LUISS

Degree Program in Economics and Business (Major in Economics)

Course of Macroeconomics

Monetary Policy in Brazil under the Taylor Rule

An empirical assessment (1999-2025)

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To the Expansionary Love Policy

of the Children of Brazil,

whose Expectations for Happiness

and Unconventional Sadness

Filled up my Heart.

Abstract

This work investigates whether the Taylor Rule effectively describes the monetary policy of the Central Bank of Brazil (BCB) from the adoption of the inflation-targeting regime in 1999 and the instauration of the Real Plan in 2004 up to the present time. It discusses how deviations can be interpreted considering Brazilian economic and political context. The analysis applies a backward-looking Taylor Rule following the Clarida, Galí, and Gertler (2000) methodology. Quarterly data on inflation, real GDP and interest rates were collected from Federal Reserve Economic Data (FRED). Regression models have provided coefficients for inflation and output gaps across four sub-periods (1999–2003, 2004–2011, 2012–2018, 2019–2025). The results show that in several phases, particularly during crises and the COVID-19 shock, the BCB's interest rate decisions diverged from what the Taylor Rule was indicating. Nonetheless, inflation targets were broadly respected, and implicit inflation targets remained within official bands. This work is innovative because it extends previous literature to incorporate post pandemic dynamics. Furthermore, it tests a hybrid approach, which combines Brazil's implicit target inflation and equilibrium interest rate with the "classic" Taylor Rule coefficients. This approach is shown to produce closer alignment with the actual policy adopted by the BCB.

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1. Introduction - The Principles of the Taylor Rule

There are many variables that Central Banks have to consider when setting monetary policy. Situations are complex and dynamic, they are influenced not only by macroeconomic conditions, but also by social and political context.
One of the most effective tools which can be used as a guide in this framework is the Taylor Rule, formulated by John Taylor in 1993. The Taylor Rule is an empirical, simple approach to set the nominal interest rate. Its fundamental principle is to anchor the policies of the Central Bank to the inflation target and to the potential output. There is a macroeconomic trade-off between choosing to optimize inflation and output.
The Taylor Rule indicates how to create a stabilizing feedback loop, suggesting that policies should be more restrictive both when inflation rises and when the output exceeds potential output (or when unemployment is below its natural level), and more easing in the reversed scenario.

The canonical Taylor Rule is typically expressed as:

$$i_t = r^* + \pi_t + 0.5(\pi_t - \pi^*) + 0.5(y_t - y^*), \quad (1)$$

where: i_t: Nominal short-term interest rate (e.g., the federal funds rate)

r^*: Real equilibrium interest rate (classically set at 2%), the desired nominal rate when both inflation and output are at their target levels.

π_t: Actual inflation rate

π^*: Target inflation rate (classically set at 2%)

$y_t - y^*$: Output gap (actual GDP minus potential GDP)

The two coefficients of inflation and output gaps (classically set at 0.5) indicate the sensitivity of interest rate adjustments to inflation and output deviations.
In order to adapt the effectiveness of the Taylor Rule to specific macroeconomic contexts, different weights can be given to the coefficients that control inflation and output. At the same time, the response can be smoothed to achieve a more balanced reaction.
Nevertheless, especially when exogenous shocks are hitting the economy, Central

Banks can choose not to follow the Taylor Rule's indications and apply autonomous tightening or easing policies that can eventually make use of unconventional tools. Due to its simplicity, the Taylor Rule plays an important role in modern Macroeconomics.
Anyway, a challenge that Central Banks have to face when they are referring to the Taylor Rule is the estimation of unobservable variables, such as the potential output, the target inflation rate and the equilibrium interest rate, which can significantly affect the outputs of the rule.
The Taylor Rule cannot be a substitute to strategic policy-making, but it still offers a structured reference in decision-making.
The effectiveness of its prescriptions is strictly affected by the credibility of the government's policies.

The Brazilian economic framework offers an interesting Historical Natural Experiment to test the Taylor Rule and to study how it would be effective in guiding decisions to shape the economy.
Cantoni and Yuchtman showed the importance of these studies in their paper *"Historical Natural Experiments: Bridging Economics and Economic History",* where they emphasize the importance of using history as testing ground to prove the effectiveness of economic theories.

The numerous failed attempts at overcoming the high inflation that peaked from the mid '80s to mid '90s and that caused the Brazilian government to lose its credibility, offer a unique testing ground; however, this analysis is going to focus on whether and how the Taylor Rule indications were followed once the Inflation Targeting regime (1999) and the Real Plan (2004) were established and once the government's fiscal reputation improved.
This work is going to study different periods of recent Brazilian history, highlighting the key political and social events that can help interpret the results of the analysis.

2. Literature Review

The method of analysis of the Brazilian economic data through the Taylor Rule will follow Clarida, Galí, & Gertler's work; studies by De Losso, Carvalho, & Muinhos, and , Muinhos, Fonseca, & Schulz will also be taken as a benchmark to compare results.

This work builds on the above by including the latest pandemic shocks, the subsequent post Covid inflation and an analysis of the present policies applied by the Brazilian Government.

2.1 "Monetary policy rules and macroeconomic stability: Evidence and some theory (2000)"

The method proposed by Clarida, Galì and Gertler is used to set up the model. The Taylor rule, which indicates the target value for the nominal interest rate as a function of the gaps between expected inflation and output and their respective target levels, is given by:

$$i'_t = i^* + \beta(\pi_t - \pi^*) + \gamma(y_t - y^*).$$

(2)

This expression offers central banks an indication on how to adjust the nominal interest rate; however, economies respond to the variations in the real interest rate: real interest rate = nominal interest rate−expected inflation (Fisher equation). Expressing the Taylor Rule in real terms yields:

$$r_t = r^* + (\beta - 1)(\pi_t - \pi^*) + \gamma(y_t - y^*), \quad (3)$$

This expression shows that the condition to have a stabilizing monetary policy reaction to inflation and output variations is to have β >1 and γ > 0. However, there is evidence that the policy reaction function given by the equation above might be too restrictive and unrealistic.

The reason for this can be found in the tendency of the Federal Reserve to smooth changes in interest rates and in the fact that it doesn't have perfect control on interest rates. Furthermore, it doesn't account for the possibility of randomness in its policy actions.

Thus, the authors assume that the presence of a lagged interest rate variable can

improve the performance of the rule.
With this assumption, the actual interest rate is expressed by:

$$i_t = \rho i_{t-1} + (1-\rho) i'_t \quad (4)$$

where $\rho \in [0, 1]$ indicates the degree of smoothing of the interest rate changes;

Substituting equation 4 in equation 1 the policy reaction function becomes:

$$i_t = (1-\rho)[r^* - (\beta - 1)\,\pi^* + \beta\pi_t + \gamma(y_t - y^*)] + \rho r_{t-1} \quad (5)$$

In order to run the regression analysis, equation 5 can be rearranged as follows:

$$i_t = (1-\rho)[r^* - (\beta - 1)\,\pi^*] + (1-\rho)\beta\pi_t + (1-\rho)\gamma(y_t - y^*) + \rho r_{t-1}, \quad (6)$$

where: $(1-\rho)[r^* - (\beta - 1)\pi^*] = cost,$ (7)

$(1-\rho)\beta\pi_t = \beta',$ (8)

$(1-\rho)\gamma = \gamma',$ (9)

The constant term that is obtained includes both π* and r*, but, since inflation target is an interesting parameter for the analysis, an additional restriction is introduced: the equilibrium interest rate is obtained as the value of the observed sample average; a reasonable compromise given the sample size.

The coefficients $cost, \beta', \gamma'$ are thus obtained by the regression (see Appendix A) and allow for the calculation of β, γ and π^*:

$$\beta = \frac{\beta'}{1-\rho}, \quad (10) \qquad \gamma = \frac{\gamma'}{1-\rho}, \quad (11) \qquad \pi^* = \frac{r^*}{\beta - 1} - \frac{cost}{(\beta - 1)(1-\rho)} \quad (12)$$

The statistical propagation of uncertainty formula has been used to calculate the standard errors of β, ϒ, π*, which have subsequently been used to obtain the test statistics, which assess the robustness of the various parameters. (see Appendix B)

2.2 “The Central Bank of Brazil’s Time-Varying Taylor Rule (2020)”

The above paper by Filipe Gropelli Carvalho and Marcelo Kfoury Muinhos has been used as a first reference to benchmark the results.

In their study, the authors adopted a modified Clarida-Galì-Galtier approach to estimate how the β coefficient varied from 2004 to 2020.

Results show β coefficients well above one up to 2010. After that, it started decreasing and It became lower than one between 2012 and 2017 and negative between 2012 and 2015. After 2016, the coefficient started recovering and gradually returned to values above one (Figure 1).

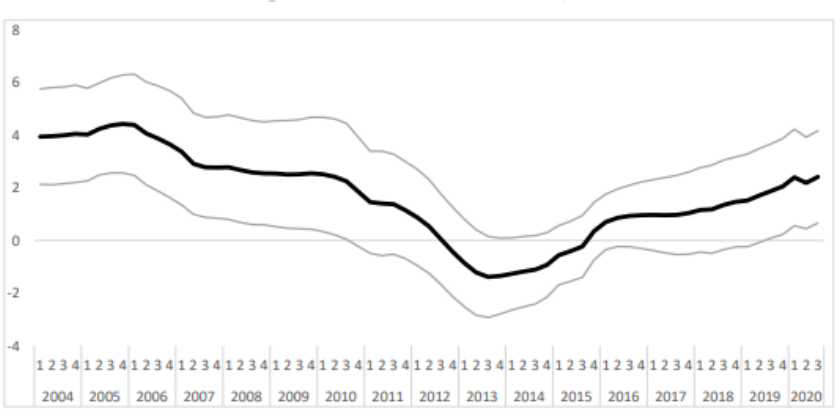

Figure 1 – Estimating β as only time-varying parameter (plus/minus two standard errors %)

The authors also focused their analysis on the variation of the implicit inflation target π* over time, observing how it matched the target values stated by the BCB and whether the differences were inside or outside a 2 sigma deviation band (Figure 2)

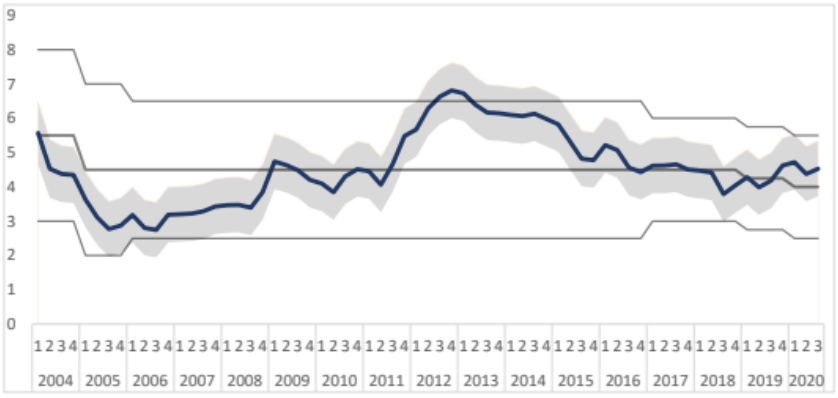

Figure 2 – Implicit inflation target and official inflation target (plus/minus) two standard errors %)

Results show that implicit inflation mostly remained between the 2 sigma deviation target bands except for the initial period, under Lula's presidential mandates, in which it reached its lowest levels, below the target.
During the Great Financial Crisis, the value started increasing and peaked between 2012 and 2013 due to the stimulative monetary policy adopted by Governor Tombini. This situation is consistent with the drop in the inflation parameter.
In the following years, after the 2015-2016 economic recession, the inflationary pressure was reduced, until the Covid-19 pandemic shock, to which the BCB responded with a stimulative approach.

2.3 "Equilibrium Interest Rate in Brazil, Convergence at Last (2020)"

In this paper, M.K. Muinhos, M. Fonseca, E.C.O. Schulz evaluated the latent variable equilibrium real interest rate (r*) over the period 2004-2020 through different methodologies.
The following graph (figure 3) shows the results emerging from their study: The equilibrium real interest rate decreases over time and stabilizes at an average value of 2,9%.

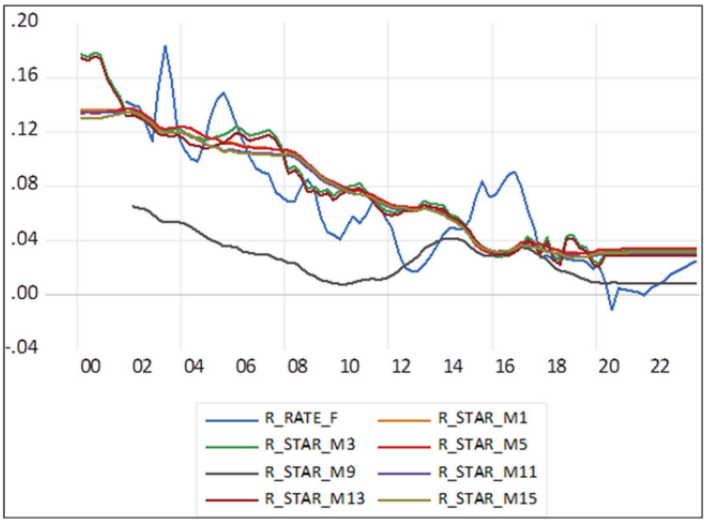

Figure 3 – The equilibrium real interest rate

The same paper also presented an evaluation of the output gap, obtained using a variety of methodologies, including the Hodrick-Prescott Filter (Figure 4)

Figure 4 – Different estimations of the output gap

2.4 ”Taylor Rule in Brazil (2005)”

The paper written by Rodrigo De Losso da Silveira Bueno (2005) was the first attempt to study the consistency of the Taylor Rule applied to Brazilian monetary policy, a few years after the inflation targeting policy had been adopted and just after the implementation of the Real Plan.

What the authors found was not totally expected: β coefficients were less than one, suggesting monetary instability, although it was clear that prices were under control.

In this analysis the authors tried to give a social-economical interpretation of this outcome.

A possible explanation is that some prices that make up the consumer’s budget were set by the government, together with some contracts of privatized firms which followed government’s agencies instructions; these were hence insensitive to interest rate movements.

Moreover, Brazilian consumers are used to growing inflation; this means that they prefer to consume today, rather than saving, to avoid losing purchasing power. Because of this, increases in interest rates have almost no effect in reducing consumption.

3. Institutional Background

Between 1986 and 1994, Brazil faced a number of failures, where different plans tried to control recurring high inflation (Figure 5).

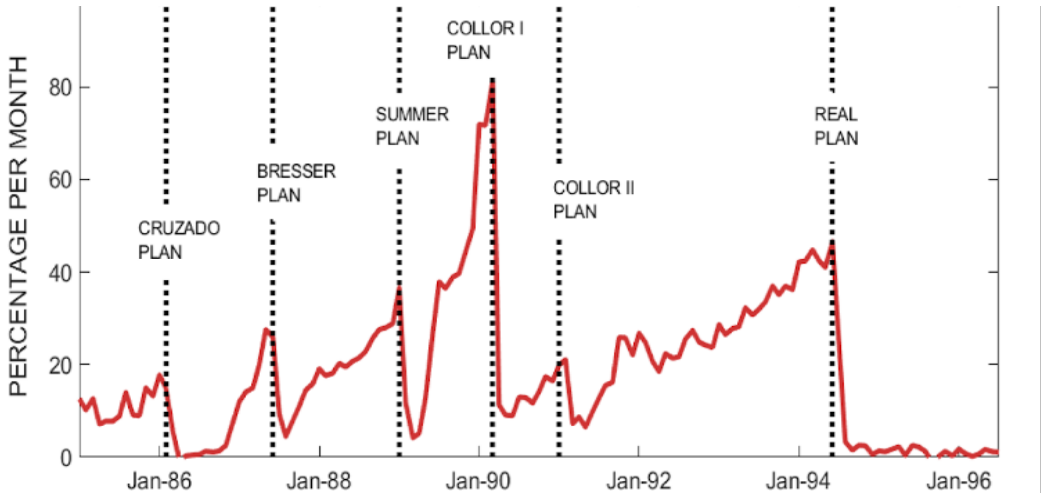

Figure 5 – Sequence of recovery plans and effects on inflation

One explanation for the difficulty of stabilizing the economy is the effect of “inertial inflation”.

Inertial inflation is a unique Brazilian feature that describes why inflation could not be controlled by monetary policies.

Inertial inflation does not identify the reasons for price increases in the money supply or expectations, but rather in the “distributive conflict” mechanism.

Due to the distributive conflict, the economy sees a continuous rise in prices and wages, as the economic actors try to protect their income share, thus leading to a self-perpetuating cycle.

Moreover, a series of external shocks and easing monetary policies accelerated inflation.

3.1 The Cruzado Plan (1986)

At the beginning of 1986, monthly inflation reached a rate of 13% and public debt was rising.

President José Sarney launched the “Cruzado Plan”, aiming to reach 0% inflation through unconventional strategies. The Cruzeiro currency was withdrawn and replaced by the Cruzado. The exchange rate was fixed at 1 USD = 13,84 Cruzados. After a one time realignment of +8%, both prices and wages were frozen.

The plan was supposed to adjust the wages in case inflation exceeded 20%.
Indexation was suspended and a tightening monetary policy was implemented.
As a consequence of these actions, inflation dropped dramatically from 22% in February 1986 to -1% in March.
The Government gained popularity and consumers responded positively.
This situation generated a consumer boom, which, thanks to the higher real wages, gave rise to an excess of demand. This caused supply shortages, due to the rising production costs which, together with the frozen prices, undermined firms' profits.
This situation also led to labour force exploitation and black markets.
As a reaction, the Government subsidized the industries to save them from going bankrupt. This led to a dramatic increase of the fiscal deficit.
President Sarney insisted on keeping the price freeze in order to maintain his popularity before the oncoming elections.
Without any fiscal reforms, the budget deficit persisted and by the end of 1986 inflation had spiked and credibility had been eroded.
In November 1986 the Cruzado 2 Plan introduced taxes on some goods and activated the wages adjustment, but these actions were only able to fuel inflation, which reached its highest value in mid-1987.
The failure of the Cruzado Plan was largely political as the Sarney administration prioritized popularity over economic logic.
The Bresser and Summer plans, which followed the Cruzado Plan, also failed due to an inconsistent fiscal discipline.

3.2 The Bresser Plan (1987)

The Bresser Plan was launched in June 1987 during President José Sarney's administration.
It had a shocking impact on Brazilian society. At its heart there was a 90 day prices and wages freeze and a devaluation of the exchange rate with the attempt to increase exports and support growth.
Interest rates were raised to reduce demand; at the same time, the fiscal policy increased taxes and cut subsidies.

Similarly to the Cruzado Plan, initially, inflation dropped from 20% in June down to about 5% in August, but by December it had risen back to 15% and led to the resignation of the finance Minister Bresser-Pereira.

3.3 The Summer Plan (1989)

In January, a further attempt to recover the situation was carried out through the Summer Plan by the Minister Mailson de Nobrega, still under President Sarney.
The Plan replaced the Cruzado with the Cruzado Novo, cutting three zeroes from the currency.
Prices and wages freeze were set indefinitely, taxes were adjusted, the exchange rate was devalued, and the interest rates were raised.
In the end, the above policy resulted in a defeat, with inflation surging to its highest levels ever.
The lesson learned was that both these plans failed in their credibility, mainly because they were lacking social and fiscal reforms that could support the control of inflation.
At this stage the public debt was huge, and the government was forced to pay very high interest rates to persuade the public to continue to buy government debt instruments.

3.4 The Collor Plan (1990)

The Collor Plan was an economic reform launched in Brazil in March 1990 by the newly elected President Fernando Collor de Mello.
Its task was to stop the accelerating hyperinflation that was growing above 80% in March 1990.
It was based on an unconventional and drastic measure: the freezing of about 80% of all private financial assets for 18 months, including savings and checking accounts, permitting occasional monthly unlocks, with the promise to return the frozen founds with interests and granting inflation stability.
The aim of this measure was to eliminate excessive liquidity and break inflationary inertia.
The plan also included a series of further measures such as:

- the introduction of the new currency, the Cruzeiro, which would substitute the Cruzado Novo at one-to-one ratio

- a new price and wage freeze
- a fiscal adjustment
- a sharp increase in the interest rate
- a large-scale privatization program, known as the National Privatization Program (PND), which caused the dismantling of several government agencies with the purpose of reducing the size of the public sector and opening the Brazilian economy to foreign markets.

Again, after an initial reduction of the inflation, the plan lost its credibility. In fact, the asset freeze generated a liquidity crisis that almost paralyzed national economic activity, leading to demand contraction, falling output and rising unemployment.
The Government had probably underestimated the psychological impact of the freeze and overestimated the economy's resilience.
Once more, a coherent fiscal strategy was missing; public spending remained high and interest payments on domestic debt increased due to elevated interest rates.
After the failure of the plan, inflation rose dramatically again.
In order to be successful, future efforts needed to combine a monetary reform with a credible fiscal adjustment and institutional support.

3.5 The Real Plan (1994)

The Real Plan chose a different approach, where transparency was a priority. Each step was announced in advance in order to rule out any unexpected situations; this was done with the aim to anchor expectations into the economy.
The plan was introduced by the Minister of finance Fernando Henrique Cardoso during the presidency of Itamar Franco. It was organized in three different phases:
The first phase involved a comprehensive and fundamental restructuring of the Government's financial policies, particularly its taxation and public spending.
It included the creation of the Emergency Social Fund (FSE), which aimed to manage tax revenues and public spending.
In the second phase, a transitional unit of account, the Real Unit of Value (URV), was introduced. Starting from March to June 1994, the URV coexisted with the Cruzeiro Real. The URV was the reference for pricing, while the Cruzeiro Real was the official currency for payments.
The introduction of the URV was meant to disrupt the backward-looking indexation,

which was the primary driver of inflationary inertia.
The URV was the reference for the price index and it was updated daily by the Central Bank of Brazil.
The value of the URV was fixed on the US dollar, with an exchange rate of 1 URV = 647,5 Cruzeiros Real.
The final phase consisted in the launch of the new national currency, the Brazilian Real (BRL), which was introduced on 1st July 1994.
It was not easy to manage the complexity of the coexistence of two different currencies. To make the transition effective, it was necessary to speed-up the physical distribution of the Brazilian Real as much as possible.
Germany, England and France contributed to the production of banknotes and coins.
The distribution of the new currency to the banks started in April 1994 and, in two months, about 1000 million banknotes and 700 million coins were distributed across the country.
In the transition period, the Cruzeiro Real continued circulating until September 1994. During this period, wages, prices, bank deposits, money transfers and contracts were gradually converted from Cruzeiro Real to URV.
On 1st July 1994, 1 URV was paired to 1 BRL, and from that moment on, all prices in the Brazilian Economy were exclusively expressed in BRL.
The disinflation that followed was substantial. The annual inflation dropped from about 5000% in June 1994 to less than 1000% by December 1994.
Notably, a contribution to the success of this plan was also possibly given by the national optimism fuelled by Brazil's victory in the 1994 FIFA World Cup and by a particularly prosperous agricultural yield.
In 1995, the annual inflation dropped consistently and reached 22%.
The Real Plan obtained a long-term sustainable reduction in inflation.
Its success was anchored to the combination of a wise institutional and fiscal reform, together with an innovative monetary policy.
A strong political leadership and a consistent strategic vision were also essential to the success of the plan.

3.6 Inflation Targeting

Since the Real Plan was launched, Brazil has experienced price stability.
This condition has been further re-enforced by a series of structural and institutional reforms.
The most important one is the adoption of the Inflation Targeting in 1999.
The National Monetary Council (CMN) is in charge of setting the inflation target and its tolerance range (Figure 6)

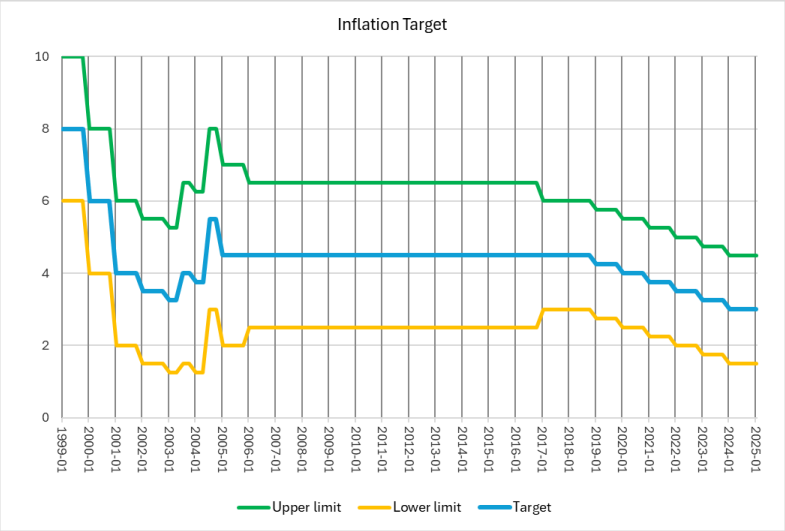

Figure 6 – Inflation Target

Stability depended on the fact that firms anchored their inflation expectations to the official target, which represented the benchmark to forecast future prices.
The success of the inflation targeting was strictly related to transparency and accountability in monetary policy.
Economic data and planned strategies were regularly published by the Central Bank of Brazil.
Credibility became a fundamental feature of the Brazilian macroeconomic environment. This framework provided a mechanism to monitor and address the deviations of inflation outside the tolerance range.
Whenever the deviation occurred, the Governor of the BCB submitted an open letter to the Chairman of the National Monetary Council, specifying the reason for the deviation, the corrective actions to be implemented and the expected time for the inflation to return within the target. This occurred in nine specific years: 2001, 2002, 2003, 2015, 2017, 2021, 2022, 2024, 2025, due to economic shocks.

3.7 The Selic Rate

The Selic Rate (Sistema Especial de Liquidaçao e Custodia) was introduced on 15th March 1999. It represents the key benchmark for interest rates throughout the economy.

It is calculated on the average rates applied to overnight landing transactions.

The Central Bank targets the desired level of Selic rate and controls over its fluctuations. The target is adjusted every 45 days during the meetings of the Monetary Policy Committee (COPOM).

The COPOM is composed of eight members from the Central Bank's board, and it has a central role in shaping the Brazilian monetary policy.

COPOM meetings last two days. On the first day, the members analyse economic indicators, inflation trends, exchange rates and global economic conditions.

The second day is dedicated to decision making and to evaluate whether to adjust the Selic rate.

At the end of the meeting, the decisions are published, and the breakdown of the votes is made public.

Figure 7 – Selic Rate management

3.8 Monetary Policy Transmission Channels

The Selic Rate affects the behaviour of economic variables such as prices and output, through five main transmission channels (Figure 8).

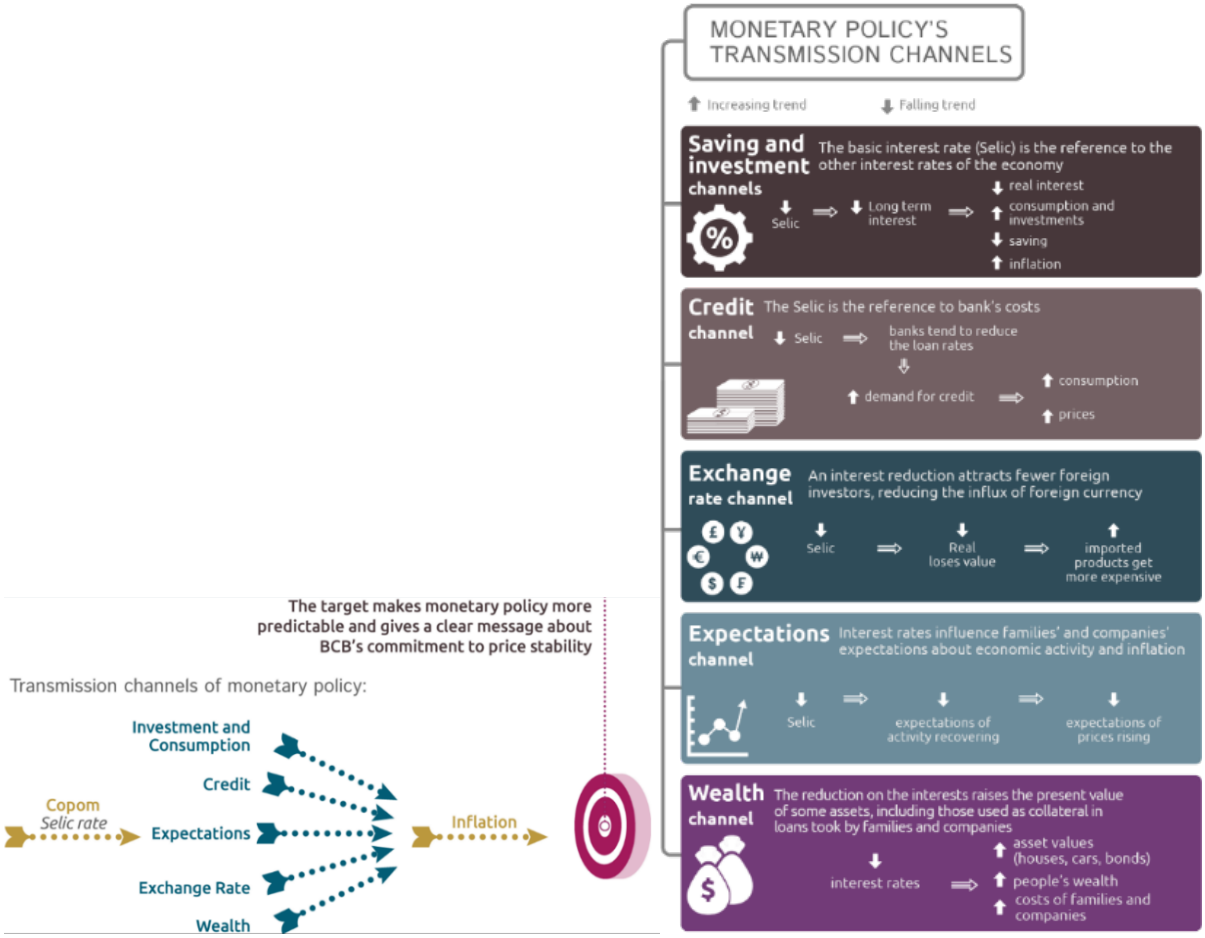

Figure 8 – Monetary policy transmission channels

3.8.1 Saving and Investment Channels

The most direct way in which monetary policy affects the economy is through the impact of the interest rate on consumption and investment decisions. When the Government lowers the Selic rate, banks respond by reducing their interest rates. The reduction in the Selic is used to stimulate economic activity, by promoting borrowing and spending. When inflation is rising and the economy is overheating, the Government should increase the Selic rate. This would make loans more costly and reduce consumption and price growth.

3.8.2 Exchange rate Channel

The Exchange Rate channel is particularly influential in economies that are more open to international trade. As the interest rate rises, the domestic currency appreciates. Such appreciation lowers the prices of imported goods when measured in local currencies, reducing net exports and ultimately smoothing inflation. Moreover, imported goods become cheaper, and this leads to a reduction of demand for local products and services. This drop in demand further lowers Inflation.

3.8.3 Wealth Channel

When the interest rate goes up, the economic activity slows down, and this results in a reduction of the financial assets' values, including those used as collaterals by families and companies. This reduces people's wealth and postpones spending and investment decisions, thus slowing the economic growth.

3.8.4 Credit Channel

Higher Selic rates increase the cost of borrowing, making credit less accessible and appealing. This contraction in availability of credit, together with the effects on collateral assets, can lead banks to further reduce credit to limit risks of insolvency and must be strictly monitored by central banks.

3.8.5 Expectations Channel

When the Central Bank adjusts the Selic Rate as a response to inflation, it signals its commitment to price stability. This guidance influences how people and companies plan consumption and investments, according to their expectations on future inflation. If the Central Bank has a solid credibility, it will be able to shape these expectations and therefore it will be able to control inflation. On the other hand, if its credibility is low, the Central Bank's ability to manage inflationary shocks will be weakened.

4. Methodology and Data

The quarterly data necessary to run the analysis of the backward-looking Taylor Rule was obtained from the FRED (Federal Reserve Economic Data, St Louis Fed) archives. The SELIC rate (Figure 9) was directly taken from the available data.

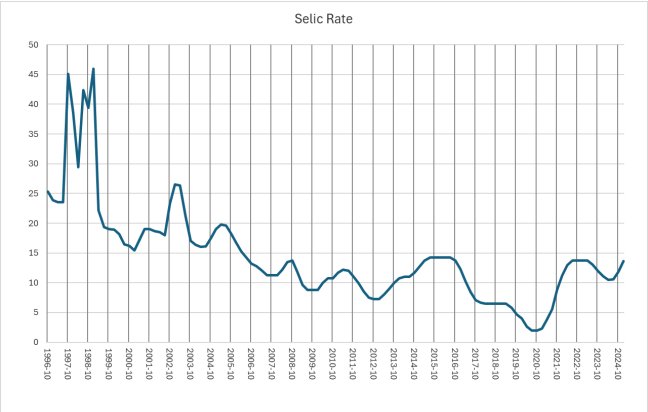

Figure 9 – Selic Rate

Inflation (Figure 11) was calculated on one year basis, taking price index values (Figure 10) as a reference.

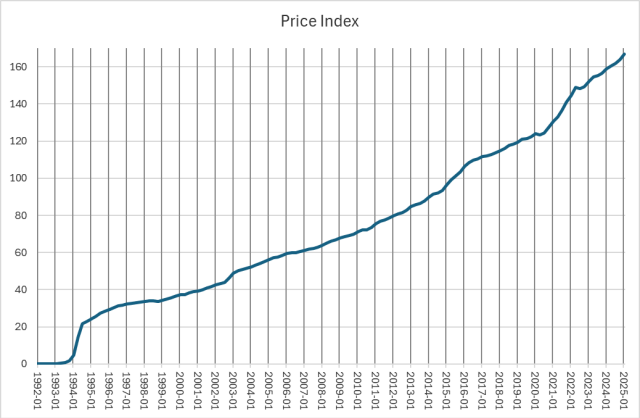

Figure 10 – Price Index

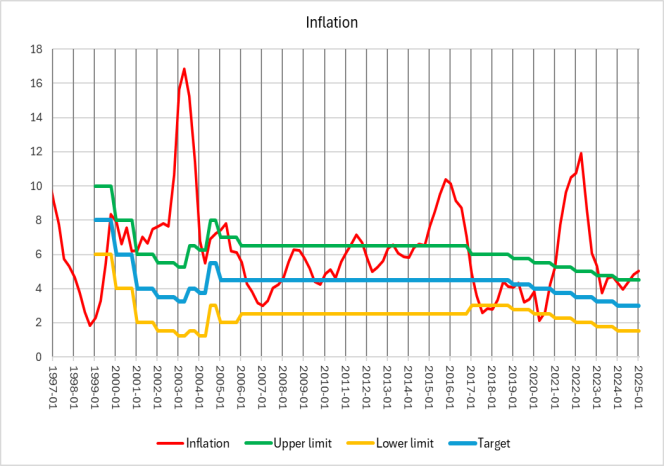

Figure 11 - Inflation

Real output data were the source to calculate the GDP growth (Figure 13) on one year basis and to estimate the Potential GDP (Figure 12).

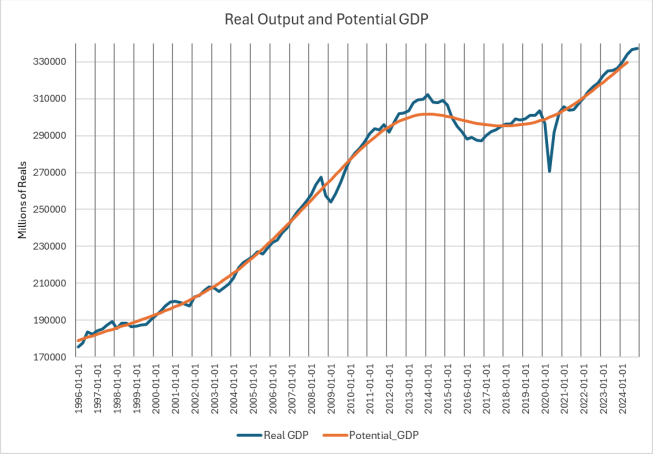

Figure 12 – Real Output and Potential GDP

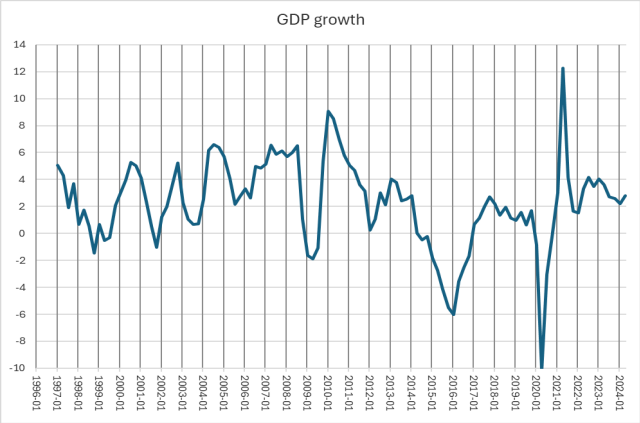

Figure 13 – GDP growth

In this work Phyton was used to estimate the potential output by applying a Hodrick-Prescott filter to the real GDP data.

The potential output (or Natural Gross Domestic Product) represents the highest value of output the economy can afford without generating any kind of upward or downward pressure on inflation. It is strictly related to unemployment; according to the Okun's law, when unemployment is below its natural level, it means that the economy is producing above its capacity (positive output gap), potentially leading to inflationary pressures.

The value of the output gap indicates whether the current economic conditions are likely to push inflation upward or downward.

However, the potential output cannot be directly observed, and thus the estimate of the output gap introduces a degree of uncertainty in the calculation.

There is no universally accepted method to calculate potential output, and central

banks may differ in their approaches.

Different statistical models can give different views of the conditions of the economy. For this reason, in order to better assess the risks associated to policy decisions, the Brazilian Monetary Policy Committee (COPOM) analyses a variety of estimation methods to evaluate a range of possible outcomes and compare different scenarios.

The output gap (Figure 14) is represented by the percentage difference between real GDP and potential output.

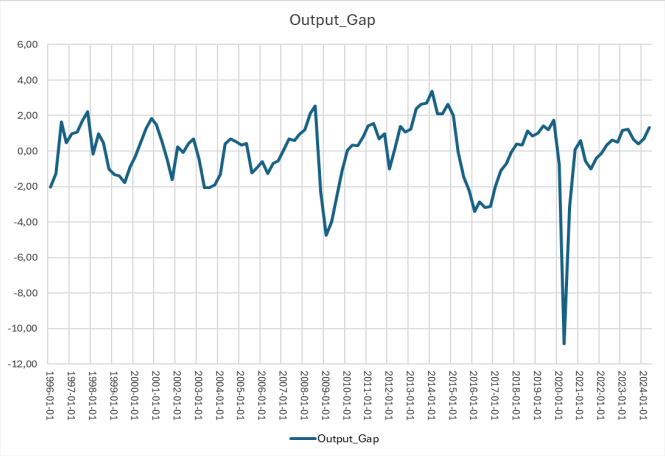

Figure 14 – Output Gap

5. Results

In this work, the collected data are divided into four different periods of analysis:
In the first period, from 1999 (when inflation targeting was adopted by the Brazilian Central Bank) up to 2003, a first reaction of the market to this new policy is analysed.
A second period, which ranges from 2004 to 2011, begins when the Real Plan was implemented and covers the first presidential mandate of Luiz Inácio Lula da Silva, including the 2008 Great Financial Crisis (GFC).
The third period, which goes from 2012 to 2018, represents one of the most critical moments in the Brazilian economy.
Finally, the fourth period, from 2019 up to 2025, goes through the pandemic crisis and ends looking at future expectations.
For each period analysed, the results are presented in a table and three graphs.
The table includes the coefficient ρ, β, ϒ and the reference values r*, π*, i* and resumes the average values, the standard errors and the test statistics obtained from the regression.
The first graph compares the Selic rate and the nominal interest rate obtained through the Taylor Rule.
The second graph shows the inflation rate and the BCB's target together with its upper and lower control thresholds.
The third graph compares the inflation rate and the real interest rate.
The results obtained are then discussed in light of the macroeconomic and political framework.
Whenever the deviations of the inflation rate exceeded the target thresholds, a brief summary of the contents of the open letters issued by the BCB is presented.

5.1 1999-2003 Analysis

	Average	Standad Error	Test Statistics
ρ	0,58	0,21	2,79
β	0,29	0,90	0,32
γ	0,05	3,22	0,02
r*	12,43	0,74	16,69
π*	6,24	25,63	0,24
i*	18,67	25,64	0,73

Table 1

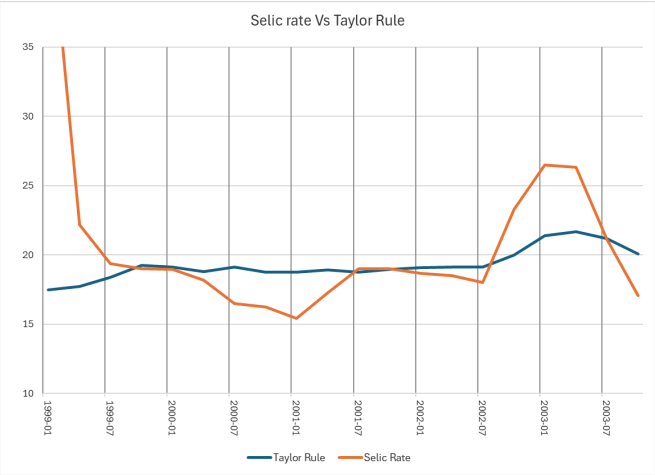

figure 15 – Selic Rate Vs Taylor Rule

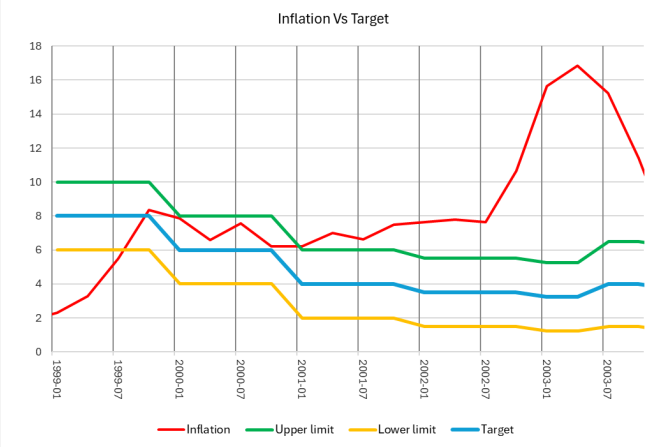

Figure 16 – Inflation Vs Target

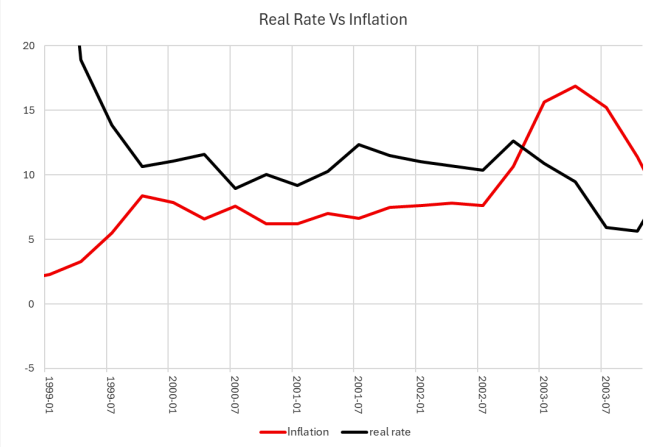

Figure 17 – Real Rate Vs Inflation

At the beginning of this period, a strong expansionary monetary policy led to a sensitive reduction of the Selic rate, together with an increase in inflation up to the target. Afterwards, the BCB gradually decreased the inflation target from 8% to 4%, but inflation remained at a steady value and exceeded the upper threshold at the beginning of 2001. The situation worsened even more in 2002, when an insufficiently tightening monetary policy boosted the increase in inflation over the two digits (Figure 16 and 17). We can in fact see that even when inflation started increasing, the real rate continued to decrease, failing to control inflationary pressure (Figure 17). This led to the issuing of open letters in 2001, 2002, and 2003.

In 2001, the deviation of inflation was mainly due to two external shocks: the Argentinian economic crisis and the September 11th terrorist attack to the World Trade Center in New York. This led to a depreciation of the Brazilian Real. Another factor that played a non-negligible role was the increase in electricity and gas costs. The letter also pointed out that residual inflationary inertia was still affecting the deviation of inflation from its target. The BCB concluded that if preventive actions hadn't been implemented, the situation could have been much worse.

In 2002, the central bank explained that inflation had surpassed the upper tolerance limit due to a combination of domestic and international factors. On the domestic side, there was a reduction in the demand for long term government securities and a trigger to positive inflation expectations; on the international side, there was a drop in foreign capital flows due to political instability with consequent depreciation of the Brazilian Real. The BCB concluded by indicating the contractionary monetary policy adopted to reduce inflation in the medium term.

In 2003, the Central Bank explained that inflation had surpassed the upper tolerance limit mainly due to the persistence of the pressures coming from the precedent year. However, the BCB pointed out that, starting from mid-2023, monetary and fiscal policies had successfully restored deflationary expectations bringing the inflation back to lower levels.

5.2 2004-2011 Analysis

	Average	Standad Error	Test Statistics
ρ	0,92	0,04	24,73
β	4,28	2,47	1,74
γ	5,38	2,86	1,88
r*	7,88	0,27	29,57
π*	6,12	5,45	1,12
i*	14,00	5,45	2,57

Table 2

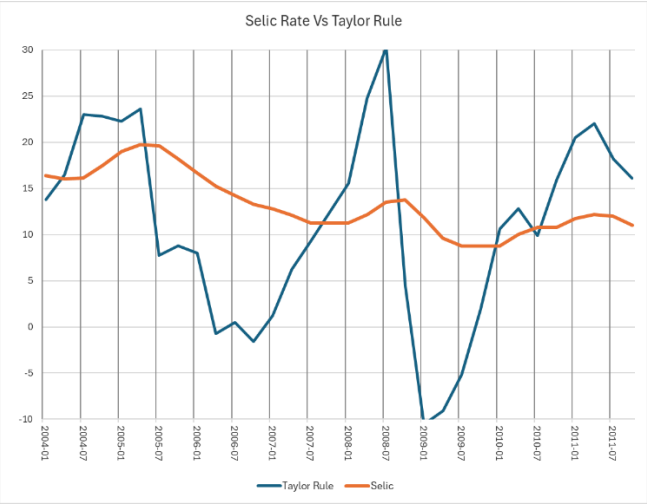

figure 18 – Selic Rate Vs Taylor Rule

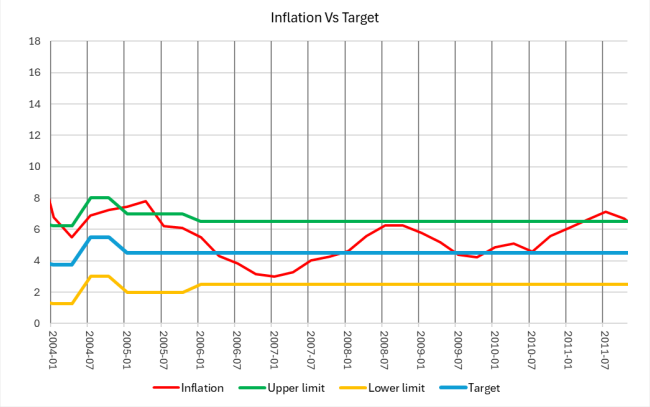

Figure 19 – Inflation Vs Target

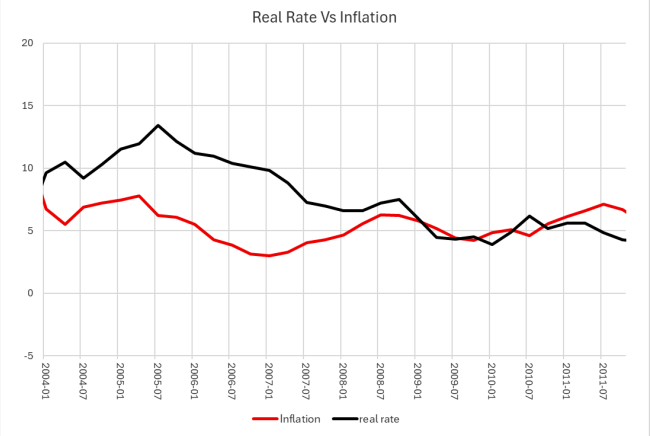

Figure 20 – Real Rate Vs Inflation

This period of analysis covers the presidential mandate of Luiz Iniacio Lula da Silva from 2003 to 2011. This period includes the Global Financial Crisis (GFC), during which a severe contraction of liquidity shocked the global financial markets. This was the result of the collapse of the U.S. housing market, which led to the bankruptcy of Lehman & Brothers in September 2008.

At the beginning of 2004, the implementation of a contractionary monetary policy completed the return of inflation within the targets. In 2009, the GFC caused a sharp reduction of the Brazilian GDP. β coefficient surged well over one; nonetheless, the monetary policy did not follow the sharp variations indicated by the Taylor rule (Figure 18 and 19). During this period, the monetary policy was mainly easing, causing a progressive reduction in the real rate, which led to an overall increase in inflation after 2007 (Figure 20).

5.3 2012-2018 Analysis

	Average	Standad Error	Test Statistics
ρ	0,89	0,04	22,92
β	3,58	1,32	2,71
γ	2,46	0,94	2,61
r*	4,15	0,15	26,85
π*	6,58	3,97	1,66
i*	10,73	3,97	2,70

Table 3

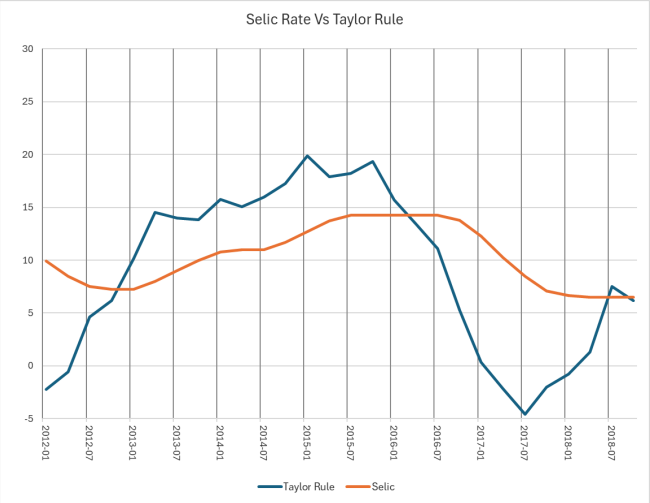

figure 21 – Selic Rate Vs Taylor Rule

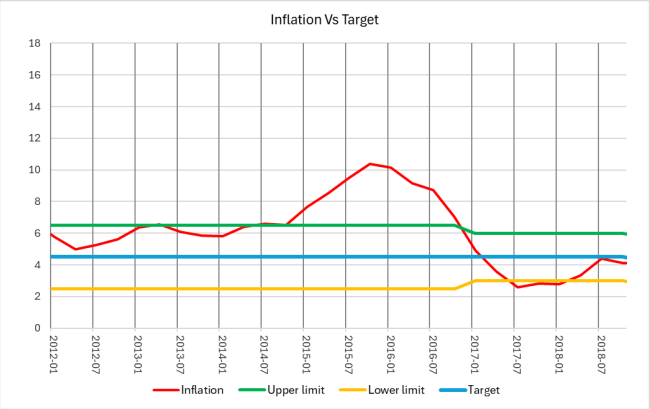

Figure 22 – Inflation Vs Target

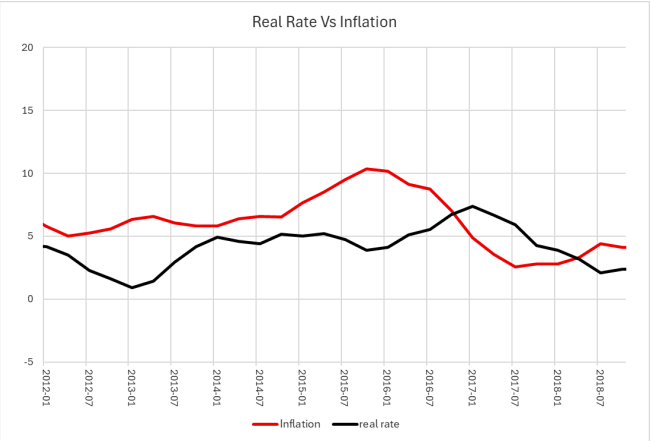

Figure 23 – Real Rate Vs Inflation

In November 2010, Alexandre Tombini was elected by President Dilma Rousseff as leader of the Central Bank of Brazil. Tombini declared that his primary objective was to keep inflation under control, aiming not to exceed the annual target of 4.5 %.
In order to boost the economy, he adopted a monetary easing policy, which reduced the interest rate from 12.5% to 7.5%. However, this coincided with a period of economic slowdown in Brazil, which spoiled the effectiveness of the monetary policy.
Between 2014 and 2016, due to a reduction in exports, weak government finances and political instability all happening at once, the Brazilian economy shrank. It was one of the worst economic periods that the country had seen in decades.
One major reason was the fact that the Government had tried to boost the economy through a mix of high public spending, tax breaks and cheap loans.
At the beginning, this policy seemed to be effective, but it quickly led to budget deficits and made investors loose trust in the economy.
At the same time, prices for the commodities that Brazil was exporting, like oil, iron, and soy, decreased significantly and this affected growth even more.
Politics also affected the situation negatively. A major corruption scandal, the Petrobras Corruption Scandal, caused political chaos, and led to President Dilma Rousseff's impeachment.
All these conditions occurring at once led businesses and consumers to reduce consumption and investments, harming the Brazilian economy for two years in a row.
The variation of the Selic rate over this period was again smoother compared to what was indicated by the Taylor rule. Inflation ranged in the upper level of the target until 2014, when the impact due to the financial crisis and a late response in monetary policy made the inflation peak up to 10% (Figure 21 and 22).
The situation improved when real rate started to increase after 2016, generating a strong reduction in inflation (Figure 23).
Afterwards, an expansionary monetary policy was required to avoid excessive disinflation. By the end of 2018 inflation was steadily anchored to its target.
This situation led to the issuing of Open Letters in 2015 and 2017

In 2015, the Central Bank explained that inflation had surpassed the upper tolerance limit mainly due to a significant increase in domestic prices and to a further depreciation of the Brazilian Real. The BCB outlined that the inflationary situation

was worsened because of fiscal uncertainty and market expectations, which negatively affected asset prices and inflation forecasts.

In 2017, the Central Bank explained that inflation had fallen below the lower tolerance limit due to a sharp reduction in food prices, which was driven by extraordinary conditions in agricultural production. Price reductions also affected industrial goods and services. The BCB indicated that the consistent expansionary monetary policy that had reduced the SELIC rate in 2017 was meant to mitigate the primary effect of these supply shocks, increasing inflation right back to the target.

5.4 2019-2024 Analysis

	Average	Standad Error	Test Statistics
ρ	0,95	0,03	32,89
β	7,22	3,93	1,84
γ	0,67	0,96	0,69
r*	2,87	0,41	7,01
π*	4,95	4,07	1,22
i*	7,82	4,09	1,91

Table 4

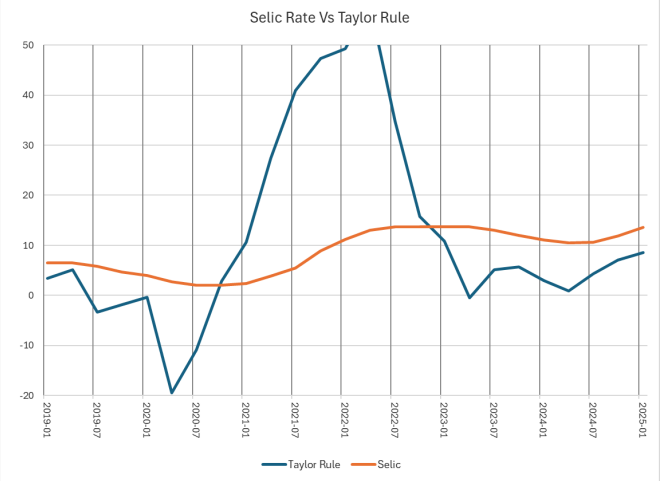

figure 24 – Selic Rate Vs Taylor Rule

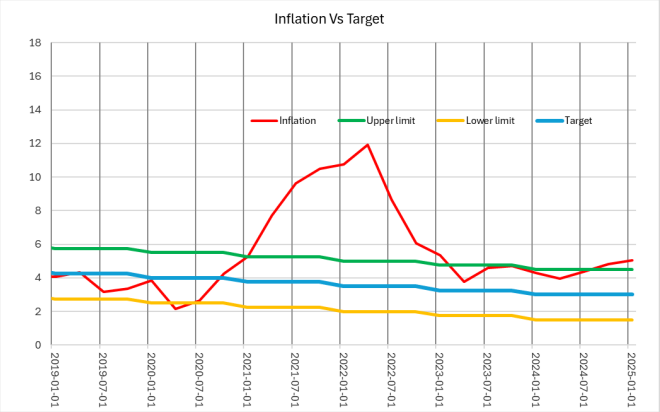

Figure 25 – Inflation Vs Target

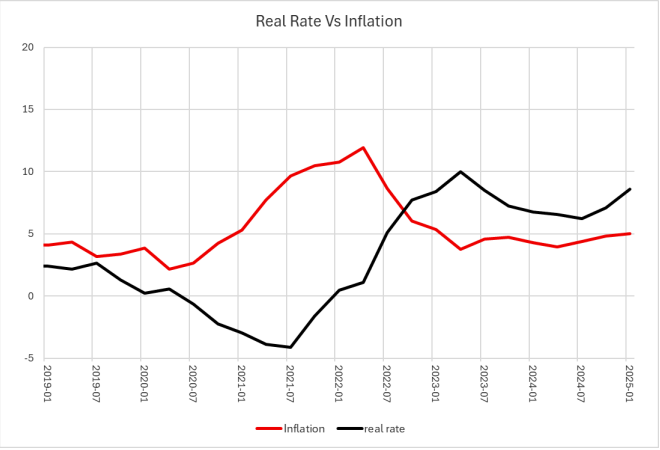

Figure 26 – Real Rate Vs Inflation

The last period of analysis is characterized by the COVID-19 pandemic crisis.
Just like the rest of the world, also Brazil was suddenly and heavily affected by the pandemic, which caused a peaking reduction in GDP in 2020-2021.
The response of the Selic rate was much smoother compared to what the Taylor Rule was indicating. The monetary policy failed to respond when Inflation started to increase in mid-2020, making the real rate drop to negative values (Figure 24 and 25).
This caused the inflation level to rise to 12%, until a contractionary monetary policy was able to increase the real rate and to reduce the inflationary pressure down to the upper side of the threshold limit (Figure 26).
This situation led to open letters being issued in 2021, 2022 and 2024.

In 2021, the Central Bank explained that inflation had surpassed the upper tolerance limit, mainly because of the pandemic crisis which caused a disruption in the supply chains and an increase in the demand for goods. This subsequently led to a reduction in the exchange rate and a marked increase in commodity prices. Moreover, there was a sharp increase in the domestic prices of fuel, electricity and bottled gas.

In 2022, the Central Bank explained that inflation had surpassed the upper tolerance limit because the economic and climate conditions had generated supply shocks which had raised food prices. The increasing prices of industrial goods were still related to post pandemic bottleneck effects. The reduction in inflation in the second part of the year was related to the reduction in prices of fuels and electricity, which benefited from tax incentives. A contribution to deflation was also given by an increase in the exchange rate and by a negative deviation of the output gap driven by a contractionary monetary policy.

In 2024, the Central Bank explained that inflation had surpassed the upper tolerance limit due to a combination of domestic and international factors. Consumption and wages had increased, generating price pressure. Fuel costs had increased due to taxes. Health services costs had also increased. The depreciation of the Brazilian Real increased prices for imported goods.
The BCB finally adopted a contractionary monetary policy to bring inflation back within the tolerance band.

5.5 1999-2025 Analysis

	Average	Standad Error	Test Statistics
ρ	0,80	0,04	21,00
β	1,71	0,55	3,10
γ	1,10	0,66	1,67
r*	6,57	0,53	12,40
π*	7,67	7,62	1,01
i*	14,24	7,64	1,86

Table 5

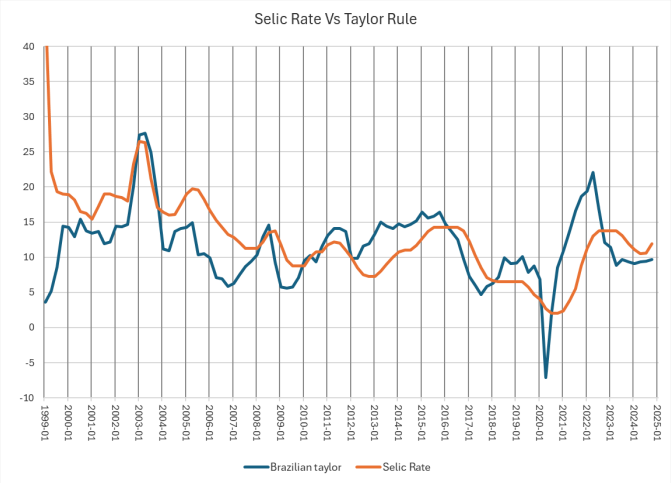

figure 27 – Selic Rate Vs Taylor Rule

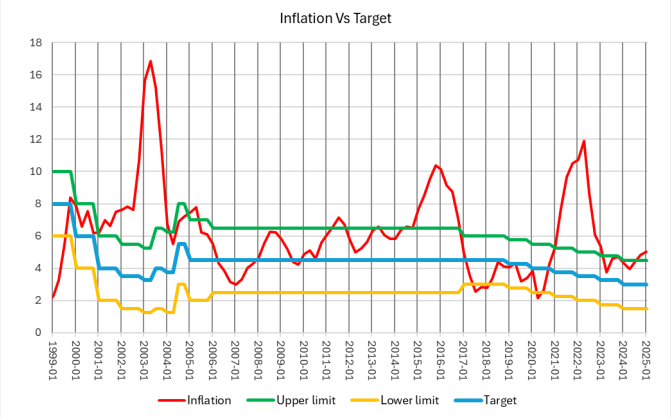

Figure 28 – Inflation Vs Target

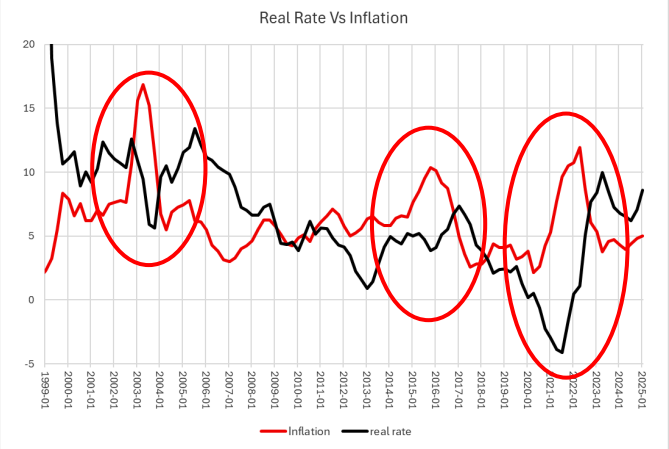

Figure 29 – Real Rate Vs Inflation

The final analysis is performed on the overall period which goes from the establishment of the Inflation targeting regime to nowadays (Figure 27 and 28). This overall picture is particularly interesting if we compare the time path of the inflation rate to the one of the real interest rate (Figure 29).
A stabilizing monetary policy ($\beta>1$ and $\gamma>0$) should generate a real rate increase to react to an increase in inflation, and vice versa. By observing how the real interest rate varied in relation with inflation movements, this global analysis highlights the periods in which the monetary policy was not able to respond to the general stabilization rule.

6. “Hybrid” Taylor Rule Analysis

This analysis has been conducted to assess how a “hybrid” Taylor Rule approach could match the nominal interest rate (SELIC) during the observed periods (Figures from 30 to 34).

In this approach, the ‘classic’ parameters β = 1.5 and ϒ = 0.5 have been used, while the calculated values for π* and r* (classically set at 2) have been retained in order to have a more reliable offset position of the curve.

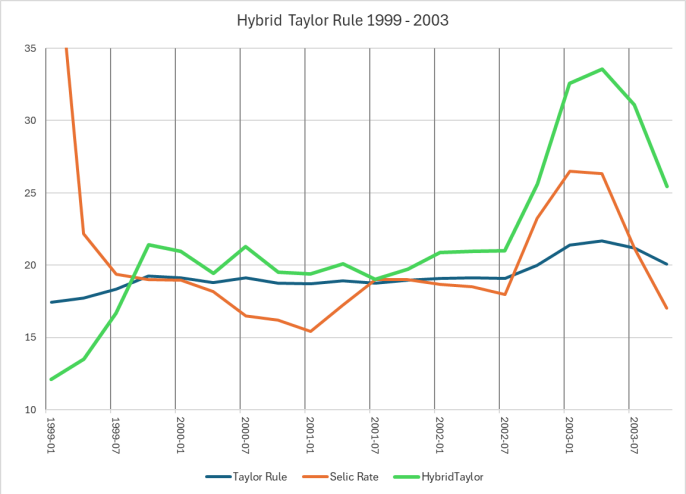

Figure 30 – Hybrid Taylor Rule 1999-2003

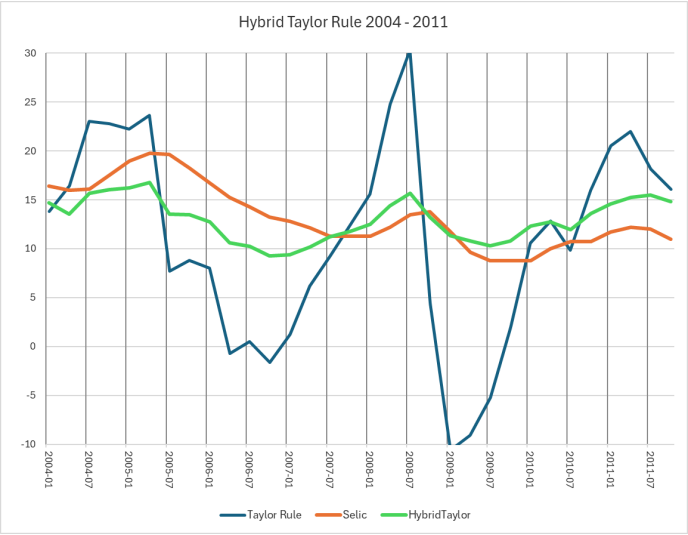

Figure 31 – Hybrid Taylor Rule 2004-2011

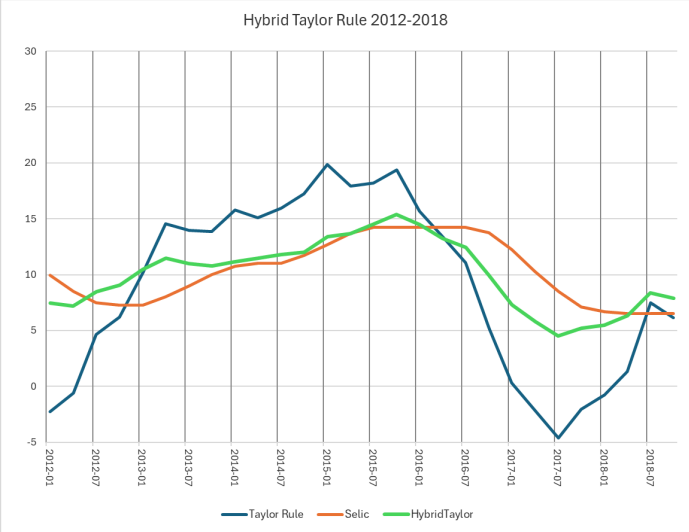

Figure 32 – Hybrid Taylor Rule 2012-2018

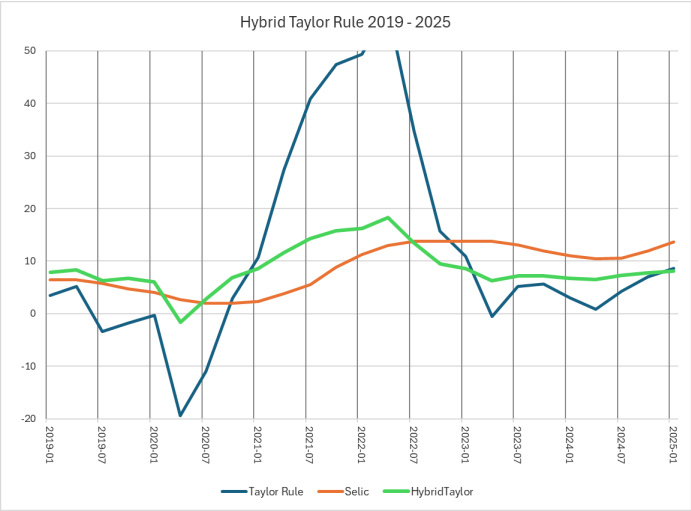

Figure 33 – Hybrid Taylor Rule 2019-2024

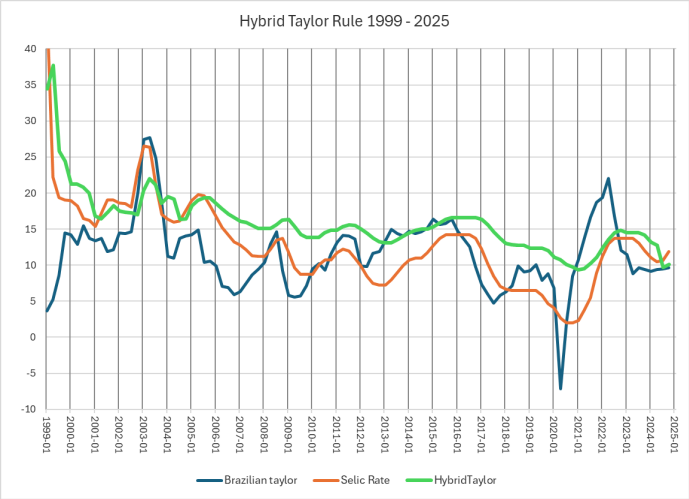

Figure 34 – Hybrid Taylor Rule 1999-2025

7. Discussion of the Results

The following tables summarize the average values (Table 6) and the test statistics (Table 7) of the regression parameters obtained in the different periods of the analysis.

	1996-2003	2004-2011	2012-2018	2019-2024
ρ	0,58	0,92	0,89	0,95
β	0,29	4,28	3,58	7,22
Y	0,05	5,38	2,46	0,67
r*	12,43	7,88	4,15	2,87
π*	6,24	6,12	6,58	4,95
i*	18,67	14,00	10,73	7,82

Table 6 – Average values

	1996-2003	2004-2011	2012-2018	2019-2024
ρ	2,79	24,73	22,92	32,89
β	0,32	1,74	2,71	1,84
Y	0,02	1,88	2,61	0,69
r*	16,69	29,57	26,85	7,01
π*	0,24	1,12	1,66	1,22
i*	0,73	2,57	2,70	1,91

Table 7 – Test statistics

The trend of the average values over time are represented in the following graph (Figure 35)

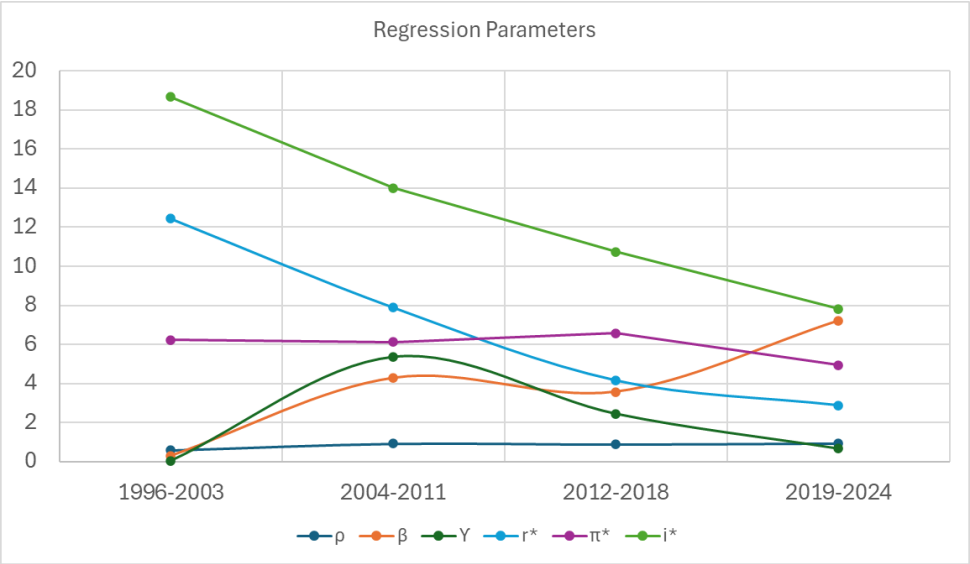

Figure 35 – Regression parameters average values in the four periods of analysis

The first period of the analysis covers the same stretch of time as the one that R. De Losso da Silveira Bueno considered in their work. The results of the two analyses agree on the β coefficient, both indicating a value lower than one.

This means that the prescriptions obtained from the Taylor Rule are much smoother than what the BCB has set for the Selic Rate.

The findings of this study with respect to the 2004-2020 period are partially consistent with those of the paper *'The Central Bank of Brazil's Time-Varying Taylor Rule'* by Filipe Gropelli Carvalho and Marcelo Kfoury Muinhos, showing values of β well above one in the earlier and later periods of the analysis, but not the same significant drop in the central period. On the other hand, the results obtained for π* are more in line with those in the paper: the values of the implicit inflation target obtained in the four periods analyzed are in fact in both cases within the target set by the BCB.

The results obtained are also consistent with the findings in the paper *"Equilibrium Interest Rate in Brazil, Convergence at Last"*, by M.K. Muinhos, M. Fonseca, E.C.O. Schulz . In the three periods ranging from 2004 to 2024, the estimated values of r* are 7,88, 4,15, 2,53, with the last value close to the convergence level of the study referenced.

The values of the parameters obtained with the regressions show good consistency, with significance values mostly close to or above 95%, making them statistically meaningful.

The result obtained through the "Hybrid" Taylor Rule analysis is particularly interesting. In most scenarios, the matching of this approach with the Selic Rate is more effective than the one obtained with the Taylor Rule based on the Brazilian economic data.
This result suggests that it would be interesting to try to apply this hybrid backward looking method to other countries, where inflation targeting has been adopted after periods of economic instability. The goal of such an analysis is to see whether and how this method could match with the monetary policies implemented by local Central Banks.

Interesting benchmark countries could be Chile, South Africa, Mexico, Turkey, Indonesia, Israel, New Zealand (which was the first country to adopt the inflation targeting in 1989), Canada, the United Kingdom, Poland, Hungary and the Czech Republic.

8. Inflation Targeting – Present Situation

Nowadays Brazil is still operating under an inflation-targeting regime.
The inflation target is set at 3.00% with a tolerance band of ±1.5%.
However, the value of inflation has remained above the upper limit of 4.5% for several consecutive months, going over 5% in February 2025 and reaching 5,5% in April 2025 (Figure 36)
This situation called for the issuing of an open letter in June 2025.
The present forecast sets average inflation at 4.4% for 2026 and 4.2% for 2027, indicating a gap between expectations and the official target.
This divergence is partly due to fiscal uncertainty. There are concerns about the sustainability of public spending and about the control of mandatory expenditures.
On the other hand, the economy is showing a positive reaction.
GDP growth for 2025 has been adjusted upward to 2.1%, supported by the labour market and by a good level of consumption.
This resilience has made it harder for monetary tightening policies to reduce the demand with the aim to lower inflation.
The BCB is responding to this situation by raising the Selic rate up to 15% (Figure 37). This represents the highest level in nearly two decades.
The intention of BCB is to maintain this rate for an extended period until the desired disinflation effect is reached.
Despite these efforts, inflationary inertia remains a concern.
Factors such as currency depreciation, climate-related food price shocks, and commodity volatility continue to exert pressure on prices.
The Central Bank’s credibility and transparency remain crucial aspects to manage expectations and guide inflation back to target over the medium term.

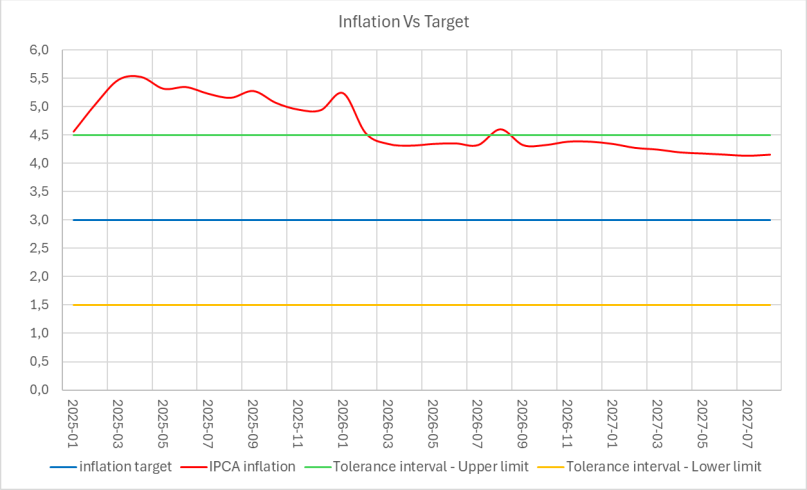

Figure 36 – Expected Inflation Rate and Inflation target up to 2027

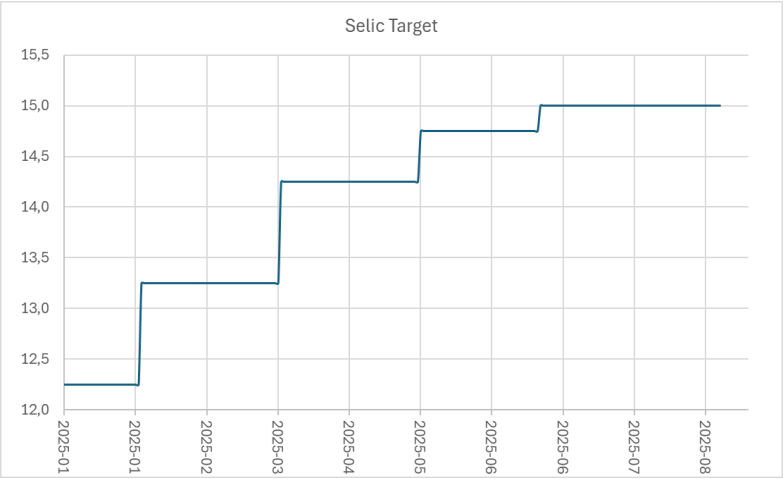

Figure 37 – Selic Rate adjustments in 2025

The analysis of the period 2019–2025 shows that the indications of both the Brazilian and the hybrid Taylor Rule are in line with the actions of the BCB, suggesting that the correct strategy is to keep a high value for the SELIC rate.

9. Conclusion

This study has investigated the role of the Taylor Rule as a benchmark for the Brazilian monetary policy between 1999 and 2025.

The empirical analysis shows β coefficients which are mostly above unity, despite the Central Bank of Brazil‘s inflation-targeting policy only partially conforming to the Taylor Rule prescriptions.

During various periods of instability, including the 2002 currency crisis, the Global Financial Crisis of 2008, the 2014-2016 domestic recession, and the COVID-19 pandemic, the policy of the BCB diverged from the prescriptions of the rule, often smoothing Selic rate adjustments.

In spite of these deviations, inflation expectations remained anchored, and implicit inflation targets largely stayed within the official tolerance bands, which proved the overall credibility of the monetary institutions.

The main contributions of this work can be summarized as follows:

First, the extension of the analysis up to 2025, incorporating the pandemic shock and subsequent inflationary pressures and thus offering an updated assessment able to complement earlier studies limited to pre-2020 data.

Secondly, the introduction of a “hybrid” Taylor Rule specification, combining country-specific estimates of the implicit inflation target and the equilibrium real interest rate with the classic Taylor coefficients $\beta=1,5$ and $\gamma=0,5$.

This approach yields results which are more consistent with the decisions made by the BCB, suggesting that such hybrid models may embody both theoretical prescriptions and real world flexibility in managing shocks.

In determining the right policy to react to recurrent external and domestic shocks, the findings underline that in emerging economies such as Brazil’s, strict adherence to rule-based prescriptions are often neither feasible nor desirable.

Hybrid formulations of the Taylor Rule can enhance transparency, strengthen expectations, and serve as a credible reference in decision-making.

It would be interesting to run comparative studies by applying this hybrid methodology to other inflation-targeting economies to further evaluate its robustness and policy relevance.

Future research could also widen this study by employing models with time-varying parameters to capture possible shifts in the policy of the BCB.

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Appendix A – Regression Analysis

$$i_t = (1-\rho)[r^* - (\beta-1)\,\pi^*] + (1-\rho)\beta\pi_t + (1-\rho)\gamma(y_t - y^*) + \rho r_{t-1}, \quad (6)$$

where: $(1-\rho)[r^* - (\beta-1)\pi^*] = cost,$ (7)

$(1-\rho)\beta\pi_t = \beta',$ (8)

$(1-\rho)\gamma = \gamma',$ (9)

Regression Statistics 1996-2003	
R	0,67
R squared	0,44
Adj. R squared	0,34
Root MSE	5,41
Obs	20

	gdl	SQ	MQ	F	F Signif.
Model	3	372,76	124,25	4,25	0,02
Residual	16	468,17	29,26		
Total	19	840,93			

	Coef	Std Err.	Test Statistics	P value	95% Conf. Interval	
Cost	7,08	6,34	1,12	0,28	-6,36	20,52
β'	0,12	0,38	0,33	0,75	-0,67	0,92
γ'	0,02	1,35	0,02	0,99	-2,85	2,89
ρ	0,58	0,21	2,79	0,01	0,14	1,02

Regression Statistics 2004-2011	
R	0,98
R squared	0,97
Adj. R squared	0,96
Root MSE	0,61
Obs	32

	gdl	SQ	MQ	F	F Signif.
Model	3	314,42	104,81	278,57	0,00
Residual	28	10,53	0,38		
Total	31	324,96			

	Coef	Std Err.	Test Statistics	P value	95% Conf. Interval	
Cost	-0,92	0,56	-1,64	0,11	-2,06	0,23
β'	0,32	0,09	3,47	0,00	0,13	0,51
γ'	0,40	0,07	5,42	0,00	0,25	0,56
ρ	0,92	0,04	24,73	0,00	0,85	1,00

Regression Statistics 2012-2018	
R	0,99
R squared	0,99
Adj. R squared	0,99
Root MSE	0,35
Obs	28

	gdl	SQ	MQ	F	F Signif.
Model	3	218,17	72,72	604,50	0,00
Residual	24	2,89	0,12		
Total	27	221,06			

	Coef	Std Err.	Test Statistics	P value	95% Conf. Interval	
Cost	-1,41	0,32	-4,43	0,00	-2,06	-0,75
β'	0,39	0,04	9,43	0,00	0,31	0,48
γ'	0,27	0,04	6,76	0,00	0,19	0,35
ρ	0,89	0,04	22,92	0,00	0,81	0,97

Regression Statistics 2019-2025	
R	0,99
R squared	0,98
Adj. R squared	0,98
Root MSE	0,57
Obs	24

	gdl	SQ	MQ	F	F Signif.
Model	3	427,56	142,52	431,49	0,00
Residual	20	6,61	0,33		
Total	23	434,17			

	Coef	Std Err.	Test Statistics	P value	95% Conf. Interval	
Cost	-1,51	0,35	-4,31	0,00	-2,24	-0,78
β'	0,39	0,04	8,88	0,00	0,30	0,48
γ'	0,04	0,05	0,75	0,46	-0,06	0,14
ρ	0,95	0,03	32,89	0,00	0,89	1,01

Regression Statistics 1999-2025	
R	0,92
R squared	0,85
Adj. R squared	0,84
Root MSE	2,41
Obs	102

	gdl	SQ	MQ	F	F Signif.
Model	3	3171,84	1057,28	181,62	0,00
Residual	98	570,49	5,82		
Total	101	3742,33			

	Coef	Std Err.	Test Statistics	P value	95% Conf. Interval	
Cost	0,23	0,69	0,33	0,74	-1,14	1,59
β'	0,35	0,09	3,79	0,00	0,17	0,53
γ'	0,22	0,13	1,76	0,08	-0,03	0,48
ρ	0,80	0,04	21,00	0,00	0,72	0,87

Appendix B - Statistical Propagation of Uncertainty

$$\frac{\delta\beta}{\delta\beta'} = \frac{1}{1-\rho}, \qquad \frac{\delta\beta}{\delta\rho} = -1(-1)(1-\rho)^{-2}\beta' = \frac{\beta'}{(1-\rho)^2},$$

$$\sigma_\beta = \sqrt{\left(\frac{\delta\beta}{\delta\beta'}\right)^2 VAR\beta' + \left(\frac{\delta\beta}{\delta\rho}\right)^2 VAR\rho} = \sqrt{\left(\frac{1}{1-\rho}\right)^2 VAR\beta' + \left[\frac{\beta'}{(1-\rho)^2}\right]^2 VAR\rho},$$

$$\frac{\delta\gamma}{\delta\rho} = \frac{1}{1-\rho}, \qquad \frac{\delta\gamma}{\delta\rho} = -1(-1)(1-\rho)^{-2}\gamma' = \frac{\gamma'}{(1-\rho)^2},$$

$$\sigma_\gamma = \sqrt{\left(\frac{\delta\gamma}{\delta\gamma'}\right)^2 VAR\gamma' + \left(\frac{\delta\gamma}{\delta\rho}\right)^2 VAR\rho} = \sqrt{\left(\frac{1}{1-\rho}\right)^2 VAR\gamma' + \left[\frac{\gamma'}{(1-\rho)^2}\right]^2 VAR\rho},$$

$$\frac{\delta\pi^*}{\delta\beta} = 1(-1)(\beta-1)^{-2}r^* - \left[-1(1)(\beta-1)^{-2}\frac{cost}{1-\rho}\right] = -\frac{r^*}{(\beta-1)^2} - \left[-\frac{cost}{(\beta-1)^2(1-\rho)}\right] =$$

$$= \frac{1}{(\beta-1)^2}\left[\frac{cost}{(1-\rho)^2} - r^*\right],$$

$$\frac{\delta\pi^*}{\delta\rho} = 0 - \left[-1(-1)(1-\rho)^{-2}\frac{cost}{(\beta-1)}\right] = -\frac{cost}{(1-\rho)^2(\beta-1)},$$

$$\frac{\delta\pi}{\delta cost} = 0 - \frac{1}{(\beta-1)(1-\rho)},$$

$$\sigma_{\pi^*} = \sqrt{\frac{1}{(\beta-1)^4}\left[\frac{cost}{1-\rho} - r^*\right]^2 VAR\beta + \left[\frac{cost}{\beta-1}\frac{1}{(1-\rho)^2}\right]^2 VAR\rho + \left(\frac{1}{(\beta-1)(1-\rho)}\right)^2 VARcost}.$$