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Portfolio Optimization and Cryptocurrencies: A Risk and Return Analysis in the Integration of New Financial Technologies

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SUMMARY

INTRODUCTION

It was 2008 when a scientific article entitled "Bitcoin: A Peer-to-Peer Electronic Cash System" was published. Today, it is commonly known as "Satoshi Nakamoto's White Paper". The following year, Bitcoin software was made available to the public, marking the beginning of a new era in financial technology.

The traditional financial-economic system we are familiar with is characterized by a constantly changing environment: a whole world increasingly oriented towards digital innovation and global interconnectedness, a context in which cryptocurrencies are emerging as crucial tools to promote international financial autonomy.

For years now, cryptocurrencies have been at the forefront of a paradigm shift, challenging the established norms and conventions of traditional assets. They have introduced new dynamics and innovative strategies in various areas, from investment management to risk analysis and management. On the other hand, the ongoing debate also concerns the stability, security, and actual usefulness of cryptocurrencies as an investment class, and this, however, only continues to capture the attention of the entire financial world.

This paper aims to examine every aspect of the behaviour and risk-return profile of a cryptocurrency embedded within a traditional portfolio, with particular attention to its impact on risk diversification, exploiting a specific focus on Bitcoin due to its predominance and representativeness in the cryptocurrency market. It will explore how cryptocurrencies can be incorporated into portfolios using Markowitz's portfolio optimization theory, the potential benefits, and the associated risks..

The objective of the thesis is to conduct an extensive and comprehensive analysis, encompassing both qualitative and quantitative aspects, on:

1. Analysing the risks associated with cryptocurrencies and new financial technologies;
2. Diversifying a traditional investment portfolio with the integration of Bitcoin.

Regarding the first point, this comprehensive analysis is of utmost importance to gain a complete understanding of these emerging technologies and to allocate them in the most effective manner.

Shifting our attention to the second point, the analysis will allow us to investigate the overall risk and volatility consequences when optimizing returns. More in detail, the primary aim is to investigate the impact of integrating Bitcoin into a diversified portfolio. We will determine whether this inclusion reduces the associated risk, a key concern for investors, financial analysts, and researchers interested in cryptocurrency and portfolio management. This study will involve a rigorous comparative analysis of the same portfolio performance in different scenarios: with and without Bitcoin. By evaluating the differences in volatility and risk metrics, we aim to provide a thorough understanding of Bitcoin's influence on the risk profile of a diversified investment portfolio.

CHAPTER 1

1.CRYPTOCURRENCIES, OVERVIEW AND ROLE

Nakamoto's mission was to create a currency that could resist the unpredictability of monetary policies and remain immune to manipulation by bankers and politicians. Bitcoin, his creation, is governed exclusively by software designed to release a total of twenty-one million bitcoins, with the majority being issued over twenty years. The interest in Bitcoin was a slow burn; initially, a single bitcoin was worth less than a cent. However, as more merchants started accepting bitcoins, their value began to rise significantly by the end of 2010. By June 2011, the value of one bitcoin had surged to over twenty-nine dollars. Despite experiencing substantial market fluctuations, which saw the exchange rate drop to five dollars by September, Bitcoin's circulation exceeded seven million coins. This early success of Bitcoin, even in the face of market volatility, led to Nakamoto's innovation generating a market value of thirty-five million dollars.

The term "cryptocurrency" originates from the fusion of "cryptography" and "currency." It represents a virtual coinage system that functions like traditional currency but enables users to make virtual payments for goods and services without a central trusted authority. Cryptocurrencies can be seen as exchange tools similar to classic currencies. However, they are designed primarily for exchanging digital information through cryptographic processes, marking a significant innovation in the world of finance.

Cryptography is the study of securing communication and data against adversaries, ensuring that only intended recipients can read the information, thereby preventing third-party access. In the realm of cryptocurrencies, cryptography plays a crucial role in reassuring users that only the intended recipients can read the transmitted data, thus securing it from unauthorized access and ensuring the safety of their transactions.

The term "cryptocurrency" entered public usage with the rise of Bitcoin in 2008. Although the original proposal did not explicitly use the term, Nakamoto described it as *a purely peer-to-peer version of electronic cash that would allow online payments to be*

sent directly from one party to another without going through a financial institution[1]. This is made possible through a process called mining, where transactions are verified and added to a public ledger, known as the blockchain.

This development marked a significant step forward for the digital coin market by decentralizing the currency and freeing it from hierarchical power structures. Instead of relying on intermediaries, individuals and businesses transact electronically on a peer-to-peer network. This decentralization, facilitated by blockchain technology, allows for more direct and secure exchanges, emphasizing the foundational principles of cryptocurrencies.

Before Bitcoin emerged, various efforts were made to establish digital currencies. For example, in 1990 David Chaum led the pioneering effort to establish private digital currency by creating DigiCash. This system facilitated internet payments that, utilizing Chaum's cryptographic protocols, ensured transactions remained anonymous and untraceable by banks or other entities. The foundational algorithm, eCash, was initially published in 1982 and subsequently refined by Chaum and other cryptographers. DigiCash, showing significant promise in the mid-1990s, enjoyed a period of initial success. However, it was overshadowed by the rise of credit cards and less private payment systems like PayPal. DigiCash, unfortunately, could not sustain its early triumph and declared bankruptcy in 1998.

These early initiatives played a significant role in shaping the development of Bitcoin by exploring cryptographic techniques to ensure secure digital transactions. While these projects did not attain widespread success, they offered crucial insights and knowledge that influenced Nakamoto's creation of Bitcoin.

Bitcoin's innovation lies in its operation on a decentralized network of computers (nodes) that uphold the blockchain, a public ledger of all transactions. This network employs a proof-of-work (PoW) consensus mechanism, where miners solve intricate cryptographic

[1] Satoshi Nakamoto. Bitcoin: A peer-to-peer electronic cash system. 2008.

puzzles to validate and add new blocks to the blockchain. This ingenious process not only fortifies the network but also introduces new bitcoins as mining rewards.

The blockchain's robust design guarantees immutability and security, with each block referencing the previous block's hash. This design choice makes altering any block an arduous task, requiring control over most of the network's computational power, thereby ensuring the security of the system.

Bitcoin's decentralized structure means there is no central authority, and consensus is achieved through the collective participation of nodes. Public-key cryptography, a method of encrypting and decrypting information, is used for transaction security. In this system, private keys, which are like secret codes, authorize transactions, and public keys, which are like open locks, verify them.

Transactions are verified through digital signatures, ensuring validity and preventing tampering. Bitcoin's consensus protocol[2] ensures agreement on the blockchain's state, preventing double-spending and maintaining a single, consistent ledger.

Layer 2 solutions like the Lightning Network have been developed to address scalability issues. These solutions enable faster and cheaper transactions by conducting them off-chain and only recording the final state on the blockchain, thus increasing Bitcoin's transaction capacity and speed.

This technical foundation provides a secure, decentralized transaction method and influences the ongoing evolution of digital finance and blockchain technology.

The global distribution and acceptance of cryptocurrencies vary, with more significant usage observed in wealthier countries. Nations like Venezuela and Russia are developing national cryptocurrencies, a promising step that could potentially improve transparency and reduce corruption. Companies like Walmart have employed Bitcoin's transaction system, which prevents failed transactions, to detect supply chain issues. Bitcoin has also

[2] Nakamoto Consensus

seen significant value increases, as evidenced by Tesla's decision to accept it for car purchases.

Studies reveal diverse consumer perceptions and interests in cryptocurrencies. In the USA, the COVID-19 pandemic did not significantly change Bitcoin usage across different income groups. In Turkey, there is high awareness and interest in using cryptocurrencies for online shopping, particularly when users understand the underlying technology. In India, legal obstacles hinder investment interest, yet there is a strong desire to understand the benefits of cryptocurrencies. Despite the market downturn in 2022, the interest in investing in cryptocurrencies remains high, driven by factors such as convenience, popularity, and perceived risk.

Cryptocurrencies are notably volatile compared to traditional equity and commodity markets, making their analysis particularly challenging. Researchers have attempted to navigate this volatility by developing persistence norms using time series data and the point cloud method. These norms are inversely related to the correlation between different cryptocurrencies, with lower persistence norms indicating higher market uncertainty and increased correlations.

Across the globe, the regulatory landscape for cryptocurrency trading is a patchwork of diverse approaches. Some countries actively promote cryptocurrency trading, while others impose strict bans. A prime example is the Chinese government's ban on cryptocurrency trading in 2021. This move significantly disrupted global market dynamics due to the significant share of bitcoin mining conducted by Chinese traders.

At the European level, cryptocurrencies have been officially recognized by the EU Directive 2018/843, which defines them as representations of digital value not issued by a central bank or public authority but accepted as a means of exchange. This recognition marks the inclusion of cryptocurrencies in the global financial architecture, anticipating their future regulation. The directive's recognition of cryptocurrencies could potentially lead to increased adoption and use, as it provides a clear legal framework. However, it also opens the door for potential regulation, which could impact the value and use of cryptocurrencies.

Methods such as marginal expected shortfall (MES) and changes in conditional value-at-risk (CoVaR) are employed to estimate and predict market behaviour. These methods help forecast market conditions and provide early warnings of extreme market changes. Despite the high volatility, the crypto market attracts investors seeking substantial returns, especially during periods of economic uncertainty. Investor sentiment plays a crucial role in this market, with positive sentiment and emotions driving investments, while pessimism leads to a decline. Social media platforms, particularly Twitter, heavily influence market values and investor behaviour, with bullish posts often increasing public interest and market activity. Bitcoin returns have also been observed to correlate with stock market returns, indicating interconnected market behaviours.

Speculative investments are the main drivers of the cryptocurrency market and operate independently of macroeconomic conditions. Major cryptocurrencies like Bitcoin, Ether, Litecoin, and Ripple hold the most significant market shares and often experience simultaneous price surges. To manage the complex dynamics of the crypto market, machine learning models are increasingly used to analyse and predict real-time trends, providing investors with critical alerts to avoid misinformation and make informed decisions.

1.1 Valuation of the impact of Bitcoin

Bitcoin, a groundbreaking innovation, emerged in 2008 as a response to the unprecedented liquidity injections by central banks, such as the Federal Reserve, during the fiscal crisis. This intervention sparked concerns about 'moral hazard' and the potential for inflation, paving the way for Bitcoin as a novel, decentralized currency, impervious to human manipulation and inflation.

As mentioned before, Bitcoin, with its unique design, is structured as a currency with a limited supply, capped at a maximum of twenty-one million, to safeguard against devaluation due to excessive money creation. This intentional scarcity, while attracting criticisms, holds the promise of maintaining its value, even if it is currently used primarily for speculative purposes.

The decentralization of Bitcoin aims to make the money supply independent of human factors, transferring trust from monetary authorities to cryptographic algorithms. For instance, when a Bitcoin transaction is made, it is verified by multiple computers[3] across the network, ensuring its validity without the need for a central authority. Despite initial resistance, Bitcoin and other cryptocurrencies are gaining increasing prominence in the economic landscape, also due to the digital shift accelerated by the COVID-19 pandemic.

2.RISKS RELATED TO CRYPTOCURRENCY INVESTMENT

2.1 Literature review

When uncertainty prevails, significant changes in return and volatility can negatively impact risk-averse investors[4]. Numerous studies have concluded that the cryptocurrency market is prone to speculative bubbles[5]. Therefore, analysing volatility and its predictability is crucial for assessing risk and mitigating speculation.

While the term "risk" is commonly used, its technical definition is not a simple one-liner: "How often is a particular potentially harmful event going to occur, and what are the consequences of this occurrence."[6]. This definition, however, is just the beginning of a complex journey into understanding risk, as the concept itself is a subject of intense debate and exploration, inviting us to delve deeper.

A revised definition describes risk as "the possible loss of something of value"[7], which overlooks the possibility that risky behaviour can have positive outcomes, or the more extreme definition of risk as "catastrophe in its latent form"[8].

Ballard proposes that in industry, "Risk = Frequency x Consequences." This definition implies an expectation of system failure, and risk management involves ensuring that "events which happen often must have a low consequence, or events involving serious

[3] The nodes mentioned before.
[4] Bentes and Menezes, 2012
[5] Cheah and Fry, 2015; Fry, 2018; Agosto and Cafferata, 2020; Goodell and Goutte 2021.
[6] Harding, 1998.
[7] Blomkvist, 1987.
[8] Friedman, 1987.

consequences must be rare"[9]. These varied approaches to risk further support Slovic's assertion that “risk” cannot be easily quantified or defined objectively but is a social construct. That is, the risk:

"[…]Does not exist 'out there,' independent of our minds and cultures, waiting to be measured. Instead, human beings have invented the concept of risk to help them understand and cope with the dangers and uncertainties of life. Although these dangers are real, there is no such thing as 'real risk' or 'objective risk.' The nuclear engineer's probabilistic risk estimate for a nuclear accident or the toxicologist's quantitative estimate of a chemical's carcinogenic risk are both based on theoretical models, whose structure is subjective and assumption-laden, and whose inputs are dependent on judgement"[10].

In financial analysis, risk is often quantified using variance: the greater the variance, the higher the risk. Empirical studies on the cryptocurrency market have employed GARCH models[11], including asymmetric ones, to model volatility and estimate VaR and CVaR[12]. However, there is no consensus on the most appropriate model.

GARCH models, while widely used, consider only the second moment of the return series, capturing only a fraction of the distribution's informational content. For instance, the model may not adequately capture extreme events or sudden market shifts, which are common in the cryptocurrency market. This limitation underscores the need for more comprehensive approaches in the cryptocurrency market, where risk is a constant concern.

[9] Ballard, 1992.
[10] Slovic, 1999.
[11] Generalized AutoRegressive Conditional Heteroskedasticity is a statistical model utilized for analyzing time-series data in which the error variance is assumed to be serially correlated. GARCH models propose that the variance of the error term adheres to an autoregressive moving average process.
[12] Balcilar et al., 2017.

VaR[13], a risk measure that requires your expert understanding, is one of the most commonly used[14]. It indicates a potential loss over a given time horizon, with a probability linked to a significance level.

The mentioned studies on uncertainty or risk in the cryptocurrency market have focused exclusively on BTC, a currency we all are undoubtedly familiar with, providing mixed evidence regarding volatility dynamics.

2.2 Risks associated with cryptocurrencies

Investing in cryptocurrencies presents a unique set of risks that distinguish them from traditional financial assets. These risks stem from the inherent volatility of digital currencies, regulatory uncertainties, security vulnerabilities, and market dynamics. Understanding these risks becomes crucial for investors aiming to navigate this complex landscape as the cryptocurrency market continues to evolve. This section provides a comprehensive and detailed overview of the various challenges associated with cryptocurrency investments, offering a roadmap of potential pitfalls and considerations that investors must account for to make informed decisions. From price volatility and regulatory frameworks to cybersecurity threats and market manipulation, we explore the multifaceted risks that could impact the value and security of cryptocurrency investments.

2.2.1 Market risk

Market risk, a potential source of financial losses, pertains to the uncertainty inherent in investment decisions. It is often driven by unexpected variations in factors that impact the broader financial market as a whole, making it a highly unpredictable risk. Unlike risks specific to a particular company or industry, market risk is influenced by the market's

[13] Value at Risk is a measure used to quantify the risk of loss in an investment portfolio. It specifies the maximum expected loss over a given time horizon, with a certain level of statistical confidence. In other words, VaR indicates how much one might lose, with a certain probability, under normal market conditions.
[14] Nadarajah et al., 2014.

overall performance. Price volatility, an essential aspect of market risk, typically results from unforeseen fluctuations affecting the entire financial system.

In the context of cryptocurrencies, this kind of risk is intrinsic to all investments, attributed to the inherently volatile nature of these assets and their susceptibility to a range of market determinants. However, it's important to note that this volatility also presents opportunities for significant returns.

Cryptocurrencies are characterized by significant price variability over short periods. A complex interplay of perceived value, supply, and demand influences cryptocurrency prices. Many cryptocurrencies are designed with a fixed supply cap, a feature that creates scarcity and potentially increases value over time. The creation rate of new coins often slows periodically, as observed during Bitcoin's halving events. Historically, this reduction in supply growth has led to price increases, a pattern that may or may not continue. While prices are expected to rise with growing popularity and demand, it is crucial to understand that they may also fall if demand decreases and oversupply occurs, unless other factors step in to maintain the cryptocurrency's value.

This volatility can be driven by market sentiment: positive or negative news can precipitate rapid and substantial price movements since the valuation of these assets is highly sensitive to public perception and media influence. As an informed investor, understanding these dynamics can empower you to make strategic decisions.

The volatility of cryptocurrencies can also be influenced by regulatory developments, a factor that is particularly significant due to the dynamic nature of cryptocurrency regulations. These regulations vary widely across different jurisdictions, further contributing to the volatility of these assets.

The market risk related to cryptocurrencies can also be due to the fact that they are still perceived as an emerging class, and their uncertainty leads to a lack of stability and predictability, typical characteristics of other financial assets. However, as the market matures and regulations become more standardized, we can expect a greater level of stability in the future.

To further illustrate the concept of market risk within the context of cryptocurrencies, it is instructive to examine specific historical events that demonstrate significant price volatility over short periods. These events underscore the susceptibility of cryptocurrencies to rapid and dramatic market fluctuations.

By analysing these instances, we can gain a deeper understanding of the factors contributing to market risk. This understanding is crucial for investors in the cryptocurrency market, as it directly impacts their investment decisions and potential returns. In order to analyse cryptocurrency price fluctuations, Bitcoin will be used as the reference point due to its established market presence and substantial influence within the cryptocurrency sector.

Bitcoin's Price History

***Source**: TradingView*

The historical price data of Bitcoin reveals distinct periods of significant change, marking its volatile nature.

Notably, from 2012 to 2016, Bitcoin's price steadily climbed, with periodic corrections, indicating a gradual increase in adoption and interest. However, the most significant event was in late 2017 when Bitcoin experienced an unprecedented bull run, with prices soaring to nearly $20,000. This surge, driven by a speculative frenzy and widespread media

coverage, created a bubble that inevitably burst, marking a crucial turning point in Bitcoin's price history.

Following the peak in late 2017, 2018 was a year of harsh correction. Bitcoin's price plummeted, losing a substantial portion of its value and entering a prolonged bearish phase. This period of decline served as a stark reminder of the risks of speculative bubbles and the cryptocurrency market's volatility, underlining the importance of cautious investment strategies.

A significant shift occurred in mid-2020 as Bitcoin began another notable upward trajectory. This new bull phase was not a result of mere speculation but was driven by increased institutional investment and the growing acceptance of cryptocurrencies as viable assets. By early 2021, Bitcoin had surpassed previous records, reaching over $60,000. Substantial trading volumes and renewed market enthusiasm characterized this period of rapid growth, indicating a shift in the perception of Bitcoin from a speculative asset to a legitimate investment.

The market experienced another critical moment in March 2020, when the onset of the COVID-19 pandemic caused a sharp and sudden crash. Bitcoin's price dropped nearly 50% in just a few days, reflecting the global financial panic. Remarkably, the recovery was swift, with prices rebounding quickly, showcasing Bitcoin's resilience and the strong belief in its long-term value.

From 2022 to 2024, Bitcoin's price history continued to demonstrate high volatility. This period included significant fluctuations, with notable peaks and troughs that illustrated the ongoing speculative trading behavior and the evolving maturity of the cryptocurrency market. Despite these fluctuations, Bitcoin's overall trend has shown a persistent interest and a dynamic growth trajectory, highlighting its role as a major digital asset.

These periods of meaningful change in Bitcoin's price history underscore the dramatic volatility of the cryptocurrency market. They illustrate the potential for both substantial gains and severe losses, emphasizing the importance for investors to carefully consider the risks and rewards associated with such a volatile asset class. By adopting cautious investment strategies, investors can navigate this volatility and make informed decisions.

2.2.2 Reputational risk

Reputational risk refers to the potential threat or damage to a business or organization's reputation or public image. This type of risk can arise in several ways:

1. Direct impact: stemming directly from the company's own actions or decisions;
2. Indirect Impact: resulting from the behaviour or actions of its employees;
3. Tangent Impact: emanating from associated third parties, such as partners or suppliers.

By effectively managing reputational risk, companies cannot only avoid potential damage to their reputation but also enhance their public image. This can be achieved through strong governance, transparency, and socially responsible and environmentally sustainable practices.

In the context of cryptocurrencies, this kind of risk can arise from a wide range of sources, including:

1. Cybersecurity Failures: major hacking incidents or breaches can severely harm a company's reputation, causing a loss of user trust and confidence;
2. Regulatory Breaches: not complying with changing cryptocurrency laws can lead to legal troubles, financial penalties, and damage to the company's reputation;
3. Ineffective Customer Support: poor customer service or communication handling can lead to dissatisfied users and public backlash;
4. Unethical Market Behaviour: involvement in market manipulation, insider trading, or other unfair practices can result in significant reputational damage;
5. Involvement in Fraud: connections to fraudulent activities can significantly damage a company's public image.

To further understand reputational risk within the cryptocurrency sector, examining specific cases where companies have encountered significant reputation challenges is essential. These instances highlight the several ways reputational risk can manifest in this volatile industry and the potential consequences for businesses. By analysing these

examples, we can gain insights into the causes and impacts of reputational damage and the strategies companies can employ to mitigate such risks.

"By utilizing the anonymizing network TOR, Silk Road ensures that users' tracks on the site are hidden. The only method of payment for these illegal purchases is an untraceable peer-to-peer currency known as Bitcoins. After purchasing Bitcoins through an exchange, a user can create an account on Silk Road and start purchasing illegal drugs from individuals around the world and have them delivered to their homes within days."[15]

This is what U.S. Senators Manchin and Schumer wrote in their letter addressed to Attorney General Eric Holder and the DEA[16].

Silk Road led the anonymous online marketplace worldwide, mainly for illegal goods and services. Its anonymity was maintained through two primary methods:

1. it could only be accessed on the "Deep Web" via the Tor network;
2. it exclusively used Bitcoin for transactions.

FBI then shut it down on October 2, 2013. The investigation began in 2011, and until 2013 Silk Road generated over $1.2 billion in revenue and $80 million in commissions. During this period, over one hundred undercover buys were made. Illegal drugs constituted the large majority of all transactions on Silk Road, but the site also provided access to weapons, forged documents, and illicit services as well.

However, other black-market sites have also emerged on the Deep Web. These platforms continue to operate as marketplaces for illegal goods and services, such as drugs and contract killings, with Bitcoin serving as the currency of choice.

Silk Road, often cited by critics of cryptocurrencies, continues to fuel the debate about Bitcoin's primary use. Senator Schumer, for instance, criticized Bitcoin for its role in Silk Road, describing it as "an online form of money laundering used to disguise the source

[15] Letter from U.S. Senators Joe Manchin and Charles E. Schumer to Eric Holder, U.S. Attorney General and Michele Leonhart, Administrator, Drug Enforcement Administration
[16] Drug Enforcement Administration

of money and the identities of those buying and selling drugs"[17]. This ongoing discussion is exemplified by Duke professor and former Federal Reserve regulator Lee Reiners, who as recently as 2021, advocated for banning Bitcoin and other cryptocurrencies due to their association with criminal activities.

Despite the closure of Silk Road, Bitcoin's value demonstrated its resilience. Initially, it experienced a slight dip, going from $140 to $110 within two hours. However, it swiftly rebounded, reaching $130 just an hour later, highlighting the robustness of this digital currency.

The Silk Road case, while leaving a negative impression on many who saw Bitcoin as a tool for criminal activity, also served as a testament to Bitcoin's practical applications. Silk Road demonstrated how a decentralized currency could operate in the real world, effectively facilitating value transfers between participants in a global market.

2.2.3 Operational risk

Operational risk is the potential loss resulting from inadequate or failed internal processes or systems, human error, or external events that occur when a company attempts to do its day-to-day business activities.

In the world of cryptocurrencies, operational risk poses a significant and complex challenge. As digital currencies become more integrated into the global financial system, it's increasingly important for investors, exchanges, and regulators to understand and address operational risk.

Operational risks associated with cryptocurrencies and any activities involving them may include fraud, cyber, conduct, financial crime, and technology risks.

As we know, cryptocurrencies are managed via digital wallets which are accessed via the use of private keys. It is essential to consider the risks related to fraud and asset security, such as the potential loss or theft of private keys, wallets containing funds, and authentication devices. Exchanges face a significant threat from theft. In recent years, we

[17] New York Senator Chuck Schumer said it at a news conference.

have seen multiple examples of cryptocurrency operators being victims of high value thefts. For instance, Parity, a digital wallet provider, and Mt Gox, a Bitcoin exchange, were robbed of $32 million and $473 million respectively. The latter amount constituted 7% of all bitcoins in circulation at that time.

At its peak, Mt. Gox reigned as the unrivalled giant of the Bitcoin world, commanding a staggering 70% to 80% of all trading volume. This colossal influence, however, painted a target on its back, attracting the attention of hackers and leading to a series of security breaches. In 2011, these breaches resulted in the unauthorized transfer of several thousand Bitcoins, a devastating blow to the exchange.

In early 2014, Mt. Gox faced increasing customer frustration due to problems with fund withdrawals, stemming from technical bugs that made it difficult to track transactions accurately. In February 2014, the exchange suspended withdrawals after detecting suspicious activity, eventually discovering the loss of hundreds of thousands of Bitcoins, with estimates ranging from 650,000 to 850,000. Although 200,000 Bitcoins were later recovered, the immense loss destabilized the market, leading to the exchange's insolvency. This event had a profound impact on the cryptocurrency market, causing a significant drop in Bitcoin's value and a loss of trust in the security of cryptocurrency exchanges. Mt. Gox filed for bankruptcy in the Tokyo District Court in April 2014 and was ordered to liquidate and devise a plan to repay creditors.

3.THE VOLATILITY OF CRYPTOCURRENCIES RELATED TO OTHER FINANCIAL ASSETS

The volatility of cryptocurrencies is a defining characteristic that sets them apart from traditional financial assets, it is significantly higher. As mentioned in the previous paragraphs, this volatility is driven by factors such as market speculation, regulatory changes, technological developments, and liquidity issues.

When comparing the volatility of cryptocurrencies with traditional financial assets such as stocks, bonds, and commodities, several key differences emerge. For instance, stocks can also be volatile, but price movements depend on more predictable factors, such as

company performance, economic indicators or overall market conditions. In addition, they benefit from established regulatory frameworks, greater market depth and more stable investor bases, all factors that can potentially moderate extreme volatility. This stability and predictability can instill a sense of security and trust in investors.

For what concerns bonds, they are considered relatively low-risk investments with lower volatility, in particular government bonds. They show fixed income and predictable returns, that contrast sharply with the unpredictable price swings seen in cryptocurrencies.

Talking about commodities, they can experience volatility due to geopolitical events[18] and supply-demand dynamics. However, this volatility is usually less pronounced compared to cryptocurrencies, and conversely, commodities like gold have historically been used as hedges against inflation and economic uncertainty.

To conclude, while high volatility in cryptocurrencies does pose considerable risks and the potential of substantial losses, it also presents the opportunity for significant returns and quick profits, even over short-time periods. This underscores the importance of understanding these dynamics, as it can empower investors to make informed decisions, emphasizing the need for careful analysis and risk management.

[18] Political actions or events that occur between two or more countries and have a significant impact on financial markets

CHAPTER 2

1.MARKOWITZ PORTFOLIO OPTIMIZATION THEORY, FUNDAMENTALS AND CONCEPTS

Harry Markowitz, a visionary American economist, is celebrated for his transformative contributions to the economic and financial world. His pioneering work, the “Modern Portfolio Theory”, was first presented in his influential paper “Portfolio Selection,” published in the Journal of Finance in 1952[19]. This theory, which he further elaborated on in his book “Portfolio Selection: Efficient Diversification” in 1959, reshaped the landscape of portfolio management.

Markowitz's profound influence on corporate finance was acknowledged with the John von Neumann Theory Prize in 1989 and the esteemed Nobel Memorial Prize in Economic Sciences in 1990, a testament to the enduring impact of his ideas.

Before Markowitz, investors typically bought stocks of companies that were expected to have the highest returns, believing these had the best prospects. However, the economist Markowitz was the first to emphasize and account for the "correlation" between different stocks, which refers to how they move in relation to each other. Markowitz demonstrated that the risk of an investment portfolio depends more on the relationship between the stocks it comprises than on the individual stocks' risk. Modern portfolio theory suggests that the relation between risk and return of an individual investment should not be considered alone. Instead, its characteristics should be evaluated based on their impact on the overall risk and return of the portfolio. That means that an investor can construct a multiple assets based portfolio that will result in greater returns without a higher level of risk. Alternatively, an investor can start with a target level of expected return and then build a portfolio that minimizes risk while achieving that return. Hence, this model is based on the assumption that an investor should always expect higher returns to compensate a higher level of risk. This is what Markowitz meant by saying that investors are “risk-adverse”.

The risk component can be measured by using some specific mathematical methods and reduced by using diversification, which aims to adequately choose a weighted set of assets

[19] Harry Markowitz. (1952). "Portfolio Selection." The Journal of Finance, Volume 7, Pages 77-91

that together show a lower risk compared to the one that would have an investment in single assets. Markowitz was the first to rigorously quantify the benefits of diversification, showing how it is possible to reduce the portfolio's overall risk without sacrificing returns, thereby achieving the "efficient" or "optimal" portfolio. This optimal portfolio is the one that offers the highest possible return for a given level of risk[20].

What Markowitz taught us, therefore, is that it is not enough to buy the stocks of the best companies; one must buy a diverse array of stocks that do not move in unison, meaning they should have a low correlation in technical terms. This knowledge empowers investors to make informed decisions and potentially enhance their investment strategies.

1.1 Expected Return

Assuming that we are monitoring an asset's return during a given period of time, let's define P_b and P_e, that respectively indicate the market price at the beginning and at the end of the period under analysis. Assuming, in addition, that also dividends (D) will be paid, the return will be:

$$R = \frac{P_e - P_b + D}{P}$$

However, the return is an aleatory variable, since it is impossible to determine the values of P_e and of D from the beginning. For that reason, we introduce the concept of expected return:

$$\mu = E(R) = \sum_{i=1}^{n} p_i * R_i$$

Hence, the expected return is a sum of probabilities. This simply indicates the weighted average of all returns, where the weights are given by the single probabilities.

The expected return of a portfolio, in the same way, is given by the weighted average of expected returns of each financial activity within the portfolio under analysis.

[20] The set of these optimal portfolios is known as the "efficient frontier"

1.2 Variance

The variance indicates in which measure the expected return differs from the actual one. The higher the variance associated to return, the higher is the risk to which it is associated.

It is calculated as follows:

$$\sigma^2 = V(R) = \sum_{i=1}^{n} (R_i - \mu)^2 * p_i$$

It always gives a positive value and, if it is equal to 0, it means that the examined returns are certain values.

1.3 Covariance

Given a specific portfolio, the covariance is a crucial measure to obtain a more appropriate risk index, since it takes into account and measures the intensity of the relationship between returns of two or more securities. Given two securities, A and B, the covariance is equal to:

$$\sigma_{AB} = COV_{AB} = \frac{1}{n} * \sum_{t=1}^{n} [(R_A - \mu_A) * (R_B - \mu_B)]$$

A positive covariance means that both returns tend to move in the same way and at the same time. A negative one means that the return move away from each other.

From an investment point of view, it can be crucial to have a low covariance since it reduces the risk.

1.4 Correlation

As covariance, the correlation measures the relationship between returns of two or more securities. However, correlation is a function of covariance and permit us to fully understand it, since it measures both the strength and direction of the relationship between the two variables mentioned before. It can be calculated as follows:

$$\rho_{AB} = Corr_{AB} = \frac{COV_{AB}}{\sigma_A * \sigma_B}$$

This measure can vary between -1 and +1. The closer to +1, the higher the correlation.

1.5 Sharpe ratio

In the context of portfolios and making investment decisions, it would be fundamental to understand the real value of the return of an investment, adjusted for its related risks.

The Sharpe ratio is the measure we are looking for, since it basically assesses the risk-adjusted return of an investment or portfolio, by measuring how much excess return you receive for extra volatility, and more in particular for each additional risk unit you assume. It will be equal to:

$$Sharpe\ Ratio = \frac{R_p - R_f}{\sigma_p}$$

where R_p is the return of the portfolio, R_f is the risk-free rate and σ_p is standard deviation of the portfolio's excess return (it is a measure of the portfolio's volatility, indicating the risk).

Remember, the higher the Sharpe ratio, the better the risk-adjusted return. Conversely, a low ratio could be a red flag, indicating potential unnecessary risks that need to be carefully considered.

Understanding the Sharpe ratio not only enhances performance assessment but also empowers you with a comprehensive understanding of your investment. It aids in gauging how effectively your investment compensates for the risk taken, thereby keeping you well-informed and in control.

As we will see later, the Sharpe ratio is a practical tool also when it comes to valuing and comparing different portfolios. It equips investors with the ability to swiftly identify the

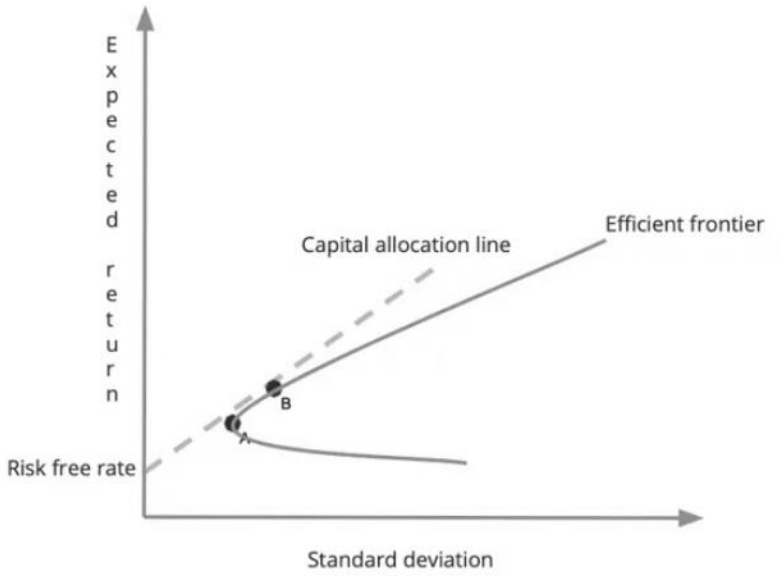
most efficient solution in terms of risk-adjusted returns, instilling confidence in their investment decisions.

2.PRINCIPLES OF DIVERSIFICATION AND OPTIMAL ASSET ALLOCATION

As discussed before, the central concept of Modern Portfolio Theory is diversification.

It is a method that goes beyond just spreading your investments. It refers to the relationship between correlations and portfolio's overall risk, and more in detail it is about strategically allocating diverse assets in a way that they are not perfectly correlated. This approach is designed to minimize risk providing you with a sense of security in your investment strategy, because having different assets means that they would react differently to the same event, that could have a negative effect on a given asset class, but can simultaneously have positive impact on another.

To truly grasp the potential of efficient asset allocation, we introduce the portfolio's efficient frontier. This graph showcases the best combination of securities within an investment portfolio, representing a range of portfolios that not only manage risk but also maximize expected returns (y-axis) for each standard deviation (x-axis) level, which indicates risk. It is a compelling illustration of the rewards that can be reaped through strategic portfolio management.



Where both represented points indicate the portfolios with the highest return for the given value of risk, but:

- point A indicates the combination that minimizes the risk (standard deviation), that is, the minimum variance portfolio;
- point B represents the combination that creates the optimal market portfolio, according to the MPT.

The main concept is that any portfolio on the curve representing the efficient frontier is better than those under it.

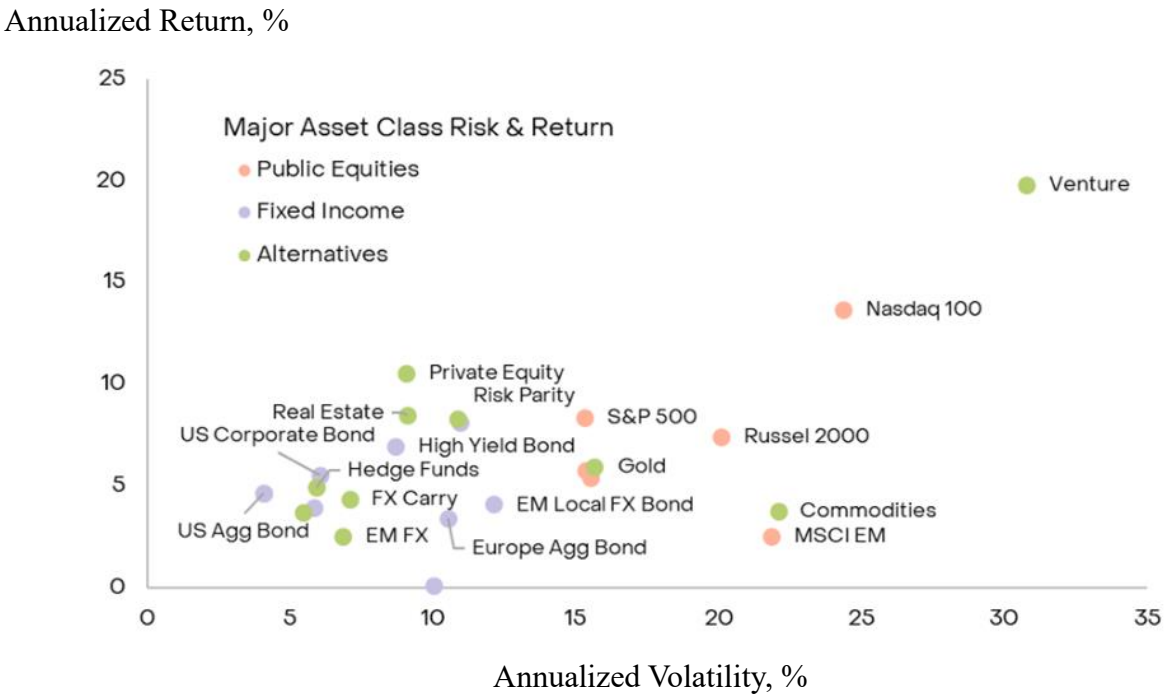
3.APPLICATION TO PORTFOLIOS THAT INCLUDE CRYPTOCURRENCIES

3.1 Reasons behind

Blockchains represent an innovative technology aimed at transforming the global economy and financial system. Investing in cryptocurrencies can help overcome modern portfolio construction challenges.

In a traditional context, investors face a well-known typical balance between risk and return: fixed-income assets generally offer lower returns with lower risk, while equity markets offer higher returns with higher risk.

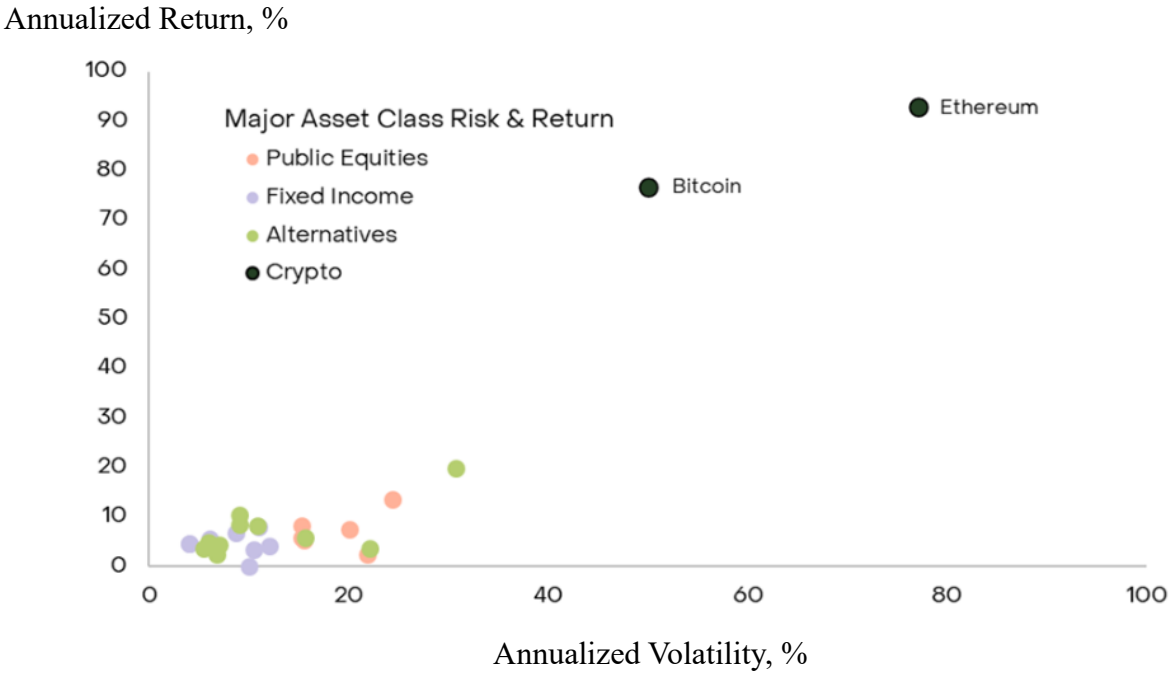
To improve this situation, many institutional investors employed some specific strategies[21]. However, these are not always accessible to individual investors.



***Source**: GRAYSCALE*

Here they come the crypto assets. This innovative asset class expands the opportunities also to individuals.

[21] In recent years many institutional investors have allocated to private markets (e.g. private equity, real estate, venture capital), or employed strategies that incorporate borrowing/leverage to enhance returns (e.g. risk parity). Source: GRAYSCALE



***Source**: GRAYSCALE*

For instance, Bitcoin has produced annualized returns of 50% with a volatility of 75%, while Ethereum is even further out on the risk/return spectrum.

Among traditional assets, venture capital investments offer the highest risk and potential reward, with historical annualized returns of 20% and volatility of 30%.

Therefore, crypto assets present a unique opportunity to diversify your portfolio, offering a broader risk and return spectrum. They hold the potential for high returns through liquid instruments readily available on exchanges, a feature not commonly found in traditional assets.

Furthermore, Bitcoin and other crypto assets have demonstrated low correlation with public equities. Correlation is a statistical measure that indicates the extent to which two or more assets move in relation to each other. If Bitcoin had high returns but also a high correlation with stocks, it would improve total returns but not risk-adjusted returns. However, since Bitcoin has both high returns and low correlation, it benefits a portfolio by enhancing overall returns and diversification.

These elements suggest that integrating Bitcoin and other crypto assets into investment portfolios can offer significant advantages in terms of return and risk management, reinforcing the role of these new financial technologies in modern portfolio construction.

3.2 The application

As mentioned before, Markowitz's Modern Portfolio Theory provides a framework for constructing portfolios that maximize returns for a given level of risk through diversification. By considering the correlations between different assets, MPT allows investors to create an "efficient frontier" of optimal portfolios.
Cryptocurrencies, with their distinct risk-return profiles and low (and sometimes negative) correlations with traditional assets, can enhance the diversification benefits of a portfolio. For instance, their potential to perform differently during periods of traditional market downturns can provide a valuable hedge against market risks, further bolstering their appeal as an investment option.

Cryptocurrencies have a proven history of delivering high returns. By including a small allocation of cryptocurrencies in a portfolio, investors can potentially enhance the overall returns. This prospect of higher returns can be particularly enticing for investors with a higher risk tolerance, looking to diversify into sectors that are poised for future growth and innovation. Such an investment not only provides exposure to the rapidly evolving blockchain technology but also to the broader digital economy, offering a promising outlook for the future.

When it comes to portfolio construction, it is crucial to have a comprehensive understanding of the most appropriate allocation of cryptocurrencies. This decision should be based on factors such as the investor's risk tolerance and investment objectives. Furthermore, regular portfolio rebalancing is essential to maintain the desired risk-return profile. Given the volatile nature of cryptocurrencies, frequent rebalancing may be necessary to ensure that the allocation remains within the targeted range. This level of

understanding and control over the investment strategy can empower investors, making them feel informed and knowledgeable.

By applying principles from Markowitz's portfolio optimization theory and adopting a disciplined approach to portfolio construction and rebalancing, investors can achieve a more resilient and diversified investment portfolio.

CHAPTER 3

1.DEFINITION OF THE MODEL

This chapter meticulously details the methodology used to analyse the potential consequences of integrating cryptocurrencies into traditional investment portfolios. The primary objective is to gain a comprehensive understanding of how cryptocurrencies can impact the overall portfolio risk and return. This rigorous analysis involved the selection of appropriately diversified assets, defining the analysis period, and using specific models to calculate expected returns, variances, covariances, as well as correlations and Sharpe ratios.

The construction of the portfolio and the analysis described before will be done using Python, a programming language chosen for its exceptional calculation accuracy and speed. This choice, guided by the need for precise and efficient calculations, ensures the reliability of the research results.

1.1 Selection of assets and period of analysis

The entire work begins with the construction of the portfolio. Of course, the first thing to do is select a set of assets.

The assets were chosen because they represent various sectors in the financial market, providing an efficient starting point for the analysis, having a broad overview, and ensuring a diversified portfolio. The historical price data for these assets was collected for the period that includes the last five entire years, starting from January 1st, 2019, to January 1st, 2024. This timeframe was selected in order to provide a comprehensive view of asset performance and to capture significant market events for both cryptocurrencies and traditional financial assets, such as the COVID-19 pandemic:2019 was a year of relative stability in global markets;

- 2020 was marked by the pandemic, causing a high market volatility and significant economic consequences;
- 2021 was the period of recovery in both traditional markets and cryptocurrencies;
- 2022-2023 continued market evolution, in particular for cryptocurrencies that saw further maturation.

As mentioned in the previous chapters, the selected cryptocurrency is Bitcoin. This choice was made due to its unique qualities and advantages, such as its decentralised nature and limited supply. Bitcoin's strong recognition, global accessibility, and continuous development have made it the most prominent and widely adopted cryptocurrency, making it a significant asset for our analysis.

For what concerns the traditional financial assets, the chosen ones include equity shares of four companies with an important global relevance.

AstraZeneca (AZN)

One of the leading pharmaceutical companies, AstraZeneca represents the healthcare sector. This sector is fundamental because it provides stability and growth potential, offering a defensive investment during the crisis, compared, for example, to the industrial sector.

Boeing (BA)

A leading company in the transport sector. Its stock is crucial to represent industries that mainly depend on economic conditions, providing exposure to manufacturing and aerospace industries.

Mastercard (MA)

A key player within the financial sector, with the payment solutions that always continues to provide, offering insights into global payments and financial transactions.

Meta Platforms (META)

A giant in technology sector, with high growth potential due to its dominance in social media and digital services, representing innovation.

2.IMPLEMENTATION OF THE MODEL

2.1 Quantitative analysis

Before starting with the calculation and the implementation of the model, the preamble of this part is that, in order to ensure the robustness of our model, we conducted several attempts on Python. These attempts were aimed at verifying the consistency of our results across different portfolio settlements. The results of these attempts confirmed the reliability of our model, ensuring that the results obtained would be the same even with other portfolios' settlements.

The first thing to do is to run a few lines in Python to import libraries and define parameters. We can now download the data for single assets in our portfolio, for the chosen period of time. We use the accessible data provider "Yahoo Finance", through the command "yf.download":

```
import numpy as np
import yfinance as yf
import pandas as pd
import matplotlib.pyplot as plt

#define the stocks in our portfolio
stocks = ['BA', 'BTC-USD', 'META', 'AZN', 'MA']

#set the period of analysis
start_date = '2019-01-01'
end_date = '2024-01-01'

#download of daily price data for each stock in the portfolio
data = yf.download(stocks, start=start_date, end=end_date)['Adj Close']
data.sort_index(inplace=True)

#display first data for confirmation
print(data.head())
```

```
[**********100%%**********]  5 of 5 completed
```

Ticker	AZN	BA	BTC-USD	MA	META
Date					
2019-01-01	NaN	NaN	3843.520020	NaN	NaN
2019-01-02	32.604118	314.645172	3943.409424	184.002899	135.536194
2019-01-03	33.061264	302.100555	3836.741211	175.701736	131.600372
2019-01-04	33.794422	317.822571	3857.717529	184.022278	137.803787
2019-01-05	NaN	NaN	3845.194580	NaN	NaN

***Source**: our calculation on Python*

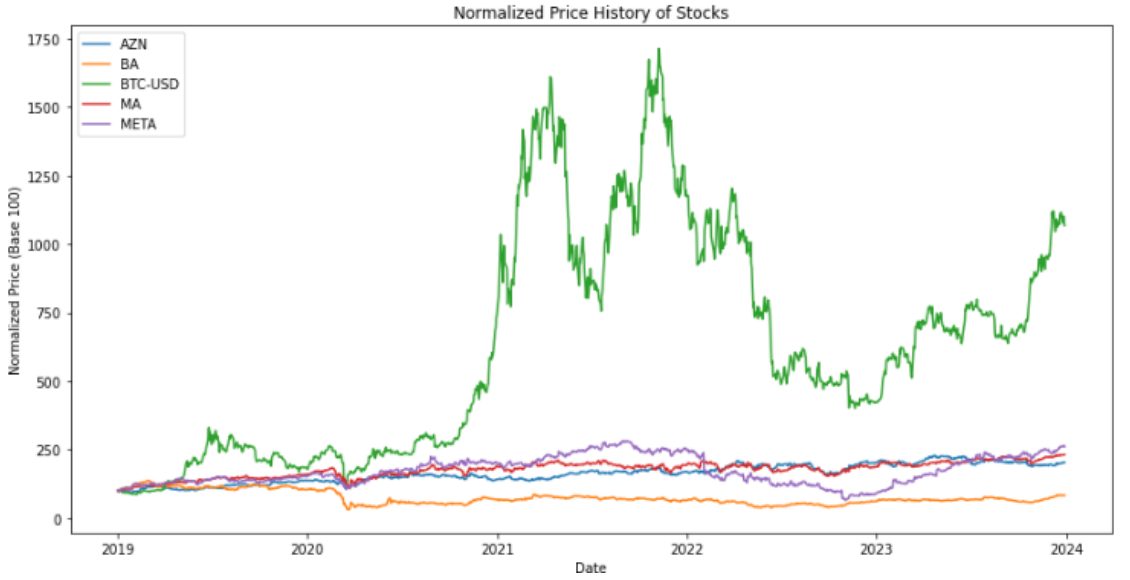
Once the data was obtained, a data-cleaning process was undertaken. This involved removing any rows with missing values, ensuring that our analysis was based on complete data sets. This process not only enhanced the credibility of our results but also assured the reader of the quality of the data used in our analysis.:

```
# Remove NaN
data_clean = data.dropna()

# Verify
print(data_clean.index.min(), data_clean.index.max())
```

***Source**: our calculation on Python*

The next step was to plot prices into a graph through Matplot library. A line plot is created for each asset to show the normalized price history. Normalizing values on base 100 was necessary to make them start from the same point and be easily comparable in that way. This graph illustrates the assets' performance over the given period of time, permitting us to notice that Bitcoin (green curve), even with its higher volatility, had the best performance:

```
#normalize data
normalized_data_clean = data_clean / data_clean.iloc[0] * 100

#plot normalized data
plt.figure(figsize=(14, 7))
for c in normalized_data_clean.columns.values:
    plt.plot(normalized_data_clean.index, normalized_data_clean[c], label=c)

plt.title('Normalized Price History of Stocks')
plt.xlabel('Date')
plt.ylabel('Normalized Price (Base 100)')
plt.legend(loc='upper left')
plt.show()
```

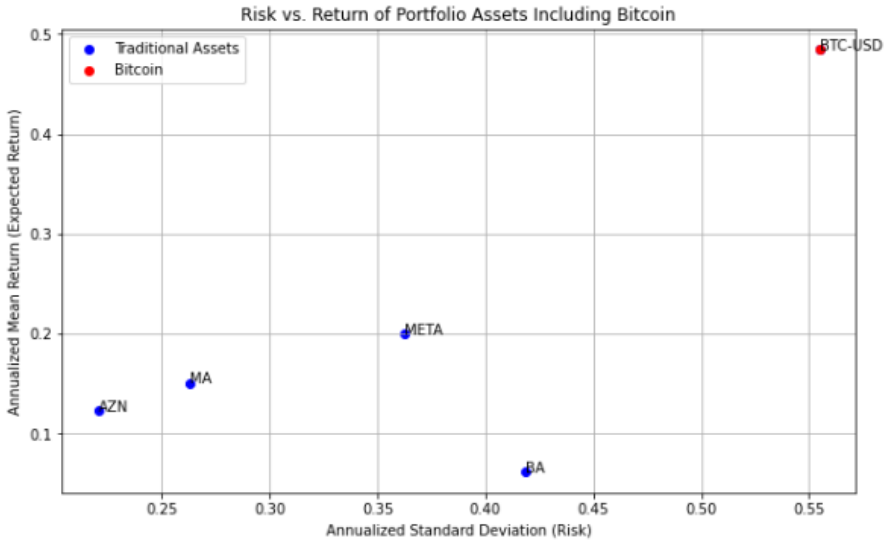


***Source**: our calculation on Python*

To gain a comprehensive understanding of the assets' performance, it is crucial to examine the scatter graph. This graph, which vividly illustrates the relationship between risk and return, is a valuable tool for making informed investment decisions. The x-axis represents the standard deviation, which signifies the risk, while the y-axis depicts the expected

returns. Both these values have been annualized, providing a clear visual representation of the assets' risk-return profile:

```python
#convert daily prices into daily returns
returns = data.pct_change().dropna()

annualized_mean = returns.mean() * 252
annualized_std = returns.std() * np.sqrt(252)

# create a scatter graph
plt.figure(figsize=(10, 6))
# Plottare tutti gli asset
plt.scatter(annualized_std, annualized_mean, color='blue', label='Traditional Assets')
# Evidenziare Bitcoin
plt.scatter(annualized_std['BTC-USD'], annualized_mean['BTC-USD'], color='red', label='Bitcoin')

# give to each point the name of assets
for i, txt in enumerate(returns.columns):
    plt.annotate(txt, (annualized_std[i], annualized_mean[i]))


plt.title('Risk vs. Return of Portfolio Assets Including Bitcoin')
plt.xlabel('Annualized Standard Deviation (Risk)')
plt.ylabel('Annualized Mean Return (Expected Return)')
plt.legend()
plt.grid(True)
plt.show()
```



***Source**: our calculation on Python*

As we can see, Bitcoin (red dot, BTC-USD) performs a higher expected return, with the risk assuming a much higher value. This demonstrates what was presented in the previous chapters, which explained the significant value of Bitcoin and its related risks due to its intrinsic nature.

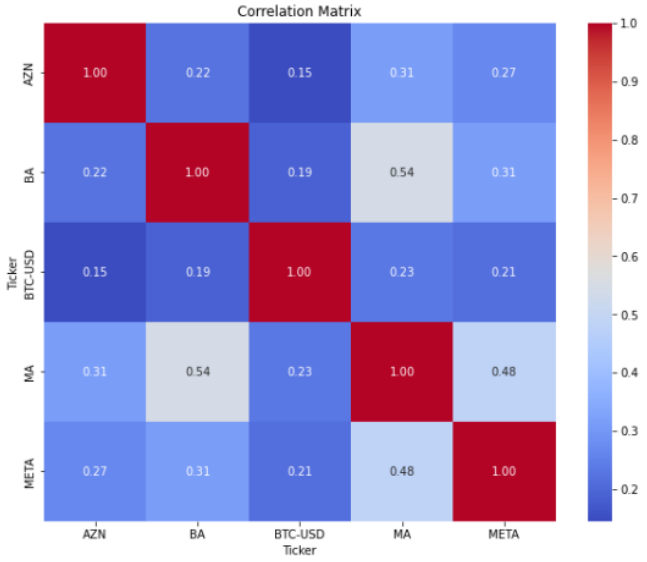
The next analysed point refers to covariances and correlations. We wanted to create a covariances matrix, where the diagonal contains the variances of assets, while other cells contain covariances.

The correlation matrix is a crucial graph, useful to quantify and represent the relationships among different assets. This relation is indicated by a value that can go from -1 to 1, where 1 represents the strongest correlation:

```
import seaborn as sns

cov_matrix = returns.cov()

corr_matrix = returns.corr()

print("Covariances Matrix:")
display(cov_matrix)

plt.figure(figsize=(10, 8))
sns.heatmap(corr_matrix, annot=True, fmt='.2f', cmap='coolwarm')
plt.title('Correlation Matrix')
plt.show()
```

```
Covariances Matrix:
```

Ticker	AZN	BA	BTC-USD	MA	META
Ticker					
AZN	0.000193	0.000082	0.000070	0.000072	0.000084
BA	0.000082	0.000695	0.000172	0.000237	0.000190
BTC-USD	0.000070	0.000172	0.001222	0.000136	0.000169
MA	0.000072	0.000237	0.000136	0.000274	0.000183
META	0.000084	0.000190	0.000169	0.000183	0.000521



***Source**: our calculation on Python*

Looking at the graph, it is evident that there are no particularly strong correlations. Here, reference is made to the central concept on which Markowitz's Portfolio Optimisation Theory is based: the choice of these specific assets was dictated by the desire to focus on a portfolio made up of securities that are not perfectly correlated with each other, in a way to obtain an appropriately diversified portfolio that can best meet the conditions for its

optimization. We can see that the highest correlation is still too far from 1; it corresponds to 0.54, which is the one between Boeing (BA) and Mastercard (MA). This value can be attributed to several factors. However, the main reason could be that, even if they seem to be unrelated companies, they have such a substantial global operativity that the overall economy significantly influences them similarly.

However, it is worth noting that Bitcoin stands out with the lowest correlation among all the traditional financial assets under consideration. This unique characteristic of Bitcoin could potentially offer significant diversification benefits when included in a portfolio.

2.2 Efficient frontier and Portfolio optimization

Once all the preliminary calculations have been made, we can move on to calculating data about the portfolio, such as its expected return and volatility. As we know, these measures are nothing more than the weighted average of each single asset's ones. Therefore, we could simply assign random weights to the single securities, assuming their sum is one.

However, what if we are not satisfied with the level of volatility of the current portfolio and want to reduce it? What if we want to take on more risk to achieve a higher expected return? How can we rearrange the weights of each security in the portfolio to achieve these objectives? Can we adjust and optimize the portfolio to a desired risk-return profile?

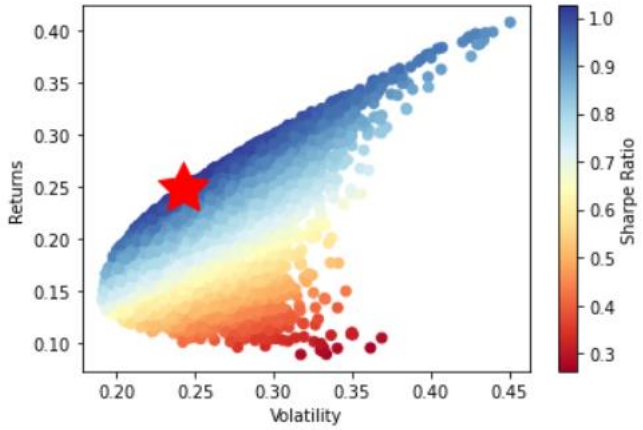
Monte Carlo simulation, a powerful tool in portfolio optimization, enables us to generate and associate different random values to the weights of each individual security in the portfolio. By running thousands of different tests, under the constraint that the sum of the weights must equal 100%, we can explore a vast array of potential portfolio scenarios. Using the Python function "np.random.random", we simulate a number of portfolios equal to 25000.

We must then calculate the expected return, expected volatility and Sharpe ratio for each of the securities in the randomly generated portfolios.

The Sharpe ratio is calculated by simply dividing the return by the standard deviation of each simulated portfolio, just because for simplicity we are assuming a risk-free rate of zero.

```
#calculating the mean and covariance of daily returns
mean_daily_returns = returns.mean()
cov_matrix = returns.cov()

#set the number of simulations
num_portfolios = 25000

#set up an array of results with space for ret, stdev, sharpe and the weights
results = np.zeros((3 + len(stocks), num_portfolios))

for i in range(num_portfolios):
    #select random weights
    weights = np.random.random(len(stocks))
    #rebalance weights to obtain sum 1
    weights /= np.sum(weights)

    # portfolio return and volatility
    portfolio_return = np.sum(mean_daily_returns * weights) * 252
    portfolio_std_dev = np.sqrt(np.dot(weights.T, np.dot(cov_matrix, weights))) * np.sqrt(252)

    #storage results in the array
    results[0, i] = portfolio_return
    results[1, i] = portfolio_std_dev

    #calculation and storage of Sharpe Ratio (risk free element excluded for simplicity)
    results[2, i] = results[0, i] / results[1, i]

    for j in range(len(weights)):
        results[j + 3, i] = weights[j]

        #converts the result array to a pandas dataframe
results_frame = pd.DataFrame(results.T, columns=['ret', 'stdev', 'sharpe'] + stocks)

#identify the portfolio with the highest Sharpe Ratio
max_sharpe_port = results_frame.iloc[results_frame['sharpe'].idxmax()]

#identify the portfolio with the lowest standard deviation
min_vol_port = results_frame.iloc[results_frame['stdev'].idxmin()]

plt.scatter(results_frame.stdev, results_frame.ret, c=results_frame.sharpe, cmap='RdYlBu')
plt.colorbar(label='Sharpe Ratio')
plt.xlabel('Volatility')
plt.ylabel('Returns')

#red star to highlight the portfolio with the highest Sharpe Ratio
plt.scatter(max_sharpe_port['stdev'], max_sharpe_port['ret'], marker=(5, 1, 0), color='r', s=1000)

plt.show()
```

***Source**: our calculation on Python*



***Source**: our calculation on Python*

In the graph, we demonstrate how even slight adjustments in the weight of each security in the portfolio can lead to significant changes in the expected return and the level of risk (standard deviation/volatility) to which the investor is exposed. This insight can help in making informed investment decisions.

Each portfolio's different and unique combinations of weights are conveniently stored in a matrix. Having this input and utilizing the “max” and “min” functions, we can quickly identify the data of portfolios with the highest Sharpe Ratio and the minimum variance within the data frame. Then, we could extract the weights relative to portfolios that contain those characteristics. This practical approach allows for efficient portfolio management.

This calculation has led us to a significant finding. However, according to the principal scope of our analysis, we wanted to focus on the portfolio with the highest Sharpe ratio, identified by the red star. Now let’s display the insights for this particular portfolio, and in which measure each asset contributes:

```
#information of portfolio with the highest Sharpe Ratio
print("Max Sharpe Ratio portfolio:")
print(max_sharpe_port.to_string())
```

```
Max Sharpe Ratio portfolio:
ret        0.248349
stdev      0.242250
sharpe     1.025176
BA         0.389444
BTC-USD    0.000123
META       0.306246
AZN        0.168618
MA         0.135569
```

***Source**: our calculation on Python*

The output prominently features the portfolio details with the highest Sharpe ratio, a key metric derived from the simulated portfolios. This underscores the value of diversification and strategic asset allocation, crucial aspects of our analysis.

The annualised return is approximately 24.83%, indicating a substantial potential gain.

As we know, the standard deviation represents the portfolio's volatility; in this case, it is approximately 24.23%. It is a relatively high level of risk, but remains in line with the high returns.

The Sharpe ratio is approximately 1.03. This means that for every unit of risk taken, the portfolio provides over one unit of return, suggesting that it offers a good risk-adjusted return. However, as we will see later, this indicator becomes even more critical and indicative in the context of doing a comparison with other similar portfolios.

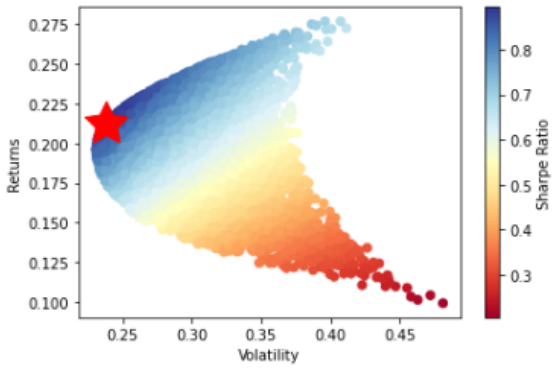
The same analysis has been conducted on the same portfolio but without Bitcoin, to fully understand the potential benefits of diversifying a traditional portfolio by using cryptocurrencies:

```
#Let's do the same thing for the portfolio without BitCoin

stocks = ['BA', 'META', 'AZN', 'MA']

data = yf.download(stocks, start='2019-01-01', end='2024-01-01')['Adj Close']
data.sort_index(inplace=True)

returns = data.pct_change().dropna()

mean_daily_returns = returns.mean()
cov_matrix = returns.cov()

num_portfolios = 25000

results = np.zeros((3 + len(stocks), num_portfolios))

for i in range(num_portfolios):

    weights = np.random.random(len(stocks))

    weights /= np.sum(weights)

    portfolio_return = np.sum(mean_daily_returns * weights) * 252
    portfolio_std_dev = np.sqrt(np.dot(weights.T, np.dot(cov_matrix, weights))) * np.sqrt(252)

    results[0, i] = portfolio_return
    results[1, i] = portfolio_std_dev

    results[2, i] = results[0, i] / results[1, i]
```

***Source**: our calculation on Python*

```
    for j in range(len(weights)):
        results[j + 3, i] = weights[j]

results_frame = pd.DataFrame(results.T, columns=['ret', 'stdev', 'sharpe'] + stocks)

max_sharpe_port = results_frame.iloc[results_frame['sharpe'].idxmax()]

plt.scatter(results_frame.stdev, results_frame.ret, c=results_frame.sharpe, cmap='RdYlBu')
plt.colorbar(label='Sharpe Ratio')
plt.xlabel('Volatility')
plt.ylabel('Returns')

plt.scatter(max_sharpe_port['stdev'], max_sharpe_port['ret'], marker=(5, 1, 0), color='r', s=1000)

plt.show()
```

```
[**********100%%**********]  4 of 4 completed
```




```
#information of portfolio (without BitCoin) with the highest Sharpe Ratio
print("Max Sharpe Ratio portfolio:")
print(max_sharpe_port.to_string())
```

```
Max Sharpe Ratio portfolio:
ret       0.212714
stdev     0.238016
sharpe    0.893695
BA        0.481761
META      0.000404
AZN       0.317621
MA        0.200214
```

***Source**: our calculation on Python*

The results highlight a significant difference in returns, with the portfolio including Bitcoin showing a robust 24.83% compared to the one without the cryptocurrency at 21.37%. This substantial increase underscores the potential benefits of including Bitcoin in a portfolio.

The inclusion of Bitcoin does lead to a slight increase in the portfolio's volatility, from 23.80% to 24.23%. However, this increase is s relatively modest compared to the increase in returns, providing reassurance about the manageable risk despite the inclusion of a volatile asset like Bitcoin.

The portfolio containing Bitcoin demonstrates a higher Sharpe ratio (1.03) compared to the portfolio without the cryptocurrency (0.89). This improvement in risk-adjusted return underscores the value of Bitcoin as an addition to a portfolio, despite its high volatility.

In the following chapter, we will delve deeper into the implications of adding Bitcoin to a portfolio.

CHAPTER 4

1.HOW THE INTEGRATION OF BITCOIN AFFECTS THE OVERALL PORTFOLIO VOLATILITY

The integration of a cryptocurrency into a diversified investment portfolio has notable implications for overall portfolio volatility. This section will include an analysis of how including Bitcoin influences the risk profile of a portfolio, based on the results obtained from our calculations.

The comparative analysis of portfolios with and without Bitcoin reveals some key insights.

First of all, it is notable how the standard deviation changes. The portfolio without Bitcoin has slightly lower volatility, indicating more stable performance. The absence of Bitcoin reduces the exposure to extreme market swings, leading to a more predictable risk profile. The portfolio with Bitcoin shows a higher level of volatility. This increased value is attributed to the cryptocurrency's well-known characteristic of high price fluctuations.

The inclusion of Bitcoin leads to an increase in standard deviation of above 2% (1.81%). This indicates that while the inclusion of Bitcoin increases the overall volatility of the portfolio, the increase is relatively modest. This modest increase in volatility suggests that the risk introduced by Bitcoin is not disproportionately high, considering its potential return benefits.

2.CHANGES IN THE RISK-ADJUSTED PERFORMANCE

Risk-adjusted performance, calculated using the Sharpe ratio, is a measure that investors use to evaluate the returns of an investment relative to the risk it carries.

Despite the higher volatility, Bitcoin has historically demonstrated substantial returns compared to traditional financial assets. Bitcoin's inclusion in a portfolio can significantly impact its risk-adjusted performance due to its high return potential. This characteristic makes Bitcoin an attractive option for investors seeking to enhance their portfolio's overall returns.

The first benefit can be seen in the returns, which increased by around 16%. The portfolio with Bitcoin has an annualized return of 24.83%, compared to 21.37% for the portfolio without Bitcoin. This difference illustrates Bitcoin's ability to boost overall portfolio returns.

Considering also the volatility, one might expect a decrease in returns adjusted for the risk. However, despite the higher volatility, the Sharpe Ratio also saw significant benefits. For the portfolio with Bitcoin, it is 1.03, compared to 0.89 for the portfolio without Bitcoin. This indicates that Bitcoin's high returns more than compensate for its volatility, resulting in superior risk-adjusted performance.

While Bitcoin's high volatility is often seen as a drawback, it can be managed effectively within a diversified portfolio. The strategic allocation of Bitcoin, even in small proportions, can lead to significant improvements in risk-adjusted returns without disproportionately increasing overall portfolio risk.

3.BITCOIN’S ROLE

Cryptocurrencies, and Bitcoin specifically, are known for their high volatility, a critical characteristic that investors need to consider. For that reason, the inclusion of Bitcoin has several effects on portfolio volatility.

The first aspect we emphasize is the concept of diversification, a cornerstone of Markowitz's MPT. Bitcoin's notably low correlation with traditional financial assets, such as those under consideration, can significantly contribute to diversification. Bitcoin's price movements are influenced by different factors compared to stocks, bonds, and commodities, making it a powerful diversifier. As discussed in previous sections, diversification aids in spreading and better managing risk, potentially lowering the portfolio's overall volatility. This is the effect we observed in our analysis.

The minimal allocation to Bitcoin indicates its role as a diversifying asset. Even a small percentage of Bitcoin can significantly enhance the portfolio's overall return while providing diversification benefits that help mitigate risk.

On the other hand, the absence of Bitcoin leads to higher concentrations in traditional assets, increasing sector-specific risks. The higher weightings in Boeing and AstraZeneca, respectively of 48.17% and 31.76%, highlight the reliance on these assets to achieve desired returns, reducing the diversification benefits.

However, given Bitcoin's intrinsic volatility, balancing its diversifying properties and price fluctuations is crucial. This is the primary factor influencing its impact on portfolio volatility.

Given these factors, a strategic allocation of Bitcoin in the portfolio is crucial. It should be included with meticulous consideration of its weight. Our analysis recommends a conservative allocation of the cryptocurrency, indicating a strategic approach. This strategy aims to leverage Bitcoin's potential for high returns while mitigating the associated risks and impact on the portfolio's overall volatility.

We wanted to maintain this small allocation, and it has been demonstrated that even a tiny proportion of cryptocurrencies can provide diversification benefits, without necessarily increasing the portfolio's volatility excessively.

Discussing returns, it is true that Bitcoin's allocation is minimal, but it contributes anyway to a higher overall return. The lower Sharpe Ratio in the portfolio that does not include the cryptocurrency indicates that, while it is less volatile, it does not achieve the same level of risk-adjusted returns. This suggests that excluding Bitcoin might lead to missed opportunities for higher returns.

Sensitivity analyses show that increasing Bitcoin's allocation can further enhance returns, although it also raises volatility. Investors must find an optimal balance to maximize benefits while managing risk.

Furthermore, it is crucial to bear in mind that while our analysis was conducted from a short-term perspective, investors should always adopt a long-term approach. While short-term volatility can be significant, the long-term benefits of Bitcoin investment, particularly in terms of returns and diversification, can be substantial.

Looking at extended investment horizons, the influence of Bitcoin's short-term volatility diminishes, making it a valuable addition to a portfolio focused on long-term growth. Additionally, Bitcoin's potential for high returns can compound over time, leading to

substantial portfolio growth and improved risk-adjusted performance. This underscores the long-term benefits of Bitcoin investment.

Strategic allocation and long-term investment horizons are essential to effectively managing the risks associated with Bitcoin while reaping its benefits.

CONCLUSION

This study, which has thoroughly examined the integration of Bitcoin into diversified investment portfolios, has yielded significant findings. We have focused on how this integration impacts overall portfolio volatility and risk-adjusted performance, and the results provide several crucial insights.

Bitcoin's unique characteristics, such as its lack of correlation with traditional financial markets, make it a compelling choice for diversification. Unlike stocks and bonds, which are influenced by economic indicators and interest rates, Bitcoin operates independently. This independence allows Bitcoin to act as a buffer against market downturns, enhancing portfolio stability.

Our findings demonstrate that including Bitcoin in a portfolio can significantly reduce systematic risks and improve risk-adjusted returns, thereby enhancing the overall portfolio performance.

Despite its high return potential, Bitcoin's integration into portfolios brings increased volatility due to its inherent price fluctuations. However, this volatility is counterbalanced by Bitcoin's high returns, leading to an improved Sharpe Ratio for portfolios that include Bitcoin.

This means that while Bitcoin introduces additional risk, its potential for higher returns justifies this risk, making it an attractive component for portfolio optimization.

The practical implications of these findings are substantial for both investors and portfolio managers. The inclusion of Bitcoin in an investment portfolio offers a unique opportunity to enhance diversification and potentially achieve higher risk-adjusted returns. Bitcoin's high growth potential and low correlation with traditional assets make it an excellent tool for reducing overall portfolio risk while potentially boosting returns.

While Bitcoin offers significant potential, it's important for investors to approach it with a well-considered strategy. Our study underscores the risk of over-investing in Bitcoin, given its high volatility. However, a measured allocation to Bitcoin can provide substantial benefits without introducing excessive risk.

It's also crucial for investors to be aware of Bitcoin's speculative nature and the potential impact of regulatory changes and security concerns on its performance. By understanding these risks, investors can make informed decisions about Bitcoin.

For portfolio managers, the integration of Bitcoin offers a unique opportunity. By strategically incorporating Bitcoin into portfolios, managers can tap into its potential for high returns and diversification benefits. This approach, however, necessitates a vigilant eye on Bitcoin's market behaviour and a keen interest in regulatory and technological developments that could impact its value.

Looking forward, several areas should be object of further exploration to understand Bitcoin's role in investment portfolios fully. The evolving regulatory landscape for cryptocurrencies will significantly shape their future performance and acceptance as mainstream investment assets. Future studies should investigate the long-term effects of regulatory changes on Bitcoin's integration into investment strategies.

Technological advancements in blockchain technology and emerging new cryptocurrencies demand our attention. These developments could reshape the cryptocurrency market, influencing Bitcoin's risk and return characteristics. It's crucial for us to stay proactive and continuously research to adapt our investment strategies to these changes, ensuring we stay ahead in this dynamic market.

Additionally, expanding the analysis to include a broader range of cryptocurrencies could provide a more comprehensive view of the market's potential for diversification and risk management. Understanding how different cryptocurrencies interact with traditional financial assets will offer deeper insights into their suitability for portfolio inclusion.

In conclusion, Bitcoin's integration into diversified investment portfolios offers both significant opportunities and challenges. While its high volatility poses risks, Bitcoin's substantial return potential and its unique characteristic of low correlation with traditional assets make it a valuable tool for enhancing portfolio performance and managing risk.

By adopting a strategic and informed approach, investors and portfolio managers can effectively harness Bitcoin's benefits while mitigating associated risks.

Future research should continue to explore the dynamic nature of the cryptocurrency market, providing ongoing guidance for incorporating these emerging assets into investment strategies.

APPENDIX A – Our calculation on Python

In [2]:
```
import numpy as np
import yfinance as yf
import pandas as pd
import matplotlib.pyplot as plt

#define the stocks in our portfolio
stocks = ['BA', 'BTC-USD', 'META', 'AZN', 'MA']

#set the period of analysis
start_date = '2019-01-01'
end_date = '2024-01-01'

#download of daily price data for each stock in the portfolio
data = yf.download(stocks, start=start_date, end=end_date)['Adj Close']
data.sort_index(inplace=True)

#display first data for confirmation
print(data.head())
```

```
[*********************100%%**********************]  5 of 5 completed
```

Ticker	AZN	BA	BTC-USD	MA	META
Date					
2019-01-01	NaN	NaN	3843.520020	NaN	NaN
2019-01-02	32.604122	314.645142	3943.409424	184.002884	135.536194
2019-01-03	33.061260	302.100525	3836.741211	175.701675	131.600372
2019-01-04	33.794418	317.822571	3857.717529	184.022247	137.803787
2019-01-05	NaN	NaN	3845.194580	NaN	NaN

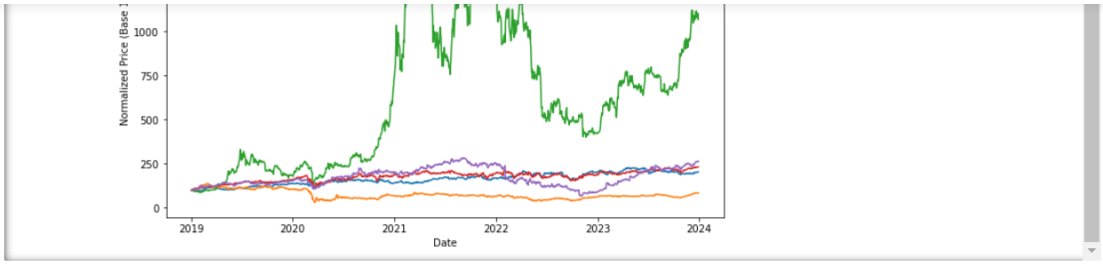
In [3]:
```
# Remove NaN
data_clean = data.dropna()

# Verify
print(data_clean.index.min(), data_clean.index.max())
```

```
2019-01-02 00:00:00 2023-12-29 00:00:00
```

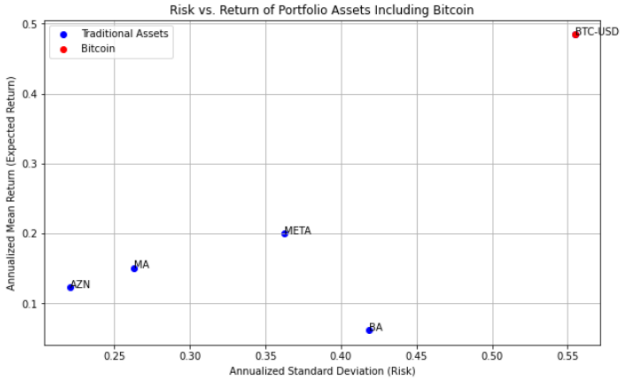
In [4]:
```
#normalize data
```

In [4]:
```
#normalize data
normalized_data_clean = data_clean / data_clean.iloc[0] * 100

#plot normalized data
plt.figure(figsize=(10, 6))
for c in normalized_data_clean.columns.values:
    plt.plot(normalized_data_clean.index, normalized_data_clean[c], label=c)

plt.title('Normalized Price History of Stocks')
plt.xlabel('Date')
plt.ylabel('Normalized Price (Base 100)')
plt.legend(loc='upper left')
plt.show()
```

In [6]:
```
#convert daily prices into daily returns
returns = data.pct_change().dropna()

annualized_mean = returns.mean() * 252
annualized_std = returns.std() * np.sqrt(252)
```



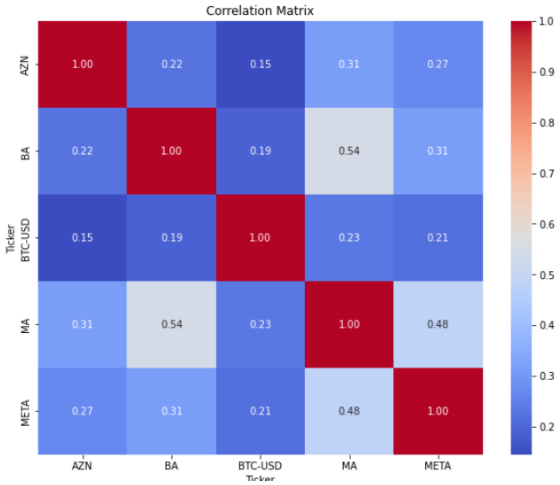
In [6]:
```
#convert daily prices into daily returns
returns = data.pct_change().dropna()

annualized_mean = returns.mean() * 252
annualized_std = returns.std() * np.sqrt(252)

# create a scatter graph
plt.figure(figsize=(10, 6))
# Plottare tutti gli asset
plt.scatter(annualized_std, annualized_mean, color='blue', label='Traditional Assets')
# Evidenziare Bitcoin
plt.scatter(annualized_std['BTC-USD'], annualized_mean['BTC-USD'], color='red', label='Bitcoin')

# give to each point the name of assets
for i, txt in enumerate(returns.columns):
    plt.annotate(txt, (annualized_std[i], annualized_mean[i]))


plt.title('Risk vs. Return of Portfolio Assets Including Bitcoin')
plt.xlabel('Annualized Standard Deviation (Risk)')
plt.ylabel('Annualized Mean Return (Expected Return)')
plt.legend()
plt.grid(True)
plt.show()
```



In [7]:
```
import seaborn as sns

cov_matrix = returns.cov()

corr_matrix = returns.corr()

print("Covariances Matrix:")
display(cov_matrix)

plt.figure(figsize=(10, 8))
sns.heatmap(corr_matrix, annot=True, fmt='.2f', cmap='coolwarm')
plt.title('Correlation Matrix')
plt.show()
```

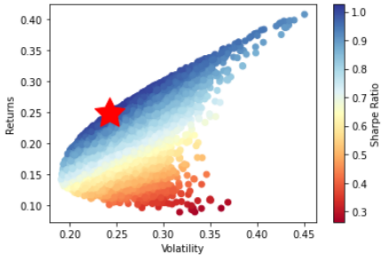
```
Covariances Matrix:
```

Ticker	AZN	BA	BTC-USD	MA	META
Ticker					
AZN	0.000193	0.000082	0.000070	0.000072	0.000084
BA	0.000082	0.000695	0.000172	0.000237	0.000190
BTC-USD	0.000070	0.000172	0.001222	0.000136	0.000169
MA	0.000072	0.000237	0.000136	0.000274	0.000183
META	0.000084	0.000190	0.000169	0.000183	0.000521



In [8]:
```
#calculating the mean and covariance of daily returns
mean_daily_returns = returns.mean()
cov_matrix = returns.cov()

#set the number of simulations
num_portfolios = 25000

#set up an array of results with space for ret, stdev, sharpe and the weights
results = np.zeros((3 + len(stocks), num_portfolios))

for i in range(num_portfolios):
    #select random weights
    weights = np.random.random(len(stocks))
    #rebalance weights to obtain sum 1
    weights /= np.sum(weights)

    # portfolio return and volatility
    portfolio_return = np.sum(mean_daily_returns * weights) * 252
    portfolio_std_dev = np.sqrt(np.dot(weights.T, np.dot(cov_matrix, weights))) * np.sqrt(252)

    #storage results in the array
    results[0, i] = portfolio_return
    results[1, i] = portfolio_std_dev

    #calculation and storage of Sharpe Ratio (risk free element excluded for simplicity)
    results[2, i] = results[0, i] / results[1, i]

    for j in range(len(weights)):
        results[j + 3, i] = weights[j]

        #converts the result array to a pandas dataframe
results_frame = pd.DataFrame(results.T, columns=['ret', 'stdev', 'sharpe'] + stocks)

#identify the portfolio with the highest Sharpe Ratio
max_sharpe_port = results_frame.iloc[results_frame['sharpe'].idxmax()]

#identify the portfolio with the lowest standard deviation
min_vol_port = results_frame.iloc[results_frame['stdev'].idxmin()]

plt.scatter(results_frame.stdev, results_frame.ret, c=results_frame.sharpe, cmap='RdYlBu')
```

```
plt.scatter(results_frame.stdev, results_frame.ret, c=results_frame.sharpe, cmap='RdYlBu')
plt.colorbar(label='Sharpe Ratio')
plt.xlabel('Volatility')
plt.ylabel('Returns')

#red star to highlight the portfolio with the highest Sharpe Ratio
plt.scatter(max_sharpe_port['stdev'], max_sharpe_port['ret'], marker=(5, 1, 0), color='r', s=1000)

plt.show()
```



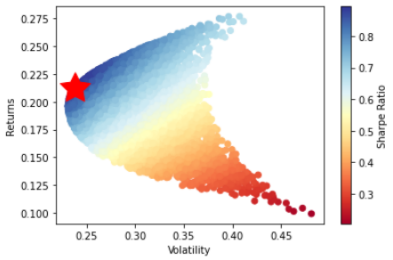
In [9]:
```
#information of portfolio with the highest Sharpe Ratio
print("Max Sharpe Ratio portfolio:")
print(max_sharpe_port.to_string())
```

```
Max Sharpe Ratio portfolio:
ret        0.248349
stdev      0.242250
sharpe     1.025176
BA         0.389444
BTC-USD    0.000123
META       0.306246
AZN        0.168618
MA         0.135569
```

In [10]:
```
#Let's do the same thing for the portfolio without BitCoin

stocks = ['BA', 'META', 'AZN', 'MA']

data = yf.download(stocks, start='2019-01-01', end='2024-01-01')['Adj Close']
data.sort_index(inplace=True)

returns = data.pct_change().dropna()

mean_daily_returns = returns.mean()
cov_matrix = returns.cov()

num_portfolios = 25000

results = np.zeros((3 + len(stocks), num_portfolios))

for i in range(num_portfolios):

    weights = np.random.random(len(stocks))

    weights /= np.sum(weights)

    portfolio_return = np.sum(mean_daily_returns * weights) * 252
    portfolio_std_dev = np.sqrt(np.dot(weights.T, np.dot(cov_matrix, weights))) * np.sqrt(252)

    results[0, i] = portfolio_return
    results[1, i] = portfolio_std_dev

    results[2, i] = results[0, i] / results[1, i]

    for j in range(len(weights)):
        results[j + 3, i] = weights[j]

results_frame = pd.DataFrame(results.T, columns=['ret', 'stdev', 'sharpe'] + stocks)

max_sharpe_port = results_frame.iloc[results_frame['sharpe'].idxmax()]

plt.scatter(results_frame.stdev, results_frame.ret, c=results_frame.sharpe, cmap='RdYlBu')
plt.colorbar(label='Sharpe Ratio')
plt.xlabel('Volatility')
plt.ylabel('Returns')
```

```
plt.scatter(results_frame.stdev, results_frame.ret, c=results_frame.sharpe, cmap='RdYlBu')
plt.colorbar(label='Sharpe Ratio')
plt.xlabel('Volatility')
plt.ylabel('Returns')

plt.scatter(max_sharpe_port['stdev'], max_sharpe_port['ret'], marker=(5, 1, 0), color='r', s=1000)

plt.show()
```

```
[*********************100%%**********************]  4 of 4 completed
```



In [11]:
```
#information of portfolio (without BitCoin) with the highest Sharpe Ratio
print("Max Sharpe Ratio portfolio:")
print(max_sharpe_port.to_string())
```

```
Max Sharpe Ratio portfolio:
ret       0.212714
stdev     0.238016
sharpe    0.893695
BA        0.481761
META      0.000404
AZN       0.317621
MA        0.200214
```

APPENDIX B – Other attempts to verify the consistency of our results across different portfolio settlements

This first alternative portfolio settlement contains "BNP Paribas" (BNP.PA), "Kering" (KER.PA), "Procter & Gamble Co/The" (PG), "Etsy Inc" (ETSY) and, obviously, Bitcoin (BTC-USD). It remains a diversified portfolio, through different sectors.

In [47]:
```
import numpy as np
import yfinance as yf
import pandas as pd
import matplotlib.pyplot as plt

#define the stocks in our portfolio
stocks = ['BNP.PA', 'BTC-USD', 'ETSY', 'KER.PA', 'PG']

#set the period of analysis
start_date = '2019-01-01'
end_date = '2024-01-01'

#download of daily price data for each stock in the portfolio
data = yf.download(stocks, start=start_date, end=end_date)['Adj Close']
data.sort_index(inplace=True)

#display first data for confirmation
print(data.head())
```

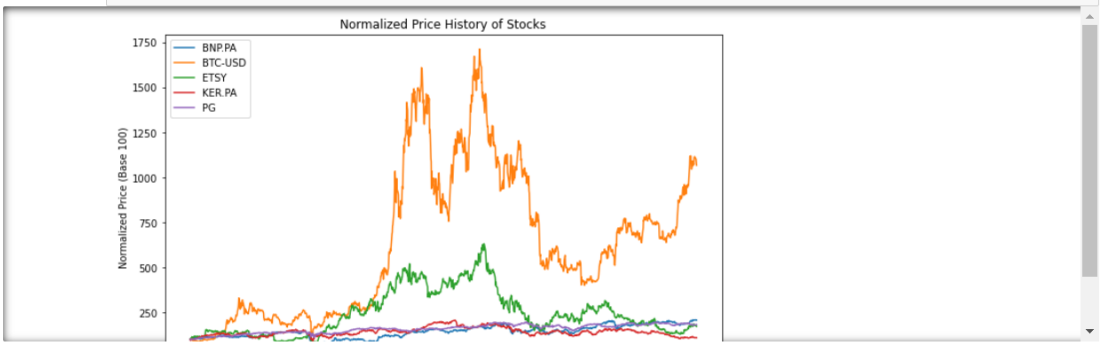
```
[*********************100%%**********************]  5 of 5 completed
```

Ticker	BNP.PA	BTC-USD	ETSY	KER.PA	PG
Date					
2019-01-01	NaN	3843.520020	NaN	NaN	NaN
2019-01-02	28.332731	3943.409424	47.000000	350.981445	79.410721
2019-01-03	28.166008	3836.741211	46.029999	331.724579	78.853935
2019-01-04	29.532448	3857.717529	49.700001	346.276245	80.463387
2019-01-05	NaN	3845.194580	NaN	NaN	NaN

In [48]:
```
# Remove NaN
data_clean = data.dropna()

# Verify
print(data_clean.index.min(), data_clean.index.max())
```

```
2019-01-02 00:00:00 2023-12-29 00:00:00
```

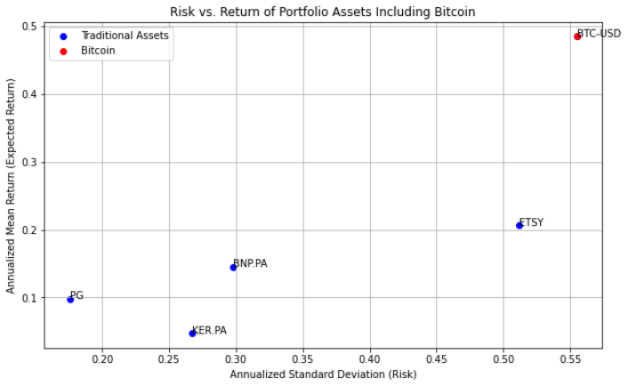
In [49]:
```
#normalize data
normalized_data_clean = data_clean / data_clean.iloc[0] * 100

#plot normalized data
plt.figure(figsize=(10, 6))
for c in normalized_data_clean.columns.values:
    plt.plot(normalized_data_clean.index, normalized_data_clean[c], label=c)

plt.title('Normalized Price History of Stocks')
plt.xlabel('Date')
plt.ylabel('Normalized Price (Base 100)')
plt.legend(loc='upper left')
plt.show()
```



In [50]:
```
#convert daily prices into daily returns
returns = data.pct_change().dropna()

annualized_mean = returns.mean() * 252
annualized_std = returns.std() * np.sqrt(252)
```

In [50]:
```
#convert daily prices into daily returns
returns = data.pct_change().dropna()

annualized_mean = returns.mean() * 252
annualized_std = returns.std() * np.sqrt(252)

# create a scatter graph
plt.figure(figsize=(10, 6))
# Plottare tutti gli asset
plt.scatter(annualized_std, annualized_mean, color='blue', label='Traditional Assets')
# Evidenziare Bitcoin
plt.scatter(annualized_std['BTC-USD'], annualized_mean['BTC-USD'], color='red', label='Bitcoin')

# give to each point the name of assets
for i, txt in enumerate(returns.columns):
    plt.annotate(txt, (annualized_std[i], annualized_mean[i]))


plt.title('Risk vs. Return of Portfolio Assets Including Bitcoin')
plt.xlabel('Annualized Standard Deviation (Risk)')
plt.ylabel('Annualized Mean Return (Expected Return)')
plt.legend()
plt.grid(True)
plt.show()
```

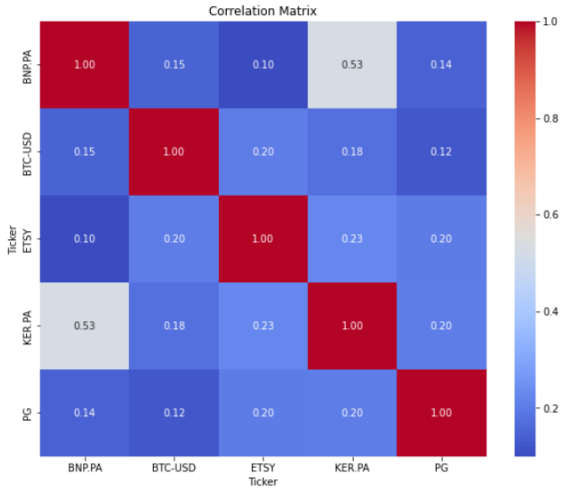
```
plt.show()
```



In [51]:
```
import seaborn as sns

cov_matrix = returns.cov()

corr_matrix = returns.corr()

print("Covariances Matrix:")
display(cov_matrix)

plt.figure(figsize=(10, 8))
sns.heatmap(corr_matrix, annot=True, fmt='.2f', cmap='coolwarm')
plt.title('Correlation Matrix')
plt.show()
```

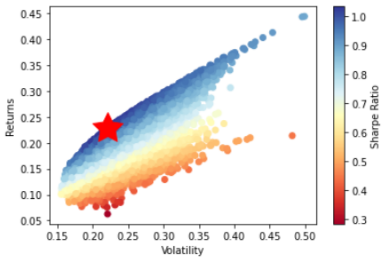
```
Covariances Matrix:
```

Ticker	BNP.PA	BTC-USD	ETSY	KER.PA	PG
Ticker					
BNP.PA	0.000352	0.000096	0.000060	0.000166	0.000030
BTC-USD	0.000096	0.001222	0.000229	0.000103	0.000048
ETSY	0.000060	0.000229	0.001038	0.000122	0.000072
KER.PA	0.000166	0.000103	0.000122	0.000283	0.000038
PG	0.000030	0.000048	0.000072	0.000038	0.000122



In [52]:
```
#calculating the mean and covariance of daily returns
mean_daily_returns = returns.mean()
cov_matrix = returns.cov()

#set the number of simulations
num_portfolios = 25000

#set up an array of results with space for ret, stdev, sharpe and the weights
results = np.zeros((3 + len(stocks), num_portfolios))

for i in range(num_portfolios):
    #select random weights
    weights = np.random.random(len(stocks))
    #rebalance weights to obtain sum 1
    weights /= np.sum(weights)

    # portfolio return and volatility
    portfolio_return = np.sum(mean_daily_returns * weights) * 252
    portfolio_std_dev = np.sqrt(np.dot(weights.T, np.dot(cov_matrix, weights))) * np.sqrt(252)

    #storage results in the array
    results[0, i] = portfolio_return
    results[1, i] = portfolio_std_dev

    #calculation and storage of Sharpe Ratio (risk free element excluded for simplicity)
    results[2, i] = results[0, i] / results[1, i]

    for j in range(len(weights)):
        results[j + 3, i] = weights[j]

        #converts the result array to a pandas dataframe
results_frame = pd.DataFrame(results.T, columns=['ret', 'stdev', 'sharpe'] + stocks)

#identify the portfolio with the highest Sharpe Ratio
max_sharpe_port = results_frame.iloc[results_frame['sharpe'].idxmax()]

#identify the portfolio with the lowest standard deviation
min_vol_port = results_frame.iloc[results_frame['stdev'].idxmin()]

plt.scatter(results_frame.stdev, results_frame.ret, c=results_frame.sharpe, cmap='RdYlBu')
```

```
plt.scatter(results_frame.stdev, results_frame.ret, c=results_frame.sharpe, cmap='RdYlBu')
plt.colorbar(label='Sharpe Ratio')
plt.xlabel('Volatility')
plt.ylabel('Returns')

#red star to highlight the portfolio with the highest Sharpe Ratio
plt.scatter(max_sharpe_port['stdev'], max_sharpe_port['ret'], marker=(5, 1, 0), color='r', s=1000)

plt.show()
```



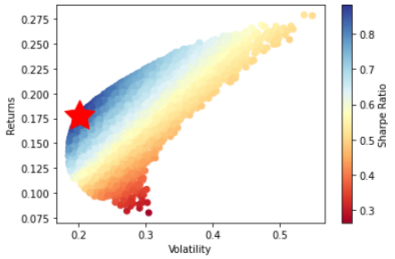
In [53]:
```
#information of portfolio with the highest Sharpe Ratio
print("Max Sharpe Ratio portfolio:")
print(max_sharpe_port.to_string())
```

```
Max Sharpe Ratio portfolio:
ret        0.228057
stdev      0.220566
sharpe     1.033962
BNP.PA     0.250751
BTC-USD    0.299380
ETSY       0.024437
KER.PA     0.003342
PG         0.422090
```

In [54]:
```
#let's do the same thing for the portfolio without BitCoin

stocks = ['BNP.PA', 'ETSY', 'KER.PA', 'PG']

data = yf.download(stocks, start='2019-01-01', end='2024-01-01')['Adj Close']
data.sort_index(inplace=True)

returns = data.pct_change().dropna()

mean_daily_returns = returns.mean()
cov_matrix = returns.cov()

num_portfolios = 25000

results = np.zeros((3 + len(stocks), num_portfolios))

for i in range(num_portfolios):

    weights = np.random.random(len(stocks))

    weights /= np.sum(weights)

    portfolio_return = np.sum(mean_daily_returns * weights) * 252
    portfolio_std_dev = np.sqrt(np.dot(weights.T, np.dot(cov_matrix, weights))) * np.sqrt(252)

    results[0, i] = portfolio_return
    results[1, i] = portfolio_std_dev

    results[2, i] = results[0, i] / results[1, i]

    for j in range(len(weights)):
        results[j + 3, i] = weights[j]

results_frame = pd.DataFrame(results.T, columns=['ret', 'stdev', 'sharpe'] + stocks)

max_sharpe_port = results_frame.iloc[results_frame['sharpe'].idxmax()]

plt.scatter(results_frame.stdev, results_frame.ret, c=results_frame.sharpe, cmap='RdYlBu')
plt.colorbar(label='Sharpe Ratio')
```

```
plt.scatter(results_frame.stdev, results_frame.ret, c=results_frame.sharpe, cmap='RdYlBu')
plt.colorbar(label='Sharpe Ratio')
plt.xlabel('Volatility')
plt.ylabel('Returns')

plt.scatter(max_sharpe_port['stdev'], max_sharpe_port['ret'], marker=(5, 1, 0), color='r', s=1000)

plt.show()
```

```
[*********************100%%**********************]  4 of 4 completed
```



In [55]:
```
#information of portfolio (without BitCoin) with the highest Sharpe Ratio
print("Max Sharpe Ratio portfolio:")
print(max_sharpe_port.to_string())
```

```
Max Sharpe Ratio portfolio:
ret       0.177729
stdev     0.201408
sharpe    0.882432
BNP.PA    0.326485
ETSY      0.114712
KER.PA    0.000622
PG        0.558180
```

Following our thesis, the inclusion of Bitcoin into a new diversified portfolio positively affects its risk-adjusted performance, with the Sharpe ratio increasing from 0.88 to 1.03.

The standard deviation saw a slight increase (from 0.20 to 0.22), more than counterbalanced by the optimization of the return, which arrives at almost 0.23.

Another simulation has been conducted in order to better verify the consistency of our study. In this new settlement, the portfolio contains "Goldman Sachs Group Inc" (GS), "Monster Beverage Corporation" (MNST), "Best Buy Co Inc" (BBY), "Booking Holdings Inc" (BKNG), and Bitcoin (BTC-USD).

In [65]:
```
import numpy as np
import yfinance as yf
import pandas as pd
import matplotlib.pyplot as plt

#define the stocks in our portfolio
stocks = ['BBY', 'BKNG', 'BTC-USD', 'GS', 'MNST']

#set the period of analysis
start_date = '2019-01-01'
end_date = '2024-01-01'

#download of daily price data for each stock in the portfolio
data = yf.download(stocks, start=start_date, end=end_date)['Adj Close']
data.sort_index(inplace=True)

#display first data for confirmation
print(data.head())
```

```
[*********************100%%**********************]  5 of 5 completed
```

Ticker	BBY	BKNG	BTC-USD	GS	MNST
Date					
2019-01-01	NaN	NaN	3843.520020	NaN	NaN
2019-01-02	44.528378	1717.305298	3943.409424	150.765152	24.379999
2019-01-03	43.614552	1658.874878	3836.741211	148.556610	24.000000
2019-01-04	43.822243	1713.166016	3857.717529	153.411819	24.895000
2019-01-05	NaN	NaN	3845.194580	NaN	NaN

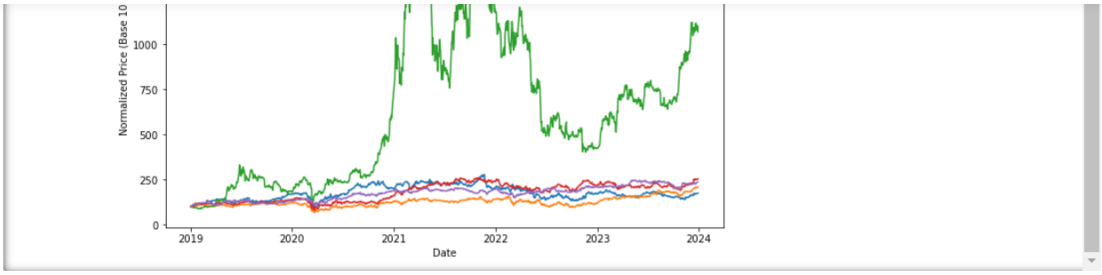
In [66]:
```
# Remove NaN
data_clean = data.dropna()

# Verify
print(data_clean.index.min(), data_clean.index.max())
```

```
2019-01-02 00:00:00 2023-12-29 00:00:00
```

In [67]:
```
#normalize data
normalized_data_clean = data_clean / data_clean.iloc[0] * 100
```

In [67]:
```
#normalize data
normalized_data_clean = data_clean / data_clean.iloc[0] * 100

#plot normalized data
plt.figure(figsize=(10, 6))
for c in normalized_data_clean.columns.values:
    plt.plot(normalized_data_clean.index, normalized_data_clean[c], label=c)

plt.title('Normalized Price History of Stocks')
plt.xlabel('Date')
plt.ylabel('Normalized Price (Base 100)')
plt.legend(loc='upper left')
plt.show()
```

In [68]:
```
#convert daily prices into daily returns
returns = data.pct_change().dropna()
```

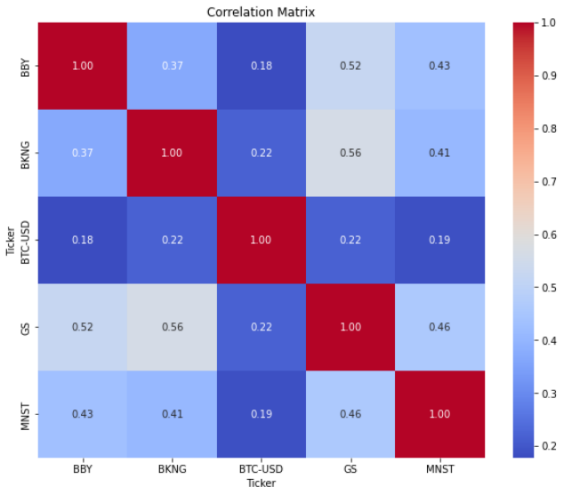


In [68]:
```
#convert daily prices into daily returns
returns = data.pct_change().dropna()

annualized_mean = returns.mean() * 252
annualized_std = returns.std() * np.sqrt(252)

# create a scatter graph
plt.figure(figsize=(10, 6))
# Plottare tutti gli asset
plt.scatter(annualized_std, annualized_mean, color='blue', label='Traditional Assets')
# Evidenziare Bitcoin
plt.scatter(annualized_std['BTC-USD'], annualized_mean['BTC-USD'], color='red', label='Bitcoin')

# give to each point the name of assets
for i, txt in enumerate(returns.columns):
    plt.annotate(txt, (annualized_std[i], annualized_mean[i]))


plt.title('Risk vs. Return of Portfolio Assets Including Bitcoin')
plt.xlabel('Annualized Standard Deviation (Risk)')
plt.ylabel('Annualized Mean Return (Expected Return)')
plt.legend()
plt.grid(True)
plt.show()
```

In [69]:
```
import seaborn as sns

cov_matrix = returns.cov()

corr_matrix = returns.corr()

print("Covariances Matrix:")
display(cov_matrix)

plt.figure(figsize=(10, 8))
sns.heatmap(corr_matrix, annot=True, fmt='.2f', cmap='coolwarm')
plt.title('Correlation Matrix')
plt.show()
```

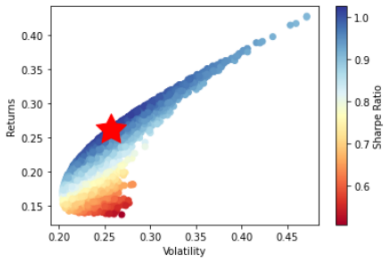
```
Covariances Matrix:
```

Ticker	BBY	BKNG	BTC-USD	GS	MNST
Ticker					
BBY	0.000421	0.000147	0.000128	0.000183	0.000127
BKNG	0.000147	0.000377	0.000149	0.000187	0.000114
BTC-USD	0.000128	0.000149	0.001222	0.000131	0.000094
GS	0.000183	0.000187	0.000131	0.000294	0.000113
MNST	0.000127	0.000114	0.000094	0.000113	0.000206



In [70]:
```
#calculating the mean and covariance of daily returns
mean_daily_returns = returns.mean()
cov_matrix = returns.cov()

#set the number of simulations
num_portfolios = 25000

#set up an array of results with space for ret, stdev, sharpe and the weights
results = np.zeros((3 + len(stocks), num_portfolios))

for i in range(num_portfolios):
    #select random weights
    weights = np.random.random(len(stocks))
    #rebalance weights to obtain sum 1
    weights /= np.sum(weights)

    # portfolio return and volatility
    portfolio_return = np.sum(mean_daily_returns * weights) * 252
    portfolio_std_dev = np.sqrt(np.dot(weights.T, np.dot(cov_matrix, weights))) * np.sqrt(252)

    #storage results in the array
    results[0, i] = portfolio_return
    results[1, i] = portfolio_std_dev

    #calculation and storage of Sharpe Ratio (risk free element excluded for simplicity)
    results[2, i] = results[0, i] / results[1, i]

    for j in range(len(weights)):
        results[j + 3, i] = weights[j]

        #converts the result array to a pandas dataframe
results_frame = pd.DataFrame(results.T, columns=['ret', 'stdev', 'sharpe'] + stocks)

#identify the portfolio with the highest Sharpe Ratio
max_sharpe_port = results_frame.iloc[results_frame['sharpe'].idxmax()]

#identify the portfolio with the lowest standard deviation
min_vol_port = results_frame.iloc[results_frame['stdev'].idxmin()]

plt.scatter(results_frame.stdev, results_frame.ret, c=results_frame.sharpe, cmap='RdYlBu')
```

```
plt.scatter(results_frame.stdev, results_frame.ret, c=results_frame.sharpe, cmap='RdYlBu')
plt.colorbar(label='Sharpe Ratio')
plt.xlabel('Volatility')
plt.ylabel('Returns')

#red star to highlight the portfolio with the highest Sharpe Ratio
plt.scatter(max_sharpe_port['stdev'], max_sharpe_port['ret'], marker=(5, 1, 0), color='r', s=1000)

plt.show()
```



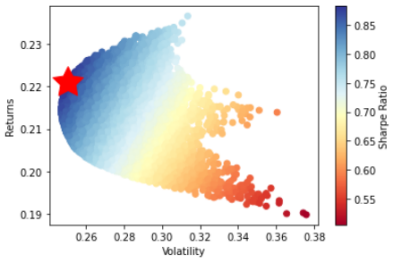
In [71]:
```
#information of portfolio with the highest Sharpe Ratio
print("Max Sharpe Ratio portfolio:")
print(max_sharpe_port.to_string())
```

```
Max Sharpe Ratio portfolio:
ret         0.262566
stdev       0.256252
sharpe      1.024639
BBY         0.008491
BKNG        0.022864
BTC-USD     0.333163
GS          0.236544
MNST        0.398938
```

In [72]:
```
#let's do the same thing for the portfolio without BitCoin

stocks = ['BBY', 'BKNG', 'GS', 'MNST']

data = yf.download(stocks, start='2019-01-01', end='2024-01-01')['Adj Close']
data.sort_index(inplace=True)

returns = data.pct_change().dropna()

mean_daily_returns = returns.mean()
cov_matrix = returns.cov()

num_portfolios = 25000

results = np.zeros((3 + len(stocks), num_portfolios))

for i in range(num_portfolios):

    weights = np.random.random(len(stocks))

    weights /= np.sum(weights)

    portfolio_return = np.sum(mean_daily_returns * weights) * 252
    portfolio_std_dev = np.sqrt(np.dot(weights.T, np.dot(cov_matrix, weights))) * np.sqrt(252)

    results[0, i] = portfolio_return
    results[1, i] = portfolio_std_dev

    results[2, i] = results[0, i] / results[1, i]

    for j in range(len(weights)):
        results[j + 3, i] = weights[j]

results_frame = pd.DataFrame(results.T, columns=['ret', 'stdev', 'sharpe'] + stocks)

max_sharpe_port = results_frame.iloc[results_frame['sharpe'].idxmax()]

plt.scatter(results_frame.stdev, results_frame.ret, c=results_frame.sharpe, cmap='RdYlBu')
plt.colorbar(label='Sharpe Ratio')
```

```
plt.scatter(results_frame.stdev, results_frame.ret, c=results_frame.sharpe, cmap='RdYlBu')
plt.colorbar(label='Sharpe Ratio')
plt.xlabel('Volatility')
plt.ylabel('Returns')

plt.scatter(max_sharpe_port['stdev'], max_sharpe_port['ret'], marker=(5, 1, 0), color='r', s=1000)

plt.show()
```

```
[*********************100%%**********************]  4 of 4 completed
```



In [73]:
```
#information of portfolio (without BitCoin) with the highest Sharpe Ratio
print("Max Sharpe Ratio portfolio:")
print(max_sharpe_port.to_string())
```

```
Max Sharpe Ratio portfolio:
ret       0.220988
stdev     0.250361
sharpe    0.882677
BBY       0.003390
BKNG      0.086364
GS        0.367215
MNST      0.543031
```

Once again, our hypothesis has been proven. In this simulation, the Sharpe ratio can be improved, increasing from 0.88 for the portfolio without Bitcoin, to 1.02 with its inclusion.

The return also increased from 0.22 to 0.26, while the standard deviation remains almost stable, changing by only a few cents.

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