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Luiss Guido Carli

"Strategic Moves on the Pitch: Enhancing Football Transfer Negotiations through Game Theory"

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Xavier Venel
Relatore

Francesco Scognamiglio
Candidato

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Abstract

In this thesis, the dynamic field of player transfers is explored, adopting an innovative game-theoretic approach to analyze and optimize negotiations between clubs. Through the application of a Nash Bargaining model specifically adapted to the football context, this study aims to outline more effective strategies for football clubs in the process of buying and selling player rights. Starting with an in-depth analysis of the basic principles of game theory, the research draws on the work of Zhaleh Memari, Maryam Esmaeili and Mojzhgan Jafari, 'How Could a Football Player Transfer Business be More Successful? A Model-Based on Game Theory Approach', as a theoretical basis to develop a conceptual framework applicable to football player transfers. The thesis consists of four main parts: an introduction to cooperative bargaining and an explanation of the context of football transfers, a practical application of this model to the football industry, followed by an extension of the model exploring the adaptation of the solution concept and applicability to different players, respectively. This research not only provides a detailed understanding of negotiation mechanisms in football but also proposes a methodological approach that could be extended to other sporting domains, thus offering new perspectives on the improvement of transfer strategies through the use of game theory.

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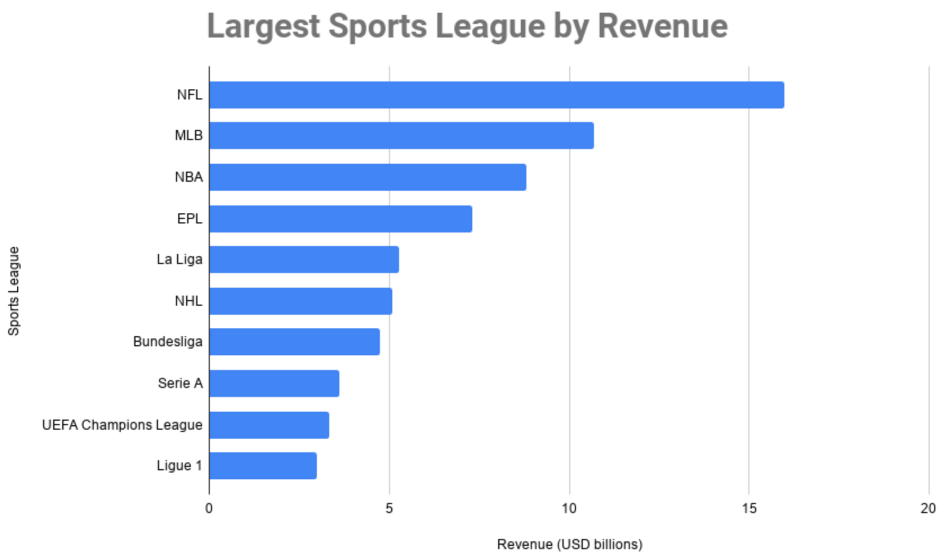
Introduction:

Football worldwide is the most profitable sport. With a global turnover of $47 billion, it accounts for 28% of the turnover generated by sport worldwide[1].



"Top 10 Largest Sports Leagues by Revenue in the World 2020"

football related league: EPL (Premier league, England); La Liga (Spain), Bundesliga (Germany), Serie A (Italy), Champions League (Europe), Ligue 1 (France).

Data based on the year 2020.

BizVibe. (n.d.). *Largest sports leagues by revenue*. [2]

This is a world that requires in-depth analysis since it attracts a sizable section of the global population and generates a significant amount of money, as can be seen by closely examining the first line and appreciating its significance. Going more specific with the data, precisely in order to have a scale of analysis that can also be national as well as global, we quote these numbers concerning football in Italy. In seven years, precisely from 2006 to 2012, without taking betting into account, professional

[1] Rome Business School. (n.d.). *Football: between eSports, crypto, NFT and metaverse*. Retrieved from https://romebusinessschool.com/research-center/football-is-the-most-profitable-sport-with-global-revenue-of-47-billion/

[2] BizVibe. (n.d.). *Largest sports leagues by revenue*. Retrieved from https://blog.bizvibe.com/blog/largest-sports-leagues-by-revenue

Italian football has guaranteed the state 5.9 billion euros.[3] Truly enormous sums, which in recent years have been getting bigger and bigger. Football, like everything else, evolves and in the recent past we have seen how fast it is moving and how the figures in any sector of the sport are rising. Football is increasingly moving into channels that go beyond the simple sport created by the English in 1863 with the birth of the 'football association'[4]. To date, several domains have been created around the sport. Entrepreneurs invest in this business because they know everything that revolves around it; the media, advertising, television rights, sponsors... These are all elements that have been created later and have accompanied the only corporate input that has always existed, ticket sales. This whole circle and recirculation of money only exists thanks to the fact that it is a sport that attracts people, that attracts fans from all over the globe, and it is they, who through their passion, transformed into views, clicks, television subscriptions and viewing of matches at the stadium, create this value.[5]

Everything we have talked about so far is one of the reasons why the sums clubs pay to secure the competitive performance of players, thus buying them from other clubs, have also risen disproportionately. In the not-too-distant past, these figures were around one million euros for a good footballer and a few tens at most for the absolute champion. An example is the transfer of Diego Armando Maradona[6], according to many the strongest footballer of all time, from Barcelona to Napoli in 1984, for 7.5 million dollars, corresponding to 13 billion lire, a record at the time.[7] In recent times, these figures are seen for mid- or low-level players and top players are paid millions upon millions. Neymar da Silva Santos Jr, the Brazilian tightrope walker more talked about for his life outside the

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[3] Truenumbers.it. (n.d.). *Lo Stato incassa 1 miliardo l'anno dalle tasse sul fatturato del calcio*. Retrieved from https://www.truenumbers.it/lo-stato-ingrassa-grazie-al-calcio/#:~:text=Quanto%20incassa%20lo%20Stato%20dal%20calcio&text=In%207%20anni%20(2006%2D12,5%2C9%20miliardi%20di%20euro.

[4] European Association of History Educators. (n.d.). *A little history of football: The beginnings*. Historiana. Retrieved from https://historiana.eu/partners/european-association-of-history-educators/a-little-history-of-football-the-beginnings

[5] Deloitte. (2024). *Deloitte Football Money League 2024*. Retrieved from https://www2.deloitte.com/uk/en/pages/sports-business-group/articles/deloitte-football-money-league.html

[6] Wikipedia contributors. (n.d.). *Diego Armando Maradona*. Wikipedia. Retrieved from https://it.wikipedia.org/wiki/Diego_Armando_Maradona

[7] La Gazzetta dello Sport. (2020, November 30). *Maradona, a Napoli 13 miliardi: oggi sarebbe impossibile*. Retrieved from https://www.gazzetta.it/Calcio/Serie-A/Napoli/30-11-2020/maradona-napoli-13-miliardi-oggi-sarebbe-impossibile-3901287964902_preview.shtml

lines than for the trophies he won, cost Sheikh Al-Khelaïfi, owner of Paris Saint-Germain, 222 million euros in 2016, making him the most expensive transfer by a long shot in the history of football.[8]

But how can one manage to put a fair price on a player's card without overpaying for it? Is there an effective way to make all parties involved happy with the terms of the transfer?

And that is precisely the focus of our project. Trying to use a Game theory model to make transfers in the football world more effective and that can solve problems related to the uncertainty of who is buying, what is a fair price for player x's price tag and all that can interfere in a negotiation between clubs.

We will first analyze the transfer market in football, giving an overview of the system, unravelling the various factors that can influence these transfers, and explaining the various economic and regulatory dynamics. Subsequently, we will discuss the fundamentals of game theory, key concepts and terminologies that can be useful to the cause, delve into the concept of Nash Bargaining, and then move on to the third phase in which we will combine these two pillars to understand how game theory concepts can be applied to the world of football transfers. Finally, a part of experimentation of the model will follow, opening us up to more fields and different applications. This will be based heavily on the research work of Professors Zhaleh Memari, Maryam Esmaeili and Mojzhgan Jafari, 'How Could a Football Player Transfer Business be More Successful? A Model-Based on Game Theory Approach', which will give us a theoretical basis on which we can build.

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[8] Taylor, D. (2017, August 4). *Neymar: how record-breaking move to PSG unfolded*. The Guardian. Retrieved from https://www.theguardian.com/football/2017/aug/04/neymar-how-record-breaking-move-to-psg-unfolded

Chapter 1: The Transfer Market in Football

The transfer process in football refers to the process by which footballers move from one team to another. In effect, each team must consist of a squad of 25 players who are either bought through a negotiation that the club undertakes with the club that owns their badge or raised in the youth sector and brought into the first team over time. We therefore note that the process of transferring a player from one team to another is a pivotal element[9] in the world of football, and what is more, we note that the more the years pass, the more the purchase price of a player's card increases exponentially.

	A	B	C	D	E	F
1	Year	1. Bundesliga	La Liga	Serie A	Premier League	total expenditure on the purchase of players (inflation not included)
2	2014/15	368.280.000	568.170.000	472.680.000	1.230.000.000	In Euros
3	2015/16	474.150.000	620.430.000	724.300.000	1.460.000.000	
4	2016/17	669.390.000	529.190.000	867.710.000	1.660.000.000	
5	2017/18	725.290.000	906.420.000	1.050.000.000	2.180.000.000	
6	2018/19	561.100.000	1.010.000.000	1.280.000.000	1.650.000.000	

Data provided by "Sole 24 Ore" assimilated and analysed in excel table[10]

Transfer is therefore a central mechanism for the movement of professional football players and has significant implications for both sporting strategies and club finances. The body that regulates the transfer system worldwide is FIFA[11]. The 'Fédération Internationale de Football Association' imposes rules and regulations that each country's federations and clubs must obsessively follow. These rules include dedicated windows for the football market, compensation for the training of players, and a solidarity mechanism. FIFA can be said to be the organ on which the entire football system is based, making an economic comparison we could liken it to the European Central Bank and its role as regulator within the EU, with the federations that can be compared to the various central banks of each state and with the football clubs that can be compared to the various commercial banks.

In order to understand this world well and also to understand how clubs act, or have acted in the past, and what strategies they have adopted, we will begin by analyzing the factors that can influence player

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[9] Spartacus Educational. (n.d.). *Football Transfers*. Retrieved from https://spartacus-educational.com/Ftransfer.htm

[10] Mancino, D. (2019, August 5). *Le spese per trasferimenti e stipendi dei calciatori sono davvero fuori controllo?* Il Sole 24 ORE. https://www.infodata.ilsole24ore.com/2019/08/05/40616/?refresh_ce=1

[11] Wikipedia contributors. (n.d.). *FIFA*. Wikipedia. Retrieved from https://it.wikipedia.org/wiki/FIFA

transfers in market windows, we will continue by analyzing the economic and regulatory dynamics, and we will conclude this analysis with four case studies of transfers that have in common the fact that they have revolutionized the world of football but diverge in why they have revolutionized it. Player transfers are influenced by a multitude of factors. As in any agreement between two parties, the elements that can directly influence the agreement are varied and complex, clearly, we will tend to mention those that are most significant and that are found in almost every negotiation that a club conducts to secure the services of a player.[12] The first element to be mentioned, and probably the most important, knowing that we are always talking about football, is the sporting aspect. Indeed, it all starts from a foundation of seeking to fill a team's technical or tactical needs. And this clearly can lead to the search for players and talent that can match the characteristics sought by the purchasing club from a sporting point of view. The second key element is clearly the economic and financial conditions of a club. The economic capacity of a club can expand or limit transfer options, and above all can strongly influence the purchase price of a player. The third element, which is also increasingly crucial as footballers are gaining more and more media power and becoming real living brands, is the possible desire and demands of the footballer himself. Indeed, the personal preferences of the footballer, including the required salary, playing opportunities and geographical preferences, play a key role. This clearly has a major impact on the negotiation as, depending on the player's salary and lifestyle demands of the club, the economic agreement between the clubs could also vary and be scaled down. Finally, the last key factor that regularly influences the clubs' approach to the football market and negotiations with other clubs are the UEFA and FIFA regulations[13] on financial fair play and the limits on the composition of the rosters of each professional team. In recent years, the football system's regulators have put in place various systems to control and monitor club finances as they realized that the system's expenditures were beginning to become completely disproportionate and

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[12] University of Warwick, Department of Economics. (n.d.). *THE DETERMINANTS OF FOOTBALL TRANSFER MARKET VALUES: AN AGE OF FINANCIAL RESTRAINT*. Retrieved from https://warwick.ac.uk/fac/soc/economics/current/modules/ec331/raeprojects/0907288-ec331-a2.pdf

[13] European Commission. (n.d.). *The Economic and Legal Aspects of Transfers of Players*. Retrieved from https://ec.europa.eu/assets/eac/sport/library/documents/study-transfers-exec-summary_en.pdf

club budgets could not always absorb them. In recent times, in fact, we note that several clubs that have always been regarded as central to the sporting world such as Barcelona, Juventus or Chelsea have in fact been sanctioned[14] [15] by this new UEFA regulation for not respecting the parameters of financial fair play or in certain cases not meeting the debt consolidation or end-of-loan deadlines imposed by the regulatory body.

In short, football transfers increasingly represent a key element in the football world both on a sporting level and on an economic and business level, considering that football clubs are real commercial enterprises. The factors that influence them are many, but these mentioned are certainly the most impactful ones and are becoming increasingly important in any negotiations between clubs. To illustrate this, we will proceed by analyzing three transfers that have turned out, over the years, to be revolutionary for the history of this wonderful sport. The element that unites them is undoubtedly the element of novelty and break with the past that these transfers have provoked in the world of football; having said that, the main characteristic of these transfers is that each one has revolutionized this world for very different reasons.

The first was the transfer of Neymar Jr. from Barcelona to Paris Saint-Germain in 2017 for EUR 222 million.[16] This is still the most expensive transfer in the history of football by far (the second is the one made by Paris Saint-Germain for Kylian Mbappé, who paid 180 million to Monaco in 2018[17] to acquire the services of the French jewel). It was a transfer that shocked the entire world of sport, as never before had anyone thought they could attach such a high price to a football player, whatever the talent and skill of the same. Indeed, in 2017 the most expensive transfer in the history of the sport

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[14] Goal. (n.d.). *Chelsea transfer ban: Why FIFA sanctioned Blues & what appeal process involves*. Retrieved from https://www.goal.com/en/news/chelsea-transfer-ban-why-fifa-sanctioned-blues-what-appeal-process/tt4ejezxvltd1k8r5symf4rkn

[15] AS. (n.d.). *FIFA's Juventus sanctions: What could they mean for LaLiga leaders Barcelona?*. Retrieved from https://en.as.com/soccer/fifas-juventus-sanctions-what-could-they-mean-for-laliga-leaders-barcelona-n/

[16] The Athletic. (2023, August 15). *Neymar PSG Saudi exit*. Retrieved from https://theathletic.com/4777471/2023/08/15/neymar-psg-saudi-exit/

[17] Sky Sports. (2017, September 1). *Kylian Mbappe joins Paris Saint-Germain from Monaco*. Retrieved from https://www.skysports.com/football/news/11820/11014378/kylian-mbappe-joins-paris-saint-germain-from-monaco

was that of Paul Pogba, who in 2016, a year earlier, went from Juventus to Manchester United for 100 million euros.[18] A year later came like a thunderbolt the purchase of Neymar Jr, the Brazilian ace whom the then newly appointed Qatari president of the Parisians wanted at all costs to build a stellar team in the French capital. And so it was that PSG shelled out this huge and disproportionate sum to Barcelona for a footballer who will always be remembered more for his undisputed and fabulous talents badly squandered by his passion for partying and the good life than for his titles won. Since this historic transfer, there have been several clubs that have spent monstrous sums on footballers, but none have yet come close to the 200 million ceiling broken by the PSG president. The official figures for the 'transfer of the century', released immediately after the signing of the contract and the exchange of documents between the clubs, say that the French club paid 222 million for Neymar Jr.'s card, to which must be added around 80 million in taxes paid to the Spanish tax authorities and 300 million gross for the five-year contract signed by the Brazilian. An operation that cost the French company, owned by Qatar Sport Investments, which holds 100 per cent of it, around EUR 600 million in total. It is therefore an investment that goes far beyond the sporting aspect and which makes it clear that behind it there was a very clear strategy on the part of the Qatari fund, which wanted to make a decisive move in order to denote their strong presence in the world of European football, and also to use Neymar as a testimonial for the 2022 World Cup that they wanted and subsequently hosted. Many media outlets have accused this deal of 'drugging' the football market by circumventing financial fair play. Indeed, the way in which the transfer took place seems unclear and unusual. It is said that the Qatari company that owns PSG, paid the player himself 300 million euros to have him buy his own card from Barcelona, so that he could circumvent financial fair play rules.

The second transfer that deserves special attention and a little more in-depth analysis is the one involving Cristiano Ronaldo first from Manchester United to Real Madrid in 2009 for €94 million and then from Real Madrid to Juventus in 2018 (a 32-year-old Ronaldo) for €116 million including

[18] Tuttosport. (2022, June 24). *Pogba, il miglior affare della Juve e di Pogba*. Retrieved from https://www.tuttosport.com/news/calcio/calciomercato/juventus/2022/06/24-94110802/pogba_il_miglior_affare_della_juve_e_di_pogba_

commission.[19] The formidable Portuguese footballer, who ranks among the top players of all time, is a true living brand. Thanks to his sporting merits, the Portuguese from Madeira has become in effect one of the most influential people on the planet, with millions and millions of followers and admirers around the globe. What draws our attention to this double transfer is not so much the amount of money, which is still very high, but the fact that Ronaldo, 10 years later, was still paid 100 million and more by Juventus. This makes us realize how much Cristiano represents an icon and having him in the team can be economically profitable as well as on a purely sporting level.[20] A figure that testifies to this is surely that Ronaldo, now over 30 years old at the time of his move to Italy, (the waning phase of his career for a normal player, he absolutely does not fall into this category) earned Juventus in the 2018-2019 season, his first of the three with the "Bianconeri", a good 44 million euros just from the sale of shirts with his name on them (over 1 million shirts globally).[21] Other numbers affirming the enormity of the Cristiano Ronaldo brand are the increase in Instagram followers on Juventus' profile from 9.8 million to 33.5 and the increase in subscribers to the Italian club's YouTube channel from 730,000 to 2.33 million.[22] The Portuguese has more than 500 million followers on social media, and holds the record as the most followed man on social media in the world. All that has been said reveals that certain footballers nowadays represent real feats and their purchase by a club can be motivated by reasons that go beyond the technical and on-field aspect.

The latest example of a transfer that has had a huge impact on the football market is surely the move of Virgil Van Dijk, the 91' Dutch defensive giant who in 2018 moved from Southampton, a team at the time in the English lower mid-table, to Liverpool, a top European and world club, for €85

[19] ET Insights. (n.d.). *Cristiano Ronaldo transfers: A journey of a lifetime*. Retrieved from https://etinsights.et-edge.com/cristiano-ronaldo-transfers-a-journey-of-a-lifetime/

[20] Chadwick, S., & Burton, N. (2008). From Beckham to Ronaldo -- Assessing the nature of football player brands. *Journal of Sponsorship*, 307-317.

[21] Benetazzo, M. (n.d.). *L'effetto Cristiano Ronaldo nella strategia Juventus* [Undergraduate thesis, Università degli Studi di Padova, Dipartimento di Scienze Economiche ed Aziendali "M. Fanno"]. Advisor: Prof. Romano Cappellari.

[22] La Gazzetta dello Sport. (2018, September 19). *Juventus, Ronaldo, PSG, Liverpool: social*. Retrieved from https://www.gazzetta.it/Calcio/Serie-A/Juventus/19-09-2018/juventus-ronaldo-psg-liverpool-social-2901133131692.shtml

million.[23] He is an example of how a top club, by making a huge but well-targeted investment, managed to change the fortunes of the defensive package with practically one man, who in just a few years became the strongest defender in the world, making the English defense virtually insurmountable.[24] In 2019, the current captain of the Reds and the Dutch national team lived his year of consecration by winning the Champions League with his Liverpool as an absolute defensive star. It is no coincidence that the same year he was awarded the Premier League's best player and UEFA's best player of the year, even coming very close to winning the coveted Ballon d'Or awarded annually by France Football. In fact, that year he came second behind Leo Messi and ahead of a sacred monster like Cristiano Ronaldo, which for a defender is quite atypical for the way France football has accustomed us to evaluating players. He would later also win an English championship, again with Liverpool, and today, years later, with 246 appearances with the Reds, he is first in the Premier League with ten days to go and is fighting for his second English title. In short, a transfer that has really radically changed a department, the defensive one, but probably also an entire team, which relies in every way on its Dutch leader.

In short, this chapter has served to give a comprehensive overview of the transfer market in football, emphasizing the importance of considering both purely sporting as well as economic and regulatory aspects in the management of player transfers.

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[23] Liverpool FC. (n.d.). *Liverpool agree world record £75m deal for Virgil van Dijk*. Retrieved from https://www.liverpoolfc.com/news/media-watch/285493-liverpool-agree-world-record-75m-deal-for-virgil-van-dijk

[24] Anfield Index. (n.d.). *Virgil van Dijk's impact on Liverpool*. Retrieved from https://anfieldindex.com/49022/virgil-van-dijks-impact-on-liverpool.html

Chapter 2: Fundamentals of Game Theory

We then move on to the other main subject of our thesis, game theory. We will briefly introduce what the discipline is generally about and then highlight and deepen the elements of this science that we consider useful and fundamental to our cause.

I would start by quoting the economics and finance section of the Treccani encyclopedia that defines game theory as follows: "A theoretical system that studies the behavior of agents in the presence of strategic interaction, i.e. when the outcome of an agent's actions also depends on the decisions taken by other agents".[25] To rewrite and understand it in other words, we can say that game theory is a branch of applied mathematics that studies the optimal decisions to be made in competitive situations. This discipline has applications in a variety of fields, including economics, politics, evolutionary biology and, as we shall see, the world of football transfers. In this context, we will examine the fundamentals of game theory, the concept of Cooperative games, Nash Bargaining, and how these can be applied to football transfers.

Starting with the fundamentals[26], it can be stated that game theory analyses the strategies used by individuals in interactive decision-making situations, where the outcome for each participant depends on the choices of all. There are three key concepts in all of this, which are the players, i.e. the real proponents of the game and those who go into the negotiation context and who will come out of it with an outcome that may make them satisfied, impartial or dissatisfied; the second pillar on which game theory resides are the strategies these players employ to succeed in obtaining, or at least try to obtain, an outcome that is congenial to them; and last but not least is the payoff. This is in fact the result obtained by each player as a result of the strategies employed by each of them.

A considerable element in what has been explained so far is certainly the Nash equilibrium. This is a fundamental concept in game theory that describes a situation in which, in a game with two or more

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[25] Treccani. (n.d.). *Teoria dei giochi*. Retrieved from https://www.treccani.it/enciclopedia/teoria-dei-giochi/

[26] Myerson, R. B. (1997). *Game Theory: Analysis of Conflict*. Harvard University Press.

players, no player can gain an advantage by changing their strategy if the others keep theirs unchanged. This concept is named after the mathematician John Nash, who in his time demonstrated that for any game with a finite number of players and strategies, there is at least an equilibrium in which the strategies of all players are mutually best responses to each other.[27]

Nash equilibrium occurs when the strategy chosen by each player is the one that maximizes his own gain, or conversely minimizes his own loss, given the strategy of the other players. In other words, a player has nothing to gain by unilaterally changing his decision. A key word that can help understand this is indifference. The player arrives at a certain situation in which it is indifferent to him to implement any strategy.

This concept is of fundamental importance in game analysis because it provides a prediction of how players will behave in certain strategic situations, assuming that they all act rationally and have knowledge of others' strategies.

Applications of Nash equilibrium are made in various fields. It can be applied to microeconomics (for example, in models of competition between firms) to political theory, to evolutionary biology to the design of mechanisms and algorithms in computer science and, of course, in sports and football transfers, where it can help to model negotiation and decision-making strategies. The latter is the area in which we will delve deeper and focus our energies.

Despite its breadth of application, Nash equilibrium has limitations. Indeed, he gives no indication of how players can reach equilibrium without explicit communication or negotiation, nor does he consider the possibility of coalitions between players. Moreover, the presence of multiple Nash equilibria in a game can make it difficult for an outside analyst to predict which equilibrium will be the one selected by the players.

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[27] Nash, J. (1950). The Bargaining Problem. *Econometrica*, 155-162.

Trying to go deeper and come closer to what will later be the key concept of game theory that will allow us to take up a model that facilitates the search for agreement between football clubs in the buying and selling of a football player's badge, let us move on to the explanation and deepening of the theme of cooperative games.[28]

In game theory, cooperative games represent a set of problems in which players can form coalitions and negotiate revenue-sharing agreements in order to improve their results over what they would achieve acting individually. This is a real departure from non-cooperative games, in which instead each player acted individually and employed an individualistic, individual strategy, the focus was fully on the player's individual strategy and Nash equilibrium. In cooperative games, on the other hand, we examine how cooperation and interaction between the various players can lead to outcomes that are beneficial to all participating players. [29]

Basing ourselves on reference texts, including Guillermo Owen's "The Theory of Cooperative Games"[30], in which the author explicates and defines the various concepts of cooperative games, we try to visualize and indicate which are the key concepts revolving around the same theme. This is clearly a type of game in which coalitions between players are the protagonists. Effectively, a set of participating players decide to cooperate in order to achieve a certain outcome that will satisfy them. In this case, cooperative game theory analyses how these coalitions are formed and how the gains made by the coalition are distributed among its members. Two other central elements are the characteristic function and the Shapley value.[31] The first of the two is a function that associates each possible coalition with a value representing the total gain that all members can obtain by joining and coalescing. The second element, on the other hand, represents a fundamental element in cooperative game theory. In effect, it is a value that distributes the gains equally to all players participating in the

[28] Nash, J. (1951). Non-cooperative Games. *Annals of Mathematics*, 286-295.
[29] Von Neumann, J., & Morgenstern, O. (1944). *The Theory of Games and Economic Behavior*. Princeton University Press.
[30] Owen, G. (1995). *The Theory of Cooperative Games*. Kluwer Academic Publishers.
[31] Shapley, L. S. (1953). A Value for n-person Games. *Contributions to the Theory of Games*, 2, 307-317.

winning coalition. This distribution is mainly based on their marginal contribution to the coalition. The last, but no less important concept that deserves a little further investigation is the core. This is a concept that describes a set of possible allocations of gains or utilities among the players in a coalition, so that no subset of players may decide, because they are incentivized, to deviate from the coalition by forming a separate group.[32] To make this concept simpler, we can say that it is a set of payoff distributions for which there are no alternative coalitions from which one can earn more, or receive greater payoffs than within existing coalitions.

To summarize, we can say that one of the main challenges of cooperative game theory is to determine how payoffs should be distributed fairly among all participants in a coalition.

Now we can finally focus on the fundamental part of game theory that will be dissected and made useful to our cause. We are in fact talking about the Nash bargaining theme.[33] It is precisely from this theme that we will be able to theorize the model that will allow us, by inserting various variables that we will shortly define, to facilitate the transfer of footballers from team to team, allowing us to frame the most suitable price for the transfer; thus, the payment of the card by the purchasing club, and also defining the most suitable salary that the footballer will be able to receive at his new club. The effectiveness of the model will not only be to identify the most appropriate expenditure, relative to the value of the player, but it will also make all parties, i.e. the players participating in the Nash bargaining, if we want to speak in specific language, satisfied with the agreement made.

To get to this, we must move on to the theoretical pillar that will make it all possible: Nash bargaining. We speak of a theoretical model developed to analyze how two or more parties can negotiate a division of resources in such a way that no one party can improve its own outcome without worsening that of another. Clearly, as the word itself indicates, Nash bargaining is related to the Nash equilibrium in game theory, even if philosophically different. It is an equilibrium in which players decide which

[32] Fudenberg, D., & Tirole, J. (1991). *Game Theory*. MIT Press.

[33] Nash, J. (1950). The Bargaining Problem. *Econometrica, 18*(2), 155-162

strategies to use in order to maximize their utility. This is made up according to the actions of the other players. The Nash bargaining solution provides a way to predict the outcome of negotiations based on principles of fairness and rationality.[34] [35]

This model rests on four fundamental principles: Pareto efficiency, symmetry, invariance with respect to affine transformations and independence with respect to relevant alternatives. These are four concepts that we will explain in a few moments and that lend themselves to various economic disciplines, including micro and macroeconomics.

Let us begin by defining Pareto efficiency. This is an important principle, as mentioned, that ensures that the outcome of the trade is optimal in terms of efficiency. That is, once the outcome of the negotiation is reached, it is not possible to make one of the parties better off without making the other negotiating party worse off. The outcome of the negotiation is such that any attempt to improve the utility of one player would necessarily lead to a reduction in utility of the other player. Thus, the result would be Pareto efficient. In Nash Bargaining, this concept is ensured by finding a trade-off that maximizes the product of the incremental utilities of the players with respect to a point of disagreement. This point of disagreement would be the point at which the bargaining fails, with no successful outcome, and thus, no agreement between the parties. In Nash bargaining one therefore tries to optimize the allocation of resources in such a way that neither party can improve its position without harming the other. It is therefore an important concept in Nash bargaining because it ensures that the agreement that is found is socially optimal.

Another pillar on which Nash Bargaining rests is the principle of symmetry. This element implies that if the negotiators are in similar positions, then the negotiable solution should not arbitrarily favor one of the participants over the others. The outcome of the negotiation should therefore reflect a basic symmetry between the players in terms of how their utility is treated, while assuming that there are no underlying differences in their positions or initial preferences. Thus, being faced with a model in

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[34] Nash, J. (1951). Non-cooperative Games. *Annals of Mathematics, 54*(2), 286-295.
[35] Binmore, K. (1992). *Fun and Games: A Text on Game Theory*. D.C. Heath and Company.

which players seek to maximize their utility at the end of the negotiation, the concept of symmetry suggests that even if players were to decide to interchange positions with each other, the outcome of the negotiation would not change, all given the equivalent initial starting position.

This principle dictates and ensures that the solution respects fairness when the players have equal bargaining power and opportunities. Next, the third of the four foundations that govern Nash Bargaining is invariance with respect to affine transformations. We speak of a principle according to which bargaining remains invariant under affine transformations of the players' utility functions. An affine transformation is a linear transformation that includes both a multiplication with a constant (C other than 0) and an addition with a constant (X other than 0). In technical terms, if U(x) is a utility function of a player, the affine transformation of U(x) would be CU(x) + X with C > 0. In the context of Nash Bargaining, this principle ensures that the bargaining solution is only determined by the relative shape of the players' preferences rather than the level of utility. In short, the solution should thus be based only on the trade relations between the players and not on the absolute values of their utility functions. This is to ensure that the solution of the players, and thus the division of the negotiated resource, does not change even if the units or starting points chosen by the players themselves change.

The last basic principle of Nash bargaining is that of independence from irrelevant alternatives. [36] This is a basic principle that states that the choice of the ideal solution within the set of alternatives should not be influenced by the introduction of new options that are not selected as the optimal option. Let us give an example to make this concept clearer and more understandable. If an option A is preferred to an option B among a certain set of choices, the introduction of an irrelevant third option C should not change the fact that A is preferred to B and should therefore not have any kind of impact on the final outcome. In the context of Nash Bargaining, this thus implies that the negotiating solution, or the agreement reached, depends exclusively on the relative preferences among the options deemed

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[36] Luce, R. D., & Raiffa, H. (1957). *Games and Decisions: Introduction and Critical Survey*. John Wiley & Sons, Inc.

feasible by the negotiators without being distorted by additional alternatives that do not enhance the utility of any of the participants. [37]

This principle thus ensures that the focus is on choices that are truly relevant to the negotiators and prevents the decision-making process from being influenced by external manipulation and influence. It thus maintains the integrity and consistency of the negotiation solution.

The only drawback, if you like, of this principle is that it is perfect for a theoretical approach and perhaps a little less suitable for a practical approach and thus a negotiation within a real situation. This happens because there is a basic rigidity that takes it for granted that preferences will remain unchanged despite the input of new proposals. In fact, it is assumed that negotiators' preferences remain stable and consistent even when new options are inserted, an assumption that may not be valid in real and complex negotiation situations, where preferences may vary and be influenced by various contextual factors.

The Nash bargaining model considers two parties who have to decide how to divide a cake, which metaphorically represents the resource to be divided. Each participant has a utility function that depends on the share of the pie received. Clearly, the goal of each party is to maximize the product of the incremental utilities with respect to a certain point of disagreement (the result that would be obtained if the negotiation failed).[38]

Having delved into the brilliant thesis of the professor of the political economy department of the University of Modena Francesca Bergamini published in 1991: "Some considerations on the solutions of a bargaining game"[39] we can visualize a Nash bargaining model in a more concrete way by fully understanding how it works. In effect, we understand that analyzing a bargaining situation between two players means hypothesizing that, given initially conflicting positions in relation to a certain

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[37] Roth, A. E. (1979). *Axiomatic Models of Bargaining*. Springer-Verlag.
[38] Codice Edizioni. (2010).*Game theory*. Retrieved from https://www.codiceedizioni.it/files/2010/07/9788875781170.pdf
[39] Bergamini, F. (1991, February). *Alcune considerazioni sulle soluzioni di un gioco bargaining*. Dipartimento di Economia Politica, Via Giardini 454, 41100 Modena, Italy.

decision-making process, they can determine, through a cooperative scheme, solutions that improve their respective utilities.

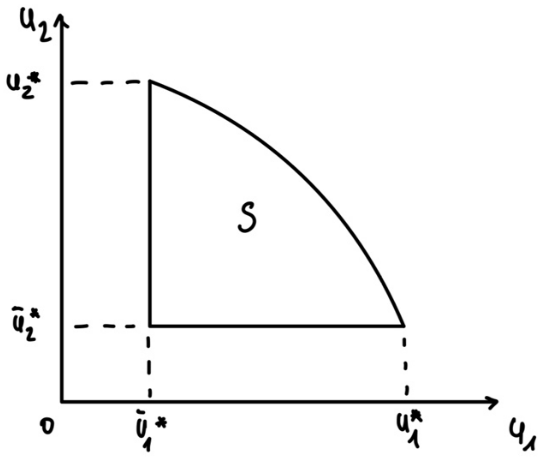
As we know, in any Nash bargaining situation two or more players are faced with two types of solutions, that of conflict, of disagreement, and that of cooperation. Professor Francesca Bergamini shows us a very interesting model.

As usual, sets and functions are defined, and we note that there are sets of conflict, denoted as $\bar{c}$ $\in$ C, where C is a set of possible alternatives; and sets of cooperation: a compact and convex set C of lotteries l ∈ L between certain events Ci ∈ C, where $\ell = ac1 + (1 - a)c2$ with a ∈ [0,1].

As a second element we propose the utility function and the set of payoffs convenient to each player. Each participant, following a rational hypothesis, evaluates the alternatives ℓ in L based on his utility function. The latter are concave and map the set L into a utility space $U1 * U2$ forming a compact convex set S called the bargaining set or payoff space.

The threat point or status quo of the game, i.e. the conflict point c-bar is associated with the point $d = (U1(\bar{c}), U2\ (\bar{c}))$ in the set S.

The solution of the bargaining problem is thus characterized by the pair (S, d). Finding a solution means determining a function that, for each pair in S associates a unique point $P* = (u1*, u2*)$ in S that satisfies certain axioms, such as pareto efficiency and invariance with respect to affine transformations of utilities (see axioms cited above). An application of this model could be the negotiation between an employer and a trade union, where the threat point represents the situation where no agreement is reached (e.g. a strike), while the bargaining set represents various possible outcomes of a negotiation on wages and working conditions. An optimal solution of the Nash Bargaining game in this context would be an agreement that maximizes the utilities of both parties with respect to the threat point, considering their utility functions that may evaluate aspects of the negotiation differently.



Bergamini, F. *Alcune considerazioni sulle soluzioni di un gioco bargaining*. Dipartimento di Economia Politica

The model is applicable in any sector in which an agreement, in which two parties must divide a resource, has to be reached. Despite its applicability to different sectors, it remains a very theoretical model and is often criticized for its complete rationality and imperfect information, which are two elements that may in some situations not fully reflect a party's way of reasoning in the negotiation. The idea of complete rationality is predicated on the idea that people maximize their utility in the best possible way by considering all the information at their disposal. In other words, a perfectly reasonable person should always choose the course of action that will result in the best possible outcome for herself. On the other side, the concept of imperfect information refers to something which contrasts with a situation of perfect information where all parties know all relevant details related to a decision or negotiation, imperfect information refers to scenarios where the parties have incomplete or asymmetric information. This means that one or more parties lack full knowledge of all the factors that could influence the decisions they are making.

In conclusion, Nash Bargaining represents an important contribution to game theory and economic theory, offering a tool for analyzing negotiations rationally and predicting outcomes based on principles of fairness and efficiency. However, its practical applications may be limited by the complexity of real-world situations and the model's assumptions.

Chapter 3: Application of the Nash Bargaining Model to Player Transfers (Analysis of the paper "How could a football player transfer business be more successful?")

We continue our journey to understand how a Nash Bargaining model could simplify a club negotiation for the purchase and sale of a football player's badge with a detailed analysis of the paper on which our discussion is based. "How could a football player transfer business be more successful? A model based on game theory approach' is a paper written by Professors Zaleh Memari, professor of sports management at Alzahra University located in the Iranian capital Tehran, Maryam Esmaeili, professor of industrial engineering at the same university, and Mojzhan Jafari, phd candidate in industrial engineering at Toosi University of Technology, also located in the Iranian capital.[40] Their study leads to the creation of a very precise model, which we will elaborate on in a few moments, based on Nash bargaining, which will allow two key elements to emerge from the various equations: the optimal buy/sell price of player x and the optimal salary that the latter should receive in the club that buys him. It is a model that holds to maximize the utility function of all parties involved in the negotiations so that a solution can emerge that can be perceived as ideal for all. In this model there are two players in the field, the buyer and the seller, and the key elements are three, the possible profit of the clubs, the salary received by the player being sold/buy and the bargaining power of each party in the negotiation.

The prospectus produced by the professors is divided into four clearly delineated parts. It starts with a general introduction on the world of football transfers. A world that is constantly growing, as extensively explained above, and about the factors that can influence purchase prices. In fact, it is explained how transfer fees are not made at random but can be defined through the use of very precise criteria, and, if adapted to a theoretical model, independent variables that can lead to an agreement that maximizes the benefits for all parties.

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[40] Memari, Z., Esmaeili, M., & Jafari, M. (2023). How could a football player transfer business be more successful? A model based on game theory approach. *Sport Business Journal*.

This is why an approach using a game theory model was considered, as it is a tool that can certainly make something normally slow and complex quicker and easier.

Next, we have a 'literature review' part, in which basically all the possible factors that can influence a transfer in the world of football are made explicit, thanks also to the different sources used. Thus, we talk about the race for the best players, the remaining contract years with a club and also the opinions of the football community. These are all influential elements when trying to put a price on player x. Social media, advisors, and outfitters, on the other hand, are the new elements that a few decades ago were not considered in the purchase of a footballer, but which today, as also seen with the example of Cristiano Ronaldo a moment ago, are also decisive, or at least very influential. To these are clearly added all those 'field' aspects such as the player's age, the appearances in recent seasons, and all the statistics and numbers gathered by the professional during his career.

The third part is certainly the one that interests us the most as it speaks specifically about the model in question. The two key elements to start with the analysis are as follows: the model is based on two players, the buyer and the seller, and is only created for outright transfers for players who have an active contract with another club. The latter element therefore excludes temporary transfers, loans, and transfers involving free agents, i.e. players without an active contract. By solving the model, one finds the optimal transfer fee that the purchasing team should pay and that the transferring team should receive for the player's tag.[41]

The system is a classic Nash Bargaining system in which there is a maximization function that corresponds to this:

$$(1)\ MaxZ\ (\pi1, \pi2, \pi3, \dots, \pi m) = (\pi1 - RP1)^{Y1} * (\pi2 - RP2)^{Y2} \dots * (\pi m - RPm)^{Ym}$$

s.t

[41] Memari, Z., Esmaeili, M., & Jafari, M. (2023). How could a football player transfer business be more successful? A model based on game theory approach. *Sport Business Journal*.

$$(2)\ \pi i \in \Omega i \qquad \forall i = 1,2,\dots,m$$

$$(3)\ \pi i \geq \text{RPi} \qquad \forall i = 1,2,\dots,m$$

$$(4)\ 0 \leq Yi \leq 1 \qquad \forall i = 1,2,\dots,m$$

$$(5)\ \Sigma Yi = 1$$

Equation (1) is the pivotal equation on which the whole model is based. We have a club profit maximization function MaxZ based on the profit of each decision maker (each player participating to the game) πi from which is subtracted, the reservation price RPi, in simple terms the disagreement point, the minimum profit that the club is willing to make for that player x before falling back on other player profiles; all this is put to the power Yi, which indicates the bargaining power that decision maker has within the negotiation. All this is multiplied by the exact same function for the number of decision makers present. So, if as in our case there are two parties involved, we will have the function $(\pi 1 - RP1)^{Y1}$ multiplied by the function $(\pi 2 - RP2)^{Y2}$.[42]

In all this, each element has very precise conditions. πi must belong to Ωi which is the feasible region of the decision makers, for all πi; then πi must be greater than or equal to RPi for all elements presents. So, this means that the profit must be greater than or equal to the minimum expected profit. Furthermore, the bargaining power (Yi) of each element must be a number between 0 and 1, and the sum of the Yi must equal one; hence Y1+Y2+...+Ym=1.

We continue the analysis with two integrating equations that help us to insert one more piece of information about the model. In fact, the profit is itself decomposed into several component:

$$(6)\ \pi b = IR - Sb - TF$$

$$(7)\ \pi s = TF + Ss - DR$$

[42] Memari, Z., Esmaeili, M., & Jafari, M. (2023). How could a football player transfer business be more successful? A model based on game theory approach. *Sport Business Journal*.

We are then given the profit functions, where the buyer club's profit πb is composed of IR, which is the increasing buyers' club income caused by a player transfer, minus Sb, which is the received wage by the player in the buyer club, minus TF which represents the transfer fee. Instead, the seller club's profit is composed of the transfer fee (TF) added to Ss, which is the received wage by the player in the seller club, all minus DR which represents the decreasing seller club's income caused by the player transfer.[43]

The following functions are further added:

$$(8)\ increased\ value\ of\ seller\ club = IR - Sb$$

$$(9)\ Lost\ value\ of\ buyer\ club = DR - Ss$$

With IR standing for increasing buyer club income caused by the player transfer.

At this point the key element is to replace equation number (6) and equation number (7) in equations (1) and (3) and the bargaining model changes in this way:

$$(10)\ MaxZ(TF) = [(IR - Sb - TF) - RPb]^{Yb} * [(TF + Ss - DR) - RPs]^{Ys}$$

S.t

$$(11)\ IR - Sb - TF \geq RPb$$

$$(12)\ TF + Ss - DR \geq RPs$$

Where $0 \leq Yb \leq 1$, $0 \leq Ys \leq 1$ and $Yb + Ys = 1$

[43] Memari, Z., Esmaeili, M., & Jafari, M. (2023). How could a football player transfer business be more successful? A model based on game theory approach. *Sport Business Journal*.

In this situation, they tell us that Ys and Yb, which corresponds to the bargaining power of the buyer and the seller, must be a number between 0 and 1, and as we note from the conformation of the equation we have simply replaced the formula of the buyer's profit that of the seller ($\pi s; \pi b$) in the original equation of profit maximization (1) with respect to the transfer market.[44]

At this point, as in a classic Nash Bargaining model, two steps need to be completed: the first is to calculate the derivative of this maximization function with respect to the transfer market (TF) (10), and subsequently equalize it to 0 to bring out the most appropriate purchase value, i.e. the final result; the second operation consists of carrying out the second derivative, again as a function of TF, and studying its concavity. According to the Treccani dictionary[45], a plane figure possesses concavity when it is not convex, that is, when there are at least two points (such as A and B in the figure) such that the segment joining them does not belong entirely to the figure. A figure with such a property is called a concave figure. To complete our definition, we will citate also the "Libretext Mathematics"[46] which states that if the function f is differentiable on an interval I. The graph of f is concave up on I if f' (f'=first derivative of f) is increasing. The graph of f is concave down on I if f' is decreasing. If f' is constant, then the graph of f is said to have no concavity. In fact, it is only if the second derivative turns out to be a concave function that the model can be called a convex optimization model, and thus in fact valid. Having said this, we know that the solution that satisfies the first-order condition will also be the global solution of the "non-convex" model.

We therefore have the first derivative with respect to TF of equation (10):

[44] Memari, Z., Esmaeili, M., & Jafari, M. (2023). How could a football player transfer business be more successful? A model based on game theory approach. *Sport Business Journal*.

[45] Treccani. (n.d.). *Concavità*. Retrieved from https://www.treccani.it/enciclopedia/concavita/

[46] LibreTexts. (n.d.). Concavity and the second derivative. In *Calculus (3rd ed., Vol. 3)*. Retrieved from https://math.libretexts.org/Bookshelves/Calculus/Calculus_3e_(Apex)/03%3A_The_Graphical_Behavior_of_Functions/3.04%3A_Concavity_and_the_Second_Derivative

$$(13)\ \frac{\partial z}{\partial TF} = [-Yb(IR - Sb - TF - RPb)^{Yb-1} * (TF + Ss - DR - RPs)^{Ys}]$$

$$+ [(IR - Sb - TF - RPb)^{Yb} * Ys(TF + Ss - DR - RPs)^{Ys-1}$$

While the second derivative always with respect to TF:

$$(14)\ \frac{\partial^2 z}{\partial TF^2} = Yb(Yb-1)(IR - Sb - TF - RPb)^{Yb-2}(TF + Ss - DR - RPs)^{Ys}$$

$$+ (-Yb)(IR - Sb - TF - RPb)^{Yb-1}Ys(TF + Ss - DR - RPs)^{Ys-1}$$

$$+ (-Yb)(IR - Sb - TF - RPb)^{Yb-1}Ys(TF + Ss - DR - RPs)^{Ys-1}$$

$$+ (IR - Sb - TF - RPb)^{Yb}Ys(Ys-1)(TF + Ss - DR - RPs)^{Ys-2}$$

At this point, we can see from the second derivative (14) that the function is indeed concave, since (-Yb), (Yb-1) and (Ys-1) are negative and that therefore the model is a convex optimization and respects the necessary properties. Next, we equalize the first derivative (13) to zero ($\frac{\partial z}{\partial TF} = 0$) and come up with the following equation:

$$(15)\ \frac{\partial z}{\partial TF} = 0 \rightarrow TF = Ys(IR - Sb - RPb) - Yb(Ss - DR - RPs)$$

The TF obtained and derived from the latter equation is the optimal result of our model. The "TF^{NBS}" (Nash Bargaining) is thus the ideal transfer value that the buying club would have to spend and that the selling club would have to earn in order to affect the transfer of the player in question maximizing the utility of both parties to the game (the two clubs in this case).

Let us interest ourselves for a few moments in the passage that sends us from equation (13) to (15) and then actually explicate the mathematical steps that the paper does not deal with.[47] We begin by recognizing the two elements on which equation (13) is based: $(IR - Sb - TF - RPb)$ and $(TF + Ss - DR - RPs)$. To reach the simplification that will lead us to equation (15), let us name "A" $=(IR - Sb - TF - RPb)$ and "B" $= (TF + Ss - DR - RPs)$.

$$(16)\ 0 = -Yb * A^{Yb-1} * B^{Ys} + A^{Yb} * Ys * B^{Ys-1}$$

We now proceed to factor by A^{Yb-1} *and* B^{Ys-1} and the equation that comes out is as follows:

$$(17)\ \ 0 = A^{Yb-1}B^{Ys-1}\,(-Yb * B + Ys * A)$$

Since it is known to us that A and B are different from 0 as they are composed of the function of each club's profit minus the reservation price or rather the point of disagreement which as per the condition must be less than the profit as indicated by equations (19), (11), and (12) which I reiterate below:

$$(10)\ MaxZ(TF) = [(IR - Sb - TF) - RPb]^{Yb} * [(TF + Ss - DR) - RPs]^{Ys}$$

S.t

$$(11)\ IR - Sb - TF \geq RPb$$

$$(12)\ TF + Ss - DR \geq RPs$$

We can therefore say that $A^{Yb-1}B^{Ys-1} \neq 0$ and thus simply equalise the other part of equation (17) to 0.

[47] Memari, Z., Esmaeili, M., & Jafari, M. (2023). How could a football player transfer business be more successful? A model based on game theory approach. *Sport Business Journal*.

$$(-Yb * B + Ys * A) = 0$$

Replacing them with the original constants we have:

$$-Yb(TF + Ss - DR - RPs) + Ys(IR - Sb - TF - RPb) = 0$$

Collecting TF as a common factor:

$$(18)\ TF(Yb + Ys) = Ys(IR - Sb - RPb) - Yb(Ss - DR - RPs)$$

Which thus corresponds to what the professors summarized in the paper with equation (15).

The paper finally concludes with two equations verifying the properties made explicit above and with the usual conclusions that emphasize the strong theoretical nature of the model. The question we ask ourselves at this point is the following: Will we be able to think about moving from a two-participant Nash bargaining model like this one to one that also includes the player being transferred? This is to make the model even more realistic knowing that the will of the player is increasingly playing a key role in transfers in this century.[48]

[48] Memari, Z., Esmaeili, M., & Jafari, M. (2023). How could a football player transfer business be more successful? A model based on game theory approach. *Sport Business Journal*.

Chapter 4: Extensions of the Model

Let us now move on to an extension of the model that is also capable of including the weight that the player may have in the negotiation of the sale-purchase of his tag between two clubs. To do this, we must then move to a Nash Bargaining model with three participants: the selling club, the buying club and the player, who then also acquires a say and a kind of decision-making power over the negotiation. This will therefore change the profit-maximizing function, which will also have the player's utility function, which we will define shortly. The model, therefore, will come up with not only the ideal transfer fee for which clubs should agree to buy and sell the player, but also the ideal wage, which we will call ideal wage (Wi), that the player should earn at the purchasing club.

We then define Wi as the ideal wage that the player should receive and stipulate that:

$$(19)\ Wi = (W - RPW)^{\gamma p}$$

So, we are saying that the ideal wage (Wi) is composed of the wage (W) minus the Reservation price wage which would be the minimum wage the buying club thinks it can pay the newly acquired player. We therefore consider in our extension of the model that the variable RPW is the last wage received by the player in the club that is selling him, to make things easier. Clearly instead, γp represents the bargaining power of the player in the negotiation.

We now insert this salary function into the TF (transfer value) maximization equation and obtain the following function:

$$(20)\ Z(TF, W) = [(IR - W - TF - RPB)^{\gamma b} * (TF + SS - DR - RPS)^{\gamma s} * (W - RPW)^{\gamma p}]$$

Here, as mentioned $\gamma p, \gamma b, \gamma s$ represent the bargaining powers of the buying, selling and bargaining club. Clearly, this basic equation carries with it rules that must be observed:

$$(21)\ IR - W - TF \geq RPb$$

$$(22)\ TF + SS - DR \geq RPs$$

$$(23)\ W \geq RPw$$

We also establish the properties of the various bargaining powers which tell us that:

$$(24)\ \gamma b + \gamma s + \gamma p = 1$$

And then we can solve the model in this way. Since there are two variables, we want to take out of this maximization equation we solve the equation first for one of the two variables, let's say for TF by setting W as a constant, and then we solve the equation for W by setting TF as a constant. The inclusion of the player as a participant in the Nash bargaining game then implies a further step in the resolution which then leads to the emergence of two outcomes: the ideal transfer fee, which maximizes the utility of the selling club and the buying club, and the ideal salary to be paid to the player by the club buying the player's badge. The resulting salary will also be an ideal salary that can satisfy the player and the purchasing club.

The way to get these two values out is the same as in the initial model: we start by making the first derivative with respect to TF or W depending on the case we are in, put it equal to zero and simplifying and solving the equation we get TF^{nb} or W^{nb}.

We begin by solving the equation by setting W as a constant and then finding the ideal Transfer Fee (TF) as realized in the previous model explained in Chapter 3.

We start with the utility maximization equation for the different players (20):

$$(20)\ Z(TF,W) = [(IR - W - TF - RPB)^{\gamma b} * (TF + SS - DR - RPS)^{\gamma s} * (W - RPW)^{\gamma p}]$$

Clearly considering the properties that follow:

$$(21)\ IR - W - TF \geq RPb$$

$$(22)\ TF + SS - DR \geq RPs$$

$$(23)\ W \geq RPw$$

$$(24)\ \gamma b + \gamma s + \gamma p = 1$$

We now set the part of W as a fixed constant and realize the first derivative of equation (20) with respect to TF:

$$(25)\frac{\partial z}{\partial TF} = [-Yb(IR - Sb - TF - RPb)^{Yb-1} * (TF + Ss - DR - RPs)^{Ys}] + [(IR - Sb - TF - RPb)^{Yb} * Ys(TF + Ss - DR - RPs)^{Ys-1}]$$

We note that the first derivative with respect to TF is the same in the two models (the one in Chapter 3 and this one above) since the player maximization function (19) $Wi = (W - RPW)^{\gamma p}$ is independent of TF in the maximization function (20)

Simplifying, we obtain:

$$(26)\frac{\partial Z}{\partial TF} = (IR - Sb - TF - RPB)^{\gamma b-1} * (TF + Ss - DR - RPS)^{\mathrm{Ys}-1}[-Yb * (TF + Ss - DR - RPs) + Ys * (IR - Sb - TF - RPb)]$$

The second derivative always with respect to TF and considering the other variables as constants is as follows:

$$(27)\frac{\partial TF^2}{\partial Z^2} = -\gamma b \cdot (\gamma b - 1) \cdot (IR - W - TF - RPB)^{\gamma b - 2} + \gamma s \cdot (\gamma s - 1) \cdot (TF + SS - DR - RPS)^{\gamma s - 2}$$

From the second derivative (27) we can see that the function is indeed concave, since (-Yb),(Yb-1) and (Ys-1) are negative, and that the model is therefore a convex optimization and respects the necessary properties. We now equalize the first derivative (26) to 0 in order to find TF^{nb}:

$$\frac{\partial z}{\partial TF} = 0 \rightarrow TF = \frac{Ys * (IR - Sb - RPb) - Yb * (Ss - DR - RPs)}{Yb + Ys}$$

Since $(24)\ \gamma b + \gamma s + \gamma p = 1$ we can say that:

$$(28)\frac{\partial z}{\partial TF} = 0 \rightarrow TF = Ys(IR - Sb - RPb) - Yb(Ss - DR - RPs)$$

Having arrived at this point, we are halfway through our journey as a Nash bargaining variable has been found. Next, we must repeat the operation done by making the derivatives as a function of W and setting all other variables, including TF, as constants. And so, we start again with derivative one, derivative two, and derivative one = 0, all with respect to W to obtain the W^{nb}.

We then begin by realizing the first derivative of Z with respect to W, considering all other variables as constants:

$$(29)\frac{\partial Z}{\partial W} = \gamma b * (IR - W - TF - RPB)^{\gamma b - 1} * (-1) * (TF + SS - DR - RPS)^{\gamma s} * (W - RPW)^{\gamma p} + \gamma p * (IR - W - TF - RPB)^{\gamma b} * (TF + SS - DR - RPS)^{\gamma s} * (W - RPW)^{\gamma p - 1} * (1)$$

At this point, we again derive the function with respect to W, taking all other variables as constants and using the derivation rules for compound functions and powers. We then find the second derivative of the function Z equal to:

$$(30)\ \frac{\partial Z^2}{\partial W^2} = \gamma b * (\gamma b - 1) * (IR - W - TF - RPB)^{\gamma b-2} * (-1) * (TF + SS - DR - RPS)^{\gamma s} * (W - RPW)^{\gamma p} - \gamma p * (IR - W - TF - RPB)^{\gamma b} * (TF + SS - DR - RPS)^{\gamma s} * (W - RPW)^{\gamma p-2} * (1) + \gamma p * (IR - W - TF - RPB)^{\gamma b} * (TF + SS - DR - RPS)^{\gamma s} * (\gamma p - 1) * (W - RPW)^{\gamma p-1} * (1)$$

As explained in the previous model, the second derivative (30) makes us realize that the function is indeed concave, since (-Yb), (Yb-1) and (Ys-1) are negative and that therefore the model is a convex optimization and respects the necessary properties. Next, we equalize the first derivative (29) to zero ($\frac{\partial z}{\partial W} = 0$) and come up with the following equation:

$$(31)\ \frac{\partial Z}{\partial W} = 0 \rightarrow \gamma b * (IR - W - TF - RPB)^{\gamma b-1} * (-1) * (TF + SS - DR - RPS)^{\gamma s} * (W - RPW)^{\gamma p} + \gamma p * (IR - W - TF - RPB)^{\gamma b} * (TF + SS - DR - RPS)^{\gamma s} * (W - RPW)^{\gamma p-1} * (1) = 0$$

$$(32)\ \frac{\gamma b}{\gamma p} = \frac{(W - RPW)^{\gamma p-1}}{IR - W - TF - RPB}$$

$$\rightarrow (IR - W - TF - RPB) * \frac{\gamma b}{\gamma p} = (W - RPW)^{\gamma p-1}$$

$$\rightarrow (IR - W - TF - RPB) = \left(\frac{\gamma p}{\gamma b}\right)^{\frac{1}{\gamma p-1}} * (W - RPW)$$

Finally, solving for W:

$$(33) W = IR - TF - RPB + \left(\frac{\gamma p}{\gamma b}\right)^{\frac{1}{\gamma p - 1}} * (W - RPW)$$

$$(34)\ W - \left(\frac{\gamma p}{\gamma b}\right)^{\frac{1}{\gamma p - 1}} * W = IR - TF - RPB + \left(\frac{\gamma p}{\gamma b}\right)^{\frac{1}{\gamma p - 1}} * RPW$$

$$(35)\ W\left(1 - \left(\frac{\gamma p}{\gamma b}\right)^{\frac{1}{\gamma p - 1}}\right) = IR - TF - RPB + \left(\frac{\gamma p}{\gamma b}\right)^{\frac{1}{\gamma p - 1}} * RPW$$

$$(36)\ W = \frac{IR - TF - RPB + \left(\frac{\gamma p}{\gamma b}\right)^{\frac{1}{\gamma p - 1}} * RPW}{1 - \left(\frac{\gamma p}{\gamma b}\right)^{\frac{1}{\gamma p - 1}}}$$

In doing so we have completed the model, having achieved the first stage, which yields the TF^{nb} and the second, with equation (36), which leads us to find the W^{nb}. We then have the ideal transfer fee for which the two clubs, the selling, and the buying club, should reach an agreement in principle for the transfer of a player X and the ideal salary W that the player should earn at his new club. Both TF and W will maximise the utility of the two clubs and the player under negotiation.

To give more value to our model and to make it more understandable and applied to reality, we give an example with numerical values. The values will be unreal but still roughly based on the figures from some of the latest transfers in the world of football. So, we can say that this numerical example will not be real but will be entirely realistic for the transfer market.

Our example will be based on the hypothetical transfer of Victor Osimhen, Nigerian center forward and Italian champion with SSC Napoli in the 2022/2023 season, from the Campania-based club to Paris-Saint Germain, the glorious French club that has dominated the national league for 12 years now and is trying, at great expense, to establish itself, so far unsuccessfully, at European level by winning the Champions League (a trophy never before lifted by any French club).

Starting from the equation of maximization of the utility functions of Napoli (selling club), PSG (buying club) and Osimhen himself (player under negotiation) we assign hypothetical values to each category:

$$(20)\ Z(TF, W) = [(IR - W - TF - RPB)^{\gamma b} * (TF + SS - DR - RPS)^{\gamma s} * (W - RPW)^{\gamma p}]$$

Let us begin by saying that the powers $\gamma b, \gamma s\ and\ \gamma p$ representing the various bargaining powers at play in the negotiation are assigned in this manner:

$$\gamma b\ (bargaining\ PSG) = 0{,}3$$

$$\gamma s\ (bargaining\ Napoli) = 0{,}4$$

$$\gamma p\ (bargaining\ Osimhen) = 0{,}3$$

This is because the bargaining power of the transferring club is always higher than that of the purchasing club as it owns the player's tag and is therefore bound by a contract. What is certainly impressive is the Yp, bargaining power of the player, which as analyzed with our previous examples, we realized has become an important factor in negotiations. This is because players have acquired and continue to acquire more and more power, media and social, and this makes their will dictate the transfer.

We continue by taking up what was written in the third paragraph:

$$(6)\ \pi b = IR - Sb - TF$$

$$(7)\ \pi s = TF + Ss - DR$$

Here above we have the profit functions, where the buyer club's profit πb is composed of IR, which is the increasing buyers' club income caused by a player transfer, minus Sb, which is the received wage by the player in the buyer club, minus TF which represents the transfer fee. Instead, the seller club's profit is composed of the transfer fee (TF) plus Ss, which is the received wage by the player in the seller club, all minus DR representing the decreasing seller club's income caused by the player transfer.

We assign πb and πs two lump-sum values based on Osimhen's market value according to the famous football transfer currency site “transfermarket” (data expressed in millions of Euros and updated to 20 December 2023) and then we pose:

$$\pi b = 110$$

$$\pi s = 110$$

Now let us stipulate that RPB and RPS, which correspond to the reserving price, the minimum value the two clubs hope to gain/lose by selling/buying Osimhen correspond to:

$$RPB = 90$$

$$RPS = 100$$

The last value to be assigned before entering the data into the Z-maximization function is the RPW, which as previously established corresponds to the salary received by the player in question at his current, i.e. pre-transfer, club. And according to the data provided by “transfermarket”, the Nigerian ace receives as much as EUR 10 million at Napoli with a contract expiring in 2026. Thus:

$$RPW = 10$$

We enter the numerical data in Z:

$$Z(TF, W) = [(110 - TF - 90)^{0,3} * (TF + 110 - 100)^{0,4} * (W - 10)^{0,3}]$$

We start by doing the derivative with respect to TF, taking W as a constant for now, which will then lead us to find the ideal TF for negotiation:

$$\frac{\partial Z}{\partial TF} = -0,3(20 - TF)^{-0,7} * (TF + 20)^{0,4} * (W - 10)^{0,3} + (20 - TF)^{0,3} * 0,4(TF + 20)^{-0,6} * (W - 10)^{0,3}$$

$$\rightarrow \frac{\partial Z}{\partial TF} = -0,3 * (20 - TF)^{-0,7} * (TF + 20)^{0,4} * (W - 10)^{0,3} + 0,4 * (20 - TF)^{0,3} * (TF + 20)^{-0,6} * (W - 10)^{0,3}$$

Equal to 0 we find the ideal transfer fee for the negotiation in question:

$$\frac{\partial Z}{\partial TF} = 0$$

$$\rightarrow -0,3 * (20 - TF)^{-0,7} * (TF + 20)^{0,4} * (W - 10)^{0,3} + 0,4 * (20 - TF)^{0,3} * (TF + 20)^{-0,6} * (W - 10)^{0,3} = 0$$

We can simplify the equation by eliminating the common factor $(W - 10)^{0,3}$, since it is constant and non-zero, and it becomes:

$$\rightarrow -0,3 * (20 - TF)^{-0,7} * (TF + 20)^{0,4} = 0,4 * (20 - TF)^{0,3} * (TF + 20)^{-0,6}$$

$$\rightarrow -0,3 * \left(\frac{1}{20 - TF}\right) = 0,4 * \left(\frac{1}{TF + 20}\right)$$

Simplifying, we obtain that:

$$\rightarrow -03(TF + 20) = 0,4(20 - TF)$$

$$\rightarrow -0,3TF + 0,4TF = 8 + 6$$

$$\rightarrow 0,1TF = 14$$

$$TF^{nb} = 140$$

So, the first part of the system is solved and out comes the TF Nash bargaining which is then the transfer value that maximizes all the utilities of the parties to the bargain, which is then 140 million. The first variable of the two has been found now we do the same procedure to derive the ideal W, thus fixing TF as a constant. We then begin by making the derivative of Z with respect to W:

$$\rightarrow \frac{\partial Z}{\partial W} = (20 - TF)^{0,3} * (TF + 10)^{0,4} * 0,3 * (W - 10)^{-0,7}$$

$$\rightarrow \frac{\partial Z}{\partial W} = 0,3 * (20 - TF)^{0,3} * (TF + 10)^{0,4} * (W - 10)^{-0,7}$$

At this point we should equal the derivative with respect to W to 0, $\frac{\partial Z}{\partial W} = 0$, to find the ideal W. However, we know that a negative power of a difference such as $(W - 10)^{-0,7}$ never becomes zero because as W approaches 10, the expression $(W - 10)^{-0,7}$ tends to infinity, not zero. There is therefore no value for which $(W - 10)^{-0,7} = 0$. The only possibility for a zero in practical terms would be the point at which the expression becomes undefined, which is at the value W=10. We can therefore say that $W^{nb} = 10$.

Conclusion:

In conclusion, we can state that a good combination of game theory and transfers in the world of football emerged from this paper. We have emphasized the importance of transfers in the world of football, due to the amount of money they move and the media interest they receive, and we have been interested in analyzing first and then constructing, two theoretical models that can simplify these negotiations, maximizing the utility of all parties involved. Clearly, these are models, which might struggle if introduced in real life as they do not take into account the myriads of factors that affect a football transfer market but focus on the ones that for us are primary and ubiquitous in negotiations.

We therefore explored the dynamic field of football transfers by adopting an innovative game-theoretic approach to analyzing and optimizing negotiations between clubs. Using a Nash Bargaining model, specifically adapted to the football context, this study offered several effective strategies in the purchase and sale of players' performances. Thus, an attempt was made to provide a comprehensive framework that could significantly improve current practices.

The results highlight how the Nash Bargaining model can serve as an effective tool to balance the interests of all parties during negotiations, maximizing, as mentioned before, the overall welfare, translated into utility, and leading to a more equitable distribution of the value generated by transfers. Moreover, the simulations performed demonstrated the applicability of the model in realistic scenarios suggesting more predictable and satisfactory outcomes for all actors involved.

Despite the promising results, as mentioned before, the study has some limitations, mainly related to the accuracy of the transfer data and the lack of some factors that may somehow influence the negotiations.

Finally, we can state that this study proposes a methodological approach that could be extended and readapted to other sporting fields, offering new perspectives on the improvement of transfer strategies through the use of game theory. We hope that the work done can serve as a launching pad for further research and the development of more sophisticated and fundamentally equitable negotiation strategies in the world of sport.

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