Labor Market Dynamics in the Aftermath of the Covid-19 Pandemic: Evidence from the Euro Area

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Abstract

US post COVID-19 labor market dynamics have been remarkably different from previous recessions, marked by a surge in vacancies and workers' resignations, also known as the Great Resignation. One potential explanation for this phenomenon is an increased value workers started to place on job amenities after the lockdowns, especially Work from Home (WFH), leading a fraction of the employed to quit their current jobs. In contrast, data from the Euro Area suggest a less pronounced labor reallocation. To explore this difference, I utilize the frictional labor market model of Bagga, Mann, Sahin and Violante, 2023, which incorporates three main features: 1) heterogeneous valuation of job amenities among workers, 2) heterogeneous supply of amenities by firms, and 3) sunk cost of entry and vacancy chains. I propose two possible mechanisms consistent with the different dynamics in the Euro Area: a higher cost of quitting to unemployment, given by a lower aggregate matching efficiency, and an initial higher value put on non-wage amenities by European workers.

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1 Introduction

US labor market dynamics in the aftermath of the COVID-19 pandemic have garnered particular attention in academia, particularly due to the stark contrast they show when compared to previous recessions. Indeed, two facts are particularly noteworthy about the US postpandemic experience: a surge in the number of workers voluntarily quitting their jobs (a.k.a the Great Resignation) and a relevant increase in job vacancies. One potential explanation proposed to address these patterns is that, after the initial lockdowns, a fraction of workers started to permanently place more value on the possibility of working from home (Bagga et al., 2023, Chen et al., 2023). Consequently, some workers employed in jobs that did not provide the amenity in question, work from home (WFH), may have started to voluntarily resign from their positions in order to seek jobs that offered WFH options.

While displaying qualitatively similar dynamics, patterns in the Euro Area are instead indicative of a lower degree of reallocation occurring in the labor market; voluntary resignations and vacancies both increased, but to an overall lesser extent than in the US. In contrast, the value that European workers seem to place on working from home after the COVID-19 pandemic seems comparable to the United States, with almost a third of the current employed reporting they would either quit or look for a WFH job if asked to return on the workplace five or more days per week (Aksoy et al., 2022).

To explore what could have led to a different degree of labor reallocation in the Euro Area as opposed to the US, I utilize the frictional labor market model contained in Bagga et al., 2023, which incorporates three main features: 1) workers' heterogeneous valuation of job amenities (in this case, WFH), 2) different levels of amenities offered by firms and 3) sunk cost of entry and vacancy chains. As a consequence of 3), vacancies in this model are not a jump variable as in the standard DMP, where vacancies have value zero and in every period are simply given by the number of entrants. In contrast, an initial sunk cost of entry makes the value of an unfilled vacancy weakly positive, leading separated jobs to actively seek for a replacement without paying any additional cost. This feature is particularly important in addressing the sudden increase in quit-induced vacancies experienced during the so-called Great Resignation, which cannot be accounted for through a standard search-matching model. As in Bagga et al., 2023, the model exhibits a transitional dynamics hit by MIT shocks to productivity (to account for the decrease in output in the months following the outbreak of COVID-19) and value of unemployment (to reflect the increase in government transfers and initial fear of the virus). More importantly, the economy is hit by a once-and-for-all shock to the value that some workers place on job amenities, which in the case in question are thought as the possibility to work from home (WFH).

I propose two mechanisms that, in light of the model, can potentially explain the different labor market dynamics in the Euro Area. The first is a higher cost of voluntarily quitting to unemployment experienced by European workers, given the lower aggregate matching efficiency in the EA labor market as compared to the US labor market.¹ Specifically, a lower matching efficiency implies a longer search period for unemployed workers, which, when combined with the fact that workers that voluntarily quit their jobs are typically ineligible for unemployment benefits, can lead to a less pronounced post-pandemic job resignation trend in Europe.²

A second potential explanation for the reduced degree of labor reallocation in the EA is an ex-ante higher value that European workers may place on other types of non-wage amenities, such as flexible work hours or longer vacations (Alesina et al., 2005). In this context, workers who value these types of amenities could already be allocated to firms that offer them; if these firms are also the ones that offer WFH facilities, a less pronounced labor reallocation ensues.

The paper is organized as follows: Section 2 summarizes the main empirical evidence for the US and compares it to the post pandemic patterns in the Euro area, Section 3 presents the model, Section 4 plots the response of the model to the three aggregate shocks to productivity, flow value of unemployment and workers' value of amenities, Section 5 describes the possible mechanisms that can give rise to a less pronounced labor reallocation in the EA, Section 7 concludes.

¹See for example Jung and Kuhn, 2014 for a comparison between the US and the German labor market. ²Legislation regarding the awarding of unemployment benefits varies among European countries. In some countries, such as Italy or Spain, unemployment benefits generally do not accrue to workers who voluntarily leave their jobs. In other countries, such as Germany, workers who quit voluntarily are usually suspended from receiving unemployment benefits for a period of time.

2 Empirical Evidence

In this section, I summarize the main empirical evidence regarding the different labor market patterns experienced by the US and the Euro Area in the aftermath of the COVID-19 pandemic, compared to previous recessions.

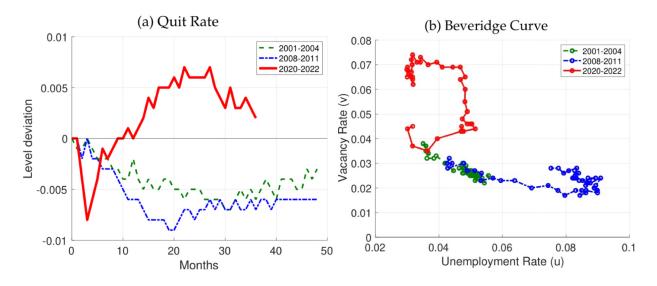


Figure 1: Dynamics of the quit rate (left) and Beveridge Curve (right) in the US. The quit rate is in deviation from its level in January 2001 (green line), January 2008 (blue line) and January 2020 (red line). Figures are sourced form: "Bagga, S., Mann, L., Sahin, A., & Violante, G. L. (2023). Job Amenity Shocks and Labor Reallocation.".

Figure 1, from Bagga et al., 2023, shows the two key features of the US post-pandemic recovery. Unlike previous recoveries, the quit rate increased relative to its pre-pandemic level, giving rise to the so-called *Great Resignation*. Moreover, the Beveridge Curve became almost vertical, losing its usual negative slope, and displaying an unprecedented rise in the vacancy rate. One explanation that has been advanced to explain this phenomenon (Bagga et al., 2023) is an increased value workers started to put on remote work arrangements after the pandemic. According to this story, after trying WFH arrangements during lockdowns, some workers started to value the possibility of working remotely more and began to voluntarily resign from jobs that did not offer the said amenity. In turn, this shift would explain the joint increase in the quit rate and the spike in vacancies, as some jobs are left vacant and become unappealing for a fraction of the population.

Patterns in the Euro Area, shown in Figure 2, seem instead to indicate a less pronounced labor reallocation. Despite an increase in the quit rate after the COVID-19 pandemic, the surge is on average lower than the one experienced in the US. The Beveridge Curve, additionally, maintained its usual negative slope, with an overall lower increase in the vacancy rate. In Appendix A.2, I also report the dynamics for the unemployment rate and the vacancy rate for the US and the Euro Area, again compared to past recoveries.

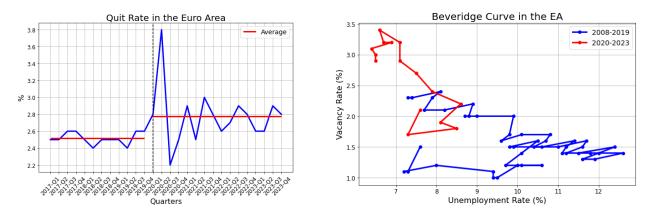


Figure 2: Dynamics for the quit rate (left) and Beveridge Curve (right) in the Euro Area. Euro Area is defined as by 2023. Data used to construct both figures come from Eurostat (LFS and jvs), the quit rate is defined as recent job leavers as a percentage of total employment.

In spite of a lower degree of labor reallocation, European workers after the pandemic appear to value WFH similarly to their US counterparts, as noted by Aksoy et al., 2022. When asked how they would respond to being required to return to the workplace five or more day per week, approximately 30% of European respondents indicated they would rather quit or look for a job offering remote work options, a share that is comparable to the one for the US.

The question that I want address, thus, is whether a shock to the value that workers place on job amenities can explain European patterns as well. More importantly, I aim to investigate whether the same shock to the value for amenities can lead to a less pronounced labor reallocation in the Euro Area, and through which potential mechanisms. To explore these questions, I utilize the frictional labor market model discussed in Bagga et al., 2023, which is described in the next Section.

3 The Model

To explore the difference between the US and EA post pandemic labor market patterns, I adopt the frictional labor market model contained in Bagga et al., 2023, which features three main characteristics: 1) heterogeneity in how much workers value job amenities (WFH), 2) heterogeneity in the amount of amenities offered by firms and 3) sunk cost of entry and vacancy chains. Feature 1) reflects the idea that only a fraction of workers in the economy may value non-wage amenities, while 2) mirrors the fact that some jobs cannot physically be performed remotely. Feature 3), instead, gives rise to the fact that, upon separation, vacant jobs still have some positive value and directly engage in the search for new employees without incurring additional costs.

As in Bagga et al., 2023, the model is written in continuous time and is characterized by a transitional dynamics hit by MIT shocks to: a) productivity (to account for the downturn in output during the first phases of the pandemic), b) value of unemployment (to account for government transfers and for an initial fear of the virus) and c) workers' value for job amenities, which in this context are the availability of work from home (WFH) options.

3.1 Environment

Agents There is a continuum of workers of measure one in the economy with linear utility over consumption and that discount the future at rate r. Workers can be of two types $x \in \{0, \overline{x}\}$, with $\overline{x} > 0$, depending on how much they value the non-wage amenity. Specifically, workers of type 0 are the ones that do not care about job amenities, while workers of type \overline{x} are those that value positively the possibility of working remotely. The fraction of workers of type 0 in the economy is exogenous and denoted by ϕ_0 . Employed workers enjoy each period a wage w_t , while unemployed workers obtain a flow value of unemployment equal to $Z_t^b b$, where Z_t^b is the aggregate shock to the flow value of unemployment.

Firms are heterogeneous in the amount of amenities they offer and are denoted by the two types $a \in \{\underline{a}, \overline{a}\}$, with $\underline{a} < 0 < \overline{a}$. Firms of type \underline{a} are those that do not offer job amenities, while type \overline{a} firms are the ones that offer WFH facilities. As a consequence, the gross flow value of a filled job is given by y + xa. Vacancy Creation and Meetings There is endogenous entry of firms in the market. Opening a vacancy requires to pay a sunk cost of entry $\kappa \left(\frac{i_t}{i}\right)^{\frac{1}{\xi}}$. Once potential entrants pay the sunk cost and enter the market, they are exogenously endowed with the job amenity (i.e., they become firms of type \overline{a}) with probability ζ . As a consequence, the free entry condition is given by:

$$\kappa \left(\frac{i_t}{\overline{i}}\right)^{\frac{1}{\xi}} = \zeta \Omega_t(\overline{a}) + (1-\zeta)\Omega_t(\underline{a}) \tag{1}$$

Where i_t and \overline{i} are the per-period and steady state value of entrants, respectively, and $\Omega_t(a)$, as will be clearer in the next sections, is the value of an unfilled vacancy of type $a \in \{\underline{a}, \overline{a}\}$. ξ , instead, measures the elasticity of entry to the expected value of opening a vacancy. Notice that equation (1) implies a number of per-period entrants along the transition equal to:

$$i_t = \overline{i} \left(\frac{\zeta \Omega_t(\overline{a}) + (1 - \zeta) \Omega_t(\underline{a})}{\kappa} \right)^{\xi}$$
(2)

Consistently with Bagga et al., 2023, the model features on-the-job search. The search effort of unemployed workers is set equal to 1, while only a fraction $s \in [0, 1]$ of employed workers actively seeks for a new job opportunity. The total number of job seekers at time t is thus given by:

$$\mathbf{s}_t = \sum_x u_t(x) + s \cdot \sum_{x,a} e_t(x,a) \tag{3}$$

And the total number of vacancies in the economy is given by:

$$\mathbf{v}_t = \sum_a v_t(a) \tag{4}$$

Job seekers and vacancies meet through the CRS meeting function $\mathbf{m}_t = m(\mathbf{v}_t, \mathbf{s}_t)$, which I assume to be Cobb-Douglas:

$$\mathbf{m}_t = A \mathbf{v}_t^{\alpha} \mathbf{s}_t^{1-\alpha} \tag{5}$$

From (5), it is possible to denote $p_t = \frac{\mathbf{m}_t}{\mathbf{s}_t}$ as the meeting rate for unemployed workers, sp_t as the meeting rate for employed workers and $q_t = \frac{\mathbf{m}_t}{\mathbf{v}_t}$ as the meeting rate for vacancies. Notice that in the case in which both types of workers form matches with both types of firms, p_t and q_t will also correspond to the job finding and the job filling rates, respectively.

Production Technology I assume, differently from Bagga et al., 2023, that all matches produce a fixed level of output y. Worker-firm matches are exogenously destroyed at rate δ per instant of time and can also be endogenously dissolved when an aggregate shock suddenly changes the joint surplus of a match.

Upon separation, separated workers enter unemployment, while separated jobs directly reenter the vacancy pool and start looking for a replacement. Unfilled vacancies, instead, exit the market exogenously at rate δ_v .

Wage Setting As in Bagga et al., 2023 and Lise and Robin, 2017, I assume the sequential auction protocol from Postel–Vinay and Robin, 2002. When hiring from unemployment, the firm makes a Take-or-Leave offer to the worker, thus initially taking all the joint surplus of the match. As the worker becomes employed and receives new offers, the two competing firms engage in Bertrand competition in order to take the worker, leading to a quit or to a renegotiation. Additionally, the wage can be renegotiated under mutual consent when an aggregate shock hits the joint surplus of an existing match, in order to avoid one-sided quits or layoffs.

Aggregate Shocks The model follows a deterministic transitional dynamics hit by unforeseen MIT shocks to: 1) productivity (denoted by Z_t^y), 2) value of unemployment (denoted by Z_t^b) and 3) the value that workers place on non wage amenities (denoted by Z_t^x). While the first two shocks are temporary, reflecting the short-term decline in output and the rise in governmental subsidies during the early stages of the COVID-19 pandemic, the third shock is considered permanent, mirroring a sustained increase in the value workers started placing on remote work following the outbreak of the pandemic.

3.2 Value Functions

Joint Surplus The gross joint surplus of a match between a worker of type x and a firm of type a at instant t is defined as:

$$\max\{S_t(x,a),\Omega_t(a)\}$$

Where:

$$S_t(x,a) = J_t(w_t, x, a) + E_t(w_t, x, a) - U_t(x)$$
(6)

Which is independent of the wage and can be written as:³

$$(r+\delta)S_t(x,a) = Z_t^y y + Z_t^x xa - Z_t^b b + \delta\Omega_t(a) + \dot{S}_t$$
(7)

Value of unfilled and filled vacancy The value of an unfilled vacancy of type *a* is given by:

$$(r+\delta_{v})\Omega_{t}(a) = q_{t} \sum_{x} \left[\mathbb{I}_{\{x\in\mathcal{H}_{t}(a)\}} (J_{t}(w_{t}^{u}(x,a),x,a) - \Omega_{t}(a)) \left(\frac{u_{t}(x)}{\mathbf{s}_{t}}\right) + \sum_{a'} \mathbb{I}_{\{(x,a')\in\mathcal{P}_{t}(a)\}} (J_{t}(w_{t}^{q}(x,a,a'),x,a) - \Omega_{t}(a)) \left(\frac{s\cdot e_{t}(x,a')}{\mathbf{s}_{t}}\right) \right] + \dot{\Omega}_{t}(a)$$
(8)

Where $\mathcal{H}_t(a)$ corresponds to the hiring set, i.e. the set of worker types x that can be hired from unemployment by a firm of type a:

$$\mathcal{H}_t(a) = \{ x : S_t(x, a) > \Omega_t(a) \}$$
(9)

 $\mathcal{P}_t(a)$ corresponds instead to the poaching set, i.e. the set of worker types x employed in firms of type a' that can be poached by a firm of type a:

$$\mathcal{P}_t(a) = \{ (x, a') : S_t(x, a) - \Omega_t(a) > S_t(x, a') - \Omega_t(a') \}$$
(10)

³For the derivation, see Appendix A.3.

The value of a match for a firm of type a matched with a worker of type x that is paid a wage w_t is given by:

$$(r+\delta+sp_t)J_t(w_t,x,a) = Z_t^y y - w_t + \delta\Omega_t(a)$$

$$+ sp_t \left[\sum_{a' \in \mathcal{Q}_t(x,a)} \Omega_t(a) \left(\frac{v_t(a')}{\mathsf{v}_t} \right) \right]$$

$$+ \sum_{a' \in \mathcal{R}_t(x,a)} J_t\left(w_t^r(x,a,a'), x, a \right) \left(\frac{v_t(a')}{\mathsf{v}_t} \right)$$

$$+ \sum_{a' \in \mathcal{N}_t(x,a)} J_t(w_t, x, a) \left(\frac{v_t(a')}{\mathsf{v}_t} \right) \right] + \dot{J}_t$$

$$(11)$$

Where $Q_t(x, a)$, the quit set, is the set of offers from firms of type a' that lead the worker of type x to quit her current job of type a:

$$Q_t(x,a) = \{a': S_t(x,a') - \Omega_t(a') > S_t(x,a) - \Omega_t(a)\}$$
(12)

 $\mathcal{R}_t(x, a)$, the retention set, is the set of offers from firms of type a' that lead to a wage renegotiation between the type x worker and the type a firm:

$$\mathcal{R}_t(x,a) = \{a': E_t(w_t, x, a) - U_t(x) < S_t(x, a') - \Omega_t(a') \le S_t(x, a) - \Omega_t(a)\}$$
(13)

 $\mathcal{N}_t(x, a)$, finally, corresponds to the neutral set, i.e. the set of offers from firms of type a' that do not lead neither to a quit nor to a renegotiation:

$$\mathcal{N}_t(x,a) = \{a': S_t(x,a) - \Omega_t(a) \ge E_t(w_t, x, a) - U_t(x) \ge S_t(x,a') - \Omega_t(a')\}$$
(14)

Value of Unemployed and Employed worker The value of an unemployed worker of type *x* is:

$$(r+p_t)U_t(x) = Z_t^b b + p_t \sum_a \left[\mathbb{I}_{\{x \in \mathcal{H}_t(a)\}} E_t(w_t^u, x, a) + \mathbb{I}_{\{z \notin \mathcal{H}_t(a)\}} U_t(x) \right] \left(\frac{v_t(a)}{\mathbf{v}_t} \right) + \dot{U}_t \quad (15)$$

And given that, when hiring from unemployment, the firm initially takes the whole joint surplus, it is the case that $E_t(w_t^u(x, a), x, a) = U_t(x)$ allowing to rewrite (15) as:

$$rU_t(x) = Z_t^b b + \dot{U}_t \tag{16}$$

Which does not depend on the workers' type x.

The value of a worker of type x employed in a firm of type a that is payed a wage w_t is instead:

$$(r+\delta+sp_t)E_t(w_t,x,a) = w_t + Z_t^x xa + \delta U_t(x)$$

$$+ sp_t \left[\sum_{a' \in \mathcal{Q}_t(x,a)} E_t \left(w_t^q(x,a,a'), x, a' \right) \left(\frac{v_t(a')}{v_t} \right) \right.$$

$$+ \sum_{a' \in \mathcal{R}_t(x,a)} E_t \left(w_t^r(x,a,a'), x, a \right) \left(\frac{v_t(a')}{v_t} \right)$$

$$+ \sum_{a' \in \mathcal{N}_t(x,a)} E_t(w_t, x, a) \left(\frac{v_t(a')}{v_t} \right) \right] + \dot{E}_t$$

$$(17)$$

3.3 Labor Market Flows

The law of motion for the unemployed workers of type x in this model is given by:

$$du_{t}(x) = \delta \sum_{a} \mathbb{I}_{\{S_{t}(x,a) \ge \Omega_{t}(a)\}} e_{t}(x,a) dt + \sum_{a} \mathbb{I}_{\{S_{t}(x,a) < \Omega_{t}(a)\}} e_{t}(x,a)$$

$$\underbrace{a}_{\text{exogenous separations}} \underbrace{e_{\text{endogenous separations}}}_{endogenous separations} \underbrace{a}_{endogenous separations} \underbrace{birgs \text{from unemployment}}$$

$$(18)$$

hires from unemployment

For workers of type x employed in jobs of type a:

$$de_{t}(x,a) = -\underbrace{\delta \mathbb{I}_{\{S_{t}(x,a) \ge \Omega_{t}(a)\}} e_{t}(x,a) dt}_{\text{exogenous separations}} - \underbrace{\mathbb{I}_{\{S_{t}(x,a) < \Omega_{t}(a)\}} e_{t}(x,a)}_{\text{endogenous separations}}$$

$$- \underbrace{sp_{t}e_{t}(x,a) \left[\sum_{a'} \mathbb{I}_{\{a' \in \mathcal{Q}_{t}(x,a)\}} \left(\frac{v_{t}(a')}{\mathsf{v}_{t}} \right) \right] dt}_{\text{flow to another job}}$$

$$+ \underbrace{sp_{t} \sum_{a'} e_{t}(x,a') \mathbb{I}_{\{(x,a') \in \mathcal{P}_{t}(a)\}} \left(\frac{v_{t}(a)}{\mathsf{v}_{t}} \right) dt}_{\text{flow from another job}} + \underbrace{p_{t}u_{t}(x) \mathbb{I}_{\{x \in \mathcal{H}_{t}(a)\}} \left(\frac{v_{t}(a)}{\mathsf{v}_{t}} \right) dt}_{\text{hires from unemployment}}$$

$$(19)$$

For vacancies of type a:

$$dv_{t}(a) = -\underbrace{\delta_{v}v_{t}(a)dt}_{\text{vacancy destruction}} + \underbrace{i_{t}(a)dt}_{\text{vacancy creation}}$$
(20)
$$-q_{t}v_{t}(a)\sum_{x} \left[\underbrace{\mathbb{I}_{x\in\mathcal{H}_{t}(a)}\left(\frac{u_{t}(a)}{\mathbf{s}_{t}}\right)}_{\text{vacancies filled from u}} + \underbrace{\sum_{a'}\mathbb{I}_{\{(x,a')\in\mathcal{P}_{t}(a)\}}\left(\frac{s\cdot e_{t}(x,a')}{\mathbf{s}_{t}}\right)}_{\text{vacancies filled from e}} \right] dt$$
$$+ \sum_{x} \left[\underbrace{\delta\mathbb{I}_{\{S_{t}(x,a)\geq\Omega(a)\}}e_{t}(x,a)dt}_{\text{exogenous separations}} + \underbrace{\mathbb{I}_{\{S_{t}(x,a)<\Omega_{t}(a)\}}e_{t}(x,a)}_{\text{endogenous separations}} \right]$$
$$+ sp_{t}e_{t}(x,a)\sum_{a'}\mathbb{I}_{\{a'\in\mathcal{Q}_{t}(x,a)\}}\left(\frac{v_{t}(a')}{\mathbf{v}_{t}}\right)dt}{\operatorname{poached jobs}}$$

Where, from the environment described above, it is clear that $i_t(\underline{a}) = (1-\zeta)i_t$ and $i_t(\overline{a}) = \zeta i_t$.

3.4 Wage Protocol

Following Bagga et al., 2023 and Lise and Robin, 2017, the wage earned by a worker of type x employed in a firm of type a is the result of the sequential auction protocol of Postel–Vinay and Robin, 2002. Specifically:

1. Upon hiring from unemployment (i.e. if $S_t(x,a) > \Omega_t(a)$), a firm of type *a* makes a Take-or-Leave offer $w_t^u(x,a)$ to the worker of type *x* in order to make the worker indifferent between employment and unemployment:

$$E_t(w_t^u(x,a), x, a) = U_t(x)$$
 (21)

Thus, initially the firm takes the whole joint surplus of the match and $J_t(w_t^u(x, a), x, a) = S_t(x, a)$.

2. If a worker of type x, employed in a firm of type a, meets a firm of type a' offering a net joint surplus $S_t(x, a') - \Omega_t(a') > S_t(x, a) - \Omega_t(a)$, the worker leaves her current job to join the new firm, taking the entire net surplus from her previous employment relationship. This means that the worker will receive from the new firm a wage $w_t^q(x, a, a')$ such that:

$$E_t(w_t^q(x, a, a'), x, a') - U_t(x) = S_t(x, a) - \Omega_t(a)$$
(22)

As a consequence, the new firm obtains a value equal to:

$$J_t(w_t^q(x, a, a'), x, a') = S_t(x, a') - [S_t(x, a) - \Omega_t(a)]$$
(23)

3. If a worker of type x, employed in a firm of type a, meets a firm of type a' offering a net joint surplus $E_t(w_t, x, a) - U_t(x) < S_t(x, a') - \Omega_t(a') \le S_t(x, a) - \Omega_t(a)$, the worker will use the outside offer in order to renegotiate her wage with the current employer. Thus, the worker will receive from the current firm a wage $w_t^r(x, a, a')$ such that:

$$E_t(w_t^r(x, a, a'), x, a) - U_t(x) = S_t(x, a') - \Omega_t(a')$$
(24)

And the current firm moves to a value equal to:

$$J_t(w_t^r(x, a, a'), x, a) = S_t(x, a) - [S_t(x, a') - \Omega_t(a')]$$
(25)

4. If a worker of type x, employed in a firm of type a, meets a firm of type a' offering a net joint surplus $S_t(x, a') - \Omega_t(a') \leq E_t(w_t, x, a) - U_t(x) \leq S_t(x, a) - \Omega_t(a)$, the worker cannot gain anything from the outside offer. The worker stays with her current employer and the wage remains unchanged.

5. Given the presence of MIT shocks, an aggregate shock can lead to renegotiation during the transition. Following Bagga et al., 2023, if at any point in time to a match of type (x, a) with current wage w_t the following occurs:

$$E_t(w_t, x, a) - U_t(x) < 0$$
 but $S_t(x, a) - \Omega_t(a) \ge 0$ (26)

The wage is raised to $w_t^+(x, a)$ in order to avoid quitting, giving:

$$E_t(w_t^+, x, a) = U_t(x)$$
 and $J_t(w_t^+, x, a) = S_t(x, a)$ (27)

6. If instead an aggregate shock hits a match of type (x, a) with current wage w_t such that the opposite occurs:

$$J_t(w_t, x, a) < \Omega_t(a) \quad \text{but} \quad S_t(x, a) \ge \Omega_t(a)$$
 (28)

The wage is lowered to $w_t^-(x, a)$ in order to avoid a layoff, giving:

$$J_t(w_t^{-}(x,a), x, a) = \Omega_t(a) \quad \text{and} \quad E_t(w_t^{-}(x,a), x, a) - U_t(x) = S_t(x,a) - \Omega_t(a) \quad (29)$$

3.5 Equilibrium

From the definition in Bagga et al., 2023, given initial distributions $u_0(x)$, $e_0(x, a)$, $v_0(a)$ and paths for the aggregate shocks $\{Z_t^y, Z_t^b, Z_t^x\}_{t\geq 0}$, an equilibrium of this economy is:

- 1. A list of value functions $\{S_t(x,a), \Omega_t(a), J_t(w_t, x, a), U_t(x), E_t(w_t, x, a)\}$ that satisfy equations (6), (8), (11), (15) and (17),
- 2. Hiring and poaching sets $\{\mathcal{H}_t(a), \mathcal{P}_t(a)\}$ and quit, retention and neutral sets $\{\mathcal{Q}_t(x, a), \mathcal{R}_t(x, a), \mathcal{N}_t(x, a)\}$ that satisfy equations (9), (10), (12), (13) and (14),

- 3. Distributions $\{u_t(x), e_t(x, a), v_t(a)\}$ that satisfy equations (18), (19), (20) and implied meetings $\mathbf{m}_t = m(\mathbf{v}_t, \mathbf{s}_t)$,
- 4. Hiring, poaching and retention wage functions $\{w_t^u(x, a), w_t^q(x, a, a'), w_t^r(x, a, a')\}$ and boundary wage functions $\{w_t^+(x, a), w_t^-(x, a)\}$ that satisfy (21), (22), (24), (27) and (29),
- 5. A measure of entrants i_t that satisfies the free entry condition (1).

3.6 Parametrization

Parameter		Value	Target to match	Target value
Parameters Set Externally				
Aggregate match efficiency	А	1		
Output produced by each match	y	1		
Elasticity of entry	ξ	15		
Elasticity of the meeting function	α	0.5	Bagga et al. (2023)	
Discount rate	r	0.05/12	Bagga et al. (2023)	
Share of \overline{a} vacancies	ζ	0.25	Bagga et al. (2023)	
Share of type 0 workers	ϕ_0	0.5	Bagga et al. (2023)	
Search effort of employed	s	0.58	Bagga et al. (2023)	
Steady state sunk entry cost	κ	2.08	Bagga et al. (2023)	
Parameters Set Internally				
Level of the amenity offered	$\underline{a}, \overline{a}$	-0.408, 0.592	Mean Amenity	0
Utility Flow from amenity	\overline{x}	0.0461	Compensating differential	0.05
Opportunity cost of work	b	0.943	UE rate	0.3
Separation rate	δ	0.015	EU rate	0.015
Vacancy destruction rate	δ_v	0.368	Share of vacated vacancies	0.5

 Table 1: Parameters and Targets

Table 1 presents the parametrization of the model, which is calibrated to the US economy and follows closely the one contained in Bagga et al., 2023. I normalize both aggregate search efficiency A and the deterministic level of output y produced by each match to 1, and I set a high value for the elasticity of entry ξ equal to 15. The elasticity of the meeting function α is set to 0.5 (consistent with Bagga et al., 2023 and Petrongolo and Pissarides, 2001)and the discount rate r is set to 5% annually. The share of vacancies endowed with the amenity is set to 0.25 in order to produce a share of approximately 37% of teleworkable jobs in the economy (Bagga et al., 2023, Dingel and Neiman, 2020), and the share of workers who positively value the job amenity is set to 0.5 (Bagga et al., 2023, Barrero et al., 2021, Barrero et al., 2023). The search effort of employed workers and the steady state sunk cost of entry, instead, are taken directly from Bagga et al., 2023 and set equal to 0.58 and 2.08, respectively.

Given the slight differences between the model presented here and the one presented in Bagga et al., 2023, I recalibrate parameters in the second block in the steady state in order to match their targets for the US.⁴ I thus set the level of the non-wage amenity offered by the firms $a \in \{\underline{a}, \overline{a}\}$ to target a mean amenity value in the economy equal to 0 and normalize their distance to 1. \overline{x} is set in order to target an initial compensating differential (defined, according to Bagga et al., 2023, as the average of $\frac{\overline{x}(\overline{a}-\underline{a})}{w}$ among the type \overline{x} workers), while b is set to target an UE rate of 0.3. The value of δ is chosen in order to produce a EU rate of 0.015 and δ_v is set to target a share of vacated vacancies (defined as the ratio of vacancies created through a separation or a quit over the per-period inflow of vacancies) to 0.5.⁵

3.7 Steady State Equilibrium Regimes

Given the parametrization presented in the last subsection, there are two equilibrium regimes in pure strategies that can potentially arise in a steady state for different values of \overline{x} and \overline{a} , depending on the sign of the net joint surplus.⁶

⁴Especially for the level of output y produced by each match which I assume deterministic, contrary to Bagga et al., 2023.

⁵Specifically, in this context the per period inflow of vacancies is composed by: (1) newly created vacancies and (2) vacancies created through a quit or an exogenous separation. The share of vacated vacancies is thus defined, consistently with Qiu, 2022, as $\frac{(2)}{(1)+(2)}$.

⁶I let both \overline{x} and \overline{a} vary in the interval (0,1). Moreover, I set $\underline{a} = -\left(\frac{0.408}{0.592}\right) \cdot \overline{a}$ in order to approximately produce a mean amenity value equal to zero in the economy.

⁷For a more detailed exposition about the different equilibrium regimes that can arise in the steady state, see Appendix A.4.

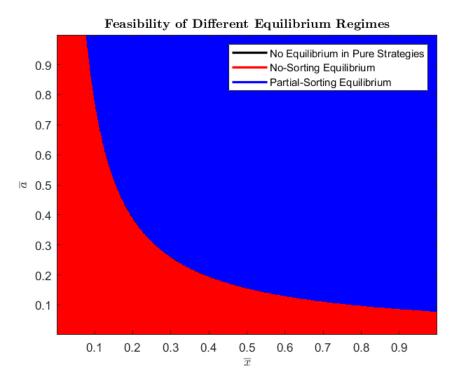


Figure 3: Steady state equilibrium regimes for different values of \overline{x} and \overline{a} . Using the proportion implied by the calibration in Section 3.6, the value of \underline{a} is set to $-\left(\frac{0.408}{0.592}\right) \cdot \overline{a}$ in order to approximately produce a mean amenity value equal to zero.

The two equilibrium regimes, displayed in Figure 3, are denoted as:

- 1. No-Sorting Equilibrium (red area in Figure 3), that arises when the product $\overline{x} \cdot a$ is small in absolute value and in which all the net joint surpluses are positive. Thus, in this equilibrium regime both types of workers $x \in \{0, \overline{x}\}$ form matches with both types of firms $a \in \{\underline{a}, \overline{a}\}$.
- 2. Partial-Sorting Equilibrium (blue area in Figure 3), that arises when the product $\overline{x} \cdot a$ is larger in absolute value and in which the net joint surplus between a worker of type \overline{x} and a firm of type \underline{a} is negative, all others being positive. Thus, in this equilibrium regime workers that value amenities (type \overline{x}) do not form matches with firms that do not provide them (type \underline{a}), as they would obtain a considerable disutility from doing so.

Since the model features on-the-job search, it is worth noticing that in both equilibrium configurations it is also always the case that employed workers that value the amenities (type \overline{x}) tend to move to high amenity jobs (\overline{a}) and that workers that do not care about the amenity (type 0) tend to move to low amenity jobs (\underline{a}).

The red and the blue areas displayed in Figure 3 do not intersect, ruling out multiplicity of equilibria for the same values of the parameters. However, at the boundary between the two areas (when the type \underline{a} firm is indifferent between forming or not forming a match with a type \overline{x} worker), possible equilibria in mixed strategies can emerge, with the match occurring with some probability between 0 and 1. Nonetheless, the region where these mixed-strategy equilibria arise is negligible, so they are not depicted in Figure 3.

Figure 3 thus clearly shows how a shock to \overline{x} can transition the economy from the No-Sorting Equilibrium to the Partial-Sorting Equilibrium, with this transition resulting in labor reallocation as some workers begin to value amenities more highly and opt not to form matches with firms not offering them.

4 Impulse Response Functions

In this Section, I plot the response of the model to each of the MIT shocks $(Z_t^y, Z_t^b \text{ and } Z_t^x)$ described in Section 3. All shocks occur at time 0 and follow the paths outlined in Table 2.

Shock	Path
Z_t^y	$1 - 0.055 \cdot e^{-0.057t}$
Z_t^b	$1 + 0.375 \cdot e^{-0.23t}$
Z_t^x	7.8

Table 2: Shocks and relative paths for $t \ge 0$

Both the size and the duration of the shocks to productivity (Z_t^y) and to the flow value of unemployment (Z_t^b) are based on the calibration of Bagga et al., 2023. Specifically, the shock to productivity is set in order to produce a drop in output of about 5% on impact and takes about 8 years to dissipate completely; the shock to the flow value of unemployment is instead set to trigger an immediate increase of roughly 37% and dissipates much faster, within about 2 years. In contrast, the shock to the value of amenities (Z_t^x) is permanent and set residually in order to generate an increase in tightness that, at its maximum, is higher of about 0.5 in log deviations from its pre-shock level.

Below, I present the response of the model to each of the shocks outlined above, which deliver dynamics that are qualitatively similar to the ones reported in Bagga et al., 2023.

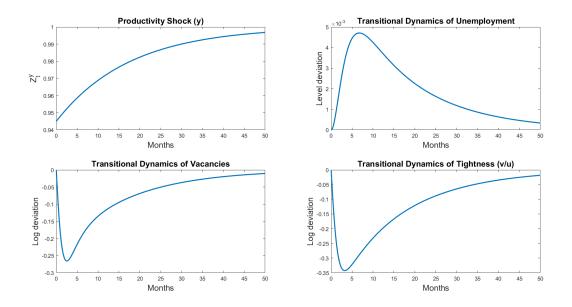


Figure 4: Impulse response functions of the model to a productivity shock (Z_t^y) .

Figure 4 illustrates the model's response to a negative productivity shock (Z_t^y) . As the output produced by each match declines, the number of entrants diminishes due to a reduced incentive to post vacancies. Consequently, this leads to a temporary rise in unemployment as the meeting rate for unemployed workers falls (tightness decreases), and which goes back to its original steady state level as the shock dissipates.⁸

⁸Notice that in Figure 4 I report tightness denoted as vacancies over unemployed workers. In the model, instead, tightness is denoted as $\frac{v}{s}$, where s is the number of total job seekers.

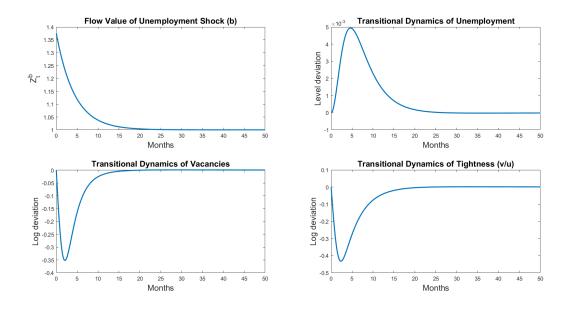


Figure 5: Impulse response functions of the model to a shock to the value of unemployment (Z_t^b) .

Figure 5 displays the response of the model to a shock to the flow value of unemployment (Z_t^b) . In this case, the incentives for workers to remain unemployed become higher, thus leading to an overall increase in unemployment. In turn, this decreases the willingness of potential entrants to post vacancies, leading to a temporary decrease in the number of vacancies in the economy.

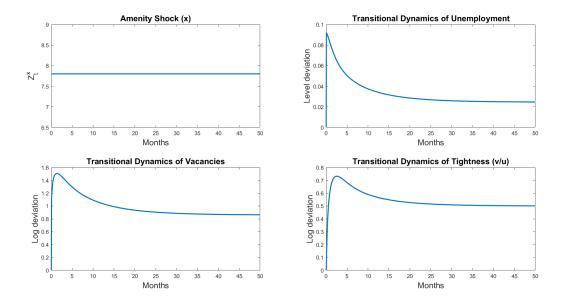


Figure 6: Impulse response functions of the model to a once-and-for-all amenity shock (Z_t^x) .

Finally, Figure 6 shows the response of the model to a once-and-for-all shock to the value of amenities. As some workers suddenly start to value amenities more, those employed in firms that are not endowed with them are immediately mismatched, giving rise on impact to a surge in unemployment. This mass of workers quitting also causes a spike in vacancies, which move to a new and higher steady state as some jobs (those that not offer amenities) are less desirable for a fraction of the population.

5 Labor Reallocation in the US and in the Euro Area: Potential Mechanisms

Before analyzing the potential mechanisms that, according to the model, could result in less pronounced labor reallocation in the Euro Area, I present the model's response to the combined impact of all three shocks discussed in the previous section $(Z_t^y, Z_t^b \text{ and } Z_t^x)$ using the parametrization for the US presented in Section 3.6. As it is visually clear from Figure 7, the once-and-for-all shock to the value of the amenity moves the economy from the No-Sorting Equilibrium regime (the one in which both type of workers form matches with both types of firms) to the Partial-Sorting Equilibrium configuration (the one in which workers that value amenities do not form matches with firms not endowed with them), thus giving rise to labor reallocation. Figure 8 illustrates the response of the model to all the three shocks outlined in Section 4.

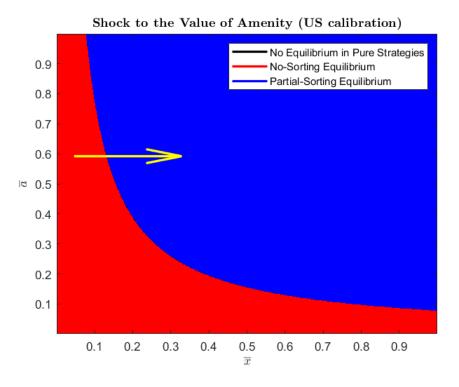


Figure 7: The impact of a permanent shock to the value of amenities (\overline{x}) on the resulting steady state equilibrium regime, US calibration.

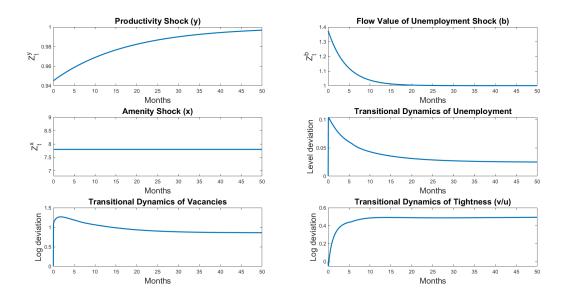


Figure 8: Impulse response functions of the model to: 1) a temporary negative shock to productivity (Z_t^y) , 2) a temporary positive shock to the flow value of unemployment (Z_t^b) and 3) a once-and-for-all shock to the value of amenities (Z_t^x) .

All the shocks contribute to a spike on impact in the unemployment rate, as the option value

of unemployment is temporarily higher and the fraction of workers that values amenities quits from low amenity firms. Initially, also workers that do not care about amenities (type 0) are mismatched from high amenity firms (type \bar{a}), as the option value of searching again becomes temporarily higher for these firms. The increase in endogenous separations leads in turn to a jump in vacancies, which move to a higher value in the new steady state as the economy transitions towards the Partial-Sorting equilibrium regime.

Having depicted what the model implies given the US parameterization from Section 3.6, I will now move to analyze two possible mechanisms that may have potentially led to a less pronounced labor reallocation in the Euro Area: 1) a higher cost of quitting to unemployment and 2) an initial higher value that European workers place on job amenities.

5.1 Higher cost of quitting to unemployment in the Euro Area

One possible mechanism that, in light of the model, could lead to the documented lower degree of labor reallocation experienced in the Euro Area is an higher cost for European workers of quitting to unemployment. In fact, the labor market in the EA is more frictional, and potentially characterized by a lower aggregate matching efficiency than in the US (Jung and Kuhn, 2014, for example, show this in the case of Germany). This increased difficulty in finding a new job would in turn give rise to phenomena such as long term unemployment, thereby increasing the overall cost of voluntarily becoming unemployed (Miyamoto and Suphaphiphat, 2020). Moreover, policies that exclude workers who voluntarily resign from receiving unemployment benefits, or impose a suspension period, as in Germany, can further exacerbate the situation.

In the model, I incorporate this higher cost of quitting to unemployment by setting a lower aggregate matching efficiency A joint with a reduced flow value of unemployment b, capturing both the increased economic and psychological costs associated with voluntary resignation.⁹ Figure 9 shows how the same shock to the value of amenities can induce a lower labor reallocation in presence of lower b and A. With a higher cost of quitting to unemployment, all the

⁹I set the value of b = 0.835 in order to produce an increase in tightness ($\frac{v}{u}$ ratio) that reaches a peak of approximately 0.5 in log deviations from its pre-shock level. A is set to 0.5 in order to produce a higher level of unemployment in the initial steady state, consistent with the evidence from the Euro Area.

net joint surpluses increase as the option value for workers is now lower; as a consequence, a shock to the value that type \overline{x} workers (those that value amenities) place on job amenities does not translate in a negative joint surplus with type \underline{a} firms, preventing the economy from reaching the Partial-Sorting Equilibrium.

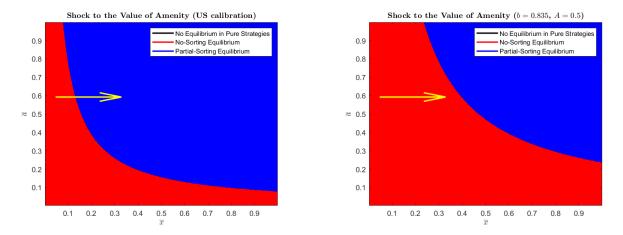


Figure 9: The impact of a once-and-for-all shock to \overline{x} on the resulting steady state equilibrium regime, US calibration (left) and case with lower b and A (right).

Below, Figure 10 also displays the response of the counterfactual with lower b and A to the three shocks defined in Section 4. On impact, both unemployment and vacancies increase because quitting for unemployment becomes temporarily preferable due to the shocks to productivity and to the option value of unemployment. As these shocks dissipate, however, workers who value amenities continue to form matches with firms that are not endowed with them, resulting in a less pronounced labor reallocation.

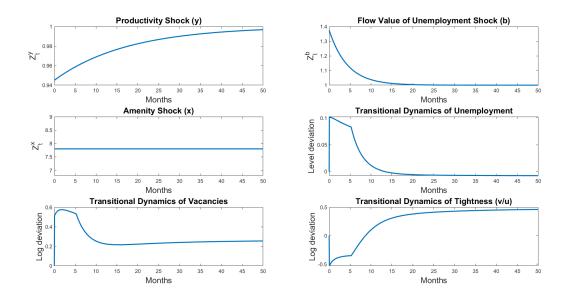


Figure 10: Impulse response functions of the counterfactual with lower b and A to: 1) a temporary negative shock to productivity (Z_t^y) , 2) a temporary positive shock to the flow value of unemployment (Z_t^b) and 3) a once-and-for-all shock to the value of amenities (Z_t^x) .

5.2 Ex-ante higher value for amenities in the Euro Area

Besides an higher cost of quitting to unemployment, another mechanism that could have led to a less pronounced labor reallocation in the Euro Area is an initial higher value placed by European workers on non-wage amenities. It is well-documented that European workers, on average, work fewer hours and enjoy longer vacations compared to their American counterparts (Alesina et al., 2005, Bick et al., 2019). While this could be in part due to higher European tax rates (Prescott, 2004), Alesina et al., 2005 argues that this divergence may have originated from a difference in institutions during the 1970s, leading European workers to obtain more utility from time off and flexibility as more people began taking longer vacations.

Thus, European workers, even before the COVID-19 outbreak, may have placed a higher value on job amenities other than remote work, such as flexible work hours or longer vacations. As a result, workers who valued these amenities might have already been employed by firms offering such flexibility. If the firms offering WFH were also those providing these other types of flexibility, less labor reallocation may have occurred in the Euro Area simply because these workers were likely already employed by such firms. In the model, I incorporate this possible mechanism through an higher initial \overline{x} , reflecting the ex-ante higher value placed by European workers on non-wage amenities, and a lower A, to account again for the higher unemployment rate in the Euro Area as compared to the US.¹⁰ Figure 11 illustrates this concept: in the counterfactual scenario with a higher initial \overline{x} , the economy may already begin in the Partial-Sorting Equilibrium. Consequently, a shock to the value of amenities would result in a less pronounced labor reallocation.

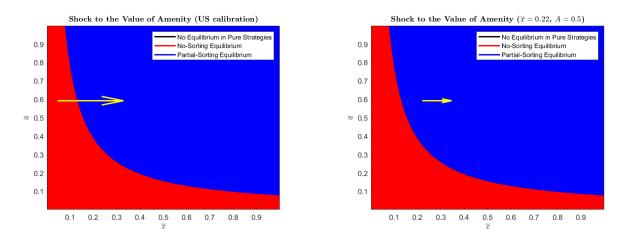


Figure 11: The impact of a once-and-for-all shock to \overline{x} on the resulting steady state equilibrium regime, US calibration (left) and case with higher initial \overline{x} and lower A (right).

An initial higher value placed by European workers on job amenities produces qualitatively similar dynamics as those outlined by a higher cost of quitting to unemployment, as shown in Figure 12. On impact, unemployment increases principally due to the shocks to productivity and to the flow value of unemployment, which induce a fraction of workers (the workers of type 0 employed in firms of type \overline{a}) to endogenously separate. This effect is however only transitory, and unemployment rapidly moves to a new, lower, steady state level.

¹⁰I set, as in the previous section with $b, \bar{x} = 0.22$ in order to produce an increase in tightness ($\frac{v}{u}$ ratio) that reaches a peak of approximately 0.5 in log deviations from its pre-shock level.

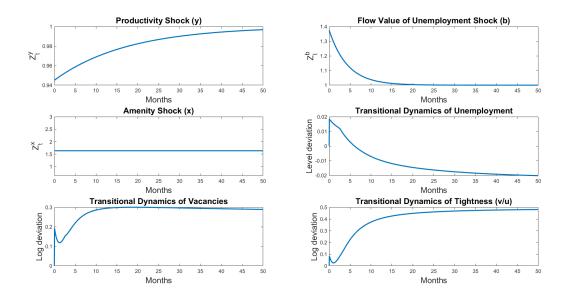


Figure 12: Impulse response functions of the counterfactual with higher initial \overline{x} and lower A to: 1) a temporary negative shock to productivity (Z_t^y) , 2) a temporary positive shock to the flow value of unemployment (Z_t^b) and 3) a once-and-for-all shock to the value of amenities (Z_t^x) .

Vacancies also increase initially, reaching a relatively higher steady-state value, though with a significantly smaller increase compared to the scenario depicted from the US calibration. Overall, labor market tightness increases but, as in the preceding subsection, a reduced labor reallocation occurs.

6 Conclusion

The recent US post-pandemic recovery has been notably different from previous ones mainly due to two key factors: 1) a spike in the number of workers voluntarily resigning from their jobs and 2) a vertical shift in the Beveridge Curve, driven by a surge in vacancies. One explanation proposed to account for these trends (Bagga et al., 2023) is that during the lockdowns workers were forced to work remotely, leading some to permanently value more job amenities, in particular work-from-home (WFH). Consequently, workers experiencing this shift in their value for amenities moved away from non-teleworkable jobs, contributing to the so-called Great Resignation and resulting in a rise of quit-induced vacancies, many of which now harder to fill given their declined desirability.

Labor market dynamics in the Euro Area display similar patterns, but are indicative of a less

pronounced labor reallocation. In order to investigate the main differences that could have led to a reduced labor reallocation in response to the same shock to the value of amenities, I adopted the model of Bagga et al., 2023, which incorporates three main features: 1) heterogeneous valuation of job amenities among workers, 2) heterogeneous supply amenities by firms and 3) sunk cost of entry and vacancy chains.

Analyzing the response of the model to a negative transitory shock to productivity, a positive transitory shock to the value of unemployment and, crucially, a once-and-for-all shock to the value of amenities (specifically, WFH), provides two potential mechanisms that could explain the reduced labor reallocation observed in the Euro Area. The first is a higher cost for European workers to voluntarily resign to unemployment: as quitting becomes more costly, workers that value the amenity still prefer to form matches with firms not offering remote work options. A second explanation is an ex-ante higher value that European workers place on other types of job amenities, such as flexible work hours and longer vacations. If firms offering these types of flexibility are on average the same as those offering WFH facilities, a reduced labor reallocation occurs because workers who value amenities are already employed in firms that provide them. In conclusion, these two explanations, as highlighted by the model, could shed light on the differing patterns between the Euro Area and the US, contributing to a deeper understanding of the mechanisms influencing the recent postpandemic labor market dynamics.

References

- Aksoy, C. G., Barrero, J. M., Bloom, N., Davis, S. J., Dolls, M., & Zarate, P. (2022). Working from home around the world (tech. rep.). National Bureau of Economic Research.
- Alesina, A., Glaeser, E., & Sacerdote, B. (2005). Work and leisure in the united states and europe: Why so different? NBER macroeconomics annual, 20, 1–64.
- Bagga, S., Mann, L., Sahin, A., & Violante, G. L. (2023). Job amenity shocks and labor reallocation.
- Barrero, J. M., Bloom, N., & Davis, S. J. (2021). Why working from home will stick (tech. rep.). National Bureau of Economic Research.
- Barrero, J. M., Bloom, N., & Davis, S. J. (2023). Long social distancing. Journal of Labor Economics, 41(S1), S129–S172.
- Bick, A., Brüggemann, B., & Fuchs-Schündeln, N. (2019). Hours worked in europe and the united states: New data, new answers. The Scandinavian Journal of Economics, 121(4), 1381–1416.
- Chen, Y., Cortes, P., Kosar, G., Pan, J., & Zafar, B. (2023). The impact of covid-19 on workers' expectations and preferences for remote work. AEA Papers and Proceedings, 113, 556–561.
- Dingel, J. I., & Neiman, B. (2020). How many jobs can be done at home? Journal of public economics, 189, 104235.
- Jung, P., & Kuhn, M. (2014). Labour market institutions and worker flows: Comparing germany and the us. *The Economic Journal*, 124(581), 1317–1342.
- Lise, J., & Robin, J.-M. (2017). The macrodynamics of sorting between workers and firms. American Economic Review, 107(4), 1104–1135.
- Miyamoto, H., & Suphaphiphat, N. (2020). Mitigating long-term unemployment in europe. IZA Journal of Labor Policy, 11(1).
- Petrongolo, B., & Pissarides, C. A. (2001). Looking into the black box: A survey of the matching function. Journal of Economic literature, 39(2), 390–431.
- Postel–Vinay, F., & Robin, J.-.-M. (2002). Equilibrium wage dispersion with worker and employer heterogeneity. *Econometrica*, 70(6), 2295–2350.

Prescott, E. C. (2004). Why do americans work so much more than europeans?Qiu, X. (2022). Vacant jobs. University of Pennsylvania.

A Appendix

A.1 Note on Variable Definitions

Data used to produce the dynamics of the quit rate and unemployment rate for the Euro Area come from Eurostat labour force survey (LFS). Specifically, following Eurostat, I use the following definitions throughout the paper:

- Quit rate: recent job leavers as a percentage of total employment. According to Eurostat, recent job leavers are those persons who report to have left their job in the 3 moths preceding the interview. Dataset downloaded from Eurostat Database on 26/05/2024 17:45:30 CEST (name file: lfsi_lea_q) at quarterly frequency, seasonally adjusted.
- Unemployment rate: number of unemployed workers as a percentage of the sum of employed and unemployed workers. Dataset downloaded from Eurostat Database on 26/05/2024 12:46:32 CEST (name file: lfsq_urgan) at quarterly frequency, unadjusted data.

Data used to produce the dynamics of the vacancy rate for the Euro Area come from Eurostat job vacancy statistics (jvs). Data used to plot the dynamics of tightness and the Beveridge Curve come jointly from Eurostat LFS and jvs. Following Eurostat, I use the following definitions:

- Vacancy rate: number of job vacancies (NACE rev. 2 sections B-S) as a percentage of the sum of occupied posts and the number of job vacancies. Dataset downloaded from Eurostat Database on 26/05/2024 13:32:11 CEST (name file: jvs_q_nace2) at quarterly frequency, unadjusted data.
- Beveridge Curve: plot of vacancy rate against unemployment rate.
- Labor Market Tightness: number of vacant jobs over number of unemployed workers. Number of vacant jobs for the Euro Area are obtained from the vacancy rate using the total number of employed (dataset name: lfsq_egan, downloaded on 26/05/2024 19:10:54 CEST) as a proxy for occupied job posts, at a quarterly frequency and unadjusted.

According to the new Framework regulation on Social Statistics (IESS), some definitions for the main indicators in Eurostat LFS have been updated starting from the first quarter of 2021. More details about this change in the definitions can be found on Eurostat website.¹¹

A.2 Dynamics of Unemployment and Vacancy Rate in the US and in the Euro Area

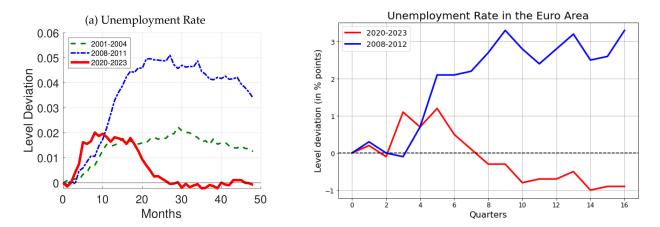


Figure 13: Dynamics for the unemployment rate in the US (left) and in the Euro Area (right). Euro Area is defined as by 2023. Patterns for the US are in deviation from their level in January 2001 (green line), January 2008 (blue line) and January 2020 (red line). Patterns in the EA are in deviation form their level in the fourth quarter of 2007 (blue line) and 2019 (red line), respectively.

Figure on the left is sourced from: "Bagga, S., Mann, L., Sahin, A., & Violante, G. L. (2023). Job Amenity Shocks and Labor Reallocation.". Data used to construct the figure on the right come from Eurostat (LFS).

 $[\]label{eq:linear} {}^{11} https://ec.europa.eu/eurostat/statistics-explained/index.php?title=EU_Labour_Force_Survey_-_new_methodology_from_2021_onwards\#Main_changes_introduced_in_2021$

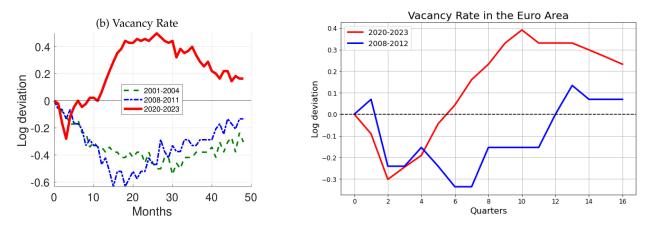


Figure 14: Dynamics for the vacancy rate in the US (left) and in the Euro Area (right). Euro Area is defined as by 2023. Patterns for the US are in deviation from their level in January 2001 (green line), January 2008 (blue line) and January 2020 (red line). Patterns in the EA are in deviation form their level in the fourth quarter of 2007 (blue line) and 2019 (red line), respectively.

Figure on the left is sourced from: "Bagga, S., Mann, L., Sahin, A., & Violante, G. L. (2023). Job Amenity Shocks and Labor Reallocation.". Data used to construct the figure on the right come from Eurostat (jvs).

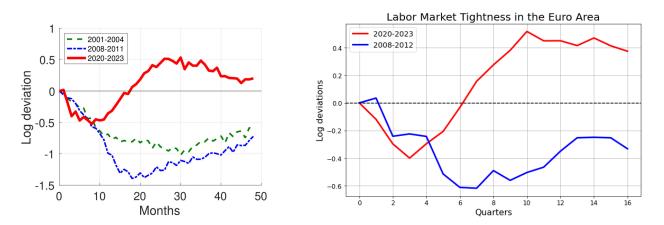


Figure 15: Dynamics for tightness in the US (left) and in the Euro Area (right). Euro Area is defined as by 2023. Patterns for the US are in deviation from their level in January 2001 (green line), January 2008 (blue line) and January 2020 (red line). Patterns in the EA are in deviation form their level in the fourth quarter of 2007 (blue line) and 2019 (red line), respectively.

Figure on the left is sourced from: "Bagga, S., Mann, L., Sahin, A., & Violante, G. L. (2023). Job Amenity Shocks and Labor Reallocation.". Data used to construct the figure on the right come from Eurostat (LFS and jvs).

A.3 Gross Joint Surplus: Derivation

Recall the gross joint surplus definition from (6) as $S_t(x, a) = J_t(w_t, x, a) + E_t(w_t, x, a) - U_t(x)$. For now, I initially guess that the joint gross surplus does not depend on the wage. Adding (11) and (17) one obtains:

$$(r + \delta + sp_t) \left(J_t(w_t, x, a) + E_t(w_t, x, a) \right) = Z_t^y y + \delta\Omega_t(a) + Z_t^x xa + \delta U_t(x)$$
(30)
+ $sp_t \left[\sum_{a' \in \mathcal{Q}_t(x, a)} \left(\Omega_t(a) + E_t(w_t^q(x, a, a'), x, a')) \left(\frac{v_t(a')}{v_t} \right) \right]$
+ $\sum_{a' \in \mathcal{R}_t(x, a)} \left(J_t(w_t^r(x, a, a'), x, a) + E_t(w_t^r(x, a, a'), x, a)) \left(\frac{v_t(a')}{v_t} \right) \right]$
+ $\sum_{a' \in \mathcal{N}_t(x, a)} \left(J_t(w_t, x, a) + E_t(w_t, x, a)) \left(\frac{v_t(a')}{v_t} \right) \right]$ + $\dot{J}_t + \dot{E}_t$

Notice however that given the wage determination protocol as described in Section 3.4 it follows that, from (22):

$$E_t(w_t^q(x, a, a'), x, a') + \Omega_t(a) = S_t(x, a) + U_t(x)$$

And from the definition of gross joint surplus in (6) it follow that, for any wage w_t :

$$J_t(w_t, x, a) + E_t(w_t, x, a) = S_t(x, a) + U_t(x)$$

Allowing me to rewrite (30) as:

$$(r+\delta+sp_t)\left(J_t(w_t,x,a)+E_t(w_t,x,a)\right) = Z_t^y y + \delta\Omega_t(a) + Z_t^x xa + \delta U_t(x) + sp_t \left[\sum_{a'\in\mathcal{Q}_t(x,a)} \left(S_t(x,a)+U_t(x)\right)\left(\frac{v_t(a')}{v_t}\right)\right] + \sum_{a'\in\mathcal{R}_t(x,a)} \left(S_t(x,a)+U_t(x)\right)\left(\frac{v_t(a')}{v_t}\right) \\+ \sum_{a'\in\mathcal{N}_t(x,a)} \left(S_t(x,a)+U_t(x)\right)\left(\frac{v_t(a')}{v_t}\right)\right] + \dot{J}_t + \dot{E}_t$$

Which becomes:

$$(r + \delta + sp_t) (J_t(w_t, x, a) + E_t(w_t, x, a)) = Z_t^y y + \delta\Omega_t(a) + Z_t^x xa + \delta U_t(x)$$
(31)
+ $sp_t (S_t(x, a) + U_t(x)) + \dot{J}_t + \dot{E}_t$

Now rewrite the value of unemployment in (16) as:

$$(r+\delta+sp_t)U_t(x) = Z_t^b b + \delta U_t(x) + sp_t U_t(x) + \dot{U}_t$$
(32)

Subtracting (32) from (31) and recalling (6) one obtains:

$$(r+\delta)S_t(x,a) = Z_t^y y + Z_t^x xa - Z_t^b b + \delta\Omega_t(a) + \dot{S}_t$$
(33)

Which does not depend on the wage and is identical to (7).

A.4 Steady State Equilibrium Regimes: Derivations

I report in this Section the derivations for the steady state equilibrium regimes presented in Section 3.7. Recall that, in a steady state, a worker of type x forms a match with a firm of type a if $S(x, a) > \Omega(a)$, which leads, using (7), to:

$$\frac{y + xa - b + \delta\Omega(a)}{r + \delta} > \Omega(a) \tag{34}$$

And simplifies to:

$$y + xa - b > r\Omega(a) \tag{35}$$

Since there are two types of workers $x \in \{0, \overline{x}\}$ and two types of firms $a \in \{\underline{a}, \overline{a}\}$, there are four cases to consider:

1. A match between a type 0 worker and a type \underline{a} firm is created if:

$$\frac{y-b}{r} > \Omega(\underline{a}) \tag{36}$$

2. A match between a type 0 worker and a type \overline{a} firm is created if:

$$\frac{y-b}{r} > \Omega(\overline{a}) \tag{37}$$

3. A match between a type \overline{x} worker and a type \underline{a} firm is created if:

$$\frac{y + \overline{x}\underline{a} - b}{r} > \Omega(\underline{a}) \tag{38}$$

4. A match between a type \overline{x} worker and an type \overline{a} firm is created if:

$$\frac{y + \overline{xa} - b}{r} > \Omega(\overline{a}) \tag{39}$$

It is evident, given $\overline{x} > 0$ and $\underline{a} < 0$, that (38) \Rightarrow (36) and (37) \Rightarrow (39). I can therefore state the two following propositions:

Proposition 1 If the type \overline{x} worker is in the hiring set of the type \underline{a} firm, then also the type 0 worker is in the hiring set of the \underline{a} type firm.

Proposition 2 If the type 0 worker is in the hiring set of the type \overline{a} firm, then also the type \overline{x} worker is in the hiring set of the type \overline{a} firm.

In this case where y > b, notice that there exists no equilibrium in which one of the two types of firm $a \in \{\underline{a}, \overline{a}\}$ is unable to hire any type of worker. This can be easily proved:

Proof. Assume that a firm of type $a \in {\underline{a}, \overline{a}}$ does not hire any type of worker. Then, from equation (36) (or (37)) it follows that when the firm of type *a* meets a worker of type 0 the following inequality must be satisfied:

$$\frac{y-b}{r} \le \Omega(a) \tag{40}$$

However, given that the firm of type a is unable hire any type of worker, from (8) it follows that $\Omega(a) = 0$, which contradicts (40).

Thus, there are four possible equilibrium regimes that can arise in a steady state:

- One in which both types of workers are in the hiring set of both types of firms (No-Sorting Equilibrium).
- 2. One in which type \overline{x} workers are in the hiring set of type \overline{a} firms only and type 0 workers are in the hiring set of type \underline{a} firms only.
- 3. One in which type \overline{x} workers are in the hiring set of both types of firms and type 0 workers are in the hiring set of type \underline{a} firms only.
- 4. One in which type \overline{x} workers are in the hiring set of type \overline{a} firms only and type 0 workers are in the hiring set of both types of firms (**Partial-Sorting Equilibrium**).

It is also possible to show that in all the four possible equilibrium regimes it is always the case that:

$$S(0,\underline{a}) - \Omega(\underline{a}) > S(0,\overline{a}) - \Omega(\overline{a})$$

and
$$S(\overline{x},\overline{a}) - \Omega(\overline{a}) > S(\overline{x},\underline{a}) - \Omega(\underline{a})$$

To show this, first notice that it cannot be that:

$$S(0, \underline{a}) - \Omega(\underline{a}) \le S(0, \overline{a}) - \Omega(\overline{a})$$

and
$$S(\overline{x}, \overline{a}) - \Omega(\overline{a}) \le S(\overline{x}, \underline{a}) - \Omega(\underline{a})$$

Proof. Using (7):

$$S(0,\underline{a}) - \Omega(\underline{a}) \le S(0,\overline{a}) - \Omega(\overline{a}) \Longrightarrow \Omega(\overline{a}) \le \Omega(\underline{a})$$

and

$$S(\overline{x},\overline{a}) - \Omega(\overline{a}) \le S(\overline{x},\underline{a}) - \Omega(\underline{a}) \Longrightarrow \Omega(\overline{a}) - \Omega(\underline{a}) \ge \frac{\overline{xa} - \overline{xa}}{r} > 0 \Longrightarrow \Omega(\overline{a}) > \Omega(\underline{a})$$

Leading to a contradiction. \blacksquare

Notice that also the following two cases can be discarded:

$$S(0,\underline{a}) - \Omega(\underline{a}) > S(0,\overline{a}) - \Omega(\overline{a}) \quad \text{and} \quad S(\overline{x},\overline{a}) - \Omega(\overline{a}) \le S(\overline{x},\underline{a}) - \Omega(\underline{a})$$
$$S(0,\underline{a}) - \Omega(\underline{a}) \le S(0,\overline{a}) - \Omega(\overline{a}) \quad \text{and} \quad S(\overline{x},\overline{a}) - \Omega(\overline{a}) > S(\overline{x},\underline{a}) - \Omega(\underline{a})$$

As they would imply, in every equilibrium regime, a contradiction with either the hiring sets or with the Bellman equation for a vacancy as defined by (8).

A.5 Solution Algorithm

In order to solve for the steady state equilibrium regimes as depicted in Figure 7, I guess and verify each of the four possible equilibrium regimes outlined in Section A.4 for different values of \overline{x} and a. I thus solve for all the equilibrium objects by finding each time the meeting rate for firms q that satisfies the free entry condition in (1).

The solution algorithm to solve for the transitional dynamics, instead, is based on the procedure reported in Bagga et al., 2023:

- 1. Set a period T at which the model is thought to converge to the new steady state.
- 2. Guess a path for the value of a vacancy $\{\Omega_t(a)\}_{t\geq 0}$, setting $\Omega_T(a)$ equal its final steady state value.

Then, at each iteration n:

- 3. Solve for $\{S_t(x,a)\}_{t\geq 0}^n$ using (7).
- 4. Solve for the path of entrants, $\{i_t\}_{t\geq 0}^n$, using (2).
- 5. Iterate forward on the distribution of unemployed workers, employed workers, and vacancies using (18), (19) and (20), starting from their initial steady state level.
- 6. Iterating backwards, update $\{\Omega_t(a)\}_{t\geq 0}$ according to (8) and get $\{\Omega_t(a)\}_{t\geq 0}^{n+1}$.
- 7. Stop when the distance between $\{\Omega_t(a)\}_{t\geq 0}^{n+1}$ and $\{\Omega_t(a)\}_{t\geq 0}^n$ is small enough.