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Blackjack as a business

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Anno Accademico 2022/2023

Blackjack as a Business

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1. Introduction

With this work I want to give a comprehensive and complete guide of the game of Blackjack. This has do be done from different aspects. First we have to understand every rule of the game, to then move to all the strategy used to mathematically beat the game. After that we will understand how all of this works from the statistical point of view to also understand how the different rules and strategies can influence the game and the player’s edge (how much advantage can be gained).

I decided to dedicate the second part to the team aspect. In the World of Blackjack, team play has a key role into reducing risk and increase profitability. Here again, decisions have to be made in a statistically supported way as well as from a managerial prospective; really important in this scenario.

This makes me see at this topic as a perfect combination of the two main fields of our degree: Computer Science being highly statistically and mathematically based and Managment.

It is worth noting that all of this has nothing to do with gambling, in fact players do not have to make decisions by their own, based on emotions: the method tells you what’s the right choice to do in every case (however, this does not guarantee one to win, it only maximises the probability to do so).

The reason why Blackjack is a beatable game (the only beatable casino game) is because is a game in which past events influence future ones. In fact, once certain cards have come out, those specific cards can no longer come out and this will have positive or negative effects on the cards that have to come out, depending on the situation.

Also, this is 100% legal and it’s not considered cheating of course. We will deepen all of these aspects.

2. The game

2.1 Rules

In Blackjack, one wins by achieving a score as close as possible to **21** — the so-called *blackjack* — yet without exceeding it. To go beyond it is to "bust" and, therefore, lose the game hand, which includes both the cards and the bets placed. Specifically, one wins when scoring higher than the dealer (in fact, this one is the true opponent, unlike in Poker where one plays against the other players). The score is obtained by summing together the values corresponding to each card, which are the following:

- Cards numbered 2 through 10: value equal to the number on the card;
- Face cards (Jack, Queen, and King): 10 points of value, as for the 10;
- Ace: while under 21, it is worth 11; when going over it, instead, it takes a value of 1

Each player is dealt two cards face-up while one of the dealer's cards remains face down.

The differences in gameplay between the dealer and the players all lie in the available options. The dealer, in fact, is obliged to draw additional cards until they reach a score equal to or greater than 17. Players, on the other hand, can continuously choose one of four options:

- Standing: ending the turn without taking any action, keeping one's cards and waiting for the dealer's moves, so as to determine the outcome of the game;
- "Hitting": receiving another face-up card from the dealer, after which one can either ask for additional cards or stand;
- Doubling down: doubling one's placed bet while receiving a single final card from the dealer and ending one's turn;
- Splitting: if a player's two initial cards hold the same score, they can be split into two separate hands by betting an additional amount equal to the initial bet. The dealer then provides each hand with a second card and each hand is played separately by choosing between the same four aforementioned choices (if a third card of the same value is received, players are usually permitted to split again). However, when aces are split, only one more card is dealt, without the possibility of making additional choices.

So, once again, the dealer can't make decisions, the player can. That being said, professional players, as we will see, also don't make decisions: they just execute the mathematically best option in every situation, both playing and betting.

2.2 How the Turn Ends

At the end of each turn, four outcomes are possible for the players:

- They win the hand: in this case, the dealer pays them an amount equal to what they bet (if they have doubled down, however, they receive twice as much);
- They lose the hand: the dealer takes their bet;

- They achieve blackjack: in this scenario, the dealer pays them 3:2 of their bet; i.e., what they bet as well as 50% of it;[1]
- They tie with the dealer: this situation is defined as a "push" and occurs when the dealer's and the player's hands have the same value. In this case, the player receives their bet back without winning or losing anything.

Another important feature to point out is that the dealer's turn occurs after that of all players. This detail is fundamental: when players bust, in fact, they lose their bet even if the same happens to the dealer, without there being a push. This is precisely where part of the dealer's mathematical advantage stems from because, in this case, they make money despite losing.

A final useful clarification to make is that Blackjack has secondary rules and distinct game variations depending on the country, the casino, and even the table at which one plays. This aspect will be elaborated further later on.

Let us now finally explore how to obtain the much-anticipated mathematical edge. Generally, counting cards by itself is thought to be enough but, unfortunately, it is not. In fact, it is necessary to know four different steps and be able to execute them all simultaneously in one's mind during the game (a peculiar multitasking exercise, basically). These four steps are as follows:

- Playing Basic Strategy (making the correct decision for each possible hand);
- Counting the cards;
- Varying one's bets ("betting deviations");
- Changing Basic Strategy depending on the cards count ("playing deviations").

Let us now analyze, learn, and deepen all these steps.

[1] To achieve blackjack, it is not enough for the total value of one's hand to be 21. Instead, one ought to reach such value specifically with an ace and a 10 (or a face card). Otherwise, the score of 21 is not considered a blackjack and the hand is simply won normally.

3. Maximizing the player's chances

3.1 Basic Strategy

"Basic strategy" refers to the set of mathematically optimal decisions for each game scenario; i.e., for each pair of cards as compared to the dealer's.

From a graphical point of view, Basic Strategy consists of a set of tables, where the possible values of one's hand are listed along the rows, those of the dealer's initial card along the columns, and the corresponding intersection indicates the recommended action to take. However, such recommended action does not guarantee one to win, but rather maximizes their probability to do so given the current hand which, in particularly unfavorable cases, can already have quite low chances to start with.

Here is the first of these tables: hard totals (without an ace) of the classic American blackjack, the most popular and the one we will refer in this work.

HARD TOTALS										
	DEALER UPCARD									
	2	3	4	5	6	7	8	9	10	A
17	S	S	S	S	S	S	S	S	S	S
16	S	S	S	S	S	H	H	H	H	H
15	S	S	S	S	S	H	H	H	H	H
14	S	S	S	S	S	H	H	H	H	H
13	S	S	S	S	S	H	H	H	H	H
12	H	H	S	S	S	H	H	H	H	H
11	D	D	D	D	D	D	D	D	D	D
10	D	D	D	D	D	D	D	D	H	H
9	H	D	D	D	D	H	H	H	H	H
8	H	H	H	H	H	H	H	H	H	H

KEY		
	H	Hit
	S	Stand
	D	Double if allowed, otherwise hit

While it might initially appear difficult to understand, a few examples should make it clearer:

If one finds themselves holding a 14 (with no ace nor a pair), whilst the dealer has an 8, the optimal move is to ask for another card and then evaluate the next move based on the new score obtained. If, instead, their current hand still has a value of 14 but the dealer has a 2, then it is recommended to stay.

There are also some cases not expressed in the table, for obvious reasons:

- From 17 upward, one should always stay, because the risk to bust with one more card is relatively high;
- From 8 downward, one should always hit, as there is no risk of going bust, whilst it is highly likely for the dealer to obtain a higher score.

It might stand out that from 9 to 11 there are several doubling down recommendations; in particular, with an 11 it seems one should always double down. The reason is, first of all, that with such low scores it is impossible to bust with one more card. At the same time, however, adding a card makes it statistically likely – although not certain – to obtain a rather high score, with consequently good chances of winning. It is, therefore, worth taking advantage of this by doubling the stakes.

Should one have doubts about the reliability of these move recommendations, they should know there is a strong foundation behind them: this and the following tables, in fact, are the result of the work of mathematicians who studied the probabilities behind it and confirmed them by running millions of simulations, establishing what the best decision for each and every situation is.

Nonetheless, it is still worth understanding the reasoning behind these choices as some can appear counterintuitive. For instance, if 12 is a low score to have, it might seem questionable to stay when the dealer has a 4, a 5, or a 6. That is because there is a detail that eludes the human eye: with the above-mentioned scores, the dealer is more likely to bust. In fact, the dealer will be forced to receive a card that will certainly not be enough to reach 17 (except when receiving an ace while having a 6); therefore, they ought to ask for a second one which might, at that point, make them exceed 21. Thus, it is not convenient for the player to ask for a card, as they would risk getting a 10 and busting; it is better to stay and wait for the dealer to lose on their own.

We will deepen the math behind all the basic strategy, and how they were obtained in the next section.

The following table shows how to behave when an ace is dealt.

SOFT TOTALS										
	DEALER UPCARD									
	2	3	4	5	6	7	8	9	10	A
A,9	S	S	S	S	S	S	S	S	S	S
A,8	S	S	S	S	Ds	S	S	S	S	S
A,7	Ds	Ds	Ds	Ds	Ds	S	S	H	H	H
A,6	H	D	D	D	D	H	H	H	H	H
A,5	H	H	D	D	D	H	H	H	H	H
A,4	H	H	D	D	D	H	H	H	H	H
A,3	H	H	H	D	D	H	H	H	H	H
A,2	H	H	H	D	D	H	H	H	H	H

KEY		
	H	Hit
	S	Stand
	D	Double if allowed, otherwise hit
	Ds	Double if allowed, otherwise stand

Here too the various moves are presented, with several double downs to exploit the ace's power, which protects the player from going bust. The legend features a peculiar entry compared to last time, however: "double if allowed". Basically, doubling down is not always allowed: in certain blackjack tables it is not permitted after having split, whilst in others it is only allowed if one's hand has a specific score (generally 10 or 11). Rules of this kind are a typical example of casinos attempting to limit the players' edge against the house.

Let us now move to the pairs that, as already anticipated, do allow for splitting. This might seem to be an advantageous choice; nevertheless, it is not always optimal.

PAIR SPLITTING										
	DEALER UPCARD									
	2	3	4	5	6	7	8	9	10	A
A,A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
T,T	N	N	N	N	N	N	N	N	N	N
9,9	Y	Y	Y	Y	Y	N	Y	Y	N	N
8,8	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
7,7	Y	Y	Y	Y	Y	Y	N	N	N	N
6,6	Y/N	Y	Y	Y	Y	N	N	N	N	N
5,5	N	N	N	N	N	N	N	N	N	N
4,4	N	N	N	Y/N	Y/N	N	N	N	N	N
3,3	Y/N	Y/N	Y	Y	Y	Y	N	N	N	N
2,2	Y/N	Y/N	Y	Y	Y	Y	N	N	N	N

KEY		
	Y	Split the pair
	Y/N	Split if "Double After Split (DAS)" is offered, otherwise do not split
	N	Don't Split the Pair

Here the player faces a double decision to make: once they recognize having a pair and decide whether or not to split it, based on the table's guidelines, they then have to determine whether to stay, hit, or double down, according to all the cases discussed beforehand. With two 5s, for instance, one should never split and, according to Basic Strategy, the ideal move is to double down (unless the dealer has a 10 or an ace). When dealt two 7s against the dealer's 4, on the other hand, it is best to split, receiving two more cards and then, according to the value of these latter, deciding what to do next.

Finally, the last scenarios to be covered are the possibility to surrender and the one to request insurance.

LATE SURRENDER										
	DEALER UPCARD									
	2	3	4	5	6	7	8	9	10	A
16								SURR	SURR	SURR
15									SURR	
14										

KEY		
	SURR	Surrender

INSURANCE OR EVEN MONEY: DO NOT TAKE

The surrender is an option rarely offered by casinos; when available, however, one should take advantage of it straight away: surrendering, in fact, means giving up a hand while losing only half of the bet placed. As can be seen from the table, there are only four cases in which it is worth applying, which are with particularly unfavorable hands. The possibility to surrender increase the general odds of the player by 0.6% so, as said, it's a good rule to take advantage of when possible.

The insurance, instead, is always offered when the dealer's face-up card is an ace. It is exactly from this scenario that the following explanation stems (this applies to the classic rules, not to the European ones; this distinction will be deepened later).

As soon as the dealer realizes having an ace as their face-up card, they are required to check if the covered card has a value of 10 (that is, if they have a blackjack). If this is the case, all players automatically lose (except those having another blackjack, for which it is a push) before even playing their hand. This outcome, in some respects, is actually a positive one for these players, as it prevents them from losing any additional bets they might make in splits and double downs. Given this possibility of losing and the presence of the ace, the players are offered the option to insure themselves against the dealer having a blackjack. This translates into a separate bet which, apart from being half of the original one, is essentially decoupled from it. At this point, after the players have decided whether to insure themselves or not, the dealer checks their covered card: if it has a value of 10, the insured players receive a compensation equal to twice their bet (which ultimately leads to a breakeven, as the initial bet is lost); otherwise, the insurance is lost but the hand continues.

Having now explained what the insurance is, it is important to point out that it should not be used. Simply put, it is not worth it: the cards with a value different from 10, in fact, outnumber the ones with such value. Nevertheless, as will be seen later, with card counting come some exceptions to this (because we know when it's more likely for a 10 to come out); for now, let us simply state that, according to Basic Strategy, insurance is not recommended.

Assuming one has learned Basic Strategy by heart, unfortunately they are yet to have obtained the edge over the house. They have definitely reduced this latter's chances of winning, but to a positive 0.5%; hence, in the long run, one will lose. This is because basic strategy does not take into account the cards that have come out. This is achieved with the next step: card counting, necessary, together with all the other steps, to achieve the turnover and become the one mathematically favored.

3.2 Card Counting

This is usually the most anticipated part of texts on the topic, as it is what is properly referred to as "card counting". It is often believed that "card counting" means memorizing all the cards that are played on the table, in order to estimate which ones are missing and, therefore, still to be dealt. This is not at all the case: counting cards literally means keeping count of them, adding and subtracting their values as they are drawn from the remaining decks.

In this context, however, the values of the cards differ from the ones explained earlier:

- Cards from 2 to 6 are counted as a +1;
- 7s, 8s, and 9s are worth 0 (meaning they can be ignored);
- 10s, face cards, and aces have a value of -1.

The objective is, then, to add up the values of all the cards that are dealt (both the players' cards and the dealer's) following this schema, from the first to the last one dealt from the current "shoe"[2], without ever losing the count.

That is essentially it: card counting is simply a matter of adding +1s and -1s together. To put this in context, blackjack is usually played with six decks, hence, players have to be really accurate into keeping the count through all of them.

Now, what this number represents and why it is so important are questions that naturally come to mind: colloquially referred to as the "Running Count", it indicates how many more (or less, if the count is negative) high cards there are among the ones still to be dealt, with respect to a neutral deck. This is because if more

[2] The container from which the cards are drawn.

low cards are drawn in the first few hands, then there will inevitably be more high cards left in the remaining hands, and vice versa. Let's see why it is from here that the counter's advantage comes from.

Firstly, the cards with a high value (10s, face cards, and aces) are advantageous for the players, but unfavorable for the dealer. The reason is as follows: they allow players to obtain high scores more quickly, including the much-cherished 21. On the other hand, they pose a challenge to the dealer as they increase their probability to bust (as a reminder, in fact, the dealer is forced to hit until they reach a score equal to or greater than 17). For each additional high card per deck, compared to a neutral deck, about 0.5% of advantage goes to the player (we will analyze it in the 5th section). Without the knoledge of how much high and low cards were delt it would not be possible to beat blackjack, in fact games that use CSM (contiuous shuffle machines) are not beatable.

The opposite also holds: low-value cards are a disadvantage to players, while being advantageous for the dealer.

In the following parts, therefore, the Running Count will be a key element to determine in which moments the chances are in one's favor, so as to adjust the bets accordingly.

Before going on, however, two more concepts ought to be presented: deck estimation and the "True Count".

Note: the system presented here is called "hi-lo", but it is not the only one; several different counting strategies exist. The reason it has been chosen here, however, is because, by achieving the best combination of simplicity and efficiency, it has become the most popular one around.

3.2.1 Deck Estimation

As suggested by its name, Deck Estimation consists of making an estimate of the number of decks in front of the player. Specifically, it refers to the discard pile, the stack of already used cards that players often ignore but which, instead, is one of the card counter's most valuable allies. As mentioned above, blackjack is usually played with six decks. Once at the table, thus, one is to examine the discard pile and roughly estimate how many decks of cards are present; this way, they will be able to calculate, through a simple subtraction, those still to be played (you can't clearly see those since they are inside a shoe).

The estimate of the number of decks in the discard pile is as valuable as the Running Count just discussed, as it is essential to derive the most important variable of all: the True Count.

3.2.2 True Count

The True Count is the most crucial piece of data among those discussed in this text, as it allows to understand how many extra high cards there are for each deck left to be played. The way to calculate it is rather simple: it is the ratio between the Running Count and the number of decks not yet played.

Let us provide an example to better clarify: suppose one is playing a hand with six decks, three of which are in the discard pile, and the Running Count is +12. The first thing to do is to find the remaining decks, by subtracting the discarded ones from the total ones (6 - 3 = 3), to then perform the following division:

$$\frac{Running\ Count}{No.\ of\ Decks\ Left} = True\ Count$$

By substituting these variables with the available data, we obtain:

$$\frac{12}{3} = 4$$

This means that, at the moment, we have four extra high cards per deck to be dealt. It is precisely on this result that the expert players base their strategy and adjusts their bet, a topic that will be addressed in the next section. It goes without saying, therefore, that a Running Count of +12 when three decks are remaining has a completely different meaning from the same Running Count when five decks are left.

One thing that should be noted in all of this is that, most of the time, the number of decks left and the True Count will not be whole numbers (contrary to the example above), but rather decimal ones; in both cases, they are always to be rounded down and never up, so as to avoid overestimating the bets and, therefore, to reduce the risk. In this regard, beginners and intermediates think in whole decks; that is to say, if in the discard pile there appear to be 3½ decks, these are considered as 3 and, therefore, also the decks left to be played are considered to be 3. Until the 4 full decks are reached, they are considered as 3. The True Count is also rounded down: if, for instance, by dividing the Running Count by the number of decks left, a 4.5 is obtained, this will be a whole 4. Advanced players, on the other hand, assess deck estimation in half-decks and, also in this case, numbers are rounded down rather than up (3¼ decks are to be considered as 3, rather than 3½).

Lastly, in tables where only two decks are used, also beginners and intermediates usually think in half-decks; otherwise, the calculation of the True Count results in an excessively approximate number.

3.3 Betting Deviations

We have now uncovered all the tools necessary to gain the mathematical edge over the house. Hence, it is time to learn to adjust one's bets based on the value of the True Count. To do so, two principles have to be reminded: high value cards (10s, face cards, and aces) are advantageous for the players, but unfavorable for the dealer and low-value cards are a disadvantage to players, while being advantageous for the dealer. This is why the higher the True Count, the more one is encouraged to increase their bet. Vice versa, if negative, a lower True Count implies accordingly lower bets should be placed.

Having said that, let us now introduce a fundamental concept in this regard: betting units.

As implied in the name, these are the units by which to lower or increase when adjusting one's bet. Suppose a betting unit of 50€, for instance; if each increment of the true count corresponds to an increment of the bet by one betting unit, then one is to play the minimum allowed at that specific table until the count goes up; at that point, instead, one is supposed to make bets equal to the True Count multiplied by the betting unit (a practical example will be provided shortly).

The choice of one's betting unit depends on their bankroll, that is how much money they can play with (keeping in mind that this sum could still be lost: even by playing perfectly, in fact, "bad luck" or rather "variance" still exists, as will be explained later on). The higher the bankroll, the higher will be the betting unit and the potential gain (as well as the potential loss).

By multiplying the values of the True Count with those of the betting unit, the so-called "bet spread" is obtained; this represents the schema indicating the variations of one's bet. Here is an illustration of the bet spread (a rather aggressive one) of a card counter with a bankroll of 20,000 € and betting units of 50 €:

True Count	**Bet**
-2	Leave the table
-1	Minimum allowed by the table (e.g.: 10 €)
0	Minimum allowed (e.g.: 10 €)
1	50 €

2	100 €
3	150 €
4	200 €
5+	250 €

Constructing one's ideal bet spread is a delicate and fundamental matter for a card counter, as a multitude of factors have to be taken into account:

- Bankroll;
- Risk aversion;
- Rules in effect where one is playing;
- Risk to be backed off, the technical term for being expelled from a casino (indeed: the more aggressive the bet spread, the more one will be noticed).

3.4 Playing deviations

For the final step of how to professionally play blackjack, let us briefly revisit Basic Strategy, the starting point from which it all started. As seen up to now, it consists of a series of moves to memorize which are statistically the best for each situation. Unfortunately, however, they are not sufficient: this is because the best strategy changes as the True Count varies. Specifically, the higher this latter, the more high cards there will be left in the decks; hence, the greater the probability of busting by hitting (and, on the other hand, of drawing optimal cards). Let us now see how to change one's strategy as the True Count varies.

Red Numbers indicate the index that the true count must meet to deviation from basic strategy

"+"after the index number indicates the deviation happens at that true count and above

"-" after the index number indicates the devation happens at the true count and below

0- indicates the deviation happens at any negative running count

0+ indicates the deviation occurs at any positive running count

HARD TOTALS										
	DEALER UPCARD									
	2	3	4	5	6	7	8	9	10	A
17	S	S	S	S	S	S	S	S	S	S
16	S	S	S	S	S	H	H	4+	0+	3+
15	S	S	S	S	S	H	H	H	4+	5+
14	S	S	S	S	S	H	H	H	H	H
13	-1-	S	S	S	S	H	H	H	H	H
12	3+	2+	0-	S	S	H	H	H	H	H
11	D	D	D	D	D	D	D	D	D	D
10	D	D	D	D	D	D	D	D	4+	3+
9	1+	D	D	D	D	3+	H	H	H	H
8	H	H	H	H	2+	H	H	H	H	H

KEY		
	H	Hit
	S	Stand
	D	Double if allowed, otherwise hit

As can be seen in the first table, the one about hard totals, some cells contain a red number: it represents the True Count value to be reached to change that cell's move. Let us now explain how it changes, starting from top. When holding a 16 against the dealer's 9, one is supposed to hit; if the True Count reaches or exceeds 4, however, one should stay. The same applies with a 10 or an ace for the dealer: in the former case, one should stay when the True Count is positive (0+, without even having to reach 1; hence, even a positive value of $\frac{1}{2}$ is enough), while in the latter they should do so when it reaches or goes above 3.

In the rows of the 15, 13, and 12, similar scenarios are at play: if in that cell, by default, one is to stay, then they should hit and vice versa. The variations of the 8, 9, and 10, instead, instruct to double where, by default, one would simply hit. Thus, if the player has a 10, the dealer has an ace, and the True Count reaches 3, then they should double instead of hitting. Let us now look at the variations for "soft totals", splits, surrender, and insurance.

SOFT TOTALS

	DEALER UPCARD									
	2	3	4	5	6	7	8	9	10	A
A,9	S	S	S	S	S	S	S	S	S	S
A,8	S	S	3+	1+	0-	S	S	S	S	S
A,7	Ds	Ds	Ds	Ds	Ds	S	S	H	H	H
A,6	1+	D	D	D	D	H	H	H	H	H
A,5	H	H	D	D	D	H	H	H	H	H
A,4	H	H	D	D	D	H	H	H	H	H
A,3	H	H	H	D	D	H	H	H	H	H
A,2	H	H	H	D	D	H	H	H	H	H

As before, the change happens at the True Count value indicated by the red number and consists in making the other choice present in that line. For instance, if one has an ace and an 8, the dealer has a 4, and the True Count is 3+, then they should double instead of staying. Alternatively, if still with an ace and an 8, but against a dealer's 6 and with a negative True Count, then one should stay instead of doubling down.

PAIR SPLITTING

DEALER UPCARD

	2	3	4	5	6	7	8	9	10	A
A,A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
T,T	N	N	6+	5+	4+	N	N	N	N	N
9,9	Y	Y	Y	Y	Y	N	Y	Y	N	N
8,8	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
7,7	Y	Y	Y	Y	Y	Y	N	N	N	N
6,6	Y/N	Y	Y	Y	Y	N	N	N	N	N
5,5	N	N	N	N	N	N	N	N	N	N
4,4	N	N	N	Y/N	Y/N	N	N	N	N	N
3,3	Y/N	Y/N	Y	Y	Y	Y	N	N	N	N
2,2	Y/N	Y/N	Y	Y	Y	Y	N	N	N	N

KEY		
	Y	Split the pair
	Y/N	Split if "Double After Split (DAS)" is offered, otherwise do not split
	N	Don't Split the Pair

LATE SURRENDER

DEALER UPCARD

	2	3	4	5	6	7	8	9	10	A
17										SURR
16							4+	-1-	SURR	SURR
15								2+	0-	-1+
14										

KEY	SURR	Surrender

INSURANCE OR EVEN MONEY: TAKE AT 3+

With splits, instead, the variation only occurs in the row of the two 10s and with relatively high True Counts, as shown in the table. It might appear silly to split a double 10, as 20 is an already optimal score and one risks ruining it this way. Yet, with such high True Counts it is likely not only for another 10 to be drawn, but even for an ace, leading to a blackjack and, therefore, to win even more. Naturally, it is also possible to draw a low card; however, mathematics demonstrates that, by making this decision thousands of times, the cases in which it will pay off will outnumber the others.

One of the most important deviations (meaning one of those that brings the greatest advantage), finally, is the one about insurance which, luckily, is also rather simple to learn. Whilst it has previously been stated that insurance should never be taken, we shall now say that, above a True Count of 3, it should (as it is more likely for the dealer to have a blackjack). Here, once again, the American rules have been used as a reference.

Let us now conclude by stressing the importance of deviations. For the advanced player, they are indeed fundamental, as they increase this latter's advantage by 20-30%, which is a large number in a field where everything is about gradually gaining small increments in each phase.

Note: the ones in the tables are the 20 most important variations, as well as those responsible for most of the advantage one can gain in this phase (about 95%). They are only a subset of playing deviations, however; in fact, the most advanced players usually memorize around 40-50, gaining a small, yet significant additional edge (bringing the 95% gained advantage closer to 100%). The reason it is a small one is that these variations occur at extremely high True Counts, thus rather rarely.

3.5 European Blackjack

Thus far, I have mentioned classic "American" Blackjack, that is because there is also the European version of the game. They are indeed slightly different versions of the game. While this difference might be subtle from the average player's point of view, it is not so for the advanced one or, more in general, from a mathematical standpoint.

In the blackjack played in most European casinos, technically called ENHC blackjack (European No Hole Card), the dealer is not dealt the second face-down card, but rather receives all their cards (except the upcard) after the players have made their choices. This means that, if they obtain a blackjack, players will only know about it at the end; this is a threat due to the increased exposure double downs and splits give the player.

The reason I preferred to explain Basic Strategy with its American version, here, is because that is what most of card counters' games are played with (as will be seen when discussing about the countermeasures casinos adopt).

It goes without saying that the European Basic Strategy is slightly different as, each time the dealer has an ace card, one must be cautious to avoid doubling down. Here it is:

HARD TOTALS

	DEALER UPCARD									
	2	3	4	5	6	7	8	9	10	A
17	S	S	S	S	S	S	S	S	S	S
16	S	S	S	S	S	H	H	H	H	H
15	S	S	S	S	S	H	H	H	H	H
14	S	S	S	S	S	H	H	H	H	H
13	S	S	S	S	S	H	H	H	H	H
12	H	H	S	S	S	H	H	H	H	H
11	D	D	D	D	D	D	D	D	H	H
10	D	D	D	D	D	D	D	D	H	H
9	H	D	D	D	D	H	H	H	H	H
8	H	H	H	H	H	H	H	H	H	H

KEY		
	H	Hit
	S	Stand
	D	Double if allowed, otherwise hit

SOFT TOTALS

	DEALER UPCARD									
	2	3	4	5	6	7	8	9	10	A
A,9	S	S	S	S	S	S	S	S	S	S
A,8	S	S	S	S	S	S	S	S	S	S
A,7	S	Ds	Ds	Ds	Ds	S	S	H	H	H
A,6	H	D	D	D	D	H	H	H	H	H
A,5	H	H	D	D	D	H	H	H	H	H
A,4	H	H	D	D	D	H	H	H	H	H
A,3	H	H	H	D	D	H	H	H	H	H
A,2	H	H	H	D	D	H	H	H	H	H

KEY		
	H	Hit
	S	Stand
	D	Double if allowed, otherwise hit
	Ds	Double if allowed, otherwise stand

PAIR SPLITTING

	DEALER UPCARD									
	2	3	4	5	6	7	8	9	10	A
A,A	Y	Y	Y	Y	Y	Y	Y	Y	Y	H
T,T	N	N	N	N	N	N	N	N	N	N
9,9	Y	Y	Y	Y	Y	N	Y	Y	N	N
8,8	Y	Y	Y	Y	Y	Y	Y	Y	N	N
7,7	Y	Y	Y	Y	Y	Y	N	N	N	N
6,6	Y/N	Y	Y	Y	Y	N	N	N	N	N
5,5	N	N	N	N	N	N	N	N	N	N
4,4	N	N	N	Y/N	Y/N	N	N	N	N	N
3,3	Y/N	Y/N	Y	Y	Y	Y	N	N	N	N
2,2	Y/N	Y/N	Y	Y	Y	Y	N	N	N	N

KEY		
	Y	Split the pair
	Y/N	Split if "Double After Split (DAS)" is offered, otherwise do not split
	N	Don't Split the Pair

EARLY SURRENDER

	DEALER UPCARD									
	2	3	4	5	6	7	8	9	10	A
17										SURR
16								SURR	SURR	SURR
15									SURR	SURR
14									SURR	SURR
13										SURR
12										SURR
7										SURR
6										SURR
5										SURR
8,8									SURR	SURR

KEY	SURR	Surrender

INSURANCE OR EVEN MONEY: DO NOT TAKE

As we can see, changes only happen when the dealer has an ace, avoiding doubling down and splitting.

Here we also have the full ENHC deviations:

Red Numbers indicate the index that the true count must meet to deviation from basic strategy
"+"after the index number indicates the deviation happens at that true count and above
"-" after the index number indicates the devation happens at the true count and below
0- indicates the deviation happens at any negative running count
0+ indicates the deviation occurs at any positive running count

HARD TOTALS

	DEALER UPCARD									
	2	3	4	5	6	7	8	9	10	A
17	S	S	S	S	S	S	S	S	S	S
16	S	S	S	S	S	H	H	4+	0+	H
15	S	S	S	S	S	H	H	H	3+	H
14	S	S	S	S	S	H	H	H	H	H
13	-1-	S	S	S	S	H	H	H	H	H
12	3+	2+	0-	S	S	H	H	H	H	H
11	D	D	D	D	D	D	D	D	4+	H
10	D	D	D	D	D	D	D	-1-	H	H
9	1+	D	D	D	D	3+	H	H	H	H
8	H	H	H	H	2+	H	H	H	H	H

KEY		
	H	Hit
	S	Stand
	D	Double if allowed, otherwise hit

SOFT TOTALS

	DEALER UPCARD									
	2	3	4	5	6	7	8	9	10	A
A,9	S	S	S	S	S	S	S	S	S	S
A,8	S	S	3+	1+	1+	S	S	S	S	S
A,7	0+	Ds	Ds	Ds	Ds	S	S	H	H	1+
A,6	1+	D	D	D	D	H	H	H	H	H
A,5	H	H	D	D	D	H	H	H	H	H
A,4	H	H	D	D	D	H	H	H	H	H
A,3	H	H	H	D	D	H	H	H	H	H
A,2	H	H	H	D	D	H	H	H	H	H

KEY		
	H	Hit
	S	Stand
	D	Double if allowed, otherwise hit
	Ds	Double if allowed, otherwise stand

PAIR SPLITTING

DEALER UPCARD

	2	3	4	5	6	7	8	9	10	A
A,A	Y	Y	Y	Y	Y	Y	Y	Y	Y	H
T,T	N	N	6+	5+	4+	N	N	N	N	N
9,9	Y	Y	Y	Y	Y	N	Y	Y	N	N
8,8	Y	Y	Y	Y	Y	Y	Y	Y	N	N
7,7	Y	Y	Y	Y	Y	Y	N	N	N	N
6,6	Y/N	Y	Y	Y	Y	N	N	N	N	N
5,5	N	N	N	N	N	N	N	N	N	N
4,4	N	N	3+/N	Y/N	Y/N	N	N	N	N	N
3,3	0-/N	Y/N	Y	Y	Y	Y	N	N	N	N
2,2	Y/N	Y/N	Y	Y	Y	Y	N	N	N	N

KEY		
	Y	Split the pair
	Y/N	Split if "Double After Split (DAS)" is offered, otherwise do not split
	N	Don't Split the Pair

LATE SURRENDER

DEALER UPCARD

	2	3	4	5	6	7	8	9	10	A
17									5+	SURR
16							4+	-1-	SURR	SURR
15								2+	SURR	SURR
14									-1-	SURR
13									3+	SURR
12										SURR
7										-1-
6										0-
5										2+
8,8									SURR	SURR

KEY	SURR	Surrender

INSURANCE OR EVEN MONEY: TAKE AT 3+

With that, the part on how to beat blackjack ends. We now have to understand how this strategies were calculated.

4. Determination of probabilities

4.1 Probabilistic Analysis of Basic Strategy

In this section, we will see how the results of Basic Strategy have been obtained and, hence, how they are calculated.

The source for this part is the book "Il grande libro del Blackjack e dei giochi da casinò", written by Dario De Toffoli and Margherita Bonaldi.[3]

In the probabilistic analysis of blackjack, both the behavior of the dealer and the one of the player need to be examined. As seen earlier, the former is constrained in their choices, whereas the latter is not, but rather has to take their own choices (except for professional players, who only need to execute them). In our probabilistic analysis, we will begin with the dealer, as the player's choices ultimately depend not only on their own probabilities of achieving certain outcomes, but also on those of the dealer, depending on this latter's face-up card.

Furthermore, this analysis ought to make use of the so-called "infinite deck" approximation, in which each card always has the same likelihood of been drawn. This is because, in the real game, past events influence future ones; that is to say, when a card is drawn, it cannot be drawn again (which is precisely what the player's edge stems from). In calculating Basic Strategy, however, this approximation is required for three reasons. Firstly, the resulting probability will differ ever just slightly from the actual one (Dario De Toffoli and Margherita Bonaldi have established it to be less than **0.5%**, hence negligible with respect to the differences in probability among the various choices available to the player). Secondly, blackjack is played with a variable number of decks, which would entail having to analyze a multitude of additional scenarios. The third reason is that it considerably simplifies this probabilistic study without deviating too much from real-world scenarios, as has been said. Moreover, from the point of view of the game itself, the changes in Basic Strategy resulting from certain cards being drawn will still be taken into account (playing deviations); however, such Basic Strategy has to first be calculated on the basis of the infinite deck approximation.

Let us recall also, that what we are finding right now is the ideal strategy when the count is zero, in other words, when no cards have come out yet, which, from a probabilistic point of view, is the exact same situation that the infinite deck is constantly in. When the difference between the actual situation at the moment and that of the inifinite deck becomes significant we proceed to apply playing deviations. Furthermore, an analysis of the probability changes as the count changes will be provided in Section 5.

4.2 The Dealer's odds

Let us then start by calculating the odds of the dealer.

The final scores they can obtain are solely six: 17, 18, 19, 20, 21, 22+ (all the cases in which they bust).

Having established that, the first calculation to be made is the one of the dealer's probability distribution; that is, the odds of achieving each of the six final scenarios, starting from the face-up card (the sum of which odds will obviously add up to 100%). Thus, it is the case of a conditional probability, as we are establishing a final result (the probability of each of the six scenarios) based on an initial value (the face-up card).

For this reason, we will analyze the case in which the dealer's card is a 10. Each of the six final scenarios can be attained in a variable number of steps. This is because, starting from the 10, a single card can be enough

[3] Cfr. Dario De Toffoli, Margherita Bonaldi, *Il grande libro del blackjack e dei giochi da casinò*, Sperling and Kupfer, 2011

to reach 17+, just like two, three, up to a maximum of six can be necessary (in the case in which the following values are drawn in this order: 2, 1, 1, 1, 1, X).

The best tool to represent this graphically is a tree diagram as the one in the figure and the following, taken again from “Il grande libro del Blackjack e dei giochi da casinò”.

FIRST CARD	SECOND CARD		THIRD CARD	
10 +	A	= 21 STOP!		
	10	= 20 STOP!		
	9	= 19 STOP!		
	8	= 18 STOP!		
	7	= 17 STOP!		
	6	= 16 +	A	= 17 STOP!
			2	= 18 STOP!
			3	= 19 STOP!
			4	= 20 STOP!
			5	= 21 STOP!
			6, 7, 8, 9, 10	= >21 STOP!
	5	= 15 +	A	= 16 + ...
			2	= 17 STOP!
			3	= 18 STOP!
			4	= 19 STOP!
			5	= 20 STOP!
			6	= 21 STOP!
			7, 8, 9, 10	= >21 STOP!
	4	= 14 +	A	= 15 + ...
			2	= 16 + ...
			3	= 17 STOP!
			4	= 18 STOP!
			5	= 19 STOP!
			6	= 20 STOP!
			7	= 21 STOP!
			8, 9, 10	= >21 STOP!
	3	= 13 +	A	= 14 + ...
			2	= 15 + ...
			3	= 16 + ...
			4	= 17 STOP!
			5	= 18 STOP!
			6	= 19 STOP!
			7	= 20 STOP!
			8	= 21 STOP!
			9, 10	= >21 STOP!
	2	= 12 +	A	= 13 + ...
			2	= 14 + ...
			3	= 15 + ...
			4	= 16 + ...
			5	= 17 STOP!
			6	= 18 STOP!
			7	= 19 STOP!
			8	= 20 STOP!
			9	= 21 STOP!
			10	= >21 STOP!

This table outlines all the possible scenarios, yet it does not indicate anything regarding their odds of happening. Nonetheless, thanks to the infinite deck approximation we know the odds of each card being drawn (1/13 for everything worth 1 to 9 and 4/13 for the 10); therefore, we can manually calculate the probability of a final outcome based on the first card.

Let us then compute the odds of the dealer obtaining a final 20 starting from a 10 as their first card. This scenario can be reached in several ways: 32, to be exact. To determine the probability of reaching 20, we need to sum the single probabilities of all these options. This is represented in the table below.

DEALER UPCARD							**PROBABILITY**
I	II	III	IV	V	VI	VII	
	10						4/13
	6	4					1/13 x 1/13
	5	5					1/13 x 1/13
	4	6					1/13 x 1/13
	3	7					1/13 x 1/13
	2	8					1/13 x 1/13
	5	1	4				1/13 x 1/13 x 1/13
	4	2	4				1/13 x 1/13 x 1/13
	4	1	5				1/13 x 1/13 x 1/13
	3	3	4				1/13 x 1/13 x 1/13
	3	2	5				1/13 x 1/13 x 1/13
	3	1	6				1/13 x 1/13 x 1/13
	2	4	4				1/13 x 1/13 x 1/13
	2	3	5				1/13 x 1/13 x 1/13
	2	2	6				1/13 x 1/13 x 1/13
10	2	1	7				1/13 x 1/13 x 1/13
	4	1	1	4			1/13 x 1/13 x 1/13 x 1/13
	3	2	1	4			1/13 x 1/13 x 1/13 x 1/13
	3	1	2	4			1/13 x 1/13 x 1/13 x 1/13
	3	1	1	5			1/13 x 1/13 x 1/13 x 1/13
	2	3	1	4			1/13 x 1/13 x 1/13 x 1/13
	2	2	2	4			1/13 x 1/13 x 1/13 x 1/13
	2	2	1	5			1/13 x 1/13 x 1/13 x 1/13
	2	1	3	4			1/13 x 1/13 x 1/13 x 1/13
	2	1	2	5			1/13 x 1/13 x 1/13 x 1/13
	2	1	1	6			1/13 x 1/13 x 1/13 x 1/13
	3	1	1	1	4		1/13 x 1/13 x 1/13 x 1/13 x 1/13
	2	2	1	1	4		1/13 x 1/13 x 1/13 x 1/13 x 1/13
	2	1	2	1	4		1/13 x 1/13 x 1/13 x 1/13 x 1/13
	2	1	1	2	4		1/13 x 1/13 x 1/13 x 1/13 x 1/13
	2	1	1	1	5		1/13 x 1/13 x 1/13 x 1/13 x 1/13
	2	1	1	1	1	4	1/13 x 1/13 x 1/13 x 1/13 x 1/13 x 1/13

The calculation to perform is as follows:

P_{TOT} = 4/13 + 5/13^2+10/13^3+10/13^4+5/13^5+1/13^6 = 34.22%

The dealer's odds of obtaining 20 starting from an initial 10 are thus 34.22%.

Utilizing the same approach, we can derive the probability of obtaining a particular final result for each initial face-up card. This method, however, is extremely slow because a particular state can be reached in several different ways (as seen with the example of the 20). Therefore, Markov chains come into play.

4.2.1 Markov Chains

A Markov Chain is a stochastic process that takes on a finite or countable number of possible values. In a Markov chain, the conditional distribution of any future state Xn+1 given the past states X0, X1, ..., Xn−1 and the present state Xn, is independent of the past states and depends only on the present state. The value Pij represents the probability that the process will, when in state i, next make a transition into state j (probabilities are nonnegative and the process must make a transition into some state).[4]

This is the case of blackjack because there is a sequence of possible events (the drawing of cards) where each of the random variables depends on the previous one (if one has a 10 and they obtain a 5, the new scenario on which to work is then 15, caused by the last draw). Naturally, the drawing of the next card is also a random event. And we can use Markov Chains only because we are supposing that we are working with an infinite deck, otherwise it would not be possible.

In our case, Markov chains are of help as they allow us to compute, much more rapidly than with the manual method, the probability of obtaining a specific final outcome starting from an initial card, with any number of draws. This is what previously had to be calculated manually and with the help of the tables for the 10.

To do so, matrices are used, representing the probability of going from one value to another with a certain number of draws. In the matrix below, the rows and the columns hold the possible values and the cells contain the corresponding probability of going from the value on the row to the one on the column (in this case, with a single card draw).

As before, computing it is immediate, as it is the case of single draws with the infinite deck approximation (1/13 for everything from 1 to 9 and 4/13 for the 10), all being expressed in percentages in the table.

Taking some examples, starting from a 3, there is a chance of 7.69% of reaching a 10 (i.e., of obtaining a 7) with a single draw. Starting again from a 3, instead, there is a probability of 30.77% of reaching 13, as the 10 is the most common value (three face cards and one 10). Many cells, on the other hand, have no chance at all of this happening because it is impossible to obtain a lower value or the same value after a draw. Odds different from 7.69, 30.77 and 0 are only those present in the last column (where multiple values are summed together, as it is the case of single scenarios) and the 100% at the end of the diagonal because, with those values, one ought not to ask for a card and should, instead, remain in that state.

This is where the advantage of using Markov chains becomes apparent: by multiplying this matrix by itself, we obtain the odds of going from a state to another through two cards. Multiplying this new matrix by the first one (the one in the picture), we obtain the probabilities for three draws, and so on; this way, there is no need to manually calculate everything (these procedures can be rapidly carried out through a spreadsheet).

[4] Cfr. Sheldon M. Ross, *Introduction to Probability Models Ninth Edition*, Academic Press, 2006

	A (11)	2	2 (12)	3	3 (13)	4	4 (14)	5	5 (15)	6	6 (16)	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22+
A (11)	0	0	0	0	0,59	0	1,18	0	1,78	0	2,37	0	0	0	0	0	4,73	4,14	3,55	2,96	2,37	10,65	10,65	10,65	10,65	33,73	0
2	0	0	0	0	0	0	0,59	0	1,18	0,59	1,18	1,18	1,78	2,37	2,96	3,55	5,33	9,47	8,88	8,28	7,69	8,28	7,69	7,10	6,51	5,92	9,47
2 (12)	0	0	0	0	0	0	0,59	0	1,18	0	1,78	0	0	0	0	0	2,37	6,51	5,92	5,33	4,73	12,43	12,43	12,43	12,43	12,43	9,47
3	0	0	0	0	0	0	0	0	0,59	0	1,18	0,59	1,18	1,78	2,37	2,96	4,73	5,33	9,47	8,88	8,28	8,88	8,28	7,69	7,10	6.51	14,20
3 (13)	0	0	0	0	0	0	0	0	0,59	0	1,18	0	0	0	0	0	1,78	2,37	6,51	5,92	5,33	12,43	12,43	12,43	12,43	12,43	14,20
4	0	0	0	0	0	0	0	0	0	0	0,59	0	0,59	1,18	1,78	2,37	4,14	4,73	5,33	9,47	8,88	9,47	8,88	8,28	7,69	7,10	19,53
4 (14)	0	0	0	0	0	0	0	0	0	0	0,59	0	0	0	0	0	1,18	1,78	2,37	6,51	5,92	12,43	12,43	12,43	12,43	12,43	19,53
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0,59	1,18	1,78	3,55	4,14	4,73	5,33	9,47	9,47	9,47	8,88	8,28	7,69	25,44
5 (15)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0,59	1,18	1,78	2,37	6,51	12,43	12,43	12,43	12,43	12,43	25,44
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0,59	1,18	2,37	2,96	3,55	4,14	4,73	14,79	8,88	8,88	8,28	7,69	31,95
6 (16)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0,59	1,18	1,78	2,37	12,43	12,43	12,43	12,43	12,43	31,95
7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0,59	1,78	2,37	2,96	3,55	4,14	35,50	12,43	6,51	6,51	5,92	17,75
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1,18	1,78	2,37	2,96	3,55	11,83	34,91	11,83	5,92	5,92	17,75
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0,59	1,18	1,78	2,37	2,96	11,24	11,24	34,32	11,24	5,33	17,75
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0,59	1,18	1,78	2,37	10,65	10,65	10,65	33,73	10,65	17,75
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0,59	1,18	1,78	2,37	10,65	10,65	10,65	10,65	33,73	17,75
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0,59	1,18	1,78	10,06	10,06	10,06	10,06	10,06	46,15
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0,59	1,18	9,47	9,47	9,47	9,47	9,47	50,89
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0,59	8,88	8,88	8,88	8,88	8,88	55,03
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	8,28	8,28	8,28	8,28	8,28	58,58
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	7,69	7,69	7,69	7,69	7,69	61,54
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	100	0	0	0	0	0
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	100	0	0	0	0
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	100	0	0	0
20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	100	0	0
21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	100	0
22+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	100

The goal is to reach a matrix where all the cells before the 17 have a value equal to zero. At such point, by summing the probabilities of each scenario (as was done manually with the example of the chances of 34.22% to obtain a 20 from an initial 10), one derives, for each starting card, the odds of obtaining each of the six final states.

This is how the probability distribution of the dealer is found, which is here represented in a table.

	16-	17	18	19	20	21	22+
A (11)	0	13,08	13,08	13,08	13,08	36,16	11,53
2	0	13,98	13,49	12,97	12,40	11,80	35,36
2 (12)	0	15,10	15,10	15,10	15,10	15,10	24,50
3	0	13,50	13,05	12,56	12,03	11,47	37,39
3 (13)	0	14,55	14,55	14,55	14,55	14,55	27,25
4	0	13,05	12,59	12,14	11,65	11,12	39,45
4 (14)	0	14,00	14,00	14,00	14,00	14,00	30
5	0	12,23	12,23	11,77	11,31	10,82	41,64
5 (15)	0	13,46	13,46	13,46	13,46	13,46	32,72
6	0	16,54	10,63	10,63	10,17	9,72	42,32
6 (16)	0	12,92	12,92	12,92	12,92	12,92	35,41
7	0	36,86	13,78	7,86	7,86	7,41	26,23
8	0	12,86	35,93	12,86	6,94	6,94	24,47
9	0	12,00	12,00	35,08	12,00	6,08	22,84
10	0	11,14	11,14	11,14	34,22	11,14	21,21
11	0	11,14	11,14	11,14	11,14	34,22	21,21
12	0	10,35	10,35	10,35	10,35	10,35	48,27
13	0	9,61	9,61	9,61	9,61	9,61	51,96
14	0	8,92	8,92	8,92	8,92	8,92	55,39
15	0	8,28	8,28	8,28	8,28	8,28	58,58
16	0	7,69	7,69	7,69	7,69	7,69	61,54
17	0	100	-	-	-	-	-
18	0	-	100	-	-	-	-
19	0	-	-	100	-	-	-
20	0	-	-	-	100	-	-
21	0	-	-	-	-	100	-
22+	0	-	-	-	-	-	100

Cases 2, 3, 4, 5, 6 are represented twice because, when that score is obtained with an ace (soft total), the odds change with respect to the cases involving hard totals.

4.4 The Player's Odds

Having established the dealer's odds, we need to understand which is the best option for the player for each game scenario. Let us begin by analyzing the option hit/stand, to then move onto the doubles and the splits.

In analyzing the hit/stand choice, it is first necessary to compute the player's odds to win, lose, or draw by choosing to stand. This is immediate because it will suffice to compare the initial value with the dealer's ones and their probabilities. Then we move on analyzing what happens when the player decides to hit and compare the final results. Everything is done manually using the probability distribution of the dealer seen in the former table.

For instance, if the player has 15 hard and the dealer has a 6, by staying he wins if the dealer busts (42.32%), he loses if the dealer makes 17 or more (57.68%) and he can't draw.

If instead, the player hits, then he:
-wins if he gets a 6 and the dealer doesn't make 21 (6.94%),
-wins if he gets a 5 and the dealer does not make 20 or 21 (6.16%),
-wins if he gets a 4 and the dealer doesn't make 19, 20 or 21 (5.34%),
-wins if he gets a 3 and the dealer doesn't make 18, 19, 20 or 21 (4.53%),
-wins if he gets a 2 and the dealer busts (3.25%),
-wins if he gets an A and the dealer busts (3.25%),
-draws if he gets a 6 and the dealer makes 21 (0.75%),
-draws if he gets a 5 and the dealer makes 20 (0.78%),
-draws if he gets a 4 and the dealer makes 19 (0.82%),
-draws if he gets a 3 and the dealer makes 18 (0.82%),
-draws if he gets a 2 and the dealer makes 17 (1.27%),
-loses (busting) if he gets 7, 8, 9 or 10 (53.85%),
-loses if he gets a 5 and the dealer makes 21 (0.75%),
-loses if he gets a 4 and the dealer makes 20 or 21 (1.53%),
-loses if he gets a 3 and the dealer makes 19, 20 or 21 (2.35%),
-loses if he gets a 2 and the dealer makes 18, 19, 20 or 21 (3.17%),
-loses if he gets an A and the dealer makes 17, 18, 19, 20 or 21 (4.44%),

Overall, summing up by groups we will have that when he asks for card, the player: wins in 29.47% of cases, draws in 4.44% of cases, loses in 66.09% of cases. We can see that these probabilities sum up to 100% as a proof.

By comparing these final results with the probabilities of staying (winning 42.32% of the times, loosing 57.68) we can see that the best choice when the player has 15 hard and the dealer has a 6 is to stay and this is in fact the suggestion that we see on the basic strategy.

Doing the same thing for all the possible cases in which a player can find themselves, a table of the probability distribution is constructed. Such a table represents all the scenarios where the player asks for a card and their respective odds. The final Basic Strategy we are aiming to prove will be obtained when comparing each of these cases and their respective probabilities with the alternative hypothesis of standing, instead. Finally, these results are compared with the dealer's odds to understand whether it is better to stand or hit, as seen in the example.

States from 10 downward are missing from the table because one always hits, as there is no chance of busting; hence, we are brought back to one of the subsequent states.

INITIAL STATE	CARD DRAWN	FINAL STATE	PROBABILITY
20	A	21	7,69%
	2,3,4,5,6,7,8,9,10	22/+	92,31%
19	2	21	7,69%
	A	20	7,69%
	3,4,5,6,7,8,9,10	22/+	84,72
18	3	21	7,69%
	2	20	7,69%
	A	19	7,69%
	4,5,6,7,8,9,10	22/+	76,92%
17	4	21	7,69%
	3	20	7,69%
	2	19	7,69%
	A	18	7,69%
	5,6,7,8,9,10	22/+	69,23%
16	5	21	7,69%
	4	20	7,69%
	3	19	7,69%
	2	18	7,69%
	A	17	7,69%
	6,7,8,9,10	22/+	61,64
15	6	21	7,69%
	5	20	7,69%
	4	19	7,69%
	3	18	7,69%
	2	17	7,69%
	A	16	7,69%
	7,8,9,10	22/+	53,85
14	7	21	7,69%
	6	20	7,69%
	5	19	7,69%
	4	18	7,69%
	3	17	7,69%
	2	16	7,69%
	A	15	7,69%
	8,9,10	22/+	46,15%
13	8	21	7,69%
	7	20	7,69%
	6	19	7,69%
	5	18	7,69%
	4	17	7,69%
	3	16	7,69%
	2	15	7,69%
	A	14	7,69%
	9, 10	22/+	38,46%
12	9	21	7,69%
	8	20	7,69%
	7	19	7,69%
	6	18	7,69%
	5	17	7,69%
	4	16	7,69%
	3	15	7,69%
	2	14	7,69%
	A	13	7,69%
	10	22/+	30,77%
11	10	21	30,77%
	9	20	7,69%
	8	19	7,69%
	7	18	7,69%
	6	17	7,69%
	5	16	7,69%
	4	15	7,69%
	3	14	7,69%
	2	13	7,69%
	A	12	7,69%

4.3.1 Soft totals

To derive the soft Basic Strategy, meaning the one where one of the player's cards is an ace, we proceed analogously; the sole difference is that, when the total score exceeds 21, the ace takes again a value of 1. We will nonetheless calculate the probability to reach each of these final scenarios based on the infinite deck approximation, so as to then compare them to the dealer's odds and see whether the scenario is more or less favorable than hitting.

STARTING POINT	DRAWN CARD	OBTAINED SCORE	PROBABILITY
20 SOFT	A	21 SOFT	7,69%
	2	12	7,69%
	3	13	7,69%
	4	14	7,69%
	5	15	7,69%
	6	16	7,69%
	7	17	7,69%
	8	18	7,69%
	9	19	7,69%
	10	20	30,77%
19 SOFT	2	21 SOFT	7,69%
	A	20 SOFT	7,69%
	3	12	7,69%
	4	13	7,69%
	5	14	7,69%
	6	15	7,69%
	7	16	7,69%
	8	17	7,69%
	9	18	7,69%
	10	19	30,77%
18 SOFT	3	21 SOFT	7,69%
	2	20 SOFT	7,69%
	A	19 SOFT	7,69%
	4	12	7,69%
	5	13	7,69%
	6	14	7,69%
	7	15	7,69%
	8	16	7,69%
	9	17	7,69%
	10	18	30,77%
17 SOFT	4	21 SOFT	7,69%
	3	20 SOFT	7,69%
	2	19 SOFT	7,69%
	A	18 SOFT	7,69%
	5	12	7,69%
	6	13	7,69%
	7	14	7,69%
	8	15	7,69%
	9	16	7,69%
	10	17	30,77%
16 SOFT	5	21 SOFT	7,69%
	4	20 SOFT	7,69%
	3	19 SOFT	7,69%
	2	18 SOFT	7,69%
	A	17 SOFT	7,69%
	6	12	7,69%
	7	13	7,69%
	8	14	7,69%
	9	15	7,69%
	10	16	30,77%
15 SOFT	6	21 SOFT	7,69%
	5	20 SOFT	7,69%
	4	19 SOFT	7,69%
	3	18 SOFT	7,69%
	2	17 SOFT	7,69%
	A	16 SOFT	7,69%
	7	12	7,69%
	8	13	7,69%
	9	14	7,69%
	10	15	30,77%
...	...	...	...

4.3.2 Doubles and Splits

In some of the cases analyzed up to now, it is advisable to double down rather than solely ask for a card; in order to understand when this is the case, one is to compute the expected value obtained from a single additional card and multiply it by two (as the bet is doubled, each win and each loss will both be twice as large). At this point, as for the previous cases, the odds are compared to the dealer's so as to establish which one is the favorable scenario.

A similar approach is taken to derive the strategy for the splits; this time, however, the procedure is longer, as it is necessary to compute the probabilities for each of the following scenarios: both hands are won, both are lost, one is won and the other is lost, one is won and the other is drawn, one is lost and the other is drawn (remember, in fact, that splitting entails playing two independent hands with two separate bets). The computational procedure, however, is equivalent to the one seen for hitting/standing, with the additional difference that it starts from a single card and, hence, the second one is always given.

5. How different rules and scenarios influence the advantage and the expected value

Under the most frequently used rules, that is, the American ones, playing perfect Basic Strategy with a running count equal to 1 reduces the house's edge to 0.5%. For every positive true count, then, a 0.5% of statistical advantage moves from the dealer to the player. This means that, with a true 1 and classic rules, the odds are balanced (however, if the rules are even slightly better, the situation already turns positive for the player). Thus, with a true 3, the player experiences a 1% advantage. Let us look at a demonstration of how additional positive cards lead to a better scenario for the player.

5.1 Advantage variations based on the count

When using a neutral deck, it was earlier explained that there is a 7.69% (1/13) chance of picking every card, except for the 10-valued ones which have a probability of 30.77% (4/13). Let us suppose to be playing at an extremely favorable positive true 4, meaning that 4 low cards have been replaced by 4 high cards; for instance, there are 4 more 10s instead of a 2, a 3, a 4 and a 5. The probabilities are then:

- 7,69% for A,6,7,8,9
- 5,77 for 2,3,4,5
- 38.46 for the 10s

If the dealer has a 6 the probability distribution of the final states is now the following:

- 17: 15,18%
- 18: 8,62%
- 19: 8,81%
- 20: 8,67%
- 21: 8,54%
- +21: 50,19%

If we compare it with the one in the final matrix of the dealer saw in the section 4.2.1 we can immediately see that it's a worse situation for the dealer and a better one for the player.

If the player has a 14 in fact, now the odds of winning are 50,19% instead of the initial 42,32%. It's the same for every other scenario: the more high cards there are in the remaining decks, the higher the chances in favour of the player. And in some cases the basic strategy has to be riadjusted (playing deviations) following the new probability distribution.

5.2 The expected value (EV)

The concepts presented thus far and those to follow serve the card counter in two ways: increasing the Expected Value and diminishing the risk. This section, in fact, is to present the diverse aspects influencing the former, both positively and negatively. Expected Value (henceforth referred to as EV) is the single most important number for a card counter as it represents the hourly earnings estimated under the specific conditions of the table in question and the precise bet spread utilized. An EV of $100 indicates this same figure to be the hourly amount of money generated by that table. As should already be clear, this does not entail that one is to generate such amount every hour, as the variance is remarkably high; in fact, the true value will rarely correspond to the previously mentioned estimate. Instead, it will generally be considerably higher or lower (in the case of losses), while it is the average over several games that will actually near such value.

Let us now also introduce the concept of Actual Value; while EV is the estimate of how much one is to generate on an hourly basis, the Actual Value indicates precisely how much has been generated, obtained by dividing the total earnings or losses by the total hours played. If a player's Actual Value diverges significantly from the EV, it means that either they are still in the short term, hence statistics has not run its course yet and they are still subject to luck, or otherwise they are not playing perfectly (something instead taken for granted when estimating the EV). As a matter of fact, a 90% perfect player (thus taking the optimal decision 90% of the time) does not have 90% of the Expected Value but rather is at a disadvantage. With just a few mistakes, in blackjack, the whole advantage is lost, therefore a high degree of precision is required.

To avoid lengthy and complex mathematical calculations, card counters use specially created programs to calculate their EV based on bet spread, rules and number of rounds per hour. The two most famous are CVCX and the PRO Betting Software of blackjack apprenticeship, both of which are paid programs.

Mathematically, what you do is multiply the advantage-disadvantage probabilities of each situation, by their probability of appearing, by the number of hourly rounds to, finally, the amounts bet.

5.3 Deck penetration

A fundamental concept in this regard is Deck Penetration (DP), which indicates how many decks are played before being shuffled up again. To illustrate, if in a six-deck game four and a half are dealt before shuffling, then the Deck Penetration equals exactly four decks and a half. This value heavily influences the EV, as the statistical advantage increases with the number of decks played. Indeed, there is a considerable difference between a table with a DP of four and one with a five-deck DP; given equivalent rules, bet spread, and all else, in fact, there will be a difference in profit of about 100% between the two (the table with DP equal to 5 will be earning twice as much).

Such difference becomes clearer with a numerical example:

- Rules: 100 rounds per hour, dealer hits soft 17s, double after splitting, no re-splitting aces, splitting a maximum of 4 hands, surrender not allowed
- Bet spread: -2 leave the table, 0-1 table mininum, 1 = 100$, 2+=true count x 100$ x 2 hands
- EV with a penetration of 4 decks = 117$
- EV with a penetration of 5 decks = 254$

Deck Penetration is therefore one of the factors predominantly influencing EV. Tables with an unfavorable Deck Penetration are ones either not profitable or even not beatable in the first place (as will be seen later when discussing the differences between the various casinos). When decks are shuffled, the counting is reset and with it the player's edge. What is more, most of the profit is made precisely toward the end of the shoe, as it is at that point that the counting is more likely to have increased. The frequency with which the shuffling is performed varies for each casino and even table, but the player can be informed of such detail through a plastic card inserted in the decks; upon being drawn, such card signals it is time to shuffle.

5.4 Different rule frameworks

As mentioned several times, blackjack rules vary considerably; this section is precisely centered on this aspect, presenting an overview of the various rule sets with their corresponding impact on the player's edge:

- Rule Player's edge
- NHCR -0,13%
- S17[5] +0,20%
- Surrender +0,62%
- DAS[6] +0,14%
- RSA[7] +0,07%
- Double deck game +0,22%
- 6:5 blackjack[8] Usually not beatable

5.5 Risk of ruin

Due to the elevated variance of the game, each player has a certain chance of losing their entire bankroll even if playing perfectly; thus, there is no guarantee for the Actual Value to approach the EV, especially in the short run. This value is also readily computed through the abovementioned software programs; it depends enormously on one's bankroll and is calculated through the standard deviation associated with a specific set of conditions. The greater the standard deviation, the higher the risk of ruin.

The application of this concept of risk of ruin is to be explored in section 6.

5.6 Order of operations

A player has to take several decisions among Basic Strategy, surrender, insurance, and deviations. It is thus crucial to understand in which order to perform them, as otherwise some of these will not be possible (asking for an additional card, for example, entails not being able to surrender nor take insurance anymore). Here is the order to follow to avoid precluding oneself the various options before they are thoroughly considered: first and foremost, if the dealer has an ace and insurance is offered, one should evaluate whether the True Count surpasses 3; afterward, if surrender is available, it is considered as an option; finally, one is to ask themselves whether the current True Count and the dealer's card reflect a deviation case, or else they are to proceed with Basic Strategy. Here follows the correct order of operations, then:

- Insurance
- Surrender
- Deviations
- Basic strategy

[5] Stand sof 17 rule: the dealer has to stand with a soft 17 instead of hitting like usual
[6] Double after splitting allowed
[7] Resplitting aces
[8] Blackjack games where the BJ is paid 6:5 instead of 3:2

5.7 Dobule hands

Several tables allow for the possibility to play more than one hand simultaneously, which constitutes an advantage for the player in the case of a positive count. Particularly, as a result of these hands' covariance, double bets can be made under the same risk which are collectively higher and thus generate a greater EV. To illustrate, a single bet of $100 entails the same risk as two separate ones of $75, which however produce a greater EV. Naturally, this occurs when the count is positive, otherwise playing multiple hands is not recommended. Card counters' aggressive bet spreads thus usually branch into double hands starting from a true 1 or a true 2, depending on the rule framework (as seen initially, at a true 1 the player is not necessarily at an advantage).

As an example, here are two bet spreads; one with double hands and the other without, including their respective EVs:

- True count: Bet ($):
- -2 leave the table
- -1 table minimum (suppose $25)
- 0 table minimum (suppose $25)
- 1 $100
- 2 $200 x 2
- 3 $300 x 2
- 4 $400 x 2
- 5 $500 x 2
- 6+ $600 x 2
- EV: 173$

- True count: Bet ($):
- -2 leave the table
- -1 table minimum (suppose $25)
- 0 table minimum (suppose $25)
- 1 $100
- 2 $200
- 3 $300
- 4 $400
- 5 $500
- 6+ $600
- EV: 104$

5.8 The countermeasures adopted by casinos

Casinos are well aware of the existence of card counters and have throughout the years adopted a series of countermeasures to weaken their position (i.e., their EV, from a statistical standpoint). Counting cards was in fact considerably more profitable in the 80-90s as compared to now. Nonetheless, it is still a possible practice to carry out, especially in the United States. In Europe and in the rest of the world, in fact, continuous shuffle machines (CSMs) are used in most casinos; the way these devices work is that, at the end of each turn, cards are inserted back into the machine and the count thus repeatedly goes back to zero. Tables employing CSMs are thus unprofitable and card counting is inhibited. In the United States, on the other hand, games are still regularly played with 1, 2, 6 or 8 decks; however, two main features have changed in the past years: the cutting card is positioned earlier than before, thus reducing the Deck Penetration, and security pays greater attention to players' strategies.

The question naturally arises of why they do not adopt CSMs themselves too, so as to completely eliminate the practice of card counting. The answer lies in the nature of gamblers: these prefer a more "manual" gameplay, as CSMs can give the impression of there being something to hide in the decks or of less favorable cards being inserted into the decks by these devices. USA casinos, therefore, prefer having to occasionally deal with card counters rather than having devoted customers disappointed by these technological disruptions.

A further doubt could emerge regarding the legality of counting cards: the practice is actually completely legal; the reason why casinos are nonetheless allowed to avoid offering their services to someone stems from them being private properties. As such, in fact, they have the right to apply a selection process to players, and it is also permitted for them to expel these latter. The only exception to this is presented by the casinos of Atlantic City where, precisely as a result of a dispute between counters and gambling establishments, the State has enacted a law prohibiting casinos from discriminating against players in this way; for this very reason, casinos have acted accordingly by adopting CSMs in almost all of their tables.

Going back to the casinos where counting is allowed, one ought to understand how security identifies card counters. The main way it is done is by noticing abrupt alterations in the bets reaching as much as a 10-20 multiplier, as well as checking whether one's increase in bets corresponds to a high count or whether they are playing perfectly. Having said that, such event (nowadays) does not represent a situation of danger for the player, as might instead be typically portrayed in movies. Casino staff will simply ask the player to leave or, in some cases, to be allowed to play but only with a fixed bet (called flat betting) as it is possible to see in this video[9].

5.8.1 online casinos

Online casinos are almost never beatable, or at least, they have a really low EV. That's because what they do is using CSM or games with a really low deck penetration (usually playing 8 decks dealing only 4 of those). The latter is often used for live online casinos[10]. They also often use software that analize players' game in order to detect suspicious behaviors so online casinos are not places for card counters.

[9] Blackjack back off: https://www.youtube.com/watch?v=6HjwzJyCQlQ
[10] Online casinos in which a real dealer deals the cards;

6. Managing a Blackjack Team like a Business

6.1 Team play

Team-played Blackjack has become quite popular thanks to the movie "21" and indeed, although rather romanticized and not completely correct from a technical perspective, it is based on a true story; it is the case of the MIT team, made from Harvard students in 1979 and operating successfully for around twenty years. Team play is in fact a major side of the world of blackjack; this section will deal with its most significant aspects.

Team play can take on various forms: a team can be composed of a couple of players just like it can be made of dozens; it can have an "improvised" structure or rather a complex management organized through roles. In blackjack, anything adding to the profit and reducing risk is sought after, and these are precisely the objectives of coming together to form a team; such objectives are achieved by working on different aspects.

First of all, before even defining the playing strategies, comes the pooling of the bankroll, meaning the quantity of money that a player (or rather *the players*, in this case) dedicates to the activity. The larger the bankroll, the higher one can afford to bet during the statistically favorable moments, hence the higher the profit. What is more, increasing the bankroll drastically reduces the risk of losing everything: if two players combine their bankrolls, reaching a joint one twice as large as before, they will not have half the risk of losing everything but rather exponentially less and less. This is because, in addition to being able to play longer, the variance is significantly reduced as a result of the increasing number of playing hours; having more players, in fact, naturally translates to more playing hours.

Indeed, it is worth refreshing the notion that, even playing flawlessly, one can still lose and by a significant amount. Professional players with a rather aggressive bet spread can incur even tens of thousands of dollars of losing streaks; moreover, if the bankroll is not ample, one risks losing a major chunk or even all of it due to these severe fluctuations. Thus, before even considering the type of gameplay to be employed, the benefits are already evident for players simply deciding to join forces by combining their finances; increasing the Expected Value (EV) and reducing risk are, in fact, the primary objectives of the professional.

Now, let us delve into how the actual gameplay at the table changes (if it does change). Teams can either choose to act with a combined gameplay, as seen in the movie "21" and analyzed shortly, or the individual players themselves can also decide to move independently. Indeed, the true advantage of team play is not merely being in the same casino and being able to implement otherwise impossible group strategies, but rather what was described earlier: the increased bankroll. Therefore, there are two types of teams, or rather two modes in which the same team can choose to act from time to time: individual play and simultaneous play. Let us analyze both.

6.1.1 Individual team play

When a team opts for the individual strategy, it focuses exclusively on the benefits coming from the increase in bankroll. Thus, no matter the number of players, they will each play independently, in different casinos. However, they will play with shared funds (the team's bankroll) and generally follow the same guidelines in terms of bet spread and strategies.

As the common objective is to reduce the variance, the winnings (as well as the losses) are distributed equally among players. Once the strategy has been established, in fact, a professional player shall not take any decision, but rather solely execute the optimal move in each scenario, as previously explained. This entails that, since everyone is playing in a mathematically perfect manner, the winnings of a single session do not depend from the players themselves, but from luck, or more precisely, variance. Two distinct players can both act perfectly, play with equivalent bet spread and rules, and within the same time frame; still, one might win tens of thousands of dollars, while the other lose just as many. Such volatility is one of the main issues faced by the players, bringing them to join forces and form a team.

Players are thus rewarded not for their direct winnings or losings but for the actual hours of gameplay based on the team's total earnings. Once a target overall profit is reached, it is split among the members of the team according to their individual contributions to the cause; again, such split is not performed based on how much they earned but on how long they played, giving for granted that they all fulfill a level of perfection at the craft. If a team is composed of three players having each contributed equally to the initial shared bankroll and having played 90, 70, and 55 hours respectively, they will receive $8372, $6512, and $5116 from a total win of 20 thousand dollars. Dividing this sum by the 215 total hours of gameplay, then, the hourly Expected Value (EV) comes to amount roughly 93 dollars, an average value for a moderate and not particularly aggressive gameplay.

It is immediately evident how paramount mutual trust is within the teams, because even if a managerial oversight can employ control and security measures, the single winning players could still falsely declare having suffered losses, so as to directly pocket the profits without sharing them with the others. A behavior of this kind sustained over the long run, however, is easily detected due to the teams usually being extremely meticulous in measuring the results of each player; since these latter are to converge with time, a malicious member would eventually find themselves having a lower actual value as compared to its peers.

That being said, although more solitary and less theatrical, individual team play turns out to be the most common and profitable approach. In order to understand why and whether it is convenient to opt for a simultaneous gameplay, let us first explore the advantages and drawbacks of such strategy.

6.1.2 Simultaneous team play

Simultaneous gameplay does not originate as an attempt to earn more, since the maximum gain is actually obtained in the individual game; rather, it arises as a strategy to counteract a reduction in earnings. It is thus the solution to a problem, the problem of bypassing security. As seen earlier, casinos can understand, or rather suspect, whether a player is a card counter and they have the right to expel him or her. In the long term, this can constitute a significant problem for this latter, as they risk having a reduced number of places to play at.

At this point, one might wonder how security is able to catch a player counting cards. Several factors are taken into consideration, but the most representative one is that of the bets, as seen before, as the substantial fluctuations they undergo in accordance with the counting can be evident. Simultaneous team play stems exactly as an attempt to mask this feature of the card counter. The principle is the following: both those betting sparingly and those betting considerably must keep such behavior throughout. The goal is for the low better to keep track of the count of their table, so that once this latter "heats up" the high better can play it.

Simultaneous gameplay, in fact, involves two types of players: the so-called "big player" and the "small players", typically in a ratio of 1:3 or 1:4. They must all operate concurrently in the same casino. The structure of it all is the following: small players always stay in different tables and their objective is to keep the count of such table playing perfectly but always betting the same quantity, save for the cases of betting deviations. This approach allows them to minimize the losses while avoiding being noticed (they are still at a loss, however, as earnings can be made exclusively by adjusting the bets). When their table's count reaches a favorable level, usually a true count of 2, the small player is to call the big one to join in and inform them of the running count (all done through specific signs rather than openly, an aspect to be deepened later). The big player is now supposed to initiate playing with an already high bet and they should adhere exclusively to such strategy. They will still follow the value of the true count, for instance betting 300 with a true count of 3, 400 with a 4, and so on; yet, from the perspective of security they will solely appear as a player betting heavily and varying their bets only within a legitimate range, as any player would. Suspicion arises mainly when the same player first bets the minimum of table, namely 25 dollars, to then suddenly rise up to several hundreds. Upon the count going below a true 1, the big player leaves the table to move toward another one signaled as “hot”.

The reason the ratio between small and big players follows a 1:3 or 1:4 proportion is that each table is only hot for a significant minority of the time, hence requiring an optimal number of small players which allows to keep the big player engaged without wasting time and thus potential earnings. Meanwhile, small players should not be too many, as otherwise the following situation plays out excessively often: two distinct tables heat up simultaneously with only one being exploitable. Such ideal ratio, therefore, lies between 1:3 and 1:4 (tables using 6 decks present a true count of 2 or greater for 16% of the time, while tables with 2 decks have it in about 26% of cases).

As mentioned above, players communicate through signals so as to cover the activity of card counting; consequently, players must not only avoid actions clearly connected to counting, but also hide their relationship with each other. It is a legal activity; however, since casinos only want losing players, one must act accordingly. Thus, in the eyes of security, the team must be completely nonexistent and unidentifiable. There are two pieces of information the small player is to pass to the big one: the heating up of a table and its running count at that same moment (the true count can be deduced by the big player on their own upon knowing the running one). No rules are typical here, as otherwise teams would be easy to spot if they all behaved alike. Every team, in fact, autonomously establishes its own “code”. In the movie “21”, small players signal the table to the big by folding their arms behind their backs; similarly, one can cross their legs, make use of their clothes (wearing or taking off a jacket or a hat), stand up for a moment. The creative element is essential here; the only guideline to follow is for the signal to be clearly identifiable by the player, yet not by security. This is also why it is worth frequently changing one’s signs.

Once the big player has reached the table, the count must be communicated to them. This can be done in several ways, among which the placement of one’s chips is often used: all of them are kept together except for a single pile, the amount of which indicates the running count. A similar approach involves utilizing one’s fingers, or otherwise one could improvise a conversation, order something to drink, or make a comment containing specific words to which the team has assigned numerical values. This latter is employed in the abovementioned film “21”, where players make use of the following code: “pool” equals “8” (the pool ball), “cat” entails “9” (cats have nine lives), “eggs” indicates “12” (egg boxes have 12 slots), “sweet” stands for “16” (sweet 16), et cetera. The appropriate word simply needs to be mentioned during an improvised conversation. This is also employed by blackjack players in the real world, although signals are more common than words.

As previously explained, playing in a team originates as a way to bypass security; yet, hourly winnings do not grow and instead slightly decrease. In fact, there will be moments with no hot table during which the big player’s time will pass without any actual gameplay, thus producing no EV. Conversely, albeit having chosen the optimal 1:3 to 1:4 ratio, there will be times where a table heats up while the big is already busy at another, constituting once again wasted EV. Moreover, to avoid the risk of making the big move too rapidly, the small player tends to call them at a true 2 or above rather than at a true 1, as otherwise the count might go back down right away; moments of this kind are again sources of wasted EV. On the contrary, an individual player is able to maximally exploit the situation: when the count rises, they play accordingly, whilst they change table when it lowers excessively. This type of gameplay, however, has the weakness of drawing attention as a result of an extremely fluctuating playing style.

In the end, there are pros and cons to both strategies. Ultimately, the choice is made based on the situation at play: casinos paying closer attention to security will require a simultaneous gameplay, while for the more permissive ones an individual team play strategy will suffice. At the same time, as a player begins raising the security's suspicion, they will have to opt more frequently for a low-profile playing style, thus engaging in simultaneous team play.

6.2 Structure of a team

Let us now tackle the organization of the team from a management perspective. If such management is to be taken to a professional level, in fact, a division of roles is necessary. In addition to the players, a team also hosts two more figures: investors and one or more managers.

6.2.1 Investors

The role of the investor arises from the fact that those approaching the discipline usually do not have an abundance of finances, yet they do have the time and will to acquire the skills needed; on the other hand, there are those who wish to take this opportunity from an economic standpoint but without being directly involved as a player. Alternatively, whoever is playing might have a personal bankroll of their own, however the scarcity of this latter constrains their ability to maximize earnings without exponentially increasing risk.

This is how players and investors thus meet in the middle: the former play with the money of the latter in exchange for a share of the profit. In the previous illustration of a team composed of three members making a $20,000 profit, let us assume that such profits have been established to be split into a 60% share for the investors and a 40% one for the players; therefore, 12 thousand are meant for the investors, while 8 thousand are to be divided among the players (to then be further split according to the hours played). The percentages through which to divide the winnings are determined by the team and typically range between 60/40 and 40/60.

The bankroll is usually formed partially by the investor/s' money and in part by the players'. That being the case, profits are distributed in proportion to each individual's contribution to the bankroll. Those investing a greater portion of capital are set for larger potential gains, yet they are also exposed to more risk. A parallel with financial markets can be made, although investing in a blackjack team entails significantly higher earnings (as well as risk), given the same time frame. Using again the example of the three-player team profiting 20 thousand dollars in a month, let us assume the total bankroll to be 200 thousand dollars, formed by four investors contributing 50,000 each; with the same division of profits considered earlier (60-40), the 12,000 are to be split among the four investors, hence receiving 3,000 each after one month of providing the capital. A monthly gain of 6% on the initial 50,000, then, characterizes the peculiarity and volatility of this type of investment, rather high given the time frame. These figures, although hypothetical, reflect reality rather well (EV of 93 dollars and variance bounded by 215 hours of gameplay), as will further be seen later with even more specific instances.

Having analyzed the roles of the players and of the investors, we now move onto the manager's.

6.2.2 Managers

As can be inferred from the previous pages, this whole process requires a crucial organizational aspect. Firstly, the management of the bankroll is delicate and complex; the same goes for the division of the profits, which has to follow precise rules clearly established in advance and considering every eventuality, especially the negative one of incurring into some unfavorable variance. Then, the choice of the casinos suitable for playing and the consequent strategy to adopt (including the choice between individual and simultaneous team play) is another aspect requiring considerable effort. Researching casinos with favorable rules, as explained earlier when discussing the rules influencing the EV, entails a range of competences unrelated to the technical ability at the game itself. All of these aspects are the responsibility of the team's manager/s, who is/are to receive once again a share of the profit in exchange, as they would in a regular company.

The hiring and training of potential new team members also deserves a mention. Expanding the team, in fact, has a direct impact on the increase in gains, which can however also be negative if such new members are not selected accurately. Every team, before enlisting a new member, conducts an extensive series of technical tests on them to be passed only upon reaching or closely nearing a level of perfection in all aspects of the game. It is also necessary to determine whether the player will be able to act perfectly even in the distinctly more stressful place that is the real casino. In fact, tests are usually carried out twice: a first time in a "private" location, while a second one directly in a casino, so as to also test one's nerves and overall mental clarity management in such a chaotic place. Furthermore, these tests are not to be considered valid indefinitely because a player can lose their technical "perfection", if not properly trained. It is then the managers' job to ensure each and every player maintains their abilities; this sphere also includes making sure that each player acts honestly, avoiding the abovementioned risk of cheating. Managers can thus decide to sporadically send anonymous observers with the objective of understanding whether the winnings or losses declared by the players correspond to the truth.

As can be inferred, all of these tasks cannot be optimally performed by someone also directly involved in the game, due to time and resource constraints as well as to avoid potential conflicts of interest. It is also true, however, that the three roles of the team can often overlap, meaning that part of the investment can also be made by the managers and by the players, while the manager can also occasionally decide to join the table aiming to generate EV.

Any managerial decision must be driven by data and statistics, which is exactly why the next section will simulate acting as the manager of a blackjack team, establishing its strategy, bet spread, and overall functioning.

6.3 Case study

Let us suppose having a team composed of 8 players, 1 manager and 4 investors (who can also sporadically take on the role of players or of the manager); these 4 investors stake $25,000 each, for a total initial bankroll of $100,000. The plan is to play for a month (four weeks) with the goal of accumulating 25 hours of actual gameplay at the table per player each week. Employing an approximation rounding down, which will be used throughout the entire case study to stay safe, let us suppose they all reach an amount of 20 hours every week (half a workweek, thus quite feasible): the resulting total game time will be of 640 hours (20 hours x 8 players x 4 weeks) of effective EV generated at the table, ignoring the time taken to reach and leave the casino, change the chips, take breaks, et cetera. That is also why it is key to set the goal at 25 hours, so that the true 20-hour target is almost certain to be achieved.

To recap, these are the parameters at play:

- 8 players, 1 manager, 4 investors;
- $100,000 of total starting bankroll;
- 640 hours of play time across 4 weeks.

The profits are to be split in the following way: 45% are to be equally divided among the 4 investors, 45% among the 8 players (supposing they all play the same amount of hours), and the remaining 10% goes to the manager. Expenses for the players' trips are to be deducted before the division of profits. At the end of the month, if the team records a loss, it all weighs solely on the investors; the loss for the players and the manager, in fact, only entails the time and the effort put into the work.

Division of profits (after deducting expenses):

- 45% to the investors;
- 45% to the players;
- 10% to the manager.

The players are supposed to all use the high-low method, with the complete strategy and the top 20 playing deviations. Playing perfectly is a prerequisite for all of them; their competence is established through a test comprising 3 shoes of 6 decks each with only 1 mistake allowed in total. The same procedure has to then be performed again in a real casino with an entire six-deck shoe.

Here is the bet spread to be used by all players in every game scenario (six decks, double decks, simultaneous gameplay):

- True count: Bet ($):
- -2 leave the table
- -1 table minimum (suppose $25)
- 0 table minimum (suppose $25)
- 1 $100
- 2 $200 x 2 hands
- 3 $300 x 2
- 4 $400 x 2
- 5 $500 x 2
- 6+ $600 x 2

It is definitely an aggressive approach, but the magnitude of the bankroll allows for it; in fact, as will be explained, the overall risk is extremely low.

A strategy of this kind will generate an hourly EV of $173 at the six-deck games and an hourly EV of $588 at the double-deck ones. The rules used to calculate this Expected Value (computed with Blackjack Apprenticeship's "Pro Betting Software") are the following (frequent American rules, corresponding to where the team is located):

- 6-deck games: 100 rounds per hour, penetration of 4.5 decks, dealer hits soft 17s, double after splitting, no re-splitting aces, splitting a maximum of 4 hands, surrender not allowed;
- 2-deck games: same rules as for the 6 decks, but with a penetration of 1.5 decks.

Maintaining the approximation of rounding down, we will consider the 173 EV as a 150 and the 588 EV as a 500. While approximating the hours played served to represent the case of players not being able to play as long as requested, this reduction of the EV instead reflects possible errors made by the players (despite the rigorous tests they are subjected to) as well as some tables being slower, with the dealer distributing less than 100 rounds per hour.

- Theoretical EV: Considered EV:
- 6D: 173$ 150$
- 2D: 588$ 500$

Something worth mentioning at this point is that some of the calculations we are making are objective, as in the case of the EV generated under a specific set of rules with a certain bet spread. Others, conversely, are subjective, namely by how much the EV will be reduced with a simultaneous game and, in general, whether the actual values will match the expected ones. As a result of this and of the high variance of the game itself, albeit 640 hours being a considerable amount, the final outcome can still potentially be much better or much worse than what is suggested here.

The strategy for the casino is the following: 80% of the time will be spent at six-deck games, whilst the remaining 20% at the double decks. The reason for this, despite the EV being considerably higher (333% more), is the fact that being backed off from a double-deck table is substantially easier and quicker, unfortunately, as well as it being inconvenient to spend too much time there at once. This 20% of the time spent at the double-deck games is to be played 50% in a simultaneous team play strategy and 50% in an individual one. The procedure for the simultaneous game is to form two groups operating in two different casinos, each formed by three small players and a big one. Going back to the six-deck games, these will instead be exploited for 70% of the time in an individual fashion and 30% in a simultaneous one, reserved to the occasions involving casinos with particularly watchful security. For the six decks, instead, this is not strictly necessary, which is why we opted for a 70/30 instead of the 50/50 employed in the double decks.

Summing up, the 640 hours are to be split as follows:

- 358 hours: 6 decks, individual;
- 154 hours: 6 decks, simultaneous;
- 64 hours: 2 decks, individual;
- 64 hours: 2 decks, simultaneous.

Supposing that the simultaneous games will be 2/3 (0.66) as profitable as the individual ones, the following total EV is then generated at the end of the four weeks:

- \$53.700 (358 x 150) from the 358 hours with 6 decks and individual gameplay;
- \$15.246 (154 x 150 x 0.66) from the 154 hours with 6 decks and simultaneous gameplay;
- \$32.000 (64 x 500) from the 64 hours with 2 decks and individual play;
- \$21.120 (64 x 500 x 0.66) from the 64 hours with 2 decks and simultaneous play.

Resulting in a total profit of \$122.066.

Then, supposing an expense of \$100 per day for each player spread between eating, commuting, and accommodation, the total spending amounts to \$22.400. One hundred dollars may seem insufficient to cover all the trip-related expenses, yet one needs to keep in mind that casinos frequently offer so-called “comps” (complimentary items) to high-stake players, meaning food and stays at their facilities. This is done as an incentive for the player to stay as long as possible, as such loyal gamers constitute a precious asset for the business (until they are found out to be card counters, of course).

Rounding \$22.400 to \$22.066, we obtain a total profit after expenses of \$100,000, that is a 100% increase over the initial sum:

- Profit after deduction of expenses: \$100,000.

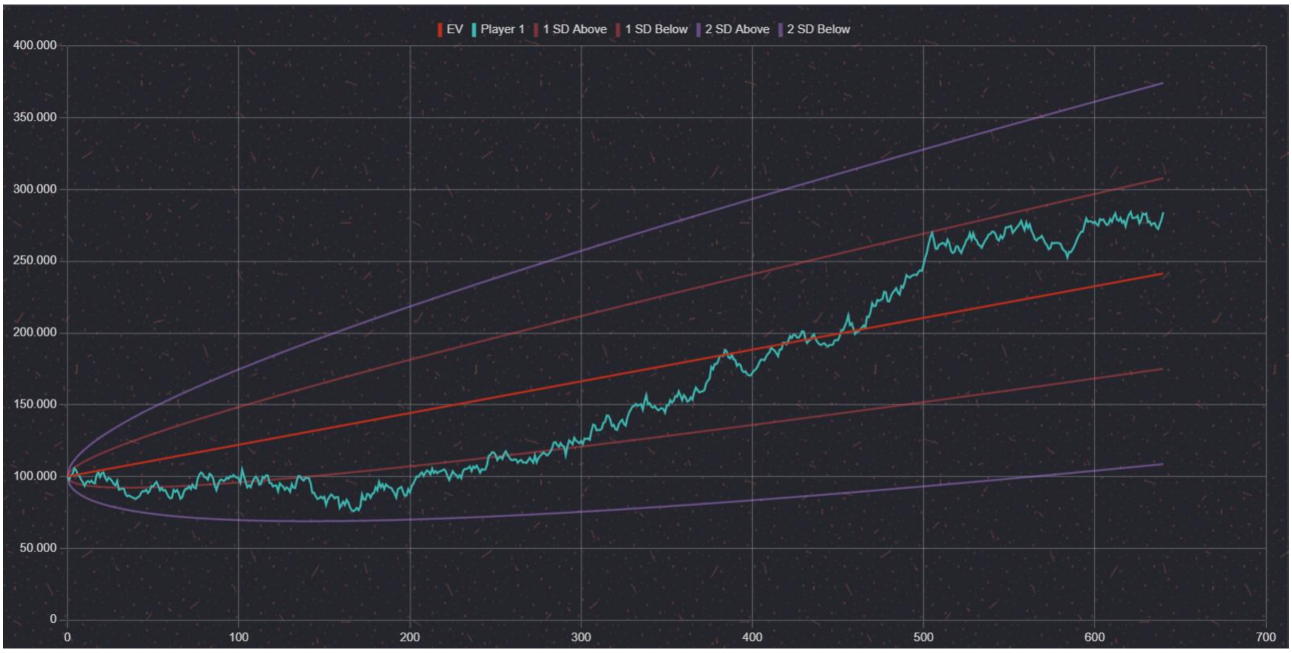
Let us now divide the profit among the members of the team:

- $10.000 to the manager;
- $11.250 to each investor (45% over the investment of 25,000);
- $5625 to each player ($70 for each hour played without any risk).

For what concerns the risk of the entire operation (solely from a statistical point of view), using once again Blackjack Apprenticeship's "Pro Betting Software" reveals that the chance of complete ruin, meaning the risk of losing the entire bankroll, is 0.83% for the six-deck games and 0.11% for the double decks. Multiplying these two risks, weighted based on the amount of time played at each type of game, results in a 0.664 x 0.022 = 0.686% risk of losing the whole bankroll.

6.3.1 Conclusions

The following graph represents a simulation of the strategy that we discussed. As we can see the EV (red line) is slightly higher than what we saw (122.066$) because in this case the EVs of both 6 decks games and double decks were not reduced. It was instead calculated with 640 hours of play and with a 0.33 reduction in efficiency for the simultaneous play. It was done using the already mentioned Blackjack Apprenticeship's "Pro Betting Software". We can see the effects of variance with "unlucky" variance in the first part and "lucky" one in the second, ending with an AV slightly higher than the EV and a resulting profit of 180k.



7. glossary

- Actual Value (AV): an indicator of precise earning, it is calculated by dividing the total earning (or total loss) by the total hours played;
- Bankroll: the capital available to play with;
- Basic strategy: set of the mathematically correct moves for each game situation;
- Bet Spread: pattern indicating the variations in the bets. It is calculated by multiplying the True Count values with those of the Betting Unit;
- Betting Deviations: variations in one's bets following the Bet Spread;
- Betting Unit: the unit that indicates by how much to reduce or increase one's bet with each change in the True Count;
- Blackjack: score equal to 21 obtained with an ace and a card of value 10; provides a reward equal to 3:2 (or 6:5 in some cases) of one's bet;
- Busting: exceeding 21 with the sum of one's card values, losing the hand and therefore the bet;
- Card Counting: summing and subtracting cards value from which the Running Count is derived;
- CSMs (Continuous Shuffle Machines): machines that replace the shoe, in which cards are shuffled after each round;
- DAS: Double After Splitting;
- Dealer: the person who deals the cards and, therefore, plays in the role of the casino;
- Deck Estimations: estimating the amount of decks (whole or not) in the discards tray and so, in the shoe;
- Deck Penetration: indicate how many decks are played before being shuffled;
- Doubling: doubling one's bet in order to get a single card from the dealer and end one's turn;
- Doubling on every hand: ability to double without any restriction (for example: doubling allowed only on 9s, 10s and 11s);
- ENHC (European No Hole Card) Blackjack: the dealer does not receive the second card but receives all its cards after the players have made their choices;
- Expected Value (AV): average hourly earnings of a table, estimated for specific conditions and play;
- Playing Deviations: playing strategies that deviate from the basic strategy following the variation of True Count;
- Pure Pairs: pairs without aces;
- Push: a situation of equality between the score of the player and the dealer, which allows the first to take back his bet without winning or losing anything;
- RSA: Resplitting Aces;
- Running Count: represents how many more high cards (or fewer, if the count is negative) there are among those still to be played;
- Shoe: the container from which cards are drawn;
- Splitting: dividing one's playing hand into two separate ones, in case the two player's cards have equal scores, playing a bet equal to the one initial one. A second card will be added to each card and both hands should be played simultaneously. When splitting aces, on the other hand, you receive only one card without the possibility of making other choices;
- Surreder: possibility of surrendering by losing half of one's bet;
- S17 (soft 17): the dealer does not hit when he has an ace and a 6;
- True Count: indicates how many more high cards there are per deck still to be played. It is calculated by doing the ratio of Running Count to remaining decks;
- 2 Decks Games: tables where only 2 decks are played;
- 6:5 Blackjack: tables where a blackjack is paid 6:5 of the bet played instead of 3:2.

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