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Fiscal Multipliers in a Liquidity Trap

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ABSTRACT

In this paper we analyze fiscal policy effectiveness when the economy experiments a shock which lowers the limit of the debt agents can borrow. Some households must "deleverage" to respect the new limit and this depresses the aggregate demand. If the shock is big enough, the nominal rate can reach the zero level, and central bank cannot further reduce it to give stimulus to the demand: thus the economy falls in a liquidity trap, characterized by output drop and deflation.

In this situation, we show that fiscal policy can be very effective in boosting GDP and price level. In particular, big multipliers emerge if the government increases public expenditure or reduces consumption tax rate.

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1. Introduction

The recent economic crisis has given birth to a wide debate about the effectiveness of fiscal policy. The discussion began in the USA in 2007, when former President George W. Bush signed \$ 158 billion in tax cuts in order to stop the first symptoms of this disease. One year later Obama settled at the Whitehouse and had to face one of the biggest economic crises of capitalism. In 2009 he pushed \$789 billion into the economy, followed by other expansionary fiscal policies still applied to date. Meanwhile, the crisis spread through the European economies and western policy-makers started to think how to boost GDP and growth. However, fiscal programs of European economies have been less expansionary than American ones, because of fiscal constraints set by the Maastricht Treaty. Now some European countries, such as Italy and Greece, are implementing restrictive policies to reduce the high burden of their respective public debts.

On the other side, central banks have set the policy rate close to zero in order to restore the credit market and help economic recovery.

Let's now have a look at how the problem is analyzed in macroeconomic literature.

The multiplier of a fiscal variable measures the effect of a unit increase of that fiscal variable on the GDP. Therefore the main issue is estimating the size of the government spending and tax multipliers, namely how much the GDP increases after a rise in government expenditure or a cut in taxation.

The problem was born in 1929, when the entire world experienced the so-called "Great Depression". British economist John Maynard Keynes argued in his "General Theory of Employment Interest and Money" (1936) that in the short run prices are sticky, slow to change. If prices are fully flexible, they can immediately decrease after a negative demand shock: fiscal policies are not necessary, the economy automatically returns to the natural level. But if prices are sticky, a

variation in the aggregate demand may not be compensated by a variation in the price level. Therefore, according to Keynes, when output is below the potential, policy-makers have to increase government expenditure to boost aggregate demand: the rise in the output will be higher than the increase in government spending, namely the multiplier is above one. The last result is strongly affected by the assumption that private consumption positively depends on current available income: a rise in government spending increases the output, which increases current income and, in turn, this induces households to consume more.

However, the Keynes model, present in each undergraduate macroeconomics book, is now considered old-fashioned with respect to new economic theories.

The main criticism to Keynes is that in his model households do not have intertemporal budget constraints: according to some economists, consumption does not depend only on current income but also on future one, because households like to smooth consumption over time. One of the implications is that if policy-makers reduce taxes today by issuing bonds, rational households know these bonds will be paid back with an increase in future taxation. This way they do not consume the higher available incomes but, rather, save them, because they know in the future they will have lower incomes. This process is called "Ricardian Equivalence"¹ and shows that households internalize the government budget constraint, vanishing any Keynesian effect of fiscal expansion by reducing their own consumption.

Starting from the 1980s, economists tried to build up macroeconomic models taking into account rational expectations; these kinds of models are microfounded, because they are based on microeconomic assumptions: there are a large number of infinite-lived households that maximize utility from consumption and leisure subject to an intertemporal budget constraint, and an infinite number of firms with access to identical technology, subject to shifts. RBC (Real Business Cycle) and DSGE (Dynamic Stochastic General Equilibrium) models belong to this class.

¹ English economist Ricardo firstly proposed this idea, in the early nineteenth century. Barro (1974) developed "Ricardian Equivalence", giving it a theoretical foundation.

The main difference between RBC and DSGE lies in the level of flexibility of wage and price, and in the degree of competition in markets.

The multiplier of government spending in an RBC^2 model is often below the unity for consumption crowding out: fiscal expansion induces a negative wealth effect, because consumers expect a rise in future taxation and, as a result, they consume less and work more.

In a DSGE framework, prices or wages are sticky; as a consequence, fiscal policy could be a good instrument to boost the aggregate demand after a negative shock. In these models the output response, after a fiscal expansion will also depend on monetary rules; an increase in GDP could be totally offset by a rise in the policy interest rate, if central bank applies an aggressive monetary rule against variation in output and inflation.

As said before, in the western economic world central bankers have set interest rates close to zero. This situation has very important implications on fiscal policy effectiveness, considering that nominal interest rates has a zero lower bound (henceforth ZLB). Eggertsson and Woodford (2003) take into account ZLB, finding multipliers well above the unity if ZLB is binding. High multipliers emerge from the works by Christiano (2004), Christiano, Echenbaum and Rebelo (2009), Eggertsson (2010a) and Woodford (2010), all considering ZLB.

Economists call "liquidity trap" any situation in which central banks are unable to modify the interest rates. This can occur when nominal interest rates are close to zero: monetary policy loses effectiveness as it can no longer lower nominal interest rates and provide a good stimulus; economy is in a trap, as any injection of liquidity has no effects on nominal rates.

In a liquidity trap situation, the economy is likely to be characterized by output below the natural level, high unemployment rate and deflation (otherwise, why did central banks reduce nominal rates so strongly?); households do not want to

² See Baxter and King (1993) for RBC model.

spend their liquidity and they prefer to save: indeed, the real interest rate is high, as prices are expected to drop due to economic difficulties; firms reduce production and prices, further increasing recession and deflation. Correct use of fiscal instruments can solve this type of crisis.

But why can "a zero interest rate" make fiscal policy more effective?

Suppose that the short-term interest rate wished by central bank is negative: monetary policy-makers set it at zero because of ZLB. In this situation, after fiscal expansion the wished interest rate may increase but remains negative; so central bankers leave the policy target at zero. It is also probable that this expansion increases the inflation expectation. As a result, the real interest rate decreases and this reduces the desire to save, thus increasing consumption and output.

In particular, Eggertsson (2010a) shows the aggregate demand can be upward sloping when ZLB is binding, namely it is positively affected by price level. He considers an economy hit by a shock which reduces both output and prices, and compels central bank to set the nominal interest rate at zero. The shock can persist in the following period with a given probability. In this situation, monetary policy is not able to offset deflationary pressure by cutting the nominal rate. Therefore the higher current inflation, the higher expected inflation³, the higher the output, because larger is the reduction of real interest rate. As pointed out by Eggertsson, in this special circumstance a positive supply shock (e.g. a reduction of labor tax rate) causes the counterintuitive effect of a decrease in the output, because it lowers inflation, thus raising the real rate. Conversely, a government spending expansion is very effective, when it succeeds in increasing inflation expectations.

A central bank constrained by ZLB is not the only situation where it is possible to obtain large multipliers. Some economists tried to break down Ricardian Equivalence: indeed, empirical evidence shows that there is quite high a correlation between consumption and current income.

³ He assumes that when the shock arrives, expected inflation positively depends on current inflation: $\pi_{t+1}^e = \mu \pi_t$, where μ is the probability that the shock will persist in the following period.

Campbell and Mankiw (1989) argue that models with homogeneity of infinitelylived consumers maximizing an intertemporal utility function are not a good instrument to analyze macroeconomic problems. They think aggregate consumption has to be viewed as generated by two groups of consumers, one consuming their permanent income and the other consuming their current income⁴.

Mankiw (2000) develops this topic, analyzing the reasons allowing a fraction of household to consume its current income without saving (he calls them "rule of thumb" consumers⁵). He claims some households may not be rational or weigh their current income too heavily when looking ahead to their future income. Another explanation could be the fact that some households cannot enter the financial market, as they face binding borrowing constraints. This framework with heterogeneity of consumers changes fiscal policy effects: a tax cut increases the current income of "rule of thumb" consumers, thus they can consume more.

Galí, López-Salido and Vallés (2005, henceforth GLSV) add "non-Ricardian consumers" to a DSGE framework. In their model, persistent expansion in government spending causes an increase in consumption. The mechanism is the following: non-Ricardian agents insulate part of aggregate consumption from the negative wealth effect generated by higher current and future taxes financing fiscal expansion; household current income is higher, since real wages and hours of work increase, generating a growth of consumption by non-Ricardians.

Another important contribution comes from the work of Forni, Monteforte and Sessa (2007). They estimate a DSGE model of the Euro Area, including two types of agents⁶ likewise GLSV, using a rich description of fiscal policy; in particular they use distortionary taxes instead of lump-sum taxes. Their results show small and short effects of a government spending expansion on consumption. On the

⁴ They estimate this kind of agent as half the population.

⁵ Literature calls this kind of consumer also as "non-Ricardian" or "hand to mouth" consumer.

⁶ They estimate the share of non-Ricardian agents at about one third.

revenue side, they find bigger effects on consumption and output, after a reduction of labor income and consumption tax.

Agents' heterogeneity was added by Galardo (2010) in an economy characterized by a liquidity trap. In this model, fiscal expansion, aimed to stimulate the aggregate demand after the shock, is fully financed with a lump-sum tax. Here the higher the share of "rule of thumb" consumers, the lower the positive impact of the fiscal policy, and the multiplier can be also negative. On the contrary, without considering non-Ricardian consumers, the multiplier is well above one. Galardo obtains this result, because the rise in the lump-sum taxes, needed to finance the increase in government expenditure, reduces the current income of non-Ricardians, who have to decrease their consumption.

Eventually we deal about Eggertsson and Krugman work (2012, EK henceforth). Like Galardo (2010), they break down the Ricardian Equivalence with heterogeneity of agents, in the context of a monetary policy constrained by ZLB. However, their model is quite different from previous literature. They consider an economy with two kinds of agents, different in their rate of time preference: impatient individuals borrow from patient ones and both individuals must respect an exogenous debt limit. This economy is subject to a shock, which lowers the debt limit: the "borrowers" have to deleverage to satisfy the new limit; in this way, the natural interest rate (the benchmark in the Taylor rule) goes down because of the fall in loans demand. If the shock is large enough, the natural interest rate becomes negative, and central bank must set the policy nominal rate at zero. Moreover, the shock causes a price level drop, which increases the burden of deleveraging and thus decreases the output. As pointed out by the authors, in this situation the role of fiscal policy is sustaining output by increasing spending. Since fiscal expansion is financed with a lump-sum tax on "patient individuals", it will lead to increased spending of impatient ones, whose consumption is a positive function of the current income. Also in this work the keys to obtain high multipliers are the breakdown of Ricardian Equivalence and monetary policy constrained by ZLB.

In this paper we analyze EK work, enriching the model with distortionary taxation: households have to pay taxes on consumption and labor income; firms have to pay taxes on profits. Following Benigno (2012) we make some modifications to EK model: we specify the utility functions and introduce a Cobb-Douglas production function. We notably focus on the case of a big deleveraging shock which generates a liquidity trap and a recession, studying the effectiveness of fiscal policy instruments in this situation. We calibrate the model in order to calculate fiscal multipliers in this economy with heterogeneity of consumers and evaluate which fiscal policies can be more effective. Finally, we analyze an extension of the model in which central bank is willing to lead prices to the preshock level: in this case also monetary policy can be effective, if it succeeds in increasing inflation expectations.

The paper is organized as follows: Chapter 2 and Chapter 3 describe the behaviors of participants in this economy. Chapter 4 deals about the market clearing conditions and Chapter 5 illustrates a linear approximation of the model. Chapter 6 explains in detail what we intend for "deleveraging shock"; and Chapter 7, the core of the paper, calibrates the model to compute the value of fiscal multipliers after the shock. Chapter 8 analyzes possible monetary policies; and Chapter 9 concludes the work.

2. The Microfoundations of the Model

This chapter analyses the microfoundations of the model, namely the behaviours of households and firms.

The model consists of: two types of consumers, a continuum of firms producing differentiated goods, a central bank fixing the nominal interest rate, and the government deciding fiscal variables. Households can be savers ("s types"), or borrowers ("b types"), and they differ in the rates of time preference: they maximize a utility function, separable between consumption and labor, under a budget constraint and a debt limit constraint. Later on, we call "s types" as savers, lenders, or unconstrained households, and "b types", as borrowers, debtors or constrained households.

Firms maximize profits over the infinite horizon, subject to a technology and to the demand of households and government. Some firms set prices freely, others set prices one period in advance. Firms have also to choose the allocation of the different kinds of workers.

The central bank decides nominal interest rate following a Taylor rule.

The government decides the amount of public expenditure, taxes (lump-sum and distortionary) and public debt.

2.1 Households

There is a continuum of households of mass one with χ_s savers and χ_b borrowers. Debtors are more impatient (namely their discount factor is lower); they prefer to consume more today and borrow to do so. On the contrary, lenders are more patient and prefer to save something to smooth consumption over time. As follows, the relationship between the discount factors of the two individuals:

$$\beta(s) = \beta > \beta(b)$$

where $\beta(i) \in (0,1)$ is the discount factor of type *i*. Like EK, from now on, we consider both A(i) and A^i as the variable A for type *i*.

Consumer *i* maximizes the following function, separable between consumption and hours of work H_t :

$$E_o \sum_{t=0}^{\infty} \beta(i)^t \left[u(C_t^i) - v(H_t^i) \right] \text{ with } i = s \text{ or } b,$$

where

$$u(C_t^i) = 1 - \exp(-zC_t^i)$$

and

$$v(H_t^i) = \frac{H_t^{i\,(1+\omega)}}{(1+\omega)}.$$

 C_t^i refers to a Dixit-Stiglitz aggregator of a continuum of differentiated goods *j*, consumed by agent *i*, which provides the producer of each good market power with elasticity of demand θ :

$$C_t^i = \left[\int_0^1 c_t(i,j)^{\frac{\theta-1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}}$$

and P_t to the corresponding price index

$$P_t = \left[\int_{0}^{1} p_t(j)^{(1-\theta)} dj\right]^{\frac{1}{\theta-1}},$$

where $p_t(j)$ is the price of good j and $c_t(i, j)$ is the consumption of good j consumed by i.

In this way, $C_t(i)$ can be interpreted as the per-capita consumption of types *i*. As follows, an expression for aggregate per-capita consumption:

$$C_t = \chi_s C_t^s + \chi_b C_t^b.$$

Households are subject to:

$$B_t(i) + (1 - \tau_t^w) W_t(i) P_t H_t(i) + (1 - \tau_t^p) \Pi_t = (1 + i_{t-1}) B_{t-1}(i) + P_t T_t(i)$$

+ $P_t(1 + \tau_t^c) C_t(i)$ (2.1)

$$\frac{(1+r_t)B_t(i)}{P_t} \le D_t > 0 \ (2.2).$$

 $W_t(i)$ is the hourly real wage received by each agent at the beginning of the period; $B_t(i)$ is the amount (in nominal terms) borrowed by agent *i*; i_t is the nominal interest rate, that is the return on one period risk-free nominal bond; r_t is the risk-free real interest rate on a one period real bond; τ_t^w , τ_t^p and τ_t^c are the tax

rates on labor, profits and consumption, equal for both types; Π_t is profit from ownership of firms, distributed in equal shares among agents; $T_t(i)$ denotes lump-sum taxes, different for the two types.

The first equation is the budget constraint: on the left hand side there are revenues, and on the right hand side there are expenditures. The second constraint is the debt limit: the total real debt (real face value plus interests) must not be higher than a positive exogenous value D_t , expressed in real terms (note that the debt limit can be different in each period.)

2.1.1 Savers

We assume that borrower is up against his borrowing constraint, while saver is not: thus for the saver the debt constraint is not binding. Saver maximizes the utility function subject to the constraints, with respect to consumption, hours and amount borrowed (lent, in this case), taking as given wages, prices and taxes. We obtain the following conditions:

$$u_{c}(C_{t}^{s}) = E_{t} \left[u_{c}(C_{t+1}^{s}) \beta \frac{P_{t}}{P_{t+1}} \frac{(1+\tau_{t}^{c})}{(1+\tau_{t+1}^{c})} (1+i_{t}) \right]$$
(2.3)
$$W_{t}^{s} = \frac{v_{H}(H_{t}^{s})}{u_{c}(C_{t}^{s})} \frac{(1+\tau_{t}^{c})}{(1-\tau_{t}^{w})}$$
(2.4),

where $u_c(C_t^s)$ and $v_H(H_t^s)$ are respectively the first derivative of $u(C_t^s)$ and $v(H_t^s)$.

(2.3) is a standard Euler Equation, a relationship between present and future consumptions.

(2.4) is the labor supply of savers: real wage has to be equal to marginal rate of substitution between consumption and labor, after distortionary taxes.

We can easily notice the role of distortionary taxation: differently from lump-sum taxes, the presence of tax rates directly affects optimal plans of savers. A higher τ_t^c implies that consumption goods are less convenient, and higher τ_t^w means a lower net wage for the household.

2.1.2 Borrowers

In order to obtain the consumption of type b, consider that (2.2) is binding and plug it in the budget constraint:

$$(1+\tau_t^c)C_t^b = -\left(\frac{1+i_{t-1}}{1+r_{t-1}}\frac{P_{t-1}}{P_t}\right)D_{t-1} + \frac{D_t}{1+r_t} + I_t^b - T_t^b$$
(2.5),

where $I_t^b = (1 - \tau_t^w) W_t^b H_t^b + (1 - \tau_t^p) \frac{\Pi_t}{P_t}$ is the real income of borrowers, after distortionary taxes.

We can already notice the big difference between (2.3) and (2.5), the equations giving the consumption of the two individuals: the consumption function of the borrower positively depends on available current income, such as the households in the standard Keynesian model; on the other part, savers' consumption depends on the expectations of future income.

As far as the labor supply of borrower is concerned, we get the same result obtained in the savers' maximization problem:

$$W_t^b = \frac{v_H(H_t^b)}{u_c(C_t^b)} \frac{(1+\tau_t^c)}{(1-\tau_t^w)} \quad (2.6).$$

Moreover, both households have to decide how to allocate their consumption expenditure among the different goods. As a consequence, $C_t(i)$ is maximized for any given level of expenditures $(1 + \tau_t^c) \int_0^1 p_t(j)c_t(i,j)dj$ yielding the following set of demand equation⁸:

$$c_t(i,j) = C_t(i)(\frac{p_t(j)}{P_t})^{-\theta}.$$

Thus, by aggregating by types we have:

$$c_t(j) = C_t(\frac{p_t(j)}{P_t})^{-\theta}$$
 (2.7).

2.2 Firms

We consider a continuum of firms of measure one, indexed by $j \in [0,1]$, each of them producing a differentiated good *j*.

⁸ See Galì (2008) for the derivation of this equation. Galì does not utilize distortionary taxation; however the final result is the same.

Following Benigno (2012), the production function is the following:

$$Y_t = AH_t$$

where H_t is an aggregator of the two types of labor:

$$H_t = H_t^{\chi_s}(s) H_t^{\chi_b}(b).$$

 Y_t is a Dixit-Stiglitz aggregator of the differentiated goods $y_t(j)$: it can be interpreted as per-capita GDP.

$$Y_t \equiv \left[\int_{0}^{1} y_t(j)^{\frac{\theta-1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}}$$

and A is a technology factor.

We assume that a fraction λ of the firms sets prices freely at all times, while a fraction $(1 - \lambda)$ charges prices one period in advance. This way, we exclude perfect price flexibility.

Firms maximize after-tax profits over the infinite horizon, subject to technology and demand schedule. We assume that all profits are paid out as dividends and that prices are exclusive of the consumption taxes, as in Eggertsson (2010a). Firms pay a hourly real wage W_t^i to each agent *i* and taxes on profits with rate τ_t^p . In order to derive the demand of good *j*, in this section we have to anticipate the good market clearing conditions:

$$y_t(j) = c_t(j) + g_t(j) = Ah_t(j)$$
$$Y_t = C_t + G_t,$$

where $g_t(j)$ is the government consumption of good, j and $h_t(j)$ are aggregate hours of work needed to produce $y_t(j)$. Like private sector demand, public demand of good j is given by:

$$g_t(j) = G_t(\frac{p_t(j)}{P_t})^{-\theta}$$
 (2.8),

where

$$G_t = \left[\int_{0}^{1} g_t(j)^{\frac{\theta-1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}}.$$

So, by (2.7), (2.8) and the market clearing conditions of good j, we obtain the following demand schedule:

$$y_t(j) = Y_t(\frac{p_t(j)}{P_t})^{-\theta}$$
 (2.9).

Thus the problem of firm *j* is:

$$Max_{p_t(j)}E_t\sum_{t=0}^{\infty}\widetilde{\phi_t}(1-\tau_t^p)[p_t(j)y_t(j)-W_tP_th_t(j)]$$

s.t.

$$y_t(j) = Y_t(\frac{p_t(j)}{P_t})^{-\theta}$$
$$y_t(j) = Ah_t(j),$$

where

$$W_t = W_t^{\chi_s}(s) W_t^{\chi_b}(b),$$

and $\tilde{\phi_t}$ is the average marginal utility of the income used to discount the profits. The first order conditions of this problem imply that the λ fraction of the firms setting their prices freely chooses the same price $p_t(1)$, such that:

$$p_t(1) = \frac{\theta}{(\theta - 1)} \frac{W_t P_t}{A} (2.10).$$

We can notice that the price is charged with a mark-up on the marginal costs.

On the other hand, the firms setting their prices $p_t(2)$ one period in advance satisfy the following first order condition:

$$p_t(2) = E_{t-1} \left[\frac{\theta}{(\theta-1)} \frac{W_t P_t}{A} \right] (2.11).$$

Notice that taxes on profits do not affect firms' choices.

Firms have to decide also the amount of hours worked by each type *i*. In aggregate, firms' cost minimization problem is the following:

$$Min_{H_t(s),H_t(b)} \left[\chi_s W_t^s H_t(s) + \chi_b W_t^b H_t(b) \right]$$

s.t: $Y_t = AH_t^{\chi_s}(s) H_t^{\chi_b}(b).$

By the first order conditions of this problem, we obtain the demand of both kinds of labor:

$$H_t^s = H_t \left(\frac{W_t^b}{W_t^s}\right)^{\chi_b} (2.12)$$

$$H_t^b = H_t \left(\frac{W_t^s}{W_t^b}\right)^{\chi_s} (2.13).$$

Note that from the previous conditions one can obtain:

$$W_t^i H_t^i = W_t H_t$$
 (2.14).

3. Economic policies

3.1 Monetary Policy

We assume that monetary authority chooses the nominal interest rate according to the following Taylor rule

$$z_t = (1 + r_t^n)(1 + \pi_t)^{\phi_{\pi}} - 1$$
 (3.1),

constrained by ZLB

$$i_t = \max(z_t, 0)$$
 (3.2),

where r_t^n is the natural interest rate, which is the real interest rate if prices are fully flexible (i.e. $\lambda = 1$ in our economy); π_t is the rate of inflation and $\phi_{\pi} > 1$ measures the degree of central bank's aggressiveness against changes in the price level.

Notice that when (3.1) implies a negative z_t , central bank sets the nominal interest rate at zero; otherwise we have $i_t = z_t$.

3.2 Fiscal Policy

Fiscal authority finances its expenditure with debt and taxes. We assume that government buys the same composite good consumed by households and uses it in such a way that it is separable from private consumption, not having any impact on savers' intertemporal optimization problem .

In this paper, differently from EK, we assume that taxes are both lump-sum and distortionary.

In every period, government chooses a sequence of $\{G_t, B_t^g, T_t^s, T_t^b, \tau_t^c, \tau_t^w, \tau_t^p\}$, subject to the following budget constraint:

$$G_t + \frac{B_{t-1}^g}{P_{t-1}} \frac{P_{t-1}}{P_t} (1 + i_{t-1}) = \frac{B_t^g}{P_t} + \chi_s T_t^s + \chi_b T_t^b + \tau_t^c C_t + \tau_t^w W_t H_t + \tau_t^p \frac{\Pi_t}{P_t}$$

Where B_t^g is government debt, other variables are defined in Chapter 2. We can see expenditures (purchase of G_t and debt payments) on the left hand side, and revenues (new debt and taxes) on the right hand side.

For any variations in T_t^b , G_t , $\tau_t^c C_t$, $\tau_t^w W_t H_t$, $\tau_t^p \frac{\Pi_t}{P_t}$, we assume that current or future T_t^s will be adjusted to satisfy government budget constraint.

In this paper, likewise EK, we do not specify a fiscal rule followed by government to implement a desired policy path. Our analysis just focuses on the effect of a change in a fiscal variable, in the presence of a deleveraging shock.

4. Market Clearing Conditions

4.1 Goods Market

As seen in Chapter 2, in the goods market the firms' supply has to be equal to the sum of private and public consumptions:

for goods

$$y_t(j) = c_t(j) + g_t(j),$$

and, in aggregate

$$Y_t = C_t + G_t (4.1),$$

with $Y_t \equiv \left[\int_0^1 y_t(j)^{\frac{\theta-1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}}$.

We remind that private consumption is the sum of the consumption of both savers and borrowers.

4.2 Labor Market

In the labor market, labour supply has to be equal to the labour demand for each type⁹:

$$W_t \left(\frac{H_t^s}{H_t^b}\right)^{\chi_s} = \frac{v_H(H_t^b)}{u_c(C_t^b)} \frac{(1+\tau_t^c)}{(1-\tau_t^w)}$$
(4.2)

$$W_t \left(\frac{H_t^b}{H_t^s}\right)^{\chi_b} = \frac{v_H(H_t^s)}{u_c(C_t^s)} \frac{(1+\tau_t^c)}{(1-\tau_t^w)}$$
(4.3).

4.3 Bond Market

We have assumed that households and government can borrow at the same nominal interest rate i_t . To obtain the market clearing condition, we have to equalize the debt of borrowers and government with savers' assets:

$$B_t^g + \chi_b B_t^b = -\chi_s B_t^s (4.4).$$

 $^{^{9}}$ We have used a re-arrangement of (2.12) and (2.13).

5. A Log-Linearization Around the Steady State

In this section we derive a log-linearized version of the model around a steady state with zero inflation. In the appendix we present a detailed description of the steady state.

The goal of this section is to obtain a linear expression for the consumption of the two types and the linear aggregate supply (AS) of this economy. We will derive the aggregate demand in the following chapters, when we analyze the effects of a deleveraging shock.

We suppose that in t-1 borrowers are in a steady state in which the debt limit is binding for them, because of their impatience to consume.

The first step is the linearization of the household optimality conditions. If not indicated differently, the lowercase letters denote the difference between the corresponding variable and its steady state, divided for the steady state of the output.

Letters with a bar denote the steady state of the corresponding variable. For instance:

$$c_t \equiv \frac{C_t - \bar{C}}{\bar{Y}}$$

By log-linearizing the labor supply of both types, we obtain the following expressions:

$$\begin{split} w_t^s &= \omega h_t^s + \sigma^{-1} c_t^s + \eta_c \widehat{\tau}_t^c + \eta_w \widehat{\tau}_t^w \ (5.2) \\ w_t^b &= \omega h_t^b + \sigma^{-1} c_t^b + \eta_c \widehat{\tau}_t^c + \eta_w \widehat{\tau}_t^w \ (5.3) \\ \end{split}$$

$$\begin{aligned} \text{where}^{10} w_t^i &\equiv \frac{W_t^i - \overline{W}^i}{\overline{W}^i}, h_t^i &\equiv \frac{H_t^i - \overline{H}^i}{\overline{H}^i} \widehat{\tau}_t^{\widehat{W}} \equiv (\tau_t^w - \overline{\tau}^w), \widehat{\tau}_t^c \equiv (\tau_t^c - \overline{\tau}^c), \sigma \equiv \frac{1}{z\overline{y}}, \eta_c \equiv (1 + \overline{\tau}^c)^{-1}, \eta_w \equiv (1 - \overline{\tau}^w)^{-1}. \end{split}$$

By a log linearization of the wage index, we obtain:

$$w_t = \chi_s w_t^s + \chi_b w_t^b.$$

The aggregator of labor becomes:

¹⁰ Observe that $0 < \eta_c < 1$ and $\eta_w > 1$.

$$h_t = \chi_s h_t^s + \chi_b h_t^b$$

and by considering that:

$$c_t = \chi_s c_t^s + \chi_b c_t^b,$$

we can obtain the following aggregate labor supply:

$$w_t = \omega h_t + \sigma^{-1} c_t + \eta_c \overline{\tau_t^c} + \eta_w \overline{\tau_t^w}.$$

By linearizing the good market clearing condition and the production function, we obtain:

$$y_t = c_t + g_t = \chi_s c_t^s + \chi_b c_t^b + g_t$$
(5.4),
$$y_t = h_t$$

So labor supply becomes:

W

$$w_t = (\omega + \sigma^{-1})y_t - \sigma^{-1}g_t + \eta_c \widehat{\tau_t^c} + \eta_w \widehat{\tau_t^w}$$
(5.5)

Now we are going to linearize the consumption functions of the two types. By (2.3) we obtain the linear Euler equation of savers:

$$c_{t}^{s} = E_{t}c_{t+1}^{s} - \sigma(i_{t} - E_{t}\pi_{t+1} - \bar{r}) + \sigma\eta_{c}E_{t}[\tau_{t+1}^{\widehat{c}} - \tau_{t}^{\widehat{c}}] (5.6)$$

where now $i_t \cong log(1+i_t), \pi_{t+1} \equiv \log(\frac{P_{t+1}}{P_t}) \ \bar{r} \equiv -log\beta.$

By approximating (2.5), we get an expression for borrowers' consumption:

 $c_t^b = \eta_c \left(\hat{l}_t^b + \beta \widehat{D_t} - \widehat{D}_{t-1} + \gamma_D \pi_t - \gamma_D \beta (i_t - E_t \pi_{t+1} - \bar{r}) - t_t^b - \gamma_C \widehat{\tau}_t^c \right) (5.7)$ where $\widehat{D}_t \equiv \frac{D_t - \bar{D}}{\bar{Y}}, \gamma_D \equiv \frac{\bar{D}}{\bar{Y}}, \gamma_C \equiv \frac{\bar{C}^b}{\bar{Y}} \text{ and } \hat{l}_t^b \equiv \frac{l_t^b - \bar{l}^b}{\bar{Y}}.$

By log-linearizing (2.14), we can rewrite the net labor income of borrowers as follows:

$$\hat{l}_t^b = \left(\eta_p^{-1} + \delta\mu_\vartheta\right) y_t + \delta\mu_\vartheta w_t - (1 - \mu_\vartheta)\widehat{\tau_t^p} - \mu_\vartheta \widehat{\tau_t^w}$$

ith $\eta_p \equiv (1 - \overline{\tau}^p)^{-1}, \, \delta \equiv (\overline{\tau}^p - \overline{\tau}^w), \, \mu_\vartheta \equiv \frac{\overline{w}}{\overline{A}} = \frac{\vartheta - 1}{\vartheta}, \, \widehat{\tau_t^p} \equiv (\tau_t^p - \overline{\tau}^p).$

Now let us linearize the equation of the production side. By log-linearizing (2.10) and (2.11), we obtain:

$$\log p_t(1) = \log P_t + w_t (5.9)$$
$$\log p_t(2) = E_{t-1}[\log P_t + w_t] (5.10)$$

So we can rewrite the last equation as follows:

$$\log p_t(2) = E_{t-1}[\log p_t(1)]$$
(5.11).

By considering the fact that a fraction λ sets the price equal to $p_t(1)$ and a fraction $(1 - \lambda)$ firms sets $p_t(2)$, we get an expression for the log-linearized price index:

$$\log P_t = \lambda \log p_t(1) + (1 - \lambda) \log p_t(2)$$
 (5.12).

By using the above equations, we can write:

$$\pi_t - E_{t-1}\pi_t = \log P_t - E_{t-1}\log P_t = \left(\frac{\lambda}{1-\lambda}\right)(\log p_t(1) - \log P_t)$$

By plugging (5.9) in the last expression, we obtain:

$$\pi_t - E_{t-1}\pi_t = \left(\frac{\lambda}{1-\lambda}\right) w_t \ (5.13).$$

Finally, in order to derive the New Classical Philips Curve, we use the labor supply equation, and we obtain:

$$\pi_t - E_{t-1}\pi_t = ky_t - k\varphi g_t + \mu_\lambda \left(\eta_c \widehat{\tau}_t^c + \eta_w \widehat{\tau}_t^w\right) (5.14)$$

where $k \equiv \left(\frac{\lambda}{1-\lambda}\right) (\omega + \sigma^{-1}), \varphi \equiv \frac{\sigma^{-1}}{(\omega + \sigma^{-1})}, \mu_\lambda \equiv \left(\frac{\lambda}{1-\lambda}\right).$

The last equation represents an AS line, a relation between prices and production: if inflation is higher (lower) than expected, output will be above (below) its steady state, everything else being equal. This line is upward-sloping with slope equal to k in a (y_t, π_t) plane. Higher output increases real wages, namely firms real marginal costs: flexible firms react by raising their prices. If we set $\lambda = 1$, eliminating "rigid" firms, the AS line is vertical and prices are fully flexible.

Moreover we can observe that any increase in the distortionary taxation causes a leftward shift of AS. Suppose a rise in labor tax rate: people want to reduce hours of work, because they earn less for each worked hour. This increases real wages, as a consequence firms supply fewer goods for more money: this way there is an increase in inflation, other things equal.

6. Deleveraging shock

6.1 The Effects of a Deleveraging Shock

This section explains what a deleveraging shock is and what effects it could cause on the economy.

In EK, a debt limit changes over time. Normally, during long periods of prosperity, characterized by an increase in asset prices, there is a relaxed attitude toward debt: nobody is worried if households and firms have a high level of debt in their portfolios. However, it could happen that this attitude becomes tighter, banks are more afraid to lend and lower the threshold level, and a period of credit crunch starts.

In the baseline model of EK, the debt limit is exogenous: the fall can be caused by sudden realization that asset prices were too high; as a result the collaterals of borrowers lose value. This event is known as the "Minsky moment" and was coined by Paul McCulley of Pimco to explain the 1998 Russian financial crisis. In an extension of their work, EK introduce an endogenous part into the debt limit, making it also dependent on the expected future available income.

What effects does this deleveraging shock have on the economy?

EK intuition is the following: when the debt limit experiences an abrupt drop, people having too high a level of debt ("borrowers" in the model) must deleverage and we assume they must do it in one period. They have two possibilities: to consume less or to work more; borrowers' dropped consumption reduces the natural interest rate by allowing savers to pick up the slack: if the reduction of the natural interest rate is big enough, the desired nominal interest rate can become negative; monetary authority sets it at zero and it cannot offset the output drop, which decreases below the potential; recession is worsened for the consequent deflation, which increases the debt burden, further reducing borrowers' real income. The last step of this process is called "Fisher Effect". In his explanation of the Great Depression, American economist Irving Fisher stated that in a crisis situation characterized by a liquidity trap a deflation can be very dangerous for the economy, as it makes debtors poorer and increases the real interest rate. The

consequence is lower spending, which generates lower output, which causes a lower level of prices, starting the vicious circle again.

In order to better understand the effects of a deleveraging shock on the economy, by following EK in this chapter we eliminate government from the model. In the next chapter, we newly introduce fiscal variables and compute fiscal multipliers when a big deleveraging shock hits the budget constraint of debtors.

The linearized equations in the model without government are:

$$y_{t} = c_{t} = \chi_{s}c_{t}^{s} + \chi_{b}c_{t}^{b} (6.1)$$

$$c_{t}^{s} = E_{t}c_{t+1}^{s} - \sigma(i_{t} - E_{t}\pi_{t+1} - \bar{r}) (6.2)$$

$$c_{t}^{b} = \hat{I}_{t}^{b} + \beta \widehat{D_{t}} - \widehat{D}_{t-1} + \gamma_{D}\pi_{t} - \gamma_{D}\beta(i_{t} - E_{t}\pi_{t+1} - \bar{r}) (6.3),$$

where now

$$I_{t}^{b} = W_{t}^{b}H_{t}^{b} + \frac{\Pi_{t}}{P_{t}} = W_{t}H_{t} + Y_{t} - W_{t}H_{t} = Y_{t}.$$

Therefore, in a linear approximation: $\hat{l}_t^b = y_t$

The aggregate supply becomes:

$$\pi_t - E_{t-1}\pi_t = ky_t.$$

A log linearization of the Taylor rule indicates:

$$i_t = \max(0, r_t^n + \phi_\pi \pi_t).$$

We assume that in t an unexpected and permanent deleveraging shock hits the economy¹¹ and borrowers have to deleverage in one period: $D_t = \overline{D} < D_{t-1}.$

Now we split our analysis between "short run" (denoted with S) and "long run" (denoted with L), and we assume that shock arrives in the short run. In the long run, economy reaches the steady state with¹²:

y_L	=	0
c_L^s	=	0
c_L^b	=	0

¹¹ Note that the "shocked" value is equal to its steady state.

¹² We remind that y_L , c_L^s , c_L^b are deviations from the steady state divided by the steady state of the output.

$$\pi_L = 0$$
$$i_L = r_L^n.$$

Notice that, because of the specification of the Taylor rule, prices remain to the short-run level. Chapter 8 introduces a new rule, implying price reversion.

Now we can compute the equilibrium output and inflation in the short run and see how they are affected by the shock.

As the shock is unexpected, AS becomes:

$$\pi_t = k y_t.$$

Using the previous equations, considering that variables in t+1 are in their longrun level, we can obtain the following relationship:

$$y_{S} = -\frac{\chi_{s}\sigma + \chi_{b}\gamma_{D}\beta}{\chi_{s}}(i_{s} - \bar{r}) - \frac{\chi_{b}}{\chi_{s}}\widehat{D} + \frac{\chi_{b}\gamma_{D}}{\chi_{s}}\pi_{S} (6.4)$$

where $\widehat{D} \equiv \frac{D_{t-1} - \overline{D}}{\overline{Y}}$ is a measure of the shock size.

Notice that (6.4) can be seen as an IS curve (or as an aggregate demand relationship, once we have plugged the Taylor rule), which is the relation between output and interest rate. We can observe that a decrease in interest rate triggers consumption of both individuals, thus increasing the output. As we have already seen, borrowers are liquidity constrained, debt limit is binding and their behavior is similar to the one of standard Keynesian household: they consume their additional income and this brings on another increase in the output, thus starting a virtuous circle.

Now let us give an expression for natural interest rate.

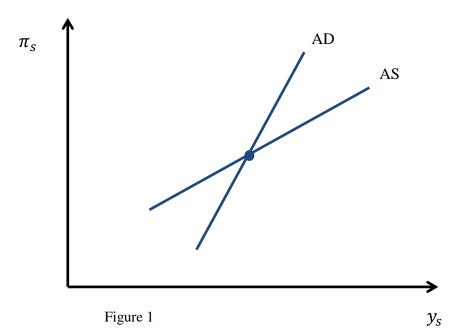
In Chapter 2 we define r_s^n as the real interest rate if prices are fully flexible. In the flexible price equilibrium (without government) $y_s = 0$ and solving for i_s , we obtain:

$$i_{s} = r_{s}^{n} = \bar{r} - \frac{\chi_{b}}{\chi_{s}\sigma + \chi_{b}\gamma_{D}\beta} \ \widehat{D} + \frac{\chi_{b}\gamma_{D}}{\chi_{s}\sigma + \chi_{b}\gamma_{D}\beta} \pi_{s} \ (6.8).$$

Here we can easily see that if shock \widehat{D} is small, the natural interest rate remains positive and central bank can lower the interest rate and compensate the output fall. Otherwise, if the deleveraging shock is big enough, the natural interest rate can be negative and because of the specification of the Taylor rule, central bank is constrained to set the nominal interest rate at zero. Note that in a steady state with zero inflation, $\bar{\iota} = r_t^n = \bar{r}$.

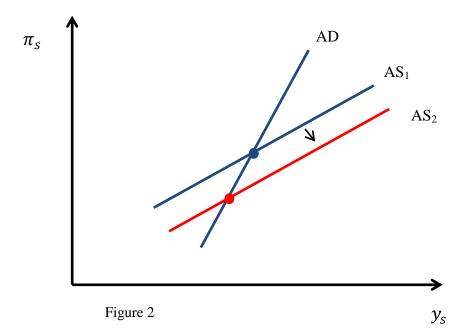
This paper focuses on the case of the liquidity trap, and assumes that the shock forces monetary authority to set the nominal interest rate at zero: so we fix $i_s = 0$ and we can observe that aggregate demand (6.4) is upward sloping¹³. Indeed, higher inflation boosts debtors' spending by alleviating the burden of the real value of the debt. Note that if ZLB were not binding, we would have the term $-\phi_{\pi}\pi_t$ in the aggregate demand, and AD would be backward sloping.

The following graph shows AS and AD when the economy is in a liquidity trap (Figure 1).



In a liquidity trap, the economic analysis changes significantly: one interesting implication of the new slope of AD is the paradox of toil, first proposed by Eggertsson (2010b). As we can observe in Figure 2, any positive supply shock not affecting the AD line reduces output and prices.

¹³ In order to have a stable short-run equilibrium, we have to assume that AD slope is higher than AS slope: $\frac{\chi_s}{\chi_b \gamma_D} > k$.



Consider, for instance, a sudden rise in the toil endurance of workers, who are willing to work more: in normal conditions, this positive shock induces firms to lower prices, households demand more consumption goods, and production increases. However, in a liquidity trap, lower prices do not boost the demand, but have the opposite effect, because they increase the real value of the debt. So, a higher desire to work causes lower production, which implies less work.

By plugging AS in AD and considering a deleveraging shock large enough to imply an $i_s=0$, we attain the following short-run equilibrium, with deflation and contraction of output:

$$y_{S} = \Gamma - \frac{\chi_{b}}{\chi_{s} - \chi_{b} k \gamma_{D}} \widehat{D} < 0$$
$$\pi_{S} = k\Gamma - \frac{k \chi_{b}}{\chi_{s} - \chi_{b} k \gamma_{D}} \widehat{D} < 0$$

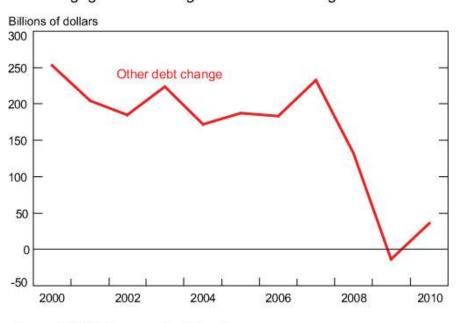
where $\Gamma \equiv \frac{\chi_s \sigma + \chi_b \gamma_D \beta}{\chi_s - \chi_b k \gamma_D} \bar{r}$.

The next chapter describes the instruments government can use to provide economy with a stimulus big enough to expand GDP and avoid deflation.

6.2 Evidence on Deleveraging

We now conclude this section with a look at the US data on the deleveraging of households.

A recent study by Brown, Haughwout, Lee and Van der Klaauw (2011) reveals USA households highly reduce their non-mortgage debts. The graph below shows the annual change in non-mortgage debt, net of charge off. Before the economic crisis, between 2000 and 2007, households had been increasing their non-mortgage borrowing by an average of over \$200 billion per year. In 2009, consumers drastically changed such behavior and their net borrowing became negative.



Nonmortgage Debt Change Other Than Charge-offs

Source: FRBNY Consumer Credit Panel.

By adding mortgage credit data, deleveraging is even more evident: authors show that between 2000 and 2007 the households' aggregate annual deficit was, on an average, \$330 billion; conversely, in 2009 the deficit turned negative (-\$150 billion). This means the cash they could spend decreased by \$480 billion in aggregate over just two years. It is important issue to understand whether deleveraging is forced or spontaneous: according to the authors, it is both. From one side, data provided by the FED show tightness of credit standard in 2007-

2010. From the other side, in a situation of fall in asset prices, families could choose to reduce their debt, in order to restore their net worth.

7. Fiscal multipliers

This Chapter is the core section of the paper. Here we are going to derive a shortrun equilibrium to analyze which instruments could be used by policy-makers to expand output.

As already said in the introduction, in a liquidity trap situation monetary policy loses its main instrument – namely, the nominal interest rate.

Some economists state this is not a problem, as central bank can increase money supply and generate higher expectations of inflation – i.e. the natural way to solve a deflation (and tackle a deleveraging shock, as we will see in the next chapter). However, as pointed out by EK, changing agents' expectations is not easy: central bank needs high credibility to persuade economic agents to follow with an inflationary policy. Moreover, over the last decades, most central banks in the world have tried to fight inflation, not to generate it.

We will now focus on fiscal instruments in the hands of government in the short run, when a deleveraging shock leads economy in a liquidity trap.

In our extension of EK model, fiscal policy-makers can also use distortionary taxation on consumption, wages and profits to lead economy to a normal level.

As we will see, the presence of constrained households breaks down Ricardian Equivalence, as they do not worry about future and their consumption directly depends on their current available income. Thus any direct increase in the aggregate demand (such as fiscal stimulus) can boost the consumption of debtors and increase the aggregate demand even more.

By considering, in the short run, a deleveraging shock forcing central bank to set $i_S = 0$, and in the long run (as already seen in the previous chapter) $y_L = c_L^s = c_L^b = \pi_L = 0$, $i_L = r_L^n = \bar{r}$ and by assuming that in the long run fiscal variables are in their steady state level, (5.7) becomes:

$$c_{S}^{b} = \eta_{c} \left(\hat{I}_{S}^{b} - \hat{D} + \gamma_{D} \pi_{S} + \gamma_{D} \beta \bar{r} - t_{S}^{b} - \gamma_{C} \hat{\tau}_{S}^{c} \right) (7.1),$$

where

$$\hat{I}_{S}^{b} = \left(\eta_{p}^{-1} + \delta\mu_{\vartheta}\right)y_{S} + \delta\mu_{\vartheta}w_{S} - (1 - \mu_{\vartheta})\widehat{\tau_{S}^{p}} - \mu_{\vartheta}\widehat{\tau_{S}^{w}}$$
(7.2).

By the last equation, we can notice that if wages increase, other things equal, the effect on borrowers' income is ambiguous and depends on the sign of $\delta \equiv (\bar{\tau}^p - \bar{\tau}^w)$. This is due to the fact that also borrowers are firms' shareholders¹⁴: higher wages imply higher labor income, but also higher labor costs for firms. Therefore on one hand, when wages are higher borrowers pay more taxes due to higher labor income; on the other hand, they pay lower taxes because of lower profits (wages are a cost for firms, thus they are tax-deductible). Therefore, if profit tax rate is higher than labor tax rate (as in our calibration), the overall effect of a rise in wages is positive. Obviously, in a model without taxation (as in the previous chapter) these effects cancel each other out, and wages do not affect the current available income.

By using the aggregate labor supply, we can rewrite (7.2) as:

$$\hat{I}_{s}^{b} = \mu_{y} y_{s} - \delta \mu_{\vartheta} \sigma^{-1} g_{s} - (1 - \mu_{\vartheta}) \widehat{\tau_{s}^{p}} - \mu_{\vartheta} (1 - \delta \eta_{w}) \,\widehat{\tau_{s}^{w}} + \delta \mu_{\vartheta} \eta_{c} \,\widehat{\tau_{s}^{c}}$$

where $\mu_y \equiv \left[\eta_p^{-1} + \delta \mu_{\vartheta} (1 + \omega + \sigma^{-1})\right]$.

Two things are remarkable: firstly we have already seen that borrowers' consumption depends on current available income; here, by plugging (7.2) in (7.1), we find that the output coefficient in the consumption function is given by $\eta_c \mu_y$ (0.79 with our calibration). This value measures the marginal propensity to consume: it tells us how much borrowers' consumption increases when output increases. We can easily notice the effects of distortionary taxes, which modifies such a coefficient. In the previous chapter we saw that, without government, borrowers' consumption increases 1:1 after a rise in the output. Here the response of consumption is lower: indeed, when output increases, borrowers have to pay more taxes:

-on profits;

-on consumption, as they consume more when output is higher;

-on wages, if $\delta < 0$, as explained above (however, our calibration implies the positivity of δ).

¹⁴ In EK only savers own firms.

Secondly, by (7.2) we can observe that the tax on labor income enters the consumption function of borrowers. as a consequence it can affect the AD line. This result is different from the one found by Eggertsson (2010a), where all households are Ricardian and their consumption function is the standard Euler equation (similar to saver's in our model); therefore it is not affected by variations in available current income and the labor tax can just affect the AS line. This result matters for the sign of the labor tax rate multiplier.

Due to the same reason, neither profits nor taxes on profits enter AD in Eggertsson (2010a).

In order to derive AD, the last missing piece is savers' consumption. In the short run, (5.6) becomes:

$$c_S^s = \sigma \left(\bar{r} - \eta_c \hat{\tau}_S^c \right) (7.3).$$

By putting together (7.1), (7.2), (7.3) and good market clearing condition, we obtain the following AD schedule: $y_{S} = \frac{\chi_{s}\sigma + \chi_{b}\eta_{c}\gamma_{D}\beta}{1 - \chi_{b}\eta_{c}\mu_{y}}\bar{r} + \frac{\chi_{b}\eta_{c}\gamma_{D}}{1 - \chi_{b}\eta_{c}\mu_{y}}\pi_{S} - \frac{\chi_{b}\eta_{c}}{1 - \chi_{b}\eta_{c}\mu_{y}}\widehat{D} - \frac{\chi_{b}\eta_{c}}{1 - \chi_{b}\eta_{c}\mu_{y}}t_{S}^{b}$ $- \frac{\chi_{s}\sigma\eta_{c} + \chi_{b}\eta_{c}(\gamma_{c} - \delta\mu_{\vartheta}\eta_{c})}{1 - \chi_{b}\eta_{c}\mu_{y}}\widehat{\tau}_{S}^{c} + \frac{1 - \chi_{b}\eta_{c}\delta\mu_{\vartheta}\sigma^{-1}}{1 - \chi_{b}\eta_{c}\mu_{y}}g_{S}$ $- \frac{\chi_{b}\eta_{c}\mu_{\vartheta}(1 - \delta\eta_{w})}{1 - \chi_{b}\eta_{c}\mu_{y}}\widehat{\tau}_{S}^{w} - \frac{\chi_{b}\eta_{c}(1 - \mu_{\vartheta})}{1 - \chi_{b}\eta_{c}\mu_{y}}\widehat{\tau}_{S}^{p}$

while AS¹⁵ is: $\pi_s = k y_s - k \varphi g_s + \mu_\lambda (\eta_c \widehat{\tau_s^c} + \eta_w \widehat{\tau_s^w}).$

Also in this case, the aggregate demand is upward sloping because inflation, by reducing the real value of the debt, gives stimulus to borrowers' spending. Now, we plug AS in AD to obtain an expression for output as a function of fiscal instruments:

¹⁵ Also here we assume the AD slope is higher than the AS slope: $1 - \chi_b \eta_c \mu_y > k \chi_b \eta_c \gamma_D$.

$$y_{S} = \frac{\chi_{S}\sigma + \chi_{b}\eta_{c}\gamma_{D}\beta}{1 - \chi_{b}\eta_{c}(\mu_{y} + k\gamma_{D})}\bar{r} - \frac{\chi_{b}\eta_{c}}{1 - \chi_{b}\eta_{c}(\mu_{y} + k\gamma_{D})}\hat{D}$$
$$- \frac{\chi_{b}\eta_{c}}{1 - \chi_{b}\eta_{c}(\mu_{y} + k\gamma_{D})}t_{S}^{b} + \frac{1 - \chi_{b}\eta_{c}(\delta\mu_{\theta}\sigma^{-1} + \gamma_{D}k\varphi)}{1 - \chi_{b}\eta_{c}(\mu_{y} + k\gamma_{D})}g_{S}$$
$$- \frac{\chi_{s}\sigma\eta_{c} + \chi_{b}\eta_{c}(\gamma_{c} - \delta\mu_{\theta}\eta_{c} - \gamma_{D}\mu_{\lambda}\eta_{c})}{1 - \chi_{b}\eta_{c}(\mu_{y} + k\gamma_{D})}\bar{\tau}_{S}^{c}$$
$$- \frac{\chi_{b}\eta_{c}[\mu_{\theta}(1 - \delta\eta_{w}) - \gamma_{D}\mu_{\lambda}\eta_{w}]}{1 - \chi_{b}\eta_{c}(\mu_{y} + k\gamma_{D})}\bar{\tau}_{S}^{w} - \frac{\chi_{b}\eta_{c}(1 - \mu_{\theta})}{1 - \chi_{b}\eta_{c}(\mu_{y} + k\gamma_{D})}\bar{\tau}_{S}^{p}$$

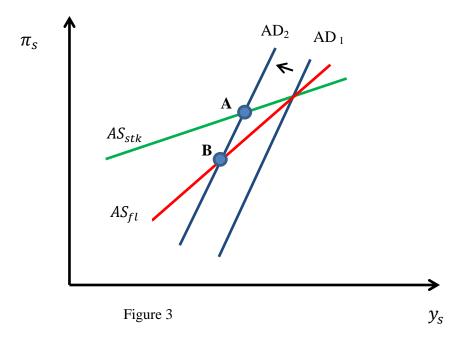
Likewise EK, we assume that any variations of T_t^b , G_t , $\tau_t^c C_t$, $\tau_t^w W_t H_t$, $\tau_t^p \frac{\Pi_t}{P_t}$ will be compensated by a variation of savers' lump-sum taxes. We leave the analysis of different funding policies to future research.

We can immediately see that if government decides to ignore the deflationary shock without intervening, economy would experiment a period characterized by crisis and deflation – i.e. a very similar result as in the last chapter:

$$y_{S} = \frac{\chi_{s}\sigma + \chi_{b}\eta_{c}\gamma_{D}\beta}{1 - \chi_{b}\eta_{c}(\mu_{y} + k\gamma_{D})}\bar{r} - \frac{\chi_{b}\eta_{c}}{1 - \chi_{b}\eta_{c}(\mu_{y} + k\gamma_{D})}\hat{D} < 0$$
$$\pi_{s} = \frac{k(\chi_{s}\sigma + \chi_{b}\eta_{c}\gamma_{D}\beta)}{1 - \chi_{b}\eta_{c}(\mu_{y} + k\gamma_{D})}\bar{r} - \frac{k\chi_{b}\eta_{c}}{1 - \chi_{b}\eta_{c}(\mu_{y} + k\gamma_{D})}\hat{D} < 0.$$

EK underline another paradox when economy is in a liquidity trap: the higher prices flexibility the higher output drop after a negative demand shock¹⁶. Indeed, the more prices decrease after a demand shock, the more borrowers reduce their spending, because their debt value is higher. In this case, flexibility does not help economy to return to normal levels. The graph below (Figure 3) represents a shock shifting AD leftward, and plots two different AS: AS_{fl} is steeper and corresponds to the case of high price flexibility; AS_{stk} is flatter and represents an economy with sticky prices. In the flexible case (point B), we can observe lower output and inflation.

¹⁶ Indeed, we can notice the equilibrium output is decreasing in k, which is the slope of the AS schedule.



We will now show that government has good instruments available to improve the situation.

We remind that y_S , g_S and t_S^b are deviation from the steady state as a fraction of \overline{Y} . On the other hand, tax rates are simply deviation from their steady state. We have not divided them by \overline{Y} , in order to have a more useful economic interpretation. The following analysis considers operator Δ as the variation with respect to the benchmark of no changes.

We calibrate the model by using parameters which are standard in American Macroeconomic literature.

As in the USA, tax rates differ between states, we do not have precise measures. Moreover, taxation is progressive, therefore we have to use approximated average values for the steady state tax rates. We set:

- $\bar{\tau}^c = 5\%$, following Eggertsson (2010a) and Uhlig (2010);

- $\overline{\tau}^w = 25\%$, which is between 20% in Eggertsson (2010a) and 28% in Uhlig (2010);

- $\bar{\tau}^p = 30\%$ (Eggertsson 2010a).

We consider $\gamma_D = 1$, namely we assume after the deleveraging shock, the borrowers debt position is equal to \overline{Y} : in Benigno (2012) the initial debt position (before the shock) is 120% of GDP.

With regard to households' shares, economic literature has estimated non-Ricardian households between 40-60% of population: we set this value at 0.5, like Campbell and Mankiw (1989).

In order to obtain a stable equilibrium, the AS schedule must be flatter than AD: by using the standard value $\lambda = 0.25$, we respect this restriction.

With our calibration, the denominator of each fiscal variable amounts to 0.25. As a consequence, it gives a multiplicative effect to each fiscal policy. This multiplicative effect is due to two channels:

M1) Any increase in output raises borrowers' consumption (i.e. standard Keynesian effect).

M2) Any increase in output raises price level, thus reducing the real debt burden and boosting borrowers' consumption.

7.1 An Increase in Government Consumption

Suppose fiscal authority decides to reverse the output fall with an increase in government expenditure. The multiplier is:

$$\frac{\Delta y_s}{\Delta g_s} = \frac{1 - \chi_b \eta_c (\delta \mu_\vartheta \sigma^{-1} + \gamma_D k \varphi)}{1 - \chi_b \eta_c (\mu_v + k \gamma_D)} = 2.54.$$

This means that for each additional Dollar of government consumption, output increases by \$ 2.54. The increase of g_s has an impact on the output via the following channels.

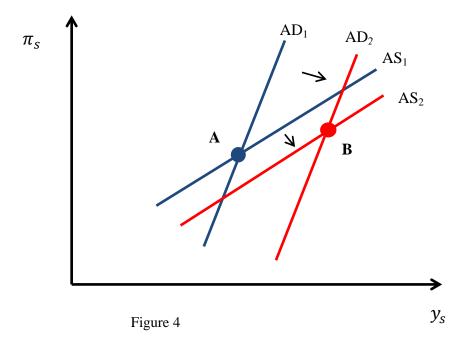
First of all, as g_s is a component of the aggregate demand, it directly affects the output positively.

The second effect is on the current available income of borrowers (see 7.2): the direct impact of g_s on y_s increases debtors' income (M1). On the other hand, with a higher g_s , other things equal, the marginal utility of consumption increases, and therefore wages decrease, lowering debtors' income (because of the positivity of δ).

The other two effects are due to AS movements: on one hand, this policy creates inflationary pressures and increases the price level, reducing the real value of debt (M2). On the other hand, the higher public spending expands the aggregate supply, and this reduces the output, as AD is upward sloping. The former effect is dominant, the multiplier is increasing in k^{17} . We can easily observe it by setting k=0. In this case, all firms charge prices one period in advance and the aggregate supply is horizontal: the multiplier is 1.59 < 2.54.

Finally, as expected, the multiplier is increasing in the number of borrowers: by letting their shares from 1/2 to 1/3, the multiplier becomes 1.52.

The graph show the movements of AD and AS: in the new equilibrium, represented by point B, output and prices are higher.



7.2 A Reduction in Lump-Sum Taxes on Borrowers

The multiplier in this case is:

 $^{^{17}}$ This is true if $\,\lambda\!<\!0.36.$ Otherwise, AS is steeper than AD, and the equilibrium is unstable.

$$\frac{\Delta y_s}{-\Delta t_s^b} = \frac{\chi_b \eta_c}{1 - \chi_b \eta_c (\mu_y + k\gamma_D)} = 1.88.$$

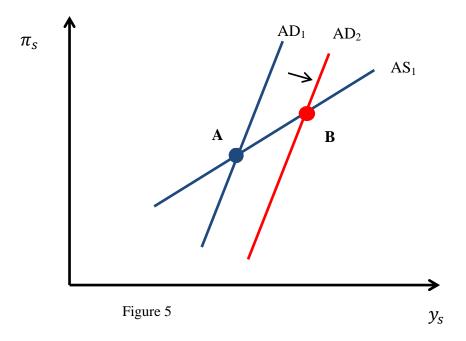
One Dollar reduction in borrowers' lump-sum taxes will increase the output by \$ 1.88.

Here the fiscal policy impacts on the output through the rise in debtors' consumption. This result is augmented by the multiplicative effect explained above, M1 and M2. If we reduce the share of borrowers to 1/3, the multiplier drastically decreases to 0.62.

In EK, the reduction of borrowers' lump-sum taxes is equal to the rise in their consumption. In our extension with distortionary taxation, the effect is lower and the multiplier $\frac{\Delta c_S^b}{-\Delta t_S^b}$ is equal to η_c =0.95.

We can easily see that a reduction of lump-sum taxes is effective only when targeted to debtors: lump-sum taxes on savers do not appear in AD, as these households are forward looking. So, as underlined by EK, who benefits from the tax cut matters, whether lenders or debtors.

Figure 5 shows the effect of this policy: only AD schedule is affected; in the new equilibrium, both inflation and output are higher.



7.3 A Reduction in Consumption Tax Rate

The tax rate multiplier gives us the amount of the output percentage variation if the tax rate changes by 1%.

In the case of a reduction in consumption tax rate, the multiplier is:

$$\frac{\Delta y_s}{-\Delta \hat{\tau}_S^c} = \frac{\chi_s \sigma \eta_c + \chi_b \eta_c (\gamma_c - \delta \mu_\vartheta \eta_c - \gamma_D \mu_\lambda \eta_c)}{1 - \chi_b \eta_c (\mu_y + k \gamma_D)} = 1.58.$$

Therefore if government cuts $\hat{\tau}_{S}^{c}$ by 1%, the output increases by 1.58%. It is evident from (7.3) that consumption tax rate has a direct impact on savers' consumption: a decrease in $\hat{\tau}_{S}^{c}$ for a household means that the store price of a unit of consumption good is lower. Indeed, like P_{t} and P_{t+1} , also τ_{t}^{c} and τ_{t+1}^{c} enter the Euler equation of savers.

This policy also affects borrowers consumption via two channels:

- Like savers, borrowers face a lower store price;

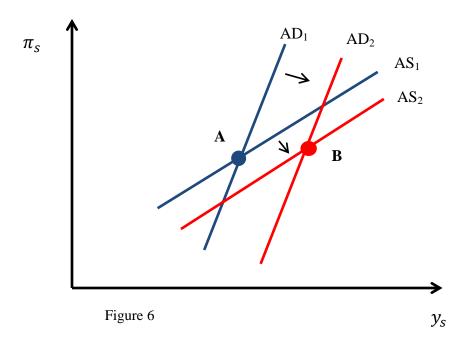
- Wages decrease: borrowers decrease consumption because δ is positive.

All the previous effects are augmented by M1.

As in the case of a rise in government spending, also AS is affected by this policy: on one side, a reduction of $\hat{\tau}_{S}^{c}$ generates higher prices (this increases output by M2); on the other side, it expands the aggregate supply (which reduces output). The first effect is dominant, and also this multiplier is increasing in k.

Here we have assumed that the price set by the firm is exclusive of consumption tax: this implies that a 1% reduction in τ_t^c means a 1% reduction of the total amount paid by households to buy a unit of consumption good. As pointed out by Eggertsson (2010a), this assumption is consistent with empirical evidence of variation in consumption taxes in the USA. Nevertheless, if we consider value-added taxes (VAT), such as the ones common in Europe, this assumption loses plausibility, as prices are normally inclusive of VAT: in this case a 1% reduction in τ_t^c corresponds to a 1% lower purchasing price only for the λ firms which set prices freely at all times.

The graphical representation is the same as in the case of public consumption increase: both prices and output are higher.



7.4 A Reduction in Labor Tax Rate

When government decides for a reduction in $\widehat{\tau_S^w}$, the multiplier is the following:

$$\frac{\Delta y_s}{-\Delta \widehat{\tau_s^w}} = \frac{\chi_b \eta_c [\mu_{\vartheta} (1 - \delta \eta_w) - \gamma_D \mu_\lambda \eta_w]}{1 - \chi_b \eta_c (\mu_y + k \gamma_D)} = 0.63.$$

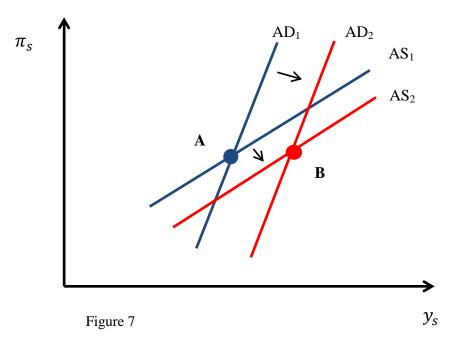
This policy is less effective than a cut in consumption tax rate: the difference is almost totally due to savers' consumption. Indeed, in this case, savers do not modify their consumption decisions.

Borrowers' income is affected as follows: on one side, a cut in labor tax rate increases borrowers' income (they have to pay lower taxes); on the other side this policy reduces it (for the reduction of wages). The first effect is dominant (1 $> \delta \eta_w$), the multiplier is positive and subject to M1.

The presence of AS introduces, as already seen, two other channels: one increases the output (by M2); the other reduces it (by supply expansion). In the case of an

increase in public government or a cut in consumption tax, the first effect is dominant and multipliers are increasing in k. However, these effects have almost the same size and the multiplier is just slightly increasing in the number of flexible firms. If k=0, the multiplier becomes 0.61.

Eggertsson (2010a) finds a negative multiplier of labor tax rate, when the economy is in a liquidity trap. In his model, all households are Ricardian and their consumption function is not affected by a reduction of labor tax rate: AD does not move after this policy. However, in our model AD does move, because a variation in the tax rate on labor affects borrowers' income, hence their consumption. Here we show the graphical representation: here prices do not raise.



7.4 A Reduction in Profit Tax Rate

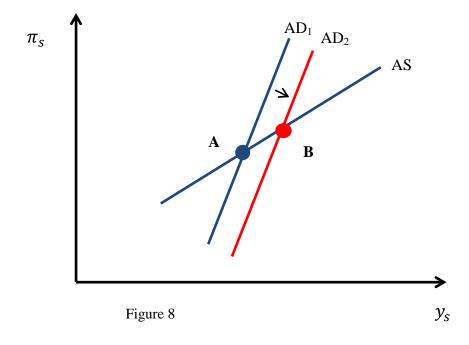
If the government reduces $\widehat{\tau_S^p}$, we obtain the following multiplier:

$$\frac{\Delta y_s}{-\Delta \widehat{\tau_s^p}} = \frac{\chi_b \eta_c (1 - \mu_\vartheta)}{1 - \chi_b \eta_c (\mu_y + k\gamma_D)} = 0.31.$$

As in the case of a reduction in borrowers' lump-sum taxes, this policy affects just the borrowers' consumption: the impact on output is positive (because $\mu_{\vartheta} < 1$) and is augmented by M1 and M2.

Notice that a reduction of the tax rate on profit has no influence on firms' behavior: this is not surprising, because a reduction in $\widehat{\tau}_{S}^{p}$ implies higher net revenues but also higher net labor costs, in the same proportion.

Figure 8 shows the effects of this policy:



8. Long-run price level

In this Chapter we are going to compute fiscal multipliers in the case of central bank controlling the long-run price level. We will see that monetary policy can be effective also in a liquidity trap, if we assume that agents believe the promises for higher inflation. Indeed, the natural solution to a liquidity trap is to generate higher expected inflation, which makes the real interest rate negative, to give stimulus to the consumption of both households.

We consider a central bank which in the short run determines the nominal interest rate, according to the Taylor rule:

$$i_S = \max(0, r_S^n + \phi_\pi \pi_S),$$

and, in the long run, it anchors prices to the pre-shock level

$$P_L = P_0 > P_S,$$

where P_0 is the price level before the shock¹⁸.

By this rule, the inflation level is:

$$\pi_L = -\pi_S$$

Since inflation is determined by monetary policy, in the long run we obtain:

$$\pi_t = E_{t-1} \pi_t \ (8.1).$$

We assume that in the long run fiscal policy fixes its instrument at the steady state level:

$$g_L = \widehat{\tau_L^w} = \widehat{\tau_L^p} = \widehat{\tau_L^c} = t_t^b = 0 \quad (8.2).$$

Notice that AS, (8.1) and (8.2) imply that $y_L = 0$

As before, we consider a big deleveraging shock hitting the economy in the short run and forcing central bank to fix the nominal rate at zero. By plugging the new long-run monetary rule, in the short run we obtain the following consumption functions:

$$c_S^s = E_S c_L^s - \sigma(\pi_S - \bar{r}) - \sigma \eta_c \hat{\tau}_S^c = -\sigma(i_L + \pi_S - 2\bar{r}) - \sigma \eta_c \hat{\tau}_S^c \quad (8.3),$$

¹⁸ Notice that after the long run, the economy reaches a new steady state with zero inflation.

$$c_{S}^{b} = \eta_{c} \left[\mu_{y} y_{S} - \delta \mu_{\vartheta} \sigma^{-1} g_{S} - (1 - \mu_{\vartheta}) \widehat{\tau_{S}^{p}} - \mu_{\vartheta} (1 - \delta \eta_{w}) \widehat{\tau_{S}^{w}} - (\gamma_{c} + \delta \mu_{\vartheta} \eta_{c}) \widehat{\tau_{S}^{c}} + \gamma_{D} (1 - \beta) \pi_{S} - \widehat{D} + + \gamma_{D} \beta \overline{r} - t_{S}^{b} \right] (8.4).$$

(8.3) shows that if there is a deflation in the short run, savers know that in the long run central bank will create inflation: consumption goods will be more expensive in the future and current real interest rate is lower, thus savers prefer to consume more in the present.

By (8.4), we observe that deflation continues to negatively affect borrowers' consumption, even though this effect is lower than before. Indeed, with this monetary rule, deflation lowers the real interest rate, allowing borrowers to increase the amount of debt they can borrow, and other things equal (in the model without price level reversion, this channel is absent). On the other side, a deflation has also the effect of increasing the burden of their debt. The latter effect always dominates the former and borrowers' consumption slightly decreases after a deflation (since $\beta = 0.99$).

Therefore, AD becomes:

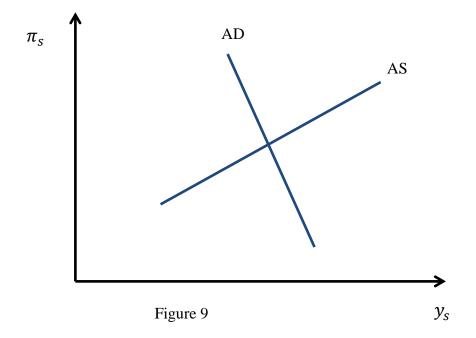
$$y_{S} = \frac{2\chi_{S}\sigma + \chi_{b}\eta_{c}\gamma_{D}\beta}{1 - \chi_{b}\eta_{c}\mu_{y}}\bar{r} - \frac{\sigma\chi_{S}}{1 - \chi_{b}\eta_{c}\mu_{y}}i_{L} - \frac{\sigma\chi_{S} - \chi_{b}\eta_{c}\gamma_{D}(1 - \beta)}{1 - \chi_{b}\eta_{c}\mu_{y}}\pi_{S}$$
$$- \frac{\chi_{b}\eta_{c}}{1 - \chi_{b}\eta_{c}\mu_{y}}\bar{D} - \frac{\chi_{b}\eta_{c}}{1 - \chi_{b}\eta_{c}\mu_{y}}t_{S}^{b}$$
$$- \frac{\chi_{S}\sigma\eta_{c} + \chi_{b}\eta_{c}(\gamma_{c} - \delta\mu_{\theta}\eta_{c})}{1 - \chi_{b}\eta_{c}\mu_{y}}\bar{\tau}_{S}^{c} + \frac{1 - \chi_{b}\eta_{c}\delta\mu_{\theta}\sigma^{-1}}{1 - \chi_{b}\eta_{c}\mu_{y}}g_{S}$$
$$- \frac{\chi_{b}\eta_{c}\mu_{\theta}(1 - \delta\eta_{w})}{1 - \chi_{b}\eta_{c}\mu_{y}}\bar{\tau}_{S}^{w} - \frac{\chi_{b}\eta_{c}(1 - \mu_{\theta})}{1 - \chi_{b}\eta_{c}\mu_{y}}\bar{\tau}_{S}^{p}.$$

Notice that now inflation negatively affects the AD schedule: with a deflation, as $(1 - \beta)$ is close to zero, the rise in savers' consumption is higher than the decrease in borrowers' consumption.

AS line does not change:

$$\pi_s = k y_s - k \varphi g_s + \mu_\lambda (\eta_c \widehat{\tau_s^c} + \eta_w \widehat{\tau_s^w}).$$

Figure 9 plots both lines in a plane: now AD is downward sloping.



By the AD equation, we can observe that fiscal instruments have the same coefficients as in previous chapters. By plugging AS in AD, we introduce, as before, two new channels: both now work in the opposite direction in respect to the previous chapter, because of the different slope of AD:

- Policies aiming to increase public expenditure or to reduce labor and consumption tax rate expand the aggregate supply; this now increases the output, as AD is downward sloping: the paradox of toil does not work anymore. This effect is not present if government reduces borrowers' lump-sum taxes or profit tax rate.

- Any fiscal expansion (also a reduction in lump-sum taxes or in profit tax rate) increases the price level above what would have been otherwise, and this now reduces aggregate consumption.

The second channel is dominant, so now $\frac{\Delta y_s}{\Delta g_s}$ and $\frac{\Delta y_s}{-\Delta \tau_s^2}$ are decreasing in the AS slope. Therefore, both multipliers are lower than before.

On the other hand, by considering a cut in labor tax rate, in the previous chapter these two effects had different signs but about the same size in absolute value, and $\frac{\Delta y_s}{-\Delta \hat{\tau}_s^W}$ was almost independent on variations in the AS slope. With a downward sloping AD, both effects change their signs, but they continue to have the same size; therefore, this multiplier does not change significantly, compared to that in the previous chapter.

Here we report multipliers' values and we compare them with the ones in the previous chapter.

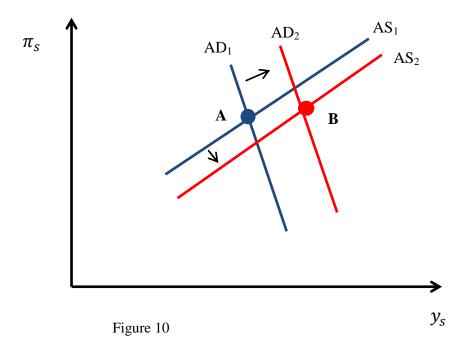
$$\frac{\Delta y_s}{\Delta g_s} = \frac{1 - \chi_b \eta_c \delta \mu_{\vartheta} \sigma^{-1} + k\varphi[\chi_s \sigma - \chi_b \eta_c \gamma_D (1 - \beta)]}{1 - \chi_b \eta_c \mu_y + k[\chi_s \sigma - \chi_b \eta_c \gamma_D (1 - \beta)]} = 1.43 < 2.54$$

$$\frac{\Delta y_s}{-\Delta \hat{\tau}_S^c} = \frac{\chi_s \sigma \eta_c + \chi_b \eta_c (\gamma_c - \delta \mu_\vartheta \eta_c) + \mu_\lambda \eta_c [\sigma \chi_s - \chi_b \eta_c \gamma_D (1 - \beta)]}{1 - \chi_b \eta_c \mu_y + k [\chi_s \sigma - \chi_b \eta_c \gamma_D (1 - \beta)]} = 0.8 < 1.58$$

$$\frac{\Delta y_s}{-\Delta \widehat{\tau_s^w}} = \frac{\chi_b \eta_c \mu_\vartheta (1 - \delta \eta_w) + \mu_\lambda \eta_w [\sigma \chi_s - \chi_b \eta_c \gamma_D (1 - \beta)]}{1 - \chi_b \eta_c \mu_y + k [\chi_s \sigma - \chi_b \eta_c \gamma_D (1 - \beta)]} = 0.61 < 0.63.$$

Now a rise in public expenditure or a cut in consumption tax rate is much less effective. As just pointed out, the reason is the different impact of the AS shift on AD schedule, because the latter is downward sloping. Indeed, if we consider a horizontal AS, multipliers are the same, both by considering an upward sloping AD (as in the previous chapter) and a downward sloping AD (as in this chapter).

The following graphical representation shows the effect of an increase in government expenditure.



As follows, the values of the other two multipliers. With both fiscal policies, the variation in output just depends on the shift of the AD schedule.

$$\frac{\Delta y_s}{-\Delta \widehat{\tau_s^p}} = \frac{\chi_b \eta_c (1-\mu_{\vartheta})}{1-\chi_b \eta_c \mu_y + k[\chi_s \sigma - \chi_b \eta_c \gamma_D (1-\beta)]} = 0.1 < 0.31$$
$$\frac{\Delta y_s}{-\Delta \widehat{\tau_s^p}} = \frac{\chi_b \eta_c}{1-\chi_b \eta_c \mu_y + k[\chi_s \sigma - \chi_b \eta_c \gamma_D (1-\beta)]} = 0.61 < 1.88$$

We have smaller values compared to the previous chapter: these policies are inflationary, and now inflation affects negatively the AD schedule because of the new specification of the monetary rule.

Finally we are going to consider a more general case in which central bank fixes the long-run price level without any tie with P_0 , in order to create a given level of inflation in "L".

$$P_L > P_S$$
 such that $\pi_L = \overline{\pi}$.

Now AD becomes

$$y_{S} = \frac{2\chi_{s}\sigma + \chi_{b}\eta_{c}\gamma_{D}\beta}{1 - \chi_{b}\eta_{c}\mu_{y}}\bar{r} - \frac{\sigma\chi_{s}}{1 - \chi_{b}\eta_{c}\mu_{y}}i_{L} - \frac{\chi_{b}\eta_{c}\gamma_{D}}{1 - \chi_{b}\eta_{c}\mu_{y}}\pi_{S} + \frac{\chi_{s}\sigma + \chi_{b}\eta_{c}\gamma_{D}\beta}{1 - \chi_{b}\eta_{c}\mu_{y}}\bar{\pi}$$
$$- \frac{\chi_{b}\eta_{c}}{1 - \chi_{b}\eta_{c}\mu_{y}}\widehat{D} - \frac{\chi_{b}\eta_{c}}{1 - \chi_{b}\eta_{c}\mu_{y}}t_{S}^{b}$$
$$- \frac{\chi_{s}\sigma\eta_{c} + \chi_{b}\eta_{c}(\gamma_{c} - \delta\mu_{\theta}\eta_{c})}{1 - \chi_{b}\eta_{c}\mu_{y}}\bar{\tau}_{S}^{c} + \frac{1 - \chi_{b}\eta_{c}\delta\mu_{\theta}\sigma^{-1}}{1 - \chi_{b}\eta_{c}\mu_{y}}g_{S}$$
$$- \frac{\chi_{b}\eta_{c}\mu_{\theta}(1 - \delta\eta_{w})}{1 - \chi_{b}\eta_{c}\mu_{y}}\bar{\tau}_{S}^{w} - \frac{\chi_{b}\eta_{c}(1 - \mu_{\theta})}{1 - \chi_{b}\eta_{c}\mu_{y}}\bar{\tau}_{S}^{p}.$$

Notice that if $\bar{\pi} = 0$, we return to the case of the previous chapter, with prices anchored to the short-run equilibrium level. If $\bar{\pi} = -\pi_s$, we return to the case discussed above.

Plugging AS in AD, we can compute a monetary multiplier, as central bank in the short run decides $\overline{\pi}$ and agents expect it not to deviate.

$$\frac{\Delta y_s}{\Delta \bar{\pi}} = \frac{\chi_s \sigma + \chi_b \eta_c \gamma_D \beta}{1 - \chi_b \eta_c (\mu_v + k \gamma_D)} = 2.85.$$

If central bank announces that long-run inflation will be higher by 1%, the output increases by 2.85%.

Therefore, if monetary authority is credible enough, in the short run it can announce it will create inflation in the long run: hence both types of households can spend more, as higher expectations on inflation reduce the current real interest rate. Also here we have the multiplicative effects (M1 and M2) examined in the previous chapter, which augment the positive impact of higher $\bar{\pi}$.

Moreover, expectations of higher inflation increase current inflation: a higher $\bar{\pi}$ acts as a rightward AD shift.

9. Conclusions

This paper analyzes a set of economic policies aiming to boost GDP, after a deleveraging shock has led the economy in a liquidity trap.

In line with the literature, we find remarkable effectiveness of fiscal policy (financed with lump-sum taxes on savers), when central bank is forced to fix the nominal rate at zero. We notably get high multipliers if government decides to increase its spending or to cut consumption tax rate. Also a reduction in lump-sum taxes is very effective, but only if targeted on borrowers.

These above results are mainly due to the increase in borrowers' consumption, which positively depends on income and price level (higher prices reduce the real value of their debt). For instance, an increase in government spending leads to higher income and higher prices, borrowers spend more and this leads to another income expansion, and so forth. If we consider a cut in labor or profit tax rate, the multiplier is smaller: in this case, the impact on borrower's consumption is not so large.

The effect of a deleveraging shock is not only the GDP contraction: it also reduces the price level, generating a deflation which makes borrowers even poorer, thus starting the Fisher effect. Government can stop the Fisher effect, as its expansionary policies have a positive impact on prices.

Therefore, we can state that when the economy experiences a deleveraging shock, fiscal policy has effective instruments to avoid a crisis period. However, policy-makers should carefully evaluate the number of borrowers and the degree of price flexibility of firms: these are two key variables to the effects of fiscal policy.

Finally, we have discussed what monetary policy can do in such a situation. In a liquidity trap, central bank cannot manage the nominal rate; however, if it is credible enough, it can increase inflation expectations, thus lowering the real rate and giving stimulus to the current spending of households.

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Appendix

A.1 The Steady State

We consider a steady state with zero inflation, in which debtors borrow up to their limit $D_t = \overline{D}$ and $i_t = r_t = \overline{r}$. The following list of relations must hold in the steady state:

The production function:

$$\overline{Y} = A\overline{H} = A\overline{H}^{\chi_s}(s)\overline{H}^{\chi_b}(b) \quad (A.1).$$

The labor demand of each type *i*:

$$\overline{H}^{i} = \overline{H} \left(\frac{\overline{W}^{j}}{\overline{W}^{i}} \right)^{\chi_{j}} (A.2) \text{ with } i, j = s \text{ or } b, and i \neq j$$

where $\overline{W} = \overline{W}^{\chi_s}(s) \overline{W}^{\chi_b}(b)$, which implies by (A.2): $\overline{W}^i \overline{H}^i = \overline{W} \overline{H}$, for each type *i*.

The labor supply of each type *i*:

$$\frac{\left(\overline{H}^{i}\right)^{\omega}}{z \exp[-z\overline{C}^{i}]} = \overline{W}^{i} \frac{(1-\overline{\tau}^{w})}{(1+\overline{\tau}^{c})}$$

which implies in aggregate:

$$\frac{(\overline{H})^{\omega}}{\operatorname{z} \exp[-z\overline{C}]} = \overline{W} \frac{(1-\overline{\tau}^{w})}{(1+\overline{\tau}^{c})} \ (A.3).$$

By considering that in steady state $\pi_t = E_{t-1}\pi_t = 0$, by the pricing equations of the firms the following relation must hold:

$$\overline{W} = \overline{A} \frac{(\theta - 1)}{\theta} \quad (A.4).$$

The households' consumption ¹⁹:

$$\bar{C}^{b} = (1 + \bar{\tau}^{c})^{-1} \left[(1 - \bar{\tau}^{p})\bar{Y} + (\bar{\tau}^{p} - \bar{\tau}^{w})(\bar{W}\bar{H}) - \frac{\bar{r}}{1 + \bar{r}}\bar{D} - \bar{T}^{b} \right] (A.5)$$
$$\bar{C}^{s} = (1 + \bar{\tau}^{c})^{-1} [(1 - \bar{\tau}^{p})\bar{Y} + (\bar{\tau}^{p} - \bar{\tau}^{w})(\bar{W}\bar{H}) - \bar{r}\bar{B}^{s} - \bar{T}^{s}] (A.6).$$

The government budget constraint:

$$\bar{G} + \bar{r}\bar{B}^g = \bar{\tau}^c\bar{C} + \bar{\tau}^p\bar{Y} + (\bar{\tau}^w - \bar{\tau}^p)(\bar{W}\bar{H}) + \chi_s\bar{T}^s + \chi_b\bar{T}^b (A.7).$$

¹⁹ Remind that a negative value of \overline{B}^s means asset.

The good market clearing condition:

$$\bar{Y} = \chi_s \bar{C}^s + \chi_b \bar{C}^b + \bar{G} \quad (A.8)$$

The bond market clearing condition:

$$\chi_b \frac{\overline{D}}{1+\overline{r}} + \overline{B}^g + \chi_s \overline{B}^s = 0 \quad (A.9)$$

A.2 Baseline Calibration

Here we list the parameters used to calculate the multipliers.

Parameter	Value	Description
Z	2	Degree of risk aversion
ω	0.2	Inverse of the Frisch labor supply elasticity
\overline{Y}	1	Steady-state output
\overline{H}	0.36	Steady state hours of work
Ē	0.7	Steady state aggregate consumption
$ar{C}^b$	0.7	Steady state borrowers' consumption
$ar{ au}^c$	0.05	Steady state consumption tax rate
$ar{ au}^w$	0.25	Steady state labor tax rate
$ar{ au}^p$	0.3	Steady state profit tax rate
\overline{D}	1	Steady state debt limit
β	0.99	Savers' discount factor
θ	6	Elasticity of substitution across varieties of goods
λ	0.25	Fraction of flexible firms
χ _b	0.5	Fraction of borrowers

This calibration implies that the steady-state wage index and the technology factor are respectively: $\overline{W} = 2.31$ and $\overline{A} = 2.77$.

Notice that we have defined the intertemporal elasticity of substitution as $\sigma \equiv \frac{1}{z\overline{Y}}$: by this calibration $\sigma = 0.5$. Finally we can observe that $\bar{C} = \bar{C}^b$ implies $\bar{C}^b = \bar{C}^s$: this is consistent with an appropriated parameterization of $\{\bar{B}^g, \bar{T}^s, \bar{T}^b\}$. For example, we can use $\{0.7, 0.03, 0\}$. In this way we respect (A.5), (A.6), (A.7) and (A.9). However this assumption is not critical and it is possible to choose other values for \bar{C}^b .