Dynamic Rebalancing Model applied to the All Weather Portfolio

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Abstract

This research paper goes through the development and evaluation of a dynamic rebalancing model thought for passive portfolios, with a focus on the empirical implementation on the Bridgewater All Weather portfolio. The study here presented aims at investigating whether it is possible to refine long-term passive portfolio strategies by looking for market rules which manage to provide better performances.

The paper presents the fundamental principles of portfolio theory, briefly exploring the origins of Markowitz Portfolio Theory and Risk Parity portfolio theory. The second section of our study introduces the Bridgewater All Weather portfolio, an investing strategy renowned for its ability to thrive across all economic seasons. Our comprehensive analysis of this portfolio serves as a benchmark against which we assess the dynamic rebalancing model's impact on the portfolio performance. The primary objective and core section of the whole study is to introduce a practical and adaptive model for rebalancing a portfolio over extended investment horizons, which dynamically and automatically moves the allocation according to certain guidelines.

Our findings reveal valuable insights into the effectiveness of this approach, shedding light on its ability to optimize returns while managing risk across different asset classes.

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Introduction

Within the intricate world of investments, portfolio management stands as a crucial discipline, requiring a delicate trade-off between the pursuit of optimal returns and the diligent management of risk. For investors, both individual and institutional, building portfolios that offer a balance between these two objectives has been a constant endeavor.

In recent years, passive investing has taken center stage, offering a compelling proposition through its simplicity, cost-effectiveness, and adherence to a set-and-forget philosophy. Yet, within this strategy lies a challenge that has represented the focal point of our study: optimization of passive portfolios over extended investment horizons.

This research paper embarks through a journey into the world of portfolio management, with the aim of developing, assessing, and refining a dynamic rebalancing model designed for passive portfolios, able to automatically adjust asset allocation. In our pursuit of this goal, we chose the Bridgewater All Weather portfolio as our primary case study, a portfolio known for its resilience and adaptability across various economic climates.

Our study begins with a brief tour through the main milestones of portfolio theory. Modern Portfolio Theory (MPT), attributed to the work of Harry Markowitz, initiated a paradigm shift in the investment world by introducing the concept of diversification. MPT argued that by investing in uncorrelated asset classes, investors could achieve risk reduction and optimize risk-return profiles.

Building upon the foundations laid by MPT, we analyze a further approach, Risk Parity portfolio theory. This innovative theory seeks to address some of the limitations of traditional asset allocation methods. By equalizing risk contributions from various asset classes, Risk Parity aims to construct portfolios with a more balanced risk profile. It is a theory acutely attuned to the dynamic nature of financial markets, recognizing that traditional allocation strategies may not succeed in offering compelling advantages in terms of risk-return.

Our investigation takes a practical turn as we move to the core section of the study, the dynamic rebalancing model, which distinguishes itself from static portfolio strategies through its adaptability and responsiveness to changing market conditions. This model, developed and tested across various asset classes, represents a proactive approach to portfolio management, by making the portfolio allocation change according to some inputs. While the All Weather portfolio aims to find the best combination of asset allocation in order to have the best performing portfolio in any economic condition, the dynamic model allows it to provide a better performance in terms of both risk and return.

As we navigate through the encouraging results deriving from the implementation of our model, we propose some further developments which might be useful to adopt in refining this rebalancing approach.

1) Portfolio theory

Portfolio allocation is the process of investing in multiple assets with the aim of achieving maximum return while reducing risk. For a good understanding of the following sections of this paper it's important to provide a broad overview of how asset allocation strategies have changed over time, in order to get a solid comprehension of the All Weather philosophy. Specifically, this section will go through two of the most popular portfolio allocation strategies: Markowitz portfolio allocation and Risk Parity portfolio allocation. Both these strategies have their strengths and weaknesses, and the choice of the most suitable approach depends on various factors such as risk appetite, investment goals and market conditions.

1.1) Modern portfolio theory

Among the various research and theories which have been developed to address the problem of building an efficient portfolio, the portfolio selection model proposed by Harry Markowitz in 1952 is one of the most important and influential theories in this field. Markowitz's approach frames the process of building an investment portfolio as a mathematical problem. The theory posits that by investing in uncorrelated stocks, the reduction in value of one stock can be offset by the increase in value of another one.

The theory developed by Markowitz for building an efficient investment portfolio is based on several key assumptions. First, investors are generally risk-averse and seek to maximize their expected utility. Second, the model assumes there are no transaction costs nor taxes, which simplifies the analysis, even though it may not reflect reality. Third, markets are assumed to be perfectly competitive, which means that all investors have access to the same information and can freely buy and sell assets. Finally, the time horizon considered is a single period, from t to $t+1$, and there are N different assets available for investment.

From these assumptions, the "Mean-Variance" principle is derived, which suggests that investors should choose assets that offer the highest expected return for a given level of risk or the lowest volatility for a given target of expected return. The model specifies that portfolio's variance is function of the correlations among assets:

$$\sigma_p^2 = \sum_i w_i^2 \sigma_i^2 + \sum_i \sum_j w_i \, w_j \sigma_i \sigma_j p_{ij}$$

The objective of Markowitz allocation is then to solve the following problem:

$$min\, \sigma_p = \sqrt{\sum_i w_i^2 \sigma_i^2 + \sum_i \sum_j w_i\, w_j \sigma_i \sigma_j p_{ij}}$$

under some constraints:

- $E(R_p) = \sum_i w_i E(R_i)$
- $\sum_i w_i = 1$
- $w_i > 0$, hence there can't be short positions.

Therefore, the mean-variance principle aims at solving this constrained optimization in order to select the investment weights that will minimize the standard deviation of the portfolio, given a target expected return.

Markowitz observed that when considering various combinations of efficient portfolios, these create a closed area known as the efficient frontier. This frontier represents the set of portfolios that provide the highest possible return for a given level of risk or the lowest possible risk for a given level of return.

Visualizing the Efficient Frontier involves plotting portfolios on a graph where the x-axis represents portfolio's standard deviation, while the y-axis represents portfolio's expected return. The curve connecting the optimal portfolios forms the Efficient Frontier. Each portfolio lying below the Efficient Frontier is considered suboptimal, as it offers lower returns for a given level of risk or higher risk for a given level of return (*Figure 1.1*).

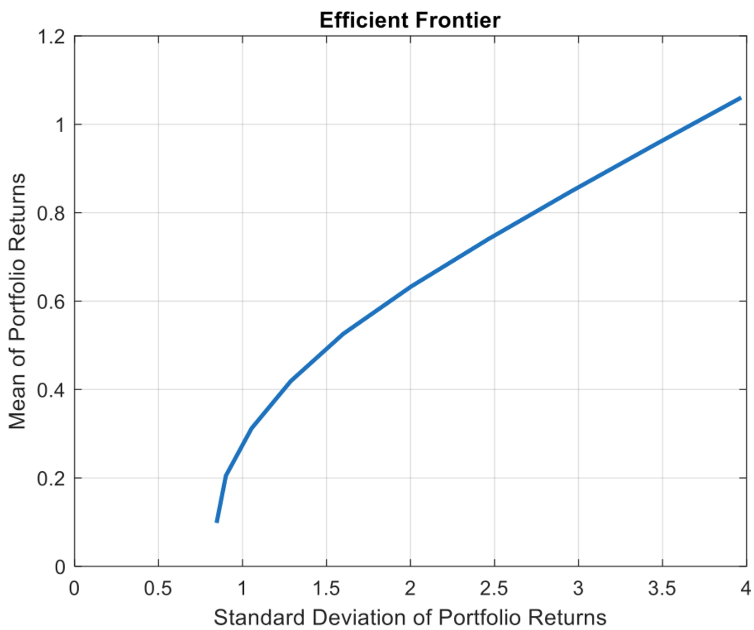

Figure 1.1 – Matlab Script

Markowitz's portfolio theory has encountered consistent critique for over half a century and has not gained complete acceptance among practitioners due to various factors. These include limitations such as variance being an inadequate risk measure, the portfolio's susceptibility to errors in parameter estimation, the theory's exclusive focus on overall risk disregarding the advantageous effects of risk diversification, and the fact that it doesn't consider factors such as liquidity or market conditions, which may affect investors' ability to buy or sell assets. Another important problem of Markowitz philosophy is that it often provides unreasonable optimal portfolios, be that a highly concentrated portfolio or the allocation of large weights on marginal markets.

1.2) Risk Parity **portfolio theory**

One approach that has gained popularity in recent years is the Risk Parity technique. Unlike modern portfolio theory, Risk Parity seeks to balance the contribution of each asset class to the overall portfolio risk. For example, a traditional 60% equity, 40% bond portfolio may not be diversified enough, as 90% of the risk of this portfolio derives from equities. Since stocks are historically three times more volatile than fixed-income securities, a traditional portfolio is not balanced in terms of risk. Risk Parity aims to eliminate this weakness by building a more diversified and balanced portfolio, ensuring that each asset class contributes equally to the portfolio's risk.

To implement this approach, there are several steps to be taken. Let's consider a portfolio of N assets, we will call w_i the weight associated to asset x_i, where $i = 1{:}N$. We want to find the weights that make each asset contribute the same to the overall's volatility.

First, it is necessary to define the total risk of a portfolio. In most cases, this is measured by the volatility of the rate of return of the portfolio. However, VaR[1] can also be used as a measure of total risk, as it allows for the incorporation of skewness and kurtosis.

The volatility of our portfolio is given by the standard deviation of our weights w multiplied by the expected return of our assets X, hence $w'X$. By recalling that Σ is the assets covariance matrix[2] and from Euler's theorem[3], we can write:

[1] VaR: Value at Risk is a risk measure in finance which estimates the maximum potential loss for an investment within a specific confidence level and time frame.

[2] Covariance matrix: A covariance matrix is a mathematical tool used in statistics and finance to measure how two or more variables move together (or apart) in a dataset. It quantifies the relationships and dependencies between variables, crucial for portfolio diversification and risk assessment.

[3] Euler's theorem: Vinìcius Z., Palomar D. (2019). Fast Design of Risk Parity Portfolios

$$\sigma(w) = \sqrt{w'\Sigma w}$$

The derived formula can be rewritten as:

$$\sigma(w) = \sum_{i=1}^{N} w_i \, \frac{\partial \sigma}{\partial w_i} = \sum_{i=1}^{N} \frac{w_i(\Sigma w)_i}{\sqrt{w'\Sigma w}}$$

Having in mind what risk contribution actually means, we know that the sum of each risk contribution must be equal to the actual standard deviation of our portfolio:

$$\sigma(w) = \sum_{i=1}^{N} RC_i$$

Hence we know that

$$RC_i = \frac{w_i(\Sigma w)_i}{\sqrt{w'\Sigma w}}$$

As we want all risk contributions to be equal, this means that:

$$RC_i = \frac{1}{N} \, \sigma(w)$$

Then weights are derived as

$$w_i = \frac{\sigma_i^{-1}}{\sum_{j=1}^{N} \sigma_j^{-1}} = \frac{\frac{1}{\sigma_i}}{\sum_{j=1}^{N} \frac{1}{\sigma_j}} \qquad \forall \, i, j$$

Where:

- σ_i is the standard deviation of asset *i*;
- $\sum_{j=1}^{N} \frac{1}{\sigma_j}$ is the sum of all assets' standard deviations.

Therefore, Risk Parity portfolios are "inverse volatility" strategies, as the higher the volatility of an asset class, the lower its weight within the portfolio.

1.3) Metrics description

This section will go through a brief description of the main performance valuation metrics which will be used to analyze the whole study.

Among the most important metrics to look at when analyzing the performance of a portfolio there is certainly the return generated by the asset allocation. Annualized return computes the average compound annual growth rate (CAGR) of a portfolio over a specified period. It helps investors understand how much their portfolio has grown or declined over time on average. A higher annualized return indicates better performance, while a lower return suggests lower growth or even losses.

$$CAGR = \left(\frac{End\ value}{Start\ value}\right)^{\frac{1}{n^{\circ}\ of\ years}} - 1$$

A higher annualized return means better performance but might be consequence of a too volatile portfolio. A 100% equity portfolio might generate way higher performance in terms of return than a diversified portfolio, but it would be too volatile and could incur in enormous losses. This is why controlling volatility is as much important as looking at annualized return. Standard Deviation measures the dispersion of returns from the mean return of the portfolio. It quantifies the degree to which returns deviate from the portfolio's average return. A higher standard deviation indicates higher volatility and, thus, higher risk.

$$SD = \sqrt{\frac{\sum_{i=1}^{N}(R_i - \overline{R})^2}{N}}$$

Where:

- R_i is the return of each individual period;
- $\bar{R}$ is the mean return of the portfolio;
- N is the number of periods (e.g., months, quarters, years).

Another important indicator within a portfolio analysis is maximum drawdown. This metrics analyzes the maximum percentage decline in the value of a portfolio from a peak to its lowest point over a specific period. Hence it measures the worst loss experienced by an investor during a specific time frame. Understanding the concept of maximum drawdown helps investors assessing the portfolio's potential risk and its ability to recover from losses. Intuitively, the lower the maximum drawdown, the more defensive the portfolio within a high volatility period.

$$MDD = \frac{Through\ value - Peak\ value}{Peak\ value}$$

Where:

- Through value is the lowest value of the portfolio during the specific period considered;
- Peak value is the highest value of the portfolio during the specific period considered.

One of the core metrics used to analyze results provided by our analysis is the Sharpe ratio, which is a risk-adjusted performance metric that evaluates the excess return of a portfolio relatively to its risk, measured by the standard deviation. It provides a measure of how well a portfolio has performed, considering the level of risk taken to achieve that return. A higher Sharpe Ratio indicates a better risk-adjusted performance, as it means that the portfolio generates a higher return for each unit of risk taken. The formula used to compute the Sharpe ratio is the following:

$$SR = \frac{R_p - RF}{SD}$$

Where:

- R_p is the portfolio's annualized return;
- RF is the risk-free rate[4];
- SD is the standard deviation of the portfolio.

Sortino Ratio is another risk-adjusted performance metric that focuses on downside risk. Unlike the Sharpe Ratio, which considers both upside and downside volatility, the Sortino Ratio only takes into account the downside volatility, usually measured as the standard deviation of negative returns. It evaluates how well a portfolio has performed relatively to its downside risk. A higher Sortino Ratio indicates better risk-adjusted returns. The formula used to compute the Sortino Ratio is the following:

[4] Risk free rate: it's the theoretical interest rate at which an investment can be done with zero risk. It serves as a benchmark for evaluating the return on investments and is typically associated with the yield on government bonds, representing a baseline for investment opportunities with no credit or default risk.

$$SOR = \frac{R_p - RF}{Downside\ deviation}$$

Where:

- R_p is the portfolio's annualized return;
- RF is the risk-free rate;
- $Downside\ deviation$ is the standard deviation of negative returns of the portfolio.

Finally, we analyzed relative metrics, in order to assess performance with respect to our benchmark. For this purpose, we deal with Tracking Error and Information Ratio. The former measures the difference in returns volatility between the analyzed portfolio and its benchmark, providing a measure of the average distance from the benchmark return, while the latter computes the expected excess return over a benchmark on the corresponding tracking error volatility. The formulas used to compute the above mentioned metrics are the following:

$$TEV = \sqrt{\frac{1}{T-1}\sum_{t=1}^{T}\left(r_p - r_b\right)^2}$$

$$IR = \frac{r_p - r_b}{TEV}$$

Where:

- r_p are the portfolio's returns;
- r_b are the benchmark's returns.

1.4) Empirical analysis

This section will focus on an empirical analysis on a simulated portfolio, using both Markowitz and Risk Parity allocation above presented. This analysis aims at investigating the performance of the two portfolios to understand whether Risk Parity approach provides a better performance with lower risk than Markowitz approach. The analysis has been executed on Matlab[5], by including 4 different asset classes: STOXX EUROPE 600 (SXXP), Gold (GLD), European long-term government bonds (LEATTREU), European short-term government bonds (LET1TREU), analyzing a time frame going from January 2004 to March 2023. The price series from which the analysis has been conducted were downloaded from Bloomberg[6] (*Figure 1.1*). The code implemented for the analysis will be attached in the appendix.

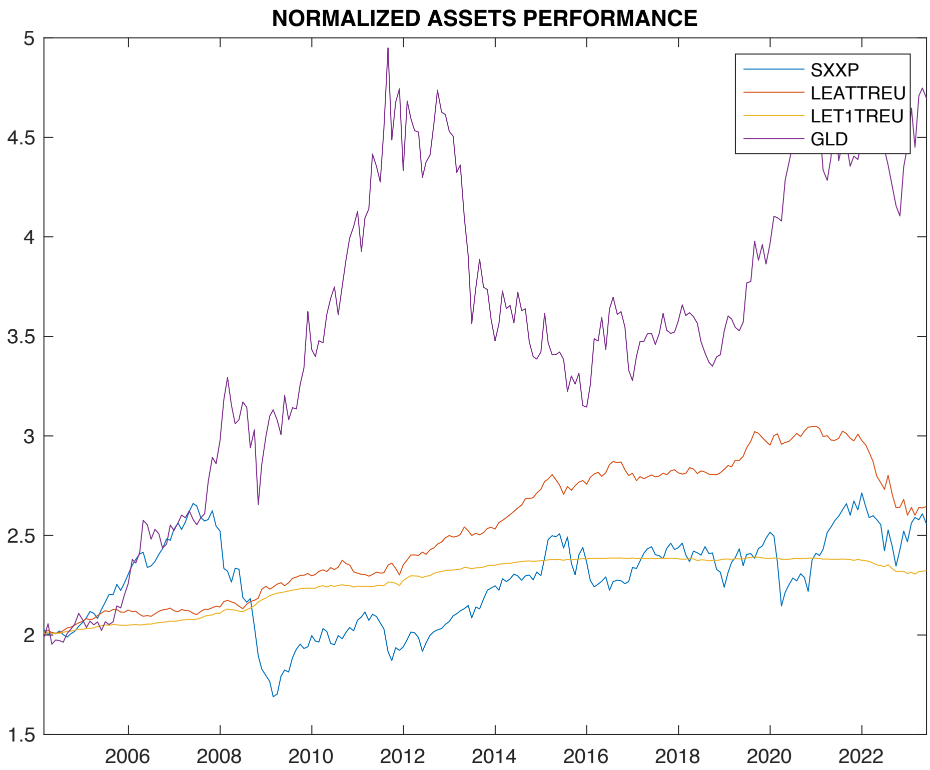

Figure 1.2 – Matlab script

[5] MATLAB: it's a high-level programming language and software environment primarily used for numerical computing, data analysis, and algorithm development.
[6] Bloomberg: it's a global financial data, news, and analytics platform widely used by professionals in finance and related industries. It provides real-time and historical market data, as well as news and analytical tools, facilitating financial research, trading, and decision-making.

This analysis starts from an initial portfolio allocation for both the portfolios as follows:

- SXXP: 40%;
- LEATTREU: 20%;
- LET1TREU: 20%;
- GLD: 20%.

The algorithm rebalances the two portfolios every three months through a loop which computes weights on each asset according to the two techniques. Here below it's presented a graph reporting the rolling window weights allocation of our portfolios, hence how weights change over time (*Figure* 1.2). As we can see, Markowitz allocation shifts immediately from the initial weights right at the first rebalancing period. By doing so, the portfolio doesn't maintain the initial level of diversification, which should be required in order to avoid periods of large volatility. On the other hand, the Risk Parity portfolio provides some slight shifts in the allocation, by maintaining a high level of diversification.

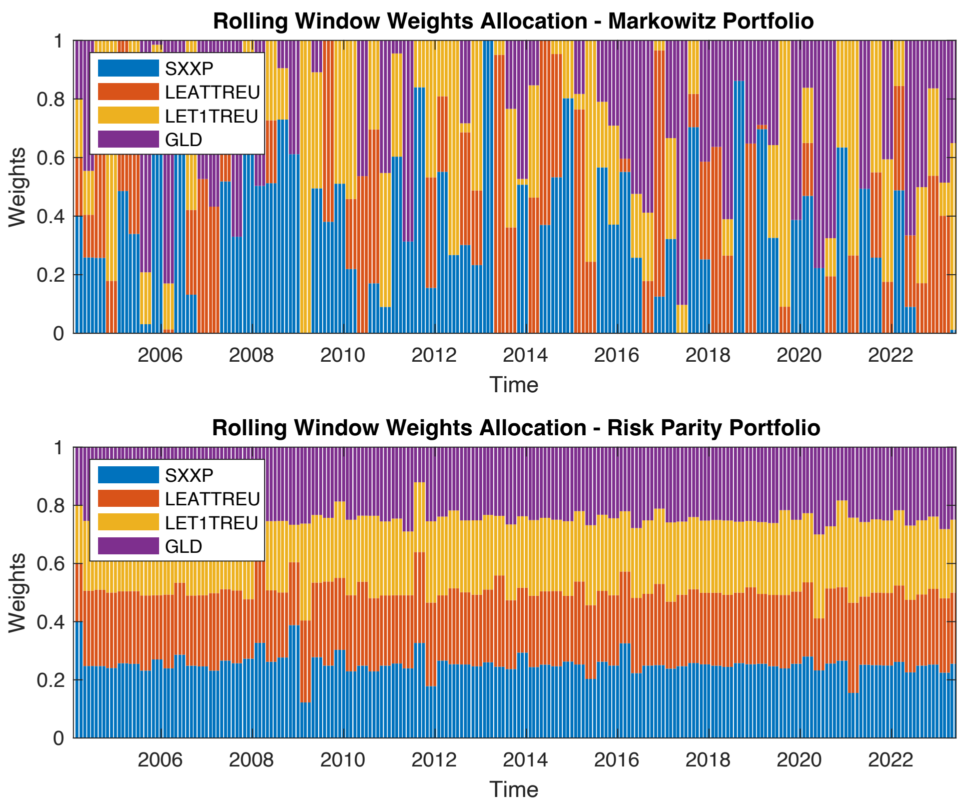

Figure 1.3 - Matlab script

Figure 1.4 plots an overview of how the two portfolios have performed over time. The Risk Parity portfolio seems to provide a lower performance in terms of return at the end of the period.

However, here we are more interested in the risk-adjusted return, as this analysis wants to investigate compelling advantages in terms of diversification and volatility. Indeed, the Risk Parity portfolio (red line) has a much smoother trend component[7], proving less noise with respect to the Markowitz one (blue line).

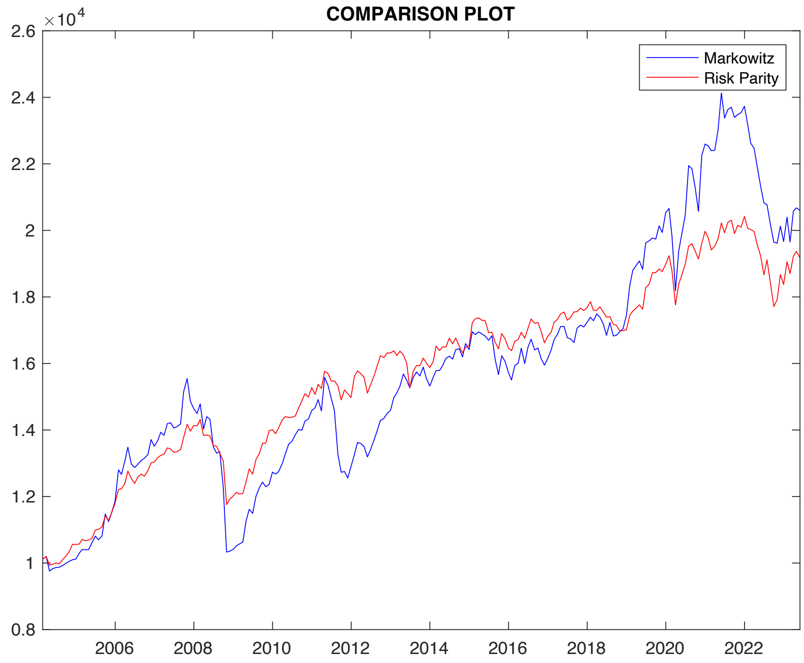

Figure 1.4 -Matlab script

Also, by looking at *Figure 1.5*, we notice that the Risk Parity portfolio always has lower drawdowns with respect to the Markowitz one, showing a more defensive allocation during difficult times.

[7] Trend component: in time series analysis it represents the long-term or underlying pattern in a dataset, capturing the consistent and sustained movements in the data over time. It helps identify the overall direction of the series, excluding short-term fluctuations and noise, and is a crucial component for forecasting and understanding the underlying behavior of a time series.

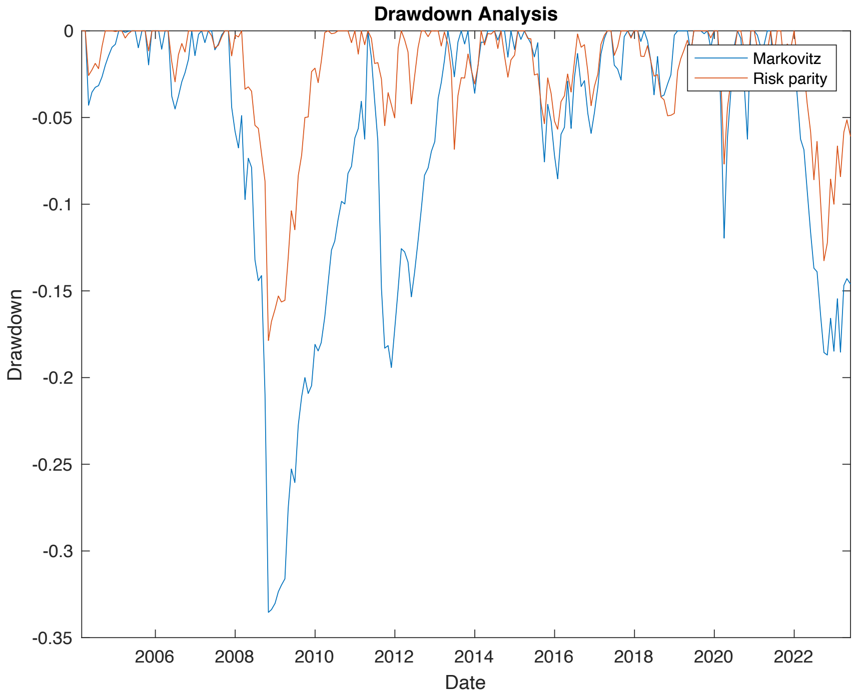

Figure 1.5 - Matlab script

By considering the entire dataset and by having a look at the performance metrics computed through the algorithm (*Table 1.1*), we conclude that the Risk Parity portfolio provides a higher Sharpe Ratio than the Markowitz one. Even though Markowitz portfolio has a higher annualized return, its standard deviation is much higher than the Risk Parity one. This is due to Risk Parity philosophy of weighting assets by controlling their risk contribution to the entire portfolio.

Metrics	Markowitz	RiskParity
"Annualized return"	0.045791	0.041062
"Standard Deviation"	0.09277	0.060492
"Sharpe Ratio"	0.27801	0.34818
"Sortino Ratio"	0.34816	0.47833
"Maximum Drawdown"	−0.31677	−0.16281

Table 1.1 – Matlab script

We should point out that the analysis here conducted is specific and not suitable for every kind of portfolio. There might be some specific cases in which portfolios built following a Markowitz allocation provide better results in terms of Sharpe ratio. However, we can say that Risk Parity philosophy manages to accomplish its main goal: reducing risk by maintaining substantial levels of diversification.

2) Bridgewater All Weather portfolio

2.1) All Weather's philosophy

In 1971 Bretton Woods agreement left the international scene thanks to what passed to history as "The Nixon Shock". The strong convertibility between gold and US dollar was canceled, due to some of the highest inflation rates of all time. This event taught Ray Dalio, the founder of Bridgewater Associates, the importance of having a broad perspective on markets and not relying solely on his own experiences. Dalio realized that he could understand how markets move by breaking them down into their components and studying the relationships between them over time. As Ray Dalio itself tells us, he investigated how markets behave looking at shifts in conditions relative to what is priced in, understanding that the greater the discrepancy between these conditions, the larger the shock.

This approach, which he called "studying the economic machine", represented the basis of the All Weather portfolio. As the name suggests, this portfolio was designed to perform well in every market environment, including those that were unexpected or surprising, by investing in a diversified mix of assets with low correlations among each other.

Hence, the portfolio is designed to balance assets based on their structural characteristics, so that economic shocks can be reduced. This strategy is therefore passive and does not require predicting future conditions. Furthermore, it's based on the Risk Parity philosophy, as each asset class considered should equally contribute to the overall portfolio's risk.

Over time, the principles and concepts that underpin All Weather began to emerge through discrete discoveries. One key insight was that different asset classes have environmental biases, meaning they perform better than others in certain economic environments. For example, bonds tend to perform best during disinflationary recessions, while stocks perform best during periods of economic growth. Understanding these biases was critical for constructing a well-diversified portfolio and laid the foundation for the inflation-growth framework which shaped the main philosophy behind this strategy.

2.2) How the economic machine works

Understanding factors driving asset class returns and benefits of a balanced portfolio requires insight into the economic mechanisms that shape today's economy. According to Ray Dalio, three primary forces drive the economy: productivity growth, the short-term debt cycle[8], and the long-term debt cycle[9].

Despite the gradual and steady rise in the productivity trend line, real-world GDP experiences notable fluctuations due to the interplay of two cycles: short-term and long-term debt cycles.

The short-term debt cycle results from rapid shifts in debt and spending growth. Central banks, like the US Federal Reserve (FED) or the European Central Bank (ECB), control these cycles by managing credit levels through interest rates, impacting aggregate demand.

Over time, recurring short-term cycles contribute to a growing debt burden, evolving into the long-term debt cycle. Though this cycle has minor yearly movements, it leads to a higher debt-to-GDP ratio. As this ratio becomes excessive, traditional monetary tools lose effectiveness, marking the peak of the long-term cycle.

These three forces provide a simplified perspective of the economy's short and long-term dynamics. They underscore how microeconomic actions collectively shape macroeconomic trends.

While Ray Dalio recognized the potential for stocks and bonds to counterbalance each other in times of growth shocks, he was also aware that increasing inflation could negatively impact both types of assets. Therefore, another pivotal aspect in developing the All Weather approach was the framing of inflation as one of the core drivers for asset allocation decisions.

It was discovered that the most successful portfolios were the ones which seemed to be resilient to surprises in inflation. This finding aligned with the historical context of inflationary and disinflationary trends. However, Ray Dalio contributed an additional perspective, proposing that the portfolio should jointly be adapted to both changes in inflation and economic growth. These dual considerations formed the bedrock for the creation of the 4 box economic scenarios diagram upon which this philosophy is built (*Figure 2.1*).

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[8] Short-term debt cycle: it refers to the cyclical patterns in borrowing and lending that occur over relatively brief periods, typically ranging from several months to a few years. It reflects fluctuations in credit availability, interest rates, and economic activity and can have a significant impact on businesses and financial markets.

[9] Long-term debt cycle: it represents the extended economic cycles that typically span several decades and involve patterns of borrowing, lending, and deleveraging in an economy. It encompasses larger-scale fluctuations in interest rates, credit markets, and economic growth, and its understanding is crucial for assessing systemic financial risks and making long-term investment and policy decisions.

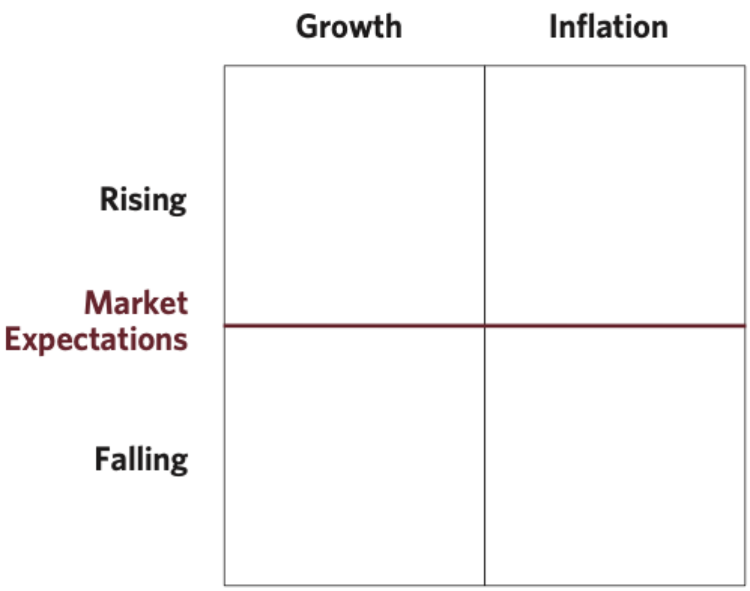

Figure 2.1 – Bridgewater

This tool became a milestone for the All Weather strategy. It was rooted in the fundamental principle of decomposing a portfolio into its constituent parts, recognizing inherent environmental biases and adjusting risk profile of asset classes accordingly. This diagram enabled Bridgewater to grasp the spectrum of potential economic environments that investors might encounter and how to achieve equilibrium by allocating equal risk across each scenario. The framework was constructed to address surprises in a general sense, encompassing unexpected developments yet to transpire, by allocating equal risk on each possible scenario (*Figure 2.2*).

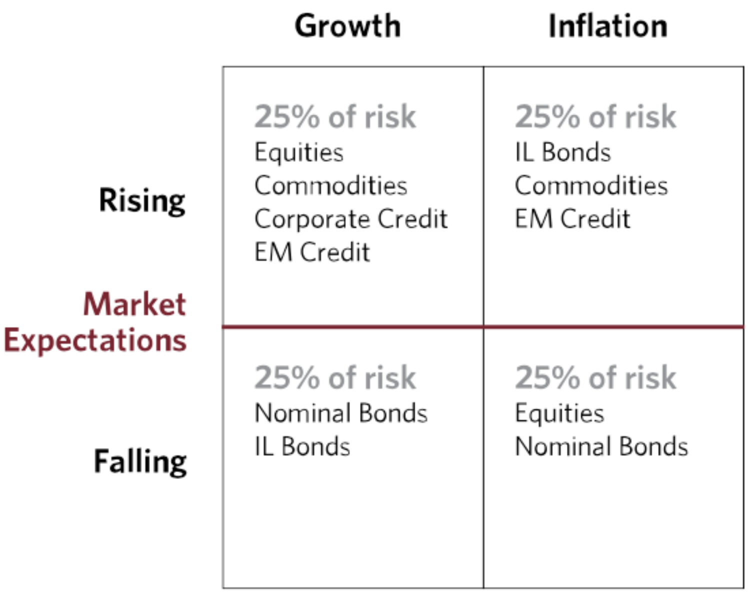

Figure 2.2 – Bridgewater

This dynamic explains the unpredictable link between nominal bonds and stocks. For instance, stocks and bonds react in opposite ways to economic growth. Stocks tend to excel when growth surpasses expectations, benefiting from increased revenue projections. In contrast, bonds perform better in times of difficult economic growth. By blending these assets, their differing responses help lower overall portfolio risk.

Similarly, the opposing reactions of bonds and commodities to inflation make their combination advantageous for risk mitigation. Lower than expected inflation favors bonds, as it can boost business profit margins through reduced input costs and central bank interest rate reductions. Conversely, commodities often perform well during periods of unexpected economic growth and inflation. Including these asset classes in the portfolio leverages their opposite inflation sensitivities to enhance balance and diversification of risk.

Regarding specific asset classes, U.S. Treasuries are viewed as low risk securities due to their absence of call, event, or default risk, as well as minimal liquidity risk. They tend to perform better when economic growth and inflation fall short of predictions, as there's a higher likelihood of central bank intervention to lower interest rates. Furthermore, Treasuries are considered a safe haven asset, providing a hedge during uncertain times, making them an attractive option for portfolio balance.

Gold, another asset of interest, is debated for its correlation with inflation and economic growth. Nonetheless, it's often seen as a hedge due to its money-like attributes and safe haven reputation, as during periods of expected higher inflation, investors might turn to gold to safeguard purchasing power.

Hence, asset class selection in the portfolio is based on projected performance relative to economic growth and inflation. By recognizing how these classes respond to underlying risk factors, the portfolio seeks equilibrium and risk reduction. Blending assets with opposing reactions helps counterbalance risks and strengthens the diversification strategy's resilience.

2.3) All Weather asset allocation

Given a broad overview of the main fundamentals of such an asset allocation philosophy, we can proceed in illustrating how the All Weather portfolio is actually composed.
Ray Dalio intentionally built the portfolio to result not over-complicated nor over-engineered, allocating 30% on equity, 55% on bonds, 15% on commodities, specifically on gold.
As the strategy is built to consider the fact that we don't know what the future holds and thus it implies the choice of thinking within a long-term time frame, only 30% of the portfolio is allocated on equity, reflecting the Risk Parity principle. Within this paper the fixed income component is divided in long-term bonds (25%), short-term bonds (25%) and corporate bonds (5%). However, some investors might decide to allocate slightly higher weights on long-term bonds rather than on short-term, depending on their risk aversion.
Hence, by investing not only on long-term bonds, but also on short-term and corporate ones, the portfolio manages to reach a higher level of diversification, yet a lower volatility, as fixed income securities with longer volatility are usually more responsive to changes in economic conditions, hence riskier. *Figure 2.3* shows a chart representation of the asset allocation.

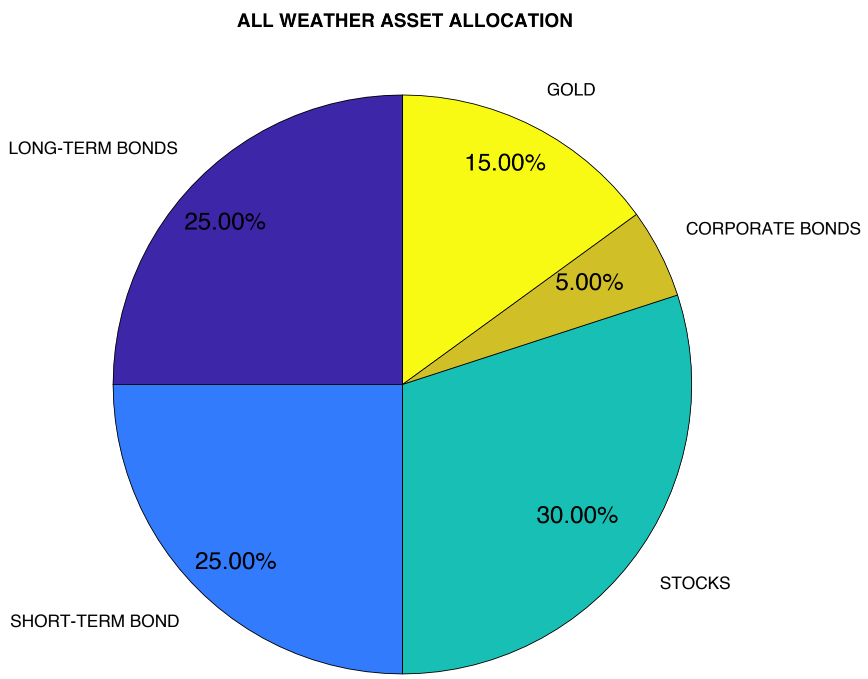

Figure 2.3 – Matlab Script

The following section will illustrate an empirical analysis of the All Weather portfolio performance by comparing it against a traditional allocation portfolio (i.e. 60% equity, 40% bonds).

2.4) All Weather backtesting

This analysis will be investigating the advantages of diversification within portfolio allocation. In this context we performed a backtesting[10] on Matlab by building up both an All Weather portfolio and a classic one. The All Weather portfolio will be composed as follows: 30% equity, 25% long-term government bonds, 25% short-term government bonds, 5% corporate bonds, 15% gold. First, we will consider only American assets in the analysis, afterwards we will employ the same algorithm on European assets. The data here used have been downloaded from Bloomberg and are monthly time series of the following assets:

- S&P 500 (SPX), 30%
- LUATTRUU, 25%
- LU13TRUU, 25%
- LGCPTRUU, 5%
- SPDR GOLD, 15%

The S&P 500 (SPX) is here used as a proxy for the US stock market. For what concerns the fixed income component, we have selected LUATTRUU and LU13TRUU Bloomberg indexes, here taken as proxies for long-term and short-term US government bonds, respectively. To further enhance diversification and hedge against market volatility, a portion of the portfolio is allocated on LGCPTRUU, which represents global corporate bonds. Corporate bonds offer exposure to the credit market and can provide additional income and risk diversification.
In order to incorporate a safe heaven asset, we allocated 15% of the portfolio in SPDR GOLD, which represents gold spot prices. As reported above, gold is usually considered as a hedging tool against inflation and provides diversification benefits due to its low correlation with other asset classes.
In contrast, the classic allocation portfolio follows a traditional approach by investing 60% on equity and 40% on bonds. This allocation is a common benchmark for a balanced portfolio, representing a moderate risk profile with exposure to both stocks and bonds. For this portfolio we picked the same assets as before, but excluding gold, global corporate bonds and short-term bonds.

[10] Backtesting: it's a quantitative analysis technique used to assess the performance of a trading or investment strategy by applying it to historical data to simulate how it would have performed in the past. This process helps evaluate the strategy's effectiveness, identify potential weaknesses or flaws, and inform decision-making for future investments or trading activities.

The algorithm first allocates the portfolio by employing initial weights reported above, and then rebalances weights every three months by adjusting them according to a Risk Parity approach, in order to make them equally contribute to the total risk of the portfolio.

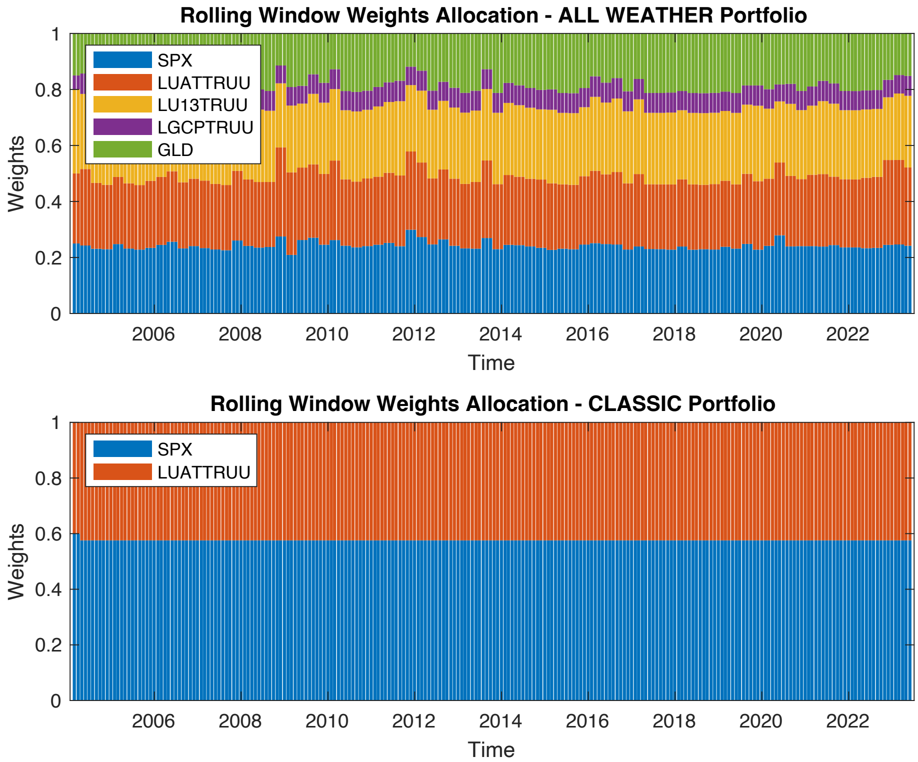

Figure 2.3 – Matlab Script

Figure 2.3 shows the rolling window weights allocation of both the portfolios, hence how weights have changed over time according to the algorithm implemented. As the All Weather portfolio is rebalanced through a Risk Parity approach, weights don't deviate much from the original ones, even though some slight changes occur. On the other hand, the classic portfolio is built in order to maintain the original 60/40 allocation.

Metrics	Classic	AllWeather
"Annualized return"	0.054605	0.049521
"Standard Deviation"	0.08502	0.056084
"Sharpe Ratio"	0.52464	0.70467
"Sortino Ratio"	0.66351	1.0432
"Maximum Drawdown"	-0.30446	-0.1358

Table 2.1 – Matlab Script

As before, we reported the main metrics of valuation for this analysis, in order to evaluate performance in terms of risk-return. The classic portfolio provides a higher annualized return with respect to the All Weather's, due to its larger exposure to equity. However, standard deviation is much lower for the All Weather portfolio, making it approximately 3% less volatile than the classic one. This gap in terms of standard deviation is due to the higher diversification that the All Weather portfolio benefits from. In particular, volatility is lowered since bonds allocation here is diversified between short-term and long-term, while in the classic one the portfolio invests only in long-term bonds, which historically are consistently more volatile than fixed income securities with shorter duration. On the other hand, the All Weather portfolio includes a significant weight on gold (15%), the most volatile asset among the ones here taken in consideration, but also uncorrelated with other asset classes, playing as a further driver of diversification. As a result, the All Weather portfolio manages to provide a Sharpe Ratio of 0,7, approximately 34% higher than the classic portfolio's.

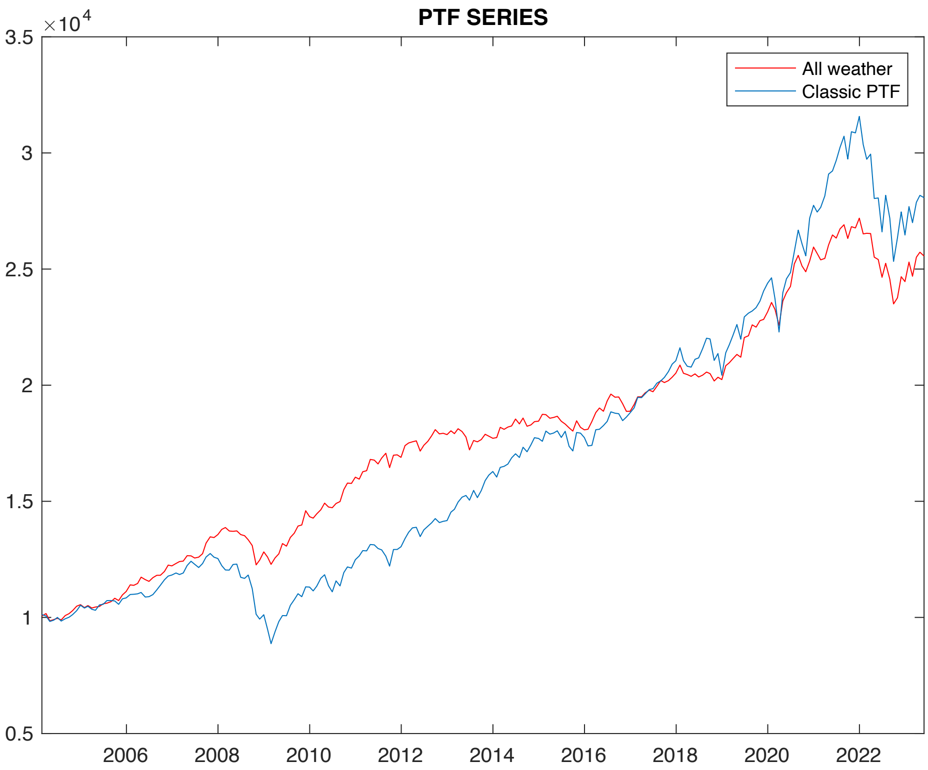

Figure 2.4 – Matlab Script

As reported in *Figure 2.4*, we can notice evidence of the All Weather portfolio having less noise and behaving more regularly through the time frame here considered. During both financial and economic crisis (2008, 2020), this portfolio manages to generate distinctive performances and offset massive losses with respect to a traditional allocation, as it's also deducible from the drawdown analysis (*Figure 2.5*).

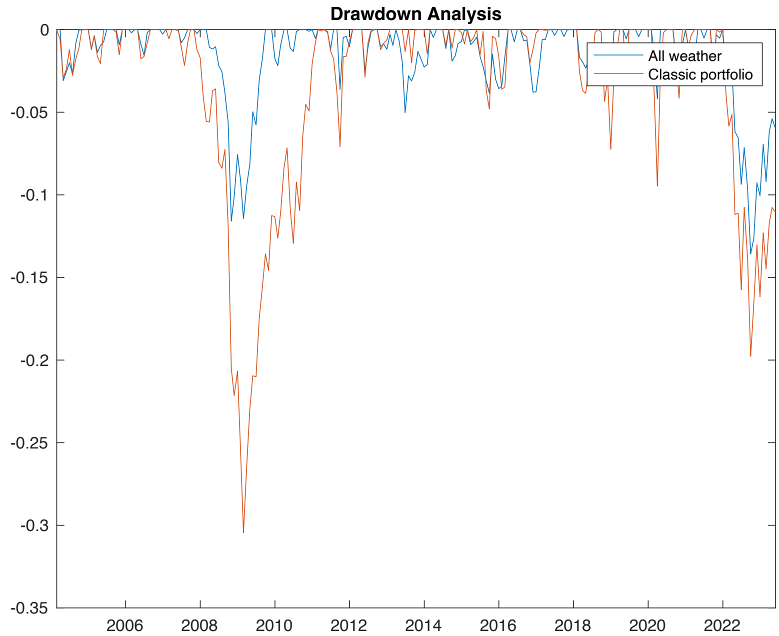

Figure 2.5 – Matlab Script

Furthermore, as we wanted to evaluate whether these results were significant not only for American asset classes, the same analysis has been conducted by replacing SPX, LUATTRUU, LU13TRUU with SXXP, LEATTREU, LET1TREU, which are nothing but the correspondent European asset classes.

Metrics	Classic	AllWeather
"Annualized return"	0.033849	0.039513
"Standard Deviation"	0.087077	0.053767
"Sharpe Ratio"	0.27388	0.5489
"Sortino Ratio"	0.3671	0.80917
"Maximum Drawdown"	-0.34046	-0.13358

Table 2.2 – Matlab Script

Table 2.2 reports the main results deriving from this implementation, which seem to be even stronger than when employing American assets. Not only here we get significant reduction in volatility, but also some gain in absolute returns, driving to a Sharpe Ratio which is double the one of the classic allocation. *Figure* 2.6 shows the portfolios performance over time.

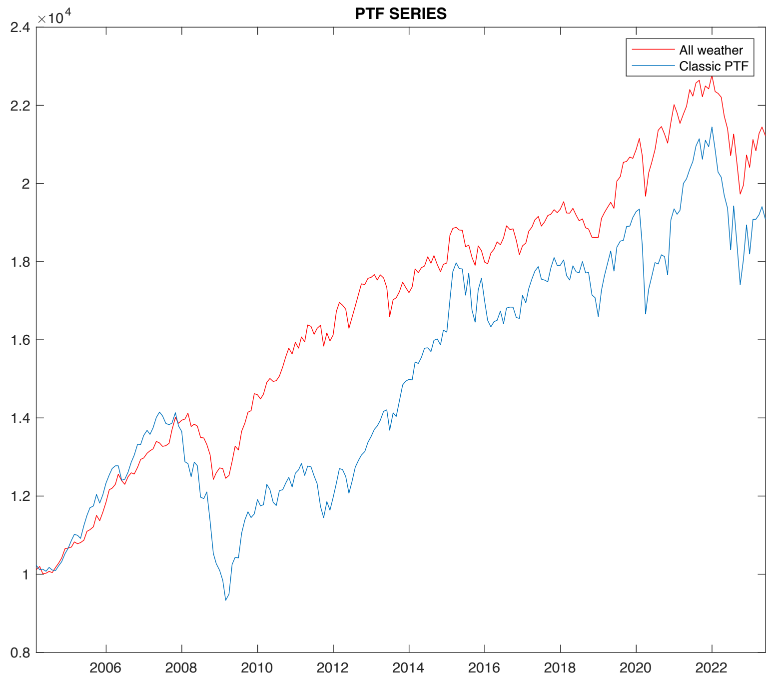

Figure 2.6 – Matlab Script

The difference in terms of risk-return here is due to the poorer performance over the last 20 years of the STOXX EUROPE 600 (SXXP), with respect to the S&P500 (SPX). The geographic switch in the equity component massively affects the classic allocation portfolio, which is largely invested in stocks.

3) Dynamic rebalancing model

During the whole analysis here conducted we've always approached the All Weather as a passive portfolio, which is built to perform well in any economic condition. This approach aims to provide consistent returns and mitigate the impact of unpredictable market fluctuations, without directly intervening on the portfolio allocation. As we've seen before, the implemented portfolio is rebalanced every 3 months with a Risk Parity approach. According to this philosophy weights don't deviate much from the ones implemented at the beginning of the analyzed time frame, in order to maintain stable levels of risk contribution for each asset.
In this chapter the analysis aims to investigate several financial markets rules that, over the long run, might be implemented within portfolio management in order to generate a better performance in terms of risk/return. By doing so we switch from a static rebalancing setting to a dynamic one. This section will go through the core analysis of the study, by first analyzing how a dynamic rebalancing portfolio works with respect to a static one, and by then presenting the core model we built by implementing the Matlab script which will be reported in the appendix.

3.1) Techniques differences

Switching from a static rebalancing portfolio to a dynamic one can offer several compelling advantages within an All Weather investment framework.
One key reason for adopting a dynamic rebalancing strategy is the recognition that market conditions and asset class performances are not static. Economic environments can change, and asset classes may experience varying levels of volatility and returns over time.
Through a dynamic perspective, instead of adhering to fixed weightings, the portfolio's allocation is regularly reviewed and rebalanced to maintain the desired risk profile and capitalize on opportunities presented by changing market conditions. This active approach allows the portfolio to take advantage of potential shifts in asset class performance and optimize risk-adjusted returns.
Furthermore, this approach applied to the All Weather can be can be particularly advantageous due to the underlying asset classes' sensitivity to different economic factors. As mentioned earlier, asset classes such as equities, bonds, and commodities have varying performance characteristics in relation to economic growth and inflation. By dynamically rebalancing the

portfolio, it becomes possible to adjust the allocation based on the expected shifts in these conditions.
For instance, during periods of economic expansion and high growth, equities may outperform other asset classes. In this scenario, a dynamic rebalancing approach would involve increasing the allocation in equities to capture the potential upside. Conversely, during periods of economic contraction or lower growth, bonds and other defensive assets may be more favorable. Dynamic rebalancing would then prompt a shift in allocation towards these asset classes to mitigate risk and preserve capital.
Moreover, dynamic rebalancing can potentially enhance risk-adjusted returns over the long-term. By capitalizing on the opportunities presented by changing market conditions, the portfolio can capture favorable returns and reduce the likelihood of significant losses.
However, we should also take in consideration the disadvantages of such a switch in our analysis. First of all the All Weather portfolio has always been thought as a passive portfolio, which takes as core philosophy the concept of allocating assets and leaving them grow in the long-term. By moving to a dynamic approach this concept no longer holds. However, the dynamic model we're referring to assumes that we set some rebalancing rules at the beginning of the period, and make them change the allocation accordingly and automatically, without any further intervention. Therefore, on the one hand such an approach makes the portfolio active, as asset classes don't maintain the same weights over time, on a different note it still maintains passive portfolios features, as it's thought to work on predefined rules and outperform its static peer in any economic condition over the long run.
Another disadvantage that in the real life might affect the performance is that by switching to a dynamic approach, the turnover [11]of the portfolio increases, as more transactions are executed. This aspect may affect the overall return, due to higher transaction costs. However, as this analysis tries to create a general model which is then adjustable accordingly by each individual investor, we won't take in consideration transaction costs.

[11] Turnover: it's a measure that quantifies the trading activity within the portfolio over a specific period. It calculates the proportion of assets bought or sold relative to the total portfolio value during that period. Turnover is a key metric for assessing the trading costs, tax implications, and overall efficiency of portfolio management, with higher turnover implying more frequent buying and selling of assets within the portfolio.

3.2) Data description

For this analysis we implemented a similar approach to what we have presented above. The asset class will be the same as the ones used in the previous chapter, but there is a further step regarding diversification: each asset class will be invested both in Europe and in the United States. This variable will be useful in considering geographic diversification to assess whether this factor is determinant in terms of portfolio performance.

The dynamic portfolio will have the same initial asset allocation: 30% equity, 50% government bonds, 5% corporate bonds, 15% gold. Specifically, geographic diversification will be implemented by equally splitting each asset class. The data here used have been downloaded from Bloomberg and are monthly price series of the following assets:

- S&P 500 (SPX), 15%
- STOXX EUROPE 600 (SXXP), 15%
- LEATTREU, 12,5%
- LET1TREU, 12,5%
- LUATTRUU, 12,5%
- LU13TRUU, 12,5%
- LGCPTRUU, 5%
- SPDR GOLD, 15%

As before, S&P 500 (SPX) and STOXX EUROPE 600 (SXXP) are used as proxies for the US stock market and the Eurozone stock market, respectively. For what concerns the fixed income allocation, we have selected both European and American government bonds. LEATTREU and LET1TREU are proxies for long-term and short-term European government bonds, respectively, while LUATTRUU and LU13TRUU represent long-term and short-term American government bonds. As before 5% of the portfolio is allocated on LGCPTRUU, which represents global corporate bonds, while the commodity component is still represented by gold, here SPDR GOLD.

The performance of this portfolio will be analyzed against its benchmark, which is nothing but the same portfolio with the static rebalancing technique implemented before in the analysis.

3.3) Rebalancing model used in the analysis and inputs description

This dynamic rebalancing model is built by investigating some rules that, over the long run, seem to provide correlation between macroeconomic indicators and assets performance. In the implemented rebalancing model three key aspects are evaluated, in order to move weights accordingly. We wanted to incorporate allocation shifts regarding the bond component, the equity side and finally the geographic diversification.

3.3.1) Bonds

Fixed income assets in All Weather portfolios represent the biggest part of the whole asset allocation, namely 55%. The input here presented will not make this overall percentage decrease, but instead will work on the allocation between short-term and long-term securities. Here we refer to the concept of duration of the portfolio, which is a key point in portfolio management, and a crucial concept to incorporate in our rebalancing model.

Duration refers to the weighted average time it takes to recover the initial investment from a fixed-income security. For example, consider a bond with a duration of 5 years. This means that, on average, it will take approximately 5 years to receive back the bond's cash flows (coupon payments and principal repayment). Duration takes into account both the timing and magnitude of these cash flows, giving investors an idea of when they can expect to receive the majority of their investment back. Intuitively, short-term bonds will have lower duration with respect to long-term ones.

In asset management duration is crucial as it is also a measure of the sensitivity of a fixed-income security's price to changes in interest rates. As bonds prices have an inverse relationship with interest rates, when interest rates rise, bond prices generally decrease, and vice versa. As we can see from *Figure* 3.1, where "GDBR10" is the index tracking the 10 years bund yield[12], duration helps investors understand the magnitude of these price changes. Whenever yields moves up or down, high duration bonds (LEATTREU in this case) will move inversely with more volatility with respect to low duration bonds (LET1TREU here).

[12] Yield of a bond: it's a measure of the total return an investor can expect to earn from holding the bond until it matures. It takes into account the bond's current market price, coupon payments, and the time remaining until maturity.

Figure 3.1 – Bloomberg Terminal

Hence, the higher the duration of a bond, the more sensitive it is to changes in interest rates. This is because cash flows from long-duration bonds are received later with respect to short-duration ones, making them riskier and more exposed to changes in interest rates over time.

From this recap on why duration plays a key role in portfolio management, it's easy to understand that the model here presented will provide different weights within the fixed income allocation of the portfolio, accordingly to what are the market conditions in each moment of the analyzed period. As we have seen that bonds with high duration are more volatile than those with low duration, the former will perform better on average while yields are going down, while the latter will lose less when yields are going up. This is due to the core reasoning behind bonds market: if yields are going up that means that there is more uncertainty, so investors will move their allocation to safer assets, in this case, shorter term securities. On the other hand, when yields decrease, markets consider economic conditions to be more stable, and investors will move their allocation accordingly to longer term securities.

Hence, short-term fixed income securities are in some way a defensive asset, in order to lose less performance, as their volatility is lower.

The model now has to predict different macro-economic conditions, in order to deviate from the initial weights. As we can see from *Figure 3.2*, one of the key drivers of bond yields is the refinancing rate[13] decided from central banks. Here we plotted on Bloomberg the 10 years

[13] Refinancing rate: it's issued by a central bank and also known as the policy rate or benchmark interest rate. It represents the interest rate at which commercial banks can borrow funds from the central bank to meet their short-term liquidity needs. It serves as a key tool for the central bank to influence overall economic conditions, including inflation and economic growth. By changing the refinancing rate, the central bank can impact borrowing costs, credit availability, and investment decisions in the broader economy. It is a crucial indicator for monetary policy and financial markets.

German government bond (GTDEM10YR), against the ECB refinancing rate (EURR20W) over the period considered (2004-2023).

Figure 3.2 – Bloomberg Terminal

As we can see there is evidence of correlation between the two time series. Specifically, when the European central bank releases interest rates hikes, the 10 years bund yield increases, while the opposite happens when the central bank releases interest rates cuts. However, we should take in consideration that this reasoning makes sense on the long run, as in real life markets move accordingly to data expectations. This means that government bonds yields move due to several factors, be that how central banks presidents speaks about their moves, or how much the released data deviate from analyst expectations. In this analysis we can't incorporate these aspects, as they depend on specific short-term market conditions. However, we can use the central bank refinancing rate as the main input to shift duration in our portfolio. As it's possible to notice in both *Figure* 3.3 and *Figure* 3.4, there's evidence of inverse correlation between refinancing rates and bonds prices, both in Eurozone and in the United States.

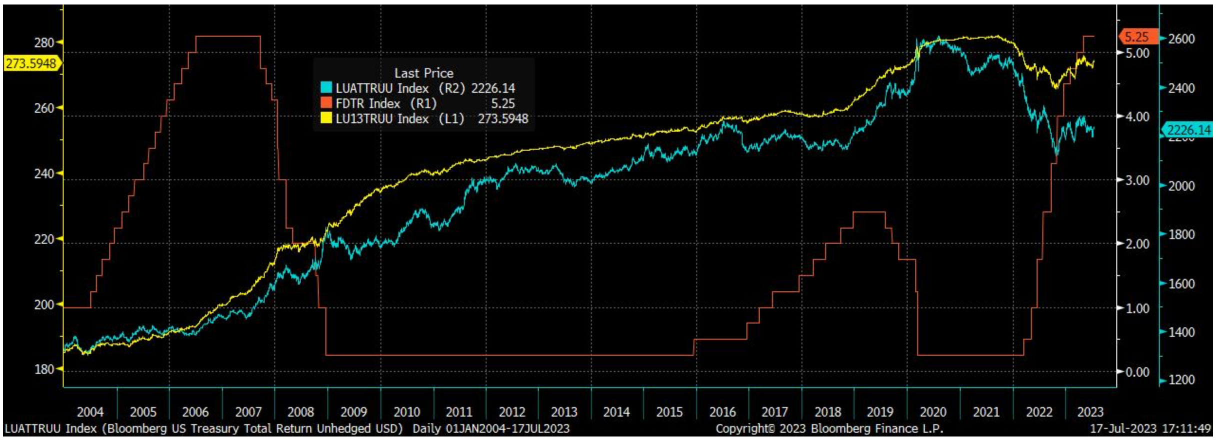

Figure 3.3 – Bloomberg Terminal

Figure 3.4 – Bloomberg Terminal

3.3.1.1) Model implementation

The algorithm takes FED and ECB refinancing rates separately and works on the differential of the time series within the time frame considered (2004-2023). To make the input as smooth as possible, we dealt with the moving average[14] at 12 months of the series, so as to avoid the input to be activated for slight changes in the differential.

Whenever a reverting pattern is found, the algorithm assumes that the reference government bond yield moves accordingly. Hence whenever the moving average of interest rates differential is positive, this means that central banks are likely to become dovish in the subsequent months.

[14] Moving average: it's a statistical calculation used in data analysis and time series forecasting. It involves taking an average of a set of data points within a moving window of a specified size, sliding that window through the dataset, and calculating a new average for each position. Moving averages help to smooth out fluctuations and highlight underlying trends or patterns in the data, making them valuable tools for identifying trends, seasonality, and short-term fluctuations in various types of data.

As this implies government bond yields to likely go down, then bond prices will probably start a bullish rally.
Our aim then is to capture performance in these periods by allocating more on high duration bonds rather than on low duration ones. On the other hand, whenever the opposite is detected, the algorithm will follow the same approach by allocating more on low duration bonds and less on the high duration ones, in order to contain losses.
In particular, the model will increase LEATTREU/LUATTRUU by 10% and decrease LET1TREU/LU13TRUU by 10%, while it will increase LE1TREU/LU13TRUU by 10% and decrease LEATTREU/LUATTRUU by 10% when the opposite situation occurs.

3.3.2) Equity

The second input we incorporate in the model refers to equity related securities. As we've shown before, the initial asset allocation of our portfolio is built to have 30% in equity, allocating 15% on the major American index (S&P500) and 15% on the major European index (STOXX Europe 600).

This model adopts a fundamental approach to portfolio allocation by incorporating real economic indicators, specifically GDP[15], to make investment decisions. Using GDP as an input provides valuable insights into the economic health and growth prospects of both US and Eurozone economies, which can significantly impact various asset classes, particularly equities. Intuitively we should increase our equity allocation whenever GDP is likely to increase. This approach aligns with the common investment principle of "buying into strength", as an expanding economy is generally associated with increasing corporate earnings and potential stock market gains. By allocating more to equities during economic expansions, our model aims to capitalize on the positive correlation between GDP growth and equity market performance.

Reversely, we should decrease our equity allocation whenever GDP is likely to decrease. This defensive approach aligns with the concept of risk management during economic downturns. Economic contractions are typically associated with reduced consumer spending, corporate profits, and market uncertainty, which can lead to declines in equity prices. By reducing equity exposure during economic downturns, the model seeks to protect the portfolio from potential losses.

As we can see from *Figure* 3.5 and *Figure* 3.6, which plot both the S&P500 index against the US real GDP rate and the STOXX Europe 600 against the European real GDP rate, there is evidence of correlation between the direction of economic growth and the performance of the major indexes.

[15] GDP: it stands for Gross Domestic Product and it's a comprehensive economic indicator that measures the total value of all goods and services produced within a country's borders during a specific period, typically a year or a quarter. It serves as a key measure of a nation's economic activity and is often used to assess its overall economic health, growth, and performance. GDP can be calculated using three different approaches: production, income, and expenditure, providing insights into various aspects of an economy.

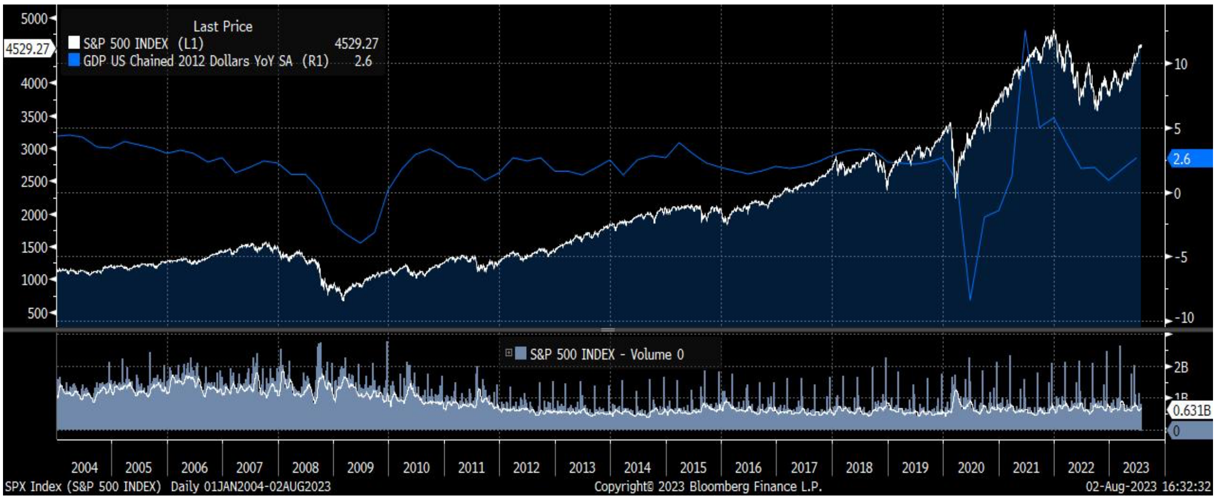

Figure 3.5 – Bloomberg Terminal

Figure 3.6 – Bloomberg Terminal

However, this might not be enough: although it seems true that whenever there is a recession stocks perform poorly, there is also evidence that the simple real GDP rate series doesn't provide sufficient information in order to move the allocation. One might intuitively think to reduce equity allocation whenever GDP rates are below zero and do the opposite whenever it is above. Nevertheless, by following this approach the model wouldn't include many movements throughout the years, as technical recession[16] happened only a few times in the time frame here analyzed.

[16] Technical recession: it refers to an economic condition characterized by two consecutive quarters of negative GDP growth. It is a quantitative indicator used to identify a period of economic contraction. Unlike a regular recession, which may involve broader economic factors and impacts, a technical recession is defined solely by the specific criteria of negative GDP growth for two consecutive quarters and is often used as an official benchmark to assess economic performance.

3.3.2.1) Model implementation

We want to incorporate changes in direction of macro trends in order to catch up any possible direction of the stock market. To provide such output we work on the differential of the GDP rate series, by analyzing how the economic indicator changes with respect to the previous period. As we want the input to work as smoothly as possible, we computed the moving average at 15 months of the differential. This further step is crucial to make the input take in consideration only significant changes in GDP rate, instead of any as it would be without implementing the moving average.

Then, the input activates whenever the moving average of the differential changes sign. When it is positive, the allocation in equity related assets within the portfolio increases of 6%, by equally decreasing other asset classes by 1,5%. Conversely, when the GDP reverts its trend, hence when the moving average of the differential becomes negative, the model reduces equity allocation by 6% and allocates it on the other asset classes.

We set 6% as weight change as we've tested it's the optimal shift in order to avoid too big deviations from a risk diversification perspective.

It's important to acknowledge that GDP is one of many factors influencing equity markets. Market sentiment, interest rates, corporate earnings reports, geopolitical events, and central bank policies are just a few other factors that can impact equity performance. However, we should still take in mind that this model provides inputs that are likely to be efficient on the long run, hence it might fail in addressing a better performance at some time, but it will work well by widening the time frame.

3.3.3) Currency

While asset allocation across different classes like stocks, bonds, and commodities is a well-known practice, the importance of currency allocation within portfolio management often goes overlooked. First of all, currency allocation provides an additional layer of diversification within a portfolio. Currencies are influenced by various factors such as economic indicators, geopolitical events, and central bank policies. By including different currency diversification in a portfolio, investors can mitigate risk associated with any single country's volatility or weakness. When one currency depreciates, others may appreciate, offsetting potential losses and reducing overall portfolio risk. By closely monitoring and analyzing global economic trends, investors can identify opportunities for currency allocation. For example, during periods of economic growth and stability, investors might allocate more capital to countries experiencing positive economic indicators. This approach enables investors to align their portfolios with long-term macroeconomic trends, potentially boosting returns.

In this model we are dealing with European and American assets, therefore our currency allocation will be on securities denominated in Euro and in US dollar.

As mentioned before, one among the key drivers of the forex market[17] is central banks policy. In particular, we have to deal with Federal Reserve and European Central Bank decisions on interest rates.

Specifically, when a central bank becomes hawkish[18], hence starts to increase the refinancing rate, then the home currency usually appreciates. The opposite happens whenever the central bank becomes dovish[19] and seems to be in the ending state of its hiking cycle. This relationship can be attributed to the attractiveness of higher interest rates for investors seeking higher returns. As a result, there is an increased demand for the home currency, leading to an appreciation in its value relative to others.

Figure 3.7 and *Figure* 3.8 show the EUR/USD against the ECB refinancing rate, and then the USD/EUR against the FED refinancing rate, to see whether it shows signs of correlation or not.

[17] Forex market: the foreign exchange (forex or FX) market is a global decentralized marketplace for trading currencies. It is the largest and most liquid financial market in the world, where participants buy, sell, exchange, and speculate on the value of various currencies.

[18] Hawkish: it refers to a policy stance that prioritizes controlling inflation and price stability over promoting economic growth. When a central bank is described as hawkish, it indicates a willingness to raise interest rates or implement other monetary tightening measures to combat inflationary pressures or perceived economic overheating.

[19] Dovish: it refers to a policy stance that prioritizes supporting economic growth and employment over controlling inflation. When a central bank is described as dovish, it indicates a willingness to implement monetary policies that promote economic expansion. This often involves lowering interest rates or using other measures to stimulate borrowing and spending by businesses and consumers.

Figure 3.7 – Bloomberg Terminal

Figure 3.8 – Bloomberg Terminal

Although in the long run the two series seem to show correlation, this is clearly not enough to shift from the initial weights regarding currencies. This comes from the fact that as we are looking for some input which affects the EUR/USD ratio, this has to take into account both the decisions of the ECB and the ones of the FED. Therefore, we have to deal with the differential between the ECB refinancing rate and the FED refinancing rate, by plotting it against EUR/USD (*Figure* 3.9). This is nothing but the difference between the interest rates issued by the two central banks.

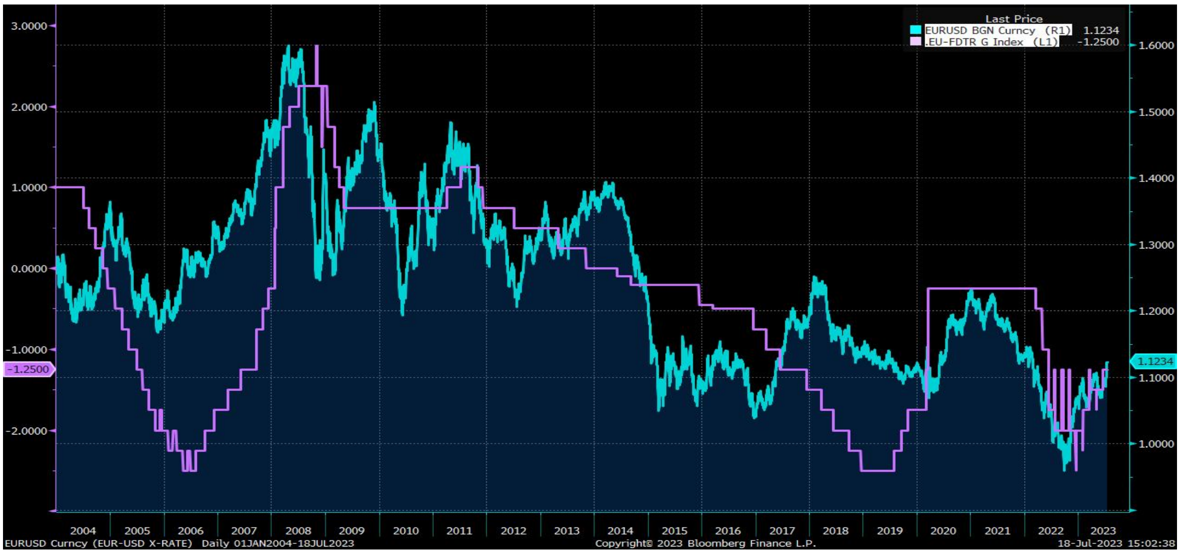

Figure 3.9 – Bloomberg Terminal

The graph illustrates much more correlation between EUR/USD exchange rate and the differential of the interest rates set by Federal Reserve (FED) and European Central Bank (ECB).

When the interest rate differential decreases, meaning that the FED is more hawkish than the ECB, and hence it raises rates at a faster pace or more aggressively, the graph shows that the EUR/USD tends to decrease, as the US dollar appreciates.

Conversely, when the interest rate differential increases or the ECB raises rates more rapidly than the FED, the graph shows that the EUR/USD tends to appreciate.

As before, it is still important to remark that while the correlation between EUR/USD and the interest rate differential is evident, other factors can influence currency movements as well. Market sentiment, economic indicators, geopolitical events, and monetary policy decisions beyond interest rates can also impact exchange rates. However as highlighted before, by only taking into account the interest rate differential variable, we are trying to make this input work specifically for a long-term time frame.

3.3.3.1) Model implementation

Our model captures EUR/USD fluctuations by detecting peaks and valleys of the differential time series between the two interest rates. The algorithm uses the MATLAB built-in function to retrieve the dates at which the time series reverts its direction. One can intuitively think that the main driver is the 0 level, meaning that whenever the interest rate differential changes sign, the allocation should change accordingly. However, markets move due to expectations, so instead of looking at the sign of the differential, we pay more attention at whenever the derivative of the differential reverts. By doing so we're considering what the real meaning of this indicator is: different directions of central banks policies.

Once the peaks and valleys dates are detected, the algorithm generates as output an allocation switch between American and European assets. As we want to incorporate diversification, this shift will be done with a threshold of 4% change on each asset class. This means that whenever a peak date is detected, hence the differential between ECB and FED rates is likely to increase, then the allocation shifts accordingly by increasing each European asset of 4%, and by decreasing each American asset of the same percentage.

4) Analysis of results

This section will focus on the main results of the analysis conducted through the dynamic rebalancing model above implemented. The chapter will additionally go through the asset allocation contribution by every input implemented.

4.1) Results description

To measure the performance of the dynamic portfolio we compared it against an equivalent portfolio but rebalanced through a static approach. Specifically, this passive portfolio will have the same initial weights as the dynamic one, but it will be rebalanced only according to the Risk Parity technique, without any input driving the allocation. Additionally, performance will be evaluated against a classic 60% equity, 40% bonds portfolio, in order to get an overview of the pros and cons of our model, with respect to a traditional approach.

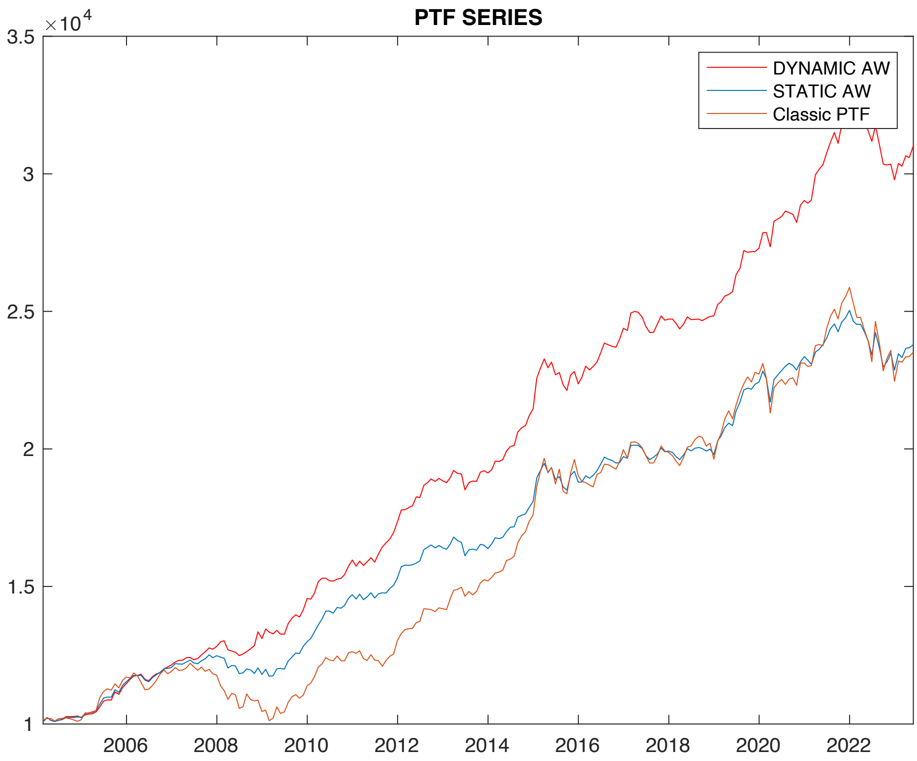

Figure 4.1 – Matlab script

As we can see from *Figure 4.1*, the dynamic All Weather portfolio grows at a substantial faster rate with respect to both the static All Weather portfolio and the classic portfolio. Specifically, its capitalized annual growth rate stands at a level of 6,1%, respectively 1.5% and 1.6% higher than the one of its benchmarks.

Metrics	STATIC_AllWeather	DYNAMIC_AllWeather	Classic
"Annualized return"	0.04564	0.061067	0.045028
"Standard Deviation"	0.042629	0.041993	0.064309
"Sharpe Ratio"	0.83604	1.2161	0.54468
"Sortino Ratio"	1.2497	2.0778	0.81452
"Maximum Drawdown"	-0.086797	-0.081494	-0.17079
"Tracking Error"	0	0.0056357	0
"Information Ratio"	0	0.218	0

Table 4.1- Matlab script

Furthermore, by looking at *Table 4.1*, we can notice that also in terms of volatility the dynamic portfolio performs better. This takes this allocation to provide significant result in terms of both Sharpe ratio and Sortino ratio. The Dynamic All Weather has a Sharpe ratio of 1.21, which is more than double the one of the classic portfolio, and approximately 45% higher than the one of its direct benchmark, the static All Weather. Furthermore, here we have results in terms of Tracking Error and Information Ratio, which provide some information about the relative performance of the dynamic portfolio with respect to the static one. As expected, they're both positive.

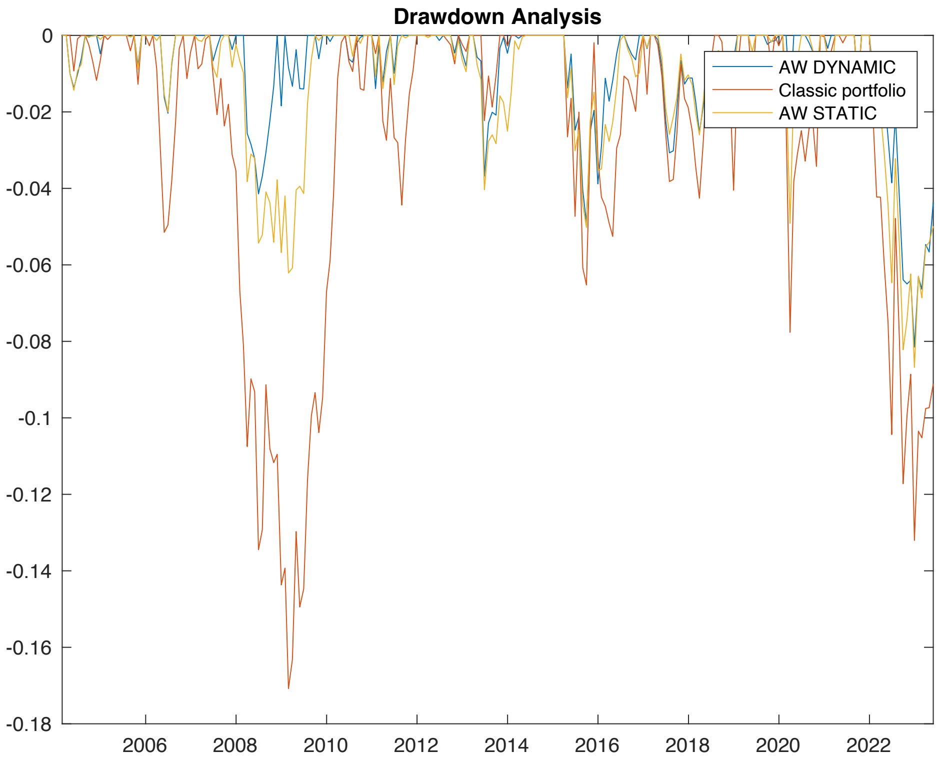

Figure 4.2 – Matlab script

By analyzing *Figure 4.2*, we can also provide significant results in terms of maximum drawdown. The Dynamic portfolio seems to always defend the best with respect to its peers. The maximum drawdown for this model stands at 8,1%, against a value of 8,3% for the static, and 17% for the classic portfolio.

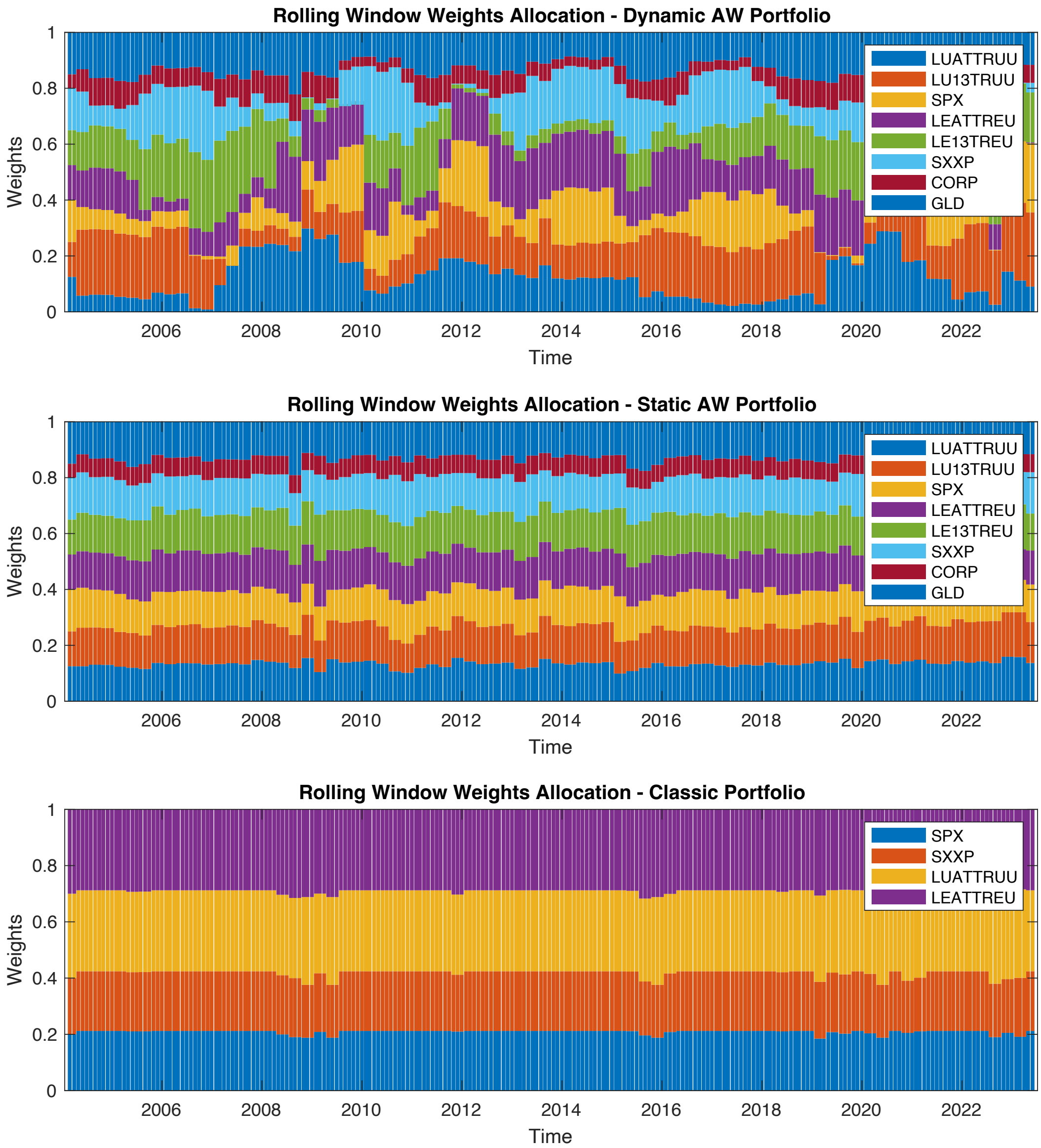

Figure 4.3 – Matlab Script

Figure 4.3 shows the rolling window weights allocation for the three portfolios. As highlighted here, our model moves the allocation at any period it's supposed to. Therefore, the turnover for this portfolio is significantly higher, as the model tries to capture any market condition and change weights accordingly.

4.2) Asset allocation contribution by inputs

After having shown the strong results deriving from the implementation of our model, we provide in this section a deeper analysis regarding the contribution of each input to the overall asset allocation. Specifically, we want to investigate how each input changes the allocation and how the model behaves when two or more inputs overlap.

4.2.1) Bonds input

As explained in the previous section the input related to the internal fixed income allocation shifts weights between long-term and short-term bonds according to decisions of central banks on interest rates. Here in *Figure 4.4* and *Figure 4.5* are shown rolling window weights allocations for both Eurozone and United States.

Weights here reported aim to provide a comprehensive understanding of how the internal bonds allocation changes according to our input. For this reason, here we have rolling window weights only related to fixed income assets within the two separate regions. The second time series of both *Figure* 4.4 and *Figure* 4.5 reports the effective functioning of the model input. Here is plotted the moving average differential of the central bank refinancing rate. By working on this time series rather than on the simple refinancing rate's (3rd graph on both *Figure 4.4* and *Figure 4.5*), the model captures shifts in steepness, so as to incorporate a more dovish/hawkish behavior. Therefore, whenever the moving average differential changes sign, the model adjusts the allocation. Green lines are plotted whenever the series changes sign from positive to negative, hence when a tightening cycle from the central bank is likely to be over, while black lines are plotted whenever the opposite happens, hence when the central bank is likely to increase interest rates.

As expected, weights shift accordingly, hence right after every green line long-term securities are increased while short-term assets are decreased, in order to capture more volatility when fixed income securities are likely to grow, while right after every black line the opposite happens, so as to defend potential losses.

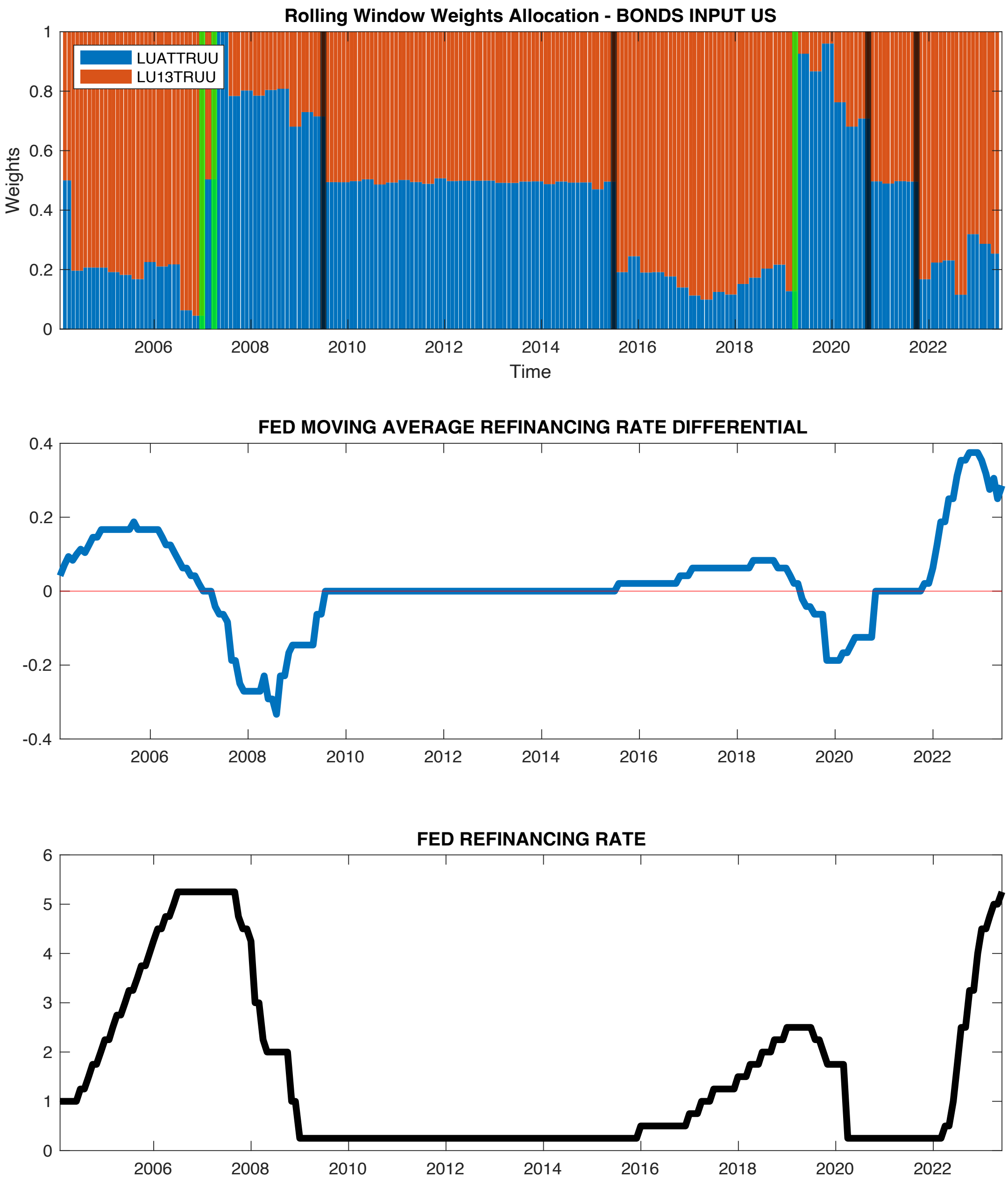

Figure 4.4 – Matlab Script

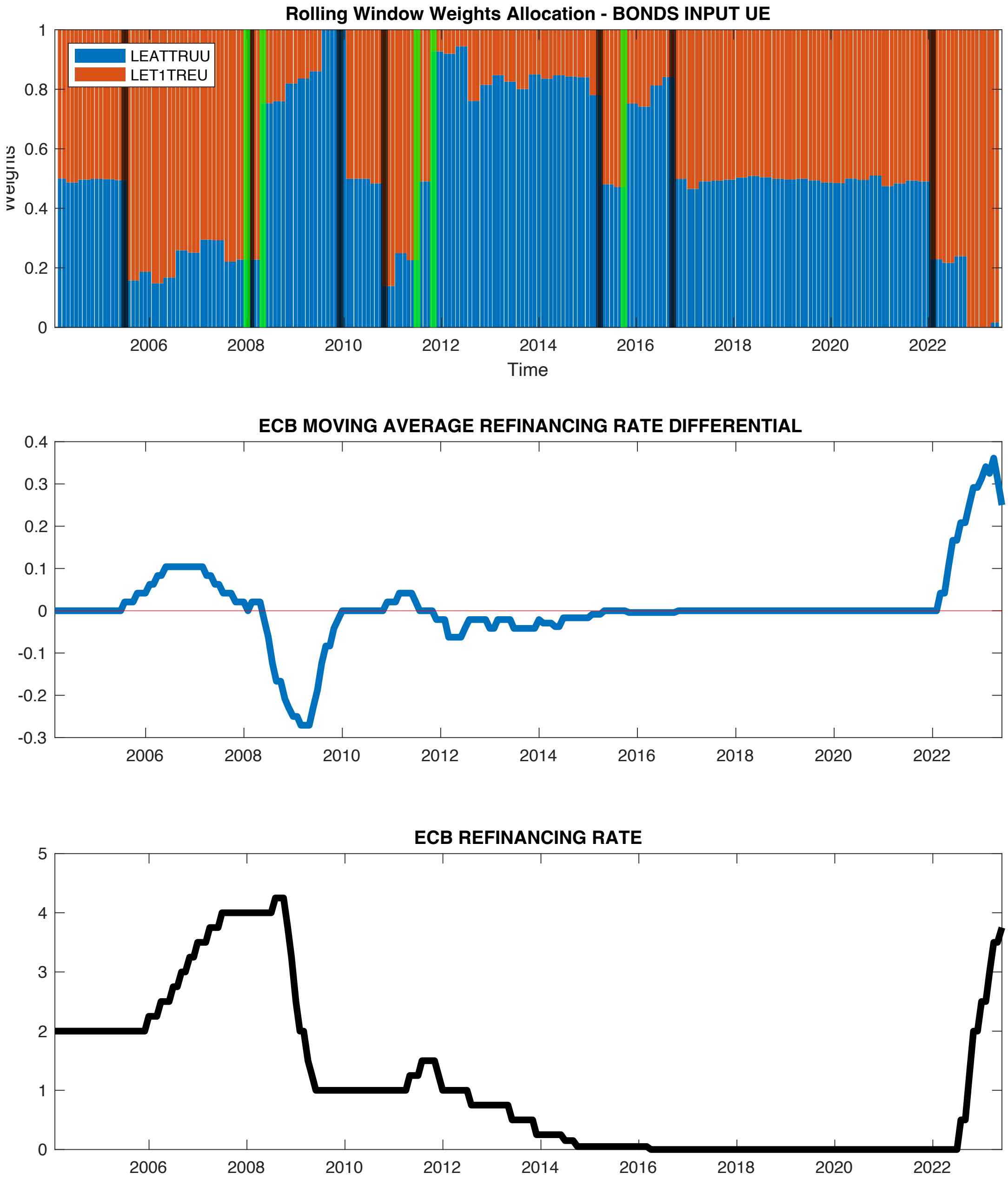

Figure 4.5 – Matlab script

4.2.2) Equity input

Here is analyzed the model input operating on equity related securities. As explained in the previous chapter this input operates due to changes in the moving average of the GDP differential. In *Figure 4.6* and *Figure 4.7* is reported the rolling window weights allocation related to this input.

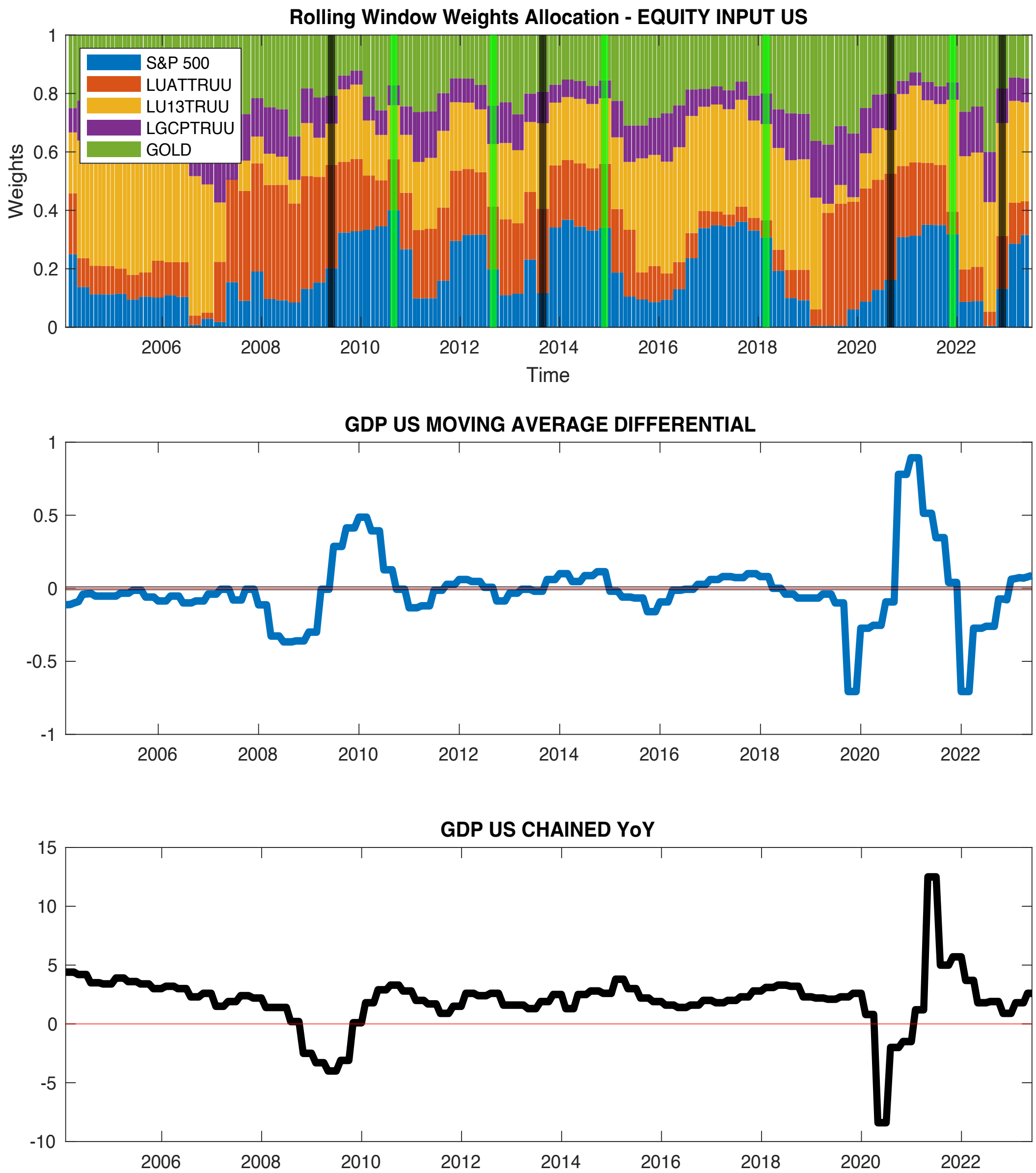

Figure 4.6 – Matlab script

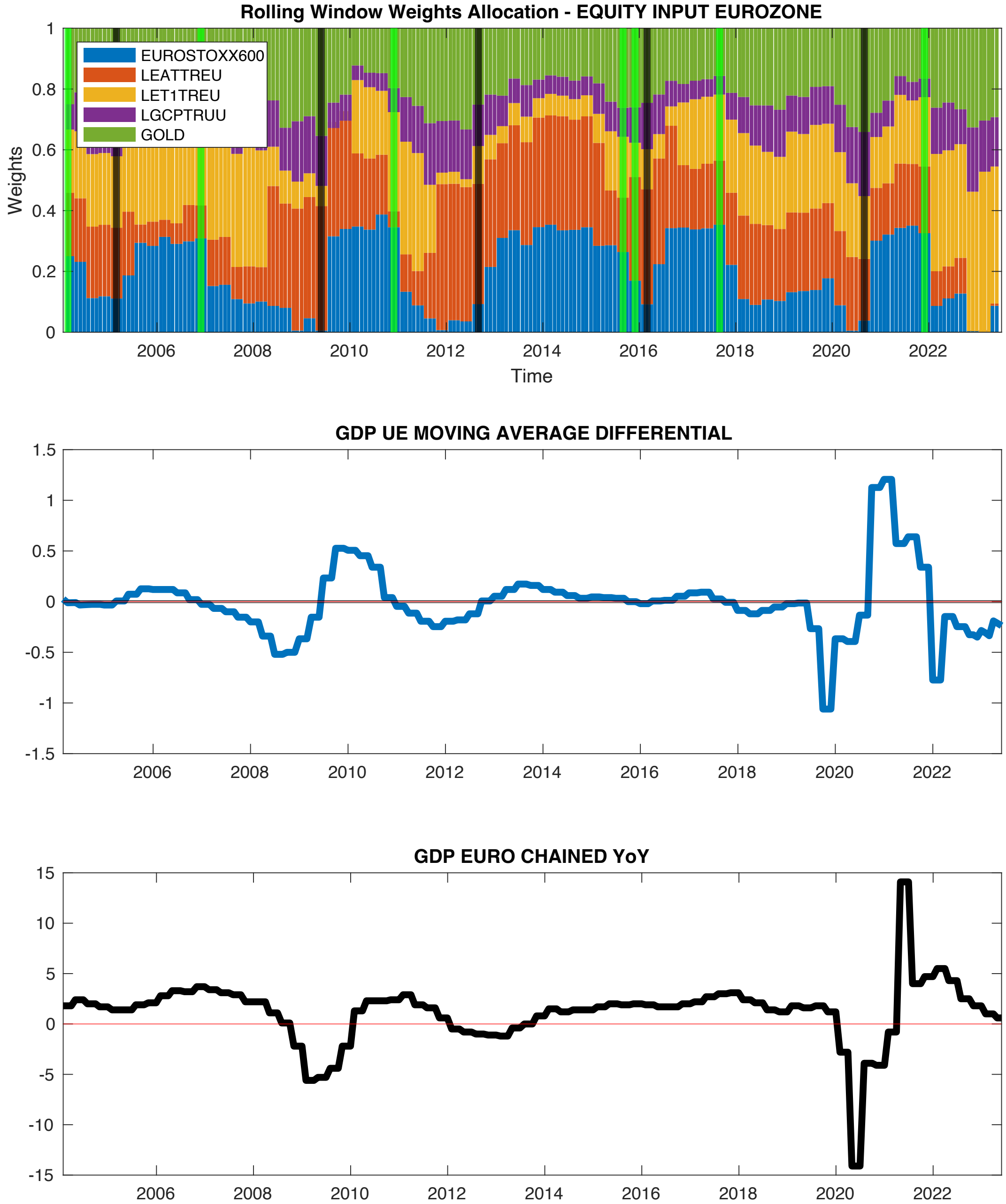

Figure 4.7 – Matlab script

As before, the first graph of both *Figure*s is implemented with vertical lines. Green vertical lines occur when the moving average of GDP differential passes from positive to negative, meaning that GDP is likely to decrease in the upcoming future, while black lines occur when the opposite happens, hence when GDP is likely to increase. Allocation is moved accordingly, therefore, after every green line equity securities are reduced while other assets are increased, while after every green line, the model provides the opposite asset allocation.

The second and third graph of both *Figure*s plot respectively the moving average GDP differential and the GDP growth rate.

4.2.3) Currency input

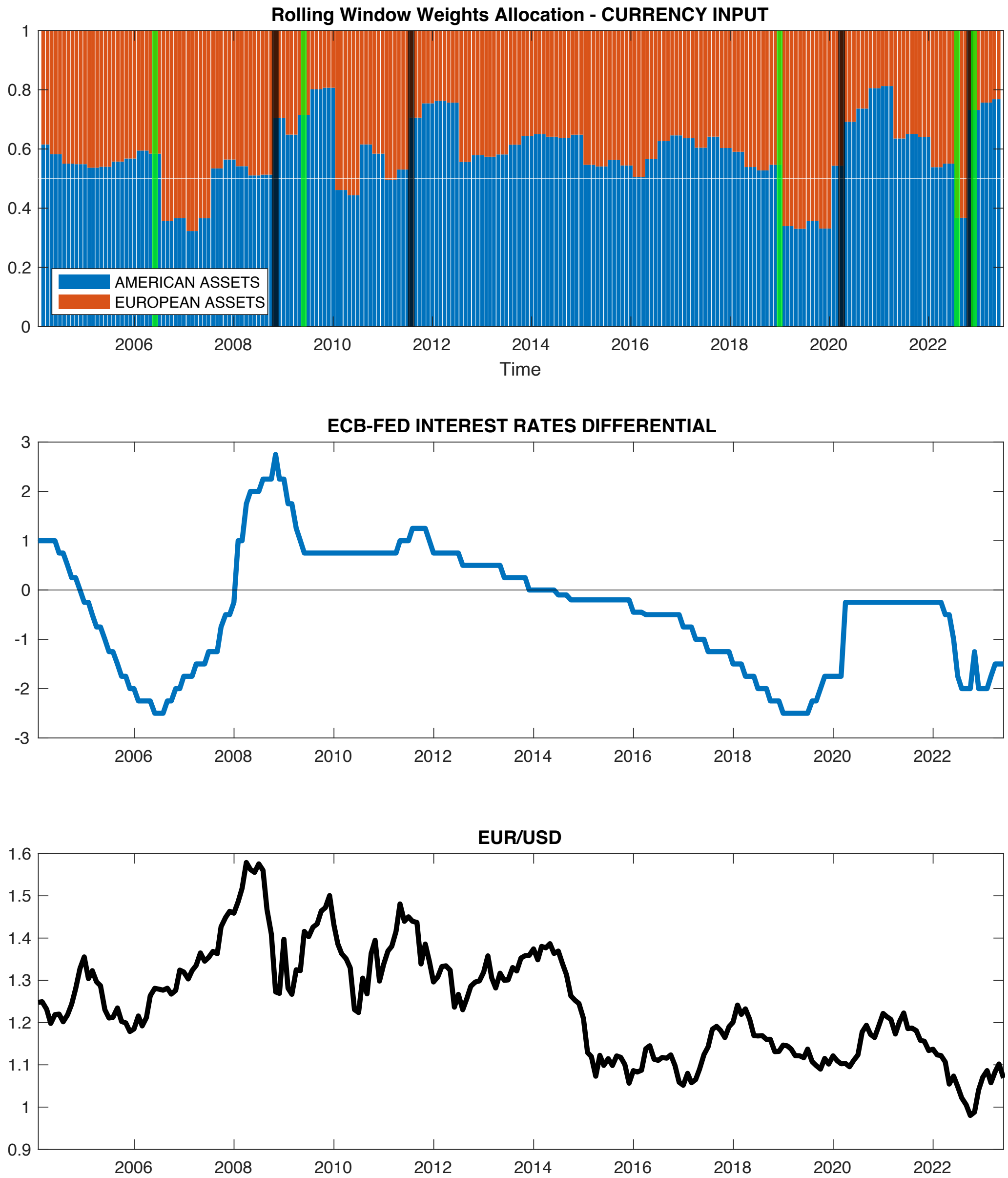

Figure 4.8 – Matlab script

Figure 4.8 shows the same rolling window weights allocation as before, but with aggregated European assets (in orange) and American assets (in blue) in order to visualize how the allocation had changed due to the currency input. Take in mind that weights here reported are not the actual weights working in our portfolio, here in fact we're taking out global corporate bonds and gold from the input implementation and consequently from the graph. As both gold and corporate bonds are US dollar denominated securities, this approach might seem counter intuitive, as it would imply always having an overweight on US dollar. However, if we included these assets within the currency input, allocation would have been extremely changed, not only

in geographic terms, but also in a structural way. As our priority is always to maintain a substantial level of diversification, we acknowledge this aspect and build the input as explained above.
Moreover, we plotted some vertical lines in order to detect when a valley or a peak in the differential between central banks interest rates occur. Specifically green vertical lines represent dates in which a valley is detected, hence when the differential is likely to increase in the subsequent periods and expectations of Euro to appreciate against US dollar, while black vertical lines represent the opposite. The white horizontal line represents the 50% level, hence when American assets and European assets have the same weights within the portfolio. As expected, we notice that right after every green line, American assets are decreased and European assets are increased, while the opposite happens right after a black vertical line.
We should also take in consideration an aspect that might not be trivial to derive from the graph. Since this model works due to three inputs, weights here derived might be affected from multiple decisions taken by the model. For instance, there are some fluctuations of European and American assets weights between 2012 and 2019 even though the differential between interest rates doesn’t reach neither a valley nor a peak. This is due to the overlap among the three inputs which might have caused some slight fluctuations. However, this is pretty much expected and doesn’t change the overall positive output generated by the currency input.

5) Suggestions for future research

Building on what we covered earlier, this section will go through some suggestion for possible future research on the study. Now, we're shifting our focus to explore some additional inputs we can bring into the model. These inputs, although not directly integrated into the initial model due to the possibility of overlapping with the core inputs, are important to consider because they could potentially enrich the model's adaptability and efficacy.

Within the current model framework, we should highlight that all the inputs we've integrated so far belong to the world of macroeconomics. This strategic decision has been driven by the aim to concentrate primarily on broader economic indicators, which hold the capacity to better adapt to a long-term period. However, it's important to recognize that the absence of financial-related inputs could potentially impose certain limitations, particularly in capturing some market trends and patterns. As we make progress, we'll go through the implications of this macroeconomic focus and contemplate how the incorporation of financial inputs could potentially heighten the model's accuracy in reflecting market dynamics. Furthermore, this chapter will introduce a further asset class to the portfolio, which might enrich the overall performance.

5.1) P/E Ratio

As we move to a financial setting in determining potential additional inputs to implement in the model, we first go through one of the most important indicators used when analyzing stocks or equity-related securities: the P/E Ratio.

The Price-to-Earnings ratio (P/E ratio) is a financial metric that is widely used by investors and analysts to assess the relative valuation of a stock. It's calculated by dividing the current market price of a stock by its earnings per share (EPS) over a specific period, often the last four quarters (trailing P/E), or sometimes the projected earnings for the next four quarters (forward P/E). Mathematically, the P/E ratio can be expressed as:

$$PE = \frac{Market\ Price\ per\ share}{Earnings\ per\ share}$$

This metric provides insights into how much investors are willing to pay for each unit of earnings generated by the company. A high P/E ratio might indicate that a company's stock is trading at a premium due to high growth expectations, which could also imply higher risk if

those expectations are not met, while a low P/E ratio may indicate that the company's stock is undervalued or that investors have lower expectations for future growth.
Therefore, in portfolio management P/E ratio is one among the tools used to determine whether an equity-related security is overvalued or undervalued with respect to the intrinsic value[20] of the underlying company. This reasoning makes sense not only for single stocks, but also for market indexes.
As we move forward with our analysis, we want to find an input which moves the allocation according to this metric. Specifically, we may want to intervene on the geographic allocation of the equity component, by deviating from the equal division between STOXX Europe 600 (SXXP) and S&P500 (SPX). Among the three inputs implemented in our model, the one which operates on currency exposure aims at shifting allocation among European and American assets. By doing so this input will change weights on every asset related to that geographic zone, yet it will not intervene directly on equity valuation. Furthermore, as the name itself suggests, the currency input is driven just by central banks interest rate differential, aiming at capturing deviations in the EUR/USD exchange rate, without considering any financial aspect.
As the PE ratio is a functional valuation metric, we will study movements in the relation between the major American and European stock indexes PE ratios. We will refer to this ratio as the "PE spread"[21].

$$PE\ spread = \frac{PE\ S\&P500}{PE\ \ STOXX\ EUROPE\ 600}$$

Intuitively, once the PE spread is above 1, it means that valuations in Europe seem to be more attractive with respect to American ones, as it benefits from a lower PE ratio than the one related to American companies, while the opposite happens when the PE spread is below 1.
Figure 5.1 shows the PE spread against the normalized performance of both the STOXX Euro 600 and the S&P 500 indexes. The black horizontal line is the 1 level, hence when valuations are exactly the same in both the regions.

[20] Intrinsic value of a stock: it represents the estimated true worth or fair value of a company's shares based on fundamental analysis. It is determined by assessing various financial factors, such as the company's earnings, cash flow, assets, and growth prospects.
[21] PE spread: it refers to the ratio between the SPX PE Ratio and the SXXP PE Ratio

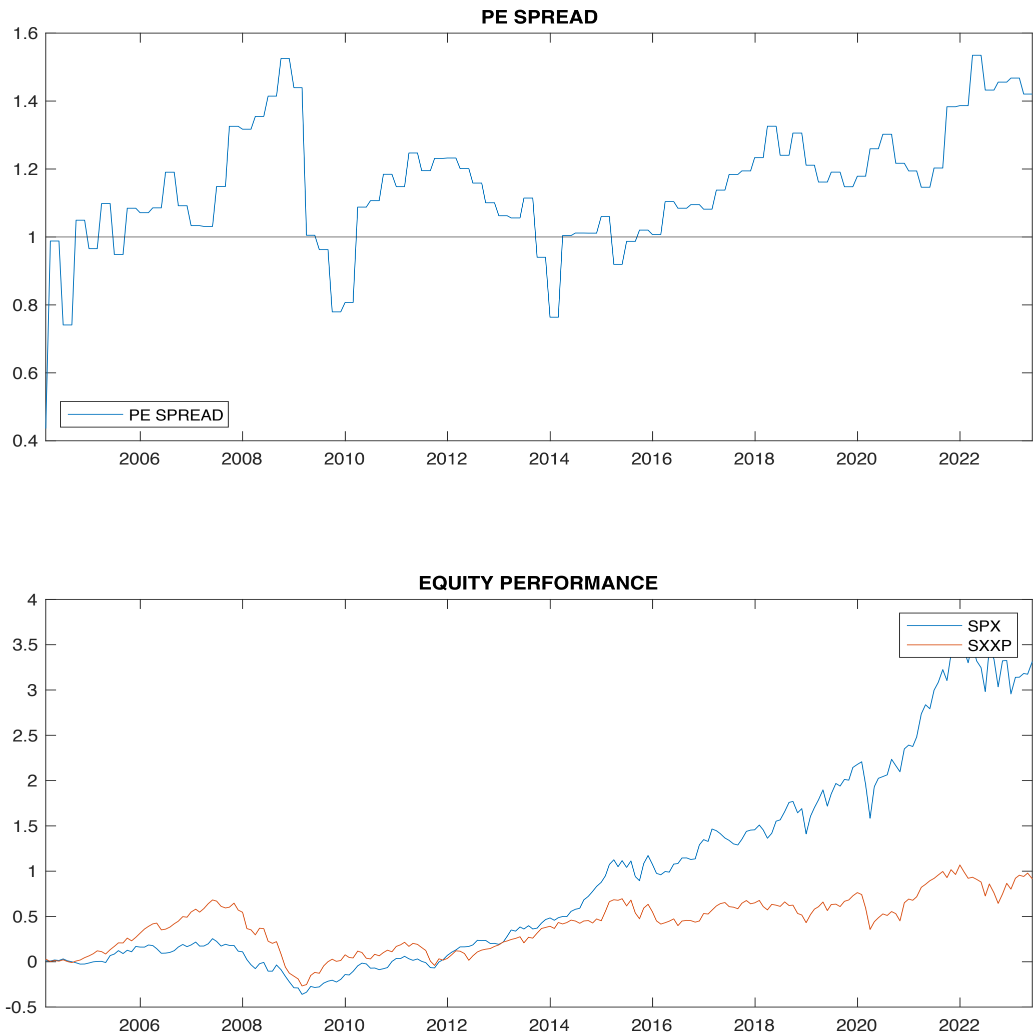

Figure 5.1 – Matlab script

From this graph we deduce that whenever the PE spread (yellow line) is below the 1 level (black line) we should expect European companies to be undervalued with respect to American's, hence we may move our allocation in order to capture potential additional returns, while the opposite occurs when it's above the 1 level. However, by looking at the graph we should notice that, over the considered time period (20 years), the PE spread has been below the 1 level only 12 quarters out of 79. This disequilibrium is not solely due to reflections in intrinsic value of stocks, as we mentioned that PE ratio may be influenced by multiple factors. American indexes have historically had higher ratios, also due to their higher percentage of "growth" companies, rather than "value".

Therefore, relying solely on this input would bring our portfolio to be over allocated on European indexes 85% of the times, lacking not only in terms of diversification, but also in terms of performance.

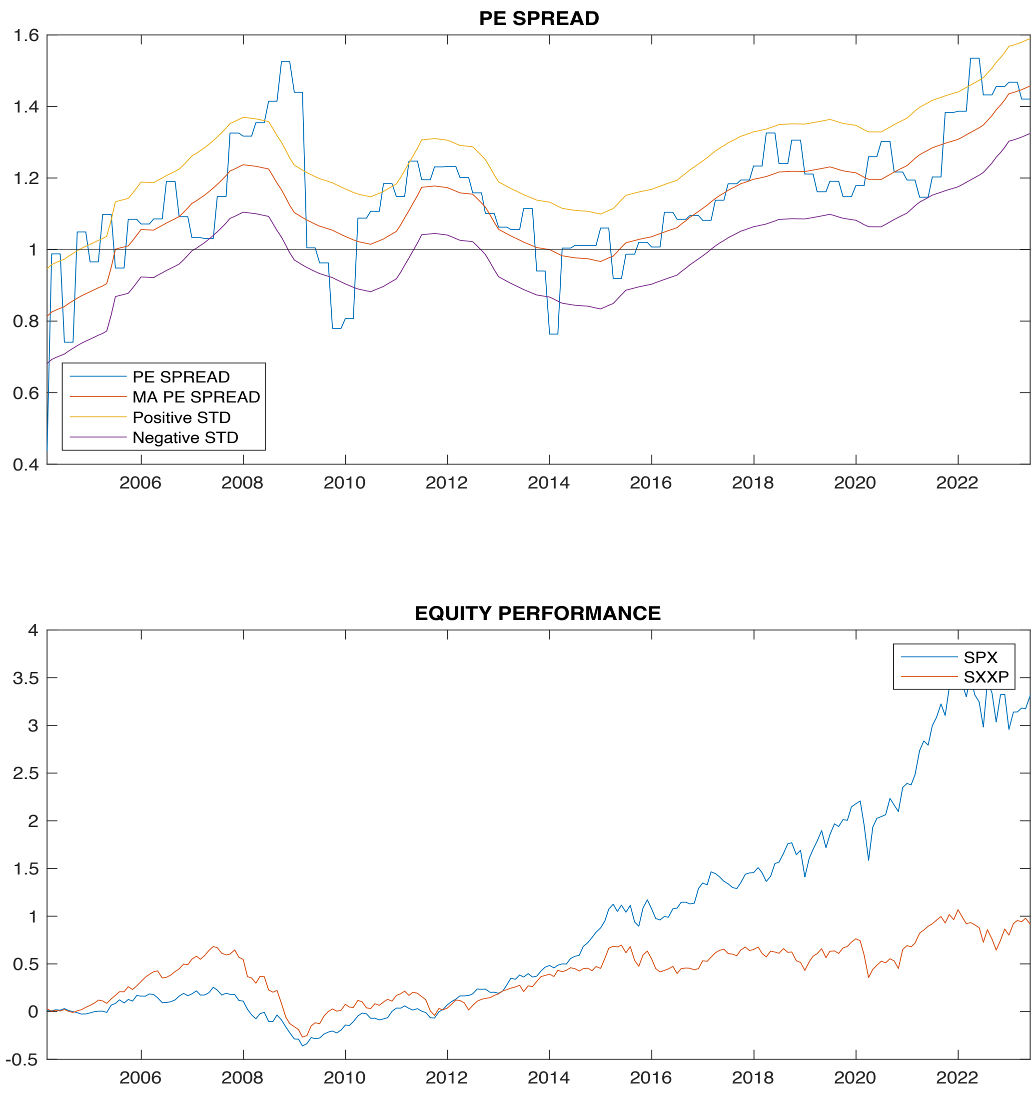

Figure 5.2 – Matlab script

A smoother approach might be once again making use of moving averages. *Figure* 5.2 plots the same series as before, but with the PE spread moving average series. Therefore, a suitable input might be shifting allocation whenever the observed PE spread deviates from the moving average by at least one standard deviation. Doing so would allow to incorporate the discrepancy between the two regions' PE ratios, by avoiding to rely on the biased level of 1 as main driver. Furthermore, yellow line and purple line serves as a brake to avoid the model to shift too frequently. In fact, by setting a threshold above and below which the input activates, we further suggest the model to shift the allocation only when the difference in the PE spread is significant.

5.2) Inflation linked bonds

While it is true that many investment strategies often incorporate inflation linked bonds (ILBs) for their hedging properties, our specific model has intentionally excluded them from asset allocation. This decision arises from the consideration of a potential overlap between this asset's properties and other inputs already implemented. We have designed our model to maintain a balanced approach, avoiding undue complexity. However, in order to provide a comprehensive understanding of this asset class, which might be useful for further studies, we present a brief overview of how inflation linked bonds work and how they might be introduced within a dynamic rebalancing model.

These bonds offer a unique advantage in protecting the portfolio against the eroding effects of inflation, especially over extended investment horizons, providing investors with returns that adjust for changes in the Consumer Price Index (CPI) or other inflation measures. By incorporating them into a portfolio, investors can effectively shield their investments from the diminishing purchasing power of currency over time.

In addition to realized inflation, the factors affecting the performance and risks of ILBs also include price changes driven by fluctuations in real yields. This element is not relevant if the bond is held till maturity. The bond's market value, however, fluctuates in relation to its nominal value prior to maturity. Inflation-linked bonds appreciate if real yields decline and depreciate if real yields rise, like nominal bonds, whose prices change in reaction to changes in nominal interest rates. The inflation-adjusted capital may be less than the nominal value if deflation takes place while an ILB is in effect. The value adjusted for deflation will serve as the basis for further coupon payments. However, many countries that issue ILBs provide a deflation floor when they mature: the investor still receives the nominal value at maturity even if deflation lowers the capital's value below the nominal value. Thus, while coupon payments are based on capital adjusted for inflation or deflation, at maturity, the investor receives the higher between the capital adjusted for inflation and the initial nominal value.

Furthermore, adding ILBs to a portfolio can significantly boost its level of diversification, lowering volatility. ILBs have traditionally demonstrated low correlation with stocks, commodities, and other asset classes and can respond differently to economic conditions. Here, it's helpful to briefly compare ILBs with nominal government bonds in order to assess their relative worth. A valuable approach is looking at the differential between nominal and real yields, also known as the break-even inflation rate. *Figure* 5.3 reports the US break-even inflation rate at 10 years against the actual US inflation (US CPI).

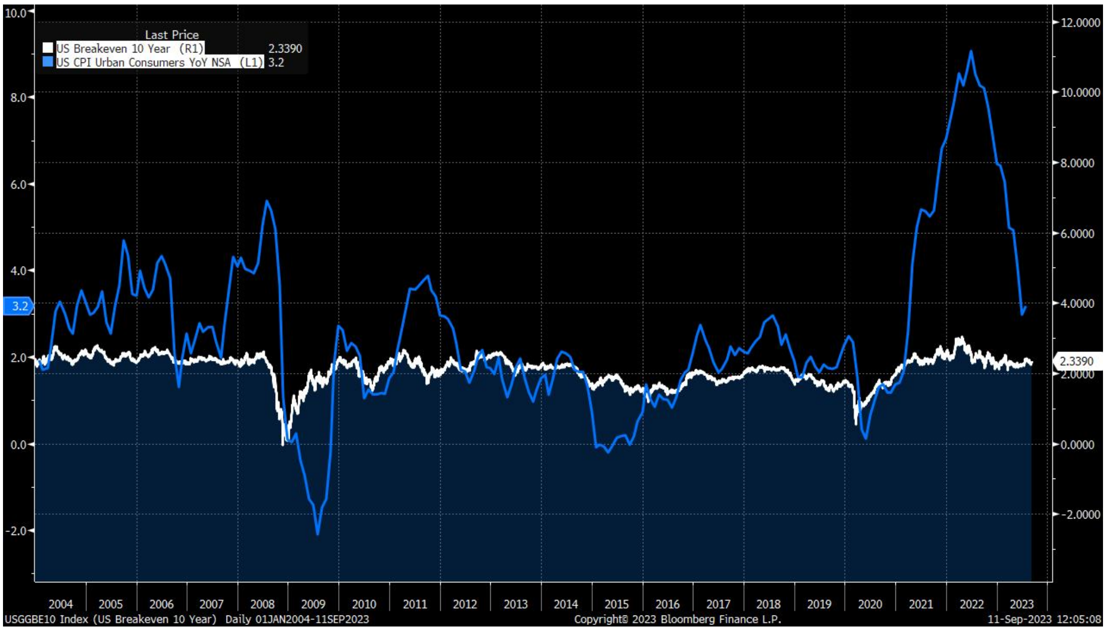

Figure 5.3 – Bloomberg Terminal

The rate differential at which the projected returns of ILBs and nominal bonds are equal serves as a proxy for market inflation expectations. Investors earn more with ILBs while bearing less inflation risk if the real inflation rate during the bond's life is higher than the break-even rate. Conversely, if the actual inflation rate is below expectations, the nominal government bond with the same maturity offers a higher return, even though with higher inflation risk.

Therefore, a possible approach to refine a long-term portfolio allocation might be to incorporate this asset class within the portfolio, by also allowing the dynamic model to shift the allocation whenever expectations of inflation deviate from the break-even rate.

Conclusions

Through this research paper we wanted to build a dynamic rebalancing model for passive portfolios over an extended time horizon, with a specific focus on the All Weather portfolio as our primary test case. The study systematically examined various aspects of portfolio theory, delving into Modern Portfolio Theory and Risk Parity portfolio theory, establishing a foundational understanding of the metrics used in the analysis, and presenting empirical insights on their underlying philosophies.

Our investigation into the Bridgewater All Weather strategy allowed us to grasp its distinctive asset allocation, which aims to withstand different economic environments. We also considered the insights from Ray Dalio’s main works to provide valuable context to the portfolio’s design and its applicability in varying market conditions.

Through backtesting, we assessed the performance of the All Weather portfolio, providing a historical perspective on its effectiveness as an investment strategy. This allocation allowed the portfolio to be significantly diversified, hence less exposed to several shocks over the time frame considered. The All Weather portfolio managed to generate a distinctive performance in terms of volatility, yet providing a significant annualized return, with respect to the classic 60/40 portfolio. This historical analysis served as a crucial benchmark for evaluating the effectiveness of our proposed dynamic rebalancing model.

The core section of our research was dedicated to developing and implementing a dynamic rebalancing model. Our model accounted for various asset classes adjustments, working due to three inputs employed on different asset classes, namely bonds, equities, and currency exposure. It was built to encompass several key aspects in the daily life of a portfolio manager, trying to capture the core drivers of asset allocation within a long-term horizon.

Analyzing the model, we unveiled essential insights into the impact of our dynamic rebalancing model, showing encouraging results in terms of performance. Not only this strategy allowed to generate a better annualized return, but it also managed to significantly reduce the volatility of the portfolio. The three inputs make the portfolio work more aggressively or defensively depending on market conditions. This section also provided a detailed breakdown of asset allocation contributions by inputs, to get the effective functioning of each driver.

In the pursuit of enhancing portfolio management practices, we also explored additional implementable inputs, expanding the possibilities for future research and model refinement, by providing an insight on a possible financial-related input and on a further asset class which might be useful to implement in hedging inflation erosion.

While our analysis yielded promising insights, it is crucial to recognize that the financial landscape is dynamic and ever-evolving. Further research and refinement of the model are warranted to adapt to changing market conditions and continue improving portfolio management strategies. As investors seek resilient and efficient portfolio management approaches, the exploration of dynamic rebalancing models remains an important way for future investigation.

Appendix A) Markowitz vs Risk Parity

```
%% RISK PARITY MODELS VS MARKOWITZ MODELS
%In this code we execute an analysis on 2 different portfolio allocation
%philosophies. For the purpose of this analysis, we want to show
%that a Risk Parity model generates better results in terms of
%diversification, and consequently on the risk-adjusted performance of the
%portfolio.

clc
clear all

datagen = xlsread('TESI DATASET');
Dates = datagen(:, 1) + 693960;
d = datestr(Dates);
dates_gen = datetime(d);

%% LOAD DATA
LEATTREU=datagen(1:end,11); % European bond long-term
LET1TREU=datagen(1:end,12); % European bond short-term
LUATTRUU=datagen(1:end,13); % American bond long-term
LU13TRUU=datagen(1:end,14); % American bond short-term
LGCPTRUU=datagen(1:end,15); % Global corporate bond

%STOCKS
SXXP=datagen(1:end,16); %STOXX Europe 600
SPX=datagen(1:end,17); %S&P500 Index
GLD=datagen(1:end,20); %Gold spot
price_data=[SXXP,LEATTREU,LET1TREU,GLD];



%% RETURNS and REBALANCING
% Compute returns
returns = price2ret(price_data,1:233,'Periodic');
cumulativereturns=cumprod(1+returns);
series=(1+cumulativereturns);

%Plot price series
Figure
plot(dates_gen(2:end),series)
legend('SXXP', 'LEATTREU','LET1TREU','GLD')
title('NORMALIZED ASSETS PERFORMANCE')



%%
numAssets = length(price_data(1,:));
numPeriods = length(price_data(:,1));

% Set rebalancing frequency (in months)
freq = 3;

% Initialize portfolio weights
weights_Markowitz = zeros(numAssets, numPeriods);
weights_rp = zeros(numAssets, numPeriods);
initWeights = [0.4;0.2;0.2;0.2];
```

```
% Set initial weights
weights_Markowitz(:,1) = initWeights;
weights_rp(:,1) = initWeights;

% Iterate over periods
for i = 2:numPeriods
    % Rebalance portfolio every rebalFreq periods
    if mod(i-1, freq) == 0
        % Compute portfolio returns over the past rebalFreq periods
        returns_window = returns(max(1, i-freq):i,:);

        % Compute mean returns and covariance matrix
        meanReturns = mean(returns_window, 2);
        covMatrix = cov(returns_window');

        % Compute optimal weights using the mean-variance optimization
(Markowitz)
        [PortRisk, PortReturn, optWeights] = portopt(meanReturns,
covMatrix,1);
        optWeights_Markowitz = optWeights;
        optWeights_Markowitz =
optWeights_Markowitz/sum(optWeights_Markowitz);
        weights_Markowitz(:,i) = optWeights_Markowitz;


        % Compute optimal weights using Risk Parity
        covMatrix_reg = covMatrix + 0.01*eye(numAssets);
        riskContribution = sqrt(diag(covMatrix)) ./
sum(sqrt(diag(covMatrix)));
        invCovMatrix = inv(covMatrix_reg);
        onesVec = ones(numAssets,1);
        invCovOnes = invCovMatrix*onesVec;
        riskParityWeights = invCovOnes./sum(invCovOnes);
        weights_rp(:,i) = riskParityWeights;



    else
        % Use previous weights
        weights_Markowitz(:,i) = weights_Markowitz(:,i-1);
        weights_rp(:,i) = weights_rp(:,i-1);
    end
end

weights_Markowitz = weights_Markowitz(:,2:end)';
weights_rp = weights_rp(:,2:end)';

%% COMPUTE RESULTS
% Compute portfolio returns
portfolioReturns_MV = sum(weights_Markowitz.*returns, 2);
portfolioReturns_RP = sum(weights_rp.*returns, 2);

% Compute cumulative returns for each portfolio
cumulativeReturns_MV = cumprod(1 + portfolioReturns_MV) - 1;
cumulativeReturns_RP = cumprod(1 + portfolioReturns_RP) - 1;

% Compute annualized returns as CAGR
numPeriodsPerYear = 12;
numYears = numPeriods/numPeriodsPerYear;
```

```
annualizedReturns_MV = (1 + cumulativeReturns_MV(end))^(1/numYears) - 1;
annualizedReturns_RP = (1 + cumulativeReturns_RP(end))^(1/numYears) - 1;

disp(['Markowitz PTF annualized return: ' num2str(annualizedReturns_MV)]);
disp(['Risk Parity PTF annualized return: '
num2str(annualizedReturns_RP)]);

% Compute annualized standard deviation of returns for each portfolio
StdDev_MV = std(portfolioReturns_MV)*sqrt(12);
StdDev_RP = std(portfolioReturns_RP)*sqrt(12);

disp(['Markowitz PTF STD: ' num2str(StdDev_MV)]);
disp(['Risk Parity PTF STD: ' num2str(StdDev_RP)]);
% Set RISK FREE RATE
rf_rate=0.02;
% Compute Sharpe ratio for each portfolio
Sharpe_MV = (annualizedReturns_MV-rf_rate) / StdDev_MV;
Sharpe_RP = (annualizedReturns_RP-rf_rate) / StdDev_RP;
disp(['Markowitz PTF sharpe ratio: ' num2str(Sharpe_MV)]);
disp(['Risk Parity PTF sharpe ratio: ' num2str(Sharpe_RP)]);

figure
scatter(StdDev_MV,annualizedReturns_MV)
hold on
scatter(StdDev_RP,annualizedReturns_RP)
ylabel('Annualized Return')
xlabel('Standard Deviation')
legend('Markowitz PTF','RISK PARITY PTF')


%% PTF SERIES COMPARISON
%Here we assume that we invest an initial sum of 1000$ and we notice that
%the Risk Parity portfolio provides a better payoff at the end of the
%period with significant lower risk.

initialInvestment = 10000;

ptf_MV = initialInvestment * cumprod(1 + sum(weights_Markowitz .* returns,
2));
ptf_RP = initialInvestment * cumprod(1 + sum(weights_rp .* returns, 2));

figure
plot(dates_gen(2:end,:), ptf_MV, 'b')
hold on
plot(dates_gen(2:end,:), ptf_RP, 'r')
title('COMPARISON PLOT')
legend('Markowitz', 'Risk Parity')

%% ROLLING WINDOW WEIGHTS ALLOCATION
barWidth = 0.85;

% Set the x-axis values
x = 1:numPeriods;

% Create a bar graph for the Markowitz portfolio weights
figure
subplot(2,1,1)
bar(dates_gen(2:end),weights_Markowitz, barWidth, 'stack');
```

```
xlim([dates_gen(1) dates_gen(end)]);
ylim([0 1]);
xlabel('Time');
ylabel('Weights');
title('Rolling Window Weights Allocation - Markowitz Portfolio');
legend('SXXP', 'LEATTREU','LET1TREU','GLD', 'Location', 'northwest');
hold on
% Create a bar graph for the Risk Parity portfolio weights
subplot(2,1,2)
bar(dates_gen(2:end),weights_rp, barWidth, 'stack');
xlim([dates_gen(1) dates_gen(end)]);
ylim([0 1]);
xlabel('Time');
ylabel('Weights');
title('Rolling Window Weights Allocation - Risk Parity Portfolio');
legend('SXXP', 'LEATTREU','LET1TREU','GLD', 'Location', 'northwest');

%% COMPARISON DRAWDOWNS PLOT
maxReturns_MV = cummax(ptf_MV);
drawdowns_MV = -(maxReturns_MV - ptf_MV)./maxReturns_MV;
durations_m = diff(find([1;drawdowns_MV(1:end-1)<0 &
drawdowns_MV(2:end)>=0;1]));
maxDD_MV=min(drawdowns_MV);
negative_ret_MV=portfolioReturns_MV(portfolioReturns_MV<0);
downside_MV=std(negative_ret_MV)*sqrt(12);
SOR_MV = (annualizedReturns_MV-rf_rate) / downside_MV;
disp(['CLASSIC PTF Sortino ratio: ' num2str(SOR_MV)]);


maxReturns_RP = cummax(ptf_RP);
drawdowns_RP = -(maxReturns_RP - ptf_RP)./maxReturns_RP;
durations_RP = diff(find([1;drawdowns_RP(1:end-1)<0 &
drawdowns_RP(2:end)>=0;1]));
maxDD_RP=min(drawdowns_RP);
negative_ret_RP=portfolioReturns_RP(portfolioReturns_RP<0);
downside_RP=std(negative_ret_RP)*sqrt(12);
SOR_RP = (annualizedReturns_RP-rf_rate) / downside_RP;
disp(['CLASSIC PTF Sortino ratio: ' num2str(SOR_RP)]);


% Plot drawdowns
figure
plot(dates_gen(2:end),drawdowns_MV)
xlabel('Date')
ylabel('Drawdown')
hold on
plot(dates_gen(2:end),drawdowns_RP)
hold off
title('Drawdown Analysis')
legend('Markowitz','Risk Parity')
%%
Metrics=["Annualized return";"Standard Deviation";"Sharpe Ratio";"Sortino
Ratio";"Maximum Drawdown"];
Markowitz=[annualizedReturns_MV;StdDev_MV;Sharpe_MV;SOR_MV;maxDD_MV]
RiskParity=[annualizedReturns_RP;StdDev_RP;Sharpe_RP;SOR_RP;maxDD_RP]
a=table(Metrics, Markowitz, RiskParity)
```

Appendix B) All Weather backtesting

```
%% ALL WEATHER VS CLASSIC ALLOCATION PORTFOLIO - US EQUITY
% In this code we execute an analysis on 2 portfolios. Here we want to
% investigate the advantages of All Weather
% portfolios, by proving they generate a better performance in terms
% of risk-adjusted return with respect to a classic allocation portfolio
(60%
% EQUITY, 40% BONDS).

clc
clear all

%% IMPORT DATES
datagen=xlsread('TESI DATASET');
Dates=datagen(1:end,1)+693960;
d=datestr(Dates);
dates_gen=datetime(d);

%% IMPORT DATA
ECB=datagen(1:end,2); %EURR02W
FED=datagen(1:end,3); %FDTR
EURR_FED=datagen(1:end,4); %Differential EURR02W-FDTR
GDP_US=datagen(1:end,5); %GDP United States
GDP_UE=datagen(1:end,8); %GDP European Union
%BONDS
LEATTREU=datagen(1:end,11); % European bond long-term
LET1TREU=datagen(1:end,12); % European bond short-term
LUATTRUU=datagen(1:end,13); % American bond long-term
LU13TRUU=datagen(1:end,14); % American bond short-term
LGCPTRUU=datagen(1:end,15); % Global corporate bond
%STOCKS
SXXP=datagen(1:end,16); %STOXX Europe 600
SPX=datagen(1:end,17); %S&P500 Index
PE_UE=datagen(1:end,18);
PE_US=datagen(1:end,19);
%GOLD
GLD=datagen(1:end,20); %Gold spot
%CHANGE
EURUSD=datagen(1:end,21); %Euro US Dollar Change

STOCKS = [SPX];
BONDS = [LUATTRUU, LU13TRUU, LGCPTRUU];
americani = [LUATTRUU, LU13TRUU, SPX];
europei=[LEATTREU,LET1TREU,SXXP];
ASSETS = [americani, LGCPTRUU, GLD];

%% ALL WEATHER ALLOCATION
 %30% SPX
 %25% LUATTRUU
 %25% LUT13TRU
 %5%  LGCPTRUU
 %15% GLD

returns = price2ret(ASSETS,1:233,'Periodic');
std_ASSETS=std(returns);
std_ASSETS=std_ASSETS*sqrt(12);
numAssets = size(ASSETS, 2);
numPeriods = size(ASSETS, 1);
SUMERT_SpX=cumprod(1+returns(:,3))-1
```

```
%% REBALANCING WEIGHTS STATIC
weights_2 = zeros(numAssets, numPeriods);
initWeights_2 = [0.25; 0.25; 0.30; 0.05; 0.15];
weights_2(:, 1) = initWeights_2;
freq = 3;

for i = 2:numPeriods
    % Rebalance portfolio every rebalFreq periods
    if mod(i-1, freq) == 0
       returns_window_2 = returns((i-freq):i,:);

       % Compute mean returns and covariance matrix
        meanReturns_2 = mean(returns_window_2, 2);
        covMatrix_2 = cov(returns_window_2);

         % Compute optimal weights using Risk Parity
        covMatrix_reg_2 = covMatrix_2 + 0.01*eye(numAssets);
        invCovMatrix_2 = inv(covMatrix_reg_2);
        onesVec = ones(numAssets,1);
        invCovOnes_2 = invCovMatrix_2*onesVec;
        riskParityWeights_2 = invCovOnes_2./sum(invCovOnes_2);
        weights__2(:,i) = riskParityWeights_2;

        % Apply the constraint that each asset allocation doesn't deviate
by more than 5%
        targetWeights_2 = initWeights_2;
        newWeights_2 = riskParityWeights_2;
        for j = 1:numAssets
            if newWeights_2(j) > targetWeights_2(j) * 1.25
                newWeights_2(j) = targetWeights_2(j) * 1.25;
            elseif newWeights_2(j) < targetWeights_2(j) * 0.75
                newWeights_2(j) = targetWeights_2(j) * 0.75;
            end
        end

        % Normalize the weights
        newWeights_2 = newWeights_2 ./ sum(newWeights_2);
        weights_2(:,i) = newWeights_2;
    else
        % Use previous weights
        weights_2(:,i) = weights_2(:,i-1);
    end
end
weights_2=weights_2';
weights_2=weights_2(2:end,:);
portfolioReturns_2 = sum(weights_2.*returns, 2);



%% CLASSIC ALLOCATION PORTFOLIO
DATA_C=[SPX,LUATTRUU];

returns_C = price2ret(DATA_C,1:233,'Periodic');
numAssets_C=size(DATA_C,2);
numPeriods_C=size(DATA_C,1);
weights_C = zeros(numAssets_C, numPeriods_C);
initWeights_C = [0.6;0.4];
```

```
weights_C(:,1) = initWeights_C;
freq=3;

% REBALANCING WEIGHTS
for i = 2:numPeriods_C
    % Rebalance portfolio every rebalFreq periods
    if mod(i-1, freq) == 0
       returns_window_C = returns_C((i-freq):i,:);

       % Compute mean returns and covariance matrix
        meanReturns_C = mean(returns_window_C, 2);
        covMatrix_C = cov(returns_window_C);

         % Compute optimal weights using Risk Parity
        covMatrix_reg_C = covMatrix_C + 0.01*eye(numAssets_C);
        invCovMatrix_C = inv(covMatrix_reg_C);
        onesVec_C = ones(numAssets_C,1);
        invCovOnes_C = invCovMatrix_C*onesVec_C;
        acweights = invCovOnes_C./sum(invCovOnes_C);
        weights_C(:,i) = acweights;

        % Apply the constraint that each asset allocation doesn't deviate
by more than 5%
        targetWeights_C = initWeights_C;
        newWeights_C = acweights;
        for j = 1:numAssets_C
            if newWeights_C(j) > targetWeights_C(j) * 1.05
                newWeights_C(j) = targetWeights_C(j) * 1.05;
            elseif newWeights_C(j) < targetWeights_C(j) * 0.95
                newWeights_C(j) = targetWeights_C(j) * 0.95;
            end
        end

        % Normalize the weights
        newWeights_C = newWeights_C ./ sum(newWeights_C);
        weights_C(:,i) = newWeights_C;
    else
        % Use previous weights
        weights_C(:,i) = weights_C(:,i-1);
    end
end
weights_C=weights_C';
weights_C=weights_C(2:end,:);
portfolioReturns_C = sum(weights_C.*returns_C, 2);

%% PTF SERIES PLOT
init_cap=10000;
cumulativeReturns_2 = cumprod(1 + portfolioReturns_2) - 1;
cumulativeReturns_C = cumprod(1 + portfolioReturns_C) - 1;
portfolio_series_2=init_cap*(1+cumulativeReturns_2);
portfolio_series_C=init_cap*(1+cumulativeReturns_C);

figure
plot(dates_gen(2:end,:), portfolio_series_2, 'r')
hold on
plot(dates_gen(2:end,:),portfolio_series_C)
legend('All Weather','Classic PTF')
hold off
title('PTF SERIES')
```

```
%% Compute annualized returns as CAGR for ALL WEATHER
nmonths = 12;
numYears = numPeriods/nmonths;

annualizedReturns_2 = (1 + cumulativeReturns_2(end))^(1/numYears) - 1;
disp(['Risk Parity PTF annualized return: '
num2str(annualizedReturns_2)]);

% Compute annualized standard deviation of returns for each portfolio
StdDev_2 = std(portfolioReturns_2)*sqrt(12);

disp(['Risk Parity PTF STD: ' num2str(StdDev_2)]);
% Set RISK FREE RATE
rf_rate=0.01;
% Compute Sharpe ratio for each portfolio
Sharpe_2 = (annualizedReturns_2-rf_rate) / StdDev_2;
disp(['Risk Parity PTF sharpe ratio: ' num2str(Sharpe_2)]);

negative_ret_2=portfolioReturns_2(portfolioReturns_2<0);
downside_2=std(negative_ret_2)*sqrt(12);
SOR_2 = (annualizedReturns_2-rf_rate) / downside_2;
disp(['ALL WEATHER PTF Sortino ratio: ' num2str(SOR_2)]);
%% Compute annualized returns as CAGR for CLASSIC
nmonths = 12;
numYears = numPeriods/nmonths;

annualizedReturns_C = (1 + cumulativeReturns_C(end))^(1/numYears) - 1;
disp(['CLASSIC PTF annualized return: ' num2str(annualizedReturns_C)]);

% Compute annualized standard deviation of returns for each portfolio
StdDev_C = std(portfolioReturns_C)*sqrt(12);

disp(['CLASSIC PTF STD: ' num2str(StdDev_C)]);
% Set RISK FREE RATE
rf_rate=0.01;
% Compute Sharpe ratio for each portfolio
Sharpe_C = (annualizedReturns_C-rf_rate) / StdDev_C;
disp(['CLASSIC PTF sharpe ratio: ' num2str(Sharpe_C)]);

negative_ret_C=portfolioReturns_C(portfolioReturns_C<0);
downside_C=std(negative_ret_C)*sqrt(12);
SOR_C = (annualizedReturns_C-rf_rate) / downside_C;
disp(['CLASSIC PTF Sortino ratio: ' num2str(SOR_C)]);
%% ROLLING WINDOW WEIGHTS ALLOCATION
barWidth = 0.95;
x = 1:numPeriods;

figure
subplot(2,1,1)
bar(dates_gen(2:end),weights_2, barWidth, 'stack');
ylim([0 1]);
xlabel('Time');
ylabel('Weights');
title('Rolling Window Weights Allocation - ALL WEATHER Portfolio');
legend('SPX','LUATTRUU','LU13TRUU','LGCPTRUU','GLD', 'Location',
'northwest');
subplot(2,1,2)
bar(dates_gen(2:end),weights_C, barWidth, 'stack');
ylim([0 1]);
```

```
xlabel('Time');
ylabel('Weights');
title('Rolling Window Weights Allocation - CLASSIC Portfolio');
legend('SPX','LUATTRUU', 'Location', 'northwest');

%% DRAWDOWNS PLOT
maxReturns = cummax(portfolio_series_2);
drawdowns = -(maxReturns - portfolio_series_2)./maxReturns;
maxDrawdown_2 = min(drawdowns);
endDates_2 = find(drawdowns == 0);
DD_check=maxdrawdown(portfolio_series_2);

maxReturns_c = cummax(portfolio_series_C);
drawdowns_c = -(maxReturns_c - portfolio_series_C)./maxReturns_c;
maxDrawdown_c = min(drawdowns_c);
endDates_c = find(drawdowns_c == 0);
DD_check_c=maxdrawdown(portfolio_series_C);

figure
plot(dates_gen(2:end),drawdowns)
hold on
plot(dates_gen(2:end),drawdowns_c)
hold off
title('Drawdown Analysis')
legend('All Weather','Classic portfolio')


%% RESULTS TABLE
Metrics=["Annualized return";"Standard Deviation";"Sharpe Ratio";"Sortino
Ratio";"Maximum Drawdown"];
AllWeather=[annualizedReturns_2;StdDev_2;Sharpe_2;SOR_2;maxDrawdown_2]
Classic=[annualizedReturns_C;StdDev_C;Sharpe_C;SOR_C;maxDrawdown_c]
a=table(Metrics,Classic,AllWeather)
```

Appendix C) Dynamic rebalancing model

```
%% DYNAMIC ALL WEATHER PTF VS STATIC ALL WEATHER PTF VS CLASSIC ALLOCATION
PORTFOLIO
% In this code we want to execute an analysis on different portfolio
allocation and
% rebalancing techniques by building up a model which deviates from the
% static All Weather portfolio of the previous code. Here we want to
% investigate whether it's possible to highlight some rebalancing rules
that,
% over the long run, allows the portfolio to generate a better performance
% in terms of risk and return. Here we will build our model by introducing
% 3 inputs that modify the portfolio when occurring during the 20 years
period
% investigated (2004-2023).
%      1) EUR/USD INPUT
%      2) BONDS DURATION INPUT
%      3) GDP-EQUITY INPUT

clc
clear all


%% IMPORT DATES
datagen = xlsread('TESI DATASET');
Dates = datagen(:, 1) + 693960;
d = datestr(Dates);
dates_gen = datetime(d);




%% IMPORT DATA
ECB = datagen(:, 2); % EURR02W
DECB=diff(ECB); %Differential
MDECB=movmean(DECB,12); %Moving Average of differential

FED = datagen(:, 3); % FDTR
DFED=diff(FED); %Differential
MDFED=movmean(DFED,12); %Moving Average of differential

EURR_FED = datagen(:, 4); % Differential EURR02W-FDTR

GDP_US = datagen(:, 5); % GDP United States
DGDP_US=diff(GDP_US);
MDGDP_US=movmean(DGDP_US,15); %Moving Average of differential

GDP_UE = datagen(:, 8); % GDP European Union
DGDP_UE=diff(GDP_UE);
MDGDP_UE=movmean(DGDP_UE,15); %Moving Average of differential
%BONDS
LEATTREU = datagen(:, 11); % European bond long-term
LET1TREU = datagen(:, 12); % European bond short-term
LUATTRUU = datagen(:, 22); % American bond long-term
LU13TRUU = datagen(:, 23); % American bond short-term
LGCPTRUU = datagen(:, 15); % Global corporate bond
%STOCKS
SXXP = datagen(:, 16); %STOXX Europe 600
SPX = datagen(:, 24); %S&P500 Index
```

```
PE_UE = datagen(:, 18);
PE_US = datagen(:, 19);
%GOLD
GLD = datagen(:, 20); %Gold spot
%CHANGE
EURUSD = datagen(:, 21); %Euro US Dollar Change

STOCKS = [SXXP, SPX];
BONDS = [LEATTREU, LET1TREU, LUATTRUU, LU13TRUU, LGCPTRUU];
americani = [LUATTRUU, LU13TRUU, SPX];
europei = [LEATTREU, LET1TREU, SXXP];
ASSETS = [LUATTRUU, LU13TRUU, SPX, LEATTREU, LET1TREU, SXXP, LGCPTRUU,
GLD];

returns = price2ret(ASSETS,1:233,'Periodic');
numAssets = size(ASSETS, 2);
numPeriods = size(ASSETS, 1);
weights_rp = zeros(numAssets, numPeriods);
initWeights = [0.125; 0.125; 0.15; 0.125; 0.125; 0.15; 0.05; 0.15];
weights_rp(:, 1) = initWeights;
freq = 3;

%% FIND PEAKS AND VALLEYS
%FOR INTEREST RATES DIFFERENTIAL
[peakValues, peakIndices] = findpeaks(EURR_FED);% Find relative maximums
(peaks)
[valleyValues, valleyIndices] = findpeaks(-EURR_FED);% Find relative
minimums (valleys)
valleyValues = -valleyValues;% Convert valleyValues back to positive values
for easy comparison
peakDates = dates_gen(peakIndices);% Find the corresponding dates for peaks
valleyDates = dates_gen(valleyIndices);% Find the corresponding dates for
valleys
STATIONARYPTS = [peakDates; valleyDates];

% Define your allocation changes based on peakDates and valleyDates
% Example allocation changes: Increase European assets at peakDates,
increase American assets at valleyDates
change = 0.05;
POS_CHANGE = 0.1;
NEG_CHANGE = -0.1;



%% REBALANCING WEIGHTS DYNAMIC
for i = 2:numPeriods
    % Rebalance portfolio every rebalFreq periods
    if mod(i - 1, freq) == 0
       returns_window = returns((i - freq):i, :);

       % Compute mean returns and covariance matrix
       meanReturns = mean(returns_window, 2);
       covMatrix = cov(returns_window);

       % Compute optimal weights using Risk Parity
       covMatrix_reg = covMatrix + 0.01 * eye(numAssets);
       invCovMatrix = inv(covMatrix_reg);
       onesVec = ones(numAssets, 1);
       invCovOnes = invCovMatrix * onesVec;
       riskParityWeights = invCovOnes ./ sum(invCovOnes);
```

```
        weights_rp(:, i) = riskParityWeights;

        % Apply the constraint that each asset allocation doesn't deviate by
more than 25%
        targetWeights = initWeights;
        newWeights = riskParityWeights;
        for j = 1:numAssets
            if newWeights(j) > targetWeights(j) * 1.25
                newWeights(j) = targetWeights(j) * 1.25;
            elseif newWeights(j) < targetWeights(j) * 0.75
                newWeights(j) = targetWeights(j) * 0.75;
            end
        end



        % EURUSD INPUT
        currentDate = dates_gen(i);
        isPeakDate = any(currentDate >= peakDates & currentDate <= peakDates
+ calyears(1));
        isValleyDate = any(currentDate >= valleyDates & currentDate <=
valleyDates + calyears(1));

        if isPeakDate
            newWeights(1:3) = newWeights(1:3) +change;% Decrease European
asset allocation
            newWeights(4:6) = newWeights(4:6) -change; % Increase American
asset allocation
        elseif isValleyDate
            newWeights(1:3) = newWeights(1:3) -change;% Increase European
asset allocation
            newWeights(4:6) = newWeights(4:6) +change;% Decrease American
asset allocation
        end



        % EQUITY INPUT
        if MDGDP_UE(i) > 0.01
            newWeights(6) = newWeights(6) + 0.06;% Increase SXXP
            newWeights(4) = newWeights(4) - 0.015;% Decrease LONG-TERM
            newWeights(5) = newWeights(5) - 0.015;% Decrease SHORT-TERM
            newWeights(7) = newWeights(7) - 0.015;% Decrease CORP
            newWeights(8) = newWeights(8) - 0.015;% Decrease GLD

        elseif MDGDP_UE(i) <= 0.01 && MDGDP_UE(i) >= -0.01
            newWeights(6) = newWeights(6);
            newWeights(4) = newWeights(4);
            newWeights(5) = newWeights(5);
            newWeights(7) = newWeights(7);
            newWeights(8) = newWeights(8);

        else
            newWeights(6) = newWeights(6) - 0.06;% Decrease SXXP
            newWeights(4) = newWeights(4) + 0.015;% Increase LONG-TERM
            newWeights(5) = newWeights(5) + 0.015;% increase LONG-TERM
            newWeights(7) = newWeights(7) + 0.015;% Increase CORP
            newWeights(8) = newWeights(8) + 0.015;% Increase GLD
```

```
        end

        if MDGDP_US(i) > 0.01
            newWeights(3) = newWeights(3) + 0.06;% Increase SPX
            newWeights(1) = newWeights(1) - 0.015;% Decrease LONG-TERM
            newWeights(2) = newWeights(2) - 0.015;% Decrease SHORT-TERM
            newWeights(7) = newWeights(7) - 0.015;% Decrease CORP
            newWeights(8) = newWeights(8) - 0.015;% Decrease GLD

         elseif MDGDP_US(i) <= 0.01 && MDGDP_US(i) >= -0.01
            newWeights(3) = newWeights(3);
            newWeights(1) = newWeights(1);
            newWeights(2) = newWeights(2);
            newWeights(7) = newWeights(7);
            newWeights(8) = newWeights(8);

        else
            newWeights(3) = newWeights(3) - 0.06;% Decrease SPX
            newWeights(1) = newWeights(1) + 0.015;% Increase LONG-TERM
            newWeights(2) = newWeights(2) + 0.015;% Increase SHORT-TERM
            newWeights(7) = newWeights(7) + 0.015;% Increase CORP
            newWeights(8) = newWeights(8) + 0.015;% Increase GLD
        end


        % DURATION INPUT
        if MDECB(i) < 0
            newWeights(4) = newWeights(4) + 0.08;% Increase LONG-TERM
            newWeights(5) = newWeights(5) - 0.08; % Decrease SHORT-TERM
        elseif MDECB(i)==0
            newWeights(4) = newWeights(4) ;% Increase LONG-TERM
            newWeights(5) = newWeights(5) ; % Decrease SHORT-TERM
        else
            newWeights(5) = newWeights(5) + 0.08;% Increase SHORT-TERM
            newWeights(4) = newWeights(4) - 0.08;% Decrease LONG-TERM
        end

        if MDFED(i) < 0
            newWeights(1) = newWeights(1) + 0.08;% Increase LONG-TERM
            newWeights(2) = newWeights(2) - 0.08; % Decrease SHORT-TERM
        elseif MDFED(i)==0
            newWeights(1) = newWeights(1) ;% Increase LONG-TERM
            newWeights(2) = newWeights(2) ; % Decrease SHORT-TERM
        else
            newWeights(2) = newWeights(2) + 0.08;% Increase SHORT-TERM
            newWeights(1) = newWeights(1) - 0.08;% Decrease LONG-TERM
        end
        newWeights = newWeights ./ sum(newWeights);
        weights_rp(:, i) = newWeights;
    else
        weights_rp(:, i) = weights_rp(:, i - 1);
    end

end
weights_rp = weights_rp';
weights_rp = weights_rp(2:end, :);
portfolioReturns_RP = sum(weights_rp .* returns, 2);
```

```
%% REBALANCING WEIGHTS STATIC
weights_2 = zeros(numAssets, numPeriods);
initWeights_2 = [0.125; 0.125; 0.15; 0.125; 0.125; 0.15; 0.05; 0.15];
weights_2(:, 1) = initWeights_2;
freq = 3;

for i = 2:numPeriods
    % Rebalance portfolio every rebalFreq periods
    if mod(i-1, freq) == 0
       returns_window_2 = returns((i-freq):i,:);

       % Compute mean returns and covariance matrix
        meanReturns_2 = mean(returns_window_2, 2);
        covMatrix_2 = cov(returns_window_2);

         % Compute optimal weights using Risk Parity
        covMatrix_reg_2 = covMatrix_2 + 0.01*eye(numAssets);
        invCovMatrix_2 = inv(covMatrix_reg_2);
        onesVec = ones(numAssets,1);
        invCovOnes_2 = invCovMatrix_2*onesVec;
        riskParityWeights_2 = invCovOnes_2./sum(invCovOnes_2);
        weights__2(:,i) = riskParityWeights_2;

        % Apply the constraint that each asset allocation doesn't deviate
by more than 25%
        targetWeights_2 = initWeights_2;
        newWeights_2 = riskParityWeights_2;
        for j = 1:numAssets
            if newWeights_2(j) > targetWeights_2(j) * 1.25
                newWeights_2(j) = targetWeights_2(j) * 1.25;
            elseif newWeights_2(j) < targetWeights_2(j) * 0.75
                newWeights_2(j) = targetWeights_2(j) * 0.75;
            end
        end

        % Normalize the weights
        newWeights_2 = newWeights_2 ./ sum(newWeights_2);
        weights_2(:,i) = newWeights_2;
    else
        % Use previous weights
        weights_2(:,i) = weights_2(:,i-1);
    end
end
weights_2=weights_2';
weights_2=weights_2(2:end,:);
portfolioReturns_2 = sum(weights_2.*returns, 2);
```

```
%% CLASSIC ALLOCATION PORTFOLIO
DATA_C=[SPX,SXXP,LUATTRUU,LEATTREU];

returns_C = price2ret(DATA_C,1:233,'Periodic');
numAssets_C=size(DATA_C,2);
numPeriods_C=size(DATA_C,1);
weights_C = zeros(numAssets_C, numPeriods_C);
initWeights_C = [0.2;0.2;0.3;0.3];
weights_C(:,1) = initWeights_C;
freq=3;



% REBALANCING WEIGHTS

for i = 2:numPeriods_C

    % Rebalance portfolio every rebalFreq periods

    if mod(i-1, freq) == 0
       returns_window_C = returns_C((i-freq):i,:);


       % Compute mean returns and covariance matrix
        meanReturns_C = mean(returns_window_C, 2);
        covMatrix_C = cov(returns_window_C);


         % Compute optimal weights using Risk Parity
        covMatrix_reg_C = covMatrix_C + 0.01*eye(numAssets_C);
        invCovMatrix_C = inv(covMatrix_reg_C);
        onesVec_C = ones(numAssets_C,1);
        invCovOnes_C = invCovMatrix_C*onesVec_C;
        acweights = invCovOnes_C./sum(invCovOnes_C);
        weights_C(:,i) = acweights;


        % Apply the constraint that each asset allocation doesn't deviate
by more than 5%
        targetWeights_C = initWeights_C;
        newWeights_C = acweights;
        for j = 1:numAssets_C
            if newWeights_C(j) > targetWeights_C(j) * 1.05
                newWeights_C(j) = targetWeights_C(j) * 1.05;
            elseif newWeights_C(j) < targetWeights_C(j) * 0.95
                newWeights_C(j) = targetWeights_C(j) * 0.95;
            end
        end

        % Normalize the weights
        newWeights_C = newWeights_C ./ sum(newWeights_C);
        weights_C(:,i) = newWeights_C;
    else
        % Use previous weights
        weights_C(:,i) = weights_C(:,i-1);
    end
end


weights_C=weights_C';
```

```
weights_C=weights_C(2:end,:);
portfolioReturns_C = sum(weights_C.*returns_C, 2);



%% PTF SERIES PLOT
init_cap=10000;
cumulativeReturns_RP = cumprod(1 + portfolioReturns_RP) - 1;
cumulativeReturns_2=cumprod(1 + portfolioReturns_2) - 1;
cumulativeReturns_C = cumprod(1 + portfolioReturns_C) - 1;
portfolio_series_rp=init_cap*(1+cumulativeReturns_RP);
portfolio_series_2=init_cap*(1+cumulativeReturns_2);
portfolio_series_C=init_cap*(1+cumulativeReturns_C);




figure
plot(dates_gen(2:end,:), portfolio_series_rp, 'r')
hold on
plot(dates_gen(2:end,:),portfolio_series_2)
hold on
plot(dates_gen(2:end,:),portfolio_series_C)
legend('DYNAMIC AW','STATIC AW','Classic PTF')
hold off
title('PTF SERIES')
```

```
%% DYNAMIC AW METRICS
numPeriodsPerYear = 12;
numYears = numPeriods/numPeriodsPerYear;

annualizedReturns_RP = (1 + cumulativeReturns_RP(end))^(1/numYears) - 1;
disp(['DYNAMIC AW PTF annualized return: ' num2str(annualizedReturns_RP)]);

% Compute annualized standard deviation of returns for each portfolio
StdDev_RP = std(portfolioReturns_RP)*sqrt(12);

disp(['DYNAMIC AW PTF STD: ' num2str(StdDev_RP)]);
% Set RISK FREE RATE
rf_rate=0.01;
% Compute Sharpe ratio for each portfolio
Sharpe_RP = (annualizedReturns_RP-rf_rate) / StdDev_RP;
disp(['DYNAMIC AW PTF sharpe ratio: ' num2str(Sharpe_RP)]);

negative_ret_RP=portfolioReturns_RP(portfolioReturns_RP<0);
downside_RP=std(negative_ret_RP)*sqrt(12);
SOR_RP = (annualizedReturns_RP-rf_rate) / downside_RP;
disp(['DYNAMIC AW PTF Sortino ratio: ' num2str(SOR_RP)]);

%% STATIC AW METRICS
numPeriodsPerYear = 12;
numYears = numPeriods/numPeriodsPerYear;

annualizedReturns_2 = (1 + cumulativeReturns_2(end))^(1/numYears) - 1;
disp(['STATIC AW PTF annualized return: ' num2str(annualizedReturns_2)]);

% Compute annualized standard deviation of returns for each portfolio
StdDev_2 = std(portfolioReturns_2)*sqrt(12);

disp(['STATIC AW PTF STD: ' num2str(StdDev_2)]);
% Set RISK FREE RATE
rf_rate=0.01;
% Compute Sharpe ratio for each portfolio
Sharpe_2 = (annualizedReturns_2-rf_rate) / StdDev_2;
disp(['STATIC AW PTF sharpe ratio: ' num2str(Sharpe_2)]);

negative_ret_2=portfolioReturns_2(portfolioReturns_2<0);
downside_2=std(negative_ret_2)*sqrt(12);
SOR_2 = (annualizedReturns_2-rf_rate) / downside_2;
disp(['STATIC AW PTF Sortino ratio: ' num2str(SOR_2)]);




%% CLASSIC PTF METRICS
numPeriodsPerYear = 12;
numYears = numPeriods/numPeriodsPerYear;

annualizedReturns_C = (1 + cumulativeReturns_C(end))^(1/numYears) - 1;
disp(['CLASSIC PTF annualized return: ' num2str(annualizedReturns_C)]);

% Compute annualized standard deviation of returns for each portfolio
StdDev_C = std(portfolioReturns_C)*sqrt(12);
```

```
disp(['CLASSIC PTF STD: ' num2str(StdDev_C)]);
% Set RISK FREE RATE
rf_rate=0.01;
% Compute Sharpe ratio for each portfolio
Sharpe_C = (annualizedReturns_C-rf_rate) / StdDev_C;
disp(['CLASSIC PTF sharpe ratio: ' num2str(Sharpe_C)]);

negative_ret_C=portfolioReturns_C(portfolioReturns_C<0);
downside_C=std(negative_ret_C)*sqrt(12);
SOR_C = (annualizedReturns_C-rf_rate) / downside_C;
disp(['CLASSIC PTF Sortino ratio: ' num2str(SOR_C)]);

%% TRACKING ERROR VOLATILITY DYN-STAT
[infoRatio, TE] = inforatio(portfolioReturns_RP, portfolioReturns_2);

%% ROLLING WINDOW WEIGHTS ALLOCATION
barWidth = 1;
x = dates_gen;

%DYNAMIC AW
figure
subplot(3,1,1)
bar(x(2:end),weights_rp, barWidth, 'stack');
ylim([0 1]);
xlabel('Time');
ylabel('Weights');
title('Rolling Window Weights Allocation - Dynamic AW Portfolio');
legend('LUATTRUU', 'LU13TRUU', 'SPX', 'LEATTREU','LET1TREU','SXXP', 'CORP',
'GLD');

%STATIC AW
hold on
subplot(3,1,2)
bar(x(2:end),weights_2, barWidth, 'stack');
ylim([0 1]);
xlabel('Time');
ylabel('Weights');
title('Rolling Window Weights Allocation - Static AW Portfolio');
legend('LUATTRUU', 'LU13TRUU', 'SPX', 'LEATTREU','LET1TREU','SXXP', 'CORP',
'GLD');

%CLASSIC PTF
hold on
subplot(3,1,3)
bar(x(2:end),weights_C, barWidth, 'stack');
ylim([0 1]);
xlabel('Time');
ylabel('Weights');
title('Rolling Window Weights Allocation - Classic Portfolio');
legend('SPX','SXXP','LUATTRUU','LEATTREU');
hold off



%% DRAWDOWNS PLOT
maxReturns = cummax(portfolio_series_rp);
drawdowns = -(maxReturns - portfolio_series_rp)./maxReturns;
maxDrawdown = min(drawdowns);
endDates_RP = find(drawdowns == 0);
```

```
DD_check=maxdrawdown(portfolio_series_rp);

maxReturns_c = cummax(portfolio_series_C);
drawdowns_c = -(maxReturns_c - portfolio_series_C)./maxReturns_c;
maxDrawdown_c = min(drawdowns_c);
endDates_c = find(drawdowns_c == 0);
DD_check_c=maxdrawdown(portfolio_series_C);

maxReturns_2 = cummax(portfolio_series_2);
drawdowns_2 = -(maxReturns_2 - portfolio_series_2)./maxReturns_2;
maxDrawdown_2 = min(drawdowns_2);
endDates_2 = find(drawdowns_2 == 0);
DD_check_2=maxdrawdown(portfolio_series_2);

figure
plot(dates_gen(2:end),drawdowns)
hold on
plot(dates_gen(2:end),drawdowns_c)
hold on
plot(dates_gen(2:end),drawdowns_2)
hold off
title('Drawdown Analysis')
legend('AW DYNAMIC','Classic portfolio','AW STATIC')



%% BONDS INPUT

% FED
barWidth = 0.99;
POS_change_MDFED = find(diff(sign(MDFED)) > 0);
MDFED_POS = dates_gen(POS_change_MDFED + 1);
NEG_change_MDFED = find(diff(sign(MDFED)) < 0);
MDFED_NEG = dates_gen(NEG_change_MDFED + 1);

x = dates_gen;
weights_DUR_US=[(weights_rp(:,1)),(weights_rp(:,2))]./(sum(weights_rp(:,1:2
)')');

figure
subplot(3,1,1)
bar(x(2:end),weights_DUR_US, barWidth,'stack')
ylim([0 1]);
xlabel('Time');
ylabel('Weights');
title('Rolling Window Weights Allocation - BONDS INPUT US');
for i = 1:length(MDFED_POS)
    xline(MDFED_POS(i), 'k', 'LineWidth', 3.5); % Vertical line at
desired_dates(i) with black color
end

for i = 1:length(MDFED_NEG)
    xline(MDFED_NEG(i), 'g', 'LineWidth', 3.5); % Vertical line at
desired_dates(i) with green color
end
legend('LUATTRUU','LU13TRUU','Location','northwest');
hold on
subplot(3,1,2)
plot(dates_gen(2:end),MDFED,'LineWidth', 4)
yline(0,'r')
```

```
title('FED MOVING AVERAGE REFINANCING RATE DIFFERENTIAL')
hold on
subplot(3,1,3)
plot(dates_gen,FED,'k','LineWidth', 4)
title('FED REFINANCING RATE')



% ECB
barWidth = 0.99;
POS_change_MDECB = find(diff(sign(MDECB)) > 0);
MDECB_POS = dates_gen(POS_change_MDECB + 1);
NEG_change_MDECB = find(diff(sign(MDECB)) < 0);
MDECB_NEG = dates_gen(NEG_change_MDECB + 1);
% Set the x-axis values
x = dates_gen;
weights_DUR_UE=[(weights_rp(:,4)),(weights_rp(:,5))]./(sum(weights_rp(:,4:5
)')');

figure
subplot(3,1,1)
bar(x(2:end),weights_DUR_UE, barWidth, 'stack')
ylim([0 1]);
xlabel('Time');
ylabel('Weights');
title('Rolling Window Weights Allocation - BONDS INPUT UE');
for i = 1:length(MDECB_POS)
    xline(MDECB_POS(i), 'k', 'LineWidth', 4); % Vertical line at
desired_dates(i) with black color
end

for i = 1:length(MDECB_NEG)
    xline(MDECB_NEG(i), 'g', 'LineWidth', 4); % Vertical line at
desired_dates(i) with green color
end
legend('LEATTRUU','LET1TREU','Location','northwest');
hold on
subplot(3,1,2)
plot(dates_gen(2:end),MDECB,'LineWidth', 4)
yline(0,'r')
title('ECB MOVING AVERAGE REFINANCING RATE DIFFERENTIAL')
hold on
subplot(3,1,3)
plot(dates_gen,ECB,'k','LineWidth', 4)
title('ECB REFINANCING RATE')




%% EQUITY INPUT
%US
barWidth = 0.99;
POS_change_MDUS = find(diff(sign(MDGDP_US)) > 0 & diff(MDGDP_US)>0.05);
MDUS_POS = dates_gen(POS_change_MDUS + 1);
NEG_change_MDUS = find(diff(sign(MDGDP_US)) < 0 & diff(MDGDP_US) <-0.05);
MDUS_NEG = dates_gen(NEG_change_MDUS + 1);


x = dates_gen;
```

```
weights_EQ_US=[(weights_rp(:,3)),(weights_rp(:,1)),(weights_rp(:,2)),(weigh
ts_rp(:,7)),(weights_rp(:,8))]./((weights_rp(:,3)+weights_rp(:,1)+weights_r
p(:,2)+weights_rp(:,7)+weights_rp(:,8)));

figure
subplot(3,1,1)
bar(x(2:end),weights_EQ_US, barWidth, 'stack')
ylim([0 1]);
xlabel('Time');
ylabel('Weights');
title('Rolling Window Weights Allocation - EQUITY INPUT US');
for i = 1:length(MDUS_POS)
    xline(MDUS_POS(i), 'k', 'LineWidth', 4); % Vertical line at
desired_dates(i) with black color
end

for i = 1:length(MDUS_NEG)
    xline(MDUS_NEG(i), 'g', 'LineWidth', 4); % Vertical line at
desired_dates(i) with green color
end
legend('S&P
500','LUATTRUU','LU13TRUU','LGCPTRUU','GOLD','Location','northwest');
hold on
subplot(3,1,2)
plot(dates_gen(2:end),MDGDP_US,'LineWidth', 4)
yline(0,'r')
yline(0.01,'k')
yline(-0.01,'k')
title('GDP US MOVING AVERAGE DIFFERENTIAL')
hold on
subplot(3,1,3)
plot(dates_gen,GDP_US,'k','LineWidth', 4)
yline(0,'r')
title('GDP US CHAINED YoY')




%UE
barWidth = 0.99;
POS_change_MDUE = find(diff(sign(MDGDP_UE)) > 0 & diff(MDGDP_UE)>0.01);
MDUE_POS = dates_gen(POS_change_MDUE + 1);
NEG_change_MDUE = find(diff(sign(MDGDP_UE)) < 0 & diff(MDGDP_UE) <-0.01);
MDUE_NEG = dates_gen(NEG_change_MDUE + 1);
x = dates_gen;
weights_EQ_UE=[(weights_rp(:,6)),(weights_rp(:,4)),(weights_rp(:,5)),(weigh
ts_rp(:,7)),(weights_rp(:,8))]./((weights_rp(:,6)+weights_rp(:,4)+weights_r
p(:,5)+weights_rp(:,7)+weights_rp(:,8)));

figure
subplot(3,1,1)
bar(x(2:end),weights_EQ_UE, barWidth, 'stack')
ylim([0 1]);
xlabel('Time');
ylabel('Weights');
title('Rolling Window Weights Allocation - EQUITY INPUT EUROZONE');
for i = 1:length(MDUE_POS)
    xline(MDUE_POS(i), 'k', 'LineWidth', 4); % Vertical line at
desired_dates(i) with black color
end
```

```
for i = 1:length(MDUE_NEG)
    xline(MDUE_NEG(i), 'g', 'LineWidth', 4); % Vertical line at
desired_dates(i) with green color
end

legend('STOXX EUROPE
600','LEATTREU','LET1TREU','LGCPTRUU','GOLD','Location','northwest');
hold on
subplot(3,1,2)
plot(dates_gen(2:end),MDGDP_UE,'LineWidth', 4)
yline(0,'r')
yline(0.01,'k')
yline(-0.01,'k')
title('GDP UE MOVING AVERAGE DIFFERENTIAL')
hold on
subplot(3,1,3)
plot(dates_gen,GDP_UE,'k','LineWidth', 4)
yline(0,'r')
title('GDP EURO CHAINED YoY')




%% CURRENCY INPUT
barWidth = 0.95;

% Set the x-axis values
x = dates_gen;
weights_CURR=[sum(weights_rp(:,1:3)')',sum(weights_rp(:,4:6)')']./(sum(weig
hts_rp(:,1:5)')');
% Create a bar graph for the Risk Parity portfolio weights
figure
subplot(3,1,1)
bar(x(2:end),weights_CURR, barWidth, 'stack')
ylim([0 1]);
xlabel('Time');
ylabel('Weights');
yline(0.5,'w')
title('Rolling Window Weights Allocation - CURRENCY INPUT');
for i = 1:length(peakDates)
    xline(peakDates(i), 'k', 'LineWidth', 4); % Vertical line at
desired_dates(i) with black color
end

for i = 1:length(valleyDates)
    xline(valleyDates(i), 'g', 'LineWidth', 3.5); % Vertical line at
desired_dates(i) with green color
end
legend('AMERICAN ASSETS','EUROPEAN ASSETS','Location','southwest');
hold on
subplot(3,1,2)
plot(dates_gen,EURR_FED,'LineWidth', 3)
yline(0,'k')
title('ECB-FED INTEREST RATES DIFFERENTIAL')
hold on
subplot(3,1,3)
plot(dates_gen,EURUSD,'k','LineWidth', 3)
title('EUR/USD')
```

```
%% RESULTS TABLE
Metrics=["Annualized return";"Standard Deviation";"Sharpe Ratio";"Sortino
Ratio";"Maximum Drawdown";"Tracking Error";"Information Ratio"];
DYNAMIC_AllWeather=[annualizedReturns_RP;StdDev_RP;Sharpe_RP;SOR_RP;maxDraw
down;TE;infoRatio]
STATIC_AllWeather=[annualizedReturns_2;StdDev_2;Sharpe_2;SOR_2;maxDrawdown_
2;0;0]
Classic=[annualizedReturns_C;StdDev_C;Sharpe_C;SOR_C;maxDrawdown_c;0;0]
a=table(Metrics, STATIC_AllWeather,DYNAMIC_AllWeather,Classic)
```

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