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Economic Growth and Fiscal Policy under Financial Stress Episodes: An Application of the VAR Model

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Abstract

This thesis analyzes the linkages between economic activity, fiscal policy, and financial stress, with focus on financial stress episodes. We used quarterly data, from 2000:1 to 2022:4, for a panel of four countries: Italy, Belgium, Czech Republic, and Poland. A VAR model is implemented to study the correlation and autocorrelation between variables, the granger causality between them, the IRF and FEVD of each one. Moreover, a comparison between countries is provided. Lastly, a forecast analysis is performed, to understand the impact of the most recent crises (COVID-19 and Ukraine-Russia war) on the economies.

KEYWORDS: VAR, Fiscal Policy, Financial Markets, Output Growth.

JEL Classification: C32, E62, G15, H30

CONTENTS

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CHAPTHER 1. INTRODUCTION AND LITERATURE REVIEW: FINANCIAL STRESS, ECONOMIC GROWTH AND FISCAL POLICY LINKAGES.

Economists, researchers, and policymakers have become more and more aware of the importance of financial markets and financial and economic stability, they recognized the significant, negative correlation between shocks in financial sector and real economy, and the negative spillover effect between economies (Edison, 2002), especially after the 2008 financial crisis, the COVID-19 crisis, and the ongoing Russia-Ukraine war. Several and more sophisticated models have been developed to find an answer to questions like: what are the effect and the mechanism of transmission of financial and fiscal shocks? What are the causes of economic fluctuations and business cycles? (Sims, 1980). Questions to which still nowadays researchers and policy makers are trying to answer. Several studies have shown that an increase in financial and economic instability could bring to an indirect effect to the real economy, through several mechanisms of transmission, such as lowering the creditworthiness of borrowers, contracting bank loans to the real economy, reducing wealth, increasing inflation and government's debt (Lo Duca and Peltonen, 2011). For this reason, a real time identification of the current state of financial system is crucial for policy purpose (Louzis and Vouldis, 2013), to reduce the macroeconomics consequence of such crises and to prevent the occurring of other crisis in the future (Cardarelli et al, 2011). Interesting in this context is the analysis of the linkages between three key variables: economic growth, fiscal policy, and financial stress, useful to understand the economic response of the economies to shocks on financial and real markets and the actions undertaken by the government to mitigate such negative consequences. Given the relevance of the topic, this paper aims to give its contribution to enhance the comprehension of the complex interaction between there three variables, within the context of Italy, Belgium, Czech Republic, and Poland. Extensive is the literature among the interaction between them. The relationship is three-sided; not only macroeconomic, and financial instability can influence economic policy, leading to a decrease of the GDP and/or an increase of the fiscal policies, but also the economic policy can affect the financial stability through the so-called "bad" policies that can bring to financial instability and economic slowdown. An example of the last mechanism is founded in the European Sovereign Debt Crisis during which the large government indebtedness undetermined the ability of governments to pay back the outstanding stock of their debt. As consequences, the economies experienced a period of financial instability and of decline in output growth (Lane, 2012). Conclusions from Caldara et al, (2008), Christiano et al, (2011), Afonso et al, (2018), emphasizes how (negative) fiscal policy shocks can bring to a decrease of GDP, decrease of private consumption, increase in uncertainty, and increase of financial instability.

However, also positive can be the effects of fiscal policies. As demonstrated by Blanchard and Perotti, (2002), using a SVAR approach for US in the postwar period, positive shock on government spending can increase economic output and private consumption. On the contrary, an increase in taxes have a negative effect on output. Conclusions confirmed by Auerbach et al, (2012). They estimated that during recession periods the increase in government expenditures replaces the lack of consumers' expenditure and firms' investments, resulting in an increase in output. Regarding Italy, Giordano et al, (2007) using a SVAR process, founded that a 1% shock on governments expenditure, increases the private real GDP by 60% after 3 quarters only. In addition, there is a positive impact on consumption and inflation. On the contrary, following Afonso and Sousa, (2009), negligible is the effect of governments spending for U.S., UK, Germany, and Italy, while significant is the effect of government revenue shocks.

Financial markets can influence fiscal policy and economic growth too. Using a threshold approach, Hollo et al, (2012), after having developed a new FSI, the CISS, have shown that high levels of financial stress depress the real economic activity, while in low stress regime the answer is negligible, for EU countries. Same conclusions are depicted by Duprey et al, (2017), for European countries and by Duprey, (2020), in the case of Canadian economy, in where, using a Bayesian TVAR, the author discovered how financial stress and worsening macroeconomic conditions amplify each other, especially in a context of high-stress conditions. Afonso et al, (2010) also founded that financial stress has worsened both economic growth and fiscal position. In their paper Cardarelli et al, (2011), dated episodes of financial stress among 17 advanced economies and analyzed their effect on real economy. They concluded that more than half of the financial stress episodes, especially banking stress episodes, have been followed by an economic slowdown or even by a recession[1]. Accordingly, Auerbach and Gorodnichenko, (2012) Fazzari, Morley and Panovska, (2015), Barro and Redlick, (2011) provide evidence of large fiscal multipliers during recession. For Hakkio and Keeton, (2009) such economic decrease in economic activity occurs through three channels: increase of uncertainty about prices, increase of financing costs and bank's tightening. Moreover, Carrillo and Poilly, (2013) Kara and Sin, (2012) analyzed the effect of fiscal stimulus on the economy, during period of credit constraint (liquidity trap) and financial instability. They founded evidence that fiscal multipliers are high during those periods and, as long as there is provision of fiscal stimulus and falling in real interest rates, the economic growth is fostered. Following the same reasoning, Baldacci et al, (2009) and Van Brusselen, (2012) studied the response of fiscal policy and economic growth to the GFC (Great Financial Crisis). The banking crisis, brought to negative consequences on GDP growth and, on the

[1] For the authors, we have a recession if financial stress is followed by a contraction of economic activity within 6 quarters.

other side, the implementation of timely countercyclical policies (fiscal expansion), helped the economies to recover, both for advanced and for emerging economies. They helped the economies to shorten the crisis duration and increased the output growth, by sustaining the aggregate demand. As demonstrated by Hamburg et al, (2010), thanks the fiscal policy adopted by governments, Germany and Italy managed to contrast the fall in GDP by more than 2 and 1 percentage points, respectively. Generalizing, depending on the business cycle, government actions have a stronger effect on output when output gap is negative (Baum et al, 2012).

Evidence suggests that also monetary policy reacts to financial stress episodes. Baxa et al, (2010), detected an increase of Central Banks' monetary policy actions for U.S., UK, Australia, Canada, and Sweden, during the GFC. A sizable fraction of the quantitative easing policy adopted after such crisis, has been a direct response to the high financial stress of the period. Moving on, Bernanke et al, (1998) studied the effect of monetary policy shocks on economic growth for US, applying a semi-structural VAR approach. Both price levels and real GDP reacts immediately positively to expansionary monetary policy shocks. However, rising in inflation, affected negatively not only the economic growth, but also the other variables, as showed by Mallick and Sethi, (2019) and Martin, (2010) in the case of India and U.S. Aarle et al, (2003), in their SVAR model, analyzed the mutual effect of monetary and fiscal policy and the interaction between macroeconomic policies and shocks in financial markets. Results at Euro-area level are compared with U.S. and Japan, showing, significant similarities. In contrast, taking the members of EMU singularly, significant differences in reactions are displayed. Following, Muscatelli et al, (2002) found a decreasing response to fiscal policy shocks, from 1980s, for countries belonging to the G7, especially U.S., Germany, France UK, and Italy. Lastly, also the short-term interest rate influences the other variables, as demonstrated by Christiano et al, (1999). As response to a contraction in the interest rate, the real GDP and the prices decline quickly, with the stronger decline within the first and second year.

As can be seen from this brief literature review, the economic evidence shows that during financial stress episodes, the economic growth decline or become negative, bringing to a slowdown of the economic system, or even a recession, the fiscal policy increase, as response to the economic slowdown and the financial stress index increase, being a measure of financial stress (Afonso et al, 2018). The aim of this paper is to test if such economic responses occur in the case of Italy, Belgium, Czech Republic, and Poland, during period of financial and/or economic crises. The paper is also going to test if there is mutual influence between these three key variables and between them and their lagged values. For this purpose, we implemented a VAR model taking quarterly data from

2000:1 to 2022:4[2]. Such period is characterized by four crises (the Great Financial Crisis (GFC), the Sovereign Debt Crisis, the COVID-19 pandemic, and the Ukraine-Russia war) that brought to significant, negative consequences on the economies under analysis.
Regarding the variables, the economic growth is measured as the percentage change of real GDP (seasonally adjusted) per-period. Financial instability is measured with the Country-Level Index of Financial Stress (CLIFS) (Duprey et al, 2017). Among the large number of FSI developed in literature, we choose the CLIFS becouse it uses a model-based approach to determine the episodes of financial stress, without taking as benchmark some preexistent financial stress episodes and without making assumptions regarding which between the financial and economic stress occur first. The fiscal policy, instead, is measured as the level of the government debt in percentage of GDP, per period. We choose the debt-to-GDP ratio as measure of fiscal policy because it provides information on both government's revenue and expenditure. Furthermore, it captures not only the economic effects of government's ordinary actions, but also of extraordinary ones, commonly undertaken by governments under periods of financial or economic stress (Afonso et al, 2018). Given these features, it is the most suitable measure of fiscal policy, for our purpose. In the regression, other two variables are added, as control variables, the (headline) inflation rate, and the money market rate, due the mutual influence between them and the previous mentioned variables. They are included to avoid the problem of omitted variables bias and the risk of obtaining incorrect results.
The paper is structured as follows. In chapter 2 a theoretical explanation of the VAR model is given. The two main assumption of the model are discussed: stationarity and stability. Following, an explanation of the forecasting procedure, of the structural analysis implemented and of the estimation technique (Maximum Likelihood Estimation Method) choose are provided. The chapter ends with an explanation the information criteria developed in literature to choose the correct number of lags of the model. Chapter 3 describes the variables involved in the process and their computing technique. We also performed an analysis of their trend from 2000:1 to 2019:4, providing economic explanations to extreme values displayed and a comparison between countries. Just from the trend analysis, some conclusions can be drawn. All the variables, per country, reached their highest and/or lowest peaks during periods of crises, with slowdowns and recessions of the economies, increase in the government debt and increase of CLIFS. Italy and Belgium, compared to Czech Republic and Poland, reacted more to the GFC and to the Sovereign Debt Crisis. In chapter 4 we enter in the core of the analysis. Results of the empirical analysis performed, using MATLAB as programming language, are discussed. We firstly tested the two assumptions of the VAR model, finding evidence of stationary,

[2] The estimation period ends up to 2019:4. Data from 2020:1 to 2022:4 are estimated performing a forecast analysis and are compared with the real ones to understand the impact of COVID-19 and Ukraine-Russia war on the economies.

stable VAR process. We also tested the normality assumption of residuals. The Lilliefors test performed rejected the normality assumption for some variables. Nevertheless, we decided to move on in the analysis[3]. A VAR (1) process is implemented, following the AIC, BIC, HCQ information criterion results. The first important conclusions are obtained from the analysis of the correlation and autocorrelation between variables, and of the autoregressive coefficients. There is evidence of a negative correlation between output growth and fiscal policy and between output growth and financial stress, while positive between fiscal policy and financial stress, as expected. Stronger is the correlation for Poland, with respect to the other countries. Moreover high, mostly positive, is the autocorrelation between each variable and its lagged value, higher for small number of lags, coherently with the VAR (1) choice[4]. Also, high is the influence of lags of the other variables on the current value of each of the variables involved in the system. Significant are the results of the granger causality, especially for Czech Republic, where economic growth, debt-to-GDP ratio and CLIFS are one step Granger-cause of each other (constituting a feedback loop). Results are significant also for Italy. All previous conclusions are confirmed by the analysis of the IRF and the FEVD. Lastly, a forecast analysis is performed. The forecast period goes from 2020:1 to 2022:4. Estimated and observed data are compared, to understand the impact of the most recent crises (COVID-19 and Ukraine-Russia war) on the economies and provide an approximation how would have performed the economies in absence of such crises[5].

Three are the main conclusion of this paper. All variables react to financial stress episodes, with an economic slowdown or even a recession, an increase of CLIFS and an increase of government debt. High is the correlation between the variables and between them and their lagged values. Crises are unexpected and disruptive phenomena for the economies.

[3] An explanation of the reasons behind such choice is provided in section 4.2.2.

[4] Weak autocorrelation is obtained only for output growth as demonstrated by the Ljung-Box Q-test.

[5] In this section we also analyzed the trend of the variables involved in the process during such period, providing detailed explanation of the extreme values reached, by all variables (following the real data).

CHAPTER 2. METHODOLOGY.

The development of the econometric models and software for the analysis of financial and macroeconomics shocks, and fiscal policy, and their consequence on the economy, started in lates 1950s at the University of Cambridge (Zivot et al, 2006). Until the beginning of 1080s for the study of the macroeconomics variables and events were used large-scale dynamics simultaneous equation models. The turning point of this analysis is obtained during the 1980s when Sims, (1980), in its seminal work "Macroeconomics and Reality", started to use the VAR model as an alternative to simultaneous equations, able to analyze the dynamic structure of variables, instead of the static one. Three are the criticisms of Sims to the previous literature. Firstly, the exogeneity assumption of the variables in simultaneous equations model is incorrect. In contrast, in VAR models almost all variables are treated as endogenous (Lüktepohl, 2009). Secondly, in these models there were lack of attention on agent's expectations. Thirdly, in the previous models there were too much "a priori" restrictions, that could bring to unrealistic results. Given these criticisms, the *Vector Autoregressive* (VAR) *model* is one of the most important and successful models used for the analysis of multivariate time series, (Zivot et al, 2006). It is one of the most suitable models for the analysis of the dynamic of financial time series, for forecasting and for the analysis of structural inference and the policy analysis. This is an innovative model since each variable of the system is in function of lagged values of itself and other variables, coherently with the idea that many variables interact the one with the others.

2.1 VAR ANALYSIS: THE MODEL

The VAR process is optimal for the analysis of the multivariate time series, being a natural extension of the univariate autoregressive model to dynamic multivariate time series[6] (Lüktepohl, 2005). The VAR(p) process is characterized by the following reduced form:

$$y_t = v + A_1 y_{t-1} + \cdots + A_p y_{t-p} + u_t \qquad \text{with } t = 0, \pm 1, \pm 2, \dots \ (1)$$

Or in matrix form:

$$\begin{bmatrix} y_{1,t} \\ y_{2,t} \\ \vdots \\ y_{k,t} \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_k \end{bmatrix} + \begin{bmatrix} A^1{}_{1,1} & A^1_{1,2} & \dots & A^1{}_{1,k} \\ A^1{}_{2,1} & A^1{}_{2,2} & \dots & A^1{}_{2,k} \\ \vdots & \ddots & \ddots & \vdots \\ A^1{}_{k,1} & \cdots & \cdots & A^1{}_{k,k} \end{bmatrix} \begin{bmatrix} y_{1,t-1} \\ y_{2,t-1} \\ \vdots \\ y_{k,t-1} \end{bmatrix} + \cdots + \begin{bmatrix} A^p{}_{1,1} & A^p_{1,2} & \dots & A^p{}_{1,k} \\ A^p{}_{2,1} & A^p{}_{2,2} & \dots & A^p{}_{2,k} \\ \vdots & \ddots & \ddots & \vdots \\ A^p{}_{k,1} & \cdots & \cdots & A^p{}_{k,k} \end{bmatrix} \begin{bmatrix} y_{1,t-p} \\ y_{2,t-p} \\ \vdots \\ y_{k,t-p} \end{bmatrix} + \begin{bmatrix} u_{1,t} \\ u_{2,t} \\ \vdots \\ u_{k,t} \end{bmatrix}$$

[6] In the multivariate time series, the dependent variable depend not only by its lagged values, but also by lagged values of other variables.

Where p is the number of lags; y_t is the set of different variables (k variables), we are studying at time t. Is a $(k * 1)$ random vector. v is a $(k * 1)$ vector of fixed intercept terms; $A_1, A_2, \ldots, A_p$ are the $(k * k)$ matrices of fixed (autoregressive) coefficients; $y_{t-1}, y_{t-2}, \ldots, y_{t-p}$ are the $(k * 1)$ vectors of lagged values of our variables of interest and u_t is the $(k * 1)$ vector of white noises (or innovation process), assumed to be i.i.d. In the VAR process, each variable is regressed on a constant term, on p lags values of itself and of other variables (Hamilton, 1994).

Regarding the white noises, they have 2 properties (Lüktepohl, 2005). Firstly, $E[u_t] = 0$. The white noises are stochastic process with zero mean. Secondly, $Cov[u_t u_s] = E[u_t u_s] = \begin{cases} \Sigma_u & if\ t=s \\ 0 & if\ t\neq s \end{cases}$. White noises have constant variance: $E[u_t u_s'] = \Sigma_u$, where Σ_u is the $(k * k)$ covariance matrix, assumed to be a positive definite, symmetric matrix. This assumption implies that the forecast errors u_t for different periods are uncorrelated, so that there are no systematic forecast errors since all useful information in the past y_t's are used in the forecast.

2.2 THE ASSUMPTIONS OF THE VAR MODEL: STABILITY AND STATIONALRITY

To have reliable results, the VAR model must respect two assumptions: stability and stationarity. To explain when stability is respected, we start by the VAR (1) model, for simplicity:

$$y_t = v + A_1 y_{t-1} + u_t \qquad (2)$$

Or in another form:

$$y_t = \left(I_k + A_1 + \cdots + A_1^j\right)v + A_1^{j+1} y_{t-j-1} + \sum_{i=0}^{j} A_1^i u_{t-i} \qquad (3)^7$$

As long as j goes to infinity, the value A_1^{j+1}goes to zero. In the limit, we can ignore the term $A_1^{j+1} y_{t-j-1}$. Also, if the modulus of all eigenvalues (λ) of A_1 are < 1, then the sequence A_1^i for $i = 0,1,2, \ldots$, is summable and the infinite sum $\sum_{i=0}^{\infty} A_1^i * u_{t-i}$ exist in the mean square.

Moreover,

$$\lim_{j \to \infty} \left(I_k + A_1 + \cdots + A_1^j\right)v = (I_k - A_1)^{-1} v$$

If all $|\lambda| < 1$ and y_t is the VAR (1) process of (2), then y_t is the well-defined stochastic process:

[7] Such form is obtained using a backward iteration. For the proof how to get to this formula see the book New Introduction to Time Multiple Series Analysis of Lüktepohl, 2005.

$$y_t = \mu + \sum_{i=0}^{\infty} A_1^i \, u_{t-i} \qquad t = 0, \pm 1, \pm 2, \dots. \qquad (4)$$ [8]

Where:

$$\mu := (I_k - A_1)^{-1} v$$

The distributions and the joint distributions of the y_t's components are uniquely determined by the distributions of the u_t process.

Stating formally the stability condition, a VAR (1) process is *stable* if all eigenvalues of A_1 have modulus less that 1. In formula, if and only if:

$$det(I_k - A_1\lambda) \neq 0 \qquad for\ |\lambda| \leq 1 \qquad (5)$$

Moreover, the first moment (the mean) and the second moment (the autocovariance) of the y_t process are:

$$E[y_t] = \mu \qquad (6)$$

And

$$\Gamma_y(h) := E[(y_t - \mu)(y_{t-h} - \mu)'] = \sum_{i=0}^{n} A_1^{h+1} \Sigma_u A_1^{i\,\prime} \qquad (7)$$

because $E[u_t u'_s] = 0$ for $s \neq t$ and $E[u_t u'_s] = \Sigma_u$ for all t.

Now we extend the previous discussion to the VAR(p) process, for $p > 1$, by rewriting the VAR(p) process in VAR (1) form. If y_t is a VAR(p) as in (1), the corresponding kp-dimensional VAR (1) is:

$$Y_t = \boldsymbol{v} + \boldsymbol{A}Y_{t-1} + \cdots + U_t \qquad (8)$$

In matrix from:

$$\begin{bmatrix} y_t \\ y_{t-1} \\ \vdots \\ y_{t-p+1} \end{bmatrix} = \begin{bmatrix} v \\ 0 \\ \vdots \\ 0 \end{bmatrix} + \begin{bmatrix} A_1 & A_2 & \dots & A_{p-1} & A_p \\ I_k & 0 & \dots & 0 & 0 \\ 0 & I_k & \dots & 0 & 0 \\ \vdots & \ddots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & I_k & 0 \end{bmatrix} \begin{bmatrix} y_{t-1} \\ y_{t-2} \\ \vdots \\ y_{t-p} \end{bmatrix} + \begin{bmatrix} u_t \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

Where Y_t is a $(kp * 1)$ vector[9]; $\boldsymbol{v}$ is a $(kp * 1)$ vector; Y_{t-1} is a $(kp * 1)$ vector; $\boldsymbol{A}$ is a $(kp * kp)$ matrix and U_t is a $(kp * 1)$ vector.

[8] This is the moving average (MA) representation of the VAR (1) process, where the joint distributions of the y_t's are uniquely determined by the sum of the mean term μ and the past and present innovation vectors u_t. This representation can be obtained only when the stability condition is respected.

[9] Each $y_t = (y_{1,t}, \dots, y_{k,t})'$

Following the previous reasoning, Y_t is stable if and only if:

$$det(I_{kp} - \boldsymbol{A}\lambda) \neq 0 \quad for\ |\lambda| \leq 1 \quad (9)$$

Using the $(k * kp)$ matrix:

$$J := [I_k : 0 : \dots : 0] \quad (10)$$

We can write the VAR(p) process y_t as

$$y_t = J\, Y_t = J\boldsymbol{\mu} + J \sum_{i=0}^{\infty} \boldsymbol{A}^i U_{t-i} \qquad (11)$$

Where:

$$det(I_k - \boldsymbol{A}_1\lambda) = det(I_k - A_1\lambda - \dots - A_p\lambda^p)$$

The *stability condition* implies that the VAR(p) process is stable if the reverse characteristic polynomial has no roots in and on the complex unit circle. So, y_t is stable if:

$$\det(I_k - A_1\lambda - \dots - A_p\lambda^p) \neq 0 \quad for\ |\lambda| \leq 1 \quad (12)$$

Before talking about the stationarity property, we first introduce the *Moving Average Representation* of a VAR process, which can be obtained only under the stability condition of the VAR(p) process. If the VAR(p) process is stable, the MA representation of y_t is:

$$y_t = JY_t = J\boldsymbol{\mu} + \sum_{i=0}^{\infty} J\boldsymbol{A}^i J' J U_{t-i} = \mu + \sum_{i=0}^{\infty} \phi_i\, u_{t-i} \quad (13)$$

Where y_t is expressed in terms of past and present errors (or innovation vectors) and in terms of the mean term; each ϕ_i can be computed recursively, using the following formula[10]: $\phi_i = \sum_{j=1}^{i} \phi_{i-j} A_j$ and $\phi_0 = I_k$.

Given this form, the mean and autocovariances of y_t are, respectively:

$$E[y_t] = \mu \qquad (14)$$

$$\Gamma_y(h) := E[(y_t - \mu)(y_{t-h} - \mu)'] = \sum_{i=0}^{\infty} \phi_{h+i} \Sigma_u \phi_i' \quad (15)$$

Moving to the *stationarity property*, a stochastic process is *stationary* if its first and second moments are time invariant (they do not depend on time t). In other words, is stationary if:

$$E[y_t] = \mu \quad \text{for all } t \quad (14)$$

[10] For the proof how to get to this formula see the book New Introduction to Time Multiple Series Analysis of Lüktepohl, 2005. N.B. because A^i are absolutely summable, also ϕ_i are.

And

$$\Gamma_y(h) = \Gamma_y(-h)' \text{ for all } t \text{ and } h = 0,1,2, \dots. \qquad (16)$$

Where μ is the vector of finite means terms and $\Gamma_y(h)$ is the matrix of finite covariances (Lüktepohl, 2005).

Condition (14) implies that all y_t have the same finite mean vector μ, for all t and condition (16) that the autocovariance of the process do not depend on t but only by the time period h the 2 vectors y_t and y_{t-h} are apart.

An important conclusion is that stability imply stationarity, so if the VAR(p) process is stable, then is also stationary. But the vice-versa is not true, an unstable process is not necessarily nonstationary.

2.3 FORECASTING

After collecting the data, the next step is to analyze the parameters of the VAR model, useful for two purposes, forecasting and structural analysis. In this section we are going to explain the forecast analysis, in the next one, instead, the structural analysis.

In a forecast analysis, the forecaster makes statements regarding the future values of the variables of interest $y_1, \dots, y_k$. Two are the requirements: a model for the data generating process (DGP), in our case the VAR(p) process, and an information set, Ω_t, assumed to contain all the available information up to and including time t, if not otherwise stated. In our case is the set containing the past and the present variables of the system under consideration: $\Omega_t = \{y_s \mid s \leq t\}$, where $y_s = (y_{1s}, \dots, y_{ks})'$. t is called the *forecasting origin* and correspond to the period where the forecast is made, the *forecast horizon* is the number of periods (in the future) for which the forecast is made, and h is the *h-step predictor* (Lüktepohl, 2005).

A forecast is chosen to minimize a loss function, such as minimizing the cost function. A forecast analysis would unavoidably end up in doing errors (*forecast errors)*, given by the difference between the true value and the predicted one. An optimal forecast analysis is the one that minimize the following loss function:

$$c^* = E_t[c(u_{t+1})|\Omega_t] = E[c(y_{t+1} - E_t[y_{t+1}]|\Omega_t].$$

Where c^* is the minimum cost necessary to suffer if predictions are wrong. In the context of VAR analysis, the forecaster tries to minimize the forecast mean squared errors (MSEs)[11] (Lüktepohl and Krätzig, 2004). In the case of unbiased predictor, the MSE is the forecast error variance.

[11] Several authors, like Granger (1969b) and Granger and Newbold (1986) explained the benefits of minimize the MSEs, since this minimizes also other loss functions other that the MSE.

There are two types of forecasts: point forecast and interval forecast.

Regarding the point forecast, we recall the equation for a stable VAR (p) process (1). Assuming that the u_t's are generated by an independent white noise process, the *minimum MSE predictor*[12], for each component of y_t, for forecast horizon h, at forecast origin t, become the conditional expected value:

$$E_t[y_{t+h}] \coloneqq E[y_{t+h}|\Omega_t] = E[y_{t+h}|\{y_s|s \leq t\}] \qquad (17)$$

The optimality of the conditional expectation implies that *the optimal h-step predictor* of a VAR(p) process y_t is:

$$E_t[y_{t+h}] = v + A_1 E_t[y_{t+h-1}] + \cdots + A_p E_t[y_{t+h-p}] \qquad (18)$$

Taking the VAR(p) representation and supposing to know the coefficients v and $A_1, \ldots, A_p$, the forecaster predicts the value of y_{t+h}, conditional to the observed value of y_t. The *h-step ahead forecast error* is the difference between the true value of y_{t+h} at time $t+h$, and the predicted value of y_{t+h} at time t:

$$u_{t+h} + A_1 u_{t+h-1} + A_1^2 u_{t+h-2} + \cdots + A_1^{h-1} u_{t+1} = y_{t+h} - E_t[Y_{t+h}] \qquad (19)$$

This computation of forecast and forecast error is no more valid if we remove the assumption of u_t independent white noise. We can give a more complete overview of the forecast process under no assumptions, by using the optimal (linear) minimum MSE predictor. Recalling the MA representation of y_t process:

$$y_t = \mu + \sum_{i=0}^{\infty} \phi_i \, u_{t-i} \qquad (13)$$

Generalizing for h periods ahead, we obtain the forecast:

$$y_{t+h} = \mu + \textstyle\sum_{i=0}^{\infty} \phi_i \, u_{t+h-i} \qquad (20)$$

The *optimum linear minimum MSE predictor* (irrespective of the properties of the white noise process) can be also expressed in terms of the MA representation:

$$y_t(h) = \mu + \sum_{i=h}^{\infty} \phi_i \, u_{t+h-i} \qquad (21)$$

From where, the h-period forecast error is:

$$y_{t+h} - y_t(h) = \textstyle\sum_{i=0}^{h-1} \phi_i * u_{t+h-i} \qquad (22)$$

[12] The minimum MSE predictor, is the one for which the loss function is minimized.

The forecast error remains unchanged if the sequence y_t has mean $\mu \neq 0$, since μ cancels.

Regarding the forecast intervals, we make assumptions on y_t and u_t distribution. We assume that $y_t, y_{t+1}, \dots, y_{t+h}$ have a multivariate normal distribution for any t and h. u_t is assumed to have a gaussian distribution, $u_t \sim N(0, \Sigma_u)$, where u_t, u_s are independent for $s \neq t$[13]. Under these assumptions, the forecast errors are normally distributed, being a linear transformation of normal vectors:

$$y_{t+h} - E_t[Y_{t+h}] = \sum_{i=0}^{h-1} \phi_i \, u_{t+h-i} \sim N\left(0, \Sigma_y(h)\right) \qquad (23)^{14}$$

The forecast error for the individual component is also normally distributed:

$$\frac{y_{q,t+h} - E_t[y_{q,t+h}]}{\sigma_q(h)} \sim N(0,1) \qquad (24)$$

Where $E_t[y_{q,t+h}]$ is the q-th component of the predictor $E_t[Y_{t+h}]$; $\sigma_q(h)$ is the square root of the q-th diagonal element of $\Sigma_y(h)$.

Denoting by z_α the upper $\alpha 100$ percentage point of the normal distribution, we get

$$1 - \alpha = \Pr\left\{-z_{\frac{\alpha}{2}} \leq \frac{y_{q,t+h} - E_t[y_{q,t+h}]}{\sigma_q(h)} \leq z_{\frac{\alpha}{2}}\right\}$$
$$= \Pr\left\{E_t[y_{q,t+h}] - z_{\frac{\alpha}{2}} \, \sigma_q(h) \leq y_{q,t+h} \leq E_t[y_{q,t+h}] + z_{\frac{\alpha}{2}} \, \sigma_q(h)\right\} \quad (25)$$

Finally, the $(1-\alpha)100\%$ interval forecast, h periods ahead, for the q-th component of y_t is:

$$E_t[y_{q,t+h}] \pm z_{\frac{\alpha}{2}} \sigma_q(h) = \left[E_t[y_{q,t+h}] - z_{\frac{\alpha}{2}} \, \sigma_q(h), E_t[y_{q,t+h}] + z_{\frac{\alpha}{2}} \, \sigma_q(h)\right] \quad (26)^{15}$$

[13] Under the assumption of Gaussian process, the distributions of the forecast and the forecast errors are known, and the forecast intervals are easy to set up.

[14] $\Sigma_y(h) = MSE\left(y_{t+h}\right) = \sum_{i=0}^{h-1} \phi_i \Sigma_u \phi_i'$ is the forecast error covariance matrix (or MSE matrix). For $h \to \infty$, the MSE matrix approach the covariance matrix of y_t.

[15] If forecast intervals of this type are computed repeatedly from a large number of time series, then about (1−α)100% of the intervals will contain the actual value of the random variable $y_{q,t+h}$.

2.4 STRUCTURAL ANALYSIS

Var models are often used to analyze the correlation among a set of variables. Important in such context is the analysis of the Granger Causality between variables, of the Impulse Response Function, and of the Forecast Error Variance Decomposition.

2.4.1 GRANGER CAUSALITY

Behind the concept of the Granger Causality there is the idea that the cause cannot come after the effect [Granger (1969a)]. Thus, taking two variables, x and z, if variable x granger-cause the variable z, the information regarding past and present values of x_t must be included in the information set to improve the predictions of z_t. Formally, denoting by $z_t(h|\Omega_t)$, the optimum MSE h-step predictor of z_t at origin t, based on the information set Ω_t and by $\Sigma_z(h|\Omega_t)$, the corresponding forecast MSE, then the process x_t *Granger-Causes* z_t if:

$$\Sigma_z(h|\Omega_t) < \Sigma_z(h|\Omega_t \setminus \{x_s|s \leq t\}) \qquad for\ at\ least\ one\ h = 1,2,\ldots.. \quad (27)$$

Where $\Omega_t \setminus \{x_s|s \leq t\}$ is the information set containing all the relevant information except the ones regarding present and past values of the x_t process.

If both x_t and z_t are Granger- Cause each other, we call the process $(z_t', x_t')'$ *feedback system.*

In practice, we cannot evaluate the optimum h-step predictor. It requires that the information set contains all the information in the universe. In empirical analysis only information regarding past and present of the process under study are included in the information set. For this reason, we evaluate the optimal linear MSE h-step predictor $z_t(h|\{z_s, x_s|\ s \leq t)$. Rewriting the statement formally, the process x_t *Granger-Causes* z_t if:

$$\Sigma_z(h|\{z_s, x_s|\ s \leq t) < \Sigma_z(h|\{z_s|\ s \leq t\}) \qquad for\ at\ least\ one\ h = 1,2,\ldots.. \quad (28)$$

In a VAR(p) process, to determine the Granger-causal relationship between the variables we refer to the MA representation (13) of the k-dimensional VAR(p) process y_t and we suppose that such process can be divided in two subprocess z_t and x_t, of dimension m and $(k-m)$, respectively. In matrix form:

$$y_t = \begin{bmatrix} z_t \\ x_t \end{bmatrix} + \begin{bmatrix} \Phi_{11}(L) & \Phi_{12}(L) \\ \Phi_{21}(L) & \Phi_{22}(L) \end{bmatrix} \begin{bmatrix} u_{1,t} \\ u_{2,t} \end{bmatrix} \qquad (13a)$$

From which, x_t is not 1-step Granger-cause of z_t if $\phi_{12,i} = 0$. Formally,

$$z_t(1|\{y_s|s \leq t\}) = z_t(1|\{z_s|s \leq t\}) \quad \Leftrightarrow \quad \phi_{12,i} = 0 \quad for\ i = 1,2,\ldots. \quad (29)$$

Or alternatively,

$$z_t(1|\{y_s|s \leq t\}) = z_t(1|\{z_s|s \leq t\}) \quad \Leftrightarrow \quad A_{12,i} = 0 \quad for\ i = 1,2, \ldots .p \qquad (30)$$[16]

Moreover, if x_t is not 1-step Granger-cause of z_t, then the same conclusion can be applied to the h-step predictor. Hence, a necessary and sufficient condition of x_t being not Granger-causal of z_t is that $\phi_{12,i} = 0$.

The same reasoning can be applied to examine the Granger-causality between two variables in a higher dimensional system. The only difference in the latter case, is that if one variable is not 1-step Granger-cause of another, then it may still be h-step causal for $h > 1$.

2.4.2 IMPULSE RESONSE FUNCTION AND VARIANCE DECOMPOSITION FORECAST ERROR

The Impulse Response Function (IRF) and the Forecast Error Variance Decomposition (FEVD) are useful tools for interpreting and estimate the relationship between economics variables in linear (and nonlinear) multivariate time series.

The IRF shows the response of current and future values of each variable in the system to a one-unit (exogenous) increase in the current value of VAR errors. It analyzes the impulse response relationship between two variables, in a system that involves other variables as well.

The FEVD is used to understand the size of the effect of the shock and how much information of one variable are provided by the other variables of the system, by decomposing the variance in series. The variance decomposition forecast error provides information regarding which shock contributes to the fluctuation of each variable in the system.

Starting with the Impulse Response Analysis, we consider the VAR (1) process firstly, and then extend the reasoning to the general VAR(p) process. Taking the VAR (1) process as in (2), we assume that $v = 0$ for simplicity, that $y_{1,t}$ increases by one unit in period zero ($u_{1,0} = 1$), and no further shocks occur in the system ($u_{2,0} = u_{3,0} = \cdots = u_{k,0} = 0$).

It is easy to demonstrate, recursively, that a unit shock in $y_{1,t}$ at time zero, after i periods, results in a vector $y_i = (y_{1,i} y_{2,i} \ldots y_{k,i})'$ corresponding to the first column of A_1^i[17]. The same reasoning is made if the shock occur to the q-th variable of y_t. A unit shock on $y_{q,t}$ at time zero, after i periods, results in a vector $y_i = (y_{1,i} y_{2,i} \ldots y_{k,i})'$ that correspond to the q-th column of A_1^i. The elements of A_1^i are called *impulse responses* and represent the effect of a unit shock in the variables of the system after i

[16] Such formulation is valid if y_t is a VAR(p) process with nonsingular white noise covariance matrix of residuals.

[17] For the proof see the book New Introduction to Time Multiple Series Analysis of Lüktepohl, 2005.

periods. As in previous sections demonstrated, $A_1^i = \phi_i$, that is the i-th coefficient matrix of the MA representation of a VAR (1) process. The MA coefficient matrices contains the impulse responses to the system.

For a VAR(p) process, also, the impulse responses are the coefficients of the matrix ϕ_i of the MA representation. The jq-th element of ϕ_i, $\phi_{jq,i}$, is the *impulse response function* of variable j to shocks on variable q. It provides the (instantaneous) reaction of the j-th variable to a unit shock in the variable q, i periods ago[18]. Of course, if $\phi_{jq,i} = 0$, there is zero impulse response, and the variable j doesn't react to unit shock on variable q. From this consideration, is easy to understand that $\phi_{jq,0}$ represent to the instantaneous impact of a change in the $u_{q,t}$ on y_t, while $\phi_{jq,1}$correspond to the instantaneous impact of a change in the $u_{q,t-1}$ on y_t, and so on[19].
A basic assumption of these IRF as previous evaluated is that a shock occurs in only one variable at time, reasonable assumption only if the variables are independent. If they are dependent, instead, is reasonable to assume that a shock in one variable can be accompanied by a shock in another variable. Under such scenario, setting all other residual to zero, may provide an incomplete, misleading picture of the relationship between variables. To solve this issue, we can evaluate the responses to orthogonal impulses Θ_i. The first step is to obtain a white noise vector with uncorrelated (orthogonal) components. The MA representation of y_t become:

$$y_t = \sum_{i=0}^{\infty} \Theta_i \omega_{t-i} \qquad (31)$$

It is obtained by first decomposing (using the Cholesky decomposition) $\Sigma_u = PP'$ where P is a lower triangular non-singular matrix with positive diagonal elements. Then by setting $\Theta_i = \phi_i P$ and $\omega_t = P^{-1}u_t$. We obtain a white noise vector $\omega_t = (\omega_{1,t}, \omega_{2,t}, \dots, \omega_{k,t})'$with uncorrelated components and unit variance $\Sigma_\omega = P^{-1}\Sigma_u(P^{-1})' = I_k$, such that $\omega_t \sim N(0, I_k)$. Is reasonable to assume that a change in one component of ω_t has no effect on other components, since they are orthogonal. The elements

[18] Provided that the effect is not contaminated by other shocks to the system.

[19] Of interest sometimes is also to analyze the cumulated effect over more than one period. We evaluate the accumulated response over n periods of variable j, to a unit shock in the q-th variable. $\Psi_n = \sum_{i=0}^{n} \phi_{jq,i}$. The long-run effects (or total multipliers), instead, is the total accumulated effects for all future periods, for $n \to \infty$.

of Θ_i are the responses of the system to such innovations[20]. $\theta_{jk,i}$ is the response of variable j, to a unit innovation (one standard deviation shock) in the q-th component, occurred i periods ago[21]. If $\theta_{jk,i} = 0$, there is Zero Orthogonalized Impulse Response. There is one last consideration to take account. Since Θ_0 is a lower triangular matrix, the order of the variables matter. They must be ordered by the analyst in such a way that just the first component of y_t may impact all the other components, while the second impact only the last $k-2$ components, and so on[22]. Obtaining so a Wold causal chain.

Moving to the FEVD, we decompose the forecast error variance into components accounted for innovations in the different variables of the system. The aim is to measure the fraction of forecast error variance of a variable that can be attributed to orthogonalized shocks to another variable of the system, or to itself. We consider the MA representation of VAR(p) process with orthogonal white noise innovations. The error of the optimal h-step forecast of the j-th component of y_t is:

$$y_{j,t+h} - y_{j,t}(h) = \sum_{i=0}^{h-1} \Theta_{j1,i}\omega_{1,t+h-i} + \cdots + \Theta_{jk,i}\omega_{k,t+h-i} \quad (32)^{23}$$

It (potentially) consists of all the innovations $\omega_{1,t}, \omega_{2,t}, \dots, \omega_{k,t}$.

The h-step MSE of $y_{j,t}(h)$, or better the corresponding forecast error variance, of variable j is:

$$\sigma_j^2(h) = MSE\left(y_{j,t}(h)\right) = E_t\left[y_{j,t+h} - y_{j,t}(h)\right]^2 = \sum_{i=0}^{h-1} \Theta_{j1,i}^2 + \cdots + \Theta_{jk,i}^2 = \sum_{i=0}^{h-1}\sum_{q=1}^{k} \Theta_{jq,i}^2 \quad (33)^{24}$$

Instead, the contribution of innovations in variable q to the h-step forecast error variance (or MSE) of variable j is:

---

[20] Also in this case is possible to evaluate the accumulated response over n periods of variable j, to a unit shock in the q-th variable. $\Xi_n = \sum_{i=0}^{n} \theta_{jq,i}$.

[21] Moreover, since the components of the white noise have unit variance, the size of the single unit innovation is one standard deviation.

[22] That is the reason why we impose P as a lower triangular matrix. We impose a restriction in the way of a "timing scheme" for the shocks, meaning that shocks enter in the equation successively, so that the q-th shock will not affect the variable of y_t, prior the $y_{q,t}$ variable.

[23] While the error of the optimal h-step forecast is: $y_{t+h} - y_t(h) = \sum_{i=0}^{h-1} \Theta_i \omega_{t+h-i}$

[24] While he h-step forecast MSE matrix of y_t is: $\Sigma_y(h) = MSE\big(y_t(h)\big) = \sum_{i=0}^{h-1} \Theta_i \Theta_i' = \sum_{i=1}^{h-1} \phi_i \Sigma_u \phi_i'$. Where, the diagonal elements of the matrix are the MSEs of the $y_{j,t}$ variables. $MSE\left(y_{j,t}(h)\right) = \sum_{i=0}^{h-1} e_j' \phi_i \Sigma_u \phi_i' e_j'$

$$\theta^2_{jq,0} + \cdots + \theta^2_{jq,h-1} = \sum_{i=0}^{h-1} \left(e_j' \Theta_i e_q\right)^2 \qquad (34)$$

Where: e_j is the j-th column of I_k and e_q is the q-th column of I_k

Finally, the portion of the *h-step forecast error variance* of variable j, given by $\omega_{q,t}$ innovations as:

$$\varrho_{jq,h} = \frac{\sum_{i=0}^{h-1}\left(e_j'\Theta_i e_q\right)^2}{\sigma_j^2(h)} \qquad (35)$$

If $\omega_{q,t}$ can be associated with variable k, then $\varrho_{jq,h}$ gives the percentage contribution of variable q to the h-step forecast error variance of variable j. If $\varrho_{jq,h}$= 0, then the h-step forecast variance of j is not influenced by innovations in variable q.

2.5 ESTIMATION TECNIQUE

So far, we have assumed that the coefficients $\mu, A_1, \dots, A_p, \Sigma_u$ were known. They are not, they need to be estimated. Several are the estimation methods. In this paper we use the *Maximum Likelihood Estimation* Method, which gives an asymptotically efficient estimator for the set of parameters (Greene, 2012). An assumption must be made: the times series data are known. We assume that a k-dimensional multiple time series is available and is generated by a stationary, stable, gaussian distributed VAR(p) process as in (1).

The log-likelihood function is[25]:

$$\ln l(\mu, \boldsymbol{\alpha}, \Sigma_u) = -\frac{kT}{2}\ln 2\pi - \frac{T}{2}\ln|\Sigma_u| - \frac{1}{2}tr[(Y^0 - AX)'\Sigma_u^{-1}(Y^0 - AX)] \qquad (36)$$

Maximizing it, we obtain the ML estimators of $\mu, \boldsymbol{\alpha}, \Sigma_u$:

$$\tilde{\mu} = \frac{1}{T}\left(I_k - \sum_t \tilde{A}_\iota\right)^{-1} \sum_t \left(y_t - \sum_i \tilde{A}_i y_{t-i}\right) \qquad (37)$$

$$\tilde{\boldsymbol{\alpha}} = \left(\left(\tilde{X}\tilde{X}'\right)^{-1}\tilde{X} \otimes I_k\right)(\boldsymbol{y} - \tilde{\boldsymbol{\mu}}^*) \qquad (38)$$

$$\widetilde{\Sigma_u} = \frac{1}{T}\left(\widetilde{Y^0} - \tilde{A}\tilde{X}\right)\left(\widetilde{Y^0} - \tilde{A}\tilde{X}\right)' \qquad (39)$$

[25] See the book New Introduction to Time Multiple Series Analysis of Lüktepohl, 2005 for the computation of the log-likelihood function and of the probability densities of $\boldsymbol{u}$ and $\boldsymbol{y}$.

Where: $Y_t^0 := (y_t - \mu, \dots, y_{t-p+1} - \mu)'$ is a $(kp\ x\ 1)$ vector; $A := (A_1, \dots, A_p)$ is a $(k\ x\ kp)$ matrix; $X := (Y_0^0, \dots, Y_{T-1}^0)$ is a $(kp\ x\ T)$ matrix; $\boldsymbol{\alpha} := vec(A)$ is a $(k^2 px\ 1)$ vector; $\boldsymbol{y} = vec(Y)$ is a $(kT\ x\ 1)$ vector; $\boldsymbol{\mu}^* := (\mu', \dots, \mu')'$ is a $(Tk\ x\ 1)$ vector and $Y^0 := (y_1 - \mu, \dots, y_T - \mu)$ is a $(k\ x\ T)$ matrix;

The estimations of μ and $\boldsymbol{\alpha}$, obtained under such condition, are identical to the LS estimators.
The ML estimator is suitable given its asymptotic properties. Taking a stationary, stable Gaussian VAR(p) process, the ML estimations are consistent and $\sqrt{T}(\tilde{\mu} - \mu)$ and $\sqrt{T}(\tilde{\boldsymbol{\alpha}} - \boldsymbol{\alpha})$ are asymptotically normally distributed, with zero mean and variance equal to $\Sigma_{\tilde{\mu}}$ and $\Sigma_{\tilde{\boldsymbol{\alpha}}}$, respectively. Moreover, taking $\tilde{\boldsymbol{\sigma}} = vech(\tilde{\Sigma}_u)$, $\sqrt{T}(\tilde{\boldsymbol{\sigma}} - \boldsymbol{\sigma})$ is also asymptotically distributed, with variance $\Sigma_{\tilde{\boldsymbol{\sigma}}}$. It is an asymptotic efficient estimator[26] (Greene, 2012).
For the forecast analysis and for the computation of the IRF and of the FEVD, estimated values are used.

2.6 INFORMATION CRITERIA

As anticipated, after checking the validity of the properties of the VAR model, the second step of the VAR analysis consists in the choice of the lags (p). The choice of p larger than what needed, could lead to the reduction of the forecast precision of the estimated VAR(p) model. As consequence, several *information criteria* have been developed in literature, to choose the appropriate number of lags p, to achieve the best trade-off between model fit and parsimony. Increasing the number of lags will of course increase the fit, but also the number of parameters that must be estimated, complicating the analysis. The three most famous criterion are the *Akaike's Information Criterion (AIC),* the *Bayesian Information Criterion (BIC),* and the *Hannah and Quinn Information Criterion (HCQ)*.

Given a VAR(m) process, the general form of the information criteria is:

$$IC(m) = \ln|\widetilde{\Sigma_u}(m)| + f(m) \qquad (40)$$

Where $\ln|\widetilde{\Sigma_u}(m)|$is the logarithm of the white noise covariance matrix and is a decreasing function of the number of lags. Instead, $f(p)$ is the *penalty function*, an increasing function in the number of

[26] An estimator is asymptotic efficient when it is consistent, asymptotically normal distributed, and has an asymptotic covariance matrix that is not larger than the asymptotic covariance matrix of any other consistent, asymptotically normally distributed estimator (Greene, 2012).

lags. Its functional form depends by the criteria adopted: AIC: $f(m) = \frac{2mk^2}{T}$; BIC: $f(m) = \frac{\ln T}{T} mk^2$; HCQ: $f(m) = \frac{2\ln\ln T}{T} mk^2$.

Where m is the order of the VAR process fitted to the data; T is the sample size; k is the dimension of the time series. mk^2 is the number of freely estimated parameters.

Independent from the criterion used, the estimate $\hat{p}(IC)$ of p is chosen to minimize $IC(m)$, to choose the order that minimize the forecast MSE (mean squared error). The AIC overestimate the true order with positive probability[27], so is not a consistent criterion[28], while BIC and HCQ are consistent.

As demonstrated by Lüktepohl, (2005), for $T \geq 16$, the BIC is the most parsimonious criterion, the AIC the less, while the HCQ is in between the two. Precisely:

$$\hat{p}(BIC) \leq \hat{p}(HCQ) \leq \hat{p}(AIC) \qquad (41)$$

In the empirical chapter we are going to use all the three criterion to choose the number of lags that best fits our data.

[27] But asymptotically it chose the correct order almost with probability one if we use a large sample.

[28] An estimator $\hat{p}$ of the VAR(p) model is consistent if for $T \to \infty$ the plim of $\hat{p}$ converge to p.

CHAPTER 3: DATA AND VARIABLES

In this chapter we are going to exploit the variables involved in the analysis. We are going to describe the trend of the variables, with focus on crises periods, using quarterly data for a panel of four countries: Italy, Belgium, Czech Republic and Poland, from 2000:1 to 2019:4. Our VAR process is composed by five endogenous variables: $y_t = f(x_t, f_t, CLIFS_t, \pi_t, i_t,)$, which represent, respectively, the economic growth, the fiscal variable, the Country-level index of financial stress, the inflation rate and the short term interest rate. In following sub-chapters each variable involved in the process, will be explained.

3.1 ECONOMIC GROWT MEASURE: THE REAL GDP

With the term *economic (output) growth* economists refers to an increase in the size of a country's economy over a period. To measure the size of the economy we take as variable the real GDP seasonally adjusted[29], for the 4 countries analyzed, which measure the total production of goods and services of a country, in "real" terms, adjusted for price changes. Moreover, being the variable seasonally adjusted, it is free of the influences of predictable seasonally patterns. We computed the output growth as the percentage change of the real GDP per-period. Data are taken from the IFS (International Financial Statistics) of the IMF (International Monetary Fund) and because it is measured in the domestic currencies, we converted it in US dollars by using the Real Effective Exchange Rate[30], taken from the IFS. The conversion is suitable to compare results between the countries.

Figure 3.1 displays large fluctuations in output growth among periods, for Italy and Belgium, while less relevant fluctuations for Czech Republic and Poland, except during the GFC. During the crisis period, all counties experienced a decline in the output, that reached its minimum, respectively of -3.93%, -3.50%, -8.83% and -11.98% at the end of 2008/beginning of 2009. Nerveless, all of them showed a great recovery after it. They came back to pre-crisis levels, after approximately two years. For Italy and Belgium, the years after have been characterized by several fluctuations, between economic growth and decline, with a strong decline between 2010 and 2012, caused by the Sovereign Debt crisis.

The slowdowns (or even recessions) of the Italian and Belgian economies have been caused by several key factors: the decline of the TFP (Total Factor Productivity), the diminishing rate of investments of households and enterprises, the sequence of banking shocks, the increase of unemployment, for

[29] It is in logarithm.
[30] We used the real one in order to take account also of the effect of the inflation.

Italy, and the lowering competition in key services sectors, for Belgium. This persistently low productivity characterizing the years of the two crises has been one of the most relevant causes of the macroeconomic imbalance of Italy, characterized by high levels of public debt and weak external competitiveness with a strong impact on SMI (Coletto, 2010). Both countries managed to overcome the economic decline thanks several investments and structural reforms undertaken, that brought to an increase in the resilience and governance of the banking system, for Italy (EC, (2015) - Italy), and to the transition to a more knowledge-intensive economy, for Belgium (EC, (2015) – Belgium).

Moving to Czech Republic and Poland, the countries experienced their strongest decline in economic growth during the 2008 crisis. The decline lasted only slightly more than one year, in fact in 2010 the economies returned back to the 2008 levels. Several factors explain the fast recovery and the fact that no strong downturns showed in the following years. For Czech Republic the main factors were the increase of domestic and foreign demand, the decline of the unemployment rate, and the increase in investments. (IMF, (2015) - Czech Republic). The Polish economy, instead, performed well throughout the crisis as result of strong fundamentals and several counter-cyclical polices implemented. The main factors have been the construction of a strong system of bilateral trade with the euro area and the persistent stability of the banking sector, well capitalized and liquid (IMF, (2012) – Poland). The Polish economy experienced only a weak downturn between 2011 and 2012 from which recovered rapidly, due an increase in demand, decline in unemployment, increase in investments and strengthening of trade. Never less, for Czech Republic, the potential growth is still far below the level required to obtain convergence towards the income levels of other (advanced) European countries. The country is still facing the risk of a non-sustainable growth. From 2018 the growth has slowed because the economy reached its capacity limits, due supply constraints. Nowadays, investments, especially in public goods, are required to boost the productive potential (IMF, (2019) - Czech Republic). Differently, Poland shows a sustainable economic convergence towards the advanced European countries, thanks all the policies undertaken in the labor market, to foster stability and increase the income convergence and the productivity level (IMF, (2015) – Poland). The overall economic growth has been supported by three mayor factors: a rebound in euro-area activity, an increase in EU transfers and by the introduction of a large social benefit programme (IMF, (2019) – Poland). Lastly, for all countries is possible to observe a slight decline from the end of 2018, caused by the COVID-19 pandemic[31].

[31] A better analysis of the impact of COVID-19 is provided in section 4.4.

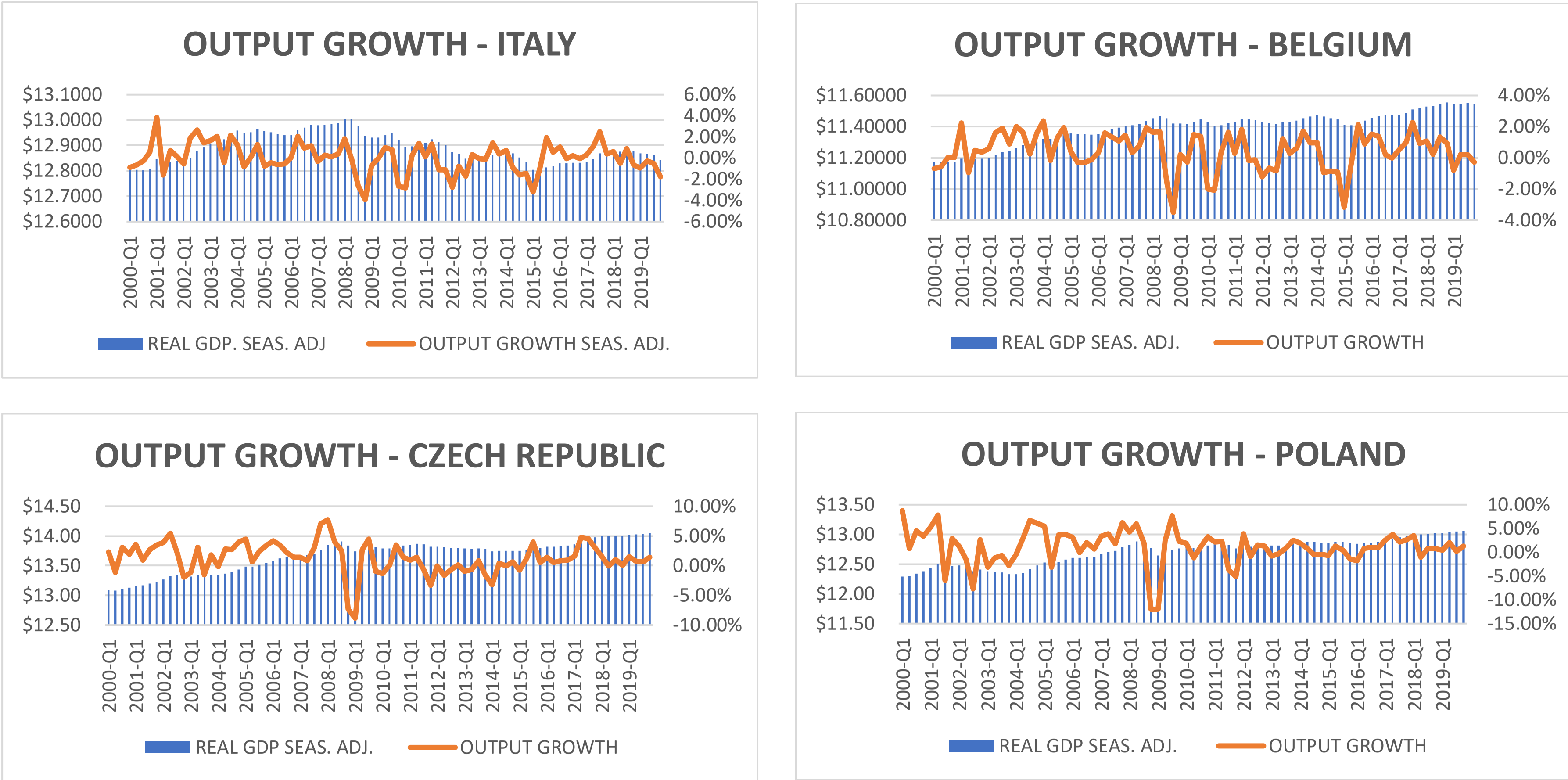


FIGURE 3.1. This figure displays the real GDP seasonally adjusted and the economic growth, for a panel of four countries: Italy, Belgium, Czech Republic, and Poland. The sample period goes from 2000:1 to 2019:4.
Data are taken from the IFS.

3.2 FISCAL POLICY MEASURE: THE GOVERNMENT DEBT-TO-GDP RATIO

The choice of the fiscal policy variable is a relevant issue due the different measures of it and the different implications of each one. There are several measures that refer to the government expenditure, or to the government revenues, only. One choice variable could be the government fiscal balance, that allows to overcome the previous problem, including both governments expenditures and revenues. But this variable doesn't take account of several government actions, such as: the recapitalization of the banking sector, the purchase of financial assets and the shock-flow adjustments. To overcome both problems this paper uses the government debt-to-GDP ratio as baseline variable. It provides information on the current and past fiscal policies. It has a closer link with the financial market than the fiscal balance, it captures the risk related to the refinancing operations and influence the interest rates. Furthermore, the level of debt/GDP ratio has been central to many policies discussion during the recent crisis (Afonso et al, 2018). It compares the country's total debt to its economic productivity[32]. Data on the fiscal balance are taken from the IFS (International Financial Statistics) and measured in US dollars, using the exchange rate, while data for the debt-to-GDP ratio are taken from the Eurostat Data Browser. The first variable represents the amount of money that a government receives from tax revenues and asset sold, minus the government spending (Eurostat, statistic explained, 2018), while the second one measures the gross debt of the government as percentage of the GDP (nominal) and provides key information regarding the sustainability of government finance (OECD, 2023). It measures the financial leverage of the economy.

In figure 3.2 both fiscal policy measures are represented. The four governments experienced a fiscal deficit[33], during almost all the period considered. Really few have been the periods of fiscal surplus, except for Belgium who repeatedly switched from periods of fiscal deficit to others of fiscal surplus and vice versa, even if the magnitude of fiscal surplus was lower that the magnitude of deficits. Following the output growth trend, all countries display their lowest level of fiscal balance during the 2008 crisis, when the governments implemented extraordinary expenditures to sustain the demand and to help important firms and banks to recover (EC, 2009). Italy, Belgium, and Poland began to recover in 2010, when the fiscal deficit started to become less negative, due several fiscal measures and packages implemented by each country (EC, (2012) - Italy; EC, (2012) - Belgium; IMF, (2012) – Poland). On the contrary, Czech Republic revealed its lower peak at the end of 2012 caused by the

[32] Is the ratio between total debt and total GDP of a country.

[33] their expenses were higher that revenues.

weak domestic demand (IMF, (2013) - Czech Republic). From end of 2015, it came back to positive levels of fiscal balance, till the start of COVID-19 crisis.

More informative is the debt-to-GDP ratio, variable that reached its highest and lowest peaks when the fiscal balance displayed its highest levels of surplus and deficit, respectively. Overall, is possible to observe an increasing trend for all countries, except Belgium. Starting by the countries belonging to the European Monetary Union (EMU), they never managed to fully comply with the European fiscal criteria. They display a debt-to-GDP ratio systematically higher that 60%, the limit set by the Treaty on the Functioning of the European Union (TFEU) (EC, 2023). Instead, Czech Republic and Poland always complied with this limit, even during the crises.

Italy reached its minimum in 2007 Q4 with a value of 103,9%. The overall reduction of the debt-to-GDP ratio obtained up to the 2007 is mainly attributable to extraordinary operations (sales of assets and debt restructuring) that brought to an increase of government revenues (Marino et a, 2008). But, as soon as the crisis stared, the ratio began to increase, mostly due the lower negative growth, the low level of inflation rate, and the increase of government spendings, but also due the temporary nature of the measures previously adopted. There has been a strong and substantial increase, starting from the 2009, strengthened in 2012, with the Debt Sovereign Crisis, until to reach the highest peak in 2015 Q2 with a value of 138,8%. Following, the ratio began to decrease due to an increase in GDP growth, decrease of interest rates, privatization programme of some state-owned enterprises, and the sale of public real estate (EC, (2015) – Italy), actions implemented to overcome the crises. Despite such reduction, however, the debt level is still high, over 130%. The high public debt has always been and still it is the major source of vulnerability for the Italian economy.

Similar to the Italian one, is the Belgian trend. The country experienced a substantial decline until the last quartile of 2007 with its minimum value of 87,3%. After the outbreak of the two crises, it started to increase, reaching its maximum peak of 111% at the beginning of 2015, and keeping levels close to 100% till the end of the sample period considered. From 2016 a slow reduction began. Hampered by low growth and inflation, the increasing trend between 2008 and 2015, has been caused by two opposing forces: the decrease in output growth, and the increase in the debt level, since the Belgian government started to support several failing financial institutions and the overall economic system. The decline started in 2016 was driven by the end of the crisis period and by several fiscal and policy reforms implemented (EC, (2015) – Belgium).

Instead, Czech Republic and Poland, have always complied with the limits sets by the TFEU. They also displayed an increasing trend, started from the GFC, reaching their highest peak in 2013 with a value respectively of 45,2% and 57.9%. After 2013, the debt level started to decrease until the end of the sample period considered. Czechian growth was caused by a persistent low inflation, but mostly by the financial tightening undertaken by the government between 2011-2013. The decline, instead, was mostly caused by strong tax revenues and lower capital and social benefit spending (IMF, (2018) - Czech Republic). For Poland, also, the resilience of the country can be explained by several factors. Firstly, by the fact that the country didn't experience a prolonged economic decline during the GFC. Other reasons were a reform of the pension funds contributions[34] and an increase of state-owned enterprises' dividends (IMF, (2012) – Poland), the adoption of ESA2010 accounting standards (IMF, (2015) – Poland), and, more recently, the introduction of new taxes on assets of financial institutions and improvements in tax compliance (IMF, (2019) – Poland).

[34] This legislation increased the retirement age to 67 years old, for both sexes.

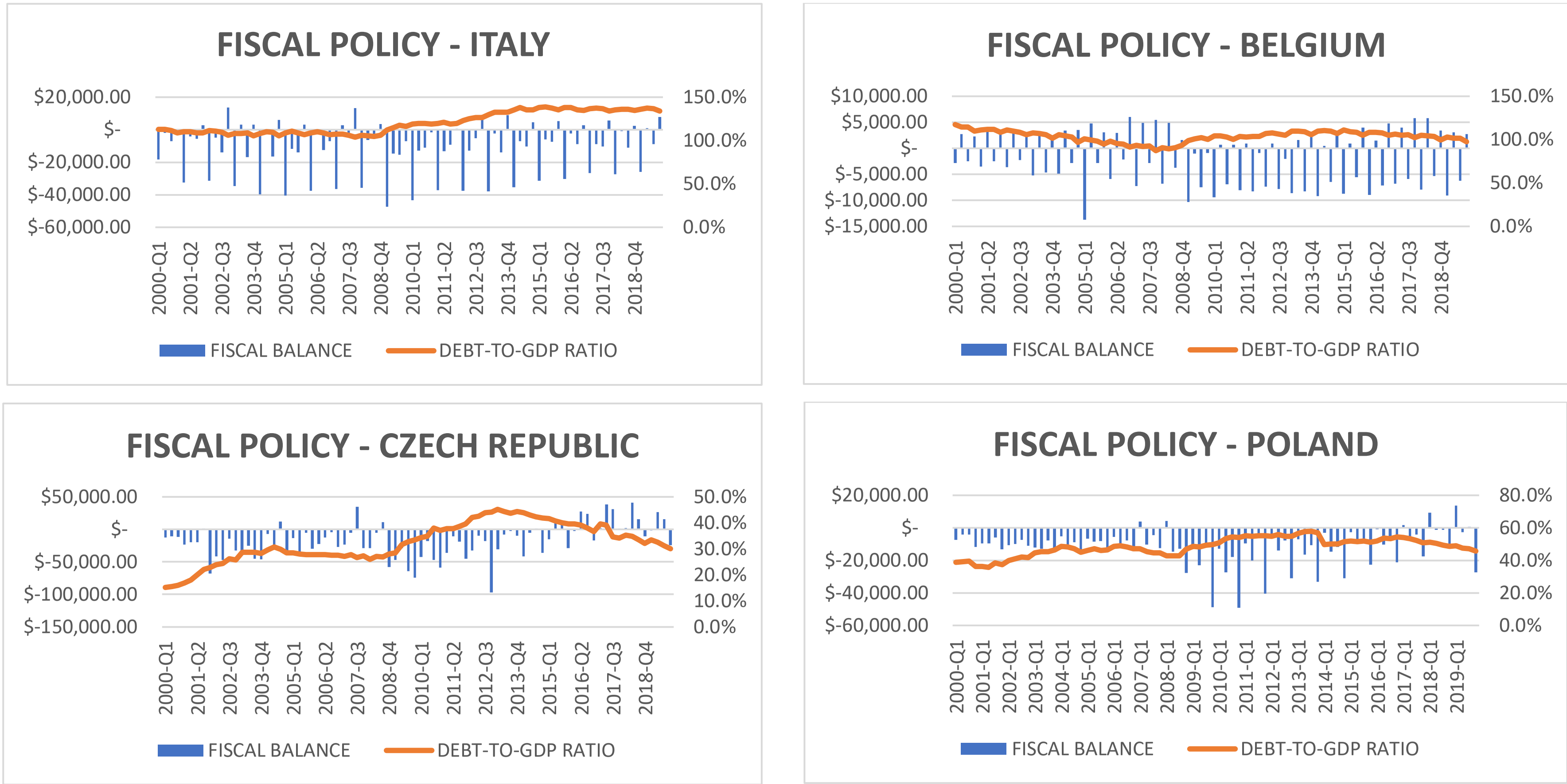


FIGURE 3.2. This figure displays the fiscal balance and the debt-to-GDP ratio, for a panel of four countries: Italy, Belgium, Czech Republic, and Poland. The sample period goes from 2000:1 to 2019:4.
Data are taken from the IFS and the Eurostat Data Browser, respectively.

3.3 FINANCIAL STRESS INDEX: THE CLIFS

Despite several measures of financial stress developed in literature, this paper uses the Country Level index of Financial Stress. The CLIFS identifies systemic financial stress episodes for each EU country, in a reproducible, objective, and transparent way, in such a way that there is comparability across countries. This index, developed by Duprey et al, (2017) differently from the others, is a model-based financial stress index, not an expert-based one, for which no assumptions on the sequence of events are made. The authors do not specify if the financial stress or the real economy stress occur first, and they do not take as benchmark known financial stress episodes. They only define *systemic financial stress episodes* as "*those events that qualify both as periods of financial markets stress and periods of real economic stress*". Systemic stress occurs when there is both low growth and high financial stress. The CLIFS includes six market-based financial stress measures, capturing three financial market segments: equity, bond, and foreign exchange market and is measured in percentage. Higher is the value the index, stronger is the financial shock in the system. Data are taken from the Statistical Data Warehouse.

The index is represented in figure 3.3. For all the four countries the highest peaks are reached during the GFC. Italy displays a value higher than 45% in 2010, Belgium higher than 40% in end of 2008, Czech Republic higher that 75% and Poland around 35%, both at beginning of 2009. High are the values during the Debt Sovereign crisis also, and, for all country except Italy, during 2001/2002. This trend is in line with the one of the other variables, especially of the output growth. In the same periods of the lower peaks of the GDP growth there are the highest peaks of the CLIFS. Strong correspondence occurs also for the fiscal policy measures. Periods in which the CLIFS displays less variations correspond to periods of less variation in economic activity and fiscal policy measures, for all countries.

Only by looking at the previous graphs is reasonable to assume that there is mutual interaction between financial stress, economic growth, and fiscal policy. Such hypothesis will be tested, using the VAR approach.

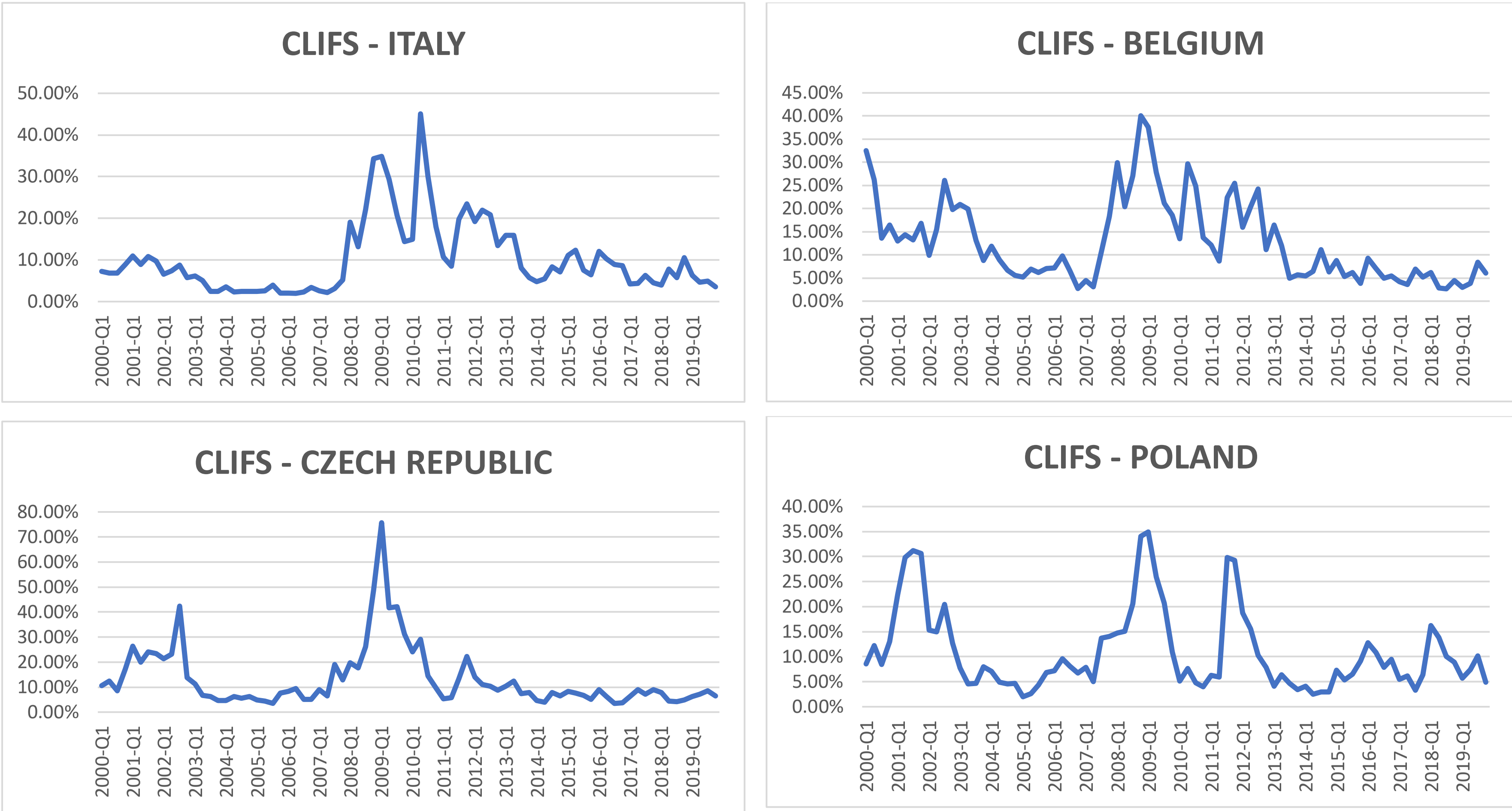


FIGURE 3.3. This figure displays the CLIFS, for a panel of four countries: Italy, Belgium, Czech Republic, and Poland. The sample period goes from 2000:1 to 2019:4. Data are taken from the Statistical Data Warehouse.

3.4 CONTROL VARIABLES

In addition to the three key variables of interest, we included in the regression two control variables: the inflation rate and the money market rate. Such controls are included to avoid the problem of omitted variables bias. *Omitted variable bias* occur when relevant variables are omitted from the regression. As result, the coefficients become bias, the effect of the omitted variable on the dependent one is attributed to the included variables. This could reduce the validity of the study. Both inflation and money market rate influence (and are influenced) the previous variables. The Inflation influences the consumer cost of living, by distorting the purchasing power, and more broadly the whole stability of the economic system while the Money Market rate provides an indication regarding the macroeconomic and liquidity conditions of the financial system and directly influences the pricing of financial assets, so the financial sector.

The data for both variables are taken from the OECD database and are measured in percentage.

3.4.1 INFLATION RATE

The inflation rate measures the change of prices over time (ECB, 2023). We have two types of inflation, the *headline inflation*, which concerns all commodities, services and goods, and the *core inflation* which concerns all commodities, services, and goods, minus the volatile fuel and food prices (Ehrmann et al, 2018). In this paper we use the headline inflation that is more representative of the total change in prices. Central Banks try to avoid both high and low inflation, reason why the ECB sets its (symmetric) inflation target at 2% (over the medium term). Such target is the one that allows a sustainable growth, brings to sustainable costs for the economy and to an adequate margin against the risk of deflation (Ehrmann et al, 2018). The same inflation target is set by Czech Republic (CNB, 2023), while the Polish one is at 2,5% (NBP, 2001).

In figure 3.4 this variable is represented. During the GFC, the inflation rates reached its highest peaks for Italy and Belgium, with values equal to 4% and 5,6% in 2008 Q3, values higher than the ECB target. They are followed by others peaks of lower intensity, but still consistent, during the Debt Sovereign crisis. These results are in line with the decrease of GDP, the increase of debt-to-GDP ratio and the increase of CLIFS. In contrast, they reached their minimums in 2009 and 2015/2016 with values of 0.1% and -0,4% for Italy and -1,2% and -0.5% for Belgium. Decrease obtained thanks the overall reduction in price pressure and the falling in oil prices during 2015 (EC, (2012), (2015) – Italy, EC, (2015) – Belgium). Inflation has been one the main causes of the low growth and the high ratio for both countries during the two crisis and of the restoring of the economy during economic recovery. Interesting are the data for the Czech Republic and Poland. Regarding the first country, the inflation, has been, almost always, positive, and close to the target rate, except for a slight increase during the GFC and the highest peak in 2018 Q1with a value of 19%. During 2018 the inflation was pushed up by the increase of food, non-alcoholic beverage, and services prices, but mostly by the increase of house price (IMF, (2018) - Czech Republic).

Neither the Polish inflation rate has been strongly affected by the GFC, showing values around 4%. During the economic recovery the rate comes back at values pre crisis, with a slight increase started at the end of 2015. The Polish inflation rate displays an overall decreasing trend from a value higher than 10% in 2000 to a value close to 2% in 2019. The peak in 2000 was a consequence of the high inflation levels that Polish economy experienced during 90s, caused by both supply and demand shocks, due the increase of fuel and services prices. After 2000 the inflation started to decline as consequence of the tightening monetary policy adopted by the government (NBP, 2000). Comparing the four countries, one of the reasons why the debt-to-GDP ratio for Eastern Countries didn't grew as much as for Italy and Belgium during the GFC is the fact that the inflation rate has not grown much.

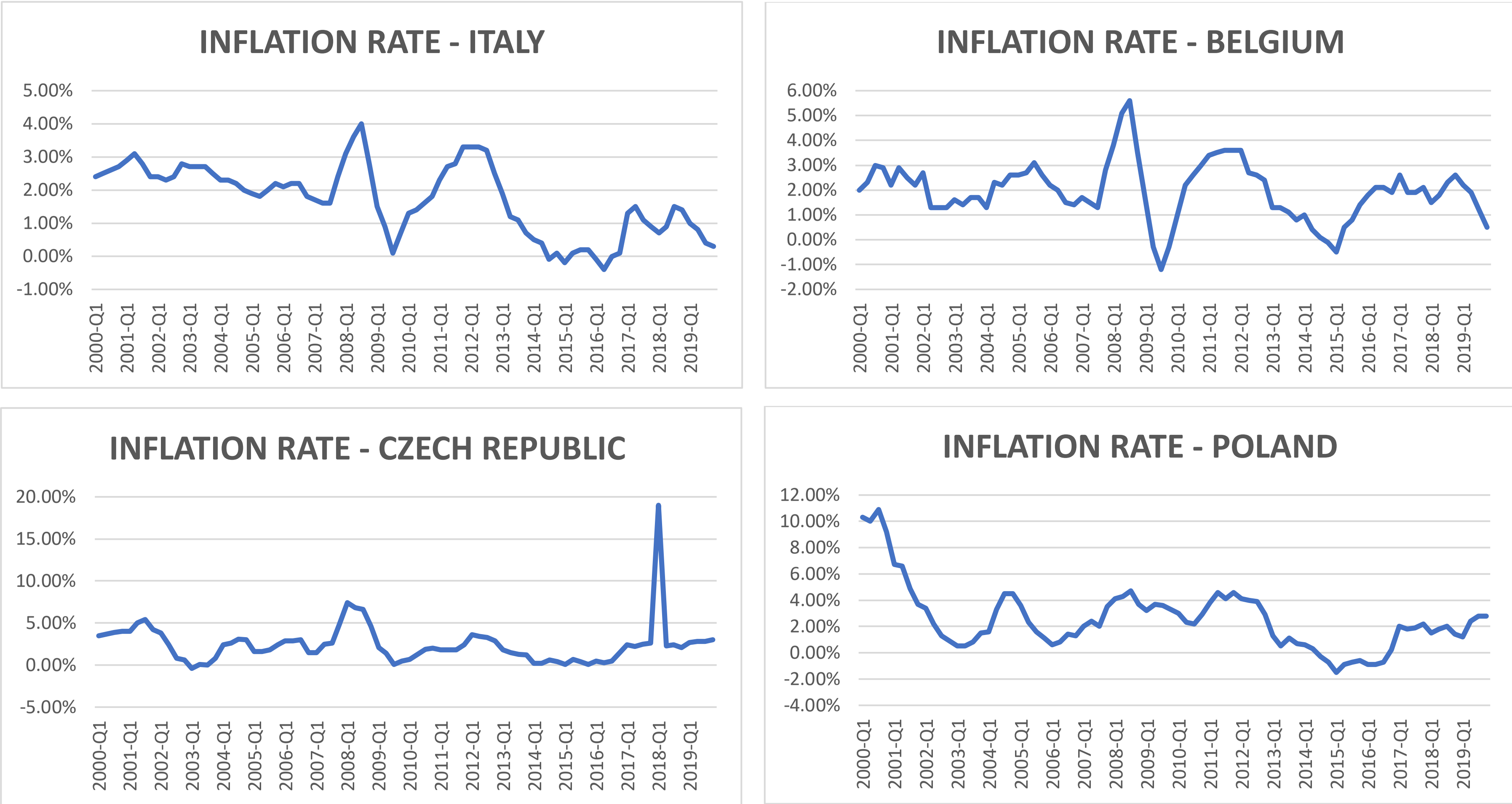


FIGURE 3.4. This figure displays the Inflation Rate, for a panel of four countries: Italy, Belgium, Czech Republic and Poland. The sample period goes from 2000:1 to 2019:4. Data are taken from the OECD database.

3.4.2 MONEY MARKET RATE

The money market interest rate represents "*the rate at which short-term borrowing are affected between financial institutions or the rate at which short term government paper is issued or traded in the market. Short-term interest rates are generally averages of daily rates, measured as a percentage. Short-term interest rates are based on three-month money market rates where available* (OECD, 2023). The higher is the rate, the higher is the risk of default, the lower is the trust in the financial system.

In figure 3.5 the money market rate is represented. It displays a decreasing trend overall. For Italy, Belgium, and Czech Republic, and in a lower measure for Poland, the strongest decline started at the end of 2008. It was due the adoption of an expansive monetary policy (reduction of the key interest rates and acquisition of government and corporate bonds on the secondary markets). As consequence, high has been the provision of liquidity in the financial system. The expansive monetary policy brought to negative levels of the rate in 2015 (Italy and Belgium). Instead, for Czech Republic and Poland, the money market never began negative, with an increasing trend started by the end of 2017, for Czech Republic and a stable trend for Poland, with a rate close to the zero percent. For no one of the country considered such rate is come back to the pre-crisis level. As in the case of Inflation rate, the trend of this variable is in line with the trend of the other variables. It reached its highest peak during the GFC, when lower were the trust in financial market, the economic growth and the liquidity level, while higher were the sovereign debt ratio and the financial stress.

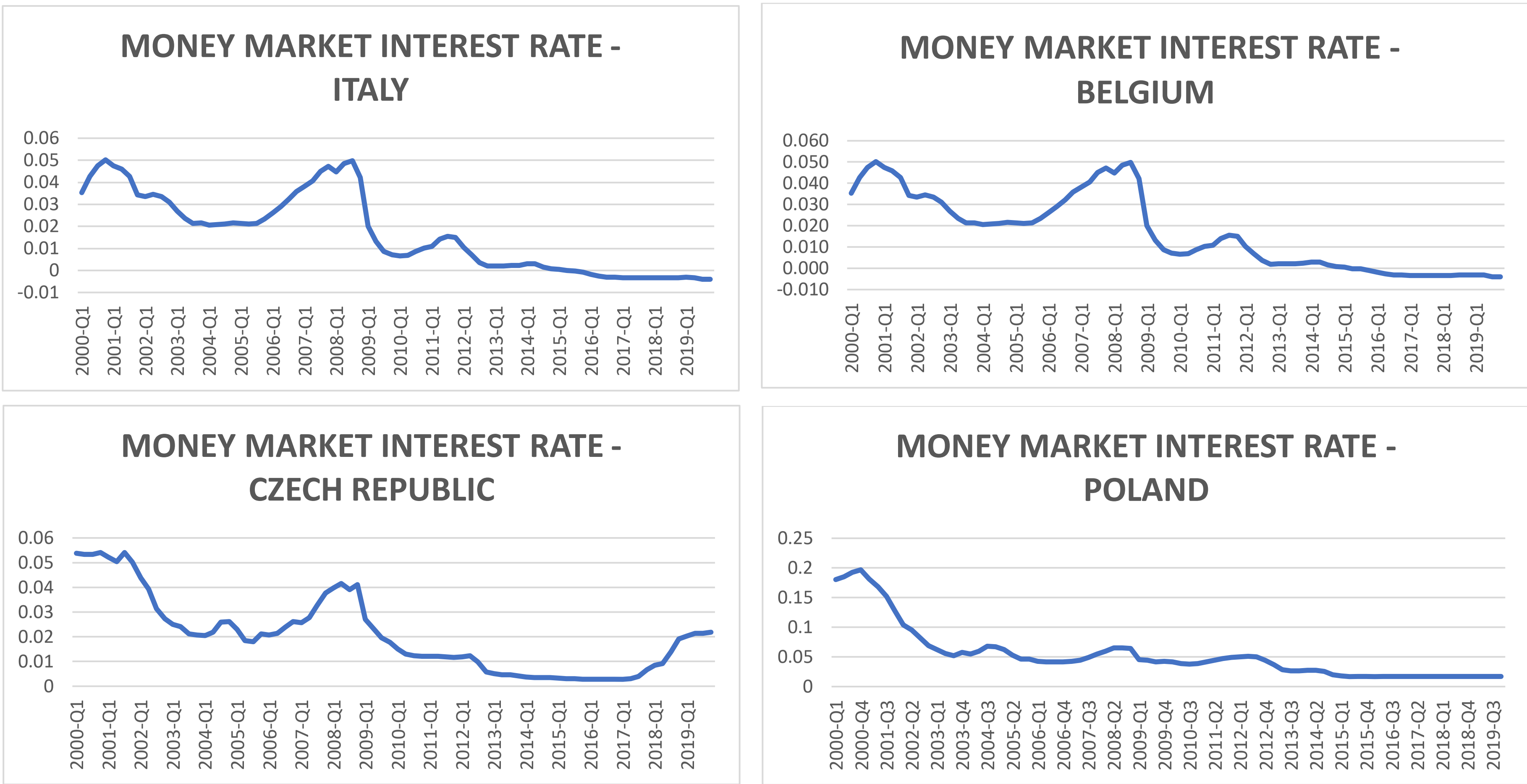


FIGURE 3.5. This figure displays the Money Market Rate, for a panel of four countries: Italy, Belgium, Czech Republic, and Poland. The sample period goes from 2000:1 to 2019:4.
Data are taken from the OECD database

CHAPTER 4: EMPIRICAL ANALYSIS AND RESULTS

In chapter 4 results regarding the implementation of the VAR model are analyzed. The software programming language used is MATLAB.

4.1 ESTIMATION OF THE VAR MODEL

We performed the model using the function *VARM*. We firstly decided to implement simultaneously a VAR(1), VAR(2), VAR(3) and VAR(4) processes, and then, to use the information criterions, in order to make the best lag choice. We have divided the data in pre-sample, estimation and forecast, to assess the quality of the data. Then we estimated the model, by using the function *ESTIMATE*, and the pre-sample period to provide lagged data[35]. The estimation period ends in 2019Q4, for a total of 76 observations, while data from 2020Q1 to 2022Q4 belongs to forecast period, and will be used to develop a forecast analysis, with the aim to understand the impact of COVID-19 crisis and Ukraine-Russia war on the economies under analysis.

From the estimation we obtain the following four output, per country. The (fully specified) Estimated Model (EstMLD)[36], the (asymptotic) standard errors of the estimated parameters (EstSE), the loglikelihoods of the fitted model (LogL) and the (multivariate) residuals of the model.

4.2 EMPIRICAL ANALYSIS

4.2.1 STABILITY AND STATIONARITY

To obtain reliable results, the first things to check are the stability and stationarity of the model. We recall that a VAR(p) process is *stable* if all eigenvalues of the matrix of the autoregressive coefficients have modulus less that 1. While a stochastic process is *stationary* if its first and second moments are time invariant [Lüktepohl (2005)]. Moreover, stability imply stationarity. The software describes the estimated model as *"AR_Stationary"*. Such output, indicates that the four autoregressive processes, per country, are stationary. To check stability, we firstly created a lag operator polynomial object using the autoregressive coefficients and then used the function to *isStable*. For each of the four countries, the function returns a Boolean value of 1, indicating that the VAR models are stable.

[35] The pre-sample period is composed by the first four periods. The same sample is used for all the four VAR estimation computed, to have all models fitted to the same data.

[36] The function estimate returns an estimated model using the Maximum Likelihood as estimation technique.

4.2.2 RESIDUALS

To obtain a well-performing model, another assumption is made, regarding the distribution of the process. The generating process is assumed to be Gaussian Normal distributed. To verify this assumption, we need to check if the residuals are normally distributed, $u_t \sim N(0, \Sigma_u)$, for all t, with zero mean and constant variance Σ_u (u_t and u_s are independent for $s \neq t$). To do so, we plotted the histogram of each series (Figure 4.1) and performed the Lilliefors test. In the case of Italy and Belgium, the test rejects the hypothesis of normal distribution of errors for CLIFS and Money Market Interest Rate, at 5% confidence level. For Czech Republic is rejected for the inflation rate also and for Poland for the Output Growth also[37]. This evidence suggest that the estimated model doesn't respect the normality assumption. Despite these results, in our opinion is not necessary to modify the model, since extreme values are caused by the presence of two relevant crises in the sample period considered and their impact is increased due the sample shortness[38]. Moreover, a non-gaussian distribution is not a great problem in our context since the ML estimator keeps its asymptotic properties even if the gaussian assumption is relaxed.

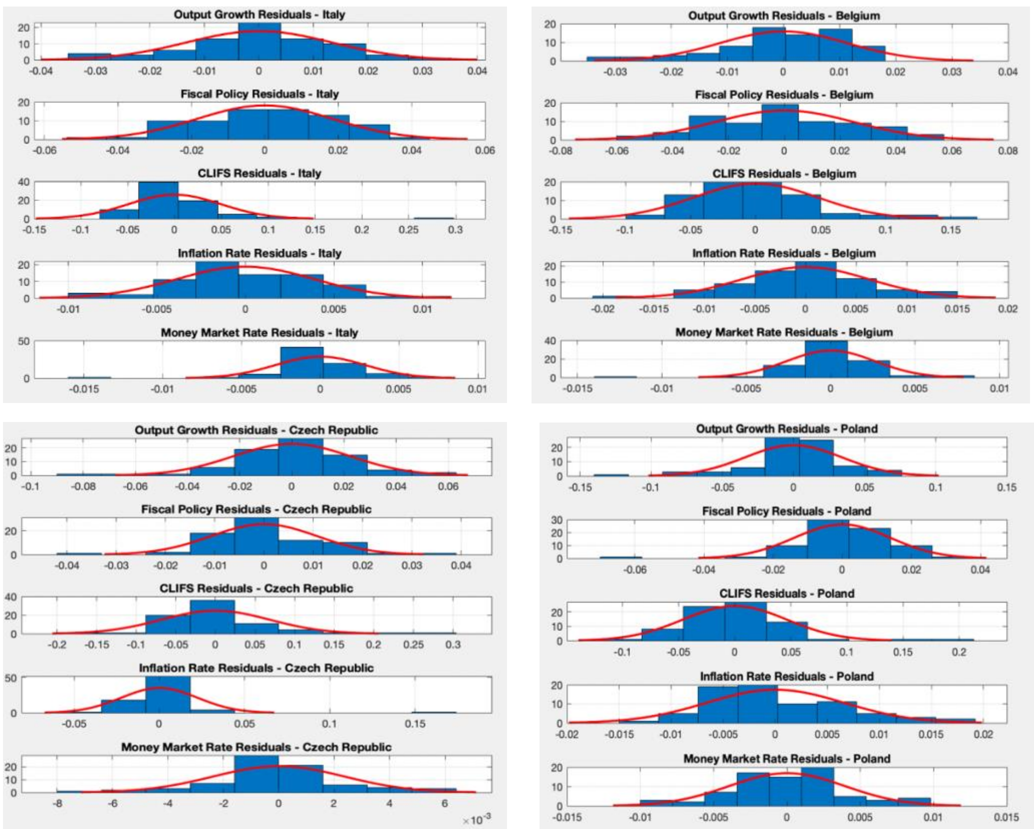


FIGURE 4.1. This figure represents the distribution of the residuals for Italy, Belgium, Czech Republic, and Poland.

[37] The histogram representations of the variables that do not respect the normality assumption suggest the presence of a non-normal kurtosis, particular leptokurtic.

[38] A larger sample including also pre-sample and post-sample crises would have performed better.

4.2.3 LAG CHOICE

Prior to the analysis of the empirical results, important is the lag choice. We implemented the three most famous information criteria: *(AIC), (BIC) and (HCQ)*, using the function *AICBIC*. The estimated results, per-country, are represented in the following table 1. Recalling that for all the three criterions, the estimate $\hat{p}(IC)$ of p is chosen to minimize $IC(m)$, following the AIC criterion, the best lag choice is 4, while following the BIC and HCQ the best is lag 1, for all countries[39]. To obtain the best trade of between model fit and parsimony, considering that the sample period is made up by more than 16 periods, and the BIC is a consistent criterion, we decided to follow the BIC and choose one as number of lags.

TABLE 1 – INFORMATION CRITERION

COUNTRY	LAG	AIC	BIC	HCQ
ITALY	1	-2,362	-2,29	-2,333
	2	-2,376	-2,245	-2,324
	3	-2,388	-2,197	-2,312
	4	-2,399	-2,149	-2,299
BELGIUM	1	-2,279	-2,208	-2,251
	2	-2,308	-2,177	-2,255
	3	-2,292	-2,101	-2,215
	4	-2,341	-2,091	-2,24
CZECH REPUBLIC	1	-2,062	-1,991	-2,034
	2	-2,064	-1,933	-2,011
	3	-2,06	-1,87	-1,984
	4	-2,038	-1,788	-1,938
POLAND	1	-2,192	-2,121	-2,163
	2	-2,203	-2,072	-2,151
	3	-2,191	-2	-2,114
	4	-2,171	-1,921	-2,071

4.3 EMPIRICAL RESULTS

4.3.1 CORRELATION AND AUTOCORRELATION

The first important results concern the direction and the degree of correlation and autocorrelation between variables and between a variable and its lagged values. The following table 2 provides the

[39] Only for Belgium the HCQ criterion indicate as best choice the lag 2.

estimated correlation results. The variables are correlated between each other, even though mostly wackily. There is a negative correlation between fiscal policy and output growth, and between CLIFS and output growth, stronger for the second pair than for the first, and very strong in the case of Poland, which is the only one that shows a strong correlation also between fiscal policy and output growth. Such results are consistent with our expectations. We expect that when output decline, in most of the cases is because there is a shock in the economy, so the CLIFS increase as consequence of it. Then the debt-to-GDP increase also because governments try to support the economy. Moreover, positive is the correlation between fiscal policy and CLIFS, as expected for what previously said. Only for Belgium such correlation is weakly negative, but close to zero. The other results are consistent with expectation too, both inflation and money market rate are positively correlated with output growth, and negatively with fiscal policy, since, as said in previous chapter, the higher is the money rate, the stronger is the signal that the economy is not performing well, and the more the economic productivity decline, the more the prices decline[40]. Close to zero is the correlation between inflation and CLIFS for all countries. A stronger negative correlation, instead, there is between CLIFS and money market rate, positive only for Poland. Positive is also the correlation between money market and inflation.
To summarize, the only variables who display a strong negative correlation are CLIFS and output growth, while a strong positive correlation there is between money market rate and inflation rate. Poland is the country in with the stronger degree of correlation.

TABLE 2 – CORRELATION

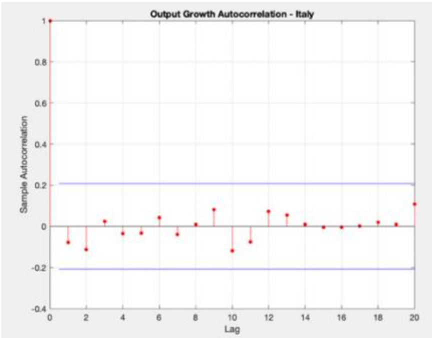
COUNTRY	CORRELATION	OUTPUT GROWTH	FISCAL POLICY	CLIFS	INFLATION RATE	MONEY MARKET RATE
ITALY	OUTPUT GROWTH	1	-0,1339	-0,3509	0,008	0,0173
	FISCAL POLICY	-0,1339	1	0,123	-0,225	-0,3537
	CLIFS	-0,3509	0,123	1	0,0056	-0,1147
	INFLATION RATE	0,008	-0,225	0,0056	1	0,4347
	MONEY MARKET RATE	0,0173	-0,3537	-0,1147	0,4347	1
BELGIUM	OUTPUT GROWTH	1	-0,1908	-0,2534	0,0137	0,0845
	FISCAL POLICY	-0,1908	1	0,1879	-0,1927	-0,3103
	CLIFS	-0,2534	0,1879	1	-0,013	-0,1928
	INFLATION RATE	0,0137	-0,1927	-0,013	1	0,3961

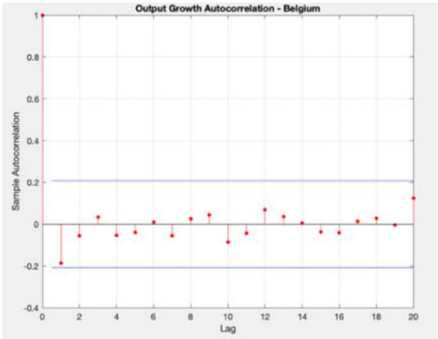
[40] However, for Poland and Czech Republic the correlations between money market and CLIFS, and between inflation and fiscal policy, respectively, are positive, even if the values are close to zero. Nevertheless, Czech Republic and Poland are also the countries for which such variables reacted less to the past crises.

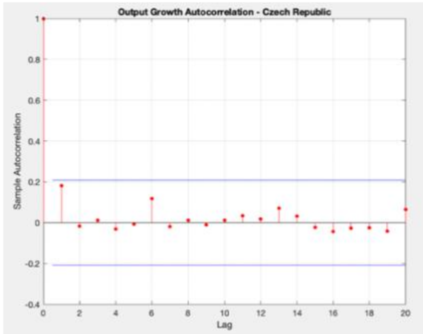
	MONEY MARKET RATE	0,0845	-0,3103	-0,1928	0,3961	1
CZECH REPUBLIC	**OUTPUT GROWTH**	1	-0,0537	-0,3717	0,0984	0,2247
	FISCAL POLICY	-0,0537	1	-0,0308	0,1666	-0,0161
	CLIFS	-0,3717	-0,0308	1	0,0124	-0,2456
	INFLATION RATE	0,0984	0,1666	0,0124	1	0,2729
	MONEY MARKET RATE	0,2247	-0,0161	-0,2456	0,2729	1
POLAND	**OUTPUT GROWTH**	1	-0,4522	-0,4427	0,3848	0,2795
	FISCAL POLICY	-0,4522	1	0,1046	-0,2912	-0,2101
	CLIFS	-0,4427	0,1046	1	-0,0848	0,1334
	INFLATION RATE	0,3848	-0,2912	-0,0848	1	0,6033
	MONEY MARKET RATE	0,2795	-0,2101	0,1334	0,6033	1

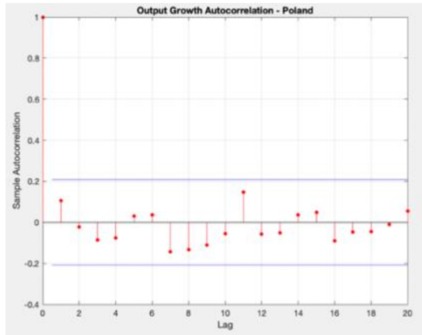
Since in a VAR process each variable depends by its lagged value, informative is the analysis of the Autocorrelation (ACF) and of the Partial Autocorrelation (PACF)[41] between the variables and their lagged values. As depicted in figures 4.2 and 4.3 the autocorrelation goes to zero as the number of lags increase and partial autocorrelation is (or become) negligible after one period, for all variables and countries[42]. Such results confirm the possibility to fit an AR(p) model and is a further proof of the fact that a VAR (p) model with lower number of lags, in our case 1, fits better the data.
Since the ACF is a more qualitative measure of autocorrelation, we performed also the Ljung-Box Q-test to obtain a more quantitative measure. The null hypothesis is that there is no residual autocorrelation, at 5% significance level, by default. For all variables, and countries, the test reject the null, suggesting that there is evidence of at least one significant autocorrelation in lags 1 through 20. The only exception is regarding the output growth, for which the null cannot be reject. Such results are coherent with the ACF and PACF functions. Low is the sample autocorrelation for output growth, except for lag 1, compared to the other variables (Figure 4.2).

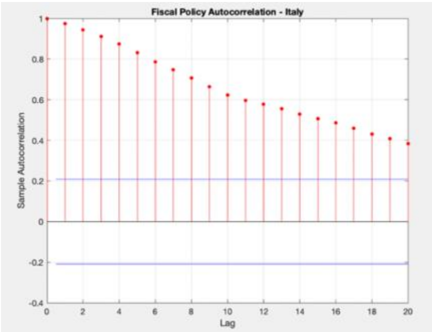
[41] The PACF displays the autocorrelation results free of the linear dependence on other variables.
[42] The output growth's PACF there is high correlation also for lag 12.

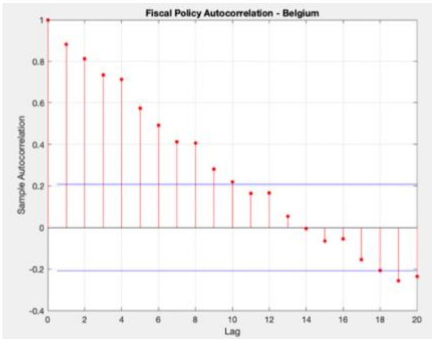

Output Growth Autocorrelation - Italy
Sample Autocorrelation
Lag

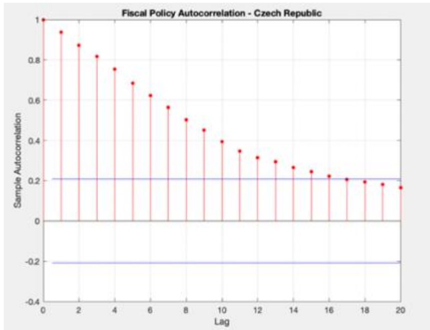

Output Growth Autocorrelation - Belgium
Sample Autocorrelation
Lag

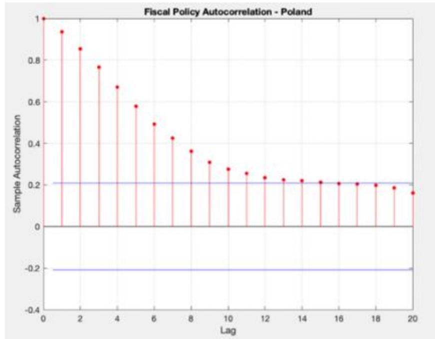

Output Growth Autocorrelation - Czech Republic
Sample Autocorrelation
Lag

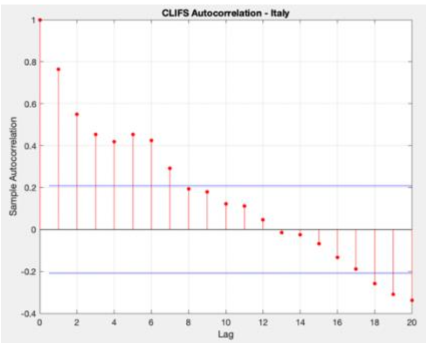

Output Growth Autocorrelation - Poland
Sample Autocorrelation
Lag

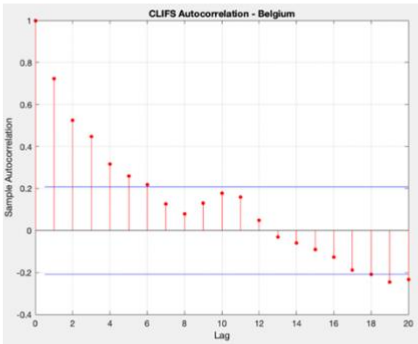

Fiscal Policy Autocorrelation - Italy
Sample Autocorrelation
Lag

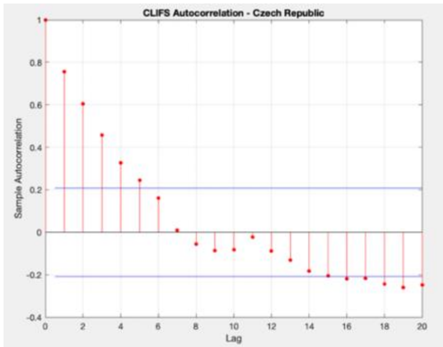

Fiscal Policy Autocorrelation - Belgium
Sample Autocorrelation
Lag

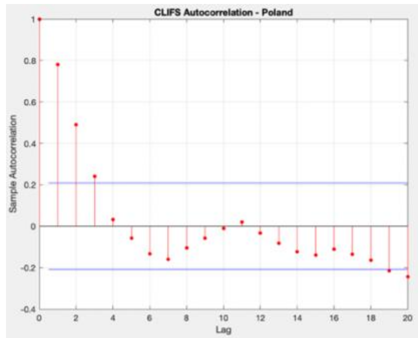

Fiscal Policy Autocorrelation - Czech Republic
Sample Autocorrelation
Lag


Fiscal Policy Autocorrelation - Poland
Sample Autocorrelation
Lag


CLIFS Autocorrelation - Italy
Sample Autocorrelation
Lag


CLIFS Autocorrelation - Belgium
Sample Autocorrelation
Lag


CLIFS Autocorrelation - Czech Republic
Sample Autocorrelation
Lag


CLIFS Autocorrelation - Poland
Sample Autocorrelation
Lag

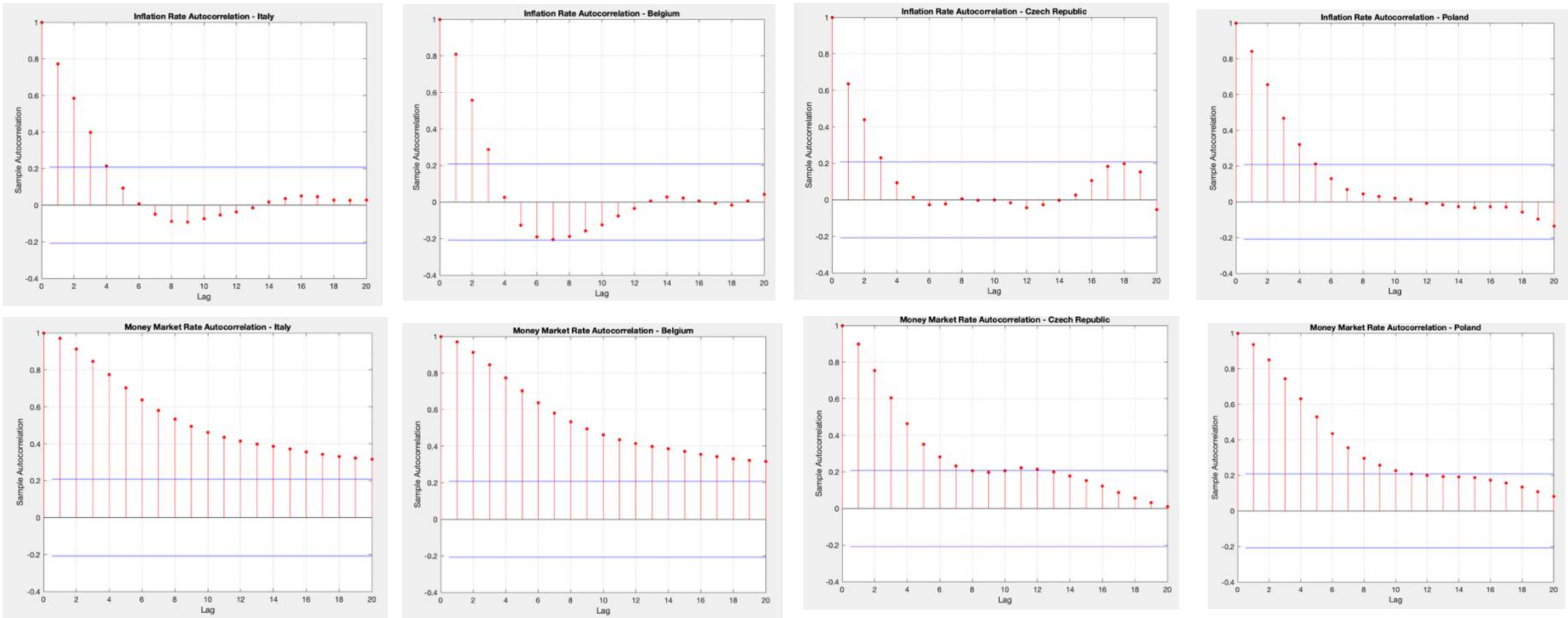
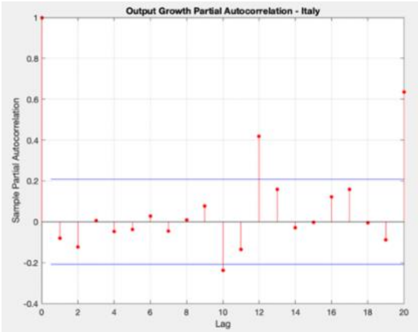
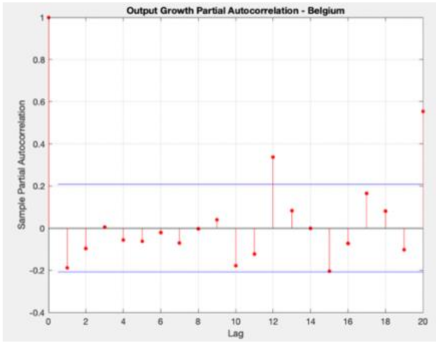
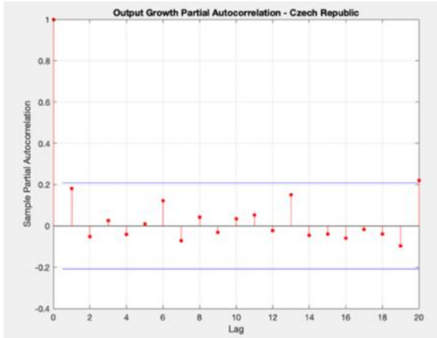
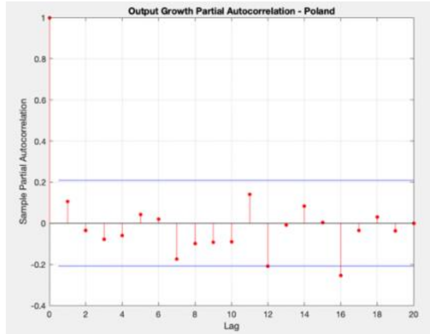


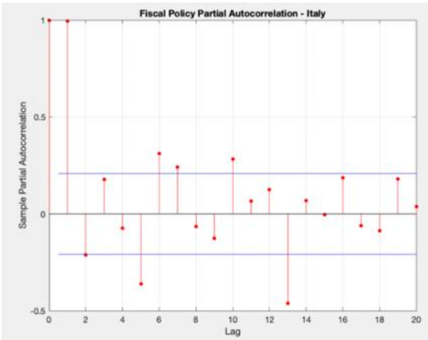
FIGURE 4.2. This figure represents the ACF, for Italy, Belgium, Czech Republic, and Poland. Values go up to 20 lags.

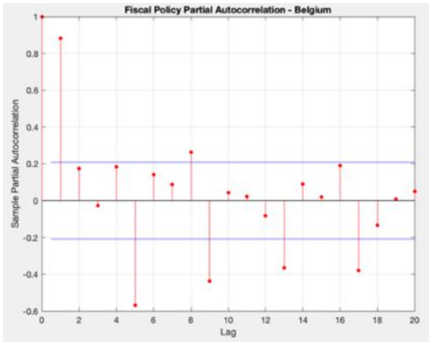

Output Growth Partial Autocorrelation - Italy
Sample Partial Autocorrelation
Lag

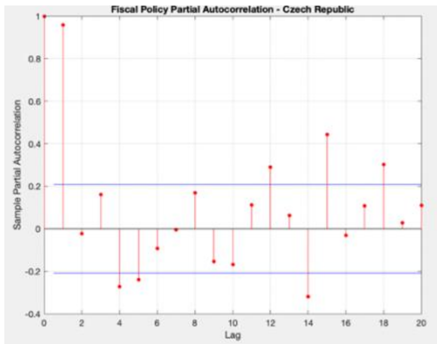

Output Growth Partial Autocorrelation - Belgium
Sample Partial Autocorrelation
Lag

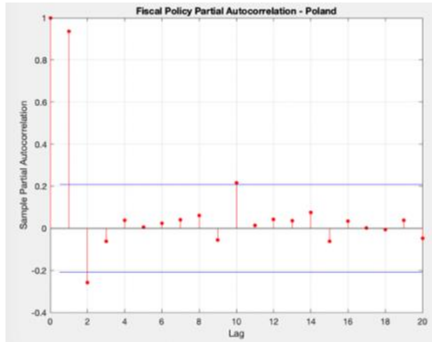

Output Growth Partial Autocorrelation - Czech Republic
Sample Partial Autocorrelation
Lag

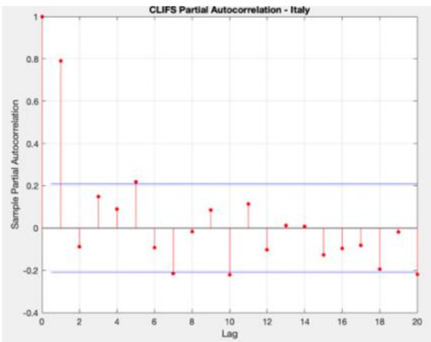

Output Growth Partial Autocorrelation - Poland
Sample Partial Autocorrelation
Lag

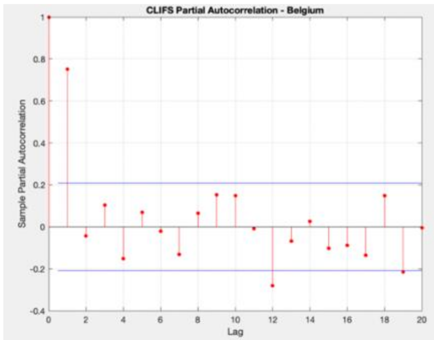

Fiscal Policy Partial Autocorrelation - Italy
Sample Partial Autocorrelation
Lag

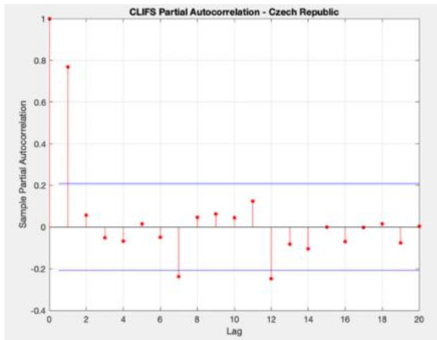

Fiscal Policy Partial Autocorrelation - Belgium
Sample Partial Autocorrelation
Lag

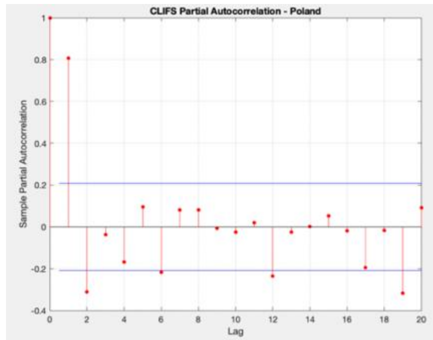

Fiscal Policy Partial Autocorrelation - Czech Republic
Sample Partial Autocorrelation
Lag


Fiscal Policy Partial Autocorrelation - Poland
Sample Partial Autocorrelation
Lag


CLIFS Partial Autocorrelation - Italy
Sample Partial Autocorrelation
Lag


CLIFS Partial Autocorrelation - Belgium
Sample Partial Autocorrelation
Lag


CLIFS Partial Autocorrelation - Czech Republic
Sample Partial Autocorrelation
Lag


CLIFS Partial Autocorrelation - Poland
Sample Partial Autocorrelation
Lag

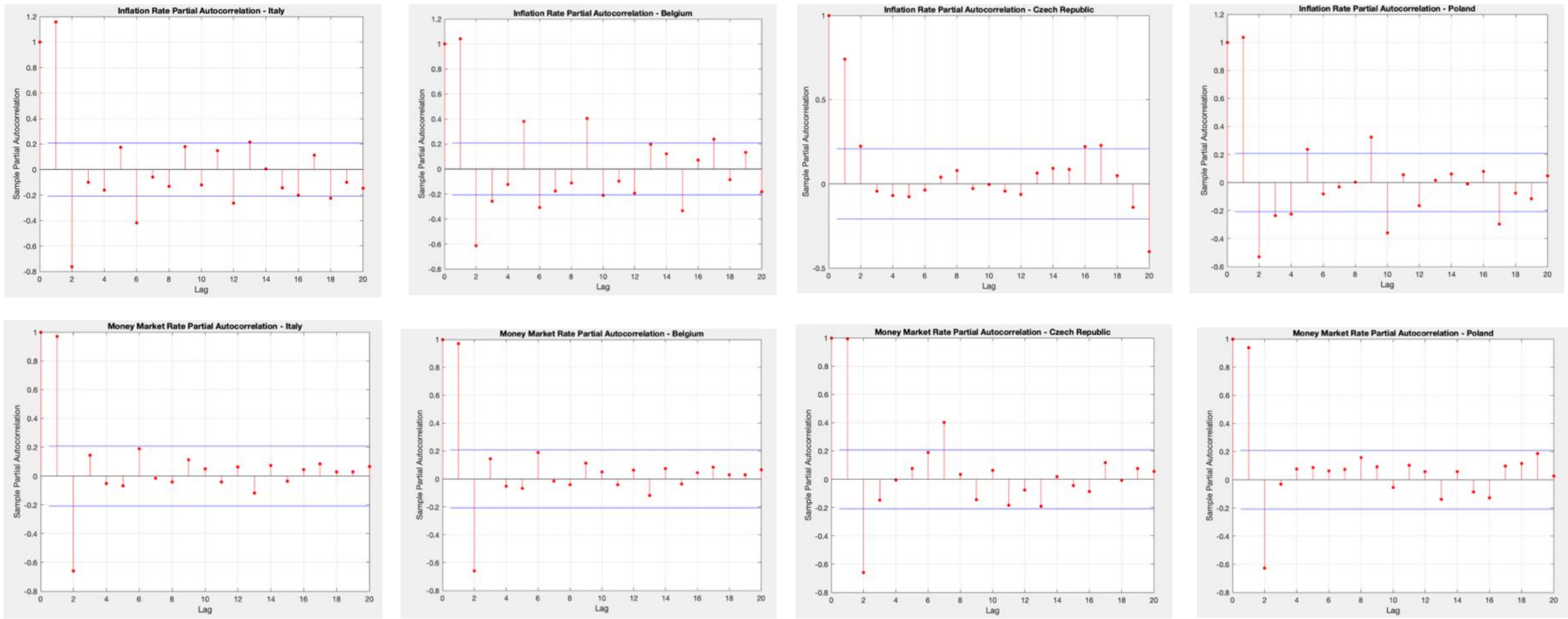


FIGURE 4.3. This figure represents the PACF, for Italy, Belgium, Czech Republic, and Poland. Values go up to 20 lags.

4.3.2 AUTOREGRESSIVE COEFFICIENTS

The VAR (1) model estimated is composed by 5 equations of 6 coefficients each (also considering the intercept), for a total of 30 estimated parameters, per country. The estimated coefficients (AR) are reported below[43]. Table 3 provides similar conclusions of Correlation and Autocorrelation. Remembering that the Autoregressive Coefficients determine the strength and the direction of the relationship between the current and the lagged values of the time series [Lüktepohl (2005)], we can observe that, for all countries, most of the actual value of each variable is determined by its lagged value, rather than by lagged values of the other components. Especially for fiscal policy, inflation, and money market rate. Regarding the mutual influence between the variables, negative is the effect of lagged value of CLIFS on output growth, meaning that positive value of CLIFS in previous period brings to a decrease in economic growth of future period. As well negative is the impact of lagged values of fiscal policy on economic growth[44]. On the contrary, almost insignificant is the impact of lagged value of output growth on fiscal policy and on CLIFS and of fiscal policy on CLIFS. On the contrary, interesting is the influence of CLIFS on fiscal policy for Belgium and Poland, that display a negative influence, while a positive one was expected[45].

TABLE 3 – AUTOREGRESSIVE COEFFICIENTS

COUNTRY	AR	OUTPUT GROWTH	FISCAL POLICY	CLIFS	INFLATION RATE	MONEY MARKET RATE
ITALY	OUTPUT GROWTH	0,3133	-0,011	-0,0086	-0,1495	0,0435
	FISCAL POLICY	-0,2072	0,9535	0,0503	0,1766	-0,361
	CLIFS	-1,4421	0,1133	0,6836	1,0377	0,9501
	INFLATION RATE	0,0583	-0,0084	-0,0022	0,8559	0,0022
	MONEY MARKET RATE	0,042	-0,0101	-0,0152	-0,0362	0,9057
BELGIUM	OUTPUT GROWTH	0,3133	0,0161	-0,0061	-0,131	0,101
	FISCAL POLICY	-0,1003	0,9009	0,0533	0,1602	-0,1851
	CLIFS	-0,7369	-0,0848	0,6699	0,7139	0,901

[43] Coefficients for the intercept are omitted and will be reported in the appendix (Table A.1, A.2, A.3, A.4), jointly to further details on the autoregressive coefficients.
[44] Close to zero only for Poland.
[45] Even though these values are close to zero.

	INFLATION RATE	0,1538	-0,01	-0,0043	0,8208	0,0116
	MONEY MARKET RATE	0,0739	-0,0105	-0,0194	-0,0125	0,9828
CZECH REPUBLIC	OUTPUT GROWTH	0,3934	-0,1773	-0,0085	-0,0706	-0,5537
	FISCAL POLICY	-0,1052	0,9661	0,03	-0,0045	0,0718
	CLIFS	-0,2259	0,6072	0,6237	0,2259	5,0015
	INFLATION RATE	0,1795	-0,0131	-0,0055	0,2807	0,2128
	MONEY MARKET RATE	0,0106	-0,0226	-0,0087	0,0138	0,8583
POLAND	OUTPUT GROWTH	0,1956	0,0441	-0,0791	0,2498	0,0324
	FISCAL POLICY	0,0105	0,9409	0,0341	-0,111	0,0193
	CLIFS	-0,1538	-0,1379	0,6144	1,047	0,0291
	INFLATION RATE	0,0203	-0,0037	0,0174	0,9026	-0,0823
	MONEY MARKET RATE	0,0355	-0,0062	-0,0089	0,0992	0,8452

4.3.3 GRANGER CAUSALITY

To obtain a quantitative measure of causality between variables we estimated the Granger Causality[46], by applying the function *GCTEST*. Such function tests the null hypothesis that one variable is not 1-step-Grange Cause of another variable, against the alternative for which the variable is 1-step-Grange Cause[47]. We recall that it is important to study this kind of causality since, if one variable cause the other, then the prediction of the latter is improved if also the first variable is included in the model. In table 4, results are provided. Czech Republic has the highest number of causalities, while Italy the lowest. Several variables are one-step-Granger-cause of each other's, creating a feedback loop[48]. Interesting are the results for Italy and Czech Republic, where the CLIFS is one step granger cause of fiscal policy, output growth cause CLIFS (Italy), and the pairs fiscal policy-output growth and CLIFS-fiscal policy are one step Granger-cause of each other (Czech Republic). Instead, for Belgium

[46] A total of 20 tests per country have been performed.
[47] We imposed a significance level of 10%.
[48] For Czech Republic respectively, economic growth and fiscal policy, fiscal policy and CLIFS, money Rate and CLIFS are granger cause of each other's. The same happen for Belgium between Money rate and CLIFS and for Poland between Inflation and Money rate.

and Poland, the three variables of interest are mostly caused, or are cause of the inflation rate and/or the money market rate. Granger-causality results are consistent with autoregressive coefficients results. Variables for which there is Granger-causality are also the variables for which the lagged values of one variable influence more the actual value of the other.

TABLE 4 – GRANGER CAUSALITY

H0 – ITALY	DECISON	DISTRIBUTION	STATISTIC	PVALUE	CRITICAL VALUE
Exclude lagged CLIFS in Fiscal Policy equation	Reject H0	Chi2(1)	3,1979	0,0737*	2,7055
Exclude lagged Output growth in CLIFS equation	Reject H0	Chi2(1)	9,6871	0,0019**	2,7055
Exclude lagged CLIFS in Money Market Rate equation	Reject H0	Chi2(1)	12,1336	0,0005***	2,7055

H0 – BELGIUM	DECISON	DISTRIBUTION	STATISTIC	PVALUE	CRITICAL VALUE
Exclude lagged Money Market Rate in CLIFS equation	Reject H0	Chi2(1)	4,7659	0,0290**	2,7055
Exclude lagged Output growth in Inflation Rate equation	Reject H0	Chi2(1)	5,5523	0,0185**	2,7055
Exclude lagged Output growth in Money Market Rate equation	Reject H0	Chi2(1)	7,3297	0,0068***	2,7055
Exclude lagged Fiscal Policy in Money Market Rate equation	Reject H0	Chi2(1)	3,1128	0,0776*	2,7055
Exclude lagged CLIFS in Money Market Rate equation	Reject H0	Chi2(1)	22,0154	0,0000***	2,7055

H0 – CZECH REPUBLIC	DECISON	DISTRIBUTION	STATISTIC	PVALUE	CRITICAL VALUE
Exclude lagged Fiscal Policy in Output growth equation	Reject H0	Chi2(1)	3,7441	0,0530*	2,7055
Exclude lagged Output growth in Fiscal Policy equation	Reject H0	Chi2(1)	3,3179	0,0685*	2,7055

Exclude lagged CLIFS in Fiscal Policy equation	Reject H0	Chi2(1)	5,3247	0,0210**	2,7055
Exclude lagged Fiscal Policy in CLIFS equation	Reject H0	Chi2(1)	4,7161	0,0299**	2,7055
Exclude lagged Money Market Rate in CLIFS equation	Reject H0	Chi2(1)	11,4688	0,0007***	2,7055
Exclude lagged Fiscal Policy in Money Market Rate equation	Reject H0	Chi2(1)	5,4899	0,0191**	2,7055
Exclude lagged CLIFS in Money Market Rate equation	Reject H0	Chi2(1)	9,1829	0,0024***	2,7055

H0 – POLAND	DECISON	DISTRIBUTION	STATISTIC	PVALUE	CRITICAL VALUE
Exclude lagged Inflation Rate in CLIFS equation	Reject H0	Chi2(1)	4,7502	0,0293**	2,7055
Exclude lagged Money Market Rate in Inflation Rate equation	Reject H0	Chi2(1)	3,3051	0,0691*	2,7055
Exclude lagged Output growth in Money Market Rate equation	Reject H0	Chi2(1)	4,6712	0,0307**	2,7055
Exclude lagged Inflation Rate in Money Market Rate equation	Reject H0	Chi2(1)	5,8989	0,0152**	2,7055

Table 4 summarize the pair of variables between which there is granger causality, for Italy, Belgium, Czech Republic, and Poland. Values go up to 20 lags.
***, **, * indicate significance level at 1%, 5%, 10% level, respectively.

4.3.4 IMPULSE RESPONSE FUNCTION AND FORECAST ERROR VARIANCE DECOMPOSITION

Interesting are the results of the Impulse Response Function and the Forecast Error Variance Decomposition, estimated using the functions *IRF* and *FEVD*, respectively. They give further information regarding the relationship between the economic variables in the linear multivariate time series. Recalling that, the IRF shows the response of current and future values of each variable in the system to a one-unit (exogenous) increase in the current value of VAR errors and that the FEVD is useful to understand the size of the effect of the shocks on each variable, they are useful tools for interpreting and estimate the relationship between economic variables in linear (and nonlinear) multivariate time series. In figure 4.4 the orthogonalized responses of each variable to unit shocks are represented[49]. The economic growth reacts significantly to shocks occurring to itself, firstly. A positive shock to output growth brings to an immediate high positive response of itself, even if such response dries out quickly. Lower in magnitude are the responses to shocks occurring to the other variables of the system. Just in the case of Czech Republic this variable reacts negatively to shocks on fiscal policy, result consistent with the Granger causality[50].

Different is the case of fiscal policy measure. It reacts positively to (positive) shocks on itself, but also on Inflation and on CLIFS, while negatively to shock on economic growth. To shocks on Money Market Rate, the debt-to-GDP ratio reacts slightly negative for Italy and Belgium and positively for Czech Republic and (in a lower magnitude) for Poland. Such results are stronger for Italy and Czech Republic, country in where the fiscal policy is granger-caused by these other variables.

Moving to the CLIFS, as well as for the other variables, it reacts immediately, strongly, positively to positive shocks on itself, even if, for Belgium and Czech Republic, such reaction first becomes negative, until it normalizes to zero, as long as the number of lags increase. On the contrary, negative is the response to shocks on economic growth. Lower in magnitude are the responses to the other variables' shocks.

Inflation response is positive and high for shocks occurred on itself[51], while slightly positively for shocks on economic growth and negatively for shocks on fiscal policy. Less relevant are the responses to shocks on the last two variables, except for Poland that reacts positively to shocks on CLIFS.

Higher in magnitude are the responses of money market rate, positive to shocks occurred on itself and on economic growth, strongly negative to shocks occurred on fiscal policy and CLIFS. Lastly, regarding shocks on Inflation, the response is positive for Poland, while mostly negative for the other countries.

Independently if, at the beginning, the responses to shocks are more or less strong, as long as number of lags increase, such responses go to zero, coherently with the choice of VAR(1) process. Moreover, IRF's results are consistent with results of the Granger Causality. The effect of shocks on a variable which is granger-cause of another, is stronger in magnitude, compared with the effect of shocks occurred on the other variables. Lastly, coherently with the autocorrelation results, each variable reacts mostly to shocks occurred on its lagged value(s).

The FEVD results further confirm the IRF's conclusion. It is depicted in figure 4.5. Almost all the output growth variation is explained by itself. Just a low portion of it is determined by the fiscal policy

[49] The function *ARMAIRF* has been used to plot the responses.
[50] Czechian fiscal policy is one-step-granger-cause of output growth, at 10% significance level.
[51]For Italy first become negative and then goes to zero.

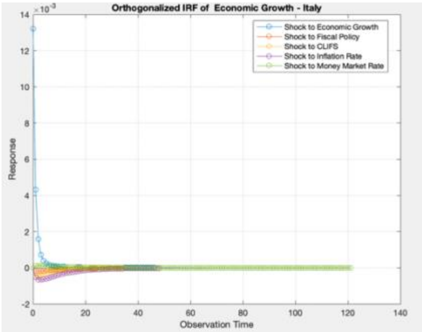
measure and by the inflation rate, especially for Czech Republic, where a minor contribution is given also by the money market rate.

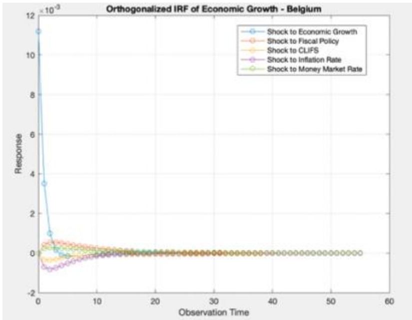
For the fiscal policy, most of the variation is explained by itself, as well. From the 5th quartile onward, increasing contribution is given by the output growth, especially for Poland. Similar pattern is described for Italy and Belgium, where more and more contribution is given also by the inflation rate and the CLIFS. Instead, for Czech Republic an increasing contribution is given by the money market rate.

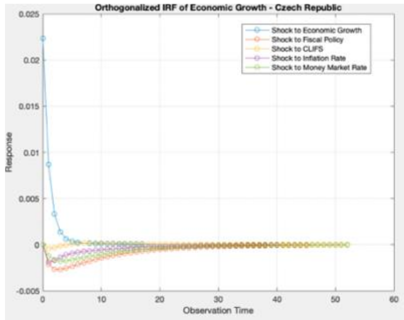
Similar to fiscal policy, is the CLIFS error variance, where, behind itself, a large contribution to its variation is given by the economic growth, especially in Italy, with an increasing contribution given by the inflation rate, also, starting by the 7th quartile. In Belgium increasing contribution is given by the fiscal policy, while for Czech Republic by the money market rate and for Poland by the inflation.

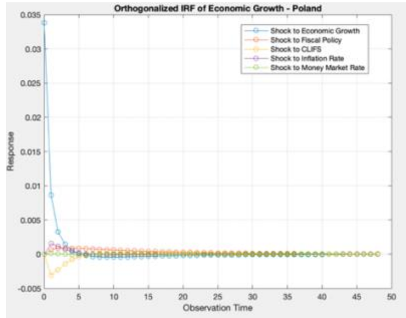
Instead, the decomposition of the inflation rate error variance for Czech Republic and Poland, is more similar the one of the output growth, the variable itself explains almost all of its variation. On the contrary, for Italy more and more contribution is given by fiscal policy and economic growth. Great influence from the latter variable is given also in the case of Belgium.

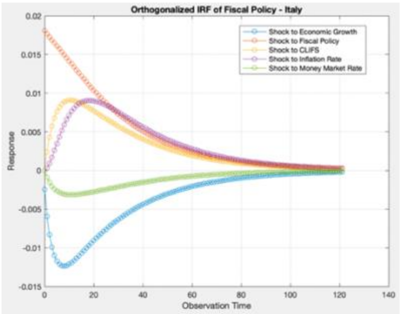
Interesting is the money market rate error variance. In all countries, from the 2nd quartile onwards, decreasing contribution is given by the variable itself, replaced, by the other four variables, in almost equal proportions, for Italy, mostly by CLIFS and economic growth in Belgium. For Czech Republic, instead, from the 2nd to the 8th quartile, increasing contribution is given by the economic growth, then replaced by the fiscal policy. Lastly, for Poland more and more contribution is given by the economic growth and by the inflation rate. Coherently with the IFR outcomes, the money market rate is the variable that react the most to shocks that occur on the other variables.

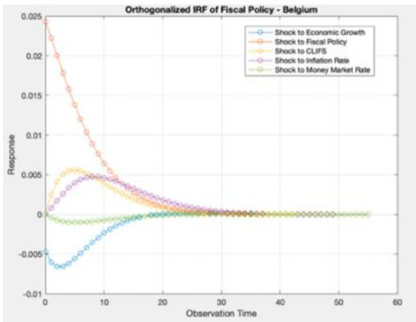

Orthogonalized IRF of Economic Growth - Italy
Shock to Economic Growth
Shock to Fiscal Policy
Shock to CLIFS
Shock to Inflation Rate
Shock to Money Market Rate
Response
Observation Time

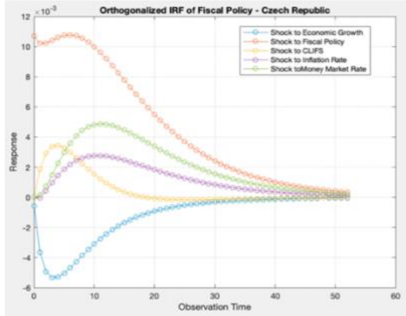

Orthogonalized IRF of Economic Growth - Belgium
Shock to Economic Growth
Shock to Fiscal Policy
Shock to CLIFS
Shock to Inflation Rate
Shock to Money Market Rate
Response
Observation Time

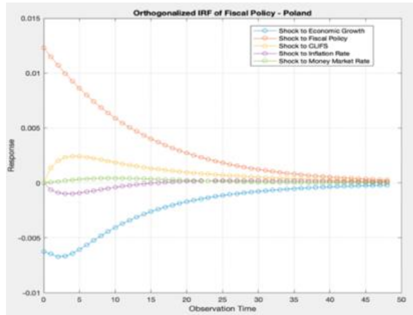

Orthogonalized IRF of Economic Growth - Czech Republic
Shock to Economic Growth
Shock to Fiscal Policy
Shock to CLIFS
Shock to Inflation Rate
Shock to Money Market Rate
Response
Observation Time

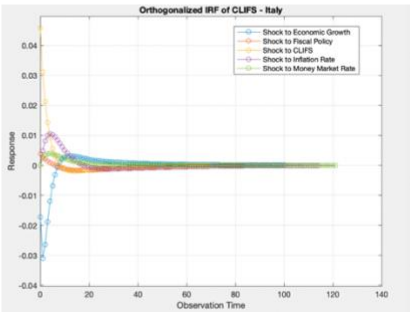

Orthogonalized IRF of Economic Growth - Poland
Shock to Economic Growth
Shock to Fiscal Policy
Shock to CLIFS
Shock to Inflation Rate
Shock to Money Market Rate
Response
Observation Time

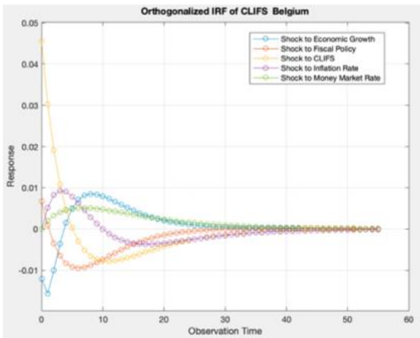

Orthogonalized IRF of Fiscal Policy - Italy
Shock to Economic Growth
Shock to Fiscal Policy
Shock to CLIFS
Shock to Inflation Rate
Shock to Money Market Rate
Response
Observation Time

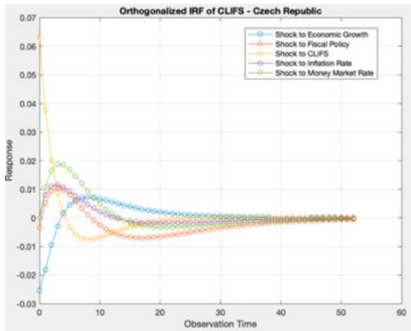

Orthogonalized IRF of Fiscal Policy - Belgium
Shock to Economic Growth
Shock to Fiscal Policy
Shock to CLIFS
Shock to Inflation Rate
Shock to Money Market Rate
Response
Observation Time

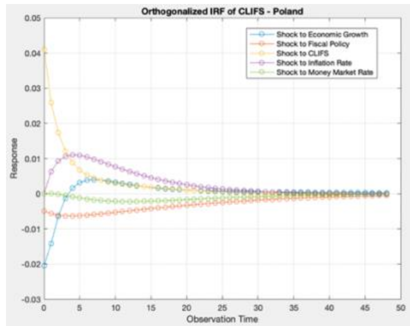

Orthogonalized IRF of Fiscal Policy - Czech Republic
Shock to Economic Growth
Shock to Fiscal Policy
Shock to CLIFS
Shock to Inflation Rate
Shock toMoney Market Rate
Response
Observation Time


Orthogonalized IRF of Fiscal Policy - Poland
Shock to Economic Growth
Shock to Fiscal Policy
Shock to CLIFS
Shock to Inflation Rate
Shock to Money Market Rate
Response
Observation Time


Orthogonalized IRF of CLIFS - Italy
Shock to Economic Growth
Shock to Fiscal Policy
Shock to CLIFS
Shock to Inflation Rate
Shock to Money Market Rate
Response
Observation Time


Orthogonalized IRF of CLIFS Belgium
Shock to Economic Growth
Shock to Fiscal Policy
Shock to CLIFS
Shock to Inflation Rate
Shock to Money Market Rate
Response
Observation Time


Orthogonalized IRF of CLIFS - Czech Republic
Shock to Economic Growth
Shock to Fiscal Policy
Shock to CLIFS
Shock to Inflation Rate
Shock to Money Market Rate
Response
Observation Time


Orthogonalized IRF of CLIFS - Poland
Shock to Economic Growth
Shock to Fiscal Policy
Shock to CLIFS
Shock to Inflation Rate
Shock to Money Market Rate
Response
Observation Time

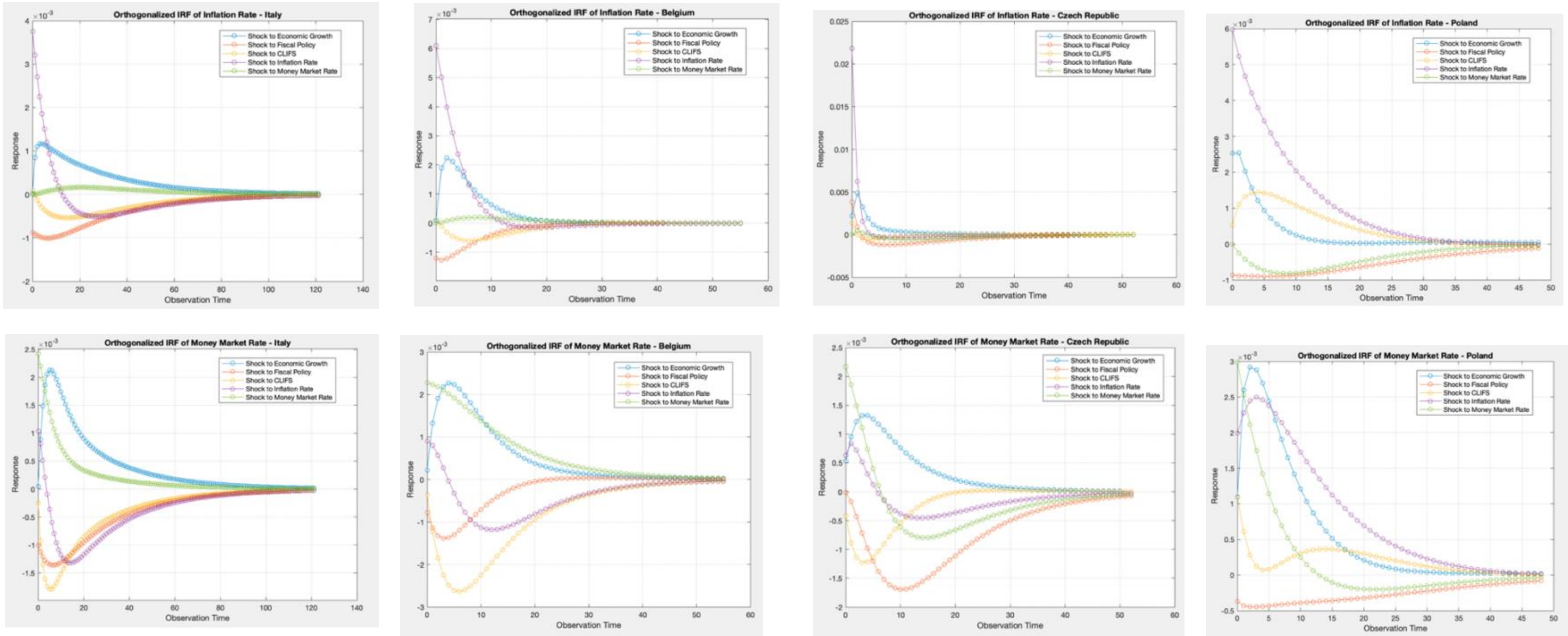
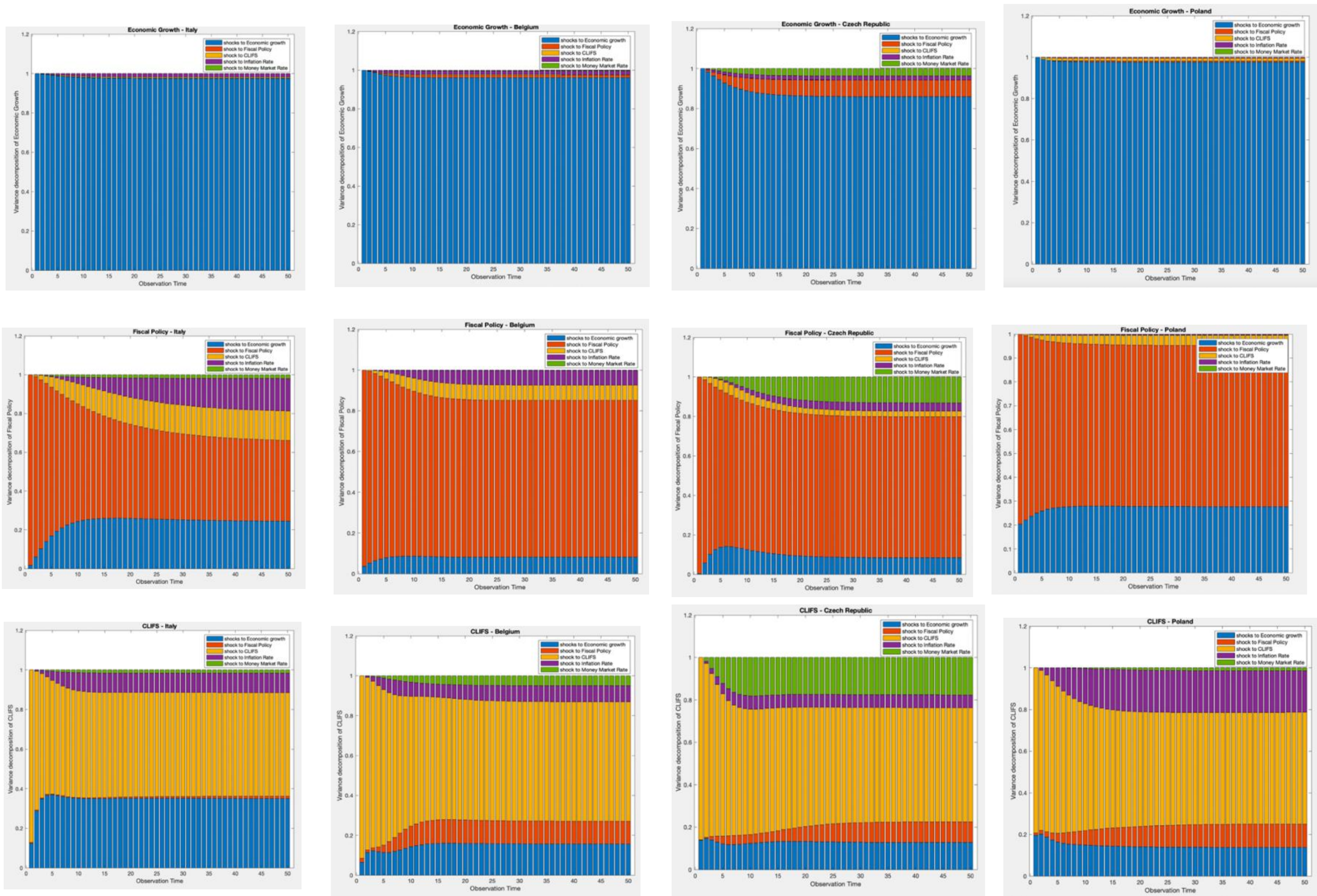


FIGURE 4.4. This figure represents the orthogonalized IRF of each variable, for Italy, Belgium, Czech Republic, and Poland.


Economic Growth - Italy
Economic Growth - Belgium
Economic Growth - Czech Republic
Economic Growth - Poland
Variance decomposition of Economic Growth
Fiscal Policy - Italy
Fiscal Policy - Belgium
Fiscal Policy - Czech Republic
Fiscal Policy - Poland
Variance decomposition of Fiscal Policy
CLIFS - Italy
CLIFS - Belgium
CLIFS - Czech Republic
CLIFS - Poland
Variance decomposition of CLIFS
Observation Time
shocks to Economic growth
shock to Fiscal Policy
shock to CLIFS
shock to Inflation Rate
shock to Money Market Rate

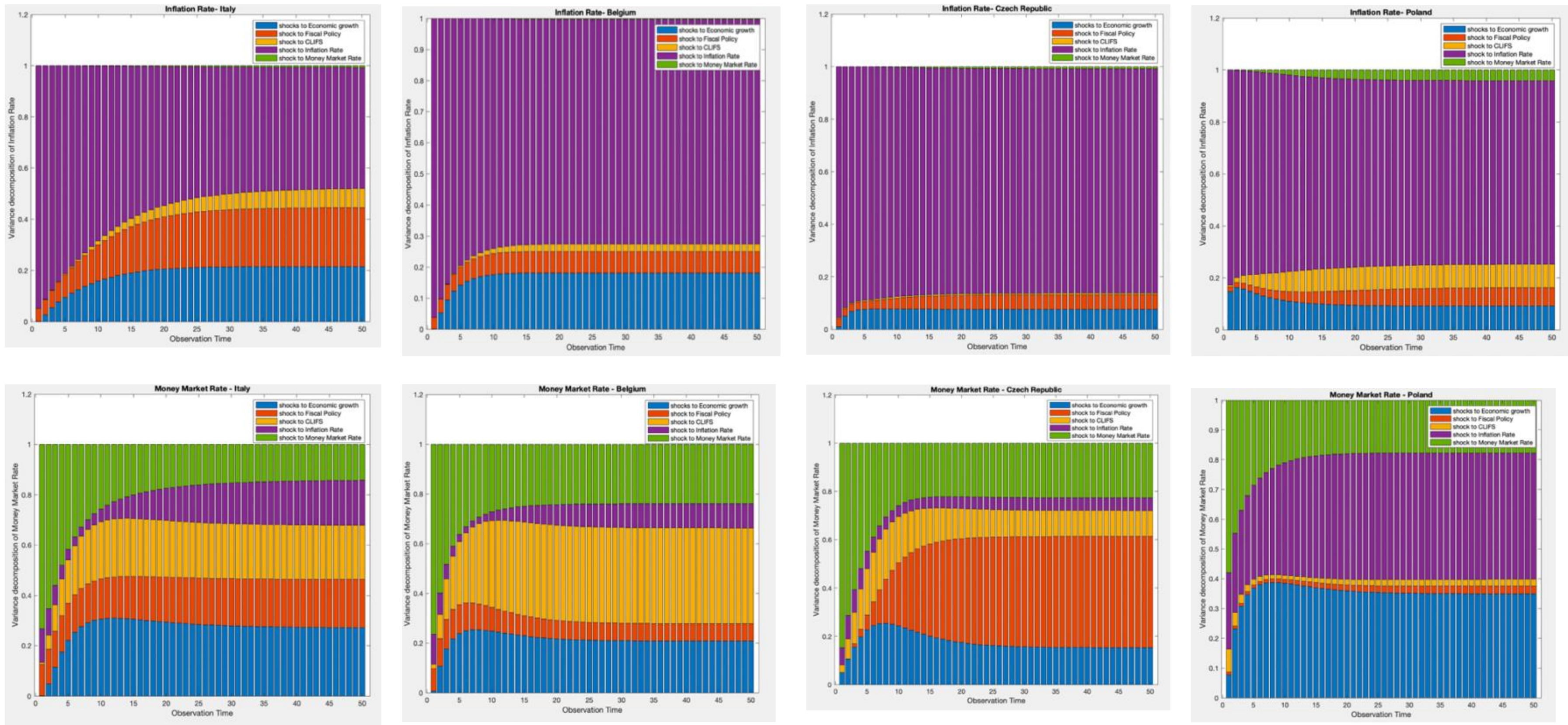


FIGURE 4.5. This figure represents the FEVD of each variable, for Italy, Belgium, Czech Republic, and Poland.

4.4 FORECAST ANALYSIS AND THE IMPACT OF COVID-19 AND UKRAINE-RUSSIA WAR ON THE ECONOMIES

In this section results regarding the forecast analysis are discussed. The forecast analysis has been performed, using the function *FORECAST*[52]. The aim of this section is to understand the impact of COVID-19 pandemic and Ukraine-Russia war on the economies and, consequentially, give an estimation of how the economies would have performed without these unexpected shocks. The forecast period goes from 2020:1 to 2022:4 and the information set used is the estimation period. In Figure 4.6 the forecast estimations are represented and compared with real data. Also, the forecast minimum MSE (mean squared errors) is computed and used to construct the 95% confidence interval[53].

The economic growth displays a similar patten for the four countries. Interesting is the difference between the predicted and the observed values from 2020Q1 to 2021Q4. The countries experienced their lowest and highest economic growth, in 2020Q2 and 2020Q3, respectively, reaching values of approximately -10% and +16% for Italy and Belgium, -12% and +12% for Czech Republic and -11% and +10% for Poland. Such trend is a consequence of the so-called *lock down*[54], imposed by the mayor European governments to fight the spread of the pandemic. After this period the economic output fully recovered, even though the growth decelerated again, due the Ukraine-Russia War. Nowadays, among the countries belonging to the Monetary Union, Italy shows excessive levels of macroeconomic imbalances, as demonstrated by the weak economic growth and the high-level of debt-to-GDP ratio (EC, (2023) – Italy). Causes are the ongoing labor market and financial markets fragilities, and the volatile weak economic productivity. Similar is the trend for Belgium due supply constraints, labor shortages, high inflation, and uncertainty, strengthened by the war. (OECD, 2022). Belgian economy recovered completely due several government measures implemented to protect employment and businesses (EC, (2022) – Belgium). Furthermore, the introduction of the Recovery and Resilience Plans and of several public spending and investments undertaken by governments, the quantitative tightening imposed by the ECB, and the green transition, brought (and are still bringing) to an increase in the productivity, reason why now the GDP is forecasted to return to a more gradual

[52] In the appendix another forecast estimation is provided (Figure A.2). It is obtained using the function SIMULATE. Then the results of the two estimations are compared to understand if there are significant differences and, in case of positive answer, which one fits better the real data.
[53] We extracted the main diagonal elements form the matrices of forecast MSE per period, that gives the error between forecasted and real value, for each variable. Using these values, we computed the standard errors, and then the 95% confidence interval, for each response series. In Appendix, results of the tests regarding the normality of the errors are provided (Figure A.1).
[54] Overall, during that period, all economies experienced a decline in the economic activity, decline in exports, reduction of mobility and consumption and increase of uncertainty.

pace of growth, for both countries. The Czechian and Polish economic growths have slowed too, after the post pandemic recovery, due a severe gas shortage, the rise in energy prices, the weak domestic demand, the rising of uncertainty and the still tight labor and housing markets conditions, factors that are hampering the catch-up with the OECD average incomes. A rebound is expected only in 2024 (OECD, (2023) – Czech Republic; OECD, (2023) – Poland).

In contrast, predicted data move in a range between -5% and +5%, almost the half of real values. This evidence suggests that these crises have been unexpected and disturbing phenomena for the economy, that could not have been predicted by economists, or at least, not fully predicted. Moreover, the impact of COVID-19 on economic growth has been stronger that the impact of the GFC, another reason why the real economic outlook could not have been estimated correctly using pasts values. Instead, after the lock down period, when the economies started to come back to a normal pace of growth, forecasted values become more like the real ones.

The debt-to-GDP ratio displays an increasing trend, from the beginning of the sample period considered (2000)[55]. The values reached during the COVID-19 period, are the highest of the whole sample period considered. Italy, starting already from high debt levels at the end of 2019 (134%), it displays a fast-increasing trend until 2021Q1 reaching a value of 159%. It is followed by a decline, until at the end of 2022 (144%). The reduction has been driven mostly by the economic recovery post pandemic, the decreasing use of non-performing loans, and by several measures introduced by the government to increase the resilience of the financial sector. Instead, the ratio increased during the pandemic and is still high due the several measures implemented to face the economic and social consequences of COVID-19 crisis and the increase in energy prices (Ukraine-Russia War), that brought to an increase of the government's expenditures. Despite the decline, the ratio is still higher than the pre-crisis level and more than double of the TFEU threshold. Nowadays the Italian debt-to-GDP level is the highest among the economies under the Monetary Union. This evidence further confirm that the high level of public debt is the major vulnerability of the Italian economy (EC, (2023) – Italy). Same pattern of Italy is displayed by Belgium and Poland. Belgium, starting by a value of 95% at the end of 2019, reached its peak in 2021Q1 (116%), followed by a decline (105% end 2022). The ratio is higher than the one fixed by the TEFU, even if lower than the Italian one. The increase has been a consequence of the gaps in the financial fiscal framework, especially of the lack of government expenditure rules. (OECD, (2022) – Belgium). The government expenditures increased a lot during the pandemic period and most of them had not a temporary feature but are still ongoing, reason why the high level of debt in percentage of GDP is not expected to stabilize in the medium

[55] Except for Belgium, for which 2022 (reals) values are like the 2000s ones.

term (EC, (2022) – Belgium). In any case, positive are the expectations for both countries for the 2024. The ratio is forecasted to further decline by 2024, mostly thanks the investments of the Next Generation EU and the implementations of measures against the tax evasion (Italy). Poland, instead, switched from 46% in end 2019 to 59% in 2021Q1, with a decrease of 10 percentage points at the end of 2022. The ratio increased due the introduction of several pandemic-related packages. Nowadays, it is decreasing, but at a slower rate as it could, because the fiscal policy implemented (together with the monetary policy) is ensuring that the high inflation doesn't become entrenched, by (temporary) lowering the VAT taxes on energy and food (OECD, (2023) – Poland). In any case, the Polish ratio is below the 60% threshold. Different is the case of Czech Republic, which shows only an increasing trend, from 30% at the end of 2019 to 45% in 2022Q3[56]. As for Poland, such increase was caused by several (temporary) targeted measures introduced during the pandemic, reinforced because of the war and still, partially, ongoing. Differently from the other countries, the Czechian fiscal policy has been appropriate, it helped to mitigate the effect of the crises and didn't bring the ratio to levels as high as the ones of the other countries. The most relevant measures adopted have been the ones aimed to help households and firms to cope with the higher energy costs (OECD, (2023) – Czech Republic).

Lastly, for all countries, except Italy, the forecasted values show an increasing trend, and the upper side of the confidence interval displays values close to the real ones. Despite this, such increase does not mirror the real one in terms of magnitude. Even though for Belgium, Czech Republic, and Poland the last values of the forecast analysis are close to the observed ones. This is not the case for Italy, probably because, this country, never experienced levels of debt as high as the ones reached during the pandemic.

The CLIFS increase also, as expected. For each country two peaks are displayed, suggesting that, even if the crises have not occurred in the financial markets, they have still brought to financial stress consequences, like the increasing of non-performing loans.

For Italy, the first peak is in March 2020 (the outbreak of COVID-19). It switched from 3.5% to 16% in just three months, followed by a decrease and, after just one year, by a sudden growth that brought the index at 25% in December 2022 (during the Ukraine-Russia war). Belgium and Czech Republic display values higher than Italy, close to 23% and 22%, in March 2020, from only 6% in December 2019 and 29%, respectively, and values close 27% in December 2022. Their second increase started exactly in March 2022, just one month later the outbreak of the war. Poland has been the country with the stronger increase. In March 2020 and March 2022, it increased of almost 10 and 14 pp., with

[56] With a decline of just 1pp. in 2022Q4.

respect to the quarters before, reaching values of 16% and 23%, respectively. In September 2022 it reached its highest value (30%). Being the CLIFS a financial stress index, such pattern was expected. There are increasing values during crises periods, while decreasing values when the economies started to recover. However, the peaks reached are not as high as the one of the GFC and the Debt Sovereign crisis, because, while the previous crises transformed into financial crises, these ones did not. The monetary policy introduced by the central banks, prevented the crises to transform into financial crises, by firstly guaranteeing liquidity in the market, with the APP and then reducing it with the quantitative tightening, following the inflationary trend (Banca D'Italia, 2021). In other words, the health of the economic systems has strengthened after the GFC, and improvements in asset qualities have been performed, (EC, (2022) – Italy). As result, the banking systems is become more resilient, with levels of liquidity over the minimum required and the reduction of the share of NPLs, that have come back to pre-pandemic level quite fast (IMF, (2023) – Czech Republic; OECD, (2023) – Poland). As in the case of previous variables, the forecasted values don't match the observed ones. For Italy, are forecasted no variations among periods, while for the other countries, just a slight increase, not enough to mirror the observed values.

Inflation rate reached its highest values during these last years (from 2020 to 2022). Italy and Belgium switched from values close to the 0% in last quarter of 2019 to 12% and 11%, respectively, at the end of 2022, with a faster and faster increase started in the last quartile of 2021. Czech Republic and Poland reached their peaks in 2022Q3 with a value close to 18% and 17%, respectively, increase started in 2022Q1. For all countries, the pandemic brought to a decline of the inflation rate (especially for Czech Republic) due to a decrease in the demand, caused by the lock down. The post-pandemic recovery and the outbreak of the war brought to the opposite effect, the increase in the inflation rate, caused by both an increase in the demand (consumers started spending their saving) and by a decrease in the supply caused by both the supply bottlenecks (raise of input costs for firms) and by the increase in energy, oil, natural gas, and food prices, due the war. Today, such values, despite the decrease, due the reduction in energy and food prices, and the actions undertaken by the central banks (quantitative tightening), are still high and over the inflation targets, due the ongoing war and uncertainty about how tighter financing conditions will affect the economies (CNB, 2023). For Italy, the key drivers of the rise in inflation, have been the energy imports, while negligible has been the impact of the raising of inflation in the other EU countries (EC, (2023) – Italy). While for Italy, there is no risk related to *wage-price indexation*[57], for Belgium, instead, is increasing the *wage-price spiral risk*, a mechanism for which if prices increase, (nominal) wages increase too, but firms, to keep the same level of profits

[57] The wage level adjusts following the price pattern.

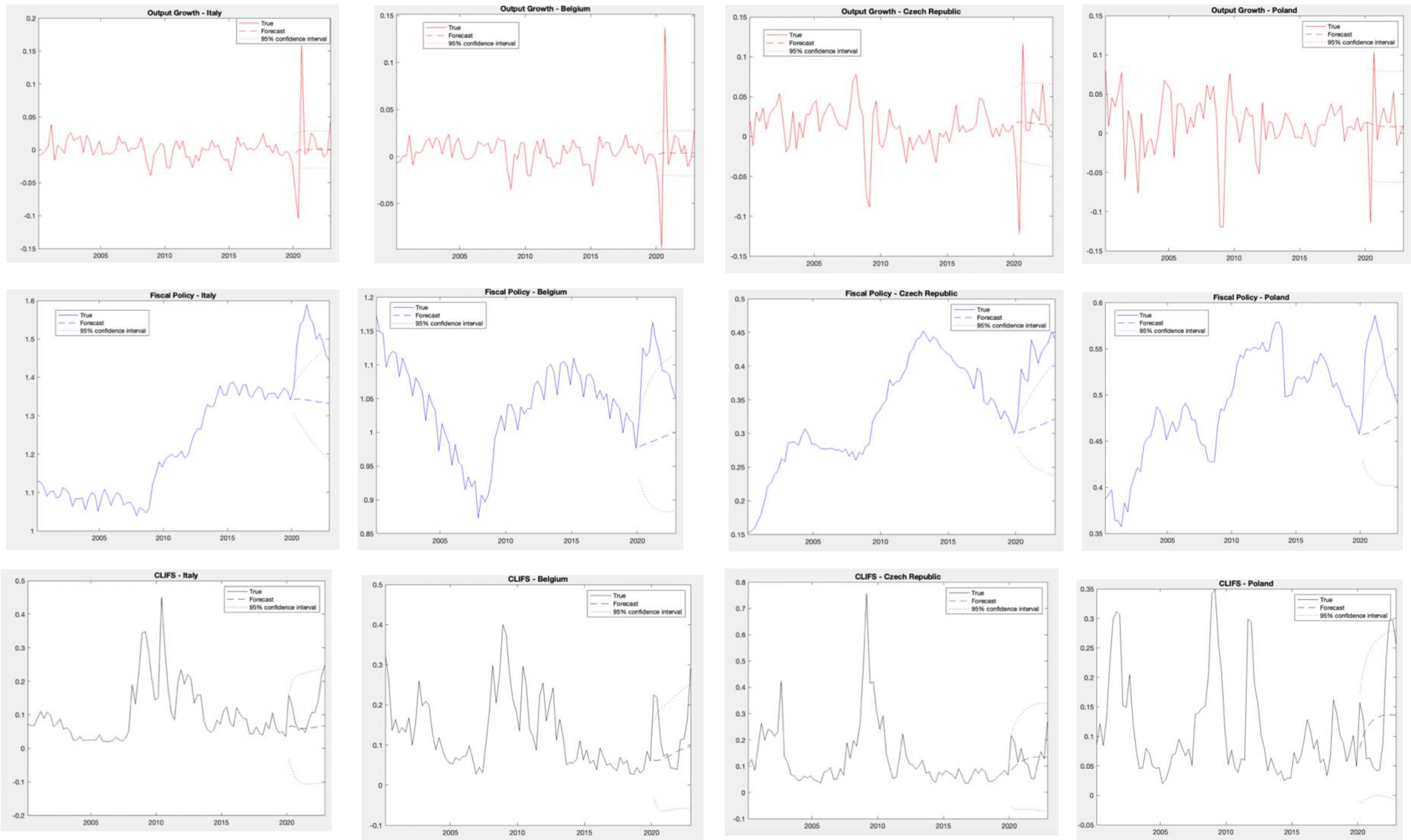
as before, increase again the prices. As consequence, wages increase more, bringing to a self-reinforcing mechanism, that could result in disruptive consequences for the economy (IMF, (2023) – Belgium)[58]. For Czech Republic a potential risk is the wage-price spiral, too. The drivers of the rising in inflation can be found both in domestic (increase of input costs) and in external factors (disruptions of GVCs and high commodity prices) (IMF, (2023) – Czech Republic). For Poland, the rise in inflation has been high also, reason why the polish APP was ended at the end of 2021, with contextual introduction of the quantitative tightening and rise of key policy interest rates (OECD, (2023) – Poland), earlier than the other central banks. Thanks all the action undertaken, the inflation rate has decreased, even tough is still high, higher than pre pandemic level.

Comparing the real and forecasted values, again the difference is significant. For Italy and Belgium, just a small increase to values close to the 2% (the ECB target) was expected. While for Czech Republic and Poland, inflation was expected to stay close to the same level of the end of 2019 (3%). There is a difference between estimated and observed value of more that 10 pp for each country.

The last variable to discuss is the money market rate. Interesting is its trend. It didn't increase a lot during the pandemic, compared to the GFC and the Debt Sovereign crisis. A strong increase is displayed just during the last quartiles of the sample period considered. Equals are the values for Italy and Belgium for which, the decline started in 2012 lasted until the last quarter of 2022, when it reached an abnormal high value of 1.77%. During COVID-19, the liquidity conditions have kept stable (EC, (2022) – Italy), due the implementation of the quantitative easing and due several credit measures adopted (Banca D'Italia, 2021). Similar is the trend for Czech Republic and Poland. They reached a value higher than 7% in 2022Q4, with an increase started at the beginning of the year. The Czechian liquidity system has been quite stable, due several liquidity provisions. The increase in the last period was caused by the war, that brought to negative consequences for liquidity, credit, and market risk. Stable has been also the Polish system, due high levels of banking capital resources and the low direct exposure of financial sector to Ukraine and Russia conflict (OECD, (2023) – Poland). For all countries, the fast increase in the last period, was caused by the quantitative tightening measures introduced, and the increase of the key interest rates. Such measures have had a double effect, a positive one, the reduction of the inflation rate, and a negative one, the reduction of the liquidity of the markets.

[58] Today, the government and the firms are bearing this cost of indexed wages, since this indexation protect consumer by the risk given by the increase in price but brings to additional costs for firms, they must pay higher salaries, and to an opportunity cost for government since it absorbs revenues that could be used for other priority areas of government spendings.

Regarding the forecasts, for all countries, before the abnormal increase in the last quarters, the estimated values are slightly higher than the reals, differently from the other variables. However, also in this case, the forecasted are different from the observed values.
To conclude, for all variables and countries, the estimated values are significantly different from the real ones, due the fact that the recent (ongoing) crises were unexpected and have been disruptive phenomena for the economies.


Output Growth - Italy
Output Growth - Belgium
Output Growth - Czech Republic
Output Growth - Poland
Fiscal Policy - Italy
Fiscal Policy - Belgium
Fiscal Policy - Czech Republic
Fiscal Policy - Poland
CLIFS - Italy
CLIFS - Belgium
CLIFS - Czech Republic
CLIFS - Poland
True
Forecast
95% confidence interval

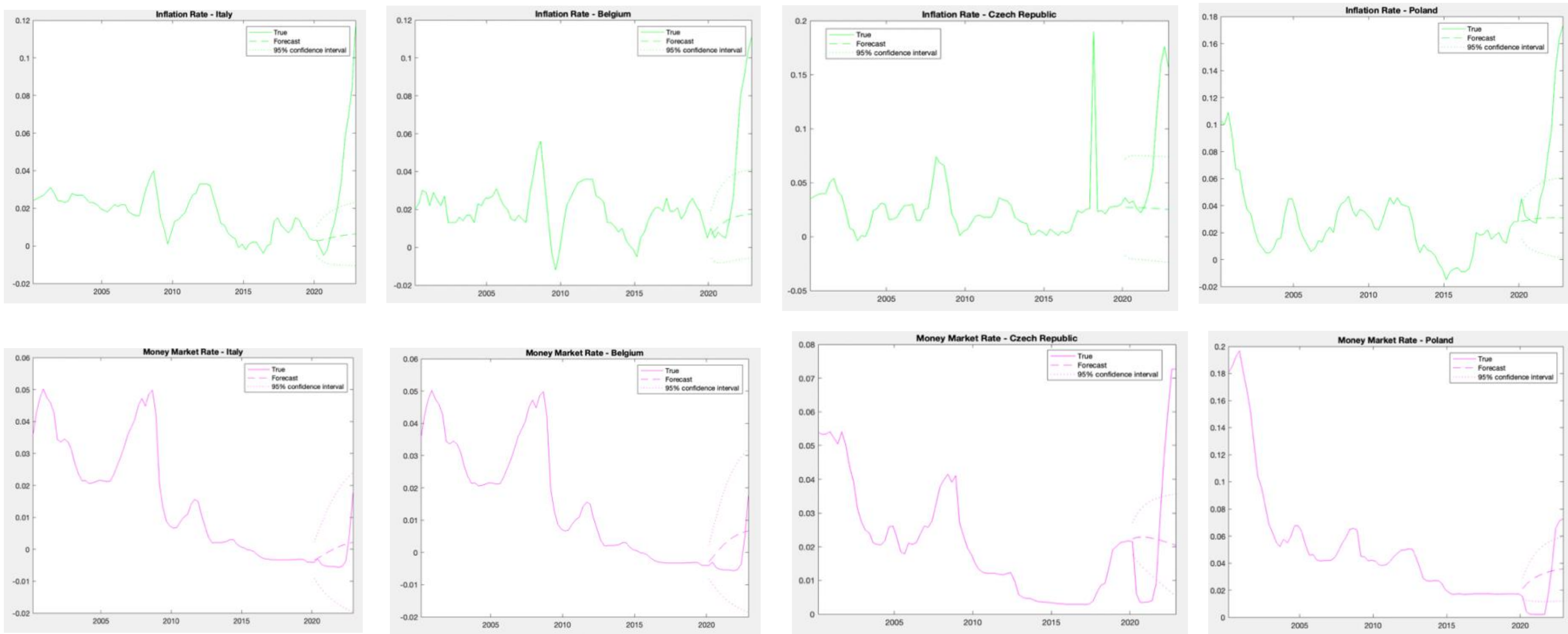


FIGURE 4.6. This figure represents the Forecast Estimation of each variable, for Italy, Belgium, Czech Republic, and Poland. Forecasted values goes from 2020:1 to 2022:4. Real data are plotted from 2000:1 to 2022:4.

CONCLUSION

This paper analyzes the interactions between output growth, fiscal policy, and financial stress episodes, using inflation rate and money market rate as control variables, in a framework of VAR process. Data goes from 2000:1 to 2022:4, for a panel of four countries: Italy, Belgium, Czech Republic, and Poland. The aim of the analysis is to understand the linkages between the variables and between the variables and their lagged values. The paper focus on periods of economic and/or financial crises, useful to understand the economic responses to shocks on financial and real markets and the actions undertaken by the government to mitigate the negative consequences. A great effort is put in the analysis of the two most recent crises, the COVID-19, and the Ukraine-Russia war, thought a forecast analysis.

Firstly, in chapter 1, we provided a brief overview of the developed literature, which highlights a three-sided relationship among the three key variables, relationship we tested in the empirical chapter. In chapter 2 is provided a theoretical explanation of the VAR model implemented with focus on the forecasting procedure used, the structural analysis implemented, and the estimation technique chosen (the Maximum Likelihood Estimation Method).

The two empirical chapters are chapter 3 and chapter 4. In chapter 3 a brief analysis of the variables involved in the process is provided. Focus is given to the variables' trend during crises period, and to the economic explanations behind the extreme values they reached.

Core or the analysis is the chapter 4, concerning the estimation of the VAR process, using MATLAB as software programming language. After checking for the assumption of the model, finding evidence of stationarity and stability, an choosing a VAR (1) process, using the BIC criterion, we moved to the analysis of the first results. We founded evidence of correlation and autocorrelation between the variables and between the variables and their lagged values (with the only exception of the output growth). We did not reject the initial hypothesis that during crisis periods, the output growth declines, while the CLIFS and the debt-to-GDP ratio increase. Instead, financial stress episodes lead to negative consequences for output growth and for fiscal policy, decreasing the first, and increasing the level of government debt as percentage of the GDP. A decrease in output growth brings to an increase in both government debt and CLIFS. On the contrary, increases in fiscal policy mitigate the negative effects on economic growth, by replacing the consumers and firms' expenditures with the government ones. Moreover, from the analysis of the autoregressive coefficients, come up that not only present values of the variables influence each other's, but also lagged ones, in the same way as previously explained. The previous results are confirmed by the Granger Causality test, and by the IRF and the FEVD analysis. Most of the variables are 1-step-Granger-Cause of each other's, especially for Czech Republic. Furthermore, for small number of lags, high are the responses of variables to a unit shocks.

Each variable reacts more to shocks occurred in its lagged value(s) and the effects of shocks of a variable which is granger-cause of another is stronger, in magnitude, compared with the effect of shocks occurred on the other variables, results coherent with autocorrelation and granger causality results, respectively. Moreover, independently if, at the beginning, the responses to orthogonalized shocks are more or less strong, as long as the number of lags increase, they go to zero, coherently with the lag one choice. Lastly, interesting are the forecast results. Being the forecasted values significantly different form the real ones, we concluded that the two most recent crises, the COVID-19, and the Ukraine-Russia war, were not expected phenomena that could not have been estimated by using past values (or not fully estimated). They lead to disruptive consequence for the economies, as demonstrated by the strong and fast output decline, the rise in government debt, and the strong rise in inflation.

To conclude, three are the main outputs of this work. Firstly, during financial and/or economic crises the output growth decline, while the fiscal policy and the financial stress index increase. Secondly, high is the correlation between these three key variables. Negative is the correlation between output growth and fiscal policy and output growth and CLIFS. Instead, positive is the correlation between fiscal policy and CLIFS. High is the autocorrelation between variables and their lagged values, also. Lastly, crises are unexpected phenomena that economists are generally not able to predict. They bring to disruptive consequences for the economies.

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APPENDIX

TABLE A.1 – ITALY: AUTOREGRESSIVE COEFFICIENTS

ITALY- AR	Value	StandardError	Tstatistic	Pvalue
Constant(1)	0,0163	0,0381	0,4273	0,6691
Constant(2)	0,0570	0,0527	1,0828	0,2789
Constant(3)	-0,1363	0,1415	-0,9631	0,3355
Constant(4)	0,0124	0,0111	1,1217	0,2620
Constant(5)	0,0151	0,0082	1,8502	0,0643
AR{1}(1,1)	0,3133	0,1197	2,6162	0,0089
AR{1}(2,1)	-0,2072	0,1655	-1,2520	0,2106
AR{1}(3,1)	-1,4421	0,4447	-3,2431	0,0012
AR{1}(4,1)	0,0583	0,0349	1,6712	0,0947
AR{1}(5,1)	0,0420	0,0257	1,6330	0,1025
AR{1}(1,2)	-0,0110	0,0284	-0,3868	0,6989
AR{1}(2,2)	0,9535	0,0393	24,2631	0,0000
AR{1}(3,2)	0,1133	0,1056	1,0731	0,2832
AR{1}(4,2)	-0,0084	0,0083	-1,0190	0,3082
AR{1}(5,2)	-0,0101	0,0061	-1,6566	0,0976
AR{1}(1,3)	-0,0086	0,0195	-0,4398	0,6601
AR{1}(2,3)	0,0503	0,0270	1,8633	0,0624
AR{1}(3,3)	0,6836	0,0725	9,4248	0,0000
AR{1}(4,3)	-0,0022	0,0057	-0,3846	0,7005
AR{1}(5,3)	-0,0152	0,0042	-3,6295	0,0003
AR{1}(1,4)	-0,1495	0,2142	-0,6982	0,4850
AR{1}(2,4)	0,1766	0,2960	0,5967	0,5507
AR{1}(3,4)	1,0377	0,7953	1,3047	0,1920
AR{1}(4,4)	0,8559	0,0624	13,7243	0,0000
AR{1}(5,4)	-0,0362	0,0460	-0,7870	0,4313
AR{1}(1,5)	0,0435	0,1996	0,2180	0,8275
AR{1}(2,5)	-0,3610	0,2759	-1,3087	0,1906
AR{1}(3,5)	0,9501	0,7412	1,2817	0,1999
AR{1}(4,5)	0,0022	0,0581	0,0375	0,9701
AR{1}(5,5)	0,9057	0,0428	21,1492	0,0000

TABLE A.2 – BELGIUM: AUTOREGRESSIVE COEFFICIENTS

BELGIUM - AR	Value	StandardError	Tstatistic	Pvalue
Constant(1)	-0,0115	0,0265	-0,4325	0,6654
Constant(2)	0,0945	0,0586	1,6121	0,1070
Constant(3)	0,1024	0,1126	0,9090	0,3634

Constant(4)	0,0131	0,0147	0,8927	0,3720
Constant(5)	0,0127	0,0062	2,0578	0,0396
AR{1}(1,1)	0,3133	0,1128	2,7782	0,0055
AR{1}(2,1)	-0,1003	0,2495	-0,4019	0,6878
AR{1}(3,1)	-0,7369	0,4792	-1,5379	0,1241
AR{1}(4,1)	0,1538	0,0626	2,4552	0,0141
AR{1}(5,1)	0,0739	0,0262	2,8210	0,0048
AR{1}(1,2)	0,0161	0,0245	0,6553	0,5122
AR{1}(2,2)	0,9009	0,0543	16,6028	0,0000
AR{1}(3,2)	-0,0848	0,1042	-0,8136	0,4159
AR{1}(4,2)	-0,0100	0,0136	-0,7361	0,4617
AR{1}(5,2)	-0,0105	0,0057	-1,8384	0,0660
AR{1}(1,3)	-0,0061	0,0171	-0,3577	0,7205
AR{1}(2,3)	0,0533	0,0378	1,4099	0,1586
AR{1}(3,3)	0,6699	0,0726	9,2218	0,0000
AR{1}(4,3)	-0,0043	0,0095	-0,4540	0,6498
AR{1}(5,3)	-0,0194	0,0040	-4,8890	0,0000
AR{1}(1,4)	-0,1310	0,1249	-1,0483	0,2945
AR{1}(2,4)	0,1602	0,2765	0,5793	0,5624
AR{1}(3,4)	0,7139	0,5310	1,3446	0,1788
AR{1}(4,4)	0,8208	0,0694	11,8273	0,0000
AR{1}(5,4)	-0,0125	0,0290	-0,4309	0,6665
AR{1}(1,5)	0,1010	0,0932	1,0832	0,2787
AR{1}(2,5)	-0,1851	0,2063	-0,8976	0,3694
AR{1}(3,5)	0,9010	0,3961	2,2747	0,0229
AR{1}(4,5)	0,0116	0,0518	0,2237	0,8230
AR{1}(5,5)	0,9828	0,0216	45,3945	0,0000

TABLE A.3 – CZECH REPUBLIC: AUTOREGRESSIVE COEFFICIENTS

CZECH REPUBLIC - AR	Value	StandardError	Tstatistic	Pvalue
Constant(1)	0,0797	0,0366	2,1801	0,0292
Constant(2)	0,0090	0,0175	0,5116	0,6089
Constant(3)	-0,2508	0,1116	-2,2477	0,0246
Constant(4)	0,0159	0,0365	0,4354	0,6633
Constant(5)	0,0105	0,0039	2,7193	0,0065
AR{1}(1,1)	0,3934	0,1156	3,4028	0,0007
AR{1}(2,1)	-0,1052	0,0554	-1,8980	0,0577
AR{1}(3,1)	-0,2259	0,3529	-0,6402	0,5220
AR{1}(4,1)	0,1795	0,1153	1,5570	0,1195
AR{1}(5,1)	0,0106	0,0122	0,8693	0,3847
AR{1}(1,2)	-0,1773	0,0879	-2,0162	0,0438
AR{1}(2,2)	0,9661	0,0421	22,9220	0,0000
AR{1}(3,2)	0,6072	0,2684	2,2628	0,0236

AR{1}(4,2)	-0,0131	0,0877	-0,1499	0,8808
AR{1}(5,2)	-0,0226	0,0093	-2,4414	0,0146
AR{1}(1,3)	-0,0085	0,0261	-0,3253	0,7450
AR{1}(2,3)	0,0300	0,0125	2,4044	0,0162
AR{1}(3,3)	0,6237	0,0796	7,8384	0,0000
AR{1}(4,3)	-0,0055	0,0260	-0,2105	0,8333
AR{1}(5,3)	-0,0087	0,0027	-3,1575	0,0016
AR{1}(1,4)	-0,0706	0,1118	-0,6315	0,5277
AR{1}(2,4)	-0,0045	0,0536	-0,0847	0,9325
AR{1}(3,4)	0,2259	0,3413	0,6619	0,5080
AR{1}(4,4)	0,2807	0,1115	2,5179	0,0118
AR{1}(5,4)	0,0138	0,0118	1,1708	0,2417
AR{1}(1,5)	-0,5537	0,4644	-1,1922	0,2332
AR{1}(2,5)	0,0718	0,2226	0,3226	0,7470
AR{1}(3,5)	5,0015	1,4174	3,5287	0,0004
AR{1}(4,5)	0,2128	0,4630	0,4596	0,6458
AR{1}(5,5)	0,8583	0,0490	17,5238	0,0000

TABLE A.4 – POLAND: AUTOREGRESSIVE COEFFICIENTS

POLAND - AR	Value	StandardError	Tstatistic	Pvalue
Constant(1)	-0,0122	0,0672	-0,1817	0,8559
Constant(2)	0,0277	0,0274	1,0113	0,3119
Constant(3)	0,0856	0,0918	0,9319	0,3514
Constant(4)	0,0051	0,0131	0,3874	0,6985
Constant(5)	0,0067	0,0078	0,8534	0,3934
AR{1}(1,1)	0,1956	0,1355	1,4434	0,1489
AR{1}(2,1)	0,0105	0,0553	0,1898	0,8495
AR{1}(3,1)	-0,1538	0,1853	-0,8301	0,4065
AR{1}(4,1)	0,0203	0,0263	0,7691	0,4419
AR{1}(5,1)	0,0355	0,0158	2,2520	0,0243
AR{1}(1,2)	0,0441	0,1225	0,3598	0,7190
AR{1}(2,2)	0,9409	0,0500	18,8152	0,0000
AR{1}(3,2)	-0,1379	0,1674	-0,8237	0,4101
AR{1}(4,2)	-0,0037	0,0238	-0,1555	0,8764
AR{1}(5,2)	-0,0062	0,0142	-0,4360	0,6628
AR{1}(1,3)	-0,0791	0,0666	-1,1874	0,2351
AR{1}(2,3)	0,0341	0,0272	1,2546	0,2096
AR{1}(3,3)	0,6144	0,0910	6,7492	0,0000
AR{1}(4,3)	0,0174	0,0129	1,3436	0,1791
AR{1}(5,3)	-0,0089	0,0077	-1,1450	0,2522
AR{1}(1,4)	0,2498	0,3372	0,7407	0,4589
AR{1}(2,4)	-0,1110	0,1377	-0,8061	0,4202
AR{1}(3,4)	1,0470	0,4610	2,2710	0,0231

AR{1}(4,4)	0,9026	0,0656	13,7682	0,0000
AR{1}(5,4)	0,0992	0,0392	2,5307	0,0114
AR{1}(1,5)	0,0324	0,2235	0,1449	0,8848
AR{1}(2,5)	0,0193	0,0913	0,2115	0,8325
AR{1}(3,5)	0,0291	0,3056	0,0951	0,9242
AR{1}(4,5)	-0,0823	0,0434	-1,8943	0,0582
AR{1}(5,5)	0,8452	0,0260	32,5285	0,0000

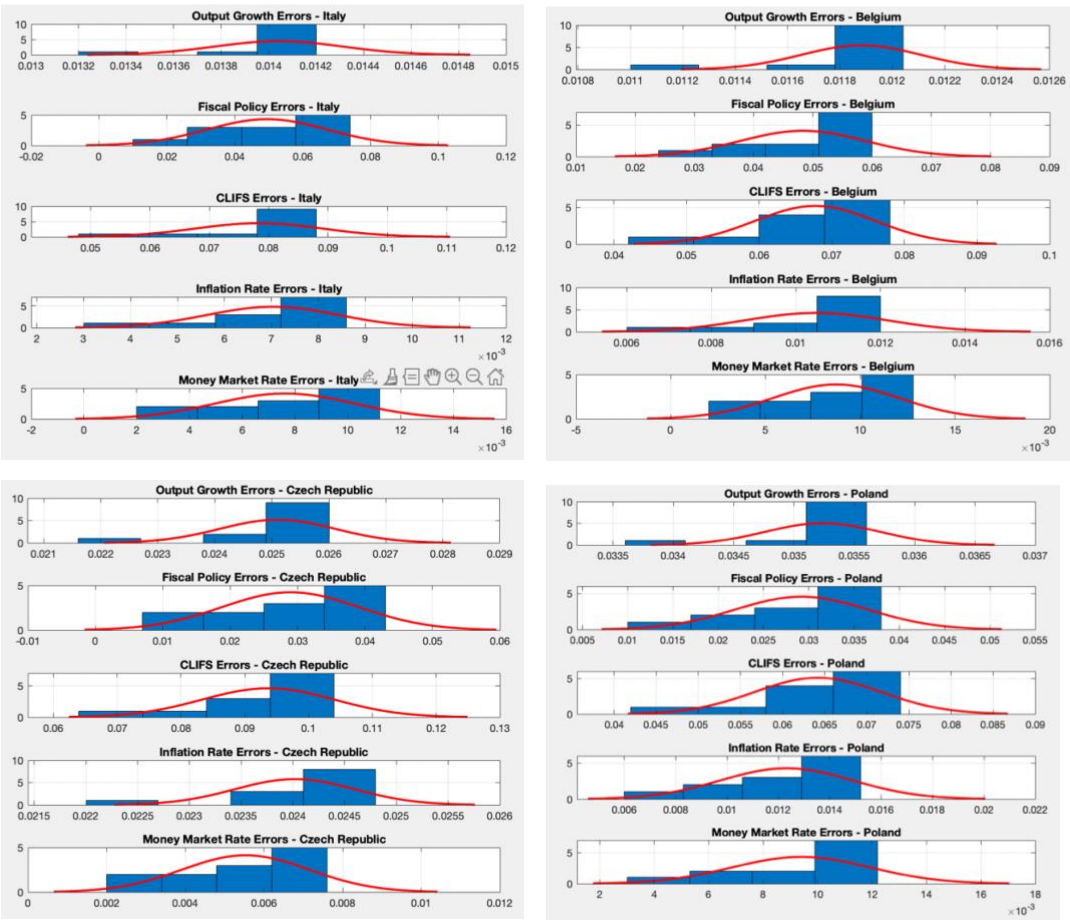
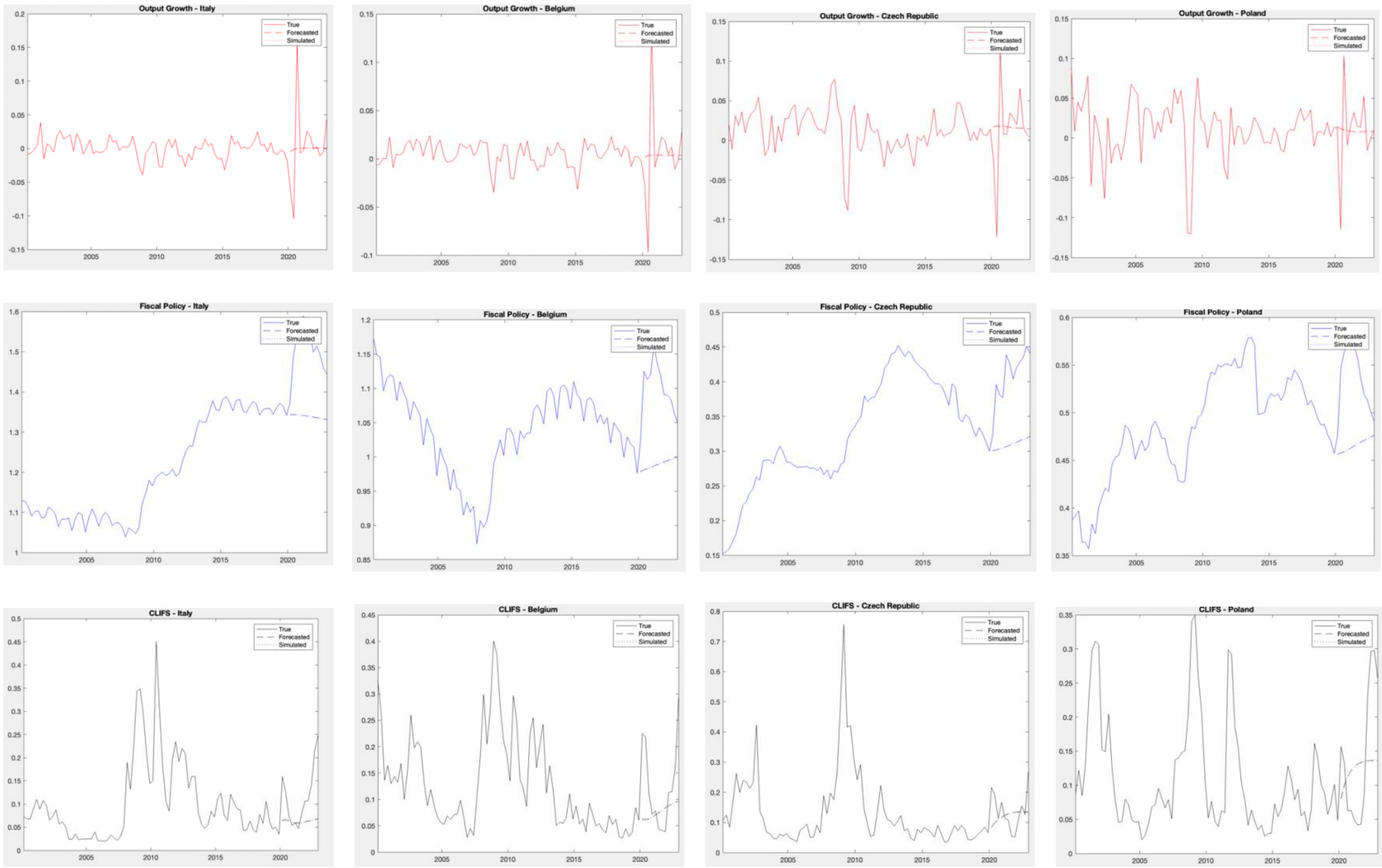


FIGURE A.1. This figure represents the distribution of the errors of the forecast analysis, for Italy, Belgium, Czech Republic, and Poland.

Note. As well as the residuals, also the errors of the forecast analysis must be normally distributed, assumption that we checked using the Lilliefors test, at 5% significance level. The hypothesis of normal distribution of output growth is not rejected only for Czech Republic and Belgium, while these countries reject the normality of Inflation Rate. Italy rejects the normality of CLIFS. As in the case of residuals, the estimated model doesn't respect the normality assumption. But, also in this case, is not necessary to modify the model, since extreme values are caused by the presence of two relevant crises in the sample period considered and their impact is increased due the sample shortness.


Output Growth - Italy
Output Growth - Belgium
Output Growth - Czech Republic
Output Growth - Poland
Fiscal Policy - Italy
Fiscal Policy - Belgium
Fiscal Policy - Czech Republic
Fiscal Policy - Poland
CLIFS - Italy
CLIFS - Belgium
CLIFS - Czech Republic
CLIFS - Poland
True
Forecasted
Simulated

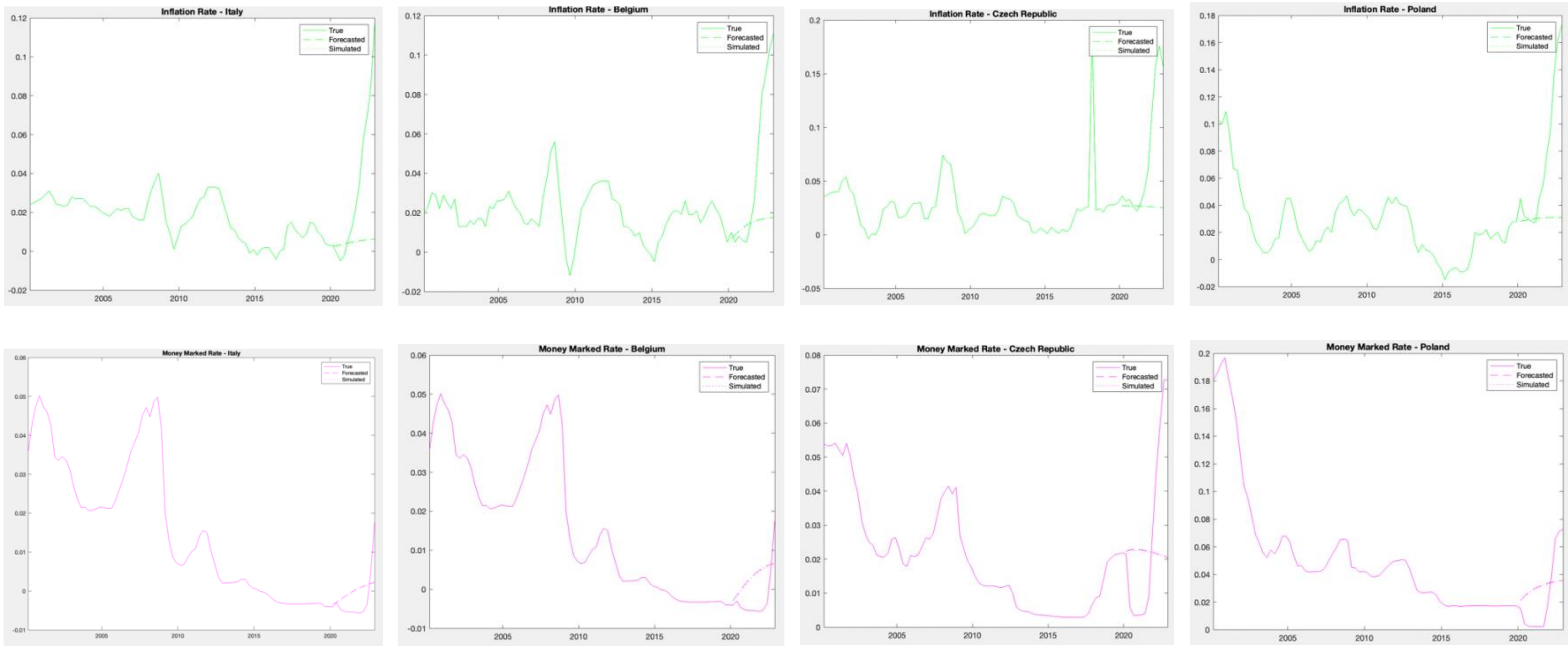


FIGURE A.2. This figure represents the forecasted data of each variable, obtained thought the functions: FORECAST and ESTIMATE, for Italy, Belgium, Czech Republic, and Poland. Estimated values go from 2020:1 to 2022:4. Real data are plotted from 2000:1 to 2022:4.

Note. The forecasted estimations obtained using the two functions, overlaps. In the case of SIMULATE the estimated values don't match the real ones, also. This evidence further suggest that the two most recent crises (COVID-19 and Ukraine-Russia war) have been unexpected and disruptive phenomena for the economies.