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Chair of Asset Pricing

The Impact of Inflation on Financial Markets: an Econometric Analysis of Asset Returns and Market Volatility

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INTRODUCTION

The thesis aims to synthesize classical portfolio theories including Markowitz's Mean-Variance Theory, Tobin's Utility Theory, and the Sharpe Ratio to develop an integrated approach for the analysis of financial market dynamics. The main objective is to evaluate the effects of inflation on portfolio optimization and asset efficiency with special emphasis on the Russian equity market via the IMOEX Index. The reason why the objective of the analysis is represented by Russia is because of the high inflation volatility that has characterized the country in the past years. Using econometric analysis specifically Ordinary Least Squares (OLS) and Quantile Regression (QR), the study investigates how inflation affects asset returns in different market conditions and distinguishes between high and low inflationary periods.

This research presents the Inflation Allocation Line (IAL) as an extension of the Capital Allocation Line (CAL) which incorporates inflation effects on optimal portfolio reallocations. Tobin's Utility Curve is utilized to analyze the interactions among risk, return, and inflation from which investor behavior can be derived based on changing macroeconomic scenarios. The Markov-Switching model is also implemented to better understand inflation's effects on financial market regime shifts and the EGARCH model is to check the inflation's volatility.

This thesis includes empirical analysis from 2014 until 2024 to detail how inflation affects equity returns as well as fixed-income securities within the Russian market over time. The research outcomes provide greater insight into how investors can make portfolio decisions based on macroeconomic changes which will help develop useful asset allocation strategies during periods of inflation.

CHAPTER ONE

MARKOVITZ THEORY: OVERVIEW

Mean-variance portfolio theory, presented by Harry Markowitz and published in his 1952 article entitled "Portfolio Selection" in the Journal of Finance represented a real breakthrough in modern financial theory. It has been the first systematic and quantitative manner of selecting an investment portfolio by means of diversification against balancing risk and return. Markowitz suggests that investors should not aim at the maximization of expected return alone but rather seek an optimal trade-off between expected return and risk, measured by the variance of portfolio returns. This framework introduced the concept of a portfolio's "efficiency", where an efficient portfolio was defined as one that had the highest expected return for a given level of risk or the lowest risk for a given expected return.

Indeed, Markowitz's theory revolutionized the way in which analysts and investors appraised portfolios, switching focus from looking at individual investments within a portfolio and their association to one another. His research served as an introduction to developing the Capital Asset Pricing Model, and the Arbitrage Pricing Theory is based upon those very notions brought forth. While the mathematical complexity and computational difficulties associated with applying his theory in the 1950s and 1960s, the advent of computers and sophisticated portfolio management software has made practical implementation of Markowitz's approaches more possible.

Nowadays, mean-variance theory serves as a basis for investment strategies not only at an institutional level but even at the level of an individual investor.

For most of these disadvantages, the model has been attacked-primarily its assumption of returns being normally distributed-and basically estimating future variances and covariances correctly presents a difficult proposition. There are other recent methods whereby addition of other risks or alternative probabilistic distribution mechanisms describing the disposition of asset return series has also made this research alive.

PRINCIPLES OF MEAN-VARIANCE PORTFOLIO THEORY

The Mean-Variance Portfolio Theory showed that an investor, in a really structured and quantitative way, may reach the ultimate balance between his expected return and risk. In portfolio theory, there are a few basic concepts: definition of a portfolio, diversification, and efficiency of portfolio; all of them together set up a paradigm for optimal choice and management of investment.

A portfolio could be thought of as a mix of different financial asset classes that an investor possesses. The constituent assets can be everything from stocks and bonds to mutual funds and all other forms of available financial instruments. Variety in the assets that make up one portfolio is important in that the underlying principle of diversification states that, with uncorrelated assets in a portfolio, one would expect overall risk to reduce. Practically, it means that diversification can smooth out returns over time by offsetting possible losses in some areas against gains in other areas.

Underpinning the entire mean-variance portfolio theory, conceptually, rests on the theory of maximizing an expected return against a given degree of risk and then, inversely, minimizing this kind of risk for one's given or agreed return level. For achieving this end, something that goes by the label "efficient frontier" is theoretically developed. The portfolios that locate themselves on this frontier are efficient because they offer the highest return for a given level of risk, or the lowest possible risk for a given level of return. The efficient frontier thus represents a series of optimal portfolios from which investors can choose based on their risk tolerance.

To define the efficient frontier, one must carry out a comprehensive analysis of variances and covariances in the return on securities since these are essential statistical measures for a good understanding of the dynamics in which the price of one security varies from the other due to changes in various market conditions. This is also one of the steps whereby such analytical process helps to bring forth not only portfolios that at any particular level of return have a least risk but also on which one could base rational decisions to reach the right matching of financial objectives and personal preferences of risks and return.

MATHEMATICAL FRAMEWORK

Portfolio optimization has a mathematical formulation that underlies the mean-variance portfolio theory. This will, in turn, be based on minimizing risk for any given level of expected return, and the relationships provided by covariance and correlation among asset returns, thereby deducing the efficient frontier in a mathematical sense.
The essence of the mathematical model of the mean-variance portfolio theory is an optimization problem in finding the minimum of the portfolio variance for some given expected return. The problem can be expressed in a quadratic programming format:

$$min\sigma_p^2 = x^T \sum x$$

subject to:

$$r^T x = \mu \sum_{i=1}^{n} x_i = 1 x_i \geq 0, \qquad for\ all\ i$$

where:

- σ_p^2 is the variance of the portfolio's return.
- x is the vector of portfolio weights.
- $\sum$ is the covariance matrix of asset returns.
- r is the vector of expected returns for each asset.
- μ is the desired level of the portfolio's expected return.
- x_i represents the proportion of total portfolio value invested in asset i.

THE USE OF CORRELATION AND COVARIANCE

Covariance and correlation represent two important tools that are usually used to understand how the prices of assets move with one another. Firstly, the covariance points out the directional relationship between the returns of two assets. It is a central element in the construction of the variance-covariance matrix, $\sum$, used in the optimization model. The correlation coefficient is a standardized covariance of the relation between -1 and 1 that defines how intuitively assets are moving in respect to each other:

$$correlation(r_i, r_j) = \frac{Cov(r_i, r_j)}{\sqrt{Var(r_i)\, Var(r_j)}}$$

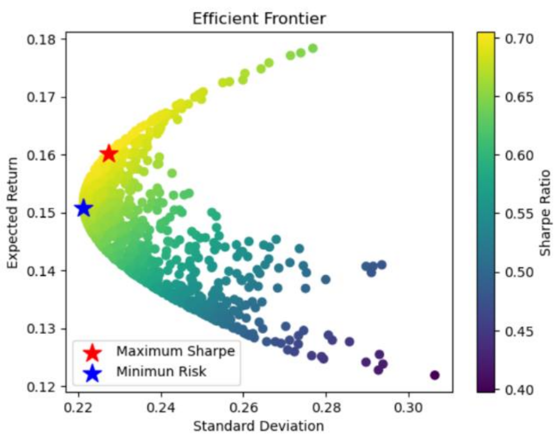
where:

- $Cov(r_i, r_j)$ is the covariance between returns of asset i and asset j;
- $Var(r_i)$ and $Var(r_j)$ are the variances of returns for assets i and asset j respectively.

DERIVATION OF EFFICIENT FRONTIER

The efficient frontier can be derived by solving the optimization problem for varying levels of μ. Over time, the efficient frontier has become synonymous with optimal portfolio construction. The efficient frontier is the set of portfolios that offers the minimum risk for every level of expected return. The frontier is constructed mathematically by varying μ, the desired level of expected return, and solving the corresponding optimization problem for each value. This produces a series of portfolios, which as a group form a curve in risk-return space. Every point in this curve portrays an efficient portfolio, since there could be no other portfolio granting a higher return for the similar amount of risk, or for a lower level of risk at a similar amount of return. Hence, the efficient frontier is such an important tool to be used by investors, helping them in picking up the most optimal portfolio for their tolerance and return objectives. It is the implementation of Markowitz's theory in a practical sense and directly applies to investment strategy, showing the risk and return trade-off. Below, it is reported an example of the efficient frontier in which has been replied 1000 random portfolios:

Figure 1.1, the Efficient Frontier.



Source: personal elaboration.

Through the expected return and the covariance matrix has been estimated the expected return weighted and the standard deviation. In addition, it has been estimated the Sharpe ratio and minimum risk portfolio. The resultant shape represents the efficient frontier that involves all the portfolios with the best possible return for every level of risk.

THE MODEL IN ASSET ALLOCATION

The so-called portfolio theory immediately changed the character of asset allocation in investment portfolios. Emphasizing the relationship between risk and return, the theory laid a formal framework of how to select and weight the assets in one's portfolio. This section sets out how Markowitz's model addresses asset selection and weighting.

SELECTING AND WEIGHTING ASSETS IN A PORTFOLIO

The process of selecting and weighting assets in a portfolio under Markowitz's model involves:

1. Asset selection involves identifying a pool of assets that should be combined into a unique portfolio. Such a selection should be made considering not only the investor's financial objectives and investment time horizon, but also his or her risk tolerance, the historical return, single volatility of assets and the correlation with each other.
2. Returns and covariances will be estimated based on investors' needs to finally determine the expected returns and the covariance matrix of assets. While the expected return will predict how the assets are likely to perform in the future, the covariance matrix indicates how the returns of assets move concerning one another.
3. After the estimation, the investors solve an optimization problem. That is to say, investors either minimize the variance of the portfolio for a given expected return or maximize the expected return for a given level of risk. The weights of the assets in the portfolio are defined after the resolution of the optimization problem, hence showing how much capital shall be allocated to each asset.

4. Investors achieve the desired level of risk by selecting a portfolio from the efficient frontier that aligns with their risk tolerance. In other words, it is merely the trade-off between the expected return and the amount of risk that the investor is willing to bear in order to receive that return.

DIVERSIFICATION

Diversification is a central theme in Markowitz's theory and a key factor that helps increase the effectiveness of a portfolio. It refers to the spreading of investments across different classes that do not move together, reducing unsystematic risk. The rationale behind diversification is underpinned by the fact that it can mitigate adverse impacts on the portfolio from the volatility of individual constituent assets. It is possible to reduce an overall portfolio's risk by diversification, if investments are spread among assets that are not perfectly correlated:

- Such a case is that when one asset class, say stocks, is in the midst of decline, the other asset class, say bonds, may stay the same or rise in their value, therefore compensating for losses and making the overall performance of a portfolio stable.
- The mathematical base for diversification is the negative covariance among classes that could also offset some risks of individual securities. This eventually leads to the total risk of a portfolio, measured by the standard deviation of returns, to be less than the risk of the assets that compose it alone, assuming non-perfect correlation of the assets.

LIMITATION OF MEAN-VARIANCE PORTFOLIO THEORY

Harry Markowitz's mean-variance portfolio theory has represented one of the cornerstones in finance, hence providing a structured approach to portfolio optimization. Despite its wide application and contribution, a number of criticisms and limitations have been pointed out with the theory, most of which relate to the assumptions made. One of the main assumptions of mean-variance theory is the normality of asset returns. This assumption implies that the returns of a portfolio are symmetrically distributed around the mean, with most of the observations clustering near the average return and fewer observations seen as you move away from the mean, following a bell-shaped distribution. Mathematically, a random variable X follows a normal distribution if its probability density function is given by:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2}$$

where μ is the mean of the distribution and σ is the standard deviation, showing the dispersion around the mean.

In the context of investment returns, assuming normality simplifies the calculation of the portfolio risk measured by variance and allows derivation of the efficient frontier more readily.

However, financial returns hardly meet the property of normality due to the presence of skewness and kurtosis. These deviations suggest that this theory will underpredict the frequency of extreme events or "black swans" and hence could result in risk being underestimated when the portfolio is built. In general, the mean-variance framework depends on the variance and covariance of asset returns to determine the risk of different portfolios. In estimating these parameters, one needs historical data, which can be quite voluminous for a large number of diversified assets. There's then further problem typically of a high computational nature estimation of the covariance matrix, given that the outstanding amount of assets in most cases tends to be relatively a multiple of the recorded historical observations.

Now, the complicating nature for most of the above renders applying these theories extremely burdensome at individual or smaller firms' levels. Besides these, another defect with the mean-variance theory is that the theoretical framework is based on using historical data in order to estimate future risk and returns. It presumes that the moments of historical means, variances and correlations will continue into the future, which often is not the case with highly dynamic and evolutionary markets. Financial markets are subject to such an array of variables, including variability in general economic conditions, political events, and changes in investor preferences, that sequentially altering performances and relationships among assets could easily occur. Making projections of future returns and risks based on the past information thus often leads to misunderstandings and underperforming portfolios. This challenge is most pronounced in times of market stress and structural breaks within the economy, which has emphasized the weaknesses of static models based on historical parameters.

TOBIN THEORY: OVERVIEW

James Tobin, a Nobel Laureate and one of the most influential economists, developed his utility theory in the context of portfolio selection during the late 1950s. This theory, often encapsulated within the broader framework known as "Tobin's separation theorem," fundamentally shifted the focus of portfolio management by incorporating the concept of utility maximization. Tobin's argument, however, lay in the fact that, unlike previous models, which had either maximized returns or minimized volatility, it provided a way of setting risk against utility derived from the investment outcome, considering the disparate appetites of investors for that risk.

Up until the time of Tobin, financial theory had evolved mainly through classical and neoclassical economic theories that often reduced investor behavior to purely profit-maximizing models. Harry Markowitz introduced mean-variance optimization in 1952, thus laying the initial way by putting into concrete form the notion of portfolio diversification based on risk and return. Tobin extended this to introduce utility as an important component and thus enhanced the process of decision-making in portfolio management. Added to this was his theory that investors select a portfolio in line with their risk tolerance and expected utility and not by quantitative measures.

Tobin's works retain their import in the field of financial economics on various grounds. First, they provided a bridge between purely theoretical economics and practical finance, particularly with regard to understanding investment behavior under conditions of uncertainty. His work underlined the meaning of considering psychological and subjective aspects of investment decisions, later to impact the development of behavioral finance. Besides, Tobin's utility theory extended the application area of economic principles to financial markets, providing, at the same time, a more suitable attitude toward risk management and toward the choice of assets with regard to both economic and human factors. This has got very important meaning for theoretical investigations and real asset management: these are guiding ideas for the making up of portfolios by means of efforts toward meeting a wide range of investor needs created within the dynamically developing economic surroundings.

FUNDAMENTAL PRINCIPLES OF TOBIN'S UTILITY THEORY

In economic and financial theory, a utility function is a relation that relates the satisfaction or utility an investor derives from various outcomes with his or her choice. Tobin's utility theory postulates that investors make decisions not simply on the basis of expected returns and standard deviations of portfolios but on the basis of how those portfolios contribute to their overall utility. The utility function is, therefore, a mathematical device that translates levels of wealth or outcomes into levels of satisfaction.

Utility functions in portfolio selection are particularly significant because they encapsulate the investor's subjective preferences regarding risk and return. It may represent various preferences that immediately put a higher value on returns up to a point where it passes beyond and, inversely, where the investor would be less sensitive to gain. This forms a successful investment strategy, since trade-offs between investors' comfort related to personal risk and the attainment of higher returns are reflected in the choices.

Utility functions help investors select a rational or optimal investment aimed at maximizing their expected utility. In a practical sense, this means choosing the mix of assets that offers the highest expected utility rather than the highest expected return. For any given level of risk, the choice will depend upon the shape of the utility function that quantifies how much satisfaction the investor derives from various levels of wealth. The usual proceeding involves computation of the expected utility for a certain number of portfolios, followed by the selection of a portfolio showing the highest utility provided. The expected utility is computed as a weighted average for all possible outcomes utilities, with weights constituting their probabilities. Such a technique considers not only the return potentials on the grounds of market scenarios but also encompasses an investor's risk attitude or personal perception of the risk tolerance principle.

RISK IN UTILITY FUNCTIONS

Risk aversion is a fundamental concept in utility theory, defined as the preference for a certain outcome over a gamble with a higher or equal expected return. Mathematically, risk aversion is represented through the curvature of the utility function. A commonly used utility function to represent risk aversion is the exponential utility function, defined as:

$$U(W) = -e^{-aW}$$

where:

- $U(W)$is the utility associated with wealth W,
- α is a positive parameter representing the degree of risk aversion; a higher value of α indicates higher risk aversion.

In this function, the second derivative *U''(W)* is positive, indicating that the investor's utility decreases when wealth increases, ensuring the principle of diminishing marginal utility of wealth. This form of the utility function is useful in illustrating why risk-averse investors favor less volatile investments since they impose a disproportionately high utility loss on potential negative outcomes relative to their utility gain from an equivalent positive change.

MATHEMATICAL FRAMEWORK

The utility maximization problem in portfolio management is defined within a decision-making framework where investors strive to maximize their expected utility while adhering to various constraints. The fundamental mathematical formulation is given by:

$$\max E[U(W)]$$

where:

- $E[U(W)]$ is the expected utility of wealth W,
- $U(W)$ is the utility function representing the investor's preference.

The wealth W is typically a function of portfolio returns, defined as $W = W_0 + \sum_{i=1}^{n} x_i R_i$, where:

- W_0 is the initial wealth,
- x_i is the proportion of total wealth invested in asset i,
- R_i is the random return of asset i,
- n is the number of assets in the portfolio.

The objective is to choose portfolio weights w_i that maximize the expected utility of the terminal wealth.

CONSTRAINTS IN UTILITY OPTIMIZATION

When solving the utility maximization problem, various constraints are typically applied to ensure that the portfolio choices are both feasible and realistic:

1. The budget constraint ensures that the total allocation of investments equals the investor's initial wealth. Formally, this is expressed as: $\sum_{i=1}^{n} x_i = 1$, where x_i represents the proportion of wealth allocated to each asset. This condition implies that all available wealth is fully invested, with no borrowing or short-selling allowed.
2. Return expectations are often included as a constraint where investors aim to achieve a specific target return. This can be expressed as: $E[W] = W_0 + \sum_{i=1}^{n} x_i E[R_i] > R_{TARGET}$ where R_{TARGET} represents the desired minimum expected return.

These constraints are critical in defining the feasible set of portfolios, effectively guiding the investor's choices toward meeting their financial objectives while balancing risk and return.

DERIVATION OF INDIFFERENCE CURVES

Indifference curves are a fundamental concept in utility theory, depicting combinations of risk and return that provide an investor with the same level of utility. In portfolio management, these curves are derived by equating the utility function to a constant value and solving for the combinations of risk and return that satisfy this condition. Mathematically, this can be expressed as:

$$U(\mu, \sigma) = c$$

where:

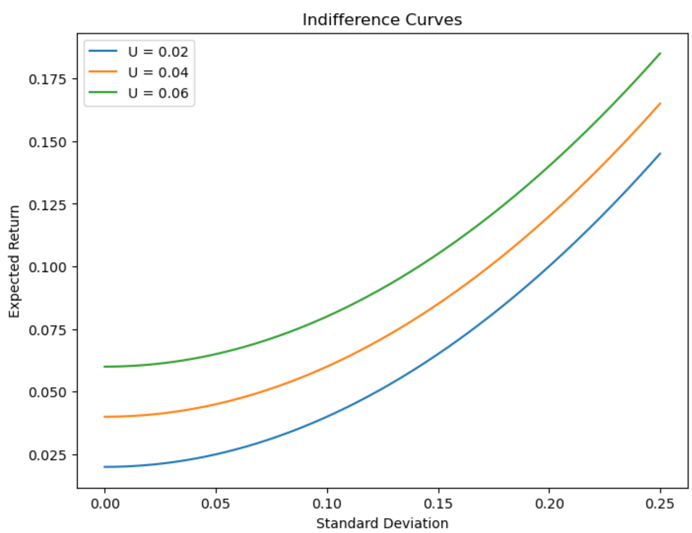
- μ is the expected return of the portfolio,
- σ is the risk, or standard deviation, of the portfolio,
- c is a constant representing a particular utility level.

The way indifference curves are shaped is closely tied to the investor's utility function and their level of risk aversion. These curves generally slope downward, which represents the trade-off between risk and return. Essentially, higher returns are needed to justify taking on additional risk, ensuring that the investor's level of satisfaction remains unchanged.

Indifference curves are particularly useful for understanding an investor's preferences and tolerance for risk. They help illustrate how varying levels of risk and return influence the investor's overall satisfaction. When paired with the efficient frontier, which shows the most efficient portfolios, these curves assist in identifying the best portfolio for the investor. The optimal portfolio lies at the point where the indifference curve touches the efficient frontier, marking the highest utility achievable within the available options.

Below, it is reported an example of three different indifference curves:

Figure 1.2, Indifference curves.



Source: personal elaboration.

Specifically, it has been estimated the expected return for the three levels of utility defined; in addition, the standard deviation, the risk, is calculated basing on the risk aversion of the investor.

TOBIN’S MODEL IN ASSETS ALLOCATION

James Tobin's utility theory plays a key role in shaping asset allocation strategies, emphasizing the importance of maximizing investor satisfaction or utility rather than solely concentrating on traditional risk and return metrics. This approach guides the process of selecting and weighting assets in a portfolio, ensuring alignment with an investor's unique preferences and risk tolerance. By incorporating practical examples and showcasing the essential role of diversification, this framework provides valuable insights into constructing portfolios that balance risk, return, and overall utility for investors.

SELECTING AND WEIGHTING ASSETS IN A PORTFOLIO

James Tobin's utility theory serves as a foundation for developing asset allocation strategies because it prioritizes investor satisfaction as the objective function instead of using risk and return as the sole criteria. The framework helps guide the process of asset selection and weighting to achieve alignment with individual investor preferences and risk preferences. The practical application of this framework combined with diversification analysis delivers significant insights for building investor portfolios that achieve optimal risk-return trade-offs.

This process can be broken down into several key steps:

1. Utility function specification: the utility function is specified to precisely represent the investor’s risk preferences while reflecting their decision-making preferences. Quadratic utility functions provide an easy computational framework because of their simple polynomial structure yet logarithmic and exponential functions deliver better representations of risk preferences and wealth diminishment.

2. Expected utility calculation: the expected utility of each portfolio represents the integration result of the selected utility function with the probability distribution of potential outcomes. The calculation assesses both the probability distribution of returns and their alignment with the investor's preference function.
3. Optimization: optimization methods select the optimal mix of assets that provide the highest expected utility. The utility function guides the process of varying each portfolio's asset ratios until an optimal mix is identified that meets the utility function requirements.
4. Constraint: constraints such as budget limitations, regulatory requirements, or investment goals need to be included. Utility-optimal portfolios must pass realistic and executable constraint tests to be considered valid.

DIVERSIFICATION

Diversification plays an important role in Tobin's asset allocation model, as it enables investors to improve their utility by distributing risk across different assets. By diversifying, investors can minimize and contain the impact of poor performance in any investment on the overall portfolio. The reduction of risk is crucial for maximizing utility, especially for risk-averse investors, since it smooths out the returns and reduces the likelihood of outcomes that would decrease utility levels.

- Utility maximization: diversification can help in achieving a smoother utility curve, reducing the portfolio's exposure to swings in returns that could significantly decrease the investor's satisfaction or utility.
- Tail risk mitigation: effective diversification also helps in mitigating tail risks, which are low-probability, high-impact events that could drastically reduce wealth and therefore utility.

LIMITATIONS OF TOBIN’S UTILITY THEORY

One of the significant challenges in applying Tobin's Utility Theory lies in the empirical measurement and practical application of utility functions. Utility, as a concept, represents an individual's subjective satisfaction or preference, which is inherently difficult to quantify and measure. In practical terms, defining a utility function that accurately reflects an investor's risk tolerance and satisfaction with various outcomes involves significant estimation.

The main problem is that utility functions are theoretically constructed based on assumptions about investor behavior, which may not necessarily hold true in real-world scenarios. The subjective nature of utility means that different investors may have different utility functions even with similar financial profiles, making it difficult to standardize these functions for broad application

In addition, utility functions are designed to model future preferences and decisions based on current understanding and past data. However, these functions often fail to adapt to changing market conditions and evolving investor preferences:

- Dynamic preferences: investor preferences and risk tolerances are not static and can change based on personal circumstances, market trends, and economic conditions. Utility functions that do not account for these dynamic preferences might lead to suboptimal decisions.
- Market volatility: financial markets are inherently volatile and influenced by numerous unforeseeable factors. The assumption that past behavior and preferences will predict future decisions under different market conditions can lead to significant discrepancies between expected and actual outcomes.

Tobin's utility theory, like many economic models, simplifies complex investor behaviors into more manageable forms. This simplification, while useful for theoretical and analytical purposes, often overlooks the complex realities of individual decision-making:

- Behavioral oversights: traditional utility models generally do not account for behavioral biases and psychological factors that significantly influence investment decisions. Phenomena such as loss aversion, overconfidence, and herd behavior are common among investors but are not typically reflected in standard utility functions.
- Rationality assumption: the theory assumes that investors are rational and that their decisions to maximize utility are always logically driven. In reality, many investment decisions are influenced by emotions, incomplete information, and irrational factors that the utility model does not fully encompass.

SHARPE THEORY: OVERVIEW

William F. Sharpe introduced the Sharpe Ratio in 1966 which represented next generation work in investment performance evaluation because it allowed for the comparison of investment returns to their volatility. The assessment of investment returns before Sharpe produced results without a review of corresponding risk exposures. Sharpe's approach shifted this perspective by quantifying how much excess return was being generated for each unit of risk taken, compared to a risk-free rate. This metric has become a fundamental tool of financial economics to standardize risk-adjusted comparisons of different investments and provide more insight into investment returns. The Sharpe Ratio was an important milestone in modern finance because it was at a time when financial market complexity was increasing and there was a demand for more advanced analytical solutions. The Sharpe Ratio has enabled portfolio management and financial analysis strategies to compare performances despite very different risk profiles.

FUNDAMENTAL PRINCIPLES

The Sharpe Ratio provided a measure that, as said before, quantifies the risk-adjusted return of an investment portfolio. The Sharpe Ratio is defined mathematically as:

$$Sharpe\ ratio = \frac{R_p - R_f}{\sigma_p}$$

Where:

R_p is the return of the portfolio,

R_f is the risk-free rate, and

σ_p is the standard deviation of the portfolio's excess returns over the risk-free rate.

The risk-free rate typically refers to the yield of government bonds or bills: it's considered risk-free since it's backed by the government's ability to tax its citizens.

The numerator, $R_p - R_f$, represents the excess return, also called *premium*, realized on an investment over a risk-free asset. This excess return is what investors seek to maximize since it compensates them for taking additional risk compared to simply holding a risk-free asset. The denominator, σ_p, represents the standard deviation of the portfolio's excess returns, the measure of the investment's volatility or risk.

OBJECTIVE OF THE SHARPE RATIO

The Sharpe Ratio is designed to answer the question of how much excess return one can earn for a unit of risk in an investment. As such, maximizing the Sharpe Ratio is a central tenet of good investment management. It is useful in assessing the performance of a portfolio and choice making. One way to look at it is that a portfolio with a higher Sharpe ratio gives more returns for the same level of risk. For diversified portfolios, Sharpe Ratio is a very important factor for comparison. It affords investors and fund managers the opportunity to compare different strategies, asset types, and performance outcomes irrespective of the risk profile. This is especially useful in enhancing portfolios or in highlighting gains in a stiff market. Focusing on risk-adjusted returns as opposed to nominal returns is something the Sharpe Ratio does, which means paying attention to risk management. This approach guarantees that risks are not only justified but also well-rewarded; promoting a risk-aware strategy that is consistent with personal financial goals.

MATHEMATICAL FRAMEWORK

The Sharpe Ratio serves as a performance metric and at the same time is a mathematical foundation for maximizing portfolio performance. As such, its formulation allows investors to adjust the weights of assets in the portfolio to optimize the returns relative to risk. It is incorporated in portfolio management during optimization problems, and in the use of variance and correlation. The optimization problem concerning the Sharpe Ratio is to maximize this ratio by changing the weights of the assets in the portfolio. The final goal is to find the asset mix that provides the highest excess return per unit of risk, as measured by the standard deviation of portfolio returns.

Mathematically, this can be expressed as:

$$\max \frac{R_p - R_f}{\sigma_p}$$

Where:

R_p is the return of the portfolio, which is a function of the weights of the various assets and their individual returns,

R_f is the risk-free rate, and

σ_p is the standard deviation of the portfolio's returns.

The weights of the assets w_i and w_j must be chosen such that they optimize this ratio. This involves solving:

$$\max\left(\frac{\sum_{i=1}^{n} w_i r_i - R_f}{\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \sigma_{ij}}}\right)$$

Subject to :

$$\sum_{i=1}^{n} w_i = 1, w_i \geq 0$$

Where:

r_i are the expected returns of the assets,

σ_{ij} are the elements of the covariance matrix, representing the covariances between the returns of assets i and j.

What the Sharpe Ratio does is to incorporate, explicitly, portfolio risk owing to the standard deviation of portfolio return, which is in turn a function of the variance of individual asset returns and the covariance between different assets. It is the variance and correlation that help determine the movements, relatively speaking, of the returns of the assets:

- Variance: it measures how much the individual asset returns vary at different levels about their mean. Higher variance indicates greater risk, and potentially greater return. In portfolio theory the aim is to maximize high returns and minimize variance.
- Covariance and correlation: covariance points out how much two assets will tend to move with respect to one another. A positive covariance implies that asset returns co-move, whereas a negative covariance implies they move in opposite directions. Correlation is but a standardized form of covariance and is important for diversifying risk. A portfolio having the same expected return may have less aggregate risk if the assets are not perfectly correlated.

This is how one, in practical terms, reduces the risk of a portfolio without bearing down too strongly on returns, by selecting a combination of assets in which the returns are not perfectly correlated. Diversification benefit from this fact forms part of the basis of modern portfolio theory and is a key ingredient in the optimization problem that the Sharpe Ratio maximizes.

ASSET ALLOCATION

The Sharpe Ratio bears a lot of importance in asset allocation, hence deciding on the type of assets to be selected and the weight of such assets in a portfolio. This key metric leads investors to build up portfolios that satisfy desired return levels but do so at an optimal level of risk. In the context of asset allocation, the Sharpe Ratio refers to the determination of investment in various classes of assets that would result in maximum return for a risk taken. The process of decision-making entails:

- Risk-adjusted performance evaluation: by comparing the different Sharpe Ratios of various assets or asset classes, investors can identify which of the assets yields the highest return per unit of risk. The higher the Sharpe Ratio, the better it is since it means a higher return for the level of volatility that occurred.
- Portfolio weight adjustments: after selecting individual assets based on their Sharpe Ratios, the next step is to establish their relative weights within the portfolio. The main goal is to optimize the Sharpe Ratio of the portfolio itself. This process involves the solution of very complex optimization problems that take into consideration elements such as asset returns, the risk-free rate, and the interrelationships between the assets, namely, covariances, to determine the optimal portfolio composition.
- Dynamic rebalancing: the weights that a particular asset bears in a portfolio are not set in stone; they get changed in response to shifting market conditions and changes in asset volatility and return. The Sharpe Ratio plays a crucial role in such rebalancing decisions since it always aims to maintain that, at any time, the portfolio is at its efficient level, i.e., the risk-adjusted return on the portfolio is at its best.

LIMITATION OF SHARPE RATIO

There are several limitations and criticisms of the Sharpe ratio, despite its usefulness. The first assumption of the ratio, as just seen in the Markovitz's theory, is that returns are normally distributed. This assumption simplifies the computation and interpretation of the ratio by using characteristics, namely, the mean and variance of a normal distribution to determine the ratio. However, financial market returns often exhibit properties that drastically violate normality:

- Skewness and kurtosis: almost every asset's returns are skewed or leptokurtic, meaning they have fatter tails than a normal distribution, with more likelihood of extreme returns. Skewness can pull values more towards one end of the distribution. This can be an issue, as it can result in under or misestimation of the risk. The Sharpe Ratio, which is mainly based on standard deviation, does not fully take this type of risk into the picture.
- Turbulent markets: return distributions become highly skewed and leptokurtic during market stress or financial crises making the Sharpe Ratio inadequate for risk-adjusted performance evaluation as it can give investors a wrong impression of the assets' risk levels. In fact, the computation of variance and covariance is the very basis for assessing the risk component of the Sharpe Ratio. Nevertheless, these calculations themselves are not without their problems, particularly in dynamic and complex markets.
- Dynamic market conditions: the financial markets are influenced by numerous factors that can cause rapid shifts in asset prices. Such dynamics make the estimation of variance and covariance challenging in order to accurately reflect the current or future market conditions.
- Estimation errors: the accuracy of variance and covariance calculations is based on the asset's historical data. If the historical period does not include major market upheavals or the data sample is too short, the estimates may not be a reliable indicator of future volatility. This limitation can lead to misjudgments in the risk assessment of portfolios, particularly if the market environment changes suddenly.
- Computational complexity: for portfolios with a large number of assets, calculating covariance matrices becomes computationally intensive and, thi

complexity, can hinder the timely adjustment of portfolios and affect the overall efficiency in managing risks.

Taking into account the limitations above, the Sharpe Ratio keeps its importance for evaluating risk-adjusted returns. The incorporation of behavioral finance perspectives and the integration of statistical measures such as skewness and kurtosis can help address the challenges posed by non-normal return distributions, providing a more comprehensive and accurate representation of investment performance.

CHAPTER TWO

MODEL'S OVERVIEW

To construct a model that can capture the effects of inflation on MOEX, Markowitz's mean-variance theory, Tobin's utility function, and Sharpe's capital allocation line are described. These theories form an important theoretical basis for understanding the optimal portfolio decisions in the presence of different economic states. With the inclusion of inflation in the model, we can assess the extent to which asset allocation strategies change for investors in response to changes in the price level under various economic conditions, including low and high inflation. Using Python, the empirical analysis was carried out and necessary data analysis, optimization, and data visualization were performed with the help of special libraries. The historical financial data was used to construct efficient portfolios with bonds and risky assets. The purpose is to graph the model's structure and the assumptions and methodologies used and then apply this to analyze the role of inflation in portfolio selection in the Russian market on MOEX.

DATA ACQUISITION

The dataset consists of monthly observations from 2014 to 2024. The assets analyzed include:

- IMOEX Index: the Moscow Exchange (MOEX) index tracks the major Russian companies and their performance across different sectors.
- Government Bonds: 10-year government bonds act as long-term fixed-income securities which are represented by this category.
- Risk-Free Asset: the one-year government treasury bill is used as the proxy for the risk-free rate.

Returns were computed differently depending on the nature of the financial asset:

IMOEX Index returns were computed using the standard return formula:

$$\frac{P_t - P_{t-1}}{P_{t-1}}$$

where P_t represents the index value at time *t* and P_{t-1} the value at the previous time step.

For bonds and risk-free assets, since the raw data for bonds and risk-free assets were initially expressed as annual yields (Y_t), a transformation was necessary to obtain the monthly equivalent return. The transformation was performed using the following formula:

$$\left(1+\frac{Y_t}{12}\right)-1$$

This intermediate step ensures that the annual yield is properly converted into a monthly return. Once converted into monthly returns, the same formula applied to the IMOEX Index was used to compute their rate of change over time.

This calculation provides the nominal return, excluding the effects of inflation.

PORTFOLIO CONSTRUCTION

The portfolio is constructed through a combination of two base assets: the IMOEX Index, for the high-risk, and Bonds, for the low-risk, or riskless, asset.

The first calculation is for both the average returns of the IMOEX Index and Bonds, and it sets a baseline for comparing how each asset performed over a specific period of time. Beyond that, a covariance matrix is calculated in an attempt to understand in relation to each other in terms of moving together. The covariance matrix is useful for portfolio risk, and a calculation can be performed for a diversification gain when assets with a range of risk profiles are mixed together.

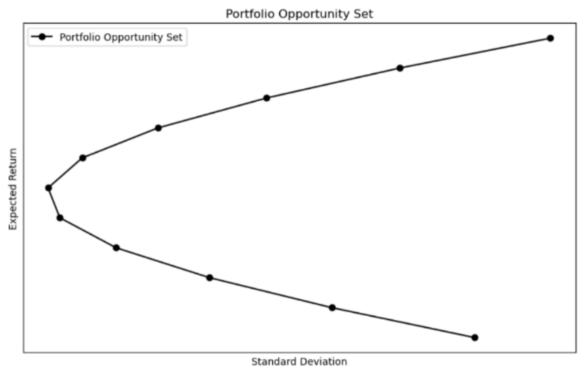
Once the base information is calculated, portfolio construction seeks out an optimal mix between the risky asset and the bond. The analysis considers a range of portfolios with a range of mixes, starting with a portfolio with 100% in bonds and 0% in the risky asset and moving through a portfolio with 100% in the IMOEX Index and 0% in bonds. Intermediate portfolios are calculated with incremental change in the proportion of the risky asset between 0% and 100%, in 10% increments, with a change in proportion in the bond to make the overall proportion 100%. For each portfolio, the return is calculated as a weighted average of individual asset returns. Portfolio volatility, which represents the overall investment risk, is calculated using the variance and covariance of the assets that compose it. In other words, to determine the portfolio's risk level, one does not simply sum the risks of the individual assets but also considers how they move together through the covariance. If two assets tend to fluctuate in the same way, the overall risk will be higher; however, if they move in opposite directions, their combination will reduce overall volatility thanks to diversification.

To evaluate performance, a portfolio is measured in terms of a calculation of a unit of risk for an excess return, with a normalized value for comparing portfolios with different risk profiles, through a value for a Sharpe Ratio. For calculation, an excess return benchmark is taken in terms of a risk-free return, calculated in terms of a mean return of a risk-free asset in a portfolio of assets. With a high value for a Sharpe Ratio, a portfolio is deemed to have a preferred risk-adjusted return, and hence a preferred portfolio for selection.

The final output is presented in a table comparing each portfolio's weight, predicted return, variance, and Sharpe Ratio. With this, one can effectively contrast a range of asset weightings and overall performance. By such analysis, one can identify the portfolio with the most satisfactory trade-off between return and risk, and make investment choices according to numerical values.

The final results are presented through a Portfolio Opportunity Set graph, which illustrates the relationship between each portfolio's expected return and its corresponding volatility, namely standard deviation. This visual representation provides a clear comparison of how different asset allocations influence overall performance. By analyzing the curve, it becomes possible to identify the portfolio configurations that offer the most favorable risk-return trade-offs, helping guide investment decisions based on quantitative metrics. The shape of the curve highlights both efficient and inefficient portfolios, enabling the selection of the optimal portfolio that maximizes returns for a given level of risk.

Figure 2.1, Portfolio Opportunity set.



Source: personal elaboration.

In addition to the Portfolio Opportunity Set, significant descriptive statistics and a correlation matrix have been added to allow for a proper understanding of the dataset and its assets' interdependencies. Descriptive statistics include significant statistics such as mean return, variance, skewness, and kurtosis, offering a quick view into individual assets' performance characteristics. The descriptive statistics expose considerable performance differences between analyzed factors. Inflation expresses the most variation in its value, with significant fluctuations over time. The IMOEX Index shows a mean return with high volatility, consistent with the inherent risk in stock markets. In contrast, yields for bonds and the risk-free rate exhibit less variation and lower volatility, reflecting their status as safer investments.

In terms of distribution, the IMOEX Index return is characterized by negative skewness, indicating a higher probability of extreme negative outcomes. Yields for bonds, the risk-free rate, and inflation display positive skewness, suggesting occasioxnal significant positive shifts. Kurtosis values reveal that inflation experiences the most extreme variations, while the distribution for the risk-free rate tends toward normal. All these observations provide a comprehensive view of risk and return dynamics, supporting portfolio construction and risk management decisions.

2.2, Main descriptive statistics.

	Mean	Std. Dev.	Skewness	Kurtosis
MOEX	0.006809	0.059795	-1.102745	5.093340
RFF	0.004255	0.085403	0.123166	-0.073308
BOND	0.006744	0.056583	1.025465	5.614018
INF	0.012226	0.135510	0.670616	13.209127

Source: personal elaboration.

The correlation matrix reveals the intensity of linear relation between returns of the IMOEX Index and Bonds. It plays a critical role in portfolio diversification, with low and even negative relations between assets contributing a lot in reducing overall portfolio risk. The correlation matrix reveals the relationships between the IMOEX Index returns, bond yields, risk-free rate, and inflation rate. The IMOEX Index shows a negative correlation with both bonds and the risk-free rate, indicating slight inverse movements, which suggests some potential for diversification. Bonds display a moderate positive correlation with the inflation rate, reflecting their sensitivity to changes in macroeconomic conditions. The risk-free rate exhibits a very weak negative correlation with inflation, indicating minimal impact from inflation fluctuations. Overall, the matrix highlights low to moderate correlations, supporting the benefits of diversification across these asset classes.

2.3, Correlation Matrix.

	IMOEX Index	Bond	Risk Free	Inflation Rate
IMOEX Index	1.000000	-0.126159	-0.096935	0.040547
Bond	-0.126159	1.000000	0.056505	0.391167
Risk Free	-0.096935	0.056505	1.000000	-0.047021
Inflation Rate	0.040547	0.391167	-0.047021	1.000000

Source: personal elaboration.

SIMPLE REGRESSION ANALYSIS

The regression analysis implies the verification of the stationary of data series. Stationarity signifies that a series possesses a stable mean, variance, and autocovariance across time, serving as a crucial assumption in numerous econometric models to guarantee accurate estimates.

To assess if the data exhibits a unit root, suggesting non-stationarity, the Augmented Dickey-Fuller (ADF) test was conducted on the IMOEX Index, bond, risk-free rate, and inflation rate. The ADF test evaluates the null hypothesis that a unit root exists in the series. If the p-value is less than 0.05, the null hypothesis is rejected, confirming that the series is stationary. The ADF test results verify that all variables are stationary, with p-values below the 0.05 threshold. This means that first differencing is not required to eliminate unit roots, allowing the regression analysis to proceed using the original dataset without modifications.

This analysis investigates the relationship between key macroeconomic factors, Inflation Rate, Risk-Free Rate, and Bond Returns, and the IMOEX Index, dividing the model in two inflation regimes, high inflation and low inflation, defined by the median value of the Inflation Rate. To provide both average and distributional insights into how these factors affect the IMOEX Index, there were employed Ordinary Least Squares (OLS) and Quantile Regression (QR).

ORDINARY LEAST SQUARES (OLS) REGRESSION RESULTS

OLS regression estimates the average effect of independent variables on the dependent variable, revealing fundamental dynamics across the two inflation regimes.

During high inflation periods, the model explains approximately 19.1% of the variability in the IMOEX Index, as indicated by the R-squared value. Notably, the Inflation Rate shows a positive and statistically significant impact ($p = 0.001$), with a coefficient of 0.2648, suggesting that rising inflation tends to boost equity returns. This could reflect the stock market's adjustment to higher price levels, possibly due to increased nominal revenues for companies or inflation-driven asset price growth. Conversely, Bond returns have a negative and significant effect with a coefficient of -0.3629 ($p = 0.009$), indicating an inverse relationship, likely stemming from a shift in investor preference from fixed-income securities to equities in an inflationary environment. The Risk-Free Rate, however, is not statistically significant ($p = 0.776$), suggesting a limited direct influence on equity returns during these periods.

In contrast, during low inflation periods, the explanatory power of the model decreases slightly, with an R-squared of 11.6%. Interestingly, the Inflation Rate exhibits a negative and significant impact ($p = 0.011$), with a coefficient of -0.1705. This implies that even moderate increases in inflation during low-inflation environments can negatively affect equity returns, possibly due to concerns about rising costs, reduced consumer purchasing power, or expectations of tighter monetary policy. Both Bond returns and the risk-free rate remain statistically insignificant, reflecting their diminished role when inflationary pressures are subdued.

QUANTILE REGRESSION (QR) RESULTS

While OLS provides insights into the average relationships, Quantile Regression offers an insight view by examining the effects across different points of the IMOEX Index distribution, from the lower (5th percentile) to the upper (95th percentile) quantiles. This approach is particularly valuable for identifying how macroeconomic factors behave during extreme market conditions, such as downturns or booms.

In high inflation scenarios, the Inflation Rate consistently shows a positive and significant effect across many quantiles, particularly around the median (0.5 quantile), where the coefficient is approximately 0.2585 ($p = 0.002$). This suggests that inflation's positive influence on equity returns is not limited to average conditions but extends across different performance levels of the IMOEX Index. Interestingly, the impact is strongest around the middle quantiles, while it diminishes at the extreme tails, indicating that inflation may have a more stable, moderate effect rather than driving extreme market movements. Bond returns maintain a negative and significant relationship in several quantiles, especially between the 0.3 and 0.6 quantiles, reinforcing the inverse relationship observed in the OLS results. The Risk-Free rate, however, remains largely insignificant across all quantiles, underscoring its limited influence in inflationary environments.

During low inflation periods, the pattern shifts. The Inflation rate demonstrates a negative and significant effect at several quantiles, particularly around the median and upper-middle quantiles (0.4 and 0.5 quantiles), with coefficients around -0.1311 ($p = 0.041$). This suggests that even modest inflation increases can dampen equity performance in stable inflation environments, likely due to heightened sensitivity to economic uncertainty or cost pressures. Unlike in high inflation periods, the negative impact of inflation is more pronounced in the middle quantiles, indicating that the average firm or market condition is more affected than extreme scenarios. Bond returns and the Risk-Free rate remain largely insignificant across most quantiles, suggesting their minimal role during periods of low inflation volatility.

REGRESSION ANALYSIS ADDING NON-LINEAR RELATION

In order to increase the explanatory power of the model, non-linear relationships were included in the regression analysis. More specifically, interaction terms of bond with risk-free rate (*Bond_Riskfree*) and with inflation (*Bond_Inflation*) were included to capture the joint impact of these variables. Other specifications included the inclusion of the square of the inflation rate (*Inflation_sq*) to capture any non-linear effects of inflation on equity returns and the use of a logarithmic transformation of the bond (*Log_Bond*) to reduce any potential distortions resulting from the scale. A binary variable (*High_Inflation*) was also created to distinguish between high and low inflation periods based on the median inflation rate so that the model can capture the specific effects of different regimes.

ORDINARY LEAST SQUARES (OLS) REGRESSION

The OLS regression, which estimates the average effect of independent variables on the IMOEX Index, reveals significant differences across high and low inflation regimes. During high inflation periods, the model explains approximately 32.4% of the variability in the IMOEX Index, as indicated by the R-squared value, reflecting a substantial improvement in explanatory power due to the inclusion of non-linear terms. The Inflation Rate shows a positive and statistically significant effect with a coefficient of 0.304 ($p = 0.043$), suggesting that rising inflation tends to boost equity returns. This could be attributed to the market's adjustment to higher price levels or inflation-driven growth in nominal revenues. In contrast, Bond returns have a negative and significant impact with a coefficient equal to -0.369 ($p = 0.010$), indicating an inverse relationship, likely driven by a shift in investor preference from fixed-income securities to equities in an inflationary environment. The interaction term *Bond_Riskfree* also shows a strong negative effect through the coefficient equal to -5.1276 ($p = 0.002$), highlighting the adverse combined influence of bond yields and the risk-free rate on equity performance. Other variables, such as the Risk-Free rate and *Log_Bond*, do not exhibit significant effects, suggesting a limited direct influence on equity returns during high inflation periods.

When inflation is low the model's explanatory power is reduced and R-squared is 15.7% which means that the impact of the macroeconomic variables on equity returns is not strong in stable inflation environments. The Inflation Rate has a negative and marginally significant effect with a coefficient of -0.279 ($p = 0.092$) , suggesting that even small increases in inflation can be negative for equity returns, possibly because of higher costs or expectations of tighter monetary policy. Both Bond returns and the interaction term *Bond_Riskfree* exhibit negative coefficients but are not statistically significant, reflecting their diminished role when inflationary pressures are subdued. The Risk-Free Rate also remains insignificant, underscoring its limited influence in low-inflation contexts.

QUANTILE REGRESSION (QR) RESULTS

Quantile Regression, even in this case, helps to identify the effect of the independent variable under both bullish and bearish market conditions.

In high inflation scenarios, the Inflation Rate consistently shows a positive and significant effect across several quantiles, particularly around the median (0.5 quantile), where the coefficient remains robust ($p = 0.035$). This suggests that inflation's positive influence on equity returns extends beyond average conditions, affecting different levels of market performance. The impact is more pronounced in the middle quantiles, while it diminishes at the extreme tails, indicating that inflation has a more stable, moderate effect rather than driving extreme market fluctuations. Bond returns maintain a negative and significant relationship in several quantiles, especially between the 0.2 and 0.6 quantiles (p-values ranging from 0.025 to 0.086), reinforcing the inverse relationship observed in the OLS results. The interaction term (*Bond_Riskfree*) also shows strong negative significance across multiple quantiles, underlining the adverse effect of combined bond and risk-free dynamics in inflationary environments. The Risk-Free Rate remains largely insignificant, highlighting its limited role under high inflation.

During low inflation periods, the pattern shifts. The Inflation Rate demonstrates a negative effect at several quantiles, particularly around the median and upper-middle quantiles (0.4 and 0.5 quantiles), though statistical significance varies (p-values around 0.217). This indicates that modest inflation increases can dampen equity performance in stable inflation environments, likely due to heightened sensitivity to economic uncertainty or cost pressures. The influence of Bond Returns and the interaction term (*Bond_Riskfree*) is less consistent and statistically insignificant across most quantiles, suggesting their minimal role during periods of low inflation volatility. Overall, the QR results confirm the non-linear and asymmetric effects of inflation and bond-related factors across different market conditions, providing a more comprehensive understanding of how these variables interact with equity returns.

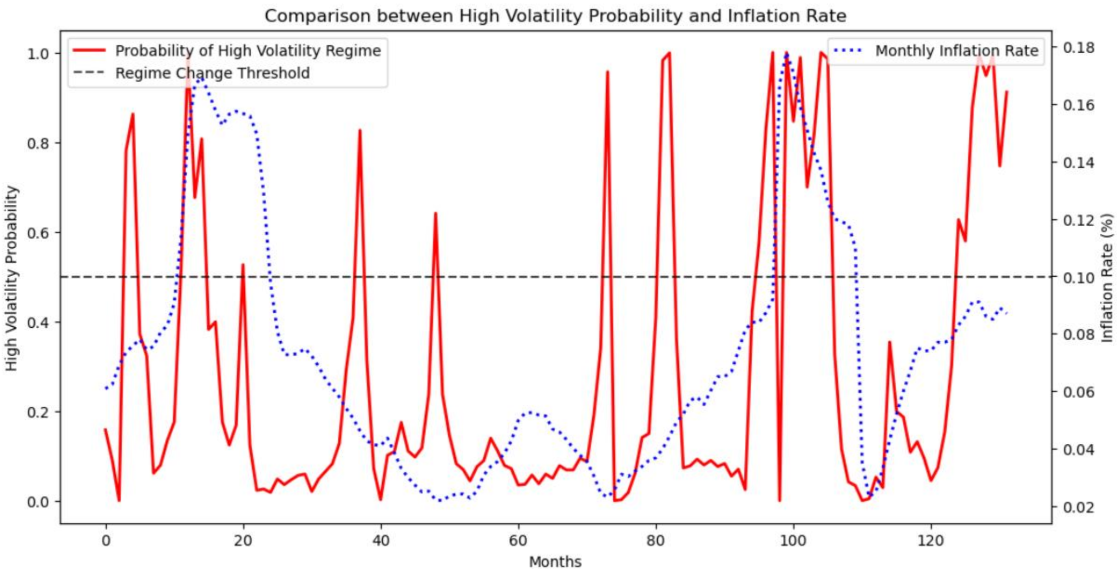
MARKOV-SWITCHING MODEL

OLS showed the overall impact of inflation on the IMOEX Index, Quantile Regression examined its diverse effects at different market capitulation levels, and both approaches used fixed and ex-ante defined categories. However, financial markets are subject to structural changes that may not match the set criteria. The Markov-Switching Model improves on this limitation by identifying market regimes within the data instead of relying on previously known thresholds. As a model of the probability of switching between high and low volatility regimes over time, the MSM is a flexible and data-driven approach to understanding how inflation affects volatility. This approach captures the non-linearities and structural changes in the system and, thus, gives a more accurate and realistic picture of the behavior of the financial markets as opposed to the static regression models. The Markov-Switching Model distinguishes two different regimes with different levels of volatility. Regime 0 of the model that identifies low volatility, shows that the Risk-Free rate is positively and strongly significant ($p < 0.001$) to have a stabilizing effect in less turbulent market conditions. On the other hand, Bond returns have a negative and statistically significant effect ($p = 0.028$), which indicates that the relationship between the two is negative in stable markets, perhaps because investors are risk averse. The interaction term *Bond_Riskfree* is negative but not statistically significant, which means that there is not much effect in this regime.

The model's explanatory power changes significantly in Regime 1 that identifies high volatility. The Risk-Free rate is still positively related to equity returns, but the p-value increases to 0.073, which may be because of high market uncertainty. Importantly, Bond returns have a stronger negative and highly significant effect ($p = 0.003$), which reinforces the strong negative association observed in the turbulent periods. The interaction term *Bond_Riskfree* has a large negative and very high statistical value ($p < 0.001$) that shows the adverse impact of both the bond and the risk-free rates on the volatility of the market during periods of high volatility.

The regime transition probabilities are very high to stay in the same regime after entering it (p[0->0] = 0.8575), while the probability of transferring from high to low volatility is relatively low (p[1->0] = 0.3548). This asymmetry suggests that markets are more likely to stick to the current situation, and once high volatility regimes are initiated, they are more likely to persist. The regime probabilities visualization also provides further insights into the temporal evolution of the probability of being in a high volatility state and can be very helpful for understanding the dynamics of market stress and stability. The graph presents the smoothed probabilities of being in a high volatility regime over time as a red line, with peaks almost reaching 1 which corresponds to periods of high market volatility and troughs almost reaching 0 which correspond to less volatile periods. The dotted blue line represents the annual inflation rate, which can be used to compare inflationary patterns with volatility. Although inflation surges are often accompanied by increased market turmoil, the Markov-Switching Model provides a dynamic specification of regime transitions based on multiple, concurrent economic and financial metrics, such as risk-free rates, bond returns, and inflation. The dashed black line at 0.5 is the regime change threshold and represents the times at which the probability of switching between high and low volatility regimes is most uncertain.

2.4, High Volatility Probability.



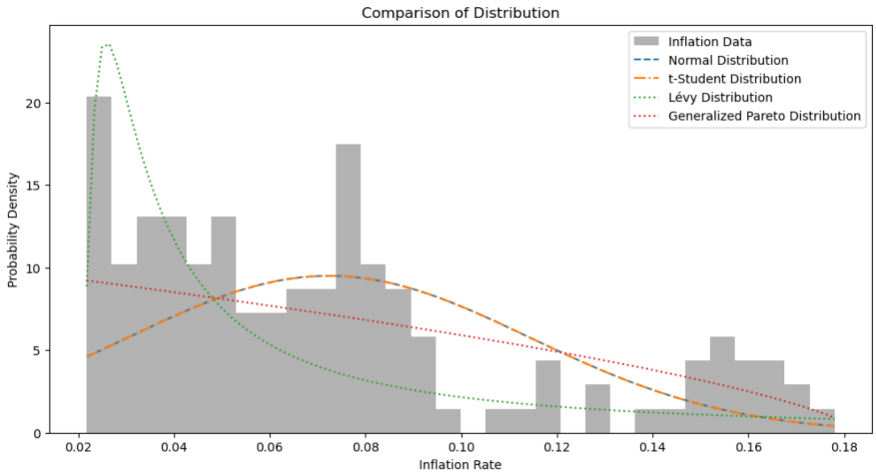
Source: personal elaboration.

INFLATION VOLATILITY'S ANALYSIS

Inflation has a very important role in determining market expectations and the behavior of investors. Although the Markov-Switching approach identifies structural breaks and

dynamic transitions between volatility states, the EGARCH model provides a similar but complementary view of the data by explicitly modelling asymmetric volatility clustering and the persistence of shocks. This allows for a more detailed analysis of how inflation and other financial factors affect market dynamics not only by means of abrupt regime changes but also by means of time-varying volatility, which provides a more refined view of the risk behavior over time. In order to accurately capture inflation related volatility, the analysis was expanded using the EGARCH model, specifically to the volatility of the inflation rate. Unlike standard GARCH models, EGARCH is particularly well-applied to model financial time series with asymmetric volatility responses, where the effect of negative and positive shocks on future volatility are different. This characteristic is particularly important in the context of inflation, in which case, unanticipated increases in the price level may lead to increased uncertainty, and, in turn, to a more persistent effect on expected prices. The main reason for choosing EGARCH with a Student's t-distribution is the distributional properties of inflation, as revealed by the best-fit distribution analysis. The Kolmogorov-Smirnov (KS) test results showed that inflation is leptokurtic, meaning that there are more extreme inflationary movements than what would be expected under the normal distribution. This is an empirical reason for using a Student's t-distribution in the EGARCH specification, because it is more appropriate to model fat-tailed returns and to capture the occurrence of extreme observations. The model also ensures that inflation volatility is not underestimated, especially in the case of extreme price movements, which other normal-based models may not pick up well.
Below is reported the comparison of distribution that justifies the t-student.

2.5, Comparison of Distribution.




```
KS Test - Normale: 0.126 0.027
KS Test - t-Student: 0.126 0.027
KS Test - Lévy: 0.274 0.0
KS Test - Generalized Pareto: 0.145 0.007
```

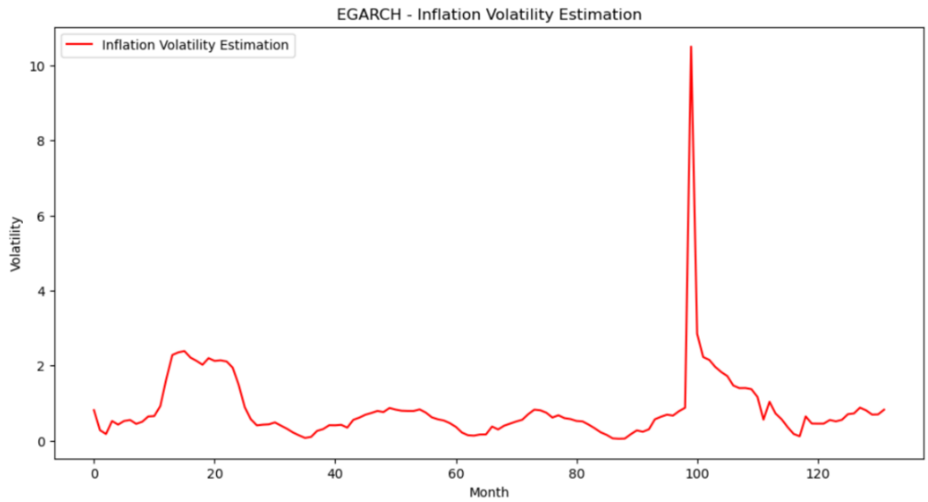
Source: personal elaboration.

Kolmogorov-Smirnov (KS) test results show that the t-Student distribution (D = 0.126, p = 0.027) is the best-fitting distribution for inflation rate data as compared to other distributions. The D-statistic that compares the empirical and theoretical distributions is lower for t-Student distribution than for Lévy (D = 0.274, p = 0.0) and Generalized Pareto (D = 0.145, p = 0.007) distributions, thus suggesting that t-Student model is a better representation of data structure. Furthermore, it has a p-value of 0.027 which, although slightly below the usual significance level of 0.05, is greater than that of the Lévy distribution (0.0) which makes it even more suitable. This result justifies the choice of the t-Student distribution in the EGARCH model, as it properly accounts for heavy tails and extreme inflation fluctuations. Since financial time series often exhibit non-normality, incorporating a t-distributed innovation term in the EGARCH model enhances its ability to capture large deviations in inflation volatility, ensuring more reliable and robust estimations.

The estimation results confirm the significant persistence of inflation volatility, with the alpha coefficient (2.4011, p < 0.001) indicating the impact of past volatility shocks, and the beta coefficient (0.8637, p < 0.001) suggesting that these effects remain substantial over time. The negative omega coefficient (-1.3599) highlights that inflation shocks are persistent and tend to have a lasting effect, reinforcing the necessity of nonlinear

modeling techniques. The visualization of the volatility series illustrates distinct inflationary spikes, highlighting periods of heightened uncertainty.
Below is reported the EGARCH model:

2.6, Inflation Volatility Estimation.



Source: personal elaboration.

REGRESSION ADDING INFLATION VOLATILITY

Following the estimation of inflation volatility using an EGARCH (1,1) model, the conditional volatility series was included as an explanatory variable in the regression model. The objective was to assess whether inflation uncertainty contributes to a better explanation of stock market returns, particularly for the IMOEX index.
A comparison of the regression results before and after adding *Inflation_Vol* indicates that the R-squared value increases from 32.4% to 36% in the high inflation regime. This suggests that incorporating inflation volatility improves the overall explanatory power of the model. Despite an increase in the model's goodness-of-fit, the coefficient for *Inflation_Vol* is marginally significant in the high-inflation regime (p =0.076) and remains largely insignificant across most quantile regressions. This suggests that while inflation volatility might contribute some explanatory power to the model, its direct impact on stock market returns is limited. The sign of the coefficient is negative, indicating that higher inflation volatility is associated with lower stock returns.
In the low-inflation regime, the inclusion of *Inflation_Vol* has a smaller impact, with R-squared remaining 15.7% and the coefficient being insignificant. This reinforces the idea

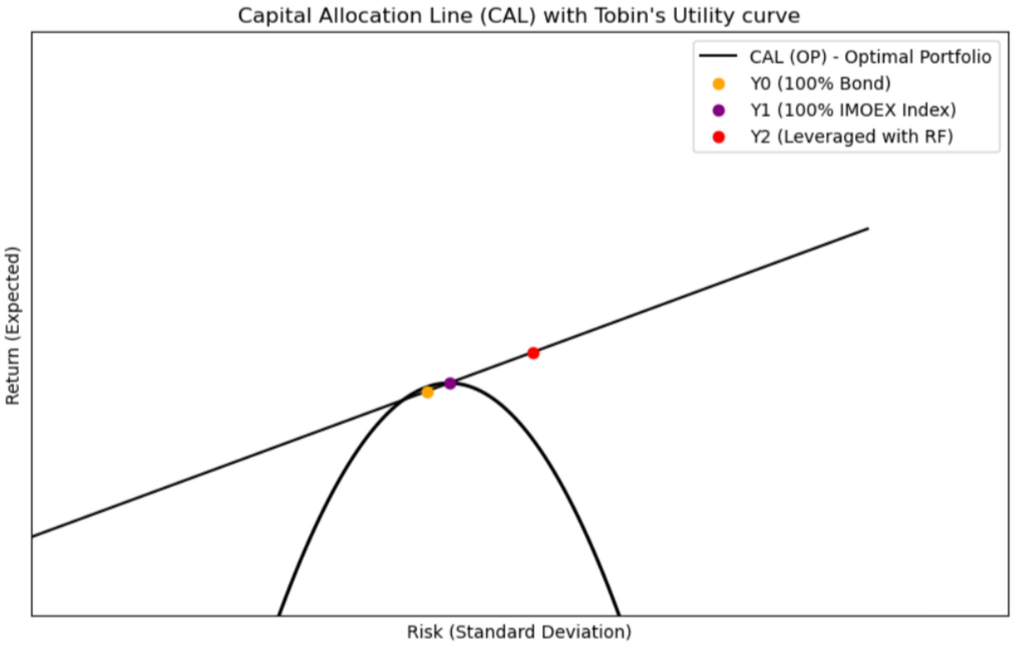
that inflation volatility is only relevant in periods of high inflation, whereas under stable inflation conditions, it does not meaningfully affect stock market movements.
From an econometric perspective, the increase in R-squared in the high-inflation regime suggests that inflation volatility may capture some variation in stock returns that was previously unaccounted for. However, the lack of strong statistical significance in the coefficient implies that this effect is likely indirect. Potentially, inflation uncertainty influences market dynamics through channels such as investor sentiment, risk premia adjustments, or liquidity conditions rather than exerting a direct influence on stock prices. This aligns with the broader economic literature suggesting that macroeconomic uncertainty can shape financial markets beyond its measurable impact on fundamental variables.

CAPITAL ALLOCATION LINE

The Capital Allocation Line (CAL) identifies the efficient frontier for an investor with access to risk-free markets and a risky portfolio of assets as the line provides the optimal expected return for the level of risk assumed. The CAL, illustrated as a straight black line, shows the various portfolio combinations that the investor can create by using or unwinding the optimal risky portfolio. The curve added to the graph represents Tobin's Utility Curve, which accounts for investor preferences in risk-taking. This curve is a quadratic function that captures the utility maximization process and helps identify the optimal portfolio for a given risk aversion level.
The three key points on the graph correspond to different portfolio choices. Y0 represents an entirely bond-based portfolio, which is the lowest-risk option but also provides the lowest return. Y1 is the market portfolio, in this case, fully allocated to the IMOEX Index, representing the tangency portfolio that maximizes the Sharpe ratio. Y2 is a leveraged portfolio that extends beyond Y1 along the CAL, incorporating borrowed funds at the risk-free rate to amplify returns, though at a higher risk.
The intersection between the utility curve and the CAL determines the investor's optimal portfolio choice. Given an investor's specific utility function, this point represents the best risk-return trade-off. The presence of Tobin’s Utility Curve enhances the interpretation of the CAL by explicitly illustrating how risk preferences influence portfolio selection. If an

investor has a low-risk aversion, the optimal portfolio choice moves higher along the CAL, incorporating more risk. Conversely, a highly risk-averse investor will select a portfolio closer to Y0, allocating more wealth to risk-free assets.

Figure 2.7, Capital Allocation Line.

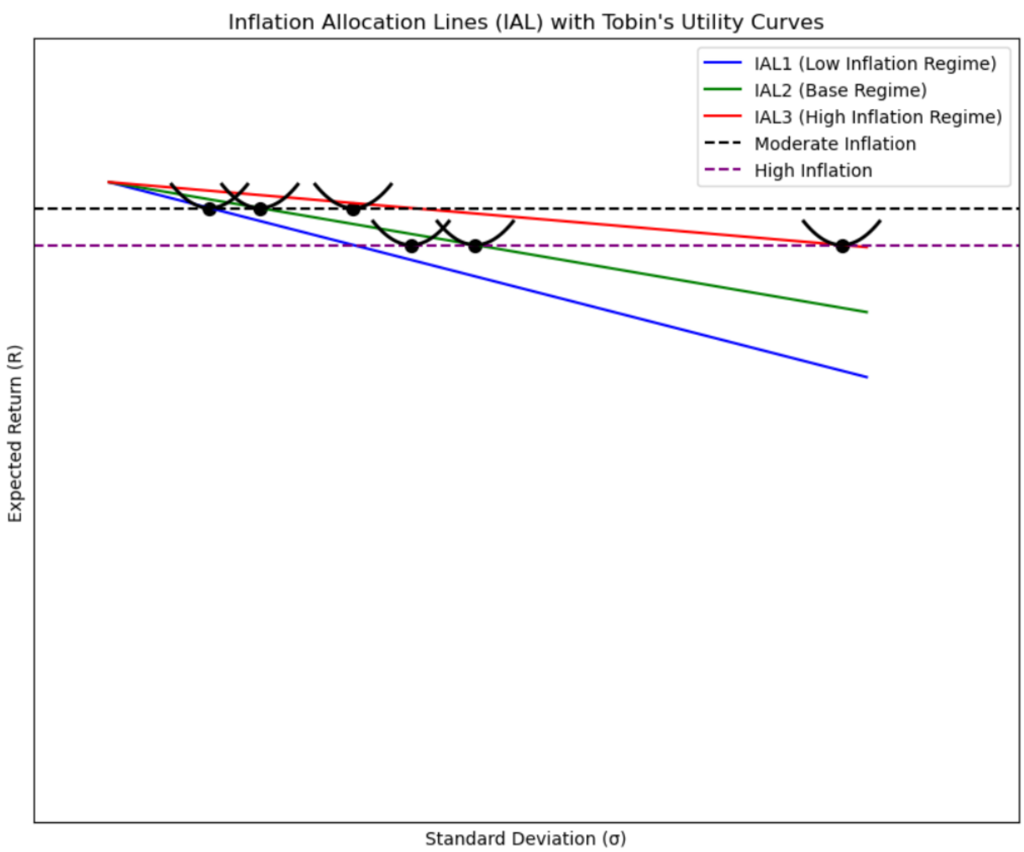


Source: personal elaboration.

INFLATION ALLOCATION LINES

The Inflation Allocation Lines (IAL) extend the traditional CAL framework by incorporating inflation as a key determinant of portfolio selection. The graph displays three distinct IALs corresponding to different inflationary regimes: low inflation, base regime, and high inflation. The downward slope of the IALs indicates that as inflation increases, the risk-adjusted return of portfolios decreases, shifting optimal allocations toward more conservative strategies. The utility curves illustrate investor preferences under different inflationary conditions, showing that high inflation regimes lead to a flattening of IALs, reducing the attractiveness of riskier assets. The dashed lines represent moderate and high inflation thresholds, emphasizing the transition points where portfolio reallocation becomes necessary. Empirical findings confirm this behavior, as seen in the regression analysis, where inflation exhibits a significant impact on IMOEX index returns. In high-inflation periods, risk-free assets and bonds play a more substantial role in portfolio construction due to their lower sensitivity to inflationary shocks. Conversely, during low-inflation periods, equity investments provide superior risk-adjusted returns, encouraging investors to increase exposure to risky assets.

Figure 2.8, Inflation Allocation Lines.



Source: personal elaboration.

ANALYSIS OVER YEARS

Over the period from 2014 to 2024, the Russian economy was characterized by high volatility not only in the inflation rate, but also in bond yields and IMOEX Index. The reasons behind this are principally connected to geopolitical factors, monetary and fiscal actions, and commodity price prophylaxis. 2014 was a drastic year for the Russian financial markets, Western sanctions were imposed on Russia and the oil price crashed which made the ruble depreciate and inflation rise to 11% and above. The Central Bank of Russia then increased the interest rates to 17% to stabilize the currency after an aggressive monetary policy was adopted. This policy shift resulted in an increase in bond yields and a sharp decline in the IMOEX Index and by the end of the year, the index had dropped 7.1%. In the following year, inflation was 15.5% because the ruble kept on falling and food prices increased due to trade restrictions but the IMOEX rose possibly because firms had learned how to operate effectively in the given environment. The results also show that sovereign risk perceptions were still high and therefore, bond yields were still relatively high. It can be seen that there is a positive correlation in the period that firms were able in some way to raise their prices. The inflation rate had dropped to 5.4% in the year 2016 due to a tight monetary policy and stability in the oil prices; a positive trend in the macro economy that led to a 26.8% growth in the IMOEX but bond yields fell due to the decrease in inflation expectations. However, the econometric analysis shows that the traditional macroeconomic factors explained a part of the variation in the stock indexes which may be the case of other or speculative factors. In 2017, the inflation was at 2.5% and the IMOEX fell 5.5% due to the volatility of oil prices and the persistence of sanctions while the bond market kept an average yield of 7.5%. In 2018 the inflation rate was 4.3% due to the hike in the VAT rate and depreciation of the ruble. However, the IMOEX rose 12% in the year in question because corporate earnings growth remained strong. The analysis of this year's data showed that the sensitivity of stock market returns to inflation shocks was decreasing. In 2019 the inflation rate was 3%, the bond market was slightly negative and the IMOEX Index was also slightly positive. It was a shock to the markets all over the world when the COVID-19 outbreak in 2020 and Russia was not spared from this. However, the rate of inflation was still stable at 3.4%; nonetheless, the IMOEX took a massive drop of almost 20% in March due to the pandemic's uncertainty. The bond yields first rose during the flight to safety and then declined after the Central Bank of Russia reduced its policy rates to spur the economy. The regression analysis of this year indicated that the IMOEX was more sensitive to the macroeconomic shocks as shown by the higher correlation between inflation and stock returns, particularly in the lower quantiles which captured the negative impacts of the crisis. In 2021, Inflation was 8.4% due to supply-side factors and domestic demand. The MOEX also kept on rising but at a slower rate, and the bond yields also rose due to the inflationary pressure. The following are the econometric results of this period: it has been found a positive link between inflation and stock returns, which supports the idea that firms were able to increase their prices. 2022 was dramatic due to the Russian invasion in Ukraine, able to determine market shocks. Inflation rose to 13.8%, the ruble dropped, and the IMOEX fell more than 35% in the first few days of the invasion. This is because the bond yields rose as the investors wanted to earn more from the Russian bonds. The statistical models for this year showed that inflation had a negative effect on the stock returns and the bond yields also had a negative effect on the market returns. In 2023, Inflation was constant between 6.3 and 7.0% due to the sanctions and the change of the trade partners. The MOEX recovery was evident with a 10% growth in the year, while the bond yields remained high. In 2024, the inflation rate was 7%, the bond yields were still high due to political risks and fiscal policies. The IMOEX also continued its recovery but the results showed that the effect of inflation on stock returns was still negative although it was not as high as in 2022.

CONCLUSION

Using the Russian economy as a specific focus, the thesis delivered an exhaustive examination of the effects of inflation on financial markets. The application of classical portfolio theories alongside Markowitz's Mean-Variance, Tobin's Utility Theory and Sharpe's Ratio and more advanced econometric models including Ordinary Least Squares (OLS), Quantile Regression (QR), Markov-Switching and EGARCH enables an understanding of how inflation impacts asset returns and, consequently, investment decision-making.

A primary finding is the asymmetric effect of inflation across different market conditions, able to detect due to quantiles' analysis. While moderate inflation can be absorbed by firms through price adjustments, high inflation introduces significant distortions in market expectations, investor sentiment, and overall asset allocation strategies. Quantile Regression results indicate that during market expansions, inflation has a weaker impact on stock returns, whereas during downturns, inflation shocks intensify financial instability, leading to capital outflows and increased market volatility. This insight highlights the importance of non-linear and distribution-sensitive models in capturing the full extent of inflation's impact on asset returns.

The Markov-Switching model further demonstrated that financial markets alternate between low and high-volatility regimes, with inflation acting as a key driver of these transitions. The persistence of inflation-induced volatility, as revealed by the EGARCH model, underscores the need for dynamic risk management strategies. It is therefore important that investors and policymakers understand that inflation is not only a macroeconomic variable but also a determinant of market sentiment and capital budgeting decisions.

As such, a country's ability to maintain economic credibility remains a key factor in attracting foreign capital even as inflation remains a factor in shaping market conditions. Stable inflation expectations, transparent monetary policies, and credible institutions can help to reduce the negative impacts of inflation on investor confidence so that capital is not flighted and market volatility is not excessive.

As a result of the effects of inflationary cycles on the global financial markets, it is important to assess the effects on portfolio balance and stability of markets. This study also shows that it is important the inclusion of inflation trends in the portfolio optimization models as investors need to be ready to change their strategies based on the macroeconomic situation.

In conclusion, inflation is not only a statistical figure but a real driver of the behavior of investors, risk spreads, and efficiency of the market. As financial markets grow and develop, the capacity to forecast and respond to inflation trends will be a critical asset for both investors and policymakers.

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