered by Morningstar's Economic Moat, focuses on the sustainability of the competitive advantage, this is what will then contribute to differentiate companies over time, as discussed above, the goal is to capture long term differences that then can be translated into premiums or discounts, instead of biases or price distortions. According to the overall score achieved by each company in all of the six dimensions, the company is then classified has having a wide moat, narrow moat or no moat.

The second tool implemented in the construction of the CABR factor, is provided by Interbrand, a consulting company that has been providing brand valuation rankings for decades. The ranking used to measure a company's brand value is Interbrand's "Best Global Brands Report,", in order for a company to be included in the ranking, 30% of its revenue must come from outside its home region and the company must be present in three or more continents. The Brand Value is calculated by multiplying the expected economic profit by the *role of brand measure.*

Previous attempts at estimating the impact of brand value over time seem to have found conclusions that would contradict the PAPM framework, in particular, Fehle et al. (2008) found a positive relation between brand value and returns, but the research only included 111 stocks and was based on a short period of time. Brand Value is here interpreted as one of the proxy for popularity, clearly, a company that has a higher brand value has a higher valuation when compared to a company with a lower brand value in a given year, but this overtime makes the company excessively popular and thus driving its demand up to a degree that goes beyond the possible economic benefits that it can return to investors,

[48] Rochet, J.-C., & Tirole, J. (2003). Platform Competition in Two-Sided Markets. Journal of the European Economic Association, 1(4), 990-1029.

ultimately leading to a lower expected return over time, due to the mechanism explained before and as confirmed by the long term study presented in the paper.

Reputation is the third aspect included in the CABR, to capture it, the Nielsens' Harris Pool Reputation Quotient (RQ) has been used. This quotient is built on the valuation given by more than 25,000 individuals, in an interview, on 20 attributes of a company subdivided into six dimensions : social responsibility, emotional appeal, products and services, , vision and leadership, workplace environment, and financial performance.[49] The way in which the reputation quotient is derived, directly reflects the popularity of a company amongst investors, the alternative tool could be the "Fortune's most admired companies" ranking, but since this is the result of the opinions of senior executives, its ranking might not indicate exactly the reputation and thus the popularity of a company amongst a larger group of people which is clearly the protagonist of a framework that describes collective behavior.

Not all the three tools mentioned distinguish between companies and parent companies, for the purpose of the construction of the portfolio factor CABR, the position in the ranking considered is that of the the parent company, an practical example from Interbrand's brand value rankings is the placement of Volkswagen Group. The final position considered in the ranking and thus whether a company will be part of popular or unpopular companies, is given by result of the aggregation of the brand value of each of the brands owned by the group that figure amongst the top 100 most valuable brands, in the case of Volkswagen Group then, the brand value of Porsche, Audi and Volkswagen is summed up and the ranking considered is that of the Volkswagen Group[50].This approach will be followed in the experimental practical implementation presented in the fourth chapter of the dissertation as well.

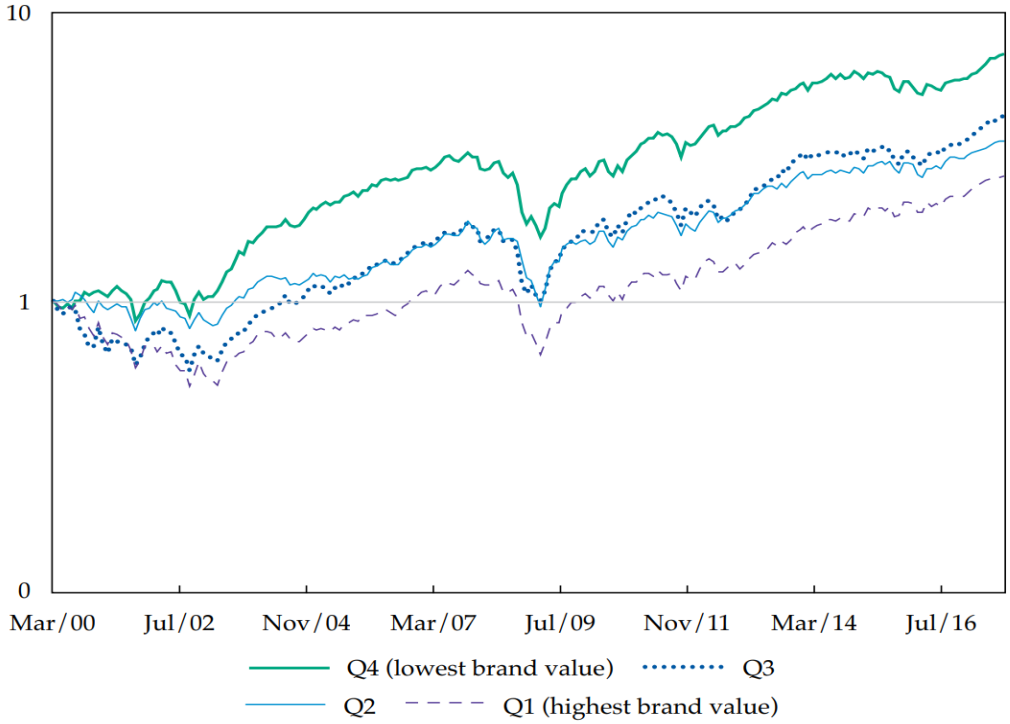
Before moving on to the construction of the factor, it is relevant to present the evidence that supports it. The research conducted by the authors of "Popularity, a bridge between Classical and Behavioral Finance" (2018) will be the main source for the evidence of the three components, namely, brand value, sustainable competitive advantage and reputation that make up the CABR factor, due to the PAPM being very recent, the literature on its evidence is still limited but the research conducted by R.G. Ibbotston, T. M. Idzorek, P. D. Kaplan and X. Xiong takes into account almost 2 decades of data and hundreds of company

[49] The Axios Harris Poll 100 - Harris Poll (theharrispoll.com)

[50] Ibbotson, R. G., Idzorek, T., Kaplan, P. D., & Xiong, J. X. (2018). Popularity: A Bridge between Classical and Behavioral Finance, pp. 84-87

and shows high consistency over time. One limiting factor in the collection of the evidence around popularity is caused by the limited number of rankings for brand value, reputation and competitive sustainable advantage, though these implemented in their research are widely recognized and used world-wide. The overview of the history of behavioral finance conducted above, explains why there is still a limited number of tools at our disposal when building the CABR, the company characteristics required for the analysis have been recognized only very recently, and still to this day, due to the complexity involved in their analysis, there is not a large amount of data around them. The period taken into consideration by the authors goes from 2000, the first year in which Interbrand's Brand Value ranking was published, to the year 2017, the last year available before the publication of the research. Starting from Brand Value, the rankings for each year are collected, the value of brands referring to a single parent company are aggregated into the parent company and the ranking are then divided into quartiles. The fourth quartile refers to the companies with the lowest brand value, whilst the first one contains companies with the highest brand value. Each quartile is a portfolio with the respective companies, the analysis is conducted with both equally weighted portfolios and market cap weighted portfolios, the first methodology shows higher returns due to the rebalancing premium but both show the same interesting result: the portfolios composed by companies in the last quartile for brand valuation outperform those composed by companies in the highest quartile for brand valuation, this lower-BV quartiles monotonically report higher returns, with Q4 > Q3 > Q2 > Q1. The arithmetic mean returns for the fourth quartile equals 13.53% whilst those in the first quartile are of 7.39%, all the other measures used to evaluate the performance of each portfolio, such as Share Ratio and Jensen's alpha, all demonstrate that Q4 has better values than Q1, net of standard deviation, which does not appear to be related with brand values, that is to say that these higher returns are not explained by a difference in the risk of the expected returns. This last aspect is particularly important to notice for the purpose of this dissertation, the lack of an increase in risk with the increase of returns, demonstrates that the PAPM, and the CABR factor specifically, is capturing a premium that would not be otherwise explained by the CAPM under a one-dimensional risk-return approach.

Growth of $1 for Equally Weighted Portfolios based on Interbrand's BV Rankings (2000-2017)

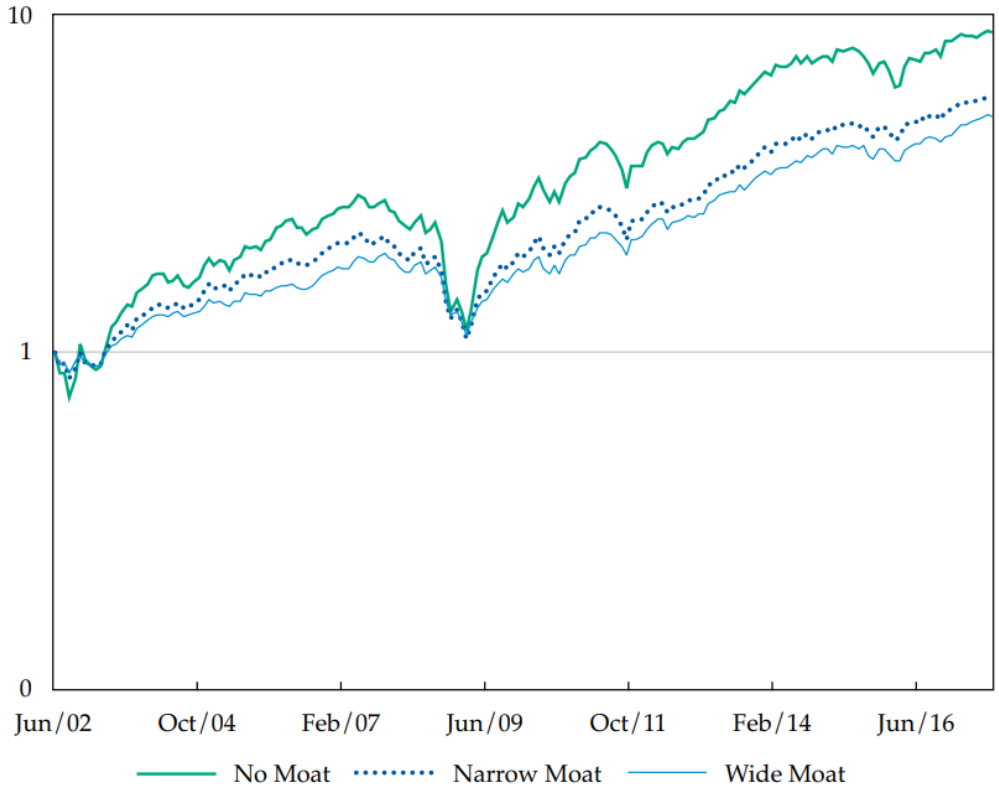


This graph shows the growth of four equally weighted portfolios representing the four quartiles for brand value, from 2000 to 2017.

The evidence collected for the second factor in the CABR, sustainable competitive advantage, goes from 2002 to 2017, as Morningstar started releasing its ranking in 2002, for each year, the companies included in the study are on average 1039. The prediction is that wide-moat companies are more popular amongst investors because their sustainable competitive advantage is perceived as a sign of long-term performance and reduced volatility. On the other hand, narrow or no moat represent companies that will be

overlooked by investors i.e. companies that will not be popular, instead of leading to lower long-term returns, will actually achieve the opposite results in the long-term. This may initially appear counterintuitive, but becomes clearer as more evidence is presented.

Growth of $1 for Equally Weighted Portfolios based on Morningstar Economic Moat Ratings (2002-2017)



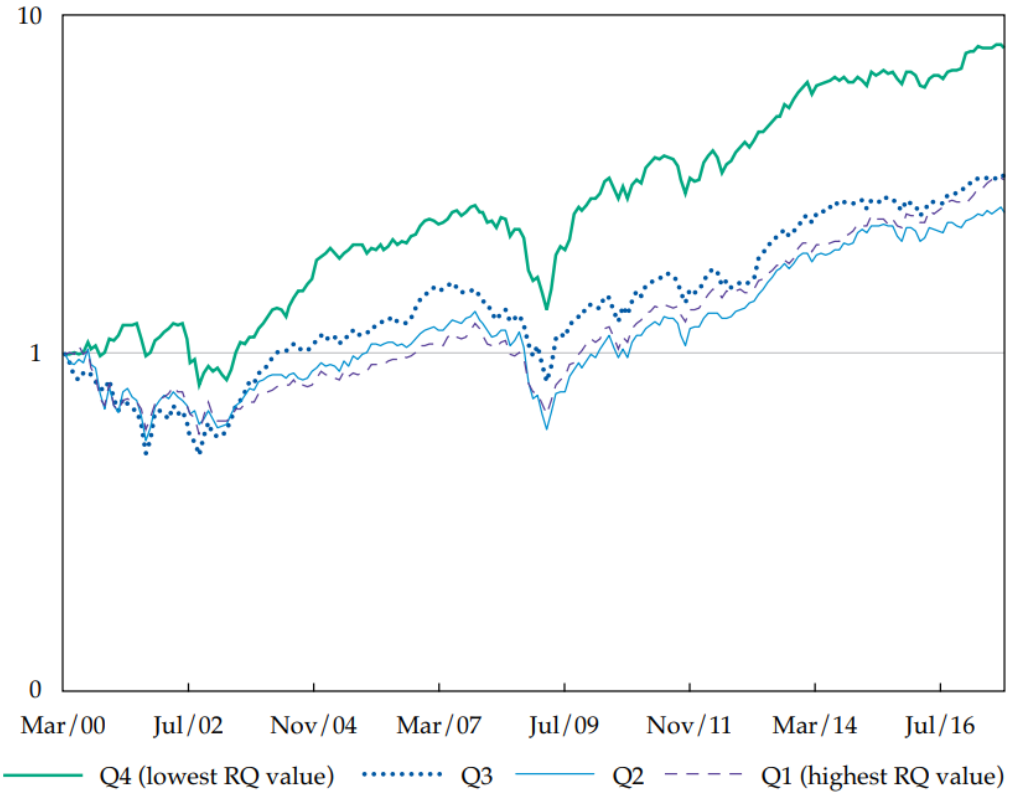
The portfolio with companies classified as having no moat, have an arithmetic mean of 18.57% , compared to 12.13% for the portfolio with wide moat companies, as observed for brand value, the long-term expected returns decrease when moving from unpopular to popular companies, therefore the portfolios composed of narrow moat companies also outperforms the one containing wide moat companies, thus No Moat > Narrow Moat > Wide Moat. This result is consistent with both the PAPM and the CAPM paradigm, because albeit it is true that less popular companies perform better over time than more popular

companies, they do so at a for a higher amount of risk, even though the Sharpe ratio values for the tree portfolios are relatively comparable with values of 0.729 , 0.758 and 0.815. While the wide-moat characteristic attracts investors with its higher expected returns and lower standard deviation, it fails to deliver higher returns, due to the companies becoming overly popular. However, it does provide a more stable return pattern, especially during economic downturns and over the long run.

Reputation is the third characteristic taken into account in the CABR factor, the Harris Pool RQ rankings are collected from 2000 to 2017 and the companies are divided into four quartiles, as seen before when investigating the evidence for brand value. The fourth quartile (Q4) therefore, will contain companies that will then be part of the portfolio with the companies with the lowest reputation, on the contrary, Q1 will contain the companies with the highest reputation, which will then populate the most popular portfolio.

Another study, conducted by Statman et al (2008), produced results similar to those observed by the PAPM authors, with regards to reputation. The only difference pertains the methodology as the companies were divided into “admired” and “spurned”, rather than forming quartiles.

Growth of $1 for Equally Weighted Portfolios based on Harris Pool RQ rankings (2000-2017)



Q4 shows an arithmetic mean return of 14.73%, whilst it is only of 8.41% for Q1. These results are consistent both with the PAPM and with CAPM because this outperformance is also reflected in a higher risk, even though the Sharpe ratio remains significantly higher for Q4 companies, 0.665 compared to 0.418, and this is the case also for both measures utilized to capture the alpha, namely Jensen's alpha and *t*-stat of alpha which are respectively of 4.01% and 1.88 for Q4 compared to -0.25% and -0.16 for the portfolio with the highest RQ value, Q1. Performing a regression for Q4 and Q1 on the Fama&French factors, it is possible to notice that Q1 has a negative HML (value) factor, whilst it's the opposite for the Q4, thus the portfolio with less popular companies is also capturing the value premium. This relation shows that as companies become more popular (moving from Q4 to Q1), when reputation is used as a proxy for popularity, they tend to become growth companies and thus their popularity will ultimately lead them to underperform over time, in fact, as showed in previous chapters, the HML factor values the value premium comparing value companies with growth companies. The following table summarizes the correlation between the characteristic composing CABR and the F&F, SMB (size) and HML (value) factors:

	Brand	*Moat*	*Reputation*
Brand	1	0.14	0.16
Moat	0.14	1	0.44
Reputation	0.16	0.44	1
SMB	0.25	0.63	0.38
HML	0.6	0.28	0.54

Brand, moat and reputation in the correlation table, were represented by factor series created by subtracting Q4 from Q1 for Brand and Reputation and by subtracting wide moat to no moat portfolios for the moat characteristic. The correlations between the factors capturing popularity (brand value, reputation, and economic moat) and the Fama–French size and value factors were relatively low, indicating that the popularity characteristics that constitute the CABR are different from one another and from the size and value factors. This result is indicative of the capacity of the PAPM to capture additional company characteristics, if the factors appeared to highly correlated with one another, that would indicate that using different factors would not provide additional benefits when considering the popularity of a company in a valuation. A high correlation to the Fama and French factors, would have rendered the factors not useful in capturing additional premiums and they would be simply redundant in an asset pricing model, instead, the low correlation among them indicates that these factors are able to capture unique aspects of a company, providing further support on the consistency of PAPM as a valuation tool.[51]

4. Practical Implementation

This chapter focuses on the experimental component of this dissertation and presents a practical example of how the PAPM can be implemented to capture a cost of equity that differs from the one that would be calculated using the CAPM and, more importantly, that differs in a way that allows the main premiums and discounts discussed above to be taken into account in a single metric, aiding the analyst especially in the valuation of companies that are expected to show a high sensitivity to premiums and discounts and that thus requires a specific focus on how they are accounted for. There are some important caveats

[51] Ibbotson, R. G., Idzorek, T., Kaplan, P. D., & Xiong, J. X. (2018). Popularity: A Bridge between Classical and Behavioral Finance, p.86-100.

to make before showing the methodology in detail. This experimental part of the dissertation builds on the work of Ibbotson, R. G., Idzorek, T., Kaplan, P. D., & Xiong, J. X, which are a team of 6 experienced researchers and practitioner that can also find support in the institution they represent (CFA) to retrieve data efficiently, whilst this dissertation was written by just one person. Despite the obvious differences caused by this aspect, the amount of data collected was still significant comprising over 2100 stock tickers spanning 12 years, from 2010 to 2022. Furthermore, random checks on the data retrieved via Python were conducted by downloading CSV files with companies data that were then analyzed on Excel, and the results were always consistent.

4.1 Data retrieving

The first step was to properly retrieve the needed data to build the PAPM, the focus has been on the CABR factor, since it's the most unique factor of the PAPM and the most relevant one when focusing on premiums and discounts.

Three tools were implemented in the paper on Popularity: Interbrand's Best Global Brands rankings, Nielsens' Harris Pool Reputation Quotient (RQ) and Morningstar's Economic Moat, out of these 3, only Morningstar' Economic Moat was not accessible because it was proprietary information only available to subscribers of their service. The two other resources were instead publicly available as both Interbrand and Harris Pool rankings are published on their respective websites. The period that was considered for the analysis goes from 2010 to 2022, a time horizon longer than five to ten years is considered to be long-term in the relevant literature, therefore that was the threshold which had to be overcome to render the analysis meaningful and representative of long-run relations between a company's characteristics and its cost of capital.

The rankings from both Interbrand's (BV) and Harris Pool's (RQ) websites were downloaded and pasted in an Excel spreadsheet which was called Popularity, the spreadsheets contains 24 sheets, containing the Brand Value rankings and the Reputation rankings for 12 years each. "Stocks and tickers were manually retrieved and organized into tables, which were called using the VLOOKUP function. Tickers were updated manually when new stocks were added to the rankings. The BV rankings comprised 100 companies for each of the 12 years, the RQ rankings were composed by 60 companies until 2014 and by 100 companies from the year 2015 onwards. It was interesting to notice how companies that were on the BV

rankings were much more stable over-time whilst the companies on the RQ rankings changed more frequently over time. It is possible to imagine that this would be the consequence of the different methodologies implied in the two rankings, BV calculates its ranking through a proprietary methodology that estimates the amount of revenue of a company that is attributable to the strength of its brand [52], so it has a more quantitative approach than the approach based on consumers' surveys implemented by the RQ rankings. The more stable nature of BV rankings could be the consequence of the inherent aspect of brand as an intangible, namely, the difficulty in recreating it in a short amount of time, on the other hand, RQ rankings seems to point more closely to irrational investor behaviors and how the preferences for the specific companies are volatile, but not the preference for the characteristic reputation in itself, as it will be possible to see later when observing the returns of these two characteristics that will make up the CABR factor utilized here. The authors of the PAPM paper adjusted the BV rankings aggregating the value of subsidiaries into the parent company, likewise, the rankings are populated solely by parent companies in this research as well.

The tickers were then divided into two separate portfolios, for example, the BV rankings for 2010 were organized into BV2010_HIGH and BV2010_LOW, as the names imply, the HIGH portfolio contains all the tickers that ranked in the first half of the BV rankings, i.e. the popular companies with a high brand value, the LOW portfolio contains instead all the tickers of the unpopular companies. The same was done for the RP rankings, so that year 2010 would have REP2010_HIGH and REP2010_LOW, therefore for each year and for both characteristics a High Popularity Portfolio and aLow Popularity Portfolio was constructed.

The daily returns of each ticker were retrieved using the yfinance library on Python, it is an open-source library which retrieves data from Yahoo Finance, after installation, it was imported into the notebook "import yfinance as yf" then the function download was called to retrieve data:

```
BV2010_H= yf.download(BV2010_HIGH, start="2010-1-1", end="2010-12-31")
```

the line of code retrieves daily information for each stock in the BV2010_H portfolio.

[52] Ibbotson, R. G., Idzorek, T., Kaplan, P. D., & Xiong, J. X. (2018). Popularity: A Bridge between Classical and Behavioral Finance, p.85-86

The ‘Adj Close’ column was selected as it is the one that contains the daily adjusted close price for each stock, the data in the column was resampled to a monthly frequency and the cumulative return for each stock on a monthly basis was calculated via:

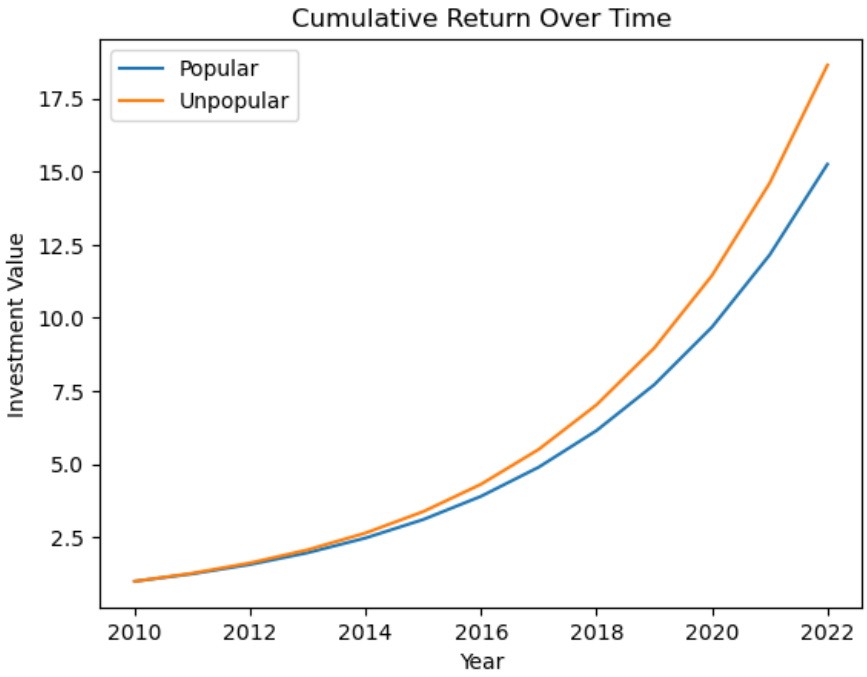
```
BV2010_H_Change= BV2010_H['Adj Close'].pct_change()
BV2010_H_Monthly_Returns = BV2010_H_Change.resample('M').agg(lambda x: (x+1).prod()-1)
```

an alternative function , the “.last()” function is more frequently used to calculate monthly returns but in that way only the last adjusted close price for each month would have been considered, this is clearly less computationally intensive but it risks to oversimplify the analysis by considering the change in the last adjusted monthly price from month to another, as representative of monthly returns, when it’s clearly not the case.
The Python notebook that was created to build the CABR factor and that was called “Popularity Discount Factor”, spans several pages, here, some of the most relevant lines will be presented.

4.2 Preliminary Results on the Popularity Premium

After the data was properly retrieved and organized in the format of DataFrames on Python, the first step was that of checking the existence of the Popularity Premiums . To do so, a slightly different methodology was implemented then the one utilized by the authors of the PAPM who divided the yearly rankings into four quartile and then compared each quartile, finding the biggest premium in the difference between the fourth and the first quartile, here instead the yearly rankings for both BV and Rdata were divided into two halves respectively and the premium is therefore considered as the difference in returns between the L (low) portfolios and the H (high) portfolio. This method of calculating the popularity premium may result in a lower premium, as the difference in brand value and reputation between companies in the second and first halves is not as pronounced as that between companies in the fourth and first quartile. Dividing the rankings into two portfolios was preferred as it is the approach used by Fama and French in their models when observing factor premiums and since the goal is that of creating a factor to empirically test the PAPM which is an expanded form of the Fama and French, as it was observed in the table above containing the main asset pricing formulas, rather than focusing on observing the premiums over time, which was already achieved by the authors of the PAPM, the aim of this work is to find an implementable discount factor, thus the data was organized accordingly.

These are the preliminary results that confirm the findings of the authors of the PAPM, it was indeed possible to observe both a Brand Value Premium and a Reputation Premium. The BV premium year over year from 2010 to 2022 was of 14.88% and the RQ premium was of 10.49%, where these percentages refer to portfolio composed by the yearly portfolios BVLow-BVHigh and RQLow-RQHigh respectively, this is the growth of 1$ in unpopular versus popular companies from 2010 to 2022:



This is a simplified display based on year over year returns. The Premium was confirmed. When examining the portfolios on a yearly basis, there was one notable exception that consistent for both characteristics. The exception was for the year 2020, where popular companies in BV rankings outperformed unpopular companies by 35.4% and by 21.5% in the RQ rankings, it is reasonable to argue that since a lot of new retail investors joined the market during the year 2020 as a result of the COVID-19 pandemic[53], there was a high increase in investments flowing especially to popular companies which are the companies investors tend to invest in, as described earlier in relation to irrational behavior.

4.3 Building the modified CABR Factor

The CABR factor was developed with the goal of ultimately adding it into a modified PAPM formula to capture premiums and discounts. The PAPM formula was modified according to

[53] https://www.schwab.com/learn/story/covid-and-market-case-study

both data availability and to better fit the focus of the dissertation.

The objective is to offer an alternative and complementary approach for capturing premiums and discounts in firm valuation. This approach can be applied to both public and private companies. When a public company is being valued, the PAPM formula can be implemented also in its extended version, and in the case of private companies, the expected return of a comparable public company, calculated using the PAPM, can be used to estimate its cost of capital. It should be noted that the ESG factor was excluded from this study, due to the lack of access to a database containing relevant information, as well as due to the absence of a viable example provided by the PAPM authors to capture it, unlike the cases for BV and RQ characteristics, for which the authors of the PAPM have explicitly mentioned the rankings that they implemented. Considering the investor behavior that was so far discussed, it is reasonable to think that including the ESG factor, can improve the explanatory capacity of PAPM. The remaining factors that were excluded, namely LIQ, RISKA and MOM primarily focus on investors' rational and irrational preferences for companies by focusing primarily on their respective stocks and the historical performance in the stock market, rather than on the preferences of investors for the intrinsic, rational and irrational characteristics of the company as a whole. Therefore, even though the explanatory power of the PAPM is reduced for public companies, the version that is now proposed, is not only still capable of capturing the main premiums and discounts on a firm level, but it is also consistent in its use both when valuing a public and when calculating the cost of equity for a private firm by implementing the comparables method. Indirectly including factors like LIQ, RISKA and MOM that pertain to the stock not to the company as whole, when calculating the cost of equity of a private company, might bring in the valuation considerations that are not coherent in the valuation of a company which is not quoted and thus its exposure to factors such as negative coskewness would introduce another level of speculation that may negatively impact the accuracy of the valuation, this observation is based on the fact that in the PAPM paper, these are considered stock-level characteristics[54], thus it is reasonable to argue that including them in a valuation tool that aims at being employable for both public and private companies might not improve and possibly reduce its validity.

The PAPM formula that was developed by the authors of the framework is : $E[R_i] = \mathrm{R_f} +$

[54] Ibbotson, R. G., Idzorek, T., Kaplan, P. D., & Xiong, J. X. (2018). Popularity: A Bridge between Classical and Behavioral Finance, p.23, p.84.

$\beta * \mathrm{E}[\mathrm{R_{MKT}} - \mathrm{R_f}] + \beta_{i2} * \mathrm{SMB} + \beta_{i3} * \mathrm{HML} + \beta_{i4} * LIQ + B_{i5} * RISKA + \beta_{i6} * \mathrm{ESG} + \beta_{i7} *$ CABR+ $\beta_{i8}\mathrm{MOM}$ and the modified formula implemented here is the following:

$E[R_i] = \mathrm{R_f} + \beta * \mathrm{E}[\mathrm{R_{MKT}} - \mathrm{R_f}] + \beta_{i2} * \mathrm{SMB} + \beta_{i3} * \mathrm{HML} + \beta_{i4} * \mathrm{BVRQ}.$

It is thus an expanded version of the Fama and French 3 factors model, it captures a firm exposure to the overall volatility of the market, to the Size factor, the Value factor and the fourth factor, which was the CABR factor in the PAPM formula, has been called BVRQ factor which stands for brand value and reputation. The BVRQ factor was built in Python, closely following the rationale used by Fama and French for their SMB and HML factors. The frequency used by Fama and French was replicated, thus the portfolios were built on a monthly basis, considering the cumulative returns over the entire month, not only the percentage change between the last adjusted closing prices for each month. The portfolios were equally weighted on a monthly basis:

```
BV2010_L_Monthly_Returns_Weighted = BV2010_L_Monthly_Returns * weightsB10L
BV2010_L_Monthly_Returns_Weighted.rename(columns={'V': 'BV2010_L'}, inplace=True)
BV2010_L_Monthly_Returns_Weighted['BV2010_L'] = (BV2010_L_Monthly_Returns_Weighted).sum(axis=1)
BV2010_L_Monthly_Returns_Weighted = BV2010_L_Monthly_Returns_Weighted[['BV2010_L']]
BV2010_H_Monthly_Returns_Weighted = BV2010_H_Monthly_Returns *weightsB10H
BV2010_H_Monthly_Returns_Weighted.rename(columns={'AAPL': 'BV2010_H'}, inplace=True)
BV2010_H_Monthly_Returns_Weighted['BV2010_H'] = (BV2010_H_Monthly_Returns_Weighted).sum(axis=1)
BV2010_H_Monthly_Returns_Weighted = BV2010_H_Monthly_Returns_Weighted[['BV2010_H']]
BV2010_L_Monthly_Returns_Weighted = BV2010_L_Monthly_Returns_Weighted.rename(columns={'BV2010_L': 'BV2010'})
BV2010_H_Monthly_Returns_Weighted = BV2010_H_Monthly_Returns_Weighted.rename(columns={'BV2010_H': 'BV2010'})
BV2010 = BV2010_L_Monthly_Returns_Weighted - BV2010_H_Monthly_Returns_Weighted
```

BV2010_L_Monthly_Returns_Weighted = BV2010_L_Monthly_Returns * weightsB10L i.e., monthly returns were weighted month by month, instead of weighing only the aggregated yearly returns as done in the simplified approach presented previously to confirm the existence of the Popularity premium over time. This adjustment precisely replicates the monthly returns of a portfolio in which the same amount of capital has been invested in each company, each month of the year. The following codes represents DataFrames[55] transformations to regroup the monthly returns of all the stocks in a single aggregated monthly return for each month, finally the BV2010 i.e. the portfolio that captures the Brand

[55] https://pandas.pydata.org/pandas-docs/stable/reference/api/pandas.DataFrame.html

Value premium for the year 2010 was calculated as:

BV2010 = BV2010_L_Monthly_Returns_Weighted - BV2010_H_Monthly_Returns_Weighted.

Date	BV2010
2010-01	-0.001587
2010-02	0.004191
2010-03	-0.018054
2010-04	0.000063
2010-05	0.014081
2010-06	0.010496
2010-07	0.016368
2010-08	0.010013
2010-09	-0.028532
2010-10	-0.008230
2010-11	-0.003455
2010-12	-0.014410

The table above represents the Python print for the Brand Value premium portfolio of 2010, this procedure was reiterated for every year to find both BV and RQ premiums and the final prints were then displayed for both the BV and the RQ premiums portfolios:

Date	Reputation	BrandValue
2010-01	-0.018068	-0.001587
2010-02	-0.016744	0.004191
2010-03	0.020382	-0.018054
2010-04	0.017407	0.000063
2010-05	-0.000518	0.014081
...	...	...
2022-08	0.005305	-0.004407
2022-09	-0.001999	0.014320
2022-10	0.036288	-0.022312
2022-11	-0.010356	0.022607
2022-12	-0.017625	0.016956

[156 rows x 2 columns]

The shape of the data is also provided under the tables, the rows are 156 as expected since they represent the number of months over all the 12 years included, the two columns represents the two portfolios and the first column is the index containing the date displayed in as “years-months” as it is displayed in the Fama and French 3 factors model to allow then to coherently merge the overall BVRQ factor in the Fama and French model. Finally, this is the shape of the data frame containing the two individual characteristics and the overall BVRQ factor which is the aggregated return of the premiums of the two characteristics, displayed here on a percentage basis to align it to the way that Fama & French present their factors:

Date	Reputation	BrandValue	BVRQ
2010-01	-0.018068	-0.001587	-1.965465
2010-02	-0.016744	0.004191	-1.255270
2010-03	0.020382	-0.018054	0.232799
2010-04	0.017407	0.000063	1.746986
2010-05	-0.000518	0.014081	1.356299
...	...	...	...
2022-08	0.005305	-0.004407	0.089706
2022-09	-0.001999	0.014320	1.232039
2022-10	0.036288	-0.022312	1.397592
2022-11	-0.010356	0.022607	1.225165
2022-12	-0.017625	0.016956	-0.066859

4.4 Answering the research question: a new discount factor

This paragraph explores the comprehensive results of all the theoretical and practical considerations explored so far, with the aim of developing a new tool that accounts for premiums and discounts into the calculation of a company's expected return. This new cost of equity is then meant to be included in a discounted cash flow (DCF) model, in place of relying solely on the capital asset pricing model (CAPM). This approach allows for a more comprehensive assessment of a company's cost of equity by considering the company's correlation with the BVRQ factor which considers the impact of a company's popularity on its expected return and value. This tool that includes considerations on premiums and discounts directly in the calculation of the weighted average cost of capital (WACC) can enhance the accuracy of the DCF model. The Python code thus was continued in the following way:

```
import getFamaFrenchFactors as gff
import pandas_datareader.data as reader
```

the Fama-French factors are publicly available, and the "getFamaFrenchFactors" library[56] (imported as "gff") provides an efficient mechanism for their retrieval in Python. In this instance, the factors comprising the 3-factor model are of interest and were obtained using this library, this is how they are displayed:

[56] https://pypi.org/project/famafrench/

```
            Mkt-RF   SMB   HML    RF
Date
2010-01      -3.36  0.40  0.43  0.00
2010-02       3.40  1.19  3.23  0.00
2010-03       6.31  1.48  2.21  0.01
2010-04       2.00  4.87  2.89  0.01
2010-05      -7.89  0.09 -2.44  0.01
...            ...   ...   ...   ...
2022-08      -3.77  1.39  0.31  0.19
2022-09      -9.35 -0.82  0.03  0.19
2022-10       7.83  0.10  8.05  0.23
2022-11       4.60 -3.40  1.39  0.29
2022-12      -6.41 -0.64  1.36  0.33

[156 rows x 4 columns]
```

the index is consistent with that of the BVRQ factor, it was therefore possible to use the function "merge.()" , to add to the model a fifth column containing the monthly returns of BVRQ factor, such that the modified PAPM discussed above was ultimately constructed and displayed as:

```
            Mkt-RF     SMB     HML      RF      BVRQ
Date
2010-01 -0.0336  0.0040  0.0043  0.0000 -0.019655
2010-02  0.0340  0.0119  0.0323  0.0000 -0.012553
2010-03  0.0631  0.0148  0.0221  0.0001  0.002328
2010-04  0.0200  0.0487  0.0289  0.0001  0.017470
2010-05 -0.0789  0.0009 -0.0244  0.0001  0.013563
...         ...     ...     ...     ...       ...
2022-08 -0.0377  0.0139  0.0031  0.0019  0.000897
2022-09 -0.0935 -0.0082  0.0003  0.0019  0.012320
2022-10  0.0783  0.0010  0.0805  0.0023  0.013976
2022-11  0.0460 -0.0340  0.0139  0.0029  0.012252
2022-12 -0.0641 -0.0064  0.0136  0.0033 -0.000669

[156 rows x 5 columns]
```

here it is shown with all the factors already scaled back i.e. divided by 100 to match the way in which monthly returns of a stock that will soon be added to the model.

The correlation amongst the 4 factor was calculated using the corr.() function in Python and yielded the following diagonal matrix:

	Mkt-RF	SMB	HML	BVRQ
Mkt-RF	1.000000	0.330326	0.089937	0.142435
SMB	0.330326	1.000000	0.040549	0.360246
HML	0.089937	0.040549	1.000000	0.327766
BVRQ	0.142435	0.360246	0.327766	1.000000

there are some observations that are interesting when we look correlation table.

The correlation of the SMB factor with the Market Factor is the highest at 33%, then BVRQ with 14%

and finally HML with 9% and the correlation of the BVRQ factor with the Size and Value factor is of 36% and 33% respectively. This is seems to be a promising result, the BVRQ factor shows only a weak positive linear relationship with "Market-Rf" and is moderately correlated with SMB and HML. These results seem to suggest that the BVRQ factor has the capacity to capture aspects of a stock return that are not being captured by the other three factors, or at least it removes any doubts on whether BVRQ is redundant or not, having a higher correlation with the other factors would have indicated that the BVRQ factor is not considering a different data distribution. These correlations describe relationship that can change overtime, conducting the analysis over a long-term period, 12 years, brings further support to these results.

The final step was to test the model via an ordinary least squares regression (OLS), it was performed in Python using the OLS function in the "statsmodel" library[57] and using the library described above, yfinance, to retrieve stock's data from Yahoo Finance. The stock adjusted closing price were collected and properly sampled on a monthly basis and the returns were calculated as they were calculated when building the BVRQ factor, thus ensuring that the methodology remained consistent. The results of the OLS regression were considered coherent and satisfactory if the modified PAPM was able to correctly capture the sensitivity of the companies expected returns to Popularity, in terms of their direction. A cost of equity that, when accounting for BVRQ characteristics, increases for unpopular companies, implying that their overall values is ultimately discounted because of their lack of popular characteristics, was the expected result of the model. On the other hand, a lower cost of capital was expected for popular companies, implying that in a DCF model this would result in a higher value for popular companies, reflecting the (valuation) premium that they were able to secure by providing the characteristic more in demand. The companies that were selected for the regression had to satisfy two characteristics. The first characteristic was on their comparability, the companies that were fit for the test would both operate in the same sector and have a comparable global reach. The second characteristic arises from the data that was included in the model, using Morningstar' Economic Moat and a proxy for ESG characteristics would have made it possible to consider an even larger set of companies, relying on Interbrand's Brand Value and on Harris Reputation Quotient rankings, still allowed to consider over 2400 historical series and hundreds of companies, for the test, the companies were chosen also based on their overall ranking over the course of the 12 years. Among the companies that satisfied both characteristics were The Coca-Cola Company and Yum!Brands. Both companies figure in the BV and RQ rankings for every year of the period considered, they are both part of the S&P500, have a similar global reach and both classify as

[57] https://www.statsmodels.org/stable/

consumer discretionary companies, operating in the food and beverage industry. Their respective ticker are KO and YUM, this is how the monthly returns of YUM are displayed in Python, merging the columns containing the modified model with two additional columns, the first one containing the monthly returns for the company and the last one containing the monthly premium returns of the company compared to the risk free rate, this is the dependent variable of the model, for Yum!Brands this column was named 'YUM-Rf'.

Date	YUM	Mkt-RF	SMB	HML	RF	BVRQ	YUM-Rf
2010-01	-0.015992	-0.0336	0.0040	0.0043	0.0000	-0.019655	-0.015992
2010-02	-0.014323	0.0340	0.0119	0.0323	0.0000	-0.012553	-0.014323
2010-03	0.136714	0.0631	0.0148	0.0221	0.0001	0.002328	0.136614
2010-04	0.112380	0.0200	0.0487	0.0289	0.0001	0.017470	0.112280
2010-05	-0.034654	-0.0789	0.0009	-0.0244	0.0001	0.013563	-0.034754
...	...	...	...	...	...	...	...
2022-08	-0.087764	-0.0377	0.0139	0.0031	0.0019	0.000897	-0.089664
2022-09	-0.044049	-0.0935	-0.0082	0.0003	0.0019	0.012320	-0.045949
2022-10	0.111999	0.0783	0.0010	0.0805	0.0023	0.013976	0.109699
2022-11	0.093020	0.0460	-0.0340	0.0139	0.0029	0.012252	0.090120
2022-12	-0.004508	-0.0641	-0.0064	0.0136	0.0033	-0.000669	-0.007808

[156 rows x 7 columns]

The results of the OLS regression performed using the "statsmodel" library in python, setting "YUM-Rf" and then "KO-Rf" as the dependent variable y and "Mkt-Rf","SMB","HML" and "BVRQ" as the independent variables collected in X, i.e. the 4 factors. The results are presented in the following table:

	YUM	KO
Mkt-Rf	0.8943	0.6095
SMB	-0.2631	-0.6779
HML	0.0209	0.1507
VBRQ	0.1079	-0.0644

both YUM and KO show a β lower than one i.e., the correlation with the market premium "Mkt-Rf", that is respectively of 0.89 for YUM and 0.61 for KO. This indicates that both YUM and KO have a low and similar correlation with the overall market returns, this is also a result of YUM and KO operating in a similar industry, the nature of the products they offer is not highly elastic with the variations of

the stock market returns due to the fact that, with all likelihood, whether there is there is a bull or a bear market, most of their clients will still want a glass of Coca-Cola and a slice of pizza or a burger from one of the many brands owned by YUM. The similar relation with the SMB factor, in both case negative, albeit more so for KO, reflecting that even though both are larger companies than average there is still a notable difference in their size, brings further support to the choice of the two companies that are now being helpful in understanding the results of the model. The factor in which they show an inverse relationship is, coherently with the expectations of the PAM theory, the VBRQ factor, the modified CABR factor calculated here.

The Coca-Cola Company constantly populated the "High" portfolios both for BV and RQ, it was part of the most popular and most reputable companies every year from 2010 to 2022, on the contrary YUM!Brands was always in the "Low" portfolios both for BV and RQ, reflecting its lower popularity. It is important that this is also intuitively recognizable, the BVRQ is capturing irrational behavior, which of the two brands comes to mind is also an important question to ask, clearly we may individually perceive a company as more popular than another, but for the majority of people it seems reasonable to argue that Coca-Cola is perceived as more popular and this will make it more likely for investors to ultimately invest in it.

The annual expected return for YUM was of 15.43% and the annual expected return for KO was of 6.36%, calculated by multiplying the coefficients found in the regression to the mean annual return of each factor over the entire period. The formula is the one that was described before:

$E[R_i] = \ R_f + \beta * E[R_{MKT} - R_f] \ + \beta_{i2}SMB + \beta_{i3}HML + \beta_{i4}BVRQ$.

Comparing the expected returns obtained with this modified version of the PAPM are strongly aligned with those predicted by the PAPM theory. Another observation that strengthens these findings comes by looking at the companies on their own as well. The expected return for YUM calculated without the BVRQ factor, is higher than its expected return calculated with the BVRQ factor. This is because YUM is an unpopular company and it is positively correlated with BVRQ, thus its expected return increases when its popularity is considered, when plugged in a DCF model, this will influence the valuation accordingly, taking into account the relative low popularity of YUM. On the other hand, the expected return for KO shows the opposite relation, it is negatively affected by the BVRQ factor, since The Coca-Cola Company is a very popular company, when its popularity is taken into account, its cost of equity lowers and this is reflected in a higher firm valuation.

The python notebook that was built here for the purpose of this dissertation is being shared online via the following link: ***https://1drv.ms/u/s!Al1OwGQZ2691zZUHYlAwYrL9c4t?e=PIM0LM***

in this way it's possible to not only see the entire code directly in a Python notebook but also to implement it for practical use. In order to calculate the cost of equity of any desired company, one has to modifying the ticker of the stock in the final section, where the regression is performed, adjusting the names of the columns accordingly.

5. Conclusions

The relevant literature on premiums and discounts was discussed, considering two different levels of analysis: firm level and shareholder level, borrowing this categorization from the work of Shannon P. Pratt (2009). A particular attention was given to Value, Size and Distress, for firm level factors, and to minority discount, LOMD and majority interest when considering shareholder level factors. Whilst keeping the analysis consistent with further considerations on shareholder level factors in a valuation setting, a particular attention was given to premiums and discounts occurring at firm level. In order to better understand premiums and discounts, the dissertation then focused on investigating the motivations behind premiums and discounts, illustrating the most recent developments in the financial literature. It was through the discussion of the Popularity Asset Pricing Model that a common framework was found to beneath all firm level premiums and discounts discussed previously. Discussing the PAPM thoroughly required a second literature review on the evolution of asset pricing models both in Classical and Behavioral Finance. The aspect that differed the most between the different theories was the way in which the investor was characterized and how this description ultimately influenced the way in which the expected investor behavior was capture and thus the way in which collective expectations were thought to affect the valuation of a company. The aspects affecting the expected return would then eventually impact the valuation of a company through its cost of equity. In order to account for different premiums and discounts and their impact on cost of equity, a modified version of the PAPM model was built in Python, following the rationale implemented by Fama and French whilst keeping the model consistent with the PAPM framework. The relation between popularity and expected return observed by the authors fo the PAPM was not only confirmed, but a construction of the PAPM in Python and its practical application to real market data was presented, this represented the first practical implementation of the PAPM and thus constitutes the original contribution offered by this work. The results were successful as the model coherently captured the sensitivity of a

company to the presence of lack of popular characteristics, thus companies that figured in the High Popularity Portfolios would show a negative BVRQ coefficient, whilst the opposite was true for unpopular companies. These sensitivities were then reflected in the cost of equity that would be increased or decreased by the BVRQ factor, as predicted by the theory. The research question was therefore answered, a framework that coherently considered the main premiums and discounts was presented and the a new valuation tool that can reflect the main premiums and discounts in firm valuation, via a different cost of equity, was presented.

This tool can be furtherly refined and its explanatory power increased, by including additional relevant data pertaining ESG and sustainable competitive advantage.

The PAPM framework and its implementation focused on firm level factors that are theoretically compatible both with private and public companies and can be coherently integrated with shareholder level analysis, to take into account also the number and type of shares that are being valued, starting from a firm value that is now adjusted by incorporating the PAPM expected return in the valuation.

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