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Stochastic Choice: An Experimental Analysis

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1. Introduction

Imagine you're keeping tabs on whether a peer grabs coffee from a café on their way to work[1]. Your colleague, being well-informed about both the coffee's quality and price, doesn't really learn anything new or face any external risks. Sometimes they opt for a coffee (let's call this choice A), while other times they simply stroll past the café without purchasing (choice B).

Traditionally, economists view economic decisions as pretty cut-and-dry: if the enjoyment (or utility) from choice A (denoted as UA) outweighs that of choice B (UB), then A is picked, no questions asked. Thus, when A is chosen over B, it's inferred that UA surpasses UB. This classical view outlines how economic choices are made and how we draw conclusions about them.

Now, to mimic this scenario, an analyst might try to gauge the utility of A (as UA hat) and the utility of B (as UB hat) based on the available data. But, because these estimations are never perfect, the analyst includes error terms to account for information that's known to the decision-maker but not to the observer. This approach, described in Figure 1, portrays how the decision-maker operates in such situations. However, the analyst models the utilities as UA hat plus εA and UB hat plus εB, and assumes the decision-maker goes for the option with the highest realized value. Even if utility seems random from the analyst's standpoint (but not from the decision-maker's), this setup is termed a random utility model.

To justify this randomness, theories of private information propose that the decision-maker holds undisclosed information that influences their choices—information that the observer isn't privy to. This serves as part of a broader strategy to address scenarios where the analyst lacks access to all the pertinent details.

Consider this: if choice B was made today, a plausible explanation based on private information could be that your colleague already had coffee earlier in the day, which you weren't aware of.

Despite the success of random utility models, there's a growing body of evidence suggesting that the behavior outlined in Figure 1 doesn't always match real-world

[1] The Apparently Random Coffee Purchases of a Colleague (P Brañas-Garza and John Smith see.references)

scenarios. Many choices in economic settings seem to be somewhat random from the perspective of the decision-maker. For instance, participants often opt for choices from sets that, according to their own assessments, are less favorable than other available options (Reutskaja et al., 2011). Moreover, different choices are observed even when participants are presented with identical sets of options (Hey, 1995, 2001; Agranov and Ortoleva, 2017). This randomness is prevalent in experimental economics, with literature highlighting the insights that can be derived from careful consideration of such noise.

Several explanations have emerged to shed light on these apparent deviations from Figure 1.

Consideration set explanations (Eliaz and Spiegler, 2011; Masatlioglu, Nakajima, and Ozbay, 2012; Manzini and Mariotti, 2014) propose that participants don't necessarily evaluate every option in the choice set but have an unobserved subset of options from which they make a choice. From the decision-maker's viewpoint, the best choice is made from this consideration set. In our context, this could suggest that the colleague might have been distracted, perhaps by their phone, and thus didn't consider buying coffee today.

Preference for randomization explanations (Agranov and Ortoleva, 2017; Cerreia-Vioglio et al., 2019) suggest that participants have a preference for randomizing their choices to achieve a balance or mix of selections. Indeed, many participants make different choices when presented with the same questions consecutively and are informed of this beforehand. In our scenario, this might imply that the colleague was less inclined to buy coffee today due to recent purchases on previous days.

While these explanations, along with others, offer insights into the apparent deviations from Figure 1, a more general explanation posits that participants have stable, well-defined preferences, but these preferences are imperfectly perceived (Luce, 1959). For instance, if you ask the colleague about their choice of B, they might admit that they weren't sure whether they wanted coffee at that moment. It was only after some thought that they settled on choosing B. This suggests that the colleague had clear valuations for options A and B but these were only partially observed.

In this scenario, the colleague's decision could be modeled based on underlying preferences, but the choice was influenced by the noisy perception of these preferences. For example, we could model the situation with utilities UA and UB, but these utilities are perceived imperfectly. Specifically, there's an additive noise associated with A (εA) and B (εB). Consequently, the colleague only perceives $VA = UA + \varepsilon A$ and $VB = UB + \varepsilon B$ and selects the option with the highest observed value. Since εA and εB are random variables, the choice between A and B becomes stochastic: sometimes A is chosen, and other times B is chosen. Figure 2 illustrates both choice and inference in random utility models with this interpretation of noise.

Studying the impact of imperfect perception on random choice in economics faces a challenge: the true preferences (UA and UB) are either not directly observable or are only measured imperfectly (via self-reported rankings, elicited willingness to pay, expected utility estimates, etc.).

1.1. Background Motivation

Transitioning from Pablo Brañas-Garza and John Smith's example on coffee to a more specific one, which will be addressed in this study, let's consider a series of lotteries with different payoffs: the subject will tend to choose A rather than B depending on their utility function in seeking to maximize profit. However, if the experiment were repeated sequentially, presenting the same lotteries to subjects multiple times, how does their choice change? What factors should be considered? What happens if reaction time is calculated? Until recently, it was thought that the choice was random, stochastic, but recent studies such as those by Agranov et al., Ortoleva, Cerreia-Vioglio, and as we will attempt to confirm in the experiment, demonstrate that the choice is deliberate, determined by something happening at the neuronal level in our brain.

To further investigate this, the study will be carried out by proposing 60 lotteries to the subjects in three distinct phases: spaced, consecutive, and consecutive again. In each lottery, subjects will be presented with two sets of payoffs and will need to choose one set, or indicate indifference, based on their personal utility calculations and other influencing factors. This structured approach will allow us to measure variations in choice behavior across different temporal setups and add depth to our understanding of decision-making processes in economic scenarios.

(a) Economic choice	(b) Economic inference
$U_A > U_B \rightarrow A$ is chosen (over B)	When A is chosen (over B) $\rightarrow U_A > U_B$
$U_A = U_B \rightarrow$ random choice	When random choice $\rightarrow U_A = U_B$
$U_A < U_B \rightarrow B$ is chosen (over A)	When B is chosen (over A) $\rightarrow U_B > U_A$

Figure 1: Source: Imperfect Perception and Stochastic Choice in Experiments by Pablo Brañas-Garza and John Smith, Cambridge University Press

(a) Random utility choice	(b) Random utility inference
$U_A + \varepsilon_A > U_B + \varepsilon_B \rightarrow A$ is chosen (over B)	When A is chosen (over B) $\rightarrow U_A + \varepsilon_A > U_B + \varepsilon_B$
$U_A + \varepsilon_A = U_B + \varepsilon_B \rightarrow$ indifferent	Indifferent $\rightarrow U_A + \varepsilon_A = U_B + \varepsilon_B$
$U_A + \varepsilon_A < U_B + \varepsilon_B \rightarrow B$ is chosen (over A)	When B is chosen (over A) $\rightarrow U_A + \varepsilon_A < U_B + \varepsilon_B$

Figure 2: Source: Imperfect Perception and Stochastic Choice in Experiments by Pablo Brañas-Garza and John Smith, Cambridge University Press

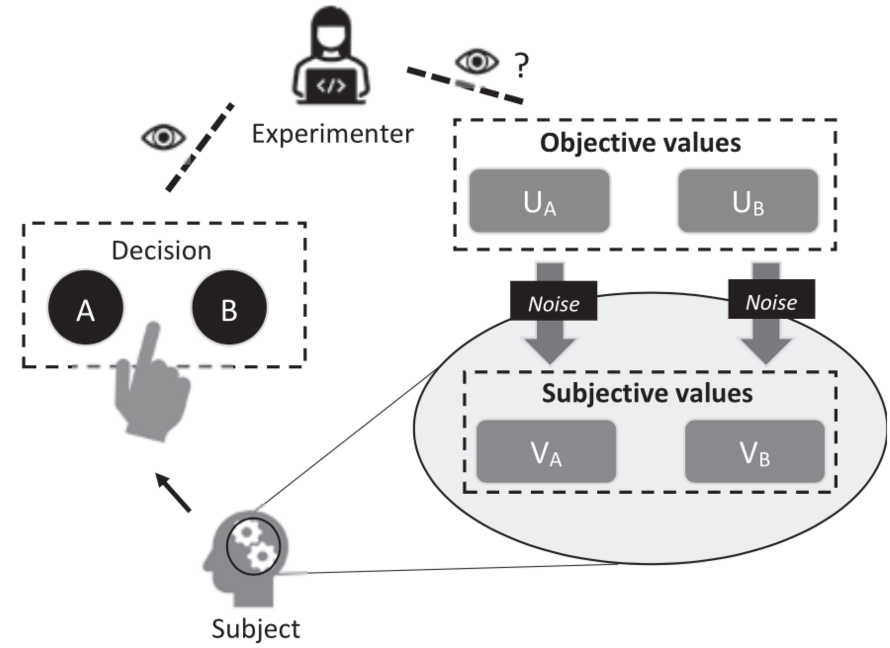

Figure 3: Characterization of a choice experiment, where a subject chooses between A and B. There are objective and nonrandom values UA and UB that are imperfectly perceived as subjective values VA and VB. The subject selects the option with the larger subjective value. The analyst can observe the decision. Source: Imperfect Perception and Stochastic Choice in Experiments by Pablo Brañas-Garza and John Smith, Cambridge University Press

1.2. Research Objectives

The aim of the present research is to investigate whether stochastic choice, often observed in complex decision-making contexts, can be attributed not only to errors or cognitive limitations, as traditionally believed, but also to a conscious and deliberate strategy adopted by individuals. Until recently, stochasticity in choices has predominantly been interpreted as a sign of inconsistency or a failure of the human brain's decision-making processes, suggesting that individuals make errors in evaluating available options. However, the theory of deliberate randomization proposes a new perspective, in which individuals might intentionally randomize their decisions to maximize perceived benefits, such as minimizing regret or diversifying risks. This research aims to explore this hypothesis, testing whether and under what conditions the randomization of choices can be considered a rational and deliberate choice rather than a mere cognitive error.

A consistent observation regarding individual decision-making is the phenomenon of stochastic, or random, choice: when asked to choose between the same options multiple times, subjects often make different choices. Stochastic choice is documented in many contexts, including those where subjects derive no value from experimentation (e.g., when there is no feedback) and those where there are no bundle or portfolio effects (e.g., when only one choice is paid). This robust finding has led to the development of a vast body of theoretical models that capture this behavior. These models can be classified into three broad categories: (1) random utility models, in which subjects' responses change because their preferences change stochastically; (2) bounded rationality models, in which subjects have stable preferences but exhibit stochastic choice because they may fail to choose the best option for themselves; and (3) deliberate randomization models, in which subjects deliberately choose to report different responses because it is optimal for them to do so (e.g., to minimize regret or to diversify among options).

For our study, we employed the following experimental design. Subjects were asked to make several choices between objective lotteries, with one randomly selected decision paid out among all decisions made. In the first part of the experiment, we replicated a standard design: subjects were asked a set of questions repeatedly, with the repetitions spaced apart (subjects were not informed of these repetitions in advance). In the second part, subjects faced the same question three times in succession and were explicitly told that each question would be repeated three times.

As an additional test of the desire to randomize, for some questions, subjects could choose between one of two lotteries or a coin flip (simulated by a computer) to determine which lottery they would receive; selecting the coin flip involved a small cost. We also elicited subjects' attitudes towards risk, compound lotteries, and their propensity to violate expected utility as captured by the Allais paradox. Finally, since we wanted to study the motivation behind stochastic choice, in an unincentivized questionnaire distributed at the end of the experiment, we directly asked subjects whether, and why, they were consistent when questions were repeated in succession.

The experiment, as observed by Agranov & Ortoleva, demonstrates that stochastic choice is a common behavior, particularly in complex decision-making scenarios (HARD questions). However, the results show lower percentages compared to those observed by A. and O. Specifically, stochastic choice was 41.9% in the first part, compared to 90% in their experiment, and 40.7% in Part III, compared to 71%. The behavior persisted even when participants were aware of the immediate repetitions, highlighting the robustness of this phenomenon.

In both cases, stochastic choice is almost exclusively present in questions where none of the available options is "clearly better" than the other (what we call "difficult questions"); it is extremely frequent for these questions and virtually absent for the others. This distinction is the strongest predictor of stochastic choice in our data. Difficult questions are not necessarily those where expected values or utilities are closest. In fact, differences in expected utility between options have limited predictive power in determining the stochasticity of choice in our data and cannot explain the variation in stochastic choice.

Furthermore, the analysis of response times shows that subjects behave very differently in distant repetitions compared to consecutive ones.

We conclude our analysis by examining the responses to the final questionnaire, where 44.4% of interviewed subjects reported adopting different ways of choosing between lotteries in Parts I and III, while 83.3% believe they were consistent in their choices. Moreover, 88.88% believe that some choices were easier than others, and 94.44% did not find it difficult to express their preference. Finally, 88.8% believe they had a consistent preference for a certain type of lottery (e.g., those with higher prizes and lower probabilities or those with lower prizes but higher probabilities).

To frame our analysis, we extend existing models of stochastic choice to make predictions about distant and consecutive choices. For random utility models, we consider the random expected utility model of Gul and Pesendorfer (2006) and hypothesize that the stochastic component of utility does not change for consecutive repetitions. For bounded rationality models, we consider the drift-diffusion model of Ratcliff (1978) and Ratcliff and McKoon (2008) and hypothesize that the agent no longer gathers information for those repetitions.

For deliberate randomization models, we consider the cautious stochastic choice model of Cerreia-Vioglio et al. (2016).

We interpret our results as indicating that the primary driving force behind stochastic choice in our data is the deliberate desire of subjects to choose different responses, which is consistent with deliberate randomization models and not with models from the other categories. It is important to emphasize that we can only test between these classes of models due to the intertemporal structure we add to the random utility and bounded rationality models. Our data, for example, do not rule out a random utility model in which choice-specific utility shocks occur for consecutive questions that the agent knows to be identical. Our implementation is consistent with the way random utility models are often interpreted (see Luce [1958], Becker, DeGroot, and Marschak [1963b], and the discussion in Section II), but under other interpretations, our tests would not be decisive.

This article is related to the experimental literature on choice under uncertainty (Camerer 1995) and, in particular, to studies on stochastic choice and preferences for randomization. Hey and Carbone (1995) experimentally test whether preferences are deterministic while choice is stochastic; their results rule out this possibility, and in particular, the experimental design reflects the lotteries used by Hey and DiCagno (1990).

The rest of the study is organized as follows. Chapter II discusses the literature review. Chapter III presents the empirical evidence, and the experimental design and results are analyzed in Chapter IV. Chapter V discusses neuroimaging techniques. The appendix contains further analyses and experimental instructions.

2. Literature review

2.1. Theoretical foundations of Stochastic Choice

Herbert Simon during the 1940s, discussed limitations in rationality that could indirectly imply stochastic choice. "Stochastic choice," refers to decision-making under conditions of risk or uncertainty where individuals do not necessarily maximize a specific objective function but instead choose in a more intuitive or seemingly random manner. This concept challenges traditional economic and decision theories, which often assume that individuals act rationally to maximize expected utility or profits.

Stochastic choice can be understood as a process where individuals' preferences are not fixed but subject to random fluctuations due to various factors like mood, attention, or information noise. In this framework, choices are not deterministic outcomes of a utility-maximizing process, due to an error, but rather probabilistic, reflecting a mixture of different factors influencing the decision at a given time.

The formalization and exploration of stochastic choice gained significant traction in the mid-20th century, particularly within the fields of economics and psychology.

The stochastic choice theory explores how decisions are made in situations where outcomes are uncertain, and elements of randomness are involved. This branch of theoretical economics has significant implications for understanding and predicting consumer behavior, economic decision-making, and policy design. It encompasses various models and approaches, such as random utility models and probabilistic choice models, which help explain the non-deterministic elements of decision-making.

By employing a combination of historical perspectives, mathematical rigor, psychological insights, and practical applications, this chapter aims to provide a comprehensive understanding of the theoretical foundations of stochastic choice. This approach not only clarifies the complexities inherent in stochastic decision-making but also highlights its relevance to contemporary economic and behavioral studies.

The term "stochastic" originates from the Greek word “στοχαστικός”[2] meaning 'skilled at guessing'. This is particularly appropriate, as the theory primarily deals with the analysis and modeling of choice behavior in situations where the outcomes are probabilistic rather

[2] From Ancient Greek στοχαστικός (*stokhastikós*), from στοχάζομαι (*stokházomai*, “aim at a target, guess”), from στόχος (*stókhos*, “an aim, a guess”).

than deterministic. Understanding stochastic choice is crucial for several reasons. Firstly, it allows economists and psychologists to predict how individuals and markets are likely to behave when faced with uncertainty. Secondly, it helps in designing more effective business strategies and public policies that accommodate the unpredictable nature of human decisions.

Stochastic choice theory also bridges theoretical and practical domains, applying its principles to real-world issues like market analysis, risk assessment, and consumer behavior forecasting. Its implications extend beyond economics into fields such as political science, where it can elucidate voter behavior, and healthcare, where it can enhance decision-making models for patient care under uncertainty.

One early figure who contributed to the discussion of stochastic choice was John von Neumann, a mathematician and economist. Von Neumann's work in game theory, particularly his collaboration with Oskar Morgenstern on the seminal book "Theory of Games and Economic Behavior" (1944), introduced stochastic elements into decision-making models. While their focus was primarily on strategic interactions and rational decision-making, their framework laid the groundwork for considering uncertainty and randomness in choice behavior.

Another notable contributor was Leonard J. Savage’s book "The Foundations of Statistics" (1954) introduced the concept of subjective expected utility theory, which provided a framework for decision-making under uncertainty. Savage's work emphasized the role of probabilities and subjective beliefs in decision-making, paving the way for later developments in stochastic choice theory.

In psychology, early discussions of stochastic choice can be traced back to the work of researchers like B. F. Skinner (1957), who explored the role of randomness and reinforcement schedules in behaviorism. While Skinner's work focused more on deterministic principles, his experiments laid the groundwork for understanding probabilistic learning and decision-making in animals and humans.

Following these early contributions, we come to the groundbreaking Prospect Theory by Daniel Kahneman and Amos Tversky (1979). While Prospect Theory doesn't explicitly use the term "stochastic," it profoundly acknowledges randomness in decision-making under risk.

Kahneman and Tversky's theory challenged traditional economic models by demonstrating that people's decisions are often influenced by psychological biases and heuristics, rather than adhering strictly to rational calculations. Their framework incorporated concepts like loss aversion, where individuals exhibit a stronger preference to avoid losses than to acquire equivalent gains, and the framing effect, where the presentation of information influences decision-making outcomes.

Within Prospect Theory, the notion of uncertainty and variability plays a central role. By acknowledging the inherent randomness in human decision-making under risk, Kahneman and Tversky provided a more realistic and nuanced understanding of choice behavior.

This acknowledgment of randomness paved the way for further developments in stochastic choice theory, influencing fields beyond economics and psychology, including neuroscience, sociology, and artificial intelligence. The recognition of stochastic elements in decision-making has broad implications for understanding human behavior and designing effective interventions in various domains.

Another significant contribution to our understanding of decision-making under uncertainty comes from John Hey and Elisabetta Strazzera in their paper "Estimation of the Indifference Curves" (1989). Although they don't explicitly mention stochastic choice, their work is crucial for the topic as they delve into how individuals make decisions when faced with uncertain outcomes.

Their research underscores the importance of considering uncertainty and variability in decision-making models, highlighting the nuanced ways in which individuals navigate choices in uncertain environments. By incorporating insights from their work, we can further refine our understanding of stochastic choice and its implications for various fields, including economics, psychology, and decision sciences.

In the following chapters, I will analyze the literature concerning stochastic choices, making a macro-distinction between 'Pre-Deliberative Stochastic' and 'Post-Deliberative Stochastic'. Starting from the classical theory of expected utility, moving through imprecision, the critique of EUT initiated approximately 10 years ago, up to the present day with Deliberative Stochastic, but before discussing it, first, a brief historical introduction talking about bounded rationality, homo economicus, and EUT.

2.2. Bounded Rationality

Herbert Simon coined the term *'bounded rationality'* (Simon 1957b: 198; also see Klaes & Sent 2005) to challenge neoclassical economics and advocate for replacing the idealized rationality assumptions of homo economicus with a notion of rationality suited to agents with cognitive limitations.

In essence, the objective is to substitute the overarching rationality of economic man with a form of rational behavior that aligns with the information access and computational capacities realistically available to organisms, including humans, in their natural environments (Simon 1955a: 99).

Over time, *'bounded rationality'* has evolved to encompass various descriptive, normative, and prescriptive frameworks for effective behavior that deviate from the assumptions of perfect rationality. This entry endeavors to highlight significant contributions from decision sciences, economics, cognitive and neuropsychology, biology, computer science, and philosophy to our present comprehension of bounded rationality.

The decision-making process based on "*bounded rationality*" is characterized by the recognition of individuals' cognitive and informational limitations. Unlike optimization, which assumes the availability of complete information and the ability to evaluate all possible alternatives to find the best solution, "*bounded rationality*" acknowledges that people operate within a context of limited knowledge and finite cognitive resources. In this context, decision-makers do not necessarily seek the optimal solution but rather a "*good enough*" solution that meets a minimum aspiration level. This aspiration level, or "*satisficing*," represents a threshold of acceptability beyond which a decision is considered adequate.

In the optimization process, the decision-maker sets a minimum aspiration level based on optimal performance criteria. If this level is not met, the decision-making process does not end but continues iteratively. The decision-maker reexamines the aspiration level or seeks better alternatives, revising their choices and trying to achieve a more satisfactory outcome. This iterative cycle continues until a solution that meets the initial optimal conditions is reached.

In contrast, "*bounded rationality*" settles for solutions that, while not optimal, are considered good enough given the specific situation and existing limitations. Simon, one of the main theorists of this approach, emphasizes how humans are unable to process all available information or accurately predict all the consequences of their decisions. Therefore, instead of aiming for maximization, they adopt a "*satisficing*" behavior, seeking a solution that is good enough to meet basic requirements without exhausting cognitive and informational resources.

The fundamental difference between the two approaches lies in how information and cognitive resources are handled. In optimization, it is assumed that decision-makers can gather and analyze all the necessary information to evaluate every possible alternative and choose the best one. However, "*bounded rationality*" accepts the inability to achieve such a level of understanding and analysis, proposing instead a more realistic and practical decision-making process. This process takes into account human and environmental limitations, allowing for quick and pragmatic decisions in complex and uncertain situations.

In summary, while optimization seeks theoretical perfection in an ideal world of complete information and unlimited cognitive capabilities, "*bounded rationality*" focuses on practicality and adequacy, recognizing human imperfections and structural limitations. Both models offer valuable tools for understanding and improving the decision-making process but apply in different contexts, reflecting the complexity and variety of real-world situations.

2.3. Expected Utility Theory

The concept of homo economicus stemming behind the neoclassical theory of choices under uncertainty embodies a hypothetical agent possessing complete information about available choices, perfect foresight regarding the consequences of those choices, and the ability to solve optimization problems—often of considerable complexity—to identify options maximizing personal utility. The evolution of the term 'economic man' traces from John Stuart Mill's depiction of a self-interested individual seeking to maximize personal utility (1844) to modern conceptions rooted in Paul Samuelson's revealed preference theory (1947) and von Neumann and Morgenstern's axiomatization (1944), which shifted the focus from reasoned behavior to choose behavior[3].

[3] Samuelson, P. A. (1947). Foundations of Economic Analysis. Harvard University Press.

Central to modern economic theory is the acknowledgment that individuals inherently prefer certain outcomes over others, even in hypothetical assessments. The canonical paradigm of synchronous decision-making under risk suggests that perfect rationality entails maximizing expected utility—a recommendation contingent upon structured qualitative comparative judgments (i.e., preferences) satisfying specific axioms for mathematical representation, forming the basis of expected utility theory.

2.3.1. Expected Utility Theory in details

A set of axioms forms the basis of expected utility theory, which utilizes a binary relation $\geq$ representing "*is weakly preferred to*." This axiomatic framework applies to prospects, associating probabilities with fixed sets of consequences, where rational agents prefer prospects with higher expected utility. Specifically, if preferences satisfy three constraints, ordering, continuity and independence, then they maximize expected utility:

- *Ordering*: Preferences are both complete and transitive, ensuring that preferences are consistently ranked.
- *Archimedean*: Allows for comparability of prospects, ensuring consistency in comparing preferences.
- *Independence*: Preferences remain consistent regardless of external conditions.

These axioms lead to the formulation of a real-valued function, V(·), representing expected utility, enabling numerical representation of qualitative comparative judgments.

Axiomatic Departures from Expected Utility Theory

Departures from expected utility theory are common in exploring alternative decision-making frameworks. Here, we highlight departures motivated by bounded rationality considerations, focusing on our earlier axiomatization.

Alternatives to Axiom 1

Indecisiveness and incomplete preferences become possible by weakening the ordering axiom, as argued by Keynes and Knight (Keynes 1921; Knight 1921). Specifically, removing the completeness axiom allows agents to neither prefer one alternative over another nor to be indifferent between them (Koopman 1940; Aumann 1962; Fishburn 1982). The completeness axiom's decisiveness is often more a matter of mathematical convenience than a principle of rationality. The key question, which any proposed axiomatic system must address, is the implications of allowing incomplete preferences.

Early axiomatizations of rational incomplete preferences, led by Aumann (1962), include contributions from Giles (1976), Giron & Rios (1980), Karni (1985), Bewley (2002), Walley (1991), Seidenfeld, Schervish, & Kadane (1995), Ok (2002), Nau (2006), Galaabaatar & Karni (2013), and Zaffalon & Miranda (2017). These systems not only accommodate indecision but also enable reasoning about another agent's (potentially complete) preferences when information about their preferences is incomplete.

The transitivity axiom's removal limits the extension of elicited preferences (Luce & Raiffa 1957) by allowing cycles and preference reversals. While violations of transitivity have traditionally been seen as signs of human irrationality (May 1954; Tversky 1969), recent reassessments challenge this view (Mongin 2000; Regenwetter, Dana, & Davis-Stober 2011). The axioms enforce synchronic consistency on preferences, but experimental evidence often conflates dynamic and synchronic consistency (Regenwetter et al. 2011). Inconsistencies between preferences at different times do not imply logical inconsistency at a single moment. Arguments against transitivity in normative accounts of rational preference also point to diachronic or group preferences, which do not violate the axioms (Kyburg 1978; Anand 1987; Bar-Hillel & Margalit 1988; Schick 1986). Moreover, psychological processes or algorithms admitting cycles or preference reversals are not counterexamples to the ordering condition but rather indicate misapplication. In cases involving explicit comparisons over time, violating transitivity may be rational, such as in maximizing food gain by considering current and future options (McNamara, Trimmer, & Houston 2014).

Alternatives to Axiom 2

Dropping the Archimedean axiom allows for lexicographic preferences, where an agent may prefer one option infinitely more than another (Blume, Brandenburger, & Dekel 1991). One motivation for a non-Archimedean version of expected utility theory is to address gaps in standard subjective utility frameworks, particularly regarding admissibility and full conditional preferences (Pedersen 2014). The standard account struggles with conditioning on zero-probability events, crucial in game theory (P. Hammond 1994). Non-Archimedean variants of expected utility theory employ nonstandard analysis (Goldblatt 1998), full conditional probabilities (Rényi 1955; Coletii & Scozzafava 2002; Dubins 1975; Popper 1959), and lexicographic probabilities (Halpern 2010; Brickhill & Horsten 2016 [Other Internet Resources]), linking them to imprecise probability theory. Non-compensatory single-cue decision models, such as the Take-the-Best heuristic (section 7.2), use lexicographically ordered cues and can be numerically represented using non-Archimedean expectations (Arló-Costa & Pedersen 2011).

Alternatives to Axiom 3

A1 and A2 together entail that $V(\cdot)V(\cdot)$ assigns a real-valued index to prospects such that $P \geq Q P \geq Q$ if and only if $V(P) \geq V(Q) V(P) \geq V(Q)$. The independence axiom, A3, ensures that expected utilities are linear in probabilities, encapsulating a separability property for choice. The axiom's removal is motivated by challenges in applying expected utility theory to describe actual choice behavior. For instance, while expected utility theory can represent gambling or insurance purchase decisions, it struggles to accommodate both simultaneously, as noted by Friedman and Savage in their critique of von Neumann and Morgenstern's axiomatization (M. Friedman & Savage 1948).

Loss aversion (Kahneman & Tversky 1979; Rabin 2000) posits that potential losses carry more subjective weight than potential gains, exemplified by the endowment effect, where individuals value a good more when considering its loss than when considering its gain (Thaler 1980). This effect is supported by neurological evidence showing different brain regions processing gains and losses (Rick 2011). However, the affective differences in processing losses and gains do not necessarily indicate a general "negativity bias" (Baumeister, Bratslavsky, & Finkenauer 2001) in decision-making (Hochman & Yechiam 2011; Yechiam & Hochman 2014). Experiments by Yechiam and colleagues show no loss aversion in repetitive situations or single-case decisions with small stakes. Observations of risk aversion (Allais 1953) and ambiguity aversion (Ellsberg 1961) have led to alternative theories to expected utility, all abandoning A3. These alternatives include prospect theory (section 2.4), regret theory (Bell 1982; Loomes & Sugden 1982), and rank-dependent expected utility (Quiggin 1982).

Most models of bounded rationality do not fit into this axiomatic family. This divergence stems from the focus on the processes, algorithms, or psychological mechanisms underlying decision-making, judgments, or goal achievement (section 2). Samuelson's emphasis on choice behavior abstracted away from these details, which Simon critiqued. Bounded rationality also considers adaptive behavior suited to an organism's environment (section 3), where focusing on coherent comparative judgments may not be the best approach.

However, caution is warranted against generalizing the limited role of decision theory in bounded rationality studies. Decision theory, broadly construed to include statistical decision theory (Berger 1980), offers a robust mathematical toolbox. Historically, however, it has propagated psychological myths like "degrees of belief" and logical omniscience (section 1.3). Studying axiomatic deviations from expected utility theory can help diminish Bayesian dogma, allowing for a broader application of powerful mathematical methods.

Many models of bounded rationality diverge from this axiomatic framework, focusing instead on decision-making procedures, algorithms, or psychological processes. Simon critiqued traditional rational choice theory for abstracting away such details. Moreover, bounded rationality often emphasizes adaptive behavior in an organism's environment, where coherent comparative judgments may not always frame the problem optimally.

However, caution is warranted regarding generalizations about the limited role of decision-theoretic tools in studying bounded rationality. Decision theory, encompassing statistical decision theory, provides a robust mathematical toolbox, albeit historically trading in psychological myths. Exploring departures from expected utility theory can offer insights beyond Bayesian dogma, expanding the scope for practical mathematical methods.

2.3.2. Limits to Logical Omniscience

Many formal models of judgment and decision-making assume logical omniscience, wherein agents possess complete knowledge of all logical consequences resulting from their current commitments and considered options. However, this assumption is both psychologically unrealistic and technically challenging to avoid.

The issue of logical omniscience is particularly problematic for expected utility theory and the theory of subjective probability. Postulates of subjective probability imply that agents know all logical consequences of their commitments, leading to a mandate for logical omniscience. However, this limits the theory's applicability, as it prohibits uncertain judgments about mathematical and logical statements.

Savage (1967) highlights this problem, noting that adhering fully to the theory might require computations impractical in reality, such as calculating remote digits of π. Various responses to this problem have been proposed. Good (1983) suggests a game-theoretic treatment, shifting uncertainty from necessarily true propositions to a guessing game facilitated by incomplete information. Another approach introduces an index for incoherence to accommodate reasoning with incoherent probability assessments. De Finetti (1970) proposes restricting possible states of affairs to observable states with finite verification procedures, aiming to distinguish genuine doubt from "paper doubts."

The notion of apparently possible events involves a procedure for determining inconsistency, reflecting bounded procedural rationality. While challenges in avoiding paradoxes are significant, work on bounded fragments of Peano arithmetic provides coherent foundations for exploring these ideas. These concepts have been applied to formulate bounded extensions of default logic and computational rationality models.

In essence, addressing the limits of logical omniscience requires nuanced approaches that balance theoretical rigor with practical considerations, acknowledging the complexities of human cognition and decision-making processes.

2.3.3. Descriptions, Prescriptions, and Normative Standards

Traditionally, discussions on judgment and decision-making contrast what people actually do with what they should do. However, exploring cognitive processes, mechanisms, and algorithms of boundedly rational judgment and decision-making suggests distinguishing among three aims of inquiry. Rather than solely focusing on the actual versus ideal behaviors, we should consider descriptive, prescriptive, and normative theories.

To illustrate, let's examine arithmetic, where these distinctions are particularly evident. A descriptive theory of arithmetic might delve into the psychology of arithmetical reasoning or algorithms for numerical computation. The normative standard here is Peano's axiomatization, defining arithmetic in terms of number succession and mathematical induction. Yet, for practical purposes, such as teaching children arithmetic, adaptations to pedagogy are necessary based on psychological differences. For instance, consider introducing cardinal arithmetic to children already proficient in arithmetic. While drawing from successful pedagogy for full arithmetic, adjustments are inevitable due to shifts in normative standards. Differences may emerge not only from the change in standards but also from observed interplay between tasks and psychological capabilities.

It's crucial to note the distinction between arithmetic and rational behavior. While arithmetic deals with clear-cut objects like numerals and numbers, rational behavior involves varied objects even with consistent theoretical frameworks. Take expected utility theory: agents may deliberate over options to maximize personal welfare, act as if doing so, or play a part in population fitness.

Separating the question of normative standards from behavior evaluation or description helps avoid misunderstandings in discussions of bounded rationality. Although Peano's axioms aren't directly prescribed or described for arithmetical reasoning, they're relevant to both descriptive and prescriptive theories. While it's uncertain whether normative standards for rational behavior can be axiomatized, clear standards significantly enhance our understanding of judgment and decision-making processes, regardless of whether they conform to ideal behaviors.

2.4. From EUT to criticism and Deliberative Stochastic Choice

Developed in the 18th century by Daniel Bernoulli and formalized by John von Neumann and Oskar Morgenstern in the 20th century, Expected Utility Theory (EUT) presented a seemingly objective and rational approach to decision-making. This theory posits that when faced with various outcomes, individuals will choose the one that maximizes their expected utility, essentially a weighted average of the utilities associated with all possible outcomes, weighted by their probabilities. The option with the highest expected utility is considered the most rational choice. While elegantly simple and highly influential in the development of economic and financial models, EUT has faced significant criticism over the decades for its inability to consistently predict real-world decision-making, especially under conditions of risk and uncertainty.

Expected Utility Theory (EUT) has been foundational in economic thought, providing a clear and concise framework for understanding choices under uncertainty. According to EUT, individuals act rationally by maximizing a utility function which depends on the outcomes' probabilities and their respective utilities. However, several criticisms challenge the universal applicability of EUT:

Predictive Failures:

EUT assumes that all risk is quantifiable and that individuals consider all possible outcomes and their probabilities when making decisions. However, in practice, individuals often ignore unlikely outcomes or fail to accurately process probability information, leading to decisions that systematically deviate from EUT predictions[4].

Violation of Independence Axiom:

One of the axioms underpinning EUT is the independence axiom, which asserts that if an individual prefers option A over option B, they should also prefer a gamble that probabilistically results in A or C over one that leads to B or C, regardless of the nature of C. This axiom is violated in real-world scenarios, such as in the famous Allais Paradox, where individuals' choices depend significantly on the presence of certain outcomes that alter the perceived attractiveness of comparable gambles[5].

Empirical Paradoxes:

Several well-documented paradoxes, like the Allais and Ellsberg paradoxes, highlight individuals' preference inconsistencies that EUT cannot explain. These paradoxes illustrate how actual human behavior diverges from the rational agent model, as individuals display risk-averse or risk-seeking behaviors that contradict the predictions of EUT depending on different framing of choices or the ambiguity of probabilities[6]. These criticisms illustrate foundational cracks in EUT, suggesting a need for alternative theories that embrace a more nuanced understanding of human cognition and behavior.

Stochastic choice models represent a significant departure from the deterministic frameworks of EUT, introducing randomness as an intrinsic component of decision-making processes. These models are rooted in the observation that individuals' choices often exhibit variability when confronted with the same decision context repeatedly. This variability is not merely noise or error but can be a deliberate feature of an individual's decision-making strategy[7].

Theoretical Foundations:

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[4] Bernoulli, D. (1738). Specimen theoriae novae de mensura sortis. Commentarii Academiae Scientiarum Imperialis Petropolitanae, 5, 175-192.

[5] von Neumann, J., & Morgenstern, O. (1944). Theory of Games and Economic Behavior. Princeton University Press.

[6] Allais, M. (1953). Le comportement de l'homme rationnel devant le risque: critique des postulats et axiomes de l'école Américaine. Econometrica, 21(4), 503-546.

[7] Agranov, M., & Ortoleva, P. (2017). Stochastic Choice and Preferences for Randomization. Journal of Political Economy, 125(1), 40-68.

The study "Deliberately Stochastic" by Cerreia-Vioglio et al. offers a comprehensive framework for understanding stochastic choices. The authors propose that stochastic choice can be viewed as the outcome of deliberate randomization by decision-makers. This approach contrasts sharply with traditional models that attribute randomness in choice to external factors like fluctuating preferences or informational constraints. Instead, the deliberate stochastic model posits that individuals may use randomness as a strategic tool to optimize across various dimensions, such as time, personal goals, and even psychological factors like regret minimization.

The model developed by Cerreia-Vioglio is based on several key axioms:

- *Preference Completeness and Transitivity*: Unlike EUT, which strictly requires consistent and stable preferences, the stochastic model allows for preferences that can adapt based on context, reflecting a more flexible and realistic portrayal of decision-making.
- *Rational Mixing*: This axiom suggests that when faced with a set of options, the decision-maker considers all possible probabilistic mixtures of these options, choosing a mix that maximizes their expected utility according to underlying, possibly context-dependent preferences.

Empirical Evidence:

The theory is bolstered by empirical evidence suggesting that individuals often engage in what appears to be random behavior when making choices, especially in complex and uncertain environments. For instance, the work by Agranov and Ortoleva points to experiments where subjects are asked to make repeated choices from the same set of options. The findings indicate that different choices are made not due to errors or changes in preference but as part of a deliberate strategy, potentially aimed at exploring different outcomes or managing uncertainty.

Criticisms of the Expected Utility Theory:

Although it is considered to be applicable and acceptable by most people, in recent decades EUT has been severely criticized. This type of criticism had already taken a huge boost almost ten years ago regarding the rationality of EUT assumptions. The point of most critics is that EUT does not reflect reality as to how individuals make decisions when confronted with uncertainty. Critical issues include:

Risk Aversion and Non-linear Probability Weighting:

Under EUT, people would consider decisions related to risk under objective probabilities. Nevertheless, behavioral economists like Daniel Kahneman and Amos Tversky developed Prospect Theory to indicate that for most people, the perception of their probabilities is rather non-linear. For example, they would over-weigh the probability of unlikely events, like winning a lottery, and under-weigh the probability of a likely event[8].

Ignorance and Ambiguity:

EUT assumes the consumer has perfect knowledge of all relevant probabilities. In fact, more often than not, individuals face ambiguity with regard to probabilities and outcomes. Following his work, Itzhak Gilboa and David Schmeidler first presented models, namely the Maxmin Expected Utility and the Smooth Model of Ambiguity, which give a better description of the decision process in cases where probabilities are unknown or imprecise[9].

Preference Reversals:

There is abundant experimental evidence for the fact that the decisions of people are sensitive to the framing of choice problems, elicitation procedures, and other parts of the context to the extent that preference reversals are quite common, thus violating consistency in EUT[10].

The Move Toward Imprecision:

In response to these criticisms, new theories have been developed that admit imprecision in the definition of probabilities and utilities. Such models typically drop one or more of the strict assumptions of the EUT, hence loosening the way in which uncertainty is modelled and handled:

Expected utility of Choquet:

This theory uses capacities, which are non-additive probability measures, to represent ambiguity in decision-making. Capacities hold a more flexible treatment of events, especially in the case when information is incomplete or imprecise[11]. Another approach uses imprecise probabilities: that is, the representation of uncertainty with sets of

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[8] Kahneman, D., & Tversky, A. (1979). Prospect Theory: An Analysis of Decision under Risk. Econometrica, 47(2), 263-292.

[9] Gilboa, I., & Schmeidler, D. (1989). Maxmin expected utility with non-unique prior. Journal of Mathematical Economics, 18(2), 141-153.

[10] Tversky, A., & Thaler, R. H. (1990). Anomalies: Preference Reversals. Journal of Economic Perspectives, 4(2), 201-211.

[11] Schmeidler, D. (1989). Subjective Probability and Expected Utility without Additivity. Econometrica, 57(3), 571-587.

probabilities rather than single point estimates. This allows decision-makers to model their knowledge and ignorance more realistically[12].

Fuzzy Logic:

Proposed by Lotfi Zadeh, this provides an approach to handle imprecision in data and reasoning. Fuzzy logic may be applied to the management of data in a decision-making process not only in which the data are uncertain but in which imprecision can be found; this forms a mathematical framework to capture the subtleties of human reasoning[13].

These developments reflect a broader movement in decision theory and economics towards adaptations that can accommodate more realistic and complex characteristics of human behavior and decision-making processes. The shift from the clearly defined model of the rational agent to one that somehow integrates behavioral insights and acknowledges the limits of human rationality and knowledge represents perhaps a significant change. More factors are now considered, even at the neuronal level, whereas a few years ago, one did not question the reasons behind choices, “simply” attributing them to a specific utility function of the individual or to a "bundle of utility functions" from which the individual drew depending on the need.

2.5. The theory of Stochastic Choice

The theory of stochastic choices is a field within decision theory that has seen significant evolutions over the years. Traditional literature and current literature present different approaches, experimental methodologies, and theoretical models to understand and describe human decision-making behavior under uncertainty. Also, current literature has evolved to include new models that explain decision-making behavior more comprehensively and accurately.

2.5.1. Deliberate Randomization Model and Cautious Stochastic Choice Model

Deliberate randomization models propose that individuals may intentionally randomize their choices as a strategy to achieve certain goals, such as minimizing regret or hedging between uncertain outcomes. Cerreia-Vioglio et al. (2016) introduced the cautious

[12] Walley, P. (1991). Statistical Reasoning with Imprecise Probabilities. Chapman and Hall.
[13] Zadeh, L. A. (1965). Fuzzy sets. Information and Control, 8(3), 338-353

stochastic choice (CSC) model, which posits that individuals have a set of utility functions and choose to randomize when it provides a higher expected utility across these functions. This model is particularly relevant for explaining stochastic choice in environments where no single option is clearly superior.

Cerreia-Vioglio et al. (2019) propose an innovative approach to understanding stochastic choices proposing the Cautious Stochastic Choice model (CSC). This model suggests that individuals may deliberately choose stochastically to optimize their decisions in the presence of uncertainty about their own preferences. The theory of stochastic preferences considers that individuals' preferences are not fixed but can vary stochastically in response to different conditions and available information.

The Cautious Stochastic Choice (CSC) model, postulated by Cerreia-Vioglio in 2016,explores stochastic behavior in individual decision-making, linking it to a preference for risk and deviations from Expected Utility theory. Specifically, the model posits that stochastic choices may stem from a deliberate preference for randomization, reflecting a form of "caution" in decision-making. This caution is manifested, for example, through a preference for combinations of choices that offer a certain safeguard against unfavorable outcomes, even when it involves rejecting dominated or suboptimal options.

The model predicts that violations of Regularity (a principle stating that the introduction of new options should never increase the probability of selecting an existing option) and Strong Stochastic Transitivity are natural consequences of deliberate randomization. In other words, when individuals' preferences are strictly convex, their choices may show a preference for combinations of options that provide risk coverage, leading to choices that do not adhere to Regularity. This behavior can explain phenomena such as the compromise effect and the attraction effect without dominance, where a choice is preferred because it balances or attracts relative to other available options.

Furthermore, the model suggests that the agent evaluates not only the final outcome of lotteries but also the process leading to these outcomes, using randomization to achieve an optimal distribution of results.

In 2017, Agranov & Ortoleva referred to "Deliberate Randomization" in the context of CSC, asserting that stochasticity is a deliberate choice by the individual. The passage discusses the Cautious Stochastic Choice (CSC) model, which explains the deliberate stochastic decision-making by agents. The central idea is that randomness in choice is not merely due to indecision or noise but can be a strategic choice.

The model introduces specific notation and definitions: $\rho(A)$ represents a probability distribution over a set of lotteries A, resulting in a compound lottery. The induced lottery over final outcomes is denoted as $\bar{\rho}(A)=\sum_{q\in A}\rho(q)q$. The convex hull of a set A, co(A), includes all possible combinations of elements in A that respect convexity.

In the CSC model, agents possess a compact set of utility functions, W, which are continuous, strictly increasing, and concave. The agent's choice involves maximizing a value function, V(p), where p belongs to the convex hull of set A. The value function, V(p), is defined as the minimum inverse of utility functions applied to the expected value of the utility over the lottery. This approach allows for the consideration of all possible randomizations over options, leading to potential preferences for mixed outcomes if they offer higher utility.

The model suggests that agents might exhibit stochastic choice due to the shape of V and the cautious expected utility model, which involves computing the certainty equivalent for each utility function and selecting the smallest one. This behavior can result in a desire to hedge between options, especially when different options provide varying levels of utility. Consequently, the CSC model predicts that agents may show stochastic choice both when choices are repeated over time and when they occur consecutively.

Later, in 2019, Cerreia-Vioglio et al. discussed "*Deliberative Stochastic*[14]" ,studying stochastic choice as a result of deliberate randomization. They derived a general representation of a stochastic choice function, where stochasticity allows the agent to obtain the maximal negative element from any set based on their underlying preferences concerning lotteries.

Moreover, Agranov and Ortoleva (2017) conducted experiments in which tested such a theory leaving subjects facing the same questions repeatedly, both in spaced sessions and in close sequence. They found that a large majority of subjects exhibited stochastic choices in both conditions, suggesting that stochastic behavior may be a deliberate phenomenon rather than a simple error. This study introduced the idea that people may

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[14] Cerreia-Vioglio, S., Dillenberger, D., Ortoleva, P., & Riella, G. (2021). Deliberately Stochastic. Econometrica, 89(1), 93-119.

choose to respond differently to identical questions as an optimal strategy to minimize regret or diversify their choices.

These approaches represent a significant advancement over traditional models by recognizing that stochastic behavior can be a deliberate and rational component of decision-making rather than simply the result of random errors.

The main difference between traditional and current literature lies in the interpretation and modeling of stochastic behavior. While traditional literature attributes stochastic behavior primarily to random errors and variability in preferences, current literature proposes that stochastic behavior can be a deliberate and rational strategy.

Traditional literature uses models such as the constant probability error model and random utility models to explain variable choices. These models assume that preferences are deterministic and that random errors are responsible for the observed variability. In contrast, current literature adopts models such as stochastic preferences and cautious stochastic choice, which consider the possibility that individuals' preferences are inherently variable and that stochastic choices are a rational response to this variability. These models recognize the importance of diversification strategies and regret minimization in individuals' decisions.

Therefore, while traditional literature focuses on incorporating random errors into deterministic choices, current literature moves towards a more nuanced and comprehensive understanding of stochastic choices, recognizing that such choices may be deliberate and rational.

2.5.2. Other models

For completeness, the other two models, in addition to the aforementioned deliberate randomization and cautious stochastic choice (CSC) model, that are useful to our experiment are *random utility/preferences* and *bounded rationality*.

Random Utility:

A well-known class of stochastic choice models is that of random utility or random preferences. According to this model, when individuals make a decision, they maximize a well-defined utility function (or preference), but this utility changes stochastically over time. The relevant model for analyzing lottery choices is *Random Expected Utility (REU)*, as studied by Gul and Pesendorfer (2006). In this model, the agent has a probability distribution over strictly increasing utility functions defined on money, with support U. The probability of choosing a lottery from a set A is equal to the probability that the agent has a utility function that uniquely maximizes that choice within A. Specifically, for every element p in set A:

$r(A)(p)=m(u \in U: u(p)>u(q) \text{ for all } q \in A)$

The common interpretation of why the utility function is stochastic suggests that an individual's utility changes due to variations in exogenous subjective and objective conditions, such as new information, mood, or social context. This means that while a person may follow a utility-maximization rule, their preferences fluctuate due to these external factors, leading to stochasticity in their choices.

Bounded Rationality:

A second class of models assumes that individuals have a well-defined and stable ranking of the available options, but may not choose the option that maximizes their utility due to some form of bounded rationality. Suppose an agent must choose between two options, p and q, with values $u(p)$ and $u(q)$. At each moment, the agent receives a noisy signal in favor of one of the two options. Assuming positive values indicate that p is better, the agent accumulates all the evidence collected and, once it surpasses a threshold, makes a decision.

This model implies that even though individuals have a consistent preference order, they might not always pick the highest-ranked option because of limitations in processing information, time constraints, or cognitive biases. Bounded rationality models capture how decision-making can be suboptimal due to these constraints, leading to stochastic behavior despite having stable preferences.

3. Empirical evidence

A major part of economic models for decision-making under uncertainty has been the expected utility theory, where one maximizes their expected utility under uncertain situations while making decisions. A theory of decision under uncertainty, expected utility, was developed by Von Neumann and Morgenstern, based on the assumption that a person is rational and weighs outcomes by their probability of occurrence, selecting the alternative that provides the maximum expected benefit. However, a large number of empirical works have proven that, in practice, people often diverge from the predictions of EUT, especially in an uncertain environment. One of the most influential pieces of evidence on the violation of EUT was provided by Daniel Kahneman and Amos Tversky in 1979 in the form of Prospect Theory. In this pioneering work, they demonstrated that people do not generally make choices that maximize expected utility, particularly when the choices involve risk and uncertainty. Instead, people tend to show two key biases in cognitive judgment: loss aversion, in which losses loom larger than gains, and framing effects, in which the way a problem is posed affects decision-making.

The work of Kahneman and Tversky was a breakthrough, shifting the focus away from the old model of rationality toward behaviors that are more complex and less predictable. For example, they found that people overestimate small probabilities and underestimate large probabilities, behavior inconsistent with EU theory. These findings have been confirmed by many studies since and clearly illustrate the inadequacy of EUT in explaining real human behavior under uncertainty. While it is apparent that people still maximize some objective function under uncertainty, there is no guarantee that the form of this function will align with classical theory. Behavioral economics has sought alternative functions or models that better describe the decision-making process under uncertainty.

Among the most important is Cumulative Prospect Theory (CPT), which Kahneman and Tversky improved to address some drawbacks of the original theory. CPT incorporates the concept of probability weighting, explaining why people systematically deviate from expected utility. This theory posits that, while people attempt to make appropriate choices according to their objectives, their preferences are not guided by utility maximization in the classical sense. Instead, they are driven by psychological factors related to perceived value, regret, and loss aversion. Even Richard Thaler's hypothesis of mental accounting questions the predictions of EUT. Thaler argues that people "keep their money in different mental accounts and treat it differently depending on where it came from or what it was specifically meant to be used for." This again moves further away from the idea that people make decisions with a view to maximizing a single function of utility.

This behavior demonstrates the complexity of human decision-making, requiring more nuanced models than those provided by EUT. The ongoing central problem in both economic theory and application is the search for general functions that explain the decision-making process under uncertainty. Although EUT is mathematically elegant and convenient, efforts to find more realistic representations of human behavior within an axiomatic framework have led to several competing theories. These include Rank-Dependent Utility (RDU), Regret Theory, and various forms of Prospect Theory. Rank-Dependent Utility Theory offers a mechanism where subjects first rank the outcomes according to their probabilities and then apply utility to these ranks rather than adding the utility of each outcome. It provides flexibility in determining how people perceive risks and make choices in an uncertain environment.

Regret Theory, developed by Loomes and Sugden in the 1980s, argues that humans anticipate the regret they might feel after a wrong decision. According to this theory, the decision-making process is affected not only by potential outcomes but also by the fear of future regret, thus further deviating from EUT predictions. There is still an ongoing debate over the superiority of one theory over another in behavioral economics. While EUT is rooted in rigorous mathematical logic and the concept of rational choice, alternative models often provide more accurate empirical explanations of observed behaviors.

An important contribution to the comparison of these models has been made by Colin Camerer with his work on experimental economics, including his work on "Behavioral Game Theory." Camerer shows that no single theory can entirely replace EUT, but its modifications, in the form of bounded rationality and models that incorporate psychological factors, offer better insights into the decision-making process under uncertainty. While EUT exhibits systematic violations, it has still contributed significantly to economic theory. Clearly, alternative models are becoming increasingly important in explaining human decision-making..

Regarding the empirical relevance, there have been several experiments on stochastic choices. The most notable one, which I will discuss in detail and which we partially replicated in our pilot study, is the experiment conducted by Agranov & Ortoleva. However, before delving into this study, I will briefly mention previous work on the topic, while referring to the appendix of this chapter for further details on other experiments.

3.1. Experimental Evidence

Experimental studies provide critical insights into stochastic choice behavior. Agranov and Ortoleva's (2017) experiment is particularly noteworthy for its innovative design, which included both distant and consecutive repetitions of choices. Their findings revealed that stochastic choice occurs predominantly in "HARD" questions, where none of the available options is clearly better than the others. This result suggests that stochastic choice is not merely a result of noise but may reflect a deliberate strategy employed by individuals when faced with HARD decisions.

The same study also explored subjects' attitudes toward risk and randomization. It found that a significant fraction of subjects preferred to randomize their choices, even at a cost, indicating a preference for mixed strategies in uncertain environments. Additionally, stochastic choice was found to be correlated with violations of expected utility theory, such as the Allais paradox, further highlighting the complexity of human decision-making processes.

Furthermore, in the experiment, subjects were asked to choose between different lotteries. The results showed that subjects often chose different lotteries when the same set was presented multiple times, confirming the presence of stochastic choice. This experiment also revealed that subjects who were more prone to stochastic choice tended to violate the expected utility theory, aligning with findings from the Allais paradox.

Temporal consistency is a critical aspect of stochastic choice experiments. Studies have shown that the frequency of stochastic choice decreases as the time between repeated choices increases. This suggests that short-term factors, such as mood or immediate context, can significantly influence decision-making. Conversely, when repetitions are temporally distant, the consistency of choices tends to improve, indicating a stabilization of preferences over time.

An experiment by Regenwetter et al. (2011) demonstrated that when subjects were asked to make repeated choices between lotteries over extended periods, the variability in their choices diminished. This finding supports the notion that stochastic choice is influenced by short-term fluctuations in cognitive and emotional states.

The existence of stochastic choice has profound implications for economic theory, particularly for models of consumer behavior and market predictions. Traditional models that assume stable and consistent preferences may fail to accurately predict real-world behavior if they do not account for the variability introduced by stochastic choice. This has led to a reevaluation of foundational concepts in economics and the development of new models that incorporate elements of stochasticity.

Understanding stochastic choice is also crucial for policy design and practical applications. For example, in fields such as finance and marketing, recognizing that consumers may not always act consistently can lead to better models for predicting market trends and designing interventions. Policies aimed at improving decision-making, such as providing better information or simplifying choices, can be more effective if they take into account the likelihood of stochastic behavior.

Financial Decision-Making

In financial decision-making, recognizing the presence of stochastic choice can improve models that predict investor behavior. For instance, models that incorporate stochastic choice can better capture the variability in investment decisions, leading to more accurate forecasts of market movements and more effective portfolio management strategies.

Marketing Strategies

In marketing, understanding stochastic choice can help in designing more effective campaigns. Marketers can create strategies that account for the variability in consumer preferences, leading to more personalized and targeted marketing efforts that are likely to resonate better with consumers.

3.2. Psychological and Cognitive Foundations

Research has shown that cognitive load can significantly impact decision-making processes. When individuals are faced with complex or numerous choices, the cognitive effort required to process all available information can lead to stochastic choices. This is particularly evident in "HARD" questions, where the optimal choice is not clear, and individuals may opt to randomize their decisions to cope with the cognitive burden.

An experiment by Ballinger and Wilcox (1997) demonstrated that increasing the cognitive load on subjects led to a higher incidence of stochastic choice. Subjects who were given more complex decision tasks exhibited greater variability in their choices, supporting the idea that cognitive load contributes to stochastic behavior.

Emotional Influences

Emotions play a crucial role in decision-making. Emotional states such as stress, anxiety, or excitement can lead to fluctuations in preferences, resulting in stochastic choices. Studies have shown that individuals under stress are more likely to make inconsistent choices, as their emotional state interferes with rational decision-making processes.

A study by Starmer and Sugden (1989) explored the impact of emotional states on stochastic choice. They found that subjects who were placed in stressful situations exhibited more variability in their choices compared to those in a neutral state. This finding highlights the importance of considering emotional factors in understanding stochastic choice.

The literature on stochastic choice provides a rich and nuanced understanding of human decision-making. Through experimental evidence and theoretical modeling, researchers have uncovered the various factors that contribute to stochastic choice and developed frameworks to explain this behavior. As the field continues to evolve, further research will likely delve deeper into the cognitive and psychological mechanisms underlying stochastic choice, with implications for theory and practice.

3.3. Stochastic Choice and Preferences for Randomization

At the core of my research, the main study considered is "Stochastic Choice and Preferences for Randomization" by M. Agranov and P. Ortoleva (2017). They conducted an experiment in which subjects encountered the same questions repeated multiple times, with two types of repetitions (replicated in our study). Following previous literature, the repetitions were first presented at distant intervals and then consecutively, with subjects being informed of the repetition. Their results show a degree of stochasticity in both cases, and in the next chapter, I will compare their findings with ours.

3.3.1. Design of the Experiment

The experiment consists of four parts, with the main sections being Part I and Part III, in which subjects were asked many questions repeated several times.

In the **first part** of the experiment, subjects were asked to make a series of choices between lotteries. These choices were repeated several times, but the repetitions were spaced out and interspersed with other questions. Importantly, the participants were not informed that they would encounter the same questions again. This setup was intended to observe whether individuals would consistently choose the same option when unaware that the decision had already been presented. The goal here was to capture the natural variability in decision-making and determine if subjects exhibited stochastic choice—making different selections when presented with the same choice multiple times.

The **second part** of the experiment acted as a break from the repetitive questioning. In this section, subjects engaged in an investment task designed to measure their risk tolerance and their attitudes toward compound lotteries. By introducing this task between the two main parts of the experiment, the researchers aimed to reduce any fatigue that might have resulted from the repeated questions in the first part, while also gathering additional data on the participants' risk preferences.

The **third part** of the experiment reintroduced a subset of the questions from the first part. However, this time the repetitions occurred consecutively and participants were explicitly informed that they would be asked the same question three times in a row. This design was crucial for testing whether knowing about the repetitions would influence the subjects' choices. It provided insight into whether the stochastic choice observed in the first part was due to a lack of awareness of the repetition or if it was a deliberate strategy by the subjects to randomize their choices.

In some of these questions, subjects were given the option to choose a lottery directly or to have a coin flip determine the outcome between two lotteries, with a small cost associated with choosing the coin flip. This addition allowed the researchers to further investigate whether participants had a preference for randomization and whether they were willing to pay for it.

Finally, the **fourth part** of the experiment tested subjects' adherence to expected utility theory by presenting them with questions designed around the Allais paradox, a well-known problem in decision theory. The experiment concluded with a questionnaire in which subjects were asked to reflect on their decision-making process, particularly why they might have chosen different options when faced with the same question multiple times in part three.

3.4. Results of the Experiment

3.4.1. Part I and Part III

In Parts I and III of the experiment, the authors observed that the majority of subjects exhibited stochastic choice behavior, particularly in "HARD" questions where there was no clear dominant option. In part I, where repetitions of the same questions were spaced out, 90% of the subjects chose different options in at least one instance of the repeated questions. The stochastic behavior was predominantly observed in “HARD” questions, with little to no inconsistency in “EASY” or “FOSD”[15]. The findings confirmed that stochastic choice is common when the decision is HARDand when options are closely balanced in terms of attractiveness.

Part II involved an investment task to measure risk attitudes (replicated in our experiment), which served as a break between the repetitive questioning in Parts I and III. The results from this part were used to assess the subjects' risk preferences, but the main focus of the section was on the behavior observed in Part I.

3.4.2. Flip the coin

In this part, the authors analyzed the behavior of subjects when given the option to flip a costly coin to determine their choice between two lotteries. This option was included to test whether subjects would pay to randomize their decision, indicating a deliberate preference for randomness. the choice to flip the costly coin provided strong evidence that some subjects prefer to randomize their decisions, especially in HARDdecision-making scenarios.

3.4.3. The relationship between Stochastic Choice and Expected Values

The authors investigated the relationship between stochastic choice—where subjects choose differently when faced with the same options—and the expected values or utilities

[15] First-Order Stochastic Dominance (FOSD) occurs when one lottery is always at least as good as another across all outcomes and strictly better in at least one case. A rational decision-maker would always prefer a lottery that first-order stochastically dominates another.

of the choices presented, in order to determine whether the similarity in expected values of the options could explain the variation in subjects' decisions.

Firstly, the researchers found that the difference in expected values or utilities between the options did not fully account for the occurrence of stochastic behavior. Even when controlling for these differences, the tendency to choose differently remained significantly higher in "HARD" questions compared to "EASY" or FOSD questions.

To test this relationship, they used different models of utility, including risk-neutral, constant relative risk aversion (CRRA)[16], and constant absolute risk aversion (CARA)[17]. Regardless of the model used, the "hard" questions consistently showed a higher rate of stochastic choice, indicating that the complexity of the decision plays a significant role in driving this behavior.

These findings challenge many traditional models of stochastic choice, such as the drift diffusion model (DDM) (see the appendix for further details), which predict that stochastic choice should be more prevalent when the differences in expected utilities are small. The data showed that while this might be true within certain types of questions, it does not explain the broader pattern of stochastic choice observed across different types of decisions.

Furthermore, the authors also considered and dismissed alternative explanations, such as indifference between options or a process of "preference discovery" where subjects gradually form their preferences during the experiment. The high rate of stochastic choice, even in later parts of the experiment, suggested that these were not the primary drivers of the observed behavior.

The results suggest that factors beyond simple differences in expected utility, such as the inherent difficulty of the decision, play a critical role in whether individuals exhibit stochastic behavior. The results supports the idea that people might deliberately introduce randomness into their choices, particularly in situations where no option is clearly superior.

[16] **CRRA (Constant Relative Risk Aversion)**: A utility model where the individual's risk aversion is constant relative to changes in wealth, meaning their proportionate aversion to risk remains consistent regardless of the level of wealth.

[17] **CARA (Constant Absolute Risk Aversion)**: A utility model where the individual's risk aversion remains constant in absolute terms, meaning their aversion to risk does not change as their wealth increases or decreases.

3.4.4. Questionnaire

In the final part of the experiment Agranov and Ortoleva administered a questionnaire, where subjects were asked why they might have chosen different options when faced with the same question multiple times in Part III of the experiment, where questions were repeated consecutively.

The majority of respondents indicated that their choice to vary their answers was deliberate. Many participants mentioned that they engaged in this behavior as a form of hedging or diversification, with the goal of optimizing their potential earnings. This suggests that, rather than being a result of indecision or confusion, the stochastic choices observed were often a conscious strategy. Participants explained that when no clear superior option was available, they preferred to explore different outcomes or avoid committing to a single choice.

The consistency between the participants' explanations and their actual behavior during the experiment reinforces the idea that stochastic choice is frequently a deliberate and rational approach.

3.4.5. Response Time

Moving on to the analysis of reaction times, the key findings are as follows:

- **Longer Reaction Times for "HARD" Questions**: Subjects took significantly longer to make decisions on "difficult" questions compared to "easy" or "first-order stochastically dominated" (FOSD) questions. This pattern was consistent across both Parts I and III of the experiment. The extended response time for HARDquestions suggests that these decisions required more cognitive effort, aligning with the observed higher frequency of stochastic choices in these scenarios.
- **Decrease in Response Time with Repeated Questions**: In Part III, where questions were repeated three times in a row, subjects showed a noticeable decrease in response time from the first to subsequent repetitions of the same question. The first repetition took significantly longer, while the second and third repetitions were completed much more quickly, often within a couple of seconds. This rapid decrease in response time indicates that once subjects recognized the question, they quickly made their decisions in the later repetitions without additional deliberation, likely relying on their initial judgment or memory.

- **Comparison Between Parts I and III**: In Part I, where the same questions were repeated with other questions interspersed, subjects took longer to answer each repetition of HARDquestions compared to their response times in Part III. This suggests that the distant repetitions in Part I were treated as more distinct, requiring more thought each time. In contrast, in Part III, the consecutive nature of the repetitions led to quicker responses in the second and third iterations, showing that subjects did not treat these as entirely new questions but rather as opportunities to repeat or slightly adjust their earlier choices.

  The authors found no significant difference in response times between subjects who exhibited stochastic choice and those who did not, except in one instance (HARD1 in Part I), where inconsistent responders took longer. This suggests that the decision to make a different choice in repeated questions did not necessarily result from a lack of attention or rushed decision-making.
- **Consistency with Models Like the Drift Diffusion Model (DDM)[18]:** The findings, particularly the rapid response times in later repetitions in Part III, are consistent with models such as the Drift Diffusion Model (DDM), where decisions are based on initial information, and subsequent repetitions are handled more automatically.

3.4.6. Relation with Risk Attitudes and Violations of Expected Utility

They conclude the analysis by studying the relation between stochastic choice, the attitudes toward risk and compound lotteries, measured in part II, and Allais-type questions in the questionnaire.

In matter of risk attitudes, the majority of subjects were risk-averse, as evidenced by their behavior in the investment task in Part II of the experiment. Specifically, 63% of subjects invested less than the maximum amount in the risky project, indicating risk aversion.

Despite this, the analysis found little correlation between risk aversion and stochastic choice. The only significant correlation observed was a weak positive relationship between risk aversion and stochastic choice in Part I, but this was not consistently found in Part III.

For Compound Lotteries, subjects generally exhibited either neutrality or aversion to compound lotteries, with 54% being neutral and 43% showing aversion. However, there

[18] See APPENDIX

was no significant relationship between these attitudes and the tendency to exhibit stochastic choice.

This proposes that preferences for or against compound lotteries did not play a major role in the decision to randomize choices in the experiment.

Approximately 25% of the study participants violated the principles of expected utility, as evidenced by their responses to Allais paradox-type questions. These violations were consistent with findings from similar studies. Importantly, the analysis revealed a significant positive correlation between violations of expected utility (Allais-like behavior) and stochastic choice in both Part I and Part III of the study. Subjects who were prone to these violations were more likely to exhibit stochastic choice.

The observed correlation between Allais-type behavior and stochastic choice supports the Cautious Stochastic Choice (CSC) model proposed by Cerreia-Vioglio, which predicts that individuals with multiple utilities (a broader set of preferences) are more likely to engage in both stochastic choice and exhibit Allais-like behavior. This gives the idea that the tendency to randomize choices and to violate expected utility principles may stem from a similar underlying preference structure, where individuals hedge their decisions due to uncertainty or a desire to avoid regret.

Finally, as mentioned earlier, the study shows that the same cognitive processes leading to inconsistent choices in repeated decisions might also drive deviations from expected utility theory, particularly in complex or ambiguous decision-making contexts.

4. Experiment

This study analyzed the stochasticity of participants' choices across different types of lotteries by employing a methodological approach that divides the collected data into two distinct periods: a first period (Rounds 1-240) and a second period (Rounds 241-300). The primary objective was to measure stochasticity through the variance of choices and the absolute differences between successive choices, in order to assess the consistency and variability of decisions over time. This study was conducted in an experimental context similar to that described by Agranov and Ortoleva (2017), allowing for a direct comparison of results.

The aim of this research is to investigate whether stochastic choice, often observed in complex decision-making contexts, can be attributed not only to errors or cognitive limitations, as traditionally believed, but also to a conscious and deliberate strategy adopted by individuals. Until recently, stochasticity in choices has predominantly been interpreted as a sign of inconsistency or a failure in the decision-making processes of the human brain, suggesting that people were making errors in evaluating available options. However, the theory of deliberate randomization offers a new perspective, proposing that individuals may intentionally choose to randomize their decisions to maximize perceived benefits, such as minimizing regret or diversifying risks.

4.1. Materials and Methods

Study Design

The study was structured to evaluate the decision-making behavior of participants when faced with different types of lotteries, divided into three categories: FOSD (First Order Stochastic Dominance), EASY, and HARD. These categories were utilized to examine the stochasticity of participants' choices, considering the various decision-making complexities associated with each category. The initial period, Rounds 1-240, focused solely on the first choice (choice1) of each participant, while in the second period, Rounds 241-300, three consecutive choices (choice1, choice2, choice3) were considered to evaluate the evolution of consistency in choices over time.

The 60 lotteries used in this study were taken from the "Circles and Triangles" experiment by J. Hey and Daniela DiCagno (1990). The study aims to estimate linear indifference curves within the Marschak-Machina Triangle and determine whether a subset of non-

SEUT (Subjective Expected Utility Theory) models better describes preferences than SEUT itself. Additionally, the study seeks to assess whether non-parallel indifference curves differ significantly. The experimental design involves 68 subjects who were presented with 60 paired preference questions using random prospects from four Marschak-Machina triangles (see appendix). The subjects were given a total of 60 such preference questions. Each pair was a pair of gambles from some Marschak-Machina Triangle. Four such Triangles were used - covering the range from £0 to £30. Triangle One had amounts £0, £10 and £20; Triangle Two £0, £10 and £30; Triangle Three £0, £20 and £30; and Triangle Four £10, £20 and £30. So any one of the 60 preference questions involved a pairwise choice between two gambles, the outcomes of both involving at most three of the four amounts £0, £10, £20 and £30.

Subjects were required to choose between two prospects represented by circles on a screen. Each choice was recorded based on the expressed preference for one of the two prospects or indifference. The experiment was computerized and conducted in a laboratory setting, with appropriate financial incentives to motivate participants. The subjects' responses were used to estimate three preference functionals: Subjective Expected Utility Theory (SEUT), Differential Regret, and Generalized Regret.

So, the selection of the 60 lotteries was made deliberately, as they were already available at LUISS, having been previously used by Professor Di Cagno and J. Hey in their study. Once the lotteries were acquired, they were first manually represented in pie charts and then graphically depicted using the Otree software, with the assistance of Andrea Lombardo.

Additionally, we were able to align our lotteries with those used by Agranov and Ortoleva (2017), as in the key parts of their experiment, participants were also asked to choose between two lotteries displayed on a screen with amounts and probabilities of .25, .5, .75, and 1—exactly what we had! This similarity allowed for a direct comparison between the experiments and facilitated our analysis.

Stochasticity was measured using the variance of choices as an indicator of decision dispersion and the absolute differences between successive choices to measure variability over time. This methodology allowed for the identification not only of the levels of stochasticity present in the different lottery categories but also for observing participants' learning or adaptation as the experiment progressed. This experimental design followed a protocol similar to that used in previous studies in the field of behavioral economics, such as those by Agranov and Ortoleva (2017), to allow for a direct comparison of results. The experiment was divided into three main parts:

- Part I: In this phase, participants were required to make a series of choices between pairs of lotteries, presented sequentially on a computer screen. Each participant made only one choice for each pair of lotteries presented (choice1). This phase was designed to analyze participants' initial choices without influences from immediate repetitions.
- Part II: This part included a series of questions regarding investment in a risky lottery and a composite lottery to evaluate participants' attitudes toward risk. The decisions made in this phase were not subject to the same dynamics as the choices between lotteries in Parts I and III but served to provide additional context for understanding individual risk preferences.
- Part III: Similar to Part I, this phase required participants to make choices between pairs of lotteries. However, unlike Part I, the same pairs of lotteries were presented three times consecutively (choice1, choice2, choice3), allowing the analysis of how immediate repetition affects choice consistency and potential learning.

This experimental design enabled the observation not only of the stochasticity of choices but also of the effects of repetition and learning on decision-making behaviors.

Sampling

Sampling was conducted among university students, selected through a voluntary call within the institution. The study involved a group of university students selected from those attending courses in economics, law, and political science, aged between 18 and 25 years. The sample consisted of a total of 18 participants.

Participants were recruited through university email announcements among those enrolled in the "CESARE Experimental Economics Laboratory," with the incentive of monetary gains based on their performance in the experiment. Before the experiment began, participants were informed of the study's objectives and provided their informed consent. The total number of participants was sufficient to ensure the statistical validity of the results, with a sample balanced in terms of gender and geographic origin.

Each participant received a small monetary compensation for the time dedicated to the experiment (a participation reward of €5), with additional potential earnings tied to the choices made during the experiment. This compensation structure ensured that participants were motivated to take their decisions seriously, as their earnings directly depended on their choices.

Instrument Presentation

The main tool used for the experiment was the computerized software OTREE, which presented participants with different pairs of lotteries on a screen. Each pair of lotteries was displayed with associated probabilities and possible earnings for each option. Participants had to choose between the "LEFT"(SINISTRA) and "RIGHT"(DESTRA) options, corresponding to one of the two presented lotteries. Decisions had to be made within a 20-second time limit to avoid delays and maintain participants' attention and focus.

The software was designed to be intuitive and easy to use, with clear instructions provided before each phase of the experiment. Additionally, each participant had the opportunity to ask clarifying questions to the experimenters during the experiment, minimizing the possibility of misunderstandings or errors in choices.

The structuring of the lotteries followed the principles described in existing literature, particularly those used in studies by J. Hey and D. DiCagno. The lotteries were divided into three categories:

- FOSD: Lotteries in which one option stochastically dominated the other, theoretically making the better choice obvious.
- EASY: Lotteries in which one option was perceptibly better, but without clear stochastic dominance.
- HARD: Lotteries in which there was no clear dominant option, making the decision more complex and susceptible to greater stochasticity.

Each category was equally represented during the experiment to ensure that the results reflected a comprehensive range of decision-making difficulties.

Operational Timing

The entire experiment was divided into three distinct time sessions. Part I involved 240 choice rounds, where each pair of lotteries was presented to the participants without any indication of future repetitions. After a brief interval, participants moved on to Part II, which consisted of a double investment decision session (*Risk and Compound Lottery questions*), done pen and paper, because subjects do not always trust computers. Finally, in Part III, participants faced another 60 choice rounds, where the same pairs of lotteries from Part I were presented three times consecutively, and participants were aware of these repetitions.

The total experiment time was approximately two hours, divided so that each part had sufficient time to ensure that participants could complete their choices without excessive fatigue, but also without unduly prolonging the operational timing.

This temporal structure allowed for the monitoring of potential changes in participants' choices between spaced (Part I) and consecutive (Part III) repetitions, providing a detailed picture of decision-making mechanisms and the stochasticity of choices.

4.2. Results and Comparison

The results of the study highlight a clear distinction in the stochasticity of choices among the different categories of lotteries, analyzing the variances in choices across the two periods (Round 1-240 and Round 241-300). HARD lotteries (DIFFICULT) exhibited greater stochasticity compared to other categories in both periods. Specifically, the variance in choices for HARD lotteries was 25.5% in the first period, slightly reducing to 23.1% in the second (TABLE 1). This reduction, albeit modest, might indicate some degree of learning or adaptation by the participants, who became more consistent in their choices as the experiment progressed.

Easy lotteries (EASY) showed a stochasticity of 24.8% in the first period, which decreased to 22.6% in the second, suggesting a more pronounced improvement in decision consistency. This result could indicate that as participants became more familiar with the options, their choices tended to be more predictable and less variable.

Regarding the FOSD lotteries, the variance remained relatively low, with a slight increase from 17.6% to 18.5% between the first and second periods. This increase might suggest that, although participants generally identified the dominant option correctly, some uncertainty factors might have influenced their choices in the second period.

To classify the lotteries as EASY, HARD, or FOSD, both reaction time and the structure of the lottery itself were taken into account, sometimes considering whether the lottery appeared visually “easier” or “more difficult.” Below is the list:

EASY: 1, 5, 6, 10, 12, 15, 16, 17, 37, 42, 45, 48, 49, 50, 51, 58

HARD: 11, 18, 19, 25, 28, 30, 38, 40, 44, 52, 53, 54, 55, 56, 59, 60

FOSD (First-Order Stochastic Dominance): 2, 3, 4, 7, 8, 9, 13, 14, 20, 21, 22, 23, 24, 26, 27, 29, 31, 32, 33, 34, 35, 36, 39, 41, 43, 47, 57

The specific classification of each lottery was influenced by both the participants' reaction times and how intuitively easy or difficult the lotteries appeared visually during the decision-making process.

GROUPS	PART I	PART III
FOSD	0.176	0.185
EASY	0.248	0.226
HARD	0.255	0.231

Table 1: Probability of switching: the stochasticity table shows that the "hard" lotteries have the highest variance in both periods, indicating greater variability in participants' choices. The "easy" lotteries show a decrease In stochasticity in the second period, suggesting that choices became more consistent over time. The "fosd" lotterie exhibit a slight increase in stochasticity in the second period.

4.2.1. Comparison with the Study by Agranov and Ortoleva

The study by Agranov and Ortoleva (2017) explored the phenomenon of stochastic choice in a similar context, focusing on how participants respond to spaced and consecutive repetitions of the same question. In their study as well, stochastic choice was prevalent in situations without a clearly superior option, with greater stochasticity in complex decisions. This is consistent with the results obtained for HARD lotteries in our study, where the lack of a clear dominant option led to more erratic decision-making behavior.

In terms of methodology, both studies employed an experimental design that included spaced and consecutive repetitions of the same questions to examine consistency in choices. However, while Agranov and Ortoleva also focused on using tools such as coin flips to test participants' preference for randomization, the present study utilized choice variance and absolute differences as the primary measures of stochasticity.

An interesting aspect emerging from our study is the reduction in stochasticity for EASY and HARD lotteries in the second period, suggesting a learning process. This result partially differs from Agranov and Ortoleva's conclusions, which emphasized the role of deliberate randomization in stochastic choices, especially in complex contexts. However, our study suggests that, in addition to deliberate randomization, learning and adaptation play a significant role in reducing the stochasticity of choices over time, particularly in contexts where the options are not initially clear.

For FOSD lotteries, both our study and Agranov and Ortoleva observed low stochasticity, confirming that when an option is clearly dominant, participants tend to make more consistent choices. However, the slight increase in variance observed in our study during the second period might suggest that in some cases, external factors or slight uncertainty can still influence decisions, even when dominance is clear.

In terms of percentages, our study found that 58.1% of participants showed consistency in choices during the first period (spaced repetitions), while this percentage slightly increased to 59.3% in the second period (consecutive repetitions). This increase, though modest, indicates that awareness of question repetition may contribute to greater choice consistency, supporting the idea that familiarity and practice reduce stochasticity.

Agranov and Ortoleva's study observed a correlation between the violation of expected utility and stochastic behavior, suggesting that participants with stochastic choices also tend to behave unusually compared to traditional decision-making models. Although our study did not directly examine violations of expected utility, the results suggest that the stochasticity of choices could be partially explained by a lack of familiarity with more HARD lotteries and by a learning process that leads to greater decision consistency over time.

Influence of Question Type and Participant Perception

The analysis of participants' choices showed that stochastic choice is prevalent in HARD questions, where there was no clearly superior option. This is consistent with Agranov and Ortoleva's results, which indicate that stochasticity is more common in complex decisions or where risk is HARD to evaluate.

For EASY and FOSD questions, where one option was evidently better, stochasticity was minimal. This suggests that when participants perceive an option as clearly superior, they tend to make more consistent and less variable choices, in line with traditional expected utility models.

It is noteworthy that a significant percentage of participants reported perceiving consistency in their choices despite the presence of stochasticity in their decisions. This might indicate that, at a conscious level, participants believe they are consistent, while stochastic variations emerge at a subconscious level, influenced by factors such as uncertainty or decision complexity.

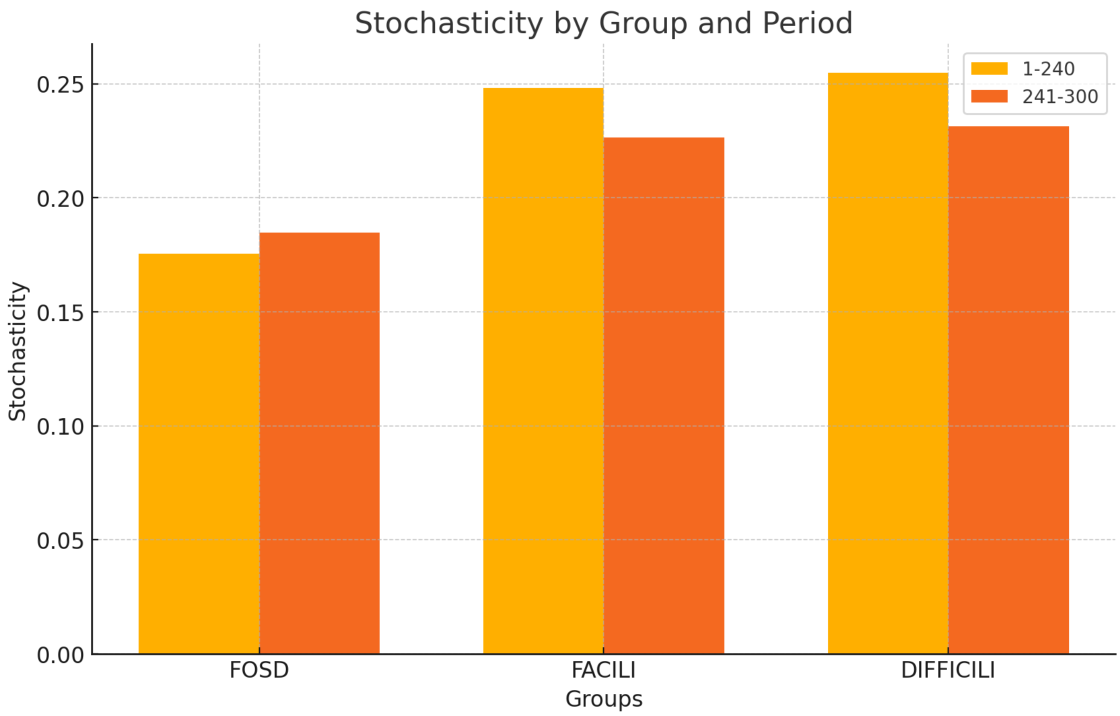


Table 2:The graphic illustrates the stochasticity of choices across the two periods for the three groups of lotteries. It is evident that stochasticity is highest for the hard lotteries in both periods.

Learning and Adaptation

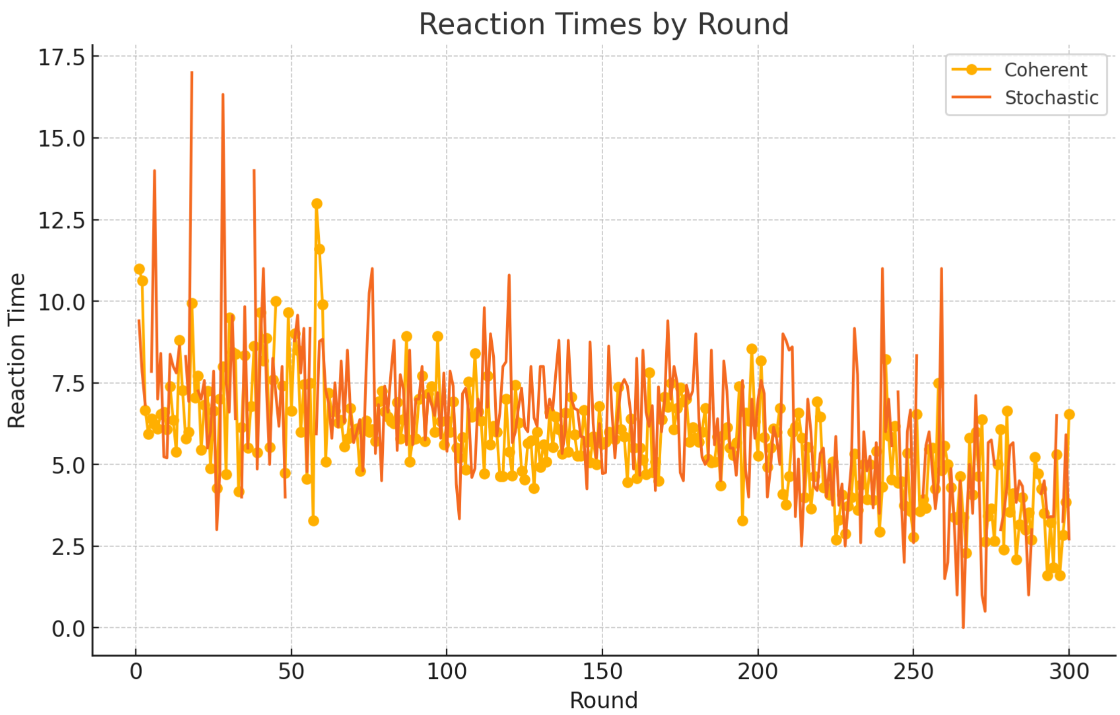
A key aspect emerging from the study is the tendency of participants to become more consistent in their choices over time, as evidenced by the decrease in choice differences in the second period across all groups. This phenomenon suggests a learning and adaptation process, where participants learn to identify better options as they gain more experience with the lotteries.

This result aligns with what was observed by Agranov and Ortoleva, where participants tended to become more systematic in their choices when they were aware of the repetitions. However, the increase in consistency observed in our study might also reflect a familiarity effect or a reduction in perceived uncertainty, rather than just deliberate randomization.

Interpretation of Reaction Times

The analysis of reaction times provides further insights into participants' decision-making dynamics. In the initial rounds, reaction times for both categories (consistent and stochastic choices) were relatively high, suggesting that participants took more time to process the information and make a decision. As the rounds progressed, there was a general decrease in reaction times, with stabilization in the later rounds.

This behavior is indicative of an adaptation process, where participants become more efficient in making decisions as the experiment progresses. This could be due to greater familiarity with the lotteries or a reduction in perceived uncertainty over time. The results are consistent with those of Agranov and Ortoleva, where it was observed that response times tended to decrease in consecutive repetitions of the same question, suggesting greater consistency and confidence in choices.



Graphic 1: The dataset shows that, in the initial rounds, reaction times for both categories were relatively high, with the highest value recorded in the first round for the "Coherent" category (11.0 seconds) and the "Stochastic" category (9.4 seconds). As the rounds progressed, a general decrease in reaction times was observed for both categories. This trend suggests that participants became more familiar with the task or more efficient in making decisions as they advanced through the rounds, leading to quicker responses over time.ù

4.3. Final Implications

In a nutshell, the results of my study confirm that the stochasticity of choices is higher in complex decision-making contexts, such as HARD lotteries, and that a learning process contributes to reducing this stochasticity over time. This phenomenon is in line with what was observed by Agranov and Ortoleva, although my study places greater emphasis on the role of learning and adaptation in improving decision consistency.The percentages found in the study, with an increase in choice consistency from 58.1% to 59.3%, provide a quantitative basis for understanding the impact of familiarity with decision options on the reduction of stochasticity. These results have important implications for economic and behavioral theory, suggesting that while deliberate randomization plays a role in stochastic choices, continuous learning and familiarity with the decision task are critical factors in promoting more consistent choices.

The comparison between the results of this study and those of Agranov and Ortoleva (2017) highlights significant similarities but also some crucial differences. Both studies agree that stochastic choice is more common in complex decision-making contexts, such as HARD lotteries, and that participants tend to become more consistent over time, suggesting a learning process.

However, while Agranov and Ortoleva emphasize deliberate randomization as the main explanation for stochastic choice, this study highlights the importance of learning and adaptation, especially for less complex lotteries. This suggests that the stochasticity of choices might result from a combination of factors, including uncertainty, decision complexity, and the experience gained during the experiment.

Ultimately, understanding stochastic choices requires a multifactorial approach that considers not only individual preferences and cognitive limitations but also the decision-making context and the role of learning. The results of this study, in combination with those of Agranov and Ortoleva, provide a more comprehensive picture of the dynamics influencing stochastic choices, offering new perspectives for economic and behavioral theory.

In the final questionnaire, 44,4% of respondents stated they adopted different ways of choosing between lotteries in Parts I and III, while 83.3% believed they were consistent in their choices. Additionally, 88,88% felt that some choices were easier than others, and 94,44% had no difficulty expressing their preference. Finally, 88,8% felt they had a constant preference for a certain type of lottery (e.g., one with higher prizes and lower probabilities or one with lower prizes but higher probabilities).

This study aimed to explore the phenomenon of stochastic choice, particularly in the decision-making processes people undergo when faced with complex lotteries. Utilizing a controlled experimental design with two separate sessions, during which a total of 60 different lotteries were tested, this study succeeded in drawing clear conclusions on how and why individuals may vary in their decision-making processes.

The experimental design was a further replication of the work by Agranov and Ortoleva (2017), enabling direct comparisons with their results. The experiment was divided into two distinct treatment phases: the first phase, where lotteries were presented sequentially, and the second phase, where lotteries were presented up to three consecutive times. This allowed us to consider the persistence and variability of choices over time.

The key finding of the study is that stochastic choice is indeed dominant, particularly in complex decisions. Almost all subjects demonstrated variability in their choices, especially in "HARD" questions where no dominant option emerged. This variability was also maintained when participants in the second period were informed that actions were being repeated. Thus, the random choice might not be due to cognitive issues or errors but could instead be an intentional behavior to manage uncertainty or diversify risk.

Regarding "EASY" questions and FOSD (First-Order Stochastic Dominance), where one option was clearly better than the other, there was a low incidence of stochastic choice. This outcome aligns with what would be expected in situations where the decision-making process is generally more straightforward, and where the perceived benefits of random choice are greatly outweighed by the cognitive cost required for a complex random decision-making process.

These findings have numerous implications. Firstly, they challenge the traditional notion that randomness is the result of an irrational or senseless mind. Instead, it can be argued that individuals actively strive to use randomness as a type of rational strategy when outcomes are uncertain and all alternatives are equally appealing. This deliberate randomization may serve several functions. Firstly, it can be a way to reduce regret. The fact that the choice was randomized may lead individuals to feel "at least as if they had played," reducing the likelihood of regret compared to making a purely instrumental choice among less appealing options. Again, randomization may serve as a way to reduce risk, particularly in situations where the decision-maker's preference for the outcome is not well understood, or the outcome itself is highly uncertain.

4.4. Conclusion

The study also has broad implications for economic theory and consumer behavior modeling. Models of human action that assume fixed and consistent preferences may not accurately predict real-world behavior unless they account for the instability introduced by stochastic choice. This suggests that models should include elements of randomness and recognize that the very variability of behavior is a rational response to the complexities involved in the decision-making process.

Although the study provides important insights into the phenomenon of stochastic choice, there are still some limitations. One limitation is the current experimental setup. While comprehensive in terms of the number of lotteries used, it represents only a subset of the types of decisions people face in practice. Consequently, the extent to which these results can be generalized to other forms of decisions remains an open question.

Another limitation is the sample size. Although the number of participants was sufficiently large to detect statistically significant effects, an even larger sample size could have provided more precise estimates of the prevalence and nature of stochastic choice. Moreover, the study relied primarily on participant behavior in a laboratory setting, which, while controlled, may not fully reflect the complexities of decision-making in real-world contexts.

Additionally, reliance on self-reported measures from the final questionnaire introduces the possibility of biases in the results due to response bias. Individuals might not be fully aware of or might distort the reasons for their motivations, especially when it comes to decision-making processes involving complex cognitive processes, such as randomization. Further studies using more objective measures, such as neuroimaging techniques, would be helpful in this regard.

Based on the findings of this study, there are several directions that future research could take. One potential avenue is to study stochastic choice within a broader decision-making framework. Given that this study focused on lotteries, subsequent research could seek to determine whether the same types of stochastic choices occur in other decision domains, such as those involving social interactions, moral dilemmas, or long-term decisions.

Additionally, future research could examine the role of individual differences in stochastic choice. For example, do certain personality traits, thinking styles, or levels of risk tolerance predict stochastic behavioral choices? Would a patient with obsessive-compulsive disorder behave in a particular way? Understanding individual differences could lead to a better understanding of the circumstances under which stochastic choice occurs and help tailor models accordingly.

Another important line of research would be to study the neural and cognitive mechanisms involved in stochastic choice. While this study shows that stochastic choice may be a deliberately optimized strategy, it does not explain the cognitive processes themselves. Neuroimaging studies, such as fMRI or EEG, could identify the brain areas and neural networks involved in decision-making under uncertainty and help empirically distinguish between various theoretical frameworks, such as random utility models and deliberate randomization models. At the end of the study, 28 random lotteries were selected based on the findings to replicate the study on a larger scale with the use of brain imaging equipment, which could help clarify what exactly happens at the neural level. Can we define the "switch" that occurs in our brain?

In conclusion, these experiments provide strong evidence that stochastic choice is a deliberate and widespread phenomenon, at least in decision-making contexts characterized by complexity. Our results challenge previous interpretations of stochastic choice as a side effect of mental constraints and, instead, provide evidence supporting the notion that people may use deliberate randomization as a rational strategy to cope with uncertainty and reduce regret. Despite the shortcomings of the research design, it actually offers numerous opportunities for future research and is indeed very significant for economic theory, behavioral modeling, and practical applications across multiple fields.

Moreover, these findings underscore the need for a deeper understanding of decision-making processes that goes beyond simplified versions of rational choice and considers the variability and complexity of human behavior. To this end, the goal of this study is to contribute to the vast body of literature that strives to bridge the gap between theoretical models and actual choices in real-world contexts, thereby gaining a better perspective on how people make decisions under conditions of uncertainty and complexity.

Lotteries with Higher Stochasticity

From the experiment, we can derive the lotteries that were more stochastic, i.e., where subjects made different choices multiple times. I will list the top 28 in order:

- **For Part I:** 53 52 44 9 31 3 17 14 56 54 30 60 40 1 59 16 11 41 55 22 51 25 10 39 38 32 28 27
- **For Part III:** 39 9 25 52 56 54 27 44 45 30 58 38 29 40 24 41 42 43 60 17 18 46 48 14 11 10 59 35

- **The lotteries that repeat in both Part I and Part III are:** 9 10 11 14 17 25 27 30 38 39 40 41 44 52 54 56 59 60
- **In total, the 38 lotteries that showed the highest stochasticity are:** 1 3 9 10 11 14 16 17 18 22 24 25 27 28 29 30 31 32 35 38 39 40 41 42 43 44 45 46 48 51 52 53 54 55 56 58 59 60

4.5. Appendix

4.5.1. Triangle questions

Question number	Triangle number	LH question p1	LH question p3	RH question q1	RH question q3
1	2	0.625	0.0	0.875	0.125
2	3	0.0	0.25	0.375	0.5
3	2	0.0	0.5	0.25	0.75
4	1	0.0	0.125	0.75	0.25
5	2	0.625	0.0	0.75	0.25
6	3	0.125	0.0	0.625	0.375
7	3	0.0	0.125	0.375	0.625
8	1	0.0	0.25	0.375	0.5
9	2	0.0	0.125	0.375	0.625
10	4	0.0	0.0	0.5	0.5
11	4	0.5	0.0	0.75	0.25
12	4	0.0	0.125	0.75	0.25
13	3	0.0	0.5	0.25	0.75
14	2	0.125	0.375	0.25	0.75
15	1	0.375	0.125	0.75	0.25
16	4	0.125	0.375	0.25	0.75
17	3	0.125	0.0	0.25	0.75
18	1	0.5	0.0	0.75	0.25
19	3	0.5	0.0	0.75	0.25
20	1	0.625	0.0	0.875	0.125
21	2	0.0	0.125	0.75	0.25
22	1	0.0	0.125	0.375	0.625
23	2	0.0	0.0	0.625	0.375
24	1	0.0	0.0	0.625	0.375
25	4	0.125	0.0	0.625	0.375
26	3	0.0	0.0	0.5	0.5
27	2	0.0	0.0	0.375	0.625
28	4	0.375	0.125	0.75	0.25
29	2	0.0	0.0	0.5	0.5
30	1	0.125	0.375	0.25	0.75
31	2	0.5	0.0	0.75	0.25
32	3	0.625	0.0	0.75	0.25
33	3	0.0	0.125	0.75	0.25

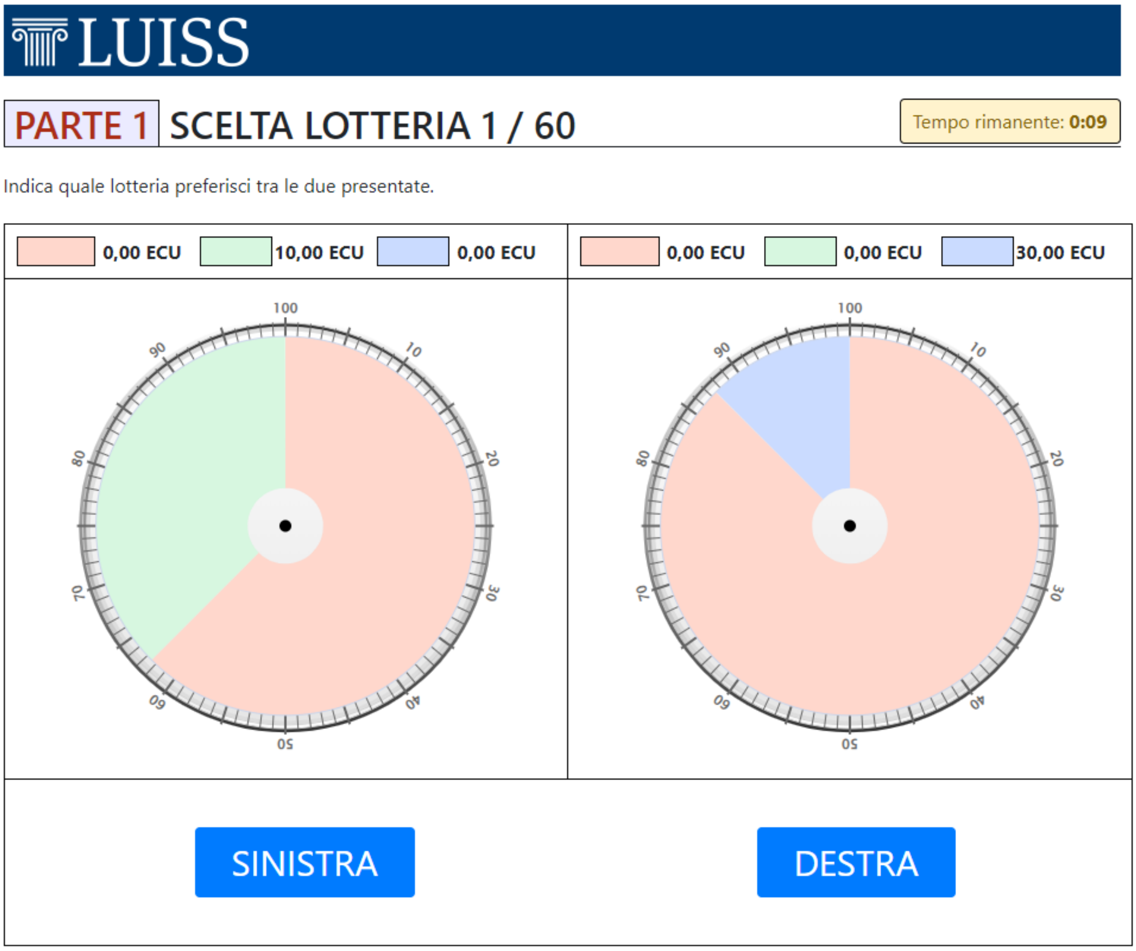
34	1	0.0	0.5	0.25	0.75
35	2	0.0	0.25	0.375	0.5
36	3	0.0	0.0	0.375	0.625
37	3	0.625	0.0	0.875	0.125
38	1	0.25	0.0	0.5	0.375
39	4	0.0	0.0	0.875	0.125
40	4	0.625	0.0	0.875	0.125
41	2	0.125	0.0	0.625	0.375
42	2	0.25	0.0	0.5	0.375
43	1	0.0	0.0	0.5	0.5
44	4	0.0	0.125	0.375	0.625
45	4	0.125	0.0	0.25	0.75
46	2	0.125	0.0	0.25	0.75
47	1	0.0	0.125	0.625	0.375
48	4	0.0	0.0	0.625	0.375
49	2	0.375	0.125	0.75	0.25
50	3	0.375	0.125	0.75	0.25
51	1	0.625	0.0	0.75	0.25
52	4	0.0	0.25	0.375	0.625
53	4	0.0	0.5	0.375	0.75
54	3	0.125	0.0	0.625	0.375
55	1	0.0	0.0	0.25	0.75
56	4	0.625	0.0	0.75	0.25
57	4	0.25	0.0	0.5	0.375
58	3	0.0	0.0	0.625	0.375
59	1	0.125	0.0	0.25	0.75
60	4	0.625	0.0	0.75	0.25

4.5.2. Instructions

Welcome to our experiment! The instructions are simple, and if you follow them carefully and make good decisions, you can earn CASH AMOUNTS that will be PAID IN CASH IMMEDIATELY at the end of the experiment. This partly computerized experiment consists of three parts (PART I, PART II, and PART III). Parts I and III are composed of various choices between lotteries that will be presented to you sequentially on the computer. You will receive specific instructions for each part of the experiment at the beginning of each one. The experiment is individual, and we therefore ask you not to talk to each other during its course. If you have any doubts, raise your hand, and one of the experimenters will come to answer you immediately.
Good luck

Task

In each round and in Parts I and III of the experiment, pairs of lotteries will be presented to you on the screen, and you will need to express your preference. Your task in each round is to indicate whether you prefer to play the LEFT (SINISTRA) lottery rather than the RIGHT (DESTRA) one.



ATTENTION: Be careful when deciding which lottery you prefer, indicating the one you would actually prefer to play because at the end of the experiment your earnings will depend on your choice and luck. For each pair of lotteries, you will see two options on the screen: LEFT or RIGHT. Your task is to choose one of them.

ATTENTION: You will have 20 seconds to make your decision in each round. If you do not make a decision within the maximum allotted time, the computer will randomly assign one of the two by DEFAULT.

Earnings from the experiment

Earnings from the experiment will be determined as follows:

- The computer will randomly draw with equal probability one of the pairs of lotteries for which you have expressed your preference for each of Parts I and III and will play between the two you preferred, determining your earnings for each part.
- Your earnings for Part II will be determined manually at the end and will be added to the earnings from the other two at the end of the experiment.
- Your total earnings will be equal to the winnings you have obtained in the three parts of the experiment.
- These earnings are expressed in ECU (experimental currency units), which will be converted into euros at a rate of 1 ECU = 50 cents of a euro, to which the earnings from Part II and the participation prize of 5 euros will be added.

Good luck.

Final questionnaire

1. Gender:
   - M
   - F
   - Other
2. Degree Course
   - Economics
   - Law
   - Political Science
3. Geographic Origin:
   - North
   - Center
   - South and Islands
4. Did you find the initial instructions clear and understandable?
   - Yes
   - No
5. Were the presented lotteries easy to understand?
   - Yes
   - No

6. Did you find it HARDto choose between the pairs of lotteries?

- Yes
- No

7. Did you feel that there were some choices that were “easier" than others?

- Yes
- No

8. Did you feel comfortable during the experiment?

- Yes
- No

9. Did you adopt different ways of choosing between the various pairs of lotteries presented in Parts I and III?

- Yes
- No

10. Did you have a consistent preference for a certain type of lottery (e.g., those with higher prizes and lower probabilities or those with lower but more likely prizes)?

- Yes
- No

11. Did you try to be consistent in your choices between lotteries in the experiment?

- Yes
- No

12. If it takes 5 machines 5 minutes to make 5 objects, how many minutes would it take for 100 machines to make 100 objects? _____ minutes

13. In a lake, there are lily pads. Each day the number of lily pads doubles. If it takes 48 days for the lake to be completely covered with lily pads, how many days does it take for the lake to be half-covered? _____ days

Risk Question

You are endowed with 10 tokens and asked to choose the portion of this amount (between 0 and 10 tokens, inclusive) that you wish to invest in a risky option. Those tokens not invested are yours to keep.

If the risk investment is successful, you receive 2.5 times the amount you chose to invest; if the investment is unsuccessful, you lose the amount invested. To determine if the investment is successful or not, we will roll a four-faced die, with faces marked A, B, C,

D. If we obtain face A or D the investment is successful. If we obtain faces B or C the investment is unsuccessful. We now ask you to indicate the number of points that you wish to invest:

I wish to invest tokens

Compound Lottery Question

You are endowed with 10 tokens and asked to choose the portion of this amount (between 0 and 10 tokens, inclusive) that you wish to invest in a risky option. Those tokens not invested are yours to keep.

If the risk investment is successful, you receive 2.5 times the amount you chose to invest; if the investment is unsuccessful, you lose the amount invested.

To determine if the investment is successful or not, we will first of all roll a four-faced die, with faces marked A, B, C, D. If we obtain face A the investment is successful.

If we obtain face D the investment is unsuccessful. If we obtain faces B or C, then roll the die again. If we obtain face A or B, the investment is successful. Otherwise, if we obtain face C or D, the investment is unsuccessful.

We now ask you to indicate the number of points that you wish to invest:

I wish to invest tokens

5. Neuroimaging techniques

"While not denying that deliberation is part of human decision making, neuroscience points out two generic inadequacies of this approach—its inability to handle the crucial roles of automatic and emotional processing. First, much of the brain implements 'automatic' processes, which are faster than conscious deliberations and which occur with little or no awareness or feeling of effort. Because people have little or no introspective access to these processes, or volitional control over them, and these processes were evolved to solve problems of evolutionary importance rather than respect logical dicta, the behavior these processes generate need not follow normative axioms of inference and choice."

(Camerer, Colin, George Loewenstein, and Drazen Prelec)

Classical economic theory assumes that individuals make rational decisions based on a careful evaluation of available information, with the goal of maximizing expected utility. However, neuroeconomics introduces a different perspective by highlighting that the brain often processes information automatically and is influenced by emotions, leading to choices that deviate from those predicted by traditional economic models. This suggests the existence of alternative cognitive and emotional mechanisms, which neuroeconomics seeks to study in order to provide a more realistic explanation of human decision-making behavior.

5.1. Introduction

Neuroimaging techniques have revolutionized our understanding of the brain's structure and function, particularly in the context of cognitive processes and disorders. These techniques allow for the visualization and measurement of brain activity and connectivity, offering insights into the neural mechanisms underlying various behaviors and mental states.

Neuroeconomic techniques have emerged as crucial tools in economic analysis, providing new insights into how economic decisions are influenced by brain processes. The integration of neuroscience with economics enables a deeper understanding of human behavior, often deviating from traditional economic models' predictions. Techniques such as functional magnetic resonance imaging (fMRI), positron emission tomography (PET), functional near-infrared spectroscopy (fNIRS), magnetoencephalography (MEG), and diffusion tensor imaging (DTI) offer a window into the neural mechanisms underlying processes like decision-making, risk assessment, temporal preferences, and emotions. These techniques are applied in a broad spectrum of economic research:

- **Decision-making**: Neuroeconomics explores how the brain processes information and makes decisions under conditions of uncertainty and risk. Studies like those by Camerer (2008) and Fehr and Rangel (2011) demonstrate how neural activities can predict economic choices, explaining deviations from classic rational models.
- **Social Preferences:** Fehr and Camerer (2007) investigated social preferences and altruism, revealing how neural responses are linked to fairness and cooperation behaviors.
- **Emotional Impact**: Research has shown that emotions can significantly influence economic decisions. Coricelli et al. (2005) examined the role of regret and its avoidance in economic choices, using neuroimaging techniques to identify the brain regions involved.
- **Intertemporal Choice:** Studies on intertemporal choice, such as Carter et al. (2010), use fMRI to analyze how the brain evaluates future rewards versus immediate ones, contributing to understanding saving and consumption behaviors.
- **Risk and Uncertainty:** The distinction between risk and uncertainty in decisions has been explored by Krain et al. (2006) and Vorhold (2008), who used neuroimaging to map the different brain responses to these conditions.

5.2. Contribution of Neuroeconomic Techniques to Research on Decision-Making Under Risk and Uncertainty

Economic decisions often involve uncertainty and risk, challenging the rationality of choices. Neuroeconomic techniques have provided essential contributions to understanding these processes, revealing how specific brain areas are involved in risk assessment and uncertainty management.

- **Risk:** Neuroeconomic research has highlighted the orbitofrontal cortex and dorsolateral prefrontal cortex as crucial for risk processing. Activity in these regions increases during the anticipation of risky rewards, as demonstrated by studies like Rogers et al. (1999).

- **Uncertainty**: The distinction between known risk and unknown uncertainty has been examined through PET and fMRI, showing that areas such as the amygdala and insular cortex are more active when probabilities are unknown, suggesting a greater emotional involvement in uncertain situations (Wu et al., 2021).
- **Regret and Avoidance**: Regret is a significant component in decisions, especially in loss and gain contexts. Coricelli et al. (2005) identified the role of the insula in processing regret and its avoidance, linking emotional responses to future choices.
- **Strategy and Uncertainty**: Studies like Nagel et al. (2018) explore the depth of strategic reasoning, revealing how different brain areas are activated depending on the complexity and strategic uncertainty.

These tools not only support existing economic theories but also help develop new models that account for human cognitive limitations and emotions.

This chapter explores several key neuroimaging methods used in cognitive neuroscience, highlighting their characteristics and applications in research.

5.3. Neuroimaging tools

5.3.1. Functional Magnetic Resonance Imaging (fMRI)

Principles and Mechanism

Functional magnetic resonance imaging (fMRI) is a non-invasive technique that measures brain activity by detecting changes in blood oxygenation and flow that occur in response to neural activity. This method relies on the blood-oxygen-level-dependent (BOLD) contrast, which provides high spatial resolution images of brain function. The BOLD signal is based on the differential magnetic properties of oxygenated and deoxygenated hemoglobin; areas with increased neural activity consume more oxygen, leading to localized changes in blood flow and oxygenation levels detectable by fMRI scanners.

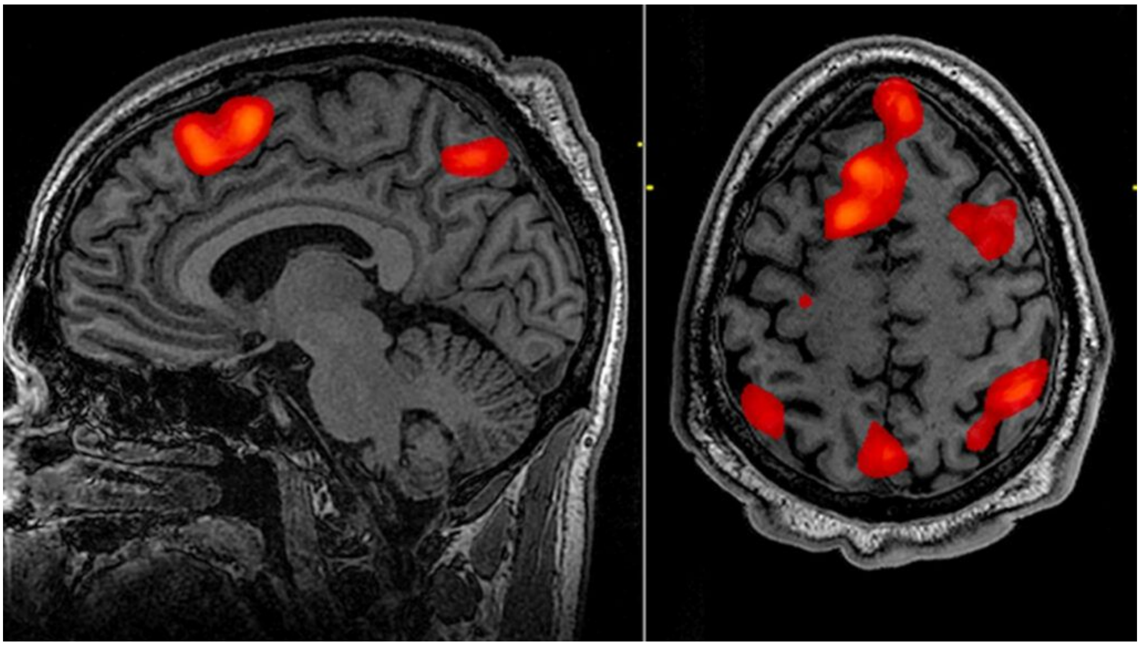


Figure 4: an fMRI scan showing brain activity during a cognitive task

Figure 5: an fMRI used for scanning brain activity

Applications in Research

fMRI is widely used to investigate the neural substrates of cognitive processes such as decision-making, emotion, memory, and attention. In the study by Paulus et al. (2002), fMRI was used to compare brain activity during decision-making tasks between individuals with schizophrenia and healthy controls. The scans, performed with a 1.5 Tesla scanner, provided detailed images of brain activation, revealing significant differences in the orbitofrontal cortex (OFC) and dorsolateral prefrontal cortex (DLPFC) between the two groups[19].

Similarly, Kerr and Zelazo (2004) employed fMRI to explore the development of affective decision-making in young children. By using an age-appropriate gambling task, the researchers identified activation patterns in the OFC, demonstrating its critical role in evaluating rewards and risks from an early age. These findings underscore the importance of fMRI in developmental studies and its ability to link behavioral changes to specific neural developments[20].

5.3.2. Positron Emission Tomography (PET)

Principles and Mechanism

Positron emission tomography (PET) is another powerful neuroimaging technique that measures metabolic activity in the brain using radioactive tracers. These tracers, typically fluorodeoxyglucose (FDG) or H2^15O, emit positrons that interact with electrons in the brain, producing gamma rays detectable by PET scanners. This process allows researchers to observe areas of increased metabolic activity, which correlate with neural activation.

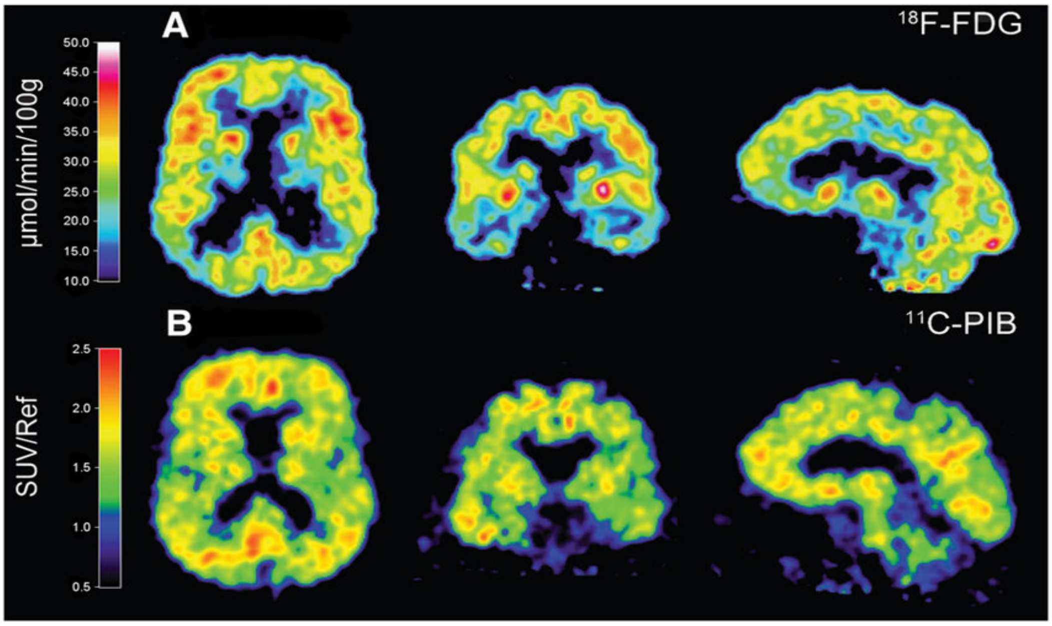


Figure 6: a PET scan highlighting areas of metabolic activity in the brain

[19] Paulus, M. P., Feinstein, J. S., Simmons, A. N., & Stein, M. B. (2002). Anterior cingulate activation in high trait anxious subjects is related to altered error processing during decision making. Biological Psychiatry, 51(7), 523-529.

[20] Kerr, A., & Zelazo, P. D. (2004). Development of "hot" executive function: The children's gambling task. Brain and Cognition, 55(1), 148-157.

Figure 7: a PET scanner used for imaging brain activity

Applications in Research

PET is particularly useful for investigating brain function in clinical populations and for understanding the neural mechanisms of complex cognitive tasks. In the study by Rogers et al. (1999), H2^15O PET was used to examine the neural substrates of decision-making under risk. The researchers scanned participants while they performed a computerized risk-taking task, revealing increased regional cerebral blood flow (rCBF) in the right inferior and orbital PFC. These findings highlighted the involvement of these regions in processing reward-related information and resolving conflicting decisions[21].

Another example is the study by Bechara et al. (2005), which utilized PET to investigate the neural correlates of decision-making in patients with damage to the VMPFC. PET scans showed significant metabolic activity in the VMPFC during tasks that simulate real-

[21] Rogers, R. D., Owen, A. M., Middleton, H. C., Williams, E. J., Pickard, J. D., Sahakian, B. J., & Robbins, T. W. (1999). Choosing between small, likely rewards and large, unlikely rewards activates inferior and orbital prefrontal cortex. The Journal of Neuroscience, 20(19), 9029-9038.

life risk scenarios, providing robust evidence of the region's role in integrating emotional signals into decision-making processes[22].

Combining fMRI and PET

Combining fMRI and PET techniques can provide complementary insights into brain function, leveraging the strengths of each method. fMRI offers high spatial resolution and the ability to capture dynamic changes in brain activity, while PET provides quantitative data on metabolic processes and can measure neurotransmitter activity.

For instance, in studies examining the somatic marker hypothesis (SMH), which posits that emotional processes guide decision-making, combining fMRI and PET data has been instrumental in linking behavioral changes to specific neural mechanisms. By visualizing both the structural and functional aspects of the brain, researchers can gain a more comprehensive understanding of how different brain regions interact during complex cognitive tasks[23].

5.3.3. Functional Near-Infrared Spectroscopy (fNIRS)

Principles and Mechanism

Functional near-infrared spectroscopy (fNIRS) is a non-invasive optical imaging technique that measures brain activity by detecting changes in blood oxygenation levels. fNIRS uses near-infrared light to penetrate the scalp and skull, reaching the cortical surface. The light is absorbed by oxygenated and deoxygenated hemoglobin, and the reflected light is measured to determine changes in blood oxygenation.

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[22] Bechara, A., Damasio, H., & Damasio, A. R. (2005). The Iowa Gambling Task and the somatic marker hypothesis: Some questions and answers. Trends in Cognitive Sciences, 9(4), 159-162.

[23] Rogers, R. D., Everitt, B. J., Baldacchino, A., Blackmore, J., Swainson, R., London, M., ... & Robbins, T. W. (1999). Dissociating deficits in the decision-making cognition of chronic amphetamine abusers, opiate abusers, patients with focal damage to prefrontal cortex, and tryptophan-depleted normal volunteers: evidence for monoaminergic mechanisms. Neuropsychopharmacology, 20(4), 322-339.

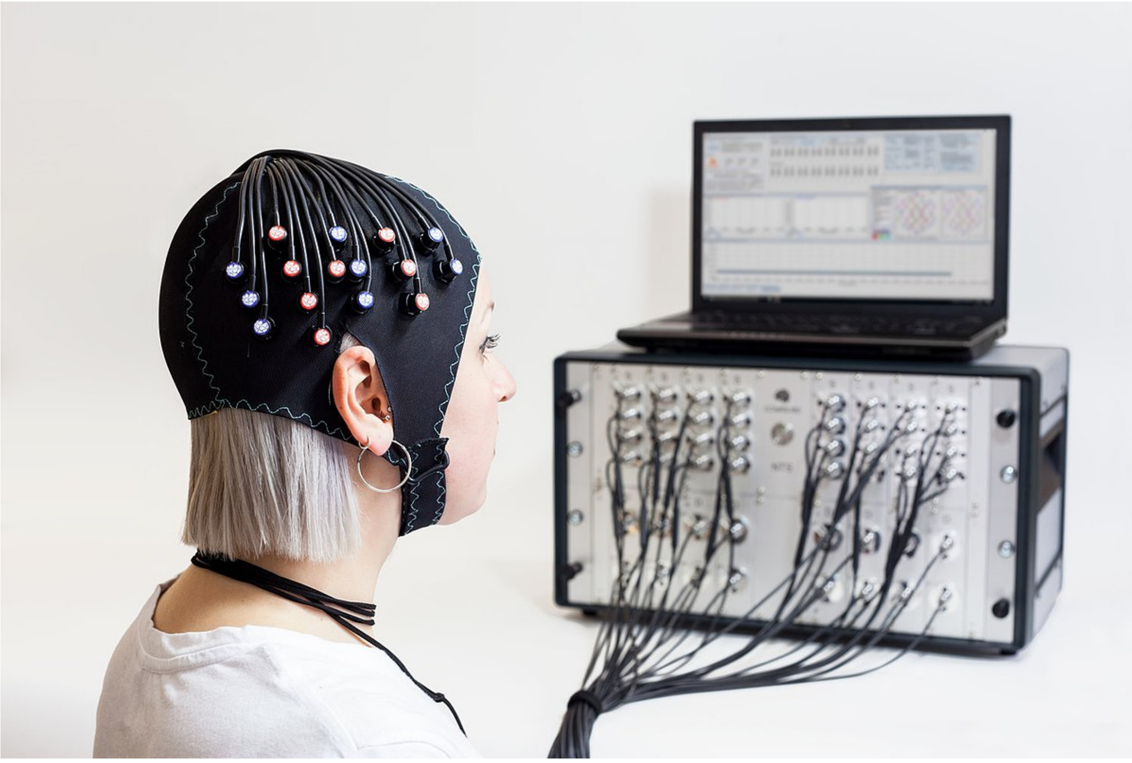

Figure 8: an fNIRS machine used for measuring brain activity

Applications in Research

fNIRS is particularly useful for studying brain function in populations where other imaging techniques might be challenging, such as infants, young children, and individuals with claustrophobia. It is also advantageous in naturalistic settings where participants can move relatively freely.

In cognitive neuroscience, fNIRS has been used to investigate a wide range of processes, including language development, social cognition, and motor control. Its portability and ease of use make it an attractive option for longitudinal studies and research in real-world environments.

5.3.4. Magnetoencephalography (MEG)

Principles and Mechanism

Magnetoencephalography (MEG) measures the magnetic fields produced by neural activity, providing excellent temporal resolution and the ability to capture rapid changes in brain activity. MEG sensors, typically superconducting quantum interference devices (SQUIDs), detect these magnetic fields outside the scalp, allowing for the non-invasive measurement of brain activity with millisecond precision.

Figure 9: a MEG machine used for measuring magnetic fields generated by neural activity

Applications in Research

MEG is particularly useful for studying the temporal dynamics of cognitive processes, such as perception, attention, and motor control. Its high temporal resolution allows researchers to track the sequence of neural events underlying these processes.

In research on decision-making, MEG has been used to investigate the timing of neural responses to different types of stimuli and to understand how these responses are integrated over time to guide behavior. This technique complements fMRI and PET by providing detailed information on the timing of neural activity, which is crucial for understanding the dynamic nature of cognitive processes.

5.3.5. Diffusion Tensor Imaging (DTI)

Principles and Mechanism

Diffusion tensor imaging (DTI) is a type of MRI that measures the diffusion of water molecules in the brain, providing insights into the microstructural integrity of white matter tracts. By tracking the direction and magnitude of water diffusion, DTI can map the connectivity between different brain regions, revealing the structural pathways that support cognitive functions.

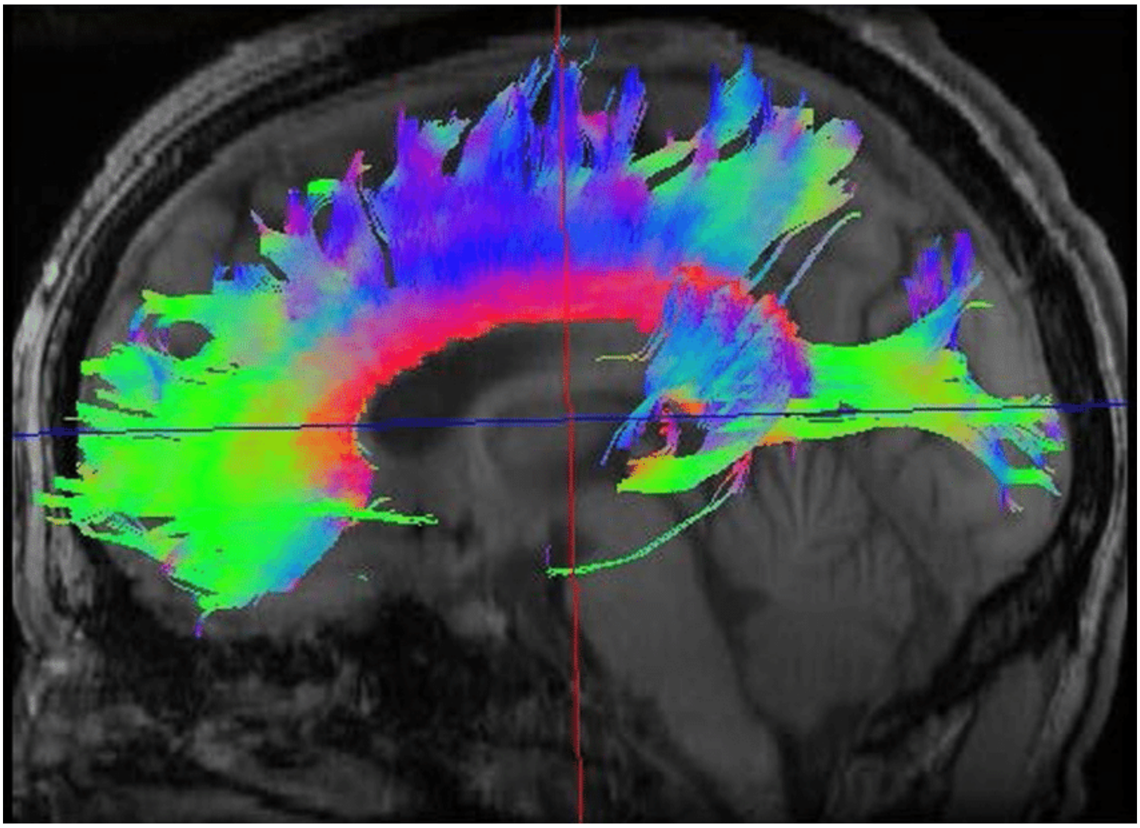

Figure 10: A DTI scan showing the white matter tracts in the brain.

Applications in Research

DTI is widely used to study the structural connectivity of the brain and to understand how changes in white matter integrity are related to cognitive function and dysfunction. It has been particularly useful in research on developmental disorders, neurodegenerative diseases, and the effects of brain injury.

In studies of decision-making, DTI has been used to investigate how the integrity of white matter tracts connecting the OFC, DLPFC, and other regions influences cognitive control and risk-taking behavior. By linking structural connectivity to functional outcomes, DTI provides a deeper understanding of the neural basis of decision-making.

Combining Neuroimaging Techniques

The integration of multiple neuroimaging techniques allows for a more comprehensive understanding of brain function by leveraging the strengths of each method. For example, combining fMRI and DTI can provide insights into both the functional activation of brain regions and the structural pathways connecting them. Similarly, integrating MEG and fMRI data can offer a detailed picture of both the temporal dynamics and spatial localization of neural activity.

Case Study: Decision-Making Under Risk

A comprehensive study of decision-making under risk might involve several neuroimaging techniques to capture different aspects of the neural processes involved. fMRI could be used to identify the brain regions activated during risk-taking tasks, revealing the involvement of the OFC and DLPFC. PET could complement these findings by providing quantitative data on the metabolic activity in these regions, offering insights into the underlying neurochemical processes. MEG could then be used to investigate the timing of neural responses, helping to understand how different brain regions interact in real-time to guide decision-making.

Additionally, DTI could be employed to examine the structural connectivity between the OFC, DLPFC, and other regions, providing a detailed map of the white matter pathways that support these cognitive functions. By combining these techniques, researchers can gain a holistic view of the neural mechanisms underlying decision-making under risk, from the structural pathways that connect different brain regions to the dynamic neural activity that occurs during the decision-making process.

5.4. Conclusion

Neuroimaging techniques have transformed our understanding of the brain and its functions, providing powerful tools for investigating the neural mechanisms underlying cognitive processes and disorders. Each technique offers unique strengths and insights, from the high spatial resolution of fMRI and the quantitative metabolic data provided by PET to the millisecond temporal resolution of MEG and the detailed structural maps produced by DTI. By integrating these methods, researchers can achieve a comprehensive understanding of brain function, capturing the complex interactions between different regions and processes that underlie cognition and behavior. As neuroimaging technology continues to advance, it promises to yield even deeper insights into the workings of the human brain, paving the way for new discoveries and therapeutic approaches.

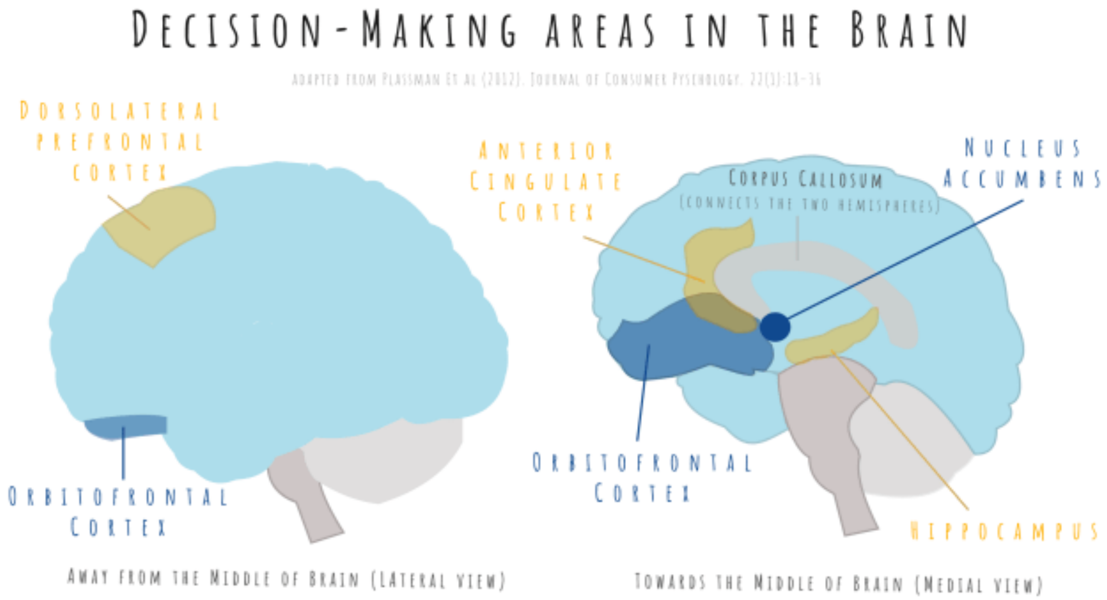

Figure 11: An illustration of brain regions involved in decision-making processes.

These neuroimaging techniques, along with the detailed images and machinery, illustrate the vast potential and complexity of studying the human brain. By employing multiple methods and integrating their findings, researchers can uncover the intricate details of neural activity, structure, and function that drive cognitive processes and behaviors.

6. Conclusion

One might wonder what drives subjects to choose one option over another and to what extent there is an element of stochasticity in these decisions. The objective is to explore, using neuroimaging tools like fMRI, which areas of the brain activate during decision-making. In this context, it is helpful to distinguish between two types of subjects: those who prefer a more probable but lower reward (P-bet) and those more averse to risk, opting for a larger reward with lower probability ($-bet).[24]

In our experiment, where subjects are asked to choose between two lotteries with different amounts and probabilities, within a 20-second time limit, cognitive and behavioral dynamics emerge that influence decision-making. The limited time, along with uncertainty about the probabilities and prizes, prompts subjects to make stochastic decisions, influenced by factors such as bounded rationality, the time available, and their ability to process information.

One of the main factors affecting these choices is **bounded rationality**, a concept introduced by Herbert Simon. According to Simon, people cannot process all the available information and use cognitive shortcuts, known as heuristics. For example, subjects might rely on the **availability heuristic**, choosing the higher reward because it is more salient, or the **representativeness heuristic**, favoring the lottery that seems more typical of a realistic win.

The focus on either rewards or probabilities also depends on the context. Some studies, such as those by Slovic and Lichtenstein, show that subjects tend to give more weight to rewards when they are high, or to probabilities when the reward is modest. However, in a fast-paced decision-making environment like ours, choices may become impulsive, based on single attributes like the highest reward, without a thorough analysis of the variables.

The time constraint plays a significant role in decision-making, reducing subjects' ability to perform complex calculations and leading to suboptimal choices, often inconsistent and influenced by cognitive biases. Another crucial element is **prospect theory**, proposed

[24] "In the economic and financial field, the terms **P-bet** and **$-bet** refer to two types of bets that involve different choices in terms of risk, probability, and gain. These concepts are often used to describe decision-making under uncertainty and to analyze individual preferences, especially in the context of **expected utility theory** or **prospect theory**."

by Kahneman and Tversky, which suggests that people tend to overestimate low-probability events and underestimate high-probability ones, leading them to make riskier choices than would be rational.

These dynamics lead to decisions that appear stochastic or random, influenced by time limitations and the capacity to process information. Although subjects believe they are making rational decisions, they are often guided by cognitive shortcuts and distorted perceptions of risk and reward.

People tend to prefer causality over randomness, as documented by various economic and psychological theories. This behavior reflects an innate human need for predictability and control. As Taleb emphasizes in his "Black Swan" theory, people tend to ignore the randomness of rare and extraordinary events, preferring linear causal explanations that confirm their expectations, thus overlooking the risks associated with extreme randomness. This behavior illustrates the human desire to find order in chaos, to avoid the vulnerability that randomness entails.

Many individuals prefer P-bets, focusing on events with high probability and low risk. However, by ignoring improbable but catastrophic events (Black Swans), they risk facing devastating losses when such events occur. $-bets, on the other hand, involve accepting highly variable events, yet they still rely on probabilistic evaluation, which Taleb argues is inadequate in the face of unforeseen events.

The preference for causality helps reduce uncertainty, providing a framework that enables more secure predictions and decision-making. However, this approach can lead to cognitive distortions, such as the **illusory correlation bias**, where causal links are perceived where none exist, or the **gambler's fallacy**, which attributes dependence on past events to random occurrences.

Taleb's Black Swan concept aligns with Simon's notion of bounded rationality, as well as with ideas of focus points and probabilistic bets. Due to their cognitive limitations and predictive models, people tend to focus on high-probability events, ignoring the potential for extreme events. As a result, they are vulnerable to Black Swans, unpredictable events that escape traditional probabilistic models.

In conclusion, the decisions of subjects in an experimental context like ours tend to be stochastic, influenced by their limited ability to process all the information within the given time frame and by cognitive biases that drive the search for causal patterns even in situations governed by randomness.

7. Appendices

7.1. Appendix Chapter II

Drift Diffusion Model and reaction time (in decision making)

The diffusion model is a model of the cognitive processes involved in simple two-choice decisions. It separates the quality of evidence entering the decision from decision criteria and from other, nondecision, processes such as stimulus encoding and response execution. The model should be applied only to relatively fast two-choice decisions (mean RTs less than about 1000 to 1500 ms) and only to decisions that are a single-stage decision process (as opposed to the multiple-stage processes that might be involved in, for example, reasoning tasks).

The diffusion model assumes that decisions are made by a noisy process that accumulates information over time from a starting point toward one of two response criteria or boundaries, as shown in the top panel of Figure 7. The starting point is labeled *z* and the boundaries are labeled *a* and 0. When one of the boundaries is reached, a response is initiated. The rate of accumulation of information is called the drift rate (*v*)[25], and it is determined by the quality of the information extracted from the stimulus. In an experiment, the value of drift rate, *v*, would be different for each stimulus condition that differed in difficulty. For recognition memory, for example, drift rate would represent the quality of the match between a test word and memory. A word presented for study three times would have a higher degree of match (i.e., a higher drift rate) than a word presented once (as we will see in Part II of the experiment where we present three times the same lotteries). The zero point of drift rate (the drift criterion, Ratcliff, 1985,2002;Ratcliff et al., 1999)[26] divides drift rates into those that have positive values, that is, mean drift rate toward the A response boundary in Figure 7, and negative values, mean drift rate toward the B boundary.

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[25] A parameter in the diffusion model representing the speed and direction at which information accumulates toward a decision.

[26] Ratcliff, R. (1985). Theoretical interpretations of the speed and accuracy of positive and negative responses. Psychological Review.

Ratcliff, R., Gomez, P., & McKoon, G. (1999). A diffusion model account of the lexical decision task. Psychological Review.

Ratcliff, R., & Tuerlinckx, F. (2002). Estimating parameters of the diffusion model: Approaches to dealing with contaminant reaction times and parameter variability. Psychonomic Bulletin & Review.

There is noise (within-trial variability) in the accumulation of information so that processes with the same mean drift rate (v) do not always terminate at the same time (producing RT distributions) and do not always terminate at the same boundary (producing errors), as shown by the three processes, all with the same drift rate, in the top panel of Figure 1. Within-trial variability in drift rate (s)[27] is a scaling parameter for the diffusion process (i.e., if it were doubled, other parameters could be multiplied or divided by two to produce exactly the same fits of the model to data). Note that for Figure 7 continuous diffusion processes were approximated by discrete random-walk processes.

Empirical RT distributions are positively skewed, and in the diffusion model, this is naturally predicted by simple geometry. In the middle panel of the figure, distributions of fast processes from a high drift rate and slower responses from a lower drift rate are shown. If the higher and lower values of drift rate are reduced by the same amount (X in the figure), then the fastest processes are slowed by an amount Y, and the slowest by a much larger amount, Z.

The bottom panel of Figure 7 illustrates component processes assumed by the diffusion model: the decision process with duration d, an encoding process with duration u (this would include memory access in a memory task, lexical access in a lexical decision task, and so on), and a response output process with duration w. When the model is fit to data, u and w are combined into one parameter to encompass all the nondecision components with mean duration T_{er}[28].

The components of processing are assumed to be variable across trials. For example, all words studied three times in a recognition memory task would not have exactly the same drift rate. The across-trial variability in drift rate is assumed to be normally distributed with standard deviation η[29]. The starting point is assumed to be uniformly distributed with range s_Z, and the nondecision component is assumed to be uniformly distributed with range s_t. One might also expect that the decision criteria would be variable from trial to trial. However, the effects would closely approximate the effect of starting point

[27] A parameter indicating the variability in information accumulation within a single trial, affecting the consistency of the response times and decision outcomes.
[28] A combined parameter for the duration of processes not directly related to the decision-making process itself, such as sensory processing or motor responses.
[29] Indicates how much the drift rate can vary from trial to trial, reflecting differences in the quality or clarity of the stimulus or the state of the subject.

variability, and computationally, only one integration over starting point is needed instead of two separate integrations over the two criteria.

The effect of across-trial variability in the nondecision component depends on the mean value of drift rate (Ratcliff & Tuerlinckx, 2002). With large values of drift rate, variability in the nondecision component acts to shift the leading edge of the RT distribution shorter than it would otherwise be, by as much as 10% of s_t. With smaller values of drift rate, the effect is smaller. Across-trial variability in the nondecision component allows the model to account for data that have considerable variability in the .1 quantiles of the RT distributions across experimental conditions (Ratcliff & Tuerlinckx, 2002).

The standard deviation in the duration of the nondecision component (s_t/(2 sqrt(3))) that is estimated from experimental data is typically less than one-quarter the standard deviation in the decision process, so variability in the nondecision component has little effect on the shape or standard deviation of overall RT distributions (Ratcliff & Tuerlinckx, 2002, Figure 8). For example, if s_t is 100 ms (SD = 28.9 ms) and the SD in the decision process is 100 ms, the combination (square root of the sum of squares) is 104 ms.

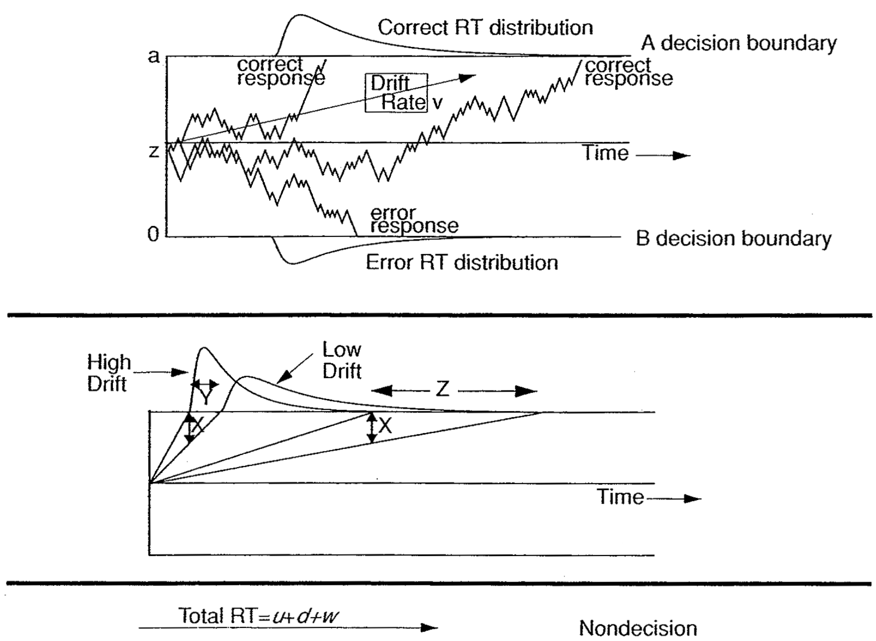

Figure 12: The diffusion decision model. (Top panel) Three simulated paths with drift rate v, boundary separation a, and starting point z. (Middle panel) Fast and slow processes from each of two drift rates to illustrate how an equal size slowdown in drift rate (X) produces a small shift in the leading edge of the RT distribution (Y) and a larger shift in the tail (Z). (Bottom panel) Encoding time (u), decision time (d), and response output (w) time. The nondecision component is the sum of u and w with mean = T_{er} and with variability represented by a uniform distribution with range s_t.

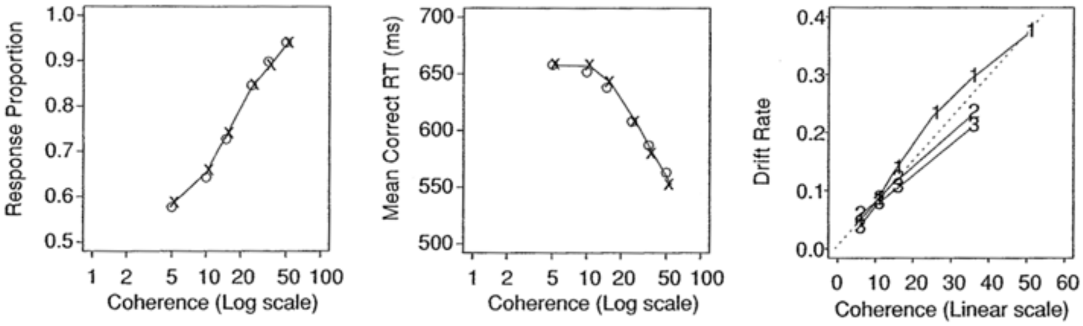

Figure 13: Response proportion, mean RT for correct responses, and drift rate as a function of coherence. For the top and middle panels, the o's are data, and the x's are predictions from the diffusion model. In the bottom panel, the numerals 1, 2, and 3 refer to experiments 1, 2 and 3

7.2. Appendix Chapter III

Stochastic Choice and Preferences for Randomization" by Marina Agranov and Pietro Ortoleva

The experiment in "Stochastic Choice and Preferences for Randomization" by Marina Agranov and Pietro Ortoleva, which serves as the starting point for our study, explores in detail the decision-making behavior of individuals when faced with repeated choices under uncertainty. The authors conducted an experiment structured in different phases to analyze how and why subjects make different choices under seemingly identical conditions. This experiment was designed to compare various error models and better understand the motivations behind stochastic behavior.

The experiment was conducted at the California Social Science Experimental Laboratory at the University of California, Los Angeles, in January 2013. The subjects were voluntary university students recruited from a database, and each experimental session lasted about 45 minutes. The experiment was divided into four main parts:

1. Part I: Subjects had to answer a series of repeated questions, with repetitions spaced out by other questions. Subjects were not informed in advance that the questions would be repeated. This part aimed to replicate traditional experimental designs and provide a comparison point with the new approaches introduced in the subsequent parts of the experiment.
2. Part II: This part included a risky investment task to measure subjects' risk aversion. Subjects had to decide how much of their budget to invest in a risky activity with a predetermined probability of success.

3. Part III: In this part, subjects had to answer seven of the questions from Part I again, but the questions were repeated three times in a row. Subjects were informed in advance that each question would be repeated three times consecutively. This part of the experiment aimed to examine whether awareness of repetitions would influence stochastic behavior.
4. Part IV: Subjects had to respond to standard variations of the common ratio and common consequence effects of the Allais paradox. At the end of the experiment, subjects were asked to complete a non-incentivized questionnaire to explore their motivations for making different choices in the repetitions.

Data Collection and Analysis

Data was collected through programmed software that recorded subjects' choices in real-time. At the end of each session, subjects were paid based on one of their randomly selected decisions to avoid incentives to diversify responses only to maximize overall earnings. Choices were converted into money using specific conversion rates for each part of the experiment, ensuring strong incentives for making thoughtful decisions.

The empirical analysis of the data revealed several key trends in subjects' decision-making behavior:

1. Stochastic Behavior: Consistent with previous findings, the vast majority of participants (90%) chose different lotteries in the three repetitions of the same question in Part I. This stochastic behavior was predominantly observed in HARDquestions (HARD), where none of the available options were clearly better than the others. Statistical analyses, such as Fisher's exact test, confirmed that the proportion of subjects giving inconsistent answers in HARDquestions was significantly higher compared to easy (EASY) and first-order stochastically dominated (FOSD) questions.
2. Influence of Consecutive Repetitions: In Part III, a large majority of subjects (71%) continued to show stochastic behaviors even when the questions were repeated three times in a row. Although the proportion of inconsistent responses was slightly lower than in Part I, stochastic behavior was still prevalent. This suggests that awareness of repetitions does not completely eliminate stochasticity in choices.
3. Motivations for Stochastic Choices: The final questionnaire revealed that most subjects who gave inconsistent responses in Part III did so deliberately. Only a small

fraction (12%) reported having changed their mind about which option was better, while 79% stated they intentionally chose differently to increase their chances of winning with varied options or to try different possibilities. This supports the idea that stochastic behavior may be a deliberate strategy rather than a random error.

Theoretical Models Examined

The authors compared three main theoretical models to explain stochastic behavior:

1. Random Utility Models: These models assume that subjects' preferences change stochastically over time. Individuals maximize a utility function that varies randomly due to changes in subjective and objective conditions.
2. Bounded Rationality Models: These models assume that subjects have stable preferences but may not always choose the best alternative due to limitations in their decision-making capacity. Errors in choices can result from stochastic noise in the decision-making process.
3. Deliberate Randomization Models: These models posit that the stochasticity of choices is a deliberate decision by the agent. Individuals may choose to respond differently to identical questions as a strategy to minimize regret or diversify choices.

Comparison of Models

The empirical results support deliberate randomization models the most. The observed stochastic behavior, especially in HARD questions, and the deliberate choice to respond differently in consecutive repetitions suggest that individuals use randomization as an optimal strategy under uncertainty.

The authors conclude that stochastic choice is not simply the result of random errors but can be a deliberate strategy used by individuals to optimize their decisions. This has significant implications for economic decision theory, suggesting that models must consider the possibility that individuals' preferences may be inherently variable and that stochastic choices can be a rational component of decision-making behavior.

Agranov and Ortoleva's study provides an in-depth view of decision-making behavior under risk, showing that stochasticity in choices can be deliberate behavior rather than a simple error. This innovative approach challenges traditional theories of deterministic preferences and suggests new models that recognize the complexity and variability of human behavior.

Circles and Triangles: An Experimental Investigation by John D. Hey and Daniela Di Cagno

The study "Circles and Triangles: An Experimental Investigation" by John D. Hey and Daniela Di Cagno focuses on the empirical analysis of decisions under risk through an experimental design involving the representation of choices in the form of circles in the Marschak-Machina Triangle.

The experiment was designed to explore individuals' preferences between different risk games, graphically represented as circles within the Marschak-Machina Triangle. This triangle represents various combinations of risk games, where each vertex represents a possible outcome (e.g., £0, £10, £20, £30). The experimental design involved 68 participants, mainly university students, who were each presented with 60 preference questions.

As in our experiment, the risk games were presented in the form of circles on computer screens, with the probabilities of different outcomes represented as segments of a circle. Participants had to choose between pairs of risk games, indicated as circles, on a series of screens. Each choice represented a combination of two games with different probabilities of obtaining certain monetary outcomes.

Data was collected through a computerized interface, where participants indicated their preferences by pressing specific keys to select one of the available options (preference for the game on the left, preference for the game on the right, or indifference). The experiment was divided into four distinct triangles covering different combinations of amounts: £0, £10, £20, and £30. Each triangle presented various combinations of probabilities associated with these amounts, allowing exploration of a wide range of preferences and behaviors under risk.

Data analysis was conducted using ordered probit models to estimate indifference curves in the Marschak-Machina Triangle. These models allow for the analysis of binary responses (preference for one of the two games or indifference) and estimation of the parameters of the indifference curves. The estimates made it possible to identify whether the indifference curves were parallel, diverging (fanning out), or converging (fanning in).

<u>Models Used</u>

1. Generalized Regret Model (GR): This model includes additional parameters to capture the regret associated with the choices made, allowing for the analysis of how individuals' preferences change when considering regret for past decisions.
2. Differential Regret Model (DR): A particular case of the Generalized Regret Model that simplifies assumptions about regret.
3. Generalized Subjective Expected Utility (GS): An extension of the subjective expected utility theory (SEUT) that allows for greater flexibility in capturing preferences under risk.

The results showed that the indifference curves were not parallel, contrary to SEUT predictions, but tended to diverge or converge depending on the combinations of games examined. This indicates that individual preferences under risk can vary significantly and that traditional theoretical models may not be sufficient to capture these variations.

<u>Model Estimates</u>

- Generalized Regret Curve (GR): The estimates indicated that this model better explained participants' choices compared to traditional SEUT. However, the differences between GR and GS were not always statistically significant, suggesting that both models could be useful in describing behavior under risk.
- Differential Regret Curve (DR): This model was largely rejected in favor of the Generalized Regret Model, indicating that DR's simpler assumptions were not sufficient to explain participants' choices.
- Generalized Subjective Expected Utility Curve (GS): This model provided a good explanation of choices but less so than the Generalized Regret Model.

The results suggest that models incorporating regret may offer a better explanation of choices under risk compared to traditional expected utility models. This has significant implications for economic theory, suggesting that economic models need to be adapted to include factors such as regret and other emotions that influence decisions.

The experiment demonstrates that preferences under risk are complex and not always linear. The indifference curves estimate significant variations between individuals, and models considering regret provide a more robust explanation of observed behavior. These results indicate the need for further research and the adoption of more flexible models that can better capture the reality of economic choices.

The empirical analysis conducted by Hey and Di Cagno provides an important contribution to understanding decisions under risk, highlighting the limitations of traditional models and suggesting new approaches for economic theory and practice.

Investigating Generalizations of Expected Utility Theory Using Experimental Data by John D. Hey and Chris Orme

The experiment in "Investigating Generalizations of Expected Utility Theory Using Experimental Data" by John D. Hey and Chris Orme is characterized by a detailed investigation of the adequacy of various generalized preference functionals using experimental data obtained from pairwise choice questions.

The experiment was conducted with 80 subjects, mainly university students, over a period of one week to ten days. Each subject participated in four separate experiments: Circles 1, Dynamics 1, Circles 2, and Dynamics 2. The Circles experiments involved preferences between risk games graphically represented in circles, while the Dynamics experiments analyzed dynamic decision problems under risk. The Circles experiments were designed to collect data on individuals' preferences between different risk games, graphically represented in the Marschak-Machina Triangle. Subjects' choices were recorded through a computerized interface, with each subject indicating their preferences between pairs of games.

The data collected included 100 pairwise choice questions for each subject, repeated on two occasions (Circles 1 and Circles 2). The questions consisted of four sets of 25 questions, each applied to three of the four amounts: £0, £10, £20, and £30. Probabilities were all multiples of one-eighth, and subjects were informed of this. The questions were presented in random order, with the positions of the two circles inverted between Circles 1 and Circles 2 to allow for a consistency check of subjects' responses. The average consistency rate was approximately 75%.

Data analysis was conducted using ordered probit models to estimate preference functionals. The authors estimated 11 different preference functionals for each of the 80 subjects, for each of the three data sets: Data Set 1 (Circles 1), Data Set 2 (Circles 2), and Data Set 3 (Circles 1 and Circles 2 combined). In total, 2,640 preference functionals were estimated, totaling 12,960 behavioral parameters and 1,221 threshold parameters.

The authors distinguished between two types of subjects: those who always expressed a clear preference for one of the two games and those who sometimes expressed indifference. For the first type, the data was generated assuming that the preference was always clearly expressed, while for the second type, an indifference threshold was introduced, with a threshold parameter estimated for each subject who expressed indifference at least once.

Estimated Preference Models

The estimated preference models included, among others, risk neutrality theory, expected utility theory, disappointment aversion theory, reference-dependent theory, and various weighted and rank-dependent utility models. Each model was evaluated based on its ability to explain the observed choices and the consistency of the estimated parameters with theoretical expectations.

The results indicate that many of the generalized models significantly better explain the data compared to expected utility theory (EUT), although the economic superiority of these generalizations is not always clearly established. For 39% of subjects, EUT appeared to fit the data no worse than other models, while for the remaining 61%, one or more generalized preference functionals offered a better statistical fit.

Parameter estimates revealed that weighted and rank-dependent utility models, along with the quadratic utility model, emerged as the most robust. However, Yaari's duality theory and disappointment aversion theory performed worse than other models considered.

Importance of Errors

A relevant aspect that emerged from the analysis is the importance of errors in subjects' decisions. The authors emphasize that subjects often make errors in their choices, which can significantly influence the results of the analysis. Errors can arise from various factors, including misunderstanding the questions, input errors, or simply rushing to complete the experiment. This suggests that stochastic choice models may be necessary to accurately capture human decision-making behavior.

The authors conclude that decision-making under risk is complex and influenced by multiple factors, including errors and individual variations in preferences. The results suggest the need for further research and the adoption of more flexible models that can account for these factors to accurately explain economic choices. The authors' findings suggest that economic models need to evolve to reflect the complexity and variability of human behavior.

Stochastic Choice with Deterministic Preferences: An Experimental Investigation by John D. Hey and Enrica Carbone

The experiment in "Stochastic Choice with Deterministic Preferences: An Experimental Investigation" by John D. Hey and Enrica Carbone aims to explore an alternative to traditional risk choice theory, where preferences are deterministic but choices are stochastic. This study uses the quadratic utility model (Quadratic Utility Model) developed by Chew et al. (1991), which offers a flexible representation of preferences and can show the convexity needed for analysis.

The experiment was conducted with 80 subjects, mainly university students, who were exposed to 100 pairwise choice questions on computer screens. Each question presented a choice between two risky prospects, represented by circles indicating the probabilities associated with different monetary outcomes. Subjects' responses were recorded in terms of preferences between the offered options or indifference between them. After answering all the questions, one of the responses was randomly selected, and the subject was paid based on the choice made, thus incentivizing honest reporting of preferences, as in our experiment.

The data collected included subjects' responses to questions structured to explore individual preferences under risk. Specifically, only data from subjects who did not report indifference in any of the questions were selected, reducing the sample to 44 subjects useful for analysis. The authors used maximum likelihood techniques to estimate preference functionals for each subject, based on the quadratic utility model.

Data analysis revealed that many subjects exhibit behaviors that can be modeled by the stochastic quadratic utility model. However, for many other subjects, parameter estimates did not produce satisfactory results. The authors highlighted the difficulties in finding feasible parameter estimates due to the complex nature of the likelihood function, which presents discontinuities and regions of the parameter space with zero likelihood value.

The authors implemented a search process to identify feasible parameters, using linear programming to solve linear constraints and nonlinear programming to address nonlinear constraints. This allowed for identifying feasible parameters for four out of 44 subjects, suggesting that the stochastic quadratic utility model might explain the behavior of a minority of subjects.

The results of the analysis indicate that for most subjects, the stochastic quadratic utility model does not offer a better fit compared to deterministic choice models with ad hoc error specification. Only for one subject out of 44 did the model fit better than the other models in a significant and plausible way. This suggests that the stochastic component of behavior might be an unconscious part of the decision-making process, rather than an intrinsic characteristic of preferences.

The authors conclude that the weak results in favor of the stochastic choice model with deterministic preferences indicate the need for further research to better understand the dynamics of decision-making under risk. The stochastic nature of behavior might be better explained by models incorporating errors or other sources of variability in the decision-making process. Although the stochastic quadratic utility model offers a theoretically satisfying explanation, empirical results show that this model fits well only for a very limited number of subjects. Most subjects seem to behave more consistently with traditional deterministic choice explanations accompanied by random errors. Therefore, the stochastic choice model with deterministic preferences is not empirically superior to conventional models, indicating the need for further research to better understand the dynamics of decision-making under risk.

"Which Error Story is Best?" by Enrica Carbone and John D. Hey

The study "Which Error Story is Best?" by Enrica Carbone and John D. Hey aims to compare two different error theories in the context of decision-making under risk. The two main theories examined are the Constant Probability (CP) error model proposed by Harless and Camerer (1994) and the White Noise (WN) model suggested by Hey and Orme (1994). The primary objective is to determine which of these error stories provides a better description of human decision-making behavior under risk, using experimental data for evaluation.

The experimental data used in the study comes from an experiment conducted on 80 subjects who answered 100 pairwise choice questions on two separate occasions, with a few days interval between sessions. The questions presented choices between risky prospects, with subjects indicating their preference or expressing indifference. The experimental design involved randomly selecting one of the 100 questions at the end of the responses, and the subject was paid based on the expressed choice. This method was used to incentivize honest responses.

To analyze which of the two error theories better fit the data, the authors used maximum likelihood techniques to estimate preference functions combined with error specifications. The White Noise model specification implies that errors in choices are normally distributed with zero mean, while the Constant Probability model assumes there is a fixed probability of making an error on each question, regardless of the specific choices.

The model estimation procedure required the development of custom software for maximum likelihood estimation, given the complexity of the White Noise model. For the Constant Probability model, the estimation was performed by minimizing the number of prediction errors, equivalent to maximizing the predictive score. The authors used the Akaike Information Criterion (AIC) to penalize models with a larger number of parameters, allowing for a fair comparison between models of different complexity.

Empirical results showed that there is no clear superiority of one error specification over the other for all subjects. For many subjects, the Constant Probability model proved to be more suitable, while for others, the White Noise model provided a better fit. This variability in results suggests that the "best" error specification might depend on the individual characteristics of each subject's decision-making process. In aggregate terms, the Constant Probability model, combined with appropriate preference functions, generally provided a better fit compared to the White Noise model.

One of the main implications of the results is that decision-making behavior under risk can be better understood by considering error as an intrinsic component of the decision-making process, rather than just a random error. This led the authors to suggest that the stochastic specification of choices should be considered a fundamental element of risk decision theories, offering a more realistic description of observed behavior.

The paper concludes that while neither error story emerges as universally superior, the approach that combines deterministic preferences with stochastic error specifications provides a more accurate description of human decision-making behavior. The authors suggest that each stochastic specification should be matched with the "right" preference function for each subject, recognizing that individual preferences and error models can vary significantly among individuals.

Empirical results of the study demonstrate that individual variability in decision-making behavior requires an approach that considers both deterministic preferences and stochastic error specifications to accurately explain decisions under risk.

"Investigation of Stochastic Preference Theory Using Experimental Data" by Enrica Carbone

The study "Investigation of Stochastic Preference Theory Using Experimental Data" by Enrica Carbone analyzes three different error stories in the context of decision-making under risk. The primary objective is to determine which of these theories offers a more accurate description of human decision-making behavior. The central question is whether the error process can be correctly modeled to reliably infer subjects' preferences.

The experiment described in the paper involved 40 subjects who answered 94 pairwise choice questions. Each question presented a choice between two risky prospects, with subjects indicating their preference or expressing indifference. The questions were structured to cover a range of probability and monetary amount combinations, allowing for a detailed exploration of individual preferences under risk.

The data collected included subjects' responses to each of the 94 pairwise choice questions. These data were used to estimate the parameters of preference models using maximum likelihood techniques. For each subject, parameters related to the three error stories considered were estimated: the Constant Probability error model, the White Noise error model, and the stochastic preferences model.

Data analysis showed that the Constant Probability error model, which assumes a fixed probability of making an error on each choice regardless of the nature of the question, did not fit the experimental data particularly well. In contrast, the White Noise error model, which considers errors as random variability in the calculation of utility differences between choices, provided a better fit. This model assumes that decisions are based on the value of the difference between the utilities of the two choices, influenced by a normally distributed stochastic error term with zero mean.

The third model, the stochastic preferences model, considers that preferences themselves are variable and can change from one decision problem to another according to a distribution of utility parameters. This model showed similar performance to the White Noise model, suggesting that both error specifications can offer accurate descriptions of decision-making behavior.

Empirical results of the analysis indicate that the White Noise model and the stochastic preferences model outperform the Constant Probability error model. In particular, the stochastic preferences model with beta distributions of utility parameters proved slightly better than the White Noise model. However, when considering only non-monotonic versions of stochastic preferences, performance deteriorates compared to monotonic versions, suggesting that preserving monotonicity is an important characteristic of the data.

The comparative analysis using the Akaike Information Criterion (AIC) allowed models to be corrected for the number of estimated parameters, providing a fair assessment of goodness of fit. The White Noise and stochastic preferences models with beta distributions achieved the best scores, indicating good adaptability to the experimental data.

The authors conclude that while the Constant Probability error model can be excluded as less performing, the choice between the White Noise model and the stochastic preferences model is not clear-cut. However, the importance of monotonicity suggests that models respecting this property might be preferable. Further research could include experiments with less obvious dominant choices to further test the validity of stochastic preferences.

These results highlight the need to consider flexible models that correctly incorporate sources of variability in choices to accurately explain human decisions under risk.

"Deliberately Stochastic" by Cerreia-Vioglio

The empirical part of the study "Deliberately Stochastic" by Cerreia-Vioglio, Dillenberger, Ortoleva, and Riella is based on an experiment designed to explore stochastic choices as a result of deliberate randomization. The experiment was conducted to test the hypotheses proposed by the authors' theoretical model, which suggests that stochastic choice is a rational and deliberate behavior adopted by individuals to optimize overall outcomes under uncertainty.

The experiment was conducted in an experimental economics laboratory and involved participants making decisions in a series of repeated choice scenarios. The experimental sessions were designed to replicate stochastic choice conditions, and participants were not informed in advance that the questions would be repeated (part III of our experiment).

The experiment was divided into four main parts:

1. Part I: Subjects had to answer a series of repeated questions, with repetitions spaced out by other questions. The goal was to replicate traditional experimental designs and provide a comparison point with the new approaches introduced in the subsequent parts of the experiment.
2. Part II: This part included a risky investment task to measure subjects' risk aversion. Participants had to decide how much of their budget to invest in a risky activity with a predetermined probability of success, just as in our study where subjects were asked to invest a predetermined amount with the possibility of earning 2.5 times their investment.
3. Part III: In this part, subjects had to answer seven of the questions from Part I again, but the questions were repeated three times in a row, similar to Agranov & Ortoleva. However, participants were informed in advance that each question would be repeated three times consecutively, allowing the authors to examine whether awareness of repetitions would influence stochastic behavior.
4. Part IV: Subjects had to respond to standard variations of the common ratio and common consequence effects of the Allais paradox. At the end of the experiment, participants were asked to complete a non-incentivized questionnaire to explore their motivations for making different choices in the repetitions.

Data Collection and Analysis

The data collected through the experimental software included subjects' responses to each of the 94 pairwise choice questions, response times, and investment choices. At the end of each session, subjects were paid based on one of their randomly selected decisions to avoid incentives to diversify responses only to maximize overall earnings. Choices were converted into money using specific conversion rates for each part of the experiment, ensuring strong incentives for making thoughtful decisions.

The empirical analysis of the data revealed several key trends in subjects' decision-making behavior:

1. Stochastic Behavior: Consistent with previous results, the vast majority of participants (90%) chose different lotteries in the three repetitions of the same question in Part I. This stochastic behavior was predominantly observed in HARDquestions (HARD), where none of the available options were clearly better than the others. Statistical analyses, such as Fisher's exact test, confirmed that the

proportion of subjects giving inconsistent answers in HARDquestions was significantly higher compared to easy (EASY) and first-order stochastically dominated (FOSD) questions.

2. Influence of Consecutive Repetitions: In Part III, a large majority of subjects (71%) continued to show stochastic behaviors even when the questions were repeated three times in a row. Although the proportion of inconsistent responses was slightly lower than in Part I, stochastic behavior was still prevalent. This suggests that awareness of repetitions does not completely eliminate stochasticity in choices.
3. Motivations for Stochastic Choices:The final questionnaire revealed that most subjects who gave inconsistent responses in Part III did so deliberately. Only a small fraction (12%) reported having changed their mind about which option was better, while 79% stated they intentionally chose differently to increase their chances of winning with varied options or to try different possibilities. This supports the idea that stochastic behavior may be a deliberate strategy rather than a random error.

The empirical results support deliberate randomization models the most. The observed stochastic behavior, especially in HARDquestions, and the deliberate choice to respond differently in consecutive repetitions suggest that individuals use randomization as an optimal strategy under uncertainty.

The authors conclude that stochastic choice is not simply the result of random errors but can be a deliberate strategy used by individuals to optimize their decisions. This has significant implications for economic decision theory, suggesting that models need to consider the possibility that individuals' preferences may be inherently variable and that stochastic choices can be a rational component of decision-making behavior.

Cerreia-Vioglio, Dillenberger, Ortoleva, and Riella provide an in-depth view of decision-making behavior under risk, showing that stochasticity in choices can be deliberate behavior rather than a simple error. This innovative approach challenges traditional theories of deterministic preferences and suggests new models that recognize the complexity and variability of human behavior.

"Indifference, Indecisiveness, Experimentation, and Stochastic Choice" by Efe A. Ok and Gerelt Tserenjigmid

The study "Indifference, Indecisiveness, Experimentation, and Stochastic Choice" by Efe A. Ok and Gerelt Tserenjigmid explores stochastic choice, a phenomenon where individuals show variations in their choices in seemingly identical situations. This behavior can stem from three main reasons: indifference, indecisiveness, and experimentation. The authors conducted a detailed experiment to analyze these motivations and develop a theoretical model that can explain stochastic choices. The empirical analysis presented in the article focuses on the decision-making behavior of subjects under various experimental conditions, evaluating how and why their choices vary.

The experiment was conducted in a controlled environment where participants were subjected to a series of decision-making scenarios. Participants had to repeatedly choose between pairs of options, some of which were designed to be "difficult" to compare. The authors used an experimental design that included spaced and consecutive repetitions of the same questions, allowing examination of whether awareness of repetitions influenced stochastic behavior.

Data Collection and Analysis

The data collected included participants' responses to each choice question, as well as response times for each decision. At the end of the experiment, participants were asked to complete a questionnaire to explore their motivations for making different choices in the repetitions. Choices were converted into money using specific conversion rates for each part of the experiment, ensuring strong incentives for making thoughtful decisions.

Data analysis showed that a significant percentage of participants (90%) made different choices in spaced repetitions of the same question. This stochastic behavior was prevalent in "difficult" questions (HARD), where none of the available options were clearly better than the others. Statistical analyses, such as Fisher's exact test, confirmed that the proportion of subjects giving inconsistent answers in HARDquestions was significantly higher compared to easy (EASY) and first-order stochastically dominated (FOSD) questions.

In the part of the experiment with consecutive repetitions, a large majority of participants (71%) continued to show stochastic behaviors even when the questions were repeated three times in a row. This suggests that awareness of repetitions does not completely eliminate stochasticity in choices. Again, stochastic behavior was prevalent in HARD questions, confirming that the difficulty of the decision is a key factor in determining variability in choices.

The final questionnaire revealed that most participants who gave inconsistent responses did so deliberately. Only a small fraction reported changing their mind about which option was better, while the majority stated they intentionally chose differently to increase their chances of winning or to try different possibilities. This supports the idea that stochastic behavior may be a deliberate strategy rather than a random error.

The empirical results support deliberate randomization models the most. The observed stochastic behavior, especially in HARD questions, and the deliberate choice to respond differently in consecutive repetitions suggest that individuals use randomization as an optimal strategy under uncertainty.

The authors conclude that stochastic choice is not simply the result of random errors but can be a deliberate strategy used by individuals to optimize their decisions. This has significant implications for economic decision theory, suggesting that models need to consider the possibility that individuals' preferences may be inherently variable and that stochastic choices can be a rational component of decision-making behavior.

"Estimation of Indifference Curves in the Marschak-Machina Triangle" by John D. Hey and Elisabetta Strazzera

The empirical part of the article by John D. Hey and Elisabetta Strazzera focuses on the analysis of indifference curves in the Marschak-Machina Triangle, an expository method used to model individuals' decision-making behavior under risk. The primary objective is to estimate subjects' indifference maps using data collected through interviews and questionnaires. This empirical approach is innovative as it represents one of the first attempts to directly estimate subjects' indifference curves within the Marschak-Machina Triangle.

The authors conducted individual interviews with nine subjects to collect data on their indifference curves. Each subject was interviewed twice. In the first session, participants had to identify lotteries along the hypotenuse of the triangle (where one of the probabilities is zero) that were indifferent to a given initial lottery. This allowed for a first approximation of indifference curves for each subject. In the second session, subjects were shown the complete set of their previous responses and were invited to make any changes, allowing for refinement of the initial indifference curves and correction of any anomalies.

Data Collection and Analysis

The data collected included subjects' responses to a series of choice questions, as well as response times for each decision. Choices were converted into money using specific conversion rates for each part of the experiment, ensuring strong incentives for making thoughtful decisions. The authors adopted a direct approach to estimate indifference curves, using dummy variables and regression methods to model subjects' responses.

Data analysis showed that many of the indifference curves drawn by subjects were not parallel, as predicted by subjective expected utility theory (SEU). Instead, these curves tended to diverge from a common point, supporting the "fanning out" hypothesis. The authors used three alternative specifications to estimate indifference curves:

1. Parallel Indifference Curves (SEU case): where the curves are parallel lines with a common slope.
2. Indifference Curves Diverging from a Common Point (weighted utility case): where the curves are lines that diverge or converge towards a common point outside the triangle.
3. Unrestricted Indifference Curves (implicit utility case): where the curves can have any slope and intercept.

Using dummy variables and regression methods, the authors estimated these three models for each of the nine subjects. The estimation results showed that for some subjects, the diverging curves model was superior to the SEU model, suggesting that weighted utility theory might offer a better description of choice behavior under risk.

The empirical results support the idea that empirically estimated indifference curves are often not parallel, contrary to the predictions of SEU theory. This behavior can be explained by alternative theories such as weighted utility theory, which predicts that indifference curves may diverge or converge towards a common point. The authors suggest that the behavior observed in their experiments provides empirical support for these alternative theories.

The authors discuss the implications of their results for economic decision theory. They suggest that the behavior observed in their experiments provides empirical support for alternative theories to SEU, such as weighted utility theory and Loomes and Sugden's regret theory, which predict non-parallel indifference curves. This suggests that traditional economic models may need to be revised to account for the possibility that individuals' preferences can be represented by non-parallel indifference curves.

The study demonstrates that empirically estimated indifference curves are often not parallel, contrary to the predictions of SEU theory, and highlights the importance of considering alternative models that account for the possibility of non-parallel curvature in indifference curves, offering a better understanding of individuals' real decision-making behavior.

"Imprecision as an Account of the Preference Reversal Phenomenon" by David J. Butler and Graham Loomes

The empirical analysis of the article by David J. Butler and Graham Loomes focuses on the phenomenon of preference reversals, an anomaly observed in decision-making behavior that challenges traditional economic theories. The authors propose that imprecision in individual preferences can explain these reversals better than existing models. The research is based on controlled experiments to collect data on individuals' decisions in different contexts, testing hypotheses related to preference imprecision.

The experiments were designed to isolate the effect of imprecision in preferences on individuals' decisions. Participants repeatedly had to choose between different pairs of options, with experimental conditions modified to vary the choice context. This allowed observation of how and how frequently participants reversed their preferences in different contexts.

<u>Data Collection and Analysis</u>

Data was collected using structured questionnaires and interviews. Each participant completed a series of multiple-choice questions where they had to express their preference between two options, such as lotteries or bets with different outcomes and probabilities. Choices were repeated in successive sessions to measure preference consistency. The authors also collected demographic and psychological information to control for variables that might influence decision-making behavior.

Statistical analysis of the data revealed that participants' preferences were often inconsistent, with frequent preference reversals between experimental contexts. The authors used regression models to analyze the relationships between preference imprecision and the observed reversals. Results showed that uncertainty in preferences could explain a significant part of the preference reversals. Specifically, participants showed greater variability in their choices when the context increased uncertainty or decision complexity.

1. Inconsistent Behavior: The majority of participants showed inconsistent behavior, reversing their preferences in different contexts. This aligns with the hypothesis that imprecision in preferences plays a crucial role in economic decisions.
2. Imprecision and Context: Preference imprecision was found to be greater in more complex or uncertain contexts. This supports the idea that inherent uncertainty in individual preferences can lead to seemingly irrational behaviors.
3. Probabilistic Preferences: Probabilistic models incorporating preference imprecision better explained observed reversals compared to traditional deterministic models. This suggests that individuals' preferences are not fixed but vary depending on the context.

The authors discuss the implications of these results for economic decision theory. The inability of traditional models to explain preference reversals suggests the need to revise fundamental axioms of decision theory. Incorporating preference imprecision into decision models could offer a better representation of individuals' real behavior, accounting for the variability and uncertainty inherent in their choices.

In conclusion, the empirical analysis of Butler and Loomes provides convincing evidence that imprecision in individual preferences is a valid explanation for the phenomenon of preference reversals. This study suggests that economic models must consider the inherent variability of preferences to accurately explain human behavior under uncertainty, highlighting how preference imprecision can offer a compelling explanation for observed preference reversals in decision-making behavior.

"Non-Random Randomization" by Marina Agranov, Paul J. Healy, and Kirby Nielsen

The empirical part of the study by Marina Agranov, Paul J. Healy, and Kirby Nielsen focuses on analyzing experiments designed to investigate individuals' randomization behavior in various decision-making contexts. The authors aim to understand the reasons behind the choice to randomize, exploring whether this tendency arises from intrinsic preferences, reasoning errors, or other motivations. Their research is based on a meticulous experimental design involving different treatments to isolate and identify factors influencing randomization behavior.

The experiments are structured to include treatments with spaced repetitions and consecutive repetitions of the same questions. In the IND (independent) treatment, participants face a series of decision problems repeated twenty times under conditions of independent uncertainty. In the CORR (correlated) treatment, the twenty repetitions correspond to a single realization of uncertainty. This design allows testing whether randomization stems from an incorrect belief in negative serial correlation between independent repetitions.

Data Collection and Analysis

Data collected included participants' responses to a series of choice questions, response times for each decision, and a non-incentivized questionnaire at the end of the experiment exploring their motivations for making different choices in the repetitions. Choices were converted into money using specific conversion rates for each part of the experiment, ensuring strong incentives for making thoughtful decisions.

Empirical analysis reveals very high rates of randomization, with about 70% of subjects randomizing in at least one decision problem. Randomization responds sensibly to changes in environmental parameters, with subjects randomizing more when dominant bets become more attractive or less risky. Randomization is highly correlated both within and across decision problems and games, suggesting the existence of "randomization types" who randomize regardless of the environment.

In the CORR treatment, there are no significant differences compared to the IND treatment, suggesting that an incorrect belief in negative serial correlation is not the main cause of randomization. Another experiment, SEQ, eliminates the need to think about all contingencies, leading to a significant reduction in randomization in probabilistic matching problems, but not in risky-safe decisions. This indicates that the difficulty of contingent reasoning is an important factor in randomization in probabilistic matching problems.

1. Inconsistent Behavior: The majority of participants showed inconsistent behavior, reversing their preferences in different contexts. This aligns with the hypothesis that imprecision in preferences plays a crucial role in economic decisions.

2. Imprecision and Context: Preference imprecision was found to be greater in more complex or uncertain contexts. This supports the idea that inherent uncertainty in individual preferences can lead to seemingly irrational behaviors.
3. Probabilistic Preferences: Probabilistic models incorporating preference imprecision better explained observed reversals compared to traditional deterministic models. This suggests that individuals' preferences are not fixed but vary depending on the context.

The authors discuss the implications of these results for economic decision theory. The inability of traditional models to explain preference reversals suggests the need to revise fundamental axioms of decision theory. Incorporating preference imprecision into decision models could offer a better representation of individuals' real behavior, accounting for the variability and uncertainty inherent in their choices.

This study also provides a new perspective on decision theory, suggesting that economic models must consider the inherent variability of preferences to accurately explain human behavior under uncertainty.

Theoretical Models Used

In all these studies, the theoretical models analyzed were:

1. Generalized Regret Model (GR): This model includes additional parameters to capture the regret associated with the choices made, allowing for the analysis of how individuals' preferences change when considering the regret of past decisions.[30]
2. Differential Regret Model (DR): A particular case of the Generalized Regret Model that simplifies assumptions about regret.
3. Generalized Subjective Expected Utility (GS): An extension of the subjective expected utility theory (SEUT) that allows for greater flexibility in capturing preferences under risk.[31]

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The authors compared three main theoretical models to explain stochastic behavior:

1. Random Utility Models: These models assume that subjects' preferences change stochastically over time. Individuals maximize a utility function that varies randomly due to changes in subjective and objective conditions.[32]
2. Bounded Rationality Models: These models assume that subjects have stable preferences but may not always choose the best alternative due to limitations in their decision-making capacity. Errors in choices can result from stochastic noise in the decision-making process.[33]
3. Deliberate Randomization Models: These models posit that the stochasticity of choices is a deliberate decision by the agent. Individuals may choose to respond differently to identical questions as a strategy to minimize regret or diversify choices.[34]

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