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Risk Parity Portfolio: A comparative analysis with traditional allocation strategies

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Introduction

Investment portfolios are the foundation and starting point for individuals and institutional investors who want to invest their own wealth efficiently. Many investment strategies have been developed over the years, each of them targeting individuals with different financial goals.
Traditional allocation strategies have long been the foundation of portfolio construction. However, these approaches often result in portfolios that are heavily dominated by equity risk, leaving investors at risk of losses during market downturns. In response to this challenge, the Risk Parity approach has emerged as an innovative strategy that shifts the focus from capital allocation to risk allocation, aiming to create a more balanced and resilient portfolio.

The objective of this thesis is to explore the Risk Parity approach as a modern investment strategy and to see how it differs from traditional strategies in terms of risk and return. Risk Parity focuses on equalizing the risk contribution of each asset class within a portfolio. By doing so, it aims to create a more diversified and stable portfolio that can perform well across different market cycles, including periods of economic uncertainty.

In Chapter 1 we present and discuss the foundations and implications of traditional allocation strategies, starting from the pillar of portfolio management, Modern Portfolio Theory by Harry Markowitz and the concept of Mean-Variance Optimization. We then move on to some common traditional allocation strategies that will be later used for the analysis in chapter 3, such as the 50/50 equity-bond split and equally weighted portfolio strategies, highlighting historical performance, practical applications, as well as the pros and cons of such strategies.
In Chapter 2, we delve into the principles of the Risk Parity approach. The idea of Risk Parity is to allocate assets in such a way that the risk is evenly spread out across the asset classes by adjusting the weights of assets based on their volatility and correlation. We also discuss the role that leverage plays in this strategy.

Finally, in Chapter 3 we will conduct an empirical analysis to compare the performance of Risk Parity portfolios with that of traditional strategies. First, we compare a portfolio made up of two asset classes, equities and bonds, constructed as a Risk Parity portfolio and as a 50/50 portfolio. We then expand the analysis to a multiple-asset portfolio made of equities, U.S. Treasuries, commodities, real estate, and inflation-protected bonds; here we construct such portfolios as a Risk Parity portfolio and as an Equally Weighted portfolio, and then compare their performance.

The results of the research highlight that Risk Parity is not necessarily superior, but serves as a complementary strategy for investors prioritizing risk management and stability over return maximization.

Chapter 1: Traditional Allocation Strategies

Asset allocation is one of the core concepts in modern portfolio theory and investment management. It refers to the process of splitting an investment portfolio into different asset classes, to balance risk and return based on an investor's goals, time horizon, and risk tolerance. Widely considered to be one of the most important factors of portfolio performance, asset allocation aims to create a diversified portfolio that maximizes returns for a given level of risk or minimizes risk for a given level of return. The way in which investors allocate their capital across different asset classes directly impacts their portfolio's risk-return characteristics.

Over the years, a wide range of allocation strategies have been developed, from simple static allocations to more complex optimization techniques. Traditional allocation strategies are often used as benchmarks in investment decision-making due to their straightforward implementation and historical reliability.
A well-diversified portfolio ensures that investors are not overly exposed to the performance of a single asset or market segment. Instead, by allocating investments across different asset classes, investors can achieve a balance between risk and potential return. Many studies have shown that asset allocation adds more to how well a portfolio does than picking out securities on their own or guessing the right time for the market.

Historically, investors have sought different methods to optimize their asset allocation strategies. The main goals of asset allocation are capital preservation and growth maximization. Conservative investors may prioritize capital preservation, seeking lower volatility and more stable returns through greater exposure to fixed income securities[1] and defensive asset classes. Conversely, investors who prioritize

[1] Investments that provide regular, fixed payments, such as bonds. They are generally considered less risky than stocks but offer lower potential returns.

growth may adopt a more aggressive approach, emphasizing equities and higher-risk asset classes to maximize long-term capital appreciation.

The importance of asset allocation is further highlighted by the cyclical nature of financial markets[2]. Economic conditions, interest rate fluctuations, inflationary pressures, and geopolitical events all influence asset class performance. Thus, a well-constructed asset allocation strategy must not only account for historical return patterns but also consider forward-looking market conditions. Over time, financial innovations have introduced more sophisticated allocation strategies that attempt to optimize risk-adjusted returns by dynamically adjusting asset class exposures in response to market signals.

One of the fundamental principles underlying asset allocation is diversification. The concept of diversification is based on the fact that asset classes have different levels of correlation from one another. For example, equities and fixed income assets have historically shown an inverse relationship; when equity markets decline, bonds often perform better as investors look for safer assets. This negative correlation allows investors to construct portfolios that achieve higher risk-adjusted returns than investing in a single asset class. Moreover, diversification helps to reduce unsystematic risk, arising from individual securities or sectors specific factors, thereby reducing overall portfolio volatility.

A comprehensive asset allocation strategy involves many layers of decision. Investors must determine strategic asset allocation, which defines long-term target allocations for each asset class based on risk tolerance, investment goals, and market expectations. In addition to SAA, tactical asset allocation (TAA) involves short-term adjustments to the weights of the assets based on current market conditions and economic outlooks. While SAA provides stability and discipline to the investment process, TAA makes the process more flexible and takes advantage of short-run market inefficiencies.

[2] Markets tend to go through periods of growth (bull markets) and decline (bear markets) over time.

Traditional allocation strategies, such as equally weighted portfolios, fixed allocation portfolios (e.g., the 60/40 or 50/50 equity-bond split), and mean-variance optimization (MVO), have long been the cornerstone of portfolio construction. These methods offer different approaches to balancing risk and return, each having its own strengths and limitations. While equally weighted and fixed allocation strategies are easier and simpler to implement, they might not always offer optimal risk-adjusted returns. Conversely, MVO aims to mathematically optimize portfolio allocation based on expected returns, volatility, and correlations, but is often criticized for its sensitivity to estimation errors.

Asset allocation is not a merely theoretical study; it has vast impacts on the real world of investment decisions. Pension funds and sovereign wealth funds are examples of institutional investors who allocate their capital across multiple asset classes to meet their long-term financial goals. Similarly, individual investors rely on asset allocation strategies to meet their retirement goals and to preserve their wealth. Understanding how different allocation strategies perform under varying market conditions is crucial in making informed investment choices.

In this chapter, we explore various traditional asset allocation strategies in greater detail. We examine their theoretical foundations, historical performance, and practical applications, providing a comprehensive framework for understanding how asset allocation shapes investment outcomes. In doing so, it will set the stage for later comparisons with the Risk Parity approach, which challenges traditional allocation methods by focusing on risk-adjusted balance rather than capital-based weighting.

1.1 Modern Portfolio Theory

One of the most significant and influential frameworks in asset allocation and portfolio management is Modern Portfolio Theory. Introduced by Harry Markowitz in 1952, MPT revolutionized investors’ perspective on diversification, risk, and return. The central premise of MPT is that an investor can create an optimal portfolio by choosing a mix of assets that maximizes expected return for a given level of risk, or conversely, minimizes risk for a given level of expected return. This optimization is achieved through the careful assessment of asset correlations, volatilities, and expected returns.

MPT assumes that investors are rational and risk-averse, meaning they will choose a portfolio that offers the highest return for a given level of risk. This assumption is the one upon which many of the modern investment strategies are built and is still a cornerstone of modern financial theory.

1.1.1 The Efficient Frontier

The concept of the Efficient Frontier is one of the fundamental contributions of MPT. This frontier represents a set of optimal portfolios that yields the highest expected return for some level of risk. Portfolios that are beneath the frontier are said to be suboptimal as they either incur excessive risk or fail to optimize returns for the level of risk taken.

In constructing the Efficient Frontier, investors evaluate various combinations of assets to identify the optimal mix. This involves calculating the expected return and standard deviation for each potential portfolio. These portfolios can then be plotted in risk-return space to reveal the Efficient Frontier as an upward-sloping boundary, usually hyperbolic in shape. Portfolios on this frontier are considered "efficient" because they maximize return for a given level of risk.

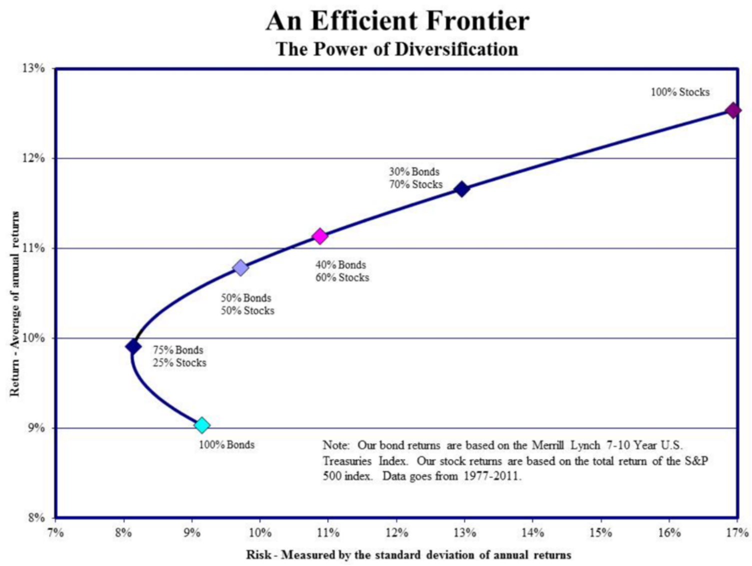

Figure 1: Visualization of the Efficient Frontier, demonstrating the trade-off between risk and return in Modern Portfolio Theory (MPT)[3].

Markowitz proved that investors can reduce portfolio volatility while maintaining returns through diversification across assets that have low or negative correlations. The mathematical foundation of this principle relies on variance and covariance calculations that assess how assets move relative to each other. By computing the expected return, standard deviation, and correlation coefficients of asset combinations, an investor can construct a portfolio that falls on the efficient frontier.

1.1.2 Active and Passive Investment Strategies

MPT serves as the foundation for both active and passive investment strategies. Passive strategies rely on MPT principles by maintaining broad market exposure to minimize risk while capturing overall market returns. Passive investing seeks to replicate market returns by maintaining broad exposure to an index or asset class.

[3] Research and publishing by Louise Cooper. Source: www.forexlive.com

The most common example of such strategy is index investing, where funds track benchmarks like the S&P 500[4], and hold it for a long time.
Passive strategies are built on the premise that markets are efficient, and consistently outperforming the market through active management is difficult. Exchange-Traded Funds[5] and index mutual funds are common vehicles used for passive investing.

Active strategies, on the other hand, attempt to outperform the market by adjusting portfolio composition based on expected returns, often deviating from MPT's prescribed efficient frontier in search of alpha[6]. Active investing involves selecting securities or asset classes in an attempt to outperform the market. Active strategies may include stock picking, tactical asset allocation, sector rotation[7], and hedge fund strategies. Fund managers use research, analysis, and predictive modeling to make investment decisions.

While both strategies have their merits, many investors adopt a hybrid approach, combining passive index funds for core holdings and active management for tactical opportunities. Choosing from active and passive investing strategies depends on individual risk tolerance, investment goals, and market conditions.

Some of the advantages of passive investing are lower costs, simplicity, and diversification. However, downsides of passive strategies are limited potential for excess returns and vulnerability to market downturns.
Advantages of active strategies include potential to outperform the market, risk management, and flexibility, while the disadvantages of active investing are higher fees, greater complexity, and it requires research and expertise.

[4] The S&P 500 is a stock market index that tracks the stock performance of the largest 500 publicly traded U.S. companies.
[5] ETFs are investment funds that trade on stock exchanges, similar to stocks. They typically track an index, commodity, or basket of assets.
[6] Alpha refers to the excess return of an investment relative to the return of a benchmark.
[7] Sector rotation involves shifting investments from one industry sector to another based on economic cycles or market conditions.

1.2 Mean-Variance Optimization

Mean-Variance Optimization (MVO) was pioneered by Harry Markowitz as part of Modern Portfolio Theory (MPT) in the early 1950s. This idea radically changed the way portfolios are built by offering a mathematical framework for selecting asset allocations that balance return and risk optimally. Before Markowitz's work, portfolio selection was largely based on intuition, past performance, and qualitative factors. His introduction of variance as a measure of risk allowed for a more systematic, quantitative approach to investing.

MVO is still very popular in institutional portfolio management, particularly in pension funds, endowments, and hedge funds seeking to optimize their risk-adjusted returns. The model gained further popularity with the development of advanced computational tools, which allow for real-time portfolio adjustments and sensitivity analyses.

The central equation for MVO can be formulated as follows:

$$\min \frac{1}{2} w^{\mathrm{T}} \sum w \quad \text{subject to } \mathrm{w}^{\mathrm{T}}\mathbf{1} = 1, \quad \mathrm{w}^{\mathrm{T}}\mu = R_t$$

where

- w represents the vector of assets weight;
- Σ is the covariance matrix[8] of asset returns;
- 1 is a vector of ones ensuring full capital allocation;
- μ represents the vector of expected returns;
- R_t is the target expected return.

Through this optimization, investors can derive the optimal portfolio that lies on the Efficient Frontier, balancing risk and return according to their preferences.

[8] A covariance matrix is a mathematical tool used in portfolio optimization to measure how different assets move in relation to each other. It helps in understanding the diversification benefits of combining assets.

MVO provides a quantitative framework for portfolio optimization, but it has a number of limitations. In particular, because MVO is extremely sensitive to estimation errors in expected covariances and returns, it is very vulnerable to changes in market conditions. Moreover, it assumes that investors are rational and that markets follow a normal distribution, and this is not always the case. To address these limitations, refinements such as Bayesian adjustments[9] and robust optimization have been introduced, improving the resilience of MVO-based portfolios in dynamic markets.

1.3 Common Traditional Allocation Strategies

Traditional asset allocation strategies serve as the foundation for investment portfolio construction. These methods have been widely used by both individual and institutional investors due to their simplicity, effectiveness, and reliability. Among the most common approaches are the equally weighted portfolio and the fixed allocation portfolio, such as the 50/50 equity-bond strategy. These strategies provide different levels of diversification, risk exposure, and expected returns, making them suitable for various investor profiles.

Although modern portfolio theory and advanced optimization techniques offer more sophisticated allocation methods, traditional strategies have stood the test of time because of their ease of implementation and effectiveness in achieving risk-adjusted returns.

1.3.1 Equally Weighted Portfolio

An equally weighted portfolio is an asset allocation strategy in which capital is distributed equally among all assets in a portfolio. Unlike strategies that allocate funds based on market capitalization or risk-adjusted measures, an equally weighted

[9] Bayesian adjustments are statistical techniques used to refine estimates in portfolio optimization by incorporating prior knowledge or beliefs about market conditions.

approach assigns the same percentage to each asset class regardless of historical returns, volatility, or any other financial metrics.

For instance, in a five-asset portfolio, an equally weighted strategy would allocate 20% of total capital to each asset class, regardless of its perceived risk or return potential. This ensures broad diversification, reducing reliance on any single asset to drive portfolio performance. The method contrasts with market-weighted portfolios, where allocations are based on the relative market capitalization of each asset.

The concept behind equally weighted portfolios is simple: by giving all asset classes the same importance, investors mitigate the potential negative effects of overexposure to a single asset or sector. This strategy is particularly appealing to investors who believe in long-term mean reversion[10], assuming that underperforming assets will eventually recover while outperforming assets may experience corrections.

Historical data suggests that equally weighted portfolios often outperform market-capitalization-weighted portfolios over the long term due to the rebalancing effect, which systematically buys underperforming assets and sells outperforming ones. Studies of the S&P 500 Equal Weight Index compared to the S&P 500 Market Cap Index have shown that equal weighting can generate higher returns, although with greater volatility.

One of the main reasons for this outperformance is the equal exposure to smaller stocks, which historically tend to offer higher risk-adjusted returns than large-cap stocks. However, during prolonged market rallies, equally weighted portfolios may underperform because they reduce exposure to dominant large-cap companies that drive major market indices.

[10] Mean reversion is the theory that asset prices and returns tend to move back toward their historical average over time.

Advantages and Disadvantages of Equally Weighted Portfolios

Let's list some of the benefits of Equally Weighted Portfolios:

- **Simplicity**: They are easily constructed without the need of advanced financial models, and they don't require complex calculations or market forecasts.
- **Diversification**: This strategy ensures exposure to multiple asset classes, reducing concentration risk.
- **Performance in volatile markets**: equally weighted portfolios can outperform capitalization-weighted portfolios in periods of high market volatility.

Disadvantages of this strategy include:

- **Frequent Rebalancing**: Since asset prices fluctuate, maintaining equal weightings requires periodic rebalancing, which incurs transaction costs.
- **Ignores Risk**: Allocating equal capital does not account for an asset's individual risk profile.
- **Underperformance in Bull Markets**: In strong equity bull markets, capitalization-weighted portfolios tend to outperform equally weighted portfolios since larger stocks grow disproportionately.

1.3.2 Fixed Allocation Portfolios (50/50 split)

A fixed allocation portfolio is an investment strategy that assigns assets with fixed weightings and relies on periodic rebalancing to maintain the target allocation. The 50/50 portfolio, one of the most well-known examples, allocates 50% of capital to equities and 50% to bonds, providing a balance between growth and stability.
The rationale behind the 50/50 strategy is to achieve an optimal mix of capital appreciation (through equities) and risk mitigation (through bonds). Equities

provide exposure to economic growth and earnings, while bonds serve as a calming factor, protecting against market downturns and reducing overall volatility.
Unlike more aggressive strategies that emphasize equities, or more conservative strategies that focus on bonds, the 50/50 approach offers a balanced alternative, making it suitable for long-term investors who prioritize capital preservation while seeking steady growth.

The 50/50 strategy, because of the historical market behavior and the stability shown by empirical studies, has gained wide acceptance within the investment field. Over the past century, equity markets have experienced significant growth, but they have also faced periodic downturns due to economic recessions, financial crises, and geopolitical turbulence.
Historically, bonds have provided a hedge against equity downturns, as interest rates tend to fall in an economic recessions, leading to higher bond prices. By maintaining a 50% allocation to bonds, investors reduce overall portfolio drawdowns[11] during market crashes while still benefiting from long-term equity growth.

The 50/50 portfolio has demonstrated strong risk-adjusted returns over time, performing well across multiple market cycles. During the 2008 financial crisis, the 50/50 portfolio experienced significantly smaller drawdowns compared to all-equity portfolios, demonstrating its effectiveness in reducing downside risk.
However, during extended bull markets, such as the 1990s technology boom or the 2010s equity bull run, the strategy lagged behind aggressive equity portfolios. Despite this, its ability to protect capital during downturns makes it a popular choice for long-term investors.

[11] A drawdown refers to a decline in the value of an investment or portfolio below its peak value.

Advantages and Disadvantages of 50/50 Equity-Bond Portfolio

Advantages of the 50/50 split include:

- **Balanced Risk**: Provides a middle ground between aggressive and conservative portfolios.
- **Diversification**: Stocks and bonds historically have a low correlation, meaning when one declines, the other often appreciates.
- **Reduced Volatility**: Compared to portfolios with higher equity exposure, the 50/50 portfolio experiences less severe drawdowns during bear markets.

However there are also downsides to this strategy such as:

- **Rebalancing**: Regular rebalancing is necessary to maintain the 50/50 split, incurring additional transaction costs.
- **Protection Against Inflation**: Bonds tend to struggle in high-inflation environments, as rising interest rates erode their real returns, so it may not fully protect investors from inflation.

Chapter 2: The Risk Parity Approach

The Risk Parity approach is a portfolio allocation strategy that seeks to balance risk contributions among different asset classes rather than allocating capital in fixed proportions. Unlike traditional allocation methods, Risk Parity portfolios allocate assets in such a way as to ensure that each asset contributes an equal share of total portfolio risk. With the risk parity approach, the portfolio becomes more resilient and diversified by adjusting the weightings according to the dynamics of asset volatility and correlation.

The origins of Risk Parity go back to the early 2000s, when firms in the investment business, such as Bridgewater Associates[12], developed the All Weather Portfolio, designed to perform well across different economic environments. The 2008 global financial crisis further highlighted the need for an asset allocation strategy that is less dependent on equity market performance. Traditional portfolios, particularly those heavily weighted toward stocks, suffered significant losses during the crisis, whereas risk-balanced portfolios held up better. As a result, institutional investors, pension funds, and hedge funds started looking into Risk Parity strategies as a way to improve risk-adjusted returns.

The key driver behind the adoption of Risk Parity is the realization that traditional portfolios tend to be dominated by equity risk[13], despite seemingly balanced capital allocations. For instance, in a 60/40 stock-bond portfolio, equities often account for more than 80% of total portfolio risk, because their volatility is significantly higher than that of bonds. With such a concentration of risk, drawdowns tend to be extended in bear markets and, consequently, the stability of the portfolio is lower.

[12] Bridgewater Associates is one of the world's largest hedge funds, known for its innovative investment strategies.
[13] Equity risk refers to the risk associated with investing in stocks, which can experience significant price fluctuations.

Risk Parity addresses this issue by adjusting asset weights according to their respective risk levels, often leading to a higher bond allocation, sometimes accompanied by the use of leverage to enhance expected returns.

The Risk Parity framework assumes that financial markets are uncertain and risky, rather than being predictable of future returns. Traditional allocation strategies often rely on expected returns to determine asset weights, but these estimates can be highly volatile and unreliable. Instead, Risk Parity shifts the focus away from return predictions and toward risk management, aiming to construct a portfolio that performs consistently across different economic cycles. This makes Risk Parity particularly appealing for long-term investors, who prioritize stability and capital preservation over speculative gains.

2.1 Principles and Rationale behind Risk Parity Portfolios

The Risk Parity strategy challenges the traditional capital-weighted approach to asset allocation by redefining how risk is distributed across a portfolio. Instead of allocating fixed proportions of capital to different asset classes, Risk Parity seeks to equalize the contribution of each asset to the portfolio's overall risk. This reallocation results in a more diversified risk exposure, reducing dependence on any single asset class and improving portfolio resilience across different economic conditions. At its core, the rationale behind Risk Parity stems from the recognition that assets exhibit different levels of volatility and correlation.

2.1.1 Performance Across Market Cycles

One of the major appeals of Risk Parity is its ability to adapt to various economic environments. Economic conditions fluctuate due to factors such as inflation, interest rates, and business cycles, affecting asset class performance in different ways. Risk Parity portfolios are designed to maintain balance across these different environments, increasing their robustness over time.

Risk Parity works well under the assumption that:

- Bonds perform well during recessions when interest rates decline.
- Equities perform well during periods of economic growth when corporate earnings expand.
- Commodities and inflation-protected securities[14] perform well during inflationary periods, preserving real purchasing power.

By distributing risk more evenly among these asset classes, Risk Parity portfolios can mitigate the cyclicality risk that traditional portfolios face when overexposed to a single economic regime. This makes them particularly appealing for long-term investors, pension funds, and institutional investors looking for stability.

2.1.2 The Role of Leverage

Since Risk Parity often results in a higher allocation to low-risk assets (such as bonds), it naturally leads to lower expected returns compared to equity-heavy portfolios. To address this, many institutional investors employ leverage to scale up portfolio returns while maintaining balanced risk exposure. This means borrowing funds to increase exposure to bonds and other lower-risk assets, ensuring that the portfolio achieves competitive returns while keeping its risk allocation intact.
Using leverage, an investor can increase exposure to low-risk assets while keeping overall risk within acceptable levels. This process can be described as:

$$w_{leveraged} = \boldsymbol{\lambda} w_{RP}$$

where λ represents the leverage factor applied to the Risk Parity weights w_{RP}.

By borrowing additional capital and increasing exposure proportionally, leveraged

[14] Bonds designed to protect investors from inflation.

Risk Parity portfolios can achieve similar return levels to traditional portfolios but with a better risk-adjusted profile.
However, the use of leverage introduces additional risks, such as interest rate risk and liquidity risk; the first stems from the fact that the cost of borrowing can rise, reducing the effectiveness of leveraged portfolios, while the latter comes up especially in times of market stress, when leveraged positions may be more difficult to unwind.
Despite these challenges, historical backtests suggest that Risk Parity portfolios, when appropriately leveraged, have delivered higher risk-adjusted returns than traditional allocation strategies.

2.2 Impacts of Correlation on Risk Contributions

A key component of risk decomposition is the correlation structure of the assets in a portfolio. Risk Parity does not just allocate based on individual asset volatility; it also accounts for how assets interact with each other. The pairwise correlations among assets influence total risk and the extent to which each asset contributes to it.
The presence of low or negative correlations between assets leads to greater diversification benefits, reducing total portfolio risk without significantly lowering expected returns. This is particularly important in multi-asset portfolios, where including bonds, commodities, and inflation-protected securities can provide valuable hedging effects[15] against equity market downturns.

To illustrate this, consider a two-asset case where the total portfolio volatility is:

$$\sigma_p = \sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2\rho w_1 w_2 \sigma_1 \sigma_2}$$

where ρ is the correlation coefficient between asset 1 and asset 2.

[15] Hedging effects occur when an investment is used to offset potential losses in another investment.

If ρ is highly positive, the two assets move in the same direction, contributing significantly to total portfolio risk.
If ρ is low or negative, diversification effects reduce overall volatility, making the portfolio more stable.

Since Risk Parity portfolios aim to balance risk rather than capital, they tend to allocate higher weights to assets that are lowly correlated with the rest of the portfolio, maximizing diversification benefits. This means that asset classes such as government bonds, gold, and inflation-protected securities often receive a higher weighting in a Risk Parity framework than in traditional allocation models.

A critical challenge in real-world implementation is that correlations are not static, so they tend to rise in periods of market stress (e.g., during financial crises, many assets exhibit stronger co-movement). As a result, Risk Parity portfolios may temporarily lose some of their diversification benefits when they are needed most. Investors often attempt to mitigate this by dynamically adjusting portfolio allocations or incorporating stress-testing techniques[16] to model correlation shifts.

[16] Stress-testing involve simulating extreme market conditions to evaluate how a portfolio might perform during periods of financial stress, such as a market crash or economic crisis.

Chapter 3: Empirical Analysis

In this chapter, we will conduct two analyses to compare and understand the performance differences between two distinct strategies: the Risk Parity portfolio strategy and the Equally-Weighted portfolio strategy.

The first analysis focuses on a portfolio composed of two asset classes. It is evaluated under two constructions: as a Risk Parity portfolio and as 50/50 equity-bond portfolio. The second analysis expands the scope to a portfolio with five asset classes, comparing its performance when constructed as a Risk Parity portfolio and as an Equally-Weighted portfolio, allocating 20% of the capital to each asset class. Before jumping into the results of the analyses, let's present the analytics to derive a risk parity portfolio.

3.1 Analytics behind the Construction of a Risk Parity Portfolio

The construction of a Risk Parity portfolio involves determining portfolio weights such that each asset contributes equally to the overall portfolio risk. This concept is fundamentally different from traditional approaches, such as equally-weighted portfolios or mean-variance optimization, which focus on capital allocation or risk-return trade-offs, respectively.

The total risk of a portfolio, typically measured by its standard deviation, depends on the portfolio weights and the covariance structure of the assets. The variance of the portfolio is given by:

$$\sigma_p^2 = \mathbf{w}^{\mathrm{T}} \boldsymbol{\Sigma} \, \mathbf{w}$$

Here $\mathbf{w} = [w_1, w_2, \ldots, w_N]^{\mathrm{T}}$ is the vector of portfolio weights, where w_1 represents the proportion of the portfolio invested in asset i, and $\boldsymbol{\Sigma}$ is the covariance matrix of asset returns, capturing both the individual variances of the assets and their pairwise correlations.

The standard deviation of the portfolio, σ_p, is the square root of the portfolio variance.
Each asset's contribution to the overall portfolio risk, known as its risk contribution (RC_i), is expressed as:

$$RC_i = w_i \cdot (\mathbf{\Sigma}\ \mathbf{w})_{\boldsymbol{i}}$$

What this relation highlights is that an asset's risk contribution depends on both its weight in the portfolio (w_i), and its marginal contribution to the portfolio variance $(\mathbf{\Sigma}\ \mathbf{w})_{\boldsymbol{i}}$. The latter term incorporates both the asset's own risk (variance) and its relationship with other assets (covariance).

The key principle of a Risk Parity portfolio is that the risk contributions of all assets are equal. This implies that:

$$RC_1 = RC_2 = \cdots = RC_N$$

In this formulation, the portfolio is designed to balance the contributions of individual assets to the portfolio's overall risk, rather than to its total capital. This equal risk contribution principle ensures that no single asset disproportionately drives the portfolio's volatility.

To construct a Risk Parity portfolio, we solve for the weights **w** that satisfy the ERC condition. This is typically framed as an optimization problem, where the objective is to minimize the deviation of each asset's risk contribution from the target risk contribution. The optimization problem can be expressed as:

$$\min_{w} \quad \frac{1}{2}\sum_{i=1}^{N}(RC_i - \overline{RC})^2$$

subject to

$$\sum_{i=1}^{N} w_i = 1, \quad w_i \geq 0 \quad \forall i$$

where $\overline{RC} = \frac{\sigma_p}{N}$ is the target risk contribution for each asset, assuming equal risk contributions across N assets. This optimization problem is non-linear due to the involvement of the covariance matrix Σ in the computation of RC_i.

In the special case of a portfolio with two assets, the Risk Parity weights can be determined analytically without the need for numerical optimization. The weights are computed by inverting the assets' standard deviations and normalizing them to sum to one

$$w_1 = \frac{\sigma_2}{\sigma_1 + \sigma_2}, \qquad w_2 = \frac{\sigma_1}{\sigma_2 + \sigma_1}$$

This relationship highlights that an asset with higher volatility (σσ) will have a smaller weight in the Risk Parity portfolio, as its contribution to risk would otherwise dominate.

For portfolios with more than two assets, the weights are typically determined using an iterative procedure due to the complexity of the optimization problem. In our analysis standard deviations (σ_i) were estimated for each asset using historical returns, the covariance matrix (Σ) was computed to account for the relationships between asset returns, and the risk parity weights (w) were initialized by inversely weighting each asset by its standard deviation. These initial weights were then normalized to sum to one

$$w = \frac{w}{\sum_{i=1}^{N} w_i}$$

In our analyses, this methodology was applied to a two-asset portfolio, where weights were computed analytically using the simplified formula, and to a five-asset portfolio, where weights were estimated iteratively using historical standard deviations and covariance matrices.

3.2 Analysis in the Two Asset Case

The objective of this analysis is to evaluate the performance of two distinct portfolio construction strategies, Risk Parity and a 50/50 equity-bond portfolio, and compare their characteristics in terms of risk and return. By applying these two strategies to the same set of asset classes over a consistent timeframe, we aim to understand how these approaches differ in their handling of risk allocation and whether Risk Parity offers advantages in portfolio management.

Let's discuss more in-depth about the asset classes and time windows chosen for this analysis.
We start by defining the asset classes; we choose the S&P 500 index representing equity, and the iShares Core US Aggregate Bond ETF (denoted by the ticker "AGG") representing bonds. The S&P 500 index is a widely recognized benchmark of the U.S. stock market. It encompasses 500 of the largest publicly traded companies and reflects the broader market's performance. The AGG provides exposure to a broad range of U.S. investment-grade bonds, including Treasuries, corporate bonds, and mortgage-backed securities.

Then we defined an estimation window and a testing window; we choose 2009/01 – 2018/12 as our estimation window, and 2019/01 – 2022/12 as our testing window. Over both windows, we calculated monthly returns using closing values, and we derive some historical statistics about the two asset classes, as shown in Table 3.1 and Table 3.2.

Statistic	Value
Mean	1.0213%
Standard Deviation	0.0441
Skewness	-0.4036
Kurtosis	0.4817
Serial Correlation	-0.1379

Table 3.1: S&P 500 Statistics

Statistic	Value
Mean	0.2001%
Standard Deviation	0.0113
Skewness	-0.6251
Kurtosis	2.1154
Serial Correlation	0.1085

Table 3.2: iShares Core US Aggregate Bond ETF Statistics

From the tables, it can be observed that the S&P 500 demonstrates a higher mean return and significantly greater volatility compared to the iShares Core US Aggregate Bond ETF, reflecting the inherent differences between equity and bond markets. The bond ETF, while offering lower returns, provides stability and lower risk, which is essential for a balanced portfolio.

3.2.1 Composition of the Risk Parity Portfolio

The Risk Parity portfolio was constructed by equalizing the risk contributions of the two assets. Unlike the 50/50 portfolio, which allocates capital equally (50% to equities and 50% to bonds), the Risk Parity strategy adjusts weights based on the assets' volatilities and correlations.

Using historical data from the estimation window, we estimate the correlation matrix to understand the relationship between the two asset classes

$$\text{Correlation Matrix} = \begin{bmatrix} & \text{S\&P500} & \text{iShares AGG} \\ \text{S\&P500} & 1 & -0.073825 \\ \text{iShares AGG} & -0.073825 & 1 \end{bmatrix}$$

The correlation between the two assets was slightly negative (-0.0738), indicating diversification benefits.

We then calculate the weights, obtaining $w_{S\&P500}^{RP} = 20.40\%$ and $w_{AGG}^{RP} = 79.60\%$. Therefore, the total capital will be allocated according to the weights, in order to equalize the contribution to the total portfolio's risk from each asset class.
This allocation reflects the inverse relationship between asset weight and volatility; because AGG showed significantly lower volatility compared to the S&P 500, it received a much higher weight to ensure its risk contribution matched that of equities. Conversely, the S&P 500, being the more volatile asset, was assigned a smaller weight to prevent it from dominating the portfolio's overall risk.
By contrast, the 50/50 portfolio maintained a fixed 50% allocation to each asset class, regardless of their volatility. This key difference illustrates how the Risk Parity approach dynamically adjusts allocations to equalize risk contributions rather than capital.
The performance of the two portfolios was evaluated using descriptive statistics and key risk-adjusted metrics over the testing window (2019/01–2022/12).

Portfolio	**Mean**	**St. Dev**	**Covariance**	**Beta**	**Downsize Risk**
Risk Parity Portfolio	0.1001%	0.0212	0.0010	0.3188	0.0161
50/50 Portfolio	0.3522%	0.0322	0.0018	0.5605	0.0204

Table 3.3: Descriptive statistics for Risk Parity and 50/50 portfolios

	Sharpe Ratio	Sortino Ratio	Treynor Ratio
Risk Parity Portfolio	0.0473	0.0623	0.0031
50/50 Portfolio	0.1094	0.1729	0.0063

Table 3.4: Performance measures for Risk Parity and 50/50 portfolios.

From the results it can be deduced that the Risk Parity portfolio exhibited lower volatility than the 50/50 portfolio; The standard deviation (0.0212) reflects the portfolio's emphasis on balancing risk, resulting in greater stability compared to the 50/50 portfolio.

The performance measures[17] further underscore these differences. While the 50/50 portfolio achieved better risk-adjusted returns across all three metrics, the Risk Parity portfolio excelled in minimizing risk.
These results highlight the inherent trade-offs in portfolio construction. The 50/50 portfolio may be preferred by investors seeking higher returns and willing to tolerate greater volatility; the Risk Parity portfolio, with its focus on risk balance, offers a more stable investment approach, appealing to those prioritizing risk management over return maximization.

The findings demonstrate that Risk Parity is not universally superior but serves as a complementary strategy, particularly for portfolios requiring diversification and stability.

3.2.2 The Process of Rebalancing

A key component of the Risk Parity approach is periodic rebalancing, ensuring that the portfolio continues to allocate risk evenly across asset classes as market

[17] The Sharpe, Sortino, and Treynor ratio measure the risk-adjusted return of an investment. The Sharpe ratio is calculated as the excess return (return above the risk-free rate) divided by the standard deviation of returns; the Sortino ratio focuses on downside risk (negative returns) rather than overall volatility; the Treynor ratio uses beta (market risk) as the measure of risk rather than standard deviation.

conditions evolve. Without rebalancing, changes in asset volatility and correlations could cause the portfolio to drift away from its intended risk allocation.

Keeping the same estimation and testing windows as before, the portfolio is rebalanced annually at the beginning of each year over the testing window. Each year, the estimation window was extended by one year, incorporating the most recent data while removing the oldest.

For instance, in 2020, the estimation window moved from 2010 – 2019 (instead of 2009 – 2018). This rolling approach ensured that risk estimations reflected current market conditions.

Using the new estimation window, the standard deviations of the S&P 500 and AGG were recalculated. The Risk Parity weights were then recomputed and used to calculate the returns for this year. This process was repeated at the start of each new year.

By contrast, the 50/50 portfolio remained unchanged throughout the testing period, maintaining a static allocation of 50% equity and 50% bonds.

After implementing annual rebalancing, the descriptive statistics and performance measures for the Risk Parity portfolio were recalculated.

Portfolio	Mean	St. Dev	Covariance	Beta	Downsize Risk
Risk Parity Portfolio	0.1979%	0.0201	0.0009	0.2870	0.0150
50/50 Portfolio	0.3522%	0.0322	0.0018	0.5587	0.0199

Table 3.5: Descriptive statistics for Risk Parity and 50/50 portfolios after rebalancing

	Sharpe Ratio	Sortino Ratio	Treynor Ratio
Risk Parity Portfolio	0.0987	0.1319	0.0069
50/50 Portfolio	0.1630	0.2636	0.0094

Table 3.6: Performance measures for Risk Parity and 50/50 portfolios after rebalancing

The Sharpe Ratio and Sortino Ratio were higher than in the non-rebalanced Risk

Parity portfolio, indicating better risk-adjusted returns. However, these values remain lower than the 50/50 portfolio's risk-adjusted returns, suggesting that in this testing period, rebalancing did not lead to superior performance compared to a simple 50/50 split.

The findings suggest that while rebalancing maintains the risk-parity structure, it does not necessarily improve risk-adjusted performance compared to a fixed 50/50 allocation.

3.3 Analysis in the Multiple Asset Case

We now extend the the analysis by comparing a Risk Parity portfolio composed of multiple asset classes, with an equally weighted portfolio constructed using the same asset classes. The objective is to examine whether the Risk Parity strategy, which allocates risk rather than capital equally, provides advantages in terms of risk-adjusted performance over a more traditional fixed equal capital allocation. The purpose of the analysis is to assess how the Risk Parity approach performs when applied to a more diversified multi-asset portfolio. Unlike the previous analysis, which focused solely on equities and bonds, this case incorporates five distinct asset classes, each offering exposure to different risk factors.

In particular, the five asset classes chosen for this analysis are:

- **Global Equities – iShares MSCI ACWI ETF (ACWI)**: Represents a broad, diversified equity market exposure including developed and emerging markets, and it serves as the primary growth component of the portfolio.
- **U.S Treasuries – iShares 20+ Year Treasury Bond ETF (TLT)**: Provides exposure to long-term U.S. government bonds, often seen as a safe-haven asset.
- **Commodities – Invesco DB Commodity Index Tracking Fund (DBC)**: Represents a diversified basket of commodities including energy, metals, and agriculture.

- **Real Estate – SPDR Dow Jones REIT ETF (RWR)**: Tracks the performance of publicly traded real estate investment trusts.
- **Inflation Protected Bonds – iShares TIPS Bond ETF (TIP)**: Offers exposure to U.S. Treasury Inflation-Protected Securities.

As for the analysis in the two asset case, we use the same estimation and testing windows: 2009/01 – 2018/12 as the estimation window, and 2019/01 – 2022/12 as the testing window.
Again, we calculate monthly returns for the asset classes using their closing values over both windows, and we derive descriptive statistics for each asset class.

Asset Class	Mean	St. Dev.	Skewness	Kurtosis	Serial Corr.
ACWI	0.9295%	0.0461	-0.2382	0.4649	-0.0777
TLT	0.2752%	0.0386	0.4297	0.5256	0.1166
DBC	0.2744%	0.0519	-0.3441	0.5757	0.1010
RWR	1.0532%	0.0604	0.1560	4.9634	-0.0687
TIP	0.2692%	0.0156	-0.3957	2.8050	-0.0967

Table 3.7: Statistics for each asset class

During the testing period, we also obtain data on the risk-free rate, the 3-month T-bills, that will be important in the calculations of the performance metrics.

3.3.1 Composition of the Risk Parity Portfolio

The Risk Parity portfolio was constructed by equalizing the risk contributions of the five asset classes. Meanwhile, the Equally Weighted portfolio maintained a static 20% allocation to each asset class throughout the testing period.
The Risk Parity strategy adjusts weights based on the assets' volatilities and correlations.
Using datas from the estimation window, we estimate the correlation matrix for the five asset classes.

$$\begin{bmatrix} & \text{ACWI} & \text{TLT} & \text{DBC} & \text{RWR} & \text{TIP} \\ \text{ACWI} & 1.0000 & -0.4268 & 0.5907 & 0.6826 & 0.1531 \\ \text{TLT} & -0.4268 & 1.0000 & -0.4228 & -0.0623 & 0.5564 \\ \text{DBC} & 0.5907 & -0.4228 & 1.0000 & 0.2183 & 0.1671 \\ \text{RWR} & 0.6826 & -0.0623 & 0.2183 & 1.0000 & 0.2410 \\ \text{TIP} & 0.1531 & 0.5564 & 0.1671 & 0.2410 & 1.0000 \end{bmatrix}$$

These values highlight the diversification benefits of the portfolio, with negative correlations between Treasuries (TLT) and both equities (ACWI) and commodities (DBC), reinforcing their role as defensive assets.

We then calculate the weights, obtaining:

Asset Class	**Weight**
ACWI	19.59%
TLT	23.38%
DBC	17.37%
RWR	14.93%
TIP	24.73%

Table 3.8: Weights in the Risk Parity portfolio

As expected, the higher weights are assigned to the less volatile stocks, U.S. Treasuries and Inflation Protected Bonds.

The performance of the two portfolios was evaluated using descriptive statistics and key risk-adjusted metrics over the testing window (2019/01–2022/12).

Portfolio	**Mean**	**St. Dev**	**Covariance**	**Beta**	**Downsize Risk**
Risk Parity Portfolio	0.4335%	0.0267	0.0012	0.4399	0.0228
Equally Weighted	0.5581%	0.0339	0.0016	0.5842	0.0279

Table 3.9: Descriptive statistics for Risk Parity and Equally Weighted portfolios

	Sharpe Ratio	Sortino Ratio	Treynor Ratio
Risk Parity Portfolio	0.1271	0.1493	0.0077
Equally Weighted	0.1370	0.1663	0.0079

Table 3.10: Performance measures for Risk Parity and Equally Weighted portfolios

As we can see from the results, the Risk Parity portfolio exhibited lower volatility (2.67%) compared to the Equally Weighted portfolio (3.39%), reinforcing the idea that risk allocation enhances stability. However, in terms of returns, the Equally Weighted portfolio delivered higher values (0.5581% vs. 0.4335%), suggesting that equal capital allocation may provide better upside potential.
Also, the risk-adjusted metrics were slightly higher for the Equally Weighted portfolio, indicating better return per unit of risk.
The Risk Parity portfolio successfully reduced risk, but the Equally Weighted portfolio offered better absolute and risk-adjusted returns. This highlights the trade-off between stability and performance in multi-asset allocation.

3.3.2 Rebalancing the Risk Parity Portfolio

Similar to what we did in section 3.2.2, we now rebalance the Risk Parity portfolio annually over the testing window. Unlike the Equally Weighted portfolio, which maintains a fixed 20% allocation to each asset class, the Risk Parity portfolio dynamically adjusts weights to maintain equal risk contribution. Given that volatilities and correlations between assets change over time, failing to rebalance would result in certain assets dominating portfolio risk, thereby violating the core principle of Risk Parity.
By rebalancing annually, the portfolio incorporates updated risk estimates, ensuring a more stable risk distribution across the five asset classes.

Using the original estimation and testing windows, at the beginning of each year, the estimation period moved forward by one year, incorporating the most recent market data.

For instance, in 2020, the estimation window moved from 2010 – 2019 (instead of 2009 – 2018).

Given the new estimation window, the standard deviations and weights for each asset class were then recalculated.

	ACWI	TLT	DBC	RWR	TIP
2020	19.64%	23.21%	17.35%	14.98%	24.82%
2021	17.81%	22.10%	13.93%	18.75%	27.41%
2022	14.67%	19.28%	12.29%	21.06%	32.70%

Table 3.11: Risk Parity portfolio's weights after rebalancing

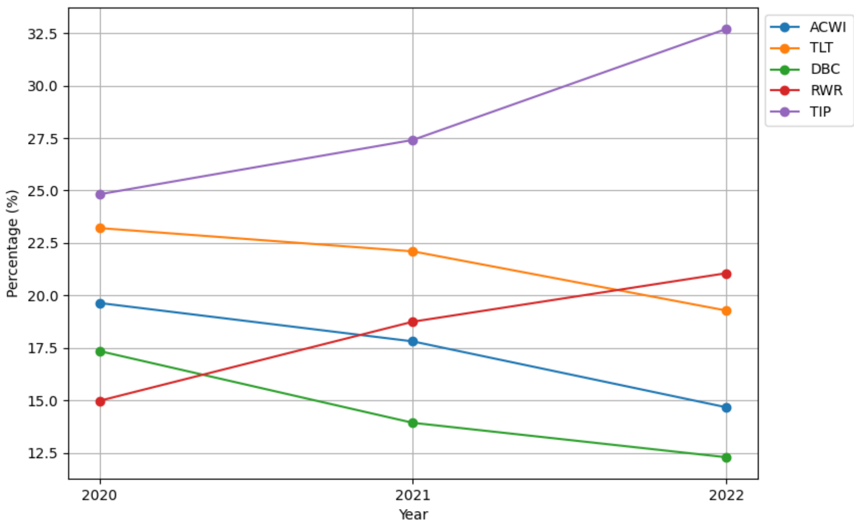

Figure 2: Visualization of the changes in allocation after rebalancing

Using the rebalanced weights, portfolio returns were computed for the next year. This process was repeated at the start of each new year in the testing period. After implementing annual rebalancing, the descriptive statistics and performance measures for the Risk Parity portfolio were recalculated.

Portfolio	Mean	St. Dev	Covariance	Beta	Downsize Risk
Risk Parity Portfolio	0.4211%	0.0274	0.0013	0.4543	0.0234
Equally Weighted	0.5581%	0.0339	0.0016	0.5842	0.0279

Table 3.12: Descriptive statistics for Risk Parity and Equally Weighted portfolios after rebalancing

	Sharpe Ratio	**Sortino Ratio**	**Treynor Ratio**
Risk Parity Portfolio	0.1196	0.1397	0.0072
Equally Weighted	0.1370	0.1663	0.0079

Table 3.13: Performance measures for Risk Parity and Equally Weighted portfolios after rebalancing

As expected, even after rebalancing, mean returns remained lower for Risk Parity (0.4211%) compared to Equally Weighted (0.5581%), while the standard deviation of the Risk Parity portfolio remained lower due to the differences in volatility between the two portfolios.

Performance Ratios were slightly lower for the Risk Parity portfolio, suggesting that the Equally Weighted strategy produced better risk-adjusted returns during this period. The Sharpe, Sortino, and Treynor Ratios were all higher for the Equally Weighted portfolio, indicating better return per unit of risk taken.

Rebalancing in Risk Parity ensured that assets continued contributing equally to portfolio risk. However, in strong market conditions, allocating capital equally (rather than risk) may be a more optimal strategy in terms of return.

Chapter 4: Conclusion

At the beginning we stated the objective of this thesis, which was to explore the Risk Parity approach to portfolio management as an alternative to traditional asset allocation strategies, by focussing on its ability to balance risk contributions across asset classes rather than allocating capital based on fixed proportions.

The empirical research focused on two different portfolio compositions, a two asset case and a five asset case. In both cases we examined the performance of Risk Parity portfolios against their traditional allocation counterparts. The results of this analyses indicated that, when constructed using the Risk Parity approach, the portfolio experienced lower volatility, reflecting its effectiveness in risk management. However, in terms of returns and risk-adjusted performance, the results were mixed. In the two asset case, the Risk Parity portfolio demonstrated lower volatility and achieved a more stable risk profile, especially during periods of market stress, as expected. However this came at the cost of lower absolute return and risk-adjusted performance, with higher values for Sharpe, Sortino, and Treynor ratios. These results highlight the trade-off between the two strategies.

The findings are further highlighted in the five asset case, in which the Risk Parity portfolio exhibited lower overall volatility compared to the equally weighted portfolio, again at the cost of lower returns. Even after rebalancing the Risk Parity portfolio, which ensured that it maintained its intended risk distribution, it did not outperform the equally weighted portfolio in terms of return maximization.

Ultimately, the findings of the thesis reinforce the concept introduced in the beginning, that there is not a universally better strategy, rather the choice of which one an individual should choose depends on the investor's specific goals and risk tolerance. As financial markets continue to evolve, portfolio construction techniques will also need to adapt, integrating elements of both risk-based and return-focused strategies to optimize long-term investment outcomes.

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