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Real Options for Investment Decision-Making: Evaluating Porsche's Strategic Flexibility

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Acknowledgments

To my beloved Family, I would like to communicate my recognition of the unwavering support they have provided during these years. Your unwavering trust in me served as a source of motivation and inspiration during challenging moments, allowing me to maintain a steadfast perspective despite obstacles that may have arisen. Your unwavering faith in my abilities served as a beacon of hope, guiding me through difficult times, and helping me stay focused on my goals.

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Dear Readers and Commissioners, I eagerly anticipate discussing my work with you and benefiting from your insightful feedback, which I consider a valuable learning experience. I also extend my greetings to **Luiss Guido Carli's staff** to accommodate their students.

My entire journey has been indelible, for I shall never forget the individuals I encountered and the comrades I made during my journey. Relocating to Rome without any prior acquaintances, I was embraced by the city's warmth and hospitality, which imbued me with a profound sense of belonging in the Eternal City.

I am endeavoring to evoke the same feelings that I had when I composed this letter. Recall that the serene atmosphere of a focused study in the Pola Library is occasionally disrupted by shared laughter or discreet jokes, particularly during chess games with ***Felice G***. These games were not just a battle of wits and strategy but also a testament to our friendship. This is similar to our gatherings at the Language Café, where we discovered our shared love for travel and passed the time with conversations as free flowing as tea was offered. During the study sessions, we also paid tribute to our commitments, such as when we completed our project work at 2 p.m. with ***Filippo M***. and ***Gianmarco N***., sharing the quiet satisfaction of a well-done job. Additionally, my outdoor experience was enriched by ***Gaia D. A***., who helped me overcome critical moments and shared personal revelations with me. Yet I still remember our walks and training sessions as if they were yesterday, ***Luca S***., and how you would always find a cozy place for us.

While studying abroad in Belgium, I had the opportunity to meet wonderful friends with whom I kept in touch. As I look back at my time there, I am immensely grateful for the special qualities and moments that each brought me into my life. Your lively energy and simplicity have been inspired by ***Carlotta C.*** Your presence brings joy and lightness to our days, making every encounter pleasant. ***Silvia G.***, your poise and openness have not only enhanced our group's dynamics but have also taught me the value of carrying oneself with grace and inclusiveness. With you, ***Arianna M.***, I enjoyed numerous extraordinary hearty laughs. Although we have not experienced Paris, I hope to return to it. Your curiosity is undeniably impressive, ***Roberta G.,*** and I am eager to know more about you soon. Your guidance is invaluable. ***Maria D.*** and I am grateful for your constant support. I apologize for not being available to you and hope to show you the same kindness in return.

Even though something might have changed in recent times, ***Mario P.F.***, you had been our source of inspiration, and you had been the only one who put me in a check mate (but just a few times!). I hope that one-day things will be smooth.

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L'intero viaggio è stato indelebile, poiché non dimenticherò mai le persone che ho incontrato e i compagni che ho conosciuto durante il mio viaggio. Trasferendomi a Roma senza alcuna conoscenza precedente, sono stato accolto dal calore e dall'ospitalità della città, che mi ha infuso un profondo senso di appartenenza nella Città Eterna.

Mi sto sforzando di evocare le stesse emozioni che avevo quando ho composto questa lettera. Ricordare l'atmosfera serena di uno studio concentrato nella Biblioteca Pola occasionalmente interrotta da risate condivise o uno scherzo discreto, durante le nostre partite a scacchi con Felice G., mi rende felice. Queste partite non erano solo una battaglia di tattica e strategia, ma anche una testimonianza della nostra amicizia. Allo stesso modo, i nostri incontri al Language Café, dove abbiamo scoperto le nostre destinazioni di scambio, solevamo trascorrere il tempo parlando e bevendo il tè offerto. Durante le sessioni di studio, abbiamo anche onorato i nostri impegni, anche la dedizione nell'aver completato il nostro progetto alle 2 di notte con Filippo M. e Gianmarco N., condividendo dopo la bella soddisfazione di un lavoro ben fatto. Inoltre, la mia esperienza da fuorisede è stata arricchita da Gaia D. A., che mi ha aiutato a superare momenti critici e ha condiviso con me rivelazioni personali. Oltre a Luca S., di cui ricordo le passeggiate e il nostro training come se fosse ieri, e come abbia sempre trovato un posto accogliente per entrambi.

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Anche se qualcosa potrebbe essere cambiato di recente, Mario P.F., sei stato la mia fonte di ispirazione e sei stato l'unico ad essere riuscito a mettermi in scacco matto (ma solo poche volte!). Spero che un giorno le cose possano riappianarsi.

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Abstract

This dissertation is willing to delineate a systematic approach to navigating *uncertainty* and evaluate a project's *strategic flexibility*. This research investigates the implementation of *Real Options Theory* (ROT) from an *investment-decision-making perspective*. First, the literature highlights the limitations of conventional *corporate finance techniques* such as Discounted Cash flow (DCF). There is a critical review of real options research to mark its strengths and denounce its weaknesses. In the second chapter, a practical framework is designed for practitioners with practical applications of option modelling in financial management. The third and last chapters exhibit a real case application in the *automotive industry*. The valuation models used rely on *asset pricing techniques*, so let's say that "*In Asset Pricing we Trust*" (*motto*).

Per Aspera ad Astra

Introduction

At the time a financial analyst evaluates an investment, they might consider the expected cash flows from the standalone value of a project, an asset, or a business venture; however, the underlying strategy that led the decision-making has value itself. According to M. Porter, strategy is “about being different”, by choosing among alternatives. When deciding whether to invest in a firm, hearing that the target company is about to launch a new product or expand into a new market will turn over a new leaf. The value generated from this choice is captured (even partially) by Real Options. Considerations of the subject can make managers foresee a plausible future evolution scenario to clarify the best business opportunities.

In the last few decades, scholars have conducted thorough research on decision-making under uncertainty. The main work that followed was Dixit and Pindyck’s elaboration of investment under uncertainty, which inspired the entire dissertation. The authors denounce the “Orthodox Theory” as the conventional methodologies used to evaluate an investment and publish the Option Approach highlighting the implications of irreversibility, uncertainty and flexibility. A strategic investment can be seen as “a portfolio of real options” as stated by Luehrman, to enable corporate decision-maker to benefit form *leveraged uncertainty* and limit *downside risk*. Trigeorgis and Smit published additional interesting research that followed the planning required for the implementation of options in a competitive context, providing cutting-edge ideas to conceptualize how to behave and see opportunities and the potential pitfalls of the strategy. Both scholars argue about “inconsistency” of the traditional valuation models and present a shift in the current perspective in order to capture uncertainty. In the Discounted Cash Flow (DFC) model, risk is captured by the opportunity cost of capital, whereas in options valuation, the standpoint is on evaluating different possibilities. While the DCF approach assumes environment to be stable, in the Real Option does not. Competitors do not sit idly. To understand how to ascertain competitive advantage, Real Options provide a huge hand to secure a viable path to move forward.

The primary goal of this dissertation is to explore the application of real options to investment decision-making to evaluate Porsche's strategic flexibility. It aims to understand how real options can be a strategic tool for managing uncertainty and flexibility, thereby contributing to the evaluation of value-creation strategies. The reader encounters three chapters in their conscientiousness, evermore "applicative."

The *first chapter* of this dissertation provides a further analysis of the literature in support of this subject, and the main research questions are as follows:

I. First Research Question: *What makes the Real Options Distant from Orthodox Theory*?
II. Second Research Question: *What are the factors that influence the adoption of Real Options in investment decision-making*?
III. Third Research Question: *What Is Uncertainty Made*?
IV. Fourth Research Question: *Are the pricing methods sound for attaching value to business prospects?*

The answers to these questions are addressed in the subsequent sections of this dissertation. Thus, reading research contributions has its purpose, especially if the topic is unknown or has low familiarity. Despite not being widely adopted by Chief Financial Officers (CFOs), their understanding has the potential to create a competitive advantage in managing and generating value, and it serves to make more informed decisions. The methodology used to overlay theory onto reality is abductive reasoning. In research methods, "abductive" refers to the conception of the best explanation based on incomplete information. Real Options involve making strategic decisions under conditions of uncertainty, yet with information asymmetry, and give a hint about most courses of action. The most important delimitation refers to the fact that *the monotonicity theorem* is contravened because conclusions can sometimes change with additional information.
In the first chapter, there is a mathematical explanation regarding the impact of uncertainty on decision-making and later the presentation of the theory of games, as well as the behavioral biases and criticisms of the pricing models. Even though the introductions of these themes seem to be ancillary to the scope of the thesis, they are useful for understanding the rationale behind

pricing alternatives. Valuation techniques are an essential contribution of *Asset Pricing,* and their reliability is based on the assumptions of market efficiency. An explanation of the pricing model will prove useful when analyzing the *case study*.

The second chapter aims to illustrate various applications of the theory in the field of corporate finance. One such application is the implementation of the option approach to assess a firm's corporate capital structure to determine the risk embedded in leveraged transactions. This contribution is reserved for practitioners.

The third chapter presents a case study of the automotive industry. The scope of this study is to provide a concrete interpretation of the theory of reality. Currently, the automotive industry is enduring one of the greatest revolutions since its mass commercialization because it is one step away from switching to electric vehicles. Although the Green Deal goals exhibit inspirational stances, is our continent ready to deal with this issue? What is still missing regarding the best results? The discussion of whether conversion to electricity is the only way forward to achieve sustainability is beyond the scope of this study. The interest is to evaluate the decisions that are incumbent on the achievement of the mission.

This case considers Porsche AG as the subject of inquiry because the top management declared that their vehicles will become *80%* + electric by 2030. According to *The Alchemy of Growth* by Coley and White the business strategy can be looked into with three horizons of growth. I discovered that reading a book is one of the most suitable methods for determining Real Options. To put it simple, there are three horizons that examine the current, the emerging and the "game changer" strategy. The goal is to create visible options among the second and third horizons that can be added to the current value. How can Porsche become fully electric if there are low-charging stations in Europe? Have they reckoned that the majority of Europeans do not own garages?

The main reason that explains why I found 3-horizions contribution relevant is that the options are not easy to be conceptualize, and it can't not be that the reality adapt to the model, but it is the model that should capture the peculiarity of the reality. Another drawback is the lack of full disclosure of *proprietary information* crucial for valuation purposes. The strategic implementation plan presented herein aims to align with Porsche AG's Mission 2030 with a renewed emphasis on sustainability. This plan outlines the steps that Porsche AG may take to achieve ambitious goals while maintaining a focus on sustainability. By adhering to these guidelines, this plan aims to provide a clearer roadmap for Porsche AG to achieve their sustainability goals and successfully implement their Mission 2030.

In the case study, the core of the investigation lies in assessing the feasibility of a Leveraged Buyout (LBO) transaction for the acquisition of Ionity GmbH, which is one of the leading charging station networks, as well as the potential expansion in a new business venture that is the yachting market by Porsche AG. The purpose of applying the real option model is hypothetical and is intended to illustrate the potential outcomes if the specified scenario were to occur. Therefore, it would be interesting to answer the following research questions in the case study:

V. Fifth Research Question: *Are Private Equity firms focused strictly on the LBO value by missing options with holding and post-exit strategies? What can be the assessment of their value?*

VI. Sixth Research Question: *How can (beta) debt behave in the context of an LBO valuation?*

VII. Seventh Research Question: *What would happen if Porsche position itself in a new business? What is the value added if the German car manufacturers expand?*

Finally, it is important to ensure that this dissertation is aimed at an academic audience and professional practitioners; thus, there is supposed to be previous knowledge about financial management. The tone used is sometimes colloquial because it ensures a smooth reading flow.

Chapter 1: Real Options Approach and Its Relevance in Investment Decision-Making

1.1. Concept of Real Options

Real options refer to a way of running corporate decisions, which are meant to be strategic and forward-looking and related to real assets. An analogy stands with financial options as derivative securities that give their holders the right (not the obligation) to purchase or sell the so-called underlying assets for a given price in the future. Assets are broadly defined as opportunities for business. A business opportunity involves value creation and is pursued by implementing a value-based strategy. In strategic management, the choice can be to invest tomorrow in an R&D program, patent, or plant expansion. However, it is worth evaluating when such capital expenditure creates greater value.

In an environment of great uncertainty, the best-known Net Present Value (NPV) might be misleading. Because NPV has been conceived as a tool for making a unique decision, made *now* or *never*, assumptions concerning external conditions will not be made. Under the options framework, it is feasible to introduce flexibility to account for irreversibility, uncertainty, and timing better. For instance, managerial flexibility affects the options of delaying, abandoning, or investing.

Since the term "Real Options" was coined by S. Myers (1977), defined as "*opportunities to purchase real assets on favorable terms*," the concept has been forged ahead with several scholars as well as under empirical evidence, and it could be, nowadays, a valuable approach for capital budgeting[1] and more generally investing in decision-making. Unlike conventional Discounted Cash Flow (DCF) methods, Real Options have been proven to be a powerful decision-making tool to allocate financial resources and maximize shareholder value by leveraging uncertainty and limiting downside risk (Arnold & Shockley, 2002).

Real Options are supposed to cope with cases of *high volatility* – increasing worldwide in recent times – to implement a model that recognizes the flexibility of a firm's investments.

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[1] *See also:* Baker, H. K., Dutta, S., & Saadi, S. (2010). Management views on real options in capital budgeting. *Journal of Applied Finance: JAF, 21(1), 18, pages 6-7.*

1.2. Orthodox Theory vs Managerial Flexibility: The Real Option Theory

To run a business, a firm must purchase real assets, which are investment decisions. However, these items do not drop free from the sky, because they must be paid by selling claims on these assets and their cash flows. These claims are financial securities. Investment decisions are crucial to value-added creation in financial management. Strategic investment decision making is a broad topic that aims at a long-term goal and entails market positioning, competitive advantage, and so on.

Capital Budgeting is the process of identifying and selecting investments in long-term assets or assets that are expected to provide benefits over the next few years within a shorter period compared to investment decisions. Nevertheless, firms must establish corporate strategies before entering into capital budgeting. The fundamental goal of this strategy is value maximization, which has been accepted as the rational basis for decision making. Conventionally, when a firm undertakes an investment, it squeezes earnings in the short run to expect the asset to generate cash flow in the near future. Hence, the "value creation' jargon is favored for profit maximization, which makes it more meaningful in microeconomics theory. The recipients of wealth are, above all, shareholders since management operates in their interests. A financial manager's job is to make decisions on behalf of the firm's investors. Managers determine how to execute actions within a firm's corporate strategy, either by looking at resource allocation constraints or in a competitive context in line with the firm's goals. Although the previous consideration, in finance, more attention is paid to the time value of money, risk, alternatives, and future opportunities. An important frame is to consider decision making in a dynamic context because a trade-off commonly arises between risk and ex ante the decision – and opportunity cost – ex post the decision.

Risk involves uncertainty, which is a deviation from a certain outcome, whereas opportunity cost considers the cost of regret when an alternative is selected. Within this framework, a firm chooses suitable investment projects and allocates resources by using capital budgeting tools[2] to achieve its goals.

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[2] *See also*: Peterson, P. P., & Fabozzi, F. J. (2002). *Capital budgeting: theory and practice* (Vol. 10). John Wiley & Sons, pages 5-10 et. Trigeorgis, L.G. (1986). *Valuing Real Investment Opportunities: An Options Approach to Strategic Capital Budgeting (flexibility).* Harvard University, pages 26-29.

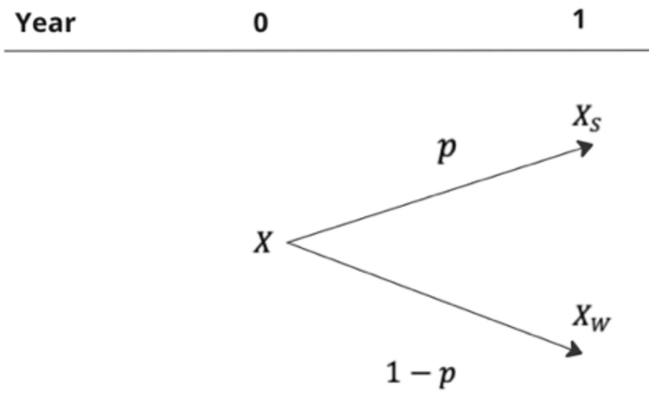
One of the simple rules that comes to mind in solving this type of problem is to discount the stream of expected cash flows, which are the benefits, and strip out the present value of the costs. The renowned rule is the Net Present Value (NPV), which consists of $NPV = PV(benefits) - PV(costs)$. Investment is undertaken if $NPV > 0$. A generic frame identifies this rule among the Discounted Cash Flow (DCF) methods as the most commonly used valuation technique. Valuation could either focus on firm-specific variables to gather information from earnings or observe more aggregate measures to reach earnings. This issue arises when calculating cash flows and discount rates. To cover a wide field of valuation, all major methods fall into three categories: 1) financial, 2) market, and 3) asset-based methods. Among the financial methods, the DCF technique estimates the value of a project by discounting the expected cash flows arising from managerial accounting at the lifetime risk rate. The discount rate is the cost of capital estimated from the CAPM (Capital Asset Pricing Model). The aforementioned approach belongs to **orthodox theory**[3] as named by Dixit and Pindyck (1994), and the discern of the subject helps to answer the *first research question*. The economic idea is to continue investing as long as the marginal cost of capital is lower than the marginal return (Jorgenson, 1963). In recent years, the dynamics of the financial market and the competition have played a major role in project success. Management seeks flexibility '*to capitalize on favorable future opportunities or mitigate losses'* (Trigeorgis, 1996). The NPV is misaligned to capture strategic value because it fails to capture the sequence of interrelated stages in an investment project. The management literature emphasizes the trade-off between commitment and flexibility, as the strategic context is uncertain by definition (Wernerfelt & Karkani, 1987). Real Options arose to bridge this gap with traditional capital budgeting methods (referred to as "orthodox theory"). Empirical research has found that the real world seems to be less sensitive to interest rate changes and tax policies; however, environmental volatility is a huge concern[4]. The NPV uses the implicit assumption that things are reversible. Otherwise, it is a single roll in cash flow. Investors are supposed to make decisions in the present or later. However, one rationale may be that the opportunity to invest is an option to buy an asset. Real

[3] The definition has been taken from Dixit, A. K., & Pindyck, R. S. (1994). *Investment under uncertainty*. Princeton university press. The authors coined the term "orthodox theory" to refer to classical decision-making theories such as the Net Present Value (NPV).
[4] *See also*: Carruth, A., Dickerson, A., & Henley, A. (2000). What do we know about investment under uncertainty? *Journal of economic surveys*, *14*(2), pages 119-154.

options offer time decisions by adding flexibility such as the true NPV, which should be the classical sum of the NPV and the value of pertinent real options:

$$\text{True NPV} = \text{Direct NPV} + \text{ Real option value}$$

Suppose a manager would desires to determine how NPV might behave with probable outcomes, considering that the market could rise (“strong market”) or down (“weak market”). Thereafter, the average NPV was computed by discounting with the cost of capital k the expected value of project $E(X)$. The gross value of one project, labeled by V, could go in the better case X^S or in the worst-case X^W under probability p. Thus,



The value V is given by:

$$V_0 = \frac{E(X)}{1+k} = \frac{[pX^S + (1-p)X^W]}{1+k}$$

Capital budgeting under uncertainty revisits decision-making from a world where cash flows are considered “safe.” The revisited NPV considers the value of the investment required I to be stripped from the project’s value.

$$NPV_0 = -I_0 + \frac{E(X)}{1+k} = -I_0 + V_0$$

This computation is closer to what is said to be a “Strategic NPV” (Trigeorgis, 1993), which is a valuable candidate for tightening the bonds between strategic decisions and corporate alternatives. The mission is to reflect on commitment, based on game theory, as well as flexibility under uncertainty, as captured by real options. The *strategic NPV*[5] is indicated by:

$$\text{Strategic NPV} = [\text{Direct NPV} + \text{Strategic value}] + \text{Flexibility value}$$

Strategic value is made up of *(a)* strategic reaction value standing for the change in market share and enterprise value and *(b*) strategic preemption value (timing effect) reflecting the potential rival's prior claim to property shifting the market structure. For a review of strategic decision making [Figure 1], the literature arranges the most common tools based on flexibility and commitment.

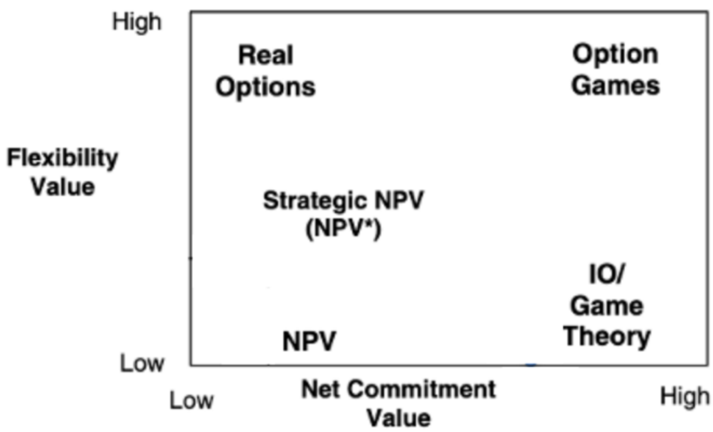


Figure 1 Positioning of Strategic decision-making tools among their applicability

According to management theory, NPV is adequate when the external context is predictable. Under uncertainty, a strategic tool is better suited to identifying the sources of value creation. In this study, the focus is mainly on the *real-options* approach.

Currently, strategic decision making is widely disseminated in the literature on **Real Options Theory** (ROT). The term option is important for determining the legacy of financial derivatives in employment as a future-choice assessment tool. Under uncertainty, a real-world decision in the real world[6] (Luehrman, 1998) is as follows:

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[5] *See also*: Smit, H. T., & Trigeorgis, L. (2017). Strategic NPV: Real options and strategic games under different information structures. *Strategic Management Journal*, *38*(13), pages 2555-2578

[6] Luehrman, T. A. (1998). Strategy as a portfolio of real options. *Harvard Business Review.*, *76*(5), 89–99.

- a call option, such as the right to purchase an asset in the future (i.e., the decision to undertake an investment) at the current price.
- a put option, such as the right to sell an asset in the future (i.e., decision disposal of an investment) at the current price.

How can this opportunity be invested in the future? This appears to be a *call option* [Figure 2].

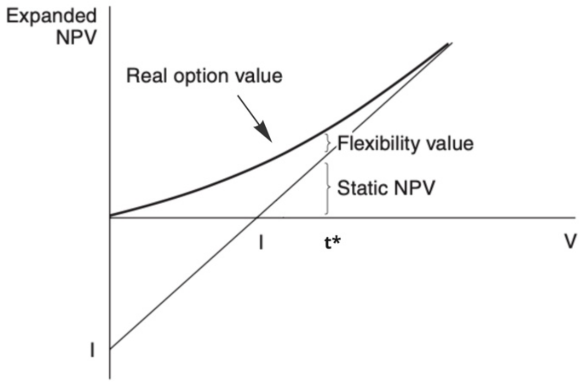

Figure 2: Opportunity to invest

The value of an option can be interpreted as the flexibility value. Thus, the new NPV rule recites:

$$Invest\ IF\ (value\ of\ capital\ raised\ costs) \gg value\ of\ the\ option\ alive$$

Connections to the strategic world focus on competitive advantage and innovation. Most of the time, firms obtain their options through patents, land, or natural resources, or through know-how, reputation, or positioning. Under contract negotiation and access to investment opportunities, a firm can undertake projects that another firm cannot undertake. For instance, the "holder" of the right obtains access to upside potential opportunities (exercising the call whereby), whereas they edge downside losses (preventing the exercise of the call).
There are several real options among the basic ones recapped[7] here [Table 1], and various other uses and extensions can occur in the real world (Trigeorgis, Dixit & Pindyck, McDonald & Siegel, Kulatilaka, Leiblein and Miller, Huchzermeir & Cohen, Kogut) to enhance managerial operating flexibility and strategic interactions. Firms can be endowed with a portfolio of options

[7] Trigeorgis, L., & Reuer, J. J. (2017). Real options theory in strategic management. *Strategic management journal*, *38*(1), pages 42-63.

with interactions between multiple alternatives. For instance, the decision to enter a new market may be a deferral or growth option as well (Folta & O'Brien, 2004). These options affect their value, which is why interactions should be considered before decision-making.

Table 1 Strategic Investment Choices Seen as Real Options

Common Real Options

Type of option	Investment choice	Nature of the option
Defer or wait	Delay the decision to invest when facing market uncertainty	Call
Growth	Potential increase in equity stake if the firms enter into a new market	Call
Expand (scale/contract)	Enlarge business ventures, outsourcing, or even cut unprofitable investments	Call
Switch	Change supplier or production across different subsidiaries.	Put
Abandon/exit	Abandon the current operations (or sale for salvage) if market conditions deteriorate.	Put

Unlike financial options, real options entail certain peculiarities that designate the *Theory*. First, *i*) these options are not traded in the organized market, *ii*) the underlying decision might be partially defined in its terms, *iii*) the benefit can be secured only for short-term durations, and *iv*) its application could raise challenges regarding asymmetries, interactions/path dependence, and incomplete property rights. The notion of shadow options (Bowman & Hurry, 1993) suggests that a firm must reveal and appraise these ties as well as the hidden opportunities that can be released in the future. Otherwise, the firm can miss prioritized access to follow-on opportunities.

Second, the ROT is deemed sound under certain hypotheses and reasoning that address its creation, preservation, and exploitation of tool features. The prevalent use of this device, as already mentioned, is due to uncertainty, and its implementation enables the mapping of investment decisions to better recognize the drivers, structure, and management of complex proceedings.

The Real Options Lifecycle can be mapped [Figure 3] in view of strategic management applications. The process begins with mapping the problem by identifying the main underlying uncertainty, irreversibility issues, and competitive pre-emption as key drivers. Decision makers seek to exploit upside potential outcomes and limit downside risk, which is exacerbated by uncertainty. trough Uncertainty embraces a variety of potential outcomes, and option value increases when uncertainty exists because it involves flexibility. Exogenous uncertainty derives from new financial information with no effort or specific action (i.e., the market), whether the endogenous requires costly measures. Moreover, managerial flexibility would be less appealing if the decision could be repealed costlessly. Thus, the irreversibility plays an important role in this process.

Finally, further consideration affects the movement of competitors or new entrants, which may take a step forward in the market. The next stages affect the *i*) identification of shadow options, *ii*) the creation of basic options (e.g., defer, grow, abandon, switch options), *iii*) the management, and *iv*) the exercise or rather the exploitation of options (so-called "killing the option").

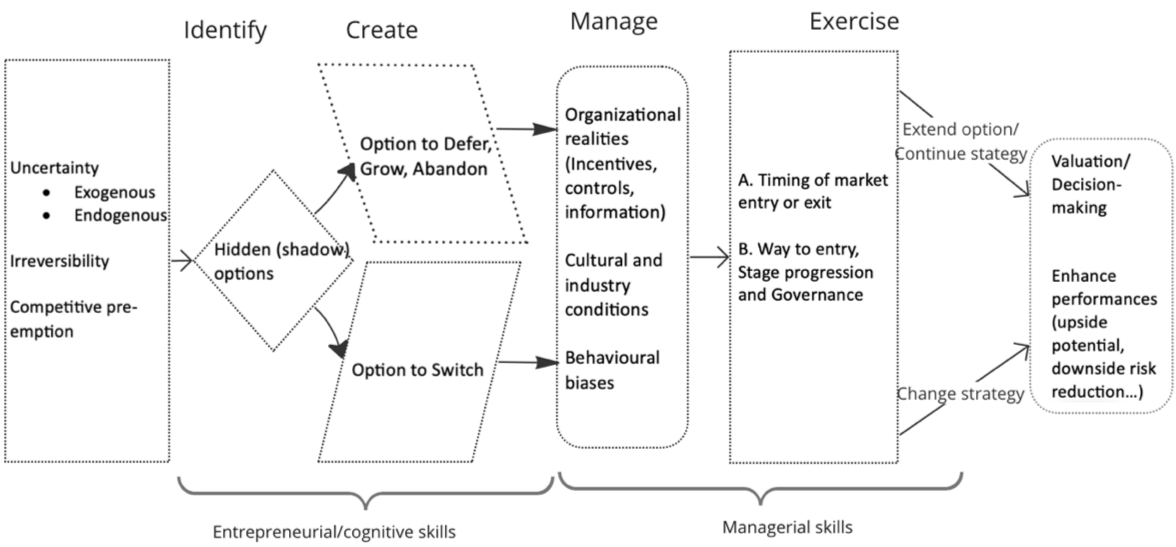

Figure 3 Mapping of Real Options

Management requires the development of an adequate organizational reality, comprehension of cultural and industry conditions, and behavioral biases. Much of the ROT literature deals with the flexibility-commitment trade-off[8]. Each option represents a form of flexibility, and its exercise involves a commitment that may not be costless to undo. Considerations are made on the *i)* timing of market entry or exit and *ii)* the way of entry. Later, if it follows to continue strategy by the expand option, there will be valuation of the option and its performance; otherwise, in the strategy, some switch (change) may occur.

Another issue is the assumptions on which the model was based. If the value of the option increases with uncertainty, the ROT encourages riskier projects and, as a norm, firms' decision-making approaches are biased when they face uncertainty. In the real world, the application of ROT can be limited when the conditions of flexibility and information accuracy are not matched. Managers should be aware of bounded rationality, limited information, and behavioral biases.

1.3. Market efficiency, optimum investment timing and *Investment-uncertainty* relationship: Which Evidence for Real Option?

A backbone of asset pricing considers the market as a place in which information can be grasped based on the player's role. When new information arises, news spreads, which affects the prices of securities (Fama et al., 1969). Therefore, the nature of market equilibrium theoretically depends on the price-setting convention adopted by the players as a result of the sounded hypotheses. The main assumption is that market dynamics reflect all information required to make the decision-making process efficient.

More precisely, this theory advocates several *efficient market hypotheses* (EMHs). However, with the advent of the new century, a growing number of critics have departed from this approach, shifting it to a behavioral approach.

In any case, defining market efficiency, where the perfect competition assumption holds, at least three hypotheses are outlined:

1) *Transparency*: Market participants have to obtain all relevant information for trading (i.e., prices reflect full information).
2) *Liquidity* – all players are able to trade at any time; thus, investors can always cash in their securities.

[8] The definition has been taken from "Amador, M., Werning, I., & Angeletos, G. M. (2006). Commitment vs. flexibility. *Econometrica*, *74*(2), 365-396".

3) *Low/no-market friction* trade occurs at no transaction costs or taxes.

Financial economics also has to examine the behavior of market participants, insofar as individuals, to be borne by rational hypotheses. In this context, all individuals (i.e., decision-makers in the current research field) have rational anticipation over *i)* expected returns, *ii)* risk aversion, *iii)* utility function, and *iv)* the indifference curve.

The above-mentioned hypotheses are derived from the expected-utility theory, which (Friedman & Savage, 1952) is based on the axioms of Von Neumann & Morgenstern. This theory shows how individuals under certain hypotheses behave when facing risk. Expected utility can be quantified by considering the individual utility function and its distribution probability. If all individuals aim at utility maximization under rational expectations (i.e., the theory approximates toward firms striving for profit (value) maximization), then the return-risk optimization principle (Markovitz, 1952) is valid; therefore, the market price converges to equilibrium. Utility theory is built on axioms that make the model applicable to decision making[9]. The utility function measures the strength of an individual's preference for alternative business opportunities: Each individual's cognizance of risk relies on the asymmetry in their utility function. Initially, the theory was established with reference to *the uncertainty*. Knight 1921) states that risk is measurable by the mean of the squared deviation of the ex-post unknown outcomes from the ex-ante average return, whereas uncertainty is not. Uncertainty, on the other hand, is related to a situation lacking full information. Empirical studies have found a negative relationship between uncertainty and investment (Ferderer, Leahy and Whited, 1996)[10], which is partially solved from the ROT perspective. The understanding of this linkage alongside the variables that impact Real Options is crucial to answer the second research questions. Investment behavior is sensitive to uncertainty and risk preferences towards risk. This study questioned how decisions are taken under risk; thus, it is essential to consider the contribution of McDonald and Siegel (1986) as a benchmark.

Scholars demand that investors invest when they are willing to bear the cost of the investment. Here, the investment expenditure is considered as the sunk cost. Dixit and Pindyck (1994) designed a topic for real option decision-making, providing the main reference for later

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[9] *See axioms on:* Schoemaker, P. J. (2013). *Experiments on decisions under risk: The expected utility hypothesis*. Springer Science & Business Media, pages 11-26.

[10] *See also*: Nakamura, T. (1999). Risk-aversion and the uncertainty–investment relationship: a note. *Journal of Economic Behavior & Organization*, *38*(3), pages 357-363.

contributions. Because the value of options depends on the underlying asset's riskiness, risk behavior is significant in understanding the pricing model leading to asset choices. In the first stance, investors are assumed to have risk-neutral aversion; thus, their utility function is linear. At time t, the manager may undertake an irreversible investment of their wealth at risk-free (i.e., no option to return the capital stock in the future) for a project whose value is contoured by X as a stream of cash flows, and it is illustrated by a geometric Brownian motion[11] (GBM):

$$d\,X_t = \alpha X_t\,dt + \sigma X_t\,dZ_t \qquad \text{or} \qquad \frac{d\,X_t}{X_t} = \alpha\,dt + \sigma\,dZ_t \qquad [1]$$

where dZ_t is the Wiener Process increment. Under the conventional NPV, the firm invests if $X - I \geq 0$. Considering the investment rule that maximizes the expected project value, the value of the investment opportunity resembles the following:

$$F(X) = \max_{\tau \in s} \mathrm{E}[(X_s - I)e^{-rs}\,ds] \qquad [2]$$

Here, $\mathrm{E}[\cdot]$ denotes expectation, s is the set of Markov moments (i.e., stopping times), r is the discount rate. The question is to identify the time τ the option would be better exercised by solving for the critical value X^* (i.e., the optimum) of the project, where X_0 is the given value of X today.

In an efficient market, prices fluctuate randomly over time and their movement is unpredictable. The hypotheses are associated with the "*random walk*" (Samuelson, 1965) concept, which characterizes the change in stock prices in the future to be independent of its previous movements. In addition to discrete-time random walks, a random variable can alternatively follow a continuous-time process. The market promptly adjusts to new information, thereby changing asset prices. At that time, a variable evolves randomly, at least in part, the forecasting of the status X_{t+1} depends only on X_t, and nothing is related before time t.

Now, the scope of the research is to investigate the effect of uncertainty over an investment; hence, the effect on the time to invest. (Sarkar, 2000; Wong, 2007). The uncertainty must be $\sigma > 0$. Investment timing is the expected time that triggers an optimal investment. According

[11] See the **appendix 1** for the derivation of the investment opportunity and optimum value process [Notes I-VI].

to CAPM, the risk-adjusted rate of return is $r = r_f + \lambda\rho\sigma$. where r_f is the risk-free rate and $\lambda = (\mu_M - r_f/\sigma_M)$ represents the price of risk, where μ_M and σ_M are the drift rate and volatility of the market portfolio, respectively. In fact, λ is also said to be the Sharpe ratio. Furthermore, the portfolio of risky assets follows the GBM. In particular, $\rho \in [-1,1]$ is the correlation coefficient between the project value and the market portfolio's return. However, the dZ Wiener increment equals $dZ = \rho dB + \sqrt{1-p^2}dZ$, where dB is the Wiener process increment for the market portfolio. In particular, $\rho^2\sigma^2$ and $(1-\rho^2)\sigma^2$ capture systemic and idiosyncratic risk, respectively. Market portfolio holders are remunerated only for systemic risk under the CAPM assumption, whereas the investment option itself shares concerns regarding non-market efficiency. The addition of idiosyncratic risk is a slight correction to the original Pindyck model, which does not include this type of risk. Alternatively, Eq. [1] can be rewritten as

$$d\,X_t = \alpha X_t\,dt + \rho\sigma X_t\,dB_t + \sqrt{1-p^2}\,\sigma X_t dZ_t \qquad [3]$$

Let $\alpha < r_f + \lambda p\sigma$, or shortly $\alpha < r$ to ensure that X^* is finite. Hence, the function expressing the value of project at time 0 can be rewritten as follows:

$$F(X) = \begin{cases} (X^* - I)\left(\frac{X_0}{X^*}\right)^{\beta} & X_0 < X^* \\ X_0 - I & X_0 \geq X^* \end{cases} \qquad X^* = \frac{beta}{beta-1}\,I, \beta > 1$$

The stochastic discount factor $(X_0/X^*)^{\beta}$ accounts for the time probability of one unit of money received at the time the optimal is reached, where b is given as the positive root of a quadratic function, as follows:

$$\beta = \frac{1}{2} - (\alpha - \lambda p\sigma)/\sigma^2 \pm \sqrt{\left[\frac{1}{2} - (\alpha - \lambda p\sigma)/\sigma^2\right]^2 + 2r/\sigma^2} \qquad [4]$$

Because the investment should occur immediately at time $X_0 \geq X^*$, the relationship between uncertainty and investment will be better investigated in the opposite case, when $X_0 < X^*$. In the latter case, the investment option is retained in the firm's portfolio until the investment trigger, X^* is reached. Postponing an investment decision equal holding an asset that pays no dividends but may appreciate over time.

Simple calculus already shows that the wedge outstanding between X^* and I increases as a function of uncertainty; therefore, the NPV principle equating both terms no longer applies. The project's critical value (i.e., the investment trigger), which makes the highest value during the exercise of the option value, shows a U-shaped pattern against the volatility of the project. However, what is the effect of uncertainty on the expected investment time? The probability density function of the investment time τ when the project's payoff reaches the investment trigger X^* from the initial value X_0, and $X = \ln(X^*/X_0)$, is:

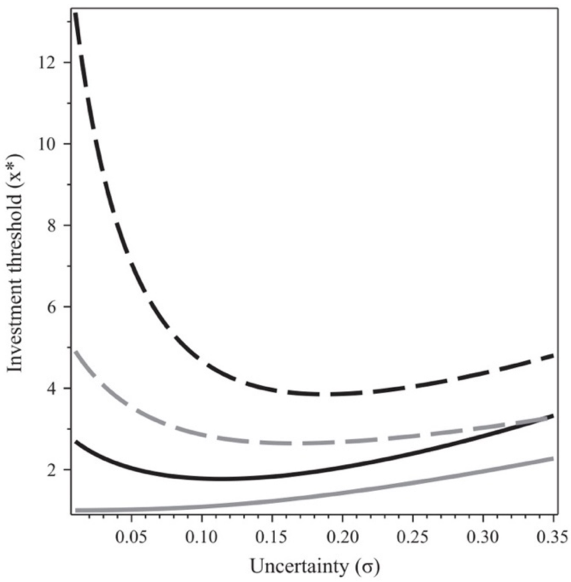
$$\Phi(\tau) = \frac{X}{\sigma\sqrt{2\pi\tau^3}} \exp\left\{-\frac{1}{2\sigma^2\tau}\left[X - \left(\alpha - \frac{\sigma^2}{2}\right)\tau\right]^2\right\} \qquad [5]$$

After the Laplace transformation of $\Phi(X, \tau)$, it is possible to derive the expected time to undertake the investment option under uncertainty as

$$E(\tau) = \int_0^\infty \tau \cdot \Phi(X, \tau)\, d\tau = -\lim_{\theta \to 0} \frac{\partial E(e^{-\theta\tau})}{\partial \theta} = \frac{\ln\left(\frac{X_0}{X^*}\right)}{\alpha - \sigma^2/2} \qquad [6]$$

The relationship of uncertainty investment can be plotted by the effect of volatility σ on the expected exercise time [Figure 4]. The results of the investigation were maintained until the $\sigma \in \left[0, \sqrt{2\alpha}\right)$. The graph shows that a low level of uncertainty reduces the optimal timing, whereas a high level of uncertainty increases it.

Figure 4: Investment-uncertainty relationship



It should be noted that the real-world scenario was slightly more complicated. In reality, the enunciated share no strict correlation[12]. Here, these assumptions are clearly reported to have them clear. Yet,

1. investors are risk-neutral towards idiosyncratic risk, meaning that they do not affect the project itself.
2. volatility can be observed at on the efficiency market information always available.
3. returns are distributed following the Normal distribution.
4. investment cost is deterministic, constant and do not behave stochastically;
5. random walk of real assets is symmetrical.

Neither of the previous assumptions can be seen to occur in managers' opportunity landscapes. Therefore, the opposite was true. Rewinding the tape, the EMHs are supposed to provide reliable information about the price of the underlying, just like the pricing environment relies on "Three Layers" as below [Figure 5].

	Hypothesis	*Technique*	*Results*
Synthetisation *Derivatives*			Market price ↑
Markets *Stocks*	- No friction - Full liquidity - Transparency	Law of one price	Market price ↑
Individuals	- Expected returns - Risk aversion - Utility function - Indifference curve →	Supply and Demand →	Price that maximizes the quantity exchange

Figure 5 The Three Layers of Financial Markets

In the finance field, this option to "purchase later" is a derivative. Therefore, the value of the derivative is related to the value of the underlying value. The process would lead to option value attribution is called "synthetization" and it is extremely relevant rely on sounded hypotheses for the markets (i.e., in this case is the "asset market"). The main issue colliding with pricing models is the assumption of market completeness, full liquidity, and frictionlessness, in which a market portfolio can exactly replicate the project value. The underlying asset here is not traded

[12] Real Options in practice A branch

into an established market most often because real options are a conceptualization of business opportunities seen just as “options” to undertake an investment.

1.4. Gambling and Behavioral biases: Which linkage with Real Options?

Investment opportunities can be seen as games that address the flexibility-commitment trade-off and illustrate decision-making in competitive markets. According to game theory, an examination must swivel at players, payoffs, and strategies. The ratio analyzes the steps under subsequent new incoming information because the theory supposes that the actions of the decision-maker affect those of the rival and that the competitors’ move impacts the firm’s own plans. The Theory of Games and Economic behavior comes after the utility theory of Von Neumann and Morgenstern (1944), when decision-making started to be looked in a strategic way. The theory strictly adheres to EMHs, with the players assumed to be rational and aware of the consequences. The utility function, which estimates individual preferences, focuses on how an individual evaluates trade-offs, where x is the wealth/gain of a decision marker. The consideration of utility theory comes out at the time when people are more averse to potential losses compared to their willingness to take potential gains. Individuals choose the alternative with the highest utility. A decision undertaken should not only consider the increase in value but also the risk incurred until the accomplishment of the project. In valuation, consideration is made, for instance, on undertaking an investment with a significant amount of leverage. Among individuals, the analysis relies on those who are not risk-seeking.

For a standard game, the setting supposes two-firms i and j (i.e., duopoly pre-emption game), each of them could undertake alternative strategy: “invest”/ “defer.” The game [Table 2] exhibit the “timing strategy” concept because the firm’s payoff affected by time. The game led to four scenarios:

Table 2: Pattern of the Game

	Defer	**Invest**
Defer	Repeat game	$[F_F(X_t), F_L(X_t)]$
Invest	$[F_L(X_t), F_F(X_t)]$	$[F_S(X_t), F_S(X_t)]$

In particular, F_{i_k,j_k} represents a firm's payoff as a function of time t. At the beginning of the game, the firm's revenue flow is given by.

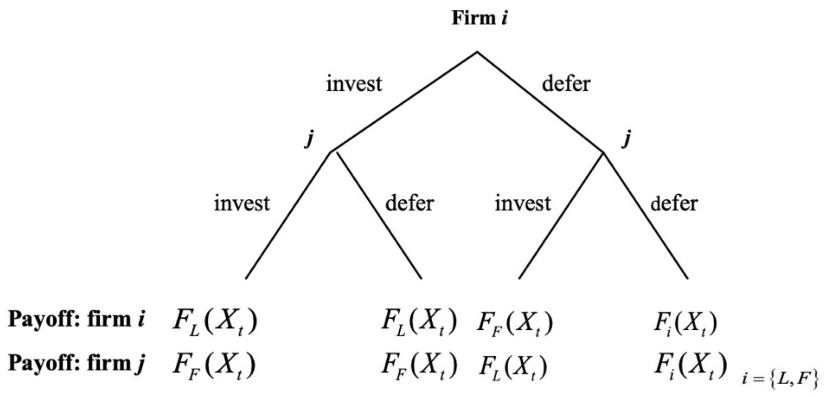
$$F_{i_k,j_k} = X_t \left[D_{K_i,K_j}\right]$$

Where X_t is the underlying asset; D_{K_i,K_j} represents the competition factor, with $k \in \{0,1\}$, for which 0 is assigned to an inactive firm and 1 to an active firm, with i denoting the leader (L) and j the follower (F). The presence of first mover's advantage (pre-emption game) is an assumption underlying the model.

However, the goal is to determine the dominant strategy at each node for the reference firm and the resultant equilibria. For the leader, the dominant strategy is $D_{1_L,0_F} > D_{1_L,1_F} > D_{0_L,0_F}$. This interpretation suggests that the leader's market share revenues are greater if it operates alone compared to when it operates with the follower ($D_{1_L,0_F} > D_{1_L,1_F}$), in turn, these revenues are higher when the leader operates with the follower than when it is inactive ($D_{1_L,1_F} > D_{0_L,0_F}$). However, there should be an additional inequality that considers $D_{1_L,1_F} > D_{1_F,1_L}$ which states that revenues from the leader's market share are more than those from the followers when both firms play actively. However, the overall inequality can be rewritten as $D_{1_L,0_F} > D_{1_L,1_F} > D_{1_F,1_L} > D_{0_L,0_F}$. In the above representation, $F_F(X_t)$ and $F_L(X_t)$ represent the payoff of the leader and the follower, respectively, while $F_s(X_t)$ represents the payoff when both are investing simultaneously. This strategy has been used in the discrete-time framework as a proxy for the continuous-time approach (Funderberg and Tirole, 1985).

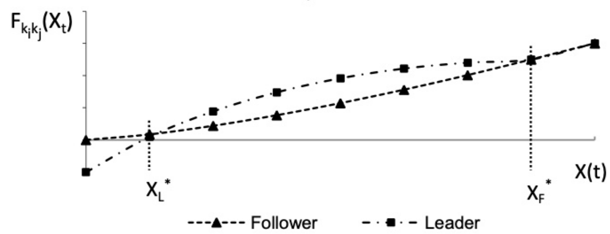
Because competition is supposed to be mainly a non-cooperative game, firms will reach a point where neither of them can gain an advantage by adopting a new strategy if the rival keeps its strategies unchanged. This set of strategies and the corresponding firms' payoffs represent a Nash equilibrium. John Nash (1949) says "A game could be like a set of strategies which the players have to follow in their moves: the equilibrium is reached when a player does not have an inventive to change its strategy unilaterally. The players should gamble jointly'. The notion of equilibrium highlights the steady status of the game, where each firm holds an adequate expectation about its rival's gamble, and thus it plays accordingly. The representation of the game follows the pattern shown [Figure 6].

Figure 6: Representation of sequential Real Option Duopoly Game



A game can be in an "strategic form" or in an "extensive form." In the first case, the players gambled simultaneously and independently, whereas in the second case, the players participated in the game by taking turns. The ROT configures games as sequential trials in which firms can either invest or defer investments. The games could be repeated unlimited times when the firm defers, but only once when the firm invests. Because the equilibria of the game are not easy to assess when the option to invest matures at some point in time when the option potentially holds infinitely, the convenience would be understood when the value of the investment reaches a threshold. The followers 'follower's and leaders' payoffs can be plotted with respect to time [Figure 7] considering the behavioral biases involving decision making.

Figure 7: Firm's payoff



Here, the X_L^* and X_F^* are the investment thresholds for the leader and followers, respectively. Games can be embedded in the Real Options model, as the option value to defer investment is driven by the fact that both firms are afraid of being preempted in the market by competitors because of the first-mover advantage principle. More competition leads to a lower value for the option to wait (i.e., defer) the investment. Game Theory is built upon the soundness of EHMs and is far from to consider the behavior biases involving making.

At the time, decisions involving individual risk prediction of prospect theory (Kahneman & Tversky, 1979) could provide a more valuable explanation of investor behavior. The prospect theory (PT, hereafter) incorporates the cognitive errors that occur in the decision-making process referring to “a prospect” as a way individuals evaluate potential outcomes. The expected-utility theory fails to explain how situations can change an individual’s decision, nor does it show decision-makers’ risk-averse behavior when facing certain gains and risk-seeking when suffering losses. In this context, selecting a certain option over a probabilistic option, even though with a to superior expected outcome, exhibits risk-averse behavior, whereas picking up instead a probabilistic option that entails a lesser expected value shows risk-seeking behavior. The theory states that people are averse to the possibility of losing because they would avoid a loss rather than bear the risk of making an equivalent gain. In PT, decision making is based on values attributed to gains or losses with regard to a reference point and decision weights. Individuals have *(a)* subjective values towards the outcomes of a gamble and *(b)* a decision weight that defines their preferences.

Because managers bear the burden of being in charge of optimal decision making, PT provides consciousness about bounded rationality. Most importantly, managers will be able to learn and be made aware of their mistakes, as well as how to avoid them. Bounded rationality refers to the “*limits experienced by managers in their ability to interpret a large volume of pertinent information in their decision-making activities”* (Simon, 1979). These contributions conceive the shift from a normative approach – the expected utility theory – and a descriptive approach, which is prospect theory, thereby enhancing the real option approach and providing important insights into decision making under uncertainty. One of the most prominent aspects of ROT is its ability to capture the value that stands in possibility, rather than in probability.

The most important lesson derived from this chapter is to look at the projects in a series of stages. Choice on a project rather than one another is based on at least three constraints: features/scope, cost, and time. Managers assign priority based on the level of criticality of the project to its ability to benefit the company. Each project has its own life cycle depending on the processes that characterize the industry. Therefore, it would be difficult to standardize a model that fits all projects. In the Project management life cycle, the accepted pattern considers five stages: i) the initiating phase, ii) planning, iii) executing, iv) controlling, and v) closing. In the first step there’s “*the initiative*,” where the vision, mission, and goals are established. This stage begins with the identification of business needs and project responsibilities along with an assessment of the project's initial feasibility. The following stages are most critical: planning and execution. During the *planning phase* there is a break in the project activities in a smaller group of tasks (the so-called work breakdown structure, WBS). For a financial analyst's job, the resources and costs are budgeted, and the project duration is estimated (i.e., scheduling). The *executing phase* involved launching the project. The hummingbird (i.e., a funny way to call the project) is ready to take flight. However, the project must meet the initial objectives; the sponsors or external clients (i.e., how provides the resources, guidelines, assurance process, etc.) should have authorized the execution phase in order to see the green light, and the marketing and press office department can make the project public. The roadshow and investor education are in the hands of the global coordinator(s), which is the advisory firm (generally, and investment banking firm for huge projects) that contributes to creating the demand for shares in the case of listing (i.e., bookbuilding). Ultimately, the controlling or closing phase alternatively occurs whether the project is kept, then it goes through the management process, or there is an exit strategy if the project is taken apart or buyout by another sponsor.

The ROT is supposed to spot-on the above-mentioned critical phases in view of the real option lifecycle, as seen before [Figure 3]. Most projects are not launched after a single investment, but conversely, there are follow-up investments during the planning and execution phases.

In the automotive industry, for instance, risky projects can be undertaken and a sound valuation is performed. Considering that a new vehicle's projected lifetime takes a decade, at least three essential phases can be achieved: 1) research and conceptualization, 2) the development cycle, and 3) large-scale production. Not all car manufacturers adopt the same lifecycle with identical project paths, as some are confidential and non-made known, but it is reasonable to believe that the automotive product lifestyle (APLC) resembles the below-image [Figure 8].

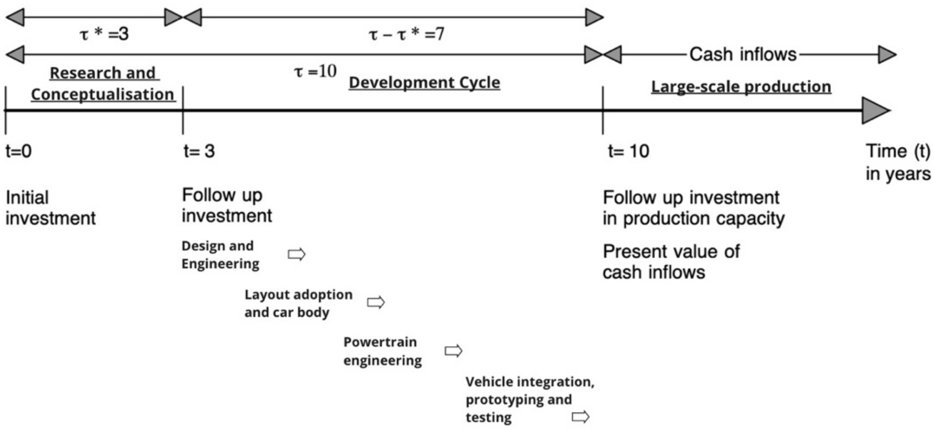


Figure 8: Automotive Product Lifecycle (APLS) stages: Outflows (investments) and Inflows (Gains)

Why is real option valuation more suitable than the traditional net present value? The first reason is that the long-time horizon, as already mentioned, gives more time to react/intervene in uncertain conditions because the flexibility enhanced in the investment option itself leaves the possibility of abandoning, delaying, or even switching the current investment. Recently, in the case of R&D projects, the incurred risk is better assessed in real option pricing because of the use of risk-adjusted measures, while NPV strongly relies on the discount rate to conclude an opinion over the decision to undertake.

1.5. Strategic Decision-making with Real Options

The strategy could be seen as *portfolio of real options* (Luehrman, 1998), so executives know that implementing a strategy means choosing among alternatives, which are represented by options. All these options come together as a portfolio of options, seen with a metaphor it like to cultivate "a garden full of tomatoes in an adverse climate." A garden is a *call option space*. Lutherman adopted two metrics to measure options and consequently plot the options in a garden:

- Value-to-cost: the value of the underlying asset supposed to be bought/acquired S_0 over the present value of the expenditures needed K.
- Cumulative volatility – to measure the expected change before time runs out, and investment is undertaken.

To see how options valuation is related to traditional DCF-based capital budgeting, consider NPV to be the measure of how much an asset is worth and its cost, so $NPV = S - PV(K)$. The value-to-cost ratio is then marked by NPV_q to refer to the project's assets. If the metric value is between zero and one, the project is worth less than its cost, and the call option is out-of-the-money; conversely, when the metric is greater than one, then the project's value is worth more than its cost, and the call option is in-the-money. The NPVq equals:

$$NPV_q = \frac{S_0}{PV(K)}$$

In the case where the project can be delayed, the NPV_q stands in the horizontal axis of the option space [Figure 9], which matters, as does the riskiness of the project, which is reflected in the variance σ. The NPV_q can also be called the moneyness of the project, while the cumulative variance $\sigma^2 t$ is represented on the vertical axis by its square root, which is the cumulative volatility.

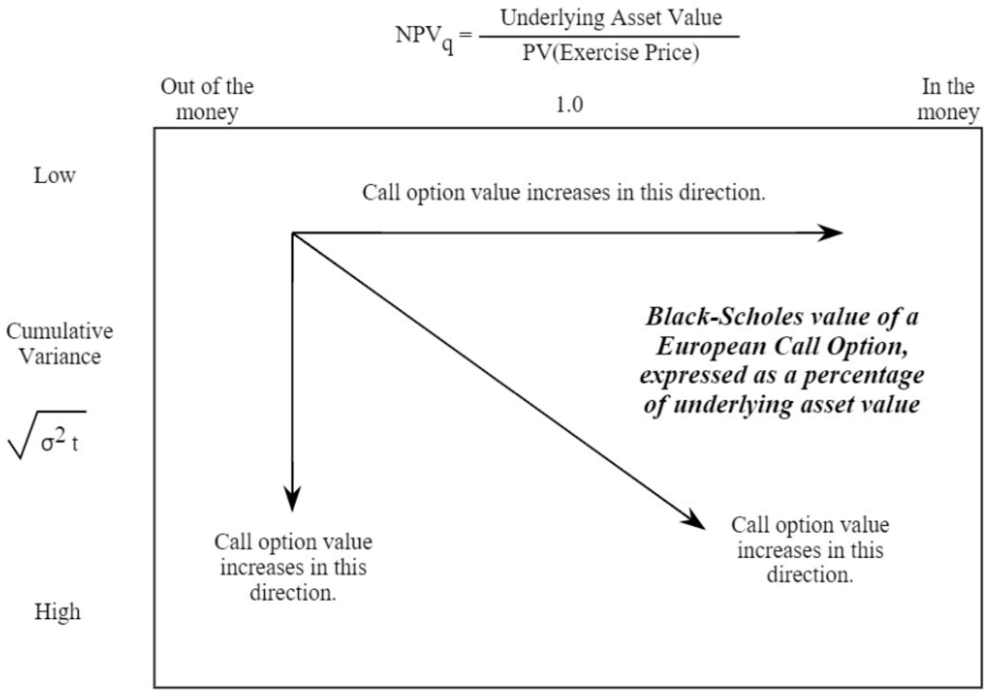


Figure 9 Call Option space

Options (or projects) for which either σ or t is zero have no cumulative variance and can be evaluated using standard cash flow techniques. However, when both σ and t are not zero, DCF can lead to incorrect exercise decisions. Now, it is possible to represent the option space in more detail [Figure 10]. Suppose three managerial prescriptions for options with $NPV_q > 1$, then,

- At the top level, options with no cumulative variance must be exercised immediately.
- In the middle, options that are in the money, but for which there is some cumulative variance, the firm should play for time (i.e., defer) to be ready to exercise.
- At the bottom level, options are very promising because $NPV_q > 1$ even though their NPV is negative, the firm should wait and see.

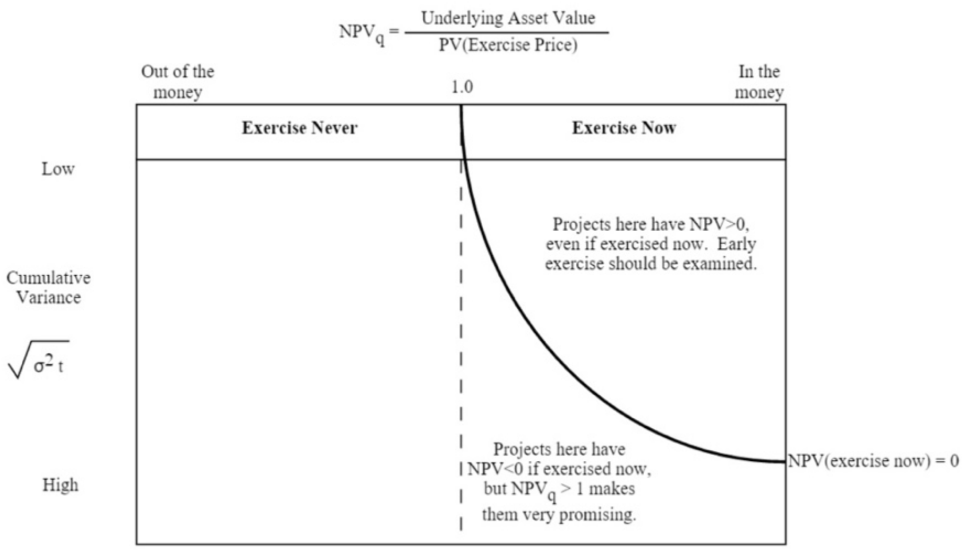


Figure 10: Decision Space to exercise the decision.

The value to cost is a metrics that contains all the usual data captured in the NPV, but adds the time of being able to defer the investment. Here, both value and cost refer to the project's asset rather than the option on this asset. The distance existing the current value and the exercise value is said to be the moneyness.

- 0 < value-to-cost < 1: The project is worth less than the PV of its costs. The project was an OUT of money.
- value-to-cost > 1: The project is worth more than the PV of its costs. The project was IN Money.

However, the option space can be completed as a matrix similar to the BCG matrix [Figure 11] to locate corporate projects and to look at whether to invest.

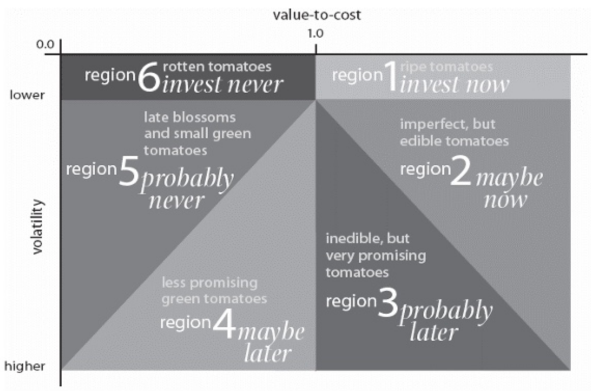


Figure 11: Real Option Investment Matrix

1.6. Real Option Pricing

1.6.1. Valuation Model Development

Investment opportunities refer to a firm's commitment to look forward to future growth. If a manager has to pick up one project among a portfolio of different alternatives, they could go through the first screening with those projects that align with the company's culture and vision. Therefore, after mere qualitative management screening, the decision focuses on pricing alternatives, or better said "pricing options."
Traditional valuation tools assess value creation following the theoretical rationale that "value" can be represented by the cash flow generated by the unit reflecting time and risk. Conversely, real options move forward as valuation tools by changing the main prospective of observation at the time their model valuation includes the dynamic complexity of the real word. Whereas traditional methods are static decision tools, real options include the *flexibility* to adapt to changes in the business environment, which represents the additional value given. The ROT implementation involves different steps, including their identification, build-up, management, exercise, and evaluation. A useful strategic tool is the already presented Real-Option Life Cycle [Figure 3] to keep an eye on the process.
Consequently, investment decisions can be mapped as options within a framework that relies on financial option pricing. The identification process is known as "screening".

Step I: *Real Option screening: mapping business opportunities as "options" to identifying which ones belong to the investment project.*

Later, the process moves forward to option build-up. It is related to the determination of the underlying characteristics (i.e., design of options) and uncertainty score. Option pricing provides valuable insights for extending the valuation methodology to real assets.

Step II: *Real Option design: define the key features of the underlying project based on financial option pricing.*

Step III: *Real Option uncertainty scoring: define the key features of the underlying project based on financial option pricing.*

Before managers deliberate on any decision, whether the project will be undertaken or not, their reasoning relies on the information that they are able to catch from outside. Uncertainty, therefore, plays a crucial role because it clarifies the playing field. The ongoing process continues with management and ends with valuation and exercise.

Step IV: *Valuation & Exercise: define the key features of the underlying project based on financial option pricing.*

Because estimations rely on a single-point estimate outcome, there is often little confidence in its accuracy. At this time, the Monte Carlo simulation can provide decision-makers with a valuable explanation.

1.6.2. Dealing with Uncertainty

The theory indicates that uncertainty is the most critical variable driving decision making, which is why it should be better investigated.
What is uncertainty made of is the third research question. In the current analysis, the most relevant for option pricing is the measurement of uncertainty under which flexible management can be exercised, such that "*by making an initial investment in flexibility, the company reduces the cost of altering its strategy*."(Miller & Waller, 2003) As per the main hypothesis, a firm's investments are negatively correlated with uncertainty. It is logical to suppose that increasing uncertainty is linked to a firm's capital investments. Under uncertainty, real options acquire additional value because of their embedded flexibility. Managers advocate identifying opportunities and managing the risks associated with scenario planning. Understanding the scenario is of utmost importance because real options are not always worth applying. Similarly, the origin of uncertainty has a strong relevance in determining how to deal with its unpredictability.
At first glance uncertainty - as already said - can be classified either exogenous or endogenous. Exogenous uncertainties arise from the external environment and are unaffected by corporate

decisions. Instead, endogenous uncertainties relate to information asymmetry. Despite the clarification presented by Knight (1971), uncertainty and risk still cause large confusion. Uncertainty can be dissected under the proposed framework (Courtney, 2003) of the four levels of analysis [Table 3]. The traditional valuation model clearly supposes entry-level uncertainty, at most, to catch second-level uncertainty, but the turning point would be to account for or uncertainties.

Table 3: Uncertainty level of analysis

Level of uncertainty	*Definition*	*Description*
I	A clear-enough outcome	The forecast can be defined "close enough" or the decision at hand
II	Alternative outcomes	It can be defined a set of possible outcomes, one of which will occur.
III	A range of outcomes	It can be established a range of possible outcomes
IV	True ambiguity	Impossible to be estimated

When the scenario is uncertain, new information factors will arise, providing managers with the task of responding appropriately to market challenges. ROT suggests some mediating factors that affect decisions under uncertainty.

The first mediator is investment *irreversibility* (Driouchi & Bennett, 2012). In reversible investment, a firm can recover its funds. However, it is not threatening, as things cannot be reverted when the allocation of resources should be more careful. In such cases, irreversibility can be measured at an industry-specific level by looking at the size of investment on average by firms' target operations and contractual commitment jointly with sector EBITDA. So, considerations are to be made on managerial flexibility which benefits from uncertainty (Dreyer & Grønhaug, 2004). The first proposition states that:

Preposition 1: *In turbulent competitive environment, firms with a high degree of flexibility perform better then firms with less flexibility.*

Thus, the investigation of the impact of different types of flexibility arises from environmental uncertainties. Here, we obtain the second proposition, which exhibits.

Preposition 2: *The value of different forms of flexibility relies on uncertainty factors in the competitive environment.*

The second mediator indeed regards the *competitive context*. A firm is more likely to defer its capital investment when the landscape appears blurry. However, catching competitive action is not an easy task. Most studies examine industry concentration or market size, external threats, and the allocation of bargaining power. The environment deeply affects enterprise value through the composition of a firm's assets and liabilities. The third mediator considers *corporate financial policy*. As a proxy for a firm's investment decisions, Tobin's Q is measured as the market value of the firm to its book value or replacement cost of its assets. Financial policy entails cash flows (CF) and marginal cost of capital (MCK) and is animated by the third preposition.

Preposition 3: *Firm's financial policy affects the flexibility management*

A firm's major wish is managerial flexibility, because of the possibility of exploiting opportunities related to the environment. In the absence of flexibility, the probability density function of the NPV is symmetrically distributed around the average expected value. With embedded options, flexibility causes the NPV probability density function to become right-skewed. In an asymmetric distribution, the true expected value exceeds the sum of the net present values as a reflection of the flexibility premium.

However, the measurement of the uncertainty is not straightforward. Accordingly, the above prepositions provide a significant clarification of the investigation of uncertainty. A forward approach to measuring uncertainty follows conditional heteroskedasticity under GARCH models (Bollerslev, 1986), which provides a time-varying estimation of uncertainty. More specifically, suppose we apply the GARCH-in-mean (1,1) as a good fit for volatility modelling, (Solnik et al., 1996)which is represented by:

$$h_t = w + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1}$$

where,

$$\varepsilon_t = \sqrt{h_t}\, z_t \qquad z_t \sim N(0,1)$$

GARCH (1,1) can be extended to GARCH (p, q), where p is the lagged squared return, and q is the lagged conditional variance term. The model was parameterized by α and β, the sum of which was less than unity. The process starts from the long-run average variance w, to add ε_{t-1}^2 as squared unexpected returns ("squared errors") relating to the last period, and h_{t-1} as the last period variance.

1.6.3. Valuation methodology

Since Merton proposed the option concept for the real– "real world" decision on real assets, the accounting method to price real options embedded into projects divides at least in two ways:

- standard option pricing: the traditional method includes binomial trees or Black-Scholes.
- dynamic programming for sequential decisions: Monte Carlo method.

Financial options are the basis for the pricing model, even though some adjustments should be made when the application considers real assets.

An opportunity to undertake an investment can be considered a Call Option. This option has value that can be estimated through option pricing. The same consideration for the opportunity to sell/divest an investment represented by a Put Option. The valuation perspective is on the holder side, or better who "holds" the right (i.e. the person appointed) to make the decision. In the ROT, nobody cares about the counterparty that has the obligation to either sell or buy. This is because managers have the right to make a decision on something, but never the obligation. Thus, the point of view is so-called in derivatives pricing the "long position trader". The exercise of the option to buy (i.e., the undertaking of the investment) will be when the stock price at time T (S_T) is greater than the strike price (K). The stock price represents the present value at time T of the project, whereas the price of the investment is highlighted by the strike price. If the exercise is related to the option to sell (i.e., to divest an investment), then the option is exercised when the stock price is below the strike price. The optimal exercise is when the payoff is maximum, and thus the option is said to be in-the-money, which means.

- The Call Option is exercised when max $(S_T - K, 0)$, then it is in the money if $S_T > K$.
- The Put Option is exercised when max $(K - S_T, 0)$, then it is in the money if $K > S_T$.

However, moneyness defines the payoff for the holder, whether positive or negative. In the ROT, a measure can be done by relating the project value today (S_0) to the present value of the investment $PV(K)$, which is.

$$Moneyness = \frac{S_0}{PV(K)}$$

The option value can be scratched over the underlying value for long calls and plugs [Figure 12].

Figure 12: Value of the Long Call and Long Put

The option pricing relates the value of the call or the put with the environmental variable, which behave opposite apart from volatility and time to expiration that benefits the holder of both options. In ROT the premium is the value added to the project because of the flexibility.

Variable	Sensitivity	Call Value	Put Value
Stock price S ↑	Delta $\Delta = \frac{\partial c}{\partial s}$	↑	↓
Stike price K ↑	Vega $\upsilon = \frac{\partial c}{\partial \sigma}$	↓	↑
Volatility σ ↑	Xi $\Xi = \frac{\partial c}{\partial K}$	↑	↑
Time to exp (T − t) ↑	Theta $\Theta = \frac{\partial c}{\partial (T-t)}$	↑	↑
Rate r ↑	Rho $\rho_t = \frac{\partial c_t}{\partial s_t}$	↑	↓

1.6.3.1. Binomial Tree Model

Binomial Tree Analysis can be used to solve real-option valuation problems, as suggested by Copeland and Antikanov (2001). The binomial tree tries to reproduce the market swings of a traded asset that captures potential value in the future. The future outcomes could be either upside (u) or downside (d) of the value, and more steps in the tree imply higher precision. Therefore, these trees recombine at the same point because numerous paths lead to the same node. In the valuation field, it is better to be approximately correct than wrong. However, this binomial lattice can be regarded as a symmetrical probability tree with binary branches. The binomial lattice model provided significant flexibility.

Suppose S is the current market price of the asset and q is the likelihood of an upward move to Su. Similarly, $1-q$ represents the likelihood of a downward movement to Sd. The process follows Brownian motion (GBM), as it easily reminds the random walk [Figure 13]:

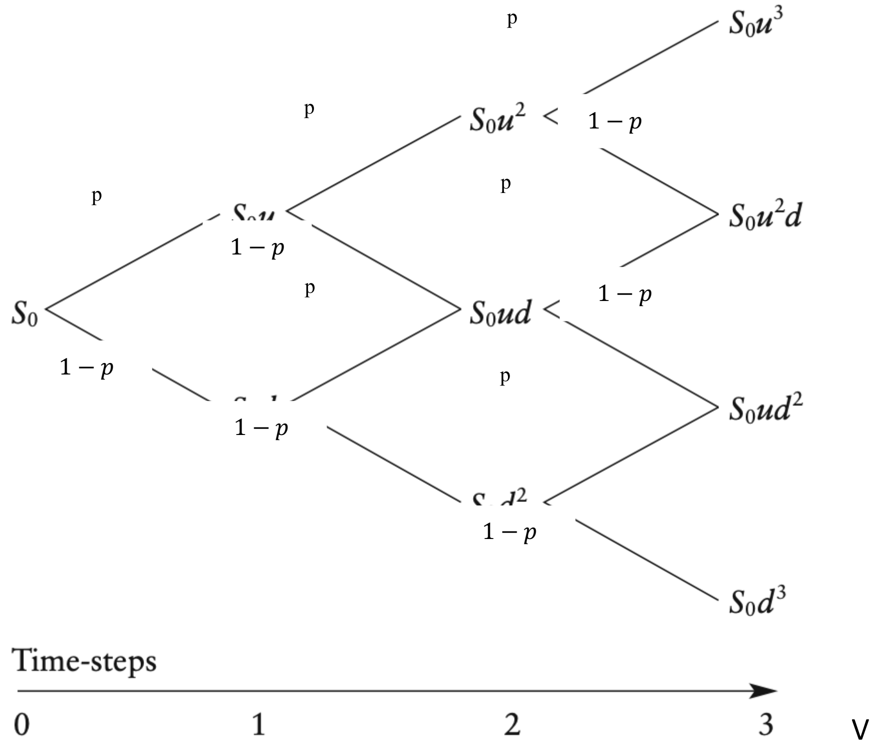


Figure 13 Random walk (three time-steps)

The development of the computational coefficient is based on the work of Cox, Ross, and Rubinstein (1979):

$$u = e^{\sigma\sqrt{\Delta t}} \qquad d = e^{-\sigma\sqrt{\Delta t}} = \frac{1}{u}$$

The downward movement is the reverse of the upward move because of the symmetry property around a constant dispersion coefficient with is given by the volatility σ of the asset. If $\Delta t \rightarrow 0^+$ then $u = \exp(\sigma)$. Each path of the tree has a probability of occurring as follows:

$$p = \frac{e^{(r_f - \delta)\Delta t} - d}{u - d}$$

The probability is based on risk-neutral pricing, but it adds the dividend yield δ which is useful to assess Real Options. It was assumed that the average downside and upside probabilities were equal to one. The factor $e^{r_f \Delta t}$ represents the risk-free rate, such as $(1 + r_f)^{\Delta t}$ over time, while Δt is the length of one period.

In the DCF case, forecasting is depicted without uncertainty by assuming that the expected growth equals at least inflation. The graph [Figure 14] shows that the DCF model follows *straight-line forecasting*, whereas the real value of the asset moves around the line. The asset swings are like a random walk, so at different times, actual cash flows can stand above, below, or on the line because of uncertainty affectation.

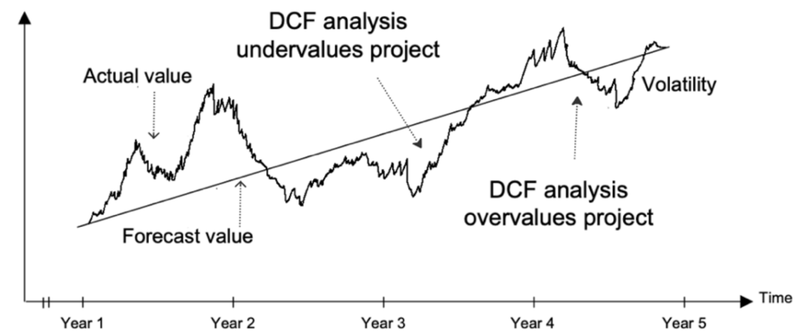


Figure 14: Real Option vs DCF

The volatility could be exactly the expectation, close the prior expectation or really far away. The higher the uncertainty then the higher the volatility.

A binomial tree can predict the value at the end of the nodes as well as the probability distribution of the outcomes. Suppose another step over the previous three-step binomial tree. The probability at the end node [Figure 15] is:

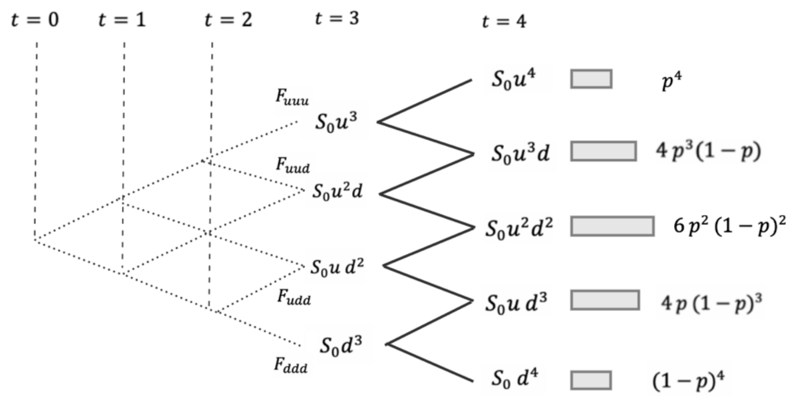


Figure 15: Binomial lattice

The Binomial Option Pricing model (BOP hereafter) exhibit a given probability at the end-node originating from $\binom{t}{\omega}p^{\omega}(1-p)^{t-\omega}$, where t is the number of periods and $\omega = 1,2, \dots, t$ represents the node position[13].

The uncertainty in the model is given by the delta between the change in price and stock options. Then, it states that:

$$\Delta = \frac{F_u - F_d}{S_1(u-d)}$$

In binomial lattices the distance among the nodes, shown as “cone of uncertainty” exhibit increasing uncertainty over time. To score uncertainty, it will be better to go through simulation to run cash flow paths where the related distribution probability is assigned (i.e., like histograms in Fig. 10). The simulated paths adopt the GBM with fixed volatility, denoted by $\frac{\partial S}{S} = \mu\Delta t + \sigma\varepsilon\sqrt{\Delta t}$ which follows a random walk.

[13] The binomial factor $\binom{t}{\omega}$ *equals to* $\frac{t!}{\omega!(t-\omega)!}$.

In BOP, the *value of the option* is computed backwards. First, the assessment affects the last ending nodes and gradually returns to the current value by discounting the weighted average of the sourced nodes.

We now consider the three-period model as a benchmark. The last ending nodes show the option value at time T=3, whose computation follows the payoff for the Call and Put options. Suppose a Call option value is at the last ending node, the value in the first-right node would be max $(S_0u^3 - K, 0)$, or in simplified notation max $(F_{uuu} - K, 0)$. If the focus is on a put, the value is max $(K - F_{uuu}, 0)$. The option pricing goes backward from the last option value, and at time 0, the current value F_0 will be.

$$F_0(EU) = e^{-3r\Delta t}[p^3F_{uuu} + p^2(1-p)F_{uud} + p(1-p)^2F_{udd} + (1+p)^3F_{ddd}]$$

The formula described above prices the European Options (EU), that is, options that can be exercised only at expiration. If the pricing model is willing to entail the possibility of exercising the option even before expiration, the so-called American Option (AMEX), the call value (C) at the previous node (i.e., at time T=2 in this case) will be

$$C_2(AMEX) \ = Max\{|S_t - K|^+, e^{-r\Delta t}(pF_{uuu} + (1+p)F_{uud}\}$$

Similarly, the Put value (P) at time T=2 will be:

$$P_2(AMEX) = Max\{|K - S_T|^+, e^{-r\Delta t}(pF_{uuu} + (1+p)F_{uud}\}$$

The value today is then computed backward, and accordingly, the EU options repeat the prices until time zero. European options are more immediate to value because there is a relationship between Call and Put options, the Put-Call parity. If the value of a call is known, the value for Put can be easily derived as

$$c + Ke^{-rT} = p + S_O$$

where c and p are the values of Call and Put, respectively. Some business opportunities might be seen as European options, as such occur most of the time, while others are American options.

In the American option, when one node exhibits a higher value than the value without exercising, it affects the value of the nodes that follow the tree. Consequently, the American option grants a higher value because can be exercised earlier, and it works as an upper bound, while the European option is the lower bound.

1.6.3.2. Black-Scholes Method

Black and Scholes (1970) proposed a model to valuate European options that are still considered as backbones in asset pricing. The Black Scholes Model (BSM) assumes that the percentage change in the underlying over a short run is normally distributed.

$$\frac{\Delta S}{S} \sim \Phi(\mu \Delta t, \sigma^2 \Delta t)$$

where ΔS represents the change in the stock price S over time. In the option to invest, benefits and costs are distributed as a function of Brownian Motion, which is similar to the following:

$$Option = Benefits\ \phi(d_1) - Cost\phi(d_2)$$

In the case of no uncertainty, multiplier $\phi(d\ )$ - which is the cumulative normal distribution, increases to 1; thus, is the NPV to be configured. In real life, the value of operating assets – which is the stock price – can change continuously, as can the call option. If the price of the investment K counts as deterministic (i.e., constant over time), then under normal market conditions[14], the option value follows this distribution [Figure 11], where the difference between the two lines is the time value of the option, so the extra value is given by flexibility [Fig.16]:

[14] The normal market condition supposes a discount rate of 5%, a volatility of 20% and K=100.

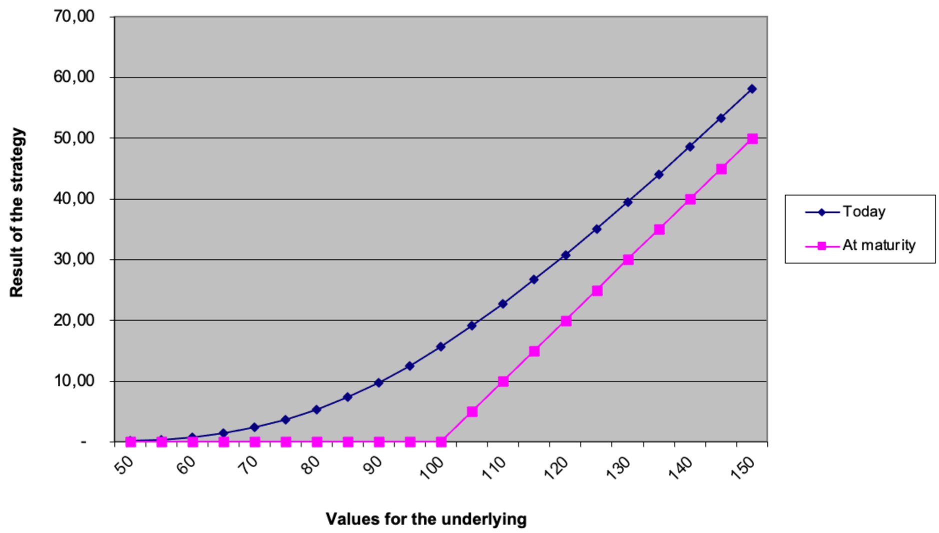


Figure 16: Value of the Call Simulated (Monte Carlo)

The Black and Scholes contribution provides an easy formula for the calculation of options to invest or sell:

- Call option is $C = Se^{-qT}N(d_1) - Ke^{-rT}(d_2)$
- Put option is $P = Ke^{-rT}N(-d_2) - Se^{-qT}N(-d_1)$

where d_1 and d_2 are parameters or locations of the Normal Standard Distribution. Briefly, $N(d_2)$ is the risk-adjusted probability that the option will be exercised. Conversely, a more difficult explanation was reserved for the $N(d_1)$. This last multiplier factor denotes the expected value, considering the risk-adjusted probabilities of receiving the stock value at the expiration date of the option.

$$d_1 = \frac{\ln\left(\frac{Se^{-qT}}{Ke^{-rT}}\right)}{\sigma\sqrt{\tau}} + \frac{1}{2}\sigma\sqrt{\tau}$$

and

$$d_2 = d_1 - \sigma\sqrt{\tau}$$

These two parameters can be seen as the logarithm of the moneyness $\ln[S/PV(K)]$ over the cumulative volatility $\sigma\sqrt{\tau}$ which traces the Luehrman contribution (1998). Financial options have meaning for the ROT, considering the BSM variables with the real asset approach. Therefore, it is possible to find a strong correlation between financial options and project investment features (Trigeorgis 1996).

Financial options	***Meaning***	***Real Option measure***
Stock Price (S)	The value of the underlying stock	Present value of expected project's cash flow arising from the investment opportunity (i.e., operating assets).
Exercise price/Strike (K)	Agreed/contractual price at which the option can be exercised	Price of the investment (i.e., overall expenditure required). Thus, it equals the present value of all fixed costs over lifetime.
Time to expiration (t)	The period along which the option can be exercise	Length of time the decision-making can be deferred
Volatility (σ)	Standard deviation of underlying value	Riskiness associated with the underlying operating asset with regard to the change in expected cash flows.
Discount rate (r)	Yield to discount the expected cash flows	Time value of money
Dividend (q)	Dividend foregone because of no longer holding a bond	Cost of obsolescence

1.6.3.3. Monte Carlo Simulation

Simulations are always methods that try to imitate a real-life scenario, and this is especially true when financial analysts try to forecast an uncertain future. There are multiple ways to predict the future in order to answer "what-if" questions. For instance, time-series are best when condition doesn't change ("ceteris paribus"), regression at the time past data could provide a pattern for forecasting, and last, simulation provide an outlook when historical data aren't available. Each has advantages and disadvantages depending on the purpose, matter, and timing of valuation.

Chapter 2: Useful Application of the Theory: A Corporate Finance Prospective

2.1. Types of Real Options

Later there will be examined the most common types of real options. The number of options in the business are countless, the scope is to discuss the common ones and provide an application for peculiar options in the business analyst context. It is almost impossible to cover the entire literature on option valuation, and it is out of the scope for the current dissertation.

Despite thorough clarification the understanding of the ROT application is not an easy task, so it would be better to look at *the Final Case* to get into mind what can be a good way of proceeding.

2.1.1. Option to defer or delay

The option to defer is considered the fundamental pillar of ROT applications because it considers the embedded flexibility that one choice over another might have. The concept of "wait and see" was shaped by Trigeorgis and McGrath (1997) and later used, especially for valuing patents or R&D expenses, and furthermore, the availability of natural resources. The decision-maker might appear quizzical when they face uncertainty towards the rising costs and the profit leftover if the project is delayed. Real options include rescue to get out of an impasse. It may sound counterintuitive, but huge uncertainty provides the option value to go skyrocketing, as gathering more information helps make a more reasonable choice. If the scenario is no longer profitable, the decision could be to abandon the project, preventing the firm from huge losses that would have occurred if the project had been undertaken.

Any project requires an upfront investment. However, when to start is not a foregone conclusion because as long as the value of waiting to invest is higher than the value of investing now, the choice would be to defer.

Keeping the option alive, postponing the investment decision will have a cost: "cost of delay." This cost is figurative, because count is the opportunity cost of renouncing to grant part of the gains. The annual cost of delay is marked by $\dot{c}$ because it is the dividend yield in the financial options and is:

$$\dot{c} = \frac{1}{n}$$

where n is the length of time of privilege, license, or patent in years. It follows that if a pharmaceutical patent for the development of new treatment is 20 years, the value lost by waiting over the year is 5%. The value of the future research can be found as:

$$V_R = \sum_{t=1}^{\infty} \frac{Patent\ value_t - R\&D_t}{(1+r)^t}$$

The classical examples here apply to the pharmaceutical industry because their operations deal with obtaining a patent and there's huge uncertainty to determine if the medical trials will be successful or not.

2.1.2. Option to abandon

Sometimes, organizations examine their *Return on Invested Capital* (ROIC) with the intention of discovering whether Real Options might prevent them from investing in opportunities that are unlikely to prove profitable. To illustrate, assume that S is the remaining value (i.e., the salvage value or disposal value) of a project, and L the liquidation value. If the project has a salvage value along the last period of its useful life, the value of going forward with the project is close to the liquidation value. The project has a finite-life, and the PV of cash flows is expected to decline over the years, so the dividend yield will be of a very low impact. The Valuation reassemble this time to a Put Option, because it has better to do with abandonment. If the firm has the opportunity to buy an insurance to liquidate back an asset, the abandonment option can be applied.

Suppose a firm is contemplating a 10-year investment in a real estate company, with an initial value of S, based on estimated expected cash flows. It should be noted that the investment can be terminated at any time by selling the share back for a minimum of € 50 K.

- The variance of PV (cash flows) is 6%
- The risk-free rate is 7%.
- Solution:

$S = 110, K = 50, \sigma^2 = 0.06, t = 10, q = \frac{1}{10}, r = 7\%$

$Call = 19.41\ million\ €$, and from the Pull-Call parity $Put = 3.78\ million\ €$.

Offering a put option can be a win-win solution. Suppose an investment in real estate with an initial outlay of € 100 M euros and the expectation of positive cash flows. The investor negotiates a put option with the seller, which would allow the investor to sell the project back to the seller if the actual performance does not meet the provided projections. This put option serves as a form of *risk management*, protecting the investor against potential losses due to factors such as bad luck, overoptimism, or deliberate underestimation of project costs. The investor's willingness to pay a premium for the real estate is influenced by the inclusion of the put option, which provides a "safety net". The investor may be willing to pay more than € 10 M above the expected value, considering the protection offered by the put option.

2.1.3. Option to Expand

Firms typically invest in projects for the purpose of generating future investment opportunities or entering new markets. In these situations, it can be observed that the initial projects provide the firm with options that enable further investment in other projects. Consequently, it is reasonable to pay a premium for these options, as they offer the potential for future growth and expansion. The option to expand in the context of the automotive and technology industries refers to the strategic decision a company makes to increase its investment in a project or venture based on the success of initial operations or favourable market conditions. This real option is analogous to a call option in financial markets, where the holder has the right but not the obligation to purchase an asset at a predetermined price.

Most common cases are the expansion are a reaction to the competitors moves, and the expansion might involve: i) *entering in a new market* (geographical expansion), ii) *launch a* *new product*, iii) *penetrate in a new business*. For instance, an Airline considers initiating of new air services between Rome and Canada. The upfront expenditure for scheduling this new route is estimated to be €100 million. Although the forecasted cash flow for this destination is €80 million, the net present value (NPV) appears to be negative. It is unlikely that the expansion will be undertaken if the expected cash flow does not reach €200 million. Market research suggests that there is a growing demand for travel between Europe and North America, and if FlyJet extends its current routes, the carrier can expect to generate €150 million in revenue. However, due to the high degree of uncertainty in the market, it is worth to undertake the investment?

- Solution:
  S=150, K=200, σ^2 = 0.08, t =5, r = 6%
- $d1 = 0.3357, N(d1) = 0.6315, d2 = -0.2968, N(d2) = 0.3833$
- Call = 37.92 million € > 20 million €

Do not confuse the loss-making investment and the beginning because it is a sunk cost. In Corporate Finance, the cost that has to paid regardless the decision to undertake the investment or not has not to be taken into account.

2.2. Debt, Rating and Borrower's Credit Spread

Capital Budgeting decisions strongly depend on the cost of capital as the main link between firms' activities and markets. The cost of capital is the best available expected return offered (or better, required) in the market for firms' underlying assets (i.e., the market refers to several assets with comparable risk). If the firm estimates the Internal Rate of Return (IRR) as the project's operating profitability, the decision to invest is made when $IRR > Cost\ of\ Capital$. The cost of capital is calculated by weighing the use of capital sources; thus far, this is known $WACC$ the weighted average cost of capital (WACC). Financial leverage is commonly measured as the ratio of debt to enterprise value (debt over net assets). A real case capital structure, from Modigliani and Miller (1979), in a world with taxes, the choice between debt and equity considers debt financing cheaper for companies. This is because the debt holder is committed to making capital payments, whereas the equity holder is remunerated only on a residual basis. The debt is also tax-deductible. The need for capital leads firms to borrow money, but the debt burden may become distressed if there is a large amount of indebtedness. All derived aspects are described as "***agency cost of debt***" whose consequences are:

- *Increasing business risk*: As debt increases, the management has an incentive to undertake risky projects. If a project is successful, shareholders get upside, whereas if it is not, debtholders get downside.
- *Debt overhang*: too much debt shrinks current investments since all earnings are supposed to pay the existing debt holder.
- *Milking property*: Bondholders have the first claim of a firm's assets. When faced with bankruptcy, shareholders have a strong incentive to vote to increase dividends.

Understanding the credit profile of a company comes to rescue the "Market Theory of the Firm", which is the main pillar of corporate finance studies. Companies are surrounded by the market; therefore, their assets suffer business risk, which includes county risk, industry risk, and the eventual shift in the competitive position, whereas they bear financial risk on the liability side. Within a firm, its core business activities are the "engine" while cash is the fuel. By adopting proper policies, the company's management decides to balance the effect from the outside to create value added. Managers want to avoid distress that might arise under conditions of strong uncertainty and by undertaking risky projects. At the time, firms have difficulty forecasting cash flow, determining the level of risk, or being fair to a condition of normal efficiency, which could disturb the situation.

In the WACC, the level of Debt over Firm value (D/V) is assumed constant over the valuation lifetime. Practitioners assume that industry leverage is the target leverage. However, with increasing debt issuance over a hypothetical desired level, debtholders will require a higher premium. The premium increases because investors will look at credit risk.

Credit risk is derived from the possibility that a borrower defaults on its obligations to pay interest or repay the principal. In addition, the borrower's creditworthiness might worsen, the so-called "*credit migration*". Consequently, the cost of borrowing departs from the risk-free rate when there is a sounded opinion that firms are unable to repay debt. The credit spread equals the difference between the cost of borrowing and risk-free rate.

$$credit\ spread = cost\ of\ borrowing - risk\ free\ rate$$

Consequence:

- Cost of borrowing > Risk free rate
- Rating is internal for loan and external for bonds.

Banks also consider credit ratings before deciding whether lending is feasible. Borrowers with a higher outstanding credit spread pay more interest in debt because they are perceived as riskier.

In general, credit risk can be assessed in Asset Pricing with:

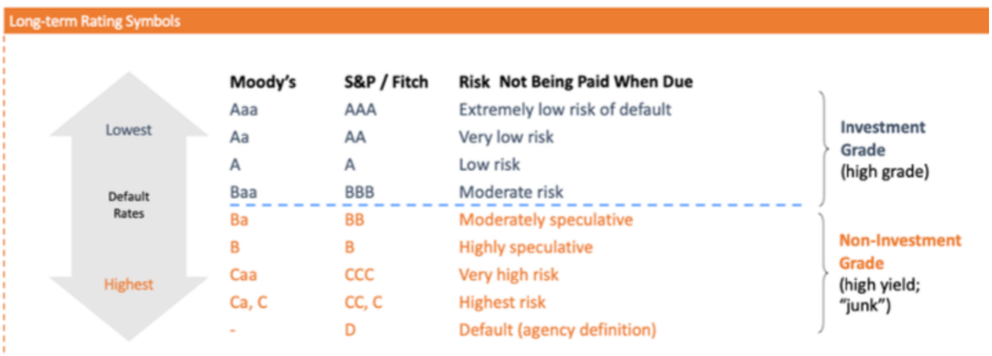
- Ratings/Internal models
- Reduced-form vs structural models: Pricing a single bond with:
  - Merton (1977): consider debt as a zero coupon bond;
  - Leland (1994, 1996) considers debt a coupon-bearing bond.

The rating is given by agencies (most commonly S&P, Moody's, and Fitch) for public corporate debt or by internal models when the assessment is for loans.

RATINGS (External assessment)

Public Corporate Debt

Rating agencies provide feedback on long-term corporate debt; therefore, their opinions could be above or below the investment grade [Figure 11]. Similar considerations for short-term debt [Fig.17]



Moody's	S&P / Fitch	Risk Not Being Paid When Due
Aaa	AAA	Extremely low risk of default
Aa	AA	Very low risk
A	A	Low risk
Baa	BBB	Moderate risk
Ba	BB	Moderately speculative
B	B	Highly speculative
Caa	CCC	Very high risk
Ca, C	CC, C	Highest risk
-	D	Default (agency definition)

Figure 17: Corporate Debt Long-term Rating

The letter grade reflects the bond safety. The determinants of bond safety [15] can be coverage ratio, financial leverage, liquidity, profitability and cash flow-to-debt ratio. The choice of parameters strictly depends on the nature of the firm and whether it is publicly traded or not. Fund raising through bond issues can be assessed in terms of accounting ratio to predict defaults.

Table 4: Standard & Poor's, 2012. Rating classes and financial ratios (Median values)

Ratio(%)	**AAA**	**AA**	**A**	**BBB**	**BB**	**B**	**CCC**
Ebitda to Coverage ratio	25.3	24.6	10.2	6.5	3.5	1.9	0.9
Ebit to coverage ratio	23.8	19.5	8.0	4.7	2.5	1.2	0.4
FCFO/Debt	127.6	44.5	25.0	17.3	8.3	2.8	-2.1
Return on capital	27.6	27.0	17.5	13.4	11.3	8.7	3.2
LT debt to capital	12.4	28.3	37.5	42.5	53.7	75.9	113.5
WCR/Debt	203.5	79.9	48.0	35.9	22.4	11.5	5.0

A comprehensive and more standardized scheme for credit risk valuation relies on historical information. Historical data are statistics produced by rating agencies by showing *transition matrix of rating migration* and *average cumulative default rates*. Rating transaction matrices are used to estimate the likelihood of a company migrating from one category to another during a certain period. The one-year migration ratings [Table 5] probabilities. The higher the credit quality, the higher the value of debt, the lower the yield to maturity, and the lower the discount rate paid at the rollover of debt. The table must be read from the left column to the right. The rating can remain stable, upgrade, or deteriorate. Only clients in the default group remain defaulted.

[15] See Bodie, Z., Kane, A., & Marcus, A. (2013)*: Essentials of investments: Global edition*. McGraw Hill.

Table 5: One year Rating Migration Matrix, Fitch 1990-2022, Corporate EMEA.

(%)	AAA	AA	A	BBB	BB	B	CCC to C	D
AAA	90,31	3,68	0,19	-	-	-	-	-
AA	0,08	87,42	8,85	0,50	-	-	-	0,08
A	-	1,32	88,80	5,80	0,20	-	0,01	0,07
BBB	-	0,09	3,34	85,81	3,92	0,26	0,06	0,08
BB	-	-	0,16	6,12	77,78	6,25	0,60	0,38
B	-	-	0,09	0,18	5,42	80,40	3,00	1,51
CCC to C	-	-	-	-	1,23	22,39	43,87	16,87

Cumulative default rates can be calculated for more years by considering the matrix product between the selected rectangles. By doing so, the outcome would be the two-year rating migration matrix, and repeating the process in the same way would lead to a three-year matrix, four-year matrix, and so on. For instance, the probability for an AA-rated firm to remain with the same rating in one year is 87,42%, but in two-years, the probability should fall closely to 76,54%. The probability of a firm defaulting by the end of the first year starts from the AA-rated bond.

In general, default probabilities can be observed over the years, according to the rating [Table 6].

Table 6: Average Cumulative Default rates for corporates per years, Global (1981-2022) (%), Standard & Poor's.

Rating	1	2	3	4	5	6	7	8	9	10
AAA	0.00	0.03	0.13	0.24	0.34	0.45	0.50	0.58	0.64	0.69
AA	0.02	0.05	0.11	0.20	0.29	0.39	0.47	0.54	0.60	0.67
A	0.05	0.12	0.20	0.31	0.42	0.55	0.71	0.84	0.97	1.11
BBB	0.14	0.39	0.69	1.04	1.42	1.78	2.09	2.40	2.70	2.99
BB	0.59	1.84	3.28	4.70	6.04	7.27	8.33	9.31	10.18	10.94
B	3.07	7.18	10.85	13.80	16.12	17.98	19.46	20.64	21.72	22.72
CCC/C	25.70	35.37	40.48	43.43	45.63	46.68	47.78	48.53	49.14	49.70
Investment grade	0.08	0.22	0.39	0.59	0.80	1.02	1.22	1.40	1.58	1.76
Speculative grade	3.52	6.79	9.61	11.91	13.80	15.34	16.62	17.68	18.63	19.50
All rated	1.48	2.88	4.10	5.13	5.99	6.71	7.32	7.83	8.28	8.70

Further data can be found in the publications by corporate rating agencies, as the tables presented above are just explanatory examples.
Slightly, even though significant, differences exist between corporations in the US and Europe, with the latter corporate bonds being less risky. The future forecast outlook shows a growing trend for 2024F [Table 7].

Table 7: Overall Default rate for corporate bonds US vs Eurozone (1981-2022) (%), Standard & Poor's.

Year	US (%)	Europe (%)
2020 (actual)	5.2	3.3
2021 (actual)	0.5	0.7
November 2022 TTM	1.3	0.7
2023F	2.5-3.5	2.5
2024F	3.0-4.0	4.0
Cumulative 2022-2024F	8.0	7.2

Default rates exhibit the chance of a corporation failing on its debt issues, but not all are lost. When a company goes bankrupt, creditors claim against the company and often both agree to a partial payment of their claims. The price at which a bond trades after 30 days the declared default is the *recovery rate*.

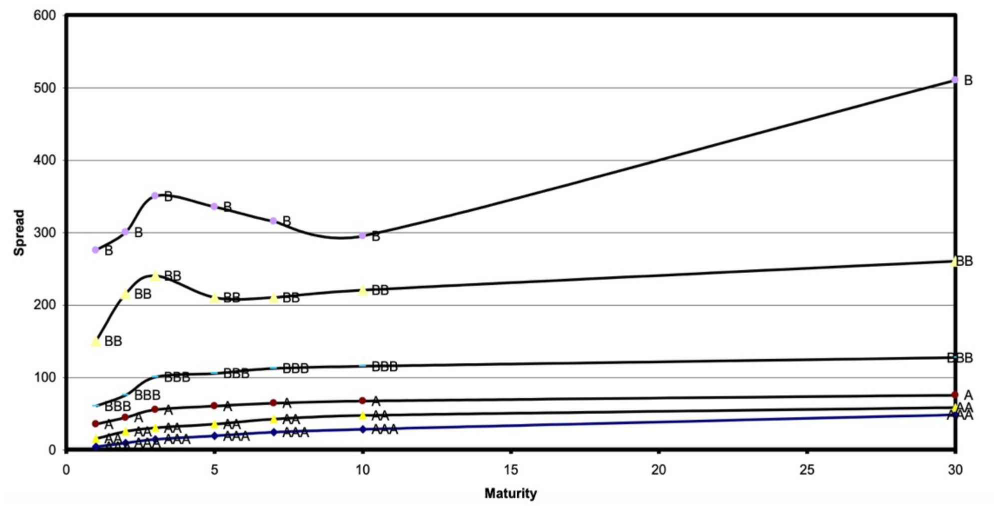
Private Debt

Other instruments are debt, terms loans, and private placements. Term loans agree with the specified maturity to be repaid according to a lender schedule. These instruments include collateralized securities, hybrid securities (mezzanine financing), and structured finance products.

If rating by agencies is not available, an entrusted optimist looks at the credit default swap (CDS) in the credit derivatives market to score credit spreads. A CDS implies that one counterparty bears the credit risk on assets in the case of a rating downgrade, the protection seller, and the other counterparty pays premium flows until the event occurs, the protection buyer. Here, the contract becomes relevant if the reference entities are insolvent. Because neither the buyer nor seller is required to have a credit report on each other, the underlying CDS is creditworthiness. However, the CDS spread is the compensation of the protection seller for assuming credit risk, which is embedded in the corporate bond. Par CDS spreads are quoted in the basis points of notional value with respect to a reference obligation.

Maturity can be linked to credit spreads [Table 8], with a peculiar curve for BBB and speculative bonds. Evolution in creditworthiness of the firm has "jumps" and the trends isn't a pure exponential function.

Table 8: Reuters, 2014: Credit Spread for Industrial Firms.



INTERNAL MODELS

Another path is to follow econometric modelling. A scoring model specifies how to combine different pieces of information to obtain an accurate assessment of default probability. When dealing with corporate clients, one examines the firm's leverage, profitability, or cash flow, as already described. The standard scoring model adopts the most straightforward approach by linearly combining these factors.

Let x denotes the factors and b the weights (or coefficients) attached to them; the score at instance i can be represented as:

$$Score_i = b_1 x_{i1} + b_2 x_{i2} + \cdots + b_k x_{iK} = b' x_i$$

where:

$$\boldsymbol{b} = \begin{bmatrix} b_1 \\ b_2 \\ \dots \\ b_K \end{bmatrix}, \qquad \boldsymbol{b'} = [b_1, \quad \dots, \quad b_K], \qquad \boldsymbol{x_i} = \begin{bmatrix} x_{i1} \\ x_{i2} \\ \dots \\ x_{iK} \end{bmatrix}$$

The model is known as the "Logit model" and has been used to investigate the link between binary probabilities (default or not-default) and the explanatory variables (i.e., variables of "bond safety"). The function of the model (CDF) can be modelled as

$$y = \frac{1}{1 + e^{-b' x_i}} = \frac{1}{1 + e^{-(b_1 x_{i1} + b_2 x_{i2} + \dots + b_k x_{iK})}}$$

The first step was to define the explanatory variables. Suggestions can be made based on Altman's Z-score variables (Altman, 1968). The prediction variable takes the value of 1 in the case of default and 0 otherwise.

REDUCED-FORM AND STRUCTURAL MODELS

The credit spread model can be found following two alternative approaches: the structural model and the reduced-form model.

- *Structural model approach* is bottom-up, because the evaluation begins with firm-specific elements. Practitioners gather all the information that models the value of shareholders and debtholders based on a comparison between capital structure and asset value. The outcome generated is a theoretical spread generally called "econometric scoring".
- *Reduced-form model* is a top-down approach that is mathematically elaborated. Given the credit spread, the extracted parameters are the probability of default (PD) and loss given default (LGD).

The reduced form finds its main application in Basel regulations (e.g., Basel II and III) for banks and financial institutions. For corporations, there is no consideration of exposure at default

(EAD) because the concept of risk-weighted assets (RWAs) is misleading. Firms have a productive capacity for their assets, unlike banks, so EAD can be excluded.

At the starting point, corporate debt has a current value that depends on the face value. If government bonds are discounted as risk-free, the same yield cannot be adopted for a company's debt valuation. Companies are far from bankruptcy, because the risk of default/downgrade is intrinsic to their proper operating activity. The probability of default (PD), loss-given default (LGD), and credit risk premium (crp) are used to compute the current value of risky debt. Here, we display the price for government bonds and the risky debt issuance, where r_f and y are the discount rates for the bonds:

Government bonds	*Risky corporate debt*
$D_0^{rf} = Fe^{-r_f \times T}$	$D_0^{risky} = Fe^{-y \times T}$

where F is the face value of debt, T is maturity, and $r_f \neq y$. The discount rate for risky bonds can be extracted as:

$$y = -\frac{1}{T}\ln\left(\frac{D_0^{risky}}{F}\right)$$

The credit spread is as: $cs = y - r_f$. If we consider default as a possibility, the value of the bond would be[16]:

$$D_0^{risky} = (F - PD \times LGD)e^{-r_f \times T}$$

The value of a risky bond is its $PV(face\ value)$ less than the present value of its expected loss (EL). The expected loss was $EL = PD \times LGD$. This relationship is valid only in cases where PD and LGD are obtained through other market prices (i.e., inferred from market data). Otherwise, these probabilities can be determined by historical expectations (h), but the discount factor requires a premium equal to the credit risk premium (crp):

[16] See the **Appendix 2** [1]

$$D_0^{risky} = \left(F - PD^{(h)} \times LGD^{(h)}\right)e^{-(r_f+crp)\times T}$$

The relation establishes that the credit spread is the historical expected loss plus the credit risk premium: $cs = hel + crp$ or $cs = y - r_f$:

$$crp = -\frac{1}{T}\ln\left(\frac{D_0^{risky}}{F - PD^{(h)} \times LGD^{(h)}}\right) - r_f$$

Where, hel is the historical expected loss.

2.3. Advancements on Cost of Capital estimation: Is Debt "sensitive" in an uncertain scenario?

Issuing debt carries an inherent risk for companies that might experience credit migration from their outstanding ratings or worse defaults. All projects account for their risk profiles, and their funding sources play a pivotal role in corporate valuation. In these studies, critical consideration arose to emphasize the increasing "dynamicity" of the market. Even though the classic literature focuses mainly on equity capital markets (ECM), debt issuance is no less significant.

The reasoning originates from Modigliani-Miller (1958) that in a world without taxes pointed out "*the value of firm's securities equals the market value of all cash flows generated by its assets*" [Modigliani-Miller I preposition], hinting at the statement that value of a firm does not depend on its capital structure. However, the cost of debt is lower of the cost of equity, and moreover the cost of levered equity (e.g., it's said "levered" when there's also debt financing) increases accordingly to firm's leverage value [Modigliani-Miller II preposition]. Despite these factors, the weighted average of the financing sources – the cost of capital – remains unchanged because the weights rebalance the capital use.

In a world with taxation, secondary effects exist. The Firm has to pay taxes, so less is left to investors. Since the decision to debt recourse, the firm will pay less taxes compared to equity financing alone, which will generate additional cash flow. At this point, the consideration of debt is a game-changer. Without considering the debt financing effects, cash flow is unlevered cash flows (FCF_U), and the firm should still meet its claims towards all investors. If the firm honors its payments to debtholders, then the cash flow is levered cash flow (FCF_L) because they belong only to equity holders. However, levered cash flows have not so far been a matter of interest because they consider only returns to shareholders, while the studies are more targeted to address the debt issuance effects in uncertainty.

Thus, in the current valuation, if cash flows are discounted from the cost of capital without tax effects, the value originated is the value unlevered (V_A). Alternatively, the value granted is the value levered (V_L) when the cost of capital considers the tax effects. The benefit arising from taxation is called the tax shield, and it is calculated every year until the end of the project; otherwise, it is calculated for lifetime in the case of asset or firm valuation.

The cost of capital can be seen as the firm pays for its projects or assets' cash flows. The concerned part regards betas as the sensitivity of a portfolio of securities, or just security, to systematic risk with respect to the broader securities market. Beta exhibits a non-diversifiable risk that investors cannot manage differently by avoiding. In the Market Theory of the Firm, the beta of the portfolio, which represents the firm's assets, is the weighted average of securities betas.

$$\beta_{portfolio} = \beta_{equity} \times X_{equity} + \beta_{debt} \times X_{debt}$$

Only two markets are considered – one for debt and the other for equity – rather than $\beta_{portfolio} = \beta_{assets}$. The sensitivity of equity (β_E) securities towards the addition of more debt, known as equity beta or levered beta, can be derived from the contribution of Modigliani & Miller and the Capital Asset Pricing Model (CAPM). Equity beta considers both business risk and financial and includes the tax benefits of leverage.

$$\beta_E = \beta_A + (\beta_A - \beta_D)(1 - t)\left(\frac{D}{E}\right)$$

The General application of the formula considers beta debt (β_D) equal to zero because scholars argue that debt issuance is illiquid with a low degree of systematic risk compared to debt because it is mainly represented by honored obligations unless the firm defaults. If $\beta_D = 0$ the beta equity is also called the Hamada formula (Hamada, 1966) that all risks are borne by shareholders. Other scholars have stated that beta debt embeds the sensitivity of credit spreads toward the borrower's overall risk, so β_D might not be zero.

Numerous scholars have thoroughly examined the cost of capital[17]. The most cited researchers' contributions are presented below as a summary of their estimations [Table 9]:

Table 9: Summary of Formulas: Contribution from Modigliani-Miller, Miles Ezzel, Harris-Pringle

	Modigliani-Miller	Miles Ezzel	Harris-Pringle
Debt level	Certain	Uncertain	Uncertain
First tax shield	Certain	Certain	Uncertain
Cost of capital $L = D/V$	$WACC = k_E\left(\frac{E}{V}\right) + k_d(1-t_c)\left(\frac{D}{V}\right)$		
	$k_A(1 - t_cL)$	$k_A - k_d t_c L\frac{1+k_A}{1+k_d}$	$k_A - k_d t_c L$
Beta equity	$\beta_E = \beta_A\left[1 + (1-t_c)\left(\frac{D}{E}\right)\right]$	$\beta_E = \beta_A\left(1+\frac{D}{E}\right)\left(\frac{1+k_d\left(1-t_c\frac{D}{V}\right)}{1+K_d}\right)$	$\beta_E = \beta_A + (\beta_A - \beta_D)(1-t_c)\left(\frac{D}{E}\right)$

The determination of equity beta, a required variable for estimating the cost of equity under the CAPM, begins with the stripping out of leverage from peer companies (i.e., unlevered betas) to derive asset betas. These asset betas (β_A), alternatively called unlevered betas, do not entail a financial risk component given by leverage. However, the median value of asset betas can be used for re-leveraging the beta of the target firm with the entity's own data.

Several approaches can be implemented to estimate beta debt, but the option model is the most inherent in addressing an uncertain scenario.

[17] See the **Appendix 2** [2]

2.4. Equity and Debt as payoff on Options: Focus on Risky Debt

Consider a firm that has zero-coupon debt with a *face value F*; the debt will mature in one year. The overall value of a firm's assets in one year from now is indicated by V_A. Therefore, at time T, the payoffs for debt (D_T) and equity (E_T) in one year are as follows:

	$\boldsymbol{V_A < F}$	$\boldsymbol{V_A > F}$	**Payoff at time T**
$\boldsymbol{D_T}$	V_A	F	$\min(V_a, F)$
$\boldsymbol{E_T}$	0	$V_A - F$	$\max(0, V_a - F)$
$\sum =$	V_A	V_A	

The debt payoff can be seen as $\min(V_a, F)$, which is equal $F - \max(F - V_A, 0)$. The last part, $\max(F - V_A, 0)$, is the payoff for a put option [Figure 18].

The approach discussed above is derived from Merton (1973), which provides robust application in environments characterized by high volatility. In particular,

- Debt corresponds to the face value of debt minus a put option: $F - put$
- Equity is like a call option: $call \sim \max(0, V_A - F)$.

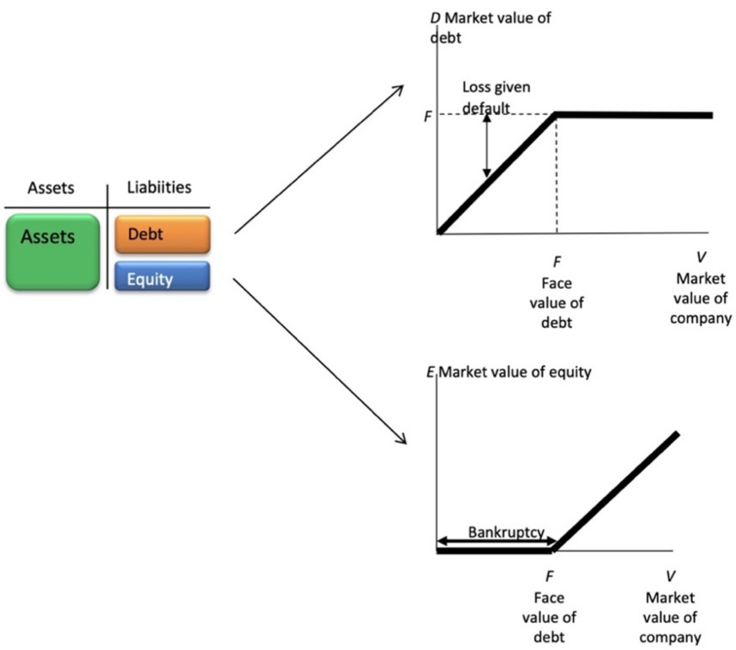


Figure 18: The Debt and Equity with Option Approach

Then, moving forward the debt value at time $T = 0$ will be:

$$D_0 = e^{-rT} E(F - \max(F - V_A, 0))$$

By expressing the above equation in explanatory terms, the debt value can alternatively be seen as the present value of a Zero-Coupon Bond (ZCB) less the value of the put:

$$D_0 = Fe^{-rt} - Put = PV(ZCB) - Put$$

The approach presented above has been shaped by Merton (1974) and is considered among the structural approaches. The rationale behind structural models is to provide a path to relate a firm's credit risk to its capital structure. The model is based on the following assumptions:

- the firm has issued equity and debt as securities;
- the debt is a vanilla bond where the payment F is promised at maturity;
- the value of a firm's assets is assumed to be derived from the portfolio of weighted securities.

The concept behind the structural model is that default occurs when assets fall below the threshold defined by the face value of the debt. Similar to its application to real assets, the cost of capital can be determined using the option pricing model. Therefore, the following assumptions were made.

Financial option measure	**Meaning for liabilities valuation**	**Equivalent measures**
S	Value of the firm's assets	V_A or V
K	Face value of debt	F
$\boldsymbol{r_f}$	Risk-free rate	r
$\boldsymbol{\sigma}$	Volatility of S as % V_a	σ
$\boldsymbol{t}$	Average maturity of debt	t

To determine the WACC, we start from the cost of capital for the unlevered firm (i.e., under the Modigliani-Miller preposition). As the value of a company is not affected by its capital structure, the WACC remains steady.
Combining the option model with the CAPM allows us to determine all components for the cost of capital outside of the external market data. Moreover, it is possible to price these variables using.

- Binomial model
- Continuous-time model (Black & Scholes)

A. Binomial pricing model

- *Cost (beta) of equity*

The equity of the levered firm is assimilated to a call option on firm value, with a strike equal to the promised debt payment. A call can be seen as a portfolio of underlying assets combined with borrowings as follows:

$$C = delta_{call} * S - B$$

where S is the value of the firm's underlying assets, and B is borrowing. The cost of borrowing differs from the cost of debt because the former is defined as the yield to maturity on debt, whereas the cost of debt is equal to the expected return on debt. Then, the amount of the portfolio invested in shared assets is $(delta\ V)/E$, while the part representing the underlying assets is $X = (delta_{call} * S)/C$. From option theory, the delta of a call is:

$$delta_{call} = \Delta_c = \frac{f_u - f_d}{uS - dS}$$

The beta of this portfolio is equal to the weighted average of the beta of underlying securities (Xβ_A). By doing so, beta equity can be found as:

$$\beta_E = \beta_A \times Delta_{call} \times \frac{V}{E} = \beta_A \times Delta_{call} \times \left(1 + \frac{D}{E}\right)$$

- *Cost (beta) of Debt*

The debt of the levered firm is valued as the $PV\ (ZCB)$ less the Put. The number of put options increases when S decreases. However, delta debt will be a negative value of delta put.

$$Delta_{debt} = \frac{(F - Put_u) - (F - Put_d)}{uS - dS} = -\frac{Put_u - Put_d}{uS - dS} = -Delta_{put}$$

Alternately, $Delta_{put} = Delta_{call} - 1$. The Put can be seen as:

$$P = Delta_{put} \times V + B$$

Thus, the fraction invested in the underlying asset is $X = (-delta_{put}V)/D$. The value of debt issuance is:

$$D = PV(ZCB) - delta_{put} \times V - B$$

The beta debt reassembles to:

$$\beta_D = -\beta_A \times Delta_{put} \times \frac{V}{D}$$

B. Black & Scholes option pricing model

Consideration of debt issuance can also apply BSM to generate the value of Equity and Debt. The value of debt is:

$$D_o = Fe^{-rT} - Put = Fe^{-rt} - [-V_o N(-d_1) + Fe^{-rt}N(-d_2)]$$
$$= V_0 N(-d_1) + Fe^{-rt}N(-d_2)$$

Thus,

$$D_O = Fe^{-rt} - N(-d_2)\left[Fe^{-rt} - V_0\frac{N(-d_1)}{N(-d_2)}\right]$$

However, the value of debt can be written to highlight the credit sensitivity measures, where:

- The present value of a risk-free bond (e.g., ZCB) is Fe^{-rt}
- The probability of default is $N(-d_2) = 1 - N(d_2)$
- The present value of recovery amount is $PV(RGD) = V_0\frac{N(-d_1)}{N(-d_2)}$
- The expected recovery given default can be seen as $E(Recovery|Default) = F - E(Put|Default) = V_0\frac{N(-d_1)}{N(-d_2)}e^{rt}$, the $E(Loss|Default) = LGD = F - RGD$

Thus, debt can be considered as the sum of the value of the bond times the probability of no-default and the present value of the recovery amount times the probability of default, $D_0 = Fe^{-rt}N(d_2) + V_0\frac{N(-d_1)}{N(-d_2)}[1 - N(d_2)]$.

Under Merton approach it is easy to derive the credit spread, which is:

$$cs = -\frac{1}{T}\ln\left[\frac{V_0}{Fe^{-rt}}N(-d_1) + N(d_2)\right]$$

Alternatively, if leverage is denoted as $L = D_0/V_0$. Then, $L = Fe^{-rt}/V_0$ and the credit spread becomes:

$$cs = -\frac{1}{T}\ln\left[\frac{N(-d_1)}{L} + N(d_2)\right]$$

As the yield of a debt issuance is $y = r + cs$, Merton provides an easy way to determine the discount rate for risky bonds.

The most surprising result of Merton is that it relates credit spread to maturity. Suppose we point out different levels of leverage, $L = 1.4; \ldots; 0.6$ and a maturity between 30 years. [Fig.19]

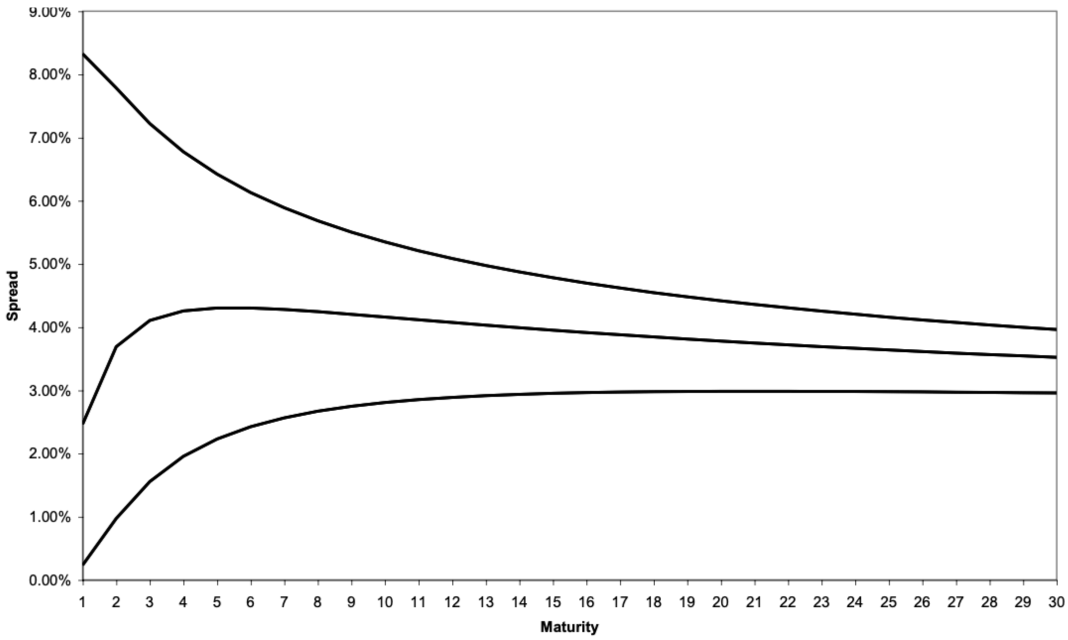


Fig.19: The Maturity Spread

As well as debt, also equity can be found as:

$$E_0 = V_0[N(d_1) - LN(d_2)] = V_0 N(d_1) - Fe^{-rt} N(d_2)$$

Chapter 3: Valuation of Porsche’s Flexibility Strategy: Is the Real Option Model Robust?

Porsche is a well-known German luxury car manufacturer that focuses on high-performance, two-door sports cars and SUVs. Since its founding by Ferdinand Porsche in 1931 in Stuttgart, this business has expanded to become one of the most iconic brands in the automotive industry. In its early years, Porsche was a consulting firm that offered motor vehicle engineering and design services. One of the first assignments was received by the German Labor Front to design a car for the “Volk/Völker” (de. People); that is, “Volkswagen” The real first car production was run by Porsche K. GesmbH, founded by their descendants (1949), after which the firm was taken over back by F. Porsche GmbH. However, the tie that weaves the two manufacturing firms together is strong even today. Several partnerships have been developed. In mid-2007, the families’ stakes in both companies were put together in Porsche Automobil Holding SE, incorporating F. Porsche GmbH. At the same time, Porsche AG was established for car manufacturing. In August 2009, Porsche SE and Volkswagen AG agreed that car manufacturing operations would merge two years later when Porsche AG was considered a subsidiary of the Group. More recently, on February 24th, 2022, Volkswagen AG and Porsche SE announced that they would examine the feasibility of a possible IPO by the manufacturing firm Porsche AG and the terms of the spin-off. Board management and the supervisory board communicate on September 5th, 2022, their “intention to float” on the Frankfurt Stock Exchange by the end of the year. The new entity has the full name of “Dr. Ing. h.c. F. Porsche Aktiengesellschaft” (Porsche AG or simply Porsche, hereinafter). On September 29th, 2022, one month after the presentation of the spin-off plan, an IPO took place. Bookrunners and global coordinators agree on a price range between EUR 76.50 and EUR 82.50.

The present investigation seeks to advance the discussion of Porsche AG's valuation by demonstrating that the real options model can be effectively applied to provide a robust valuation of the firm in the face of various alternatives. The implementation of options is at the discretion of the firm because these opportunities are seen as the firm’s ability to avoid stumbling around complex industries and uncertain environments. The main research question is whether Porsche's strategy is sufficiently flexible to face a transformation change. The Automotive industry has experienced a lightning-fast change: How can Porsche swim in a sea full of sharks?

3.1. Porsche Corporate Strategy: Horizon One

An overview of the strategy that leads Porsche to create value is observed under three layers, which have been called horizons in "*The Alchemy of Growth*' (M. Baghai, S. Coley, D. White 1999). Ultimately, the pattern makes it possible to estimate project portfolios, growth, capital expenditure, and so on. The application of *Real Options* has a better fit with the *second and third horizons* because of the opportunity to enhance new projects, abandon, switch, or extend them. However, further analysis could not prevent comprehension of the Current Strategy, as highlighted by Horizon One.

A. PORSCHE'S HORIZON ONE: Maintain & Defend Core Business

Horizon One aims to consolidate current competitive advantage. The analysis is based on a firm's strategic positioning and choices. Core activities have a strong operational and managerial scope because their goal is to maintain sustainable cash flows and profits.

3.1.1. Strategic Positioning: *target customers, market share and macro-environment*

The automotive industry is a major contributor to global economy. Considering technology- and resource-orientation, the entire sector is experiencing transformational change. Here, we focus on niche Luxury Sports Cars.
Porsche belongs to a small niche market segment in the automobile industry, namely the luxury segment. According to McKinsey, this segment includes car models with a manufacturer's suggested retail price of higher than $80,000. Those who purchase these cars might not be interested in fulfilling their mobility needs, at least not only, but the matter should be addressed

as a status symbol of wealthy individuals (High Net Worth Individuals, HNWIs). To be considered HNWI the net worth should reach US$ 1 million or more.[18]

The leading sources of data are *market luxury players* and customers. Both provide relevant insights into market share and net worth of individual growth.

Competition in the luxury automotive segment is driven by the strength of the brand, performance, and product appeal (i.e., style, innovation, advertising, and shared values). By looking at the positioning, Porsche stands above the premium segment represented by BMW, Mercedes-Benz, Tesla, and Audi, while it remains below high-luxury cars, such as Ferrari, Lamborghini, Bugatti, Bentley, and Rolls-Royce. However, the discriminating factor was the number of cars produced versus customization. To provide an indication, Porsche manufacturing is roughly 20x the vehicle production of Ferrari (2022Y). On average, Porsche cars cost 226% more than the market average. Car passenger deliveries are steadily growing, and the overall number has exceeded 300,000 units (2021), driven by strong geographical expansion. Porsche's revenue market share in the global market grew from 0.84% in 2017 to 1.04% in 2021 (five years later), despite the firm converting only 0.2% of global automotive sales. Among luxury car manufacturers, Porsche has a market stake of 1 % (in units). Because the luxury market share is significantly greater than the entire car market, the focus is on the luxury car market. Porsche gets on the podium for the sports cars market share worldwide, with a 31.6% stake (i.e., Porsche 911). However, the number of deliveries grew up steadily to 4.78% (2018-2023F). [19]

The luxury car segment is expected to drive most of the growth of the car market. Porsche is located in the segment "$80.000 to $149.000" where most popular models collocate. Almost 1.375.000 vehicles in this segment have been sold by 2021 (i.e., 1/6 of the vehicles are Porsche manufacturing).

Market Share Revenues (worldwide)	
2017	0.84%
2021	1.04%

Market Share Units (Luxury), 2023F-2030F.	
Sceptical scenario	17.9 %
Optimistic scenario	21 %

[18] Meanwhile, to be ultra-high-net-worth individual (UHNWI) the net worth should reach US $30 million or more, including primary residence.
Source: (Knight Frank, 2023)

[19] Calculations had been adjusted for 2023 (i.e., F=forecasted) because Q1-Q3 data are already available. Data are taken from Porsche Investors' relation reports.

The estimated CAGR (2021-27) in units amounts to 4.720 % when the market share remains unchanged, whereas it would reach 7.3 % when the market share follows HNWI trends in HNWIs. The next business segment “$150.000 to $299.000” has a sale volume of 140.000 units, with a CARG (2021-31) of 10% (according to McKinsey, 2023).

The latest data show that deliveries are growing in the Americas (predominantly the USA and Canada); however, the largest market for Porsche is in Europe, with a slice of 3.0% over the total.[20] China has been leading geographical growth, even though deliveries have recently experienced a slight slump. The most threatening is the arrival of new Chinese brands such as BYD and Geely, or start-ups such as Xiaomi, which could saturate the Chinese and Asian markets before market penetration in Europe or the USA.

Industry attractiveness is provided by the presence of HNWIs which account for the top 1% world's adult population (UBS, 2022), 59.4 million individuals (i.e., 0.6 ml of them are UHNWIs). According to estimates (Knight Frank, 2022), the number of high-net-worth individuals is expected to increase by 8.5% by 2026. The target customers of Porsche were HNWIs. Potential buyers can be estimated by examining the overall number of HNWIs as target customers [Table 10]. [21]

Table 10: Distribution of High-Net-Worth individuals (HNWIs) around the world and estimation of growth (on average). The data were displayed in thousands (000).

	2022	2023	2024	2025	2026	2027	Geo share (%) change	CAGR 5Y
Americas	29.364	31.748	34.326	37.114	40.128	43.386	-0,7%	8,12%
Europe	18.929	20.430	22.051	23.800	25.688	27.725	-0,7%	7,93%
Asia & Pacific	20.108	21.976	24.017	26.248	28.686	31.351	1,1%	9,29%
Middle East	1.006	1.135	1.280	1.443	1.627	1.835	0,3%	12,77%
Rest of the World	136	141	147	153	159	165	-	3,99%
overall HNWIs	69.543	75.431	81.821	88.757	96.287	104.462	**growth**	**8,47%**

Souce: Knight Frank, 2022

A report published by the KPMG (Report, Kpmg 2023)[22] has shown the major challenges that come to be disruptive from the market in the near future. The first concern relates to the fall in

[20] Source: “*Porsche – Market Data Analysis and Forecast*”: Statista, 2022. The European market corresponds with the continental Europe (Almost the countries part of European Union including Switzerland and Norway). Data re-elaborated. See the Appendix 3 {Note 3.a}.

[21] Table 10 shows the Geo Share change (%) which indicate the increase in geographical share over the total amount of HNWIs. To put it simple, it indicates which area of the world is increasing its relevance.

[22] Source: “*The European automotive industry, Unlikely to return to normal*: KPMG, 2023.

passenger car sales during the 2020-21 COVID-19 pandemic, when business analysts agreed that the automotive industry would rebound between 2023 and 2025. A conservative view seems to stretch out that demand might fail to materialize due to global uncertainty but is more likely to change customer preferences. New generations are less desirous of owning a car and are willing to use public transit and micro-mobility. In the less 30-years age group, the use of a private vehicle deploys for 42%, rising to 49 % in 30-45 years, while staying bullish after the 45 years with roughly 80% of users (Survey, McKinsey & Co. 2023)[23]. Mobility will become multimodal with ownership preference switching to leasing options or another form of contractual subscription. Nearly 55% of Gen Z consumers were open to sharing their private vehicles with others and moving towards flexible ownership. Additionally, smaller and more compact cars are preferred. Although the number of vehicles in Europe has expanded in recent years, the number of cars will peak at 308 million, but this number will drop to 294 million by 2030. Manufacturers are implementing stricter emission rules to meet regulatory targets. In Europe, compliance with the *Paris Climate Agreement*. The development of low- and zero-emission zones is a growing trend supported mainly by liberal governments that are intended to promote sustainable urban environments. Their will is not exclusively related to limiting pollution, but more broadly the '15-minute *city concept'* where the goal is to reduce car dependence. The European industry faces high supply chain risks owing to the short sourcing of electronic components and semiconductors. Raising conflicts, or worst wars, exhibits a willingness for the dominant country to gain influence in the control of rare-earth elements (REEs). In the wake of the will to mutate global order, a huge concern is leading to increasing threats.

In light of the above, one must wonder if all HNWIs are likely to buy a luxury car. The answer is simple: No, it is not. Certainly, the target customer base is those who have a deep passion for fast-driving cars and who are willing to pay premium prices. A *conservative scenario* could be proposed, separating worthy individuals by gender and age group, with the assumption of the same purchase preferences by generation affinity [Table 11] (*ceteris paribus*). By 2030, there is expected to be a more balanced gender ratio (men/women), even though fewer HNWIs are women than men, and a decrease in willingness to own a car by 10% (trend in 10Y). Meanwhile, there will be a switch in wealth over the course of the years with a leading path from Millennials

[23] Source: "*Europe's Gen Z and the future of mobility*", McKinsey & Company, 2023.

and Gen X. Consequently, the expected growth of the potential customers ranges from 8.5% to 6% [24].

Table 11: Estimating potential customers by age group differences in preferences. Data were elaborated by Capgemini, BCG, and McKinsey in thousands.

Conservative Scenario

Year 2022	AGE	Gen Z <30	Millennials 30-44	Gen X 45-59	B. boomers 60+	HNWIs
Willingness to own a car	PROB	42%	49%	80%	80%	Total
Men		1,052	3,548	11,037	15,474	31,111
Women		759	2,559	6,538	9,166	19,022
						50,133
Year 2030						
Willingness to own a car (hp: decrease by 10%)	PROB	38%	44%	72%	72%	Total
Men		2,084	7,262	19,972	16,806	46,123
Women		1,811	6,312	14,403	12,120	34,646
						80,769
CAGR 8Y		**6.1%** Medium Prudence scenario				
Year 2030						
Willingness to own a car (hp: decrease by 20%)	PROB	34%	39%	64%	64%	Total
Men		1,852	6,455	17,753	14,938	40,998
Women		1,610	5,611	12,803	10,773	30,796
						71,795
CAGR 8Y		**4.6%** High Prudence scenario				

After the industry has been detailed, horizon 1 is interested in the macro-environment, which is the field in which threats and opportunities arise. Indeed, due to current geopolitical tensions, technology, and regulations, a perfect storm of disruption ("*threats and opportunities*") is bearing down on the original automotive equipment manufacturer (OEM). The automotive context has shown that OEM profitability is shrinking because of huge investments in electric powertrains and autonomous driving, which has decreased earnings profit (EBT) due to higher depreciation and amortization (D&A) [Table12].

[24] The "conservative scenario" is based on hypothesis with data taken from Credit Suisse and UBS in Global Wealth Report (2022). The decrease in *willingness to own a car* is an assumption likely to occur. For instance, with a decrease by 25% the CAGR falls to 3,8%. The worst case (no longer CAGR) is reached with a drop of 44% (breakdown point).

Table 12: Major OEMs financials: Strategy& PWC, 2020

	2017	2019	Growth
Revenues (B €)	130.1	136.8	+ 5.2%
EBITDA (%)	12.3 %	12.5 %	+ 0.2 %
EBT (%)	7.5 %	5.4 %	- 2.1 %

Autonomous driving (AD) is based on technological advancements and strategic partnerships. The industry is moving towards high levels of automation, such as *Level 3* (conditional automation) and *Level 4* (high automation) systems, which will soon become available for the majority of cars. Advancements in 5G technology have also played a crucial role in AD development. By 2030, autonomous driving will generate 150$ billion to 225$ billion, but this will double in the following years. A quarter of car buyers are highly interested in AD, with two-thirds of them available for an upfront payment of 10.000$ for premium features (data, McKinsey 2022).

The shift towards Electric vehicles (EVs) is likely to compress the traditional automotive market. EV production requires fewer resources (e.g., −27 % component-intensity parts and − 67% labor intensity hours). However, EVs are still more expensive than combustion-engine vehicles owing to their high battery costs. This would lead to the reorganization of cycle management and the entire supply chain. The main concern is for the suppliers. Lower production volumes, together with a forced shift to different capabilities, can put pressure on suppliers and distress them. This is relevant, because many suppliers are merged or acquired by automotive groups. The data shows that 28 % of suppliers are in a distress zone (Report, Kmpg 2023). A bad liquidity position should squeeze working capital funds and capital expenditures, which are crucial for innovation.

3.1.2. Strategic Choices

Porsche has a unique business model and operates in the automotive industry to deliver high-performance cars that offer an exceptional driving experience. Differentiation is a value-based strategy that focuses on the niche.

Porsche placed a strong emphasis on engineering and creating cars to improve performance. However, more than everything else, Porsche delivers "*uniqueness*" strengthened by iconic luxury design, driving experience, and lifestyle. Strong relevance is given to customers with their dealerships that communicate "The Porsche Experience," where strong emphasis is placed on customization services to meet individual customer preferences and dedicated personal assistance.

The group sales revenue is 37.6 bn € (Porsche Data, 2022), with an operating profit of 6.8 bn € making the financial accounts outstanding. The automotive EBITDA margin increased from 24.5% in 2021 to 25.2% in 2022. Revenues originate from vehicle and after-sales services. The major source of car revenues for Porsche comes from SUVs and sports cars, which are respectively "*Cash Cows*" and "*Stars*" for the German manufacturer (according to BCG Matrix). The realized revenue amounts to EUR €33.1 ml (Porsche AG, 2021), where 33 % accounts for the sports cars segment and 57 % for SUVs, and the remaining segment is covered by sedans. Leaks seem to suggest that Porsche is about realizing soon a new SUV in the segment "prestige," thus it will be above "Cayenne." [Table.13]

Table13: Car Models sold: Porsche Data, 2021. The Car Prices include German prices for the internal market (VAT).

					
Cayenne: mid-size crossover (SUV)	*Macan*: compact crossover (SUV)	*Panamera*: Mid-size lift back sedan	Taycan: Executive sedan sporty car	*911*: Sport coupe, convertible, rear-engine	*718 Cayman:* Sport fastback coupe
Unit sold (world-wide) per model (2021)					
83.071	88.362	30.220	41.296	38.494	20.502
Unit sold (world-wide) per model (2022)					
95.607	86.721	34.106	34.822	40.414	18.214

Porsche promotes itself in motorsports, golf, tennis events, and exhibition centers (10 locations worldwide). A brand partnership is also a common path for sponsorship, with TAG Heurs, Michelin, and Bose as examples. Porsche is also engaged in donation and corporate social responsibility with an overall amount of 22.1 m € (2022) raised for educational, cultural, and environmental projects.

Thereafter, a SWOT analysis of the current Porsche Strategy is presented. Reading these issues leads to a better understanding of the firm's position and ability to change its commitments.

SWOT Analysis

Strengths	Operational	Strong research and development (R&D) activities further enhance Porsche's portfolio of offerings. The Car manufacturer enhance its business activities through new engine advancements and new car model launces (i.e., Taycan is now full-elettric). There's a strong commitment in sustainably issues with the ambition to become the first luxury-car manufacturer to make 80 % elettric car by 2030. In the Battery-elettric vehicles (BEVs) there're improvement is performance, especially in oil cooling, as well as in efficiency with 800 kW recuperation and in charging with 350kW/900-volt turbo charging. The Sustainability management aims at six areas: decarbonization, circular economy, diversity, partner to society, supply chain responsibility, governance, and transparency. In 2019, Porsche become the first car-manufacturer to join the Value Balancing Alliance to lower the externalities of the industry to promote cooperation. The company implemented a *lean organization*, which means "transforming through continuous improvement" bringing together adaptability and interconnection. Porsche turns to "Production 4.0" moving towards increasing connectivity, automation, and AI. The vision declared for this purpose is "Smart, Lean, and Green." Porsche was the first OEM to adopt automated guided vehicles in a production line under continuous flow. Last but not least to mention, the core asset for Porsche related with the skilled workforce and the leading position in engineering and IT system.
	Global presence & Marketing	Porsche has a wide global presence. The company has established offices in several locations worldwide to distribute its products efficiently and to reach out to a large customer base. The most important value added is the loyal customer base because the brand grants from its good reputation around the globe.

	Financials	The firm benefits from an outstanding financial position with a 24.5 % EBITDA margin (2021 data), so 10 % above the OEM industry average. This performance allows to be committed with strong investments with a 10 bn € in cumulative gross impact to support the industry transformation. Porsche shows resilience also in disruptive times with a return on sales of 9.7 % during '09 Financial and 14.6 % in SARS-Cov2 times.
Weaknesses	Operational	Main tipping point is identified in some defects in its products which has led to the product recall. Product recalls not only affects the firm's growth and revenues but also its long-term performance by shrinking costumer confidence. Because of the need of huge investment to keep its high standing position in the luxury segment, the operational risk happens to be the main concern.
	Strategic initiative	The company took various strategic initiatives to expand its operations. The strategy 2030 has a great focus in emerging markets such as China, and perhaps in the future India. Strong relevance will be reserved to Porsche Ventures.
Opportunities	Environment	The favourable outlook of the global automotive sector presents prospects for Porsche in all of its operational markets. According to the Organization for Economic Cooperation (OECE) and Development/International Energy Agency reports, the demand for vehicles with low emissions, such as plug-in hybrid electric vehicles (PHEVs) and battery electric vehicles (BEVs), is anticipated to have reached approximately 20 million by the year 2023 and will reach more than 100 million by the year 2050. The Energy Information Administration (an agency of the US Government) predicts that sales of unconventional vehicles will represent a significant proportion of the total sales of Light Duty Vehicles (LDVs) by the year 2035, with these sales accounting for approximately 40% of the market. China has the potential to emerge as a leading market for electric cars and plug-in hybrids, positioning it as a forerunner among major markets in this domain. It

		aims to have five million electric cars on its roads by 2024.
	New Technologies	Car competitiors have been engaged in the development of various innovative technologies, including autonomous driving, platooning, active safety measures that support assisted driving, as well as strategies for reducing emissions and fuel consumption, and exploring alternative fuels. Continuous improvements in automotive fuels have concentrated on alternative and renewable energy sources, with the goal of enhancing energy security, reducing emissions and improving vehicle performance. New fuels are biofuels and ethanol, Hydrogen, or more advanced Internal combustion engines. A number of these cutting-edge technologies have already been introduced to certain markets, and it is anticipated that this trend will continue to gain traction in the near future, creating opportunities for growth.
Threats	**Regulation & Change in Customer's preferences**	The European Commission enforces consistent emissions standards on vehicles sold in all 27 European Union (EU) member states, as well as in other countries that follow the guidelines set by the United Nations Economic Commission for Europe (UN ECE). Member states of the European Union (EU) have the authority to award tax benefits to automobile manufacturers for vehicles that comply with emissions standards prior to the deadline. The potential emergence of new requirements may give rise to the development of innovative technologies, which could also result in the escalation of the price for diesel engines, as they are currently more expensive than gasoline engines. On the other side of the ocean, also U.S. federal government has put in place guidelines for vehicle emissions that consist of pre-production and post-assembly testing, emission and performance warranties, and a responsibility to recall and fix any vehicles that fail to meet emissions standards. Regulation can affect the decision to purchase a car because the mass public votes politicians more aware of Climate Change.
	Competition	The automotive sector in emerging markets is witnessing increased competition, not only from conventional European rivals, but also from Japanese, Indian and Chinese automotive

	manufacturers who have made significant technological advancements and are focusing more on exports. These companies possess a competitive edge in terms of their initial pricing and are able to consistently attract customers.
Wars & Geopolitical issues	Rising raw material costs could potentially impact Porsche's business operations. Essential raw materials such as iron and steel castings, forgings, alloy wheels, fuel injection systems, batteries, electrical wiring systems, and electronic information systems are among the key components that could be affected. The main issue is here related with the control of theses resources that they are far away from the manufacturing sites. It employs various interior components, including plastic finishes, glass, and consumables, as well as various fuels. Its primary production process employs natural gas, electricity, and other energy sources. At the same time, there has been a substantial rise in processed metal and steel prices. Increasing input costs could have significantly affect on the operational costs of the company, which may not be easily transferred to the customers as intense competition could lure away its customers through low price offerings. As a result, the margins of the company could be affected.

3.1.3. Strategic Forecasting

Porsche's annual reports show that cars are sold per year, while external estimations provide data for market trends[25]. In Table 14-15, the market provides the current outlook and Porsche's position, while Table 16 shows the company's future outlook.

[25] The Provider used for external data is named MarketLine, the license has been offered by Luiss Guido Carli University. The internal data are Porsche realize, see the Appendix 2 {Note 3.b.}.

	Source	2019	2021	2022
Porsche unit sold	Investors report	**280,800**	**301,915**	**309,884**
Global unit sold	External: MarketLine	176,141,733	146,304,333	149,577,258
Growth in unit sold (%)		-	-16.9%	2.237%
Porsche Market Share to global, unit		0.159%	0.206%	0.207%
Global unit sold (Battery-elettric only)	External	1,740,752	4,605,871	7,914,462
Global Plug-in Hybrid Elettric Vehicles unit sold	External	3,095,988	6,294,955	7,650,558
Global elettric vehicles share (%)			3.148%	5.3%
Global Plug-in Hybrid Elettric Vehicles share (%)			4.303%	5.115%
Combined (%)			7.451%	10.406%
Luxury cars unit sold	External		**1,600,400**	**1,693,040**
Growth in luxury cars (%)			n.a.	5.789%
Porsche Market Share to luxury, unit (%)			**18.86%**	**18.30%**
Elettric car luxury				
HNWIs	External (estim.)		n.a.	69,543,000
Growth in HNWIs (%)			n.a.	8.470%
New Customer reached (global) to luxury			n.a.	n.a.
New Customer reached (Porsche) to luxury			n.a.	n.a.

Table 14: Current outlook of the market and Porsche positioning

HNWIs	**Source:**	**Segment:**	**2019**	**2021**	**2022**
Sales Group	*Annual reports*		*28,518,000,000*	33,138,000,000	37,630,000,000
Vehicles	*Annual reports*	*Automotive*	*n.a.*	**27,183,000,000**	**31,692,000,000**
Vehicles % Sales Automotive				**89.7%**	**91.6%**
Vehicles % Sales Group				**82.0%**	**84.2%**
Second-hand vehicles	*Annual reports*	*Automotive*	*n.a.*	1,223,000,000	1,305,000,000
		Financial serv.	*n.a.*	1,472,000,000	1,509,000,000
Second-hand vehicles % Sales Group				8.1%	7.5%
Second-hand vehicles % Sales Automotive				4.0%	3.8%
Rental and leasing services	*Annual reports*	*Financial serv.*	*n.a.*	1,225,000,000	1,293,000,000
Rental & Leasing services % Sales				4%	3%
Rental & Leasing services % Financial services				71%	71%
Hedges on sales	*Annual reports*	*Automotive*	*n.a.*	(300,000,000)	(1,290,000,000)
Interest/similar income from financial service	*Annual reports*	*Financial serv.*	*n.a.*	242,000,000	323,000,000
Other revenue (consulting & developement sales)	*Annual reports*	*Automotive*	*n.a.*	2,183,000,000	2,884,000,000
Other financial revenues % Sales Group				1.6%	4.3%
Other revenues % Sales Group				6.59%	7.66%
Sales Automotive (only)				**30,289,000,000**	**34,591,000,000**

Table 15: Current Financial prospective of Porsche AG

	2023	CAGR	2024	2025	2026	2027	2028	2029	2030
Porsche units sold	320,221	**Constant market share**	339,014	359,056	380,443	403,281	427,687	453,787	481,720
Porsche units sold	320,221	**Growing market share**	365,157	393,372	423,661	456,168	491,047	528,461	568,585
Global units sold	155,597,084	2.556%	161,284,332	167,028,233	171,944,145	176,524,899	181,036,805	185,664,033	190,409,531
Growth in units sold (%)	4.025%		3.655%	3.561%	2.943%	2.664%	2.556%	2.556%	2.556%
Porsche Market Share to global, units	0.206%		0.226%	0.236%	0.246%	0.258%	0.271%	0.285%	0.299%
Global units sold (Battery-elettric only)	11,296,375	21.25%	16,075,089	20,697,485	25,459,414	29,605,388	35,896,912	43,525,465	52,775,184
Global Plug-in Hybrid Elettric Vehicles units sold	8,368,336	1.27%	8,627,220	8,490,236	8,374,984	8,914,262	9,027,649	9,142,478	9,258,767
Global elettric vehicles share (%)	7.3%		10.0%	12.4%	14.8%	16.8%	19.8%	23.4%	27.7%
Global Plug-in Hybrid Elettric Vehicles share (%)	5.4%		5.3%	5.1%	4.9%	5.0%	5.0%	4.9%	4.9%
Combined (%)	12.638%		15.316%	17.475%	19.678%	21.821%	24.815%	28.367%	32.579%
Luxury cars (units) sold	1,791,707	4.94%	1,896,859	2,008,996	2,128,660	2,256,447	2,393,005	2,539,041	2,695,330
Growth in luxury cars (%)	5.828%		5.869%	5.912%	5.956%	6.003%	6.052%	6.103%	6.155%
Luxury cars sold over total sales (%)	1.152%		1.176%						
Porsche Market Share to luxury, units (%)	17.9%	Constant market share	17.9%	17.9%	17.9%	17.9%	17.9%	17.9%	17.9%
			Assumption: Constant Market Share 2023Y-2030Y						
HNWIs	73,019,172		79,203,620	85,911,867	93,188,277	101,080,971	109,642,147	118,928,422	129,001,209
Growth in HNWIs (%)	8.199%		8.470%	8.470%	8.470%	8.470%	8.470%	8.470%	8.470%
New Customers reached (global) to luxury	98,667		105,152	112,136	119,665	127,787	136,557	146,036	156,289
New custumers reached (global) to market	1136.15		1236.69						
New Customers reached (Porsche) to luxury	18,662		20,242	21,957	23,817	25,834	28,022	30,395	32,969
	18.9%	2.33%	19.25%	19.58%	19.90%	20.22%	20.52%	20.81%	21.10%
		Growing market share	Assumption: Growth in market Share						

Table 16: Forecasting of the Porsche's unit sold scenarios. Legend: the pink colour indicates that are external estimation, whereas on green there are pointed out assumptions based on external data. The light blue colour shows assumptions based on market data like the HNWIs growth. The dark grey colour is reserved to target estimation. [16]

Vehicles (& Automotive) revenues

The investor report published by Porsche provides an overview of a company's current financial data for the automotive industry. The German manufacturer has two divisions: one focused on automotive production and financial services. Vehicle revenues made 85 % of revenues out the overall group sales, while the remaining slice was predominantly financial.
In light of the above, the research focuses on the automotive sector and, finally, on the financial implications over the group, data on [Table 17].
Strategic forecasting aims to estimate future sales trends because these revenues reflect a company's business strategy. Market trends have a strong impact on sales for every B2C company, and this is a crucial point for assessing strategic flexibility.
Basic accounting suggests that sales revenue is $Unit\ sold\ \times\ Price\ per\ Unit$. However, none of these methods are available. As a result, reasoning is developed on two sides as long as the tree begins to branch out. The better option would be to go backward, as in crabs. To clarify this, the potential demand for Porsche cars follows the current relation:

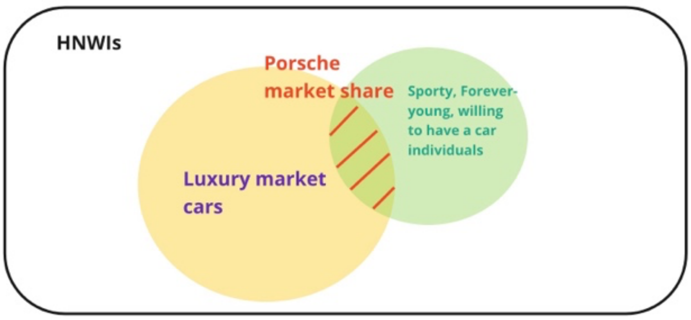


Table 17: The Porsche Market Share

$$Potential\ market = Market\ size\ (population) \times\ target\ market\ (\%) \times\ Adoption\ rate\ (\%)$$

The potential market is made up of wealthy individuals interested in luxury cars who are willing to purchase a new car. Estimating the adoption rate for this specific rate is based on many assumptions. This is a much simpler method.
Let us suppose two states of the world that relate the trends in sales: bull and bear markets. In the former case, market share grows over time, while in the latter case, it is supposed to remain

constant. A simple question arises: Who are Porsche's target customers? The HNWIs. However, growth in market share cannot overpass the growth of HNWIs.

One side there are the target customers, on the other side the number of cars sold in the luxury segment. According to reports from Marketline.com, there has been significant growth in the number of luxury cars sold.

In the world there sold roughly 1.8M of luxury cars (data, 2023), Porsche current market share to luxury (i.e., means "related to" luxury sector) is 17.9 % (data, 2022). In a simple scenario (bear market), market share remains constant over time. For instance, 2024Y units sold comprise 17.9% of the $Luxury\ cars\ sold$ (2024).

For a bull market, further assumptions are needed. Thus, new customers can achieve equal changes in the number of luxury cars. To put it simple, the "excess" in units sold of luxury cars correspond to the potential new audience. By considering increase in size of luxury market, the new market share can become 18.9 % (data 2023) [26]. The better scenario (bull market) of growth is that each new HNWIs could be considered a target clientele; thus, the new market share will increase with the growth of wealthy individuals. Based on these expectations, Porsche will cover 21 % out of the total luxury market by 2030. Once these assumptions are established, the units sold can be calculated for growing market share in the same way.

The outlook is presented alongside the forecasted units sold [Table 18]; however, the task is still incomplete because there should be estimated vehicle revenues and revenues for the group. Each model has different features and several vehicle configurations, and consequently, the selling prices are different, but the use of an average price is a solid assumption.

The average price for new vehicles is 106.482 eur (without VAT), whereas that for second-hand vehicles is 69.213 eur (without VAT). Prices increase over time due to inflation (low case 0.5% 2024-2030). Despite the previously declared data, reasonable assumptions cannot forget to consider the future trends of customer behavior. [paragraph 3.1.1.] "Strategic choices" section, the market analyst predicts a disruptive scenario for automotive. Electric transformation is among the main reasons that will lead to a price increase, so the price is expected to increase. These higher selling prices also influence more revenue. Further explanation is reserved for horizon 2.

[26] The new market share is the previous constant market share increased by the growth in luxury cars (growth 5.8%, data 2023).

Currently, the calculation of vehicle revenues leads to a constant market scenario share and a growing market share scenario.

Other from automotive sales: Rental and leasing services, income from financial services and consulting

Revenues from complementary services roughly accounted for 3/20 of Group AG's sales. Today, the share of revenues seems meager, but market trends suggest that to give a glance. The forecasting process is not within our purview, but we can infer that the prediction is likely to adhere to the hypotheses put forth in the previous year.

Hedge On Sales

Porsche implemented a strategy to limit the risk incurred when sales trading in foreign currency moves in the opposite direction.
Porsche's income statement shows a *hedge on sales* position that aims to protect against basis risk. Porsche AG Group declared the use of derivatives to hedge future cash flows. At the time of hedging, the instrument used was measured as a fair value. The estimation is computed using Python as a hedging instrument for the state of the economy (S&P500) and volatility index (VIX). The number of futures needed is 134.0 units for the position value hedged at 26,906,000 (e.g., 69 % of revenue sales).

EBITDA

The Cost of Goods Sold (COGS) is related to the sales forecast because the cost of producing changes directly with the level of sales[27]. Later, Selling, General & Administrative expenses (SG&A) include a) non-staff overheads and personal expenses, advertising, shipping, and promotions, part of the distribution expenses, and b) administrative expenses. Distribution expenses have a variable nature, which is constantly increasing[28].The Variable part is assumed to be the 2.5% of automotive sales.[29] Other operating income is made mainly by income from derivatives within hedge accounting, foreign exchange gains, cost allocations, and gains on assets because of the disposal and reversal of impairment losses. Other operating expenses

[27] Cost of good solds: COGS=Sales-Gross profit=Sales × (1-% GPM), where GPM is the gross profit margin
[28] Distribution expenses will be the: Distribution expenses = Fixed Costs + Variable Part
[29] The selling and general expenses: SG&A = Selling expenses + administrative expenses. See Appendix 3 {Note 3.c}

include the other side of the coin, the expenses for derivatives, and the losses from foreign exchange [Table.18].

These operating incomes and expenses are not considered in the EBITDA because even though linked to operations, they are not strictly related to the core business. Moreover, forecasting is difficult because off-balance-sheet derivative contracts are not available. This is because these values cannot be completely excluded.

Table18: Porsche Sales Group

	2023	F	2024	2025	2026	2027	2028	2029	2030
Sales Group	42,371,380,000	Bull Market	52,002,144,696	56,255,390,703	60,842,799,691	65,789,348,027	71,121,819,861	76,868,949,193	83,061,572,163
		Bear Market	48,382,693,507	51,480,991,495	54,800,290,747	58,358,715,952	62,176,099,598	66,274,163,066	70,676,714,898
Vehicles									
Vehicles % Sales Vehicles (Automotive)		*Bull Market*	44,079,676,189	47,722,995,845	51,654,593,373	55,896,124,223	60,470,818,239	65,403,602,039	70,721,233,275
	37,617,793,791	*Bear Market*	40,923,818,801	43,559,819,999	46,385,210,944	49,415,641,122	52,668,244,701	56,161,797,097	59,916,889,191
		Auto % Sales group (bull)	84.8%	84.8%	84.9%	85.0%	85.0%	85.1%	85.1%
Vehicles (%)/Sales Group	85%	Auto % Sales group (bear)	84.58%	84.61%	84.64%	84.68%	84.71%	84.74%	84.78%
BEV and PHEV vehicles	*9,593,956,799*	*Bull Market*	17,968,551,839	25,663,711,955	30,413,778,137	35,601,630,968	41,052,726,803	47,152,167,612	54,574,116,543
		Bear Market	16,682,104,388	23,424,905,614	27,311,211,305	31,474,050,191	35,755,677,265	40,489,367,368	46,236,627,137
Combustion Motor vehicles	28,023,836,993	*Bull market*	26,111,124,350	22,059,283,890	21,240,815,236	20,294,493,255	19,418,091,436	18,251,434,426	16,147,116,732
		Bear market	24,241,714,413	20,134,914,386	19,073,999,639	17,941,590,931	16,912,567,436	15,672,429,729	13,680,262,054
BEVs and PHEVs (%) over Sales	25.5%		41%	54%	59%	64%	68%	72%	77%
vehicles (automotive part)	1,085,611,633	Bull Market	1,241,668,782	1,341,621,238	1,449,259,042	1,565,141,660	1,689,867,483	1,824,077,704	1,968,458,939
		% over sales auto	3.03%	3.08%	3.12%	3.17%	3.21%	3.25%	3.29%
		Bear Market	1,152,772,267	1,224,583,214	1,301,417,395	1,383,682,315	1,471,823,215	1,566,327,827	1,667,730,194
		% over sales auto	2.82%	2.81%	2.81%	2.80%	2.79%	2.79%	2.78%
Hedges on sales	-1,260,548,555	Bull Market	(513,086)	(513,086)	(513,086)	(513,086)	(513,086)	(513,086)	(513,086)
Other revenues	3,084,894,174	Bull Market	3,775,610,032	4,087,675,715	4,424,433,612	4,787,738,604	5,179,580,426	5,602,094,147	6,057,571,673
		Bear Market	3,505,297,547	3,731,082,158	3,973,088,800	4,232,657,916	4,511,257,120	4,810,494,606	5,132,134,069
Sales Automotive (only)	**40,527,751,044**	Bull Market	**49,096,441,918**	**53,151,779,712**	**57,527,772,941**	**62,248,491,401**	**67,339,753,062**	**72,829,260,803**	**78,746,750,801**
		Bear Market	**45,581,888,616**	**48,515,485,372**	**51,659,717,139**	**55,031,981,353**	**58,651,325,036**	**62,538,619,530**	**66,716,753,454**
leasing services: Increase by 5.6%	1,364,774,694		1,440,533,616	1,520,497,931	1,604,901,081	1,693,989,468	1,788,023,169	1,887,276,700	1,992,039,814
vehicles (Financial service part)	1,281,021,727	Bull Market	1,465,169,163	1,583,113,061	1,710,125,669	1,846,867,158	1,994,043,630	2,152,411,690	2,322,781,548
		Bear Market	1,360,271,275	1,445,008,193	1,535,672,526	1,632,745,132	1,736,751,394	1,848,266,836	1,967,921,629

3.2. Porsche Transformational Strategy: Horizon Two

B. PORSCHE'S HORIZON TWO: Bring up and Feed emerging business

Horizon Two aims to develop the next competitive advantage. Porsche faced unfamiliar strategic choices and capabilities. Here, the current business is moved in new directions or extended based on the Real Option Model. The analysis develops through firm growth strategies and organic and non-organic growth.

Porsche is committed to becoming sustainable throughout the value chain. However, the German manufacturer has initiated an extensive decarbonization programme, and it has engaged in a circular economy and has declared that battery-electric powertrains will become mainstream'. Porsche wishes to achieve carbon neutrality across the entire value chain by 2030. In 2019, Porsche Taycan realized the first full-electric series with the goal of achieving 80% for all vehicles sold. Vehicle electrification is only the first step directed toward the objective target, followed by *i)* green electricity, *ii)* zero-impact factory vision, *and iii) carbon-neutral supply chain.*

At the investors' conference, the company declared a strategic ambition to sell vehicles within a 50%+ BEVs (including the PHEVs and plug-in hybrid) share in 2025 to reach the target of 80% by 2030 (full electric). Current reflects a significant shift towards electrification in the automotive industry. This mission is not just a naïve response to growing consumer demand for sustainable goods but also aligns with global efforts to reduce carbon emissions and flight against climate change. The transition to electric vehicles (EVs) is a critical component of Porsche's sustainability strategy. However, it is also sustainable from a financial perspective. How can a Porsche venture enter an uncertain context? What happens if the demand drops? If a new era of hostility leads to a natural resource control race, rather than cooperation, and our geopolitical sphere of influence comes to scratch, what can happen later?

Another critical issue is the infrastructure. Porsche's investment in the charging infrastructure and battery technology is crucial for supporting the widespread adoption of EVs. Collaborations with partners to develop premium charging stations and investments in high-performance battery cells are part of Porsche's efforts to address common barriers to EV adoption, such as range anxiety and charging convenience.

By setting such ambitious targets, Porsche is positioned as a leader in the electrification of the luxury and sports car segments. This strategy involves significant investments in electric vehicle technology, charging infrastructure, and development of new electric models. Currently, the number of electric vehicles sold is 13.7% (data, 2021) out of the total cars sold, unless one considers the slight decrease in 2022 (i.e., Porsche reported 11.7 % share).

As a proxy for market sentiment, there is news and all releases from the players in the market. News spreads and affects the decision-making of the players. This is similar to the recent press release from Volkswagen to cut the EV output in a German production plant as demand craters (Bloomberg, September 26, 2023). The carmaker declared that the prevision was due to the downturn of the German economy and increasing competition from Tesla and Chinese autos.

Porsche's declaration of becoming 4/5 reliant on BEVs means that in the future, the German manufacturer will pronominally be an electric car vehicle manufacturer, such as Tesla Inc.

The outlook can be visualized by interpolating (linear interpolation) the target goal to derive a constant trend in the adoption of BEVs. It is clear that Porsche needs to support a large effort [Table 19].

	2023	CAGR	2024	2025	2026	2027	2028	2029	2030
Porsche units sold	320,221	**Constant market share**	339,014	359,056	380,443	403,281	427,687	453,787	481,720
Porsche units sold	320,221	**Growing market share**	374,273	403,192	434,237	467,556	503,305	541,653	582,779
Global units sold	155,597,084	2.556%	161,284,332	167,028,233	171,944,145	176,524,899	181,036,805	185,664,033	190,409,531
Growth in units sold (%)	4.025%		3.655%	3.561%	2.943%	2.664%	2.556%	2.556%	2.556%
Porsche Market Share to global, units	0.206%		0.232%	0.241%	0.253%	0.265%	0.278%	0.292%	0.306%
Porsche Share in BEVs	24.5%		37%	**50%**	56.0%	62.0%	68.0%	74.0%	**80%**
Non BEVs (amount)	241,874								
BEVs (amount)	78,347	Constant market share	126,226	179,528	213,048	250,034	290,827	335,803	385,376
	116%	Growing market share	139,354	201,596	243,173	289,885	342,248	400,823	466,223

Table 19: Porsche Mission towards BEVs and PHEVs, Investors' Presentation, 2022.

The forecasting relation shows that between two points passes through one straight line, $y = mx + b$. The points are (Q_1, Y_1) and (Q_n, Y_n), and the slope is $m = \frac{Q_n - Q_1}{Y_n - Y_1}$. The quantities (i.e., market share) are market by Q, while Y is the number of years. The quantity in between is found as:

$$Q_i = Q_1 + \frac{Q_n - Q_1}{Y_n - Y_1}(Y_i - Y_1) = Q_i = Q_1 + m(Y_i - Y_1)$$

The data written in green are illustrated in Porsche's 2030 strategy as target goals. Quantities in between are required, because the BEVs units are sold by Porsche. Under the current assumptions for Porsche, the main concern is related to reliance on the EVs market in the future. How much being so much depends on whether one niche of the market can be forward-looking? The share of BEVs can also be calculated in terms of values (%). However, further research is required. The average selling price of vehicles is growing. Electric vehicles sell at a higher price because the manufacturing cost overruns.

The demand of electric vehicles cannot even depend on government incentives because the higher selling price does not fit within the spending parameters. In the year 2022, the average cost of an electric vehicle was found to be 15% higher than that of a comparable combustion motor vehicle. If the electric conversion strategy is implemented, the average price jumps from 108.497 eur (owing to inflationary conditions) to 112.478 eur (data, 2024), where a full-electric vehicle is expected to sell at 118.372 eur. EVs Prices should fall in late 2026 (data, S&P Global) as researchers' expectations over battery prices become more affordable as long as the Li-Ion (KwH) increases capacity. Looking at Li-ion batteries, the average price was down to 128$/KwH for battery pack and for cells to 89$/KwH for cells (2022 Data, Bloomberg).

The premium price for EVs components will decrease exponentially to zero by 2030. When an electric conversation is completed, customers are not supposed to be curbed by higher selling prices when deciding to purchase an EVs. The forecast sinks beneath the investigation by a reliable market analysis provider (e.g., MarketLine) [Table 20]. The 2023 data can be built based on partial (e.g., quarterly, semesterly) information release, and in the years 2024-2027 (market in green), Porsche should concentrate on the most expensive effort to dominate the market when more sharks will swim in the sea. The red side of the table assumes cheaper selling prices for BEVs automotive, so the revenues from BEVs and PHEVs % to sell automotive should show the pace.

Table20: Impact of the BEVs and PHEVs over the Sales (%) automotive

	2023 F		2024	2025	2026	2027	2028	2029	2030
Vehicles									
Vehicles % Sales Automotive		*Bull Market*	42,097,609,624	45,577,105,434	49,331,916,545	53,382,724,658	57,751,715,075	62,462,693,574	67,541,214,635
	35,705,844,523	*Bear Market*	38,131,757,692	40,587,915,546	43,220,541,876	46,044,218,469	49,074,910,500	52,330,112,418	55,829,010,274
BEV and PHEV vehicles	*9,593,956,799*	*Bull Market*	17,160,586,153	24,509,729,217	29,046,206,096	34,000,784,308	39,206,768,660	45,031,944,803	52,120,161,771
		Bear Market	15,543,954,131	21,826,722,212	25,447,881,510	29,326,707,297	33,316,216,852	37,726,947,064	43,082,095,321
Combustion Motor vehicles	26,111,887,725	*Bull market*	24,937,023,472	21,067,376,217	20,285,710,450	19,381,940,351	18,544,946,415	17,430,748,771	15,421,052,864
		Bear market	22,587,803,561	18,761,193,334	17,772,660,366	16,717,511,172	15,758,693,649	14,603,165,354	12,746,914,953
Revenues from BEV and PHEV (%) over Sales automotive	26.9%		41%	54%	59%	64%	68%	72%	77%

Owing to its commitment to electrification, Porsche is committed to safeguarding its use in future technology. Throughout the M&A deals and strategic cooperation held by Porsche, the corporate firm has implemented a device to promote an innovation ecosystem not only to gain access to new technologies, but also to improve ESG standards. Porsche Ventures is the company's venture capital ("private equity") that engages in early stage or growth-stage companies with 42 current active direct investments (in May 2022) and from 1 to 4 million euros of stake exposure in each firm. The most promising companies for accomplishing Porsche's strategy targets are listed below. Among these projects, IONITY has been selected as one of the most prominent.

Company	Type of Partnership	Core business & Mission
Actnano Inc., Dover (USA)	Indirect investment by Porsche Automobiles Holding SE with 3.59% stake	Nanotechnology enterprise for automotive electronics. Their flagship product, nanoGUARD, is a fluorine-free coating solution that offers protection for PCBA against moisture, humidity, condensation and salt.

Bcomp (Switzerland)	Porsche has a stake of 3.71%	Sustainable natural fiber composites for high-performance applications. Their products aim at a reduction up to 85% lower Co2 footprint.
Cellforce Group GmbH (Germany)	Joint venture with CustomCells GmbH ▪ Before: 83.75% majority stake (2021) ▪ Takeover (May 2023) by Porsche Automobil Holding SE	Energy storage developer. The company's special lithium-ion battery cells are manufactured under innovative construction technology. The goal is to charge the battery cars up to 80% of capacity in less than a quarter of an hour.
Ionity GmbH	*Joint venture* established by Porsche with another automobile firms	Ionity is high-power charging network founded by a joint venture to with also Porsche belong. The mission would have "no border" when it comes to electric mobility.
Nozomi Networks (USA)	Porsche has a stake of 0.73%	Operational technology and IoT security provider. The company is engaged in several industries.
Rimac Group D.o.o (Croatia)	Joint venture with Bugatti-Rimac is owned by Rimac Group (55%) and Porsche AG (45%). Indirect investment by Porsche Automobil Holding SE with 24% stake into Rimac Group.	Automotive manufacture that specializes in electric sports cars and hypecars with outstanding performances. The unit sold are actually limited in their selling.
TriEye (Israel)	Minority interest	Technology sensor solutions producer. Innovative autonomous driving systems under low visibility conditions with CMOS-based short wave infrared sensing.

3.2.1. Ionity GmbH

A crucial barrier that needs to be addressed for wider EVs acceptance is improving the status of EV charging networks. Hundreds of EV charging networks exist across North America, Europe, and China, with smaller newcomers expected to be rolled up into larger ones.

At present, no charging network for electric vehicles (EVs) generates a profit because of the substantial capital and operating costs involved in establishing a wide network and encouraging widespread usage as EVs become more prevalent on roads. Despite these challenges, the number of EV charging points has significantly increased over the past decade. In 2011, there were only 13,040 charging points globally; however, by 2020, this number had grown to over 1 million. To encourage customers to switch from combustion-powered vehicles, it is crucial to maintain this growth rate and to demonstrate that the necessary infrastructure is in place. A valid success story is the Tesla Supercharger network on the road, which is a source of competitive advantage of the leading cause.

Ionity Business Model

Ionity is a joint venture that was incorporated in July 2016 and began public operations in late 2018 to serve as a high-power charging station network for electric vehicles to facilitate long-distance travel across Europe. Value creation components lead to a service delivery model based on partnerships and CRM[30]. Ionity's business model relies on the rapid expansion of electric vehicle sales, and as such, it is continually evolving to keep pace with market development. By focusing on this emerging market segment, Ionity aims to establish itself as a major player in the electric vehicle charging infrastructure industry. The Lean Start-up Approach (Jones and Roos, 1990) is a model that can be utilized to gain a deeper understanding. This approach emphasizes the importance of following user demands without inventory.

The current outlook of the profit and loss statements shows, as predictable, that all main profit indicators are loss-making [Table 21]. However, huge investments are required for infrastructure and energy costs to follow market trends. EBITDA is negative in the first phase and is cash absorbing but has a very high increase potential.

[30] CRM – Customer Relationship management (CRM). To enhance the experiences of clients, CRM systems can encourage them to make repeat purchases and take responsibility for addressing any complaints.

Table 21: Current Profit and Loss report for Ionity GmbH

Data in thousands

	2020	2021	2022
Operating Revenues	10,951	37,742	89,119
of which Deliveries	9,319	33,909	85,709
(-) Costs of consumptions (VC)	(6,968)	(16,215)	(43,029)
(=) Contribution margin	3,983	21,527	46,090
Cost of employees (FC)	(9,287)	(12,125)	(17,351)
SG&A (MX)	(25,208)	(34,756)	(58,604)
EBITDA	(30,512)	(25,354)	(29,865)
Depreciation & Amortization	(19,296)	(24,213)	(28,506)
EBIT	(49,808)	(49,567)	(58,371)
Financial position	(4,650)	(942)	(5,230)
Profit (Loss) before taxes	(54,458)	(50,509)	(63,601)
Tax benefit (T=30%)	16,337	15,153	19,080
NET INCOME	(38,121)	(35,356)	(44,521)

Strategic Forecasting

Strategic forecasting aims to identify potential clients of Ionity and revenues. The primary activities of Ionity's operations are the construction and installation of charging stations, which represent core business. However, the subscriptions to charging points or non-affiliated clients' charges are the sole source of revenue; energy prices, maintenance costs, and personnel wages are the main source of costs. An increase in Ionity's corporate stake can later be seen as a business opportunity. During this forecasting, the analysis goes a bit into physics. Because the battery capacity is measured in kWh, prices are reported for the same unit. Ionity exhibits prices and fees for using its charging services. Ionity offers a subscription for 5.99 eur/month to get access to their entire charging system at discount rates. The revenue per charging is the usable capacity × price per kWh. There are special prices for Porsche subscribers (only for three years) because of the partnership between Porsche and Ionity. Current prices sell at discounts because of the market penetration strategy adopted by Ionity. The price of a subscription is about to raise to 11.99 eur (data, Ionity Press Release, 2024), and the start-up is declared to bring the fees to pricelist of 17.99 eur. However, this last statement is hardly believed because it seems to be a marketing strategy. Thus, it is assumed that subscription fees sell for 11.99 € [Table 22]. In the future (2025 ongoing) prices for direct costumers rise up to 0.79 eur/kWh.

Table 22: Revenues and number of clients (per charger) and breakdown of revenues. Prices data displayed are provided by Ionity, the coefficient of efficiency is the dispersion energy rate in electrochemical process. Usable capacity for batteries should be around 96 kWh.

Usable capacity (KwH)	96	Avg. time needed (minutes)		24
	Price per kWh		Price per kWh	Revenues per charging
5.99 eur/ mouth	subscribers		0.49	47.04
without subscribtion	direct		0.69	66.24
Price per porsche clients			0.30	28.80
	Time for charging capacity (kWh)		50	minutes up to 80%
	Charging power (kW)		150	20
			350	9
	Classical battery capacity (kWh)		100	
	from 10% up to 80%		70	Time needed (minutes)
	Coeff. of efficiency	0.9	with supercharger	14
			normal	33
			average time	24
	MAX Charging capacity per point of charger (daily) 12 h to minutes		Max clients per point of charger (daily)	Max charges per point of charger (annually)
	720		31	11,016
	Average values	Daily	Clients/day	1.24
	Source: Balance sheet			Number of clients per charger
	Revenues per charger	Annual	25,343	447

Monthy subscription (price), eur	Surplus in operating revenues	3,410,000
11.99	Annual price of the subscription	72
	Factor of subscription rate/Surplus	0.880
	Surplus in operating revenues (subscriptions)	3,000,800.00
	Numbers of subscribers (total amount)	41,747
Growth (%)	Max number of charges to offer (annual)	1,513,224
12%	Subscibers	2.8%
-2%	Directcustumers	73.0%
3%	Pamership	24.2%

Further data provided by Ionity considers the time required for different charging powers. A charger power of 150 kW employs 20 min to fulfil a battery capacity of 50 kWh, whereas 350 kW requires only 9 min for the same capacity. The advertising for electric cars is to increase the battery capacity from 10% to 80%. This is because Li-ion batteries do not charge constantly, as the physics process assumes at least two phases: rapid growth at the beginning and later at a slower pace. The charging function for this electrochemical process reassembled a cubic function limited to extremes. For the concern of a business analyst, it would be sufficient to consider the time to be in function of power[31].

Assuming 12 h per day (i.e., 720 min), a single charger can serve a maximum of 31 clients per day. However, Ionity was far from these numbers. The network for Ionity shows 3,382 charging points (data, 2022), although the firm announced that it would reach 7,000 units by 2025. Linear

[31] The charging time has been estimated by: charge time (minutes) =

$$\frac{\text{Battery capacity (kWh)}}{\text{charger power (kW)} \times \textit{coeff. of efficiency}} \times 60$$

interpolation assumes that the units are in between, and exponential growth occurs later. Undoubtedly, substantial investments will be necessary by the end of 2025 as the market is expected to reach its peak growth phase, as indicated by industry specialists.
The revenue (Deliveries 2022Y) per charging station is *considered* to be the number of Deliveries (2022) over number of charging stations.

The number of clients currently using Ionity services is 477 per charging point[32], on average (that is, means 1.24 clients daily). Now, the analysis should look at the electric car market in Europe because the geographical expansion of Ionity is on the Old Continent. Europe currently accounts for 20% of the total amount of sales. The number of charges (annual) per vehicle is one hundred times (two charges per week) per year on average. It is possible to calculate the number of charging times for all vehicles (per year), where Ionity represents the times, they can potentially gain profit. Regrettably, the adoption rate is anticipated to be a mere 20%, as per data from market research revealing that a substantial majority of electric vehicle owners, roughly 80%, opt to charge their cars at home because they fact that is cheaper.
The strategy of Ionity is to focus its presence near or on highways, as there are already a significant number of national and local providers in urban areas. The market share can then be calculated, and at the same time, the forecasted revenues. [Table 23] depicts the market outlook and positioning of Ionity. The total value of deliveries is calculated once the maximum number of charges to offer and the average price per charge because it is their mathematical product.[33]
The average price per charge is 56.64 eur (assuming equal proportion among subscribers and direct clients), as shown in table [Table 22].

[32] The number of clients is: number of clients = Revenues per charging station/Revenues per charge (average)

[33] Max number of charging stations = Clients per charging point × High-power charging points

Table 23: IONITY Market Share

MARKET	2022	2023	2024	2025	2026	2027	2028	2029	2030
Electric cars market size (units)	**15,565,020**	**19,664,711**	**24,702,309**	**29,187,721**	**33,834,398**	**38,519,650**	**44,924,561**	**52,667,943**	**62,033,951**
Evs	7,914,462	11,296,375	16,075,089	20,697,485	25,459,414	29,605,388	35,896,912	43,525,465	52,775,184
PHEVs	7,650,558	8,368,336	8,627,220	8,490,236	8,374,984	8,914,262	9,027,649	9,142,478	9,258,767
Europe zone	3,268,654	4,193,920	5,178,770	5,801,489	6,766,880	7,703,930	8,984,912	10,533,589	12,406,790
%Europe	21%	21%	21%	20%	20%	20%	20%	20%	20%
Charges annual per vehicle (100 per year)	326,865,423	419,392,000	517,877,000	580,148,900	676,687,960	770,393,000	898,491,213	1,053,358,858	1,240,679,018
Adoption rate (20%)	65,373,085	83,878,400	103,575,400	116,029,780	135,337,592	154,078,600	179,698,243	210,671,772	248,135,804

IONITY	**2022**	**2023**	**2024**	**2025**	**2026**	**2027**	**2028**	**2029**	**2030**
High-power charging points	3,382	4,588	5,794	7,000	9,162	11,666	14,853	18,912	24,080
New chargers to add per year		1,206	1,206	1,206	2,162	2,504	3,188	4,059	5,168
Clients (new and old) per charging point	383	421	463	509	560	616	678	746	820
Speed to reach new clients (client/year)	10%								
Max number of charges to offer (annual)	1,293,916	1,930,850	2,682,232	3,564,582	5,132,202	7,188,020	10,067,341	14,100,039	19,748,123
Consumption kWh	124,215,942	185,361,625	257,494,285	342,199,862	492,691,374	690,049,877	966,464,725	1,353,603,696	1,895,819,805

Monthy subscription (price), eur	Surplus in operating revenues	3,410,000	11,240,474	17,482,849	26,012,826	62,912,801	98,643,369	154,658,283	242,466,641
12	Annual price of the subscription	72	144	144	144	216	216	216	216
	Factor of subscription rate/Surplus	88.0%	88.0%	88.1%	88.1%	88.2%	88.2%	88.3%	88.3%
	Surplus in operating revenues	3,000,800	9,895,192	15,396,695	22,919,211	55,458,942	87,005,625	136,498,886	214,149,751
	Numbers of subscribers (total amount)	41,747							
Growth (%)	charges to	1,293,916	1,930,850	2,682,232	3,564,582	5,132,202	7,188,020	10,067,341	14,100,039
12.0%	Subscibers	3.2%	4%	4%	5%	5%	6%	6%	7%
-1.7%	Direct custum	72.5%	72%	70%	69%	68%	67%	65%	64%
2.6%	Parnership	24.2%	25%	26%	26%	27%	28%	28%	29%

Potentialsubscribers (Market share)	3.2%	3.6%	4.0%	4.5%	5.1%	5.7%	6.4%	7.1%	8.0%
Revenues from subscribe	3,000,800	10,038,988	15,619,090	23,248,020	56,248,521	88,233,732	138,407,045	217,110,959	340,569,140
Other operating revenues	360,096	1,201,486	1,863,759	2,764,806	6,664,280	10,409,636	16,251,238	25,355,682	39,533,670
Total deliveries (value)	**92,398,022**	**136,952,010**	**188,884,406**	**249,107,277**	**355,743,364**	**493,913,655**	**685,317,313**	**950,228,551**	**1,316,512,616**
Average Price per charge	71	71	70	70	69	69	68	67	67
	55	136,952	188,884	249,107	355,743	493,914	685,317	950,229	1,316,513

Ionity further releases declared that the slice of revenues not related to deliveries (i.e., car charges) is made up of subscriptions (i.e., factor of subscription rate, 0.88) and partnership royalties [Table 23]. The revenue generated from subscriptions is expected to increase, despite a decrease in the average price per charge. This is because subscriptions and partnerships enable holders to purchase at discounted prices.[34] However, the cost of consumption relies mainly on energy costs; thus, their estimation is the price per kWh × Units of energy (kWh).

The price of energy for business (kWh) is taken from the market, and reports show an average price of 0.265 eur/kWh (Data, forecast Germany). However, in the future some energy intensive companies might require applying for subsidised energy (price cap) to reach 0.08 eur/kWh (e.g., still not the case for Ionity).

The cost for employees can be estimated by examining the number of employees per charger station. Currently, the workforce comprises 155 individuals, which corresponds to a ratio of 1:20, meaning that there is one employee for every 20 charging stations. From 2020 to 2022, wages (annual) paid to employees varied between €137,000 and €112,000. Future expansion has led to more employees being linked to the number of locations. Ionity is currently operational in 600 locations, and it is anticipated that additional 96 locations will be added immediately. The ratio is 1:4. On average, there were six charging stations for the location. Ionity focuses on providing charging stations along roadways rather than to urban areas, which makes it difficult to further lower the ratio.

Selling, General, and Administrative Expenses have experienced a significant rise owing to increased advertising, promotional activities, and an expansion in administrative staff. It is anticipated that expenses will eventually stabilize at a ratio of approximately 22% of sales.

[34] The total deliveries (value) = $Number\ charges\ (max\ ) \times\ Average\ price\ per\ charge$. The units of energy ($consumption\ kWh$) are determined by: Number of charges × Battery usable capacity (kWh)

Working Capital Requirement (WCR) management

WCR is seen as being sales driven. Liquidity forecasts for start-ups experience a huge absorption of cash, mainly due to a negative EBITDA and CAPEX increase. Even though lowering working capital needs will increase a project's net present value, critical constraints will be to have low receivables with a potential effect on revenues.

The forecasting moves to the Balance Sheet, where the reclassification for the managerial outlook" [Table 24] shows the WCR and capital expenditures (Capex).

The needs for working capital include $Receivables - Payables + Inventories$. The estimation of receivables and payables is based on benchmark values, which are $Receivables/Sales$ and $Payables/COGS$, while a further analysis is applied for inventories. Under the business model for Ionity, it would be wrong to consider a strong link between sales and inventories, as suggested by the EOQ approach.

Ionity shows that, except for the first year, a negative cash conversion cycle is seen as a strength point because clients pay before the firm has to honour commitments towards suppliers.

Table 24: Balance Sheet of Ionity, data in thousands (000). IAS 1 Framework

	2020	2021	2022		2020	2021	2022
				Long-term debt	5,719	33,250	29,819
Non current assets	**231,115**	**253,118**	**302,350**	*Term Loan A*			29,819
				Term Loan B			
Intangible assets	1,558	1,443	1,337	*Revolving Debt*			
Tangible assets	229,557	251,675	301,013	*Unsecured Bonds*			
				Other Term Debt	85,000	98,000	185,090
Other non-current assets	none	none	none				
Goodwill			none				
Defered financial fees							
Current assets	**167,028**	**150,017**	**171,303**				
				Other non-current liabilities	8,452	14,665	35,988
Inventories	none	660	3,550	**Non Current liab.**			
Account receivable	5,176	13,453	17,822				
Other current assets	156,133	147,187	125,937				
				Current liabilities	**292,801**	**260,911**	**248,337**
Cash and Cash eq.	5,719	14,665	35,915	Account payable	8,348	23,676	23,365
				Current Liab	284,453	237,235	224,972
Total assets	**403,862**	**417,800**	**509,568**	**Total liabilities and equity**	**403,862**	**417,800**	**509,568**

The Economic Order Quantity (EOQ) model [Table 25] was used to determine the optimal order quantity for inventory management[35]. Consequently,

[35] The main assumption that Q is below is found to be: $Q = \sqrt{\frac{2 \times D * S}{H}}$

Table 25: Estimation of Inventory needs (2023F-2027F)

	2023	2024	2025	2026	2027
Volume ordered (D)	1,206	1,206	1,206	2,162	2,504
Average inventor (A)	603	603	603	1,081	1,252
Cost per unit (C)	38,943	39,722	40,516	41,326	42,153
Inventory charge (i)	6%	6%	6%	6%	6%
Holding cost (H=i*C)	2506.2	2556.3	2607.4	2659.6	2712.8
Inflation	2%				
Ordering costs (S)	27,085	27,627	28,179	28,743	29,318
EOQ	161	161	161	216	232
Inventory value	6,269,786	6,395,182	6,523,086	8,926,498	9,779,475

The average investment per charging unit is € 38,9 K (Data, Ionity), with an ordering cost of € 27 K (Estimation, Ionity).[36]

The need for working capital led to the following outcomes [Table 26]: Ionity's need for working capital is linked to operations and organic growth strategies, as a leading charging network. The top management has declared its intention to establish at least 1206 (at least until 2025) charging stations (see: demand of unit, D) annually, with the aim of surpassing 7000 target units. The objective of this initiative is to significantly expand charging infrastructure, which will ultimately contribute to the growth and development of the EV industry.

[36] Ordering Costs – The additional cost for placing an order: shipping, internal acceptance plans, contracting, etc. The total ordering cost are the fixed cost to support operations plus the variable costs times the units delivered: $S = FX + VC * N$.

Table 26: Estimation of WCR (2020Y-2030F). Data in thousands (000)

Working Capital requirement	2020	2021	2022	Average Benchmark	2023	2024	2025	2026	2027	Used Benchmark	2028	2029	2030
Average collection period	203	145	75.9	Benchmark (AVG)	76.7	76.7	76.7	76.7	76.7	Benchmark (Min)	77	77	77
Account receivables	5,176	13,453	17,822		28,760	39,666	52,313	74,706	103,722		154,882	214,752	297,532
Recivables/sales	89.6%	69.8%	27.3%	62%						23%			
Average payable period	437	533	198		266	266	266	266	266		266	266	266
Account payables	(8,348)	(23,676)	(23,365)		(42,186)	(58,603)	(73,987)	(106,524)	(141,735)		(198,510)	(264,126)	(369,928)
Payables/COGS	74.3%	83.0%	41.4%	73%						73.0%			
Inventories	0	660	3,550		4,109	5,667	7,473	10,672	14,817		10,280	14,253	19,748
Average inventory period	0	35	80		26	26	27	27	28		14	14	14
Inventories/sales	0.00%	1.75%	3.98%	3%	3.0%	3.0%	3.0%	3.0%	3.0%	1.5%			
WCR	(3,172)	(9,563)	(1,993)		(9,318)	(13,270)	(14,201)	(21,146)	(23,196)		(33,349)	(35,121)	(52,648)
Change in WCR		(6,391)	7,570		(7,325)	(3,953)	(930)	(6,945)	(2,050)		(10,153)	(1,773)	(17,527)
Cash Conversion cycle	(235)	(354)	(42)		(164)	(164)	(163)	(163)	(162)		(176)	(175)	(176)

Depreciation and Capex

Capital expenditures, commonly referred to as capex, encompass the procurement of fixed assets and exert a substantial influence on both the balance sheet and income statement. On the one hand, capex impacts the balance sheet through the addition of fixed assets, while on the other hand, it affects income statements through depreciation. For this reason, capital expenditure less depreciation is called "net investment." The relation is enhanced by:

$$\text{Fixed Assets}_0 + \text{Capex-Depreciation} = \text{Fixed Assets}_{\text{end}}$$

Thus, the Capex can be estimated by:

$$\text{Capex} = (\text{Fixed Assets}_{\text{end}} - \text{Fixed Assets}_0) + Depreciation$$

The depreciation assumptions for D&A are centered on a straight-line method over a period of five years. Given that each charging station serves as the primary asset for the Ionity network, the annual depreciation value amounts to €7,8K [Table 27].

Table 27: Depreciation and Amortization (D&A) of the machineries

			2020	2021	2022	2023	2024	2025	2026	2027	2028	2029	2030
Depreciation per charging station (000)	7.8	Data in thousands											
	Added Depr. (annual)	Added Depr. (5 years)	(19,296)	(19,296)	(19,296)	(19,296)	(19,296)						
				(4,917)	(4,917)	(4,917)	(4,917)	(4,917)					
2022	4,293	21,465			(4,293)	(4,293)	(4,293)	(4,293)	(4,293)				
2023	9,393	46,965				(9,393)	(9,393)	(9,393)	(9,393)	(9,393)			
2024	9,393	46,965					(9,393)	(9,393)	(9,393)	(9,393)	(9,393)		
2025	9,393	46,965						(9,393)	(9,393)	(9,393)	(9,393)	(9,393)	
2026	16,841	84,203							(16,841)	(16,841)	(16,841)	(16,841)	(16,841)
2027	16,841	84,203								(16,841)	(16,841)	(16,841)	(16,841)
2028	24,827	124,136									(24,827)	(24,827)	(24,827)
2029	31,611	158,055										(31,611)	(31,611)
2030	40,249	201,244											(40,249)
Depreciation			(19,296)	(24,213)	(28,506)	(37,899)	(47,292)	(37,389)	(49,313)	(61,860)	(77,294)	(99,512)	(130,368)

The primary assumptions regarding Capex are that Ionity will allocate additional resources towards research and development to enhance intangible assets and invest in equipment to expand its network.

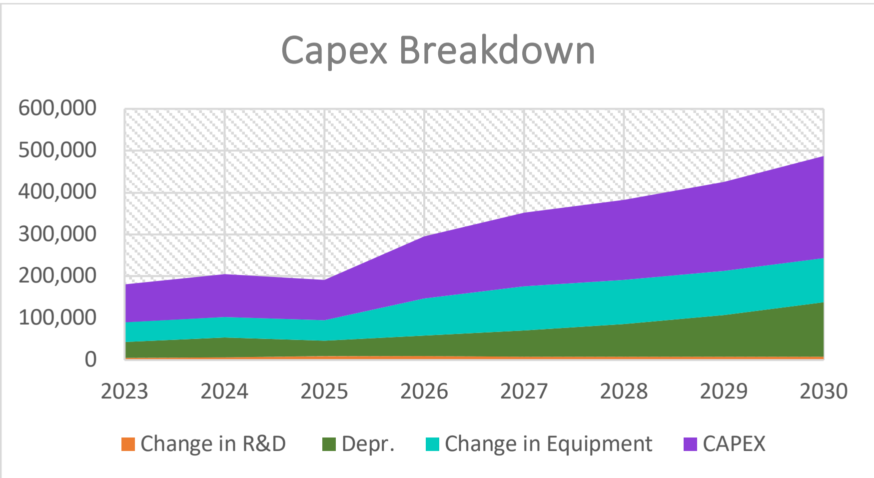
$$\text{Capex} = \Delta\text{Tangible assets} + \Delta\text{Intangible assets} + \text{Depr}$$
$$= (\text{Equipment}_{\text{end}} - \text{Equipment}_{0}) + New\ R\&D + Depreciation$$

The Damodaran database indicates that the electric equipment industry allocates a portion of its revenues towards research and development (R&D) expenses. According to the database, the industry invests approximately 3.9% of its revenue towards R&D, with a growth rate of 3.3% over time. In terms of equipment value, this can be calculated as the cost per unit multiplied by the ordered volume (D). Capex can be easily estimated [Table 28A, 28B]. Note that the Return on Invested Capital (ROIC) is negative (2020Y-2026F), but it is expected to become positive in the late prospect forecast.

Table 28A: Capex estimation

		2020	2021	2022	2023	2024	2025	2026	2027	2028	2029	2030
Fixed Assets (FA)	Intangible	1,558	1,443	1,337	7,191	15,614	26,845	37,700	48,192	58,334	68,138	77,614
	Tangibleas	229,557	251,675	301,013	347,977	395,882	444,744	534,101	639,633	745,165	850,697	956,229
	FA	231,115	253,118	302,350	355,168	411,496	471,589	571,800	687,825	803,499	918,834	1,033,842
	Depreciation	19,296	24,213	28,506	37,899	47,292	37,389	49,313	61,860	77,294	99,512	130,368
	Capex		**46,216**	**77,738**	**90,718**	**103,620**	**97,482**	**149,524**	**177,885**	**192,968**	**214,848**	**245,376**
	Capex/Revenues		122%	87%	61%	50%	35%	36%	30%	23%	18%	14%
	Capex/Depr		191%	273%	239%	219%	261%	303%	288%	250%	216%	188%
	Net Invested Capital (NIC)	227,943	243,555	300,357	358,691	413,735	475,038	575,668	695,131	815,087	943,172	1,064,594
	ROIC	-15.30%	-14.25%	-13.60%	-6.60%	-4.34%	2.72%	10.15%	17.74%	26.27%	35.36%	46.47%
	R&D % Revenues	A. Damodaran		3.95%	**5,854**	***8,424***	***11,230***	***10,855***	***10,493***	***10,142***	***9,803***	***9,476***
				Growth	3.34%							
					37,899	**47,292**	**37,389**	**49,313**	**61,860**	**77,294**	**99,512**	**130,368**
					43,753	**55,716**	**48,619**	**60,168**	**72,353**	**87,436**	**109,316**	**139,844**
	R&D % NIC			15.37%								
	DAMODARAN				**63,920**	**79,763**	**63,060**	**83,170**	**104,333**	**130,364**	**167,837**	**219,879**
	Capex/Depr			**168.7%**								
					14,553	20,265	27,017	41,112	58,189	82,486	117,123	166,608
	Net Capex/Revenues			**9.8%**	37,899	47,292	37,389	49,313	61,860	77,294	99,512	130,368
	(+Depr)				**52,451**	**67,557**	**64,406**	**90,425**	**120,049**	**159,780**	**216,635**	**296,976**
				CAPEX	52,451	67,557	63,060	83,170	104,333	130,364	167,837	219,879
				Equipment	46,965	47,904	48,862	89,357	105,532	105,532	105,532	105,532
				CAPEX	**90,718**	**103,620**	**97,482**	**149,524**	**177,885**	**192,968**	**214,848**	**245,376**

Table 28 B: Capex Visualization



Long-Term Budgeting Forecast

As projected, Ionity's negative NOPAT [Table 29] is anticipated to persist until 2025, with the main factor still being low operating revenues, which are largely influenced by prices and the number of charges offered. If the cost of consumption is fixed (% of sales), the company has the authority to determine its market penetration strategy. To expand its customer base, Ionity ought to allocate additional resources during the growth phase, although this may lead to higher operating costs and further investments. By conducting a sensitivity analysis[37], it can be observed that doubling the number of charges by 2027 while maintaining the same average prices would result in a 60% increase in revenue. Alternatively, increasing prices to an average of 59.5 € would have the same effect.

Ionity depends on public subsidies and financial support because otherwise, the current financial prospect would be even worse, and if these interventions were to cease, the firm could soon face a distress scenario [Table 29].

Table 29: Long-term Forecast of the Income Statement

Data in thousands

		1	2	3	4	5	6	7	8
	2022	**2023**	**2024**	**2025**	**2026**	**2027**	**2028**	**2029**	**2030**
Operating Revenues	89,119	148,192	206,367	275,120	418,656	592,557	839,976	1,192,695	1,696,615
of which Deliveries	85,709	136,952	188,884	249,107	355,743	493,914	685,317	950,229	1,316,513
(-) Costs of consumptions	(43,029)	(57,789)	(80,278)	(101,351)	(145,923)	(194,157)	(271,932)	(361,817)	(506,751)
Contribution margin	**46,090**	**90,403**	**126,090**	**173,769**	**272,733**	**398,400**	**568,044**	**830,878**	**1,189,865**
Contribution margin/Sales	54%	66%	67%	70%	77%	81%	83%	87%	90%
Cost of employees	(17,351)	(16,019)	(20,063)	(23,392)	(36,952)	(48,037)	(62,448)	(81,181)	(105,535)
Gross Margin	28,739	74,384	106,027	150,377	235,780	350,362	505,596	749,697	1,084,330
SG&A (MX)	(58,604)	(70,325)	(84,390)	(94,517)	(103,023)	(112,295)	(122,402)	(173,800)	(247,232)
Selling and General (	66%	47%	41%	34%	25%	19%	15%	15%	15%
EBITDA	**(29,865)**	**4,060**	**21,637**	**55,860**	**132,757**	**238,067**	**383,194**	**575,897**	**837,098**
Depreciation & Amortization	(28,506)	(37,899)	(47,292)	(37,389)	(49,313)	(61,860)	(77,294)	(99,512)	(130,368)
Depreciation &	32%	26%	23%	14%	12%	10%	9%	8%	8%
EBIT	**(58,371)**	**(33,839)**	**(25,655)**	**18,471**	**83,445**	**176,207**	**305,900**	**476,384**	**706,730**

[37] See Appendix 3 {Note 3.d}.

3.2.2. New Fuels: Hydrogen

Hydrogen offers numerous advantages, including its ability to serve as a clean and efficient fuel source and its versatility as a reliable power source.
The objective of advancing fuel cell electric vehicles (FCEVs) in transportation has been to optimize fuel efficiency and decrease carbon and other pollutant emissions. Like electric vehicles, FCEVs do not emit CO2. Although the capital costs for FCEVs are lower compared to those of conventional internal combustion engines (ICE), the cost of a hydrogen combustion engine is roughly 1.5 times higher than that of a conventional ICE. However, the higher efficiency of hydrogen compared to gasoline in spark ignition engines has recently led to increased interest in hydrogen combustion technology.
Fuel Cell Research and Development exhibits options-like attributes, which involve significant upfront capital investment, with the commercial value being highly speculative and uncertain. Porsche might employ joint R&D ventures as a hedging strategy in response to increasing market and technological unpredictability. By engaging in these collaborations, companies can explore numerous technologies simultaneously, creating a diverse portfolio of early-stage R&D options that may become future growth opportunities. These agreements require the reconciliation of various motivations, assumptions, and expectations in order to establish a contractual arrangement that manages the expectations of both parties amidst high uncertainty. The R&D on fuel cell own call option-like features: large capital expenditure required up-front but uncertain future commercial value.

3.3. Porsche Unconventional Strategy: Horizon Three

C. PORSCHE'S HORIZON THREE Create genuinely a New business

Horizon Two is for visioners and unconventional thinkers. The path aims to explore options for future competitive advantage when the environment is changed, and Porsche must deal with uncertain choices and capabilities. Here, the current business model will be transformed, and thus the change might be disruptive. This analysis considers scalability to be a leading factor. A disruptive strategy is typically achieved when entering an existing market or creating a new one. For instance, launching a new product or service is a significantly different business model.

To provide so-called disruptive strategies, the concept of Red and Blue Oceans will be considered (Wattenbergh, 2019).

A new Business Model

Porsche, known for its high-performance sports cars, has the potential to disrupt the luxury yacht market by utilizing its brand prestige, technological expertise, and dedication to sustainability. By launching a line of high-end yachts, Porsche could expand its footprint in the luxury market and focus on innovation and luxury. This move aligns with the growing trend towards electrification and sustainability in the luxury sector, providing a unique combination of performance, luxury, and environmental awareness.

Introducing luxury yachts would enable Porsche to expand its brand experience from land to sea, providing customers with a novel means to appreciate the brand's values. By harnessing its existing customer base and brand loyalty, Porsche could introduce exceptional yacht experiences, such as private viewings, test sailings, and tailored customization options, comparable to its automotive offerings. This approach would further strengthen Porsche's reputation as a luxury brand that surpasses conventional automotive limitations.

The importance of scalability is paramount in this new initiative. Porsche's strategy for entering the luxury yacht market with electric-powered yachts presents several opportunities for scalability. For one, the modular design and technology can be adjusted to fit yachts of varying sizes and types, enabling Porsche to cater to a wider market segment. Additionally, the collaboration with established yacht builders offers a scalable production model that can respond to changes in market demand. Lastly, by prioritizing sustainability and electric mobility, Porsche aligns with global trends, ensuring long-term scalability as regulations and consumer preferences shift towards more eco-friendly options.

Firm specific capabilities

Firm-specific capabilities are those tangible and intangible assets that can enhance Porsche to start building in-house yachts. At the first step, there's the evaluation of the proprietary technology, that see Porsche to acquisition of patents in a leading position among the OEMs manufacturers.

The State of Yachting

Yachts are classified into several categories, into board, outboard and sailing yachts. The state of yachting report that a positive trend for the industry, with a CAGR of 10% (2021Y-2023Y), with Italy holding a leading position with roughly 350 units build, and The US and Taiwan to have the highest growth (%) as the CAGR outpaces 15% (2022Y-2023Y). Yacht clients can purchase yachts though dealers, brokers or directly with the salesforce of yacht companies. The current research only focuses on new building yachts all the time it's logged as sale to costumers. At the beginning of 2023, a number of 668 yachts under construction, but only 134 were offered for direct sale. The size of the yachts most prominent is between 30-39.99 M, that covers over 50 % of the sales.

Main trends in the global industry

- Growing customer base: the UHNWI population, the main target for yachts, is constantly increasing in the upcoming years.
- Enhance the propensity of potential clients to invest in luxury maritime travel, which allows them to travel privately while journeying.
- High growth potential in these emerging markets, as China and Middle East.

3.4. The Cost of Capital of Porsche AG and Corporate Structure

The calculation of the cost of capital involves specific factors for each of its contributions. As the cost of capital is calculated as the weighted average of the cost of equity and debt, both components are essential to the valuation process. The cost of capital (hereinafter, WACC) is essential because represents what the firms needs to make collectively on all its investments [Damodaran, 2001].

Cost of Equity

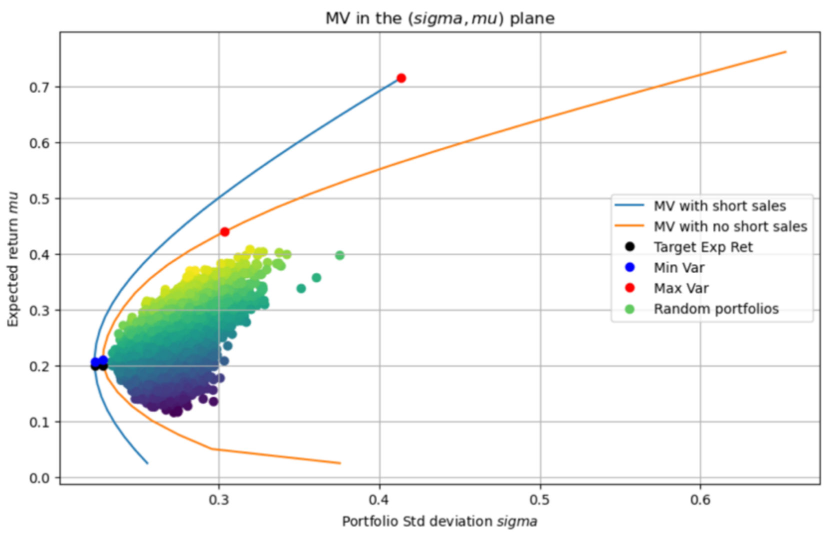
An investor who has made an equity investment (i.e., shareholders) seeks a return that for the firm's side corresponds to the cost of equity. To estimate the cost of equity, hereinafter k_e, the CAPM comes to the rescue.

Based on Markowitz studies on modern portfolio theory and diversification and EMHs, CAPM has been created [*Sharpe 1964, Lintner 1965*] for measuring the systematic risk, symbolized by beta (β), alongside to price risky securities to generate expected returns.

In Markowitz model's, an investor chose a portfolio at time $t-1$ which is "mean-variance" efficient, meaning that the portfolio minimizes the variance given an expected return or maximizes the expected return once the variance is known. Diversification is trivial and it can be reached by adding assets from sectors/industries. The CAPM allows to test the previous prediction by determining the return on similar investments with same sensitivity to market risk. The return provided in known as risk-adjusted expected return.

Investment opportunities can be illustrated to show the trade-off between risk and expected return for minimum variance portfolio [Graph 6]. On the curve there are designated the investment opportunities that have been considered as "optimal". The plotting on the chart exhibits all the efficient portfolios within the efficient frontier with short-selling or without short-selling[38].

Graph 1: ***Investment opportunities: Markowitz's Efficient Portfolio.*** *The portfolio considered are made up by the peers' stocks. Computed with Python.*



[38] Short selling (or Short Sales) – means to sell securities that one does not currently own with the intention to purchase back them in the future at a lower price. In CAPM, short selling is unrestricted, which means that investors are free to sell securities short without limit or any kind of constrain.

Further development looks at the Fama-French Model (1992) for estimate risky projects. The model is built considering that investors might hold other portfolios than the CAPM, pushing out strategies that creates extra-returns. Fama-French adds two factors which are the market capitalization (SMB) and book-value-to-market (HML) portfolios.

CAPM vs Multifactor models

Asset pricing alternatives for equity investments	CAPM	Simply put, the CAPM generates the expected returns of once their risk is known. The capital asset expected $R_{i(t)}$ is: $R_{i(t)} = r_{f(t)} + \beta_{i,mkt}\left(R_{mkt} - r_{f(t)}\right),$ $\beta_{i,mkt} = \frac{cov(R_i, R_m)}{Var(R_m)}$ where, $r_{f(t)}$ is risk-free rate of return, $\beta_{i,mkt}$ the sensitivity security i compared to the market and R_{mkt} the expected of the market index. Betas close to one means the lio/stock return has a perfect correlation with the market.
	3-factors	Here, the expected return is: $R_{i(t)} = r_{f(t)} + \beta_{i,mkt}\left(R_{mkt} - r_{f(t)}\right) + \beta_{smb}SMB + \beta_{hml}HML$ where, the factors betas are respectively related with the ırement of the market risk, the size risk and the value risk. ticular, • SMB: small minus big, the difference between return on fied portfolios of small and large capitalization stock • HML: difference between return on diversified ios of high (distressed firms) and low (non-distressed firms) alue-to-market.

Corporate finance practitioners suggest computing the equity beta in CAPM (or likewise for Fama-French) by looking at the betas for comparable firms (i.e., "the peers"), from which extract the unlevered beta.

1) Selection of peers

The selection of peers is selected by degree of comparability: 1) sector/industry; 2) competitive condition; 3) size and 4) profit margins.
To limit the subjective biases the selection has been based on, the acceptance of a comparable firm rather than another, relies on a scoring method. The decision-making process is driven by an analysis of the business performance over time, which are:

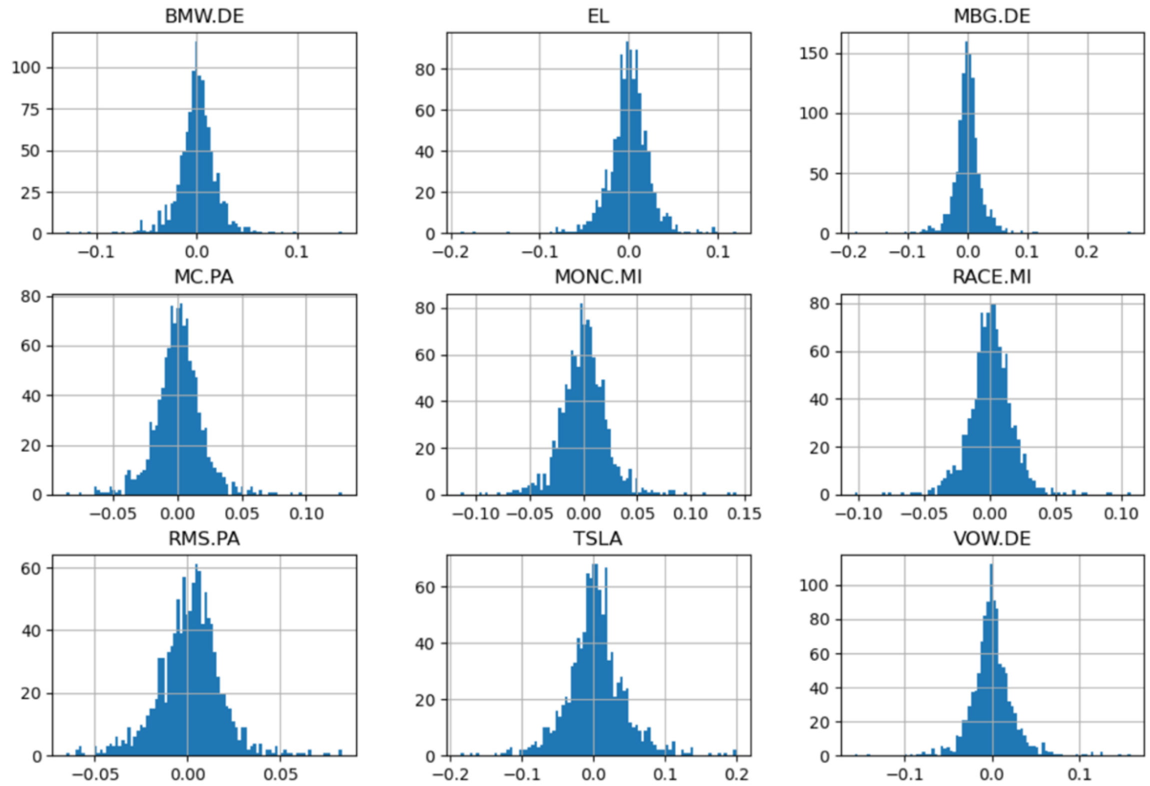
- Average performance (2017Y – 2022Y) named "AP": consider the average value for the business measure of interest.
- Last performance (2022Y) named "LP": consider the last value available for the business measure of interest.

The application of the scoring model allows to decide on selection of peers once the qualitative requirement is met. Porsche benefits from its outstanding financial performance, and as a consequence of that, the threshold values are shaped on its business performances. The intention is to maintain consistent standards for all listed companies in the same industry, but this will not be a rule. Instead, it will serve as a recommendation.[39]

After selecting the peers, the distribution of returns on stocks can be visualized as histogram returns over a specified period. The period selected for analysis spans from December 31, 2019, to December 31, 2023. Here [Table 30], the height of the histograms signifies the frequency of returns, while the x-axis showcases the percentage returns.

[39] Appendix 3 – Peers testing {Note 3.e.}

Table 30: Histogram returns computed with Python for selected peers. Computed with Python.



Peers are evaluated for *i)* liquidity, *ii)* profitability, *iii)* earnings power and iv) *return on equity.* In the case of Porsche AG, comparisons are with the automotive industry as well as the luxury segment which entails premium brands, mainly in fashion and jewellery.

The firms that succeed to the peers testing are BMW, Ferrari, Mercedes-Benz, Volkswagen, LVMH-Moet, Hermes, Moncler, and Estee Lauder. Here below [Table 31] all peers selected are assessed for their Net financial debt (also said, Net Financial Position, NFP). Furthermore, the capacity of peers to fulfill their financial commitments is highlighted by their Interest Coverage Ratio (IRC). NFP would be useful for debt computations.

Table 31: Net Financial Debt For the peers identified and Interest Covered Ratio (ICR)

Table of the net financial debt and interest coverage ratio of the peers

2022								
Net debt (EUR)	BMWG.DE	RACE.MI	MGBn.DE	LVMH.PA	VOWG.DE	EL	MONC.MI	HRMS.PA
Short term debt & current position of (financial debt, trades and other payables)	36,501,000,000	979,393,000	47,141,000,000	9,299,000,000	81,470,000,000	254,000,000	30,156,000	-
Capitalized leases	2,764,000,000	57,423,000	2,645,000,000	2,632,000,000	1,102,000,000	14,000,000	163,914,000	270,000,000
Debt long term	49,870,000,000	1,790,880,000	62,051,000,000	23,156,000,000	121,060,000,000	5,144,000,000	718,709,000	1,664,000,000
Cash and cash equivalents	(16,870,000,000)	(1,388,901,000)	(17,679,000,000)	(7,300,000,000)	(29,172,000,000)	(3,957,000,000)	(882,254,000)	(9,225,000,000)
Short term investment	(5,164,000,000)	(7,068,000)	(6,679,000,000)	(3,522,000,000)	(37,206,000,000)	-	(11,351,000)	
Net financial debt (EUR)	**67,101,000,000**	**1,431,727,000**	**87,479,000,000**	**24,265,000,000**	**137,254,000,000**	**1,455,000,000**	**19,174,000**	**(7,291,000,000)**

Table of the net financial debt and interest coverage ratio of the peers

2021								
Net debt (EUR)	BMWG.DE	RACE.MI	MGBn.DE	LVMH.PA	VOWG.DE	EL	MONC.MI	HRMS.PA
Short term debt & current position (financial debt, trades and other payables) incl. capitalized leases	39,975,000,000	604,283,000	52,300,000,000	10,401,000,000	76,640,000,000	32,000,000	289,191,000	249,000,000
Debt long term	61,613,000,000	2,113,892,000	73,543,000,000	24,052,000,000	130,743,000,000	5,537,000,000	624,732,000	1,553,000,000
Cash and cash equivalents	(16,009,000,000)	(1,344,146,000)	(23,120,000,000)	(8,021,000,000)	(39,723,000,000)	(4,958,000,000)	(932,719,000)	(6,696,000,000)
Short term investment	(5,800,000,000)	(19,350,000)	(6,744,000,000)	(2,544,000,000)	(35,116,000,000)	-	(722,000)	
Net financial debt (EUR)	**79,779,000,000**	**1,354,679,000**	**95,979,000,000**	**23,888,000,000**	**132,544,000,000**	**611,000,000**	**(19,518,000)**	**(4,894,000,000)**

2022	BMWG.DE	RACE.MI	MGBn.DE	LVMH.PA	VOWG.DE	EL	MONC.MI	HRMS.PA
Net Financial Expenses	230,000,000	54,196,000	367,000,000	382,000,000	442,000,000	167,000,000	26,273,000	46,000,000
EBIT	14,028,000,000	1,221,207,000	17,637,000,000	21,068,000,000	21,807,100,000	3,544,000,000	774,547,000	4,820,000,000
ICR	**60.99**	**22.53**	**48.06**	**55.15**	**49.34**	**21.22**	**29.48**	**104.78**

2021								
Net Financial Expenses	236,000,000	38,397,000	375,000,000	242,000,000	1,818,000,000	173,000,000	23,337,000	32,000,000
EBIT	13,457,000,000	1,068,592,000	4,605,000,000	17,121,000,000	18,925,600,000	3,031,000,000	579,220,000	3,595,000,000
ICR	**57.02**	**27.83**	**12.28**	**70.75**	**10.41**	**17.52**	**24.82**	**112.34**

Now, the analysis is directed to estimate the average value of beta unlevered. Each comparable firm is assessed for its market sensitivity, as it would be necessary to calculate the beta equity for each. Before, there's something else to bring up.

2) Find the Benchmark index

A financial analyst wonders *what market am I referring at?* The CAPM assumes the pricing is commensurate to market factors, so the estimations of both risk free and equity risk premium are on the way. The main concern at hand is determining the best index benchmark for assessing the market's sensitivity. If not "the best", at least one indicator with a good fit.

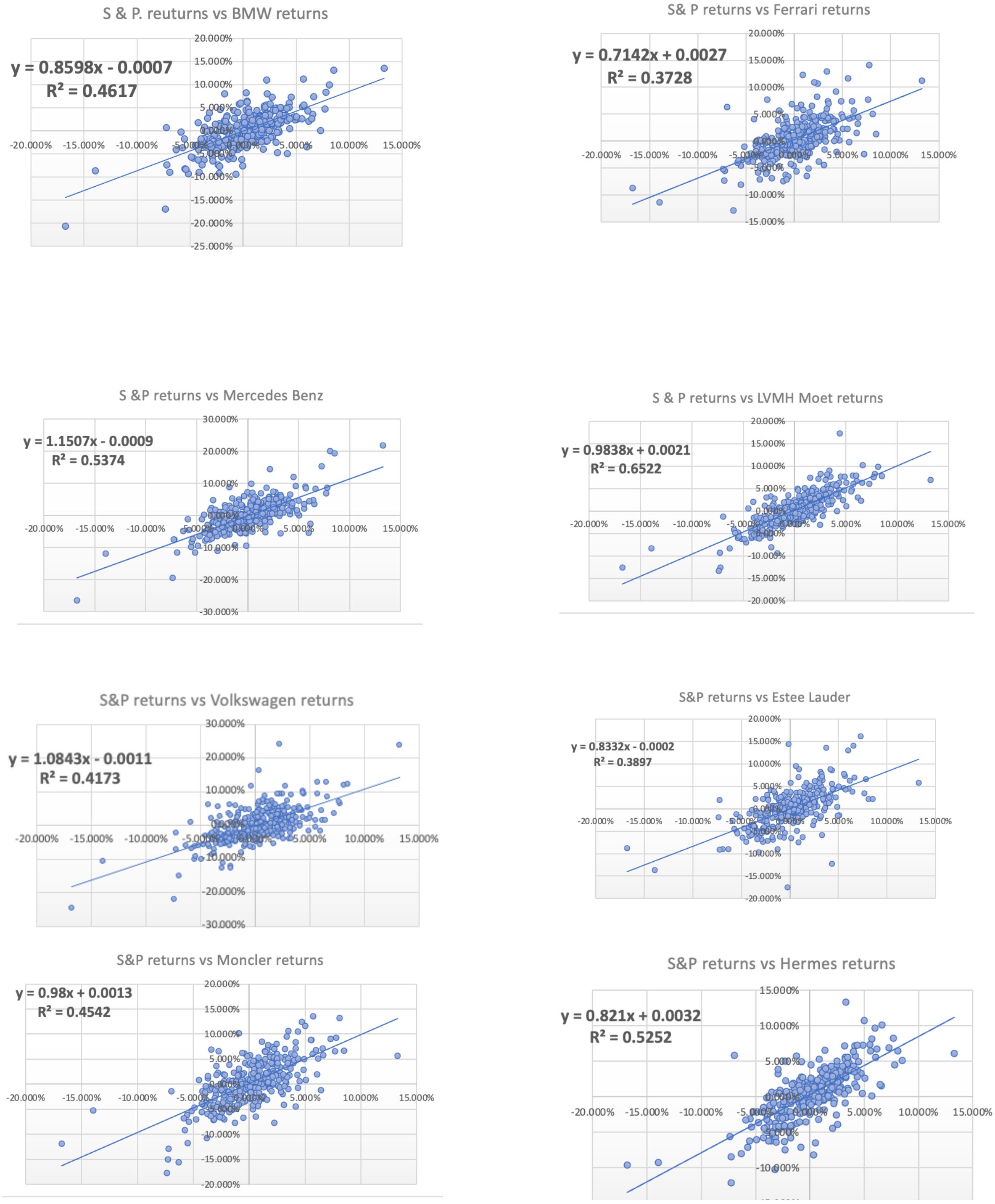
The Risk-Free Rate

The CAPM assumes a premium over a riskless investment for which the investor knows the expected return with certainty. Thus, an investment to be riskless should meet the condition of being "default immune" and to have no uncertainty about reinvestment rates. Those requirements can be meet by plain vanilla bonds issued by sovereign states. A valid proxy is the Federal Republic of Germany Government Bond of 10 years tenure. The risk-free rate comes from the average in the log returns (i.e., yields) of the last 9 months. Consequently, the current (31/01/24) risk-free rate is 2.482%.

The Expected Return of the Market Index and the Equity risk premium (ERP)

The next decision concerns the selection of an appropriate index to represent the luxury market. Standard & Poor's Global Luxury Index is a good fit, such it is made of 79 constituencies, including Hermes, LVMH-Moet, Ferrari and Mercedes-Benz among the top ten elements. S&P Luxury index's estimated return for the last 7 years is 10.64% (i.e., average weakly return). One way to assess the responsiveness of the peer's stock to the index is to conduct a regression analysis [Table 32]. The R square indicates the correlation between the stock performance and its benchmark index. To put it simple, it shows the goodness of the fit. The decision to consider as peers also firms from different industry acknowledges consumer behaviors associated to luxury brands. Despite a clothes designer seems to have no affinity to a car manufacturer, the dynamic of luxury sector leads to valuation and investment strategies tailored to the luxury market. By adopting the S&P Luxury benchmark, it is determined that the estimated ERP is 8.16%. German manufacturers show high dependence with DAX Index that includes the 40 largest listed company in the Frankfurt Börse. Because the index return is 5.28%, the ERP for the Federal Republic of Germany is 2.8%. Indeed, the use of a strictly automotive index such as STOXX Europe 600 Automobiles and Parts shows high correlation with BMW, Mercedes and Volkswagen but a low link with the other peers. For instance, R squared is 0.87 for Mercedes Benz, but 0.14 in the case of Estee Lauder. Alternatively, a wide benchmark like the MSCI World Price Index leads to the same weakness related to low correlation with peers' returns. Luxury market is a niche with peculiar characteristics and dynamics.

Table 32: Regression analysis and R squared: ***Peers and S&P Luxury benchmark.***



3) Determine the average unlevered beta

The section preceding the analysis strictly for Porsche is focused on establishing the unlevered beta, which is a crucial component in the determination of a company's cost of equity. The beta equity is close to one for the selected peers. Some of them are also more volatile than the market. On average the beta equity is 0.93, as reported in [Table 33].

Table 33: Betas levered and the unlevered beta.

Table of peers (beta unl Data in €

Company name	Ticker Refinitiv	Ticker Yahoo Finance	Currency	Share Price (31 gen)	Common share outstainding (listed)	Market Cap EUR	Net Debt EUR	EV	D/E	D/V	R^2	Beta Levered	Beta Unlevered	Tax Rate
Bayerische Motoren Werke AG Group (BMW)	BMWG.DE	BMW.DE	EUR	97.98	646,078,518	63,302,773,194	67,101,000,000	130,403,773,194	106%	51%	0.46	0.860	0.49	30%
Ferrari NV	RACE.MI	RACE.MI	EUR	347.50	193,923,499	67,388,415,903	1,431,727,000	68,820,142,903	2%	2%	0.37	0.714	0.70	24%
Mercedes Benz Group AG	MGBn.DE	MBG.DE	EUR	64.34	1,069,837,477	68,833,343,270	87,479,000,000	156,312,343,270	127%	56%	0.54	1.151	0.61	30%
LVMH Moet Hennessy LV SE	LVMH.PA	MC.PA	EUR	778.10	501,076,940	389,887,967,014	24,265,000,000	414,152,967,014	6%	6%	0.65	0.984	0.94	25%
Volkswagen AG	VOWG.DE	VOW.DE	EUR	131.85	501,295,263	66,095,780,427	137,254,000,000	203,349,780,427	208%	67%	0.42	1.084	0.44	30%
Estee Lauder Companies Inc	EL	EL	USD	124.37	357,055,550	44,194,677,206	1,455,000,000	45,649,677,206	3%	3%	0.39	0.833	0.81	25%
Moncler SpA	MONC.MI	MONC.MI	EUR	57.74	268,824,374	15,521,919,355	19,174,000	15,541,093,355	0.1%	0%	0.45	0.980	0.98	25%
Hermes International	HRMS.PA	RMS.PA	EUR	2,023.00	105,460,940	213,347,481,620	(7,291,000,000)	206,056,481,620	3%	4%	0.53	0.821	0.80	25%
Average (weighted)						116,071,544,748	38,964,237,625		64.1%	58.4%		0.93	0.72	27%
Median												0.98	0.76	
Tax rate source: https://pages.stern.nyu.edu/~adamodar/New_Home_Page/datafile/countrytaxrates.html														

4) Estimate the Cost of Equity per Porsche AG

The process of delevering leads to a beta unlevered of 0.72, on average. After, the computation related the assessment of sensitivity, the CAPM application give back a cost of Equity of Porsche AG is 11.0% [Table 34].

Table 34: Cost of Porsche's Equity

WACC Porsche AG	
Cost of Equity (Ke)	
Risk free rate	2.48%
Beta unlevered	0.72
D/E	64.1%
Beta relevered	1.05
Tax rate	30%
Market risk premium	8.16%
Ke	**11.0%**

Cost of Debt

The expense associated with borrowing funds, commonly referred to as the cost of debt, is determined by the interest coverage ratio (ICR) in addition to the risk-free rate. As per the estimation of Damodaran, a high ranking for Porsche, such as an AAA rating, would result in a slight premium. Non-financial firms with an AAA rating typically have a premium of 0.59 percent, which implies that the cost of debt for such a firm would be accordingly.

Cost of Debt (Kd)	
Kd before tax	3.07%
Tax rate	30.00%
Kd after tax	**2.15%**

Table 34: Cost of Porsche's Debt

The cost of capital for Porsche AG is 9.7%.

Corporate Structure

The recent IPO followed after the spin-off of luxury vehicle division from the Volkswagen AG group has changed the company shareholding. At the time of the announcement, Volkswagen AG's shareholding below reported, is divided into a group of equity holders and free float capital – owing different voting rights (December 31, 2021). The Company's Board holds – through Porsche GmbH, the entire (100%) Porsche AG. The likely spin-off of Volkswagen's luxury vehicle division arises great interest among stakeholders.

The new entity, Porsche AG, has been established with 911.00 ml of capital share according to the Prospectus. One half of the capital share are preferred share, and the other half is represented by ordinary shares. The admission to trading at Frankfurt Börse affects the 25% of non-voting preferred shares, which are split into base shares and over-allotment shares. The ordinary shares have not been listed, and the stake is owned by Volkswagen. So, the firm remained in the consolidated annual reports.

Table 35: The Corporate structure of Porsche AG before IPO

Volkswagen AG	100.00%	
	Stake	Voting Rights
Porsche Automobil Holding SE	31.40%	53.30%
Foreign Institutional investors	27.00%	-
State of Lower Saxony	11.80%	20.00%
Private Institutional Investors	16.00%	9.70%
Qatar Holding	10.50%	17.00%
German Instituional investors	3.30%	-
	100.00%	100.00%

Preferred Shares	455.500.000
Public Offering (IPO)	
Floated Preferred Shares (25%)	113.875.000
Base Shares	99.021.740
Over-Allotment Shares (15%)	14.853.260
Ordinary Shares	455.500.000
Share capital amount	911.000.000

3.6. The Current Valuation of Porsche AG and IONITY GmbH

Under the previous assumptions regarding the scenarios for Porsche's strategic implementation of the switch to the electric market, we can examine their flexibility by considering the best-case and normal-case scenarios.

Table 36: Value of Porsche AG with DCF

The value and its scrutiny with discounted cash flows

Free cah flow Bear market	2022	2023	2024	2025	2026	2027	2028	2029	2030
Free cash flow		2,653	3,056	3,582	3,757	3,944	4,143	4,354	4,580
Terminal growth rate									2.20%
Terminal value cash flow (from 2028 to infinite)									116,360

Free cah flow Bull market	2022	2023	2024	2025	2026	2027	2028	2029	2030
Free cash flow		2,653	4,552	4,949	5,401	5,890	6,418	6,987	7,601
Terminal growth rate									2.20%
Terminal value cash flow (from 2028 to infinite)									116,360

Undertaken with precision and care, the valuation has been executed with Capital Budgeting with uncertainty[40]. Consequently, if both the best case and the normal case will have same likelihood, the value of the cash flows will prove to be their average. Weather the uncertainty increases the value of the option, here there's not a proper option, or at least not yet. The best case and the normal case depend strongly on the market trends, that is why the market had a in deep analysis.

[40] See the Appendix 3: the Capital Budgeting under uncertainty {Note 3.f.}

Table 37: The Enterprise Value and The Equity Value Porsche AG – Date 31 Dec 2023

	BEAR SCENARIO		BULL SCENARIO	
	HP: Book value of the debt is a good proxy for Market Value of the debt	Data in millions of €		
Valuation				
	Enterprise value	**75,084**	Enterprise value	**83,848**
	Finacial Debt	15,651		15,651
	Cash & Cash Equivalent	3,719		3,719
	Net Financial Debt	11,932		11,932
	Equity Value	**63,152**		**71,916**

WACC										
	Wacc	9.70%								
Discounting	Bear market									
	Discount factor	0	1	2	3	4	5	6	7	8
	Discount rate	1.000	0.912	0.831	0.758	0.691	0.630	0.574	0.523	0.477
	Discounted cash flow	-	**2,418**	**2,539**	**2,714**	**2,595**	**2,483**	**2,377**	**2,278**	**2,184**
	PV(FCF)	**19,589**					-			
	Discounted Terminal Value						-			**55,496**
Discounting	Bull market									
	Discount factor	0	1	2	3	4	5	6	7	8
	Discount rate	1.000	0.912	0.831	0.758	0.691	0.630	0.574	0.523	0.477
	Discounted cash flow	-	**2,418**	**3,783**	**3,749**	**3,730**	**3,708**	**3,683**	**3,656**	**3,625**
	PV(FCF)	**28,352**								
	Discounted Terminal Value									**55,496**

On the other side, the current valuation of Ionity presents a promising outlook for future growth, as the electric market holds immense potential. Projections extending ten years into the future indicate that the terminal value will offer the highest degree of value added.

The cost of Ionity is considered to be at a premium due to its high level of *indebtedness*, so the WACC stands around 12.8%. Ionity has recently announces (data, Ionity Press Office) to be committed for a € 700 M investment for power station expansion within 2025. The non-discounted value of the capital expenditures overpass € 2B (time horizon 2030).

Table 38: The Ionity DCF valuation - A

Data in thousands										Growth		4%	2.50%
			1	2	3	4	5	6	7	8	9	10	
	2020	**2022**	**2023**	**2024**	**2025**	**2026**	**2027**	**2028**	**2029**	**2030**	**2031**	**2032**	**2032N**
EBITDA	**(30,512)**	**(29,865)**	**4,060**	**21,637**	**55,860**	**132,757**	**238,067**	**383,194**	**575,897**	**837,098**	**870,582**	**905,406**	858,026
Depreciation & Amortization	(19,296)	(28,506)	(37,899)	(47,292)	(37,389)	(49,313)	(61,860)	(77,294)	(99,512)	(130,368)	(166,703)	(218,583)	(224,047)
Depreciation & Amortization	176%	32%	26%	23%	14%	12%	10%	9%	8%	8%	8%	8%	
EBIT	**(49,808)**	**(58,371)**	**(33,839)**	**(25,655)**	**18,471**	**83,445**	**176,207**	**305,900**	**476,384**	**706,730**	**703,879**	**686,823**	**703,993**
		Tax rate:											
NOPAT		**30%**	**(23,688)**	**(17,959)**	**12,930**	**58,411**	**123,345**	**214,130**	**333,469**	**494,711**	**492,716**	**480,776**	**492,795**
(+)Depretiation			37,899	47,292	37,389	49,313	61,860	77,294	99,512	130,368	166,703	218,583	224,047
Gross Flows			14,211	29,333	50,319	107,724	185,205	291,424	432,981	625,079	659,419	699,359	716,843
(-)Delta WCR			7,325	3,953	930	6,945	2,050	10,153	1,773	17,527	2,106	2,190	
(-)Capex			(90,718)	(103,620)	(97,482)	(149,524)	(177,885)	(192,968)	(214,848)	(245,376)	(282,028)	(334,234)	(342,590)
FCFO			**(69,182)**	**(70,334)**	**(46,233)**	**(34,855)**	**9,370**	**108,609**	**219,906**	**397,230**	**379,496**	**367,315**	**374,253**

Due to the current negative EBITDA, the multiple must be considered "figurative" as it demonstrates an EV to be much higher than the EBITDA.

Table 38: The Ionity DCF valuation - B

2030 Prospective		
FCF t		834,096
PV (K)		**(700,000)**
NPV		**134,097**
Project Value (without TV)	**€**	**134,097**
TV (10 YEARS)		**1,094,398**
EV (with Terminal Value)		**1,228,495**
IRR		**2.9%**
	ABANDON	
the WACC is higher than the return on the project		

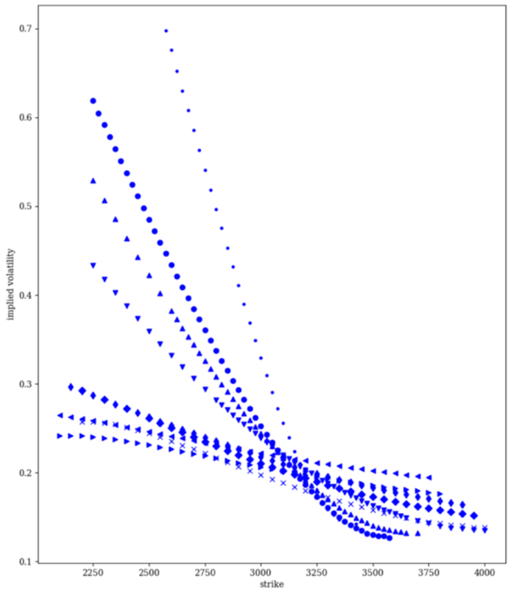
Ionity's current valuation renders a takeover unlikely, and thus, Porsche is likely to abandon the project based on its discounted cash flow analysis.

3.7. The Uncertainty in the Market: Implied Volatility

The estimation of the cash flows volitivity is essential to capture part of the uncertainty in the market. Higher uncertainty implies upside gains other than losses, which enhance the value of the option. Due to the unique characteristics of the Black-Scholes model, it is essential to recognize that the *Vega*, which represents the first derivative of the formula with respect to volatility, is consistently positive.

$$C_0^* = BSM\ Call\ Option\ Price\ (S_0, K, t, r, \sigma_{implied})$$

This positive value signifies a direct relationship between call values and volatilities, and consequently, a single solution for the equation. The studies will be conducted on the volatility of the *EURO STOXX* 50 stock index. The model can be settled to study the forecasting on discrete time [Graph 2].



Graph 2: Implied Volatility with BSM

3.8. The Options Application

APPLICATION IN THE HORIZONT 2: The Case of Ionity's takeover.

The Case examines the acquisition of Ionity, which in the future might be one of the most extensive charging stations networks, with the involvement of "Porsche Ventures" as the venture capital entity. The target firm could be valued under an *LBO model* to embed the flexibility of exiting when favourable conditions occur. The options available for Private Equity firm (PE)[41] is planning their exit strategy: exit now or later. Here, either American or European option can be used because PE have no restriction in deciding when to exit.
The analysis there is the build of the pre-LBO model: three-statement financial projections. The entire implementation relies on assumptions based on recent transactions by ***Porsche Ventures.*** PE [Table 39] firms undertake investments to keep in their portfolio for around 10 years, but they could go further and extend the period extra, later the investment vehicle decide to exit.

[41] A Private Equity Firm is a vehicle that undergoes buyout transactions using a large amount of debt.

Once the funding source are gathered the management company – the counselling branch of the PE – start to scout for potential target, and if it is suitable, invest in them.

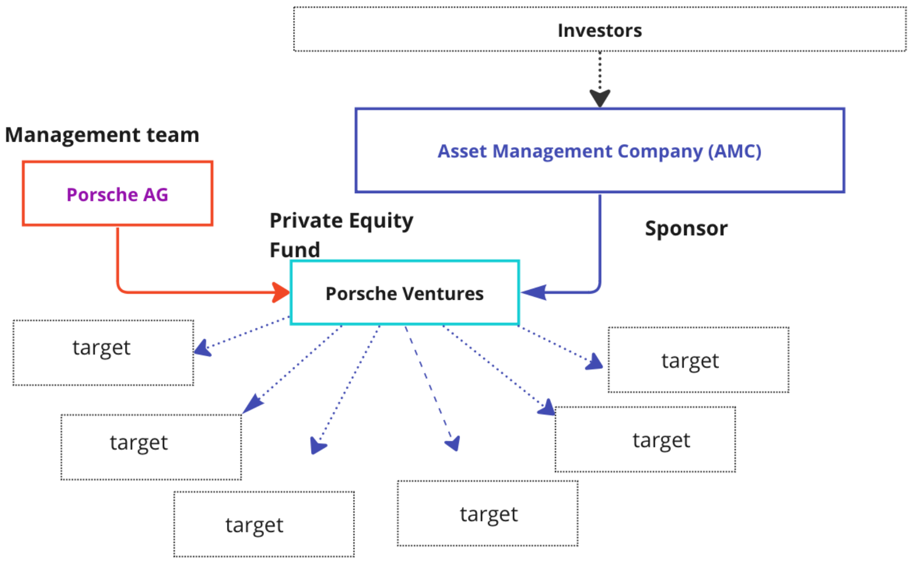


Table 39: The Private Equity Structure (Porsche Ventures)

At the time when Porsche was contemplating granting access to its electric car network, the venture capital division elected to investigate the feasibility of acquiring Ionity. Our examination is comprised of two components: the holding period and the post-exit period.

The value of the target will be:

$$Target\ value = LBO\ value + Holding\ option + Post\ exit\ option$$

Static Pre-LBO Valuation

The process of performing a Leveraged Buyout (LBO) valuation begins with the preparation of a Pro Forma Income Statement[42]. This statement is formulated based on extensive market research, which is meticulously detailed in the forecasting sections. The primary objective of

[42] See the Appendix 3: **presentation of the Proforma Income Statement and Proforma Balance Sheet** {Note 3.g.**}**

this exercise is to ascertain the *net income* for the first year following the buyout, while simultaneously taking into account the subsequent "erosion" of equity that will be incurred by Ionity. Same considerations extend to Balance Sheet Pro-Forma, where there are pointed out the required adjustments following the buyout. To sum up, the first steps are:

- Build the historical and Forecasted Income Statement;
- Define Opening Balance Sheet and Projection of the Items in the future;
- Elaborate the Cash Flow Statement.

Most important advancements are reserved to the Cash Flow Statement Table because it led to the determination of the Cash Flow to debt repayment [Table 40].

Data in thousands

Cash Flow Statement

	Year 2023	Year 2024	Year 2025	Year 2026	Year 2027	Year 2028	Year 2029	Year 2030	Year 2031	Year 2032
					Projection Period					
Operating Activities										
EBIT less taxes	(23,688)	(17,959)	12,930	58,411	123,345	214,130	333,469	494,711	492,716	480,776
Plus: D&A	37,899.0	47,292.0	37,389.0	49,312.5	61,860.1	77,294.2	99,512.3	130,368.0	166,702.9	218,582.8
Changes in Working Capital Items										
(Inc.) / Dec. in Accounts Receivable	(10,938)	(10,906)	(12,647)	(22,394)	(29,016)	(51,160)	(59,870)	(82,780)	(11,901)	(12,377)
(Inc.) / Dec. in Inventories	(559)	(1,558)	(1,807)	(3,199)	(4,145)	4,538	(3,974)	(5,494)	(790)	(822)
(Inc.) / Dec. in Prepaid and Other Current Assets										
Inc. / (Dec.) in Accounts Payable	18,821	16,417	15,384	32,538	35,211	56,775	65,616	105,802	14,797	15,389
Inc. / (Dec.) in Accrued Liabilities										
Inc. / (Dec.) in Other Current Liabilities										
(Inc.) / Dec. in Net Working Capital	7,325	3,953	930	6,945	2,050	10,153	1,773	17,527	2,106	2,190
Cash Flow from Operating Activities	**21,536**	**33,286**	**51,249**	**114,669**	**187,255**	**301,577**	**434,754**	**642,606**	**661,524**	**701,549**
Investing Activities										
Capital Expenditures	(90,718)	(103,620)	(97,482)	(149,524)	(177,885)	(192,968)	(214,848)	(245,376)	(282,028)	(334,234)
Other Investing Activities	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0		
Cash Flow from Investing Activities	**(90,718)**	**(103,620)**	**(97,482)**	**(149,524)**	**(177,885)**	**(192,968)**	**(214,848)**	**(245,376)**	**(282,028)**	**(334,234)**
FREE CASH FLOW	**(69,182)**	**(70,334)**	**(46,233)**	**(34,855)**	**9,370**	**108,609**	**219,906**	**397,230**	**379,496**	**367,315**

Figure 40: Cash Flow Statement of Ionity (FCF)

In the context of financial structure, the use of *Leveraged Buyout* (LBO) analysis by an analyst is to establish a feasible financing structure for the intended acquisition. In practical application, the objective is to provide the financial sponsor with a tailored financing structure that optimizes returns while maintaining the status of marketable securities. The entire process is iterative as not all information is available at the moment of the analysis.

A table of sources and uses [Tables 41-42] is the key path to understand the funds required for a buyout transaction. The pricing is characterized by the spread (in basis points) over the EURIBOR (L), which is typically elevated during these periods.

Transaction Summary					
Sources of Funds					
		Amount	% of Total Sources	Multiple of EBITDA 1/0/00	Pricing
Revolving Credit Facility	€	94,000	18.4%	(3.1x)	L+525 bps
Term Loan A			- %	- x	
Term Loan B	€	200,000.00	39.2%	(6.7x)	L+500 bps
Term Loan C			- %	- x	
2nd Lien			- %	- x	
Senior Notes		100,000.0	19.6%	(3.3x)	
Senior Subordinated Notes			- %	- x	
Equity Contribution		80,200	15.7%	(2.7x)	
Rollover Equity			- %	- x	
Cash on Hand		35,915.0	7.0%	(1.2x)	
Total Sources		**$510,115**	**100.0%**	**(17.1x)**	

Table 41: Source of Funds - LBO

Uses of Funds			
		Amount	% of Total Uses
Purchase ValueCo Equity		$268,981.0	52.7%
Repay Existing Debt		214,909.0	42.1%
Financing Fees	9%	19,500.8	3.8%
Other Fees and Expenses	2.50%	6,724.2	1.3%
Total Uses		**$510,115**	**100.0%**

Table 42: The uses of Funds - LBO

The acquisition of businesses typically involves the repayment of debt, in addition to fees such as financing fees and other expenses. These costs are taken into account during the buyout process. The Debt Schedule [Table 43] relies on mandatory payments and optional based of more flexible financing alternative. Revolving Credit requires repayment at the maturity date of all outstanding advances, a term loan is a closed contract agreed in advance. Other debt is senior unsecured notes issues for financing the transaction.

data in thousands

Debt Schedule	Projection Period									
	Year 1 2023	Year 2 2024	Year 3 2025	Year 4 2026	Year 5 2027	Year 6 2028	Year 7 2029	Year 8 2030	Year 9 2031	Year 10 2032
Forward EURIBOR Curve	3.85%	2.93%	2.30%	2.30%	2.30%	2.30%	2.30%	2.30%	2.30%	2.30%
Cash Flow from Operating Activities	€ 21,535.97	€ 33,286.20	€ 51,249.43	€ 114,668.86	€ 187,254.92	€ 301,577.22	€ 434,754.03	€ 642,606.49	€ 661,524.46	€ 701,548.97
Cash Flow from Investing Activities	(90,718.0)	(103,620.0)	(97,482.0)	(149,524.0)	(177,885.0)	(192,968.0)	(214,848.0)	(245,376.0)	(282,028.4)	(334,234.4)
Cash Available for Debt Repayment	**€ (69,182.03)**	**€ (70,333.80)**	**€ (46,232.57)**	**€ (34,855.14)**	**€ 9,369.92**	**€ 108,609.22**	**€ 219,906.03**	**€ 397,230.49**	**€ 379,496.03**	**€ 367,314.57**
Total Mandatory Repayments	(10,000.0)	(10,000.0)	(10,000.0)	(10,000.0)	(10,000.0)	(10,000.0)	(10,000.0)	(10,000.0)	-	-
Cash From Balance Sheet	-	-	-	-	-	-	-	-	220,512.1	600,008.2
Optional Debt Repayment	**($79,182.0)**	**($80,333.8)**	**($56,232.6)**	**($44,855.1)**	**($630.1)**	**$98,609.2**	**$209,906.0**	**$387,230.5**	**$600,008.2**	**$967,322.7**

Table 43: Debt Schedule (Ionity)

Revolving Credit Facility												
Revolving Credit Facility Size	€	94,000.00										
Spread		5.250%										
LIBOR Floor		- %										
Term		8 years										
Commitment Fee on Unused Portion		0.50%										
Beginning Balance			$94,000.0	$173,182.0	$253,515.8	$309,748.4	$354,603.5	$355,233.6	$256,624.4	$46,718.4	-	-
Drawdown/(Repayment)			79,182.0	80,333.8	56,232.6	44,855.1	630.1	(98,609.2)	(209,906.0)	(46,718.4)	-	-
Ending Balance			**$173,182.0**	**$253,515.8**	**$309,748.4**	**$354,603.5**	**$355,233.6**	**$256,624.4**	**$46,718.4**	**-**	**-**	**-**
Interest Rate			9.10%	8.18%	7.55%	7.55%	7.55%	7.55%	7.55%	7.55%	7.55%	7.55%
Interest Expense												
Commitment Fee			(198.0)	(596.7)	(938.2)	(1,190.9)	(1,304.6)	(1,059.6)	(288.4)			
Administrative Agent Fee			0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2
Term Loan B Facility												
Size	€	200,000.00										
Spread		4.250%										
LIBOR Floor		- %										
Term		8 years										
Repayment Schedule		5.0%	Per Annum, Bullet at Maturity									
Beginning Balance			$200,000.0	$190,000.0	$180,000.0	$170,000.0	$160,000.0	$150,000.0	$140,000.0	$130,000.0	-	-
Mandatory Repayments			(10,000.0)	(10,000.0)	(10,000.0)	(10,000.0)	(10,000.0)	(10,000.0)	(10,000.0)	(10,000.0)	-	-
Optional Repayments			-	-	-	-	-	-	-	(120,000.0)	-	-
Ending Balance			**$190,000.0**	**$180,000.0**	**$170,000.0**	**$160,000.0**	**$150,000.0**	**$140,000.0**	**$130,000.0**	**-**	**-**	**-**
Interest Rate			8.10%	7.18%	6.55%	6.55%	6.55%	6.55%	6.55%	6.55%	6.55%	6.55%
Interest Expense			**15,795.0**	**13,283.0**	**11,462.5**	**10,807.5**	**10,152.5**	**9,497.5**	**8,842.5**	**4,257.5**		
Senior Notes												
Size		$100,000.0										
Coupon		8.000%										
Term		10 years										
Beginning Balance			$100,000.0	$100,000.0	$100,000.0	$100,000.0	$100,000.0	$100,000.0	$100,000.0	$100,000.0	$100,000.0	$100,000.0
Repayment			-	-	-	-	-	-	-	-	-	-
Ending Balance			**$100,000.0**	**$100,000.0**	**$100,000.0**	**$100,000.0**	**$100,000.0**	**$100,000.0**	**$100,000.0**	**$100,000.0**	**$100,000.0**	**$100,000.0**
Interest Expense			8,000.0	8,000.0	8,000.0	8,000.0	8,000.0	8,000.0	8,000.0	8,000.0	8,000.0	8,000.0

Figure 44: Debt Payment estimation – LBO

The purchase price for a loss-making firm in the technological infrastructure start-up sector typically ranges around 15 times EBITDA (15x). The sponsor is evaluating the purchase price based on a negative operating income scenario. When applying a Discounted Cash Flow (DCF) analysis, the Enterprise Value would likely be higher due to the consideration of future cash flows and growth potential. The Cash Flow Statement ultimately serves to demonstrate the liquidity at hand.

Cash Flow Statement										
					Projection Period					
	Year 2023	**Year 2024**	**Year 2025**	**Year 2026**	**Year 2027**	**Year 2028**	**Year 2029**	**Year 2030**	**Year 2031**	**Year 2032**
FREE CASH FLOW	**(69,182)**	**(70,334)**	**(46,233)**	**(34,855)**	**9,370**	**108,609**	**219,906**	**397,230**	**379,496**	**367,315**
Financing Activities										
Revolving Credit Facility	79,182.0	80,333.8	56,232.6	44,855.1	630.1	(98,609.2)	(209,906.0)	(46,718.4)	-	-
Term Loan B	(10,000)	(10,000)	(10,000)	(10,000)	(10,000)	(10,000)	(10,000)	(130,000)	-	-
Cash Flow from Financing Activities	**$69,182**	**$70,334**	**$46,233**	**$34,855**	**($9,370)**	**($108,609)**	**($219,906)**	**($176,718)**	**-**	**-**
Excess Cash for the Period	-	-	-	-	-	-	-	$220,512.1	$379,496.0	$367,314.6
Beginning Cash Balance	-	-	-	-	-	-	-	-	220,512.1	600,008.2
Ending Cash Balance	**-**	**-**	**-**	**-**	**-**	**-**	**-**	**$220,512.1**	**$600,008.2**	**$967,322.7**
Cash Flow Statement Assumptions										
Capital Expenditures (% of sales)										

Figure 45: Cash Flow Statement – Financing Activities (Ionity)

Perform Returns Analysis: LBO Valuation

Generally, sponsor will require a target IRR between 15% to 20 %. IRRs are the key driver to project the target's *financial performance.* IRR stands for the "opportunity cost of capital" that investor will require for entering into a illiquid investment.

In thousands												
Returns Analysis												
			Projection Period									
		Pro forma 2022	**Year 1 2023**	**Year 2 2024**	**Year 3 2025**	**Year 4 2026**	**Year 5 2027**	**Year 6 2028**	**Year 7 2029**	**Year 8 2030**	**Year 9 2031**	**Year 10 2032**
Entry EBITDA Multiple	(15.0x)											
Initial Equity Investment		80,200										
EBITDA		(29,865)	4,060	21,637	55,860	132,757	238,067	383,194	575,897	837,098	870,582	905,406
Exit EBITDA Multiple	3.0x											
Enterprise Value at Exit			**12,179**	**64,911**	**167,581**	**398,272**	**714,201**	**1,149,583**	**1,727,690**	**2,511,295**	**2,611,747**	**2,716,217**
Less: Net Debt												
Revolving Credit Facility			173,182	253,516	309,748	354,604	355,234	256,624	46,718	0	0	0
Term Loan B			190,000	180,000	170,000	160,000	150,000	140,000	130,000	0	0	0
Term Loan B												
Term Loan C												
Existing Term Loan												
2nd Lien												
Senior Notes			100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000
Senior Subordinated Notes												
Other Debt												
Total Debt			**463,182**	**533,516**	**579,748**	**614,604**	**605,234**	**496,624**	**276,718**	**100,000**	**100,000**	**100,000**
Less: Cash and Cash Equivalents			-	-	-	-	-	-	-	(220,512.1)	(600,008.2)	(967,322.7)
Net Debt			**463,182**	**533,516**	**579,748**	**614,604**	**605,234**	**496,624**	**276,718**	**320,512**	**700,008**	**1,067,323**
Equity Value at Exit			**(451,003)**	**(468,605)**	**(412,167)**	**(216,331)**	**108,967**	**652,959**	**1,450,972**	**2,190,783**	**1,911,739**	**1,648,894**

Figure 46: Return Analysis - LBO

The results of the financial analysis indicate that the earliest possible time to exit is in the year 2028Y, based on the internal rate of return (IRR). Additionally, the most suitable time period for exercising the option to exit is estimated to be between 2029Y and 2030Y. No changes should be made to the citations, references, or in-line citations, and the numbers within the text should not be altered. Here the exit multiple is expected to be 800% higher than entry multiple.

Figure 47: The IRR - LBO

Cash Return	**-5.6x**	**-5.8x**	**-5.1x**	**-2.7x**	**1.4x**	**8.1x**	**18.1x**	**27.3x**	**23.8x**	**20.6x**
	Year 1 2023	**Year 2 2024**	**Year 3 2025**	**Year 4 2026**	**Year 5 2027**	**Year 6 2028**	**Year 7 2029**	**Year 8 2030**	**Year 9 2031**	**Year 10 2032**
Initial Equity Investment	(80,200.00)	(80,200)	(80,200)	(80,200)	(80,200)	(80,200)	(80,200)	(80,200)	(80,200)	(80,200)
Equity Proceeds	(531,203.43)	-	-	-	-	-	-	-	-	-
		(548,805)	-	-	-	-	-	-	-	-
			(492,367)	-	-	-	-	-	-	-
				(296,531.19)	-	-	-	-	-	-
					28,767	-	-	-	-	-
						572,759	-	-	-	-
							1,370,772	-	-	-
								2,110,583	-	-
									1,831,539	-
										1,568,694
IRR	**NA**	**NA**	**NA**	**NA**	**(18.5%)**	**39%**	**50%**	**50%**	**42%**	**35%**

Assessment on the Risky Debt: Option Valuation in the earliest occasion to exit

Under the auspices of Chapter II, the valuation of the target firm can be ascertained through the use of the option approach, particularly in situations where there is ambiguity surrounding the volatile market conditions.

Data exhibits that on 2028, the expected enterprise value is € 1,150 M, while the debt will be roughly a bit less than a half. Under the below forecast. Suppose that the current value is the asset value (e.g., also DCF valuation could be used). If the intention is to estimate the impact of debt on the LBO transaction, the current analysis could be useful to be read.

Data in millions

Asset Value	€	1,150	
Debt	€	497	
rf		3%	
volatility		21%	
t		**6**	**years**
delta t		1	
Beta asset		0.93	

u	1.23367806
d	0.810584246
1+rf	1.03
p	51.27%
1-p	48.73%

The probability of default is quite significant, even though a small value of the Loss Given Default (LGD).

Figure 48: The Binomial Lattices: The Risky Debt in a LBO transaction

2022 0	2023 1	2024 2	2025 3	2026 4	2027 5	2028 6	Ups	Probabilities	Payoffs Debt	 Equity
€ 1,150	€ 1,418.11	€ 1,749.49	€ 2,158.31	€ 2,662.66	€ 3,284.87	€ 4,052.47	6.00	**1.8%**	497	3,556
	€ 931.77	€ 1,149.50	€ 1,418.11	€ 1,749.49	€ 2,158.31	€ 2,662.66	5.00	**10.4%**	497	2,166
		€ 755.28	€ 931.77	€ 1,149.50	€ 1,418.11	€ 1,749.49	4.00	**24.6%**	497	1,253
			€ 612.21	€ 755.28	€ 931.77	€ 1,149.50	3.00	**31.2%**	497	653
				€ 496.25	€ 612.21	€ 755.28	2.00	**22.2%**	497	259
					€ 402.25	€ 496.25	1.00	**8.5%**	496	0
						€ 326.06	0.00	**1.3%**	326	0

Probabiliy of Default	**32.0%**	Expected Values	**494.3**	**858.4**
Loss Given Default	**7**	Present Values	**420**	**729**

The debt value today is € 420 M, while the equity at maturity $t = 6$ will be: $EV_{(6)} - Debt_{(6)}$. The estimation goes backward like $Exp(-rt) * (pEV_{6_{NODE\,1}} + (1-p)EV_{6_{NODE\,2}})$

Figure 49: Debt Value and Equity Value of Ionity

Debt Value	1	2	3	4	5	6
420	433.62	445.55	457.81	470.40	483.33	496.62
	429.49	445.54	457.81	470.40	483.33	496.62
		436.85	457.77	470.40	483.33	496.62
			439.49	470.31	483.33	496.62
				431.86	483.16	496.62
					402.25	496.25
						326.06

Equity value	1	2	3	4	5	6
729	984	1,304	1,701	2,192	2,802	3,556
	502	704	960	1,279	1,675	2,166
		318	474	679	935	1,253
			173	285	448	653
				64	129	259
					0	0
						0

The Cost of Capital can be derived by the beta equity and beta debt. Remember that,

$$\beta_e = \beta_a Delta_{call} \frac{V}{E} \qquad \beta_d = -\beta_a Delta_{put} \frac{V}{D}$$

In this context, beta debt is essentially negligible but not completely non-existent. As the likelihood of default increases, particularly due to higher volatility which negatively impacts a company's creditworthiness, the beta debt tends to rise accordingly.

Figure 50: Delta Equity and Delta Debt (Ionity LBO)

Delta Equity	1	2	3	4	5
0.99	1.00	1.00	1.00	1.00	1.00
	0.98	1.00	1.00	1.00	1.00
		0.94	1.00	1.00	1.00
			0.85	1.00	1.00
				0.61	1.00
					0.00

Delta Debt	1	2	3	4	5
0.01	0.00	0.00	0.00	0.00	0.00
	0.02	0.00	0.00	0.00	0.00
		0.06	0.00	0.00	0.00
			0.15	0.00	0.00
				0.39	0.00
					1.00

Beta debt appears to be negligible. It is essential that the sum of delta equity and delta debt is equal.

Figure 51: Beta Equity and Beta Debt – (Ionity LBO)

Beta equity	1	2	3	4	5
1.45	1.34	1.25	1.18	1.13	1.09
	1.69	1.52	1.37	1.27	1.20
		2.08	1.83	1.57	1.41
			2.81	2.46	1.93
				4.41	4.41
					3.17

Beta debt	1	2	3	4	5
0.02	0	0	0	0	0
	0.04	0	0	0	0
		0.09	0	0	0
			0.19	0	0
				0.41	0
					0.93

The relation between the two betas can be found with Harris-Pringle. The link shows up that…

$$\beta_E = \beta_A + (\beta_A - \beta_D)(1 - t_C)\left(\frac{D}{E}\right)$$

By considering tax shield the true value of beta equity at time zero should be close to 1.21 value.

Figure 52: Beta equity with tax shield

Beta equity with tax shield

	1	2	3	4	5
1.2	1.2	1.1	1.1	1.0	1.0
	1.3	1.2	1.2	1.1	1.1
		1.5	1.4	1.3	1.2
			1.8	1.6	1.4
				2.3	2.2
					4.1

Same calculation can be done with Black-Scholes Model.

Figure 53: Black-Scholes Model

Black-Scholes Merton Model

Continuous Rate		2.71%
Risk Free Debt	€	**422**
d1		**2.21**
d2		**1.69**
Equity	€	**730.88**
Debt	€	**418.62**
N(-d2)=Loss Given Default		4.54%
N(-d1)		1.372%
Expected recovery-given default	€	408.46
Expected LGD	€	11.59

Decision-making in LBO: Embedded Real Options

At the time the PE decides to exit from its investment, the "exit value" is the value attributed to the target by a potential buyer. Multiple valuation for determine the exit value has been preferred because of high bargaining power from the buyer at the time Ionity is a loss-taking firm.

In the current case, the underlying asset is the firm's assets market value of the firm. If the PE decides to exit before the investment maturity, there isn't a related cost for exercising the option (i.e., by considering any transaction costs negligible). The real "opportunity cost" is the leftover required return from the investors. As a consequence, the holding option will be exercised if the target value (i.e., the underlying) is greater than the firm value at some point in the time schedule. To put it simpler, if PE decides to leave earlier then the investment vehicle quits from its right to sell at expiration date. The exercise price at node x can be sum up as the value of the equity determined by the IRR, and the addition of the leftover debt in the LBO model subtracted by the cash flows (FCF).

$$V_x^{hold} = Equity_{x-1}\left(1 + IRR_{target}\right)^{\Delta t} + [Debt_{x-1} - (FCF)\Delta t]$$

Similarly, the post-exit option is valued at the required equity less the debt, and the cash flows are expected to maintain the capital structure's financial stability. In this scenario, it is assumed that the cash flows will remain constant following the exit. Dividends are added considering the policy that Ionity has the intention to distribute *25 %* of the net income. Further assumption in the constant capital structure will be interesting to add the debt-equity ratio.

$$V_x^{post-exit} = Equity_{x-1}\left(1 + IRR_{target}\right)^{\Delta t} - DIV_x + \mathrm{debt_x}$$
$$= Equity_{n-1} + \left(1 + IRR_{target}\right)^{\Delta t} - DIV_n)\frac{Debt_t}{Equity_t}$$

Determining the time to expiration is not an easy task because in the real-world real options could be "shared" in the sense that also other peers might undertake the same decision. Here, the options are proprietary of the investment vehicle. In the holding option the expiration time follows the "holding period" itself. Reversely, in the "post-exit period" it can't be exerted following the theory because the expiration date would be seen as meaningless. However, in the real-world the PE can't wait for "free" from the commitment. For this reason, the time horizon of the post-exit option is the agreed time (i.e., the most common agreed lifespan) for the PE firm to have in its portfolio this investment.

The Holding Option

The Exercise price is calculated with the result from the Multiple Valuation in LBO, as the current framework provide the required return in an easy way. To compute the holding period, marked as $X(holding)$, the process is split in two stages:

- The calculation of the value of equity of each period extrapolated with the required IRR, $Equity_{x-1}(1 + IRR)^{0.5}$. The IRR is the average of the required returns for transactions when compared to the transaction comparables (Comps).

- The calculation of the debt value alongside the transaction period is $Debt_{x-1} - (0.5 \times FCF)$. The cash flows represent the mandatory debt payment to the debtholders.

Yet, the $X(holding)$ is the sum of both the contributions. The assumptions are that...

Value of target's assets	**447,975**
volatility	21%
rf	3%
t	8
delta t	0.5
IRR	20%
Equity	
Debt	

u	1.160084
d	0.862007
1+rf	1.03
p	55.5%
q	44.5%

Thus,

	2022		2023		2024		2025	
	1	2	1	2	1	2	1	2
Exercise price								
Equity	268,981	294,654	322,777	353,585	387,333	424,302	464,799	509,162
Debt	214,909	209,909	204,909	199,909	194,909	189,909	184,909	179,909
X(holding)	483,890	504,563	527,686	553,494	582,242	614,211	649,708	689,071

2026		2027		2028		2029		2030
1	2	1	2	1	2	1	2	1
557,759	610,994	669,311	733,193	803,173	879,832	963,808	1,055,798	1,156,569
174,909	169,909	164,909	159,909	154,909	149,909	144,909	139,909	134,909
732,668	780,903	834,220	893,102	958,082	1,029,741	1,108,717	1,195,707	1,291,478

Figure 54: Ionity asset's market value

2022		2023		2024		2025		2026		2027		2028		2029		2030
1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1
447,975	519,689	602,882	699,394	811,356	941,241	1,091,919	1,266,718	1,469,499	1,704,742	1,977,644	2,294,234	2,661,504	3,087,568	3,581,838	4,155,233	4,820,419
	386,157	447,975	519,689	602,882	699,394	811,356	941,241	1,091,919	1,266,718	1,469,499	1,704,742	1,977,644	2,294,234	2,661,504	3,087,568	3,581,838
		332,870	386,157	447,975	519,689	602,882	699,394	811,356	941,241	1,091,919	1,266,718	1,469,499	1,704,742	1,977,644	2,294,234	2,661,504
			286,936	332,870	386,157	447,975	519,689	602,882	699,394	811,356	941,241	1,091,919	1,266,718	1,469,499	1,704,742	1,977,644
				247,341	286,936	332,870	386,157	447,975	519,689	602,882	699,394	811,356	941,241	1,091,919	1,266,718	1,469,499
					213,210	247,341	286,936	332,870	386,157	447,975	519,689	602,882	699,394	811,356	941,241	1,091,919
						183,788	213,210	247,341	286,936	332,870	386,157	447,975	519,689	602,882	699,394	811,356
							158,426	183,788	213,210	247,341	286,936	332,870	386,157	447,975	519,689	602,882
								136,565	158,426	183,788	213,210	247,341	286,936	332,870	386,157	447,975
									117,720	136,565	158,426	183,788	213,210	247,341	286,936	332,870
										101,475	117,720	136,565	158,426	183,788	213,210	247,341
											87,472	101,475	117,720	136,565	158,426	183,788
												75,402	87,472	101,475	117,720	136,565
													64,997	75,402	87,472	101,475
														56,028	64,997	75,402
															48,296	56,028
																41,632

The on-the right computation shows the evolution of Ionity market value under the Multiple assumptions.

By employing the approach of considering the EV at the buyout's inception, it is feasible to replicate the binomial lattice model. The theoretical framework dictates that the Holding option value is determined by tracing backward from the present date. In 2030, the exercise year's value is €24.742 M, and it is slightly higher at €24.991 M for the 2029 exercise year. Indeed, it appears that 2029 holds as the best choice. In reality, the value of the option is a bit higher than the current estimation because the American Options would be the best fit for the case, while the current implementation looks at the European Options.

Nevertheless, the result means that the fact that it is possible to exit before the planned time is worth a value to the PE firm. Later, it can be showed the evolution of the binomial lattices for the suggested year.

Figure 55: LBO Holding option 2029 vs 2030

2022		2023		2024		2025		2026		2027		2028		2029		2030
1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1
24,792	37,532	56,265	83,455	122,354	177,124	252,888	355,667	492,133	669,150	893,161	1,169,608	1,502,834	1,897,035	2,358,565	2,898,320	3,528,941
	10,423	16,469	25,805	40,061	61,555	93,504	140,232	207,318	301,603	430,907	603,315	825,982	1,103,700	1,438,231	1,830,655	2,290,360
		3,520	5,834	9,607	15,708	25,480	40,959	65,164	102,447	158,851	242,345	362,680	530,398	754,371	1,037,321	1,370,026
			850	1,485	2,585	4,482	7,735	13,278	22,655	38,377	64,454	107,126	175,754	283,623	447,829	686,166
				110	204	378	699	1,294	2,394	4,431	8,200	15,176	28,085	51,977	96,192	178,021
					0	0	0	0	0	0	0	0	0	0	0	0
						0	0	0	0	0	0	0	0	0	0	0
							0	0	0	0	0	0	0	0	0	0
								0	0	0	0	0	0	0	0	0
									0	0	0	0	0	0	0	0
										0	0	0	0	0	0	0
											0	0	0	0	0	0
												0	0	0	0	0
													0	0	0	0
														0	0	0
															0	0
																0

2022		2023		2024		2025		2026		2027		2028		2029
1	2	1	2	1	2	1	2	1	2	1	2	1	2	1
24,991	38,563	58,866	88,785	132,126	193,703	279,271	395,211	547,896	742,776	983,532	1,272,177	1,611,340	2,008,525	2,473,121
	9,595	15,605	25,161	40,174	63,433	98,873	151,819	229,060	338,518	488,184	684,048	927,481	1,215,191	1,552,787
		2,687	4,641	7,976	13,627	23,118	38,897	64,792	106,595	172,632	273,886	422,483	625,699	868,928
			414	765	1,417	2,622	4,852	8,980	16,619	30,756	56,919	105,338	194,946	360,782
				0	0	0	0	0	0	0	0	0	0	0
					0	0	0	0	0	0	0	0	0	0
						0	0	0	0	0	0	0	0	0
							0	0	0	0	0	0	0	0
								0	0	0	0	0	0	0
									0	0	0	0	0	0
										0	0	0	0	0
											0	0	0	0
												0	0	0
													0	0
														0

The Post-Exit Option

Using the same simulation but with adjusted assumptions, the post-exit option can be also calculated. The new assumption considers a lower value for volatility because in the long run after the "likely exit" either the capital structure or the environment scenario is considered more stable.

Value of target's assets	**1,108,717**
volatility	14%
rf	3%
t	8
delta t	0.5
IRR	20%
Equity	
Debt	

u	1.1017211
d	0.9076707
1+rf	1.03
p	61.8%
q	38.2%
Div	91.7
D/E	0.80
Dividend rate	0.25

	2029		**2030**		**2031**		**2032**	
	1	2	1	2	1	2	1	2
Exercise price								
Equity	963,808	1,055,798	1,156,569	1,266,958	1,387,883	1,520,350	1,665,459	1,824,419
Debt	144,909	144,817	144,726	144,634	144,542	144,450	144,359	144,267
X(post exit)	**1,108,717**	**1,200,616**	**1,301,295**	**1,411,592**	**1,532,425**	**1,664,800**	**1,809,818**	**1,968,686**

If the Ionity value after the LBO transaction evolves alike…

Figure 56: Ionity Future (Post-Exit) Value Option

	2029		**2030**		**2031**		**2032**	
	1	2	1	2	1	2	1	2
Ionity Value								
	1,108,717	1,221,496	1,345,748	1,482,640	1,633,455	1,799,612	1,982,671	2,184,350
		1,006,350	1,108,717	1,221,496	1,345,748	1,482,640	1,633,455	1,799,612
			913,434	1,006,350	1,108,717	1,221,496	1,345,748	1,482,640
				829,097	913,434	1,006,350	1,108,717	1,221,496
					752,547	829,097	913,434	1,006,350
						683,065	752,547	829,097
							619,998	683,065
								562,754

The private equity firm has "retained" a value for the potential decision to delay its exit after the initial planned date. The value of the option is € 7,384 M.

Figure 57: Post-Exit Value

Post-Exit Value	2029		2030		2031		2032	
	1	2	1	2	1	2	1	2
	7,384	11,958	19,365	31,360	50,784	82,238	133,176	215,664
		0	0	0	0	0	0	0
			0	0	0	0	0	0
				0	0	0	0	0
					0	0	0	0
						0	0	0
							0	0
								0

Decision-making in LBO: The End Value with Real Options

At the time the embedded real option is valued, the real value of a leveraged buy-out transaction can be determined as the combination between either the holding option or the post-exit option with the value attributed to static LBO. The value of the operation will be around € 1,76 B.

$$LBO\ value = 1{,}727{,}690 + 24{,}991 + 7{,}384 = € \ 1{,}760{,}065\ K$$

APPLICATION IN THE HORIZONT 3

Real options provide a strategic framework for evaluating investment opportunities, including those in new disruptive businesses. This framework offers a means of navigating the uncertainties that are inherent in such ventures and can be applied to situations where innovations are introduced that significantly alter market dynamics, customer behaviours, or the competitive landscape. The use of real options in this context allows decision-makers to make more informed choices about how to proceed with investments, taking into account the potential risks and rewards associated with disruptive innovations. Growth and expand option are the most common in an unfolding market scenario.

For the Third Horizon Porsche's look might look at brand extension. Since Porsche manufacturer is well identified in the luxury segment, a HNWIs might consider buying a sailing vessel branded "Porsche".

First step, the strategic decision has to entail the value of the brand with the *Relief from Royalty* method. The assumptions [Table 57] rely on ten years of residual useful life (RUL), the royalty rate for luxury brand as 10.4% (Data, 2023 RoyaltyStat), the tax benefit (Tab) and the cost of capital (CoC) for intangibles[43].
The objective of the ROT approach is to consider the growth opportunities inherent in a brand that are frequently overlooked in conventional valuation methods. By growth opportunities can be seen option to expand in new geographical areas ("brand expansion") or to expand in the market by launching new products ("brand extension")
The value of the brand is determined by the total amount of "savings" as a result of not having to pay a royalty fee in order to utilize the brand with the assumption of zero long term growth plus the embedded options of possible expansion (Gonzalez Lodoño, Zulauga Carmona and Maya Ochoa, 2012).

The pursuit of brand expansion and the entry into a disruptive new scenario is anticipated to enhance the worth of the brand. The timeline for the implementation of this business decision is envisioned to be in the year 2025 [Table 58].

Assumptions

Table 58: The Relief from Royalty - assumptions

Royalty rate	10.4%	*RUL*	10
Discount rate	9.7%	**CoC intangible**	15.53%
Tax rate	30%	Tax rate	30%
Long-term growth	2%		
		Tab factor	**1.189**

Relief from Royalty method

[43] See the appendix 3 {Note 3.h.}

Table 59: Discounted Cash Flows from Relief from Royalty method

	2022	2023	2024	2025	2026	2027	2028	2029	2030	2031	2032
Sales	31,692	37,618	40,924	43,560	46,385	49,416	52,668	56,162	59,917	63,167	66,593
% Growth		19%	9%	6%	6%	7%	7%	7%	7%	5%	5%
income		3,912	4,256	4,530	4,824	5,139	5,477	5,841	6,231	6,569	6,926
Taxes		(1,174)	(1,277)	(1,359)	(1,447)	(1,542)	(1,643)	(1,752)	(1,869)	(1,971)	(2,078)
After taxes royalty income		**2,739**	**2,979**	**3,171**	**3,377**	**3,597**	**3,834**	**4,089**	**4,362**	**4,599**	**4,848**
Discount factor		0.87	0.75	0.65	0.56	0.49	0.42	0.36	0.32	0.27	0.24
PV (royalty income)		2,370	2,232	2,056	1,895	1,748	1,612	1,488	1,374	1,254	1,144

The current value of the brand is €2.9 billion, and it is expected to reach €3.2 billion by 2025. Almost 30 % of the trademark is represented by a terminal value to capture the long-term outlook. The name Porsche count for 1/3 of the *Enterprise Value* (EV), excluding the Tab factor.[44]

If Porsche were to begin manufacturing yachts, it is conceivable that some of its existing customers or potential buyers would opt to purchase one. In 2025, Porsche, the German manufacturer, will be faced with the decision of whether to expand or not to expand. Expanding would entail increased capital expenditures, but it would also present the opportunity for greater sales in the future, which could result in an enhancement of the brand's value.

The yacht market demonstrates an increase in value, indicating a promising route for growth [Table 59]. The assumption in the Long-Run would be that Porsche Yacht – the new subsidiary established – will start its operation in 2026.

The expected value from the project is € 304 M. The brand extension will be carried out mainly in North America and Middle East. The subsequent step entails the implementation of Real Options, which involves setting a no growth scenario to which the value of future brand growth opportunities can be added. The decision making involves a trade-off among:

- No growth: assume there is no additional investment in the project (i.e., other than maintenance expenses);
- Growth: higher capital expenditures to have future sales increase.

[44] *The EV of Porsche is estimated to be 75,084 millions of Euro (Estimation in Horizon 1).*

			1	2	3	4	5	6	7	8	9	10	
	2024		2025	2026	2027	2028	2029	2030	2031	2032	2033	2034	
Market size													
Units Sold	709		768	832	901	976	1,058	1,146	1,241	1,345	1,457	1,578	
Focuses segment (11% Growth)	242		256	271	288	305	323	343	363	385	408	433	
Price	6.3		6.4	6.6	6.7	6.9	7.1	7.3	7.4	7.6	7.8	8.0	
Market size	1,510		1,641	1,783	1,937	2,105	2,287	2,485	2,700	2,933	3,187	3,463	
Target share				10%	10%	17.5%	20.0%	20.0%	25.0%	25.0%	25.0%	25.0%	
Sales (Millions)				178.3	193.7	368.4	457.4	496.9	674.9	733.3	796.7	865.6	
Cost of sales	47.9%	Raw materials (9		(85.4)	(92.8)	(176.4)	(219.1)	(238.0)	(323.3)	(351.3)	(381.6)	(414.6)	
	14.6%	Contractor cost		(26.2)	(28.4)	(53.9)	(66.9)	(72.7)	(98.7)	(107.2)	(116.5)	(126.5)	
	13.4%	Staff costs (%)		(23.9)	(26.0)	(49.4)	(61.3)	(66.6)	(90.4)	(98.3)	(106.8)	(116.0)	
Gross Income				42.8	46.5	88.6	110.1	119.6	162.5	176.6	191.9	208.5	
Operative expenses	14.2%			(25.3)	(27.5)	(52.3)	(64.9)	(70.6)	(95.8)	(104.1)	(113.1)	(122.9)	
EBITDA				17.5	19.0	36.3	45.1	49.1	66.7	72.5	78.7	85.6	
Depreciation	4.5%			(8.0)	(8.7)	(16.6)	(20.6)	(22.4)	(30.4)	(33.0)	(35.9)	(39.0)	
EBIT				9.5	10.3	19.7	24.6	26.7	36.3	39.5	42.9	46.6	
NOPAT	30.0%			6.6	7.2	13.8	17.2	18.7	25.4	27.6	30.0	32.6	33.4
(+) Depr				15	16	30	38	41	56	61	66	72	
					0.912	0.831	0.757	0.691	0.574	0.523	0.523	0.477	
				0.83	0.76	0.69	0.63	0.57	0.52	0.48	0.43	0.40	0.40
Expected cash flows				**12.18**	**12.07**	**20.99**	**23.77**	**23.55**	**29.18**	**28.90**	**28.63**	**28.36**	**33.43**

PV(FCF)	2024-2027	**24**	PV(FCF)	2027 on going (Additional value)	**96**	PV (Total)	**120**
TV PV		**130**		TV PV (in addition to previous)	**54**	Terminal value (Total)	**184**
Value of the project from positioning		**154**		Value added from expansion	**149**	VALUE OF THE PROJECT	**304**

Figure 60: The Current Status of Yacht market. Data taken from average annual statements of the firms, exhibit in millions.

The intention from Porsche Yacht is to launch the first vessels for the sale in early 2027. As such the estimate investment required for the positioning in the market is € 124.8 M, which involve the First Phase were the newCo start joint agreement with already operating companies. The expected value added is € 154.4 M. Later, Porsche Yacht decides to further extend its operations into the business with in-house production of the vessels.

Figure 61: Capital Expenditure for Horizon 3: Expenditures for establishing a Shipyard. Data based on Damodaran. Data in millions

2024	**2025**	**2026**	**2027**	**2028**	**2029**	**2030**	**2031**	**2032**	**2033**	**2034**
Total expenditures per year (Data in USD)										
15.29	**22.937**	**48.9**	**76.5**	**160.6**	**160.6**	**160.6**	**160.6**	**160.6**	**160.6**	**160.6**
Design	Design	Other expenses								
15.29	7.65	7.6	7.6	7.6	7.6	7.6	7.6	7.6	7.6	7.6
	Other expenses	Manufacturing								
	15.29160	41.3	68.8	152.9	152.9	152.9	152.9	152.9	152.9	152.9

It is imperative to distinguish between the concepts of the option to expand and the decision to extend, as they pertain to different strategic considerations within the realm of real options. The literature on real options delineates the option to expand as pertaining to the valuation of the decision to initiate a new venture or product. This is contrasted with the decision to extend, which specifically refers (in this specific case) to the decision by Porsche Yachts to potentially increase its operations subsequent to its market positioning. The former is associated with the economic value of launching new endeavours, while the latter deals with strategic decisions regarding operational scalability post-establishment in the market.

Please note that the capital expenditure [Table 61] is supplementary to Porsche's existing capital expenditures. The option to expand does not occur on the same time of the positioning. The possible date for positioning can be 2026 (i.e. date for the start of operations), but the follow extension is scheduled for 2028.

Positioning 2026: PV (2024)		**Expand 2028 ongoing: PV (2024)**	
$134.78	Capex PV (USD)	$598.01	Capex PV (USD)
€ 124.80	Capex PV (EUR)	**€ 553.72**	Capex PV (EUR)

In case the project of positioning and extension are going to be undertaken jointly, the NPV is negative. However, the option sells at € 6 M. Because NPV $< Option$ the current value of the option is value of waiting 10 years. The decision would be to not undertake the project now because it would grant a negative value [Table 62].

$$NPV = -(K_1 + K_2) + \sum \frac{CF}{(1+w)^i} = -678.5 + 304 = -395 < Option\ to\ Wait$$

However, Porsche Yachts is willing to evaluate if the positioning within the 2028 could be feasible or not. The option gives the right to launch the new business ("option to expand") to start its operation within 2026. Positioning entails joint agreements for the realisation of yachts with other firms already operating in the business and it is mainly related with the design and brand recognition.

Table 62: Decision to undertake the projects jointly.

Cash Flows from the project	303.9	S			
Positioning	124.8	K1	9.70%	**Cost of capital (w)**	
Expansion cost	553.7	K2			
rf	3%			u	1.161834
σ(stock returns)	15%			d	0.860708
Delta t	1				
				p*	48.73%
				1-p*	51.27%

The decision to undertake the project can be seen with the binomial lattice [Table 63]. The representation shows that the decision to "positioning" or "extend" occurs in specific cases in the future.

Table 63: Decision to undertake the project jointly – Binomial lattice Data in millions

2024	2025	2026	2027	2028	2029	2030	2031	2032	2033	2034
										692
									595.8	EXTEND
								513	EXTEND	513
							441	EXTEND	441.37297	EXTEND
						380	EXTEND	380	EXTEND	380
					327	EXTEND	327	EXTEND	327	EXTEND
				281	EXTEND	281	EXTEND	281	EXTEND	281
			242	EXTEND	242	EXTEND	242	EXTEND	242	EXTEND
		208	POSITIONING	189	EXTEND	208	EXTEND	208	EXTEND	208
	179	POSITIONING	179	EXTEND	162	EXPAND	179	EXTEND	179	EXTEND
154	POSITIONING	154	POSITIONING	140	EXTEND	140	EXTEND	154	EXTEND	154
	133	ABANDON	133	ABANDON	133	ABANDON	133	ABANDON	133	ABANDON
		114	ABANDON	114	ABANDON	114	ABANDON	114	ABANDON	114
			98	ABANDON	98	ABANDON	98	ABANDON	98	ABANDON
				85	ABANDON	85	ABANDON	85	ABANDON	85
					73	ABANDON	73	ABANDON	73	ABANDON
						63	ABANDON	63	ABANDON	63
							54	ABANDON	54	ABANDON
								47	ABANDON	47
									40	ABANDON
										34

Recognizing the shortcomings of collaborating on projects, the managers nonetheless sought to determine whether the proposal to adopt a particular positioning is feasible. If the choice falls in positioning, the expected cash flows reduce to € 154 M [Table 60]. Here, the NPV is positive, and the option has a value too.

Table 64: Decision of Positioning – Option Value. Data in millions

Positioning Option Value

2024	2025	2026	2027
			117
		83	**POSITIONING**
	55	**POSITIONING**	**55**
30	**POSITIONING**	**30**	**POSITIONING**
	8	**ABANDON**	**8**
		0	**ABANDON**
			0

If Porsche were to enter the yachting market, the brand would generate an additional € 59 M in revenues[45]. Currently, the choice of whether to expand the business rests with the company, and the value of making this decision now is €30 M [Table 64]. Yet, it means that the decision to extend can be valued as an *option to abandon*.

Table 65: Decision to Extend – Option value. Data in millions

PV (CF)	€	304
K	€	554
rf		3%
volatility		15%
t		10
delta t		1
d1		-0.39
d2		-0.87
N(-d1)		0.653537981
N(-d2)		0.807639792
Put Option	€	133

Therefore, there're can be the following interpretation of the results. If the calculation of the NPV occurs today, it would be negative for the extension project. The advantages of extension grant € 304 million (discounted), while the expenditures amount to roughly one time and a half, at € 554 million. The expansion option is likely to be out of the money, at least the

[45] The decision considers the NPV + Option to expand, the NPV is the difference between benefits and costs for the project and the option is an add-in to the project value.

considerations fall into the decision to wait ten years. In any case the extension decision is feasible today.

However, Porsche can announce its decision to expand towards the yachting market, start positioning, and at the end "dispose" the extension project. The value of a brand is primarily intangible and is determined by the market. The announcement made by the company can have an impact on the brand's value. Here, the options are compounded. If Porsche Yachts make the *decision to positioning,* the managers do not wait (i.e., managers "kill the option"), and they will have later the option to abandon.

The decision would increase brand value:

$$\begin{aligned}&\textit{Value of the brand}\\&+ \textit{Decision to positioning}\\&+ (\textit{Option to wait})\\&+ \textit{Option to Abandon}\end{aligned}$$

$$\textit{Likely value of the brand} = 29{,}172 + 59 - 6 + 133 \;\; = € \, 29,364 \, M$$

The estimated value of the brand is believed to be approximately € 29,364 million, including an additional € 186 million from option valuation.

3.9. Conclusions

Real options are a strategic tool used for decision-making under uncertainty, allowing companies to evaluate and manage the flexibility they have in their investment decisions.

By incorporating real options analysis into their decision-making process, companies can better understand the value of their investments and make more informed decisions. Even though dissertation on Real Options generally involve the pharmaceutical or real estate sectors, the idea was to provide peculiar applications in M&A and investment valuation. However, it is important to note that real options are not a tool "*per se*" (lat.) as other valuation methods such as DCF, APV, or Multiple valuation are still used. Real Options are a complement to traditional valuation methods by providing additional insights to the decision-makers. In practical application, this has proven to be a lesson well-learned.

In private equity transactions are carried out with a significant amount of leverage, resulting in the acquisition being referred to as a leveraged buyout. It is essential to have a comprehensive understanding of the debt intake on and the capital structure during the holding period. At present, Ionity is experiencing losses as electric vehicles have not yet gained widespread popularity (except from Norway). If public authorities do not install charging stations along European roadways, the private sector may need to step in to meet this need. The automotive industry is currently characterized by a high degree of uncertainty. Use real options in private equity is brilliant, but some shrewdness in required. The exit option is the value of flexibility, that is monetizable as long as someone is willing to pay for it, otherwise the value remain "theoretical". Without a doubt, the true value of real options is not recognised in financial reports. Nevertheless, it does possess a market value that encompasses the potential flexibility strategy. In this case, the value of flexibility is reserved for a potential buyer.

In order to fully understand and utilize real options, it is helpful to consider three horizons. Without these horizons, it may be difficult to conceive the creation of real options. In this current dissertation, the Porsche's evolutionary strategy is hypothetical. We don't know if *Porsche Ventures* will ever takeover Ionity (or any other charging station network) or initiate a new R&D for hydrogen fuel or decide in the future to expand by creating a new business, but our knowledge about Real Options can give us a hint to understand how to behave to face transformational strategy. If event that can potentially disrupt the market arises, what measures can be taken to address the situation?

The current value of Porsche is € 75 B, in case their market share to luxury remains constant over time. However, the Enterprise Value could reach € 83,8 B in case of a growing market share. Porsche should ensure that electric car users can re-charge their vehicle fast and "every-time available". The importance of maintaining a commitment to car station availability is paramount. By the year 2030, Porsche projected to sell between 385,000 and 454,000 units, resulting in a total value of €48 billion to €53 billion. Given these projections, it is clear that staying abreast of the trends in the electric market is of utmost importance. Due to Ionity's current financial state as a loss-incurring enterprise, its shares are being sold at a discounted price. As a result, the buyouts may present an attractive opportunity for gaining control over the

charging network while also expanding the company’s business operations. The value of Ionity after the takeout could overpass € 1.7 billions. It is possible that a similar situation may arise if Tesla chooses to spin off its Tesla Supercharger division.
In the estimation of the value of Porsche at the third horizon, a crucial distinction is made between the company’s decision to enter the yachting market and to wait for potential future expansion. Another aspect is the worth that can be assigned to the brand. When making decisions, Porsche’s internal deliberations remain undisclosed, like in the first case. The public outside never will know if Porsche is really indented to extend its operations after the positioning. It is likely that Porsche will primarily engage in branding initiatives and form partnerships with established companies in the yachting industry at this time. Therefore, the market value assigned to the company is heavily influenced by the behavior of high-net-worth individuals (HNWIs) in response to news of a new business venture. In the latter case, the announce influence strongly the brand.

In any case, the goal of the dissertation was “*to explore the application of real options to investment decision-making to evaluate Porsche’s strategic flexibility*”. The intention behind the case study was to offer a practical use and comprehension of Real Options that could serve as a guide for practitioners or, at the very least, for students who are eager to learn about options in corporate finance. The ongoing changes in the world seem to suggest that decision-making will be more and more riffed with uncertainty and Real Options are here to stage a nice chill ambush to smooth the things over (*idiomatic*)!

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Further materials are uploaded online –

https://github.com/Mattesjdd/Dissertation-contents.git

or

https://drive.google.com/drive/folders/1i6NBSs_joAMjTIKRO21tZQDbjB71Uhyc?usp=share_link

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MarketLine.com*

LSEG Data & Analytics (ex-Refinitiv)*

Orbis M&A transactions*

OECD data

Newsroom.porsche.com

Royaltyrange.com

RoyaltyStat.com

*The licensing for the use of these databases is provided by Luiss Guido Carli

List of Appendices

Appendix 1

[1] Definition: A stream of cash flows follow a GBM, so that:

$d\,X_t = \alpha X_t\,dt + \sigma X_t\,dZ_t$ where α is the mean change in X per unit time (or drift), while σ is the standard deviation, dZ is the random increment of the Wiener process. Thus, $dZ \sim \varepsilon_t\sqrt{dt}$ and $\varepsilon_t \sim N(0,1)$.

[2] Definition: A firm wishes to undertake an investment when the expected present value of the option to invest is maximum: $F(X, t) = \max_{\tau \in s} \mathrm{E}[(X_s - I)\exp(-rs)\;ds]$. The project payout won't be discounted at a risk-free rate, but at a bearing risk rate. So, the discount rate follows the CAPM.

The CAPM (Merton, 1973) has been implemented to determine the risk-adjusted rate of return on the project. Same to the stream of cash flows, also the market portfolio is governed by a Brownian motion:

$dP_m(t) = \mu_m P_m(t)\,dt + \sigma_m P_m(t)\,dB(t)$ where the $B(t)$ is the standard Wiener process for the capital market, which is correlated with the previous random increment dZ in [1]. Here, the μ_m and σ_m are the drift rate and standard deviation, respectively, for $P_m(t)$ which enters the model [2] as a risk measure with parameter λ. In particular, λ is the extra return over the risk-free rate over market volatility: $\lambda = \frac{\mu_m - r_{free}}{\sigma_m}$. The risk measure parameter is located in expression [2] inside the discount rate, because $r = r_{free} + \lambda p \sigma$.

[3] Proof: The equation has been set out in Clark (1970) and Ocone and Kaearzas (1991), X_t can be extended as an Itô process. By recalling that $dZ = \rho dB + \sqrt{1-p^2}dZ$, by substituting into [1], then the Brownian motion becomes:

$dX_t = \alpha X_t\, dt + \rho\sigma X_t\, dB_t + \sqrt{1-p^2}\,\sigma X_t dZ_t$. This equation can be summed up as $dX_t = X_t\left[\alpha dt + \sigma^x\left(pdB_t + \sqrt{1-p^2}\, dZ_t\right)\right]$, where the generic standard deviation is market by σ^x so that the correlation parameter p might be broken down as:

$p = \frac{\sigma_m}{\sqrt{\sigma_m^2+\sigma_z^2}}$, $(\sigma^X)^2 = \sigma_m^2 + \sigma_z^2$.

The components σ_m and σ_z measure the exposure of the underlying toward systematic and idiosyncratic risk, respectively.

[4] Proof: The investment option satisfies the following differential equation (derived with Itô's Lemma):

$$dF = \left(\frac{1}{2}\sigma^2[X(T)]^2\,\frac{\partial^2 F(X,t)}{\partial^2 X} + (\alpha)[X(T)]\,\frac{\partial\, F(X,t)}{\partial\, X} + \frac{\partial\, F(X,t)}{\partial\, t}\right)dt + \frac{\partial\, F(X,t)}{\partial\, X}\,p\sigma[X(T)]\,dB(t)$$
$$+\frac{\partial\, F(X,t)}{\partial\, X}\sqrt{1-p^2}\,\sigma[X(T)]\,dZ(t)$$

Consider a dynamic portfolio at time t with 1) investment option dF and 2) selling option (i.e., go short) of n units in the market portfolio. The return from portfolio is:

$$dF - nP_M(t) = \left(\frac{1}{2}\sigma^2[X(T)]^2\,\frac{\partial^2 F(X,t)}{\partial^2 X} + (\alpha)[X(T)]\,\frac{\partial\, F(X,t)}{\partial\, X} + \frac{\partial\, F(X,t)}{\partial\, t}\right)dt + \frac{\partial\, F(X,t)}{\partial\, X}\,p\sigma[X(T)]\,dB(t)$$
$$+\frac{\partial\, F(X,t)}{\partial\, X}\sqrt{1-p^2}\,\sigma[X(T)]\,dZ(t) - n\mu_m P_M(t)dt - n\sigma_M P_M(t)dB\,(t)$$

Which can be ordinated to:

$$= \left(\frac{1}{2}\sigma^2[X(T)]^2\,\frac{\partial^2 F(X,t)}{\partial^2 X} + (\alpha)[X(T)]\,\frac{\partial\, F(X,t)}{\partial\, X} + \frac{\partial\, F(X,t)}{\partial\, t} - n\mu_m P_M(t)\right)dt$$
$$+\left(\frac{\partial\, F(X,t)}{\partial\, X}\,p\sigma[X(T)] - n\sigma_M P_M(t)\right)dB(t) + \frac{\partial\, F(X,t)}{\partial\, X}\sqrt{1-p^2}\,\sigma[X(T)]dZ(t)$$
$$= \left\{\frac{1}{2}\sigma^2[X(T)]^2\,F''(X,t) + (\alpha)[X(T)]\,F'(X,t) - n\mu_m P_M(t)\right\}dt$$
$$+\{p\sigma X(t)F'(X,t) - n\,\sigma_M P_M(t)\}dB(t) + F'(X,t)\sqrt{1-p^2}\,\sigma X(t)dZ(t)$$

By substituting $n = \frac{p\sigma X(t)F'(X,t)}{\sigma_m P_m(t)}$ into the return yield the equation look as:

$$= \left\{\frac{1}{2}\sigma^2[X(T)]^2 F''(X,t) + (\alpha)[X(T)]\,F'(X,t) - \frac{p\sigma[X(t)]F'(X,t)}{\sigma_m}\mu_m\right\}dt$$

$$+ \left\{p\sigma X(t)F'(X,t) - \frac{p\sigma X(t)F'(X,t)}{\sigma_m P_m(t)}\ \sigma_M P_M(t)\right\}dB(t) + F'(X,t)\sqrt{1-p^2}\,\sigma X(t)dZ(t)$$

Therefore,

$$= \left\{\frac{1}{2}\sigma^2[X(T)]^2 F''(X,t) + \left(\alpha - \frac{\mu_m p\sigma}{\sigma_m}\right)[X(T)]\,F'(X,t)\right\}dt + F'(X,t)\sqrt{1-p^2}\,\sigma X(t)dZ(t)$$

The risk associated with a dynamic portfolio over dt is diverse. However, the expected rate of return for the portfolio equals the risk-free rate. To prevent arbitrage opportunities, the first part is as follows:

$$\left\{\frac{1}{2}\sigma^2[X(T)]^2 F''(X,t) + \left(\alpha - \frac{\mu_m p\sigma}{\sigma_m}\right)[X(T)]\,F'(X,t)\right\}dt = r\left\{F(X,t) - \frac{p\sigma}{\sigma_m}F'(X,t)X(t)\right\}dt$$

Thus,

$$\frac{1}{2}\sigma^2[X(T)]^2 F''(X,t) + \left(\alpha - \frac{\mu_m p\sigma}{\sigma_m}\right)[X(T)]\,F'(X,t) - r\left\{F(X,t) - \frac{p\sigma}{\sigma_m}F'(X,t)X(t)\right\} = 0$$

$$\frac{1}{2}\sigma^2[X(T)]^2 F''(X,t) + \left[\left(\alpha - \frac{\mu_m p\sigma}{\sigma_m}\right) - r\frac{p\sigma}{\sigma_m}\right]X(t)\,F'(X,t) - rF(X,t) = 0$$

In case we would like to better arrange the content into the square bracket the

$$\alpha - \frac{\mu_m p\sigma - rp\sigma}{\sigma_m} = \alpha - \frac{(\mu_m - r)p\sigma}{\sigma_m} = \alpha - \lambda p\sigma$$

Finally, the value of the investment option should resemble this equation () which is a second-order linear differential equation:

$$\frac{1}{2}\sigma^2[X(T)]^2 F''(X,t) + (\alpha - \lambda p\sigma)X(t)\,F'(X,t) - rF(X,t) = 0$$

The solution of the equation follows the expression of the power function AX^{β} and β is the solving equation:

$$\frac{1}{2}\sigma^2\beta\,(\beta-1) + (\alpha - \lambda p\sigma)\beta - r = 0 \qquad \frac{1}{2}\sigma^2\beta^2 - \beta\left[\frac{1}{2}\sigma^2 - (\alpha - \lambda p\sigma)\right] - r = 0$$

Which has two roots solutions: $ax^2 + bx + c = 0$

The delta is $\Delta = \left[\frac{1}{2}\sigma^2 - (\alpha - \lambda p\sigma)\right]^2 - 4\left(\frac{1}{2}\sigma^2\right)(-r) = \left[\frac{1}{2}\sigma^2 - (\alpha - \lambda p\sigma)\right]^2 + 2\sigma^2 r$ and calculate also

$\frac{\Delta}{\sigma^2} = \frac{1}{\sigma^2}\sqrt{\left[\frac{1}{2} - (\alpha - \lambda p\sigma)\right]^2 + 2\sigma^2 r} = \sqrt{\frac{1}{\sigma^4}\left[\frac{1}{2} - (\alpha - \lambda p\sigma)\right]^2 + \frac{1}{\sigma^4}2\sigma^2 r} = \sqrt{\frac{1}{\sigma^4}\left[\frac{1}{2} - (\alpha - \lambda p\sigma)\right]^2 + \frac{2r}{\sigma^2}} =$

$\sqrt{\left[\frac{1-2(\alpha-\lambda p\sigma)}{2\sigma^2}\right]^2 + \frac{2r}{\sigma^2}}$. So $x_1, x_2 = \frac{\left(\frac{1}{2}\sigma^2 - (\alpha - \lambda p\sigma)\right) \pm \sqrt{\Delta}}{2\frac{1}{2}\sigma^2} = \frac{\frac{1}{2}\sigma^2 - (\alpha - \lambda p\sigma) \pm \sqrt{\Delta}}{\sigma^2}$

The final solution to differential equation is:

$$\beta x_1, x_2 = \frac{1}{2} - (\alpha - \lambda p\sigma)/\sigma^2 \pm \sqrt{\left[\frac{1}{2} - (\alpha - \lambda p\sigma)/\sigma^2\right]^2 + 2r/\sigma^2}$$

However, only the positive root is considered in [4], as it solves $AX^{\beta_{x_1}}$.

[5] Proof: Suppose $T = \tau$ the time when the project reaches the optimal investing trigger X^* for an initial value X_0. Implementing the Theorem 5.3 from Karlin and Taylor which states:

Theorem: "Let $X(t)$ be a Brownian motion with a positive drift, x the component of the process, and z the desired level the process reaches. Let also $z > X(0) = x$ and be given, so let T_z be the first time the process reaches the z desired level". The T_z has a probability density function:

$$f(x,t,z) = \frac{(z-x)}{\sigma\sqrt{2\pi t^3}} \exp[-\frac{(z-x-\mu t)^2}{2\sigma^2 t}], \qquad t > 0$$

For us, the distance $z - x = X$ so that $X = \ln(X^*/X_0)$, while the drift is $\mu = \alpha - \frac{\sigma^2}{2}$. The T_z is marked as $\Phi(\tau)$, the probability density function can be rewritten as:

$$\Phi(\tau) = \frac{\ln(X_0/X^*)}{\sigma\sqrt{2\pi\tau^3}} \exp\left\{-\frac{1}{2\sigma^2\tau}\left[\ln(X_0/X^*) - \left(\alpha - \frac{\sigma^2}{2}\right)\tau\right]^2\right\}, or$$

$$\Phi(\tau) = \frac{X}{\sigma\sqrt{2\pi\tau^3}} \exp\left\{-\frac{1}{2\sigma^2\tau}\left[X - \left(\alpha - \frac{\sigma^2}{2}\right)\tau\right]^2\right\}$$

Thereafter, by adopting the Laplace transform $\mathcal{L}\{f(t)\} = F(\Theta)$, then $F(\Theta) = \int_0^\infty e^{-\theta\tau} f(t)dt$, and the input function is $\Phi(\tau)$, the expectation operation is:

$E(e^{-\Theta\tau}) = \int_0^\infty e^{-\theta\tau}\Phi(\tau)dT = \exp\{\ -[\sqrt{(\mu^2 + 2\sigma^2\theta} - \mu]\ (\mathrm{X})/\sigma^2\ \}$ because

$$F(\Theta) = \int_0^\infty e^{-\theta\tau}\Phi(\tau)\, dt = \int_0^\infty \frac{X}{\sigma\sqrt{2\pi\tau^3}} e^{-\theta\tau} e^{-\frac{1}{2\sigma^2\tau}[X-(\mu)\tau]^2}\, dt$$

$$= \int_0^\infty \frac{X}{\sigma\sqrt{2\pi\tau^3}} e^{-\theta\tau - \frac{[X-(\mu)\tau]^2}{2\sigma^2\tau}} dt \cong e^{\frac{-\left[\sqrt{(\mu^2+2\sigma^2\theta} - \mu\right]X}{\sigma^2}}$$

[6] Definition: Based on the output from Laplace, the expected time to exercise the investment can be found as

$$E(\tau) = \int_0^\infty T\Phi(\tau)dT = -\lim_{\Theta\to 0} \frac{\partial E(e^{-\Theta\tau})}{\partial\Theta} = \frac{X}{\mu} = \frac{\ln(X^*/X_0)}{\left(\alpha - \frac{\sigma^2}{2}\right)}$$

APPENDIX 2

[1] The value of a bond if default is a possibility will be the face value F times the probability of recovery $(1 - PD)$ and adding the residual amount of the loss once the default occurs.

$$E[D_T] = F(1 - PD) + PD \times E[R|default] = F(1 - PD) + PD\ (F - E[Loss|Default])$$
$$= F - P_{dft} \times LGD$$

[2] Proof: The WACC can be written as a function of cost of asset k_A.

$$k_A \frac{V_U}{V_L} + k_d \frac{tD}{V_L} = k_E \frac{E}{V_L} + k_d \frac{D}{V_L} \qquad k_A \frac{V_L - tD}{V_L} = k_E \frac{E}{V_L} + k_d (1 - t) \frac{D}{V_L}$$

$$k_E \frac{E}{V_L} + k_d (1 - t) \frac{D}{V_L} = WACC \qquad thus, the\ cost\ of\ capital\ is \qquad WACC = k_A \left(1 - t \frac{D}{V_L}\right)$$

APPENDIX 3

Note 3.a: Units of Porsche's deliveries divided by geographical areas, and related table of growths. Not all geographical areas have the same growth, USA and Europe will remain in a leading position, but the Asia Giants will probability outpace the Western.

Deliveries (in units)	2018	2019	2020	2021	2022	2023F
Worldwide	256,255	280,800	272,162	301,915	309,884	323,629
Europe	77,216	88,975	80,892	86,160	92,197	102,075
Germany	27,541	31,618	26,629	28,565	29,512	33,085
Americas	70,461	75,367	69,629	79,166	79,260	85,983
USA	52,202	61,568	57,294	70,025	70,049	71,729
Asia, Africa and Middle East	108,578	116,458	121,641	131,098	138,427	135,572
China	80,108	86,752	88,968	95,671	93,286	80,997

	2019	2020	2021	2022	2023F	Growth
Worldwide	9.58%	-3.08%	10.93%	2.64%	4.44%	4.78%
Europe	15.23%	-9.08%	6.51%	7.01%	10.71%	5.74%
Germany	14.80%	-15.78%	7.27%	3.32%	12.11%	3.74%
Americas	6.96%	-7.61%	13.70%	0.12%	8.48%	4.06%
USA	17.94%	-6.94%	22.22%	0.03%	2.40%	6.56%
Asia, Africa and Middle East	7.26%	4.45%	7.77%	5.59%	-2.06%	4.54%
China	8.29%	2.55%	7.53%	-2.49%	-13.17%	0.22%

Note 3.b: Annual reports for Porsche AG.

Income Statement of Porsche AG

In millions	2018	2019	2020	2021	2022
Sales revenues	25,784	28,518	28,695	33,138	37,630
		11%	1%	15%	14%
Cost of sales	(18,629)	(21,256)	(21,155)	(24,281)	(26,871)
Gross profit	7,155	7,262	7,540	8,857	10,759
Distribution expenses	(1,901)	(2,044)	(1,881)	(2,111)	(2,353)
Administrative expenses	(1,103)	(1,029)	(1,255)	(1,426)	(1,655)
Other operating income	813	846	953	1,079	1,894
Other operating expenses	(675)	(1,173)	(1,180)	(1,085)	(1,662)
Operating profit	4,289	3,862	4,177	5,314	6,983
	138	(327)	(227)	(6)	232
Share of profit/loss of equity-	3	(1)	(10)	(22)	(7)
Interest income	408	416	406	421	460
Interest expenses	(92)	(148)	(129)	(113)	(114)
Other financial result	(56)	(75)	(47)	129	(40)
Financial result	263	192	220	415	299
Profit before tax	4,552	4,054	4,397	5,729	7,282
Income tax	(1,253)	(1,434)	(1,231)	(1,691)	(2,112)
current	(1,268)	(1,427)	(998)	(1,528)	(1,927)
deferred	15	(7)	(233)	(163)	(185)
Profit after tax	2,801	3,118	3,166	4,038	5,170
Average taxation	27.5%	35.4%	28.0%	29.5%	29.0%

Consolidated Balance sheet (audited in accordance with IFRS) of Porsche AG

In millions	2018	2019	2020	2021	2022
Asset side					
Intangible assets	4,929	5,085	5,437	6,190	7,473
Propery, plan and equipement	6,928	8,624	8,695	8,763	8,924
leased assets	3,776	3,829	3,614	3,954	3,854
equity-accounted investments	368	298	167	573	623
other equity investments	98	146	217	313	636
financial services reveivables	1,656	1,841	2,414	3,461	4,382
other financial assets	8,398	8,350	8,870	8,596	753
other receivables	125	179	164	113	100
deferred tax assets	730	1,355	817	867	742
Non-currrent assets	27,008	29,707	30,395	32,830	27,487
Inventories	3,889	4,013	4,108	4,517	5,504
trade receivables	759	842	1,081	1,199	1,290
financial services receivables	730	842	1,122	1,081	1,538
other financial assets	2,292	2,415	2,761	5,353	5,493
other receivables (note 4)	468	490	606	579	728
tax receivables	81	95	163	155	87
securities	297	451	755	982	1,827
cash, cash equivalents and time deposits	2,635	3,511	4,500	4,686	3,719
Current assets	11,151	12,659	15,096	18,552	20,186
Total assets	**38,159**	**42,366**	**45,491**	**51,382**	**47,673**

Libilities side					
Subscipted capital	45	45	44	45	911
capital reserves	11,453	12,726	13,754	14,225	3822
retained earnings	4,876	4,991	6,302	9,146	12,387
other reserves	97	- 339	118	- 489	- 101
equity before non controlling interests	16,471	17,423	20,219	22,927	17,019
non controlling interests	6	5	5	8	8
Equity	16,477	17,428	20,224	22,935	17,027
Provisions for pensions and similar obligations	3,792	5,438	5,932	5,525	3,668
other provisions	778	996	939	1,184	1,138
Deferred tax liabilities	650	681	685	782	1,605
Financial liabilities	3,644	5,375	5,668	6,599	6,016
Other financial liabilities	399	657	285	633	872
other liabilities	402	492	473	645	734
Non-current liabilities	9,665	13,639	13,982	15,368	14,033
Provisions for taxes	96	129	111	126	167
other provisions	1,951	2,118	1,849	2,189	2,812
financial liabilities	2,215	2,239	2,657	3,128	3,464
trade payable	3,134	2,582	2,335	2,447	2,899
other financial liabilities	3,441	3,082	2,959	3,638	5,299
other liabilities	1,087	1,077	1,331	1,486	1,908
tax payable	93	72	43	65	64
Current liabilities	12,017	11,299	11,285	13,079	16,613
Liabilities & Equity	**38,159**	**42,366**	**45,491**	**51,382**	**47,673**

Note 3.c: Operative expenses for Porsche AG. The operating expenses are valued considering fixed and variable outflows.

In thousands	2,019		2,020	2,021	2,022
Distribution expenses	**(1,901,000)**		**(1,881,000)**	**(2,111,000)**	**(2,353,000)**
Sales Group	25,784,000		28,695,000	33,138,000	35,126,280
Sales automotive	**21,916,400**		**24,390,750**	**27,183,000**	**31,692,000**
Sales automotive % Group	**85%**		**85%**	**85%**	**85%**
Number of units sold (000)	**281**		**265**	**302**	**310**
Change in distrubution expenses			(20,000)	230,000	242,000
			(16)	**37**	**8**
		VAR	(335,443)	(549,987)	(776,377)
1% ***Inflation***		FIXED	**(1,545,557)**	**(1,561,013)**	**(1,576,623)**
		Change in variable costs		**64%**	**41%**
		Variable cost per 1000 units	**(1,266)**	**(1,822)**	**(2,505)**
		Delta Ctu	(7,098)	(6,992)	(7,593)
		Fixed cost per 1000 unit	(5,832)	(5,170)	(5,088)
			(1,881,000)	(2,111,000)	(2,353,000)
Distrib. Expenses/Sales automotive			7.7%	7.8%	7.4%
Administrative expenses	(1,103,000)		(1,095,000)	(1,426,000)	(1,655,000)
Adm./Sales (%)	4.3%		3.8%	4.3%	4.7%
Other operating expenses	(675,000)		(897,000)	(1,085,000)	(1,662,000)
Operating/sales Group (%)	2.62%		3.13%	3.27%	4.73%
General and amministrative costs			**(1,095,000)**	**(1,426,000)**	**(1,655,000)**
Selling costs			**(1,881,000)**	**(2,111,000)**	**(2,353,000)**
		of which variable	(335,443)	(549,987)	(776,377)
Total SG&A			**(2,976,000)**	**(3,537,000)**	**(4,008,000)**

Note 3.d. Sensitivity Analysis

Case for 2027F		Average Price per charge						
		€ 52.0	€ 52.0	€ 53.5	€ 55.0	€ 56.5	€ 58.0	€ 59.5
Number of charges offered (in millions)	0.7	33	34	35	36	37	38	39
	1.7	84	86	89	91	94	96	99
	2.7	134	138	142	146	150	154	158
	3.7	185	190	196	201	207	212	218
	4.7	235	242	249	256	263	270	277
	5.7	286	294	303	311	320	328	337
	6.7	336	346	356	366	376	386	396
	7.7	387	398	410	421	433	444	456
	8.7	437	450	463	476	489	502	515
	9.7	488	502	517	531	546	560	575
	10.7	538	554	570	586	602	618	634
	11.7	589	606	624	641	659	676	694
	12.7	639	658	677	696	715	734	753

Note 3.e: Peer evaluation

Data in millions				tax rate average:		30%	*Liquidity*	***Profitability***		**Earning power/Dupont**					
	Sales	Gross Margin	EBITDA	**EBIT**	**NOPAT**	**Net Income**	WCR/TA	EBITDA (%)	EBIT (%)	Net Profit Margin	Asset turnover	Leverage: TA/S	After tax ROE	**Is it worth as comparable firm? (scoring)**	
BMW Group														YES	
Average (2017-2022)	108,698.2	25,220.8	17,141.8	9,762.3	6,833.6	9,096.0	4%	15.8%	9.0%	8.0%	0.50	3.5	14%	4.50	/7 stability
Last Performace	142,610.0	31,192.0	22,594.0	14,028.0	9,819.6	17,941.0	3%	16%	9.8%	13%	0.60	2.9	22%	4.750	/7 reliability
Standard dev							2%	3%	3%	3%	0.05	0.35	6%	9.3	/14
Ferrari NV				tax rate average:		24%								YES	
Average (2017-2022)	3,905.0	1,986.1	1,304.0	915.6	695.9	731.2	16%	33%	23%	18.7%	0.7	4.1	56%	4.50	/7 stability
Last performance	5,095.3	2,446.3	1,767.4	1,221.2	928.1	932.6	14%	35%	24%	18.3%	0.7	3.04	39%	4.500	/7 reliability
Standard dev							3%	2%	2%	2%	0.09	1.5	24%	9	/14
Mercedes Benz Group AG (Daimler)				tax rate average:		30%								YES	
Average (2017-2022)	151,658.2	31,660.0	25,659.0	10,173.0	7,121.1	10417.50	7%	17%	7%	6.8%	0.6	4.1	15%	4.00	/7 stability
Last performance	150,017.0	34,127.0	31,502.0	14,501.0	10,150.7	17,637.0	6%	21%	10%	11.8%	0.58	3.25	22%	4.500	/7 reliability
Standard dev							1%	3%	6%	3%	0.09	0.5	6%	8.50	/14
Aston Martin Lagonda Global Holding				tax rate average:		21%								NO	
Average (2017-2022)	1,184.6	403.7	172.6	(25.7)	(25.7)	(245.5)	(0.1)	13%	-4%	-22.4%	0.5	6.4	-26%	0.00	/7 stability
Last performance	1,625.3	530.2	236.7	(138.7)	(138.7)	(621.9)	0.04	14.6%	-26.2%	-38.3%	0.46	4.15	-73.0%	2.000	/7 reliability
Standard dev							7%	13%	19%	27%	0.13	3.81	53%	2.00	/14
Volkswagen AG				tax rate average:		30%								YES	
Last performance	245,057.8	47,243.8	43,052.8	18,379.6	12,865.7	12,399.3	20%	18%	7%	5.0%	0.5	3.8	10%	4.50	/7 stability
Last performance	279,232.0	53,070.0	50,037.1	21,807.1	15,265.0	14,867.0	17%	17.9%	41.1%	5.3%	51.0%	3.37	9.2%	3.000	/7 reliability
Standard dev							2.0%	1%	1%	1%	0.04	0.25	2%	7.5	/14
				tax rate average:		29%									
Tesla Inc.															
Average (2017-2022)	37,436.5	8,570.4	5,820.6	3,419.5	#REF!	2,498.3	8%	12%	3%	0.5%	0.8	3.6	-2%	NO	
Last performance	81,462.0	20,853.0	17,375.0	13,832.0	#REF!	12,583.0	17%	21.3%	66.3%	15.4%	1.13	1.89	33.0%		
Standard dev							11.9%	8%	11%	11%	0.23	1.52	26%	0.50	/7 stability
														4.75	/7 reliability
														5	/14

Data in millions					Average tax	29%	*Liquidity*	***Profitability***		**Earning power/Dupont**					
	Sales	Gross Margin	EBITDA	**EBIT**	**NOPAT**	**Net Income**	WCR/TA	EBITDA (%)	EBIT (%)	Net Profit Margin	Asset turnover	Leverage: TA/S	After tax ROE	**Is it worth as comparable firm? (scoring)**	
LVMH Moet					Average tax	29%								YES	
Average (2017-2022)	55,197.0	36,906.2	16,674.0	14,165.0	10,123.4	8,836.0	7.7%	29.5%	25.7%	15.4%	0.59	2.4	23%	6.00	/7 stability
Last Performace	79,184.0	54,196.0	26,679.0	22,127.0	16,595.3	14,751.0	6.0%	34%	27.9%	19%	0.61	2.5	28%	5.500	/7 reliability
Standard dev							3%	4%	4%	3%	0.0	0.018	4%	11.50	/14
Hermes International					Average tax	29%								YES	
Average (2017-2022)	7,562.0	4,890.1	3,276.2	2,844.2	2,195.1	1,876.2	44%	42.6%	37.6%	24.1%	0.75	1.5	26%	6.00	/7 stability
Last Performace	11,602.0	7,662.0	5,427.0	4,876.0	3,636.1	3,330.0	51%	47%	42.0%	29%	0.74	1.4	30%	6.000	/7 reliability
Standard dev							4%	3%	4%	3%	0.0	0.0	3%	12.00	/14
						24%									
Moncler					Average tax	24%								YES	
Average (2017-2022)	1,721.8	1,321.2	682.6	496.2	377.1	373.6	23.7%	0.4	0.3	0.2	0.7	1.7	0.3	3.00	/7 stability
Last Performace	2,602.9	1,987.8	1,039.5	775.0	589.0	606.7	14.0%	0.4	0.3	0.2	0.6	1.7	0.2	5.500	/7 reliability
Standard dev							8%	5%	2%	2%	0.2	0.1	6%	8.50	/14
Estee Lauder					Average tax:	29%								YES	
Average (2017-2022)	14,769.3	11,376.2	3,177.0	2,586.8	1,858.6	1,681.0	20%	21%	17%	11%	1.0	3.1	33%	3.50	/7 stability
Last Performace	17,737.0	13,432.0	4,271.0	3,544.0	2,658.0	2,390.0	17%	24%	20%	13%	0.8	3.2	36%	4.750	/7 reliability
Standard dev							2%	0.02	0.02	0.04	0.16	0.49	0.13	8.25	/14

Note 3.f: Porsche Capital Budgeting with uncertainty

	2024	2025	2026	2027	2028	2029	2030	2030 Normalized
Group Sales	52,002	56,255	60,843	65,789	71,122	76,869	83,062	
	48,383	51,481	54,800	58,359	62,176	66,274	70,677	72,232
Sales automotive	44,080	47,723	51,655	55,896	60,471	65,404	70,721	
	40,924	43,560	46,385	49,416	52,668	56,162	59,917	61,235
	85%	85%	85%	85%	85%	85%	85%	
	85%	85%	85%	85%	85%	85%	85%	
COGS	(37,942)	(41,046)	(44,393)	(48,002)	(51,893)	(56,086)	(60,604)	
	(35,302)	(37,562)	(39,984)	(42,580)	(45,366)	(48,356)	(51,568)	(52,703)
Gross Profit	14,060	15,210	16,450	17,787	19,229	20,783	22,457	
	13,081	13,919	14,816	15,778	16,810	17,918	19,109	**19,529**
administrative expenses (net	(4,484)	(4,820)	(5,181)	(5,568)	(5,985)	(6,433)	(6,915)	(7,067)
D&A adding	4,060	4,299	4,553	4,825	5,114	5,424	5,756	5,882
Other operating income (expenses) Net	(0.5)	(0.5)	(0.6)	(0.6)	(0.6)	(0.7)	(0.7)	(1)
EBITDA	13,636	14,688	15,822	17,043	18,358	19,774	21,297	
	12,657	13,397	14,188	15,034	15,939	16,909	17,949	18,344

EBITDA margin (average)	26%	26%	26%	26%	26%	26%	26%	
D&A	(4,060)	(4,299)	(4,553)	(4,825)	(5,114)	(5,424)	(5,756)	(5,882
	9,576	10,389	11,268	12,218	13,243	14,349	15,542	
EBIT								
	8,597	9,098	9,635	10,209	10,825	11,485	12,193	12,461
	6,703	7,272	7,888	8,553	9,270	10,045	10,879	
NOPAT ###								
	6,018	6,369	6,744	7,146	7,577	8,039	8,535	8,723
	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	
(+) Non-cash adjustements (inl. I	4,060	4,299	4,553	4,825	5,114	5,424	5,756	
(-) Capital expenditures	(6,943)	(7,388)	(7,864)	(8,375)	(8,923)	(9,511)	(10,143)	
	811	463	500	540	582	628	677	
(-) Change WCR								
	(79)	302	324	348	374	401	432	
	4,552	4,949	5,401	5,890	6,418	6,987	7,601	
Free cash flow to the firm								
	3,056	3,582	3,757	3,944	4,143	4,354	4,580	

Note 3.g: Pro Forma Ionity Income Statement

	2022	Adjustments		2022 pro forma	2,023	2,024	2,025	2,026	2,027	2,028	2,029	2,030	2,031	2,032
Non current assets	**302,350**			**302,350**	**355,168**	**411,496**	**471,589**	**571,801**	**687,825**	**803,499**	**918,835**	**1,033,843**	**1,149,167**	**1,264,819**
Intangible assets	1,337			1,337	*7,191*	*15,614*	*26,845*	*37,700*	*48,192*	*58,334*	*68,138*	*77,614*	*87,406*	*97,526*
Tangible assets	301,013			301,013	347,977	395,882	444,744	534,101	639,633	745,165	850,697	956,229	1,061,761	1,167,293
Other non-current assets	none													
Goodwill	none	**233,066**		233,066	233,066	233,066	233,066	233,066	233,066	233,066	233,066	233,066	233,066	233,066
Defered financial fees		19,501		19,501	13,651	9,555	6,689	4,682	3,278	2,294	1,606	1,124	787	551
Current assets	**171,303**			**171,303**	**158,844**	**171,308**	**185,762**	**211,354**	**244,515**	**291,137**	**354,981**	**443,256**	**455,947**	**469,146**
Inventories	3,550			3,550	4,109	5,667	7,473	10,672	14,817	10,280	14,253	19,748	20,538	21,359
Account receivable	17,822			17,822	28,760	39,666	52,313	74,706	103,722	154,882	214,752	297,532	309,433	321,810
Other current assets	125,937			125,937	125,976	125,976	125,976	125,976	125,976	125,976	125,976	125,976	125,976	125,976
Cash and Cash eq.	35,915		**(35,915)**	0	0	0	0	0	0	0	0	220,512	600,008	967,323
Total assets	**509,568**			**706,719**	**747,078**	**815,871**	**890,418**	**1,016,222**	**1,165,406**	**1,327,702**	**1,506,882**	**1,930,677**	**2,438,188**	**2,934,353**
Shareholder funds (Equity	**10,334**	**73,476**	**(10,334)**	**63,142**	**33,898**	**10,251.18**	**15,537**	**54,675**	**142,092**	**297,038**	**540,760**	**907,668**	**1,273,080**	**1,629,537**
Long-term debt	29,819			**179,091**	**463,182**	**533,516**	**579,748**	**614,604**	**605,234**	**496,624**	**276,718**	**100,000**	**100,000**	**100,000**
Term Loan A	29,819		**(29,819)**											
Term Loan B		**200,000**		200,000	*190,000*	*180,000*	*170,000*	*160,000*	*150,000*	*140,000*	*130,000*	*0*	*0*	*0*
Revolving Debt		**94,000**		94,000	173,182	253,516	309,748	354,604	355,234	256,624	46,718	0	0	0
Unsecured Bonds		**100,000**		100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000
Other Term Debt	185,090		**(185,090.0)**											
Other non-current	35,988			35,988	35,988	35,988	35,988	35,988	35,988	35,988	35,988	35,988	35,988	35,988
Non Current liab.					5,557	11,244	17,126	23,354	30,143	37,678	46,185	51,685	57,185	62,685
Current liabilities	**248,337**			**428,498**	**208,454**	**224,872**	**242,018**	**287,602**	**351,950**	**460,374**	**607,231**	**835,335**	**971,935**	**1,106,143**
Account payable	23,365			23,365	42,186	58,603	73,987	106,524	141,735	198,510	264,126	369,928	384,725	400,114
Current Liab	224,972			405,133	166,268	166,269	168,031	181,078	210,215	261,864	343,104	465,407	587,210	706,029
Total liabilities and equity	**509,568**			**706,719**	**747,078**	**815,871**	**890,418**	**1,016,222**	**1,165,406**	**1,327,702**	**1,506,882**	**1,930,677**	**2,438,188**	**2,934,353**
CHECK				**CHECK**	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE

Data in thousands

	1	2	3	4	5	6	7	8	9	10	
										4%	2.50%
	2023	**2024**	**2025**	**2026**	**2027**	**2028**	**2029**	**2030**	**2031**	**2032**	**2032N**
EBITDA	**4,060**	**21,637**	**55,860**	**132,757**	**238,067**	**383,194**	**575,897**	**837,098**	**870,582**	**905,406**	858,026
Depreciation & Amortization	(37,899)	(47,292)	(37,389)	(49,313)	(61,860)	(77,294)	(99,512)	(130,368)	(166,703)	(218,583)	(224,047)
Depreciation &	26%	23%	14%	12%	10%	9%	8%	8%	8%	8%	
EBIT	**(33,839)**	**(25,655)**	**18,471**	**83,445**	**176,207**	**305,900**	**476,384**	**706,730**	**703,879**	**686,823**	**703,993**
Cash Interest Expenses											
Revolving Credit	(198)	(597)	(938)	(1,191)	(1,305)	(1,060)	(288)	0	0	0	0
Term Loan B	(10,000)	(10,000)	(10,000)	(10,000)	(10,000)	(10,000)	(10,000)	(10,000)	0	0	0
Existing Debt											
Senior Notes	(8,000)	(8,000)	(8,000)	(8,000)	(8,000)	(8,000)	(8,000)	(8,000)	(8,000)	(8,000)	0
	(18,198)	(18,597)	(18,938)	(19,191)	(19,305)	(19,060)	(18,288)	(18,000)	(8,000)	(8,000)	0
Interest Expense											
Earning Before '	**(52,037)**	**(44,252)**	**(467)**	**64,254**	**156,902**	**286,841**	**458,096**	**688,730**	**695,879**	**678,823**	**703,993**
% Income tax (exp	15,611	13,276	140	(19,276)	(47,071)	(86,052)	(137,429)	(206,619)	(208,764)	(203,647)	(211,198)
Net Income	(29,244)	(23,646)	**7,048**	**52,183**	**116,557**	**206,594**	**324,963**	**489,211**	**487,216**	**475,276**	492,795
Net income ma	-20%	-11%	3%	12%	20%	25%	27%	29%	28%	26%	26%
Accoumulated Net Income	**Retained**	**75%**	5,286	39,137	87,417	154,946	243,722	366,908	365,412	356,457	369,597

Note 3.h: Cost of capital of intangibles

	Fair Value	**Weight**	Cost of capital of intangible
Data in millions (Eur)			
Intangible Asset	**7,473**	28%	**15.53%**
Tangible Assets	**12,778**	47%	4.81%
Financial assets	**6,394**	24%	9.70%
NWC	**3,995**	15%	3.07%
Funds	**(3,719)**	14%	2.48%
Net Invested Capital	**26,921**	100%	**9.7%**

PYTHON CODES

Note 3.i: Python Code for Optimal Portfolio of assets

```
import yfinance as yf
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
from pylab import rcParams
from datetime import date
from scipy.optimize import minimize
from matplotlib import cm
from matplotlib.ticker import LinearLocator, FormatStrFormatter
from mpl_toolkits.mplot3d import Axes3D
```

Python

```
# Settings for data download

start_date = "2019-09-01"      # Start date for the download
end_date = date.today().strftime("%Y-%m-%d")
```

Python

```
tickers = "BMW.DE RACE.MI MBG.DE MC.PA VOW.DE EL MONC.MI RMS.PA TSLA"
asset_list = tickers.split()
print(asset_list)

portfolio = yf.download(tickers, start=start_date, end=end_date)
portfolio.head()
```

Python

```
portfolio = portfolio["Close"]
portfolio.head()
```

Python

```
ret = (portfolio-portfolio.shift(1))/portfolio.shift(1)
#the return is (the price at time t less the price at time t-1)
#over the price at time t
# the price is pointed out by the portfolio
ret.head()
```

Python

```python
# Compute the absolute retun and the variance for each equity

scarling_factor = 252
mu = ret.mean()*scarling_factor
print(mu)

sigma = ret.cov()*scarling_factor
sigma
```

Python

```python
weights= np.ones(len(asset_list))*(1/len(asset_list))  #Equaly weighted po
print(weights)

w0 = weights
weights = np.reshape(weights,(len(asset_list),1))
mu = np.reshape(mu.to_numpy(), (len(asset_list),1))
sigma=sigma.to_numpy()
```

Python

```python
def ptfRet(x,mu):
    xT = np.transpose(x)
    return xT@mu

def ptfVar(x, sigma):
    xT = np.transpose(x)
    return xT@(sigma@x)

# x are the weights, mu the expected return, sigma the volatility

def plot_weights(weights):
    x = np.arange(len(asset_list))+1
    heights = np.squeeze(weights)
    plt.subplots(figsize=(10,4))
    plt.bar(x, heights)
    plt.xticks(x, asset_list)
    plt.title("Portfolio weights")
    plt.show()
    pass
```

Python

```python
mu0 = 0.2

# portfolio_return = mu0, portfolio_return - mu0 = 0


#Markowitz system
linConstraints = ({'type':'eq', 'fun':lambda x : ptfRet(x,mu)-mu0}, {'type':'eq', 'fun':lambda x: np.sum(x) -1})
res = minimize(ptfVar, w0, args=sigma, method='SLSQP', constraints=linConstraints, options={'ftol':(1e-09)})
print(res.x)
```

```
### 6. Plot the efficient Frontier and efficient Portfolio
```

```
#No shortsales (NS), Shortsales allowed (S)
muMin_NS = np.min(mu) # min vector of expected return
muMax_NS = np.max(mu) # max vector of expected return
muRange_NS = np.linspace(muMin_NS, muMax_NS, 30)  # Build a range of mu values

muMin_S = np.min(mu) # min vector of expected return
muMax_S = ptf3_expret # Portfolio max sharpe ratio
muRange_S = np.linspace(muMin_S, muMax_S, 30)  # Build a range of mu values

#Analytic method
ones = np.ones((len(asset_list),1))
onesT = np.transpose(ones)
sigma_inv = np.linalg.inv(sigma)
mu_t = np.transpose(mu)
a = (onesT@sigma_inv)@ones
b = (mu_t@sigma_inv)@mu
c = (mu_t@sigma_inv)@ones

ptfVarRangeA = []
for el in muRange_S:
    ptfVarRangeA += [(a*el**2-2*c*el+b)/(a*b-c**2)]
```

```
#Numerical method (we do this for portfolios in which shortsales are not allowed
ptfVarRangeNS = []
for el in muRange_NS:
    bound = (0.0,1.0)
    bounds = tuple(bound for asset in range(len(asset_list)))
    linConstraintMu = ({'type':'eq','fun':lambda x : ptfRet(x,mu)-el}, {'type':'eq', 'fun':lambda x: np.sum(x) - 1})
    res = minimize(ptfVar, w0, args=sigma, method='SLSQP', constraints=linConstraintMu, bounds=bounds, options={'ftol':1e-09})
    w = res.x
    ptfVarRangeNS += [ptfVar(w,sigma)]

    muRange = np.vstack(muRange_NS)
ptfVarRangeA = np.vstack(ptfVarRangeA)

prets = []
pvols = []
for p in range(25000):
    weights = np.random.random(len(asset_list))
    weights /= np.sum(weights)
    prets.append(ptfRet(weights,mu)[0])
    pvols.append(np.sqrt(ptfVar(weights,sigma))) # variance or volatilty?

prets = np.array(prets)
pvols = np.array(pvols)

rcParams['figure.figsize']= 10,6
plt.plot(np.sqrt(ptfVarRangeA),muRange_S, label="MV with short sales")
plt.plot(np.sqrt(ptfVarRangeNS),muRange_NS, label="MV with no short sales")
plt.plot(ptf1_vol,ptf1_expret, "ko", label= "Target Exp Ret")
plt.plot(ptf1_2vol,ptf1_2expret, "ko")
plt.plot(ptf2_vol,ptf2_expret, "bo", label= "Min Var")
plt.plot(ptf2_2vol,ptf2_2expret, "bo")
plt.plot(ptf3_vol,ptf3_expret, "ro", label= "Max Var")
plt.plot(ptf3_2vol,ptf3_2expret, "ro")
plt.scatter(pvols, prets, c = prets / pvols, marker ="o", cmap = "viridis", label="Random portfolios")
plt.legend(loc='right')
plt.grid()
plt.title('MV in the $(  sigma,  mu)$ plane')
plt.ylabel('Expected return $  mu$')
plt.xlabel('Portfolio Std deviation $sigma$')
plt.show()
```

Note 3.I: Python Code for GARCH (1,1)

```
#importing packages
import numpy as np
import pandas as pd
import yfinance as yf
import matplotlib.pyplot as plt
import scipy.optimize as spop
```

Python

```
#specifying the sample
ticker = '^VIX'
start = '2018-12-31'
end = '2023-12-31'
#downloading data
prices = yf.download(ticker, start, end)['Close']
#calculating returns
returns = np.array(prices)[1:]/np.array(prices)[:-1] - 1
#starting parameter values - sample mean and variance
mean = np.average(returns)
var = np.std(returns)**2
def garch_mle(params):
    #specifying model parameters
    mu = params[0]
    omega = params[1]
    alpha = params[2]
    beta = params[3]
    #calculating long-run volatility
    long_run = (omega/(1 - alpha - beta))**(1/2)
    #calculating realised and conditional volatility
    resid = returns - mu
    realised = abs(resid)
    conditional = np.zeros(len(returns))
    conditional[0] =  long_run
    for t in range(1,len(returns)):
        conditional[t] = (omega + alpha*resid[t-1]**2 + beta*conditional[t-1]**2)**(1/2)
    #calculating log-likelihood
    likelihood = 1/((2*np.pi)**(1/2)*conditional)*np.exp(-realised**2/(2*conditional**2))
    log_likelihood = np.sum(np.log(likelihood))
    return -log_likelihood
#maximising log-likelihood
res = spop.minimize(garch_mle, [mean, var, 0, 0], method='Nelder-Mead')
#retrieving optimal parameters
params = res.x
mu = res.x[0]
omega = res.x[1]
alpha = res.x[2]
beta = res.x[3]
```

```
    return -log_likelihood
```

```
#calculating realised and conditional volatility for optimal parameters
long_run = (omega/(1 - alpha - beta))**(1/2)
resid = returns - mu
realised = abs(resid)
conditional = np.zeros(len(returns))
conditional[0] =  long_run
for t in range(1,len(returns)):
    conditional[t] = (omega + alpha*resid[t-1]**2 + beta*conditional[t-1]**2)**(1/2)
#printing optimal parameters
print('GARCH model parameters')
print('')
print('mu '+str(round(mu, 6)))
print('omega '+str(round(omega, 6)))
print('alpha '+str(round(alpha, 4)))
print('beta '+str(round(beta, 4)))
print('long-run volatility '+str(round(long_run, 4)))
print('log-likelihood '+str(round(log_likelihood, 4)))
#visualising the results

plt.title('VIX Volatility Prediction')
plt.legend(['True Daily Log Returns', 'Predicted Volatility'])
plt.figure(1)
plt.rc('xtick', labelsize = 10)
plt.plot(prices.index[1:],realised)
plt.plot(prices.index[1:],conditional)
plt.show()
```

Note 3.m: Python Code for Implied Volatility with BSM

```python
import numpy as np
import pandas as pd
from scipy.optimize import fsolve
from scipy.stats import norm
import matplotlib as mpl
import matplotlib.pyplot as plt
mpl.rcParams['font.family'] = 'serif'
%matplotlib inline
```

Add Code Cell

Python

```python
# define ingredients for B&S as separate functions:
def N(d):
    return norm.cdf(d)

def d1f(St, K, timetomaturity, r, sigma):
    ''' Black-Scholes-Merton d1 function.
        Parameters see e.g. BSM_call_value function. '''
    d1 = (np.log(St / K) + (r + 0.5 * sigma ** 2)* timetomaturity) / (sigma * np.sqrt(timetomaturity))
    return d1

# B&S Call and Put value:
def BSM_call_value(St, K, timetomaturity, r, sigma):
    ''' Calculates Black-Scholes-Merton European call option value.
```

```python
    ...
    d1 = d1f(St, K, timetomaturity, r, sigma)
    d2 = d1 - sigma * np.sqrt(timetomaturity)
    call_value = St * N(d1) - np.exp(-r * timetomaturity) * K * N(d2)
    return call_value
```

```python
def BSM_put_value(St, K, timetomaturity, r, sigma):
    ''' Calculates Black-Scholes-Merton European put option value.
```

```python
# to compute implied volatility
def impl_vol(IsCall,market_price,St,strike,timetomaturity,r,sigma_estimate):
        ''' Return implied volatility given option price. '''
        if (IsCall):
            iv = fsolve(lambda sigm : BSM_call_value(St, strike, timetomaturity, r, sigm) - market_price, sigma_estimate)[0]
        else:
            iv = fsolve(lambda sigm : BSM_put_value(St, strike, timetomaturity, r, sigm) - market_price, sigma_estimate)[0]

        return iv
```

```
# Pricing Data
pdate = pd.Timestamp('30-09-2023')

#
# EURO STOXX 50 index data
#

# URL of data file
es_url = 'http://www.stoxx.com/download/historical_values/hbrbcpe.txt'
# column names to be used
cols = ['Date', 'SX5P', 'SX5E', 'SXXP', 'SXXE',
        'SXXF', 'SXXA', 'DK5F', 'DKXF', 'DEL']
# reading the data with pandas
es = pd.read_csv(es_url,  # filename
                 header=None,  # ignore column names
                 index_col=0,  # index column (dates)
                 parse_dates=True,  # parse these dates
                 dayfirst=True,  # format of dates
                 skiprows=4,  # ignore these rows
                 sep=';',  # data separator
                 names=cols)  # use these column names
# deleting the helper column
del es['DEL']
# the Euro-Stoxx index value on the pricing date
S0 = es['SX5E']['30-09-2023']
# assume this is the (flat) interest rate on that date
int_rate = -0.05
```

```
#
# BSM Implied Volatilities
#
def calculate_imp_vols(data):
    ''' Calculate all implied volatilities for the European call options
    given the tolerance level for moneyness of the option.'''
    data['Imp_Vol'] = 0.0
    tol = 0.30  # tolerance for moneyness
    for row in data.index:
        t = pd.Timestamp(data['Date'][row])
        T = pd.Timestamp(data['Maturity'][row])
        ttm = (T - t).days / 365.
        forward = np.exp(int_rate * ttm) * S0
        if (abs(data['Strike'][row] - forward) / forward) < tol:
            data.loc[row,'Imp_Vol'] = impl_vol(True,data['Call'][row],S0,data['Strike'][row],ttm,int_rate,0.2)
    return data

#
# Graphical Output
#
markers = ['.', 'o', '^', 'v', 'x', 'D', 'd', '>', '<']
def plot_imp_vols(data):
    ''' Plot the implied volatilites. '''
    maturities = sorted(set(data['Maturity']))
    plt.figure(figsize=(10, 12))
    for i, mat in enumerate(maturities):
        dat = data[(data['Maturity'] == mat) & (data['Imp_Vol'] > 0)]
        plt.plot(dat['Strike'].values, dat['Imp_Vol'].values,
                 'b%s' % markers[i], label=str(mat)[:10])

    plt.xlabel('strike')
    plt.ylabel('implied volatility')
```

```
options_data=calculate_imp_vols(options_data)
options_data
```

```
plot_imp_vols(options_data)
```