JOBLESS RECOVERY: A GENERAL EQUILIBRIUM MODEL WITH A COLLATERAL CONSTRAINT

Summary

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Academic year 2012/2013
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1 Introduction

A recurring phenomenon has been observed in the recovery from a few crisis episodes from the past decade and from the 2008 financial crisis in particular. At a time when output has already recovered its pre-crisis level, employment tends to lag behind, so that the recovery takes place without a reduction in unemployment, i.e. a jobless recovery occurs. Many different explanations have been provided for this fact, most of which rely on wage rigidities assumptions and other labor market imperfections. An alternative theory primarily focuses on credit constraints rather than on labor market issues. According to this theory, firms can borrow against their physical capital (their tangible assets), while they are not able to obtain a loan on the basis of their human capital (their employees). After a crisis, firms need liquidity in order to start reinvesting and restructuring. At the same time, banks are less prone to lend and raise their requirements for granting loans. Indeed, they are aware of the higher probability of dealing with a distressed firm, that might not be able to repay its debt. As a consequence, all firms, whether distressed or not, are required to post an higher amount of collateral, so that the creditor is protected against the possibility of default on the part of the debtor. Capital intensive projects are hence favored over labor intensive ones, because of the characteristic of inalienability of human capital. Production is adjusted in such a way that a lower level of employment is needed to produce a unit of output. This is the mechanism that causes output to recover, and unemployment to remain high.

This paper develops a general equilibrium model to analyze this phenomenon. In the first part, a frictionless neoclassical growth model for a decentralized equilibrium is presented, following Shimer (2012). Successively, a credit constraint on the resources of the firm is introduced and finally, rigid wages are assumed. In the end, a number of conclusions will be drawn from the results obtained.

2 Literature Review

Throughout time, the jobless recovery phenomenon has been analyzed from a wide range of points of view, looking for its determinants and considering its consequences and possible remedies. The main cause for high unemployment levels - after a recovery from a crisis and in general - was typically considered to be the structure and rigidities of labor markets. Most economists believed that it was the lack of rigidities in the US labor market to ensure a low rate of unemployment, while the higher European rate was the consequence of its job market’s frictions. But in fact, a European labor market as a whole does not even exist, so that a comparison with the unified American market for jobs makes little sense. Moreover, looking at the average unemployment level alone may not
provide an accurate picture: a number of European countries do exhibit higher unemployment rates than the US, but some others do not, and not all of the countries with more flexible labor markets also exhibit lower unemployment levels (see e.g. the United Kingdom). This hints at the possibility that labor market rigidities may not be a sufficient explanation. When high unemployment levels became a feature of the flexible American labor market during the recovery from the 2008 financial crisis, it was clear that a different approach to explain this fact was needed.

Acemoglu (2001) notices how neither institutional changes nor macroeconomic factors are thoroughly convincing explanations for the European high and persistent unemployment level. In particular, there were not many major institutional reforms in the 1970’s that could motivate the surge in unemployment occurring after the 1980’s, and macroeconomic shocks of different types were never large enough to account for that either. Therefore, a credit market explanation for this fact is proposed, showing that European firms belonging to credit-dependent sectors have a lower share of employment with respect to comparable American firms, although growth in the two cases is approximately the same.

Calvo, Coricelli, and Ottonello (2012) also provide a financial market explanation for output growth occurring with high unemployment levels. Their paper considers both jobless and wageless recovery phenomena: these are the two possible outcomes of a financial crisis. When the value of the collateral of a firm experiences a sudden drop, countries with low inflation and rigid wages face a jobless recovery, while countries with high inflation will go through a wageless recovery. A cross country regression is performed on a split sample of high and low inflation countries, including both developed and emerging economies. Data show that in case of non-financial recessions, neither of the two phenomena affect the recovery process. A simple partial equilibrium model also supports the empirical analysis, adding an exogenous collateral constraint to the standard maximization problem of the firm\(^1\). During a financial crisis, the constraint becomes binding at optimum, but while it has a full impact on the amount of labor costs the firm can bear, it only partially influences the amount of capital used for production. The principle of inalienability of human capital puts capital and labor costs on two different levels, so that labor costs result in being more restricted than capital by the constraint on collateral.

The concept of human capital inalienability is inherited from a long-standing strand of literature. Hart and Moore (1994), for example, propose a model for debt financed projects, showing that in a number of cases, a profitable investment opportunity will be missed because firms are subject to credit constraints. In fact, an entrepreneur always has the option to repudiate a debt contract by withdrawing human capital from the project. In that case, the investor will only be able to retaliate

\(^1\)This is the kind of constraint that will be used in the analysis that follows.
by liquidating the project’s physical assets, and this creates an upper bound on the total amount of indebtedness of the entrepreneur.

Almeida and Campello (2007) provide further insight into the influence of financial frictions on real investment decisions. They define a “credit multiplier” for which pledgeable assets support borrowing for further investment in pledgeable assets. They identify a non-linear relationship between sensitivity to cash flows of the project and tangibility of the assets of firm.

A seminal paper by Kiyotaki and Moore (1995) shows how credit market frictions give rise to a credit multiplier that amplifies business cycle fluctuations. These “credit cycles” are characterized by major spillover effects, higher persistence and amplification of shock. A key assumption is that a creditor cannot force the borrower to repay his debt except for the secured part of the loan: this naturally gives rise to credit constraints. Amplification and persistence are in place through both a static and an inter-temporal credit multiplier.

Along the same line, Bernanke, Gertler, and Gilchrist (1999) propose a principal-agent model with macroeconomic relevance. They identify the solvency of the borrower as one of the sources for macroeconomic fluctuations, and modify a classic real business cycle model to account for an informational asymmetry between entrepreneurs and investors. This inefficiency produces agency costs, due to which internal financing is less expensive than external financing. When there is a recession, the net worth of the borrower is reduced and agency costs rise, causing investment fluctuations and shock persistence.

3 Model Setup

The proposed model is a classic real business cycle model, where two sources of frictions are introduced to reproduce the jobless recovery phenomenon: wage rigidity and a collateral constraint on the borrowing of the firm. In fact, given a credit constraint, if wages were fully flexible, they would adjust at a lower level so as to avoid a jobless recovery, causing a “wageless recovery” instead. Since these two frictions need to be incorporated into the model, it can be shown that there is no equivalence between the “planner’s problem” and the competitive equilibrium\(^2\). For this reason, a decentralized equilibrium needs to be analyzed, where households maximize their utility function by choosing the optimal path for consumption and leisure subject to an inter-temporal budget constraint, while firms maximize their expected profits, subject to a collateral constraint. The problem will be analyzed in three steps: the standard model without constraints and with fully flexible wages is first considered, then a credit constraint will be included, and only at the end wage

\(^2\)I.e. the welfare theorem does not apply here.
rigidities will also be assumed.

3.1 Baseline model

A representative household is considered. The number of components of the household is normalized to 1 and they live infinitely. They discount utility from future consumption at the rate of $\beta$. Labor is indivisible. A fraction $n_t$ of the household members is employed and consumes $C_{e,t}$, while a fraction $1 - n_t$ is unemployed and consumes $C_{u,t}$. Total consumption for the household is thus

$$C_t = n_tC_{e,t} + (1 - n_t)C_{u,t} \quad (1)$$

The problem for the representative family can be stated as:

$$\max \sum_{t=0}^{\infty} \beta^t \left( n_t \frac{C_{e,t}^{1-\sigma} (1 + (\sigma - 1)\gamma)^\sigma}{1 - \sigma} + (1 - n_t) \frac{C_{u,t}^{1-\sigma}}{1 - \sigma} \right)$$

subject to

$$a_t + w_t n_t = \frac{q_{t+1}^{0}}{q_0^{0}} a_{t+1} + (n_tC_{e,t} + (1 - n_t)C_{u,t})$$

The parameter $\sigma$ measures risk aversion and is the inverse of the inter-temporal elasticity of substitution. It also determines the complementarity between labor and consumption, hence defining whether the employed will consume more than unemployed individuals. $\gamma > 0$ is the disutility that the employed must bear from working. The budget constraint simply states that resources at time $t$ (on the left-hand side of the equation) can be either consumed or saved and invested for the next period. In particular, $q_0^t$ is the time 0 price of a unit of consumption at time $t$, $a_t$ is the amount of assets of the household at time $t$, set at time $t-1$, given an initial endowment at time 0 of $a_0$, and $w_t$ is the wage.

Form the Lagrangian and derive first order conditions by computing derivatives with respect to $C_{e,t}$, $C_{u,t}$, $n_t$ and $a_{t+1}$.

$$\left( \frac{C_{e,t}}{1 + (\sigma - 1)\gamma} \right)^{-\sigma} = \lambda_t \quad (2)$$

$$C_{e,t}^{\sigma} = \lambda_t \quad (3)$$

$$\beta \frac{\lambda_{t+1}}{\lambda_t} = \frac{q_{t+1}^{0}}{q_0^{0}} \quad (4)$$
\[ w_t = \frac{\sigma}{\sigma - 1} \left( C_{e,t} - C_{u,t} \right) \tag{5} \]

As it is usually the case, \( \lambda_t \), the Lagrangian multiplier, corresponds to the marginal utility of aggregate consumption, i.e.

\[ \lambda_t = \left( \frac{C_t}{1 + (\sigma - 1)\gamma n_t} \right)^{-\sigma} \tag{6} \]

Combining equations (2) and (3) with (5), what emerges is that the wage is equal to the marginal rate of substitution between consumption and leisure:

\[ w_t = \sigma \gamma \lambda_t^{-\frac{1}{\sigma}} \tag{7} \]

Let us now analyze the maximization problem for a representative firm, whose aim is to maximize the present value of its profits by deciding a time path for capital and employment. The production function is a Cobb-Douglas, with labor augmenting technology evolving overtime according to

\[ A_{t+1} = \left( 1 + g \right) A_t \]

\[ K_{t+1} = K_t \left( A_t n_t \right)^{1-\alpha} + (1 - \delta) K_t - K_{t+1} - w_t n_t \]

\[ q_t^{t+1} \left[ \kappa_t^{\alpha-1} + (1 - \delta) \right] = q_0' \tag{8} \]

\[ (1 - \alpha) A_t \kappa_t^\alpha = w_t \tag{9} \]

where \( \kappa_t = \frac{K_t}{A_t n_t} \) is capital per efficiency unit of labor.

It is now possible to combine the result for the household and the firm to obtain a Euler equation and an employment equilibrium condition for the whole economy:

\[ \lambda_t = \beta \lambda_{t+1} \left( \alpha \kappa_{t+1}^{\alpha-1} + 1 - \delta \right) \tag{10} \]

\[ (1 - \alpha) A_t \kappa_t^\alpha = \sigma \gamma \lambda_t^{-\frac{1}{\sigma}} \tag{11} \]

The resource constraint for the entire economy is

\[ K_{t+1} = K_t^\alpha \left( A_t n_t \right)^{1-\alpha} + (1 - \delta) K_t - C_t \tag{12} \]
After some manipulation, two equations for the marginal utility of consumption and capital can be obtained:

\[ \lambda_t = \beta \lambda_{t+1} \left[ \alpha \left( \frac{(1-\alpha)A_{t+1}^{\frac{1}{\sigma}} \lambda_{t+1}^{\frac{1}{\sigma}}}{\sigma \gamma} \right)^{\frac{1-\alpha}{\alpha}} + 1 - \delta \right] \]

\[ K_{t+1} = \left[ \left( \frac{(1-\alpha)A_{t}^{\frac{1}{\sigma}} \lambda_{t}^{\frac{1}{\sigma}}}{\sigma \gamma} \right)^{\frac{1-\alpha}{\alpha}} \left( \frac{1-\alpha + \alpha \sigma}{\sigma} \right) + 1 - \delta \right] K_t - \lambda_t^{-\frac{1}{\sigma}} \]

These two equations completely describe the dynamics of the economy. From here, an equilibrium for the model can be found, by solving a linear system of two first order difference equations in two unknowns (\( \lambda \) and \( K \)). This is possible after log-linearizing the two equations around a balanced growth path where labor augmenting technology, capital, output and consumption (of both employed and unemployed individuals) all grow at rate \( g \), employment is constant, and the marginal utility of consumption \( \lambda_t \) grows at \((1 + g)^{-\sigma} - 1\). The log-linearized expressions result in:

\[ \hat{\Lambda}_{t+1} = \frac{\alpha \sigma}{1 - \alpha + \alpha \sigma - \beta(1-\alpha)(1-\delta)(1+g)^{-\sigma}} \hat{\Lambda}_t \]

\[ \hat{k}_{t+1} = \frac{(1-\alpha + \alpha \sigma)(1+g)^{\sigma} - \beta(1-\delta)(1-\alpha)}{\beta \alpha \sigma (1+g)} \hat{k}_t + \frac{(1-\alpha + \alpha \sigma)(1+g)^{\sigma} - \beta(1-\delta)(1-\alpha) - \beta \sigma \alpha^2 (1+g)^{\sigma} \hat{\Lambda}_t}{\beta \alpha^2 \sigma^2 (1+g)} \]

### 3.2 Credit constraint

Let’s now analyze the case in which the problem for the household is left unchanged, while firms face a constraint on the amount of borrowing they can undertake. In particular, the resources needed for production in period \( t \) are \( K_{t+1} + w_t n_t \), but they can be no higher than the total amount of collateral that the firm owns. This is equal to the exogenous credit constraint \( Z_t + (1-\theta)K_{t+1} \) where \( Z_t \) is the extrinsic collateral and \((1-\theta)K_{t+1}\) the proportion of total capital that serves as intrinsic collateral, \((0 \leq \theta \leq 1)\). The parameter \( \theta \) measures the extent to which capital enters in the credit constraint, i.e. if \( \theta = 0 \) then capital is its own collateral and is not subject to the credit constraint at all (only human capital will be restricted). The problem of the firm becomes:
\[ \mathcal{L} = \sum_{t=0}^{\infty} q_0^t \left\{ K_t^\alpha (A_t n_t)^{1-\alpha} + (1 - \delta) K_t - K_{t+1} - w_t n_t + \mu_t [Z_t - \theta K_{t+1} - w_t n_t] \right\} \]

The interesting case is when the constraint is binding at optimum, i.e. it holds with equality. In this case, first order conditions are

\[ q_0^{t+1} \left( \alpha \kappa_{t+1}^{\alpha-1} + 1 - \delta \right) = q_0^t (1 + \mu_t \theta) \] (13)

\[ (1 - \alpha) \kappa_t^\alpha A_t = w_t (1 + \mu_t) \] (14)

\[ \theta K_{t+1} + w_t n_t = Z_t \] (15)

Put equations 13 and 14 together, then use the Euler equation (4) to eliminate prices

\[ \beta \lambda_{t+1} \left( \alpha \kappa_{t+1}^{\alpha-1} + 1 - \delta \right) = \lambda_t \left[ (1 - \theta) + \theta (1 - \alpha) \kappa_t^\alpha \left( \frac{A_t}{w_t} \right) \right] \] (16)

Substituting for both employment and wages, one obtains:

\[ \beta \lambda_{t+1} \left( \alpha \left( \frac{A_t \lambda_t^{\frac{1}{\sigma}} (Z_t - \theta K_t)}{\kappa_t^{1-\alpha}} \right)^{1-\alpha} + 1 - \delta \right) = \lambda_t \left[ (1 - \theta) + \theta (1 - \alpha) \left( \frac{A_t \lambda_t^{\frac{1}{\sigma}} (Z_t - \theta K_{t+1})}{\kappa_t} \right)^{-\alpha} \left( \frac{A_t \lambda_t^{\frac{1}{\sigma}}}{\kappa_t^{1-\alpha}} \right) \right] \] (17)

Eliminate wage, employment and consumption from the resource feasibility of the economy (12) which is left unchanged from the baseline model

\[ K_{t+1} = \left[ \left( \frac{A_t \lambda_t^{\frac{1}{\sigma}} (Z_t - \theta K_{t+1})}{\kappa_t^{1-\alpha}} \right)^{1-\alpha} + (1 - \delta) \right] K_t - \lambda_t^{-\frac{1}{\sigma}} - \frac{(\sigma - 1)}{\sigma} (Z_t - \theta K_{t+1}) \] (18)

These are the two fundamental relationships of the model: after their log-linearization, it is possible to solve for an equilibrium. In particular, when the model is calibrated and resolved by means of a statistical software, it is possible to see what the reaction of the variables to different kinds of shocks is.
3.3 Rigid wages

A key feature of the neoclassical growth model is that the reaction of employment following a negative shock on the stock of capital is an increase on impact from its steady state value. In order to have this tendency reversed, the introduction of some form of rigidities for wages is needed. In particular, it is assumed that households have no bargaining power as far as wages are concerned. They need to take whatever wage level is proposed to them, and supply labor at that price. Thus, wages grow according to an exogenous path: \( w_{t+1} = (1 + g) w_t \).

This equation will substitute the wage equation (7) that would result from families maximizing their employment level. The maximization problem for firms, on the contrary, is left unchanged. As \( t \) goes to infinity, the wage equation has no solution and tends to explode. However, it can be made steady-state compatible by dividing it for the law of growth of labor-augmenting technology. This results in: \( \frac{w_t}{A_t} = \frac{w_0}{A_0} \).

If this result is plugged into equation (9), then the expressions for the unconstrained rigid wage model can be easily derived.

\[
K_{t+1} = \left[ \left( \frac{(1 - \alpha)A_0}{w_0} \right) \frac{1-\alpha}{\alpha} + 1 - \delta \right] K_t - \left[ 1 + (\sigma - 1)\gamma \left( \frac{(1-\alpha)A_0}{w_0} \right) \frac{1}{\sigma} \frac{K_t}{A_t} \right] \lambda_t \frac{1}{\sigma}
\]

\[
\lambda_t = \beta \lambda_{t+1} \left[ \alpha \left( \frac{(1 - \alpha)A_0}{w_0} \right) \frac{1-\alpha}{\alpha} + 1 - \delta \right]
\]

It can be proven that a shock on capital has now a different impact on the considered variables. In particular, wages are now forced to keep growing at the rate of \( g \). The reduction in capital causes an equal reduction in consumption, output, investment and employment, after which all variables except for the latter start growing at the rate of \( g \) again, while employment remains at a permanently depressed level, so that a jobless recovery occurs.

Introducing rigid wages in the constrained model, the two main equations become:

\[
\beta \lambda_{t+1} \left( \alpha \left( A_0 \frac{(Z_{t+1} - \theta K_{t+2})}{K_{t+1}} \right) \frac{1-\alpha}{\alpha} + 1 - \delta \right) = \lambda_t \left[ (1 - \theta) + \theta(1 - \alpha) \left( \frac{(Z_t - \theta K_{t+1})}{K_t} \right) ^{-\alpha} \left( \frac{A_0}{w_0} \right) ^{1-\alpha} \right]
\]
\[ K_{t+1} = \left[ \left( \frac{A_0}{w_0} \frac{(Z_t - \theta K_t + 1)}{K_t} \right)^{1-\alpha} + (1 - \delta) \right] K_t - \lambda_t^{\frac{1}{\sigma}} \left[ 1 + (\sigma - 1) \gamma \left( \frac{A_0}{w_0} \left( \frac{Z_t - \theta K_t + 1}{A_t} \right) \right) \right] \]

Whenever \( \theta < 1 \), it can be proven that a neutral shock (i.e. a shock that does not favor either labor or capital) such as a shock on technology, will support a more capital-intensive production. Output and capital grow more than employment, which tends to lag behind. On the contrary, when wages are flexible, they fall in consequence of a shock, thus bearing all the adjustment.

4 Main results

This section shows the outcome after all different specifications of the model analyzed so far are calibrated and solved using a statistical software. In particular, two types of negative shocks are considered, reducing either capital or the collateral value \( Z \) by 10%.
Figure 1 shows the baseline model: it displays the reaction of considered variables when capital starts below its steady state path. Investment increases, so as to make up for the lower capital stock, which then starts accumulating at a faster pace. Hence, savings take over consumption, which decreases quite substantially. As already mentioned, the reaction of employment is one of increase, so that output also grows slightly.

When employment is subject to the constraint, the variables of the model react only slightly differently with respect to the unconstrained case. In particular, wages still decline in consequence to the shock, and employment still increases, but to a lesser extent. Also, income barely increases in this case, but the main tendency of all the variables is left unchanged.

When the shock hits the collateral constraint (figure 2), the reaction of the variables is extremely different instead: both income and employment shrink quite substantially.

Investment and consumption are also reduced, and the consumption trend is downward sloping: this means that the economy is moving further away from the previous steady state, before recovering its normal growth pace. Similarly, capital does not jump immediately on impact of the shock; rather, it tends to move away from its steady state for many periods after. This kind of shock will thus be much more persistent than a simple transitory shock on capital. Wages also tend to shift down on impact of the shock and then continue to further decline, while employment starts to
The introduction of rigid wages in the otherwise frictionless model gives rise to the jobless recovery phenomenon. In particular, after the shock on capital, all variables fall by exactly the same amount as the initial shock. After this, they continue growing at their usual level (the shock has now permanent effects). This means that output starts growing again at the rate of $g$, while employment remains constant, and will never catch up with output. On the contrary, the introduction of rigid wages in the constrained model does not create a jobless recovery situation when the shock on capital is considered. Finally, the shock on credit constraint together with rigid wages is able to reproduce a situation of jobless recovery, where both employment and output fall on impact of the shock, but employment falls more. Although employment starts growing faster than output, it does not catch up throughout five years. Capital starts deteriorating after the impact of the shock, while investment slowly begins to recover. Therefore, when the shock hits the economy, a unit of output contains more capital than employment. Only very slowly will this configuration shift towards a less capital and more labor intensive production. Consumption falls and continues falling for many periods after the shock: a perfect storm takes place.

Figure 3: Shock on credit constraint, model with wage rigidities and constraint on employment
5 Conclusions

A simple frictionless real business cycle model is not able to reproduce the major facts observed in the recessions of the past decade. However, introducing some forms of constraints and rigidities substantially improves the explanatory ability of such a model. Rigid wages alone are enough to reproduce a situation of jobless recovery, although on a much smaller scale than that which has been observed during the past crisis. A negative shock on a collateral constraint, combined with inflexible wages, performs this task better. Indeed, the root of the 2008 financial crisis can be traced back to the bursting of the subprime mortgage bubble. After that, the value of collateral owned by firms and banks plunged to record low levels. Due to this, a shock that hits the collateral value first, and then propagates, eroding the stock of capital, is more accurate and truthful to reality.

This shows how the current labor market condition is not exclusively nor primarily the consequence of labor market rigidities and frictions. Rather, the credit market situation has a strong impact on the performance of other markets, and major spillover effects exist from one sector to the other. Moreover, credit conditions give rise to fluctuations in the business cycles and constitute a major source of shock amplification and propagation.

The main conclusion that can be drawn from this is that the old adage that finance and real economy are separate and distinct from one another is no longer valid or meaningful; a more in-depth scrutinization of the former is needed if one desires to maintain the latter in order.
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